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Statistical modelling of the ocean environment – A review of recent developments in theory and applications

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A B S T R A C T

Probabilistic modelling and statistical analysis of environmental conditions is important for the design and assessment of ships and other marine structures. It will give a necessary input to structural reliability assessments and risk analysis and provides a means to identify design conditions that structures are expected to withstand in their lifetime. In this paper, recent developments in statistical modelling of relevant metocean variables describing the environment at sea will be reviewed and presented. This includes a review of statistical modelling applied to such data, for example wave-parameters, but also some theoretical and methodological developments from other fields of applications will be reviewed. The paper is divided into different sub-sections addressing various aspects of statistical modelling of the ocean environment, such as long-term and short-term statistics, extreme value analysis, non-stationary analysis and covariate effects, multivariate analysis and joint distributions, spatial statistics and machine learning applications. This distinction into sub-topics may be somewhat arbitrary, and some papers address several of these topics, e.g. non-stationary, multivariate extreme value statistics for spatial data, but it is believed to be useful to still keep separate sections for the main aspects.

It is believed that this overview of recent developments in statistical modelling of the ocean environment will be useful for anyone involved in design or risk assessment of marine structures, and that it may contribute to push the state of the art and industry practice.

1. Introduction

Modelling of the ocean environment (and hence its forecast and hindcast) can be done in two different ways: deterministic and statistical. Deterministic modelling, i.e. predicting a specific event in time and space is possible in some circumstances, but is usually prohibitive. This is because of the complexity of oceanic phenomena, i.e. a great variety of physical processes superposed, and the random nature of most of them which is a result of both turbulence and our ability to resolve variations of large-scales due to superposed small-scale physics. This is particularly so with respect to metocean phenomena, i.e. physical processes at the ocean interface where the Atmospheric Boundary Layer interacts with the upper ocean, mostly through winds, waves and currents which create or facilitate dynamic, mass, moisture, heat and other fluxes at the interface.

Yet, metocean processes are of the utmost interest for marine industries and ocean engineering, because most human activities at sea, whether this is merchant vessels or offshore platforms or even underwater pipelines, are located at or near the ocean surface. Hence, the statistical modelling becomes and remains the main tool in ocean engineering when dealing with the environment. This includes operation and design in the marine industries and ocean engineering, risk and reliability assessment, among other issues and problems.

This review paper is based on examination of relevant publications on statistical modelling, conducted by the authors within ISSC

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(International Ship and Offshore Structural Congress) mandated for the period of 2018–2021. The review is conditionally sub-divided into Sections Long-term and short-term statistics, Extreme value analysis, Non-stationary analysis and covariate effects, Multivariate analysis and joint distributions, Spatial statistics and machine learning applications.

2. Long-term and short-term statistics

Information about the statistics of relevant metocean variables is of great importance for the design of ships and other marine structures. Typically, the ocean environment can be described by long-term statistics of relevant sea state variables, for example significant wave height (H_S), or by short-term statistics of wave parameters conditional on the sea state, for example individual wave heights within a stationary sea state. A common assumption is then that the ocean environment can be described as a piecewise stationary process, with stationary sea states of duration a few hours. Then, if the interest is in the long-term distribution of some wave parameter, say individual wave height, this can be found by combining the long-term distribution of the sea state, say **X**, and the short-term distribution of the wave parameters of interest, say **Y** and integrating over all sea states

$$f_{\mathbf{Y}}(\mathbf{y}) = \int f_{Y|X}(\boldsymbol{y}|\boldsymbol{x})\mathbf{f}_{\mathbf{X}}(\boldsymbol{x})d\boldsymbol{x}. \tag{1}$$

Often, the long-term distribution of sea state variables may be discretized in the form of a scatter diagram, in which case the integral above translates to a sum. However, this implicitly neglects the serial correlation at various scales, and how to combine short-term distributions of individual waves with long term distributions of sea states remains an active area of research.

In this section, various developments in modelling either the long-term distribution of sea state variables, short-term distribution of wave parameters or the combination of these to form long-term distribution of wave parameters will be reviewed. It should be noted that some statistical models have been well established in the industry to describe sea state and individual wave variables, e.g. a Weibull (often in its translated 3-parameter form) for significant wave height or a Rayleigh distribution (linear waves), Tayfun distribution or Forristall distribution (non-linear waves) for individual wave heights. Particular attention is given to developments related to shallow and intermediate water wave statistics, since this has historically not been given as much attention and it may be questionable how well standard statistical models describe such data. Note that the sea state variables and the wave variables may be multivariate, but multivariate modelling will be covered in section 3.

The translated 3-parameter Weibull distribution is often assumed to model significant wave height. However, a recent study suggests that another 3-parameter Weibull distribution, the exponentiated Weibull distribution with an additional shape parameter may fit the data better, especially if fitted by weighted least squares methods that emphasize the data in the tail of the distribution more in the model fitting [1]. Several candidate models for the probabilistic description of significant wave height are examined in Ref. [2], and compared to a model not previously applied to ocean data, i.e., the extended generalized inverse Gaussian distribution. According to the reported results, the proposed new distribution outperforms the other model candidates included in the study, for several, but not all datasets that were applied.

Typically, a statistical distribution is fitted to measurement data, but due to the often-limited availability of long-term in-situ measurements, statistical models are often fitted to wind-wave hindcast data. However, an approach to integrate model data and measured data in statistical modelling of significant wave height is proposed in Ref. [3], where model data are used as indicators and buoy data from nearby locations are used for bias correction and uncertainty evaluation.

It is acknowledged that some wave conditions are more hazardous to marine structures than others, for example waves with high steepness and waves from crossing sea states or unexpectedly severe wave conditions due to rapid development of the conditions. Hence, long-term statistics of potentially hazardous sea states are addressed in Ref. [4], limited to crossing sea states, unusually steep waves and rapidly developing sea states, as opposed to modelling all sea states. Specifically, an enhancement factor defined to describe rapidly developing seas, wave steepness and directional spreading is analyzed. The distributions of these variables are reported in terms of box-plots and scatter diagrams for selected locations and seasons, but there is no parametric modelling of the distributions. The statistics associated with steep waves and steep sea states are also addressed in Ref. [5], where spectral wave steepness is modelled as a lognormal distribution conditioned on significant wave height.

A rather different approach for modelling wave heights is proposed in Ref. [6], based on fractals and fractal theory. The methodology is extended in Ref. [199]. They find that records of significant wave height exhibit weak fractality and suggest that their proposed method can be used to estimate design wave conditions for long return periods.

Long-term distributions of individual wave or crest heights require combining the short-term distribution of wave or crest heights in a sea state with the long-term distribution of sea states. Three different ways of estimating long-term distributions of individual wave and crest heights are discussed in Ref. [8], assuming independent individual waves, independent highest wave height in a sea state and independent highest wave height in a storm, respectively. They found that methods that ignore the serial correlation of sea states tend to be biased and that storm-based approaches are more accurate. A generalized equivalent storm model for the long-term statistics of individual wave heights and crests is proposed in Ref. [9]. Methods to estimate the probability of extreme individual wave heights are also proposed in Ref. [218] based on peaks-over-threshold analysis on random maxima in different sea states and based on the distribution of significant wave heights and wave model runs are also presented in Ref. [10].

A novel approach to combine the long-term statistics of sea states and the short term conditional distributions of extreme structural response is proposed in Ref. [11]. It applies a sequential sampling technique and a Gaussian process emulator for the response in order to achieve long-term extreme structural response assessment, accounting for both long-term and short-term variabilities.

The statistical properties of individual wave heights and crests with a particular focus on rogue waves are investigated in Ref. [12].

The occurrence of rogue waves in measured data is compared to that predicted from statistical distributions such as Rayleigh, Forristall and Tayfun (for crests), and it was concluded that wave and crest heights generally follow the Forristall distributions relatively well. The exception is data from buoy measurements, which are known to underestimate wave crests. The physical constraints for exceeding probabilities of deep water rogue waves were studied in Ref. [13]. Statistics of rogue waves in crossing sea states were also investigated in Ref. [14] and it is shown that the maximum crest elevation in such situations depends on the crossing angle, with an effect that is opposite to the nonlinear effects. Extreme waves in crossing sea states are also discussed in Ref. [15]. The study in Ref. [16] suggest that the extreme wave statistics in a mixed sea, composed of a wind-sea component and a following swell could appear to be milder than the extreme wave statistics of the wind sea alone. However, analysis of the two sea states separately reveals that the extreme wave statistics of wind sea can be nearly unaffected by the presence of a following swell. Statistics of extreme waves in single and mixed sea states were also studied in Ref. [17], and the shape and height of extreme wind waves are analyzed in Ref. [18], based on space-time extremes.

The short-term distribution of individual wave periods in combined seas is investigated in Ref. [19], where parametric mixture distributions are suggested. The mixture models are compared with theoretical and parametric models for a number of different types of mixed sea states, including in-situ measured data and simulated data exhibiting two-peaked spectra. The paper suggest that the mixture distribution models yield improved modelling of the individual wave periods in combined seas.

The statistics of trough depths are perhaps less studied compared to wave crests. However [20], proposes to model individual trough depths in a sea state by way of a transformed Rayleigh distribution. They show that this gives a better fit to the data than the standard Rayleigh distribution, that does not account for nonlinear effects. Wave trough exceedance probabilities in nonlinear seas are also studied in Ref. [21], based on asymptotic expansions of the up-crossing rates and a transformation relating the non-Gaussian sea surface process with a Gaussian process.

Rogue waves have received much attention lately, and a statistical theory for rogue waves, based on large deviation theory is proposed in Ref. [22]. It allows for estimation of the far tail of the probability density function for the surface elevation and hence to estimate extreme event probabilities. Moreover, the method describes the precursors of rogue waves enabling early detection and prediction of the likelihood of extreme events within a given time window [23].

2.1. Shallow water statistics

There has recently been much interest in the statistics of shallow water waves, and it is an open question to what extent statistical models used to describe deep ocean waves apply to waves in shallower waters. A recent study based on deep waters measurements presented in Ref. [24] suggests that the Forristall distributions for individual wave and crest heights generally fits the deep-water data well, but that it is less accurate in steeper sea states corresponding to high wind speeds. It is suggested that more research should be carried out at more shallow water depths. Results from a new laboratory study, based on data generated in different experimental facilities, are reported in Ref. [25], where possible departures from commonly applied statistical distributions for crest heights due to different sea state steepness and water depths are investigated. They found that nonlinear effects beyond second order are important in intermediate water depths. However, the dissipative effects of wave breaking which increases with increased steepness reduces the nonlinear effects and the relative importance of wave breaking increases as the water depth reduces. Notwithstanding, they report that nonlinear amplifications of the crest heights are largest in the shallower effective water depth (Effective water depth is defined as the product of the peak of the wavenumber spectrum, k_p, and water depth, d). They conclude that systematic departures from the commonly applied Forristall model are evident and that an important challenge for future work is to derive a simple crest height distribution that incorporates such nonlinear effects.

Statistical analysis of data from large scale experiments of unidirectional waves propagating over a variable bottom profile presented in Ref. [26] indicates that whereas the Rayleigh distribution performs well for individual waves in deep waters, it underpredict the probability of large waves in shallower waters. A number of statistical distributions were fitted to the shallow water data, and it was suggested that the generalized Boccotti distribution [27] performs best for the shallow water data, particularly for large waves with $H > H_S$.

The fact that the steepness and asymmetries of extreme waves increase with shallower water depths is also found by Ref. [28], which proposes an empirical parametrization of wave steepness and asymmetries in nearshore environments.

The effect of variations in bottom topography on the distribution of wave heights is investigated in Ref. [29], suggesting that this may qualitatively influence the wave statistics. They found that abrupt depth changes can lead to random Gaussian seas becoming close to a gamma distribution a short distance downstream. This may give more frequent extreme waves, since the gamma distribution generally has heavier tails than the normal distribution. Statistical transitions from near Gaussian sea to highly skewed statistics in shallow water waves with an abrupt depth change are also studied in Refs. [30,31] where statistical dynamical models are proposed to explain and predict such effects.

The study presented in Ref. [32] aims at obtaining a more accurate description of the long-term wave climate in shallow waters by combining short- and long-term statistics in deep waters. First, the joint long-term statistics of wave height and period in deep waters are found by combining the conditional short-term joint distribution of these parameters with the long-term joint distribution of the sea state's significant wave height and mean zero-crossing wave period. Then, the joint distribution in shallow waters is estimated by considering the wave transformation of each individual wave as the waves propagate from the open sea towards shallower waters. They find that wave statistics in shallower water differ from those in deeper water, but ends up with the same parametric family for intermediate waters as in deep water for the long-term distribution of sea states, i.e. Weibull or Gamma distributions for the significant wave height and conditional lognormal for mean zero-crossing period. However, the distributional parameters change. A truncated, translated Weibull distribution is proposed for significant wave heights in shallow waters in Ref. [33], where wave heights are restricted from growing too high due to the water depth. Different versions of truncated Weibull distributions have previously been applied to model wind speeds in e.g. Refs. [34,35].

3. Extreme value analysis and extreme wave statistics

Often in marine engineering applications it is the extremes of the environmental conditions that are of most interest, and extreme value analysis is often needed in order to extrapolate the tail of a statistical distribution to describe events occurring with a frequency that is small compared to the length of observations. Hence, statistical extreme value analysis is a useful tool that, even though it is well established in the industry, still has a number of unsolved challenges and remains an area of active research. In this sub-section, a review of recent developments in extreme value modelling will be presented, with a particular focus on applications to ocean climate variables. A recent review of some approaches to statistical modelling of extreme ocean environments are presented in Ref. [36], see also [37].

There are obviously large uncertainties in extreme value estimation, both aleatory and epistemic and the reliability of extreme value estimates is of great concern. Hence, in Ref. [38] a statistical approach for assessing the reliability of return value estimates from a particular extreme value estimation method is proposed, based on a variability criterion. The variations in return value estimates of ocean waves are also addressed in Ref. [39], where estimates from different methods and for both measured buoy data and reanalysis data are compared. They found, inter alia, that the influence of a single storm in the data can give a large difference in the extreme value estimates compared to the differences due to varying lengths of data. The effect of parameter estimation method and the available sample size for extreme value analysis of metocean conditions are also studied in Ref. [40]. The uncertainty of extreme value estimates of ocean waves from different sources is also discussed in Ref. [41], and a Bayesian approach to account for these various sources of uncertainty is proposed in Ref. [42]. In fact [42], suggests that Bayesian uncertainty analysis should be the preferred framework for estimation of uncertainty, and they propose a framework consisting of a statistical emulator that should try to predict hindcast simulator output and a statistical discrepancy model to predict the differences between hindcast output and the true wave environment.

Traditionally, there are three main approaches to univariate extreme value analysis, sometimes referred to as the initial distribution approach, where a probability distribution is fitted to all the available data and extreme quantiles are estimated based on this distribution; the peaks over threshold (POT) approach, where a distribution is fitted directly to the tail using only data that are above a certain threshold; and the block maxima (BM) approach, where a distribution is fitted for the tail using only block maxima, see e.g. [43–45]. The choice of approach is typically a traditional bias-variance tradeoff, and extreme value methods such as POT and BM will typically be less biased for the tail behavior but will have much larger variance due to the reduced sample size. The effect of a difference in sampling between the BM and POT methods is investigated in Ref. [46], suggesting that annual maxima may be too few to yield reasonable extrapolation but that the POT method can be reasonable when the threshold is suitable. However, threshold selection remains a challenging task in POT modelling, where the choice of threshold may significantly influence the results. The differences in extreme wave height estimation in practical engineering practice, by following various guidelines and industry practices, are investigated in Ref. [47].

From theory, it is known that the peaks over a sufficiently high threshold follow, asymptotically, the generalized Pareto distribution (see e.g. Ref. [48]). The exponential distribution is a special case of the generalized Pareto distribution with one less degree of freedom. The appropriateness of the generalized Pareto distribution for modelling significant wave height data above a threshold is addressed in Ref. [49], suggesting that it is more appropriate than for example a 2-parameter Weibull distribution or the exponential distribution. However, it is stressed that the results are highly sensitive to the choice of threshold, and a new threshold selection methodology is suggested based on the second order derivatives of the cumulative density function. The idea is that the point of the probability density function with maximum curvature represents a shift from the bulk of the data to the tail. The Weibull-Pareto distribution is proposed for modelling extreme wave heights above a threshold in Refs. [50–52].

A new fitting method for estimating the parameters of the generalized Pareto distribution for exceedances over thresholds are presented in Refs. [50–52], based on transformations of order statistics, namely the weighted nonlinear least squares method. Simulation studies and real data analyses indicate that this method compares well with other methods for parameter estimation of the generalized Pareto distribution. Extreme value estimation based on small samples of low quality is addressed in Ref. [53], proposing a practical approach based on Bayesian inference with the group likelihood rather than the standard likelihood and assuming near-uniform priors on the parameters. The group likelihood incorporates data uncertainty due to for example measurement errors. The effect of return value estimates from a peaks over threshold analysis according to how parameter uncertainty is handled is discussed in Ref. [54], illustrating that there are notable differences.

Threshold selection in peaks-over-threshold modelling remains an active area of research, even though there are several well established approaches to determine a suitable threshold. Some of these require manual interpretation and for example graphical inspection of various plots and leave room for subjectivity. Moreover, manual inspection means that threshold selection cannot be included in automated scripting of extreme value analysis. In order to remedy this, a novel automated threshold selection method is proposed in Ref. [55], based on the characteristics of extrapolated significant wave height. The method investigates the differences in extrapolated return significant wave heights for neighboring thresholds to identify a stable threshold range. The highest threshold within the stable threshold range is then automatically selected as a suitable threshold. A comparison with some established threshold selection methods is presented, indicating reasonable agreement in resulting return value estimates. See also [56] for a proposed automated method for extracting IID (independent and identically distributed) samples from time series for subsequent extreme value analysis.

One obvious drawback of the peaks-over-threshold approach is that it is wasteful. All data below the selected threshold are disregarded, even though they may contain useful information. In order to alleviate this problem, an approach applying multiple threshold in parameter estimation is proposed in Ref. [57]. These multiple levels are introduced as a means to incorporate more observations to reduce the variance of parameter estimates. In Ref. [58] a Bayesian cross-validation scheme is proposed to address the bias-variance trade-off in threshold selection by comparing thresholds based on extreme level predictive ability. They use Bayesian model averaging to combine inferences from many thresholds in order to reduce the sensitivity of the choice of a single threshold and to incorporate the uncertainty in the threshold choice. The approach is applied to data of significant wave height.

A new four-parameter extreme value distribution is proposed in Ref. [59], which is a generalization of the Fréchet distribution for block maxima. There also exist other approaches to extreme value analysis, and some methods that have been applied to ocean waves include the equivalent storm approach [60] and the average conditional exceedance rate (ACER) approach. A k-th order Markov model for extremes is proposed in Ref. [61], which can account for temporal dependencies.

4. Multivariate analysis and joint distributions

Ships and other marine structures are typically affected by several environmental variables, and the joint effect of these on the environmental loads needs to be taken into account. Failure to accurately account for the dependence between the variables may lead to overly conservative or non-conservative assessment of the structural reliability. Hence, multivariate statistical models for the joint behavior of selected variables will give more accurate descriptions of environmental loads and responses and are important for improved design and operation of ships and other marine structures. Typically, assuming either independent or fully dependent variables may give wrong results even if the marginal, univariate models are appropriate [62]. However, it is increasingly challenging to find good distribution models with increasing number of variables, and even in the bivariate case, joint statistical modelling remains challenging. Thus, there has recently been considerable attention and research on multivariate analysis and joint statistical models for sea-state variables relevant for ship design and a brief review will be presented in this sub-section of the report. The evolution of joint probability methods used for coastal engineering applications is described in Ref. [63], including recent developments in multivariate statistical approaches such as joint exceedance curves and response based methods.

Examples of sea state variables that are often modelled jointly are significant wave height, mean wave period and mean wave direction. However, models that also include other variables such as wind speed and direction and sea level are also sometimes needed. More recently, joint distribution of waves and currents has been studied, although the availability of current data is still scarce and makes it difficult to establish good models for these variables [64,65].

There are different ways of establishing a multivariate statistical model and three common approaches are to assume a parametric multivariate distribution, the so-called conditional modelling approach and the copula-based approach, see e.g. Ref. [66]. Non-parametric approaches are also sometimes used [67], but these will have difficulties in extrapolation and cannot be expected to model the extremes accurately. With the former approach there exist some multivariate distributions that are often used, such as the multivariate normal or log-normal distributions, and the model parameters can then be fitted to the data. Typically, this involves estimation of the covariance matrix as well as location parameters and variable transformations can be applied to fit a multivariate normal distribution to non-Gaussian data (see e.g. Ref. [68]). However, this approach is somewhat restrictive and not too frequently used in practice. An approach to multivariate modelling based on multivariate probability distribution class is proposed in Ref. [69], conditioned on log-concavity of the joint probability density function [70]. Although the numerical example given in Ref. [69] is not for an environmental variable vector, it is assumed that this approach could also be used to model joint environmental variables, and the method is versatile enough to cover many multivariate probability models and it facilitates fitting a model to data with limited amounts of data.

The joint distribution of several variables can also be modelled by a hierarchical model as a product of marginal and conditional distributions [71]. Hence, a multivariate model for variables $X_1, X_2, \ldots, X_n$ can be modelled in the following form

$$f(X_1, X_2, \ldots, X_n) = f_1(X_1) f_{2|1}(X_2|X_1) \cdots f_{n|1,\ldots,n-1}(X_n|X_1, X_2, \ldots, X_{n-1}).$$

Estimation of the model then involves estimating the marginal model for the primary variable and the various conditional models for the remaining variables. Sometimes, conditional independence between some of the variables can simplify the modelling, but it remains challenging to define the conditional models for all relevant variables. A new set of marginal and conditional distribution models for metocean variables is suggested in Ref. [72], including conditional models for wind sea and swell variables as well as wind and water levels. They assume a sparse dependency table, where many of the variables can be modelled as independent from many of the other variables. They report a reasonable fit to the data. A conditional model was also assumed for modelling the joint distribution of significant wave height and current speed in Ref. [73]. The joint distribution of significant wave height and spectral peak period is modelled by a conditional model in Ref. [74], where a hybrid lonowe (lognormal and Weibull) distribution is used for the marginal significant wave height and a conditional lognormal distribution for wave period. A conditional model based on Weibull-lognormal and lognormal distributions are used to model significant wave height and spectral wave steepness in Ref. [75]. Different distributions were tried out to model the joint distribution of wave height and period using the conditional modelling approach in Ref. [76].

Hierarchical conditional models have also been used to model the joint distribution of circular-linear variables such as wind speed and direction, as outlined in Refs. [77,78]. They assume a mixture of von Mises distributions for the marginal model of wind direction (the circular variable) and a conditional Weibull distribution for wind speed conditioned on the direction. Similar models extended to the trivariate case for a combination of circular and linear variables are proposed in Ref. [79].

Different bivariate time series models for significant wave height and spectral wave period are investigated in Ref. [80], taking the joint behavior of the variables into account together with temporal dependencies. They assume a conditional model for the joint distribution and transform the data to a standard normal space using the Rosenblatt transform and then apply a seasonal transform before various time-series models are applied to the transformed data. Vector autoregressive (VAR) models, vector ARMA (VARMA; vector autoregressive moving average) models and Markov models are explored and it is concluded that the VAR and VARMA models perform well.

The use of copula to establish multivariate statistical models is an alternative that has received increasing attention in recent years. Essentially, a joint distribution of variables $X_1, X_2, \ldots, X_n$ can be modelled by way of their marginal distributions and a copula describing their dependence structure in the following way (see e.g. Ref. [81]),

$$f(X_1,\ X_2, \ldots, X_n) = f_1(X_1) f_2(X_2) \cdots f_n(X_n) c(F_1(X_1), F_2(X_2); \ldots, F_n(X_n)).$$

The joint model can then be established by estimating the marginal models $f_1(X_1), f_2(X_2), \ldots, f_n(X_n)$, independently, and the copula density $c(\cdot)$.Such methods have been applied to environmental sea states variables [50–52,82], although it has been shown that straightforward use of standard symmetric copulae may not be appropriate, meaning that asymmetric copula-constructions are needed [66,83,84]. A combination of parametric and non-parametric marginal distributions and c-vine copulas is used to model multivariate wave and wind variables in Refs. [85–87].

Even though there exist several parametric copulas to choose from, it may not be straightforward to find the best one, even in the bivariate case. The copula approach can also be extended to 3 or more dimensions, for example by pair-copula constructions [88] or vine copulas [89], including also circular variables [90,91], but modelling becomes increasingly challenging as the number of variables increases. A pair-copula based model for the trivariate probability distribution of typhoon-induced wind, wave and the time lag between them was outlined in Ref. [92]. More complicated models based on mixtures of copulae have also recently been suggested [93, 94], offering an interesting approach to construct more complex multivariate models using copulas. Copulas are also used together with the principle of maximum entropy for establishing the joint distribution function for wave height and period in Ref. [7], and for modelling significant wave height in different locations in Ref. [95].

Different copula-based models for modelling the joint distribution of circular-linear wind variables are explored in Ref. [96], and compared to the so-called Johnson-Wehrly model [97]. It concluded that the Johnson-Wehrly model performs best and that this is a useful model for joint bivariate models of wind speed and direction. This model has also previously been applied to wave height- and direction data in Ref. [98].

4.1. Multivariate extreme value analysis and environmental contours

The analysis and description of extreme values is especially challenging in the multivariate case, and it is even ambiguous what a multivariate return value is [99], see also e.g. Refs. [62,100]. There are various statistical modelling approaches for multivariate extremes, and e.g. the conditional extremes model has recently been promoted as a good approach [37,101]. A critical review of this approach along with a comparison to classical multivariate extreme value models is given in Ref. [102]. Several applications of the conditional extremes model to metocean data are reported, including non-stationary models, in e.g. Refs. [103–106].

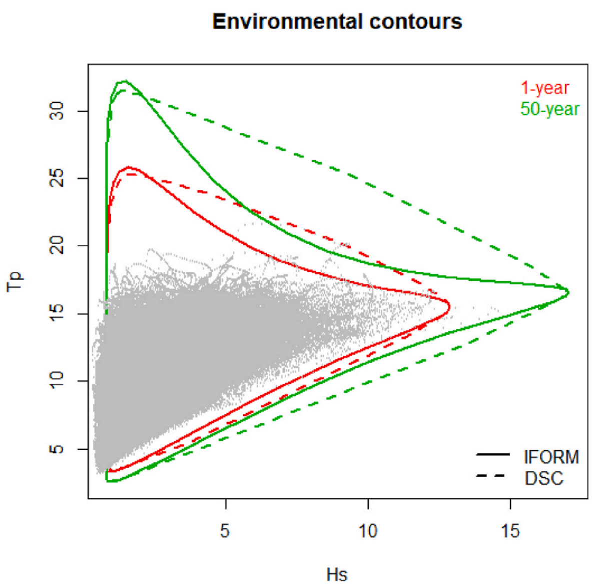

Fig. 1. Examples of environmental contours for significant wave height and peak wave period.

An event-based approach for the modelling of joint extremes of waves and sea levels is presented in Ref. [107], focusing on an event-based sampling from the bivariate time series and then joint modelling of the extreme samples by way of extreme value copulas. Various sampling methods are explored, and it is demonstrated that this has great influence on the results. The bivariate approach is also extended to higher-dimensional cases.

The environmental contour method is one approach for describing joint extremes which, given a joint statistical model, is often used for structural reliability assessment of ships and other marine structures. Traditionally, contours based on iso-density or IFORM (inverse first-order reliability method) have been used (S. [108,109], but recently a number of other approaches to environmental contours have been proposed, see e.g. Refs. [62,110–113], and also the direct sampling method [114] is now recommended in DNV's recommended practice on environmental conditions [115]. The main difference between the direct sampling approach and the IFORM approach is that the linearization due to the first-order approximation is performed in the physical variable space rather than in the standard normal space, see e.g. Refs. [116,117] for a detailed comparison of these approaches.

Different approaches to environmental contours have different definitions, but they typically correspond to certain exceedance probabilities. Examples of environmental contours are shown for significant wave height and wave period in Fig. 1, showing both IFORM contours and direct sampling contours (DSC).

The effect of sampling variability on the uncertainties of environmental contours is studied in a simulation experiment reported in Ref. [118], suggesting that this is an important aspect to consider when constructing environmental contours based on finite datasets. Other uncertainties in environmental contours are assessed in Ref. [119].

Recent applications of the environmental contour method for different problems are discussed also in e.g. Refs. [85,120–123] and [124]. Environmental contours based on non-parametric bivariate kernel density estimation is also proposed in Ref. [125], but it is advised to use such methods with great care when the interest is in the tails of the distributions, as is the case when constructing environmental contours. Recently, a free software has been released allowing for easy computation of environmental contours [126], but it is noted that this software has serious limitations with regards to the choice of parametric distribution functions and fitting methods that can be used. Different contour methods are compared with response-based methods for extreme ship response analysis in Ref. [127], suggesting that results are in general agreement.

There have been several proposed developments of contour methods for describing joint extreme conditions recently. Buffered environmental contours are presented in Ref. [128], based on buffered failure probabilities. Similar adjustments corresponding to a number of unwanted events are presented in Ref. [129]. Environmental contours based on a particular version of inverse SORM (second-order reliability method) is proposed in Ref. [130], to give more conservative contours than the IFORM method, and contours based on inverse directional simulation are presented in Ref. [131]. A new way of estimating direct sampling contours is proposed in Ref. [132], where environmental contours are constructed as Voronoi cells. A variance reduction technique is proposed in Ref. [133]. A novel approach to construct environmental contours without the need for fitting a joint distribution is developed in Ref. [134], based on fitting a number of univariate distributions to data projected in various search directions.

Most applications of environmental contours focus on bivariate problems but are in principle extendable to arbitrary dimensions. An extension of the direct sampling approach to higher dimensions, with examples of 3-dimensional problems is presented in Ref. [135]. A similar approach is also taken in Ref. [136]. In some cases, one of the variables in a multivariate problem may be circular (e.g. direction or period/season), and environmental contours for such situations are proposed in Refs. [78,137]. Environmental contours for a three-dimensional problem where one of the variables are circular are presented in Ref. [91], based on three-dimensional vine copulas.

Given the fact that environmental contours have been an active area of research in recent years, a benchmarking exercise was announced at OMAE 2019 [138], where researchers and practitioners were invited to construct contours for datasets that were made available. Several responses to this call were presented at OMAE 2020 and summarized in (A [139]. This summary revealed that there is significant variability in contour results from different practitioners, due to differences in data handling, statistical modelling and contour methods. One particular issue that was highlighted as important in this benchmark study was how to account for serial dependence in the statistical modelling, something that is further investigated in Ref. [140]. A quantitative assessment of different environmental contour approaches based on selected structural responses is reported in Ref. [141].

5. Non-stationary analysis and covariate effects

The statistics of environmental variables will typically be dependent on many factors such as season of the year, location, long-term trends (e.g. due to climate change) and prevailing wind or wave directions. That is, the IID assumption will generally not be met and data describing the environment will typically not be stationary. These non-stationarities could be important and should be incorporated in the statistical models [142]. One way of doing this is by including covariate effects in the statistical models. A covariate is an explanatory variable that can explain parts of the variation in the target variable. Another is to perform pre-processing of the data to remove the non-stationary effects and to assume stationary models on the residuals or pre-processed data. Yet another approach could be to use time-series and spatio-temporal models to account for autocorrelations and dependencies in space and time. The effect of non-stationarities would be important for both univariate and joint models, and for extreme value models and distribution models for all the data.

Stationary and non-stationary extreme value models for significant wave height are compared in Ref. [143]. The non-stationary models account for the seasonal variations and results suggest that non-stationary models perform better. A number of simulation studies are presented in Ref. [144], where stationary extreme value models are fitted to non-stationary data, non-stationary models are fitted to stationary data and non-stationary models are fitted to non-stationary data, to assess the performance of stationary and non-stationary extreme value models. They conclude that non-stationary extreme value models can give improved estimates of return values, provided that the models are consistent with the data-generating model. However, in general, the relative performance of stationary and non-stationary extreme value models will be problem specific, and in some cases stationary models may be sufficient to obtain omnicovariate return values.

A number of recent publications address the problem of accounting for non-stationarities in extreme value modelling of sea state variables. A review of methods for nonstationary extreme events is presented in Ref. [145], and some approaches to model and make inference of the effect of covariates for extreme ocean environments are critically compared in Ref. [146]. A simple approach to account for non-stationarities due to seasonal effects is presented in [100], where data are pre-processed by seasonal normalization in order to make the IID assumption more reasonable, and then fitting stationary statistical models to the pre-processed data. The effect of seasonality can then be put back in for estimation of return values for particular seasons, or for omni-seasonal estimates. Spatio-temporal trends in significant wave height are based on nonparametric methods such as the Theil-Sen estimator and the line of best fit in Ref. [147], see Ref. [148].

Extreme value models for ocean environments with covariates are addressed in several papers. The directional time evolution of extreme significant sea states is modelled assuming a nonstationary Markov extremal model in Ref. [149]. Directional-seasonal extreme value analysis of storm peak significant wave height is presented in Ref. [103], where a piecewise gamma-generalized Pareto distribution is assumed, for body and tail, respectively, where the effect of covariates is based on discrete bins in the covariate space but smoothed by way of splines across bins. Bayesian inference is used with conjugate priors. The effect of long-term climatic trends may also be incorporated in extreme value models by using time as a covariate (see e.g. [150]. for a block maximum approach and [151] for a peaks-over-threshold approach). A non-stationary generalized extreme value model with a cyclic time-covariate with a period of around 30 years was used to model extremes in Ref. [152]. The need for non-stationary extreme value analysis for significant wave height in the Mediterranean Sea was explored in Ref. [153].

Non-stationary extreme value models for multivariate extremes are also being promoted. Non-stationary conditional extremes models composed from piecewise stationary models in covariate bins are presented in Ref. [104], and non-stationary marginal models with stationary conditional extremes models are suggested in Ref. [105]. The effects of several covariates such as direction, season, surge and tide on the joint distribution of extreme significant wave height, individual wave and crest heights and total water level are modelled in Ref. [154]. A joint model for several storm wave climate variables based on combining marginal models with copulas for describing the dependencies on time and ENSO (El Niño – Southern Oscillation) variations, and using a vine copula to model different storm summary statistics is presented in Ref. [155]. A multivariate non-stationary model for marine storms using time and climate indices as covariates and assuming copulas for the dependence modelling is presented in [156].

Non-stationary joint time-series of significant wave height and mean zero-crossing wave periods that captures seasonal and inter-annual patterns are modelled in Ref. [157], assuming a model with several components including renewal processes, Fourier series with random coefficients, ARMA processes and copulas. A regime switching approach is applied to account for switches in main wave direction.

6. Spatial and temporal statistics

The oceans, by their nature, cover a large area and one is often interested in the spatial and temporal variability of sea states and other environmental variables and to model the spatial and temporal dependencies of relevant parameters, as well as their extremes. Hence techniques from spatial statistics and time series modelling are relevant and useful for statistical modelling of the ocean environment, and several applications of spatial modelling have been applied to ocean environment data. Spatio-temporal modelling consider dependencies in both space and time and combines spatial models and temporal models. In the following, a brief review of relevant literature will be presented.

6.1. *Spatial statistics*

A review of some methods for spatial analysis of extremes is given in Ref. [158], including regressing distributional parameters on spatially varying covariates and regional frequency analysis. They develop an approach for modelling the spatial variability of extreme significant wave height utilizing both long-term measurements and high resolution hindcast. A spatial model for extremes is proposed in Ref. [159], where the extremes are modelled by a generalized extreme value model and where the parameters vary in space according to a clustering of the locations and a spatial Markov model for the clusters. The generalized extreme value (GEV) model is also combined with a spatial model in Ref. [160].

The regional frequency analysis is a method to utilize spatial data by pooling data from locations that can be regarded as homogeneous in order to effectively increase the sample size in estimating the probability distribution (or equivalently, the quantile function) within the region. The location-specific probability distributions can then be found from this common regional distribution function (growth curve) by applying a site-specific scaling factor (the index flood). Regional frequency analysis has traditionally been applied in hydrology, and a few recent applications of this method have been reported for ocean wave data [161–165]. and wind data [166]. These studies indicate that regional frequency analysis is a useful tool for spatial modelling of ocean environment data and that improved estimates of extreme return values can be obtained, if the underlying assumptions of homogeneous regions are reasonable. Confidence bounds for extreme quantile estimates obtained by regional frequency analysis were constructed in Ref. [167], suggesting that uncertainties are small. An application of bivariate regional frequency analysis for significant wave height and wave period is presented in [219], demonstrating that the regional frequency analysis approach can be extended and is useful for multivariate extreme value analyses.

Spatial extreme statistics can be modelled as so-called max-stable processes in order to characterize the spatial dependence, and spatial models based on such processes are applied for modelling storm peak significant wave height in Ref. [168]. However, as pointed out in Ref. [169] space-time processes are typically only observed at discrete points, and the influence of interpolation to fill such gaps in the marginal distributions is discussed. Spatial extreme events have also been modelled by the conditional extremes model, where the distribution of extreme events over a spatial region can be conditioned on the spatial process being extreme at observed locations within the region [170]. Such conditional extremes spatial models are applied to ocean storm severity data, i.e. significant wave height, in Ref. [171], see also [172] where the conditional dependence of a spatial process measured at one or more locations are conditioned on extreme values of the process at other locations.

A spatial model for extreme significant wave height in cyclone-dominated regions is proposed in Ref. [173], which combines models for the space-time maximum (STM) with models for the exposure (E). The space-time maximum is defined as the largest significant wave height observed anywhere in the spatial region during the time period of a cyclone, and such space-time maxima above a threshold are modelled by a generalized Pareto distribution. Since space-time maxima are used, data from all cyclone events are used and not only data from a single location. This is then combined with the exposure for a particular location, which is defined as the storm severity as that locations as a fraction of the space-time maximum. A marginal distribution is then estimated for the exposure at all locations, providing a spatial model over the domain. The joint STM-E model is then found by assuming that exposure is independent of the space-time maximum. The model is applied to data from Gulf of Mexico in Ref. [174].

A non-stationary spatial model for significant wave height using stochastic partial differential equations (SPDE) is proposed in Ref. [175]. They combine a SPDE representation of a Gaussian Matérn field (a Gaussian random field with a Matérn covariance function) with a deformation approach (a bijective mapping characterizing the non-stationarity and anisotropy of the random field) to capture both non-stationarities and anisotropies. It is shown that this model agrees well with significant wave height data from the north Atlantic Ocean. This model is extended to jointly model significant wave height and wave period over space in Ref. [176]. Other multivariate stochastic differential equation random fields for multivariate spatial modelling are discussed in Ref. [177].

A probabilistic graphical model is a model where a graph expresses the conditional dependence structure between the random variables, and it is normally defined in terms of a set of nodes and a set of edges of pairs of nodes. A simple example of a graphical model for four random variables is shown in Fig. 2. Graphical models have recently been proposed as alternatives to spatial models, see e.g. Refs. [178,179], and a model for spatial extremes based on ensemble of trees of pairwise copulas are presented in Ref. [180]. High-dimensional dependence modelling using vine copulas and graphical methods are suggested in Ref. [181]. Spatial and temporal clustering of extreme wave events are outlined in Ref. [182], where main spatial footprints were identified around the coast of UK.

6.2. Time series analysis

In time-series analysis one wants to model the temporal evolution of stochastic variables, and several techniques are available for such temporal dependencies or correlations. For multivariate time-series, one needs to model both the cross-correlations, or dependencies between variables, and the temporal dependencies, or autocorrelations, in the time series.

A generic approach to modelling and simulating time series with specified marginal distribution and correlation structures are proposed in Ref. [183]. This is based on establishing proper transformations and a finding a parent Gaussian autoregressive model that yields a time series with the desired properties after transformation. Such methods could be applied to time-series of metocean data in order to obtain statistical models that not only describes the marginal distribution, but also the serial correlation, see also [184].

A fuzzy number is a generalization of a real number, which rather than taking a single value but rather refer to a set of possible values. Each possible value is associated with a weight or membership function, and fuzzy numbers are often used to incorporate

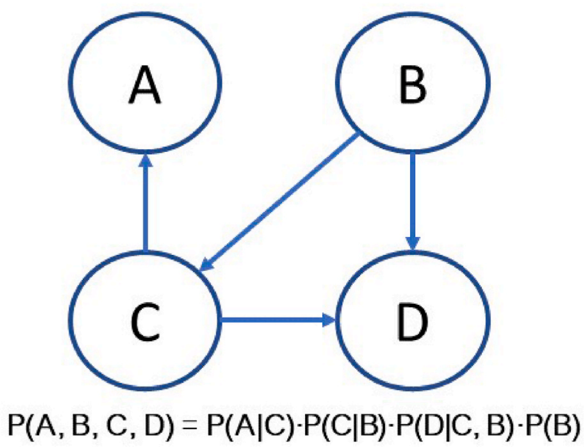

Fig. 2. A simple example of a graphical model.

uncertainty. Fuzzy time-series have been proposed for modelling non-stationary time series of wave and wind data in (C. [185], and extensions and applications at various time scales of such modelling have been presented in Refs. [186,187]. Whereas a conventional time series is considered as a realization of a random process, a fuzzy time series is considered a realization of a fuzzy random process, i.e. a sequence of fuzzy random variables. A fuzzy time series is then modelling the temporal relationship between such fuzzy variables. Typically, fuzzy time series modelling involves fuzzification of the input variables (data; crisp values), inference and defuzzification to transfer fuzzy output to crisp values.

A shapelet transform was applied to time-series of ocean wave data to classify and identify breaking waves in Ref. [188].

Bivariate time-series of significant wave height and wave period using vine-copulas and assuming the Markov property for the temporal evolution are presented in Ref. [189]. Joint time series modelling of wave height, period and directional data are also presented in Ref. [157].

7. Machine learning applications

Advanced statistical models and algorithms for describing or predicting random behavior based on sampled data are often referred to as machine learning. Typically, machine learning is used for regression and classification tasks, relating various responses or outputs to input data, in what is commonly referred to as supervised learning, or in pattern recognition and clustering of unlabeled data in what is referred to as unsupervised learning. As alternatives to more traditional statistical models for regression and classification, machine learning has recently been used in a number of applications related to the description and prediction of the ocean environment. A brief review of some recent applications will be given herein. A recent survey on machine learning methods for various sea wave parameters can be found in Ref. [190].

Artificial neural networks are powerful algorithms that can be used to model highly nonlinear relationships between inputs and outputs, and several recent applications of such models to predict wave parameters are reported. Significant wave height predictions based on neural networks using wind speed and previous observations of significant wave height is presented in Ref. [191], see also [192–194]. Neural networks, as well as support vector machines are applied in Ref. [195] to predict near-future significant wave height based on previous measurements of significant wave height and wind measurements. Sequential neural networks, with the capability of updating the network as it learns, are applied to predict wave heights based on several input variables including previous wave heights and wind speed in Ref. [216]. Hybrid models combining neural networks with the mind evolutionary algorithm and the genetic algorithm, respectively, are explored in Ref. [197]. Machine learning models are trained by numerical wave model output to forecast wave conditions in Ref. [198], where a neural network is used to model significant wave height and a support vector machine is used to model wave period. Neural networks are applied for wave ensemble averaging based on 20 ensemble members of a wave forecast model in Ref. [215], reporting a benefit corresponding to gaining five forecast days compared to using the arithmetic ensemble mean averaging. Convolutional neural networks have also been used to predict wave conditions from acceleration data [217], deep neural networks have been used to estimate significant wave height in real time from raw ocean images [200], and recurrent and sequence-to-sequence networks have been used to forecast significant wave height [201].

Although different variants of neural networks are perhaps the most commonly used machine learning technique, there are several other approaches that have been used to predict ocean wave conditions or other related variables, see e.g. Ref. [202], (C.-C [203]. These include fuzzy k-nearest neighbor models [204], Group Method of Data Handling (GMDH) models [205], deep learning models [206,207], hybrid models [208], support vector machines [209], random forests [210], sequential sampling and Gaussian processes regression [211] and ensembles of neural networks [196,212]. A genetic algorithm is proposed to estimate JONSWAP spectral wave parameters from measured data in Ref. [213]. Ensembles of computationally lightweight surrogate models for forecasting ocean waves were combined with aggregation techniques in Ref. [214].

8. Summary and conclusions

This paper has presented a review of recent developments within the statistical modelling and description of the ocean environment. The review has focused on different sub-disciplines related to short-term and long-term statistical modelling of environmental variables, extreme value analysis, multivariate statistics, non-stationary statistics and space-time statistics. These are all relevant for the description of the ocean environment for the purpose of designing of assessing marine structures. In addition, a review of application of machine learning methods in describing the ocean environment is included, which is regarded as a type of statistical modelling.

This review focuses on recent developments, within the last few years, and reveals that there is a lot on research on both theoretical developments and applications of statistical modelling of the ocean environment. It is believed that awareness of such recent developments is valuable for other researchers as well as engineers and naval architects involved in design and assessment of marine structures. Hopefully, this review can contribute to push state-of-the art and common industry practice in probabilistic methods in design and assessment of marine structures, which can again lead to more optimized design and operation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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