2.2

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Mathematics education in Norwegian academic-track upper secondary school

The Norwegian school system consists of a seven-year primary school (grades 1–7, ages 6–13), followed by lower secondary school (grades 8–10, ages 13–16) and upper secondary school (grades 11–13, ages 16–19). While in primary and lower secondary school there is no streaming into different tracks, in upper secondary school, which is our main focus of attention here, there is a track structure which will be described in section 2 of the chapter.

1 General overarching goals underlying the design of upper secondary mathematics education in Norway

In order to properly describe the situation for mathematics education in Norwegian upper secondary school, in particular the academic tracks, it is necessary to start by considering the wider picture of school politics and curricula in Norway.

The vast majority of Norwegian children, all the way from grade 1 (5/6 year-olds) to grade 13 (18/19 year-olds), attend public schools free of charge for the parents. In these schools, curricula are designed through political processes led by the Norwegian government. Hence, in Norway, the educational system offers relatively large possibilities for shaping school practices politically. This freedom also applies to the different tracks in upper secondary school, among which some are academically inclined.

During the past 50 years, there have been several curriculum changes in Norway, which has led to quite large changes for the mathematics subject. We will mention the following:

- The 1974 curriculum **M74** (M for *Mønsterplan,* meaning "Pattern curriculum")
- The 1987 curriculum **M87**
- The 1994 reform **R94** (R for *Reform*)
- The 1997 curriculum **L97** (L for *Læreplan*, meaning "Curriculum")
- The 2006 curriculum **LK06** (K for *Kunnskapsløftet*, meaning "Knowledge promotion")
- The 2020 curriculum **LK20**

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The ideological basis for school mathematics in Norway, including mathematics in upper secondary school, has changed back and forth with these reforms. LK20 is not a reform, but a "renewal" of LK06, which we will elaborate on. The present curriculum LK20 for Norwegian school mathematics can be considered as a kind of hybrid between the curricula L97 (KUF, 1996b) and LK06 (KD, 2006).

Typically, the various reforms have been carried out in a stepwise fashion, so that, for instance, reforms in primary and lower secondary schools have been followed by a corresponding reform in upper secondary school a year or two later. Hence, upper secondary school curricula will be a consequence of the curricula changes in the lower grades. By autumn 2022, all grades in school will follow LK20.

The concepts or ideas connecting the reform across different school levels, have typically been defined partly by overarching goals, and partly by changes in pedagogical and subject-specific theoretical positioning. The overarching goals or ideologies particular to *academic-track upper secondary school mathematics* follow these fluctuations in the goals for school mathematics in general.

Looking at Norwegian school mathematics in general, some of the curriculum changes have resulted from changes imposed across disciplines. An example of this is the focus on problem solving and working on projects across subjects in the curriculum M87 (KUD, 1987). Another example is the *process-oriented approach* in the curriculum L97, and a third one is the competence-based approach found in the most recent Norwegian curricula LK06 and LK20.

Other changes have been internal to mathematics as a school subject. These have originated from changes in the subject itself. Examples of this are the so-called modern mathematics found in the curriculum M74 (KUD, 1974) and the subsequent "back to basics" wave (greater focus on basic arithmetic skills) which in Norway was reflected in M87.

To explain the "ideological" basis for the current Norwegian upper secondary mathematics curriculum, we will consider in somewhat more detail the three most recent Norwegian curricula, namely L97, LK06 and LK20, from the perspective of mathematics.

Norwegian curricula in the 1990s: L97 and R94. Process orientation and responsibility for own learning

Compared with its predecessor M87, which physically was a relatively modest book, the paper version of L97 for the 10-year primary school looked very impressive (KUF, 1996a). It was large format, hardbound and with lots of colour images, most of them displaying artworks. In the mathematics part of the curriculum, all of the five pictures were related to arts and crafts. Two of the five pictures concerned knitting. The general part of the curriculum described a holistic view of human beings, with subsections entitled "The spiritual human being", "The creative human being", "The working human being", "The liberally-oriented human being", "The social human being", "The environmentally aware human being", "The environmentally conscious human being", finishing with "The integrated human being". This curriculum focused on individual personality formation and what is customarily referred to by the word *allmenndannelse* in Norwegian, meaning "general education" for you as a human being, independently of your future profession.

In the Norwegian mathematics education community, the curriculum reform L97 was generally celebrated as a victory for progressive mathematics teaching methods (Herbjørnsen, 1998). In L97, strong emphasis is placed on the individual student's responsibility for professional and social development. L97 placed great emphasis on students' responsibility for their own learning. This principle can be linked to an underlying constructivist view of learning, i.e. a view of learning where emphasis is placed on the students creating their own knowledge (Herbjørnsen, 1998).

L97 was also *process oriented*. The mathematics curriculum listed the main elements that the students were supposed to *work with* at each of the 10 school grades. Each part consisted of the sentence "In the education the students shall…" followed by a list of things that the students should *work with* or *experience*. As an example, under the item «Graphs and functions» for 9th grade it reads (our translation): *The students should*

- *continue reading and drawing graphs that describe situations from everyday life*
- *practice using letters to symbolise variable numbers and quantities, and express simple functional relations in ordinary language and in mathematical symbols, and in particular explore linear functions* (KUF, 1996a, p. 181).

Just as the principle of "responsibility for one's own learning" did, this process orientation corresponded well with the idea of teaching based on an underlying constructivist view of learning. If one reads L97 from such an angle, it builds up to a classroom situation where the teacher primarily becomes a supervisor rather than a leader of the learning process.

In parallel with the introduction of L97 for primary school, a corresponding curriculum reform was also implemented in upper secondary school. This is often referred to as Reform 94 (KUF, 1994, 1999). Structurally, R94 represented a culmination of changes that had taken place in upper secondary school since the 1980s. Previous stages in the development included the curricula of 1992 (KUD, 1992) and 1996 (KUD, 1986), which should be compared to the upper secondary plan from 1976 (KUD, 1976).

In R94, the last traces of the previous division into specialized *linjer* (study programmes) in the academically oriented tracks of upper secondary school, disappeared. For upper secondary school mathematics, this meant for example that there no longer was any *naturfaglinje* (science programme) where one could assume a collaboration with physics at 12th and 13th grade. Previously, mathematics courses could rely on physics courses for interpretations and applications of topics such as vectors and calculus, and in return the physics courses could build on mathematics concerning the theoretical development of these subjects. Clearly, the decoupling of these subjects in R94 can be linked ideologically to the emphasis on individual responsibility expressed in L97.

In terms of content, a comparison of the plans from 1976 and 1992 shows that the mathematics in the academically oriented tracks moved away from mathematical rigor and formal, logical development. In other words, the development here was quite parallel to the development that took place in primary school during the same period.

Norwegian curricula after the year 2000: Competencies and measurability

The curriculum L97, and partly also the corresponding R94 in upper secondary school, promoted activity-based learning. A general criticism, however, was that these activities were not sufficiently linked to the theoretical work in the subject.

Another problem was that the slogan *responsibility for own learning* eventually resulted in a form of schoolwork organization that was widely criticized. Students were given *arbeidsplaner* (work plans) that typically covered a week or more (Bergem, 2016b; Dalland & Klette, 2014). The work plan listed what the students were supposed to work with in the relevant period, across all subjects. The students could choose the working order themselves. One consequence of this was that students in the same classroom often worked with completely different things, often even in different subjects. This made collaboration difficult. Since typically one teacher was present in the classroom at a given time, it was also far from certain that she or he had the competence required for giving advice in all the different subjects.

These emerging problems set the stage for the next major ideological shift in Norwegian curricula. The turning point came in 2003. In Norway, the results from the international comparative surveys PISA and TIMSS from this year are often referred to as the "PISA shock" (Kjærnsli et al., 2004; Mullis et al., 2004). These surveys showed that Norwegian students did not perform nearly as well as one would expect based on the country's resource levels, compared with other countries. For instance, this was the case in mathematics. We will describe this more closely in section 3.

From about 2003 onwards, the Norwegian discourse on school mathematics switched away from the "soft" and child-oriented L97-approach, focusing instead on *competence-based* approaches to curricula. This development was clearly inspired by the theoretical framework of PISA (OECD, 2003), which in turn was strongly related to the mathematical competence framework developed in the Danish KOM project (Niss, 1999; Niss & Højgaard Jensen, 2002).

As a result, the new 2006 curriculum for grades 1–13 (LK06) was purely competency based. The new curriculum emphasized measurability of competencies and so in many ways represented an approach diametrically opposed to the one in L97. The standard formatting phrase "The students should [work with]" from L97 was now replaced with "The goal of the training is that the students should be able to", followed by bullet points.

This time the curriculum did not come in the form of an impressive-looking book with colour pictures. On the contrary, LK06 was a plain black and white text document listing how the students should perform at different points in their education. This reflected the shift away from the abovementioned emphasis on aesthetics and "art" found in the previous plan.

Along with the new requirements for measurability also came stronger documentation requirements and feedback requirements for teachers. The teachers had to break down the curriculum's competence goals into measurable sub-competencies, for which they were then required to develop mechanisms for measuring and documenting student achievements to school authorities, parents, and other stakeholders. At the same time, national tests in mathematics and some other subjects were introduced on 5th and 8th/9th grades, along with national tests in lower grades aimed at locating general learning difficulties and needs for special attention among children in lower grades. In upper secondary school, there continued to be no national tests of these kinds. Following LK06, textbooks also introduced their own additional test regimes, which were sold as part of the packages the schools could buy. These packages included half-year tests, full-year tests and textbook chapter tests.

To sum up, it has been argued that LK06 led to an intensified test regime in school mathematics in Norway. It has been claimed that this represents an ideological shift away from the child-oriented traditional approach of Scandinavian schools, highlighted by L97 in the case of mathematics in Norway, but going much further back historically (Sjøberg, 2007; Braathe & Ongstad, 2001; Otterstad & Braathe, 2016). It has also been argued that the political will to do well in international studies can affect both education policy, curricula, and the individual teacher in a negative way. The result may be a national "teach to the test" effect where the goals of the teaching are dictated by international tests and national test regimes designed to match the international ones (Sjøberg, 2007).

It is also reasonable to say that the changes in LK06 represent a shift in the direction of greater emphasis on school mathematics as a *foundation for professional education*. The enforced requirements of measurability were partly justified politically by referring to reports on declining mathematical competence among beginning students at the university level (NMR, 2015; Grønmo and Hole, 2017, chapter 12).

Also, at the beginning of the new century, both an Official Norwegian Report (NOU, 2003) and a white Paper (KD, 2003–2004) suggested that students in school should be allowed to move ahead in school subjects, and also be given access to taking exams at university level. For example, the Department of Mathematics (MI) at the University of Oslo (UiO) has given access to gifted students in mathematics for 30 years, and this has been a permanent arrangement since 2004.

In the years leading up to 2020, a new process of curriculum revision in school was initiated in Norway. In the first phase of the work, the so-called *Ludvigsen committee* played a central role. In 2014 and 2015, this committee delivered two Official

Norwegian Reports about the future school in Norway (NOU, 2014, 2015). Here, the concept of *dybdelæring* ("in-depth learning") was highlighted as a new basic idea. The definition of the concept was somewhat vague and contradictory, but it was clear that *dybdelæring* had to do with going in depth, both within a given subject and across different subjects. Emphasis was also placed on long-term progress across different grades and long-term learning trajectories.

The curriculum revision in LK20, which followed the Ludvigsen reports, was initially described as a process for narrowing the syllabi, giving room for greater emphasis on so called *core elements*. Furthermore, it was set as a guideline that no new school subjects should be introduced in LK20, hence it is not considered a true "reform". LK20 is also called *Fagfornyelsen* ("subject renewal"). A defining idea was simplification. Separate subject groups were then set up with the task of defining core elements in each of the school subjects. The core elements in mathematics will be listed in section 2.

When the core elements for each subject had been described, the ball was passed on to *subject specific curriculum groups*, including a mathematics group. These groups designed the final curriculum for all grades 1–13, including the academic tracks of upper secondary school. The groups were given quite detailed templates specifying what the finished plan should look like.

At the same time, it was also decided that basic training in programming and algorithmic thinking should be introduced into the mathematics curriculum in all grades. This meant that other subject areas had to be taken out or reduced in the curriculum, and the "narrowing" became more challenging. Furthermore, it was required that the curriculum, like LK06 before it, should be competence based. The mathematics learning goals stated for the various school grades in LK20 are thus competence descriptions. However, the requirement on competence orientation made by the Ministry of Education did not apply to the core elements. For the mathematics subject, this has resulted in the core elements being mainly process oriented.

As a result, the present curriculum LK20 for Norwegian school mathematics can be considered as a kind of hybrid between the L97 and L06 curricula. The process-oriented approach of L97, motivated by personal development and general education of "modern citizens", is partly represented by the core elements of mathematics, as these are expressed in LK20. On the other hand, for each grade the LK20 mathematics plan lists learning goals in the form of measurable competencies, just like LK06 did.

To sum up, the ideological basis for school mathematics in Norway, including mathematics in upper secondary school, has changed back and forth during the last decades. While the emphasis on competence and measurability in LK06 certainly can be labelled as academically oriented, the emphasis on the development of "integrated human beings" inherited from L97, is more oriented towards personal developments, independently of future professional or educational choices.

On the other hand, overarching goals or ideologies *particular to* academic-track upper secondary school mathematics, are difficult to spot in the Norwegian discourse. Rather, the mathematics in these tracks has followed the changes in general ideologies “passively”, maybe in a kind of damped manner, without any particular attention given to it. This fact is interesting, and it may be summed up as the first of our main conclusions in this report:

- Mathematics education in upper secondary school, and in particular mathematics in the academically inclined tracks, has not been a main area of focus in the Norwegian discourse on school mathematics in the past decades. Rather, curricula and other educational policy elements regarding this segment of mathematics education in Norway have been sleeping passenger on a train driven by the discourse on mathematics in lower grades.

2 Shaping the overarching goals specifically for mathematics instruction and visibility of these goals in concrete mathematical content

The students who end up in academic-track mathematics after LK20, start on a general study programme in the first year of upper secondary school. They then choose between two mathematics subjects, 1T (T for theoretical) and 1P (P for practical). Figure 1 illustrates the structure of the mathematics subjects in the three years of upper secondary school in LK06. The structure is the same in LK20, except 2T no longer exists. The columns in the figure indicate the possible subjects each year, and the arrows show the possible choices for the students. All students must have two years of mathematics. The subject Mathematics X is optional, and is for students who need to fill up their schedule or want to take it as an extra subject.

The academic-track mathematics subjects are S1 and S2 (S for *samfunnsfag*, social sciences) and R1 and R2 (R for *realfag*, natural science and mathematics).

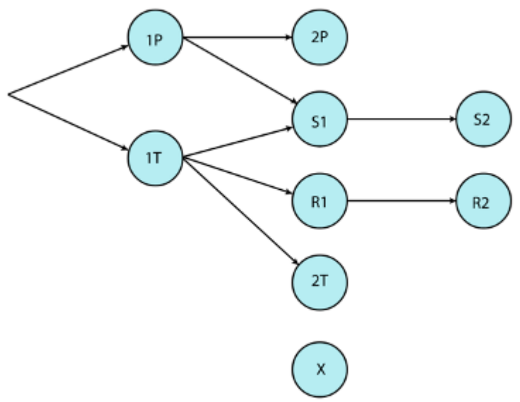


Figure 1: The structure of the mathematics subjects in upper secondary school in LK06 (Borge et al., 2014). T stands for theoretical, P for practical, S for social sciences, and R for “realfag”, natural sciences. The subject Mathematics X is optional.

The core elements in mathematics in LK20 became (Utdanningsdirektoratet, 2020) (our translation):

- Inquiry and problem solving
- Modelling and applications
- Reasoning and argumentation
- Representation and communication
- Abstraction and generalization
- Mathematical topic areas

Note that the first five of these are not core elements in the *subject of mathematics* in the sense that they describe central topic areas in the subject, such as numbers, geometry, algebra, etc. Instead, they are core elements in *work* with the subject. Thus, they are *process oriented*. In LK20, the core elements in mathematics apply to all the grades 1–13, including the academic tracks of upper secondary school.

One of the reasons for focusing on the work with the subject rather than the specific mathematical content in the core elements was to make the students work more on methods and ways of thinking to get a better understanding of mathematics, hereby also making the teachers change their instruction methods.

In LK06 and LK20, the curriculum also included *grunnleggende ferdigheter* (basic skills) in the subjects mathematics and Norwegian. In mathematics, these skills are oral and digital skills, and to be able to write, read, and calculate.

The concrete mathematical content becomes visible in the learning goals, which are competence based. The goals come in one bulleted list for each subject. In the academic-track upper secondary mathematics subjects each subject has 12–14 goals linking together the core elements and the basic skills with the mathematical content. The goals follow a certain template: they use certain kind of verbs, e.g., explore, explain, present, analyse, and each goal also consists of two competencies.

We will elaborate more on the way the mathematical content is presented in the new curriculum LK 20 below, but to give an idea of the mathematical topics included in the subjects S1, S2, R1 and R2, we list some of it below (Utdanningsdirektoratet, 2020) (our translation):

- **S1:** limits, differentiation, powers, logarithms, continuity, combinatorics, probability
- **S2:** series, integrals, fundamental theorem of calculus, exponential growth, logistic growth, statistical distributions, stochastic variables, economic models, statistical hypothesis testing
- **R1:** limits, differentiation, continuity, powers, logarithms, exponential growth, logistic growth, inverse functions, parametric equations, 2-dimensional vectors
- **R2:** series, recurrence relations, integrals, parametric equations, 3-dimensional vectors, radians, trigonometric functions, mathematical proof

These subjects all comprise 5 hours per week.

The requirements in mathematics for entrance to further studies at colleges and universities vary. Some institutions require R2 (in particular, university studies involving mathematics), whereas others only require R1, or S1+S2, which is considered as equivalent when it comes to entrance requirements. Different institutions have had trial periods with different entrance requirements.

When it comes to the concrete mathematical content, the mathematics learning goals stated for the various school grades in LK20 are less specified, both in relation to mathematical domains and the mathematical content itself, than in LK06. As an example, we use some of the learning goals in the course R2, the most advanced mathematics course from upper secondary school.

In LK06, the learning goals in R2 were presented under four *hovedområder* (main areas): geometry, algebra, functions and differential equations. The learning goals in algebra state (KD, 2006) (our translation):

The goal of the training is that the student should be able to
- *find and analyse recursive and explicit formulae for number patterns with and without digital aids, and complete and present simple proofs linked to these formulae,*
- *complete and explain proof by induction,*
- *sum finite series with and without digital aids, derive and use the formulae for the sum of the n first terms in arithmetic and geometric series, and use this to solve practical problems,*
- *calculate with infinite geometric series with constant and variable quotients, decide the convergence region for these series and present the results.*

In LK20, the learning goals come in one list, hence they are not divided into main areas. The goals related to the learning goals in algebra from LK06 now state (Utdanningsdirektoratet, 2020) (our translation):

The goal of the training is that the student should be able to
- *explore properties of different series and explain practical applications of properties of series,*
- *explore recursive relationships by using programming and present own approaches,*
- *analyse and understand mathematical proofs, explain the "carrying" ideas in a mathematical proof and develop own proofs.*

We see that words like "recursive and explicit formulae", "arithmetic and geometric series", "infinite geometric series" are replaced by less concrete phrasings like "recursive relationships" and "different series".

One of the reasons for making the learning goals less concrete is that there should be "no ceiling", i.e. the teacher (who interprets the learning goals) can go as far as she or he feels is reachable for her or his students. However, the learning goals provide no floor either.

Below is an example from one of the textbooks in R2 under LK20, showing an exercise incorporating the core element "inquiry" (Borge et al., 2022) (our translation):

> Let a and b be two positive numbers. *The arithmetic mean* between a and b is defined to be the number $m = (a + b)/2$. *The geometric mean* of a and b is defined as the number $\sqrt{ab}$.
>
> Explore the connection between arithmetic and geometric mean and arithmetic and geometric series.

When the learning goals become less concrete, the interpretation becomes more difficult. Especially considering that the exam is a national exam. At the time of this writing, there is a committee working on new types of exam questions for all mathematics exams in Norway, designed for the new curriculum. The format of the exam is also under discussion. In the S- and R-subjects, the exam has been 5 hours, consisting of two parts. In part 1, the students cannot use digital aids, whereas in part 2 all aids are allowed, except open internet and other tools that can be used for communication.

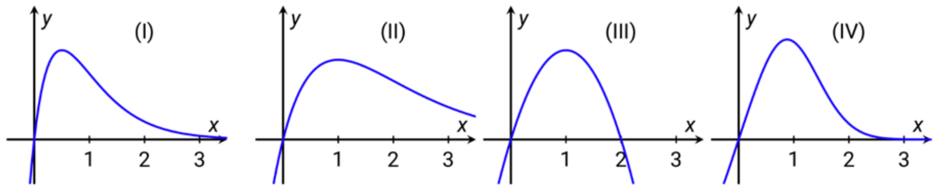
Some examples of exam questions have been released, and there have been hearings. We give an example from R1 (part 1) (Utdanningsdirektoratet, 2022a):

Funksjonen f er gitt ved

$$f(x) = 4x \cdot e^{-x}$$

En av grafene nedenfor er grafen til f.

Begrunn hvilken av grafene nedenfor som er grafen til f.



In this exercise, one of the graphs shown is the graph of the given function *f*. The task for the student is to "Justify which of the graphs below is the graph of *f*."

Since the work with exam questions is ongoing, it is difficult to say where it all will end up.

Possibly the outcome of this process will help making the interpretation of the new curriculum a bit easier. But still, we may formulate our second main conclusion as follows:

- The degree to which curricula and other educational policy documents determine what actually should happen in the Norwegian mathematics classrooms, has been decreasing over the past decades. This development is particularly pronounced in upper secondary school. Much is left open to interpretations made by less politically controlled groups such as exam development committees and publishers.

3 Empirical findings concerning the teaching reality and the knowledge and skills achieved in academic-track upper secondary mathematics

A good source for data about students' achievements at upper secondary level is the IEA TIMSS Advanced study (IEA, 2022). This study collects data on student achievement and attitudes as well as other educational factors in upper secondary school, in an international perspective. The TIMSS Advanced Mathematics framework (Grønmo et al., 2015) can be seen as defining a common ground for the participating countries concerning what should constitute academic mathematics in upper secondary school.

TIMSS Advanced has conducted an international study in mathematics at the end of upper secondary school in 1995, 2008, and 2015 (Mullis et al., 2009; Mullis et al., 2016b). While Norway participated both in mathematics and physics in 2008 and 2015, in 1995 Norway participated only in the international study in physics. However, in 1998 Norway conducted an identical study in mathematics, using the same procedures and instruments as used internationally in 1995. The results for Norway are therefore not published in the international report in 1995, only in national reports (Angell et al., 1999; UiO, 2022b). Nine countries participated in TIMSS Advanced in 2015: France, Italy, Lebanon, Norway, Portugal, Russia, Slovenia, Sweden, and USA. The results for Norwegian students in mathematics is pretty low compared with the other participating countries. The percentage of the actual age cohort that studied mathematics at the highest level in each country, the so-called coverage index, is also lower for Norway than for most of the other countries (Mullis et al., 2016a). While France and Slovenia have about the same mean mathematics achievement as Norway, their coverage indexes are much higher. The overage index in France is over 20 percent, in Slovenia about 34 percent, whereas the Norwegian coverage index is just above 10 percent. This tells us that France and Slovenia educate a significantly higher percentage of their population to an advanced "academic-track" level in mathematics, giving their students an indisputable better basis for a number of further educations at the university level.

TIMSS Advanced compare the percentage of students in each country that reach three defined levels for mathematical competence: Intermediate level, High level, and Advanced level (for more about the definition of these levels, see the international report (Mullis et al., 2016b; Grønmo et al., 2015)). Table 1 shows the percentage of students that participated in the study that reach each level in Norway, Slovenia, and France, and the percentage of the actual age cohort reaching each level.

Table 1: Percentages of students at intermediate, high and advanced level in France, Norway and Slovenia, TIMSS Advanced 2015

	Intermediate level		High level		Advanced level	
	Percent of TIMSS Adv. students	Percent of age cohort	Percent of TIMSS Adv. students	Percent of age cohort	Percent of TIMSS Adv. students	Percent of age cohort
France	43	9	11	2	1	0
Norway	41	4	10	1	1	0
Slovenia	42	14	14	5	3	1

The percentage of students participating in the study that reached Intermediate level is about the same in Norway as in Slovenia and France, while comparing the percentage of the actual age cohort reaching this level is respectively 14 % and 9 % in Slovenia and France, and only 4 % in Norway. The pattern is the same for the percentage of students reaching High level in these three countries.

In the international report, presenting results from all participating countries, it is important to notice that the percentage of age cohort that reach High level in Norway was lower than in all other countries in TIMSS Advanced 2015 (Mullis et al., 2016b). The national report concluded that Norway, to a lesser degree than other countries, failed to give students with talent and interest for mathematics the mathematical competence needed for many further studies at the university level (Grønmo & Hole, 2017).

TIMSS Advanced also indicates interesting information about how different domains of mathematical content are prioritized across countries. Students in TIMSS Advanced are tested in three different mathematical domains: Algebra, calculus, and geometry (Grønmo et al., 2015). Achievement data in each of these separate domains may help us understand what type of competence students in a country have achieved last year of upper secondary school. By that indicating what type of competence a country sees as an essential basis for further academic studies. TIMSS Advanced compares a country's achievement in each of these domains with the mean level of achievement for the country. By that, we get a measure for the relative focus given in a country on each of these domains. Norway's mean is 459 points, but 13 points lower in algebra, and 4 points and 14 points higher in calculus and geometry respectively. All differences are statistically significant. Norway has a lower relative focus on algebra than any of the other countries that participated in TIMSS Advanced 2015. France, Slovenia, Portugal and Russia all had a significantly higher focus on algebra (Mullis et al., 2016b; Grønmo & Hole, 2017). Data from TIMSS Advanced 2015 show that one content domain, namely algebra, stands out as the domain where Norwegian students' performance is especially weak. This is a result consistent for Norwegian students in all TIMSS studies at all levels in school from 1995 till today (UiO, 2022c). Algebra is a domain where Norwegian students achieve especially low. Taking into account that algebra may be seen as a mathematical language which is a very important building block for all forms of university level education involving mathematics (Grønmo, 2018), this is problematic. Research in Norway on problems students meet in mathematics at the university level, support the conclusions that lack of basic algebraic knowledge that the students were supposed to learn in upper secondary school can be a main reason for students' failure in beginning university courses in calculus (Hole et al., 2021).

Another problematic issue concerning academic-track mathematics in Norway is the percentage of girls choosing the advanced mathematical courses in upper secondary school. As already mentioned, the general coverage index (percentage of age cohort) taking advanced mathematics tracks is pretty low in Norway compared with most of the other countries in TIMSS Advanced. For Norwegian female students, the coverage index is even lower (Mullis et al., 2016a). Only 8 percent of the actual age cohort of female students in Norway take advanced mathematics last year of upper secondary school. In France and Slovenia, respectively 20 and 40 percent of females take these courses. Norwegian politicians seem to like portraying Norway as an exemplary country concerning equality between females and males, but the coverage indexes mentioned here show that this is not the case in advanced mathematics in school. The Norwegian difference in coverage index between boys and girls is even more pronounced in upper secondary school physics, which is the other school subject tested in TIMSS Advanced. Unfortunately, we must conclude that recruitment of females into mathematics and science has not worked well in Norway. Even more problematic, the arrows seem to be pointing in the wrong direction, with fewer students and a decrease in achievement for each year (Grønmo & Hole, 2017).

In the PISA study in 2018, Norwegian girls scored a little higher than Norwegian boys (OECD, 2022; UiO, 2022a). This may be interpreted as a contradiction to our conclusion about the situation between boys and girls in mathematics in Norway. However, when comparing results from different studies we have to take into account the framework and items used to measure the students' achievement. Norway has participated in various international studies in lower secondary school, which is a good thing. We participate both in PISA and TIMSS, and we also have national tests of students in mathematics and reading in lower secondary school, with a participating rate over 90 percent. By carefully examining and comparing the results from all these tests, we get a better picture of the situation and the challenges we have to meet in our educational system. PISA tests what they define as *mathematical literacy* at the end of lower secondary school. Mathematical literacy focuses on use in daily life. Items in PISA, since related to daily life, include in general quite a lot of text to be read describing a daily life situation, much more than traditional items in mathematics. In Norwegian national tests, boys achieve better than girls in mathematics, the opposite is the case in reading (Utdanningsdirektoratet, 2022b). The PISA results for Norway are therefore likely to be influenced by the large amount of reading needed to answer items in what they define as mathematical literacy. The results in tests in TIMSS in lower secondary school are also in contradiction to the

PISA results that girls tend to perform better than boys. In TIMSS Advanced we see that boys outperformed girls both in percentage taking the course, and in results on the tests. PISA tests students to a little degree in mathematics needed for further studies (Hole et al., 2018). Already in 2005 over 200 mathematics teachers and researchers (Astala et al., 2005) point to the problematic issue that PISA results are interpreted as students achieving high in that study have a good foundation for academic mathematics. They argue that that is not at all the case. PISA gives adequate data for Norwegian students' knowledge in mathematics to be used in daily life, but we have other and more relevant data, for what type of mathematics students will need in upper secondary school and for further studies. Mathematics learned in lower secondary school is important, both for using mathematics in daily life and for giving a basis in algebra for further learning. The Norwegian challenge to be met is to implement a better balance between these two goals for mathematics in school.

With relatively few students choosing academic mathematics in upper secondary school compared to other countries in TIMSS Advanced 2015, and the level of achievement in general, we need to take a closer look at what students learn and are exposed to in lower levels in school. As mentioned in section 1 of this report, in the Norwegian curriculum all the grades 1–13 (primary, lower, and upper secondary) are seen as steps on a ladder. In lower secondary school, there are consistent results from 1995 up until today showing that Norwegian students achieve especially low in algebra (UIO, 2021). The Norwegian TIMSS and PISA results in mathematics around 2003 were considered a "shock" by both educators, politicians, and the public in Norway (Grønmo et al., 2004; Kjærnsli et al., 2004). While there have been some indications that Norwegian students achieve slightly better in mathematics overall after 2003, this is not the case in algebra (Bergem, 2016a). When comparing the focus on algebra in lower secondary school with the focus on more daily life mathematics as statistics, Norwegian students showed the largest difference in favour of statistics compared with algebra out of any of the countries. The result in algebra for Norway is consistently low over time and in different studies. This is the case, even if it already in 1995 was pointed out that Norwegian students were achieving problematically low in algebra (UiO, 2022c).

It is also relevant to look at analyses combining data from studies of student achievement to data from studies related to other aspects of school mathematics. In the Norwegian context, the TEDS-M study of competencies of mathematics teacher students is interesting (Tatto et al., 2012; Grønmo & Onstad, 2012). There is much discussion in Norway concerning lack of well-educated teachers, but this discussion has also mainly been focused on grades 1–10. Data from TIMSS Advanced concerning age and education levels of teachers in Norwegian upper secondary school have been alarming for several years (Grønmo et al., 2010). Several of the international studies have documented that mathematics teachers in Norway have a relatively low competency in mathematical subject content when compared to other countries, and that they are offered relatively few additional in-service courses in mathematics when teaching in school (Grønmo et al., 2016; Mullis et al., 2016a). Also, the fact that this challenge needed to be met was pointed out in the national reports already in 1995 (UiO, 2022c).

In TEDS-M, patterns in achievements were also found which confirmed the regional clusters concerning achievement which has consistently been found in studies such as PISA and TIMSS, on all school levels. In Nordic and English-speaking countries, students achieve relatively low on items related to academic mathematics such as algebra, and better on items closer to daily life, such as statistics and probability. In East-European and East-Asian countries, the picture is the opposite. These regional clusters are stable over time and consistent across studies using different frameworks. It appears that there are quite different cultures of mathematics education in different regions of the world (Grønmo, 2018; Blömeke et al., 2013). For more about these types of cluster analyses, see (Olsen, 2006).

As far as Norway is concerned, we may summarize the above in our third main conclusion of this report:

- Compared to other countries, the culture in mathematics education in Norway does not favour academically oriented mathematics such as algebra. Algebra may be particularly important in academic-track upper secondary mathematics, but in society today it is also needed in several vocational studies. To learn algebra, as other languages, the foundation laid in compulsory school is important for learning later. Norwegian school mathematics is geared more towards mathematics relevant for people in their everyday lives than for further learning. This fact is also visible in mathematics teacher education. A better balance is needed in Norwegian schools between daily life mathematics and mathematics for further learning. There is also a significant recruitment problem concerning academic-track mathematics and physics in upper secondary school, particularly for females.

4 Strengths and challenges arising concerning mathematics education in Norwegian academic-track upper secondary school

The challenges that arise concerning mathematical education in the upper secondary school has to be reflected upon based both on the educational goals as presented in the curriculum, and on the empirical findings on knowledge and skills achieved by the students. Also, as mentioned above, it is important to take into account the situation at lower levels in school. Although the Norwegian results in PISA are respectable (Kjærnsli & Olsen, 2013), we consider the relevance of this to academic-track upper secondary mathematics as quite limited, since PISA is primarily testing everyday mathematics (Hole et al., 2018). See section 3.

Concerning challenges, we have already listed the following:

1) A general lack of focus on upper secondary school mathematics, with no separate policy for academic tracks.
2) Weakened mechanisms for political influence on the mathematics actually taught in the classrooms, caused by curricula which are less concrete and specified.
3) Low student achievement in academically oriented areas of mathematics, in particular algebra, compared to other countries (results consistent over several decades, different studies and different levels).
4) We need a better balance between daily life mathematics and academically oriented school mathematics in Norway.

On the positive side, the core elements in the new curriculum concerning abstraction, generalization, modelling, exploration and so on may potentially give Norwegian teachers and students added possibilities for reforming mathematics education in a positive way.

Concerning strengths, it should also be remarked that Norway generally scores very high on school climate, school safety, student engagement and attitudes, and teacher job satisfaction (Mullis et al., 2016b). This, along with other data from various international studies where Norway has participated, may be perceived as a good foundation for improving Norwegian schools on all levels, including upper secondary mathematics academic tracks.

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