Partial internal models for a Norwegian life insurance company

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Abstract

In this thesis we wish to explore and develop Solvency II-compliant computational tools that will provide arguments for the insurance company to employ internal models. Applying these tools we will replace some of the stress tests of the standard model (standard formula) with partial internal models when computing the solvency capital requirement (SCR). We will then compare the SCR numbers of the partial internal models with the SCR numbers of the corresponding stress tests.

Since the standard model is meant to apply for all insurance companies in all countries subject to the EU regulations for insurance companies, it has to be an average model for a diverse universe of insurance companies. For an average model meant to guarantee solvency with a 99.5% probability for all "SCR compliant" insurance companies, it is our hypothesis in this thesis that the standard model will have to be calibrated quite conservatively. We will test this hypothesis.

In this thesis we introduce multivariate simulation as an alternative to the standard model/formula. With this numerical method the insurance company may compute SCR estimates that are tailor-made to its actual balance sheet risk exposures. This way we aim to provide partial internal models that not only will compute SCR numbers that better reflect the actual risk exposures, they may also assist the insurance company with a better understanding of its own risks. This last part will come in handy when the insurance company is performing the mandatory "Own risk and solvency assessment" (ORSA) and "Forward looking assessment of own risks" (FLAOR).

With such partial internal models the insurance company may be able to free up capital that in effect will provide an extra financial flexibility for the company. This freed up capital could for example be paid out as an extra dividende to the shareholders/owners, or it could be used as available SCR for an increase in the risk level from existing or additional insurance activities. Either way, assuming fiduciary responsibility of the insurance company, its expected return on investment is improved.

In this thesis we will look at partial internal methods for a fictive Norwegian life insurance company, but the main priciples of the models apply to any insurance company subject to Solvency II. As we in particular look at balance sheet items exposed to market risk, the partial internal models developed in this thesis should to a large extent be applicable to other types of insurance companies as well, with the necessary modifications reflecting differences in insurance activites.

Acknowledgements

My motivation for this thesis came from working on a mandatory assignment for a Master's course on life insurance (STK4500) at the University of Oslo (UiO) in 2018. This assignment required the students to compute the solvency capital requirement (SCR) of a fictive Norwegian life insurance company applying the stress tests of the standard model/formula of Solvency II. Working my way through endless pages of legal regulatory requirements outlining a method for computing risk exposures within a mathematical structure, I found this structure to lack any significant level of mathematical stringency. The standard method/formula seems to approach the diversification of financial portfolios in a haphazard way, and the stress tests seemed to be too coarsely constructed to measure risk in any reliable and meaningful way. My reaction to all this was; "Clearly, there has to be a better way."

Luckily, the answer was close by. In the very same Master's course we studied actuarial techniques and numerical simulation methods relevant to the life insurance company. I would like to thank the academic staff behind the "Finance, insurance and risk programme", at the University of Oslo, for a very interesting and instructive curriculum. In particular, I would like to thank Førsteamanuensis Ingrid Hobæk Haff for giving inspirational lectures in life insurance (STK4500). Also, a special thank you to Professor Emeritus Erik Bølviken for his excellent textbook "Computation and Modelling in Insurance and Finance". It is a treasure trove for the budding actuarian or financial quant.

While the above set my course for this thesis, it was my supervisor Professor Arne Bang Huseby who set may ways straight when I strayed off. Thanks to his focus on scope and his easygoing manner and helpful insight I took great plasure in discussing the topics of my thesis with him as well as many other topics. Also, I had the pleasure of dicussing actuarial topics and Solvency II regulations with Professor Emeritus Erik Bølviken. Once again he managed to broaden my horizon.

Along the course of a Bachelor and a Master's programme one is set to meet people that will have a significant impact on one's academic and personal outcome from such studies. The academic staff at the Department of Mathematics is a given. But just as important were my fellow students. In particular, Anders Matheson, Vegard Antun, Even Teigland and Johnny Ringstad, who all had a significant impact on my academic progress as well as making life more interesting on and off campus.

Finally, I would like to thank you, the reader of this document, for making

the effort to do just so. My hope is that when you are done reading you will have found what you were looking for, if only just fragments of it.

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CHAPTER 1

Introduction - Solvency Capital Requirement (SCR)

1.1 Solvency II

Introduction

The rules and regulations of Solvency II, provide life insurance companies with a standard method for computing the solvency capital requirement (SCR), known as the *standard model*. This method comprises a set of stress tests for the insurance company to apply to its balance sheet items, to compute the SCR for sub-groups of its insurance and market investment activities. These stress tests will provide the SCR for a number of sub-groups of the balance sheet. Aggregation of the SCR for certain risk sub-groups, i.e., from sub-groups of the balance sheet to higher balance sheet levels, is performed by applying the *standard formula* (see below). As an alternative to the standard model, Solvency II also allows the insurance company to replace the standard model, or parts of it, with an internal model, or a partial internal model, if the company can qualify that such an internal model will provide a better estimate of the SCR.

In this thesis we wish to explore and develop computational methods and tools that will provide the life insurance company with arguments for employing internal models. Instead of the schematic simplifications of the stress tests and the standard formula, we wish to develop a computational method and tools which provide more accurate SCR computations. Computations that are more tailored to the actual risk exposures of the individual insurance company.

The reason for why we want to do this is two-fold. One is that there is a gap in mathematical stringency between the vision of Solvency II – to keep the probability of insolvency below 0.5 %, and the less stringent approach the standard model. The SCR computational methods are an inegral quantitative tool for financial risk management and reporting, and we wish to improve its performance. Second, we believe that this "gap of stringency" has led to too conservative stress tests and correlations (for the standard formula) in order to make abolutely sure that *all* insurance companies that are "SCR compliant" will indeed avoid the risk of being insolvent and thus avoid going bankrupt.

We belive that replacing the standard model with a better performing internal model will improve the risk management, i.e., the understanding of the risk exposures and handeling them. Also, we belive that for some insurance

companies internal models are likely to reduce the SCR significantly, freeing up capial for the company to be used otherwise to improve its return on capital.

Thus, we will in this thesis introduce partial internal models to replace the stress tests for some of the risk sub-modules of the insurance company's balance sheet. By doing so we wish to demonstrate that this brings the SCR computations stringently more in line with the vision of Solvency II, i.e., it's a better risk management tool, and with the silver lining of improving profitability.

Before we get to the partial internal models, we will first have to take a look at the stress tests and the standard formula of the standard model. That will be the topic for the remainder of the introduction in this chapter. Next, we will introduce a reference case in Chapter 2, i.e., the necessary balance sheet information in terms of a fictive life insurance company. This information will put us in position to apply the stress tests to compute the SCR. In Chapter 3 we will compute the SCR for each risk sub-module, i.e, for each stress test relevant to the reference case of our fictive insurance company. We then aggregate the SCR from the risk sub-modules all the way up to the basic operations of the company, BSCR. This will then serve as the reference case for the partial internal models to be treated in Chapters 4 and 5.

Solvency II regulation

The regulations for solvency capital requirements for European insurance companies (Solvency II) are outlined in an EU Directive (the "Directive") [Eur09], and are further detailed in a Delegated Regulation (the "Delegated Regulation") [Eur14] and Technical Specifications. To be compliant with such regulations, one will need to have a good handle on how to compute the solvency capital requirement (SCR) as set out in these documents. They are not an easy read. For a brief and basic, but to the point, exposition of the main principles behind computing SCR numbers as outlined in these rather intricate documents, the reader is referred to the more easy to read, yet succinct, exposition of the main principles of Solvency II for life insurance companies provided by Professor Erik Bølviken, [Bøl17]. In this section we will just provide a brief overview for how the standard model and the standard formula for computing the SCR has been constructed. The overview, icluding its figures, are taken from Bølvikens exposition. Description of the actual stress tests applied in our reference case is found in Chapter 2. Any reader who is well versed in this world of SCR and stress tests for a life insurance company, may want to jump ahead to the next chapter.

The purpose of the standard model is to provide a simple method for quantifying how much solvency capital the insurance company is required to hold on its books in order to keep the risk of insolvency for the next financial year below a 0.5 % probability. Implicit in this requirement is the assumption of "going concern", i.e., no "fire sale" of assets or any other out of the ordinary transactions, etc.. To quantify this risk of insolvency one will need to quantify the risk inherent in all assets and liabilities that may have an adverse effect on the solvency capital, i.e., on the insurance company's equity, the basic own funds (BOF), over the next financial year. This approach is the classical case of asset liability management (ALM), and we will need to quantify the risk of adverse effects on the value of all assets and liabilities on the balance sheet.

1.2 The insurance company balance sheet

To provide for a computational method for the SCR that will be tractable for all insurance companies, and probably also for the purpose of all supervisory authorities, the regulatory authorities have applied a top-down breakdown approach to the insurance caompany's balance sheet (see Figure 1.1). This breakdown takes a financial accounting perspective and basically groups risk into the different business activities, or areas, of the company. On the *asset side*, on the left, this will be the investment activities, represented by total *Asset* (A). In essence, these assets are the net total of all investment portfolios. On the right we have *liability and equity*. First, we have the core activity of any insurance company, the insurance activities - receiving premiums today against uncertain future liabilities. This activity is quantified by the amount *best estimate* (BE), which is the expected value of all future cash flows in and out of the accounts under existing insurance contracts. Second, there is the *risk margin* (RM), which under Solvency II is defined as the extra amount another insurance company would have to be paid to be willing to take over responsibility for all obligations under the BE. Third, most insurance companies have *other liabilities* (OL), other than the obligations of its insurance contracts. These would most certainly be pending tax obligations, but could also be obligations such as ordinary debt, shareholder benefits outstanding or any other loan arrangement. Aggregating all these liabilities we get total liabilities, also known as the *technical provisions* (TP) in insurance,

$$TP = BE + RM + OL.$$

Finally, we have the equity of the insurance company, also known as *basic own funds* (BOF), which is the residual of assets less technical provisions,

$$BOF = A - TP.$$

Any company that has run out of equity will be insolvent, thus negative BOF must be avoided. This is the principal aim of the Solvency II regulation, and it is imposed by applying detailed risk measures to compute the solvency capital requirement (SCR). This quantification of risk is for the next financial year, where the company has certain amounts at current time $t = 0$ for assets and technical provisions (A_0, TP_0), while these amounts are uncertain for future time $t = 1$. Thus, the company will have to estimate (A_1, TP_1). The standard model of Solvency II apply stress tests to quantify the risk exposures to its different blance sheet items. When computed, the stress tests will then provide the solvency capital required (SCR) for those balance sheet items. These SCR numbers are then aggregated by using the standard formula. The effect of the SCR number computed for the insurance company as a whole will then reflect either a decrease in assets or an increase in technical provisions, and, cosequently, a reduction in BOF (equity). Alternatively, the insurance company may use their own internal risk models, if the company has qualified their models with the supervisory authories.

The transition from (A_0, TP_0) to (A_1, TP_1) must account for *premia received* (on existing contracts), *obligations paid*; insurance (pension, death, accidents, etc.) and other, changes in investment *portfolio values* (gain or loss), and, *other cash flows* (overhead, dividende, taxes, etc.).

A (Total Assets) Investment Portfolio	**BOF** (Basic Own Funds) *Assets – Liabilities*
	OL (Other Liabilities)
	RM (Risk Margin)
	BE (Best Estimate)

Figure 1.1: Solvency II: The life insurance balance sheet.

Naturally, there will likely be other items to account for in addition to those listed here, but these are under normal operations the most important ones. Among those, assets and the best estimate are expected to contribute to most of the risk due to both their riskiness and by representing the largest nominal amounts on the balance sheet.

1.3 The vision of Solvency II

In building a model that will quantify the solvency capital requirement (SCR), the model will have to be stochastic and it must be based on probabilities. The *conditional probability* that would be most describing of the problem is

$$P\left[A_1 < TP_1 | A_0 = TP_0 + SCR\right] = \varepsilon \quad \text{where} \quad \varepsilon = 0.005. \tag{1.1}$$

This is the probability of negative BOF in one year, given that the insurance company holds the current value of assets (A_0) and an additional solvency capital, SCR, on its balance sheet today. Here ε is the "acceptable" risk level of insolvency (going bankrupt) for the next year. There is also the *minimum capital requirement* (MCR)

$$P\left[A_1 < TP_1 | A_0 = TP_0 + MCR\right] = \varepsilon \quad \text{where} \quad \varepsilon = 0.15,$$

where MCR is a much smaller value than SCR. Failing to comply with this latter requirement will automatically lead supervisory authorities to suspending the authorisation of the insurance company. In this text we will only discuss the computation of the solvency capital requirement (SCR).

Solving Equation (1.1) is not straightforward and a closed form solution may be impossible. This is likely a driving factor behind the standard model and the standard formula; the design of the stress tests, calibration of shock factors and correlations, and, probably also the provisons for internal models.

1.4 Execution of Solvency II - the standard model

The standard model of Solvency II provides a method that approximates the actual risk exposure of the insurance company. The method is a recursion over an oriented graph, and risk levels are approximated by the solvency capital requirement (SCR) computed using the *stress tests* of the standard model. Each iteration of the recursion is an aggregation of the SCR numbers to a higher balance sheet level, and is executed by applying the *standard formula*.

To be able to establish meaningfull and tractable *stress tests* as an approximation to the risk level for all relevant quantities on the balance sheet over the next financial year, the accounting items of the balance sheet, described above, are further broken down into more homogeneous sub-groups (see Figure 1.2). We now move from the accounting terms of Figure 1.1 to the Solvency II terms of the solvency capital requirements (SCR) for those balance sheet items. In this thesis we will focus on the basic own funds (BOF) in Figure 1.1, i.e., the owners' equity in the company. Thus, we will in this exposition disregard other liabilities (OL) and risk margin (RM) which are expected to be of less importance risk wise. The risk term corresponding to BOF is in Solvency II terms referred to as the SCR of Basic Operations (BSCR), as seen in Figure 1.2.

In this thesis we will narrow in on BSCR and leave out SCR from Operatioal activities and that of Adjustments.

The BSCR is further broken down into two **risk modules**, as they are labeled in the Delegated Regulation [Eur14], namely *market risk* (SCR^{mark}) and *liability risk* (SCR^{liab}) (Layer 2). These two risk modules are further broken down into sub-groups (Layer 3), which are labelled **risk sub-modules**. This is where the stress tests reside. For the stress tests to work fairly well as approximations to the actual risk exposures each risk sub-module must be assumed to consist of fairly homogeneous risk exposures, or the risk exposures must "average out" for an "average-based" stress test to be representative with respect to the risk exposures.

Recalling Equation (1.1), since conditioning on $A_0 = TP_0 + SCR$, and since $BOF = A - TP$, we have that the SCR takes the place of BOF at time $t = 0$. SCR expresses how large BOF needs to be in order to withstand the total risk exposures to asset and liability values over the next financial year, in order for the insurance company to stay solvent with 99.5% probability. Under Solvency II, SCR is the "minimum buffer capital" required. To compute the SCR we will therefore have to compute the effect of all risk exposures to the asset and liabilitiy values for the insurance company as a whole at the 99.5% probability for solvency. As earlier mentioned, in this thesis we will limit our exposition to assets and the best estimate values.

The current value of the *assets* (A_0) is established by market values observed (or, in some cases, estimates of these), while the uncertain future value in one year (A_1) is exposed to risk coming from many different risk sources; bonds and other interest rate instruments are for example exposed to changes in *interest rates*, stocks and other share price related instruments are exposed to changes in *share prices*, and correspondingly for other asset classes in the investment portfolios. In this thesis we denote each such source of risk, e.g., a share price or an iterest rate, as **risk factors**. The risk factors are the random variables that are used by the stress tests to compute the SCR of each stress test.

The *best estimate* (BE) is the expected value of the insurance liabilities of the life insurance company. It is computed for all existing contracts grouped into groups of contracts with fairly homogeneous terms, and for each group the best estimate is computed:

$$BE = \sum_{k=1}^{K} \frac{L_k}{(1+r_k)^k}, \tag{1.2}$$

where L_k is the expected, net cash flow out of the company accounts in year k, and r_k is the expected risk-free interest rate in year k (used for discounting risk-free cash flows). The uncertain future value of the best estimate in one year (BE_1) is exposed to risk factors affecting L_k, such as changes in the mortalities of the policy holders - an increase in mortality (term insurance) and that of people living longer than expected (longevity; pensions), and from possible changes to the risk-free interest rates r_k.

The *market risk module* for assets (A) is comprised of several risk sub-modules which are aggregated applying the *standard formula*. The *liability risk module* for life insurance risk also has several risk sub-modules which are aggregated in the same way. Further aggregation of the SCR to more aggregate

Top Company surplus
SCR

Layer 1 Operational   Basic Operations   Adjustment
SCR^{op}   **BSCR**   **Adj**

Layer 2 Investments   Insurance activities
SCR^{mark}   **SCR^{liab}**

Layer 3 Interest rate (**SCR^{int}**)   Mortality (**SCR^{mort}**)
Equity (**SCR^{equi}**)   Catastrophy (**SCR^{cat}**)
Property (**SCR^{prop}**)   Longevity (**SCR^{long}**)
Spread (**SCR^{spread}**)   Disability (**SCR^{disab}**)
Currency (**SCR^{curr}**)   Expences (**SCR^{expe}**)
Concentration (**SCR^{conc}**)   Lapse (**SCR^{laps}**)
Revision (**SCR^{rev}**)

Standard formula:

$$SCR_y = \left(\sum_i \sum_j \rho_{ij} \times SCR_i \times SCR_j \right)^{1/2}, \qquad i,j = 1, 2, \ldots, n.$$

Figure 1.2: A simplified Solvency II representation of life insurance risk.

balance sheet levels is also done by applying the standard formula:

$$SCR_y = \left(\sum_{i=1}^{n} \rho_{ij} \times SCR_i \times SCR_j \right)^{1/2}, \qquad (1.3)$$

where the subscript y represents the aggragate SCR-level, subscripts i, j represent the different sub-level groups, and $\rho_{ij} = cor(SCR_i, SCR_j)$ the correlation between the different sub-level groups, with $-1 \leq \rho_{ij} \leq 1$ as usual. Equation (1.3) is the *general version* of the standard formula. For certain aggregations there may be a *customised standard formula*, but the general priciples of the general standard formula applies to all dependent risk modules. Independent risk modules are simply added.

With the standard formula at our disposal we may now compute the SCR bottom-up from the basic risk sub-modules/stress tests (Layer 3), through the risk modules of assets and liabilities (Layer 2) - and all the way to the BSCR (Layer 1) of the company's basic operations. This is the risk in BOF.

When the undelying risk variables of this method of aggregation, i.e., the SCR numbers computed by the stress tests, are all assumed to be Gaussian, this aggregation method is appropriate. However, the Gaussian assumption may not be appropriate, and consequently dependency between the underlying risk variables may not be well described by correlations. This is the case of of non-linear risk, since correlation is a description of dependency in linear relationships and works poorly when ralationships are strongly non-linear. See [Bøl17] for more details.

1.5 The stress tests

To estimate the risk exposure inherent in the balance sheet items of each risk sub-module, stess tests are applied under the standard model.

For *assets* (A) these stress tests are for different *market risk factors*: *interest rates*, *equity* (share prices, share indices, etc.), *property* (real estate prices, rental prices), *spread* (on interst rate instruments), *currency*, and *concentration* ("too many eggs in one basket").

For the *best estimate* (BE) the *liability risk factors* are *mortality* (term insurance), *catastrophy* (general insurance), *longevity* (life insurance), *disability* (pensions, term insurance), *expenses* (overhead), *lapse* (more clients leaving than expected), and *revision* (legal rules being changed).

We will give thorough description these stress tests in Chapter 3 when we compute the SCR for each risk sub-module applying these stress tests. Suffice here to say that these stress tests are approximations to the actual risk exposures expected (and observed). They are all rather schematic in design, which in many cases make them very "crude tools" when computing the SCR numbers. Consequently, the stress tests may often lead to risk estimates (SCR) that are way-off given the actual risk exposures. If the stress test are to be a "guarantor for solvency", they will consequently have to be designed quite conservatively, as will also the correlations for the standard formula.

1.6   Internal models

Given the conservative nature of the purpose behind Solvency II, one should not be surprised to be observing SCR numbers that are overshooting the actual risk exposure. Such an observation should motivate insurance companies to explore the benefits of internal models.

In the case of implementing an internal model one will replace one or several of the risk modules or risk sub-modules of the standard model, with corresponding internal model modules. In this thesis we will focus on the market risk module and discuss partial internal models to replace *interest rate risk* ($SCR_{interest\ rate}$), *equity risk* (SCR_{equity}) and *currency risk* ($SCR_{currency}$).

PART I

The life insurance company and Solvency II

CHAPTER 2

Reference case - a fictive Norwegian life insurance company

2.1 Introduction

In 2018 a mandatory assignment for a Master's course on life insurance (STK4500), at the University of Oslo, required the students to compute and explain the solvency capital requirement (SCR) of a fictive Norwegian life insurance company. When computing the SCR the students were instructed to apply the standard model stress tests and the standard formula approach of Solvency II. The results of these computations were so illustrative in terms of the solvency capital required that it will serve as a *reference case* for this thesis when we consider the option of replacing parts of the standard model with partial internal models. Here, we will make one change to the above mentioned assignment by introducing a risk sub-module for currency risk. Otherwise the input and framework from the assignment are the same. The reference case involves assumptions about the insurance and market investment activities of our fictive Norwegian life insurance company.

The reference case will only involve a select group of the risk sub-modules of the market risk module (SCR_{market}) and the liability risk module (SCR_{life}) for a life insurance company. The reader is referred to [Bøl17] for a description of the other risk modules.

In this chapter we will establish all input and assumptions necessary for the reference case. In the Chapter 3 we will compute the SCR for the reference case under the standard model. With this our reference case will be complete. Then, in Chapter 4, we will extend the input and assumptions about our fictive life insurance company to include details necessary for our partial internal models. In doing so we will keep with every input and assumption of the reference case, only providing more nuanced details. Finally, we will then compute the SCR for a select group of partial internal models. We will focus on risk exposures related to *market risk*, and will establish partial internal models that will replace the stress tests in the risk sub-modules of *interest rate risk*, *equity risk*, and *currency risk*.

2.2 Life insurance liability assumptions

The insurance liability of our fictive life insurance company will come from its *term insurance* and *pension insurance* activities only. These two activities will capture what should be the two most important liability risks of any life insurance company, i.e., *mortalisty risk* and *longevity risk*. Thus, we will in this thesis ignore *catastrophe risk*, *disability risk*, *expenses risk*, *lapse risk*, and *revision risk*, for a simplistic exposition of risk exposures.

The actual activity level for these two liability groups, and thus the actual liability risk exposures, are assumed to follow prespecified probability distributions describing the insurance portfolio policy structures and the mortality rates applicable for both insurance portfolios.

Mortality exposure

The *term insurance* provided by this life insurance company is insurance against death. All *contracts* (policies) are accumulated in *Portfolio 1*, and are assumed to be equal; they were agreed at $l_0 = 30$ years of age (of the policy holder), they last for $K = 25$ years, with premia π_1 paid in advance, and, the one-time benefit arrears payment of $s_1 = 2$ (if any) is transferred to the beneficiary upon death of the policy holder.

The *survival probabilities*, and thus, the *mortalities*, are assumed to follow the *Gompertz–Makeham* model,

$$p_l = e^{-\theta_0 - (\theta_1/\theta_2)(e^{\theta_2}-1)e^{\theta_2 l}},$$

with $\theta_0 = 0.00078$, $\theta_1 = 0.0000376$, and, $\theta_2 = 0.092759$.

The age of the term insurance policy holders is assumed to follow the *age distribution*,

$$N_{1k} = C_1 e^{-\gamma_1 |k - \mu_1|}, \qquad k = 1, \ldots, 25,$$

where N_{1k} is the number of individuals in Portfolio 1 who set up their contract k years earlier, and,

$$C_1 = \frac{J_1}{\sum_{k=0}^{25} e^{-\gamma_1 |k - \mu_1|}}, \qquad \gamma_1 = 0.10, \qquad \mu_1 = 20,$$

with J_1 being the total number of policy holders,

$$J_1 = \sum_{k=0}^{25} N_{1k} = 150,000.$$

Longevity exposure

The *pension* portfolio is assumed to be made up of identical policies. All *contracts* (policies) are accumulated in *Portfolio 2* and are assumed to be equal; they were drawn up at the age of $l_0 = 30$ years (policy holder), last to the end of life defined as $l_e = 120$, and, with a retirement age of $l_r = 67$. The annual pension received after retirement, until the death of the policy holder, is $s_2 = 0.3$ received in arrears. Prior to retirement a premium π_2 is paid in advance.

The *survival probabilities* are assumed to follow the same *Gompertz–Makeham* model as for the above term insurance portfolio, while the age for these policy holders is assumed to follow the *age distribution*,

$$N_{2k} = C_2 e^{-\gamma_2 |k - \mu_2|}, \qquad k = 1, \dots, 65,$$

where N_{2k} is the number of individuals in Portfolio 2 who set up their contract k years earlier, and,

$$C_2 = \frac{J_2}{\sum_{k=0}^{25} e^{-\gamma_1 |k - \mu_1|}}, \qquad \gamma_1 = 0.15, \qquad \mu_1 = 24,$$

with J_2 being the total number of policy holders,

$$J_2 = \sum_{k=0}^{65} N_{2k} = 200,000.$$

Other liability assumptions

To compute the liability of term insurance and pension insurance we will need the discount rates derived from the risk-free interest rate curve.

Every month EIOPA offer their projections for the risk-free interest rate curve with a forward-looking horizon of 150 years. These projections include both the spot curve for each of the 150 years as well as for the shocked curve, and they are provided for each country subject to Solvency II regulations. There is a shocked curve for both an upward and a downward shift. Instead of estimating the risk-free interest rate curve oneself, and then compute the shocked cure, as specified in Article 167 of the Delegated Regulation, one may use the interest rate curves offered by EIOPA. We will use these EIOPA interest rate curves when we compute the interest rate SCR under the standard model.

2.3 Market investment assumptions

The market risk for our fictive Norwegian life insurance company is assumed to come from the following risk sub-modules listed by Solvency II; *interest rate risk*, *equity risk*, *property risk*, *spread risk* and *currency risk*. Thus, we ignore *concentration risk* in our exposition of market risk for the purpose of this thesis.

We assume that the company has a market portfolio of investments with a current market value for total assets of NOKmn 300,000, of which 30% is invested in equity, 10% in property and 60% in bonds. For the purpose of a clear and concise exposition of risk exposure effects, we assume that only the equity investments carry currency risk exposures, with 80% of the equity investments invested in foreign currency assets.

Interest rate exposure

We shall assume that BOF is adversely affected by a decrease in the market risk-free interest rates for all insurance liabilities, i.e., both the term insurance portfolio and the pension insurance portfolio are fully exposed to decreasing interest rate rates. Thus, the interest rate risk sub-module of Article 167 of the

Delegated Regulation apply to this situation. For the standar model SCR, we shall use the EIOPA prjections for the risk-free interest rate spot curve and the shocked curve with a downward shift, mentioned in the previous section.

Equity exposure

We shall assume that the equity investments are all of type 1 as specified by Article 168 of the Delegated Regulation. 20% of the equity invesments are assumed to be of a *strategic* nature, and the remainder being *non-strategic*. Further, we shall assume that all equity investments are long positions and that there are no risk mitigating positions. Thus, BOF will be adversely affected by decreasing share prices for the entire equity portfolio. Therefore, the standard equity risk sub-module of Article 169 of the Delegated Regulation apply to this situation.

Property exposure

We shall assume that all property investments are of an immovable nature. For the purpose of a simplified illustration of certain risk effects, all property investments are assumed to be in the local currency. Property investments are subject to the property risk sub-module of Article 169 of the Delegated Regulation.

Spread exposure

We shall assume that all investments subject to spread exposure are traditional bond investments. Thus, we assume no credit derivatives nor securitized products. For further simplification, we assume that bond investments are in the local currency only, and that they are divided 50/50 between two bonds, one with an AA rating and duration 5.5, and, the other with an AAA rating and duration 10.5. The spread risk may then be computed as the bond risk specified by Article 104 of the Delegated Regulation,

Currency exposure

We shall assume that 80% of the equity investments are subject to currency exposure. Otherwise, there are no other currency exposures. Article 188 of the Delegated Regulation specify the computation of currency risk.

CHAPTER 3

Standard model SCR

3.1 Introduction

Under Solvency II, the computation of the solvency capital requirement (SCR) is regulated in the document DIRECTIVE 2009/138/EC (the "Directive", [Eur09]) and in the document COMMISSION DELEGATED REGULATION (EU) 2015/35 (the "Delegated Regulation", [Eur14]).

3.2 SCR under the standard model

Recalling Equation (1.1), the probability of insolvency shall, in principle, not be allowed to exceed 0.5%. Also, since conditioning on $A_0 = TP_0 + SCR$, and recalling that $BOF = A - TP$, this equation replaces BOF_0 with SCR. Thus, the bearing principle of the Solvency II regulation, as stipulated in the relevant EU regulation, the SCR computed shall serve as an estimate of how large basic own funds (BOF) needs to be to provide protection against adverse economic events over the course of the next financial year. Adverse events that without sufficient protection might lead to insolvency of the insurance company. Also, recall that $TP = BE + OL + RM$, and that we in this thesis ignore the effects on SCR from other liabilities (OL) and risk margin (RM). This means that when we in this thesis are seeking to compute the SCR, we are in fact looking at adverse economic events that may lead to a reduction in asset values (A) or an increase in the technical provision (TP) through an increase in the best estimate (BE). This is also the principle behind the stress tests of the standard model and the standard formula approach of Solvency II.

In the remainder of this chapter we compute the SCR for our fictive Norwegian life insurance company applying the standar model stress tests, and using the input and assumptions listed in the Chapter 2. All computations are performed applying the statistical computer software R, and the scripts for these computations can be found in Appendix B .

3.3 BSCR - SCR for basic own funds (BOF)

The Basic Solvency Capital Requirement ($BSCR$), is aggregated from the risk modules of BSCR by applying the standard formula given in Article 87 of the Delegated Regulation and the correlations specified in Annex IV of the Directive.

$$BSCR = \sqrt{\sum_{i,j} Cor_{(i,j)} \cdot SCR_i \cdot SCR_j} + SCR_{intangibles}.$$

Here, the sum $\sum_{i,j}$ covers all possible combinations (i,j) of the risk modules of $BSCR$, in this thesis limited to life insurance risk (SCR_{life}) and market risk (SCR_{market}). The correlation between the two risk modules is $Cor_{(life,market)} = 0.25$, given in Article 136 of the Delegated Regulation. Ignoring risk from *intangibles* in this thesis, yields a BSCR of

$$BSCR = \sqrt{SCR_{life}^2 + SCR_{market}^2 + 2 \cdot Cor_{(life,market)} \cdot SCR_{life} \cdot SCR_{market}}$$
$$= \left((48,023.34)^2 + (113,361.2)^2 + 2 \cdot (.25) \cdot 48,023.34 \cdot 113,361.2\right)^{1/2}$$
$$= 133,712.4.$$

The Basic Solvency Capital Requirement is NOKmn 133,712.

The SCR_{life} and SCR_{market} are computed in the following sections.

3.4 Liability risk (SCR_{life})

The life insurance liability risk (SCR_{life}), is aggregated from the risk sub-modules of life insurance by applying the standard formula given in Article 136 of the Delegated Regulation (and Annex IV of the Directive).

$$SCR_{life} = \sqrt{\sum_{i,j} Cor_{i,j} \cdot SCR_i \cdot SCR_j}.$$

Here, the sum $\sum_{i,j}$ covers all possible combinations (i,j) of the risk sub-modules under life insurance risk, in this thesis limited to mortality risk (SCR_{mort}) and longevity risk (SCR_{long}). Applying the correlation between the two, $Cor_{(mort,long)} = -0.25$, yields the total life insurance risk

$$SCR_{life} = \left(SCR_{mort}^2 + SCR_{long}^2 + 2Cor_{(mort,long)} \cdot SCR_{mort} \cdot SCR_{long}\right)$$
$$= \left((1,009.843)^2 + (48,265.85)^2 + 2 \cdot (-.25) \cdot 1,009.843 \cdot 48,265.85\right)^{1/2}$$
$$= 48,023.34.$$

The solvency capital requirement for life insurance risk is NOKmn 48,023.

The SCR_{mort} and SCR_{long} are computed in the following sections. Common for both these computations is that they need to estimate the change in the best estimate from a shock to the mortalilities of the policy holders. Aggregating all discounted future net cash flow out of the account from existing contracts we get the expected net present value of the total liabilities of an insurance portfolio, i.e., the best estimate (BE):

$$BE = \sum_{k=0}^{\infty} d_k L_k,$$

where $d_k = 1/(1 + r_k)^k$ are the discount factors for time k. These discount factors are derived from the market risk-free interest rates, r_k (the annualised forward rates for future time k).

Assuming no reinsurance, the SCR for any type of liability is then given by

$$SCR_{liab} = BE^S - BE = \sum_{k=1}^{K} d_k L_k^S - \sum_{k=1}^{K} d_k L_k, \qquad (3.1)$$

where L_k^S and L_k are the contributions to the best estimate at time k under stressed and normal conditions (mortalities), respectively.

Mortality risk

First, the premium for the policy holders is established through equivalence pricing,

$$\mathcal{PV}_0 = -\pi_1 \sum_{k=0}^{K-1} d^k \,_kp_{l_0} + s_1 \sum_{k=1}^{K} d^k \,_kq_{l_0},$$

where $d^k = 1/(1 + r)^k$ are the discount terms, $r = 2\%$ is the technical rate and $k = 0, \ldots, 25$. The survival probabilities $\,_kp_l$ and the mortalities $\,_kq_l$ represent the probability of surviving another k periods given age l and the probability of dying during the previous period, respectively. Here $q_l = 1 - p_l$ and $\,_{k+1}p_l = (1 - q_{l+k}) \,_kp_l$. This is the recursion over k time periods. The equation $\,_{k+1}q_l = q_{l+k} \cdot \,_kp_l$ connects the mortalities and survival probabilities over time periods. Then, $-\pi_1 \,_kp_{l_0} + s_1 \,_kq_{l_0}$ is the expected cash flow from the contract at time k (remember, for $k > K = 25$, $\pi_1 = 0$ and $s_1 = 0$). The discount terms d^k bring all annual expected cash flows from the contract at time k back to year zero, taking care of the *present value* part. Therefore, (3.4) represents the expected net present value of this contract.

Under equivalence, i.e., when $\mathcal{PV}_0 = 0$, we compute the premia π_1 by equating expected cash flow in with the expected cash flow out, i.e., by solving the following equation

$$\mathcal{PV}_0 = 0$$
$$\pi_1 = s_1 \frac{\sum_{k=1}^{K} d^k \,_kq_{l_0}}{\sum_{k=0}^{K-1} d^k \,_kp_{l_0}}$$

Applying the survival probabilities provided by the Gompertz–Makeham model and for the given benefit $s_1 = 2$, we compute the premia to be

$$\pi_1 = 0.005743617,$$

i.e., a premium of NOK 5,743.6 annually.

With the premia π_1 in hand, as well as the given benefit $s_1 = 2$, and, the age distribution N_{1m} for $m = 0, 1, \ldots, 25$, we need to find an expression for the liability for this portfolio at any given time $k = 1, \ldots, K = 25$.

We know that all policy holders in this portfolio entered into their contract at the age of l_0. For a policy holder of age $l_0 + m$ at $k = 0$, the liability related to this contract after k years must be

$$-\pi_{1\ k}p_{l_0+m} + s_{1\ k}q_{l_0+m}.$$

This expression for the term insurance liability will apply for all contracts entered into $k = 1, \ldots, K - 1$ years ago. For the contract entered into K years ago the last premium has already been paid and the liability for this contract is only the possibility for a one-time payment in case of death,

$$s_{1\ K}q_{l_0}.$$

The number of policy holders of age $l_0 + m$ at $k = 0$ is N_{1m}. To get the total portfolio liability we multiply the liability expressions above by the number of policy holders and then aggregate for all ages $m = 0, 1, \ldots, K$,

$$L_k = -\pi_1 \sum_{m=0}^{K-k} N_{1m\ k}p_{l_0+m} + s_1 \sum_{m=0}^{K-k} N_{1m\ k}q_{l_0+m}, \quad k = 1, \ldots, K - 1,$$

and (3.2)

$$L_K = s_1 N_{10\ K}q_{l_0}, \quad k = K.$$

Applying the premia π_1 and all other given information given we compute the solvency capital requirement for the term insurance liabilities by estimating the BE as given by the liability expressions above, before and after a *shock* in the mortalities.

The stress test

The stress test of the *mortality risk sub-module* (term insurance) is specified in Article 137 of the Delegated Regulation as,

$$q_l^s = (1 + S) \times q_l \quad \text{where} \quad S = 15 \ \%,$$

where the shock S represents a general *increase* in mortalities. An increase in mortalities will have an adverse effect on BOF from term insurance (but not from pensions, so these are not included here).

The SCR computation

Now, applying Equations (3.2), first on the original mortalities and then on the mortalities after a shock, we get L_k and L_k^S, respectively. These quantities are then applied in Equation (3.1) to produce the SCR_{mort}:

$$SCR_{mort} = 1,009.843.$$

The solvency capital requirement for mortality risk is NOKmn 1,010.

This is a relatively small number under the circumstances. This is due to the relatively short time span of these contracts, the age group (30-55) has relatively low mortalities, and the discount rate used for the best estimate is quite high.

Longevity risk

To produce the SCR due to longevity risk (SCR_{long}), we proceed in the same way as for SCR_{mort}, with only a few minor alterations.

First, we need to compute the premium for the life insurance policies. For contracts in Portfolio 2 (pension insurance), payments happen as long as the policy holder is alive, and cease upon the death of the policy holder. Thus, survival probabilities $_k p_l$ are the only probability needed, representing the probability of surviving another k periods given age l. This time $-\pi_{1\,k}p_{l_0}+s_{1\,k}p_{l_0}$ is the expected cash flow from the contract at time k, where, for $k > l_e - l_0 = 90$, $\pi_2 = 0$ and $s_2 = 0$. The following equation represents the expected present value of this contract,

$$\mathcal{PV}_0 = -\pi_2 \sum_{k=0}^{l_r-l_0-1} d^k \,_k p_{l_0} + s_2 \sum_{k=l_r-l_0}^{l_e-l_0} d^k \,_k p_{l_0},$$

where $d^k = 1/(1+r)^k$ and $r = 2\%$ as before, and now $k = 0, \ldots, l_e - l_0$.

Under equivalence ($\mathcal{PV}_0 = 0$) we compute the premia π_2

$$\pi_2 = s_2 \frac{\sum_{k=l_r-l_0}^{l_e-l_0} d^k \,_k p_{l_0}}{\sum_{k=0}^{l_r-l_0-1} d^k \,_k p_{l_0}} = 0.06220195.$$

This time we are only interested in the survival probabilities. These are the same as those used for Portfolio 1, only for a longer time period. We compute the annual premium to be NOK 62,202.

Again With the premia π_2 in hand, as well as the given benefit $s_2 = .3$, and the age distribution N_{2m} for $m = 0, 1, \ldots, 65$, we need to find an expression for the liability for this portfolio at any given time $k = 1, \ldots, K = 65$.

Using the same approach as for Portfolio 1 we get

$$L_k = -\pi_2 \sum_{m=0}^{l_r-l_0-1-k} N_{2m\,k}p_{l_0+m} + s_2 \sum_{m=l_r-l_0-k}^{\min(l_e-l_0-k,65)} N_{2m\,k}p_{l_0+m},$$

for $k = 1, \ldots, l_r - l_0 - 1$, and

$$L_k = s_2 \sum_{m=0}^{l_e-l_0-k} N_{2m\,k}p_{l_0+m},$$

for $k = l_r - l_0, \ldots, l_e - l_0$.

These quantities for L_k are then used to compute the SCR_{long} the same way as we did for SCR_{mort}, only with a different shock factor S.

The stress test

For the *longevity risk sub-module* (pension) the stress test is

$$q_l^s = (1 - S) \times q_l \quad \text{where} \quad S = 20\ \%, \tag{3.3}$$

where the shock S is a general *decrease* in mortalities, which has an adverse effect on BOF from pension schemes (but not from term insurance).

Computing the SCR

Applying the premia π_2 and all other information given in the equations and expressions above, and once more assuming no reinsurance, we compute the solvency capital requirement for the pension portfolio

$$SCR_{long} = 48,265.85. \tag{3.4}$$

The solvency capital requirement for longevity risk is NOKmn 48,266.

3.5 Market risk (SCR_{market})

The market risk (SCR_{market}), is aggregated from the risk sub-modules of market risk by applying the standard formula given in Article 164 of the Delegated Regulation.

$$SCR_{market} = \sqrt{\sum_{i,j} Cor_{i,j} \cdot SCR_i \cdot SCR_j},$$

Here, the sum $\sum_{i,j}$ covers all possible combinations (i,j) of the risk sub-modules under market risk, in this thesis limited to interest rate risk, equity risk, property risk, spread risk, and, currency risk. Applying the SCR-vector for all these market risk sub-modules

$$SCR_{sub} = (SCR_{interest\ rate}, SCR_{equity}, SCR_{property}, SCR_{spread}, SCR_{currency})$$
$$= (71,753, 32,040, 7,500, 13,950, 18,000)$$

and the correlation matrix for these risk sub-modules,

$$(\rho_{sub}) = \begin{pmatrix} 1 & .5 & .5 & .5 & .25 \\ .5 & 1 & .75 & .75 & .25 \\ .5 & .75 & 1 & .5 & .25 \\ .5 & .75 & .5 & 1 & .25 \\ .25 & .25 & .25 & .25 & 1 \end{pmatrix}$$

yields the total market risk from the following matrix multiplication

$$SCR_{market} = (SCR_{sub}\ (\rho_{sub})\ t(SCR_{sub}))^{1/2}$$
$$= 113,361.2.$$

The solvency capital requirement for market risk is NOKmn 113,361.

The elements of SCR_{sub} are computed in the following sections. With respect to market risk the basic own funds (BOF) can be affected by adverse events that either reduce asset values or increase best estimate values.

With the exception of interest rate risk, all market risk sub-modules are computed based on current asset values. These asset values are usually given by readily available market values. In the few cases where such market vaules are unavailable, estimated market values can be used.

The computation of interest rate risk is based on the best estimate value for the insurance portfolios, and the market risk is due to the uncertain nature of the risk-free interest rate curve.

Common for all risk sub-modules of market risk is that the risk level for each sub-module is a direct consequence of an immediate adverse movement (increase or decrease) in the market prices that determine the asset values. Even though some sub-modules may be quite complex and involved, the SCR computations of these sub-modules are usually more straightforward than those for the liability risk.

Interest rate risk

The solvency capital requirement (SCR) for interest rate risk, $SCR_{interest\ rate}$, is given as the difference $BE^S - BE$, i.e., the the difference in best estimate ($BE = BE_1 + BE_2$, for the two insurance portfolios) after a shock (S) is introduced to the risk-free interest rate curve less the best estimate under ordinary conditions. Assuming no reinsurance, we have

$$SCR_{interest\ rate} = BE^S - BE = \sum_{k=1}^{K} \frac{L_k}{(1 + r_k^S)^k} - \sum_{k=1}^{K} \frac{L_k}{(1 + r_k)^k},$$

where L_k is the liability estimated at year k and r_k^S and r_k are the market risk-free interest rates at time k, under stressed and normal conditions, respectively.

The stress test

BOF is adversely affected by a negative shift in the risk-free interest rate curve, both from term insurance and the pension schemes. Thus, Article 167 of the Delegated Regulation will dictate the stress test for interest rate risk for both portfolios. Article 167 specifies an instantaneous decrease in basic risk-free interest rates at different maturities. Maturities for 1 through 20 years and for 90 years are specified, while the value of the decrease for maturities between 20 and 90 years shall be linearly interpolated. For maturities shorter than 1 year, the decrease shall be 75 %. For maturities longer than 90 years, the decrease shall be 20 %.

As mentioned in Section 2.2, EIOPA offer their projections for the risk-free interest rate curve which are updated monthly. We will use these projections, and compute the best estimate applying the EIOPA spot curve ("$RFR_spot_{n}o_{V}A$") as discount rates, and the stressed (shocked) best estimate applying the EIOPA shocked curve ("$Spot_{N}O_{V}A_shock_{D}OWN$").

Computing the SCR

The interst rate risk ($SCR_{interest\ rate}$) computed this way is

$$SCR_{interest\ rate} = 71,753.45$$

The solvency capital requirement for interest rate risk is NOKmn 71,753.

Equity risk

The equity risk come from equity related financial instruments, such as shares and equity derivatives. The terms of such instruments can be quite complex and involved, thus, the Delegated Regulations have several Articles handeling different aspects of such instruments.

The stress test

With the simplified assumptions for the equity investments of our fictive life insurance company, we may apply the standard equity risk sub-module of Article 169 of the Delegated Acts. The equity risk SCR_{equity} is then computed as the instantaneous decrease equal to 22 % in the value of investments that are of a strategic nature, and an instantaneous decrease of 39 % otherwise.

$$SCR_{equity} = A_1 \times S_1 + A_2 \times S_2,$$

where A_1 and A_2 are the market values of the strategic and non-strategic equity investments, respectively, while $S_1 = 22\%$ and $S_2 = 39\%$ are the corresponding instantaneous decreases in market values.

Computing the SCR

With NOKmn 90,000 invested in equity, of which 20% is strategic and 80% non-stretegic, the SCR computed for equity risk is then

$$SCR_{equity} = 18,000 \times .22 + 72,000 \times .39, = 32,040.$$

The solvency capital requirement for equity risk is NOKmn 32,040.

Property risk

The property risk sub-module is specified in Article 174 of the Delegated Regulation.

The stress test

The property risk $SCR_{property}$ shall be computed as the instantaneous decrease of 25% in the value of immovable property.

$$SCR_{property} = P \times S$$

where P is the aggregate market value of immovable property investments, and, $S = 25\%$ is the instantaneous decrease in market values.

Computing the SCR

With property investments of NOKmn 30,000, the property risk is

$$SCR_{property} = 30,000 \times .25 = 7,500.$$

The solvency capital requirement for property risk is NOKmn 7,500.

Spread risk

The spread risk relates to interest rate-valued financial instruments. The terms of such instruments can be quite complex and involved, thus, the Delegated Regulations have several Articles handeling different aspects of such instruments.

The stress test

For the spread risk (SCR_{spread}) of our simplified bond portfolio we may use the SCR computations specified in Article 104 of the Delegated Regulation,

$$SCR_{spread} = SCR_{bonds} = \sum_{i=1}^{J} A_i \times b_i \times D_i,$$

where A_i is the asset value of a bond with credit quality step i, while b_i is the stess factor and D_i is the modified duration for the same bond.

Computing the SCR

With bond investments of NOKmn 180,000, divided 50/50 between two bonds; NOKmn $90,000$ in a bond with AA rating and duration 5.5, and NOKmn $90,000$ in a bond with AAA rating and duration 10.5, the Solvency Capital Requirement for spread risk is

$$SCR_{spread} = 90,000 \times 1.1 \times 5.5 + 90,000 \times 0.9 \times 10.5, = 13,950.$$

The solvency capital requirement for spread risk is NOKmn 13,950.

Currency risk

Article 188 of the Delegated Regulation specifies the SCR computation for currency risk.

The stress test

the currency risk ($SCR_{currency_i}$) shall be be equal to the larger of the loss in basic own funds resulting from an instantaneous increase of 25% or from an instantaneous decrease of 25% for that currency. The currency risk is then computed aggregating over all I currencies,

$$SCR_{currency} = \sum_{i=1}^{I} SCR_{currency_i} = \sum_{i=1}^{I} \max \left(A_i^s \times S^s, A_i^l \times S^l \right),$$

where A_i^s is the asset value for short exposures to currency i and A_i^l for the corresponding long exposures, while $S^s = 25\%$ is the instantaneous increase and $S^l = 25\%$ the instantaneous decrease in the currency rate.

Computing the SCR

With 80% of the equity investments, i.e., NOKmn $72,000$ exposed to currency fluctuations, the SCR for currency risk is

$$SCR_{currency} = 72,000 \cdot 0.25 = 18,000.$$

The solvency capital requirement for currency risk is NOKmn 18,000.

3.6   SCR summary of the reference case

In Table 3.1 we have listed all SCR numbers computed earlier in this chapter applying the standard model and the standard formula.

		BSCR	133,712			
Market SCR	**113,361**			**Liability SCR**	**48,023**	
Interest rate SCR	71,753			Mortality SCR	1,010	
Equity SCR	32,040			Longevity SCR	48,266	
Property SCR	7,500					
Spread SCR	13,950					
Currency SCR	18,000					

Table 3.1: Solvency Capital Requirement in NOKmn (standard formula).

3.7   Standard model/formula issues

As we can see from the exposition of the stress tests in this chapter, they are all very "crude and schematic" in the way the risk exposures are estimated. This is quite evident from EIOPA's document on the "underlying assumptions in the standard formula" ([EIO14b]).

One quite glaring issue is immediately noticeable. A static shock parameter is applied, as in most of the stress tests, with the purpose of representing the risk for all assets or liabilities in that risk sub-module, or for a large collection of these. This fact makes these stress tests inflexible with respect to handling the diversity in riskiness that a lot of asset classes and liability portfolios possess. Insurance companies with below average risky assets and liabilities, will experience a too strict solvency capital requirement (SCR) under the standard model, and vice versa. Also, this way of grouping risk does not allow for any diversification within the group. Consequently, this way of grouping risk assumes as correlation of close to 1 for all pairs of assets in each risk sub-module.

Another issue with the standard model is how SCR numbers are aggregated using the standard formula. This formula has the distinct resemblance to the risk aggregation of modern portfolio theory, as developed by Harry Markowitz. Modern portfolio theory assumes that returns follow a Gaussian distribution. This assumption is also stated by EIOPA, along with the offhand notion of an adjustment to the correlations as a "fudge factor" to qualify their assumption ([EIO14b, p. 7]). As early as in the 1960s, Benoit Mandelbrot and Eugene Fama

showed the inadequacy of this assumption and proposed the use of more general stable distributions instead. Also, Nassim Nicholas Taleb has later weighed in "if you remove their Gaussian assumptions and treat prices as scalable, you are left with hot air."

To address the above issues, according to EIOPA ([EIO14b, p. 10]), the standard model/formula needs to be carefully calibrated; "The SCR calibration objective, corresponding to the VaR of basic own funds subject to a confidence level of 99.5 % over a one-year period, is applied to each individual risk sub-module."

Below we discuss some other issues that are of particular importance when considering the suitability of these stress test with respect to their ability to estimate the actual risk exposures involved. We will address these issues when we construct our partial internal models in Chapter 4 and employ them in Chapter 5.

Interest rate risk

Even though the shock to interest rates are made relative to the interest rate level, it seems that the stress test for interest rate risk assigns a higher risk (large SCR) the lower the rate curve (level) is. When trying out different interest rate regimes (rate curves), assuming parallell shifts, this effect is quite staggering. This is a counsequence of cash flow dicounting.

Interest rate risk in liability values can be factorised into two factors; the effect of a certain change (say 100 basis points) in the interest rates, and the probability of such a change. Both of these factors depend on the specific interest rate level.

A certain change in the interest rates, say a parallell negative shift of 100 basis points to the interest rate curve, has a greater impact on liability values the lower the interest rate curve is, (assuming that interest rates never are negative for any maturity).

Assuming that interest rates follow a mean reversion process, i.e., that there is a pull towards the mean level for that interest rate, interest rates are more likely to drop the higher they are, and vice versa. With this logic we should observe that an insurance company, through the dicounting of its liability cash flow (best estimate), will experience a higher probability for interest rate risk the higher the interst rate level is.

To capture both these effects, and get the computations right, the risk-free interest rates used to discount expected liability cash flows should be simulated as a mean reverting stochastic processes.

Equity risk

Grouping equity investments into "strategic" and "non-strategic" investments are at best a naive approach. It allows for no diversification within each group, which are both clearly quite diverse with respect to the riskiness of the assets included in each group. Also, it allows for an extremely risky and agressive invesment profile to be "sheltered" if its actual risk exposure is more risky than the shock parameter, $S = 22\% and S = 39\%$, respectively.

Currency risk

Currency risk is the risk of holding investments and liabilities in a foreign currency domination. Before such holdings can be accrued in the consolidated profit and loss account, in the balance sheet stament, or in the cash flow stament, the insurance company will have to translate its foreign currency holdings to its local currency. Not knowing the future currency rate represents currency risk. But since the foreign currency value of such holdings also are uncertain, e.g., equity risk or property risk, there is an extra risk element to the currency risk. *Currency risk is multiplicative.*

For asset j, $j \in \{1, \ldots, J\}$, with currency exposure from currency k, $k \in \{1, \ldots, K\}$, with currency rate against the local currency NOK, FX_k, we have

$$A_{1,j} = A_{0,j}(1 + R_{1,j}),$$
$$FX_{1,k} = FX_{0,k}(1 + R_{1,k}),$$

where $A_{t,j}$ is the asset value of asset j at time t, $R_{1,j}$ is the return (change) of asset j from time $t = 0$ to time $t = 1$, and, similarly for the currency rate $FX_{t,k}$ and its return (change) $R_{1,k}$. The uncertain asset value at time $t = 1$, for this asset in the local currency, NOK, is then

$$A_{1,j}^{\text{NOK}} = A_{1,j} \times FX_{1,k}$$
$$= A_{0,j}(1 + R_{1,j}) \times FX_{0,k}(1 + R_{1,k}).$$

This multiplicative effect makes the stress test for currency risk especially crude and schematic. One glearing deficiency is that the stress test applies the shocks to the historic asset values (A_0). Thus, the stress test does not account for the fact that assets and liabilities inherently are risky themselves, disregarding any asset risk. Or it must do so by including the multiplicative effect in the shock parameter (S), making it a "fudge factor", and fudge factors rarely allow for any precision at all. Or as EIOPA states, "The design of the currency risk sub-module is intended to take into account currency risk arising from all possible sources, and the underlying assumption of the market risk module design is that currency effects appear only in this sub-module, i.e. currency effects have been stripped out in the calibration of the other market risk sub modules."

The workings of the stress test for currency is something like

$$E(XY) = E(X) + E(Y) + \text{error},$$

where X is the future asset value and Y is the future currency rate. For such a stress test to have a any reliable degree of accuracy, one must assume that by setting the model parameters "correctly", i.e., as the weighted average for the entire risk sub-module, the error term will be negligible,

$$E(XY) \approx E(X) + E(Y).$$

There is also the matter of aggregating risk, which by the EIOPA assumptions is something like:

$$\text{sd}(XY) = \text{sd}(X) + \text{sd}(Y) + \text{error},$$

where it seems like the error term has been handeled in the "calibration of the other market risk sub modules".

A proper way to handle this multiplicative effect, is by multivariate simulation. By simultaneously simulating both the asset value and the relevant currency rate, while allowing for any dependencies, for each simulation scenario one will then have access to both $A_{1,j}$ and $FX_{1,k}$, allowing for a straight forward computation of $A_{1,j}^{\mathrm{NOK}}$.

PART II

Partial internal models

CHAPTER 4

The partial internal model

4.1 Introduction

The way the standard model is designed, with very simplistic stress tests that estimate the solvency capital requirement (SCR) in terms of crude averages for broad asset classes or liability portfolios, our hypothesis is that the standard model is too conservative in its risk measurement, and that, on average, the stress tests are likely to *overshoot the actual risk exposures*. We believe that the standard model is too conservative in terms of the parameter values that it employs, both for its stress test shock factors in each of the risk sub-modules to arrive at the sub-module SCR, and for the correlations used by the standard formula to aggregate those risk estimates all the way up to the SCR for the company as a whole. Reading EIOPAS documentation on the "underlying assumptions in the standard formula" supports this hypothesis.

Given that the standard model SCR estimates are likely to overshoot the actual risk exposures, the profitability conscious insurance company will certainly entertain the possibility of employing a more accurate method/model for computing the SCR in its reporting to the supervisory authorities. Solvency II provides for this as the rules allow the insurance company to replace the standard model with what is called an *internal model*. Therfore, in this thesis we wish to provide an alternative method to the standard model, that is better suited to measure the actual financial and actuarial risk exposures of a Norwegian life insurance company. More specifically, for practical reasons we wish to introduce what is called *partial internal models*, i.e., models that will replace only a few select risk sub-modules of the standard model. With these partial internal models we wish to test our hypothesis by computing partial internal model SCR estimates that can be compared to the SCR estimastes of the standard model. To make this comparison as direct and transparent as possible, althoug we propose a very different method for computing the SCR, we will as far as possible keep the standard model assumptions behind the SCR computations in Chapter 3 . We will use the same reference case of our fictive life insurance company, just extend it with more detailed information when necessary. We will also use the same assumption about a Gaussian distribution function for all the random variables that are the actual sources of risk (i.e., the risk factors). This last assumption is dubious at best, but it makes the comparison of computational methods more direct, and in our proposed partial internal models this can quickly and easily be replaced by employing more

appropriate distribution functions.

Also, we wish to demonstrate that our partial internal models have the necessary features and properties to qualify as partial internal models with the supervising authorities. Especially, we wish to demonstrate that;

- these model provide improvements in terms of modelling the risk effects on the probability distribution of the basic own funds (BOF),

- the models apply up-to-date actuarial techniques,

- the models are fairly easy to understand and use, and,

- the models may provide an opportunity to improve the performance of the risk management system of a Norwegian life insurance company.

4.2 Internal model requirements

Introduction

At the discretion of the insurance company, the internal model can be either be a "*full monty*" internal model, computing the SCR for all risk sub-modules as well as for more aggregate levels all the way up to the SCR for the company as a whole, completely replacing the standard model, or it can be a *partial internal model*, replacing only certain parts of the standard model. But before the insurance company can employ the internal model in its reporting of its SCR, the model must be approved by the supervising authorities. Such an approvel requires a quite involved application process.

In order for an alternative method for computing the SCR, to keep with the vision of Solvency II (see Section 1.3), it needs to solve Equation (1.1). Solvency II lists a number of requirements for such a model to qualify as an internal model. However, from a mathematical standpoint, an internal model must at a minimun provide a method that is closer to solving Equation (1.1) in a mathematically stringent way than what the standard model does. That is, the internal model needs to provide a method that is mathematically superior to the stress tests of Solvency II as described in Section 1.4 .

One way to construct such a model is to replace the stress tests with a more accurate way of measuring risk exposure effects on balance sheet values. The stress tests introduce shocks to balance sheet values at quite aggregate levels of the balance sheet. This is in most cases a very crude way of measuring such risk exposures and may not reflect the actual risk exposures in an adequate way. Also, the standard model, with its standard formula, aggregates risk from different stress tests in a way that is far from mathematically stringent when the purpose is to examine risk at the 99.5%-percentile for solvency.

A method that readily lends itself to solving Equation (1.1) numerically, is multivariate simulation of the random variables that de facto put balance sheet values at risk. These random variables are the actual risk drivers, and constitute the risk exposures to balance sheet values which the stress tests of Solvency II are meant to reflect. Multivariate simulation of all such risk factors in one fell swoop will, when constructed properly, provide a numerical method for computing the SCR that is mathematically stringent – also at the 99.5%-percentile for solvency. However, it may very likely be too big a task to replace

the standard model of Solvency II all at once in one big push. The Directive, [Eur09], and the Delegated Regulation, [Eur14], provide for partial internal models, where one or more of the risk sub-modules of the standard model may be replaced by a corresponding partial internal model at the discretion of the insurance company. Integration techniques on how to integrate the partial model(s) with the remaining risk sub-modules from the standard model into one integrated model, is also included.

Solvency II requirements

For an insurance company to be able to include an internal model as part of its risk management system with respect to its risk (SCR) reporting to the supervisory authorities, it must first apply to these authoroties for the approval of such an internal model. To receive an approval for the internal model the insurance company must provide detailed documentation of the appropriateness of the internal model with respect to its ability to measure the actual risk exposure of its market investments and insurance activities, as detailed in the Delegated Regulation.

This process is quite involved and the applicant must provide an internal model that satisfies a comprehensive set of requirements that will ensure that the model will produce solvency capital requirement numbers that reflect the actual inherent risk exposures to the insurance company's balance sheet, and that the model will be a significant contribution to the insurance company's risk management process. All this will have to be documented in detail. Important topics of this documentation will be:

- *Materiality*: The results/output of the model (SCR estimates) must be able to influence the decision-making or the judgement of the users of that information.

- *Demonstration of use*: The internal model must be widely used and play an important role in the insurance company's system of governance, including the top management and supervisory bodies of the insurance company.

- *Statistical quality standards*: The internal model shall be able to assign probability distribution forecasts to changes in the amount of basic own funds (BOF), or a representative proxy in its place. This involves adequate, applicable and relevant actuarial techniques, based on up to date information and progress in actuarial science and generally accepted market practice.

- *Calibration standards*: The intenal model shall be set up in such a way, including choice of time period, that it is representative with respect to the protection provision for policy holders, i.e., it shall not introduce a material error in the Solvency Capital Requirement nor lead to a lower Solvency Capital Requirement than is in accordance with the requirements set out in Article 101(1) of the Directive.

- *Integration of partial internal models*: Annex IV of the Directive and Chapter 5 of the Delagated Regulation provides a default integration technique (standard formula), while Annex XVIII of the Delegated

Regulation offers alternative techniques. But in the end, the insurance company must argue and provide an integration technique that is appropriate.

- *Profit and loss attribution*: The insurance company shall review, at least annually, the causes and sources of profits and losses for each major business unit. This involves demonstrating how the categorisation of risk chosen in the internal model explains the causes and sources of profits and losses.

- *Validation standards*: The insurance company shall on a regular basis perform arms-length validation of the internal model. This entails monitoring the performance of the model, reviewing the appropriateness of its specifications, and testing its results against experience. By applying an effective statistical process for validating the internal model the insurance company shall demonstrate to the supervisory authorities that the resulting capital requirements are appropriate, provide an analysis of stability of the model, and an assessment of the accuracy, completeness and appropriateness of the data used by the internal model.

- *Documentation standards*: The insurance company shall document the design and operational details of their internal model. The documentation shall provide a detailed outline of the theory, assumptions, and mathematical and empirical bases underlying the internal model. Also, the documentation shall indicate any circumstances under which the internal model does not work effectively.

4.3 Mathematical tools for internal models

Introduction

We will in the following sections provide mathematical arguments for how an insurance company can construct an internal model based on multivariate simulation of all random variables that constitute a source of risk that may affect the probability distribution of its basic own funds (BOF). By applying a multidimensional copula the multivariate simulation will be able to handle dependencies between these risk factors. In the following we will also present the numerical method itself, which we believe has the required features and properties to qualify as an internal model. First we start out with the mathematics.

Sklar's Theorem

The following mathematical outline is based the textbook "An Introduction to Copulas" by R.B. Nelsen. For further details on the mathematics than what is outlined her, the reader is referred to [Nel06].

In 1959, the American mathematician Abe Sklar introduced the notion of and the name of "copulas" into probability theory and proved the theorem that bears his name ([Skl59]). This article is written in French, but Sklar has since then elaborated on the theme ([Skl73], [Skl96]). Our construction of partial internal models hinges on this important theorem as it provides us with

an elegant way to "couple" (i.e., combine) independent marginal distribution functions into a multivariate distribution function with dependent margins. It is the copula that handles the dependencies between the margins.

For the bivariate case, this theorem is stated as

Theorem 4.3.1 (Sklar's Theorem)**.** *Let X and Y be random variables with distribution functions F and G, respectively, and joint distribution function H. Then there exists a copula C such that*

$$H(x,y) = C(F(x), G(y)). \tag{4.1}$$

If F and G are continuous, C is unique. Otherwise, C is uniquely determined on $\mathrm{Ran}F \times \mathrm{Ran}G$. Conversely, if C is a copula and F and G are distribution functions, then the function H defined by (4.1) is a joint distribution function with margins F and G.

What this theorem tells us, is that we may take two independent random variables X and Y, with marginal distribution functions F and G, respectively, and "couple" them into a bivariate distribution function. All we need for this copula to be unique is that these margins are continuous, but the copula will always be uniquely determined on the range of F and G. No matter what these margins look like, as long as they are continuous, the theorem guarantees us that there exists a copula function C that will do the job.

For the multivariate case the theorem is stated as

Theorem 4.3.2 (Sklar's Theorem in n-dimensions)**.** *Let H be an n-dimensional distribution function with margins $F_1, F_2, \ldots, F_n$. Then there exists an n-copula C such that for all x in $\overline{\mathbb{R}}^n$,*

$$H(x_1, x_2, \ldots, x_n) = C\left(F_1(x_1), \ldots, F_n(x_n)\right). \tag{4.2}$$

If $F_1, F_2, \ldots, F_n$ are all continuous, then C is unique; otherwise, C is uniquely determined on $\mathrm{Ran}F_1 \times \ldots \times \mathrm{Ran}F_n$. Conversely, if C is an n-copula and $F_1, F_2, \ldots, F_n$ are distribution functions, then the function H defined by (4.2) is an n-dimensional distribution function with margins $F_1, F_2, \ldots, F_n$.

This result is exactly what we need for our partial internal model. We wish to have full freedom when designing the distribution functions for the random variables that represent the actual risk exposures to the insurance company's balance sheet. Being able to simulate each such risk factor individually, and then join them together with a copula into a joint probability distribution, is what makes us able to go from the distribution functions of the individual risk factors, to the joint distribution function of the insurance company's basic own funds (BOF). We may then use our simulated risk factors to replace the stress tests of the standard model.

Another important result about copulas is that they are invariant under strictly increasing transformations.

Theorem 4.3.3. *Let X and Y be continuous random variables with copula C_{XY}. If α and β are strictly increasing on $\mathrm{Ran}X$ and $\mathrm{Ran}Y$, respectively, then $C_{\alpha(X)\beta(Y)} = C_{XY}$. Thus C_{XY} is* invariant *under strictly increasing transformations of X and Y.*

This result allows us not only to use the copula on the original random variables, but may also apply the copula to random variables that are strictly increasing transformations of the original variables. E.g., the logarithmic function applied on a Gaussian distribution function, is a strictly increasing transformation of the Gaussian random variable, and the Gaussian copula may then be applied to the log-normal distribution function.

With Sklar's Theorem at our disposal, we may first concentrate on designing the univariate distribution functions to use in the simulation of each random variable that is a source of risk, before we apply the appropriate copula to take care of any dependencies between all these risk factors.

The univariate distributions

To design the simulation of each single risk factor, the random variable that represents an actual risk exposure to the insurance company, we divide these variables into two distinct types of random processes, the *random walk* processes, and the *stationary* processes. In the following sections we describe the mathematics of these processes. The description is based on [Bøl14], and will be kept rather brief.

Random walk

A series $\{Y_k\}$ is a **random walk** if generated by the recursion

$$Y_k = Y_{k-1} + X_k, \quad k = 1, 2, \ldots \qquad (4.3)$$

where $X_1, X_2, \ldots$ are independent increments. Usually a random walk signifies increments that are identically distributed. It is convenient to represent them as

$$X_k = \xi + \sigma \varepsilon_k, \quad k = 1, 2, \ldots \qquad (4.4)$$

where the random sequence $\varepsilon_1, \varepsilon_2, \ldots$ is identically and independently distributed with zero mean and unit variance, but not necessarily Gaussian. The expectation $\xi = E(X_k)$ is sometimes called the *drift*, and, σ is the standard deviation of the increments.

The random walk is widely used in the financial industry. Random variables such as share prices and commodity prices typically follow a random walk process. In particular the random walk on a logarithmic scale has become a standard model for equity, i.e., for modelling share prices and the price of financial instruments derived from share prices. If S_k is the share price at time k, then $Y_k = \log(S_k)$ in (4.3) and we now have a stochastic process for the share price

$$S_k = S_{k-1} e^{X_k} = S_{k-1} e^{\xi + \sigma \varepsilon_k}, \quad k = 1, 2, \ldots, \quad \text{starting at } S_0 = s_0. \quad (4.5)$$

This is known as a **geometric random walk**, and if the ε_k are distributed $N(0,1)$ it is called a **geometric brownian motion**.

We note that

$$X_k = \log\left(\frac{S_k}{S_{k-1}}\right) = \log(1 + R_k),$$

where R_k is the return in period k and increments under the geometric random walk are then log-returns.

Important behaviour of the random walk can be understood directly from the defining equations (4.3) and (4.4). Recursing from $k = K$ to $k = 0$, we get

$$Y_K = Y_0 + K\xi + \sqrt{K}\sigma\eta \quad \text{where } \eta = \frac{\varepsilon_1 + \ldots + \varepsilon_n}{\sqrt{K}} \qquad (4.6)$$

Here η is another standardized error term; i.e., $E(\eta) = 0$ and $\text{sd}(\eta) = 1$. The geometric random walk equivalent is

$$S_k = e^{Y_K} = S_0 e^{K\xi + \sqrt{K}\sigma\eta}.$$

Note that if this is the model for equity, the investment eventually exceeds the initial value S_0 by huge amounts if the drift is positive (however small). In real life this is almost never the case even though it may seem so when, e.g., considering some of the large industrial or global tech companies of today. However, in the long run (when we are all dead), most companies will experience that their expected growth (ξ) and the variation in this growth (σ) change over time. These parameters are stochastic in real life due to technolgical shifts, competition, etc.. But given the short time frame of one year for computing the solvency capital requirement, and also the short time horison of a few years for most risk management models and systems, we may be quite happy operating with static model parameters.

Simulating random walk is rather easy. The follwing algorithm illustrates that such a simulation can be implemented with only a few lines of coded instructions. In our code examples and elsewhere in this thesis, we will, when the distinction is necessary or appropriate, use an asterisk ($*$) attached to simulated variables.

Algorithm 2.1: Equity portfolio under a geometric random walk

```
0 Input: s.0, and model parameters; xi and sig
1 S.k* <- s.0                       % Initial asset value
2 For (k = 1, ..., K) do            % Time loop
  {
3   Generate veps ~ N(0,1)
4   S.k* <- S.k* exp(xi + sig veps*)   % Update asset value
  }
5 Return (S.1*, ..., S.K*)
```

Algorithm 2.1 details the steps for a share price on a logarithmic scale, following a geometric random walk, defined in Equation (4.5).

Stationary processes

A much used modelling priciple for stationary process random variables is the autoregrassion of order 1, AR(1). Such a process oscillates randomly around the expected level, i.e., the mean of the process. Therefore, such processes are also called **reversion to mean**, or just **mean reversion**. The reason for this mean reversion is that there are "forces" governing the stationary process, which constantly pull the process back towards the mean. Stationary patterns are irregularly cyclic; there is no systematic growth or decline, nor do the oscillations become dampened or reinforced.

Stationary processes are also widely used in the financial industry. Random variables such as interest rates and currency rates typically follow a stationary process.

Two prominent AR(1) models that have been widely used to model or understand stationary processes, namely the Vasiček model (1977) and the Black–Karasinski model (1991).

The **Vasiček model** is defined by the equation

$$r_k = r_{k-1} + (1-a)(\xi - r_{k-1}) + \sigma\varepsilon_k \quad \text{for} \quad k = 1, 2, \ldots \tag{4.7}$$

where r_k is the value of the random variable at time k, ξ is the expected (mean) value for the random variable, a is a constant model parameter determining the "force of pulling" r_k towards ξ if $|a| < 1$, σ is the volatility of the process, and, $\varepsilon_1, \varepsilon_2, \ldots$ are independent and identically distributed with mean 0 and standard deviation 1.

Rewriting (4.7), letting $Y_k = r_k$ and $X_k = r_k - \xi$, we get

$$Y_k = \xi + X_k, \quad \text{where} \quad X_k = aX_{k-1} + \sigma\varepsilon_k, \quad \text{for} \quad k = 1, 2, \ldots, \tag{4.8}$$

where the expression on the left defines the *level* of the process, while the *dynamics* of the process is governed by the expression on the right. This is the autoregressive series of first order, AR(1).

The behaviour of first-order autoregressions is of great interest, in particular, what is the importance of the initial vaule of the process variable, x_0? The start of the process is simply stated

$$X_0 = y_o - \xi = x_0, \tag{4.9}$$

where y_0 is the observed value of Y_k at time $k = 0$. We condition on x_0 and are intersted in the behaviour of $E(X_k|x_0)$, sd$(X_k|x_0)$, and, var$(X_k|x_0)$. The driver process $\{X_k\}$ is recursive, and tracking it back to its initial value, we get

$$X_k = \sigma \sum_{i=1}^{k} a^{i-1}\varepsilon_{k+1-i} + a^k x_0 \tag{4.10}$$

Since the ε_k-terms are are independent and identically distributed, it is easy to show that

$$E(X_k|x_0) = a^k x_0 \quad \text{and} \quad \text{sd}(X_k \mid x_0) = \sqrt{\frac{1 - a^{2k}}{1 - a^2}}\sigma. \tag{4.11}$$

For the process to be mean reverting, only $|a| < 1$ is of interest, which makes the a^k-terms taper off towards zero, thus

$$\lim_{k\to\infty} E(X_k|x_0) = 0 \quad \text{for} \quad |a| < 1.$$

The standard deviation, on the other hand, at first experience a sharp increase before stabilising at some level.

$$\lim_{k\to\infty} \text{sd}(X_k|x_0) = \sigma_x = \frac{\sigma}{\sqrt{1 - a^2}} \quad \text{for} \quad |a| < 1.$$

In addition we have that

$$\lim_{k\to\infty} \mathrm{cov}(X_k, X_{k-l}) = a^l \mathrm{var}(X_{k-l}) \to a^l \sigma_x^2 \quad \text{for} \quad l \geq 0.$$

We notice that all these conditional variables stabilise in the long run when $|a| < 1$. After an initial stage where the process is pulled towards the mean from where it starts (the **transient** stage), the process gradually slides into the **stationary** stage. How long that takes depends on the model, more specifically the parameters a and σ. By the time the process is well into its stationary stage, the value x_0 at the beginning has been forgotten, as has σ.

The **Black–Karasinski model** with the exponential function as transformation to a non-linear scale is defined by the equation

$$r_k = \xi e^{-\sigma_x^2/2 + X_k} \quad \text{where} \quad \sigma_x = \frac{\sigma}{\sqrt{1 - a^2}}. \tag{4.12}$$

We find the initial value for the driver process $\{X_k\}$ by solving (4.12) for X_0 when $k = 0$,

$$x_0 = \log(\frac{r_0}{\xi}) + \frac{1}{2}\sigma_x^2, \tag{4.13}$$

Here r_0 is the observel value of r_k at time $k = 0$.

The transient stage passes gradually into a steady one as before, and eventually

$$E(r_k) = \xi \quad \text{and} \quad \mathrm{sd}(r_k) = \xi\sqrt{e^{\sigma_x^2} - 1} \tag{4.14}$$

which are the ordinary formulae under log-normal distributions.

Simulating a stationary process is also quite simple, The only thing new compared to the random walk, is the inclusion of how to handle the driver process.

Algorithm 2.2: Simulating Black–Karasinski

```
0 Input: xi, a, sig, r.0, and,
         sig.x = sig / sqr(1 - a^2)
1 X* <- log(r.0/xi) + sig.x^2/2      % Alt.: X* ~ N(0,sig.x)
  P* <- 1
2 For (k = 1, ..., K) do
  {
3   Draw veps* ~ N(0,1)
4   X* <- a X* + sig veps*           % Update driver process
5   r.k* <- xi exp(-sig.x^2/2 + X*)
    P* <- P* (1 + r.k*)
  }
6 Return (r.1*, ..., r.K*) and (r.0:K* = P* - 1)
```

Algorithm 2.2 shows how the driver process is simulated and then converted to realisations of r_k in the Black–Karasinski model. The logic is based on the stochastic recursion (4.8) right, which is reproduced in Line 4 and converted to interest rates in Line 5. There are several ways to initialise this algorithm. You start from the *current state of the economy* by using the expression on Line 1 (Black–Karasinski) or replacing it with $X^* \leftarrow r_0 - \xi$ (Vasiček). Simulations may also be run from the *stationary state* (see the comment on Line 1) or even from its *mean value* (set $X^* \leftarrow 0$ in Line 1).

The copula

In this thesis we wish to compare the SCR results of the standard model with those of our partial internal models. Consequently, we wish to keep as many of the model assumptions behind the standard model/formula as possible when we apply our partial internal models. What we propose in our partial internal models is a change in the mathematical method, i.e., from a schematic method of "averages" to a mathematically stringent numerical method. E.g., the underlying distribution functions for the random variables representing risk exposures to the insurance company are all assumed to be Gaussian ([EIO14b, p. 7]) for the standard model/formula. Keeping our partial internal models in line with what EIOPA states as a "central assumption" underlying the standard formula, we will apply the Gaussian copula as the copula of choice in our partial internal models. This choice is for comparison purposes only.

A word of caution is in order; the Gaussian copula is not able to provide model control of dependencies when the model variables take on values far off from their expected values. The random variables gradually become independent the more they stray off from their mean. This is of particular concern when we are looking at SCR numbers, i.e., outcomes that are extremely deep down into the left tail of the BOF probability distribution. Selecting a more appropriate copula to handle such dependencies is advised. However, this is not straightforward to do, especially when we are handling larger numbers of random variables in our multivariate simulation. Also, the Gaussian distribution is symmetric and it takes negative as well as positive values. More often than not, this is not the case in insurance, especially not in general insurance where distributions for claims and claim frequency assume positive values only and tend be be heavily skewed to the right. However, in life insurance the Gaussian assumption may work fairly well, both for insurance and market risk. Of course, with the exception, as mentioned, the issue of vanishing tail dependency. Thus, we stick to the Gaussian copula for the purpose of expositions in this thesis.

The Gaussian copula is a distribution function over the d-dimensional unit hypercube $[0,1]^d$. It is constructed from a multivariate normal distribution over $\mathbb{R}^d$ by using the probability integral transform. For a given correlation matrix $R \in [-1,1]^{d \times d}$, the Gaussian copula with parameter matrix R can be written as

$$C_R^{\mathrm{Gauss}}(u) = \Phi_R\left(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\right),$$

where Φ^{-1} is the inverse cumulative distribution function of a standard normal, and, Φ_R is the joint cumulative distribution function of a multivariate normal distribution with mean vector zero and covariance matrix equal to the correlation matrix R. While there is no simple analytical formula for the copula function, in numerical simulation the Gaussian copula can be put in action by applying a Cholesky decomposition on the covariance matrix, where the resulting matrix C is such that

$$R^d = CC^T,$$

where C and C^T are both of triangular form with zeros above or below the main diagonal.

The general multivariate Gaussian model for J Gaussian assets is then

$$X_j = \xi_j + \varepsilon_j \quad \text{where} \quad \varepsilon_j = c_{j1}\eta_1 + \ldots + c_{jJ}\eta_J, \tag{4.15}$$

for assets $j = 1, \ldots, J$, where ξ_j is the model parameter for the mean and the c_{ij}'s are parameters taken from the Cholesky decomposition of the correlation matrix, $C = (c_{ij})$. Also, $\eta_1, \ldots, \eta_J$ are independent and (indentially) $N(0,1)$ for $j = 1, \ldots, J$. In this model setup, we get our Cholesky decomposition matrix from the covariance matrix, which works just as well. In our algorithms, we use a standard R library to run the Cholesky decomposition.

Multivariate simulation by applying copulas

We now wish to extend the unvariate simulation to the simulation of several, dependent variables. To do this we apply the Gaussian copula and the Cholesky decomposition of the covariance matrix.

For a portfolio of J log-normal assets we now extend Algorithm 2.1 for a single asset, to Algorithm 2.3 for assets with a multivariate Gaussian distribution.

Algorithm 2.3: Equity portfolio under a geometric random walk

```
0 Input:
  s.10, ..., sJ0, model parameters xi.j. sig.j
  and C = t(chol(cov)) = (c.ij)      % Cholesky decomposition
1 For (j = 1, ..., J)
  {
2   S.j* <- s.j0                      % Initial portfolio
  }
3 For (k = 1, ..., K) do              % Time loop
  {
4   Generate eta.1*, ...., eta.J*     % iid N(0,1)
5   V.p.k* <- 0
6   For (j = 1, ..., J)
    {
7     S.j* <- S.j* exp(xi.j + c.j1 eta.1* + ...+ c.jJ eta.J*)
8     V.p.k* <- V.p.k* + S.j*
    }
  }
9 Return (V.p.1*, ..., V.p.K*)
```

Algorithm 2.3 details the steps for an equity portfolio with J log-normal assets which are not rebalanced. The portfolio value $\mathcal{V}_k$ then evolves according to

$$\mathcal{V}_k = S_{1k} + \ldots + S_{Jk} \quad \text{where} \quad S_{jk} = S_{jk-1} e^{\xi_j + \sigma_j \varepsilon_{jk}}, \quad j = 1, \ldots, J. \quad (4.16)$$

The investments are initially worth $S_{10} = s_{10}, \ldots, S_{J0} = s_{J0}$ and the portfolio $\mathcal{V}_0 = s_{10} + \ldots + s_{J0}$. The logic follows (4.16) closely.

For simulation of stationary processes, Algorithm 2.2 (Black–Karasinski) can be extended in the same way (as is done in the algorithms for currency rate and interest rate simulation in Appendix C).

For a more detailed exposition of the univariate random processes of random walk and stationary processes, and the application of the Gaussian copula, the reader is referred to [Bøl14].

4.4 Constructing the partial internal model

Selecting the risk factors - univariate random variables

As earlier mentioned, any internal model shall replace a risk module or sub-module of the standard model in such a way that all risk exposures in the risk module are covered. Also, it must transform the risk exposures from measured risk to the appropriate effect on the distribution function for the basic own funds (BOF). Replacing all risk modules in one go is likely to represent a Herculean task for any insurance company. Thus, introducing partial internal models, replacing one or more risk sub-modules at a time, is probably the way to go. Starting with the risk sub-module, or a select group of risk sub-modules, that represents the greatest potential for SCR reduction, one may gradually be able to replace all or most risk modules of Solvency II. At least one should aim at addressing the most important risk modules in terms of the amount of risk contribution to the overall Basic Solvency Capital Requirement (BSCR).

This gradual approach is indeed the case for how we in this thesis intend to replace risk sub-modules of the standard model. We will in the following introduce a select group of partial internal models that will replace market risk sub-modules only. This is done due to limitations in our access to the internal workings of the insurance risk exposures of the insurance company, and due to the complexity of certain market risk sub-modules. However, with this select group of partial internal models we will be able to provide a demonstration of the full structure behind our method for constructing internal models. And with this select group as a starting point, the road to a complete internal model is simply paved with the introduction of additional partial models, all based on the same methods and principles covered in this thesis, until all risk modules are replaced.

We will now take a closer look at some of the stress tests of the standard model, as described in Section 3.4 , and suggest random variables that will represent the actual risk exposures to the balance sheet values in a way that is demonstrably superior to these stress tests. We will focus on the following sub-modules of the market risk module; *interest rate risk*, *equity risk* and *currency risk*. With these risk sub-modules we will be able to introduce simulations of both *random walk* and *stationary processes*, but more importantly, we will be able to address some of the serious shortcomings of the standard formula approach of Solvency II, mentioned in Section 3.7 .

Market risk comes in its entirety from risk exposures that are subject to market valuations, e.g., share prices, interest rates and currency rates. Mostly, it will be the assets in the different investment portfolios that are exposed to these risk factors, though the best estimate is exposed to risk-free interest rates through the discounted expected net cash flows from the liabilities.

The investment portfolios can easily be broken down into its individual investments, and for most investments there will be specific market information (asset prices) that decide the value of the assets. Most market prices are available on a regular basis (most often daily, or even more frequently), but sometimes estimates will have do (particularly for illiquid assets). When such information is available, we have a good starting point for stochastic simulations of the risk exposures. We may then quite easily estimate the model parameters that we need to calibrate the stochastic processes (models) that we use to numerically

simulate the the actual risk exposures of a standard model risk sub-module. A multivariate simulation model of all random variable that represent risk exposures in a risk sub-module, will then constitute a partial internal model to replace the stress test of this risk sub-module. This multivariate model will then provide a numerical method for measuring market risk at $0,005$-percentile for insolvency (or the 99.5%-percentile for solvency) that is mathematically stringent for the risk sub-module as a whole.

Interest rate risk

The stress test for interest rate risk is modelled as a shock to the risk-free interest rate curve. The "shock curve", i.e., the risk-free interest rate curve after a shift that represents an adverse effect on the basic own funds, is specified in the Delegated Regulation, Article 167 ([Eur14]). The interest rate shock curve covers a time horizon of 90 years (tenors $= \{\leq 1, 2, \ldots, 10, 12, 15, 90\}$).

Replacing this stress test one could simply simulate the risk-free interest rate as a stationary random variable with annual increments (as described above in this Chapter). For each iteration of this simulation we compute the best estimate (BE) as the dicounted expected cash flows from the liabilities affected by the shock, using the the simulated risk-free interest rate curve to determine the discount factors. These simulated best estmate values ($BE*$) are then sorted in an ascending order. For m simulations, the sorted best estimate value in the 99.5% placeholder, `SE.sort*[.995*m]`, will then replace the stressed best estimate of the stress test for the interest rate risk sub-module. This way we will stringently handle the issue of mean reversion mentioned in Section 3.7. We use the EIOPA spot curve for Norway to estimate the initial value (r_0) and long term value (ξ) for our stationary model, while we estimate its volatility (σ) from the quoted `Euribor 12m` rates (as a proxy for the risk-free interest rate).

Even though our model for simulating the term structure of the risk-free interest rate is a rather simple, it is easy to use and understand, and it is designed to address the issue of mean reversion for the standard model stress test. For more comprehensive ways of modelling the interest rate curve, such as the Heath, Jarrow and Morton (HJM) model, the Cox, Ingersoll and Ross (CIR) model, the Jamshidian model, or the Brace, Gatarek and Musiela (BGM) model, the reader is referred to "Interest Rate Modelling" by Jessica James and Nick Webber ([JW00]).

Equity risk

The standard model stress test for the equity risk sub-module aggregate all equity investments into two quite broad categories; *strategic* and *non-strategic* equity investments. For each category an adverse shock is assigned to asset values, i.e., a reduction in asset values of 22% and 39%, respectively. Together with the corresponding correlations, these shocks are calibrated to express the effect of equity risk on the distribution function of BOF at the 99.5%-percentile ([EIO14b, p. 8]). This is irrespective of the quite inhomogeneous risk level of the different equity investments that can be put into each of these two categories. This inhomogeneity can be along a number of different risk dimensions; growth vs. mature dividende companies, experienced company vs. newcomer, liquid vs. illiquid stocks, etc.. Therefore, EIOPA assumes that

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the insurance company holds an "equity portfolio that is well diversified with respect to geography (developed market countries), stock size (large, mid, small, micro cap[italisation]), sectors and investment style (growth, value, income etc.)."

A more direct and superior way to handle such risk exposures is to measure the risk level of each individual equity investment (stocks, derivatives, etc.). An alternative to handling each investment individually, is to group them into groups that are significantly more homogeneous than strategic vs. non-strategic investment. This can typically be done by using the many different equity indices that exist; geographically – Europe, US, Asia, etc.; size-wise – Dow Jones, S&P, FTSE, Nasdaq, etc. vs. small cap, growth or emerging market indices; industry-wise – energy, transportation, consumer goods, etc.; or any other applicable grouping that provide for *more homogeneous* risk groupings. Quoted indices is a natural option as they have already been prepared for us.

In this thesis we will use a few select geographical indices that cover a large portion of the total equity investment portfolio for many a life insurance company. This selection is done based on the general empirical result of the Markowitz portfolio theory that a close to optimal level of diversification can be obtained with a as few as six to eight well diversified single asset investments in the investment portfolio. Adding further asset investments to the portfolio will then provide only a small marginal contributions to portfolio diversification. Unsystematic risk specific to each investment is quickly diversified, while systematic market risk will remain no matter how many asset investments we include in the portfolio ([Bøl14]). Using indices that cover numerous single investments, the number of indices needed to obtain the same level of diversification is expexted to be lower than for single investments.

We suggest six geograpical indices as introduced in Table (4.2), providing the correlation matrix neede to handle any dependencies between these indices. These indices are OSEBX ("Hovedindeksen", quoted in NOK), OMX ("Stockholm 30 Index", quoted in SEK), UKX ("FTSE 100 Index", quoted in GBP), SXXP ("STOXX Europe 600 Index", quoted in EUR), SPX ("S&P 500 Index", quoted in USD), and NKY ("Nikkei 225 Index", quoted in JPY).

Currency risk

Currency risk comes from all investments denominated in a foreign currency, and for which a currency rate against the base currency (NOK) is needed to report the asset value in NOK in the insurance company's financial reports.

The Delegated Regulation ([Eur14]) states that "The capital requirement for currency risk ... shall be equal to the sum of the capital requirements for currency risk for each foreign currency.", and further "..., foreign currencies shall be currencies other than the currency used for the preparation of the insurance or reinsurance undertaking's financial statements ('the local currency')."

In this thesis we assume for illustrative purposes that the equity investments are the only sources for currency risk exposures. Thus, equity investments other than the NOK-denominated investments in the OSEBX index are assumed to represent currency risk exposure. In this thesis we assume currency risk exposures from the the following currency rates: SEKNOK, GBPNOK, EURNOK, USDNOK, and JPYNOK.

To handle any dependencies between these currency rates, a correlation matrix for these currency spot rates is presented in Table (4.4).

Equity risk and Currency risk combined

To combine several risk sub-modules of Solvency II into one integrated, multivariate internal model is simply the matter of expanding the correlation matrix to include all random variables that constitute the actual risk exposures of the integrated model.

The total risk exposure effect for any asset that is subject to currency risk is multiplicative (see Section 3.7). To account for this multiplicative risk exposure we need the expanded correlation matrix of all individual assets in the asset classes that involve currency risk (also those assets denominated in the "local currency", or "base currency") as well as the corresponding currency rates. In this thesis that means that we will include all equity investments and the corresponding currency rates SEKNOK, GBPNOK, EURNOK, USDNOK, and JPYNOK.

The extended correlation matrix can be seen in Table 4.5. With this expanded correlation matrix simultaneous risk modelling of this larger number of risk factors is now readily available. More on this in Chapter 5.

Multivariate simulation

To replace the stress tests of the different risk modules or sub-modules of Solvency II, we will use multivariate simulation of the random variables that constitute the actual risk exposures of each risk sub-module. This will allow us to construct an internal model whether it is a partial model to replace a single risk sub-module, a complete replacement of all sub-modules under Basic Solvency Capital Requirement (BSCR), or any combination in between of replacing several but not all risk sub-modules.

First, we will demonstrate the replacement of the individual risk sub-modules, i.e., *interest rate risk*, *Equity risk* and *Currency risk*, separately. This replacement will be based on separate multivariate simulations of random variables that represent (i) all interest rate risk exposures, (ii) all equity risk exposures, and (iii) all currency risk exposures. This will illustrate the power of multivariate simulation on the risk sub-module level. when computing the solvency capital requirement.

Next, we will demonstrate the power of multivariate simulation when it comes to integrating two or more risk sub-modules, into one single model, i.e., circumventing the shortcomings of the standard formula. Since the currency risk is multiplicative (see Section 3.7), and since we assume that only equity investments carry any currency risk exposure, we integrate the partial internal models for equity risk and currency risk.

This will show that if we are able to replace all risk sub-modules of the BSCR, we may dispense with the standard formula alltogether, it will simply no longer be necessary. The numerical approach to computing the solvency capital requirement that we have just introduced, will take care of business on its own.

And the best part, we will still be able to maintain, analyse and report any risk sub-module that we wish to investigate or include in our risk management

and report systems, whether it is a sub-module for Solvency II reporting or any other risk module that we wish to design for the internal risk manangement system. The multivariate simulation approach leaves us full flexibility in our risk modelling. It is simply a question of writing a script that utilise the multivariate simulation of all risk factors involved, to construct a risk model that represents the probability distribution of the variable that we wish to investigate. More on this in Chapter 5.

The partial internal models - summary input and assumptions

To provide the necessary calibration of our partial internal models, we have used time series of daily market price quotes from 1 January 2000 through 17 August 2022. These quotes are the time series for the risk-free interest rate, for each equity index and for each currency rate mentioned earlier in this chapter. For each time series we have computed the expected value, the daily returns, the average daily returns, and the standard deviation of daily returns. All of these daily quantities have then been annualised. We have also computed the correlation matrix for all required combinations of time series pairs. Finally, we have used the qoutes on 17 August 2022 as spot values. All these estimates are summarised below.

Interest rate risk

Interest rate risk is related to an adverse change in the risk-free interest rate curve. In our case, this means an immediate downward shift in the risk-free interest rate curve. Such a shift will have an adverse effect on the distribution function for the basic own funds (BOF), through an increase in the best estimate (BE). This is so since the BE is the dicounted net cash flows from the insurance liabilities using the risk-free interest rates as discount rates.

In this thesis we will simulate the risk-free interest rate curve for the entire duration of the liability cash flows. We will do this by simulation the one-year risk-free interest rate for 150 years going forward (the EIOPA horizon), from which the interest rate curve is the direct result of simulating the risk-free interest rate, and the discount rates are the result of geometric growth of the one-year rates.

We will use EIOPA's estimates for the short and the long end of the risk-free spot curve as the initial value and the expected long-term level for the risk-free interest rates, whereas we will use the Euribor 12m interest rate as a proxy when estimating the volatility of the risk-free one-year interest rate. This way we keep with the EIOPA/Solvency II assumptions as much as possible, and only deviate from these for the purpose of simulating the risk-free interest rate curve. This way we may compare how the simulated risk level (shock) of the risk-free interest rates affect the BE to the effect of the pre-sepecified shocked curve.

As of 31 July 2022, EIOPA's estimates for the risk-free interest rate curve is represented by the dashed black line in Figure 4.1 . For the short end we have $r_1 = 3.115\%$ with a sudden drop over the next few years. Then EIOPA estimates the risk-free interest rate to slowly increase towards the long-term level of $r_{150} = 3.368\%$, represented by the blue line. The orange line represents the minimum level of the EIOPA spot curve. The Euribor 12m volatility is

estimated to be $\sigma_r = 0.1347$ based on the time series represented by the solid red curve. The time period from second half of 2008 through first half of 2022 seems to be quite an extraordinary stretch of time for the Euribor 12m (and for most similar European rates), with extremely low interest rates, even negative since late 2016. This is a result of challenging times in several European countries (The PIG crisis - Portugal, Italy, Greece), with the European Central Bank providing rescue packages to these countries as well as trying to stimulate economic activity through ever decreasing interst rate levels. Finally, this long-term unsustainable situation has turned, as increasing interest rates are now deemed necessary in order to fight the galloping inflation that is currently troubling most developed countries. Thus a mean reversion process for the risk-free interest rate at the levels of pre 2008, may seem to be appropriate. However, given the extreme uncertainty of the current geopolitical situation in Europe, the risk is that this may rapidly change. Still we will simulate the risk-free interest rate as a mean reversion process, and we will use EIOPA's projections for the initial value and the long term level, and the estimated volatility of the Euribor 12 given the above historical market quotes.

Euribor 12m – Historical interest rates
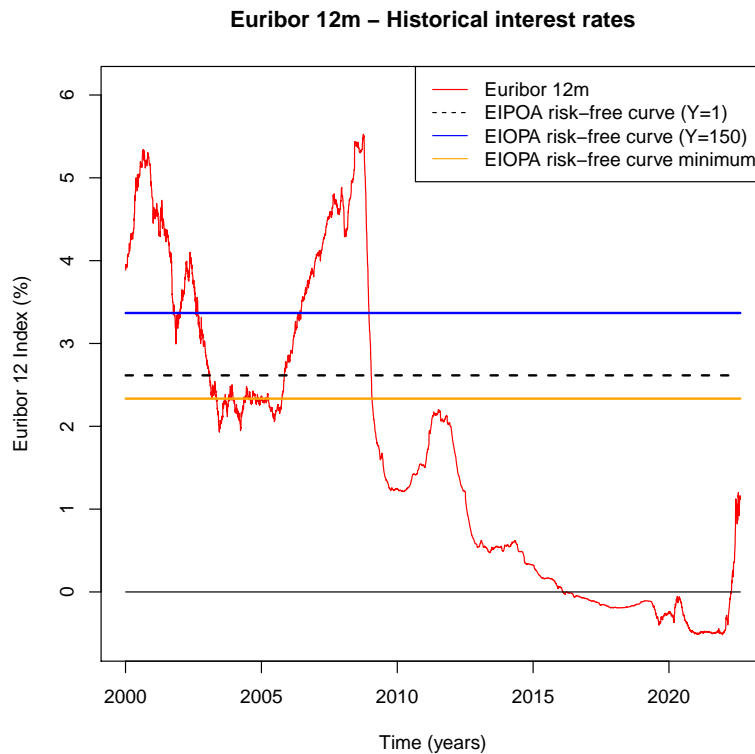
Figure 4.1: Euribor 12m, daily quotes, 1 January 2000 - 17 August 2022.

Equity risk

For equity risk we have used time series for the following equity indices (with corresponding currency denomination): OSEBX ("Hovedindeksen", quoted in NOK), OMX ("Stockholm 30 Index", quoted in SEK), UKX ("FTSE 100 Index", quoted in GBP), SXXP ("STOXX Europe 600 Index", quoted in EUR), SPX ("S&P 500 Index", quoted in USD), and NKY ("Nikkei 225 Index", quoted in JPY).

With time series of daily market prices for the above indices from 1 January 2000 through August 17 2022, see Figure 4.2 we have estimated the parameters needed for simulating these index prices as dependent geometric brownian motions. We can clearly identify an upward trend for all indices, although some more pronounced than others. Also, the increments for each index seem to be independent and identically distributed. Thus, assuming a geometric brownian motion when simulating future values for these indices seems to be appropriate. For such simulation we have estimated the parameters as presented in Tables 4.1 and 4.2 .
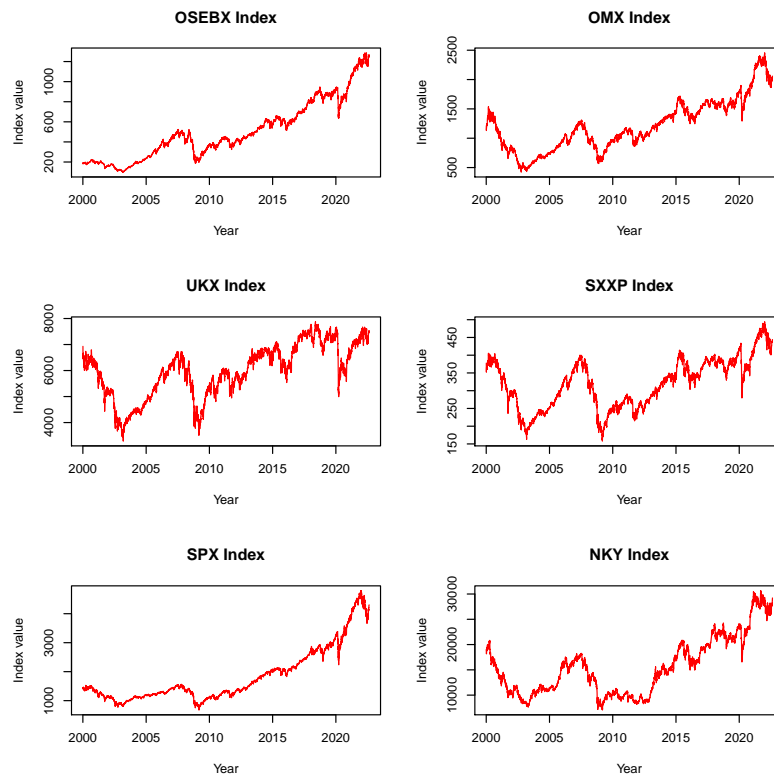
Figure 4.2: Equity indices, daily quotes, 1 January - 17 August 2022.

(ρ)	OSEBX	OMX	UKX	SXXP	SPX	NKY
E(Ret)	0.1072	0.0493	0.0212	0.0258	0.0673	0.0456
sd(Ret)	0.2204	0.2302	0.1876	0.1930	0.1971	0.2298

Table 4.1: Expected return and corresponding std. dev. for six equity indices.

(ρ)	OSEBX	OMX	UKX	SXXP	SPX	NKY
OSEBX	1.0000	0.6875	0.7099	0.7421	0.4422	0.3248
OMX	0.6875	1.0000	0.7850	0.8603	0.5141	0.2810
UKX	0.7099	0.7850	1.0000	0.9243	0.5402	0.2977
SXXP	0.7421	0.8603	0.9243	1.0000	0.5792	0.3217
SPX	0.4422	0.514	0.5403	0.5792	1.0000	0.1454
NKY	0.3248	0.2810	0.2977	0.3217	0.1454	1.0000

Table 4.2: Correlations for five equity indices.

Currency risk

Since we assume that only equity investments are exposed to currency risk, we are interested in the currecy rates against NOK (the base currency) for the currency denominations of the equity indices listed in the previous section. For currency risk we have used time series for the following currency rates: SEKNOK, GBPNOK, EURNOK, USDNOK, and JPYNOK.

With time series of daily quoted currency rates from 1 January 2000 through August 17 2022 (see Figure 4.3), we have estimated the parameters needed for simulating these currency rates as dependent mean reversion processes. Observing these time series there is no apparent trend for any of these currency rates, although for some there may seem to be a time-varying long-term mean level. However, for the short time estimation of the one-year risk of the SCR, using the latest observed "long-term level" may be appropriate. Then both the long-term mean level, $\xi = E(Y_k)$, and the autocorrelation function

$$\rho_l = \mathrm{cor}\,(Y_k, Y_{k-l}) = \frac{\gamma_l}{\gamma_0} = \frac{\mathrm{cov}(Y_k, Y_{k-l})}{\mathrm{var}(Y_k)},$$

are independent of time k. Here Y_k and Y_{k-l} are observations of the random variable of interest, lagged l time periods apart.

Thus, assuming a mean reverting process when simulating future values for these currency rates seems to be appropriate. For such simulation we have estimated the parameters as presented in Tables 4.3 and 4.4 , based on our time series for the selected currency rates.

Equity risk and Currency risk combined

For the multiplicative effect of equity risk and currency risk, we use the above information for both equity risk and currency risk, only replacing the two correlation matrices with the extended correlation matrix for both equity indices and currency rates combined. This extended correlation matrix is estimated from the combined time series, and is presented in Table 4.5 .
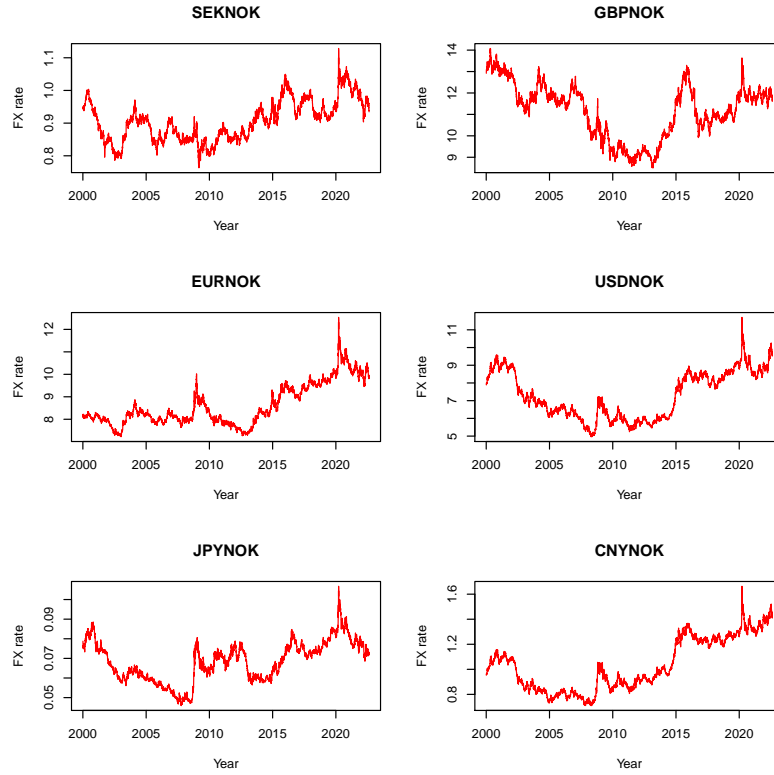
Figure 4.3: Currency rates, daily quotes, 1 January - 17 August 2022.

(ρ)	SEK	GBP	EUR	USD	JPY
E(Rate)	0.9063	11.1567	8.6080	7.3102	0.0683
sd(Ret)	0.0780	0.1026	0.0820	0.1234	0.1453
Spot rate	0.9346	11.7115	9.8393	9.6810	0.07198

Table 4.3: Expected rate and std. dev. (returns) for five currency spot rates (against NOK).

(ρ)	SEK	GBP	EUR	USD	JPY
SEK	1.00000	0.39863	0.60891	0.36922	0.32201
GBP	0.39863	1.00000	0.61871	0.67242	0.53356
EUR	0.60891	0.61870	1.00000	0.62799	0.60381
USD	0.36922	0.67242	0.62799	1.00000	0.71966
JPY	0.32201	0.53356	0.60381	0.71966	1.00000

Table 4.4: Correlations for five currency spot rates (against NOK).

(ρ)	OSEBX	OMX	UKX	SXXP	SPX	NKY	SEK	GBP	EUR	USD	JPY
OSEBX	1.000	0.687	0.710	0.742	0.442	0.325	-0.064	-0.181	-0.283	-0.267	-0.357
OMX	0.688	1.000	0.785	0.860	0.514	0.281	-0.010	-0.147	-0.254	-0.212	-0.307
UKX	0.710	0.785	1.000	0.924	0.540	0.298	-0.0534	-0.242	-0.306	-0.254	-0.371
SXXP	0.742	0.860	0.924	1.000	0.579	0.322	-0.041	-0.109	-0.333	-0.215	-0.356
SPX	0.442	0.514	0.540	0.579	1.000	0.145	-0.047	-0.191	-0.325	-0.291	-0.444
NKY	0.325	0.281	0.298	0.322	0.145	1.000	-0.053	-0.039	-0.116	-0.103	-0.189
SEK	-0.064	-0.010	-0.053	-0.041	-0.047	-0.053	1.000	0.399	0.609	0.369	0.322
GBP	-0.181	-0.147	-0.242	-0.109	-0.191	-0.039	0.399	1.000	0.619	0.672	0.534
EUR	-0.282	-0.254	-0.306	-0.333	-0.325	-0.116	0.609	0.619	1.000	0.628	0.604
USD	-0.270	-0.212	-0.254	-0.215	-0.291	-0.103	0.369	0.672	0.628	1.000	0.720
JPY	-0.357	-0.307	-0.371	-0.356	-0.447	-0.189	0.322	0.534	0.604	0.720	1.000

Table 4.5: Correlations for five equity indices and corresponding currency spot rates (against NOK).

CHAPTER 5

Partial internal model SCR

5.1 Introduction

Combining the mathematical tools of Section 4.3 with the company and market information described in Chapters 2 and 4 , we are now in a position to build our partial internal models. Every partial internal model that we propose in this thesis is based on multivariate simulation, and will represent a numerical alternative to the schematic approach of the standard model/formula.

The underlying assumptions of the standard formula offered by EIOPA ([EIO14b]) clearly demonstrate that the construct underpinning the standard model/formula lacks mathematical stringency. When the underlying assumption of Gaussian distribution for the random variables of the stress tests is acknowledged to be spurious, the response is to make "adjustments" to the related correlation factors so that the final effect on the distribution function of the basic own funds (BOF) is such that the solvency capital requirement (SCR) can be "properly computed". In their own words; "Due to imperfections that are identified with this aggregation formula (e.g. cases of tail dependencies and skewed distributions) the correlation parameters are chosen in such a way as to achieve the best approximation of the 99.5 % VaR for the overall (aggregated) capital requirement." Another important issue with the the standard model/formula is the matter of diversification. It is recognised in the standard formula when aggregating the SCR of risk sub-modules; "Diversification effects are taken into account when capital requirements are aggregated by using correlation matrices. For aggregating the individual risk sub-modules and modules to obtain the overall SCR, linear correlation techniques are applied. The setting of the correlation coefficients is intended to reflect potential dependencies in the tail of the distributions, as well as the stability of any correlation assumption under stress conditions." As we can see from EIOPAS explanations, the correlation matrices provided by Solvency II are actively used to adjust for the shortcomings of the standard model/formula.

To sum up, one is hard pressed if asked to use the standard model/formula to explain, with any degree of accuracy, how a random variable that represents a risk exposure to the BOF will affect the distribtion of the BOF. E.g., what is the marginal risk contribution to the BOF from adding a new equity investment to the total equity portfolio? The standard model/formula is not a transparent construct. In fact, every risk sub-module described by a stress test has the potential of being opaque and confusing. In addition, the level of mathematical stringency for the standard model/formula is low, if present at all.

Contrary to this is our numerical method of multivariate simulation that we apply in our partial internal models. We are free to choose the distribution function for every single random variable that represents a risk exposure to the BOF, and it will be easy to simulate each random variable the way we want. This flexibility in modelling will maintain a high degree of transparency for our partial internal models. Aggregation of risk by applying copulas allows us to handle dependencies with stringency, and still maintain a high degree of transparency for the final output of the models.

By contrasting the standard model/formula with our multivariate simulation models, we will obtain a better understanding for the shortcomings of the standard model/formula. This way, we will also demonstrate the flexibility as well as computational ease and modelling power of multivariate simulation. In general, working with a transparent, flexible and mathematically stringent model will lead to a better understanding of the inherent risk exposures of the life insurance company.

In the following we will introduce individual partial internal models for the risk sub-modules of *Interest rate risk*, *Equity risk* and *Currency risk* as well as a combined model for *Equity and Currency risk*. These models will show that multivariate simulation will allow us to stringently handle the multiplicative nature of currency exposures (see Section 3.7). This is not the case with the standard model/formula. One of EIOPAs underlying assumptions for the standard model treatment of currency risk simply states "The [currency risk] sub-module takes into account currency risk arising from all possible sources, and the underlying assumption of the market risk module design is that currency effects only appears in this sub-module, i.e. currency effects have been stripped out in the calibration of the other market risk sub- modules." Again, the standard model/formula has been adjusted to address a shortcoming of the standard model, instead of dealing with the issues head-on with mathematical stringency.

As our partial internal models are all based on numerical methods they will have to be presented in their natural working environment, as scripts in a programming language. The scripts for our partial internal models are all written in the programming language of the statistical software R, and can be found in Appendix C . Our partial internal models provide us with a mathematically stringent computation of the solvency capital requirement (SCR) along the mathematical outline of Section 4.3 . We will denote the SCR computed by our partial internal models as **PIM-SCR** and add an extension like **.eqt** when we wish to indicate the the risk source for our partial internal model, i.e., **PIM-SCR.eqt** is the PIM-SCR for the equity risk sub-module. We will in this chapter compare the PIM-SCR with the corresponding SCR based on the standard model/formula of Solvency II.

Note: All simulated quantities reported in this thesis are subject to change every time a new simulation is executed. Therefore, to keep the algorithms (scripts) behind this document alive, we will not include any `set.seed()` in these algorithms to allow for reproducible results. Instead we accept that there will be a slight difference between the stochastic quantities reported in this thesis and what the algorithm show (e.g., in the run examples found in Appendix D).

5.2 The partial internal models - SCR

Introduction

Multivariate, simulation-based partial internal models are easily available through quite simple "risk mapping" scripts that quantify how the random variables that represent the actual risk exposures to the insurance company will affect the insurance company's basic own funds (BOF). The correlation matrix of the partial internal model will take care of any dependencies between these risk factors, and allow us to factor diversification effects into our partial internal models.

The partial internal models are constructed in R-scripts which basically are detailed "risk mappings" of the inherent risk exposures of each risk sub-module onto the insurance companys's distribution function for its basic own funds (BOF). The general construction is the same for all our partial internal models. Briefly summarised, the scripts run as follows:

First there is the **model set-up** which includes setting all model parameters such as the SCR 0.5-percentile, i.e., the probability of insolvency (see Equation (1.1) in Section 1.3). the number of time periods and their lengths (y, sp, T), the number of iterations simulated (m). Also, we load the market information of asset price time series. From these time series we compute expected asset values, asset returns, asset volatilities and correlations between all asset pairs. We transform the correlation matrix into a covariance matrix and make sure that it is positive definite before we run a Cholesky decomposition on it. For univariate simulation we skip the correlation/covariance matrix. Finally, we prepare the data containers to keep the estimated risk outputs from the model.

When the model set-up is complet, we run the **simulation** for each of the timesteps ($t = 1, \ldots, 5$) producing a total of m iterations of $T = 5$ time steps. Each iteration simulate a stochastic process for each of the rando variables (risk factors) included in the model, and the correlation matrix allows us to handle any dependencies between these random variables. In the case of equity asset price this will be *geometric random walk* processes, whereas for currency rates and interst rates this will be *stationary* processes in the form of autoregressive processes (AR(1)). The multivariate simulation of asset prices for all assets in a risk sub-module will allow us to compute stochastic asset portfolio values and/or returns. The change in this portfolio value over the next 12 months (financial year) represents the PIM-SCR for the risk sub-module of the partial internal model. This way we get a mathematically stringent representation of all dependencies included in the partial internal model. With the right choice of copula, we may also handle dependencies for SCR numbers deep into the left tail of the basic own funds distribution distribution.

Finally, the simulated values stored in the data containers specified in the scripts allow us to easily **sort PIM-SCR** values for each time step in an ascending order, thereby providing a ranking of porfolio values that immediately produces the 99.5%-percentile for solvency (the additive inverse of insolvency, $1 - .005$).

The following computations of the solvency capital requirement (SCR) for each partial internal model are based on the information given in Chapter 2 , the standard model, and, Chapter 4 , internal models.

Interest rate risk

The output from running the R-script for the interest rate partial internal model can be found in Appendix D . The script itself without any output can be found in Appendix C . Here we briefly report the general interest rate risk profile affecting basic own funds through the best estimate, including the **PIM-SCR.ir** value (the absolute value of the 0.5-percentile for insolvency in year 1), estimated by this partial internal model. Figure 5.1 is a graphical representation of the interest rate risk profile, wheras Table 5.1 presents the numbers behind the figure. The black lines in the figure are EIOPA's projection for the risk-free interest rate in Norway as of 31 July 2022. The solid black line is the spot curve, and the dashed black line is the the shifted term structue after the shock of the interest rate stress test. The colored lines in the figure are different percentiles for the risk-free rate simulated by our partial internal model.
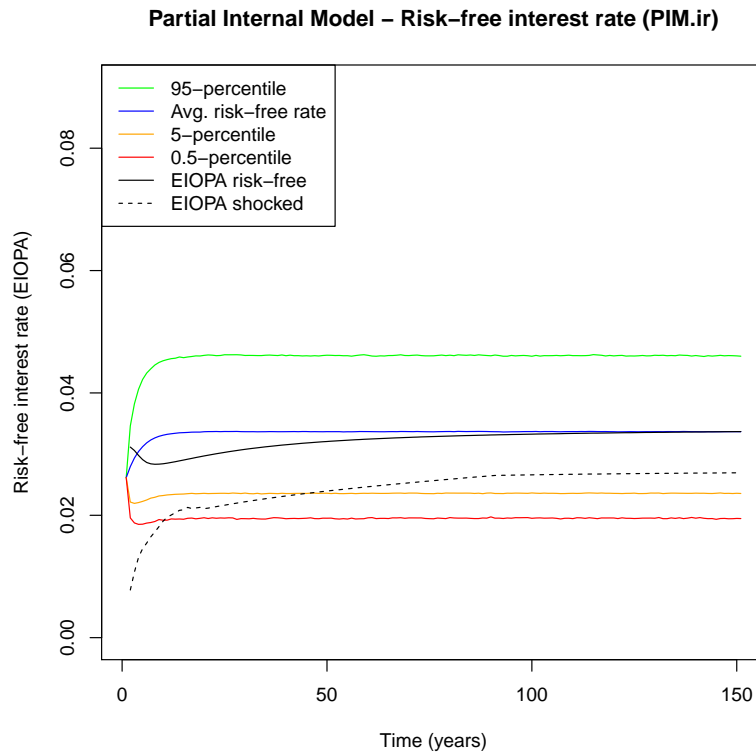
Figure 5.1: 150-year forward-looking Interest rate risk profile.

Now, we cannot read the PIM-SCR.ir from this table. None of the curves in the figure are directly linked to this risk quantity. What we actually do is to use the term structure from each simulated interest rate scenario to determine the simulated discount factors. Then, for each simulated scenario, we will discount the expected net cash flows from the liabilities to get to a simulated value for the best estimate (BE). Thus, we get a simulated estimate of the BE for each

(risk-free rate, %)	$t = 1$	$t = 2$	$t = 3$	...	$t = 10$	...	$t = 50$	...	$t = 150$
95 %	3.383	3.702	3.959	...	4.677	...	4.957	...	4.986
mean	2.735	2.823	2.905	...	3.218	...	3.372	...	3.367
5 %	2.168	2.080	2.040	...	2.071	...	2.146	...	2.150
0.5 %	1.908	1.761	1.705	...	1.621	...	1.672	...	1.706
EIOPA spot	3.115	3.059	2.979	...	2.852	...	3.211	...	3.368
EIOPA shocked	0.779	1.071	1.311	...	1.968	...	2.404	...	2.694

Table 5.1: 150-year forward-looking Interest rate risk profile.

term structure scenario. Sorting all these simulated BE values, we find the 0.05-percentile value for the BE, which represents the stressed BE value ($BE.s$). We also compute the estimate of the BE using EIOPA's spot term structure for the risk-free interest rate ($BE.EIOPA$). The difference between these two BE quantities represents the partial internal model SCR:

PIM-SCR.ir = BE.s − BE.EIOPA = 405, 218.6 − 370, 692 = 37, 227.

The PIM-SCR.ir produced by the interest rate partial internal model is NOKmn 37, 227. This is a reduction of some NOKbn 34.5 (48%) from the standard model SCR of NOKmn 71, 753.

The stressed BE value from the partial internal model is a result of simulating the term structure of the risk-free interest rate as a mean-reverting stochastic process. As such it is dependent on a number of factors, the most important ones being the *mean reversion constant* (the `a.bk` parameter in the Black–Karasinski model), the *initial value* for the risk-free interest rate (the starting point of every term structure scenario), the expected long-term level for risk-free interest rate, and the *estimated volatility* for the risk-free interest rate. We have calibrated our model based on the current market environment with short-term risk-free interest rates that are slightly lower than EIOPA's short end of the term structure, but with a slightly steeper short-to-mid section of the term structure than what EIOPA projects. We base this calibration on a view that the historic and current low levels of the short end of the term structure for the risk-free interest rates will see a steepening tilt going forward. Most federal/national banks are now expected to frequently increase the overnight rates over the next year or two, and with larger than usual increments, in order to fight the imminent inflationary pressure. This difference in term structure assumptions account for some of the above reported SCR reduction, but the mean reversion effect is also important. The lower the interest rate level the more pronounced the mean reversion effect will be.

Equity risk

The output from running the R-script for the equity partial internal model can be found in Appendix D . The script itself without any output can be found in Appendix C . Here we briefly report the general equity risk profile from equity exposures, including the **PIM-SCR.eqt** value (the absolute value of the 0.5-percentile for insolvency in year 1), estimated by this partial internal model. Figure 5.2 is a graphical representation of the equity risk profile, wheras Table 5.2 presents the numbers behind the figure. The colored lines in the

figure represent certain percentiles of the simulated changes in equity value from equity exposures. In particular, the blue line is the expected change, and the red line represents the 0.5-percentile for insolvency related to this partial internal model. The absolute value for the first year given by this red line represents the SCR estimate provided by the partial internal model,

$$\text{PIM-SCR.eqt} = 28,576.$$

Partial Internal Model – Equity (PIM.eqt)
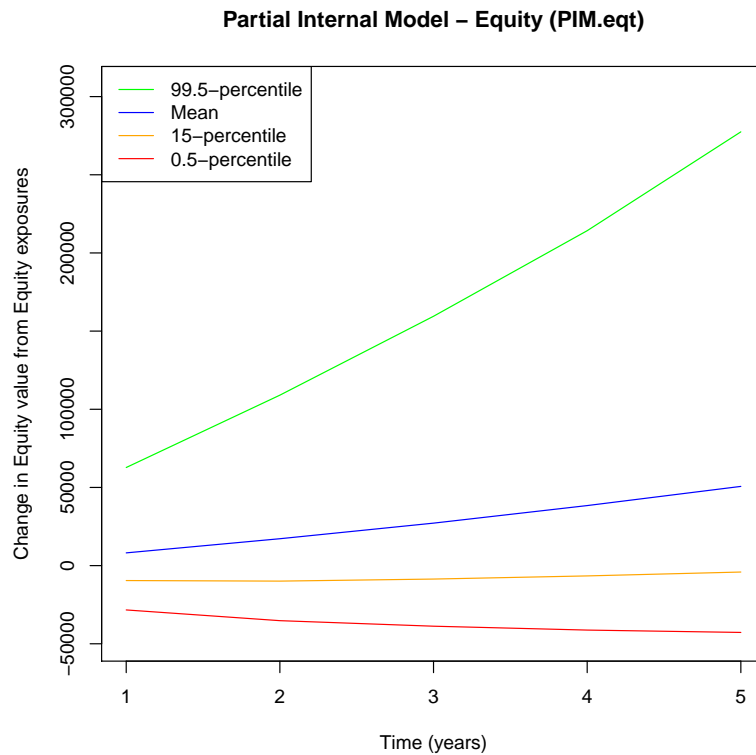
Figure 5.2: 5-year forward-looking Equity risk profile (NOKmn).

Percentile (insolvency)	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
99.5 %	62,475	107,603	157,397	212,121	273,662
mean	8,079	17,006	26,786	37,661	49,735
15 %	-9,617	-9,946	-8,859	-7,023	-4,466
0.5 %	**-28,576**	-35,024	-39,025	-41,184	-43,087
SCR standard model	**-32,040**				

Table 5.2: 5-year forward-looking Equity risk profile (NOKmn).

The PIM-SCR.eqt produced by the equity partial internal model is NOKmn 28,576. This is a reduction of NOKbn 3.5 (11%) from the standard model SCR of NOKmn 32,040.

Currency risk

The output from running the R-script for the currency partial internal model can be found in Appendix D . The script itself without any output can be found in Appendix C . Here we briefly report the general equity risk profile from currency exposures (we assume that only equity investments have currency exposure), including the **PIM-SCR.ccy** value (the absolute value of the 0.5-percentile for insolvency in year 1), estimated by this partial internal model. Figure 5.3 is a graphical representation of the currency risk profile, wheras Table 5.3 presents the numbers behind the figure. The colored lines in the figure represent certain percentiles of the simulated changes in equity value from currency exposures. In particular, the blue line is the expected change, and the red line represents the 0.5-percentile for insolvency related to this partial internal model. The absolute value for the first year given by this red line represents the SCR estimate provided by this partial internal model,

$$\text{PIM-SCR.ccy} = 12,479.$$
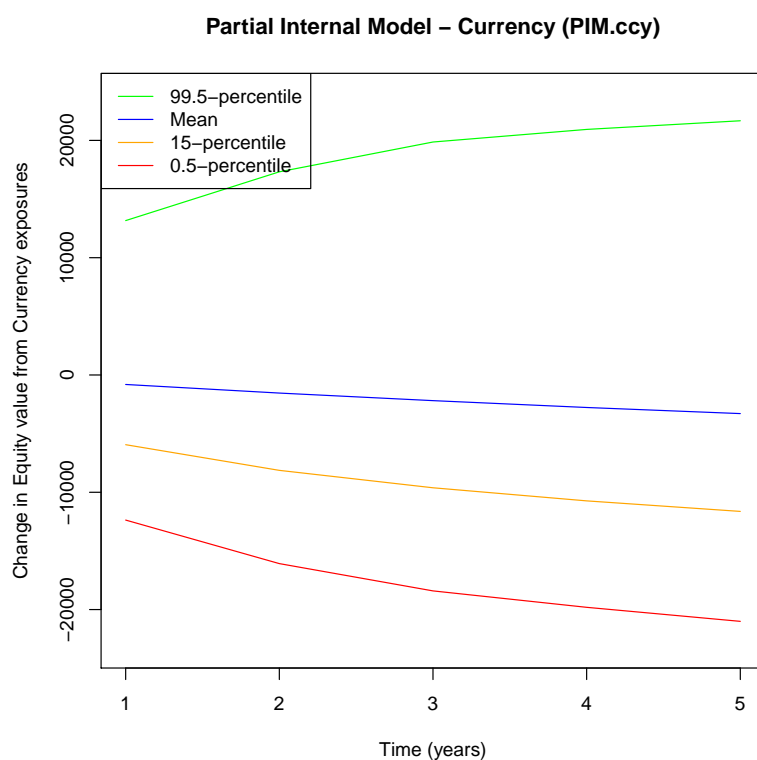
Figure 5.3: 5-year forward-looking Currency risk profile (NOKmn).

The PIM-SCR.ccy produced by the currency partial internal model is NOKmn 12,479. This is a reduction of NOKbn 5.5 (31%) from the standard model SCR of NOKmn 18,000.

Percentile (insolvency)	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
99.5 %	13,132	17,021	19,576	20,846	21,608
mean	-832	-1,545	-2,190	-2,757	-3,294
15 %	-5,918	-8,094	-9,638	-10,696	-11,595
0.5 %	**-12,479**	-16,090	-18,350	-19,922	-21,008
SCR standard model	**-18,000**				

Table 5.3: 5-year forward-looking Currency risk profile (NOKmn).

Equity and Currency risk combined

The output from running the R-script for the multiplicative equity and currency partial internal model can be found in Appendix D . The script itself without any output can be found in Appendix C . Here we briefly report the general equity risk profile from equity and currency exposures (we assume only equity has currency exposure), including the **PIM-SCR.eqt.ccy** value (the absolute value of the 0.5-percentile for insolvency in year 1), estimated by this partial internal model. Figure 5.4 is a graphical representation of the multiplicative equity and currency risk profile, wheras Table 5.4 presents the numbers behind the figure. The colored lines in the figure represent certain percentiles of the simulated changes in equity value from both equity and currency exposures. In particular, the blue line is the expected change, and the red line represents the 0.5-percentile for insolvency related to this partial internal model. The absolute value for the first year given by this red line represents the SCR estimate provided by this partial internal model,

$$\text{PIM-SCR.eqt.ccy} = 380.$$

Percentile (insolvency)	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
99.5 %	307	501	732	999	1,276
mean	-13.7	-18.3	-17.7	-15.6	-11.0
15 %	-80	-119	-152	-180	-213
0.5 %	**-380**	-627	-861	-1,138	-1,408
SCR standard model	**-34,308**				

Table 5.4: 5-year forward-looking Equity and Currency risk profile (NOKmn).

The PIM-SCR.eqt.ccy produced by the combined equity and currency partial internal model is NOKmn 380.

This is a massive reduction of almost NOKbn 34.5 (99%) from the standard model SCR of NOKmn 34, 308 (applying the standard formula between the two, with correlation set to 0.25). One important issue to notice is that the standard formula operates with a correlation of 0.25 between the equity risk sub-module and the currency risk sub-module. Our estimated correlation matrix shows negative correlation for all combinations of equity-currency-pairs, ranging from -0.01 to -0.44. Also, the correlations between pairs of currency rates are

Partial Internal Model – Equity and Currency (PIM.eqt.ccy)
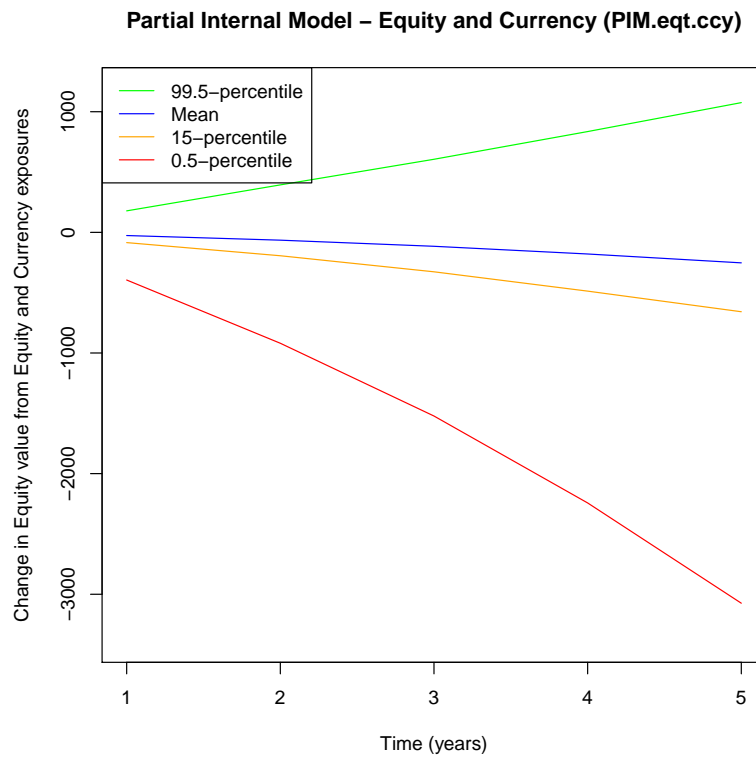
Figure 5.4: 5-year forward-looking Equity and Currency risk profile (NOKmn).

moderate, ranging from 0.32 to 0.71. The negative correleations represent a huge potential for diversification in the computation of the combined equity and currency SCR. Clearly, insurance companies which are invested in assets that are negatively correlated to the currency rate in which they are denominated, may expect to see a significant reduction in its SCR from currency risk.

CHAPTER 6

Conclusions and final remarks

6.1 Conclusions

In this thesis we have introduced multivariate simulation as an alternative method to the standard model/formula of Solvency II. With this numerical method the insurance company may compute solvency capital requirement (SCR) estimates that are tailor-made to its actual balance sheet risk exposures. This method is useful when constructing partial internal models as it is very flexible and versatile and can be applied to any of the stress tests of the standard model/formula. Not only will such partial internal models compute SCR numbers that better reflect the actual risk exposures of the insurance company, they may also assist the insurance company with a better understanding of its own risks. This last part will come in handy when the insurance company is performing the mandatory "Own risk and solvency assessment" (ORSA), [EIO14a], and "Forward looking assessment of own risks" (FLAOR), [EIO13].

SCR reduction

We see from the SCR estimates produced by the partial internal models in Chapter 5, that partial internal models which better reflect the actual risk exposures, may lead to a significant reduction in the life insurance company's capital requirement, compared to the corresponding SCR estimates of the standard model/formula presented in Chapter 3 . This is due to the conservative nature and calibration of the standard model/formula as discussed throughout this thesis.

For the interest rate risk sub-module this SCR reduction is some NOKbn 34.5 (48 %), while the SCR reduction is more moderate for the equity risk sub-module and the currency rate risk sub-module, NOKbn 3.5 (11 %) and NOKbn 5.5 (31 %), respectively.

While the reduction in SCR is modereate for the equity and currency rate risk sub-modules separately, combining the two due to the multiplicative nature of currency risk (its a risk "on top of" another risk, see Section 3.7), the effect on the SCR can be formiddable. For the combined equity and currency partial internal model we get a whopping reduction of NOKbn 34 (99 %). This reduction is predominantly explained by the diversification effect you get from the negative correlation estimates that we get for every pair of equity index and currency rate relevant to our partial internal model for equity and currency risk combined. While the standard model/formula operates with a

positive correlation of 0.25 between the equity risk sub-module and the currency risk sub-module, we have estimated a negative correlation in the range of -0.01 to -0.45 for all (Equity index, Currency rate)-pairs relevant to our model. The negative correleations represent a huge potential for diversification in the computation of the combined equity and currency SCR. Also, the correlations between pairs of currency rates are moderate, ranging from 0.32 to 0.71. In conclusion, the reductions in SCR are moderate but significant for equity risk and currency risk. Most likely this is the result of insurance companies having well diversified investment portfolios, just as assumed by the standard model/formula. Therefore, the shock factors for the stress tests of these risk sub-modules, although crude, may still be fairly accurate. The cocrrelation of 0.25, however, between these two risk sub-modules set by the standard formula, is clearly too conservative for our fictive life insurance company. This correlation effect is more pronounced for insurance companies that have a base (local) currency that is subject these negative correlations between any investment and its corresponding currency risk.

The reduction in the SCR for interest rate risk is also significant, and about the same size as for the combined equity and currency risk. The SCR estimate for the interest rate risk provided by our partial internal model hinges on the mean reversion parameter (`a.bk` in our script). We have set this parameter value to 0.75, which is a conservative estimate given the current inflationary situation in most countries and what this situation means in terms of expected frequent and large interest rate hikes over the next year or two. However, the EIOPA spot curve for Norway is even more conservative, indicating a parameter value of 0.85. This is when the SCR estimates are the same for both the standard model and our partial internal model. The two expected spot curves which form the basis for the dicount rates when determining the best estimate, can be seen in Figure 5.1, where the black solid is the expected EIOPA spot curve, while the blue line is the expected spot curve from our partial internal model. For a parameter value of 0.65 our partial internal model SCR estimate is down to some NOKbn 21, a reduction of approximately NOKbn 50 (71%). This reduction in SCR from applying the partial internal model for interest rate risk is partly due to the mean reversion effect discussed in Section 3.7 , and partly due to the calibration of the partial internal model as mentioned above. The mean reversion property of interest rate processes tells us that the lower the interest rate level the higher the probability for an upward shift in the term structure. By testing how the interest rate level affects the magnitude of the SCR, we found that the standard model returned an increasingly larger SCR as we fed it increasinly lower interest rate levels. This is all well and good since this is the nature of discounting cash flows. But at some at some point we would expect that any mean reversion effect would dominate the discounting effect, but that did not happen. It seems that the standard model/formula and EIOPA's risk-free interest rate projections are not able to reduce the effect of the shock parameter enough for lower intereste rate levels, even though the shoch is a percentage reduction of the interest rate level.

All in all, for our fictive life insurance company it seems worth while to explore the possibilities of employing partial internal models in its risk reporting to the supervisory authorities.

6.2 Final remarks

A word of caution

Our partial internal models are just that - models. As such thay are only a simplified representation of real world situations. As for all models, and particularly stochastic ones, the results they produce are heavily dependent upon the assumptions behind the model and all input used.

We have in this thesis made a particular effort to combine real life relevancy of the models with simplicity and transparity in their application and interpretation. In our calibration of the models, we have combined historical market information with current market expectations. However, much of the input is based on historical market price/rate quotes, and history is no guarantee for reliable future predictions. The results that we derive have to be evaluated carafully in light of this. Therefore, the output from these models are to be interpreted as arguments to explore the possibility of employing internal models, and a starting point for further development of such models, rather than a definitive computation of the solvency capital requirement.

As for any model, it is just a tool. And as for most tools, be careful with what you apply it on. Applying a hand saw to a nail may not be a good idea. And if you have a hammer, not everything is a nail. Considering the use of copulas, a central tool in our partial internal models, they can be misused as all tools can. The CDO debacle that ignited the subprime mortgage crisis that occurred between 2007 and 2010, was partly due to the misuse of copulas. When everythig goes south at the same time, and financial and other markets collapse simultaneously, dependencies between most market prices and other related random variables all seem very quickly to be perfectly correlated. Historically calibrated dependencies become obsolete in an instant as panic spreads and everybody is on the sell side. Exit copulas, correlation matrices and any other measure of dependency that is historically calibrated.

However, as we are working with insurance risk, our best estimate will have to do. Also, when estimating the solvency capital requirement, the SCR quantites that we get from our models shold be evaluated both for the short term and the long term. Short term, a qualitative evaluation should consider what could possibly happen during the next year that could render our SCR estimates obsolete? This will be important when making short term capital structure decisions for the insurance company's balance sheet. Long term, a qualitative evaluation should consider what could possibly happen during over the long stetch that could render our long term estimates, such as liability cash flow projections and market investment expectations, obsolete? Such long-term evaluations should have an appropriate impact on the strategic decison making of the insurance company, such as making necessary changes to the insurance and market investment activities.

Future work

The method and the models discussed in this thesis are far from fully developed. This is a consequence of keeping many of the assumptions behind the standard formula to be able to better compare our partial internal models with the standard model/formula. In particular, the assumptions about the underlying

distribution function for the different random variables that represent risk exposures to the basic own funds are very unlikely to be Gaussian. This will be especially true for general insurance, where the underlying distributions typically are heavily skewed to the right. Therefore, further work should be done to explore and apply other distribution functions for the underlying random variables as well other copulas than the Gaussion to handle dependencies.

In this thesis we have put our methods and partial internal models to work only on the market risk module. Applying this method on the insurance risk module constructing relevant and useful partial internal models for liability risk sub-modules, either in life insurance or in general insurance, will add significant support for our method proposed in this thesis. Also, it will add an important boost to the construction a complete internal model for the basic own funds of the insurance company. With a complete internal model for basic own funds there will no longer be a need for the standard formula when computing the Basic Solvency Capital Requirement. The complete internal model will then render the standard formula obsolete as risk aggregation is a direct consequence of simulating all random variables (risk factors) simultaneously.

Appendices

APPENDIX A

Solvency II - standard method/standard formula

The following R-script for the standard model/formula of Solvency II shows how this model is constructed to compute the solvency capital requirement (SCR).

RUN example for this script can be found in Appendix A. The RUN example will show the actual computed SCR values.

```
# SCR computations
# - under the standard model/standard formula

rm(list=ls(all=T))                              # clear workspace

# Source Files:
source("Asset_Input_F.r")
source("Liability_Input_F.r")
source("Solvency_II_Input_F.r")
source("Liabilities_Input_F.r")
source("Functions_F.r")
source("Liabilities_F.r")

# OBJECTIVE
# Compute BSCR = SCR
# ! Assuming no Operational nor Adjustment risk
# => Aggregate Liability and Market risk

# LIABILITY RISK
# - Only considering Mortality risk and Longevity risk
#   - Mortality: Term insurance
#   - Longevity: Pension insurance
# ! Assuming no reinsurance

# Preparing all discounts:
K.max <- max(K.1,K.2)
K.max
# Technical rate:
K.tr = max(l.e.2,K.max)
d.tr <- 1/(1 + r.tr)^(0:K.tr)
```

```
# Market Interest Rates (before and after Solvency II shock):
file.rf <- "../Thesis_R_IRB/EIOPA_rf_curves_no_VA.txt"
RF <- read.table(file.rf,header=T,row.names=1)
rf.EIOPA.spt <- RF$Spot_curve
rf.EIOPA.shock <- RF$Shocked_curve_down
# Discount factors:
d.k <- 1/(1 + rf.EIOPA.spt)^(1:K.max)                    # ordinary
d.k.S <- 1/(1 + rf.EIOPA.shock)^(1:K.max)         # after shock
# Plotting risk-free rates (before and after Solvency II shock):
pdf("RiskFreeRates.pdf")
  plot(1:K.max,rf.EIOPA.spt[1:K.max],type="l",
       main="Risk free interest rates",
       ylim=c(0,.05),col=1,
       xlab="Time - k years froward",ylab="Rates")
  lines(1:K.max,rf.EIOPA.shock[1:K.max],col=2)
  legend("bottomright",legend=c("Rates","Rates after shock")
          ,lty=rep(1,2),col=1:2)
dev.off()


# Preparing Liability Quantities:
# Preliminary computations - Portfolio I:
l.e.1 <- l.0.1 + K.1                  # Age at contract expiration
s.prb.1 <- lifetab(t.0, t.1, t.2,       # Survival probabilities
                   l.0.1, l.e.1)
s.prb.S.1 <- lifetab(t.0, t.1, t.2,  # Survival probab. w/ shock
                   l.0.1, l.e.1, S.mort)
N.1 <- ndist(gam.1, mu.1, K.1, J.1) # Age distribution portfolio
# Preliminary computations - Portfolio II:
s.prb.2 <- lifetab(t.0, t.1, t.2,       # Survival probabilities
                   l.0.2, l.e.2)
s.prb.S.2 <- lifetab(t.0, t.1, t.2,   # Survival prob.. w/ shock
                   l.0.2, l.e.2, S.long)
N.2 <- ndist(gam.2, mu.2, K.2, J.2) # Age distribution portfolio


# Some initial illustrating plots:
# Age distribution - both portfolios
pdf("AgeDistribution.pdf")
  par(mfrow=c(2,1))
  plot(1:(K.1+1),N.1,type="l",
       xlab="Time - k years ahead",ylab="no. of policy holders",
       main="Age Distribution - Term Insurance")
  plot(1:(K.2+1),N.2,type="l",
       xlab="Time - k years ahead",ylab="no. of policy holders",
       main="Age Distribution - Pension Insurance")
dev.off()


# COMPUTATIONS PROPER

### LIABILITY Computations:
## Portfolio I - Term Insurance
```

```
# Premium (pi.1):
pi.1 <- s.1 * d.tr[2:(K.1 + 1)] %*%
        (s.prb.1$q[1:K.1]*s.prb.1$kp[1,2:(K.1+1)]) /
        (d.tr[1:K.1] %*% s.prb.1$kp[1,1:K.1])
pi.1
# The liablities (L = L.k, for k = 1,...,K=25):
L.trm <- LiabTerm(pi.1, s.1, K.1, N.1, s.prb.1)
write.table(L.trm[1:K.1],file="L_term.txt",sep="\t",row.names=T,
            col.names="Liability_term")
            #col.names=c("Year","Liability_term"))
BE.trm = L.trm %*% d.k[1:K.1]
#BE.trm
L.trm.S <- LiabTerm(pi.1, s.1, K.1, N.1, s.prb.S.1)     # w/ Shock
# Plot:
pdf("Liabilities_Term_Insurance.pdf")
  plot(1:K.1,L.trm.S,type="l",col="red",
       ,main="Portfolio I - Term Insurance",
       xlab="Time - k years ahead",
       ylab="Liability")
  lines(1:K.1,L.trm,col="blue")
  legend("topright",legend=c("Liability","Liability w/ Shock"),
         lty=rep(1,2),col=c("blue","red"))
dev.off()
# SCR Portfolio I:
SCR.mort <- (L.trm.S - L.trm)%*%d.k[1:K.1]
SCR.mort
## Portfolio II - Pension Insurance
# Premium (pi.2):
pi.2 <- s.2 * d.tr[(l.r - l.0.2 + 1):(l.e.2 - l.0.2 + 1)] %*%
    (s.prb.2$kp[1,(l.r - l.0.2 + 1):(l.e.2 - l.0.2 + 1)]) /
    (d.tr[1:(l.r - l.0.2)] %*% s.prb.2$kp[1,1:(l.r - l.0.2)])
pi.2
# The liablities (L = L.k, for k = 1,...,K=65):
L.pen <- LiabPen(pi.2, s.2, K.2, N.2, l.r, s.prb.2)
write.table(L.pen[1:K.2],file="L_pension.txt",sep="\t",row.names=T,
            col.names="Liability_term")
#L.pen
BE.pen = L.pen[1:K.2] %*% d.k[1:K.2]
#BE.pen
L.pen.S <- LiabPen(pi.2, s.2, K.2, N.2, l.r, s.prb.S.2) # w/ Shock
# Plot:
pdf("Liabilities_Pension_Insurance.pdf")
  plot(1:K.2,L.pen.S[1:K.2],type="l",col="red",
       xlab="Time - k years ahead", ylab="Liability",
       main="Portfolio II - Pension Insurance")
  lines(1:K.2,L.pen[1:K.2],col="blue")
  legend("topright",legend=c("Liability","Liability w/ Shock"),
         lty=rep(1,2),col=c("blue","red"))
dev.off()
# SCR Portfolio II:
```

```
(SCR.long <- (L.pen.S[1:K.2] - L.pen[1:K.2])%*%d.k[1:K.2])
# Best estimate
#(BE = BE.trm + BE.pen)
#

# SCR LIABILITIES:
SCR.lia.vector <- c(SCR.mort,SCR.long)
(SCR.liab <- sqrt(t(SCR.lia.vector) %*%
                  SCR.lia.cor %*%
                  SCR.lia.vector))

# MARKET Computations:
# Interest Rate Computations:
# BE:  the expected values of all futurenet cash flows
#      out of and into accounts under existing contracts.
#      - BE is an expected, net obligation.
# L.k: the expected, net cash flow out of the accounts, year k.
# NB! Only include contracts adversely affected by the shock.
# Total liabilities:
l.t = length(L.trm)
l.p = length(L.pen)
max.l = max(l.t,l.p)
diff.l = l.p - l.t
if (diff.l >= 0) {
  L.trm = append(L.trm,rep(0,diff.l))
} else {
  L.pen = append(L.pen,rep(0,diff.l))
}
L.tot = L.pen + L.trm
write.table(L.tot[1:K.max],file="L_tot.txt",sep="\t",row.names=T,
            col.names="Liability_tot")
# SCR Interest Rates:
(BE <- L.tot[1:K.max] %*% d.k[1:K.max])
#(BE <- (L.trm + L.pen)[1:K.max] %*% d.k[1:K.max])
(BE.S <- L.tot[1:K.max] %*% d.k.S[1:K.max]) # w/ shock
(SCR.int <- BE.S - BE)
# SCR Equity:
(SCR.eqt <- S.eqt.1*A.eqt.1 + S.eqt.2*A.eqt.2)

# SCR Property:
(SCR.prp <- S.prp*A.prp)

# Spread Computations:
(SCR.spr <- ((1 - S.spr.1)*B.1.share +
            (1 - S.spr.2)*B.2.share)*spr.share*A)

# SCR Currency:
(SCR.fx <- S.fx*A*eqt.share*(1 - port.eqt[1]*(1 - eqt.to.fx[1])))

# SCR Market: Standard formula -  Market Risk sub-modules:
```

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```
SCR.mkt.mod <- c(SCR.int,SCR.eqt,SCR.prp,SCR.spr,SCR.fx)
(SCR.mark <- sqrt(t(SCR.mkt.mod) %*% SCR.mkt.cor %*% SCR.mkt.mod))

# SCR Basic Operations (Basic Own Funds)
BSCR.vector <- c(SCR.liab,SCR.mark)
#BSCR.corr <- matrix(c(1,corr.lima,corr.lima,1),nrow=2)
(BSCR <- sqrt(t(BSCR.vector) %*% BSCR.cor %*% BSCR.vector))
```

APPENDIX B

Standard method/standard formula - SCR computations

The following RUN example shows the actual computed SCR values for the standard model/formula for our fictive Norwegian life insurance company, described in Chapters 2 and 3, and for which the R-script can be found in Appendix A.

```
R version 3.4.4 (2018-03-15) -- "Someone to Lean On"
Copyright (C) 2018 The R Foundation for Statistical Computing
Platform: x86_64-pc-linux-gnu (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> # SCR computations
> # - under the standard model/standard formula
>
> rm(list=ls(all=T))                          # clear workspace
>
> # Source Files:
> source("Asset_Input_F.r")
> source("Liability_Input_F.r")
> source("Solvency_II_Input_F.r")
```

```
> source("Liabilities_Input_F.r")
> source("Functions_F.r")
> source("Liabilities_F.r")
>
> # OBJECTIVE
> # Compute BSCR = SCR
> # ! Assuming no Operational nor Adjustment risk
> # => Aggregate Liability and Market risk
>
> # LIABILITY RISK
> # - Only considering Mortality risk and Longevity risk
> #    - Mortality: Term insurance
> #    - Longevity: Pension insurance
> # ! Assuming no reinsurance
>
> # Preparing all discounts:
> K.max <- max(K.1,K.2)
> K.max
[1] 65
> # Technical rate:
> K.tr = max(l.e.2,K.max)
> d.tr <- 1/(1 + r.tr)^(0:K.tr)
> # Market Interest Rates (before and after Solvency II shock):
> file.rf <- "../Thesis_R_IRB/EIOPA_rf_curves_no_VA.txt"
> RF <- read.table(file.rf,header=T,row.names=1)
> rf.EIOPA.spt <- RF$Spot_curve
> rf.EIOPA.shock <- RF$Shocked_curve_down
> # Discount factors:
> d.k <- 1/(1 + rf.EIOPA.spt)^(1:K.max)                  # ordinary
Warning message:
In (1 + rf.EIOPA.spt)^(1:K.max) :
  longer object length is not a multiple of shorter object length
> d.k.S <- 1/(1 + rf.EIOPA.shock)^(1:K.max)         # after shock
Warning message:
In (1 + rf.EIOPA.shock)^(1:K.max) :
  longer object length is not a multiple of shorter object length
> # Plotting risk-free rates (before and after Solvency II shock):
> pdf("RiskFreeRates.pdf")
>   plot(1:K.max,rf.EIOPA.spt[1:K.max],type="l",
+        main="Risk free interest rates",
+        ylim=c(0,.05),col=1,
+        xlab="Time - k years froward",ylab="Rates")
>   lines(1:K.max,rf.EIOPA.shock[1:K.max],col=2)
>   legend("bottomright",legend=c("Rates","Rates after shock")
+           ,lty=rep(1,2),col=1:2)
> dev.off()
null device
          1
>
> # Preparing Liability Quantities:
```

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```
> # Preliminary computations - Portfolio I:
> l.e.1 <- l.0.1 + K.1              # Age at contract expiration
> s.prb.1 <- lifetab(t.0, t.1, t.2,      # Survival probabilities
+                   l.0.1, l.e.1)
> s.prb.S.1 <- lifetab(t.0, t.1, t.2,  # Survival probab. w/ shock
+                   l.0.1, l.e.1, S.mort)
> N.1 <- ndist(gam.1, mu.1, K.1, J.1) # Age distribution portfolio
> # Preliminary computations - Portfolio II:
> s.prb.2 <- lifetab(t.0, t.1, t.2,      # Survival probabilities
+                   l.0.2, l.e.2)
> s.prb.S.2 <- lifetab(t.0, t.1, t.2,   # Survival prob.. w/ shock
+                   l.0.2, l.e.2, S.long)
> N.2 <- ndist(gam.2, mu.2, K.2, J.2) # Age distribution portfolio
>
> # Some initial illustrating plots:
> # Age distribution - both portfolios
> pdf("AgeDistribution.pdf")
>   par(mfrow=c(2,1))
>   plot(1:(K.1+1),N.1,type="l",
+         xlab="Time - k years ahead",ylab="no. of policy holders",
+         main="Age Distribution - Term Insurance")
>   plot(1:(K.2+1),N.2,type="l",
+         xlab="Time - k years ahead",ylab="no. of policy holders",
+         main="Age Distribution - Pension Insurance")
> dev.off()
null device
          1
>
> # COMPUTATIONS PROPER
>
> ### LIABILITY Computations:
> ## Portfolio I - Term Insurance
> # Premium (pi.1):
> pi.1 <- s.1 * d.tr[2:(K.1 + 1)] %*%
+         (s.prb.1$q[1:K.1]*s.prb.1$kp[1,2:(K.1+1)]) /
+         (d.tr[1:K.1] %*% s.prb.1$kp[1,1:K.1])
> pi.1
             [,1]
[1,] 0.005743617
> # The liablities (L = L.k, for k = 1,...,K=25):
> L.trm <- LiabTerm(pi.1, s.1, K.1, N.1, s.prb.1)
There were 50 or more warnings (use warnings() to see the first 50)
> write.table(L.trm[1:K.1],file="L_term.txt",sep="\t",row.names=T,
+             col.names="Liability_term")
>             #col.names=c("Year","Liability_term"))
> BE.trm = L.trm %*% d.k[1:K.1]
> #BE.trm
> L.trm.S <- LiabTerm(pi.1, s.1, K.1, N.1, s.prb.S.1)     # w/ Shock
There were 50 or more warnings (use warnings() to see the first 50)
> # Plot:
```

```
> pdf("Liabilities_Term_Insurance.pdf")
>   plot(1:K.1,L.trm.S,type="l",col="red",
+        ,main="Portfolio I - Term Insurance",
+        xlab="Time - k years ahead",
+        ylab="Liability")
>   lines(1:K.1,L.trm,col="blue")
>   legend("topright",legend=c("Liability","Liability w/ Shock"),
+          lty=rep(1,2),col=c("blue","red"))
> dev.off()
null device
          1
> # SCR Portfolio I:
> SCR.mort <- (L.trm.S - L.trm)%*%d.k[1:K.1]
> SCR.mort
        [,1]
[1,] 1009.843
> ## Portfolio II - Pension Insurance
> # Premium (pi.2):
> pi.2 <- s.2 * d.tr[(l.r - l.0.2 + 1):(l.e.2 - l.0.2 + 1)] %*%
+     (s.prb.2$kp[1,(l.r - l.0.2 + 1):(l.e.2 - l.0.2 + 1)]) /
+     (d.tr[1:(l.r - l.0.2)] %*% s.prb.2$kp[1,1:(l.r - l.0.2)])
> pi.2
           [,1]
[1,] 0.06220195
> # The liablities (L = L.k, for k = 1,...,K=65):
> L.pen <- LiabPen(pi.2, s.2, K.2, N.2, l.r, s.prb.2)
> write.table(L.pen[1:K.2],file="L_pension.txt",sep="\t",row.names=T,
+            col.names="Liability_term")
> #L.pen
> BE.pen = L.pen[1:K.2] %*% d.k[1:K.2]
> #BE.pen
> L.pen.S <- LiabPen(pi.2, s.2, K.2, N.2, l.r, s.prb.S.2) # w/ Shock
> # Plot:
> pdf("Liabilities_Pension_Insurance.pdf")
>   plot(1:K.2,L.pen.S[1:K.2],type="l",col="red",
+        xlab="Time - k years ahead", ylab="Liability",
+        main="Portfolio II - Pension Insurance")
>   lines(1:K.2,L.pen[1:K.2],col="blue")
>   legend("topright",legend=c("Liability","Liability w/ Shock"),
+          lty=rep(1,2),col=c("blue","red"))
> dev.off()
null device
          1
> # SCR Portfolio II:
> (SCR.long <- (L.pen.S[1:K.2] - L.pen[1:K.2])%*%d.k[1:K.2])
         [,1]
[1,] 48265.85
> # Best estimate
> #(BE = BE.trm + BE.pen)
> #
```

```
>
> # SCR LIABILITIES:
> SCR.lia.vector <- c(SCR.mort,SCR.long)
> (SCR.liab <- sqrt(t(SCR.lia.vector) %*%
+                   SCR.lia.cor %*%
+                   SCR.lia.vector))
         [,1]
[1,] 48023.34
>
> # MARKET Computations:
> # Interest Rate Computations:
> # BE:  the expected values of all futurenet cash flows
> #      out of and into accounts under existing contracts.
> #       - BE is an expected, net obligation.
> # L.k: the expected, net cash flow out of the accounts, year k.
> # NB! Only include contracts adversely affected by the shock.
> # Total liabilities:
> l.t = length(L.trm)
> l.p = length(L.pen)
> max.l = max(l.t,l.p)
> diff.l = l.p - l.t
> if (diff.l >= 0) {
+   L.trm = append(L.trm,rep(0,diff.l))
+ } else {
+   L.pen = append(L.pen,rep(0,diff.l))
+ }
> L.tot = L.pen + L.trm
> write.table(L.tot[1:K.max],file="L_tot.txt",sep="\t",row.names=T,
+            col.names="Liability_tot")
> # SCR Interest Rates:
> (BE <- L.tot[1:K.max] %*% d.k[1:K.max])
         [,1]
[1,] 333465.2
> #(BE <- (L.trm + L.pen)[1:K.max] %*% d.k[1:K.max])
> (BE.S <- L.tot[1:K.max] %*% d.k.S[1:K.max]) # w/ shock
         [,1]
[1,] 405218.6
> (SCR.int <- BE.S - BE)
         [,1]
[1,] 71753.45
> # SCR Equity:
> (SCR.eqt <- S.eqt.1*A.eqt.1 + S.eqt.2*A.eqt.2)
[1] 32040
>
> # SCR Property:
> (SCR.prp <- S.prp*A.prp)
[1] 7500
>
> # Spread Computations:
> (SCR.spr <- ((1 - S.spr.1)*B.1.share +
```

```
+                 (1 - S.spr.2)*B.2.share)*spr.share*A)
[1] 13950
>
> # SCR Currency:
> (SCR.fx <- S.fx*A*eqt.share*(1 - port.eqt[1]*(1 - eqt.to.fx[1])))
[1] 18000
>
> # SCR Market: Standard formula -  Market Risk sub-modules:
> SCR.mkt.mod <- c(SCR.int,SCR.eqt,SCR.prp,SCR.spr,SCR.fx)
> (SCR.mark <- sqrt(t(SCR.mkt.mod) %*% SCR.mkt.cor %*% SCR.mkt.mod))
         [,1]
[1,] 113361.2
>
> # SCR Basic Operations (Basic Own Funds)
> BSCR.vector <- c(SCR.liab,SCR.mark)
> #BSCR.corr <- matrix(c(1,corr.lima,corr.lima,1),nrow=2)
> (BSCR <- sqrt(t(BSCR.vector) %*% BSCR.cor %*% BSCR.vector))
         [,1]
[1,] 133712.4
>
>
> proc.time()
   user   system elapsed
  0.320   0.021   0.309
```

APPENDIX C

Partial internal models - R-scripts

The following R-scripts for our select group of partial internal models show how these models are constructed to compute the solvency capital requirement (SCR).

RUN examples for these scripts can be found in Appendix D. These RUN examples will show the actual computed SCR values.

C.1   Interest rate risk

```
####################################################################
# Temporary run file for the FX simulation                        #
####################################################################

rm(list=ls(all=T))                              # clear workspace

# Initialisation:
## Solvency II:
scr.e <- .005          # Solvency Capital Required - risk level
mcr.e <- .15           # Solvency Capital Required - risk level
## General model input:
y <- 150                               # No. of years simulated
sp <- 1                  # No. of sub periods simulated per year
m <- 100000         # No. of iterations simulated per time step
T = y*sp                            # No. of time steps simulated
## Vesicek
a.v <- .75
## Black--Karasinski:
a.bk <- .75    #c(.3,-.1,.6,.6,.5)

## Loading data
#- Market information - time series:
file.ts <- "EQT_FX_IR_Input_20220817.txt"
TS <- read.table(file.ts,header=T)
eur.12m = TS$EUR012M_Index/100
n.TS = length(eur.12m)
(exp.eur.12m = mean(eur.12m))
eur.12m.ret = eur.12m[2:n.TS] / eur.12m[1:(n.TS-1)]
```

```
(sd.eur.12m = sd(eur.12m.ret))
(xi.ts <- exp.eur.12m)
(spt.ts <- eur.12m[n.TS])
(sig.ts <- sd.eur.12m)
#- EIOPA interest rate assumptions Norway
file.rf <- "EIOPA_rf_curves_no_VA.txt"
RF <- read.table(file.rf,header=T,row.names=1)
rf.EIOPA.spt <- RF$Spot_curve
rf.EIOPA.shock <- RF$Shocked_curve_down
n.RF = length(rf.EIOPA.spt)
spt.rf.adj <- -.005
(spt.rf = rf.EIOPA.spt[1] + spt.rf.adj)
(xi.rf = rf.EIOPA.spt[n.RF])
(min.rf = min(rf.EIOPA.spt))
(sig.rf = sd.eur.12m)                          # Proxy
sig.x.rf = sig.rf / sqrt(1 - a.bk^2)        # Black--Karasinski
K = length(xi.rf)    # ???

## Data containers
#R.rf <- array(numeric(),c(m,T+1,K))           # Univariate return
R.rf <- matrix(numeric(m*(T+1)),nrow=m)          # Univariate return
P.rf <- matrix(numeric(m*(T+1)),nrow=m)          # Univariate return
R.rf[,1] <- spt.rf
P.rf[,1] <- 1
#R.rf[,1,] <- spt.rf

## Simulation interations (m)
for (i in 1:m)
{
  X = log(spt.rf/xi.rf) + sig.x.rf^2/2    # Observed initial value
  for (t in 2:(T+1))
  {
    # Marginal distribution variation
    eps <- rnorm(1)
    # AR(1) driver process - Vasicek
#    X = a.v*X + sig.rf*eps
    # AR(1) driver process - Black--Karasinski
    X = a.bk*X + sig.rf*eps
#    X = a.bk*X + U.chol
    # Log-normal returns, AR(1) - Black--Karasinski
    R.rf[i,t] = xi.rf*exp(-sig.x.rf^2/2 + X)
    P.rf[i,t] = P.rf[i,t-1]*(1 + R.rf[i,t])
  }
}

# Preparing SCR results (for plotting):
R.rf.avg = apply(R.rf,2,mean)
R.rf.srt = apply(R.rf,2,sort)
R.rf.005 = R.rf.srt[scr.e*m,]
#R.rf.mcr = R.rf.srt[mcr.e*m,]
```

```
R.rf.05 = R.rf.srt[.05*m,]
R.rf.95 = R.rf.srt[.95*m,]

# TEST - plotting
#pdf("RF_rates.pdf")
#  matplot(t(R.rf[,,1]),type = "l",main="Risk-free rates")
#dev.off()

## Plotting some graphs
#- SCR risk-free rate 150 years simulated vs. EIOPA projection
pdf("SCR_RF_150y.pdf")
  plot(1:(T+1),R.rf.95,type="l",col="green",
        main="Partial Internal Model - Risk-free interest rate (PIM.ir)",
        xlab="Time (years)",ylab="Risk-free interest rate (EIOPA)",
        ylim=c(0,.09))
  lines(1:(T+1),R.rf.avg,col="blue")
  lines(1:(T+1),R.rf.05,col="orange")
  #lines(1:(T+1),R.rf.mcr,col="orange")
  lines(1:(T+1),R.rf.005,col="red")
  lines(2:(T+1),RF$Spot_curve,col="black")
  lines(2:(T+1),RF$Shocked_curve_down,lty=2,col="black")
  legend("topleft", lty=c(1,1,1,1,1,2),
          col=c("green","blue","orange","red","black","black"),
          legend=c("95-percentile",
                    "Avg. risk-free rate",
                    "5-percentile",
                    "0.5-percentile",
                    "EIOPA risk-free",
                    "EIOPA shocked"))
dev.off()

#- Euribor 12m:
pdf("Euribor_12m_Historical.pdf")
  plot(as.Date(TS$Dates),TS$EUR012M_Index,type="l",col="red",
        main="Euribor 12m - Historical interest rates",
        xlab="Time (years)",ylab="Euribor 12 Index (%)",
        ylim=c(min(TS$EUR012M_Index),max(TS$EUR012M_Index))*1.1)
  lines(as.Date(TS$Dates),rep(spt.rf*100,n.TS),
          col="black",lty=2,lwd=2)
  lines(as.Date(TS$Dates),rep(xi.rf*100,n.TS),
          col="blue",lty=1,lwd=2)
  lines(as.Date(TS$Dates),rep((min.rf+spt.rf.adj)*100,n.TS),
          col="orange",lty=1,lwd=2)
  lines(as.Date(TS$Dates),rep(0,n.TS),col="black") #,lty=1,lwd=1)
  legend("topright", lty=c(1,2,1,1),
          col=c("red","black","blue","orange"),
          legend=c("Euribor 12m",
                    "EIPOA risk-free curve (Y=1)",
                    "EIOPA risk-free curve (Y=150)",
                    "EIOPA risk-free curve minimum"))
```

```
dev.off()

# Table input:
R.rf.avg[c(2,3,4,11,51,151)]*100
R.rf.005[c(2,3,4,11,51,151)]*100
R.rf.05[c(2,3,4,11,51,151)]*100
R.rf.95[c(2,3,4,11,51,151)]*100


# The Best Estimate (BE) stressed at the 0.5-percentile:
## We compute the BE for each iteration of the simulation
## and stores the result.
#- Load data:
file.tot <- "../Thesis_R_SII_sf/L_tot.txt"
Liab.tot <- as.matrix(read.table(file.tot,header=T,row.names=1))
#Liab.tot = Liab.pen + Liab.trm
colnames(Liab.tot) <- "Liability_tot"
max.l = length(Liab.tot)

# Best Estimate from EIOPA SII risk-free curve spot
d.rf = 1/(1 + rf.EIOPA.spt[1:(max.l)])^{1:max.l}
(BE.EIOPA = t(Liab.tot) %*% d.rf)
# Best Estimate from EIOPA SII risk-free curve shocked - down
d.rf.s = 1/(1 + rf.EIOPA.shock[1:(max.l)])^{1:max.l}
(BE.s.EIOPA = t(Liab.tot) %*% d.rf.s)

# SII SCR:
(SCR.int.EIOPA = BE.s.EIOPA - BE.EIOPA)

# Simulated BE estimates (discounted cash flows)
d.R.rf = 1/P.rf[,2:(length(Liab.tot)+1)]
length(Liab.tot)
dim(d.R.rf)
BE.sim = d.R.rf %*% Liab.tot
BE.sim.srt = sort(BE.sim)
BE.sim.srt[(1 - scr.e)*m]
(SCR.ir.sim = BE.sim.srt[(1 - scr.e)*m] - BE.EIOPA)
```

C.2 Equity risk

```
####################################################################
#                 Partial Internal Model -- EQUITY              #
####################################################################

rm(list=ls(all=T))                                # clear workspace

# Libraries:
source("Asset_Input.r")
source("ParamEst.r")
source("PosDefTest.r")

# Initialisation:
## Solvency II:
scr.e <- .005          # Solvency Capital Required - risk level
mcr.e <- .15           # Solvency Capital Required - risk level
## Model parameters - General input:
y <- 5                             # No. of years simulated
sp <- 1                  # No. of sub periods simulated per year
m <- 100000         # No. of iterations simulated per time step
T = y*sp                           # No. of time steps simulated

#- Loading data
#-- Time series
filePathName <- "EQT_FX_IR_Input_20220817.txt"
TS <- read.table(filePathName,header=T)   #,row.names=1)
(m.TS = dim(TS)[1])
(n.TS = dim(TS)[2])
TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100
#-- EQT holdings:
#- Plotting EQT time series
pdf("EQT_Indices.pdf")
  par(mfrow=c(3,2))
  plot(as.Date(TS$Dates),TS[,2],type="l",col="red",
      main="OSEBX Index",xlab="Year",ylab="Index value")
  plot(as.Date(TS$Dates),TS[,3],type="l",col="red",
      main="OMX Index",xlab="Year",ylab="Index value")
  plot(as.Date(TS$Dates),TS[,4],type="l",col="red",
      main="UKX Index",xlab="Year",ylab="Index value")
  plot(as.Date(TS$Dates),TS[,5],type="l",col="red",
      main="SXXP Index",xlab="Year",ylab="Index value")
  plot(as.Date(TS$Dates),TS[,6],type="l",col="red",
      main="SPX Index",xlab="Year",ylab="Index value")
  plot(as.Date(TS$Dates),TS[,7],type="l",col="red",
      main="NKY Index",xlab="Year",ylab="Index value")
dev.off()

#- Parameter estimation - Stochastic model
m.par = ParamEst(TS[,2:n.TS])
```

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```
#- Model set-up - market information:
(xi.eqt <- m.par$exp.ret[1:6])                      # Expectations
(sig.eqt <- m.par$sd.ret[1:6])                      # Volatilities
##- Model parameters
J = length(xi.eqt)
val.eqt <- A*eqt.share*port.eqt          # Initial equity values

# Correlation matrix:
cor.mat.eqt = m.par$cor.ret[1:6,1:6]
# Positive definite correlation matrix
(cor.mat.eqt <- PosDefTest(as.matrix(cor.mat.eqt)))
# Covariance matrix:
cov.mat.eqt = diag(sig.eqt) %*%
              cor.mat.eqt %*%
              diag(sig.eqt)

## Cholesky decomposition:
C = t(chol(cov.mat.eqt))

##- Data containers
R.eqt <- array(numeric(),c(m,T,J))              # Univariate return
P.R.eqt <- array(numeric(),c(m,T,J))            # Portfolio return
P.R <- matrix(rep(0,m*T),nrow=m)        # Portfolio absolute return

### Simulation:
for (i in 1:m)                                  # m iterations
{
  ## Marginal distribution variateion
  eta <- rnorm(J)
  ## Cholesky decomposition applied
  R.eqt[i,1,] = exp(xi.eqt + C %*% eta)
  for (t in 2:T)
  {
    ## Marginal distribution variateion
    eta <- rnorm(J)
    ## Cholesky decomposition applied
    R.eqt[i,t,] = R.eqt[i,t-1,] * exp(xi.eqt + C %*% eta)
  }
}

## Portfolio absolute return
for (j in 1:J)
{
  P.R = P.R + (R.eqt[,,j] - 1)*val.eqt[j]
}

# SCR and other percentiles: sorting ascending
P.R.srt = apply(P.R,2,sort)
(SCR = P.R.srt[m*scr.e,1:t])
```

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```
(MCR = P.R.srt[m*mcr.e,1:t])
(R.avg = apply(P.R,2,mean))
(R.85 = P.R.srt[m*(1 - mcr.e),1:t])
(R.995 = P.R.srt[m*(1 - scr.e),1:t])
#(R.max = P.R.srt[m,1:t])

## Plotting some graphs
pdf("SCR_EQT_5y.pdf")
  plot(1:T,R.995,type="l",col="green",
       main="Partial Internal Model - Equity (PIM.eqt)",
       ylab="Change in Equity value from Equity exposures",
       xlab="Time (years)",
       ylim=c(min(SCR)*1.1, max(R.995)*1.1))
  lines(1:T,R.avg,col="blue")
  lines(1:T,MCR,col="orange")
  lines(1:T,SCR,col="red")
  legend("topleft", lty=c(1,1,1,1),
         col=c("green","blue","orange","red"),
         legend=c("99.5-percentile",
                  "Mean",
                  "15-percentile",
                  "0.5-percentile"))
dev.off()
```

C.3 Currency risk

```
####################################################################
#               Partial Internal Model -- CURRENCY               #
####################################################################

rm(list=ls(all=T))                              # clear workspace

# Libraries:
source("Asset_Input.r")
source("ParamEst.r")
source("PosDefTest.r")

# Initialisation:
## Solvency II:
scr.e <- .005          # Solvency Capital Required - risk level
mcr.e <- .15           # Solvency Capital Required - risk level
## General input:
y <- 5                               # No. of years simulated
sp <- 1                   # No. of sub periods simulated per year
m <- 100000          # No. of iterations simulated per time step
T = y*sp                           # No. of time steps simulated
### Black--Karasinski:
a.bk <- c(.7,.8,.9,.9,.95)

#- Loading data
#-- Time series
filePathName <- "EQT_FX_IR_Input_20220817.txt"
#filePathName2 <- "../../MT_R_Input/EQT_FX_IR_Input_20220817.txt"
#-- EQT holdings:
port.shares.eqt <- read.table("Portf_Eqt.txt",
                               header=T,row.names=1)
TS <- read.table(filePathName,header=T)   #,row.names=1)
(m.TS = dim(TS)[1])
(n.TS = dim(TS)[2])
TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100

#- Plotting EQT time series
pdf("FX_Rates.pdf")
  par(mfrow=c(3,2))
  plot(as.Date(TS$Dates),TS[,8],type="l",col="red",
      main="SEKNOK",xlab="Year",ylab="FX rate")
  plot(as.Date(TS$Dates),TS[,9],type="l",col="red",
      main="GBPNOK",xlab="Year",ylab="FX rate")
  plot(as.Date(TS$Dates),TS[,10],type="l",col="red",
      main="EURNOK",xlab="Year",ylab="FX rate")
  plot(as.Date(TS$Dates),TS[,11],type="l",col="red",
      main="USDNOK",xlab="Year",ylab="FX rate")
  plot(as.Date(TS$Dates),TS[,12],type="l",col="red",
      main="JPYNOK",xlab="Year",ylab="FX rate")
```

```
   plot(as.Date(TS$Dates),TS[,13],type="l",col="red",
        main="CNYNOK",xlab="Year",ylab="FX rate")
dev.off()

#- Model set-up:
#- Parameter estimation - Stochastic model
#-- EQT
val.eqt <- A*eqt.share*port.eqt                 # Initial values
#-- FX
m.par = ParamEst(TS[,2:n.TS])
(spt.fx <- m.par$spot[7:11])                      # Spot rates
(xi.fx <- m.par$exp.act[7:11])                  # Expected rates
(sig.fx <- m.par$sd.ret[7:11])                    # Volatilities
sig.x.fx = sig.fx / sqrt(1 - a.bk^2)        # Black--Karasinski
cor.mat.fx <- m.par$cor.ret[7:11,7:11]
### Currency exposures: -- val.eqt[1] = NOK
fx.expo.bccy <- val.eqt[2:length(port.eqt)]        # in base ccy
fx.expo.fccy <- fx.expo.bccy/spt.fx          # in foreing ccy

# Positive definite correlation matrix
(cor.mat.fx <- PosDefTest(as.matrix(cor.mat.fx)))
# Covariance matrix:
cov.mat.fx = diag(sig.fx) %*%
                cor.mat.fx %*%
                diag(sig.fx)

# Set-up:
## Model parameters
(K = length(xi.fx))
## Data containers
R.fx <- array(numeric(),c(m,T,K))             # Univariate return
P.R.fx <- array(numeric(),c(m,T,K))          # Portfolio returns
P.R <- matrix(rep(0,m*T),nrow=m)      # Aggregate absolute return
FX.test <- array(numeric(),c(m,T+1,K))                  # FX rates
for (k in 1:K) {FX.test[,1,k] <- spt.fx[k] }

# Cholesky decomposition
C = t(chol(cov.mat.fx))

# Simulation:
## Simulation interations (m)
for (i in 1:m)
{
  X = log(spt.fx/xi.fx) + sig.x.fx^2/2    # Observed initial value
  for (t in 1:T)
  {
    # Marginal distribution variation
    eta <- rnorm(K)
    # Cholesky decomposition applied
    U.chol = C %*% eta
```

```
    # AR(1) driver process - Black--Karasinski
    X = a.bk*X + U.chol
    # Log-normal returns, AR(1) - Black--Karasinski
    R.fx[i,t,] = (xi.fx*exp(-sig.x.fx^2/2 + X))/spt.fx
    # Xtra: Returns FX rates
    FX.test[i,t+1,] = xi.fx*exp(-sig.x.fx^2/2 + X)
  }
}

## Portfolio absolute return
#- Absolute return per univariate
for (k in 1:K)
{
  # The following only considers FX exposure, with EQT static!
  # (Not considering the multiplicative effect: EQT*FX)
  P.R = P.R + (R.fx[,,k] - 1)*fx.expo.bccy[k]
}

# SCR
P.R.srt = apply(P.R,2,sort)
(SCR = P.R.srt[m*scr.e,1:t])
(MCR = P.R.srt[m*mcr.e,1:t])
(R.avg = apply(P.R,2,mean))
#(R.85 = P.R.srt[m*(1 - mcr.e),1:t])
(R.995 = P.R.srt[m*(1 - scr.e),1:t])

#- Plotting SCR graph:
pdf("SCR_FX_5y.pdf")
  plot(1:T,R.995,type="l",col="green",
       main="Partial Internal Model - Currency (PIM.ccy)",
       ylab="Change in Equity value from Currency exposures",
       #main="SCR Currency",
       xlab="Time (years)",
       #ylab="FX exposure",
       ylim=c(min(SCR)*1.1, max(R.995)*1.1))
  lines(1:T,R.avg,col="blue")
  lines(1:T,MCR,col="orange")
  lines(1:T,SCR,col="red")
  legend("topleft", lty=c(1,1,1,1),
         #col=c("green","blue","red"),
         col=c("green","blue","orange","red"),
         legend=c("99.5-percentile",
                  "Mean",
                  "15-percentile",
                  "0.5-percentile"))
dev.off()

#- TEST - plotting - only a few iterations!
if (m <= 100)
{
```

```r
  FX.test.avg <- matrix(rep(0,K*(T+1)),nrow=K)    # Data container TEST
  for (k in 1:K) { FX.test.avg[k,] = apply(FX.test[,,k],2,mean) }
pdf("FX_rates_test.pdf")
  par(mfrow=c(3,2))
  matplot(t(FX.test[,,1]),type = "l",main="SEKNOK")
  lines(1:(T+1),rep(spt.fx[1],(T+1)),col="black",lty=3,lwd=2)
  lines(1:(T+1),rep(xi.fx[1],(T+1)),col="black",lty=2,lwd=2)
  lines(1:(T+1),FX.test.avg[1,],col="black",lty=4,lwd=3)
  matplot(t(FX.test[,,2]),type = "l",main="GBPNOK")
  lines(1:(T+1),rep(spt.fx[2],(T+1)),col="black",lty=3,lwd=2)
  lines(1:(T+1),rep(xi.fx[2],(T+1)),col="black",lty=2,lwd=1)
  lines(1:(T+1),FX.test.avg[2,],col="black",lty=4,lwd=3)
  matplot(t(FX.test[,,3]),type = "l",main="EURNOK")
  lines(1:(T+1),rep(spt.fx[3],(T+1)),col="black",lty=3,lwd=2)
  lines(1:(T+1),rep(xi.fx[3],(T+1)),col="black",lty=2,lwd=1)
  lines(1:(T+1),FX.test.avg[3,],col="black",lty=4,lwd=3)
  matplot(t(FX.test[,,4]),type = "l",main="USDNOK")
  lines(1:(T+1),rep(spt.fx[4],(T+1)),col="black",lty=3,lwd=2)
  lines(1:(T+1),rep(xi.fx[4],(T+1)),col="black",lty=2,lwd=1)
  lines(1:(T+1),FX.test.avg[4,],col="black",lty=4,lwd=3)
  matplot(t(FX.test[,,5]),type = "l",main="JPYNOK")
  lines(1:(T+1),rep(spt.fx[5],(T+1)),col="black",lty=3,lwd=2)
  lines(1:(T+1),rep(xi.fx[5],(T+1)),col="black",lty=2,lwd=1)
  lines(1:(T+1),FX.test.avg[5,],col="black",lty=4,lwd=3)
  #mtext("My 'Title' in a strange place", side = 3, line = -2, outer = TRUE)
dev.off()

pdf("FX_returns_test.pdf")
  par(mfrow=c(3,2))
  matplot(t(R.fx[,,1]),type = "l",main="SEKNOK")
  matplot(t(R.fx[,,2]),type = "l",main="GBPNOK")
  matplot(t(R.fx[,,3]),type = "l",main="EURNOK")
  matplot(t(R.fx[,,4]),type = "l",main="USDNOK")
  matplot(t(R.fx[,,5]),type = "l",main="JPYNOK")
  #mtext("My 'Title' in a strange place", side = 3, line = -2, outer = TRUE)
dev.off()
}
```

C.4 Combined equity and currency risk

```
####################################################################
#          Partial Internal Model -- EQUITY & CURRENCY        #
####################################################################

rm(list=ls(all=T))                              # clear workspace

# Libraries:
source("Asset_Input.r")
source("ParamEst.r")
source("PosDefTest.r")

# Initialisation:
## Solvency II:
scr.e <- .005          # Solvency Capital Required - risk level
mcr.e <- .15           # Solvency Capital Required - risk level
## General input:
y <- 5                               # No. of years simulated
sp <- 1                # No. of sub periods simulated per year
m <- 100000          # No. of iterations simulated per time step
T = y*sp                             # No. of time steps simulated
### Black--Karasinski:
a.bk <- c(.7,.8,.9,.9,.95)

#- Loading data
#-- Time series
filePathName <- "EQT_FX_IR_Input_20220817.txt"
TS <- read.table(filePathName,header=T)   #,row.names=1)
TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100

#- Parameter estimation - Stochastic model
m.par = ParamEst(TS[,2:dim(TS)[2]])

#- Model set-up:
#-- Model parameters
#--- EQT
xi.eqt <- m.par$exp.ret[1:6]                      # Expectations
val.eqt <- A*eqt.share*port.eqt              # Initial values
#--- FX
spt.fx <- m.par$spot[7:11]                     # Spot FX rates
#xi.fx <- m.par$exp.ret[7:11]                    # Expectations
xi.fx <- m.par$exp.act[7:11]                  # Expected rates
sig.fx <- m.par$sd.ret[7:11]                  # Volatilities FX
sig.x.fx = sig.fx / sqrt(1 - a.bk^2)      # Black--Karasinski
#--- EQT & FX
sig.eqt.fx <- m.par$sd.ret[1:11]         # Volatilities EQT & FX
cor.mat.eqt.fx <- m.par$cor.ret[1:11,1:11]
J = length(xi.eqt)
K = length(xi.fx)
```

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```
## Currency exposures: -- val.eqt[1] = NOK
fx.expo.bccy <- val.eqt[2:length(port.eqt)]          # in base ccy
fx.expo.fccy <- fx.expo.bccy/spt.fx                  # in foreing ccy

#-- Positive definite correlation matrix
(cor.mat.eqt.fx <- PosDefTest(as.matrix(cor.mat.eqt.fx)))
#-- Covariance matrix:
cov.mat.eqt.fx = diag(sig.eqt.fx) %*%
                 cor.mat.eqt.fx %*%
                 diag(sig.eqt.fx)
## Cholesky decomposition
C = t(chol(cov.mat.eqt.fx))

##- Data containers
#- Univariate, dependent asset returns
R.eqt <- array(numeric(),c(m,T,J))                          # EQT
R.fx <- array(numeric(),c(m,T,K+1))              # FX (+1 = NOK)
#R.fx[,,1] <- 1                                          # NOK
R.fx <- array(numeric(),c(m,T,K))                        # FX
#- Protfolio returns
P.R.eqt.fx <- matrix(rep(0,m*T),nrow=m)             # EQT & FX

#- Simulation:
#-- Driver process:
#-- Simulation proper - m iterations:
for (i in 1:m)
{
  #-- Driver process:
  X = log(spt.fx/xi.fx) + sig.x.fx^2/2    # Observed initial value
  ## First time step
  t <- 1
  ## Marginal distribution variateion
  eta <- rnorm(J+K)
  ## Cholesky decomposition applied
  U.chol = C %*% eta
  ## Log-normal returns
  R.eqt[i,t,] = exp(xi.eqt + U.chol[1:J])              # EQT ret
  X = a.bk*X + U.chol[(J+1):(J+K)]              # Driver updated
  R.fx[i,t,] = xi.fx*exp(-sig.x.fx^2/2 + X)/spt.fx     # FX ret
  # Consecutive time steps:
  for (t in 2:T)
  {
    ## Marginal distribution variateion
    eta <- rnorm((J+K))
    ## Cholesky decomposition applied
    U.chol = C %*% eta
    ## Log-normal returns
    R.eqt[i,t,] = R.eqt[i,t-1,]*exp(xi.eqt + U.chol[1:J])  # EQT
    X = a.bk*X + U.chol[(J+1):(J+K)]                 # Driver FX
    #!!!! NOT - 1 !!!!
```

```
    R.fx[i,t,] = xi.fx*exp(-sig.x.fx^2/2 + X)/spt.fx          # FX
  }
}
FX.1 <- array(rep(1,m*T),c(m,T))                              # NOK
R.fx = array(append(FX.1,R.fx),c(m,T,K))           # full FX array

#- Aggregated portfolio returns:
for (k in 1:K)
{
  # The multipliticative nature of currency exposure
  P.R.eqt.fx =  (R.eqt[,,k] - 1)*(R.fx[,,k] - 1)*fx.expo.bccy[k]
}

# SCR and other percentiles: sorting ascending
P.R.eqt.fx.srt = apply(P.R.eqt.fx,2,sort)
(SCR = P.R.eqt.fx.srt[m*scr.e,1:t])
(MCR = P.R.eqt.fx.srt[m*mcr.e,1:t])
(R.avg = apply(P.R.eqt.fx,2,mean))
#(R.85 = P.R.srt[m*(1 - mcr.e),1:t])
(R.995 = P.R.eqt.fx.srt[m*(1 - scr.e),1:t])

## Plotting some graphs
pdf("SCR_EQT_FX_5y.pdf")
#  #par(mfrow=c(1,2))
  plot(1:T,R.995,type="l",col="green",
       main="Partial Internal Model - Equity and Currency (PIM.eqt.ccy)",
       ylab="Change in Equity value from Equity and Currency exposures",
       #main="SCR Equity and Currency (multiplicative)",
       xlab="Time (years)",
       #ylab="Portfolio value",
       ylim=c(min(SCR)*1.1, max(R.995)*1.1))
  #lines(1:T,R.85,col="green")
  lines(1:T,R.avg,col="blue")
  lines(1:T,MCR,col="orange")
  lines(1:T,SCR,col="red")
  legend("topleft", lty=c(1,1,1,1),
         #col=c("green","blue","red"),
         col=c("green","blue","orange","red"),
         #col=c("green","green","blue","orange","red"),
         legend=c("99.5-percentile",
                  "Mean",
                  "15-percentile",
                  "0.5-percentile"))
dev.off()
```

Partial internal models - SCR computations

The following RUN examples show the actual computed SCR values for the partial internal models described in Chapter 4, and for which the R-scripts can be found in Appendix C.

D.1 Interest rate risk - run example

```
R version 3.4.4 (2018-03-15) -- "Someone to Lean On"
Copyright (C) 2018 The R Foundation for Statistical Computing
Platform: x86_64-pc-linux-gnu (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> ##############################################################
> # Temporary run file for the FX simulation                  #
> ##############################################################
>
> rm(list=ls(all=T))                           # clear workspace
>
> # Initialisation:
```

```
> ## Solvency II:
> scr.e <- .005          # Solvency Capital Required - risk level
> mcr.e <- .15           # Solvency Capital Required - risk level
> ## General model input:
> y <- 150                              # No. of years simulated
> sp <- 1                  # No. of sub periods simulated per year
> m <- 100000          # No. of iterations simulated per time step
> T = y*sp                              # No. of time steps simulated
> ## Vesicek
> a.v <- .75
> ## Black--Karasinski:
> a.bk <- .75    #c(.3,-.1,.6,.6,.5)
>
> ## Loading data
> #- Market information - time series:
> file.ts <- "EQT_FX_IR_Input_20220817.txt"
> TS <- read.table(file.ts,header=T)
> eur.12m = TS$EUR012M_Index/100
> n.TS = length(eur.12m)
> (exp.eur.12m = mean(eur.12m))
[1] 0.01681691
> eur.12m.ret = eur.12m[2:n.TS] / eur.12m[1:(n.TS-1)]
> (sd.eur.12m = sd(eur.12m.ret))
[1] 0.1347448
> (xi.ts <- exp.eur.12m)
[1] 0.01681691
> (spt.ts <- eur.12m[n.TS])
[1] 0.01146
> (sig.ts <- sd.eur.12m)
[1] 0.1347448
> #- EIOPA interest rate assumptions Norway
> file.rf <- "EIOPA_rf_curves_no_VA.txt"
> RF <- read.table(file.rf,header=T,row.names=1)
> rf.EIOPA.spt <- RF$Spot_curve
> rf.EIOPA.shock <- RF$Shocked_curve_down
> n.RF = length(rf.EIOPA.spt)
> spt.rf.adj <- -.005
> (spt.rf = rf.EIOPA.spt[1] + spt.rf.adj)
[1] 0.02615
> (xi.rf = rf.EIOPA.spt[n.RF])
[1] 0.03368
> (min.rf = min(rf.EIOPA.spt))
[1] 0.02834
> (sig.rf = sd.eur.12m)                          # Proxy
[1] 0.1347448
> sig.x.rf = sig.rf / sqrt(1 - a.bk^2)          # Black--Karasinski
> K = length(xi.rf)    # ???
>
> ## Data containers
> #R.rf <- array(numeric(),c(m,T+1,K))           # Univariate return
```

```
> R.rf <- matrix(numeric(m*(T+1)),nrow=m)              # Univariate return
> P.rf <- matrix(numeric(m*(T+1)),nrow=m)              # Univariate return
> R.rf[,1] <- spt.rf
> P.rf[,1] <- 1
> #R.rf[,1,] <- spt.rf
>
> ## Simulation interations (m)
> for (i in 1:m)
+ {
+   X = log(spt.rf/xi.rf) + sig.x.rf^2/2     # Observed initial value
+   for (t in 2:(T+1))
+   {
+     # Marginal distribution variation
+     eps <- rnorm(1)
+     # AR(1) driver process - Vasicek
+ #     X = a.v*X + sig.rf*eps
+     # AR(1) driver process - Black--Karasinski
+     X = a.bk*X + sig.rf*eps
+ #     X = a.bk*X + U.chol
+     # Log-normal returns, AR(1) - Black--Karasinski
+     R.rf[i,t] = xi.rf*exp(-sig.x.rf^2/2 + X)
+     P.rf[i,t] = P.rf[i,t-1]*(1 + R.rf[i,t])
+   }
+ }
>
> # Preparing SCR results (for plotting):
> R.rf.avg = apply(R.rf,2,mean)
> R.rf.srt = apply(R.rf,2,sort)
> R.rf.005 = R.rf.srt[scr.e*m,]
> #R.rf.mcr = R.rf.srt[mcr.e*m,]
> R.rf.05 = R.rf.srt[.05*m,]
> R.rf.95 = R.rf.srt[.95*m,]
>
> # TEST - plotting
> #pdf("RF_rates.pdf")
> #  matplot(t(R.rf[,,1]),type = "l",main="Risk-free rates")
> #dev.off()
>
> ## Plotting some graphs
> #- SCR risk-free rate 150 years simulated vs. EIOPA projection
> pdf("SCR_RF_150y.pdf")
>   plot(1:(T+1),R.rf.95,type="l",col="green",
+        main="Partial Internal Model - Risk-free interest rate (PIM.ir)",
+        xlab="Time (years)",ylab="Risk-free interest rate (EIOPA)",
+        ylim=c(0,.09))
>   lines(1:(T+1),R.rf.avg,col="blue")
>   lines(1:(T+1),R.rf.05,col="orange")
>   #lines(1:(T+1),R.rf.mcr,col="orange")
>   lines(1:(T+1),R.rf.005,col="red")
>   lines(2:(T+1),RF$Spot_curve,col="black")
```

```
>   lines(2:(T+1),RF$Shocked_curve_down,lty=2,col="black")
>   legend("topleft", lty=c(1,1,1,1,1,2),
+          col=c("green","blue","orange","red","black","black"),
+          legend=c("95-percentile",
+                   "Avg. risk-free rate",
+                   "5-percentile",
+                   "0.5-percentile",
+                   "EIOPA risk-free",
+                   "EIOPA shocked"))
> dev.off()
null device
          1
>
> #- Euribor 12m:
> pdf("Euribor_12m_Historical.pdf")
>   plot(as.Date(TS$Dates),TS$EUR012M_Index,type="l",col="red",
+        main="Euribor 12m - Historical interest rates",
+        xlab="Time (years)",ylab="Euribor 12 Index (%)",
+        ylim=c(min(TS$EUR012M_Index),max(TS$EUR012M_Index))*1.1)
>   lines(as.Date(TS$Dates),rep(spt.rf*100,n.TS),
+         col="black",lty=2,lwd=2)
>   lines(as.Date(TS$Dates),rep(xi.rf*100,n.TS),
+         col="blue",lty=1,lwd=2)
>   lines(as.Date(TS$Dates),rep((min.rf+spt.rf.adj)*100,n.TS),
+         col="orange",lty=1,lwd=2)
>   lines(as.Date(TS$Dates),rep(0,n.TS),col="black") #,lty=1,lwd=1)
>   legend("topright", lty=c(1,2,1,1),
+          col=c("red","black","blue","orange"),
+          legend=c("Euribor 12m",
+                   "EIPOA risk-free curve (Y=1)",
+                   "EIOPA risk-free curve (Y=150)",
+                   "EIOPA risk-free curve minimum"))
> dev.off()
null device
          1
>
> # Table input:
> R.rf.avg[c(2,3,4,11,51,151)]*100
[1] 2.796326 2.935686 3.041749 3.323457 3.366714 3.366289
> R.rf.005[c(2,3,4,11,51,151)]*100
[1] 1.957076 1.875131 1.851797 1.920517 1.953613 1.946488
> R.rf.05[c(2,3,4,11,51,151)]*100
[1] 2.216245 2.194571 2.207847 2.332693 2.354915 2.356539
> R.rf.95[c(2,3,4,11,51,151)]*100
[1] 3.460539 3.821269 4.058743 4.544620 4.614868 4.600296
>
>
> # The Best Estimate (BE) stressed at the 0.5-percentile:
> ## We compute the BE for each iteration of the simulation
> ## and stores the result.
```

```
> #- Load data:
> file.tot <- "../Thesis_R_SII_sf/L_tot.txt"
> Liab.tot <- as.matrix(read.table(file.tot,header=T,row.names=1))
> #Liab.tot = Liab.pen + Liab.trm
> colnames(Liab.tot) <- "Liability_tot"
> max.l = length(Liab.tot)
>
> # Best Estimate from EIOPA SII risk-free curve spot
> d.rf = 1/(1 + rf.EIOPA.spt[1:(max.l)])^{1:max.l}
> (BE.EIOPA = t(Liab.tot) %*% d.rf)
                  [,1]
Liability_tot 333465.2
> # Best Estimate from EIOPA SII risk-free curve shocked - down
> d.rf.s = 1/(1 + rf.EIOPA.shock[1:(max.l)])^{1:max.l}
> (BE.s.EIOPA = t(Liab.tot) %*% d.rf.s)
                  [,1]
Liability_tot 405218.6
>
> # SII SCR:
> (SCR.int.EIOPA = BE.s.EIOPA - BE.EIOPA)
                  [,1]
Liability_tot 71753.45
>
> # Simulated BE estimates (discounted cash flows)
> d.R.rf = 1/P.rf[,2:(length(Liab.tot)+1)]
> length(Liab.tot)
[1] 65
> dim(d.R.rf)
[1] 100000     65
> BE.sim = d.R.rf %*% Liab.tot
> BE.sim.srt = sort(BE.sim)
> BE.sim.srt[(1 - scr.e)*m]
[1] 370459.7
> (SCR.ir.sim = BE.sim.srt[(1 - scr.e)*m] - BE.EIOPA)
                  [,1]
Liability_tot 36994.59
>
> proc.time()
   user  system elapsed
 26.536   0.421  26.941
```

D.2   Equity risk - run example

```
R version 3.4.4 (2018-03-15) -- "Someone to Lean On"
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Platform: x86_64-pc-linux-gnu (64-bit)

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Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> ################################################################
> #              Partial Internal Model -- EQUITY               #
> ################################################################
>
> rm(list=ls(all=T))                              # clear workspace
>
> # Libraries:
> source("Asset_Input.r")
> source("ParamEst.r")
> source("PosDefTest.r")
>
> # Initialisation:
> ## Solvency II:
> scr.e <- .005         # Solvency Capital Required - risk level
> mcr.e <- .15          # Solvency Capital Required - risk level
> ## Model parameters - General input:
> y <- 5                            # No. of years simulated
> sp <- 1                # No. of sub periods simulated per year
> m <- 100000          # No. of iterations simulated per time step
> T = y*sp                         # No. of time steps simulated
>
> #- Loading data
> #-- Time series
> filePathName <- "EQT_FX_IR_Input_20220817.txt"
> TS <- read.table(filePathName,header=T)   #,row.names=1)
> (m.TS = dim(TS)[1])
[1] 5903
```

```
> (n.TS = dim(TS)[2])
[1] 14
> TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100
> #-- EQT holdings:
> #- Plotting EQT time series
> pdf("EQT_Indices.pdf")
>   par(mfrow=c(3,2))
>   plot(as.Date(TS$Dates),TS[,2],type="l",col="red",
+        main="OSEBX Index",xlab="Year",ylab="Index value")
>   plot(as.Date(TS$Dates),TS[,3],type="l",col="red",
+        main="OMX Index",xlab="Year",ylab="Index value")
>   plot(as.Date(TS$Dates),TS[,4],type="l",col="red",
+        main="UKX Index",xlab="Year",ylab="Index value")
>   plot(as.Date(TS$Dates),TS[,5],type="l",col="red",
+        main="SXXP Index",xlab="Year",ylab="Index value")
>   plot(as.Date(TS$Dates),TS[,6],type="l",col="red",
+        main="SPX Index",xlab="Year",ylab="Index value")
>   plot(as.Date(TS$Dates),TS[,7],type="l",col="red",
+        main="NKY Index",xlab="Year",ylab="Index value")
> dev.off()
null device
          1
>
> #- Parameter estimation - Stochastic model
> m.par = ParamEst(TS[,2:n.TS])
>
> #- Model set-up - market information:
> (xi.eqt <- m.par$exp.ret[1:6])                   # Expectations
OSEBX_Index   OMX_Index   UKX_Index  SXXP_Index    SPX_Index   NKY_Index
 0.10722182  0.04931923  0.02119051  0.02580511   0.06725830  0.04558064
> (sig.eqt <- m.par$sd.ret[1:6])                   # Volatilities
OSEBX_Index   OMX_Index   UKX_Index  SXXP_Index    SPX_Index   NKY_Index
  0.2204044   0.2301674   0.1876397   0.1929969    0.1971469   0.2297777
> ##- Model parameters
> J = length(xi.eqt)
> val.eqt <- A*eqt.share*port.eqt          # Initial equity values
>
> # Correlation matrix:
> cor.mat.eqt = m.par$cor.ret[1:6,1:6]
> # Positive definite correlation matrix
> (cor.mat.eqt <- PosDefTest(as.matrix(cor.mat.eqt)))
            OSEBX_Index OMX_Index UKX_Index SXXP_Index SPX_Index NKY_Index
OSEBX_Index   1.0000000 0.6874574 0.7098918  0.7421424 0.4422036 0.3247565
OMX_Index     0.6874574 1.0000000 0.7849716  0.8603476 0.5141085 0.2809940
UKX_Index     0.7098918 0.7849716 1.0000000  0.9243429 0.5402493 0.2977320
SXXP_Index    0.7421424 0.8603476 0.9243429  1.0000000 0.5791742 0.3217279
SPX_Index     0.4422036 0.5141085 0.5402493  0.5791742 1.0000000 0.1453886
NKY_Index     0.3247565 0.2809940 0.2977320  0.3217279 0.1453886 1.0000000
> # Covariance matrix:
> cov.mat.eqt = diag(sig.eqt) %*%
```

```
+                   cor.mat.eqt %*%
+                   diag(sig.eqt)
>
> ## Cholesky decomposition:
> C = t(chol(cov.mat.eqt))
>
> ##- Data containers
> R.eqt <- array(numeric(),c(m,T,J))              # Univariate return
> P.R.eqt <- array(numeric(),c(m,T,J))             # Portfolio return
> P.R <- matrix(rep(0,m*T),nrow=m)       # Portfolio absolute return
>
> ### Simulation:
> for (i in 1:m)                                   # m iterations
+ {
+   ## Marginal distribution variateion
+   eta <- rnorm(J)
+   ## Cholesky decomposition applied
+   R.eqt[i,1,] = exp(xi.eqt + C %*% eta)
+   for (t in 2:T)
+   {
+     ## Marginal distribution variateion
+     eta <- rnorm(J)
+     ## Cholesky decomposition applied
+     R.eqt[i,t,] = R.eqt[i,t-1,] * exp(xi.eqt + C %*% eta)
+   }
+ }
>
> ## Portfolio absolute return
> for (j in 1:J)
+ {
+   P.R = P.R + (R.eqt[,,j] - 1)*val.eqt[j]
+ }
>
> # SCR and other percentiles: sorting ascending
> P.R.srt = apply(P.R,2,sort)
> (SCR = P.R.srt[m*scr.e,1:t])
[1] -28356.68 -35218.58 -38759.28 -41234.06 -42737.76
> (MCR = P.R.srt[m*mcr.e,1:t])
[1] -9558.153 -9920.168 -8609.807 -6593.402 -4106.561
> (R.avg = apply(P.R,2,mean))
[1]  8141.352 17201.127 27167.737 38396.999 50667.570
> (R.85 = P.R.srt[m*(1 - mcr.e),1:t])
[1]  26067.59  44662.17  63319.97  84564.23 106625.67
> (R.995 = P.R.srt[m*(1 - scr.e),1:t])
[1]  62744.56 109032.29 159496.15 214228.94 277458.99
> #(R.max = P.R.srt[m,1:t])
>
> ## Plotting some graphs
> pdf("SCR_EQT_5y.pdf")
>   plot(1:T,R.995,type="l",col="green",
```

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```
+          main="Partial Internal Model - Equity (PIM.eqt)",
+          ylab="Change in Equity value from Equity exposures",
+          xlab="Time (years)",
+          ylim=c(min(SCR)*1.1, max(R.995)*1.1))
>   lines(1:T,R.avg,col="blue")
>   lines(1:T,MCR,col="orange")
>   lines(1:T,SCR,col="red")
>   legend("topleft", lty=c(1,1,1,1),
+          col=c("green","blue","orange","red"),
+          legend=c("99.5-percentile",
+                   "Mean",
+                   "15-percentile",
+                   "0.5-percentile"))
> dev.off()
null device
          1
>
>
>
> proc.time()
   user  system elapsed
  2.848   0.072   2.910
```

D.3 Currency risk - run example

```
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R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> #################################################################
> #                Partial Internal Model -- CURRENCY             #
> #################################################################
>
> rm(list=ls(all=T))                              # clear workspace
>
> # Libraries:
> source("Asset_Input.r")
> source("ParamEst.r")
> source("PosDefTest.r")
>
> # Initialisation:
> ## Solvency II:
> scr.e <- .005         # Solvency Capital Required - risk level
> mcr.e <- .15          # Solvency Capital Required - risk level
> ## General input:
> y <- 5                              # No. of years simulated
> sp <- 1                 # No. of sub periods simulated per year
> m <- 100000          # No. of iterations simulated per time step
> T = y*sp                            # No. of time steps simulated
> ### Black--Karasinski:
> a.bk <- c(.7,.8,.9,.9,.95)
>
> #- Loading data
> #-- Time series
> filePathName <- "EQT_FX_IR_Input_20220817.txt"
> #filePathName2 <- "../../MT_R_Input/EQT_FX_IR_Input_20220817.txt"
```

```
> #-- EQT holdings:
> port.shares.eqt <- read.table("Portf_Eqt.txt",
+                                     header=T,row.names=1)
> TS <- read.table(filePathName,header=T)    #,row.names=1)
> (m.TS = dim(TS)[1])
[1] 5903
> (n.TS = dim(TS)[2])
[1] 14
> TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100
>
> #- Plotting EQT time series
> pdf("FX_Rates.pdf")
>   par(mfrow=c(3,2))
>   plot(as.Date(TS$Dates),TS[,8],type="l",col="red",
+        main="SEKNOK",xlab="Year",ylab="FX rate")
>   plot(as.Date(TS$Dates),TS[,9],type="l",col="red",
+        main="GBPNOK",xlab="Year",ylab="FX rate")
>   plot(as.Date(TS$Dates),TS[,10],type="l",col="red",
+        main="EURNOK",xlab="Year",ylab="FX rate")
>   plot(as.Date(TS$Dates),TS[,11],type="l",col="red",
+        main="USDNOK",xlab="Year",ylab="FX rate")
>   plot(as.Date(TS$Dates),TS[,12],type="l",col="red",
+        main="JPYNOK",xlab="Year",ylab="FX rate")
>   plot(as.Date(TS$Dates),TS[,13],type="l",col="red",
+        main="CNYNOK",xlab="Year",ylab="FX rate")
> dev.off()
null device
          1
>
> #- Model set-up:
> #- Parameter estimation - Stochastic model
> #-- EQT
> val.eqt <- A*eqt.share*port.eqt               # Initial values
> #-- FX
> m.par = ParamEst(TS[,2:n.TS])
> (spt.fx <- m.par$spot[7:11])                        # Spot rates
[1]  0.934600 11.711500  9.839300  9.681000  0.071979
> (xi.fx <- m.par$exp.act[7:11])                  # Expected rates
SEKNOK_Curncy GBPNOK_Curncy EURNOK_Curncy USDNOK_Curncy JPYNOK_Curncy
   0.90632841   11.15666070    8.60795822    7.31020335    0.06831601
> (sig.fx <- m.par$sd.ret[7:11])                   # Volatilities
SEKNOK_Curncy GBPNOK_Curncy EURNOK_Curncy USDNOK_Curncy JPYNOK_Curncy
   0.07802089    0.10264366    0.08200303    0.12342602    0.14525993
> sig.x.fx = sig.fx / sqrt(1 - a.bk^2)      # Black--Karasinski
> cor.mat.fx <- m.par$cor.ret[7:11,7:11]
> ### Currency exposures: -- val.eqt[1] = NOK
> fx.expo.bccy <- val.eqt[2:length(port.eqt)]       # in base ccy
> fx.expo.fccy <- fx.expo.bccy/spt.fx               # in foreing ccy
>
> # Positive definite correlation matrix
```

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```
> (cor.mat.fx <- PosDefTest(as.matrix(cor.mat.fx)))
              SEKNOK_Curncy GBPNOK_Curncy EURNOK_Curncy USDNOK_Curncy
SEKNOK_Curncy     1.0000000     0.3986287     0.6089071     0.3692150
GBPNOK_Curncy     0.3986287     1.0000000     0.6187021     0.6724230
EURNOK_Curncy     0.6089071     0.6187021     1.0000000     0.6279909
USDNOK_Curncy     0.3692150     0.6724230     0.6279909     1.0000000
JPYNOK_Curncy     0.3220071     0.5335550     0.6038117     0.7196613
              JPYNOK_Curncy
SEKNOK_Curncy     0.3220071
GBPNOK_Curncy     0.5335550
EURNOK_Curncy     0.6038117
USDNOK_Curncy     0.7196613
JPYNOK_Curncy     1.0000000
> # Covariance matrix:
> cov.mat.fx = diag(sig.fx) %*%
+              cor.mat.fx %*%
+              diag(sig.fx)
>
> # Set-up:
> ## Model parameters
> (K = length(xi.fx))
[1] 5
> ## Data containers
> R.fx <- array(numeric(),c(m,T,K))            # Univariate return
> P.R.fx <- array(numeric(),c(m,T,K))          # Portfolio returns
> P.R <- matrix(rep(0,m*T),nrow=m)     # Aggregate absolute return
> FX.test <- array(numeric(),c(m,T+1,K))                    # FX rates
> for (k in 1:K) {FX.test[,1,k] <- spt.fx[k] }
>
> # Cholesky decomposition
> C = t(chol(cov.mat.fx))
>
> # Simulation:
> ## Simulation interations (m)
> for (i in 1:m)
+ {
+   X = log(spt.fx/xi.fx) + sig.x.fx^2/2    # Observed initial value
+   for (t in 1:T)
+   {
+     # Marginal distribution variation
+     eta <- rnorm(K)
+     # Cholesky decomposition applied
+     U.chol = C %*% eta
+     # AR(1) driver process - Black--Karasinski
+     X = a.bk*X + U.chol
+     # Log-normal returns, AR(1) - Black--Karasinski
+     R.fx[i,t,] = (xi.fx*exp(-sig.x.fx^2/2 + X))/spt.fx
+     # Xtra: Returns FX rates
+     FX.test[i,t+1,] = xi.fx*exp(-sig.x.fx^2/2 + X)
+   }
```

```
+ }
>
> ## Portfolio absolute return
> #- Absolute return per univariate
> for (k in 1:K)
+ {
+   # The following only considers FX exposure, with EQT static!
+   # (Not considering the multiplicative effect: EQT*FX)
+   P.R = P.R + (R.fx[,,k] - 1)*fx.expo.bccy[k]
+ }
>
> # SCR
> P.R.srt = apply(P.R,2,sort)
> (SCR = P.R.srt[m*scr.e,1:t])
[1] -12369.59 -16081.96 -18408.26 -19809.51 -21005.57
> (MCR = P.R.srt[m*mcr.e,1:t])
[1]  -5943.068  -8132.462  -9616.492 -10732.889 -11632.103
> (R.avg = apply(P.R,2,mean))
[1]  -808.7142 -1542.6829 -2188.2571 -2772.3340 -3291.6839
> #(R.85 = P.R.srt[m*(1 - mcr.e),1:t])
> (R.995 = P.R.srt[m*(1 - scr.e),1:t])
[1] 13162.37 17320.55 19862.86 20936.62 21673.17
>
> #- Plotting SCR graph:
> pdf("SCR_FX_5y.pdf")
>   plot(1:T,R.995,type="l",col="green",
+        main="Partial Internal Model - Currency (PIM.ccy)",
+        ylab="Change in Equity value from Currency exposures",
+        #main="SCR Currency",
+        xlab="Time (years)",
+        #ylab="FX exposure",
+        ylim=c(min(SCR)*1.1, max(R.995)*1.1))
>   lines(1:T,R.avg,col="blue")
>   lines(1:T,MCR,col="orange")
>   lines(1:T,SCR,col="red")
>   legend("topleft", lty=c(1,1,1,1),
+          #col=c("green","blue","red"),
+          col=c("green","blue","orange","red"),
+          legend=c("99.5-percentile",
+                   "Mean",
+                   "15-percentile",
+                   "0.5-percentile"))
> dev.off()
null device
          1
>
> #- TEST - plotting - only a few iterations!
> if (m <= 100)
+ {
+   FX.test.avg <- matrix(rep(0,K*(T+1)),nrow=K)   # Data container TEST
```

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```
+   for (k in 1:K) { FX.test.avg[k,] = apply(FX.test[,,k],2,mean) }
+   pdf("FX_rates_test.pdf")
+     par(mfrow=c(3,2))
+     matplot(t(FX.test[,,1]),type = "l",main="SEKNOK")
+     lines(1:(T+1),rep(spt.fx[1],(T+1)),col="black",lty=3,lwd=2)
+     lines(1:(T+1),rep(xi.fx[1],(T+1)),col="black",lty=2,lwd=2)
+     lines(1:(T+1),FX.test.avg[1,],col="black",lty=4,lwd=3)
+     matplot(t(FX.test[,,2]),type = "l",main="GBPNOK")
+     lines(1:(T+1),rep(spt.fx[2],(T+1)),col="black",lty=3,lwd=2)
+     lines(1:(T+1),rep(xi.fx[2],(T+1)),col="black",lty=2,lwd=1)
+     lines(1:(T+1),FX.test.avg[2,],col="black",lty=4,lwd=3)
+     matplot(t(FX.test[,,3]),type = "l",main="EURNOK")
+     lines(1:(T+1),rep(spt.fx[3],(T+1)),col="black",lty=3,lwd=2)
+     lines(1:(T+1),rep(xi.fx[3],(T+1)),col="black",lty=2,lwd=1)
+     lines(1:(T+1),FX.test.avg[3,],col="black",lty=4,lwd=3)
+     matplot(t(FX.test[,,4]),type = "l",main="USDNOK")
+     lines(1:(T+1),rep(spt.fx[4],(T+1)),col="black",lty=3,lwd=2)
+     lines(1:(T+1),rep(xi.fx[4],(T+1)),col="black",lty=2,lwd=1)
+     lines(1:(T+1),FX.test.avg[4,],col="black",lty=4,lwd=3)
+     matplot(t(FX.test[,,5]),type = "l",main="JPYNOK")
+     lines(1:(T+1),rep(spt.fx[5],(T+1)),col="black",lty=3,lwd=2)
+     lines(1:(T+1),rep(xi.fx[5],(T+1)),col="black",lty=2,lwd=1)
+     lines(1:(T+1),FX.test.avg[5,],col="black",lty=4,lwd=3)
+     #mtext("My 'Title' in a strange place", side = 3, line = -2, outer = TRUE)
+   dev.off()
+
+   pdf("FX_returns_test.pdf")
+     par(mfrow=c(3,2))
+     matplot(t(R.fx[,,1]),type = "l",main="SEKNOK")
+     matplot(t(R.fx[,,2]),type = "l",main="GBPNOK")
+     matplot(t(R.fx[,,3]),type = "l",main="EURNOK")
+     matplot(t(R.fx[,,4]),type = "l",main="USDNOK")
+     matplot(t(R.fx[,,5]),type = "l",main="JPYNOK")
+     #mtext("My 'Title' in a strange place", side = 3, line = -2, outer = TRUE)
+   dev.off()
+ }
>
>
> proc.time()
   user  system elapsed
  5.964   0.286   6.230
```

D.4 Combined equity and currency risk - run example

```
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Platform: x86_64-pc-linux-gnu (64-bit)

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Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> ###################################################################
> #          Partial Internal Model -- EQUITY & CURRENCY          #
> ###################################################################
>
> rm(list=ls(all=T))                              # clear workspace
>
> # Libraries:
> source("Asset_Input.r")
> source("ParamEst.r")
> source("PosDefTest.r")
>
> # Initialisation:
> ## Solvency II:
> scr.e <- .005         # Solvency Capital Required - risk level
> mcr.e <- .15          # Solvency Capital Required - risk level
> ## General input:
> y <- 5                          # No. of years simulated
> sp <- 1              # No. of sub periods simulated per year
> m <- 100000          # No. of iterations simulated per time step
> T = y*sp                        # No. of time steps simulated
> ### Black--Karasinski:
> a.bk <- c(.7,.8,.9,.9,.95)
>
> #- Loading data
> #-- Time series
> filePathName <- "EQT_FX_IR_Input_20220817.txt"
> TS <- read.table(filePathName,header=T)   #,row.names=1)
```

```
> TS$JPYNOK_Curncy = TS$JPYNOK_Curncy/100  # JPYNOK quoted per 100
>
> #- Parameter estimation - Stochastic model
> m.par = ParamEst(TS[,2:dim(TS)[2]])
>
> #- Model set-up:
> #-- Model parameters
> #--- EQT
> xi.eqt <- m.par$exp.ret[1:6]                      # Expectations
> val.eqt <- A*eqt.share*port.eqt              # Initial values
> #--- FX
> spt.fx <- m.par$spot[7:11]                        # Spot FX rates
> #xi.fx <- m.par$exp.ret[7:11]                      # Expectations
> xi.fx <- m.par$exp.act[7:11]                      # Expected rates
> sig.fx <- m.par$sd.ret[7:11]                    # Volatilities FX
> sig.x.fx = sig.fx / sqrt(1 - a.bk^2)        # Black--Karasinski
> #--- EQT & FX
> sig.eqt.fx <- m.par$sd.ret[1:11]        # Volatilities EQT & FX
> cor.mat.eqt.fx <- m.par$cor.ret[1:11,1:11]
> J = length(xi.eqt)
> K = length(xi.fx)
> ## Currency exposures: -- val.eqt[1] = NOK
> fx.expo.bccy <- val.eqt[2:length(port.eqt)]        # in base ccy
> fx.expo.fccy <- fx.expo.bccy/spt.fx            # in foreing ccy
>
> #-- Positive definite correlation matrix
> (cor.mat.eqt.fx <- PosDefTest(as.matrix(cor.mat.eqt.fx)))
              OSEBX_Index     OMX_Index     UKX_Index   SXXP_Index     SPX_Index
OSEBX_Index    1.00000000  0.687457437   0.70989178   0.74214238   0.44220357
OMX_Index      0.68745744  1.000000000   0.78497157   0.86034764   0.51410849
UKX_Index      0.70989178  0.784971566   1.00000000   0.92434293   0.54024935
SXXP_Index     0.74214238  0.860347639   0.92434293   1.00000000   0.57917424
SPX_Index      0.44220357  0.514108491   0.54024935   0.57917424   1.00000000
NKY_Index      0.32475646  0.280993968   0.29773204   0.32172785   0.14538862
SEKNOK_Curncy -0.06355255 -0.009584498  -0.05338005  -0.04110138  -0.04656204
GBPNOK_Curncy -0.18112799 -0.147106849  -0.24209528  -0.10909238  -0.19073310
EURNOK_Curncy -0.28246710 -0.253907476  -0.30616657  -0.33340125  -0.32478534
USDNOK_Curncy -0.26999596 -0.212090182  -0.25384632  -0.21523139  -0.29124161
JPYNOK_Curncy -0.35696918 -0.306682360  -0.37113336  -0.35597009  -0.44366624
               NKY_Index SEKNOK_Curncy GBPNOK_Curncy EURNOK_Curncy
OSEBX_Index    0.32475646  -0.063552546   -0.18112799    -0.2824671
OMX_Index      0.28099397  -0.009584498   -0.14710685    -0.2539075
UKX_Index      0.29773204  -0.053380046   -0.24209528    -0.3061666
SXXP_Index     0.32172785  -0.041101381   -0.10909238    -0.3334013
SPX_Index      0.14538862  -0.046562039   -0.19073310    -0.3247853
NKY_Index      1.00000000  -0.052971104   -0.03918553    -0.1156096
SEKNOK_Curncy -0.05297110   1.000000000    0.39862871     0.6089071
GBPNOK_Curncy -0.03918553   0.398628712    1.00000000     0.6187021
EURNOK_Curncy -0.11560964   0.608907112    0.61870206     1.0000000
USDNOK_Curncy -0.10341076   0.369215048    0.67242303     0.6279909
```

```
JPYNOK_Curncy -0.18903285   0.322007063   0.53355502   0.6038117
              USDNOK_Curncy JPYNOK_Curncy
OSEBX_Index      -0.2699960    -0.3569692
OMX_Index        -0.2120902    -0.3066824
UKX_Index        -0.2538463    -0.3711334
SXXP_Index       -0.2152314    -0.3559701
SPX_Index        -0.2912416    -0.4436662
NKY_Index        -0.1034108    -0.1890329
SEKNOK_Curncy     0.3692150     0.3220071
GBPNOK_Curncy     0.6724230     0.5335550
EURNOK_Curncy     0.6279909     0.6038117
USDNOK_Curncy     1.0000000     0.7196613
JPYNOK_Curncy     0.7196613     1.0000000
> #-- Covariance matrix:
> cov.mat.eqt.fx = diag(sig.eqt.fx) %*%
+                 cor.mat.eqt.fx %*%
+                 diag(sig.eqt.fx)
> ## Cholesky decomposition
> C = t(chol(cov.mat.eqt.fx))
>
> ##- Data containers
> #- Univariate, dependent asset returns
> R.eqt <- array(numeric(),c(m,T,J))                      # EQT
> R.fx <- array(numeric(),c(m,T,K+1))            # FX (+1 = NOK)
> #R.fx[,,1] <- 1                                          # NOK
> R.fx <- array(numeric(),c(m,T,K))                        # FX
> #- Protfolio returns
> P.R.eqt.fx <- matrix(rep(0,m*T),nrow=m)            # EQT & FX
>
> #- Simulation:
> #-- Driver process:
> #-- Simulation proper - m iterations:
> for (i in 1:m)
+ {
+   #-- Driver process:
+   X = log(spt.fx/xi.fx) + sig.x.fx^2/2    # Observed initial value
+   ## First time step
+   t <- 1
+   ## Marginal distribution variateion
+   eta <- rnorm(J+K)
+   ## Cholesky decomposition applied
+   U.chol = C %*% eta
+   ## Log-normal returns
+   R.eqt[i,t,] = exp(xi.eqt + U.chol[1:J])            # EQT ret
+   X = a.bk*X + U.chol[(J+1):(J+K)]               # Driver updated
+   R.fx[i,t,] = xi.fx*exp(-sig.x.fx^2/2 + X)/spt.fx   # FX ret
+   # Consecutive time steps:
+   for (t in 2:T)
+   {
+     ## Marginal distribution variateion
```

```
+     eta <- rnorm((J+K))
+     ## Cholesky decomposition applied
+     U.chol = C %*% eta
+     ## Log-normal returns
+     R.eqt[i,t,] = R.eqt[i,t-1,]*exp(xi.eqt + U.chol[1:J])  # EQT
+     X = a.bk*X + U.chol[(J+1):(J+K)]                    # Driver FX
+     #!!!! NOT - 1 !!!!
+     R.fx[i,t,] = xi.fx*exp(-sig.x.fx^2/2 + X)/spt.fx       # FX
+   }
+ }
> FX.1 <- array(rep(1,m*T),c(m,T))                              # NOK
> R.fx = array(append(FX.1,R.fx),c(m,T,K))           # full FX array
>
> #- Aggregated portfolio returns:
> for (k in 1:K)
+ {
+   # The multipliticative nature of currency exposure
+   P.R.eqt.fx =  (R.eqt[,,k] - 1)*(R.fx[,,k] - 1)*fx.expo.bccy[k]
+ }
>
> # SCR and other percentiles: sorting ascending
> P.R.eqt.fx.srt = apply(P.R.eqt.fx,2,sort)
> (SCR = P.R.eqt.fx.srt[m*scr.e,1:t])
 [1]  -394.9698  -919.3731 -1522.5209 -2243.0536 -3074.2191
> (MCR = P.R.eqt.fx.srt[m*mcr.e,1:t])
 [1]  -84.92181 -193.88861 -326.97280 -487.16369 -658.13158
> (R.avg = apply(P.R.eqt.fx,2,mean))
 [1]  -26.32210  -64.75855 -115.51648 -179.27582 -252.88160
> #(R.85 = P.R.srt[m*(1 - mcr.e),1:t])
> (R.995 = P.R.eqt.fx.srt[m*(1 - scr.e),1:t])
 [1]  178.1991  393.5549  605.7523  835.5093 1075.3295
>
> ## Plotting some graphs
> pdf("SCR_EQT_FX_5y.pdf")
> #  #par(mfrow=c(1,2))
>   plot(1:T,R.995,type="l",col="green",
+       main="Partial Internal Model - Equity and Currency (PIM.eqt.ccy)",
+       ylab="Change in Equity value from Equity and Currency exposures",
+       #main="SCR Equity and Currency (multiplicative)",
+       xlab="Time (years)",
+       #ylab="Portfolio value",
+       ylim=c(min(SCR)*1.1, max(R.995)*1.1))
>   #lines(1:T,R.85,col="green")
>   lines(1:T,R.avg,col="blue")
>   lines(1:T,MCR,col="orange")
>   lines(1:T,SCR,col="red")
>   legend("topleft", lty=c(1,1,1,1),
+         #col=c("green","blue","red"),
+         col=c("green","blue","orange","red"),
+         #col=c("green","green","blue","orange","red"),
```

```
+            legend=c("99.5-percentile",
+                     "Mean",
+                     "15-percentile",
+                     "0.5-percentile"))
> dev.off()
null device
          1
>
>
> proc.time()
   user  system elapsed
  4.585   0.069   4.698
```

APPENDIX E

Input files and support functions

E.1 Input files to the Solvency II SCR computations

Solvency II input

```
# Solvency II

#source("SII_Standard_Formula_F.r")
source("Functions_F.r")

# Solvency II Input - Liability and Market Risk:

# Basic Operations (BOF - Basic Own Funds)
cor.lima <- .25                          # corr(liability, market)
# Correlation matrix (market risk, liability risk)
BSCR.cor <- matrix(c(1,cor.lima,cor.lima,1),nrow=2)

# LIABILITY risk:
S.mort <- .15               # Shock Portfolio I; mortality shift
S.long <- -.2               # Shock Portfolio II; longevity shift
cor.molo <- -.25
SCR.lia.cor <- matrix(c(1,cor.molo,cor.molo,1),nrow=2)

# MARKET risk:
# Interest rate shock:
# - Oblig STK4500
#k <- c(1:10, 12, 15, 90)
#r.shock.k <- c(.75, .65, .56, .50, .46, .42, .39,
#                .36, .33, .31, .29, .27, .20)
# - Delegated Acts, Article 167 (Term structure decrease)
k.s <- c(1:20, 90)
r.shock.k <- c(.75, .65, .56, .50, .46,
               .42, .39, .36, .33, .31,
               .30, .29, .28, .28, .27,
               .28, .28, .28, .29, .29,
               .20)
r.shock.mat <- matrix(c(rbind(k.s,r.shock.k)), nrow=2)
S.r.k <- interPol(r.shock.mat, 1:K.rf, 2)$y
```

```
# Equity risk shock:
S.eqt.1 <- .22
S.eqt.2 <- .39
# Property shock:
S.prp <- .25
# Spread shock:
S.spr.1 <- 1 - B.1.bomega*B.1.dur    # Shock bond 1
S.spr.2 <- 1 - B.2.bomega*B.2.dur    # Shock bond 2
# Currency shock:
S.fx <- .25
# Market risk correlation:
rho <- .5               # = 0 if company risk due to rate increas3
cor.ireq <- rho                     # corr(interest rate, equity)
cor.irpr <- rho                    # corr(interest rate, property)
cor.irsp <- rho                     # corr(interest rate, spread)
cor.irfx <- .25                    # corr(interest rate, currency)
cor.eqpr <- .75                   # corr(interest equity, property)
cor.eqsp <- .75                    # corr(interest equity, spread)
cor.eqfx <- .25                   # corr(interest equity, currency)
cor.prsp <- .5                    # corr(interest property, spread)
cor.prfx <- .25                  # corr(interest property, currency)
cor.spfx <- .25                    # corr(interest spread, currency)
# Correlation matrix (market risk sub-modules)
SCR.mkt.cor <- matrix(c(1,cor.ireq,cor.irpr,cor.irsp,cor.irfx,
                        cor.ireq,1,cor.eqpr,cor.eqsp,cor.eqfx,
                        cor.irpr,cor.eqpr,1,cor.prsp,cor.prfx,
                        cor.irsp,cor.eqsp,cor.prsp,1,cor.spfx,
                        cor.irfx,cor.eqfx,cor.prfx,cor.spfx,1),
                      nrow=5)
```

Asset input

```
###################################################################
# Company Input - ASSETS:   Solvency II   and   IRB         #
###################################################################


### EXPOSURE MAP -- ASSETS

##  Asset Input:
A <- 300000             # Total Assets (Investments Portfolio)
eqt.share <- .3                     # Share of total assest
pr.share <- .1                      # Share of total assest
sp.share <- .6                      # Share of total assest
#eqt.to.fx <- c(1,1,1,1,1)       # FX exposure as % of equities
                                # SEK, GBP, EUR, USD, JPY @NOK


##  SII Input:
#-  Equity input:
str.share <- .2
```

```
oth.share <- 1 - str.share
A.eq.1 <- A*eqt.share*str.share          # Strategic investments
A.eq.2 <- A*eqt.share*oth.share            # Other investments
#-   Property input:
A.pr <- A*pr.share                       # Property investments
#-   Spread input:
B.1.share <- .5
B.2.share <- 1 - B.1.share
B.1.bomega <- .011                       # Rating AA  (omega = 1)
B.2.bomega <- .009                       # Rating AAA (omega = 0)
B.1.dur <- 5.5                                 # Durasjon
B.2.dur <- 10.5                                # Durasjon
#- Currency input:


##  IRB Input:
#-  Loading data
#-  EQT univariates:
port.eqt <- c(.3,.2,.07,.2,.2,.03)         # Equity indices invested
                                         # NOK, SEK, UK, EUR, US, Asia
```

Liability input

```
####################################################################
# Company Input  -  ASSETS:   Solvency II   and   IRB          #
####################################################################

### EXPOSURE MAP -- LIABILITIES

### Liability input:
# Common portfolio input:
# - Survival Probabilities
#    - Gompertz--Makeham
# - Payments;
#    - into accounts (advance - start period);
#    - out of accounts (arrears - end period)
r.tr <- .02                                   # Technical rate
a.rf <- .01                      # Market rate, model coefficient
#b.rf <- .025                     # Market rate, model coefficient
b.rf <- .15                      # Market rate, model coefficient
#theta.rf <- .125                 # Market rate, model coefficient
theta.rf <- .1                   # Market rate, model coefficient
t.0 <- .00078            # Gompetrz--Makeham, model coefficient
t.1 <- .0000376          # Gompetrz--Makeham, model coefficient
t.2 <- .092759           # Gompetrz--Makeham, model coefficient
K.rf <- 90                       # Interst rate shock time span
# Portfolio I - Term Insurance
# Input - Portfolio I:
# - all contracts are equal
s.1 <- 2               # Claim payable upon death brfore maturity
```

```
K.1 <- 25                               # Contract life (maturity)
l.0.1 <- 30                        # Age at initiation of contract
pi.1 <- 0               # Premium (annual, until maturity/death)
gam.1 <- .1                          # Age distribution coefficient
mu.1 <- 20                           # Age distribution coefficient
J.1 <- 15e4                          # Age distribution coefficient
# Portfolio II - Pension Insurance
# Input - Portfolio II:
# - all contracts are equal
s.2 <- .3        # Pension (annually; from retirement until death)
K.2 <- 65                               # Contract life (maturity)
l.0.2 <- 30                        # Age at initiation of contract
l.r <- 67                                       # Retirement age
l.e.2 <- 120                   # Max age (expiration of contract)
pi.2 <- 0             # Premium (annual; from l.0 until retirment)
gam.2 <- .15                         # Age distribution coefficient
mu.2 <- 24                           # Age distribution coefficient
J.2 <- 2e5                           # Age distribution coefficient
```

E.2 Support functions for Solvency II SCR computations

The following support functions for Solvency II SCR coputations were provided as part of the mandatory assignment in STK4500, UiO, 2018, that this thesis builds on for standard model SCR comparison.

Liability input fuctions - lifetables, demographics and discounts

```
lifetab <- function(t0,t1,t2,l_0,l_e,Shock=0)
{
  # Input:
  # - l_0; age of policy holders when the contracts are drawn up,
  # - l_e; maximum age,
  # - t0, t1, t2; Gompertz--Makeham parameters.
  # Output:
  # - l_0,
  # - l_e.
  # - kp; the lifetable,
  # - q; mortalities,
  # Indexing:
  # - kp[l,k]; the suvival probabilities,
  #   -> the probability of surviving k-1 years
  #       given the initial age (l_0 + l - 1).
  # - q[l]; mortalities,
  #   -> the probability of dying at age (l_0 + l - 1).
  ll <- l_0:(l_e + l_e)
  # Gompertz--Makeham
  # - one-step mortalities at age l; m.l
  q <- pmin(pmax((1 - exp(-t0 - (t1/t2)*(exp(t2) - 1)*exp(t2*ll)))*(1 + Shock), 0), 1)
  kp <- matrix(0, l_e + 1, l_e + 1)
  # zero-step survival probability; sp.0.1 (start of recursion
  kp[,1] <- 1
  # the remaining survival probabilities; sp.k.l
  for (l in 1:(l_e - 1))
    {
      for (k in 2:(l_e + 1 - l))
      {
        kp[l,k] <- kp[l,k-1]*(1-q[l+k-2])    # Recursion (12.10)
      }
    }
  return (list(l.0=l_0, l.e=l_e, kp=kp, q=q))
}

ndist <- function(gamma,mu,K,J)
{
  # The distribution of policies according to
  # 'how long they have been in the company'.
  # Input:
  # - Parameters describing the distribution of the policies.
```

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```
  # Output:
  # - N[k]: number of policies at different age
  #           - policies set up k years ago.
  # Indexing:
  # - as described in function "lifetab".
  p <- exp(-gamma*abs(0:K - mu))
  N <- p*(J/sum(p))
  return (N)
}

riskfree <- function(a,b,theta,K=200,Shock=NULL)
{
  # Modelling the risk-free rates of interest.
  # Input:
  # - a,b,theta; parameters for the interest rate curve
  # - K; longest remaining life for pololicies,
  # - Shock; interest rate curve shift (stressor).
  # Output:
  # - rf; interest rate curver over K periods
  rf <- a + b*(1 - exp(-(1:K)*theta))
  if(length(Shock) > 0)
   {
     rf <- rf*pmax(1 - Shock,0)
   }
  return (rf)
}
```

Liability computations

```
# Insurance Liabilities


# Liabilities (time=k)
# - Portfolio I - Term insurance
LiabTerm <- function(pi.trm, s, K, N, s.prb)
{
  L <- rep(0, K)
  for (k in 1:(K-1))
  {
    L[k] <- -pi.trm * N[1:(K-k+1)] %*% s.prb$kp[1:(K-k+1),k+1] +
            s * N[1:(K-k+1)] %*% s.prb$q[1:(K-k+1)] * s.prb$kp[1:(K-k+1),k+1]
    #k.q.l = q.l+k-1 * k-1.p.l
  }
  L[K] <- s*N[1]*s.prb$q[s.prb$l.0+K]*s.prb$kp[s.prb$l.0,K+1]
  return (L)
}

# Liabilities (time=k)
# - Portfolio II - Pension insurance
```

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```
LiabPen <- function(pi.pen, s, K, N, l.r, s.prb)
{
  L <- rep(0, K)
  for (k in 1:(l.r-s.prb$l.0-1))
  {
    # Contributions
    L[k] <- L[k] - pi.pen * N[1:(l.r-s.prb$l.0-k)] %*% s.prb$kp[1:(l.r-s.prb$l.0-k),k+1]
    # Benefits
    L[k] <-  L[k] + s * N[(l.r-s.prb$l.0-k+1):(min(s.prb$l.e-s.prb$l.0-k,K)+1)] %*%
          s.prb$kp[(l.r-s.prb$l.0-k+1):(min(s.prb$l.e-s.prb$l.0-k,K)+1),k+1]
  }
  for (k in (l.r-s.prb$l.0):(s.prb$l.e-s.prb$l.0))
  {
    # Benefits
    L[k] <- L[k] + s * N[1:(s.prb$l.e-s.prb$l.0-k+1)] %*% s.prb$kp[1:(s.prb$l.e-s.prb$l.0-k-
  }
  return (L)
}
```

E.3 Other support functions

The following support functions for the partial internal models have been made by the author in the course of working on this thesis in order to make routine computations automated.

Parameter estimation

```
####################################################################
#        Parameter Estimation for PARTIAL INTERNAL MODEL        #
####################################################################

# Libraries:
source("PosDefTest.r")

# Algorithm proper:
#ParamEst <- function(filePathName,frequency="d")
ParamEst <- function(timeSeries,frequency="d")
{
  # Input:
  # - filePathName: the file path and name to the source file
  #   (file with time series of market prices)
  # - frequency: daily/weekly/monthly/annualy time series
  # Ourput:
  # - estimates of;
  #   - expected value of the time series
  #   - standard deviations and correlations of log-returns
  #     of these time series

  # Initialisation:
  ts <- timeSeries
  freq <- frequency                               # data frequency

  # Setting annualisation factor:
  ann.fac <- 52*5                            # daily to annual
  if (freq != "d")
  {
    if (freq == "w") { ann.fac = 52 }          # weekly to annual
    else if (freq == "m") { ann.fac = 12 }   # monthly to annual
    else { ann.fac = 1 }                           # annual
  }

  # Reading time series (ts):
  #ts <- read.table(filePathName,header=T,row.names=1)
  n.ts <- dim(ts)[1]

  # Returns:
  ts.ret = ts[2:n.ts,]/ts[1:(n.ts-1),] - 1
  #ts.log.ret = log(ts[2:n.ts,]/ts[1:(n.ts-1),])     # log return
```

```
  # FX Spot/Expectation/Standard deviation/Correlation:
  spot = t(ts[n.ts,])
  #spot.ret = ts.log.ret[n.ts - 1,]
  exp.act = colMeans(ts)                    # expected actual value
  exp.ret = colMeans(ts.ret)*ann.fac          # expected return
  #exp.log.ret = colMeans(ts.log.ret)*ann.fac
  sd.act = apply(ts, 2, sd)*sqrt(ann.fac)
  sd.ret = apply(ts.ret, 2, sd)*sqrt(ann.fac)
  #sd.log.ret = apply(ts.log.ret, 2, sd)*sqrt(ann.fac)
  cor.ret = cor(ts.ret)
  #cor.log.ret = cor(ts.log.ret)

  # Positive definite correlation matrix
  #cor.ret.pd = PosDefTest(cor.ret)
  #cor.log.ret.pd = PosDefTest(cor.log.ret)

  return(list(spot=spot,
              exp.act=exp.act,
              #spot.ret=spot.ret,
              exp.ret=exp.ret,
              #exp.log.ret=exp.log.ret,
              sd.act=sd.act,
              sd.ret=sd.ret,
              #sd.log.ret=sd.log.ret,
              cor.ret=cor.ret))
              #cor.ret.pd=cor.ret.pd,
              #cor.log.ret=cor.log.ret))
              #cor.log.ret.pd=cor.log.ret.pd))
}
```

Positive definite correlation matrix

```
PosDefTest <- function(mat)
{
#  cor.mat <- read.table("Cor_Mat_Eqt.txt",
#                        header=T)
  origMat <- mat

#origMat <- array(c(1, 0.9, 0.7, 0.9, 1, 0.3, 0.7, 0.3, 1),
#                 dim = c(3,3))
#  origEig <- eigen(origMat)
#  origEig$values
## [1]  2.296728  0.710625 -0.007352

  cholStatus <- try(u <- chol(origMat), silent = FALSE)
  cholError <- ifelse(class(cholStatus) == "try-error", TRUE, FALSE)

  newMat <- origMat
```

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```
# Method
# Rebonato and Jackel (2000),
# as elaborated by Brissette et al. (2007)
  iter <- 0
  while (cholError) {
      iter <- iter + 1
      cat("iteration ", iter, "\n")
      # replace -ve eigen values with small +ve number
      newEig <- eigen(newMat)
      newEig2 <- ifelse(newEig$values < 0, 0, newEig$values)
      # create modified matrix eqn 5 from Brissette et al 2007, inv = transp for
      # eig vectors
      newMat <- newEig$vectors %*% diag(newEig2) %*% t(newEig$vectors)
      # normalize modified matrix eqn 6 from Brissette et al 2007
      newMat <- newMat/sqrt(diag(newMat) %*% t(diag(newMat)))
      # try chol again
      cholStatus <- try(u <- chol(newMat), silent = TRUE)
      cholError <- ifelse(class(cholStatus) == "try-error", TRUE, FALSE)
  }
## iteration  1
## iteration  2
# final check
#  eigen(newMat)$values
## [1]  2.290e+00  7.096e-01 -1.332e-15

#  chol(newMat)

#  origMat
  return(newMat)
}
```

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