# Why Plato Thinks That Geometry Is Beautiful

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## Abstract

There is hardly anyone, who knows anything about Plato and his philosophy, who would claim that Plato did not think that geometry is beautiful. Therefore, I focus primarily on two questions. The first question is why Plato thinks that geometry is beautiful and the second question is how much both practical and theoretical geometries are beautiful according to him. As for the former, I claim that since geometry is a constitutive part of ethical education, it is thus substantially beautiful. As for the latter, I connect the Ladder of Love and the Cave, and I argue that theoretical geometry is as beautiful as the Love for knowledge and that practical geometry is as beautiful as the Love for laws and institutions.

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## Contents

# Why Plato Thinks That Geometry Is Beautiful

## 1. Introduction

In this essay, I will argue that Plato thinks that geometry is beautiful because it is a constitutive component of ethical education[1]. This argument will be the first and the most general reason of why Plato considers geometry to be beautiful. The second is that theoretical geometry is as beautiful as the Love for knowledge in *Symposium's* Ladder of Love. The third argument is that practical geometry is as beautiful as the Love for laws and institutions.

First, I start with the general introduction of *Symposium* dialogue. Then I present speech of Socrates in which he introduces the Ladder of Love as well as the nature of human desire, Love and beauty. I talk about the possible translations and meanings of the Greek word *kalon*, beauty, so then I can analyze the speech with help of Ferrari's *Platonic Love.* I conclude the first chapter by providing direct evidence that geometry is, in Plato's eyes, beautiful in the first place. I use an example from *Philebus* as well as some interpretation of the Ladder. Fundamentally, the end of the first chapter outlines possible reasons *why* is geometry beautiful apart from that it is. This, I believe, will help with the orientation and the right anticipation of further development of the arguments.

Second, I present the broader context of the *Republic,* including the Divided Line and allegory of the Cave. The Cave allegory is together with the Ladder of Love an important measure used to place both practical and theoretical geometry in relation to the Good. I argue that there are three transitions, first, the release of the prisoners T1, second, the ascent out of the Cave T2 and third, the descent back to the Cave T3. I argue that education is responsible for T1 and T2 and that mathematics, especially theoretical geometry and arithmetic, is responsible for T2 in particular. In order to show that, I analyze the educational system Socrates present in book VII *Republic*. At the end, I have two important conclusions. First, I argue that there is threefold ontological distinctions of reality, images of images, images, Forms, but a fourfold division of the Divided Line (*eikasia, pistis, dianoia, noesis*). I argue that the difference between practical and theoretical geometry is the psychological

[1] In Plato's Academy, before the temple of the Muses, it was written that: 'Let no one ignorant of geometry enter'. University of Illinois Urbana-Champaign, "'Let No One Ignorant of Geometry Enter'".

affection (*pistis* versus *dianoia*), its origins (man-made origin versus divine image) and then its sensible and intelligible nature but that the ontological status is the same: both are images. Second, I claim that especially theoretical mathematics helps to calibrate soul's measurements and it is a constitutive component of ethical education.

Third, in the last chapter, I discuss how beautiful the virtue of good ordering of cities and household is using reference in *Symposium.* I claim that the products of this virtue, laws and institutions, are fundamentally good because they were put in place by enlightened philosopher-rulers. And since it was said that the constitutive component of ethical education is geometry and that one of the products of this education is justice and 'good ordering of cities', then I claim that geometry must, generally speaking, be beautiful as well. Then, I argue that since Socrates uses word *episteme* while talking about the vast sea of knowledge one contemplates at the level of the Love for knowledge, theoretical geometry falls under this category. The word *episteme* has two meanings: knowledge of the Forms and theoretical component of *techne*, a craft, the second important term in this dichotomy. I argue that Socrates uses *episteme* in relation to theoretical geometry specifically in the latter meaning of the word, and thus, it is one of the reasons to include theoretical geometry under the Love for knowledge. Finally, I claim that practical geometry is as beautiful as the Love for laws and institutions. The main reasons are that practical geometry is in its nature related to sensible realm and that the geometers mistaken the diagrams they draw for the reality and thus I use *pistis – dianoia* dichotomy argument again. Consequently, I argue that the ideal philosopher-ruler creating laws has theoretical knowledge, *episteme*, and that the laws and institutions he enacts are both practical and sensible manifestation of this theoretical knowledge. This will lead me to say that the last place where beautiful sensible things can manifest is under the Love for laws and that practical geometry also has to be there because it is the study of sensibles for the sake of sensibles. Moreover, I claim that another reason to include practical geometry under this level is that the transition between practical and theoretical geometry is continuous thus it must be as close to theoretical geometry as possible. I finish the essay with summary of the main arguments.

# 2. Symposium

## 2.1. Symposium, the Desires and the Beautiful

Aside from the main question *why* geometry is beautiful, the more fundamental question is whether geometry is beautiful at all in Plato's view. Thus, in this first passage, I will try to provide some direct evidence that aesthetically, ethically as well as in terms of utility, in all the basic meanings of the word *kalon*, geometry, and generally mathematics, is considered beautiful by Plato. But first, I think that it is necessary to introduce Diotima's Ladder of Love as presented in *Symposium*.

In short, *Symposium* is set at the house of Agathon, a tragic poet celebrating his recent victory in 416 BC at one of the great dramatic festivals. He hosts an all-male party, or *symposium*, dedicated to a speech contest (*encomia*) of praising *Eros*, the god of love[2]. Yet the speeches are devoted to the nature of erotic relationships, they are fundamentally about the nature of *eudaimonia*, generally translated as happiness or flourishing[3]. M. C. Howatson and F. Sheffield writes that "Plato's concern with desire and its role in the good life in a number of works suggests that he believed that one's ability to act well and to lead a worthwhile and good life depends, in part, on desiring the right kinds of things and acting on that basis. What, or whom, one desires determines the choices one makes and thereby affects one's chances of leading a worthwhile and happy life"[4]. *Symposium* is then an ethical work at its core. Furthermore, the speeches about *Eros* make a very distinctive contribution to an understanding of the *nature of human desire* and thus of the nature of beauty. The concept of desire and love is very close to each other in its meaning so then when Socrates talks about the Ladder of Love, it is also about things humans *desire*, or *love* so to speak.

Socrates' speech is largely based on reconstruction of Diotima's words. Diotima is described as a seer or priestess, a wise woman. She reveals to Socrates that there are six types of love. The types are sorted in succession into the Ladder representing the path of love as an ascent

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[2] The participants contributing in the contest include the intellectual elite of the times: a heroic poet Phaedrus, a law expert Greek states Pausanias, a representative of science, physician in particular Eryximachus, a comedy writer and poet Aristophanes, Agathon and Socrates. Each speech about *eros* makes a very distinctive contribution to an understanding of the nature of human desire and the end (*telos*) of loving relationships.

[3] "*Eudaimonia* was considered not just to be a subjective feeling of pleasure, or contentment, or the mere satisfaction of an individual's desires (whatever these may be). What is under consideration here is whatever it is that makes a life worthwhile, that is, the success, or flourishing, of a human being who can be considered to be living well." Howatson and Sheffield, *The Symposium*, Introduction XVI.

[4] Howatson and Sheffield, *The Symposium*, X.

starting with the Love for a body (B1), proceeding with the Love for all bodies (B2), the Love for souls (B3), the Love for laws and institutions (B4), the Love for knowledge (B5), and ending with the Love for beauty itself (B6).

Furthermore, Socrates' speech is fundamentally about human nature which is, according to him, a desire for happiness, or, alternatively, the kind of good which brings happiness to one's possession. Socrates claims that happiness is a real end (*telos*) of desire that it is desired for its own sake and that there is nothing above happiness to be desired next. No matter whether the good which is supposed to satisfy the desire is understood as inclusive (valuing more good things to reach the end) or exclusive (dropping other goods, valuing only one good to meet the end) the criterions for the good(s) are: i) desired for its own sake ii) and endurance. To the latter, Socrates says that we want immortality with the good (207a) because the goal is to stretch happiness over indefinite period of time, ideally eternally. Socrates then finds this good with these properties in the account of Love which I provide in the next sub-chapter.

## 2.2. Socrates' Speech

Socrates' speech starts with a clarification of what Love is, first as represented by *Eros.* It is said that *Eros* is not a God but rather a great spirit who *lacks* beautiful and the Good and thus he desires them because he lacks them. It is said that Gods are already in possession of these qualities; Love thus cannot be a god. Love is like a philosopher, say Diotima, who is also in between ignorance and wisdom – he desires wisdom (which he does not have yet) but he cannot be ignorant because the ignorant does not search for something he does not know he does not have. Similarly, it is with the Love. Love lacks the beautiful yet Love is not ugly. One might think that since Love lacks the beautiful, it *has to* be ugly. Diotima says otherwise: "Love is not good and not beautiful [but] that is no reason for thinking he has to be ugly and bad" (202b). Love is in between these two qualities.

So the argument is that Love is always love *of something* and that something is what Love (or even lover) lacks that is the beautiful. As it was said, all humans ultimately in various forms desire the kind of good which *brings* them happiness (*eudaimonia*)[5], it implies that humans must create a good life, it is not *given* to them. Lovers as well as *Eros* (personification of

[5] "This claim is often seen as part of a larger Platonic thesis referred to as *psychological eudaimonism.*" M. C. Howatson and F. Sheffield, *The Symposium,* XV.

Love) desire this good life. The way this good life is achieved is through “procreating and giving birth in the beautiful“ (206e). It is a creative process of replacement of old things with new things in a beautiful environment. This refers to not only procreation of children which falls under the category of bodily desires but also to the ideas, art, craftsmanship etc., a category of desires of the soul.

The idea is that since lovers desires the possession of the good *unlimitedly* (ideally infinitely long), then “it is immortality together with the good that must necessarily be desired” (207a). Immortality is thus an object of love. Diotima claims that the objects of love are somehow always involved in procreating and giving birth which is the only way for humans to approach divine immortality and the beautiful because no procreation is done in the ugly. An obvious example is procreation of children. Since humans are mortals, one way of capturing the immortal good is giving life to another generation of humans. The procreations do occur also in the soul, Diotima says. For example, soul’s habits, worries, joys, beliefs are not changeless but in the process of procreation: replacement of the old for the new, infinitely many times. “In this way everything mortal is preserved, not by remaining entirely the same forever, which is the mark of the divine, but by leaving behind another new thing of the same kind in the place of what is growing old and passing away” (208b). This process is what secures the happiness: “it is the good things that result from an encounter with beauty that promise happiness”[6].

Diotima then talks about the love of honor as a foundation for (dangerous) actions. Some people are strongly affected by honor, respect, fame and they desire to reach the immortal legacy of themselves for themselves. By saying this, she is proving her point that all people desire what somehow partakes in immortal in one way or the other. “Those whose pregnancy is of the body … are drawn more towards women, and they express their love through the procreation of children, ensuring for themselves, they think, for all time to come, immortality and remembrance and happiness in this way.” (208e). The others who are pregnant in the soul more than in the body, as some of the honor-loving people are, procreate “wisdom [*phronesis*] and the rest of virtue” (209a). These people include poets and craftsmen but ‘the most important ones’ are philosophers who seek “the good ordering [*diakosmesis*] of cities and households; … [with] the names for this kind of wisdom are

[6] Howatson and Sheffield, *The Symposium,* XIX.

moderation and justice" (209a). Importantly, the procreation of the soul has more beauty and immortality precisely because the 'wisdom and the rest of virtue' are carried in the mind which results in stronger association with the beautiful than in case of mortal children. What should be stressed out of this passage is that it is the beauty and *lack* of beauty which guide the desire to secure good things, create a good life and happiness for all men. I will talk later about the nature of the relationship between the good and beautiful. However, while securing the good, which is bringing the happiness, beauty seems to be "pursued in each case because it is a visible manifestation of something good and, as lovers of the good, beauty thereby prompts us to secure some good for ourselves"[7].

## 2.3. The Ladder of Love

Now after Diotima makes the statement that one kind of Love is more beautiful than the other, the logical questions are: i) how many kinds of Love are there and ii) what is the order for these various kinds of Love? She proceeds with creating what has become known as the Ladder of Love or 'ascent of desire'. The first kind of Love is the Love of a particular body, B1. It is said that the lover has to start in youth and with one individual body and that a beautiful discourse is procreated during the relationship. The realization should follow after the beautiful discourse between the partners and the Love for all bodies, B2, emerge from the experience. Reasoning is that any one body is as beautiful as any other body. If the form (*eidos*) of one body is the aspect to be strived for, then "it is folly not to regard the beauty in all bodies as one and the same" (210b), Diotima says to Socrates. The intruding passion for one body following this realization is naturally put in question and weakened – the lover does not live for only one body but also for all beautiful human bodies out there.

Next it comes to the realization that the beauty of souls, B3, is more valuable than the beauty of all bodies. Bodies are, after all, subject to decay, the glitter of youth easily disappears. The lover starts to appreciate more stable souls even to that extent that Diotima says that if a body is flawed but the soul is virtuous then it will be enough to love and care for that person. The lover with the beautiful souls will engage in beautiful discourse again, the one which improve young men bodies, minds and souls. Since the product of these beautiful souls is the 'good ordering of cities', Diotima argues that a consequence of the Love for souls will be "to contemplate the beautiful as it exists in human practices and laws,

[7] Ibid, XIX.

to see that the beauty of it all is of one kind" (210c). These laws and practices, B4, are products of beautiful souls who have given birth to them.

This will ultimately lead to another kind of Love: the Love for knowledge, B5. The lover contemplates knowledge in its various forms and branches leaving the love relating to humans in one way or the other for good. That means that either be it a particular body, soul or practice, he will never contemplates about beauty as strictly tied up to a specific thing. He will pay attention to the boundless 'sea' of beautiful and while contemplating the sea of beautiful "he will give birth to many beautiful … discourses and thoughts in a boundless love of wisdom" (210e). In the end, the product of this discourse is the Form of beauty itself, B6, and the Love for love itself. Diotima claims that for the sake of this final love, all the previous loves have to be studied. The Form is eternal thus having neither beginning nor ending and it is also not undertaking any change whatsoever. Moreover, this Form is not beautiful relative to place, time, to people judgments or relative to point of views. No. Diotima says that the Form "exists on its own, single in substance and everlasting. All other beautiful things partake of it, but in such a way that when they come into being or die the beautiful itself does not become greater or less in any respect, or undergo any change" (211b). The Form is the final end, the final Good we were looking for. It is truth itself. All the things under the Form of Beauty have been truer as the progress went in succession towards the Form itself. Whereas at the beginning, things were highly changeable, unstable and uneven just from their sensible nature, the Form is the opposite of it. The Ladder has progressed towards uniform, stable and changeless intelligible Form.

Diotima also makes very interesting argument: a person capable of contemplating the divine substance of beauty is virtuous. My understanding is that if a person is capable of contemplating the Form of Beauty, the whole ascent has been completed thus all developments and experiences has shaped the person's character making him virtuous. Sheffield and Howatson even argue that Socrates' claim seems to be "that the activity of contemplating the Form of Beauty is itself a virtuous activity"[8]. I agree that the contemplation of the Form itself is an intellectual virtuous activity but, in my opinion, it is due to the possession of the special intellectual capacity. The reason is that Diotima hints that the person capable of this final contemplation has the faculty "by which it has to be

[8] Ibid, XXII.

viewed" (212a) and that he "sees the beautiful with that by which it has to be viewed, … [then] he will give birth not to mere images of virtue but to true virtue, because it is not an image that he is grasping but the truth [by contemplating the Form]" (212a). So the contemplation of the Form seems to be a result of having the *ability* (or capacity or quality) to do so and whose possession makes a person virtuous. Diotima says to us that to contemplate the Form one needs to progress through all the levels in the Ladder. This suggests the development of this capacity or maybe better "mere" refinement of inborn capacity. After all, the progress on the Ladder is not necessary and the ascent depends largely on the lover's ability to extract the right knowledge from each discourse[9]. So whereas Sheffield and Howatson's arguments rely on Socrates hinting that "there is the life which a human being should live, in the contemplation of Beauty itself " (211d) suggesting the importance of the activity, I do not deny the importance of the activity. However, I stress the importance of the capacity, refined throughout the ascent, making the contemplation of the Form possible at the first place. Either way, Diotima ends her speech saying that the person possessing the true virtue can be loved by gods and become as close to immortality as a human can be.

### 2.3. The Analysis of the Ladder

Now it is important to extract key points from *Symposium* and provide some analysis helpful for the essay's purposes which is to analyze the Ladder so then a placement of practical and theoretical geometry can be conducted. G. R. F. Ferrari in his *Platonic Love* writes that the Ladder shifts between two "frameworks" of interest, "from the beautiful target of … [the] discourse to its beautiful topic"[10]. The point Ferrari is making is that the Love for a beautiful body stimulates the lover to have a beautiful discourse (*logous kalous*) with his counterpart. And that is the point of "departure" for the lover. Ferrari writes that given the other levels of the Ladder, the discourse will be "limited to enthusiasm for the physical beauty and prowess of the beloved"[11]. Apart from that, Ferrari also claims that the starting point of this lover is already higher than the ones who procreate children rather than the any kind of knowledge. However, the decisive moment of the journey will be how the lover reacts to this discourse

[9] In sub-chapter 2.3., I interpret Ferrari's analysis of the Ladder and he claims also that the ascent is dependent on what implications the lover takes from the discourse(s). In other words, the ascent is not necessary but dependent on philosophical qualities of the soul and the knowledge he takes from the discourse.
[10] Ferrari, "Platonic Love," 257.
[11] Ibid, 256.

and his new knowledge. “His “beautiful” words have beauty as their topic - not the beauty of this body alone, but also bodily beauty in general, because to praise something is to insert it in its comparison class.”[12] This will have consequences for the initial relationship: the lover of the beautiful body inspired by his counterpart to contemplate the beauty of all bodies has shifted his interest from ‘beautiful target to beautiful topic’. So then his enthusiastic love for one body has weakened. If this transition is accomplished, the lover has got a potential to progress to the highest levels of the Ladder since the same transitions, in principle, between beautiful target and topic occurs similarly between B3-B4 and B5-B6, Ferrari claims.

For example, not being attached to people or any products of them, object of interests of the Love of knowledge has changed from sensible objects to intelligible but the relationship between the pairs, B5 and B6, is the same. To explain, Ferrari claims that the Love for knowledge “causes … [the lover] to give birth once again to beautiful discourse - now the discourse of philosophy”[13] and that “now, as before, the initiate's concern is transferred from the beauty that enticed him to the beauty that he has generated”[14]. At B5, the lover initiates the discourse of philosophy and contemplates the sea of knowledge in its great multiplicity which he does, in my opinion, by theoretical mathematical sciences such as theoretical geometry and arithmetic. In the process, he looks up, as Ferrari writes, to see the Idea of Beauty in its uniformity, unity and eternality. Ferrari argues that the lover went “from simply “doing” beautiful philosophy (considering what is beautiful in the varieties of knowledge - the beauty that attracted him, 210c7) to grasping the beauty of his philosophy (the beauty that he engendered, 210d5)”[15].

So then the whole Ladder consists of three pairs: three different levels which share the same relationship between each other, or “frameworks”: from the beautiful discourse to beautiful topic which is the knowledge generated from the discourse, the ‘product’ of the discourse. Moreover, the second important takeaway is that the Ladder progresses from the most relative to no relative at all, to the beauty of the Form. Each level in the Ladder is continuously truer and more harmonious than the level beforehand. This quality will be an important feature in the next sub-chapter as well as in the whole essay.

[12] Ibid.
[13] Ibid, 258.
[14] Ibid.
[15] Ibid, 259.

## 2.4. Kalós, Beauty and Geometry

Aside from the main question *why* geometry is beautiful, the more fundamental question is whether geometry is beautiful at all in Plato's view. After presenting the Ladder and some of its fundamental qualities, I provide some direct evidence that aesthetically, ethically as well as in terms of utility, in all the basic meanings of the word *kalon*, geometry, and generally mathematics, are considered beautiful by Plato. I start with refining the word 'beautiful'.

The nature of the word beautiful (*κᾰλός, kalós*) which is closely related to Love and desire providing the main incentives to secure the good things can have three meanings: i) aesthetic meaning - beautiful, attractive, good-looking; ii) moral meaning – honorable, noble and in that case it is close in meaning to *agathos*: the general adjective for good in the sense of being suited to a desirable purpose or function, or being morally good. The corresponding abstract noun is *arête* (virtue, goodness); and iii) utilitarian meaning – with regard to use, good, of fine quality[16] meaning that a particular entity is genuinely good, beautiful in sense of being useful, meeting its desirable purpose or end. Howatson and Sheffield write that "the moral sense is often found in the *Symposium*, where 'what is good' sometimes has the abstract sense of 'the good'. If what is good is also attractive, *agathos* comes close in meaning to *kalos*"[17]. In *Symposium*, the beautiful also refers to aesthetic qualities especially in case of causing attraction and desire for the objects to be in our possession physically. The object are considered to be beautiful and, therefore, attractive to us. The last meaning, utilitarian, can also be found in *Symposium,* regarding laws and practices if they fulfill their purpose, so then they are being beautiful in the utilitarian sense. Later in the essay, I will come back to the meanings of 'beautiful' in relation to geometry and geometrical figures.

Next, I want to provide some direct evidence that geometry is beautiful to Plato. Since *Symposium* is dependent on interpretation and thus requires context, I start with the passage from *Philebus* in which Socrates talks about geometrical figures. In *Philebus*, while discussing nature of pleasures, Socrates is engaged in a dialog with Protarchus arguing for the existence of higher pleasures such as those of mind. An important question is asked by Protarchus: what are the pleasures which should be conceived to be true? Socrates answers that the true pleasures are the ones which are painless and unconscious "arising from what

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[16] LSJ - Ancient Greek dictionaries, "Καλός".
[17] Howatson and Sheffield, *The Symposium,* 64.

are called beautiful colours, or from forms, most of those that arise from odours and sounds”[18] (51b). This is then further specified:

*“For when I say beauty of form, I am trying to express, not what most people would understand by the words, such as the beauty of animals or of paintings, but I mean, says the argument, the straight line and the circle and the plane and solid figures formed from these by turning-lathes and rulers and patterns of angles; perhaps you understand. For I assert that the beauty of these is not relative, like that of other things; but they are always absolutely beautiful by nature and have peculiar pleasures in no way subject to comparison with the pleasures of scratching; and there are colours which possess beauty and pleasures of this character.” (51c-d)*

So in search for higher pleasures, Socrates makes case for geometrical figures arguing that they evoke certain higher pleasures.  The beauty of form is closely related to straight lines, circles and, in general, to geometrical figures. Plato thinks that the reason these pleasures are higher is that the objects are truer and have ‘absolute beauty by nature’. The beauty of them is not relative, it is not a subject to any change which is in contract with animals and paintings, sensible object subject to generation and corruption. In relation to meanings of *kalós*, these geometrical figures could be mainly beautiful aesthetically. Socrates is appealing to their changeless nature, absoluteness and even to regularity. In *Timaeus*, Plato introduces so-called Platonic Solids - convex regular polyhedrons in three-dimensional Euclidean space. There Socrates stresses the regularity of the Solids as an important quality[19]. Since regularity is connected to order (what is regular has to be somehow arranged in a pattern), then Socrates pointing to geometrical figures as beautiful could be largely in the aesthetic sense of *kalós*. And since geometry studies primarily these geometrical figures then in this sense geometry can be considered beautiful.

Now to find the direct evidence for geometry to be beautiful in *Symposium*, the Love for knowledge seems to be the right place to start. The exact content of this love is not specified but Socrates talks about this knowledge with the term *epistēmē* (*ἐπιστήμη*). Specifically he says that “after this his guide must lead him to contemplate knowledge in its various branches, so that he can see beauty there too” (210d), he uses the term *ἐπιστημῶν*

[18] Plato, *Philebus*. “… which are given by beauty of colour and form” in translation by Benjamin Jowett.
[19] Lloyd, “Symmetry and Beauty in Plato,” 455.

*κάλλος*, *epistēmõn kállos*. The term *epistēmē* is usually translated as knowledge, or science, and it relates to the system of structured understanding similar to scientific knowledge. Plato contrasts *episteme*, real knowledge with *doxa*, a belief or opinion, and this fundamental distinction is broadly speaking identified with distinction between intelligible and sensible world. This term, *epistēmē,* is also distinct from the term *techne,* a craft or applied practice. It is debatable whether Plato uses these terms, *epistēmē* and *techne,* interchangeably but it seems that in some of his dialogues he does so.[20] Damon Young says that *techne* "was not concerned with the necessity and eternal *a priori* truths of the cosmos, nor with the *a posteriori* contingencies and exigencies of ethics and politics.… Moreover, this was a kind of knowledge associated with people who were bound to necessity. That is, *technē* was chiefly operative in the domestic sphere, in farming and slavery, and not in the free realm of the Greek polis"[21]. At this point, I want to identify the knowledge in the Love for knowledge with geometry. In the next chapters, I discuss Socratic education in the *Republic*, distinction between theoretical and practical geometry and between *techne* and *epistēmē* in greater details but for now, it is safe to say that the general term geometry refers to knowledge though the nature of the relationship will be specified later. This is thus the second direct evidence for geometry being in fact beautiful.

To support the second evidence, *Philebus* can offer some additional reasons to consider geometry to fall under the Love for knowledge in the Ladder. Plato talks about geometrical figures in *Philebus* as not relative and simply true. The same dynamics can be observed in the Ladder which progresses from very relative and subjective domain, from a body, to the Form of beauty, absolute, eternal, immutable entity which is beautiful to everybody capable of contemplating it. And since Plato in *Philebus* closely relates the Form of beauty with abstract geometrical figures, though not directly with the Forms themselves, it is reasonable that geometrical figures would be right under the second most beautiful category, the Love for knowledge. And since geometrical figures are subject of study of geometry, this would support the second evidence I provided above.

Next argument for considering geometry beautiful comes from *Symposium* though more interpretation and some assumptions are needed. Diotima says that "by far the most

[20] Stanford Encyclopedia of Philosophy, "Episteme and Techne".
[21] Young, "Philosophy East and West," 190.

important and beautiful expression of this wisdom [of the soul] is the good ordering of cities and households; and the names for this kind of wisdom are moderation and justice“ (209a). Here she talks about virtues and what good souls do – they devote themselves to *ordering* of cities. In this case, the ‘ordering’ is a practical expression of virtues, specifically justice. In the next chapter, I am going to argue that ethical education is the fundamental objective of the *Republic’s* curriculum and that, especially, theoretical components of the education such as theoretical geometry and arithmetic, are constitutive part of this ethical education. Since geometry is fundamental in education of virtues responsible for ordering of cities which are ‘the most beautiful’, then geometry, broadly speaking, has to be somehow beautiful as well. In this sense, the beauty of geometry would mainly refer to its moral meaning of the word. The precise nature of the relationship will be discussed later but for now, this can be seen as a general argument to consider geometry as beautiful in Plato’s view.

In many cases, this overview has already touched upon the reasons *why* geometry is beautiful. The direct evidence can be considered *Philebus* because *Symposium* requires context since the knowledge in the Love for knowledge is not directly specified there. Nevertheless, the precise nature of the knowledge will be provided later in the essay. For now, to summarize, geometry can be beautiful in all of the meanings of the word *kalós*. First, I argue that geometry is connected to the virtue of justice, to the ‘good ordering of cities’, and therefore, it is beautiful in rather the moral meaning of the word. In the next chapter, I will talk precisely how geometry is part of ethical education using references from the *Republic*. Second, in the aesthetic meaning of *kalós,* since geometrical figures, which are subject of study of geometry, were ‘simply true and not relative’, they were necessary stable. The geometrical figures were also regular, as, for example, Platonic Solids are, and they were not a subject to generation and corruption. All of these properties evoke aesthetically pleasing beauty because of their high degree of ordering. About this kind of aesthetic beauty characterized especially by regularity and containing geometrical figures, I will not talk more in the essay. Lastly, the utilitarian meaning can be twofold. First, the ordering of cities is very useful virtue if the Laws and institutions fulfill their purpose for the Good. For that craft of ordering of cities to develop, I will argue that geometry is fundamental.  Second, practical geometry of land measuring, or generally the kind of geometry having practical and visible consequences for the people, was seen by public as more useful than theoretical geometry. Thus this practical geometry could be seen as *kalós* in this utilitarian sense as well if the *telos* of the craft would be met that is if practical geometry provided the service it was meant to provide. About the practical geometry, its utility and beauty, I will talk later on in the essay.

In the following chapter containing the analysis of the *Republic*, I will discuss the role of mathematics and geometry in particular within Platonic educational system and within his philosophy of Divided Line and the Cave. It will be clearer that one of the main values of mathematics lies in its ability to systematize and *unify* knowledge before the students can contemplate the Good. Other important findings will be a distinction between theoretical and practical geometry and the placement of these scientific fields in the Line and the Cave as well.

# 3. The Divided Line and the Allegory of the Cave

## 3.1. The Republic and the Divided Line

In the following sub-chapters, I provide some general context for the *Republic* so I can introduce the Divided Line and the allegory of the Cave. In book VII of the *Republic,* Socrates talks extensively about the education and its role on character-development. Together with Paul Pritchard's book *Plato's philosophy of mathematics,* I determine the position, psychological affection and origins of practical and theoretical geometry. Then I use M. F. Burnyeat's paper to show that geometry is a constitutive component of ethical education. I carry all these findings into the final chapter to determine: i) why generally the whole geometry is beautiful and ii) how much practical and theoretical geometries are beautiful according to Plato.

The *Republic* is Plato's most prominent work discussing questions of justice (*δικαιοσύνη*), the just order of the Ideal State as well as the just order of a character. Throughout the whole book, Socrates with other participants of the dialog, mostly Glacoun, argues for the constitution of the Ideal State which is as just as possible and which utilizes the best of each human to address national as well as each individual needs the best way possible. The Socratic State is divided into three classes: producers, auxiliaries (or Guards), and guardians (or philosophers). Socrates argues that the soul of every individual has a three-part structure analogous to the three classes of society: a rational part of the soul, which seeks after the truth, a spirited part of the soul, which desires honor and is responsible for our feelings of courage and anger, and an appetitive part of the soul, which desires all sorts of things, money in particular. The guardians are philosophers who have proved themselves in a war, have conducted lengthy precisely defined education and have passed through many difficult mental and physical challenges. They are educated enough to ensure that the social order is understood and maintained so that the Ideal State is preserved. Their soul seeks after truth. They are philosophers. One of the main objectives of education is to raise philosopher-rulers capable of governing justly. This is one of the reasons motivating Socrates to establish the educational structure in the *Republic* which is largely dependent on the study of geometry as will be shown later.

Now, I want to introduce the Line so that we can understand some of the important divisions and transitions in reality-perception. The analogy of the Divided line (*γραμμὴ δίχα τετμημένη*) precedes the Cave analogy and it can serve as the first relevant measure to understand Plato's 'psychological affections' (*παθήματα*), the way our soul sees the world. This analogy is one of the big narratives in the *Republic.* Since Socrates knows his audience, narratives are important tools to transmit a message to interlocutors. He knows his task, namely, "to lead souls from the dreamy world of appetitive transgression toward justice … [since] only then can the Good be seen—and, paradoxically, only by seeing the Good can rightful boundaries be honored"[22]. Therefore, he *starts* with the *images* in a sense of a story (an 'image', narrative) told to children as well as in an ontological (and metaphysical) sense going from the lowest images of reality to the highest Forms (the meaning of will be explained with the Line later). Socrates does this because some of the interlocutors are at level of appetitive transgression (as for example Thrasymachus) and Socrates is aware that they need "to draw their souls upward on their journey"[23]. The use of images is a result of that. The use of narratives, *images,* has this deeper meaning.

The analogy itself goes as follows: Plato says in *the Republic* 509d–511e that the Line is divided into four pieces with different ratios. The point is for now that the ratios are precisely defined and are not random. The first two sections (L1-L2) represent visible world whereas the last two sections (L3-L4) denotes the world of being. The affections of the Soul

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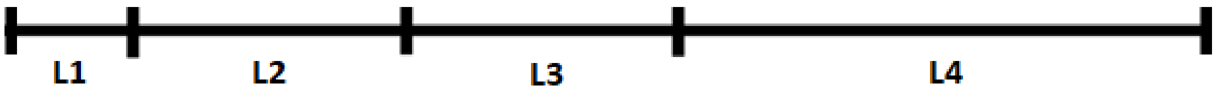
[22] Mitchell, *Plato's Fable*, 45.
[23] Ibid.

are described in succession going from lower levels of reality and truth to more real and truer: from conjecture (L1, *εἰκασία*) to belief (L2, *πίστις*) to thought (L3, διάνοια) and finally to understanding (L4, *νόησις*). To each of this group belongs a corresponding group of things causing the specific knowledge: L1 consisting of "images" such as shadows and reflections; L2 are physical objects themselves (*ὁρατά*); L3 are abstract objects, ideas, especially those of mathematics and then at L4 are the Forms themselves grasped by dialectics. The Form of the Good is the highest entity of all; this Form is not only the basis for understanding all other forms but also for understanding everything else. The Good gives meaning to everything top-down (the *Republic,* 508 a–c). However, to contemplate the Good, people progress from bottom-up starting at L1 and going through all other levels (the *Republic,* 514-515). This is why I said before that Socrates starts with images in this, *inter alia*, ontological sense. The final haven, ultimate goal is the Good contemplation.




### 3.2. The Allegory of the Cave

The Divided Line can allow us to approximate the Cave allegory because the Line runs through the allegory as well. In the first part of the ascent, prisoners are immobilized and can observe only shadows (C1) of objects mingling on the wall and are unconsciousness of any other realities. The psychological affection is *eikasia* with imagination being predominant mode of thinking. The sources of shadows are objects carried behind the wall of prisoners by fake "prophets" who might be sophists and poets[24]. Behind the objects lies fire, the only source of light so that the shadows are created on the wall in front of the prisoners. Basically, the plot starts when a prisoner is *forced* to see the original objects creating the shadows. This turning point, T1, (*περιάγειν*) or conversion (*μεταστραφή*) [25] is the beginning of the journey of understanding. At first, the prisoners' eyes need to adjust after being exposed to the sharp light of fire to see the originals of the shadows, the puppets (C2).

In the next stage, the prisoner is further forced to go up outside of the Cave and thus the second transition, T2, occurs. Once again, the moment of eye adjustment follows since the

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[24] Burnyeat, "Plato on Why Mathematics is Good for the Soul," 44.
[25] Ibid, 43.

sun light is even sharper than the twilight of the Cave. Therefore, the prisoner gradually sees only shadows, reflections of the actual people and objects (C3), then finally the objects themselves whereas at the end, he looks at stars and the Moon (all C4). At the end, he is capable of seeing the Sun (C4) as it is, not through reflections but rather directly and contemplates about it as it is. The order is made explicit in *the Republic* and has its purpose of which I talk hereafter.

In the second part, the descent back to the Cave represents the third transition, T3. It is an important continuation of the allegory. Socrates argues that prisoners in the Cave compete among themselves to properly predict the future of the order of the objects and honor the ones who are the best in this discipline. Our "enlightened" prisoner who stepped back would hardly partake in these foolish games according to Socrates and would "undergo everything rather than live as they do down there" (516e) because he got to know the real world outside. Moreover, as suddenly being dragged down to the Cave, he would suffer from temporary blindness due to the exposure to darkness and would appear to be ridiculous in the game of predicting the right order of shadows and sounds attached to them (which are created by object-carriers). Socrates memorably asserts that men would say of him "that in going up to the top he had come back with his eyesight ruined and that it wasn't worth even attempting to go up there " (517a). At this stage, Socrates makes it clear: he tells us to connect this allegory to the Divided Line and to the previous analogy of the Sun (507b–509c).

To put it simply, the description of the analogy of the sun starts with Socrates not being able to define goodness at first. So he proposes to Glaucon to describe first 'the child of goodness' (*ἔκγονός τε τοῦ ἀγαθοῦ*) who he identifies with the actual sun. Socrates claims that as the sun illuminates the sensible world, basically granting humans the ability to see things and be seen, so then similarly the idea of the Good illuminates the intelligible world with truth and allows us to contemplate the intelligible objects. This suggests that the Idea of the Good is the Sun (C4) outside the Cave and the fire is the sun (or the child of goodness) inside the Cave for that to be coherent with what Socrates is telling us. Moreover, it also suggests that if C1-C4 objects were not partaking to some extent in the Good either directly or via the child of goodness, fire, they could not be of the interest of psychological affections of prisoners because they would not see them at all. Now if these analogies are related to the real world situations, the logical question is how the ascent to the Good is possible, by what means precisely. This is the topic for the following sub-chapters.

### 3.3. The General Framework

Since the Divided line has showed us various degrees of the Soul affections and corresponding types of objects, the rulers have to be the wisest and smartest among all people and that is only the case when they contemplated the Good. As I said before, Socrates treats the Cave as a representation of the real world, so the question is what causes the transitions T1 and T2 in practice?

I think that the *general answer* to the question is *education*, mathematical disciplines[26] in particular being responsible for T2 transition. However, Socrates argues at length for mathematics being the bridge between the Cave and the outer world. He thinks that the Guards before potentially becoming the philosophers need to conduct several years of proper mathematical training at the different stages of their lives. They start learning in infancy by playing. After two years of military training, the auxiliaries at age of twenty get *ten years* of mathematical study; everything from plane geometry to solid geometry or arithmetic. Moreover, the auxiliaries are expected to do all the possible "reincarnations" of mathematics such as arithmetic, geometry, harmony or astronomy. The goal is nothing less than to work towards the theory that will *unify* the different fields through mathematics in a *systematic* manner (which is precisely what not many Greek citizens thought to be useful (no immediate instrumental usefulness) or even possible (the sciences were not established fields of study))[27]. However, nothing is said that the Guards will be making any significant discoveries within those fields; all this training is to become the best possible ruler(s) of the Ideal State. At the end of this process stands the understanding of the Good grasped and reasoned about by dialectic, the highest science in the Line. Then the argument is that if everything goes well, the philosophers "will think of the mathematical structures they

[26] I use the term mathematical disciplines to refer to all sciences proposed by Socrates in the education: arithmetic, geometry, harmony or astronomy. I use the term mathematics especially for arithmetic and geometry both practical and theoretical branches because, as I will argue, they are the least complex, at the beginning of the transition and the ones most fundamental. Moreover, harmony and astronomy in its theoretical form are related to metaphysics of *Timaeus,* another differentiating aspect of the first two.
[27] Burnyeat, "Plato on Why Mathematics is Good for the Soul," 66.

internalised on the way up as abstract schemata for applying their knowledge of the Good in the social world"[28].

### 3.4. The Educational System in Detail

Since Socrates is saying that the development towards *noesis* is possible and that one is *becoming* to be a philosopher then the question still remains the same: *how* and *why* is it precisely possible?

In Book VII of the *Republic*, Plato provides the detailed description of the education system and explaining at length why mathematical disciplines are responsible for transition T2. Socrates lists several criterions for the science he is looking for. First criterion is that the science turns the soul "from what is coming-to- be to what is" (521d) without implementing the faculty of sight and focusing on the intelligible realm of eternal abstract entities. However, Socrates notes the second criterion: usefulness both *practical* usefulness in war since the education is suited for protectors of the system and *theoretical* for reasons to be said later. The third criterion includes universal application "which all crafts, thinking, and knowledge make use of " (522c) and, fourth, the tendency towards the Good.

Mathematics, arithmetic in this instance, meets these criterions: *number and calculations* are involved in all sciences and crafts, Socrates claims. However, what kind of arithmetic are we talking about? Socrates makes case for twofold distinction: practical (i.e. counting physical objects) and theoretical (i.e. abstract arithmetical objects) arithmetic. Contrary to the common beliefs at that time, Socrates thinks that theoretical arithmetic is more useful in terms of character and ethical development for the students and potentially leading to the capacity to contemplate the Good.

One of the reasons for this argument is the base distinction of world of being and of becoming. Socrates discriminates two objects: "first those things among our perceptions that do not require the mind to examine them because they are adequately apprehended by sense perception, and those that demand that it should be used to investigate in every way because the sense perception is producing nothing sound" (523b). In other words, Socrates is deriving the origins of the abstract objects from the objects of sense that cannot be adequately judged only by senses. This means that also practical calculations *are important*

[28] Ibid, 73.

on the way *towards the Good* and in the development of mathematical thought as long as they lead towards completely abstract objects. The example is given after Glacon hesitates. For sight, Socrates claims, a finger is a finger – no intelligence is involved since sight only sees things. The touch perceives the qualities of objects as other senses do the same. "When the sense has been designed to identify something hard it has also been designed to make contact with something soft: it relays to the soul that it has sensed that the same thing is both hard and soft" (524a). For the soul, this is a perplexing problem and in order to get the *right measurements* and understand the nature of things (if an object is heavy or light etc.), the "calculation and understanding" (524b) is called to help.

Thus I argue that even practical arithmetic has an elevating effect on the soul thanks to the invocation of the abstract numbers. It is the first step towards the world of being but not the only one possible. Socrates is building his order of sciences for his educational system starting from less complicated to more difficult and, therefore, geometry is his next station: geometry amounts to both numbers *and* abstract mathematical objects. Practical exercise of geometry is helpful from similar reasons as practical arithmetic: it draws Souls attention to intelligible realm. However, Socrates argues that to draw the geometrical figures is "worthless" as long as they are thought to be the main objects of interest[29].

He thinks that practical geometry is necessary starting point but ultimately the focus of the education lies in geometry of abstract entities, geometrical figures, and not on geometry of sensible diagrams or land measuring. One of the reasons is that the knowledge of geometrical figures is knowledge of eternal, more real entities which Socrates claimed already in *Philebus*. They are closer to the truth as opposed to perishable, ever-changing and transitory objects of the sensible realm.

The specific example is given concerning a unit which has particular favorable properties for Socrates. In *Elements* Book VII, Euclid spends a lot of time providing precise definitions of the most fundamental mathematical concepts, including a number, or, in better words, *a unit*. Greeks understood a number as a multitude of units that is a number *four* consisted from *four* separate but completely equal units. The unit which equaled one was defined by Euclid

[29] Socrates even makes a sarcastic complaint on the contemporary practice of geometry: "They talk as if they were doing something and making all their terms to fit their activity: they talk about making the square, applying and adding, and similarly with everything else; but in my view the subject as a whole is studied for the sake of knowledge." (527a)

as being “in accordance with which each of the things that exist is called one”[30]. When Socrates refers to one (as a numeral) or contemplates about arithmetic, he has *usually* in mind the same concept of a unit which was later precisely defined by Euclid.

Burnyeat can provide some additional insights: given anything that exists either as a sensible or intelligible object and setting aside all the *many* features it can have, the *one* can be abstracted from the object thus representing an indivisible Euclidean unit (for example a bike can be unit as well as many parts such as handlebars, tires etc.). The indivisibility is realized in *thought* and not anywhere else by setting such and such conditions. Moreover, as I already outlined, Greek arithmetic does not know anything as the number three, only a set of three units, completely alike. So then Burnyeat presents an example of practical calculation with these units saying that “numerical equality is equinumerosity, not identity”[31] so then four plus four is not identical with the number eight but rather a pair of quadruplets consists of as many units as octuplets. The calculation thus always has in mind a unit, ever-lasting, changeless mathematical entity which existence is realized in thought.

Even more interesting for the purposes of the main question is the understanding of a unit in geometry. Euclid defines a geometrical unit as a line AB of *random* length. This line is said to represent our *one* in numerical notation. As Socrates comments, there is no doubt that the line can be divided into smaller parts no matter how long or short the unit line is, the critics can always say that the line *one* is divisible into smaller pieces. The critics then say that since AB is divisible, or, in other words, it can be made smaller (which is not true about a geometrical point, for example), it is not *that* unit. However, Plato writes that these claims do not falsify the theorem, only changes the measures. This means that if AB as a unit is made smaller just for the sake of it then *only the measure were changed*. Socrates claims that what needs to be invited here is the *thought* in order for the soul to abstract from many to one and grasp the line as a unit, as one thing, though dividable in principle (as in the bike example). In principle, if the exercise is conducted, then Socrates continues in this part of Book VII saying that geometry will “be the soul’s transport to the truth … and be productive of philosophical thought”(527b) all starting with the invitation of thought itself.

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[30] Burnyeat, “Plato on Why Mathematics is Good for the Soul,” 31.
[31] Ibid.

Regarding the order of the education so far, Socrates places first arithmetic, then plane geometry (a study of two-dimensional geometrical figures) and solid geometry (a study of dimension and depth) as a third science even though the science itself had not been well established and such an idea was very revolutionary[32]. The order is made explicit by Socrates and it follows the general rule of going from easier to more complex studies.[33] However, the curriculum is not complete because Socrates lists astronomy and harmony in the structure too. This essay focuses primarily on geometry, both practical and theoretical, potentially on mathematics which includes theoretical and practical arithmetic. I consider these to be fundamental in the transitions T2 as it was shown above. Plus, these sciences are definitely less complex then astronomy and harmony given where Socrates lists them in the education. Moreover, astronomy and harmony seems to be, in a way, applied sciences though they are also supposed to be studied primarily theoretically, Socrates says. For now, I proceed with the placement of practical and theoretical geometry, and I will come back to these disciplines later in this chapter.

### 3.5. The transition T2

I outlined the principles behind the educational system and how and why arithmetic and geometry are the sciences largely responsible for T2. In general, I said that transitions T1 (prisoners realizing existence of the Cave's fire) and T2 (prisoners leaving the Cave) are caused by education, T2 by mathematics. However, can the Cave allegory justify or explain mathematics to be the main focus of the education? Can the Cave provide additional insights into the matter?

First, Socrates told us to connect the allegory of the Cave with the Divided Line. Therefore, it might be helpful to look at the Divided Line as a measure to determine the exact position of both practical as well as theoretical geometry in relation to the Good. Paul Pritchard in *Plato's Philosophy of Mathematics* provides detailed analyses of the Divided Line. I used his notations for the text to be coherent so the content of L1-L4 is the same: L1 are images such as shadows and reflections, L2 as objects natural world (animals) as well as man-made objects. Pritchard writes that important is the relation between L1 and L2; "the relation of L1

[32] Ibid, 1.

[33] Apart from the invitation of thought, Socrates lists another minor benefit for the Guards – both students of theoretical arithmetic and geometry are far quicker in learning than their comrades – another reason to include them in the education.

and L2 … is the relation of image to original."[34] This is consistent with the Cave; prisoners watch shadows (C1) and the originals (C2), the puppets, are seen after the first awakening. Pritchard writes that the same relationship holds between L3 and L4. Socrates puts in contrast mathematical reasoning (L3), which uses hypothesis to proceed to conclusions, with dialectical reasoning (L4). The 'mathematical' conclusions are not and cannot be the first unhypothetical principles which are ultimately the Forms. In contract, the dialectical reasoning is characterized by starting with 'genuine' hypothesis to finish at what Socrates calls 'unhypothetical', the first principle of everything, the Good.

The reason to believe that between L3 and L4 is the same relationship as between L1-L2 is, first, the Cave and, second, the passage in the *Republic.* Shadows and reflections outside the Cave (C3) are images of actual real objects, of the Forms (C4). Since Socrates told us to connect the Cave with the Line and the sun analogy then L3 is an image to L4, an original. Second, in the *Republic* (511), Socrates says that "intelligible that I was talking about, where a soul is forced to use hypotheses in its search for it, without working toward a first principle because it is unable to escape from its hypotheses to a higher level, but by using as images the very same things of which images were made at a lower level and, in comparison with those images, were thought to be clear and valued as such" (511a). In other words, he says that images at L2 are of the same kind as images at L3 though they are now used directly as images of the Forms. This is a crucial argument of this section: the ontological status of L2 and L3 is the same. Thus, to conclude: C1 are the shadows, C2 the originals (puppets), C3 are the shadows and reflections after the ascent and C4 are the Forms that is animals, heavens, Sun. Then we got levels of the Line and they correspond to the Cave levels, thus L1-C1, L2-C2, L3-C3, L4-C4. The content of L1 is shadows and reflections and L2 is objects of natural world. L3 and L4 belong to intelligible realm: the content of the former is reserved for mathematical objects and thinking, the latter for dialectics, and the highest knowledge of the Forms.

Moreover, regarding the relationships, the whole sensible realm (L1+L2) is an image to the original of intelligible realm (L3+L4), Pritchard writes, and it can be found in the text in the *Republic* (511). All of this findings, some of them based on interpretation, can formally be

[34] Pritchard, *Plato's Philosophy of Mathematics,* 91.

written as such: i) (*L1+L2) : (L3+L4) :: (L1:L2)* and ii) *L1:L2 :: L3:L4*[35]. Now Pritchard writes that "the result of applying these conditions will be to produce a line in which the sections L2 and L3 are equal"[36]. This equality can also be produced from what is said about the Line in the *Republic.* There are intelligible and sensible realms compared to a line which is then divided into two sections – one part representing intelligible, the other sensible world. Pritchard writes that better manuscripts tell us that this division is unequal. Nevertheless, "we are instructed to divide the line again, each section in the same ration as the original division. Thus, if the four resultant sections are L1, L2, L3 and L4, where L1 and L2 together represent the visible, and L3 and L4 the intelligible, we [we will get the two conditions above]"[37]. The bottom line is that L2 and L3 are equal: based on the explicit notes in the text, based on the formal application of the conditions about the Line and also based on the interpretation of the text (putting together the Cave, Line and Sun analogy).


This is significant conclusion for further argumentation. Moreover, for the sake of consistency with the two conditions, Pritchard claims that both L2 and L3 should be the image of L4 and that L1 should be the image of both L2 and L3[38]. Now the question is if all of this formal adjustment of the conditions is coherent with the actual analogies.

So if, as Pritchard argues, we have *threefold* ontology within the Line, that is image of images (L1), images (L2-L3) and the originals (L4), what differs L2 from L3 (or similarly C2 from C3) if ontologically they are the same? The answer Pritchard provides is that the difference is the state of mind. The difference between L2 and L3 is that prisoners know that while going through transition T2 i) they have "woken into another dream"[39] and that ii) objects at L3 are only images of originals but they are seen clearer now. This is what Socrates basically tells us in the *Republic* 511 when he states the *lower images* (L2) *were thought* to be clear but in contract to the higher level (L3), it was mere confusion. The nature of the transition T1 between L1 and L2 is characterized by waking up from a dream (from a "reality" of L1) but

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[35] Ibid.
[36] Ibid, 91.
[37] Ibid.
[38] This is because Pritchard says that since L2 and L3 are equal in size then: L1:L2 :: L1:L3 and because L1:L3 :: L2:L4 (which he says is produced by adjusting the proposition ii) then we get L1:L2 :: L2:L4.
[39] Pritchard, *Plato's Philosophy of Mathematics,* 92.

falsely considering ‘objects’ at L2 as originals. This is once again consistent with the Cave in which many would be stuck if not being dragged out of the Cave. The free prisoners at C2 would be comfortable with their current “reality”.

Consequently, the nature of the transition T2 is similar; it is as awakening from a dream. Whereas transition T1 is characterized by waking up from the dream thinking the *true reality* is the objects at L2, the nature of T2 is the realization that the mind has woken into another dream knowing this time that *it is only a dream* (only an image). The reason is that what was before considered a reality (L2) is clearly seen as mere image because the true reality of the Forms is intuitively comprehended. This is perfectly coherent with the Cave. After the ascent from the Cave, the prisoner sees first shadows and reflections due to physical limitations of his vision. It would be foolish of him to think at that point that the reflections are the true reality. In contract, to assume that the objects at C2 are real was perfectly understandable assumption because there was not world outside the Cave, was it?

Thus, Pritchard writes that “though the states of mind … are four, the ontology is threefold … forms, images of the forms, and images of the images of forms”[40]. Then there are clearly four levels in the Line, the four ‘psychological affections or states of mind but only three ontological levels. This is important statement but further clarification is needed.

To summarize all the divisions so far, the broadest distinction is between the world of becoming and the world of being. To this distinction, the different capacity is due: the objects of knowledge (*έπιστήμη*, *episteme*) differ from the objects of opinion (*δόξα*, doxa). Then there is the Line that is the four kinds of psychological affections: L1-L4: Eikasia (εἰκασία), Pistis (πίστις), Dianoia (διάνοια), Noesis (νόησις) . The ontology of them was said to be threefold: forms, imagines of the forms, and images of the images of forms but more arguments is needed to solidify the argument. Moreover, it was said that the faculties responsible for acquisition of the ‘understanding’ is twofold: the one directed at intelligible objects (objects of knowledge), the other on sensible objects (objects of opinion). It means that the same faculty which is capable of seeing directly the Forms at L4 can get to know the images at L3 because both are intelligible, thus object of knowledge and thus subject to *episteme*. Equally, objects at L1-L2 are sensible and are subject to opinion and *doxa.*

[40] Ibid, 94.

Now the question of how precisely is mathematics responsible for conversion T2 *and* where practical and theoretical geometry can be found in the Line can be settled. Prisoners in the Cave looking at the shadows are seeing images of the images with corresponding state of mind *eikasia* and employing the faculty responsible for sensible things. As I already said, this means that the puppets are images of the Forms and that the corresponding state of mind at level C2 is *pistis*. In order for the allegory and the Line to be consistent, objects (reflections and shadows outside the Cave) should ontologically be the same but at the same time be seen through the faculty responsible for seeing intelligible objects *and* be understood better at the level of *dianoia*. This is in contract to C2 where faculty employed is *doxa* and it is within the realm of becoming. It was already said that they differ in the state of mind, in the psychological affection and that in the Line these two parts have the same length. However, some further clarification is needed to solve the puzzle that "after we leave the cave [shadows and reflections] will be at the same ontological level as the images which cast the shadows"[41].

A passage in *Sophist* (265e-266d) might help. Socrates draws a distinction between images of divine origins which, in case of the Cave, corresponds to C3, shadows outside the Cave, and man-made shadows corresponding to C1, the shadows inside the Cave, as well as originals of divine origins relating to C4, the Forms, and man-made origins referring, in language of the Cave, to C2, to the puppets. If this new framework is applied to the Cave, it can add yet another reason why objects at C2 and C3 are ontologically the same but in some respects different at the same. The result of applying this framework is that C2 would be man-made originals and C3 would be divine images. This is once again coherent with the Cave because the puppets were or, in principle, could have been created by man. At the same time, they are originals in the sense that they create an image, a shadow. Consequently, objects at C3 are without a doubt of divine origins, in the intelligible realm, and they are in fact images. Thus, Pritchard argues that "Plato uses examples of human imitation for C2, but examples of divine imitation for C3"[42]. On the broader scale, objects which occupy the Cave are all man-made whereas the objects outside the Cave, illuminated by the Sun, are of divine origins. This is again consistent with what we are told about the analogy in the *Republic*.

[41] Ibid*,* 101.
[42] Ibid, 102.

The last piece of evidence to consider C2 and C3 ontologically equal can be found in the *Republic.* Socrates talks about geometry and he says that objects drawn by geometer *have shadows*[43]*. The paintings* drawn by geometer are sensible man-made objects and in the interpretation by the Cave, they would occupy level C2. Consequently, he continues saying "these very things they are forming and drawing, of which shadows and reflections in water are images, they now in turn use as their images and aiming to see those very things which they could not otherwise see except in thought" (510e). In other words, the sensible geometrical drawings have (metaphorical) shadows. These shadows refer to objects C3 in the Cave meaning they are images of divine origins and of the Forms. Moreover, Socrates states that both sensible objects (the paintings) and the shadows of these paintings are useful as long as reminds geometers of the Forms. Thus, they both fulfil the role of an image because their main purpose is to *remind* us the Forms. This can only be done if they both are actual images of the Forms and, therefore, are both ontologically the same. "Whether they are divine or human productions, the mathematician's is not in these things … but in the things they image [that is in the Forms themselves],"[44] concludes Pritchard.

Now I want to argue that objects L2 are subject of study of the practical geometry. First, I think it is reasonable to claim that practical geometry is of human origins and for human needs. The reason is that the practical geometry contains such practices which have immediate instrumental usefulness for people meeting theirs practical human needs. Moreover, apart from the diagrams being factually drawn by men, Socrates says that objects of human origins are products of productive art (or craft) such as house, and geometry is explicitly considered a craft in many Platonic dialogues[45].

Second, as it was said, objects at L2 are confused with reality. Since L2 corresponds with C2, it means that the prisoners with this kind of confusion have passed through the T1 transition and have woken from a dream of the lowest level C1. They now mistake the objects as the final reality which is in the nature of their corresponding psychological affection: *pistis*. And since they are in fact stuck in the Cave at C2, then believing that what they see is the reality

[43] The shadows here are meant as metaphors. Socrates is not saying that the paintings drawn by geometers have actual shadows but rather that these paintings have shadows (an image) in intelligible form. The whole point of geometry is to study sensibles for the sake of intelligibles meaning the sensible drawings are there to remind the intelligible shadows and ultimately the forms themselves.
[44] Pritchard, *Plato's Philosophy of Mathematics*, 103.
[45] I discuss this in the charter 4.

is very reasonable assumption for them. At this point, they would be superior to their enchained comrades in guessing what comes next as a shadow on the wall, thus being honored by them. They would feel good about themselves when, in reality, they do not know what they do not know. In order for them know what they do not know, the transition T2 needs to occur. Then they will realize that they have woken from a dream into what now is conscious dreaming: they now know that the objects in the Cave were images though considered "to be clear and valued as such [as originals]" (511a). After T2, however, they realized that objects at C3 and C2 are the same, both images of the Forms and that C2 objects were far from being the final reality. They know now what was before unthinkable. They are familiar with the images of the Forms and they know that the truest reality comes with the Forms which they cannot see quite yet. Their mental state has shifted from *pistis* to *dianoia*.

Now I will examine what Socrates tells us about practical geometers. "They talk as if they were doing something and making all their terms to fit their activity: they talk about making the square, applying and adding, and similarly with everything else; but in my view the subject as a whole is studied for the sake of knowledge." (527a). The practical geometers thus study geometry (as a study of sensibles) for the sake of *sensibles* and not for the sake of intelligibles as theoretical geometers would do[46]. They are 'making all their terms to fit their activity'. They are enclosed in a circle, in the activity they conduct and they do not see the metaphorical, intelligible shadows their drawings have (510e). Thus, the practical geometers are yet to transition. So then practical geometry, I argue, is occupied with objects at L2 because i) the objects are within sensible realm, ii) practical geometers study for the sake of sensibles exclusively, iii) and because that is the case, they fit the description of *pistis*. Furthermore, practical geometry is of human origins and because geometers think the way they do at the level of *pistis*, they see the drawings as man-made *originals*, as *the* truest reality, not being aware of divine originals at all.

If practical geometry is occupied with objects at L2, so then the placement of theoretical geometry is easier. First, the transitions and in general the ascent is continuous. Second, the transition has to change something by definition. I argue that the change occurs not ontologically but in psychological affection (*pistis to dionoia*), in the origins (man-made

[46] Mueller, "Mathematical method and philosophical truth," 190.

origins to divine image) and from sensible to intelligible realm. Third, Socrates tells us that the sensible diagrams have intelligible shadows (in the metaphorical sense) and that they have to be divine (because they are intelligible and because 'divine image' is classified as shadows and reflections in *Sophist*). This secures us the change from sensible to intelligible realm I talked at the beginning. Next, Socrates also tells us that these intelligible shadows in this metaphorical sense are images: there "a soul is forced to use hypotheses [that is at L3] … by using as images the very same things of which images were made at a lower level [L2]" (511a). Thus we have divine images reminding us the existence of the Forms which are not *yet* seen but the existence of them is present. *Dionoia* fits the description of this psychological affection: there it was argued that the prisoner has woken from a dream into another, conscious, dreaming. It is a conscious dreaming because the prisoner knows that L2 and L3 are images and thus necessary reminds him the original, the Form. The theoretical geometers study sensibles, the diagrams they draw, for sake of intelligibles which then remind them the Form. This also Socrates tells us when he notes that "in my view the subject as a whole [geometry] is studied for the sake of knowledge" (527a). Ultimately, the whole geometry is studied for the sake of knowledge but that is not always the case. Ideally, the practical geometers will transition becoming the proper geometers that is to say the theoretical ones. Thus, I argue that theoretical geometry is then the study of objects at L3.

On the nature of the transitions, I said that both of them are *continuous* but I did not justify it. Socrates at many places stresses out the continuity within and outside of the Cave allegory. For instance, after the Cave departure, he says that the prisoner will gradually see shadows *then* reflections and *then* animals etc. ending with the Sun. The continuity is stressed even with respect to the overall structure of the education from less to more complex disciplines. Moreover, as I already said, the continuous is even the way Socrates structures the dialog: he knows that his job is to lead "souls from the dreamy world of appetitive transgression" thus starting with an image (in a sense of image of image), because Socrates, in my opinion, sees himself as a puppet carrier in some of these situations. Puppets being the images (the stories, arguments) Socrates presents to his interlocutors, such as Thrasymachus. They see them as images of (his) image (or story), they see shadows mingling on the wall because that is where people start their journey as stated in the *Republic*. Socrates knows that the psychological affections are in succession and thus skipping one part is not possible and continuity is a necessity. Socrates makes this explicit in the Line as well in the Cave.

At this point, it is time to address some objections regarding the nature of the ontological status of L2 and L3 as being the same because that is one of the fundaments of this section. I needed to defend that the ontological status is the same and thus it can be studied by one science, geometry in my case, but also to find differences to explain why practical and theoretical geometry differ. Now it could appear from what was said that since mathematics can be studied within as well as outside the Cave, the corresponding state of mind could be for both levels *dianoia* which would lead, as Ross writes in his *Plato's Theory of Ideas*, that "there is not distinction in the cave symbol answering to the distinction between εἰκασία and πίστις"[47]. Ross's assumption is that Plato connects mathematical studies and objects (images outside the Cave) *only* with *dianoia.* And since Ross also sees that Plato admits that mathematics can be studied even within the Cave, then it leads Ross to conclude that εἰκασία and πίστις has not distinct objects assigned to them in the Cave symbolism. Ross thinks that both εἰκασία and πίστις belongs to the lowest levels (to shadows, C1). Moreover, he writes that "the difference between εἰκασία and πίστις was introduced to serve as an illustration of the difference between two stages in the life of intelligence, and once it has served its turn it is tacitly dropped as unimportant"[48]. However, Pritchard claims that, indeed, Plato writes that the Guards will be studying *logistike* "for the purpose of war, and turning the disposition of the soul itself away from transience to truth and reality" (525c) and that in *Laws* Plato says that education is in the form of play and its start with practical arithmetic problems. All of this supports the idea both Ross and Pritchard agree on: mathematical studies starts within the Cave with practical mathematics. However, the quoted passage 525c states that this effectively leads to theoretical, abstract objects, via the transition T2. Thus, this weakens Ross's main assumption that *dianoia* has to be assigned only to mathematical studies. "If this study were to *begin* at the stage of *dianoia,* it would have no conversion to effect; it would be preaching to the converted,"[49] Pritchard writes.

In fact, the absolute essence of T2 is realization that objects at L2 are not originals as previously though *and* that the abstract entities of mathematics are themselves only images

[47] Ross, "The *Republic* and the *Phaedrus*," 75.
[48] Ibid, 75.
[49] Pritchard, *Plato's Philosophy of Mathematics,* 106-107.

as well. This transition corresponds to the change on the Line from *pistis* to *dianoia,* as Pritchard argues meaning that mathematical studies are possible within the Cave and with *pistis* - contrary to what assumed Ross. Pritchard addresses also the second Ross's assumption that both *εἰκασία* and *πίστις* belongs to C1. I won't go into exact details but the claim is that in *Philebus*, Socrates says that *mousikē* is "'full of guess work and imitation', and is introduced as less clear even than 'unpurified mathematics'"[50] with the next on scale is theoretical mathematics and then dialectics. Among other things[51], this leads Pritchard to claim that "*mousikē* extends through C1 and C2, mathematics … from C2 to C3"[52]. Thus, I think that Ross' assumption of identifying *dianoia* only with mathematical studies is not correct. I spent a great deal of time arguing on the nature of transition T2 defending the idea that practical mathematics can be studied within the Cave and associated with *pistis.*

### 3.6. Unification of the Sciences and the Purpose of Education

Now after the placement of practical and theoretical in relation to the Good, within the educational system as well as within the Line and the Cave allegory, it is time to deliver the second important question: what is the purpose of having so much (especially theoretical) mathematics in the curriculum? Why Burnyeat thinks that mathematics is part of ethical education for Guards? My argument was that if the study of geometry constitutes fundamentally the ethical education then it has to be, broadly speaking, substantially beautiful because, in Plato's view, virtue of, specifically, good ordering is a fundamental good.

In order to answer the question, it is necessary to complete the educational system of the Guards: they learn arithmetic and plane and solid geometry. The fourth science fulfilling the criterions listed above[53] is astronomy though understood very specifically. First, there is astronomy for the sake of practical utility that is, for example, for weather predictions. Socrates laughs at such understanding of the nature of the science. Second, it is presented as a study of physical heavenly objects but Socrates again disputes this view. He aims to study

[50] Ibid, 107.
[51] In 402c, Socrates says that "one is not properly educated in *mousikē* before one can recognize the forms of the virtues" (Pritchard, *Plato's Philosophy of Mathematics,* 106). The forms of virtues are puppets at C2 meaning that *mousikē* is supposed to loosen the bonds again, Pritchard writes, and thus it has to start at C1.
[52] Pritchard, *Plato's Philosophy of Mathematics*, 107-108.
[53] That is the tendency towards the intelligible realm, towards the Good, universality and Socratic usefulness of evolving the capacity to contemplate the Good.

astronomy theoretically. The justification is that the motions of heavenly objects are the fairest among visible things, yet still they are *inferior* to “the real ones … represented by real speed and real slowness in real number and in all the real geometrical shapes” (529d). The argument is similar to what has already been discussed since the superior objects are the ones studied with reason, by thought and intelligence within the intelligible realm. And though ‘heavenly objects are the fairest’ among the sensibles, they are still inferior to the absolute intelligible objects. The thought can abstract from non-perfect, non-divine motions of real planets and can provide more definite answers for the Soul which might get confused from mere observations.

The last science, harmony, is as Socrates notes similar to astronomy because “just as our eyes have fixed on astronomy, so our ears have fixed on harmonic motion and these sciences are related to each other” (530d). Socrates criticizes the conventional way of doing the science again since the scholars focus on the sounds and consonances and compare them and that is not true nature of the science, Socrates thinks. They never reach the fundamental questions such as why some numbers are harmonious and others are not (531c). Socrates thinks that mathematical harmony is the truest harmony to determine why some numbers are concordant while the others are not. The general conclusion is that only if harmony is studied with regard to the *beautiful* and *good*[54], then it is not useless.

However, intuitively speaking, harmony does not seem to fit into the general structure of the education. Burnyeat sees harmony as a “snag” not fitting into the order of “a steady increase in complexity: from extensionless to extended magnitude, from two to three dimensions, from solid figures as such to spheres in motion”[55]. It is necessary to note two main things, Burnyeat argues: i) harmonics is simpler than its predecessors but yet it is the first science focused primarily on ratios and ii) that the ratios are not of the Pythagorean nature of seeking numbers in heard concords but rather the ratios of non-sensible motion[56]. Burnyeat writes that the “redirected harmonics, like … [the] redirected astronomy, will need some non-sensible kind of motion to focus on. And what could this be but the movements of

[54] “Socrates implies that moving to the more abstract level is a prerequisite for harmonics to help us understand values like beauty and goodness.” Burnyeat, “Plato on Why Mathematics is Good for the Soul,” 47.
[55] Burnyeat, “Plato on Why Mathematics is Good for the Soul,” 68.
[56] In this passage, Socrates directly calls out Pythagoreans who correctly investigates numbers of the harmony but they do not search for natural harmonies or do not contemplate on why some are harmonious then others – they lack the judgment.

thought in the World Soul which the Timaeus casts as the objects of Platonic astronomy? ... For Platonic harmonics explains the good structure of the World Soul, which is expressed in the movements of thought studied by Platonic astronomy."[57] The precise reasons for implementation of astronomy and harmonics are to be studied in *Timaeus* because that is the study of Platonic solids and World Soul. It relates to the very metaphysics of the world of Being and Becoming and relation of World Soul and Individual Soul. In the *Republic,* Socrates is satisfied with the respect given by the interlocutors who are amazed by the amount of knowledge the Guards are suppose to have to complete their education.

For the purposes of the essays main question, the structure of the whole educational system just presented is important for following reasons: i) it places the sciences in specific order showing the evolution, ii) in general, it provides reasons for why theoretical disciplines are considered more useful than the practical ones and iii) it helps to explain why Socrates places theoretical over practical in the development of the education, in relation to the Good and in complexity showing the continuous progress.

The whole educational structure is strictly defined by Socrates which essentially leads him to say that "if our method of dealing with all these topics [the sciences] we've mentioned gets to their common relationship and works out how they relate to each other, it has some bearing on the direction in which we want our efforts to be spent, and is not wasted; but that if not, it is wasted" (531d). This means that only if students reach the point of understanding the unity and intercommunion of all sciences in their education, it has a value. Moreover, this intercommunion is a necessary condition in proceeding to dialectics, the highest science, and understanding of the Good itself. This is the "direction" Socrates refers to in 531d. Socrates is aware that it is an enormous task hardly done by anyone at all at that time and this might be one of reasons for why the mathematical studies occupy the Guards attention for so long.

Apart from the difficulty reasons and the potential contemplation of the Good, the question remains: why Plato thinks that so much mathematics is important? I think that the studies are not conducted for themselves (or primarily for mind-sharpening) but rather for *the right judgments* in life in general. As rulers in the Ideal State, the right decisions will be decisive.

[57] Burnyeat, "Plato on Why Mathematics is Good for the Soul," 73-74.

As I was trying to show, Socrates is dealing with the problem of one or many through mathematics – since the Soul must be confused based on sensory inputs, the thought is naturally invited, the intelligible instrument or the 'eye of the soul', as Socrates puts it, is employed and the result of that process is a definite judgment based on numbers and calculations. Once this journey starts and prisoners has reached the outer world and are capable of seeing the Forms which essentially means that they understand the *unity* of all listed sciences and contemplation of the Sun has been done, then as Burnyeat writes "soul has become assimilated to objective being [and] can take it [the knowledge] as a model for reorganising the social world"[58]. Burnyeat presents following argument:

*"In its immediate context this is about the rulers' knowledge of the Forms. But one cannot reproduce Forms on earth. What one can reproduce, at least approximately, are structures that exemplify Forms like Justice and Temperance. If, as I have been arguing mathematics is the route to knowledge of the Good because it is a constitutive part of ethical understanding, the corollary is that, when they return to the cave, the philosophers will think of the mathematical structures they internalised on the way up as abstract schemata for applying their knowledge of the Good in the social world."*[59]

The Rulers need to have the right judgments for keeping the social order at place. The order with all of its character-shaping norms, structuralized education, structuralized state system and the Soul which gives everybody what is due needs to be understood, valued and protected. The mathematical education *with definite structure* plays a key role in making this possible; it is part of an ethical education. The structure itself is an object of understanding: "not only should we grasp each mathematical discipline as an orderly body of knowledge developed out of a set of first principles [the Forms] (its hypotheses), but we should understand the several disciplines as themselves [geometry, arithmetic, harmonics, astronomy] forming a unified system, a family … in which the prior and simpler provides the basis for a series of more and more elaborate developments."[60]

[58] Ibid, 72.
[59] Ibid, 73.
[60] Ibid, 67.

Burnyeat thus argues that the objective of the platonic educational system is "to mathematicise ethics and politics and, simultaneously, to moralise mathematics"[61]. Mathematics is the instrument of tuning up the Soul with the right judgments (=moralization of mathematics) which essentially leads to internalizing mathematical objects and applying the Form-like schemata in politics or ethics (mathematization of politics and ethics). Moreover, Burnyeat writes that "the fundamental concepts of mathematics are the fundamental concepts of ethics and aesthetics as well, so that to study mathematics is simultaneously to study, at a very abstract level, the principles of value"[62].

The alternative insights about the whole purpose of educational system and the role of right judgments for ethical development can provide Mitchell's interpretation of the Cave allegory. Mitchell thinks that the prisoners "opine that all things that can be measured can be managed, and therefore turned to their own "advantage""[63]. The *bodily desire* drives the prisoners to look for the measures which will measure everything because of the desire of possessing everything. They look for a *definite* measure in *indefinite* sensible world. One of the "definite" measures can be *power* since everything can be reduced to it – honor, wealth, freedom, as, for example, Thrasymachos argued. The justification for it, as Mitchell thinks, is that "power debunks the pretense of all things "higher" than itself and mocks the idea of the Good, the idea of Higher Thing of a different order altogether. By such debunking, kindred defective measures are revealed to have their roots in power itself; and the idea of a divine measure is mocked for being beyond the wildest flights of fancy"[64]. The alternative account of measures lies outside the Cave – the divine measures, Mitchell writes. These measures cannot be manipulated, changed or opposed and any attempt of changing them is a proof of misunderstanding them. If rightfully understood, a man is aware of their true durability and eternality, and he rests in serenity, Mitchell claims.

Socrates is aware of the general distinction of these measures. He needs to show that the power is an insufficient measure. He does it by claiming that "the measure of power is unable to give an adequate account of itself and is reduced to silence"[65]. The power account

[61] Ibid, 76.
[62] Ibid.
[63] Mitchell, *Plato's Fable*, 122.
[64] Ibid, 123.
[65] Ibid, 125.

can, in fact, provide us with arguments on why honor, freedom or wealth are “backed” by power in order to be viable and can give us some vague insight on the mechanics at play but, as Mitchell writes, it has nothing to say on the most important ethical questions: *what it is a good life* and *how to live well.* Mitchell claims that “to do that requires the light of the Good, by which reason is empowered, justice made possible, understanding made precise, and action made authorial”[66]. The divine reason is in this respect untouchable. It is a substance of a higher order. The prisoners, however, will most likely adopt the first option for its immediateness and availability – we cannot forget that prisoners are playing a foolish guess-game in which the most powerful man is the “smartest” one – the one being first to guess the shadow correctly. And since their Cave and mind are enclosed, physically and mentally, all they will probably think is a desire for owning it all, a desire for power. The question of living well which would ultimately lead us to better judgments, according to Mitchell, is not conceivable. Socrates knows it. That is why he claims that prisoners need to be *forced* to look at the fire and be *dragged* (*helkesthai*) outside the Cave[67].

Though Mitchell does not comment on mathematics as being *the instrument* in turning the Soul’s eye from sensible to intelligible and as *the instrument* of Soul’s calibration to divine measures (because that is not his subject of interest), his interpretation of ill and divine measure is showing us what is at stake for Socrates in employing mathematics: the ultimate question of living well, an ethical and the probably most important question of Socratic philosophy. His account also shows the corruption of the soul if things go according to suggestions of some of the participants in the dialogue. It is an important comparison of ill and right ethical development of the right judgments.

### 3.7. Conclusion of this Chapter

To sum up, as I was trying to show from the analysis of the Book VII, Plato advocates that especially at the beginning, both practical and theoretical arithmetic and geometry helps to turn the Soul in the right direction away from the sensible world, world of becoming, to the world of being through natural process of employing the numbers, calculation and reason. It was said that the sensory inputs are too confusing for the Soul and cannot provide true *definite* answers whereas numbers can provide not only answers but also calibrate the Soul

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[66] Ibid.
[67] Ibid, 135.

ethically, establishing the right judgments employed in life later on. An important distinction between practical and theoretical geometry has been made and, by analysis of the Divided Line, I suggested that practical geometry is further from the Good because of being man-made original associated with *pistis* within the sensible world, in particular at level L2. However, practical geometry was an image of the Form together with theoretical geometry, thus being ontologically of the same kind. The difference between them was that theoretical geometry was of divine origins within intelligible realm at L3 with higher stage of understanding *dianoia,* and being broadly under the faculty of *episteme* in contract to *doxa*, a domain responsible for sensible world among other things. The bottom line is that both were images of the Forms, they were equal ontologically and differed among other reasons by the psychological affection. Thus I placed practical and theoretical geometry in relation to the Good as well as determined the purpose of the educational system: ethical development of the students. The question of what a good life is can be, to some extent, answered by especially theoretical mathematical studies. It will be the task of the final chapter to determine how much and how precisely the whole geometry as well as practical and theoretical geometry partake in the beautiful.

# 4. Why Plato Thinks That Geometry Is beautiful.

## 4.1. The Good and Beautiful and Their Relationship

I think that I determined where both practical and theoretical geometry belong to the Divided Line and the allegory of Cave. In the first chapter, I put forward the Ladder, its distinct levels and the Idea of Beauty, the Form. I proposed some direct evidence that Plato thinks of geometry as being beautiful with reference to *Philebus* and some approximation of the interpretative arguments which I am going to present now in greater detail. I also talked substantially about the Idea of Good in relation to the Line and the Cave. Now since the objective is to determine how beautiful both geometries, practical and theoretical, are, the central question is the connection between the Form of Beauty and Good. If these Forms are equal or similar either ontologically or structurally, only based on that finding, a connection between the Ladder and the Cave can be made. Thus, it will become easier to place geometries within the Ladder of Love since I already argued how both practical and theoretical geometry relate to the Good in the Cave. Apart from this connection, I will look for some common features such as level of orderliness, purpose of the levels in the allegories, qualities and properties such as eternality and relativity to place correctly geometries within the Ladder to determine the beauty of them. Thus, I start with the relation between the Good and beautiful to make connection between the Ladder and the Cave.

So what is the nature of the connection between the Good and Idea of Beauty? Ferrari thinks that this nature is complicated. He writes that “admittedly, what … [the lover] communes with at the summit [of the Ladder] is the Beautiful itself, not the Good itself; and the relation between the beautiful and the good, here as elsewhere in Plato, is problematic”[68]. He thinks that “in view of such passages as 201c [Symposium] and Phaedrus 250c-d, let us say that the beautiful is thought of as the quality by which the good shines and shows itself to us. We can then claim that the ascent to the Beautiful itself is indeed also an ascent to the Good itself but described so as to bring out at every turn what it is about the good that captivates us”[69]. In other words, Ferrari is saying that the ascent leads to the Good and the Beautiful interchangeably but that the Beautiful is the way the Good manifests itself along the way. It is visible, captivating aspect of the Good. The beautiful is a quality of the Good as Ferrari writes.

Richard Kraut in his *The defense of justice in Plato's Republic* writes about the common qualities a form has. He says that given the broader context of Plato’s philosophy harmony typically equals good which means then in relation to the body that harmony of body parts equal health because health is something fundamentally good. Similarly, “the goodness of Forms consists in the fact that they possess a kind of harmony, balance, or proportion; and their superiority to all other things consists in the fact that the kind of order they possess gives them a higher degree of harmony than any other type of object”[70]. Then all the Forms are superior in terms of their orderliness and anyone having access to them possesses the greatest good there is, Kraut says. However, this does not imply anything about the relation between the Good and Beautiful except of them having few same qualities: orderliness and goodness.

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[68] Ferrari, “Platonic Love,“ 260.
[69] Ibid.
[70] Kraut, “The defense of justice in Plato's Republic,” 322.

To access the nature of the relationship, I will look into *Symposium* for help. At the beginning of Socrates speech providing first insights into the problem of Love, he says that “if Love is lacking in what is beautiful, and what is good is beautiful, then … [the lover] will also be lacking in what is good’” (210c). In Diotima’ speech, Love is always lack of the beautiful and Good, and it is the desire for these two qualities what mainly characterizes Love. However, it is important not to confuse “that love is always of the beautiful and [that] good does not imply that love is beautiful and good,”[71] Ferrari writes.

All of this does not definitely resolve the issue around the relationship. An interesting part (204d-205b) would favor Ferrari interpretation that the beautiful is visible attribute of the Good. After Diotima asks Socrates “what will he gain by possessing beautiful things” he does not know the answer. After the question is rephrased to “what does the lover of good things actually desire,” Socrates does not hesitate with answer: “to possess the good thing” which will result in happiness. Thus the matter is settled, both of them think. The final end of Love has been found. From this intercourse, Ferrari’s interpretation seems attractive to me. Socrates, as many other lovers, did not know that the beautiful is manifestation of the Good things which we desire because of what they bring to us – happiness. He only recalled it after question-reformulation which literally meant connecting the beautiful, as it appears to us, to the Good, as we desire it, not knowing that what attract us are the beautiful qualities. The beautiful makes the Good appealing, an object of desire.

Therefore, I will accept Ferrari’s bottom-line arguments: the beautiful manifests itself at various levels of the ascent and ultimately leads us to the Good and happiness. If that is true then the ascent within the Divided Line and the Cave is motivated by the beautiful attracting us to the Good. This corresponds with Mitchell’s as well as Burnyeat’s findings I wrote about. One of the main objectives of the whole allegorical ascent of the Cave was to answer crucial Socratic question of *what a good life is* and *how to live well.* Mitchell claimed that “the light of the Good, by which reason is empowered, justice made possible, understanding made precise, and action made authorial”[72] is required for the question to answer. Apart from that, Burnyeat argued for ethical aspect of mathematical education of establishing the right judgments in relation to our Souls. Now given the context of *Symposium* and Ferrari’s

[71] Ferrari, “Platonic Love,” 252.
[72] Mitchell, *Plato’s Fable,* 125.

argument, the *telos* of all of this ascending is happiness because that where both allegories, the Ladder and the Cave, lead us to – towards the Good, while being attracted by the manifestation of the good by the beautiful. All this progression is settled in happiness.

Moreover, Kraut grants all Forms the status of greatest good and not only to the Form of the Good or Beautiful. He writes:

*"… the Forms are a good - in fact they are the greatest good there is. In order to live well we must break away from the confining assumption that the ordinary objects of pursuit-the pleasures, powers, honors, and material goods that we ordinarily compete for - are the only sorts of goods there are. We must transform our lives by recognizing a radically different kind of good - the Forms - and we must try to incorporate these objects into our lives by understanding, loving, and imitating them, for they are incomparably superior to any other kind of good we can have."*[73]

Since both allegories lead us to the Forms, in particular to the Form of the Good, the placement of practical and theoretical geometry is at least to some extent possible. I do not dare to determine whether the beautiful and the Good are equal or not. Apart from having some of the same qualities, I chiefly argue that the beautiful is a visible manifestation of the Good, motivating both ascents. And since the general framework of connecting the two main allegories has been concluded, in the following sub-chapter, I will try to determine how beautiful geometry is according to Plato.

## 4.2. Why Is the Whole Geometry Beautiful?

My plan was from the very beginning to identify practical and theoretical geometry in relation to the distinct levels on the Ladder of Love and thus determine the 'level' of beautifulness. But first, I said that the general argument for considering the whole geometry beautiful will be its role in ethics. So then first, I am going to argue for that now before placing practical and theoretical branches within the Ladder itself.

The Ladder, as well as the Cave, share the broad distinction between sensible and intelligible realms. Given Ferrari's 'two frameworks' argumentation (from beautiful discourse to beautiful topic), I would argue that the transition between intelligible and sensible realms in

[73] Kraut, "The defense of justice in Plato's Republic," 319.

the Ladder occurs between levels B4 and B5. The determination of the turning point can be deducted from the properties of particular levels. For example, the level B3 concerned with the beauty of souls produces a beautiful discourse which ultimately leads the lover to the Love for laws. As Ferrari writes, the transition from B3 to B4 "is when the beloved person falls from view that Diotima ceases to use the term "to love" (*eran*, 210a7, 210c1) and "lover" (*erastes*, 210b5) to describe the initiate's relation to what he finds beautiful"[74]. This is because the love is not longer attached to people or a person but to the product of them. I will claim later on that the Love for laws and institutions is characterized with some practical-sensible as well as theoretical-intelligible aspects. On the next level B5, the lover's mind is not longer occupied with person or people or not even with products (such as laws) of these people – from B5 to B6 the love is connected *only* to intelligible, abstract objects and not to faces. Given these circumstances, I argue that the turning point between the sensible and intelligible is given by the transition between B4-B5 though, importantly, the transition from B3 to B4 bears some important qualities to keep in mind.

Apart from that, Diotima seems to distinguish between two categories of people: those pregnant in body (expressing their love through procreation of children) and those in the soul. As I argued above, the whole Ladder seems to be for people already pregnant in their souls, however marginally, because the final product from the discourse at B1 is beautiful topic - knowledge. As I said, the reaction to this discourse determines whether the soul has a philosophical potential to progress further or not. So then, within the Ladder, the product of the Love for a body is not a child but rather a philosophical and psychological transition related to the soul. Thus, people being either partly or fully pregnant in their souls at B1 have more beauty and more immortality than the ones pregnant primarily in the body, procreating only children. The reason for that is that while ignoring specified levels of the Ladder, the offspring of pregnancy of the soul is "wisdom and the rest of virtue" (209a). This is very broad specification ranging from *phronesis*, a practical wisdom or virtue, to *arête,* more of moral virtue or excellence. More detailed and important is the next sentence where Diotima says that "by far the most important and beautiful expression of this wisdom is the good ordering [*diakosmesis*] of cities and households; and the names for this kind of wisdom

[74] Ferrari, "Platonic Love," 258.

are moderation and justice" (209a). Now Diotima says that such an *expression* of the wisdom is *the most beautiful* one.

Now I claimed that the nature of transition T3 is the descent back to the Cave presented as moral obligation by Plato results from being a good philosopher-ruler who knows the importance of order both institutional and psychological. Plato tells us there that "at one point … when philosophers look to the harmonious arrangement of the Forms, they develop a desire to imitate that harmony in some way or other (500c)"[75]. It was said that one of the culminations of the educational system is to become a philosopher capable of ruling the Ideal State. This requires the understanding of unity of presented sciences, both geometries including, and establishment of the harmonious order within the soul. After that, the philosopher can descent back and replicate the harmonious order in the *polis.* "The person who is willing to do her part in a just social order, and whose willingness arises out of a full understanding of what justice is [the Form itself], will see the community of which she is a part as an ordered whole, a worldly counterpart to the otherworldly realm of abstract objects she loves"[76]. After transition T3, this connection between social harmony, or social justice, and the harmony of abstract objects is apparent to the philosopher-rulers. As Burnyeat argued, mathematics is fundamental instrument in both: the right calibration of the soul to the harmonious whole as well as in ethical education of potential future rulers. And since Diotima says that "the most important and beautiful expression of this wisdom [justice] is the good ordering of cities and households" (209a) and geometry is a constitutive part of developing this wisdom, then geometry has to be beautiful as well. This beauty could reflect the ethical beauty of the meaning of the word *kalos* than the others, utilitarian and aesthetic meanings.

In my opinion, the product of the virtue, the laws, can be further specified within the Ladder. The social order is guided and regulated through laws and institutions. Since the nature of the laws is not specified in *Symposium* nor, in great detail, who can produce these laws, I am going to argue that to classify the laws of 'good ordering' which are product of the virtue of justice under the category of the Love for laws and institutions. Some hints about what could be 'bad political ordering' can be substituted from *Gorgias.* There Socrates claims that

[75] Kraut, "The defense of justice in Plato's Republic," 328.
[76] Ibid, 329.

rhetoric is “a semblance of a branch of politics” (463d). The real politics is concerned with justice, truth and good which is also an objective for laws and institutions. On the other hand, the aim of rhetoric is flattery, it has no business in determining what is right or wrong which is in contrast with law-court which has in mind truthfulness and justice. Is that then possible to say that laws cannot be unjust and that they always have to be beautiful? I do not know. But since Socrates admits that rhetoric purposefully mislead people to take advantage of them, then at least in reality of ancient Greece, all laws and institutions were not just (thus beautiful) because of the false, misleading rhetoric which influenced people’s mind. However, the laws we are trying to place under the Love of laws and institutions are just, harmonious and good because they were put in practice by philosophers so then they must be good. Therefore, these just and good laws can be sorted under this category.

Furthermore, in *Symposium,* Diotima talks about Lycurgus who reformed legislation of Sparta. He was “the procreator of your laws” (209e), Diotima says, and through his actions he secured a great respect and almost divine reputation. Before talking about Lycurgus, Diotima talks about honor and how it motivates people to conduct dangerous actions; these people are motivated by desire for immortal legacy, they are pregnant in their souls. In this relation, Ferrari writes that the Ladder can be understood from the perspective of three fundamental desires. He says the lover is “being led first by sexual desire [B1-B2], then by the ambition for honor [B3-B4], and finally by the love of learning [B5-B6].”[77] Even though it is not explicitly said in *Symposium,* Lycurgus could have been motivated by honor and still be a great legislator. So even people not having purely philosophical and completely harmonious souls and thus not being the perfect philosopher-rulers of the Ideal State as discussed in *the Republic* can create the beautiful laws and institutions as Lycurgus surely did. So then to conclude, I argue that both philosopher-rulers as well as honor-loving people can create laws as beautiful as the Love for laws and institutions. Moreover, I think that the philosophers create *always* the best and just laws thanks to their knowledge of the Forms whereas honor-lover only *can*. I leave the question open whether the laws have to be just by definition which would essentially mean the laws of honor-lovers are just and good as well. The bottom-line is that from what is said in *Symposium* even honor-lovers can create beautiful laws.

[77] Ferrari, “Platonic Love,“ 260.

### 4.3. How Beautiful Is Theoretical Geometry?

Now, since I determined that the whole geometry is beautiful because of its constitutive role in ethical education and I place the products, laws, of the developed virtue resulting from the education in the Ladder, the theoretical geometry can be placed as accurately as possible within the Ladder as well. I am going to argue that theoretical geometry is as close to the Good in the Cave allegory as it is in the Ladder and that under the Love of knowledge, among other sciences, falls also theoretical geometry.

First, Socrates spends a great deal of time explaining the objective of the educational system that is the unification of knowledge of all suggested sciences. The unification, occurring in the Cave allegory at C3, is a necessary condition to be promoted towards the Good. This change is characterized by a change in thinking, something, I have not talked about in this specific way that much. Ian Mueller in his *Mathematical method and philosophical truth* writes about features of mathematical thinking: i) “reasoning about sensible objects … to understand … intelligible ones, [ii)] … the laying down of hypotheses, presented as the assumption of certain objects (the odd and even, the figures, the kinds of angle), but in fact involving assumptions about the nature of these objects and the ways they can be manipulated [and iii)] … the downward development of these hypotheses, including, but not necessarily restricted to, deduction”[78]. Mueller thinks that since “mathematicians reason about sensible things, they must make hypotheses and they must move downward from them, as they must speak about acting on sensibles”[79]. He thinks that Plato argues that mathematics really must move only downward in the hypotheses development. The reason is that mathematics uses hypotheses resulting from the need to use sensibles but these hypotheses are never justified. Mueller thinks that this might only be the nature of definition of mathematics because modern mathematics does “perform analyses on propositions below the level of their ultimate hypotheses”[80]. For Plato, however, the questions outside of mathematics are questions about hypothesis, like ‘what is a figure’. This requires dropping the first principle ‘i)’: a shift from arguing about sensible object to purely intelligible ones. To be clear, Socrates promotes mathematics because of its ability to lead the soul to ask the questions about hypothesis but mathematics itself is not capable of answering them.

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[78] Mueller, *Mathematical method and philosophical truth,* 188.
[79] Ibid, 189.
[80] Ibid, 189.

Mueller writes that “mathematicians reason about sensibles for the sake of intelligibles; they use sensibles. Dialecticians reason about intelligibles for the sake of intelligibles; whether or not images occur in their minds or they refer to sensible things, they do not reason about sensible things, they do not use them.”[81] Both ultimately lead to unhypothetical first principle, the former by unification of the sciences, the latter by itself. Its unhypothetical aspect is anchored in its ability to defend oneself by itself – no further justification is needed. Unhypothetical first principle is the Idea of the Good.

I argue that similarly this mathematical framework is needed for a person to proceed from the Love for knowledge, B5, to the Love for itself, B6. The ‘vast sea of knowledge’ is contemplated at B5 as Diotima says in *Symposium,* it is a discourse primarily but not exclusively with oneself during which a person, as Ferrari writes, “is looking back from the height he has scaled, and sees beauty as a whole, but a whole of great multiplicity. Now he turns his face to the peak, and comes to see beauty as a unity”[82]. In other words, he contemplates sensibles for the sake of intelligibles in the discourse with himself. Then he naturally ask question outside of the sensible hypothesis, *what is beauty*, to which he will need the help of dialectics. Then the transition to the final stage can happen.

Furthermore, as Socrates argues in the *Republic,* geometry will lead the soul to the truth and the spirit of philosophy will be born. The whole education is trying to raise rulers who are philosophers, seekers of the truth, and that is done through lengthy mathematical training. Thus the spirit of philosophy is probably born at C3 after leaving the Cave though some of the philosophical aspects and philosophical potential can be track back to the Cave. As Ferrari argues, in the last two stages B5-B6, lovers are motivated by the love of learning; they are truth-seekers which is one of the signs of the philosophical soul. It thus seems to me that the last discourse of the Love for knowledge raises philosophers though, similarly, some philosophical potential can be tracked all the way down to C1[83].

Aside from the same or very similar psychological affection or frameworks one uses to access the knowledge and the birth of philosophical soul, the last reason to include theoretical geometry in the knowledge at B5 is the term *episteme*. I have touched upon this:

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[81] Ibid, 190.

[82] Ferrari, “Platonic Love,“ 258-9.

[83] It would the *reaction* to newly acquired knowledge out of the first discourse: whether that knowledge is taken into account or not can hint the potential of the lover as I argued at the beginning.

*episteme* is usually referring to systematic scientific knowledge whereas other terms, such as *techne*, to crafts or applied sciences. I assumed that Plato uses *epistēmē* purposefully in *Symposium* and the 'vast sea of knowledge' one can contemplate refers, primarily to theoretical geometry and arithmetic.

Richard Parry in his article *Episteme and Techne* write that "the relation between knowledge (*epistêmê*) and craft or skill (*technê*) is complex and surprising [and that] there is no general and systematic account of either but rather overlapping treatments, reflecting the context of different dialogues."[84] However, there are certain aspects of this relation which are consistent throughout Plato' philosophy. Since I focus primarily on the *Republic* and *Symposium,* I will elevate Plato interpretations with respect to these to dialogues though reflecting the other interpretations as well.

Parry writes that Plato usually talks about geometry as about *techne*, a craft, with others being farming, calculation, medicine, political craft among many others. Each of this discipline is associated with a practitioner though not all of them have a product as its outcome. Parry writes that medicine provides a care for humans, a quasi-product or maybe better a service, many others have a specific product but some are 'problematic' in this regard as, for example, calculation which does not have a product nor it is a service.

As was said above, there are passages where Plato uses the terms interchangeably: "in *Protagoras* (356d-e) Socrates refers to measuring as both a craft and a kind of knowledge"[85]. However, there are some universal traits of *techne* to be found; *techne* is used "as a way of explicating central themes, such as virtue, ruling, and the creation of the cosmos"[86]. Thus, very complex account of *techne* is developed. First, Socrates says that a craft has *ergon*, a function, which is to say a specific goal, the purpose which the activity aims to achieve. Thus, *ergon* is the goal of the craft which is typically separated from the activity itself. For example, in *Gorgias*, Socrates claims the goal of calculation is persuasion "which deals with the amount of an odd or an even number" (453e). According to Parry, this is the "result separate from the activity of calculation"[87].

[84] Stanford Encyclopedia of Philosophy, "Episteme and Techne".
[85] Ibid.
[86] Ibid.
[87] Ibid.

So then, *technê* becomes about knowing how to do (*epistasthai*) certain activities and *episteme* would refer to the theoretical aspect of *technê*. This theoretical aspect of *technê* includes the theoretical knowledge of goals of each activity. In case of calculation, it would be the knowledge that its *telos* is the persuasion. Plato also emphasizes that part of this theoretical component is an ability of the practitioner to reason and articulate oneself of why he does what he does.[88]

In *Theaetetus,* Parry argues, *episteme* gets more solid foundation. Since knowledge of the Forms has become *ergon*, the end in itself and since the knowledge of the Forms can only be theoretical because the objects are intelligible, then *episteme* is used in relation to this theoretical knowledge. *Damon Young* said that *techne* "was not concerned with the necessity and eternal a priori truths of the cosmos"[89] which now appears to be the domain of *episteme.* For example, "in *Republic* V*,* Socrates introduces an altogether different notion of the knowledge that philosophers will have — one whose object is forms"[90]. This knowledge is *episteme.* In *Republic* (428 b–d), Socrates talks about *episteme* in the previous sense of the word, that is as the theoretical component of political craft. *Epistêmê* in that sense is theoretical knowledge of the goal of that craft as well as the ability to articulate what is the best for the object of that craft, that is the city, both internally and externally. I think that this twofold usage of *episteme* is intentional corresponding with claims above.

The Divided Line can bring more clarity into the matter. After the knowledge of the Forms is identified with *episteme*, the intelligible world, *noêton,* is divided into two subsequent parts: mathematical or deductive reasoning (*dianoia*), a domain for mathematics, and the grasping of the unhypothetical beginning point (*nous*), a domain for dialectics. Later, in *Republic* V, knowledge of the Forms, *episteme,* "will be deductive and logical like mathematics; unlike mathematics, its deductions will be based on foundations that need no further justification. In part it will be something like mathematical deduction based on fundamental reality,"[91]

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[88] In *Charm.* 171b7–9), Socrates aims to investigate the physician in his theoretical knowledge, procedures and whether those are true and right. Importantly, Plato sometimes emphasizes other distinctions such as *techne* and *empeiria*, and then *techne* gets to include this theoretical component on its own and thus the distinction between *epistêmê* and *techne* is not stressed as such.
[89] Young, "Philosophy East and West," 190.
[90] Stanford Encyclopedia of Philosophy, "Episteme and Techne".
[91] Ibid.

writes Parry. This fundamental reality is the reality of the Forms and its *ergon* seems to be dialectical thought itself, thus being purely theoretical.

Now, finally, it can be explained how theoretical and practical mathematics relates to the concept of *episteme.* Mueller emphasizes two differences between *dianoia* and *nous*: "dianoia is compelled to study its objects by proceeding from a hypothesis toward an ending, but noesis studies its objects by proceeding from a hypothesis to an unhypothetical beginning (principle) [and second] … dianoia uses sensible things as images, but noesis uses no images and proceeds through Forms in a systematic way"[92]. One of the main arguments of the former chapter was to show that there is threefold ontology: images of images, images and originals, but fourfold psychological affection. What differed practical and theoretical mathematics was difference between *pistis* and *dianoia*. I have given an example of geometer who draws diagrams which are sensible and which have intelligible, divine shadows. These divine shadows corresponded with reflections and shadows after the ascent from the Cave. Now, in *Republic* 533b-c, Socrates claims that "geometry and the studies associated with it … do apprehend something of being, but … they are dreaming about it. They cannot have a waking vision of it as long as they use hypotheses and keep them fixed, unable to give an account of them. For when the starting point is not known and the finishing point and what comes in between are woven together out of what is not known, there is no way that such a consistency will ever become knowledge (*ποτὲ ἐπιστήμην γενέσθαι, potè epistḗmēn genésthai*)"[93]. Then Socrates adds that 'geometry and the studies associated with it' can become knowledge, *episteme,* only with dialectical method which removes the hypothesis "to the actual first principle" (533d), thus achieving the unification of the sciences.

Now this suggests few important things. First, no theoretical geometry or arithmetic can become knowledge, *episteme,* in the sense of knowledge of the Forms as long as they are not unified by dialectics. That is consistent with what was already said. This is also consistent with the Line (*dianoia*) as well as with the geometer example that even theoretical mathematics is only an image of the Form though of the divine origins.

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92 Mueller, *Mathematical method and philosophical truth*, 184.
93 Ibid, 188.

However, I want to suggest that knowledge in the second, 'weaker' meaning of *episteme* that is in the sense of ability to reason and explain the goal of the craft, theoretical geometry is *episteme* then. If that is true then the Love for knowledge, *ἐπιστημῶν κάλλος* or *epistēmō͂n kállos,* is primarily occupied with theoretical components of knowledge, especially theoretical geometry and arithmetic as well as other theoretical disciplines in the curriculum.

It was said that one of the key differences between theoretical and practical is dichotomy of *pistis - dianoia,* a confusion of an image for an original versus seeing image as an image and intuitively acknowledging the Forms. Moreover, I also argued that Socrates claims that some geometers draw diagrams for the sake of practice confusing "necessities of geometry with those of daily life; whereas knowledge is the real object of the whole science" (527a). In my opinion, this is the Socratis reference to the geometry in which geometers confuses the sensible images they draw with intelligible objects they resemble. I argue that those people possess "only" knowledge in the sense of *techne*, they "only" know procedures or 'how to do' (*epistasthai*).

They are unable to reason about the true goals of their craft and thus reach the theoretical component, *episteme,* which this craft, as any other, can offer to its students. In *Laws,* a distinction is given between a doctor of free men and slaves. Parry writes that "the slave doctor relies on experience (*empeiria*) and has no account to give for his procedure [whereas] the free doctor not only has an account, he communicates it to his patients as a way of eliciting their cooperation in the course of treatment (720 b–d)"[94]. The doctor of slaves laughs at the free doctor for instructions he has given to his patients thinking as if the doctor of free men was trying to turn them into doctors themselves (857 d-e). I argue that this mockery is due to the differences between *pistis* and *dianoia*: slave doctor sees his craft as the only reality, the original, and he makes fun of others who think different, a pattern found in the Cave allegory as well. On the other hand, the free doctor has *episteme,* he knows the theoretical component in form of the goal of his craft and is capable of articulating and reasoning about this knowledge to either himself or his patients. Similarly, the practical geometer knows without the doubt the procedures but is unable of communicating the real goals of geometry exactly as Plato claims in the *Republic.* So then

[94] Stanford Encyclopedia of Philosophy, "Episteme and Techne".

both geometry and arithmetic are crafts – that is stated throughout the dialogues, and the theoretical components of them is *episteme,* a kind of knowledge which is subject to the Love of knowledge. Since Plato uses the term *episteme* while talking about the Love of knowledge, I suggest that that includes theoretical geometry as well as arithmetic.

### 4.4. How Beautiful Is Practical Geometry?

Now the remaining thing is placement of practical geometry. I said above, there is a continuous transition between *pistis* and *dianoia* as well as between practical and theoretical geometry which is suggesting that practical geometry should be as close to theoretical knowledge within the Ladder as possible. This would mean that the Love for laws and institutions would include practical geometry among other things. And even though the evidence for that seems to be a bit more scattered, I am going to suggest that.

First, I present a reason why I do *not* think that practical geometry is included in the Love for knowledge. The spirit of philosophy is born with geometry as I already argued before, though this geometry seems to be only theoretical. Since the philosophical spirit is characterized by search for truth and it seems to be born only after the departure from the Cave, then practical geometry cannot be at the level of the Love for knowledge. The reason is that practical geometry still belongs to the Cave. One reason is that it is sensible and man-made, as discussed above. And since the Love for knowledge definitely belongs to intelligible world, it cannot include practical geometry.

The reason why I think that the Love for laws includes practical geometry is that, in this Cave, the desire in 'learning' is at best motivated by honor, in lower instances by power. These desires can correspond to the fundamental desires within the Ladder, as Ferrari writes, where first is sexual desire which could be easily reducible to power, then honor, and lastly truth. The practical geometry, characterized by *pistis,* confuses fundamentally of what is reality and what is only image. Now I think that it is perfectly reasonable to assume that the geometer who confuses "necessities of geometry with those of daily life" (527a) are mainly motivated by honour, especially when the whole context is considered. It was said that practical geometry had been the part of science considered to be useful by Greeks. It can then be reasonably assumed that philosophers, such as Socrates, might not have gotten as much as respect as geometers who were viewed by public as more useful given their profession of land measuring was instrumentally useful from the public's point of view. The truth-seeker does not care about that but honor-lovers care about their perception, ‘image’ and legacy as Diotima claims. So then given the continuous transition between practical and theoretical, and given the difference in motivations, the Love for laws where people are mainly motivated by honor should, in terms of beauty, involve practical geometry.

Now I argued that the descent back to the Cave, T3, is motivated by an ability to imitate the harmonious Forms in ruling the *polis* and in maintaining the right social order. I argued that the laws can create two spirits: the ones motivated by truth and the ones motivated by honor too. However, I said that only the philosopher-rulers have the capacity to imitate the Forms in the social order. One of the things, I did not mentioned before is that the descent is descent back to sensible world, in allegory, back to the Cave. After the descent, philosopher-rulers imitate the harmonious Forms but due to limitations of sensible world, they have to use images. Now it is important to note that their knowledge is *episteme.* Plato tells us this in book VI, *Republic*, that “the philosopher has a knowledge (*gnôsis*) of the reality of each form, thus a clear paradigm in his soul. Like painters, philosophers look to (*apoblepontes*) the truest paradigm, always referring to it and contemplating it as accurately as possible; in this way they establish here the laws respecting the fine, the just, and the good”[95]. So then a political *craft* has a theoretical aspect which is, in this case, the knowledge of the Forms. This theoretical knowledge, *episteme,* is important for imitation of the harmony within the social system.

Now I suggest that the theoretical knowledge these philosopher-rulers have can be classified higher in the Ladder than under the Love for laws but the actual practical manifestation that is the laws and institutions they create, fall under the Love for laws. It is a product of beautiful Souls and I think it is reasonable to assume that since this ability of “good ordering” is the highest virtue as Diotima said, it will be as close as possible in the Ladder to the Good yet still in, at least partly, in sensible world because *it manifests in the sensible world*, a criterion the Love for laws fulfils ideally being in between practical and theoretical components of knowledge.

[95] Stanford Encyclopedia of Philosophy, “Episteme and Techne”.

Now I think that both philosopher-rulers and practical geometers create images of the Forms and that both are man-made. The former are the laws and institutions, the latter are the diagrams and lines. The difference between them is psychological affection and in this regards, philosopher is superior to the practical geometer. However, they both create man-made images of the Forms in the sensible world. If, ontologically speaking, they create the same thing not only in terms of *image* creation of the Form but also that both in terms of origins as man-made, then it follows that practical geometry has to be as beautiful as is the manifestation of this virtue of ordering of cities and households. Thus, practical geometry as well as arithmetic is as beautiful as Love for laws and institutions. This is my suggestion and the answer to essays questions: theoretical geometry is as beautiful as the Love for knowledge and practical geometry as the Love for Laws and institutions.

## 5. Conclusion

Now I want to summarize the main arguments of this essay. First, I argue that the whole geometry is beautiful because it is a constitutive component of ethical education. The reason is that Socrates argues that the most important and beautiful expression of this wisdom is justice in which nature is 'good ordering of cities' and since geometry is vital in the development of the capacity responsible for contemplation of the Good which leads to this wisdom of justice, I argue that the whole geometry is substantially beautiful. To determine precisely how much geometry is beautiful, I differentiate theoretical and practical geometry. And I claim, second, that theoretical geometry is as beautiful as Love for knowledge in *Symposium's* Ladder of Love. Third, I assert that practical geometry is as beautiful as Love for laws and institutions.

In the first chapter, I search for direct evidence that geometry is beautiful. I provide three pieces of evidence: i) geometrical figures, ii) the Love for knowledge, iii) the good ordering. First, in *Philebus*, the geometrical figures such as the straight line and the circle which are subject to geometry are said to be very beautiful having some of the same properties as the Forms yet not identical with them. Second, since Socrates uses the term *epistēmē* while talking about the knowledge in the Love for knowledge, I argue that the term refers to scientific knowledge and thus to geometry as well though the precise nature of the term I specify later. Third, the final 'evidence' is closer to assumption since I assume that geometry is a constitutive part of ethical education and thus it is beautiful because Diotima says so while talking about justice as the most beautiful expressions of wisdom and virtues.

In the second chapter, I answer two fundamental questions: i) how and ii) why is the ascent in the allegory of the Cave possible? To the latter, I claim that there is a capacity in the soul referred by Socrates as 'eye of the soul' which is employed once the soul get confused. This capacity employs thoughts and intelligence to get definite answers as a reaction to chaotic inputs from senses. The definite answers are determined by calculations, numbers and by abstraction from many to one. To the former, I argue that the general answer is by education, in relation to the transition T2, by mathematical disciplines. Specifically, the transition occurs from practical to theoretical geometry. I argue that while practical geometry is further from the Good because of being man-made original associated with *pistis* within the sensible world at L2, theoretical geometry is of divine origins within intelligible realm at L3 with higher stage of understanding *dianoia* and being broadly under the faculty of *episteme* in contract to *doxa*, a domain responsible for the sensible world. However, both levels share the ontological fundament, an image of the Form.

In the third chapter, I discuss the nature of relationship between the Good and beautiful saying that the beautiful is the way the Good manifests itself along the ascent. This is enough for me to generally connect the Cave and the Ladder so that the beauty of geometry can be determined in relation to the Good because that is what both allegories ultimately share. First, I claim that the whole geometry is beautiful because mathematics is fundamental instrument in both: the right calibration of the soul to the harmonious whole as well as in ethical education of potential future rulers. The future rulers then recognize the institutional and psychological order which they strive to replicate in the social world. This 'ordering of cities' is the most beautiful of expression of wisdom and since geometry is constitutive part of it, it is beautiful as well. Moreover, I claim that the practical expression of this virtue, the laws and institutions, fall under the Love for laws and institutions and the placement of this specific manifestation of the virtue is settled. Second, I argue that theoretical geometry is as beautiful as the Love for knowledge because: i) both L3 and B5 use mathematical deduction based on hypothesis, ii) the spirit of philosophy is born at L3 and B5, iii) *episteme* understood as theoretical component of *techne* connects L3 and B5 as well. Third, I claim that practical geometry is as beautiful as the Love for laws because: i) the transition is continuous, ii) it is attached to sensible world, iii) it is for honor-seekers and iv) both philosopher-rulers and practical geometers create images of the Forms and that both are man-made.

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