Parts and Loops

A Qualified Defence of Unrestricted Fusion

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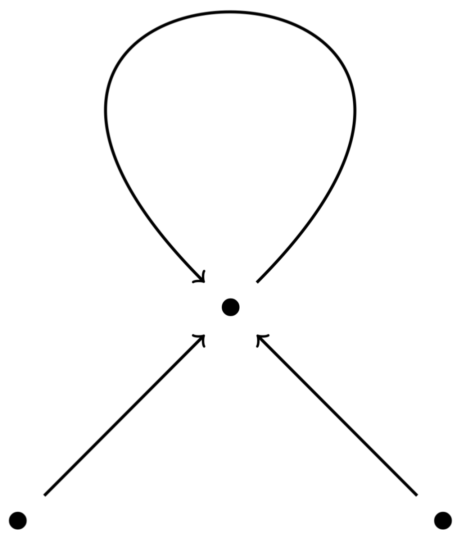

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Abstract

There are reasons to believe that the correct principle governing which composite objects there are is the Unrestricted Fusion principle of classical mereology: for any satisfied condition there is an object composed of the things satisfying that condition. However, this principle appears to be inconsistent with so-called mereological junk. A world is junky if everything at that world is a proper part.

Unrestricted Fusion guarantees the existence of an object for any condition, including a universally satisfied trivial condition. The resulting object, the universe, exhausts all other objects so that everything is part of it. However, if the world were junky, then there would be an object of which the universe would be a proper part. However, since the universe exhausts everything, there can be no such object. Contradiction.

Still, junk is compatible with Unrestricted Fusion on a suitable weakening of classical mereology. Independent examples motivate a relaxation of the partial ordering of its proper and improper parthood relations, so that some objects consistently may be proper parts of themselves. The resulting non-wellfounded mereology takes proper parthood as its primitive relation and lets it obey only transitivity. Improper parthood can be defined in a standard way, and a standard strong supplementation principle added to complete the theory.

The resulting theory satisfies the same criteria used to argue for the possibility of junk. Moreover, it is consistent, independently motivated, and allows for Unrestricted Fusion even in the presence of junk: if the world were junky, the universe would exist as a proper part of itself. Alternatively, if such non-wellfoundedness is too disagreeable, one could claim, by parity of reasoning, that the criteria employed to establish the possibility of junk are flawed. If so, then the case for junk fails, and Unrestricted Fusion can be upheld.

Acknowledgements

First and foremost, I am grateful to my supervisor professor Peter Fritz. His intellectual generosity and incisive commentary, grounded in deep philosophical as well as technical insight, have been of great value. His supervision, from the autumn semester of 2021 to the spring semester of 2022, has greatly improved the final result.

Two more intellectual debts are due: First, to the late professor Katherine Hawley of the University of St. Andrews who first introduced me to philosophical questions of parthood and composition. Second, to professor Einar Duenger Bøhn under whom I read metaphysics at the University of Oslo and for whose coursework I first broached some of the thoughts and arguments developed herein.[1] This dissertation would not have come to be without their inspiration.

[1]While mostly heavily revised and expanded, section 2.1 in particular draws on coursework for FIL4110 (Spring 2012). So, too, do some of the examples in section 3.1, and the first and third objections considered in chapter 4.

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Introduction

There are many things in the world. At least some of these things are composites in that they have more than one part. Mereology is the theory of parts and composites, or more precisely of the relations of parthood and composition between them. This dissertation forms an investigation of mereology by way of a qualified defence of the mereological principle that any objects whatsoever compose some further object. This principle, *unrestricted fusion*, *universalism*, or following Lewis (1986) *unrestricted composition*, is among the most controversial and most discussed principles of mereology. It is a central tenet of all mereologies that rise to the strength of what is now called classical mereology.

Objections have often been made to unrestricted fusion because of its apparent profligate commitment to objects. Think of any satisfiable condition and there is potentially an object composed of the things satisfying that condition. Think of any satisfied condition and according to unrestricted fusion there actually is an object composed of the things satisfying that condition. Examples include the object composed of one's left leg and the Sphinx's head, or the object composed of the still-attached upper half of a trout and the still-attached lower half of a turkey, famously introduced by Lewis (1991, 7) as the 'trout-turkey'. If unrestricted fusion is true, there are all sorts of strange object like this. For some, this is too much to swallow. Therefore, they argue that weaker principles of composition, restricted ones, on which only some objects compose a further object are true instead. Some even claim that there are no composite objects at all. This, they say, fit better with our intuitive notion of objects and their properties. To this, I submit my disagreement.

The question of when it is the case that some things compose a further thing was raised to prominence in contemporary metaphysics by van Inwagen (1990) as the *Special Composition Question*: 'Under what conditions do two or more material objects compose a further, composite object?'. It, roughly, allows for three answers:

1) none, 2) some, or 3) any. Those who uphold unrestricted fusion are adherents of the third answer. They claim that for any satisfied condition, there is an object composed of the things satisfying that condition.[2]

The second answer is really a category of answers corresponding to theories holding some intermediate restriction on what satisfied conditions entail the existence of a fusion. Under some conditions a further, composite object is formed, and under others composition fails. Such theories are interesting in their own right, as when van Inwagen (1990) claims that only those things whose collective activity constitutes a life form composite objects, or when Merricks (2005) claims that consciousness is required. Unfortunately, these theories all tend to allow for borderline cases in which it is unclear whether there is a fusion of the things satisfying a condition. In the absence of a non-arbitrary determination as to whether there is such a fusion, the intermediate positions appear unstable as argued by the so-called *argument from vagueness* of Lewis (1986, 212-213) and expanded by Sider (2001, 120-132).

Mereological nihilism is the position that there are no composite objects at all. As such, it answers the Special Composition Question in the negative: There are no conditions under which composition occurs. If the intermediate positions are unstable, then nihilism and unrestricted fusion are the main contenders for best answer to the Special Composition Question. In that context, unrestricted fusion appears plainly to be the best candidate for making sense of which ordinary objects exist: all of them versus none of them. There is thus a pragmatic case to be made for unrestricted fusion. Only *it* is consistent with the literal truth of our judgments of ordinary objects.

I will not have more to say in general defence of unrestricted fusion. I take the argument from vagueness to establish that intermediate positions are unstable, so that either nihilism or unrestricted fusion must be the correct answer to the Special Composition Question. Moreover, given this choice I contend that unrestricted fusion is by far the best answer. Taking this position without further argument may be controversial, and is a limitation of the scope of this dissertation. As such, the arguments of this dissertation are intended for those already sympathetic to unrestricted fusion. It is in this sense that the dissertation forms a *qualified* defence.

[2]This principle is held, among others, by Leonard and Goodman (1940), Armstrong (1997), Lewis (1986), Schaffer (2010), and Sider (2001). What exactly qualifies as classical mereology, is sometimes terminologically complicated, see Bohn (2009a, 61-63) and Hovda (2009).

Defense against what? Bohn (2009b) presents a version of the so-called 'junk argument' against the the necessity of unrestricted fusion. It appeals to the metaphysical possibility of mereological junk. If a world is junky, then everything in that world is a proper part of something. Junk appears to be inconsistent with unrestricted fusion. Unrestricted fusion normally guarantees the existence of a unique object that exhausts all other objects, the so called universal object or, for short, the universe. However, if the world were junky, then there would be an object of which universe would be a proper part. However, since the universe exhausts everything, there can be no such object. Contradiction.

This dissertation forms an argument that a case can be made that junk nonetheless is consistent with unrestricted fusion. Affirming unrestricted fusion in the face of the junk argument has a cost in the coin of other commitments of classical mereology. This dissertation will therefore eventually recommend a retreat from full classical mereology, but the aim is to preserve as much of the strength of classical mereology as is consistent with our everyday understanding of composite objects. Unrestricted fusion is essential, if not alone, in granting that strength. Therefore, this dissertation will examine the consequences of taking the contradiction of the junk argument at face value and consider how the universe consistently could be a proper part of itself. This solution may sound extreme, but I will argue that it is consistent, delivers the required results, and is not *ad hoc*. The argument is made over four main chapters:

- Chapter 1 is devoted to an introduction to and review of classical mereology. Standard definitions of parthood and proper parthood are introduced and principles of supplementation and fusion are discussed.

- Chapter 2 introduces and discusses the junk argument and concludes that if Bohn's three-step test for possibility is accepted, it appears to pose a legitimate threat of inconsistency in the presence of unrestricted fusion.

- Chapter 3 introduces, motivates and discusses a non-classical mereology in which the asymmetry of proper parthood is relaxed so as to allow for parthood loops. I argue that relative to this mereology, junk is consistent with unrestricted fusion. Moreover, the mereology in question is independently motivated by several examples. These examples satisfy Bohn's three-step test and thus puts the case for this mereology on par with the case for junk.

- Chapter 4 considers, and attempts responses to, some objections both to the development of this mereology as well as to the metaphysical picture resulting from it.

There are some important limitations inherent in the theoretical framework that forms the basis of this dissertation: Classical first-order logic with identity. Classical behaviour for quantifiers and all connectives, as well as identity, is assumed. It is thus important to the development of the argument of this dissertation that the universe *consistently* should be able to be a proper part of itself. No attempt is made to modify the underlying logic and no attempt is made to investigate a paraconsistent approach to mereology.[3]

The expressive limitations of mereology as a first order theory pose a philosophical peculiarity insofar as the dissertation is to be taken as a defence of unrestricted fusion. Given the first-order context, the principle that any things whatsoever has a fusion, must be formulated with an axiom schema. The first-order language of mereology consists only of the first-order quantifiers, the truth functional connectives, a primitive mereological predicate governed by additional axioms, and a countable infinity of first-order variables. Additionally, 'φ' is used as a metalinguistic variable for stating conditions in the axiom schema. One may, or may not, also include up to a countable infinity of ordinary predicates, such as '... is a cat'. While these may be useful for some purposes, they are not required for pure mereological theory. Thus the language of mereology has a vocabulary no larger than a denumerable infinity.

However, suppose there is a denumerable infinity of atoms, i.e. $\aleph_0$ many atoms. Then, if any things whatsoever have a fusion, intuitively there should be $2^{\aleph_0}$ many composites.[4] Given its denumerable vocabulary the theory can only specify denumerably many conditions, but by Cantor's theorem $2^{\aleph_0}$ is not denumerable. So, uncountably many fusions cannot be specified by the theory. Moreover, given the intuitive size of the domain of composites, one would think that, in the infinite case, a theory committed to unrestricted fusion should guarantee a domain with size at least $2^{\aleph_0}$. However, as is well known by the downward Löwenheim-Skolem

[3] For some inquiry into paraconsistency and mereology, see Priest (2014) and Weber and Cotnoir (2015).

[4] If a set A contains n atoms, there will be precisely $2^n - 1$ fusions, since there are as many fusions as there are subsets of A except for the empty set. Mereology does not allow for a null object.

theorem, no first order theory with a denumerable vocabulary can guarantee that; every such theory with an infinite model has a denumerable model.[5]

What, then, is the point of defending unrestricted fusion within a first order theory when, in the infinite case, there will be uncountably many fusions 'left out'? Perhaps a defence of unrestricted fusion should be made against a stronger logical background including plural or higher-order quantification? Within the limitations of first-order theory, the defence of unrestricted fusion will in large part turn out to be a defence of the existence of the universe. I will argue, then, that if junk is possible, then a case can still be made for the existence of the universe by the same standard for possibility as for junk. In the present case, this is sufficient for a defence of unrestricted fusion as the junk argument appears only to present a counterexample to the existence of the universe, not to any other fusion.

As is usual, iff is an abbreviation for if and only if. Unary connectives bind stronger than binary ones, and conjunction and disjunction bind stronger than material implication and equivalence. Outer universal quantifiers are implicit for unbound variables unless otherwise noted. Context will disambiguate between use and mention.

[5] See also Cotnoir and Varzi (2021, 232).

Chapter 1

Classical Mereology

Understanding the junk argument requires a basic understanding of mereological concepts, and resolving the tensions raised by it requires philosophical choices about the properties of the parthood relation. In order to lay the groundwork for the argument and upcoming discussions, this chapter will provide a short introduction to the central concepts of classical mereology based on the first-order framework and notation used by Leonard and Goodman (1940).[1]

1.1 Parthood

Mereology is the theory of parthood relations. That is, mereology is the theory of the relations between parts as such, but also of the relations between parts and the wholes that are composed of them. As such, mereology is a theory both of parthood and composition. Generally the term classical mereology is reserved for extensional theories that reach the strength of the *Calculus of Individuals* of Leonard and Goodman (1940). Even though mereology is most often thought of as the theory of parthood relations this relation is not always taken as primitive. Leonard and Goodman (1940) take disjointness as their primitive, while Goodman (1951) takes overlap. Still, parthood is often taken as the primitive notion, as by, for example, Hovda (2009), Varzi (2008) and Kleinschmidt (2017). Simons (1987) also takes parthood as his primitive, but works formally with the relation called

[1]Strictly speaking, Leonard and Goodman (1940) formulated their theory in the language of first-order logic augmented with the language of set theory for the critical axiom for Unrestricted Fusion. No set theoretical language will be employed in the following presentation which is more akin to that of Goodman (1951), cf. Cohnitz and Rossberg (2014).

proper parthood that is often thought better to match with the relation expressed by the English 'part'. As will be made clearer later, which mereological relation is taken as primitive can have implications for the ensuing system depending on what properties that primitive relation is taken to obey, cf. Parsons (2014).[2]

As in Hovda (2009, 57, fn.2) the departing point for the below development of mereology is the full language of the first-order predicate calculus enhanced with the addition of a special dyadic predicate, in this case '$\leq$', governed by additional axioms. On this view, there is really nothing logical about mereology other than the fact that it is developed within the constraints of the language of first-order logic. While some authors believe that mereology is closer to logic than on this view, their views require specific commitments about the deeper nature of, at least, some mereological concepts. One much discussed such view is that of Composition as Identity (CAI), which is briefly discussed in section 2.2.

Classical mereology takes parthood to be a partial order. That is, classical mereology holds that parthood is reflexive, transitive and antisymmetric. A relation R is reflexive iff xRx, transitive iff $xRy \wedge yRz \rightarrow xRz$, and antisymmetric iff $xRy \wedge yRx \rightarrow x = y$. Applying these definitions to parthood yields:

($\leq$-Reflexivity) $$x \leq x$$

($\leq$-Transitivity) $$x \leq y \wedge y \leq z \rightarrow x \leq z$$

($\leq$-Antisymmetry) $$x \leq y \wedge y \leq x \rightarrow x = y$$

Let the theory of partial orders, PO, be the first-order theory (with identity) with $\leq$-Reflexivity, $\leq$-Transitivity and $\leq$-Antisymmetry as axioms.

(PO) {$\leq$-Reflexivity, $\leq$-Transitivity, $\leq$-Antisymmetry}

It is sometimes complained that it goes against the meaning of 'part' in English that something could be a part of itself. After all, the objector might say, I am not part of myself, I am all of myself. This unease stems from a lack of disambiguation of parthood in English (and, presumably, other natural languages) that conflicts with parthood being a weak partial order allowing for symmetric

[2]The discussion in this chapter draws particularly on Simons (1987, ch. 2), Hovda (2009), Varzi (2016), Linnebo and Florio (2021, ch. 5) and Cotnoir and Varzi (2021, ch. 2).

instances. Parthood in mereology is really a relation of part or whole which entails self-parthood. Mereology has a relation to offer the objector which corresponds to a strict partial order, the perhaps more familiar relation of *proper parthood*.

Taking $\leq$ as primitive, let proper parthood be denoted by $<$ and defined as either of the following, where $x \nleq y$ is shorthand for $\neg(x \leq y)$:

(PP1) $$x < y := x \leq y \wedge x \neq y$$

(PP2) $$x < y := x \leq y \wedge y \nleq x$$

The first of these says that something is a proper part if it is a part not identical to the whole, whereas the second says that a proper part is asymmetric parthood. Since parthood is defined as a partial order these two definitions are equivalent relative to PO in the sense that each can be derived from the other:

(PP$\leftrightarrow$) $$(x \leq y \wedge x \neq y) \leftrightarrow (x \leq y \wedge y \nleq x)$$

The proof for PP$\leftrightarrow$ relies on the $\leq$-Antisymmetry of parthood for left-right, but only on the logic of identity for right-left.

Proof. For left-right, suppose for reductio that $y \leq x$. Then, by $\leq$-Antisymmetry, $x = y$, but that contradicts the second conjunct of the hypothesis. So, $x \nleq y$. For the other way around, suppose for reductio that $x = y$. But if so, then both $x \leq x$ and $x \nleq x$. Contradiction, so $x \neq y$. □

Both definitions disallow the counterintuitive aspect of something being a part of itself. A part is never a proper part of itself, by PP1 on pain of non-self identity and by PP2 by stipulation of asymmetry.

1.2 Supplementation and Extensionality

The requirement that parthood be a partial order is generally not thought to be sufficient for a theory of parthood, as there are apparent counterexamples. The simplest is that the above definitions of proper parthood allow for an object to have a single proper part as in fig. 1.1. According to Simons (1987, 1.4) that 'goes against what we mean by "part."' From this thought, it is natural to introduce principles of *supplementation* that tell us something of what other parts must

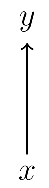

Figure 1.1: A single proper part

supplement a proper part in order to obtain a whole. One such principle is the following, which states that if x is a proper part of y, then there must be another proper part of y which is not identical to x:

(Company) $$x < y \rightarrow \exists z(z < y \wedge z \neq x)$$

While this principle rules out the case where there is only a single proper part, it is too weak to rule out an infinitely descending chain of single proper parts where every proper part on each level of decomposition only has one proper part as in figure 1.2.

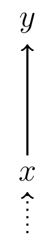

Figure 1.2: Company

It might therefore be preferable with a stronger principle that states that if x is a proper part of y, then there must be another proper part of y which is not part of x:

(Strong Company) $$x < y \rightarrow \exists z(z < y \wedge z \not\leq x)$$

However, as Simons (1987, 27) points out this allows for a model such as the one in Figure 1.3 where all proper parts overlap. However, if a whole truly has proper parts one would think that there should at least be two of them that do not overlap. Consider a chair. Since the seatback is a proper part of the chair, there is another proper part of the chair that does not overlap the back, e.g. the seat or the legs.

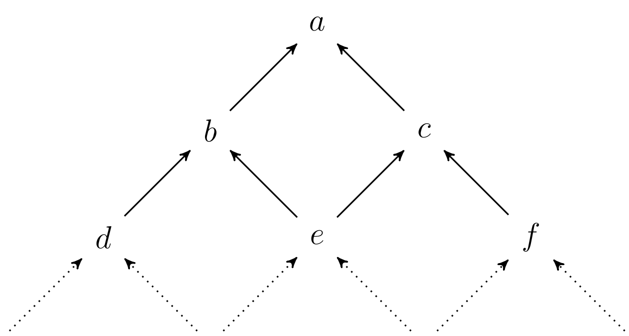

Figure 1.3: Strong Company

In mereological terms, with parthood primitive, something overlaps something else if the two objects have a part in common. If they do not overlap, we say that they are disjoint or, in other words, they do not have a part in common.

(Overlap) $$x \circ y := \exists z(z \leq x \wedge z \leq y)$$

(Disjoint) $$x \wr y := \neg(x \circ y)$$

With these resources we can formulate the thought that a whole can not have a single proper part that overlaps all other proper parts:

(Weak Supplementation) $$x < y \rightarrow \exists z(z < y \wedge z \wr x)$$

Informally, if something has a proper part x, then it also has another proper part z that does not overlap x. This satisfies our thought that the chair has at least one proper part that does not overlap the seatback. Weak Supplementation is said to be weak because it together with PO does not force the resulting mereological theory to be extensional. It allows for non-extensional models as can be seen by considering fig. 1.4. Here each of the proper parts of a and b serves as a supplement for the other to satisfy Weak Supplementation, yet a and b are distinct despite being composed of the same proper parts.

There is also a stronger supplementation principle that says something about the conditions obtaining if something is not part of something else, i.e. if x is not a part of y. When this condition obtains, then Strong Supplementation says that

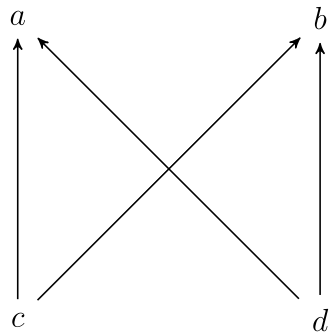

Figure 1.4: Weak Supplementation without Strong Supplementation

there is something that is a part of x, but disjoint from y.

(Strong Supplementation) $$x \not\leq y \to \exists z(z \leq x \wedge z \wr y)$$

This principle can also be motivated by a colloquial example: Consider a room with a chair and a table. Since the table is not part of the chair, there is a part of the chair that does not overlap the table: in this case the entire chair. However, this example shrouds some of the generality inherent in Strong Supplementation. As pointed out in Cotnoir and Varzi (2021, 25) there are three cases to consider when x is not a part of y: '(i) the case where y is a proper part of x; (ii) the case where y is not a proper part of x but where x and y overlap; and (iii) the case in which x and y are completely disjoint.' The example above really only covered the latter case. For case (i), consider the chair again. It is not part of the seatback, so there is something that is part of the chair, but disjoint from the seatback, e.g. the legs. Case (ii) is similar, except that in this case y is not wholly contained within x as a proper part. Let x be the upper $\frac{2}{3}$ of the chair and y be the lower $\frac{2}{3}$. Neither is a part of the other, but the middle third overlaps both x and y. Strong supplementation thus tells us that there is something which is part of x, the upper $\frac{2}{3}$, but disjoint from y. That would be the uppermost third of the chair, the upper half of x.

Consider again the model in fig. 1.4. It does not satisfy Strong Supplementation: We have $a \not\leq b$, but there is no $z \leq a$ that is disjoint from b, as both c and d are proper parts of b. This is not a full argument for extensionality, but it raises the question of whether Strong Supplementation rules out all non-extensional models. Does it? First note that even the unsupplemented theory is somewhat extensional in that $\leq$-Antisymmetry is an extensionality principle for improper parthood:

($\leq$-Extensionality) $$x \leq y \wedge y \leq x \rightarrow x = y$$

There is another extensionality principle that holds by virtue of $\leq$-Antisymmetry:

($\circ$-Extensionality) $$\forall z(z \circ x \leftrightarrow z \circ y) \rightarrow x = y$$

$\circ$-Extensionality is equivalent to $\leq$-Extensionality, as its antecedent can be rewritten as $x \leq y \wedge y \leq x$ by unpacking the definition of overlap. So, both of these extensionality principles hold even without any principle of supplementation, but a strong supplementation principle is required in order to secure the extensionality of proper parthood:

($<$-Extensionality) $$(Co(x) \wedge Co(y)) \rightarrow (\forall z(z < x \leftrightarrow z < y) \rightarrow x = y)$$

This extensionality principle requires that x and y be composite in the sense that a composite has proper parts:

(Composite) $$Co(x) := \exists y(y < x)$$

Without this requirement the extensionality of proper parts would entail that any atom, that is an object with no proper parts, would be identical to every other atom. That is there would be only one non-composite object. This is normally taken to be undesirable.

In a two-step argument, Simons (1987) shows that Strong Supplementation entails $<$-Extensionality. First he considers the so-called Proper Parts Principle:

(Proper Parts) $$Co(x) \wedge \forall z(z < x \rightarrow z < y) \rightarrow x \leq y$$

This says that if x is composite and all its proper parts are also proper parts of y, then x itself is a part of y. It is easy to see that conjoining two instances of Proper Parts will result in $<$-Extensionality by applying $\leq$-Antisymmetry to the consequent.[3] Simons (1987, 28-29) also shows that Strong Supplementation entails Proper Parts. As such, $\leq$-Antisymmetry along with Strong Supplementation

[3] See also Cotnoir and Varzi (2019, 111-112).

ensures that extensionality holds for the central mereological concepts of proper and improper parthood, as well as overlap. This satisfies the tenet of Goodman (1951, 26) that there should be no distinction of entities without distinction of content. This is echoed by Lewis (1991, 78) when he claims 'there is no difference without a difference-maker: x and y are identical unless there is something to make the difference between them by being part of one but not the other.'

There is also a related principle of Complementation:

$$\text{(Complementation)} \qquad x \nleq y \to \exists z \forall w (w \leq z \leftrightarrow w \leq x \wedge w \wr y)$$

When the conditions of the antecedent as discussed above obtains, this principle asserts the existence of an object z whose every part w is also part of x and disjoint from y. Informally, this object can be thought of as the 'remainder' of x if y is removed. In case (iii) above illustrated by the case of the table and the chair, there is nothing that is 'removed' from x, so the original object x itself serve as the remainder. For case (i) and (ii), z will 'include every part of x that is disjoint from y and exclude those parts of x that overlap y' (Cotnoir and Varzi, 2021, 25). For case (1): The chair is not part of the table, so there is something whose every part is a part of the table and disjoint from the chair. For case (ii), consider again the divisions of the chair from above. Again there is something whose every part is part of the top $\frac{2}{3}$, and disjoint from the lower $\frac{2}{3}$. The remainder in this case, then, will be the top third.

Complementation rules out models that Strong Supplementation does not as argued by Varzi (2016, sect. 3.3). For example, the model in fig. 1.5 satisfies Strong Supplementation: a is not a part of e, so there are z that are parts of a, and disjoint from e, namely d and f. However, Complementation is not satisfied: The complement of a with e removed is d and f, but in this model there is no thing that is composed of exactly those parts as required. We can also see that Complementation rules out the model from fig. 1.4 as $b \nleq a$, but there is nothing composed of all those parts of b disjoint from a as there are no parts disjoint from a at all. As such, there is no complement of b in a. So, the principle fails.

Drawing on the counterexamples above, the supplementation principles can be ordered by logical strength. If φ entails ψ, but ψ does not entail φ, then φ is strictly stronger than ψ. Relative to PO, Complementation is strictly stronger than Strong

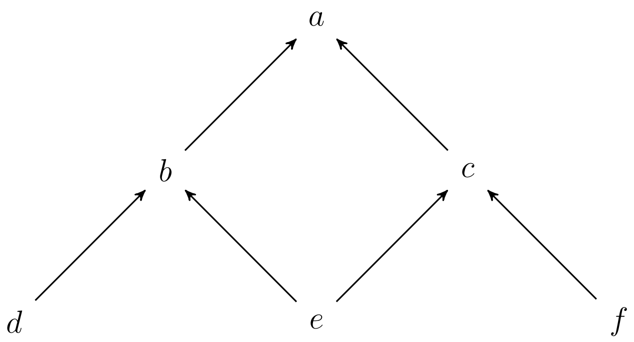

Figure 1.5: Strong Supplementation without Complementation

Supplementation, which again is strictly stronger than Weak Supplementation.[4] Weak Supplementation is strictly stronger than Strong Company, which is strictly stronger than Company.[5]

1.3 Fusion

Classical mereology is characterised by its commitment to the existence of arbitrary sums. That is, some things, whatever and wherever they might be, have a sum. The principle governing this behaviour is generally known as unrestricted composition or unrestricted fusion.

(Unrestricted Fusion) $$\exists x\varphi(x) \to \exists z Fu^i_\varphi(z)$$

This can be read: if the condition φ is satisfied, there is a z that is the fusion of the φs. But what is a fusion? It can be defined in several ways. Because of that, the Fusion-predicate above is indexed with i to the relevant definition of fusion. Here is one in terms of least upper bound:

(Fusion$_1$) $$Fu^1_\varphi(z) := \forall x(\varphi(x) \to x \leq z) \land \forall y(\forall x(\varphi(x) \to x \leq y) \to z \leq y)$$

The definition can usefully be given a reading in natural language. The definiendum can be read as z is the fusion of the φs, with fusion subject to the relevant index.

[4] See Simons (1987, 29) for a proof of Weak Supplementation from Strong Supplementation, and Linnebo and Florio (2021, 97) for a proof of Strong Supplementation from Complementation. See also Varzi (2016) for details.

[5] The entailment from Weak Supplementation to Strong Company to Company is easy to check by unpacking the definitions in the respective consequents.

The definiens of Fusion_1 is read: every φ is part of z and z is part of any (other) object y of which the φs are part. The first conjunct states that everything which is φ is part of z, so that z is an upper bound of the φs. Since the second conjunct requires z to be part of any other object that has the φs as part, this means that z is minimal among the upper bounds. Fusion_1 is thus a definition of fusion in terms of least upper bound, see Cotnoir and Varzi (2021, 29). There are also other definitions of fusion which rely on the defined notion of Overlap. One of these is similar, if not in notation, to that deployed by Lesniewski (1927):

$$(\text{Fusion}_2) \qquad Fu^2_\varphi(z) := \forall x(\varphi(x) \to x \leq z) \land \forall y(y \leq z \to \exists x(\varphi(x) \land x \circ y))$$

This states that a fusion of the φs is such that it contains every φ as part and every part of the fusion overlaps a φ. Similarly to Fusion_1, the first conjunct of Fusion_2 requires that a fusion be an upper bound of the things it fuses.

The third definition of fusion is akin to the one used by Leonard and Goodman (1940):

$$(\text{Fusion}_3) \qquad Fu^3_\varphi(z) := \forall y(y \circ z \leftrightarrow \exists x(\varphi(x) \land y \circ x)$$

This definition says that z is a fusion of the φs if anything that overlaps z also overlap a φ and *vice versa*. This definition of fusion leaves out the explicit requirement that a fusion be an upper bound of the things it fuses.[6] However, in the presence of Strong Supplementation, it follows that also a Fusion_3 is an upper bound of the things it fuses.

What is the relationship between these definitions of fusion? Relative to all other principles they are equivalent in that they all can be used to formulate classical mereology, but in weaker contexts they may come apart. Because of $\leq$-Antisymmetry, the uniqueness of fusions holds for all definitions in the context of classical mereology:

$$(\text{Unique Fusion}_i) \qquad Fu^i_\varphi(z) \land Fu^i_\varphi(w) \to z = w$$

Where 'Fu^i_φ' denotes either of the three definitions of fusion when i is replaced by

[6] This is, as argued by Cotnoir and Varzi (2021, 166), seen in the model in fig. 1.4 on page 12 where, 'a turns out to be a [Fusion_3] of a and b even though b is not part of a (and vice versa)'. See also pages 22 and 62 for some further discussion, cf. Cotnoir and Varzi (2021, 166).

the appropriate index. Each is dependent on different auxiliary principles to reach the strength of classical mereology. For example, relative to PO, Fusion_1 needs to be paired with Complementation, while Fusion_3 needs Strong Supplementation in order to axiomatise classical mereology. Fusion_2 is the 'strongest' in that it can be used alongside Weak Supplementation and a specific definition of proper parthood to specify the full theory.[7]

Equipped with these definitions of fusion, we can assert the claim of classical mereology of a fusion for any condition φ for each of the types of fusion:

(Unrestricted Fusion_1) $$\exists x\varphi(x) \rightarrow \exists z Fu^1_\varphi(z)$$

(Unrestricted Fusion_2) $$\exists x\varphi(x) \rightarrow \exists z Fu^2_\varphi(z)$$

(Unrestricted Fusion_3) $$\exists x\varphi(x) \rightarrow \exists z Fu^3_\varphi(z)$$

Unrestricted Fusion is an axiom schema stating that if any condition φ is satisfied, then there is a z such that z is the fusion of the the things satisfying φ; there is a fusion for any (non-empty) condition φ. Since there is no restriction on the conditions under which objects are fused, the principle is generally referred to as *unrestricted fusion*.[8]

1.4 Axiomatisations

Taking parthood as primitive and using the above definitions for Overlap and PP1 (or PP2), classical mereology can be formulated with parthood as a partial order and two additional axioms:

1. $\leq$-Reflexivity, $\leq$-Transitivity, $\leq$-Antisymmetry
2. Complementation
3. Unrestricted Fusion_1

However, classical mereology need not be formulated as above, and given the equivalence relative to PO of many of the alternative definitions, other formulations of the theory may be given. One example is the formulation of classical mereology

[7] The proofs are straightforward, but involved, see Cotnoir and Varzi (2021, 159-193) for proofs and discussion.

[8] As noted above, following Lewis (1986) it is sometimes referred to as *unrestricted composition*. Sometimes, following van Inwagen (1990) it is also referred to as *Universalism*.

which uses the same definitions for Overlap and PP1, but exchanges the definition of fusion to that employed by Leonard and Goodman (1940), namely Fusion$_3$:

1. $\leq$-Reflexivity, $\leq$-Transitivity, $\leq$-Antisymmetry
2. Strong Supplementation
3. Unrestricted Fusion$_3$

Another example draws on the original approach of Lesniewski (1916, 1927), which was also taken up by Tarski (1935). While this also specifies classical mereology, there are some definitional differences as Lesniewski and Tarski took proper parthood instead of improper parthood as their primitive mereological relation and required that it be transitive and asymmetric, and hence irreflexive. They then defined parthood as:

(P) $$x \leq y := x < y \vee x = y$$

This specification is in fact tantamount to holding that (improper) parthood is antisymmetric, reflexive and transitive.[9] They could then formulate classical mereology using the definitions P, Overlap and Fusion$_2$

1. $<$-Asymmetry, $<$-Transitivity
2. Weak Supplementation
3. Unrestricted Fusion$_2$

Common to all formulations of classical mereology is the commitment to Unrestricted Fusion, and even though more minimal axiom sets may be given, the commitment to parthood as a partial order is at least implicit in all. Some principle of decomposition in terms of a supplementation principle is also implicitly or explicitly present in axiomatisations of classical mereology. What is common between all axiomatisations of classical mereology can be further elaborated: In short, the result, first discovered by Tarski (1935), is that all axiomatisations of classical mereology specify a complete Boolean algebra with the zero element removed.[10]

[9]This is the well-known result that every strict partial order defines a weak partial order and *vice versa*. See also chapter 3 for a proof that $<$-Asymmetry and $<$-Transitivity suffices to show $\leq$-Antisymmetry, $\leq$-Reflexivity and $\leq$-Transitivity.

[10]See Cotnoir and Varzi (2021, 35-40) for a proof of the result.

Classical mereology has enjoyed broad support as a useful instrument for theorising about parts and wholes. Still, pretty much every commitment of classical mereology has been questioned at some point. Most controversial in contemporary metaphysics is perhaps the commitment to extensionality and the commitment to Unrestricted Fusion, with its proliferation of *prima facie* implausible objects such as Lewis's trout-turkey. In the introduction, I made an initial case for holding Unrestricted Fusion to be true. I will not have much more to say in general defence of the principle outside of it being the most plausible candidate principle guaranteeing the existence of all the semantic values needed for the literal truth of our ordinary talk of objects. The ensuing discussion of this dissertation will be concerned with a challenge to this principle and an assessment of the metaphysical cost, as it were, of maintaining the principle in the face of the challenge from the so-called junk argument.

Chapter 2

The Junk Argument

2.1 The Challenge to Necessity

Bohn (2009b) presents a challenge to unrestricted fusion. It is normally accepted that whatever principle of composition is true, it must be so necessarily. Hence, if true, unrestricted fusion is necessarily true. Then, necessarily, if the condition φ is satisfied, then there is some y that is the fusion of the things satisfying φ.

(Unrestricted Fusion) $$\exists x \varphi(x) \rightarrow \exists z F u^i_\varphi(z)$$

However, by appeal to the conceptual possibility that everything is a proper part of something, Bohn purports to offer a *reductio* of the necessity of unrestricted fusion. His first premise, for *reductio*, is:

(2.1) Unrestricted fusion is necessarily true.

Bohn (2009b, fn. 2) takes care to point out that the necessity in question is *de dicto*. So, he is not entertaining the question of whether a composite object has its parts necessarily, but rather the question of whether it necessarily is the case that any collection of objects compose a further object.[1] By 2.1 it follows that any class of objects composes a further object.[2] Central to Bohn's argument is the case of

[1]The claim then is $\Box(\exists x \varphi(x) \rightarrow \exists z F u^i_\varphi(z))$, not $\exists x \varphi(x) \rightarrow \Box \exists z F u^i_\varphi(z)$. For more on the interaction of mereology and quantified modal logic, especially as it pertains to whether objects have their part necessarily, see Uzquiano (2014).

[2]Bohn speaks of a class composing an object, but van Inwagen (1990) does not believe in classes and so formulates his mereology in terms of plural predication. For current purposes, a principled objection to class-talk makes little difference. All locutions of a class composing

universal fusion: since every class composes a further object, in particular the class of every object composes some further object. This object is called the universal object or the universe, U.

(2.2) There must be a universal object, U

The universal object has one important property for Bohn's purposes: since every object is a part of U, it appears that there is nothing left for U to be a proper part of. The universe exhausts all objects. Everything, unrestrictedly, is part of the universe:

(Universe) $$\exists x \forall y (y \leq x)$$

This follows readily from upper bound requirement of the first conjunct of Fusion_1 by letting φ be the condition $x = x$ and likewise for Fusion_2. Fusion_3 does not, on its own, require that a fusion be an upper bound of the things it fuses, but this follows in the presence of Strong Supplementation, cf. Cotnoir and Varzi (2021, 166). Thus, Universe also holds by virtue of Unrestricted Fusion_3 in the context of classical mereology. Intuitively, the result also holds in terms of the overlap requirement of Fusion_3: If φ is taken to be a trivial condition then Unrestricted Fusion_3 says that there is a z that overlaps everything. It is hard to see how this could obtain if not everything was also part of z.[3]

Bohn (2009b, 28) remarks that 2.2 must hold regardless of whether the cardinality of the world is finite or infinite. To hold that only finite worlds have a universal object is to hold that fusion is restricted, contrary to our initial assumption.[4] Bohn then moves to the primary premise of his argument: consider a world in which

an object can be translated into an idiom of plural predication in which 'the class α composes y' is equivalent to 'the xs compose y', provided every x in α is one of the xs, cf. Boolos (1984, 1985). The idiom of this dissertation is generally to avoid class talk as well as plural predication, and rather to speak of a the things satisfying a condition statable in the first-order language of mereology as forming a fusion.

[3]In the presence of $\leq$-Reflexivity and $\leq$-Transitivity, Strong Supplementation is equivalent to a principle of overlap supervenience: $\forall x \forall y (\forall z (z \circ x \rightarrow x \circ y) \rightarrow x \leq y)$. In turn, this principle entails the requirement that a Fusion_3 be an upper bound of the things it fuses: $\forall z (Fu^3_\varphi(z) \rightarrow \forall x (\varphi(x) \rightarrow x \leq z))$, see Cotnoir and Varzi (2021, 167) cf. Gruszczyński (2013).

[4]Nonetheless, some authors have argued that a principle of unrestricted dual sum is candidate for a necessarily true principle of fusion, see for example Contessa (2012). This principle would guarantee the existence of all finite, but no infinite fusions: $\forall x \forall y \exists z (x + y = z)$. See Cotnoir (2014) for discussion.

everything is a proper part of some further thing. Bohn (2009b, 28) suggests conceiving of the possibility that everything 'is spatially extended and just one half of something else that is also spatially extended.' He also considers a more elaborate possibility:

> Our universe is a miniature replica universe housed in a particle of a bigger replica universe, which is again a miniature replica universe housed in a particle of an even bigger replica universe, and so on ad infinitum. (Bohn, 2009b, 28)

If this possibility obtains, then, Bohn (2009b, 28) says, the world would be 'junky'. He then presents his '[o]fficial definition: A world w is junky $=_{\text{df.}}$ anything in w is a proper part.' This can be represented in the formal mereological language from chapter 1, which in natural language can be read as Bohn's official definition with the relativisation to a world w dropped:

(Junk) $$\forall x \exists y (x < y)$$

This suggests that the model satisfying Junk would look like the model in fig. 2.1. Here every object is a proper part and every object that has a proper part has another disjoint proper part so as to satisfy Weak Supplementation, which Bohn, following Simons (1987), claims is constitutive of the meaning of proper parthood.[5] Bohn then claims that we have conceived of a world being junky, and suggests that this gives a *prima facie* reason to believe that junky worlds are possible.

Conceivability is a good, but not infallible guide to possibility. One might conceive of a scenario, only to discover on closer inspection that the scenario was ill-conceived after all. For example, it appears perfectly possible to conceive of an infinitely divisible gold bar. Imagine a gold bar cut in half and keep repeating *ad infinitum*. However, gold is not infinitely divisible. When the pieces get sufficiently small they cease to be gold.[6] It appears equally easy to imagine a perfectly tuned piano, but it is impossible to tune a piano perfectly because 'the ratio 2:1 (an octave) is not a power of 3:2 (a fifth). Pianos cannot be tuned so that all octaves

[5]This commitment is discussed more fully in chapter 3. Also, in order to simplify the figure, the model in fig. 2.1 is atomistic, but junk is also consistent with atomless models. In that case each of a_0, a_1 and a_2 would require infinitely descending proper parthood chains below, cf. fig. 3.9.

[6]It is a matter of physics to say exactly how large these pieces would be, but when their mass gets below approximately 197 atomic units they are no longer made up of gold.

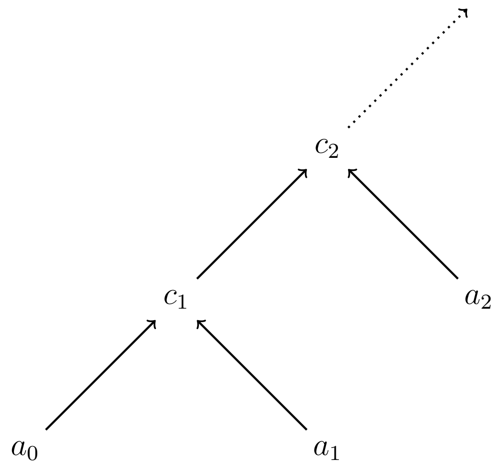

Figure 2.1: Atomistic Weakly Supplemented Junk

and fifths are perfect.' (Weber, 2021, 27). In these cases, what one conceived was not possible after all.

Could this be the case in conceiving about junky worlds? Bohn thinks not, and he gives two reasons. For one thing, several prominent philosophers have held junky worlds to be possible. In his support, Bohn quotes Leibniz, G.E. Moore, and A.N. Whitehead. Quoting Simons (1987, 83), Bohn points out that Whitehead did not even argue for the possibility of junk, but apparently just found it plainly obvious that 'the world is "open" above (...)'. This appeal to authority plays the role of circumstantial evidence in favour of the possibility of junk.

The second reason rests on the existence of junky models for many non-classical mereologies. This, Bohn (2009c, 29) claims, 'entails that there are no logical contradictions lurking in the background.'. This is supposed to still concerns about junk not being possible after all. Bohn thus gives three reasons in favour of the possibility of junk: i) conceivability, ii) reputable philosophers thought junk possible, and iii) junk is logically consistent. From this Bohn (2009b, 29) infers that 'we are hard pressed to deny the mere possibility of the world being junky.'[7]

(2.3) The world might be junky.

Since, by 2.1, unrestricted fusion is necessarily true, there should be a universal

[7]In spite of this three-step test, Watson (2010) charges that Bohn has failed in conceiving of a junky world. See Bohn (2010) for a rejoinder that concedes that the argument from conceivability offers 'independently *good* though not infallible evidence for possibility.'

object even if the world were junky. Consider then the fusion, U of all objects in a junky world. Does it exhaust all the objects? No, for by the 'official definition' of junky worlds everything is a proper part, and this must also apply to U. So, U was not a universal object after all. As Bohn puts it, a junky world w cannot contain a universal object, for if it did there would exist something at w that was not a proper part of anything, namely U, and that would contradict the world being junky. Instead of a universal object, Bohn (2009b, 29) claims, a junky world contains an '*infinite plurality* of objects such that each thing is a proper part of something else in the plurality.' So, it follows from 2.3 that

(2.4) There might not be a universal object U.

But 2.4 contradicts 2.2, so the initial hypothesis, 2.1, the necessity of unrestricted fusion must be denied.

From the denial of the necessity of unrestricted fusion, Bohn claims, it follows either that some principle of restricted composition is necessarily true or that composition is contingent. Bohn (2009b, 30) believes that the latter is more plausible because the most plausible principle of restricted composition, that all and only finite pluralities have a fusion, is inconsistent with the world being *gunky*. In a gunky world, everything *has* a proper part:

(Gunk) $$\forall x \exists y (y < x)$$

If everything has a proper part, then any object is infinitely divisible and hence any fusion will have to be of infinite cardinality. Since it is possible, maybe even probable, that the actual world is gunky, Bohn (2009b, 30) rejects the option of a necessarily true principle of restricted composition.[8] He concludes instead that 'which fusions exist and which don't is a contingent matter, varying from world to world.'

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[8]Why is the world potentially gunky? Modern physics have repeatedly discovered ever smaller elementary particles; witness the discovery of the electron and later of quarks. It would be difficult to rule out the possibility of a future discovery of even smaller particles.

Additionally, consider the position of the mereological nihilist in a gunky world. The nihilist believes that there are no composites, only mereological simples. Since Gunk will ensure that there are only composites, there can be no simples. Hence, the nihilists's 'ontology would drain away into a bottomless pit.' (Schaffer, 2007, 184).

2.2 Assessing the Junk Argument

It is somewhat puzzling that Bohn concludes from the denial of the necessity of Unrestricted Fusion that which fusions exist might vary from world to world. After all, it is consistent with Unrestricted Fusion that which fusions exist might vary from world to world.

Consider a simple example of two atomic worlds: w_1 and w_2.[9] Let w_1 contain two atoms and let w_2 contain three atoms. By Unrestricted Fusion, given n atoms, there are $2^n - 1$ composite objects.[10] So, w_1 contains three composite objects, while w_2 contains seven. It appears quite clear that those fusions at w_2 composed of a greater number of parts than those at w_1 are distinct from those at w_1. Fusions varying from world to world is thus as uncontroversial as there being worlds of different cardinality. The world could have contained more or fewer things than it does, and, if so, the number of fusions would vary accordingly. There is also another way in which fusions varying from world to world is consistent with Unrestricted Fusion. Let w_1 be a world containing dogs, but no cats; let w_2 be a world of cats, but no dogs. When $\varphi(x)$ is the condition satisfied iff x is a dog or x is a cat, $Fu^i_\varphi(z)$ would differ between w_1 and w_2 on account of the qualitative difference between cats and dogs.

It appears more fitting to say that contingency attaches not to which fusions there are, but to the principle of composition that governs which fusions there are. That is to say, the *principle of composition* is contingent. On this reading, there could be multiple true principles of composition, each true of a different world, or, as Bohn might put it: a composition principle P_1 could be true of a world w_1, while another P_2 could be true of another world w_2. In one world, unrestricted fusion is true, in another world some principle of restricted composition is true, and in some third world, perhaps, even nihilism is true. Bohn's conclusion, then, is a form of pluralism about the principle of composition.

Bohn presents this as the most plausible option, but the position has peculiar consequences considering the analysis of possibility in terms of possible worlds.

[9] A world is atomic if every composite object is, at the lowest level of decomposition, composed of atoms, i.e. objects that have no proper parts: $At(x) \leftrightarrow \neg\exists y(y < x)$. That is, every composite 'bottoms out' in atoms. It might be argued that the world is not atomic, but the junk argument is about possibility and it appears at least as possible that there are atomic worlds as there are junky worlds.

[10] Unrestricted fusion ensures that the number of composite objects of a class α is equal to the cardinality of $\mathcal{P}(\alpha) - 1$, since mereology does not include a null object.

Consider what it entails that the principle of composition could vary from world to world. On a standard possible world semantics for modal statements p is possible if there is an accessible world w such that p is true at w. In other words, Bohn believes that the principle of composition could vary with what possibilities happen to obtain in the actual world. If it turned out that there were enough things of the right sort, suddenly the universe might, strictly speaking, not exist. It would be preferable to be able to affirm Unrestricted Fusion as a necessary truth.

The junk argument follows a simple structure, premise 2.1 (and its implication 2.2) are first used to establish that there is a possibility that Unrestricted Fusion rules out. Then 2.3 (and the implied 2.4) are used to make a case for the possibility that is ruled out by the first premise. Following the discussion in the literature, we can refer to the first premise as an incompatibility premise, and the second as a possibility premise. The argument thus exhibits the following structure:

(Incompatibility)
If Unrestricted Fusion is necessarily true, then there can be no junky worlds.

(Possibility) There is a junky world.

The conclusion that Unrestricted Fusion is not necessarily true then follows immediately from Incompatibility and Possibility by *modus tollens*. Both Cotnoir (2014) and Calosi (2021) analyse the structure of the argument along similar lines. This understanding of the argument constrains the possible responses: One should either reject Incompatibility by showing how Unrestricted Fusion can somehow be compatible with junk, or one should have to reject the possibility of junk in the face of Bohn's argument from conceivability to possibility.

In later papers, Bohn (2014a,b, 2019) has opted for the strategy of denying the possibility of junk by appeal to an understanding of fusion as identity, most often referred to as Composition as Identity:

(CAI) $$Fu^{=}_{xx}(z) := xx = z$$

Where xx is a plural variable ranging over one or more objects. As such, Composition as Identity will require the resources of plural quantification in order to account for identity across counts (many-one identity). Bohn (2014b) argues that classical mereology can be derived from CAI, so that classical mereology really is just a manifestation of the logic of identity. As the logic of identity cannot vary from one possible world to another, Bohn (2014b, 157) cf. Bohn (2009a) claims that 'the result is a form of *necessarily* true (...) universalism.' Given such a characteristic of the logic of identity, accepting CAI appears to be a plausible way of resisting the possibility of junk.

While this route of argument is interesting and perhaps also promising, Composition as Identity has a number of issues that would need to be resolved. The chief of which is its apparent profligate violations of Leibniz's Law. After all, everything that is one, is not many – but that is exactly what CAI appears to claim. Without extra resources CAI appears to collapse the distinction between composite objects and their parts. Moreover, even barring that objection, it is not obvious that CAI settles the Special Composition Question in favour of Unrestricted Fusion. Cameron (2012) argues that CAI establishes the biconditional claim that some things form a whole if they are identical to it, it does not say when these identities obtain. Thus, CAI appears to require the further assumption that any things (plural) are identical to some thing (singular) in order to lead to Unrestricted Fusion.

The inference to $\exists y(xx = y)$ from $xx = yy$ is what Sider (2007, 61) refers to as 'the dodgy move'. For one thing, it could be argued that there is no relation of 'plural identity', there is only a relation of mutual plural inclusion:

$$xx \approx yy := \forall z(z \prec xx \leftrightarrow z \prec yy)$$

While this definition will guarantee the extensional equivalence of two pluralities, it is not obvious that it truly specifies a relation of plural identity. Even if sense can be made of plural identity, what is required for CAI is cross-count, many-one identity as discussed by Baxter (1988a,b). Such a generalised conception of identity, as in Cotnoir (2013a, 318), requires identity to be polymorphic, taking either singular or plural terms. Regardless of whether sense can be made of such a relation, the chief concern still remains: If composition is identity, then it is 'dodgy' to assume that there is a singular object for every plurality, because that is to beg the question of Unrestricted Fusion.

Further exploration of CAI would go beyond the stated limitation of this dissertation: that identity behaves as in classical first-order logic and no attempt will be made to revise the underlying logic. In chapter 3, instead of elaborating and attempting to resolve these issues, I will instead explore a compatibilist response to the junk argument in a classical first-order context.

Chapter 3

Mereology with Loops

This chapter is devoted to exploring a compatibilist response to the junk argument, and thus proceeds to deny its Incompatibility premise. Is there an acceptable formulation of a mereological theory that allows for the existence of junk as well as Unrestricted Fusion? In this chapter, it is argued that there is, but only at the cost of relaxing the constraints on the parthood relation as a partial order on which classical mereology rests. Consider what would be required for Unrestricted Fusion and junk to be compatible. Since everything in a junky world is, by definition, a proper part, it appears that the universe must also be a proper part. There is only one plausible candidate object of which the universe could be a proper part, namely itself. Can something be a proper part of itself?

That would go against the mereological commitments of Bohn (2009b, 27), who briefly spells them out in a footnote: 'There is a natural distinction to be drawn', Bohn claims, 'between the principles that have existential import and the ones that don't. I assume at least some mereological principles without existential import are analytically true, and in virtue of that are necessarily true too.' He then goes on to state that these are the axioms of the system Simons (1987, 25-31) calls Minimal Extensional Mereology (MEM). The axioms of this system, Bohn (2009b, 27) claims, is 'a *minimum* of mereological necessary truths'. This system includes axioms for the asymmetry and transitivity of proper parthood and 'a weak supplementation principle'.

(<-Asymmetry) $$x < y \rightarrow y \nless x$$

(<-Transitivity) $$x < y \wedge y < z \rightarrow x < z$$

From these principles, the irreflexivity of proper parthood immediately follows:

(<-Irreflexivity) $x \not< x$

While Bohn does not specify the ‘weak’ supplementation principle, Simons (1987), in his original formulation of MEM, employs Weak Supplementation.

(Weak Supplementation) $x < y \rightarrow \exists z(z < y \land z \wr x)$

Bohn draws a line between the mereological principles that have existential import and those that do not, and holds that only the latter can be analytically true. At the same time he claims that Weak Supplementation, with its existential import, is analytically true, see also Cotnoir (2018, 4239-4240, fn. 27).

Bohn also cites Rosen (2006) in his support for his choice of analytic principles of mereology, but Rosen (2006, 19) does not endorse the asymmetry of proper parthood as analytic. He rather holds that the ‘analytic core’ of mereology consists of ≤-Reflexivity and ≤-Transitivity. Notably, he does not endorse ≤-Antisymmetry as analytic. Perhaps because a case can be made against its analyticity. At least Bohn and Rosen agree that Unrestricted Fusion is not analytic, but in this context that is neither here nor there: I will later argue against the analyticity of Weak Supplementation, but the case for that does not require a case for the analyticity of Unrestricted Fusion.

At any rate, Bohn endorses Weak Supplementation. As Weak Supplementation alone is not sufficient for extensionality, Simons (1987, 30) employs the definition of Overlap and relies on a principle of ‘maximal common part’ for overlapping objects:

(Max Common Part) $x \circ y \rightarrow \exists z \forall w(w \leq z \leftrightarrow w \leq x \land w \leq y)$

Intuitively, Max Common Part is weaker than Unrestricted Fusion, because the former only asserts the existence of a further object when two objects overlap. This can be seen if we compare the objects whose existence are asserted by Max Common Part and Unrestricted Fusion in a simple case of two partly overlapping objects. In this case, Max Common Part asserts the existence of a further object z, whose every part is also part of both x and y as indicated by the shaded area of fig. 3.1. This object is the mereological product of x and y. Since every part of z

must also be part of both x and y, this clearly satisfies Max Common Part.

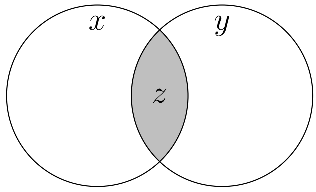

Figure 3.1: Max Common Part

Unrestricted Fusion, on the other hand, also asserts the existence of a further object for the non-overlapping parts. If the extension of φ is taken to be x and y, then $Fu^i_\varphi(z)$ will correspond to the shaded area in fig. 3.2, corresponding to the mereological sum of x and y. In fact, the main reason that Unrestricted Fusion is more existentially profligate than Max Common Part is due to its assertion of the existence of an object overlapping all of both x and y even when x and y does not overlap each other as in fig. 3.3. In this case, Max Common Part would not assert the existence of a further object at all.[1] In this sense, Simons's MEM is clearly weaker than classical mereology. Loosely speaking, Unrestricted Fusion asserts the existence both of larger and more objects.

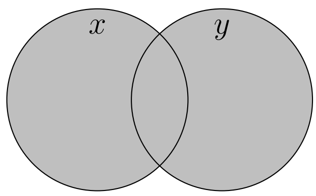

Figure 3.2: Fusion$_i$ with Max Common Part.

Simons shows that the mereology resulting from $<$-Transitivity, $<$-Asymmetry, Weak Supplementation and Max Common Part is strong enough to derive Strong Supplementation.[2] MEM is therefore minimal because it offers extensionality without primitive commitment to the stronger principle of Strong Supplementation.

[1] Max Common Part is vacuously satisfied in the case of fig. 3.3 as the antecedent, $x \circ y$, comes out false .

[2] See also Cotnoir and Varzi (2021, 111, fn 35).

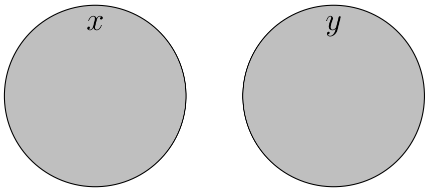

Figure 3.3: Fusion$_i$ without Max Common Part

This suggests that the label *minimal* for MEM is misleading, as it is a strong, extensional theory with significant existential commitments – if still weaker than classical mereology. Bohn presumably endorses MEM because he desires extensionality, but equally desires to include a supplementation principle no stronger than Weak Supplementation, which he believes analytic in spite of its existential import. However, PO and Weak Supplementation alone is not sufficient to guarantee extensionality. Thus, Bohn requires an additional principle to ensure extensionality. Max Common Part does that job, and ensures that MEM is weaker than classical mereology because MEM does not guarantee a fusion for every condition, cf. Koslicki (2008, 19).[3]

From a philosophical standpoint, while certainly less profligate than Unrestricted Fusion, it does not appear particularly unprofligate to accept the existence of a Max Common Part for any pair of overlapping objects. Bohn could perhaps have endorsed what Varzi (2016) calls *Minimal Mereology* which consists of PO and Weak Supplementation, but this theory is not extensional. For the purposes of this dissertation, then, what Bohn takes to be the *minimum* of necessary mereological truths are charitably taken to be only those that he explicitly names: the asymmetry and transitivity of proper parthood and Weak Supplementation.[4] There is not much of a case to be made for the analyticity of Max Common Part.

Bohn does not speak of the properties of (improper) parthood, because he follows Simons (1987) in taking proper parthood as his mereological primitive. The choice of primitive makes little formal difference in classical mereology, but there may be philosophical reasons to prefer one above the other. The presentation in

[3]While the primary focus of Tsai (2009, 2011) is the decidability of mereological theories, the papers also include a useful ranking of the strength of various mereological theories that also indicate that MEM is not a 'weak' theory.

[4]As is dicussed below, these explicit commitments are in fact exactly the axioms of Varzi's *Minimal Mereology.*

chapter 1 took parthood as a (weak) partial order and defined proper parthood as a strict partial order by PP1 or PP2, but that choice is not forced. One could take proper parthood ($<$) as primitive and let it obey $<$-Asymmetry and $<$-Transitivity, so that proper parthood is a strict partial order. One can then define (improper) parthood:

(P) $$x \leq y := x < y \vee x = y$$

The choice of $\leq$ or $<$ as primitive makes no difference to the resulting theory. From $<$-Asymmetry and $<$-Transitivity it can be shown that $\leq$ obeys PO.

Proof. The transitivity of $\leq$ is obviously inherited from $<$ by definition P as both $<$ and $=$ are transitive. The required reflexivity is also inherited from $=$ in the case where $x \leq x$. For $\leq$-Antisymmetry, suppose that $x \leq y$ and $y \leq x$. Then, by P, either $x < x$ due to the transitivity of $<$, or $x = y$. By $<$-Asymmetry we have $x \not< x$, so $x = y$ by first-order logic (disjunctive syllogism). □

From the $\leq$-Transitivity, $\leq$-Antisymmetry and $\leq$-Reflexivity of $\leq$, that is from PO, it can be shown that $<$ obeys $<$-Asymmetry and $<$-Transitivity.

Proof. By PP$\leftrightarrow$ it suffices to show that the transitivity and asymmetry of $<$ follows from either of PP1 or PP2. Transitivity is obviously inherited on PP1. For asymmetry, let $x < y$, and suppose for reductio that $y < x$. However, by transitivity of $<$, that violates PP1 which gives us $x \neq x$. So, $y \not< x$. □

From this we conclude that the differences between the presentation of mereology based on parthood in chapter 1 and Bohn's commitment to the transitivity and and asymmetry of proper parthood are merely superficial. Minimal Extensional Mereology (MEM) is then the theory resulting from PO and two additional axioms:

(MEM) $$\text{PO} \cup \{\text{Weak Supplementation, Max Common Part}\}$$

If it is analytic that proper parthood obeys asymmetry, then nothing can be a proper part of itself. However, *pace* Bohn's claim for the analyticity of the principles of MEM, there appears to be reason to question the asymmetry of proper parthood. This can be done by considering examples where the antisymmetry of improper parthood appears to be violated.

3.1 Parthood Loops

Several authors have made cases for mereologies that put $<$-Asymmetry into question. If their arguments have merit, there appears to be a *prima facie* case against its status as an analytic, and so necessary, truth. As such, that would form an argument against Bohn's claim that the asymmetry and transitivity of proper parthood as well as Weak Supplementation must be constitutive of any theory of parthood.

The examples that have appeared several times in the literature and recently been discussed thoroughly by Cotnoir (2010, 2013b, 2018) and are also considered by Cotnoir and Varzi (2021, Ch. 3). I first consider some examples that consider parts of concrete objects. I then consider some that draw on parts of abstract objects. Finally, I discuss what the examples can be taken to show in light of Bohn's three-step test for possibility. I argue that the examples fare as well as his case for the possibility of junk.

Concreta

In book 10 of the Hindu holy text *Bhāgavata Purāṇa*, Krishna's mother peeks inside Krishna's mouth, and within she sees the whole universe. At face value this implies that the universe is a proper part of Krishna. Moreover, Krishna is an object in the world, and is thus also a proper part of the universe. There is no reason given by the example to suggest that Krishna and the whole universe are identical. There thus appears to be a counterexample to $\leq$-Antisymmetry as illustrated in fig. 3.4.

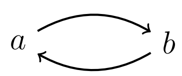

Figure 3.4: Parthood Loop

Insofar as parthood is transitive, it also appears that this example makes a case for how something could be a proper part of itself. Krishna is a proper part of the universe, but the universe is also a proper part of Krishna. Since Bohn (2009b, 27) accepts $<$-Transitivity, this example also serves as a *prima facie* counterexample to his claim that proper parthood is asymmetric as illustrated by fig. 3.5.

Cotnoir and Bacon (2012, 188) and Cotnoir and Varzi (2021, 65) note that Sanford (1993) claims that the Aleph in Jorge Luis Borges's short story *The Aleph*

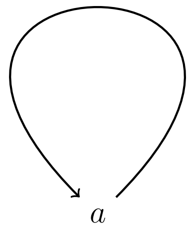

Figure 3.5: Reflexive Proper Part

exhibits a case of proper self-parthood and a violation of antisymmetry. The Aleph is a small cube sitting on the nineteenth step of the dining-room cellar stair in the house of Beatriz Viterbo: 'The Aleph's diameter must have been about two or three centimetres, but Cosmic Space was in it, without diminution of its size.' As the narrator of the story goes on: '[I] saw the Aleph from all points. I saw the earth in the Aleph and in the earth the Aleph once more and the earth in the Aleph [...].'.

This case appears exactly parallel to the case of Krishna and the universe: The Aleph and the Earth are both parts of each other, but not identical. Sanford (1993, 222) argues that the Aleph contains everything. He reasons thus that the Aleph is part of the household effects of Viterbo's house, but the household effects are also part of the Aleph. Yet, they are not identical. In the presence of transitivity this is also a counterexample to the irreflexivity and asymmetry of proper parthood, as the latter quotation from Borges already indicates.[5]

Priest (2014, 179) presents the concept of Indra's net of jewels from Buddhist philosophy by quoting Cook (2010):

> Far away in the heavenly abode of the great god Indra, there is a wonderful net which has been hung by some cunning artificer in such a manner that it stretches out indefinitely in all directions. In accordance with the extravagant tastes of deities, the artificer has hung a single glittering jewel at the net's every node, and since the net itself is infinite in all dimensions, the jewels are infinite in number. There hang the jewels, glittering like stars of the first magnitude, a wonderful sight to behold. If we now arbitrarily select one of the jewels for inspection and look closely at it, we will discover that in its polished surface there are reflected all the other jewels in the net,

[5]For more in-depth discussion of the Aleph, see also the unpublished Parsons (2013).

> infinite in number. Not only that, but each of the jewels reflected in this one jewel is also reflecting all the other jewels, so that the process of reflection is infinite

There is then the net of jewels of which every jewel is a proper part, but according to Buddhist tradition every jewel appears to contain every other jewel of the net. Thus, it seems that the entire net is contained within each jewel. Pick an arbitrary jewel: it is a proper part of the net, and the net is a proper part of the jewel. Since there, again, is no reason to believe the net and the jewel identical, $\leq$-Antisymmetry is violated. In the presence of transitivty the net is also a counterexample to the asymmetry of proper parthood. Jones (2009) provides a similar mereological interpretation of the case.

The example of the Net of Indra says that every jewel is *reflected* in every other. This could indicate that, contrary to the interpretation of the case by Jones (2009), there is no mutual proper part containment, but rather mere photic reflection of the jewels in each other's shiny surfaces. If so, the example might not constitute violations of $\leq$-Antisymmetry and $<$-Asymmetry. The same can perhaps also be said of the example of the universe inside Krishna's mouth. Perhaps the universe is not contained in Krishna, but that there is a very special mirror in his mouth that reflects the entirety of the universe. Borges's Aleph might also be taken to be such a very special mirror. For the sake of argument, grant this interpretation of the examples. Does this mean that the examples fail to support the required conclusion? Not necessarily, since Bohn's junk argument relies only on the possibility of junk, the response requires only the possibility of the truth of the examples. As such, it is not necessary that the examples support the conclusion that $\leq$-Antisymmetry and $<$-Asymmetry are violated on any reading, but that there is at least one reading on which the conclusion is supported. Given the interpretations by Jones (2009) and Priest (2014), it is clear that such a reading exists.[6]

If this is not convincing enough, Cotnoir and Varzi (2021, 66) present another example from a Hindu holy text, the *Upanishads*, which does not as readily give way to a reading that does not support mutual parthood without identity. The relevant passage discusses the relationship between Brahman and persons: Within the heart of each person is a very small lotus-like abode, and the further that '[t]he whole of the heaven and the whole earth can be found inside this little space.' (*Chandaya Upanishad*, ch. 4, sect. 1) On the face of it, this case is not about

[6] I am grateful to Peter Fritz for raising this objection.

reflection, but plainly about proper containment.

Similar examples can be found in other religious texts. Cotnoir and Varzi (2021, 66) point to an example from the Gospel of John of Christian theology: ‘I am in the Father, and (...) the Father is in me’ (14:10). Cotnoir (2017) argues that such religious examples of mutual indwelling cannot be dismissed as mere fiction, but that, ‘[p]hilosophically, their conceivability and metaphysical possibility appear to be a substantive question that raises genuine challenges to the antisymmetry of parthood’, cf. (Cotnoir and Varzi, 2021, 66).

Another, less theologically loaded, example stems from a relatively common solution to the puzzle of the statue and the clay: there is a lump of clay shaped like a statue. They are perfectly coincident, yet the clay and statue differ in their modal properties. The lump can survive deformation, the statue cannot. The statue and the clay does not seem to share all their properties, so by Leibniz’s law they are not identical. According to Thomson (1998, 155), parthood for physical objects consists, at least in part, in occupying the same space at the same time. Therefore, as the lump and the statue coincide, she claims that the statue is part of the clay and the clay part of the statue. Thomson concedes that the clay constitutes the statue, but, she claims, constitution does not imply identity. In this example, too, there is a claim of something being reciprocal proper parts, but not identical. That is a violation of $\leq$-Antisymmetry. Not surprisingly, the transitivity of proper parthood will imply proper self-parthood and present another counterexample to the asymmetry of proper parthood.

Abstracta

So far the examples to question $\leq$-Antisymmetry and $<$-Asymmetry have drawn on examples of concrete, if not quite ordinary, objects. Moving to the realm of abstracta, Yablo (2016, 143) presents a purported counterexample to antisymmetry by considering the proposition expressed by the following sentence:

(3.1) The universe is large.

According to Yablo, the universe is part of the proposition that the universe is large, and the proposition is part of the universe. Yet, there is no reason to suppose the two identical. There appears to be a failure of antisymmetry. Again, if the proposition is taken to be a proper part of the universe, then by $<$-Transitivity there is a violation of <-Asymmetry. Cotnoir and Varzi (2021, 66-67) point out that a commitment to the universe is not required to generate similar counterexamples, but can also be drawn from the sorts of propositions central to the solution to the liar paradox from Barwise and Etchemendy (1987). Consider the propositions expressed by the following sentences:

(3.2) The proposition expressed by 3.3 is true.

(3.3) The proposition expressed by 3.2 is contingent.

If the constituents of propositions are parts of propositions, then it appears that the proposition expressed by 3.3 is part of that expressed by 3.2 and *vice versa*, but they obviously differ in content and must therefore be distinct. Both of these examples rely on the identification of the relation between a proposition and its constituents with parthood. That is a controversial identification which nonetheless has been defended by a number of philosophers. For example, Gilmore (2014), Kearns (2011) and Tillman and Fowler (2012) have claimed that propositions have mereological parts at least as long as one's account of propositions is broadly Russellian.

What Do the Examples Show?

The above examples can be used to argue for a few different claims. First, the examples seem to indicate a failure of antisymmetry for improper parthood:

(≤-Antisymmetry) $$x \leq y \wedge y \leq x \rightarrow x = y$$

As pointed out by Cotnoir (2013b, 837), this principle has been treated with 'all undue respect' such as when Simons (1987, 11) claims that the partial ordering axioms are 'partly constitutive of the meaning of "part", which means that anyone who seriously disagrees with them has failed to understand the word.' The above examples seem to give *prima facie* reason to question this orthodox insistency on at least one of the ordering axioms, even as it remains to be seen whether complete sense can be made of a mereological theory that does not satisfy all of them.

In the presence of the transitivity for proper parthood, the examples also appear to indicate a failure of the asymmetry of proper parthood:

(<-Asymmetry) $$x < y \rightarrow y \not< x$$

If the examples are taken at face value, then, it appears that we must accept either that proper parthood may fail to be transitive even when (improper) parthood is, or we must accept the possibility of parthood loops.

The examples are neither incontrovertible evidence against the antisymmetry of parthood, nor against the transitivity or asymmetry of proper parthood. However, such proof is not required to respond to Bohn's argument. Bohn has argued for the possibility of junky worlds by appealing to the conceivability of a world in which everything is a proper part. As a step toward arguing for the possibility of a junky world in which Unrestricted Fusion holds, we need only make a similar case as Bohn for the possibility of a world in which $<$-Asymmetry fails. Bohn proposed a three-step test: i) conceivability, ii) reputable philosophers believe it possible, and iii) consistency in the sense of having a (non-classical) mereological model. The above examples motivate the first two of these, while the next sections will argue that a mereology with a (non-classical) model can be developed that satisfies Bohn's definition of a junk as well as permits Unrestricted Fusion.

3.2 Rejecting Asymmetry

The examples in section 3.1 seemed to violate $\leq$-Antisymmetry and supported either a case against $<$-Transitivity or against $<$-Asymmetry The development of a theory in this section will proceed on the assumption that is preferable to deny asymmetry rather than transitivity. The reason for this is that rejecting transitivity does not seem to help resolve the inconsistency between Junk and Unrestricted Fusion.

To echo Bohn's formulation for junky worlds, let a world be loopy if it contains something that is a proper part of itself:

(Self-Loop) $$\exists x(x < x)$$

Self-Loop is motivated by the examples in 3.1, but is the claim coherent? Not if proper parthood is defined as PP1 or PP2 from section 1.1:

(PP1*) $$x < x := x \leq x \wedge x \neq x$$

(PP2*) $$x < x := x \leq x \wedge x \nleq x$$

Self-Loop is inconsistent with both definitions irrespective of the ordering principles. PP1* implies non-self identity and PP2* implies a straight contradiction of x both being and not being a part of itself. What if proper parthood is taken as primitive? Let proper parthood ($<$) be our primitive mereological concept and let it obey only $<$-Transitivity. Recall definition P:

$$x \leq y := x < y \vee x = y \tag{P}$$

If proper parthood is not taken to obey asymmetry in general, then this definition licenses no immediate inference to contradiction.

What of the properties of the improper parthood relation when proper parthood is only taken to be transitive? By P, improper parthood will obviously inherit transitivity from proper parthood and the second disjunct will guarantee reflexivity in the appropriate cases. So we have $\leq$-Transitivity and $\leq$-Reflexivity. However, $\leq$-Antisymmetry will not be satisfied. Consider, according to our examples, two distinct objects a and b such that $a < b$, $b < a$. By definition P we will have $a \leq b$ and $b \leq a$. Since a and b were stipulated to be distinct, we have $a \neq b$ so that $\leq$-Antisymmetry is not satisfied. Thus a mereology which rejects $<$-Asymmetry does not in general satisfy $\leq$-Antisymmetry, see also Cotnoir and Bacon (2012, 192). Improper parthood, then, fails to be a partial order on this account, but is instead merely a preorder.[7] This is not a surprise, as the examples motivated rejecting $\leq$-Antisymmetry, as well as motivating the rejection of $<$-Asymmetry in the presence of $<$-Transitivity.

3.3 Supplementation Revisited

The transitivity of proper parthood and a definition of (improper) parthood is obviously not sufficient to formulate an acceptable mereology for reasons of the resulting theory being unsupplemented. That some things may be proper parts of themselves, does not license a retreat from supplementation. It appears still unacceptable to allow for a proper part of something without some remainder. MEM included Weak Supplementation in order to arrive at an adequately supplemented mereology. However, in the current context, Weak Supplementation will not do because it entails both the irreflexivity and asymmetry of proper parthood.

[7] A preorder is any relation that is both reflexive and transitive.

Proof. For irreflexivity suppose that $x < x$. Then by Weak Supplementation there is a z such that $z < x$ and $z \wr x$. However, it is impossible for something to have a part that is disjoint from itself. So, $x \not< x$.

For asymmetry, assume $x < y$ and $y < x$. Then according to Weak Supplementation there is a z that is part of y and disjoint from x. However, due to $<$-Transitivity every part y is also part of x. Hence, every part z of y overlaps x. So, there is no z disjoint from x. Contradiction.[8] □

Since this argument does not rely on $\leq$-Antisymmetry, Weak Supplementation is not an appropriate supplementation principle for our purposes where we are looking to allow reflexive instances of proper parthood, viz. $x < x$. If the examples of the previous section can be taken at face value, then it is peculiar that the purportedly analytic principle of Weak Supplementation would entail their falsity. I take this as evidence against the analyticity of Weak Supplementation.

What, if any, supplementation principle is appropriate? As noted in chapter 1, there are several other candidates for supplementation principles that in classical mereology are stronger than Weak Supplementation. Will any of these work? Cotnoir and Bacon (2012, 193-194, 197) show that Strong Supplementation is consistent with the negation of $<$-Asymmetry. In the context of classical mereology, Strong Supplementation entails Weak Supplementation, but without $<$-Asymmetry the proof does not go through.[9]

As noted by Cotnoir and Bacon (2012, 193), the antecedent of Strong Supplementation is $x \not\leq y$, which by P entails both that $x \not< y$ and $x \neq y$. That is a weaker constraint than the antecedent of Weak Supplementation which is $x < y$, so that Strong Supplementation does not 'rule out all reflexive parts, nor does it rule out all mutual parthood structures.' However, it does rule out models in which a reflexive part has only a single proper part (other than itself) as in fig. 3.6. In this model, a does strictly speaking not have a single proper part as both $a < a$ and $b < a$. Still, since $a \not\leq b$, Strong Supplementation requires that there be a z that is part of a, but disjoint from b. Since a overlaps b, it cannot serve as this

[8] See Cotnoir (2016, 121, fn1).

[9] See section 1.2 for further discussion of the ranking by strength of supplementation principles. See also Cotnoir (2014, 654). Obojska (2013, 346-347) proves that when improper parthood is taken as primitive, and proper parthood defined by PP2, Strong Supplementation entails Weak Supplementation even in the absence of $\leq$-Antisymmetry. Of course, due to the built-in wellfoundedness of PP2, that system will not allow for proper parthood loops on pain of contradiction, see PP2*.

supplement, and there is no other z to supplement a. Therefore, it appears that Strong Supplementation puts an appropriate restriction on the permissible models by requiring a 'remainder' such as in fig. 3.7. Here the disjoint c and b supplement each other as proper parts of a in accordance with Strong Supplementation.

Neither the weaker supplementation principle of Company, nor that of Strong Company are appropriate because they do not allow for the simplest non-wellfounded models such as in fig. 3.4 and fig. 3.5. Since the antecedent of Company and Strong Company is $x < y$, they would require these simple parthood loops to be further supplemented, whereas Strong Supplementation, with the weaker antecedent $x \nleq y$, is vacuously satisfied by these models. Since it was these simple models that initially motivated the non-wellfounded theory, it would be disagreeable to have a supplementation principle ruling such models out.

Figure 3.6: Reflexive Proper Part without Strong Supplementation

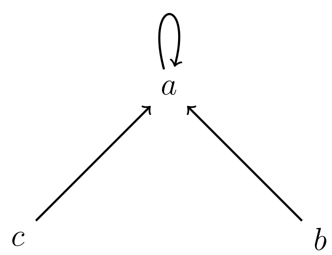

Figure 3.7: Strongly Supplemented Reflexive Proper Part

Recall that this mereology was motivated by examples of distinct mutual parts. In the presence of the transitivity of $<$ the existence of such parthood structures is inconsistent with extensionality. Consider $\leq$-Antisymmetry, which is the extensionality principle for improper parthood. Assume that there are distinct mutual proper parts, a and b, such that $a < b$ and $b < a$. By definition P, $a \leq b$ and $b \leq a$. Since, by hypothesis, $a \neq b$, $\leq$-Antisymmetry must fail on pain of inconsistency. Similar arguments can be made for both the extensionality of overlap and for proper parthood. For proper parthood, consider the distinct a and b above. By $<$-Transitivity, $a < a$ and $b < b$. Moreover, every part of a is a part of b and *vice versa*. However, since both a and b have proper parts, then by $<$-Extensionality, they would be identical which contradicts our hypothesis. The same argument could be made for the extensionality of overlap: since every part of a is part of b and *vice versa*, they must overlap the same things and thus should be identical by $\circ$-Extensionality.[10] The sort of examples that motivated this mereology are also counterexamples to extensionality, so the mereology developed here cannot be extensional without undermining its own motivation. It should therefore come as no surprise that none of the standard extensionality principles can be consistently upheld if one accepts the existence of distinct mutual parts.

In the context where $\leq$-Antisymmetry holds, Strong Supplementation entailed $<$-Extensionality as argued in section 1.2. Moreover, $\leq$-Antisymmetry on its own entailed both $\leq$-Extensionality and $\circ$-Extensionality. Obviously, the latter two principles will fail in the non-antisymmetric context. What about extensionality for proper parts? That will fail too, as the argument from the Proper Parts Principle relied on conjoining two instances and applying $\leq$-Antisymmetry to the consequent.

In fact, there is reason to believe that $\leq$-Antisymmetry is responsible for all extensionality principles in mereology, cf. Cotnoir (2010, 401-402) and Cotnoir and Bacon (2012, 198). However, recall the equivalence in classical mereology between the antecedents of $\leq$-Extensionality and $\circ$-Extensionality, this proof relies only on the definitions P and Overlap and therefore holds also in the context without $\leq$-Antisymmetry.

($\circ$-Ext*) $$\forall z(z \circ x \leftrightarrow z \circ y) \rightarrow x \leq y \wedge y \leq x$$

The proof of Proper Parts from Strong Supplementation did also not rely on $\leq$-Antisymmetry. So, Proper Parts will still hold, but without antisymmetry we will be unable to simplify the consequent when conjoining two of its instances. That leaves us with this:

($<$-Ext*) $$(Co(x) \wedge Co(y)) \rightarrow (\forall z(z < x \leftrightarrow z < y) \rightarrow (x \leq y \wedge y \leq x))$$

[10]See also Cotnoir and Bacon (2012, 191-192) for more considerations on extensionality in this sort of mereology.

So in the current context, we can neither prove that composite objects whose parts overlap each other are identical, nor can we prove that composite objects that share all their proper parts are identical. Still, we can prove that when either of these conditions obtain, the objects in question are mutual improper parts; they are improper parts of each other, but not identical. Again, this is hardly surprising given the examples that motivated the rejection of antisymmetry. The examples involved cases of objects being part of each other without being identical; the statue and the clay being a case in point.

Cotnoir and Bacon (2012, 195-196) rely on the definitions Fusion$_1$ and Fusion$_3$ in formulating their mereology without $<$-Asymmetry because they can be shown equivalent given $<$-Transitivity and Strong Supplementation. Note that Fusion$_1$ was the definition of fusion in terms of *least upper bound*. When $\leq$-Antisymmetry fails, however, uniqueness can no longer be guaranteed, so the definition then only expresses the notion of *minimal upper bounds*, see Cotnoir and Bacon (2012, 195) cf. Hovda (2009, 61). Due to the failures of extensionality in this context, the uniqueness of fusions will fail too. As with the other extensionality principles, however, it is only the last step of reasoning from mutual improper parthood to identity that is disallowed. Therefore, only weaker principles of 'uniqueness' for fusions will hold:

(Unique Fusion$_1$*) $$Fu^1_\varphi(z) \wedge Fu^1_\varphi(w) \rightarrow x \leq y \wedge y \leq x$$

(Unique Fusion$_3$*) $$Fu^3_\varphi(z) \wedge Fu^3_\varphi(w) \rightarrow x \leq y \wedge y \leq x$$

3.4 Axiomatisations and Discussion

Cotnoir and Bacon (2012, 197) offer two axiomatisations that both specify the same mereology which allows for parthood loops. For the first axiomatisation, they use the definitions P, Overlap, and Fusion$_3$:

1. $<$-Transitivity
2. Strong Supplementation
3. Unrestricted Fusion$_3$

For the second, they employ Fusion$_1$, but retain definitions P and Overlap:

1. $<$-Transitivity
2. Complementation
3. Unrestricted Fusion$_1$

The formulations are very similar to those of classical mereology in that Unrestricted Fusion$_1$ goes with Complementation, while Unrestricted Fusion$_3$ goes with Strong Supplementation. In fact, as is noted by Cotnoir and Bacon (2012, 197) the connection between the above mereology and classical mereology 'is quite simple: one needs only to drop the asymmetry of proper parthood from a standard axiomatisation of classical mereology.' The only principle of classical mereology that resists parthood loops is the asymmetry of proper parthood, and dropping this requirement will return the above mereology.

Since such parthood loops will allow for infinite proper parthood chains, there may be cases in which there is no $<$-minimal element. An element a is $<$-minimal of a set A if there is no $x \in A$ such that $x < a$. If every subset of A, whose elements are related by $<$, have a $<$-minimal element, then the order of '$<$' is wellfounded on A, otherwise it is not wellfounded. As such, the mereology axiomatised above is a non-wellfounded mereology. Cotnoir and Varzi (2021, 64) argue that this mereology is 'sound and complete with respect to a well-defined class of non-wellfounded algebras that generalize in a natural way to complete Boolean algebras (minus zero) of classical mereology.'[11]

[11] Cotnoir and Bacon (2012, 199-203) provide a proof of this through model constructions of so-called non-wellfounded algebrae. The proof is briefly sketched in Cotnoir and Varzi (2021) as the following: 'A model is a quintuple $\langle \mathcal{A}, \sqsubset, D, \leq, ' \rangle$. $\langle D, \leq \rangle$ is a model for classical mereology, $\mathcal{A}$ is any non-empty set including D, and $'$ is a map from from $\mathcal{A}$ into D such that $d' = d$ for every $d \in D$, and $\sqsubset$ is the irreflexive kernel of the relation $\{\langle x, y \rangle \in \mathcal{A} \times \mathcal{A} : x' \leq y'\}$. When $\mathcal{A} = D$, the map $'$ reduces to the identity map and the algebra reduces to a well-founded model of classical mereology'

Since the mereologies in this dissertation are axiomatised in a first-order language, each will have a countable model by the (downward) Löwenheim-Skolem theorem. However, since no countably infinite complete Boolean algebra exists, the metatheoretic results depend on a second-order axiomatisation of non-wellfounded mereology. Therefore, the schema for unrestricted fusion is replaced by a second order axiom:

$$\forall F(\exists x Fx \rightarrow \exists z \forall y(y \circ z \leftrightarrow \exists x(Fx \wedge y \circ x)))$$

Also, as noted in the introduction, mereology as a first order theory has expressive limitations that greatly limit the number of fusions that follows from the schema for Unrestricted Fusion. The condition for each instance must be statable in a first-order language. Since the vocabulary of this language must be denumerable, uncountable many fusions will be left out in an infinite domain. This is so because if a domain has $\aleph_0$ many atoms, intuitively there should be $2^{\aleph_0}$ fusions. Therefore, the theory should guarantee models with size at least $2^{\aleph_0}$, but as noted above no

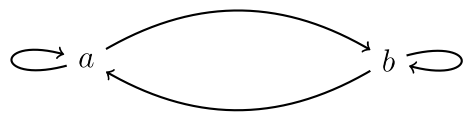

Figure 3.8: Parthood Loop with Reflexive Parts

Due to allowing for reflexive loops of proper parthood, non-wellfounded mereology formally allows for junk as defined by Bohn, since it allows for models in which something is a proper part as in fig. 3.8.[12] Moreover, Unrestricted Fusion$_1$ and Unrestricted Fusion$_3$ also hold in this mereology. This shows that is is formally possible to have a mereology that satisfies Junk as well as Unrestricted Fusion. In fact, it appears to show that Unrestricted Fusion may very well hold in the kind of scenario that Bohn used to argue against its necessity. If Unrestricted Fusion holds in that scenario, Bohn's case against it appears to falter.

Junk, as Bohn defines it, is compatible with unrestricted fusion after all . In section 3.1, I argued that the examples that motivate non-wellfounded mereology satisfy two steps of Bohn's three-step test for possibility: i) it is conceivable that the asymmetry of proper parthood may fail, and ii) reputable philosophers have so argued. Sections 3.2 and 3.3 argued that non-wellfounded mereology also satisfies step (iii) of Bohn's (2009b, 29) possibility test, namely that the idea is 'logically consistent in the sense that there are junky models of many non-classical mereologies. That is, there are no logical contradictions lurking in the background.' Since the resulting mereology is consistent with Unrestricted Fusion, we can, by Bohn's own method, conclude that Junk is consistent with Unrestricted Fusion in the junky scenario. This suggests that Bohn's argument does not succeed as a counterexample to the necessity of Unrestricted Fusion.

Of course, the model in fig. 3.8 is much too simple to represent Bohn's model of infinite nesting universes, but more complex models may be introduced that might better fit the metaphysical picture sketched by Bohn. Consider for example the model in fig. 3.9. It has an infinite upward chain of strongly supplemented proper parts that culminates in a reflexive universal object. Non-wellfounded mereology is also consistent with the infinitely descending proper parthood chains of Gunk. The satisfaction of both Junk and Gunk at the same time is sometimes referred to

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first-order theory with a denumerable vocabulary can do that, cf. Cotnoir and Varzi (2021, 232).

12 Where are the supplements according to Strong Supplementation? Since there is no x such that $x \nleq a$ or $x \nleq b$, the principle holds vacuously.

as Hunk:

(Hunk) $$\forall x \exists y \exists z (y < x \land x < z)$$

Since the non-wellfounded mereology is consistent with both Junk and Gunk, it is also consistent with Hunk; both fig. 3.8 and fig. 3.9 are models for this possibility. As noted above, the model in 3.8 appears too simple to represent the metaphysical picture sketched by Bohn, but I contend that the model in fig. 3.9 fits the picture.

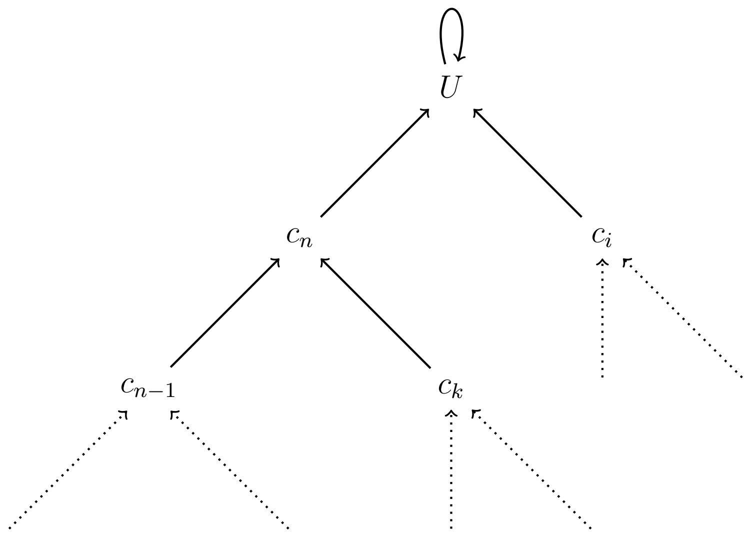

Figure 3.9: Gunk and Junk with Universal Object

Due to the failure of extensionality in non-wellfounded mereology, the uniqueness at the 'top' of this model is not guaranteed. The model in fig. 3.10 is also permissible. Here, again, there is an infinite chain of strongly supplemented proper parts, but they end in two objects a and b that are both proper parts of each other as well as of themselves. Here, a is a universal object of which every other object in the model is part. Since $a < a$ it also satisfies Junk. What about b? It is a fusion of the exact same parts as a and will thus also count as a universal object. Due to the failures of extensionality in this mereology the universe need not be unique, and there are two universal objects. They share all their parts, but since $\leq$-Antisymmetry does not hold we can not infer their identity.

It might appear peculiar that there could be more than one universal object,

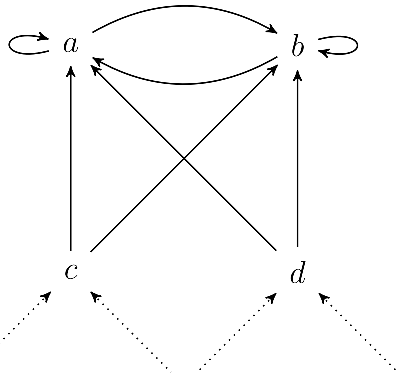

Figure 3.10: Two Universal Objects

viz. more than one universe, but does the lack of extensionality pose a problem for the adherent of unrestricted fusion? One could perhaps ask if the existence of two universal objects constitutes a violation of the fact that *everything* should have a fusion. Perhaps one could insist that there should be a fusion of them both. However, both a and b are fusions of both a and b. So, in fact, the two universal objects have a fusion it is just that each of them counts as that fusion. This peculiarity of non-wellfounded mereology is what allows for the existence of a universal object as a proper part of itself in the junky case. As such, this peculiarity is also an advantage.

Moreover, insofar as the primary objective is to secure Unrestricted Fusion, the lack of extensionality need not be problematic. In fact, extensionality principles are among the most hotly contested principles in mereology, and there are copious counterexamples to all of them. The statue and the clay of Thomson (1998) has already been discussed. Simons (1987, 114) argues that all the same members may be part of the Library Committee as well as the Philosophy Department Football Team without collapsing the two entities. There are other solutions to these puzzles than to insist on a mereological reading of 'part' in these cases, and it is often held that social groups are not fusions. Still, Hawley (2017) argues for a mereological interpretation of such cases. While this is not the place for a thorough discussion of all counterexamples to extensionality, the existence of these examples suggests that the lack of extensionality may be an advantage.

Still, if one would like to guarantee the existence of a unique universe such as in fig. 3.9, one could force the defined improper parthood relation to obey $\leq$-Antisymmetry. This would accordingly also vindicate the uniqueness of fusions as in classical mereology. As was noted above, postulating antisymmetry for the (improper) parthood relation undermines the examples that motivated non-wellfounded mereology in the first place, but it is consistent as it is satisfied by the model in 3.9. As improper parthood, '$\leq$', already obeys transitivity and reflexivity, postulating its antisymmetry would make it a non-strict partial order. It would then also define a corresponding strict partial order. In a classical setting, this strict partial order would be proper parthood, but in the current setting proper parthood, as denoted by '$<$', is neither reflexive nor irreflexive. The strict partial order would then only be a subrelation of proper parthood that would isolate its irreflexive kernel. Postulating improper parthood to be antisymmetric when proper parthood is not irreflexive therefore appears to be to take half a step, as it were, back to classical mereology.

However, if one recognises that classical mereology conflates identity and mereological equivalence there is also another option. Making this distinction explicit, it is possible to capture the structures of classical mereology by working from within non-wellfounded mereology. This approach is discussed in section 4.2.

Chapter 4

Objections and Replies

This chapter is devoted to presenting, and responding to, some objections to the theory, and the resulting metaphysical picture, developed in the preceding chapter.

4.1 Non-Wellfoundedness Goes Beyond Classical Mereology

The rejection of $<$-Asymmetry is a significant step beyond the bounds of classical mereology. Staunch defenders of classical mereology may find this a step too far, dig their heels in, and claim that the theoretical advantages of classical mereology are so many that any motivation to step away from it should be denied. This position is arguably held by Schaffer (2010, 64-65) who claims that junk is metaphysically impossible in no small part because it is inconsistent with classical mereology. Schaffer appears to see the universal object as akin to a realist possible world in the Lewisian tradition. As such, junk cannot exist at a world 'because a world that contained junk would be an entity not a proper part of another entity at that world. A world would top-off the junk.'[1]

One may reasonably hold this position, but recall that non-wellfounded mereology is a response to those that are sympathetic to unrestricted fusion, but believe that junk is possible. If you find yourself sympathetic to the possibility of junk, you have already left classical mereology behind; models of classical mereology do not allow for junk. The step towards non-wellfoundedness is thus not a categorically

[1] If, instead, worlds are taken to be maximal consistent propositions the argument that worlds would 'top off' junk does not appear equally convincing.

different step away from classical mereology than the step to junk in the first place. Therefore, this step should not be denied merely because it is in a non-classical direction. Instead, the advantages and disadvantages of rejecting the asymmetry of proper parthood should be judged on its own merits and against competing alternatives. I submit that if junk is accepted, the rejection of the asymmetry of proper parthood appears to provide exactly what is desired, namely the preservation of Unrestricted Fusion.

4.2 Non-Wellfounded Mereology is Not a Theory of 'Part'

One might insist that the English 'part' denotes the relation of parthood as specified by classical mereology; non-wellfounded mereology is not a theory of that relation, but of some other made-up relation.

It is true that the properties of $x \leq y$ would differ between classical and non-wellfounded mereology. However, as far as classical mereology is concerned, 'part' is a theoretical term. It has little probative value to insist that a theoretical term denotes its own extension. Rather, instead of insisting that 'part' denotes classical parthood, one should pose a more substantive question: Does the English 'part' specify only the relations expressed by '$<$' and '$\leq$' in classical mereology, or does it also specify the relations expressed by those same symbols in non-wellfounded mereology.[2]

If the examples of section 3.1 are to be taken at face value, this suggests that the English 'part' must also be taken to express proper and improper parthood as expressed by '$<$' and '$\leq$' in non-wellfounded mereology. Informally speaking, the understanding of parthood in the non-wellfounded mereology discussed in chapter 3 is broader than that of classical mereology because its relation of proper parthood is neither reflexive, nor irreflexive. The relation of proper parthood in classical mereology can be understood as the irreflexive kernel of the proper parthood relation of non-wellfounded mereology. This suggests that there is a sense in which

[2]Relatedly, there is also a substantive question of whether the classical or non-wellfounded relations are most fruitful in metaphysical theorising. It appears that they are useful for different applications. Whereas the classical relations are useless in theorising about junk, the non-wellfounded relations do not provide simple identity conditions for composite objects. Differently inclined metaphysicians might draw different conclusions about whether that is an advantage or disadvantage of non-wellfounded mereology.

the notion of parthood from non-wellfounded mereology can be used to capture the classical notion of parthood. As such, non-wellfounded mereology can, in a sense soon to be made precise, express 'part' even if that word only denoted classical parthood.[3]

In what formal sense can classical mereology be captured by non-wellfounded mereology? Following the approach taken by Cotnoir (2010, 403-405), I will provide a formal sketch of how non-wellfounded mereology can capture the structures of classical mereology. If a relation is a preorder on a domain A, then it can induce a partial order on the quotient set of A. Since $\leq$ in non-wellfounded mereology is a preorder, i.e. transitive and symmetric, this is straightforward to do. First let the relation $\sim$ be defined as:

(D$\sim$) $$x \sim y := x \leq y \land y \leq x$$

This relation is intended to express mereological equivalence and is easily shown to be an equivalence relation.

Proof. For reflexivity, $x \sim x$ when $x \leq x$ and $x \leq x$. Reflexivity is inherited from $\leq$. For symmetry, suppose that $x \sim y$. Then $x \leq y$ and $y \leq x$, but this is equivalent to $y \leq x$ and $x \leq y$, so $y \sim x$. For transitivity, suppose that $x \sim y$ and $y \sim z$. By D$\sim$, we have $x \leq y$ and $y \leq z$. Then, by $\leq$-Transitivity we get $x \leq z$. Similarly, we also have $z \leq y$ and $y \leq x$, so also $z \leq x$. Then both $x \leq z$ and $z \leq x$, so by D$\sim$, $x \sim z$. □

Being an equivalence relation, $\sim$ will partition A into the set of equivalence classes $\{[a] \mid a \in A\}$ where $[a] = \{x \in A \mid x \sim a\}$. We represent this quotient set of all equivalence classes on A with respect to $\sim$ as $A/\sim$. As Cotnoir (2010, 403) notes, these equivalence classes only make a difference in the cases where antisymmetry fails. If an object, z, has no mutual parts, then its equivalence class under $\sim$ will be the singleton set of z. Now, we can define a relation,'$\preceq$', on $A/\sim$

[3] Classical proper parthood satisfies a strict partial ordering (transitive and asymmetric), while non-wellfounded proper parthood is merely transitive. Classical improper parthood is also a partial order (reflexive, transitive and antisymmetric), while that of non-wellfounded mereology is a preorder (reflexive and transitive). See also Pietruszczak (2020, 13-14, 274-276) for discussion on parthood as a preorder or as a partial order.

that corresponds to the relation that was a preorder on A:

(D$\preceq$) $$x \leq y \text{ on } A \text{ iff } x \preceq y \text{ on } A/\!\sim.$$

To show that '$\preceq$' is well-defined, we must show that it respects arbitrary witnesses: $[a] \preceq [c]$ iff $[b] \preceq [d]$ where $a \sim b$ and $c \sim d$.[4]

Proof. For left-right, suppose that $[a] \preceq [c]$. Then by definition of $\preceq$, $a \leq c$. By definition of $\sim$, we have $c \leq d$. This gives $a \leq d$ by transitivity of $\leq$. By definition of $\sim$, we also have $b \leq a$. Since $a \leq d$, we get $b \leq d$ by transitivity. The definition of $\preceq$ then gives the desired result: $[b] \preceq [d]$. Similarly for right-left. □

Now, '$\preceq$' will be a partial order on $A/\!\sim$. This is so because if $x \sim y$ on A, then x and y will be members of the same equivalence class in $A/\!\sim$. And if x and y are in the same equivalence class, then obviously $[x] = [y]$.

($\preceq$-Antisymmetry) $$[x] \preceq [y] \wedge [y] \preceq [x] \rightarrow [x] = [y]$$

In fact, the resulting antisymmetry is really just a paraphrase of the familiar property of quotient sets that if $x \sim y$, then $[x] = [y]$, because the antecedent entails $x \leq y$ and $y \leq x$ by definitions D$\sim$ and D$\preceq$.

The relation $\preceq$ can then be used to define a strict partial ordering to represent classical proper parthood. The definitions are familiar:

(PP$_\prec$1) $$[x] \prec [y] := [x] \preceq [y] \wedge [x] \neq [y]$$

(PP$_\prec$2) $$[x] \prec [y] := [x] \preceq [y] \wedge [y] \npreceq [x]$$

These will now satisfy a version of PP$\leftrightarrow$:

(PP$_\prec\leftrightarrow$) $$([x] \preceq [y] \wedge [x] \neq [y]) \leftrightarrow ([x] \preceq [y] \wedge [y] \npreceq [x])$$

Proof. Immediate by the proof of PP$\leftrightarrow$, see p. 9.[5] □

Cotnoir (2010, 404) then goes on to argue that it follows from this result that 'all of the classical mereological structure is consistent in the new domain.', including

[4] Cotnoir (2010, 403, fn. 18) notes this requirement, but with an apparent typo in the presentation.

[5] A proof is also found in Cotnoir (2010, 404)

'all the standard supplementation axioms, unrestricted fusion, and the like.'[6] The non-wellfounded mereology in A has Unrestricted Fusion as an axiom, which entails both that every element of A has a Boolean complement, but also that every subset of A has a minimal upper bound relative to $\leq$. This structure carries over to $A/\!\sim$. Since, additionally, $\preceq$ partially orders $A/\!\sim$, the structure $\langle A/\!\sim, \preceq \rangle$ will model classical mereology.

To see how non-wellfounded mereology can capture the structures of classical mereology, note that to regain a sort of quasi-antisymmetry within non-wellfounded mereology it is only necessary to recognise that identity in classical mereology behaves as mereological equivalence in non-wellfounded mereology. Since $x \sim y$ in A iff $[x] = [y]$ in $A/\!\sim$, the equivalence relation $\sim$ is the relation between elements of A that represents the identity relation between the equivalence classes of $A/\!\sim$.[7] In order to be explicit about its surrogacy for identity between equivalence classes, the equivalence relation $\sim$ can be renamed in order to represent classical mereology in A.

(D$=_c$) $$x =_c y := x \sim y$$

From this, we have the 'antisymmetry' of $\leq$: $x \leq y \wedge y \leq x \rightarrow x =_c y$. It might appear confusing that non-wellfounded mereology would operate with both '$=$' and '$=_c$'. However, in capturing classical mereology on A, one would only need to employ $=_c$, because the identity of x and y obviously entails the identity of their equivalence classes under $\sim$: If $x = y$, then obviously also $x \sim y$, so that $[x] = [y]$.

The contention is then that through treating mereological equivalence as the identity relation of classical mereology, the notion of parthood in non-wellfounded mereology is able to capture the classical relations of parthood along with the full structure of classical mereology. This, along with the examples of section 3.1, are reasons to believe that the relations expressed by '$<$' and '$\leq$' in non-wellfounded mereology also express the relation expressed by the English 'part', even if 'part' denotes the parthood relation of classical mereology. As such, non-wellfounded mereology is also a theory of 'part'.[8]

[6] See also Cotnoir and Bacon (2012, 201-202, fn. 28).

[7] Thus Cotnoir (2010, 404) quips that 'the anti-extensionalist can agree that classical mereology is almost entirely correct, with the sole exception that it runs together mereological equivalence and identity.'

[8] In addition to the above, Cotnoir (2010, 405) claims that the partial order '$\preceq$' on $A/\!\sim$ can also be represented in A by specifying a relation '$\leq_c$', such that $x \leq_c y$ holds between elements

4.3 It Is Ad Hoc to Reject the Asymmetry of Proper Parthood

An objector might say that it is *ad hoc* to reject the asymmetry of proper parthood. The objector may require a more principled reason for making such a drastic move other than that it provides a promising response to a potential counterexample to unrestricted fusion.

I submit that the response is not *ad hoc*. Bohn purports to make a *reductio* of unrestricted fusion by appeal to junky worlds. However, on having reached a contradiction, logic does not prescribe what to do next. Bohn chooses to deny a mereological principle, the necessity of unrestricted fusion. I have merely denied a different mereological principle, the asymmetry of proper parthood.[9]

Logic only helps derive the contradiction. As argued by Harman (1986), extralogical principles of reasoning are required to choose what to reject. Bohn claims that the universal object could not be a proper part of anything, because he believes that Weak Supplementation is analytic. Since Weak Supplementation entails the asymmetry of proper parthood, rejecting the latter is a non-starter for Bohn. If indeed Weak Supplementation were analytic, then that would be a defensible position. However, the examples from section 3.1 provide *prima facie* independent motivation for the rejection of the asymmetry of proper parthood. The rest of chapter 3 provides an argument that the idea is consistent and useful. This indicates both that Weak Supplementation is not analytic, and, accordingly, that the rejection of $<$-Asymmetry is not *ad hoc*.

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of A iff the relation $x \preceq y$ holds between the equivalence classes of $A/{\sim}$:

(D-$\leq_c$) $$x \leq_c y := \forall w \forall z (w \sim x \wedge z \sim y \rightarrow w \leq z)$$

However, it appears that this definition does not in general capture a partial order. Consider a two element countermodel where a and b are two distinct mutual parts, so that $a \leq b$, $b \leq a$ and $a \neq b$. If x and y in D-$\leq_c$ are instantiated by a and b respectively, the definition still appears to be satisfied. The antecedent of the definiens will be satisfied because $\sim$ is symmetric, so that the conditional is non-trivially true. However, in this case $\leq_c$ will not be antisymmetric.

In order to secure the antisymmetry of $\leq_c$ it appears we would require a restriction on the quantifiers to range over only those elements whose equivalence classes have more than one member. Moreover, the philosophical point of regaining the structures of classical mereology does not appear to require this definitions, because treating $\sim$ as identity already makes $\leq$ obey 'antisymmetry'. I am indebted to Peter Fritz for discussion on this issue.

[9]Of course, the rejection of the asymmetry of proper parthood goes along with a rejection of the antisymmetry of improper parthood.

4.4 Where Are These Proper Parts of Themselves?

The theory in chapter 3 allows for the existence of things that are proper parts of themselves. An objector might claim that it is plain to see that the world contains no such things.

Non-wellfounded mereology, if true, allows for reflexive parts in the sense that a and b may be parts of each other without being identical, and it allows for parthood loops, viz. a is a proper part of a. The theory is neutral as to how many of these objects there are, and it certainly does not require that all objects be proper parts of themselves. As is shown by the model construction of Cotnoir and Bacon (2012, 199-202) the theory allows for a spectrum, as it were, where on one end everything is a proper part of itself and on the other end nothing is, so that the theory then reduces to classical mereology. As such, non-wellfounded mereology is consistent with only the universal object being a proper part of itself as in fig. 3.9. Moreover, the universal object need only be a proper part of itself if the possibility that the world is junky obtains. There is no contention from the theory itself that ordinary objects be proper parts of themselves. That would be a further metaphysical commitment to which the theory, nor this author, need subscribe. There should be no expectation to find such objects out in the world in ordinary circumstances.

4.5 The 'Official Definition' Does No Longer Fit the Setting

Cotnoir and Varzi (2021, 222) note that the model in fig. 3.4 is 'clearly not what philosophers have in mind' when discussing junk. They go on to claim that a similar consideration applies to the more complex models, e.g. those 'culminating in a top element that is a proper part of itself' (Cotnoir and Varzi, 2021, 228). As argued by Cotnoir (2014, 655), '[p]roponents of junk are arguing for the existence of certain metaphysical structures; they are not (at least not primarily) arguing for the existence of worlds that satisfy a particular formal definition.'

In the context where asymmetry fails, Cotnoir (2014) believes that Bohn's definition of Junk does no longer capture the intended structures. He goes on to note that this is often the case when moving between formal systems. Cotnoir contends that the metaphysical picture of junk, that of infinitely upward nested universes, warrants a stronger principle in the setting without $<$-Asymmetry:

(Strong Junk) $\forall x \exists y (x < y \wedge y \nleq x)$

In a strongly junky world, everything is a proper part of something that is not one of its own parts. This builds asymmetry into the definition of junk. As such, Strong Junk rules out cases where something is a proper part of itself and thus appears inconsistent with unrestricted fusion even in the non-wellfounded setting. Strong Junk will be inconsistent so long as the universe is required to be an upper bound of everything. Consider the universal object u that is the fusion of everything, that is every x satisfying the condition $x = x$. To satisfy the definition of Strong Junk, we have $u < u$ and $u \nleq u$, the latter which by P entails $u \nless u$. Contradiction. This is a well-targeted objection to the solution to the inconsistency of junk and unrestricted fusion developed in chapter 3 and it can be very serious. While the theory developed in chapter 3 will not allow for Strong Junk, it is not obvious what the notion of junk truly should be in different contexts. More work would have to be done to examine what notion of junk is appropriate in different theoretical contexts.

Conclusion

The existence of the universe can be upheld even if the world were junky, because non-wellfounded mereology allows the universe consistently to be a proper part of itself. Thus, with suitably relaxed ordering principles for a primitive proper parthood relation, the possibility of Junk is consistent with the necessity of Unrestricted Fusion. The case for relaxing the partial ordering for proper parthood of classical mereology to a merely transitive order satisfies the exact same criteria that Bohn (2009b) used to argue the case for the possibility of Junk: conceivability, consistency, and being believed by reputable philosophers.

Non-wellfounded mereology is consistent in the sense of having a well-defined class of models, including all models of classical mereology. It is not an *ad hoc* solution to Bohn's challenge to Unrestricted Fusion as it is also motivated by independent examples of both concrete and abstract objects. The examples make a case for the possibility of non-wellfounded mereology by the same standard by which Junk is possible. The cases in the examples are clearly conceivable, and there are several reputable philosophers that believe in the possibility of non-wellfounded parthood structures. If junk is possible, then an equally strong case can be made for a non-wellfounded mereology where Unrestricted Fusion is consistent with Junk.

If, after having considered the parity of reasoning between the case for junk and that for non-wellfounded mereology, one still finds non-wellfounded mereology to be objectionable, then that is a reason to reject Bohn's three step test for possibility. Consequently, junk may be discarded as impossible, or, at least, one may reasonably hold that a case for its possibility has not been made. This would amount to taking the development of non-wellfounded mereology as a *reductio* of the possibility of junk. This would be an alternative route to rejecting the possibility of Junk than that taken by Schaffer (2010).

The inconsistency of Unrestricted Fusion and Junk originates in the requirement that a fusion be an upper bound of the things that it fuses. This is an explicit requirement both in Fusion_1 and Fusion_2, while Fusion_3 requires Strong Supplementation as an additional principle in order to generate the contradiction. This suggests that while relaxing the partial ordering of parthood is necessary to resolve the contradiction for Fusion_1 and Fusion_2, there may be other options for a system employing Fusion_3. However, it is generally thought undesirable that a fusion not be required to be an upper bound of the things it fuses, because a fusion that is not an upper bound can, counterintuitively, fuse objects not part of the fusion. These options have therefore not been further explored in this dissertation. However, if a suitable philosophical account of such fusions is available, then an investigation into such a theory and its resulting metaphysical picture could be of interest. The approach might be able to offer a different kind of Unrestricted Fusion consistent with Strong Junk. Still, it does not appear plausible to be able to develop an agreeable mereology where a fusion may fuse objects not part of it.

As noted in the introduction, the non-wellfounded mereology applied in this dissertation, first developed by Cotnoir and Bacon (2012), is developed within the framework of first-order theory with its inherent expressive limitations. Non-wellfounded mereology has so far received only modest attention, and the metaphysics of non-classical mereologies are in general little explored, cf. Cotnoir (2013b, 842). This dissertation has in part formed an investigation into the metaphysics of non-wellfounded mereology by an application of the theory to the case of mereological junk. The theories of non-wellfounded mereologies formulated with plural or higher-order quantification, and their corresponding metaphysics, should pose an interesting area for further technical and philosophical study.

A decision was made in chapter 3 to proceed on the assumption that parthood, proper as well as improper, is a transitive relation. This decision was made by taking the examples that motivated non-wellfounded mereology at face value on the assumption of transitivity. However, the examples were primarily counterexamples to the antisymmetry of improper parthood. As such, one could take the examples to motivate a different kind of non-classical mereology in which proper parthood fails to be transitive even when improper parthood is. It does not appear that taking this route would help resolve the incompatibility of Junk and Unrestricted Fusion, but further study of the metaphysics of the resulting system could have inherent theoretical interest.

The theoretical framework of non-wellfounded mereology is closely related to classical mereology. It is a weaker, more general theory that is not intended to replace classical mereology for all reasoning about parts. The arguments and examples in favour of non-wellfoundedness do not suggest that the entire universe should have to be non-wellfounded. The theory of non-wellfounded mereology allows for a wellfounded part of the universe. As discussed in chapter 4, the structures of classical mereology can be captured using only the expressive resources of non-wellfounded mereology.

The discussion on the antisymmetry of improper parthood in the non-wellfounded setting, at the end of chapter 3, suggested that intermediate theories are possible between the non-wellfounded mereology discussed and applied in this dissertation and classical mereology. One such theory was briefly discussed, where proper parthood is merely transitive, but (improper) parthood a partial order. This theory will also resolve the challenge of the junk argument since it will allow for parthood loops. Still, this theory would require further philosophical development as it appears, at least partly, to undermine the examples that served as motivation for non-wellfoundedness. In addition, the partially ordered (improper) parthood relation on this view would give rise to a corresponding strict partial ordering. However, if we insist that proper parthood is merely transitive, one may reasonably ask what relations this strict partial order is. Is it also a parthood relation? If so, what is its theoretical role? As discussed by Canavotto and Giordani (2020, §3) it appears that this combination of properties for proper and improper parthood might give rise to a proliferation of parthood relations. The resulting system would warrant further examination.

Despite the inherent theoretical interest in the development and application of non-wellfounded mereology, the immediate upshot of its defence of Unrestricted Fusion in the face of junk is modest: the defence succeeds in guaranteeing the semantic values required for the literal truth of our statements about the universe even if the world were junky. Accordingly, Junk need not be taken as a successful challenge to the necessity of Unrestricted Fusion.

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