Generative Design With an Aesthetical Approach Using Multiobjective Optimisation

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Thesis submitted for the degree of
Master in Informatics: Robotics and Intelligent Systems
60 credits

Department of Informatics
Faculty of Mathematics and Natural Sciences

UNIVERSITY OF OSLO

Spring 2022

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http://www.duo.uio.no/

Printed: Reprosentralen, University of Oslo

Abstract

Generative design provides an iterative process that allows for the optimisation of $3D$ models. It is used in architecture to effectively explore a wide range of solutions and enable creativity in the design process. The evolutionary approach used in generative design is imitated by the use of a test environment or artificial intelligence. A set of rules and constraints is determined for the optimisation to follow. Otherwise, generative design is free from limitations and assumptions. Principles of beauty can be found everywhere, in nature as well as art and architecture, but also in mathematics and computational algorithms. It can be represented in ideas of symmetry, proportions and properties like the Golden ratio that can give a sense of order and consistency.

In this thesis, the different ways generative design could be used to alter and optimise a design in accordance with external factors were explored. During a generative design process, both practical objectives, as well as aesthetical ones, were applied to a frame truss design that represented a stool. Strength and stability were examples of measurements for usability. The aim was a stool that was both practical to use and pleasing to the eye through the use of a multiobjective optimisation strategy. By proving that an optimisation that both considered practical and aesthetical objectives could be used on a simple frame truss design, it was presumed that this method could be used on more complex designs.

Acknowledgements

First, I want to thank my supervisor, Mats Høvin, for inspiring me to work with an area of research that interested me immensely, for listening to my ideas and for facilitating my need to be creative. I have greatly appreciated all the interesting, insightful conversations and constructive feedback.

I also want to thank my family and close friends for supporting me through this project and for bearing with me in this period of my life where most of my time has revolved around this project. Thank you for the help with proofreading, taking pictures and all the questions and remarks that helped improve my work.

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Chapter 1

Introduction

During the Renaissance, the principles of perfection found in classical aesthetics were virtually unchallenged. It was believed that the principles of perfection applied in nature as well as art. They adhered to the idea that perfect symmetry depicted in architecture was a reflection of the imperfect symmetry found in nature [21].

Nature has always been a source of inspiration, and the fields of computer science and artificial intelligence are no exceptions. This is true for several fields within informatics, especially within Artificial Intelligence and robotics. Generative design and Evolutionary computation are both areas where mechanisms inspired by the behaviour of organisms and nature are utilised. Within Generative design, algorithms can be used to explore different variations of a $3D$ design in the attempt to meet a set of requirements. This exploration can result in new and innovative solutions to both new and old problems as the design is evolved and adapted in complex ways to fit its environment. An example of how generative design can be combined with classical design can be seen in the Queen Elizabeth II Great Court in the British Museum in London, show in figure 1.1. The roof is supported on a rectangular outer boundary and a triangular grid of steel members was constructed to provide structural stiffness [26].

In this thesis, different methods using generative design to alter and optimise the design of a stool in accordance with external factors were explored. The design was evaluated both in relation to usability and its design. As an example of how usability could be evaluated, strength was used as an important factor. When it came to the visual elements of the design, mathematical principles were used as inspiration and for guidance.

Figure 1.1: Queen Elizabeth II Great Court with a roof made with the use of generative design [15]

1.1 Motivation for the Research

This section covers the motivation for the research done for this thesis as well as the contribution to the wider field of research. The use of a generative design process has clear advantages in relation to the structural optimisation of the design of a stool as many different configurations can be evaluated consecutively. A well-constructed generative design algorithm can consider more different configurations and variations than the human brain. The algorithm will also examine the problem unbiased and thus propose solutions that can be distinctly unlike traditional solutions to a known problem. This can result in designs that are very unlike what is expected, but there is also a possibility that conventional and traditional designs are optimal after all. New insight and innovative solutions to design can be obtained or old knowledge can be backed up with knowledge from an unprejudiced algorithm.

The significance of aesthetics is an integral part of generative design where the aim is to combine aesthetics with function. The main goal of this thesis was to show how optimising for aesthetics did not have to compromise the function or the strength of the end product. The aim was to create designs where not only the practical aspect of design had been considered in the optimisation process but also aesthetical aspects by applying mathematical models to the optimisation.

1.2 Research Goals

The primary goal of this thesis was to determine if visually interesting designs were obtainable in a multiobjective optimisation when practical and aesthetical objectives were combined. This was explored using a generative design process to adapt and evaluate the $3D$ design of a stool. The stool was represented by a frame truss design. Multiple objectives of optimisation were combined to analyse the influence each objective had on the evaluation of the stool and how a harmonious stool design could be produced when multiple objectives were combined. More specific secondary goals were set with the purpose of investigating the optimisation process further and reaching a conclusion on whether the primary goal of combining practical objectives of optimisation with aesthetical objectives gave visually interesting designs.

> **Primary goal:** Investigate if visually interesting $3D$ designs could be achieved when simultaneously optimising for both practical and aesthetical objectives.

When working on the thesis these secondary goals were explored to find whether the main goal was attainable:

- **Goal 1:** Determine which objectives were realistic to utilise in the optimisation process. This included both structural and aesthetical objectives.
- **Goal 2:** Assess which objectives could be combined in a multiobjective optimisation for the generative design process to make a visually interesting design. Which objectives were not compatible was also assessed.
- **Goal 3:** Apply the objectives of optimisation on a stool represented by a frame truss. A frame truss design was used as a foundation for evaluation and could later be evolved to become a larger and more complex frame truss.
- **Goal 4:** Explore the effect of each objective on the frame truss as well as combinations of two or more objectives. This analysis was used as the last step to analyse whether the main goal was attainable.

1.3 The Structure of the Thesis

This thesis was divided into 8 chapters. Chapter 1 introduced the topic of the thesis as well as the motivation and goals of the research. The topics explored and the background was presented in chapter 2. Chapter 3 explored the relationship between mathematics, art and aesthetics, and the measurements used for the optimisation were defined. Strategies of optimisation were discussed in chapter 4. The choice of framework used was discussed as well as two algorithms considered for the optimisation. Each objective of optimisation was investigated separately in chapter 5, and the methods used for the evaluation of each objective were described. Experiments combining the objectives discussed in chapter 5 were presented in chapter 6. The different methods and measurements were compared and the advantages, as well as the disadvantages of each case, were presented in chapter 7. Chapter 8 concluded the research according to the defined research goals presented and presented possible future research.

Chapter 2

Background

In this chapter, the background for the structural optimisation of the stool was presented. Truss theory and the use of mathematics in architecture and art were introduced. The strategy of concept seeding was described as well as multiobjective optimisation. Two different methods for optimisation, Pareto front optimisation and Weighted fitness optimisation, were described.

Structural optimisation can be described as a method of making a structure sustain loads in the most optimal way [3]. The structure used for the optimisation and simulation of the stool design consisted of a truss design. More specifically a frame truss design was chosen. In this chapter, truss design theory was presented as well as the two types of truss designs that are used within engineering: Truss and frame truss.

2.1 Truss Design Theory

A truss is a structure that is mainly utilised within engineering and can be used for support and to provide structural integrity that distributes external forces resulting from either tension or compression. The purpose of a truss is to distribute weight evenly and handle changing stress whilst keeping its shape. It consists of beams connected by joints to make the whole construction behave as if it is one object. These joints are also referred to as nodes. When stress is applied to any part of the truss, it is distributed through the whole structure as it travels through the beams. This lessens the pressure on any single beam of the truss and in effect, it makes the overall structure stronger than each individual component. When constructing a truss beams can be connected in any shape that can support external stress, but generally triangular shapes are chosen because of their ability to distribute stress and not be distorted as they have a stable geometry. A truss will require

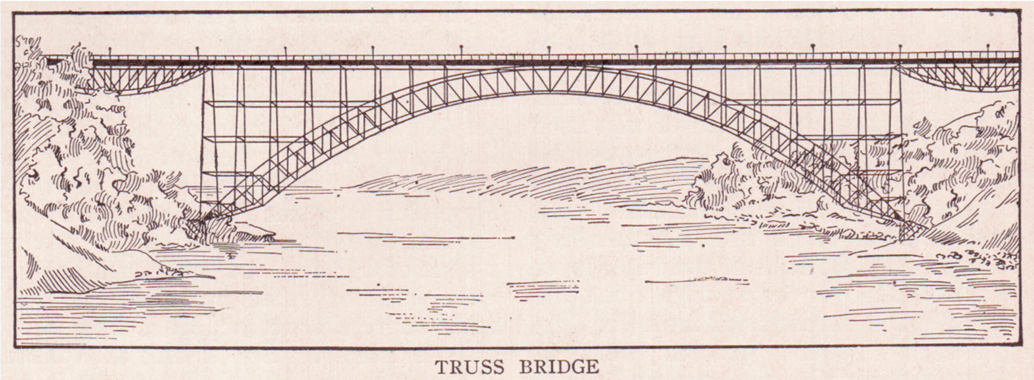


Figure 2.1: Example a bridge utilising a frame truss design [12]

less material than a compact structure that can support the same amount of stress. This makes a truss suitable for supporting considerable amounts of weight over a large span whilst using less material than a similar compact structure.

Trusses can have a multitude of different configurations and they are used for a large variety of purposes. They are commonly used in bridges, towers and roofs, but also in the frame of bicycles. An example of a truss bridge that utilises both triangular and rectangular shapes is shown in figure 2.1.

The main benefit of the use of a truss design instead of a solid design is the reduction in material used compared to the strength achieved. Trusses can also run over long spans, reduce deflection and support heavy loads.

2.1.1 The Difference Between a Truss and a Frame Truss

The points where two or more edges meet are referred to as joints. A truss has revolute joints that allow the edges to move in accordance with each other. Nevertheless, a truss has by definition no rotary moment as the triangular shapes are used to stiffen the structure and make the truss rigid. The edges can be contracted and pulled, but they can not be rotated as the truss is often stiffened by supporting beams forming triangles. When the edges that form the triangles are removed the truss collapses if stress is applied.

A truss where the joints are rigid is called a frame truss as it constitutes a rigid frame. This can, for example, be obtained by welding the joints together. The joints in a frame truss will contribute with rotary moment as the edges are welded together, and the stress applied might contribute to bending of the whole frame truss. Because of the welded joints, a frame truss will not collapse when weight is applied even if it does not contain triangular shapes that stiffen the structure.

2.2 Mathematics in Architecture and Art

Mathematics has always been used as a guide to building all types of structures. Examples of this can be found in both ancient and modern buildings. It has been used in music, photography, paintings and sculptures. Nature has been a source of inspiration and patterns and configurations where mathematical principles like symmetry, spirals, geometric figures, fractions and ratios, are present can be found everywhere. These patterns can among others be described by series like the Fibonacci numbers, recursive systems like the Lindenmayer system, and fractals where smaller parts resemble the whole. As an example of this, fractals can be found in crystals made from frost on cold surfaces and show intricate patterns, and in the Romanesco broccoli where the shape of the individual parts of the broccoli resembles the shape of the whole. The concepts from geometry used for the optimisation were defined in section 3.3.

2.3 Concept Seeding

The concept seeding approach was developed by John Frazer [9] from the late 1960s onwards and was used to capture and codify developed architectural concepts in a generic form. His theory was that a generative system might be able "to generate designs that embody the architectural concepts" [10].

This approach requires a concept seed and a set of rules. A concept seed captures certain architectural ideas but needs further development to become a design, and rules are set to develop the seed into a design. Then designs are generated using a generative system.

By making small modifications to the seed, the concept seeding approach creates a method to explore many different variations of a design. A plethora of possible variations of the same seed are made and can be evaluated and compared. With this approach, it is feasible to evaluate numerous different variations of the design to obtain the best possible solution or solutions. Figure 2.2 shows how different variations of stools can embody different architectural ideas whilst still evolving from the same origin, the same seed.

2.4 Multiobjective Optimisation

Multiobjective optimisation is an area within optimisation concerned with choosing the best solution when evaluating multiple objectives simultaneously. In multiobjective optimisation problems, "the quality of a solution is

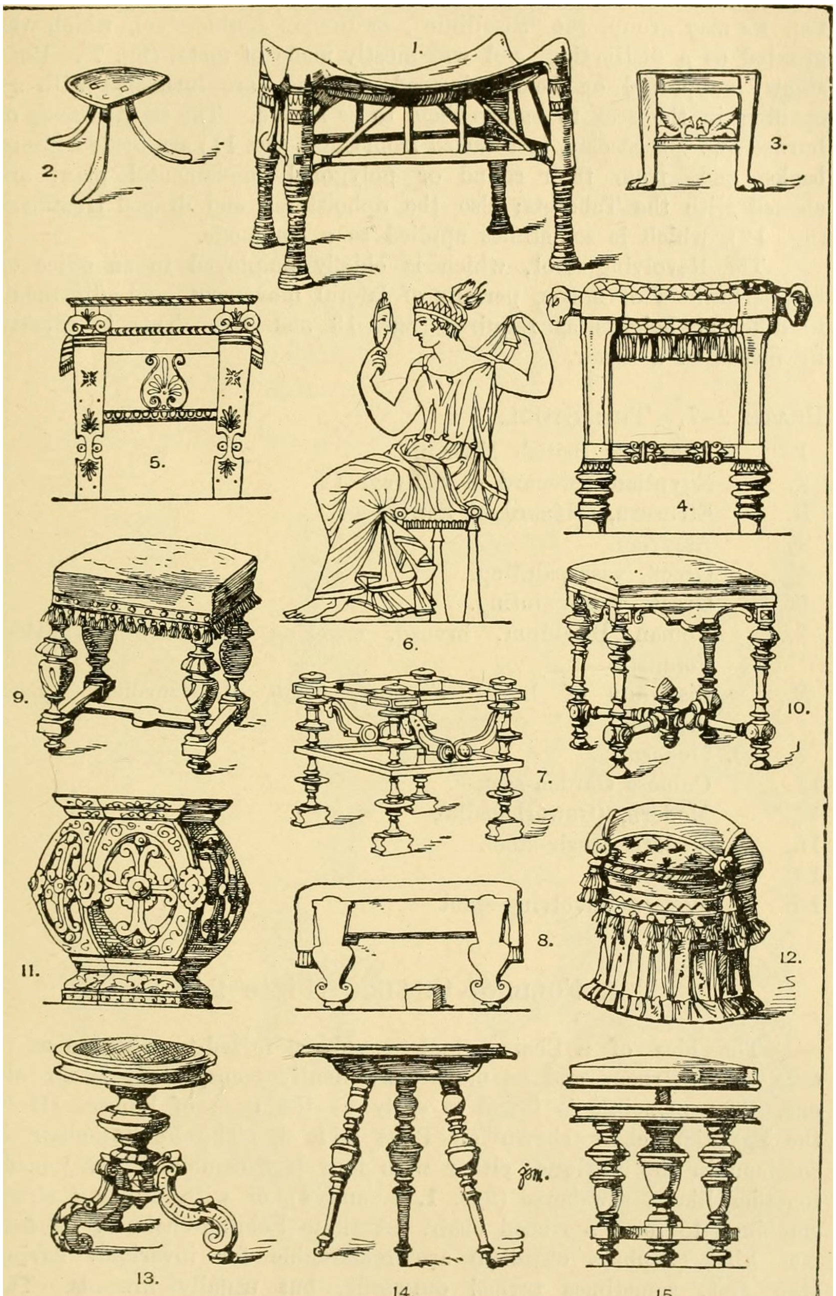


Figure 2.2: Stool designs with classical ornamentation [8]

defined by its performance in relation to several, possibly conflicting, objectives" [4].

Using an objective function is a method to assign a value to the quality of a solution based on the objective and give it an evaluation. This is also called the fitness or the quality of the solution. The purpose of optimisation is to find the solution with the best fitness for the problem, either by minimising or maximising the evaluation. During a multiobjective optimisation, multiple objectives are optimised simultaneously to make an optimal decision. When objectives contradict compromises are made to reach a satisfactory conclusion. This might result in suboptimal results for individual object functions, while still obtaining an optimal result for the evaluation of the multiobjective problem.

The fitnesses of a multiobjective optimisation problem can be organised in a fitness landscape. Fitness landscapes have been used to visualise the distribution of fitness since Sewall Wright introduced it in 1932 [27]. A landscape consisting of peaks and valleys is used to visualise the fitness of solutions. The optimal solutions are found at the extremes of the global maximum or minimum. Evolutionary optimisation techniques often utilise strategies made to avoid getting stuck in local maxima or minima, represented by smaller peaks or smaller valleys in the fitness landscape.

2.4.1 Pareto Front Optimisation

Through multiobjective optimisation, a set of solutions called the Pareto set or the Pareto front is found. Each solution to the optimisation problem is evaluated separately according to a set of objectives. The Pareto set is the set of solutions that are better than all other solutions in at least one objective. Their evaluations can not be improved for any single objective without at the same time negatively affecting the evaluation of any of the other objectives. The solutions obtained in a Pareto front optimisation "lie on the edge of feasible regions of the search space" [4] when constraints are present. The solutions are considered to be of equal quality.

The Pareto set was visualised in figure 2.3 where the optimisation was represented by the evaluation of two objectives. The solutions were evaluated by the objectives x and y, which could represent any kind of objective of optimisation. Each solution to the optimisation problem was represented by a dot. The Pareto set was represented by red dots and the solutions not in the Pareto set were represented by blue dots.

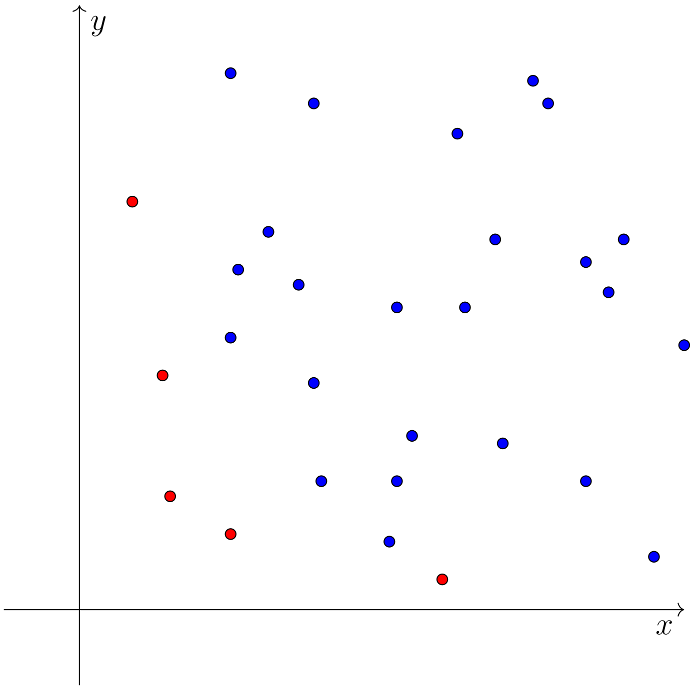


Figure 2.3: Illustration of the Pareto front (represented by red dots) for a minimisation problem

2.4.2 Weighted Fitness Optimisation

By using a method of scaling the fitness evaluation of each objective in multiobjective optimisation, a single score for the fitness of the optimisation can be found. This method is called Weighted fitness optimisation or simply scalarisation. The objectives are given individual importance and weighted accordingly. Each solution of the optimisation problem is assigned an overall fitness that is determined by the fitness f of each objective weighted by an assigned $\omega \in [0, 1]$. The weight ω is determined according to the importance of the trait. Thus the Weighted fitness function for a specific problem is given by:

$$g = \sum_{i=1}^{n} \omega_i f_i$$

In Weighted fitness optimisation, the choice of good parameters is important to get a representative result and for the importance of the relevant evaluations to neither get exaggerated nor overlooked. For each optimisation, different aspects of the optimisation can be highlighted based on how they are weighted.

When using Weighted fitness optimisation, it is not possible to get a visual representation resembling what can be found using Pareto front optimisation. Nevertheless, this method provides the possibility of evaluating each objective used for the fitness and analysing its significance in the optimisation.

Chapter 3

Measurements of Evaluation

In this chapter, the measurements used for evaluation during the multiobjective optimisation were introduced and outlined. They consisted of both mathematical and structural concerns that were related to the design of the stool. The connection between what can be measured and what can be experienced concerning the expression of beauty was also investigated.

In chapter 5, the measurements presented in this section were discussed further in regards to the practical setting of the design of a stool. The stool was optimised and analysed in relation to each objective.

3.1 The Bridge Between Mathematics and Art

When exploring the relationship between mathematics and art, clear parallels can be drawn between how mathematics can be found in nature, art and architecture. These comparisons are simplifications made to describe relationships and patterns found in nature. They are not absolute, but as these theories have been a part of the human perception of beauty they are an essential part of this discussion.

The Roman architect Vitruvius wrote that “Without symmetry and proportion there can be no principles in the design of any temple” [23] in his work *De architectura* that has been regarded as one of the most influential works on architecture since the Renaissance [16]. According to Vitruvius buildings embodied the attributes of stability, utility and beauty. This can be translated to other structures than buildings and has in this case been utilised for the design of a stool.

Figure 3.1: An example of a three-legged stool made from wood [25]

3.2 Stool Design

To explore the effect of each objective of optimisation, a stool design was chosen as it had both a practical function, but it could also have a decorative one. A stool is a common piece of furniture and has a simple structure to make the optimisation problem as fundamental as possible. Figure 3.1 shows an example of a simple stool design.

When defining what a stool is for the purpose of this generative design process the shape in its most basic form was discussed. It is simply a structure to sit on that supports weight, but simultaneously stools can differ vastly in their design. Stools are among the earliest and simplest types of furniture used for sitting on and they can have a varying number of legs and come in varying shapes and sizes. A chair has many of the same characteristics as a stool, but a backrest is usually attached at a $\sim 90°$ angle to the seat and can also feature armrests.

For the optimisation, the specifications of the design of the stool were set to be as fundamental as possible. The stool was therefore given a square seat and four legs. The design could have been chosen in a multitude of different ways and different configurations of the stool were tested out. However, all the observations made when it came to the objectives of optimisation that were explored are also valid for structures with differently shaped seats and different numbers of legs.

The skeleton of the stool used for the optimisation was based on a truss

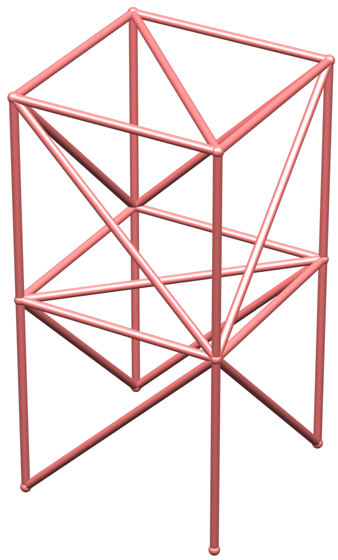


Figure 3.2: Original frame truss of the stool used for optimisation

design, and more specifically a frame truss design was chosen. The benefits of a frame truss design were discussed in section 2. The stool design used as a seed or base for optimisation was represented by the frame truss as shown in figure 3.2. From this point onwards the joints of the truss were referred to as nodes whilst the beams were referred to as edges. The stool consisted of edges in the form of the legs and supporting beams to stiffen the structure. It was proposed that altering the design could make a stronger structure and thus also a stronger stool.

The stool was chosen to have three layers that each consisted of four nodes. Only three layers were used to show how different the designs could end up when different objectives and constraints were applied in the optimisation while still keeping the design relatively simple. This made the design easier to interpret than if it had more layers and thus more complexity. It also limited the run time of the optimisation.

The stool consisted of a bottom layer with four nodes that were locked in their position, a middle layer where the nodes were able to change position during optimisation and a top layer that made up the rigid seat. Only the nodes of the middle layer were able to change position in this configuration whilst the rest of the nodes were kept at the same position. Optimising the position of the bottom layer nodes was also tested, but this ended up giving structures where the legs gravitated towards the middle of the stool when optimising for strength. Choosing a structure where only the nodes in the middle layer were moved in the optimisation ensured optimisation according to the given constraints whilst the overall shape was kept and ensured a functional stool.

The three layers of the stool were connected with edges. All nodes were connected with the nodes positioned right above and below them. There were also edges connecting the nodes diagonally to strengthen the structure by stiffening the truss. The edges were arranged to contain mirrored symmetry over the plane that contained one of the nodes of the bottom layer as well as the opposite node of the bottom layer and was perpendicular to the ground. It was, however, not symmetrical around the plane that contained the two other nodes of the bottom layer and was perpendicular to the ground. This ended up giving an interesting shape that was symmetrical without obviously being so.

Lastly, the stool needed a seat. Here the seat was set to have a quadratic form to make the structure of the frame truss as fundamental and simplistic as possible for the optimisation. It was not explored whether this was the optimal shape of the seat when it comes to strength and optimising for weight put on the chair. The width and depth of the seat were chosen to be the same and the height of the chair was chosen to be twice the width of the seat.

3.3 Defining Beauty

When looking at beauty in a qualitative setting, defining what makes something perceived as beautiful is essential. For a better understanding of how this can be defined, mathematical concepts related to beauty, art and architecture were described and discussed.

3.3.1 The Golden Ratio

The Golden ratio is one of the most well-established properties of mathematics and was famously discovered by ancient Greek mathematicians as it appears frequently in geometry. It was found to neither be a whole number nor a fraction, but rather an irrational number. The Golden ratio is famous for the way it appears throughout mathematics and in nature in areas such as proportions, spirals, geometry and architecture amongst others.

The Golden ratio is the ratio between two quantities where the ratio of the two quantities is the same as the ratio of the sum of the quantities to the larger one of them. It is uniquely defined by:

Figure 3.3: A visual representation of the Golden ratio

$$\frac{a+b}{a} = \frac{a}{b} := \phi$$

Where the Golden ratio is represented by ϕ with the value:

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180\ldots$$

While other irregular numbers can be quite closely estimated by a fraction, this is not true for the Golden ratio. As an example, π can be estimated to be 22/7, but it can also be estimated using fractions:

$$\pi = 3 + \frac{1}{7 + \frac{1}{15+\frac{1}{1+\frac{1}{292+\cdots}}}}$$

After only a few steps the added fraction is so small that it does not add much extra information to the estimation. On the other hand, when the Golden ratio is estimated in the same way an interesting pattern appears:

$$\phi = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\cdots}}}$$

Thus ϕ can be written as a continued fraction:

$$\phi = 1 + \frac{1}{\phi}$$

Which gives:

$$\begin{aligned} \phi^2 &= \phi + 1 \\ \phi^2 - \phi - 1 &= 0 \\ \left(\phi - \frac{1}{2}\right)^2 &= \frac{5}{4} \end{aligned}$$

And thus the Golden ratio is defined as:

$$\phi = \frac{1 \pm \sqrt{5}}{2}$$

In fact, two values and not just one can define the irregularity of nature. The positive value can be used for scaling up and the negative can be used for scaling down.

The Golden ratio is often seen in nature as well as in geometry. Therefore, it has been a subject of much interest and it has been studied by mathematicians as far back as by the Ancient Greeks in the 5th century BC who found that the Golden ratio often showed up in geometry and that many patterns in nature conformed to the same mathematical laws using fractions and ratios.

A frequent pattern where the Golden ratio is present is the Golden spiral, a logarithmic spiral with a growth of a ratio of ϕ. The Fibonacci spiral where arcs are drawn connecting opposite corners of squares with sides equal to the sum of the two previous sides is a close approximation to the Golden spiral. Logarithmic spirals can for example be found in the arrangement of leaves on the stem of a plant. This is called phyllotaxis and can be generated from Fibonacci ratios. An example of the Aloe polyphylla plant showing the property of the Golden spiral can be found in figure 3.4a. The Nautilus shell, as seen in figure 3.4b, is frequently attributed to the Golden ratio but is rather an example of a different logarithmic spiral. Clement Falbo argued that this is clear from just viewing such a shell in his paper *The Golden Ratio—A Contrary Viewpoint* [5].

(a) Aloe polyphylla [24]

(b) Nautilus shell [2]

Figure 3.4: Two examples of logarithmic patterns

Psychologists like Gustav Fechner found that humans preferred rectangles that were based around the Golden ratio [6]. This theory has been disputed by Mario Livio amongst others. In his book *The Golden Ratio* Livio concluded that there is "hardly any formal, accepted description of aesthetic judgement in mathematics and how it should be applied" [18]. Even though its role in the human perception of beauty has been contested, the Golden ratio has interested humans since its discovery and still does. Therefore, it has been explored further in this thesis.

3.3.2 The Silver Ratio

Closely related to the Golden ratio another mathematical ratio has been defined, the Silver ratio or the Silver mean. Along with the Golden ratio and the Bronze ratio, they make up the Metallic ratios. Only the Golden and the Silver ratio have been discussed in this work. These three ratios are irrational mathematical constants and are made up of continued fractions.

Figure 3.5: A visual representation of the Silver Ratio

The Silver ratio is represented by ρ and can be described using a line that is divided into three parts. Two longer parts of the same length, and one shorter. The ratio between the whole line and one of the larger segments is the same as the ratio between one of the larger segments and the smaller segment as shown in figure 3.5.

The Silver ratio is defined as:

$$\rho = \frac{2a+b}{a} = \frac{a}{b}$$

Which gives:

$$\frac{a}{b} = \frac{2a+b}{a} = 2 + \frac{b}{a}$$

$$\rho = 2\frac{1}{\rho}$$

$$\rho^2 = 2\rho + 1$$

The value of ρ can be found by using:

$$\rho^2 - 2\rho - 1 = 0$$

And gives only one positive solution:

$$\rho = \frac{2 + \sqrt(4 * (-2) * (-1))}{2 * 1} = 1 + \sqrt(2)$$

$$\rho = 2.414\ldots$$

All the metallic ratios are defined by continuous fractions, and the silver ratio can be defined as:

$$\rho = 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \cdots}}}$$

3.3.3 Equilateral Triangles

As the Golden ratio has been found to not be a good representation of perceived attractiveness, other measurements of attractiveness in triangles have been suggested. Jay Friedenberg found that the compactness of a triangle was thought to be more appealing than whether the triangle followed the Golden ratio in the paper *Aesthetic Judgement of Triangular Shape: Compactness and Not the Golden Ratio Determines Perceived Attractiveness* [11].

In his paper, Friedenberg described how few studies investigating the perception of beauty influenced by the Golden ratio explored its influence on how triangles are perceived. Through surveys, Friedenberg found that participants did not prefer triangles where the ratio of the length of the longest side to the short side of the triangle followed the Golden ratio as discussed in section 3.4.5. He found that the participants rather gravitated toward triangles where the lengths of the sides were of equal lengths. These are fittingly called equilateral triangles and an equilateral triangle can be found in figure 3.6. Participants also greatly preferred triangles that pointed upwards. Equilateral triangles can be perceived as more compact and stable and Friedenberg hypothesised that they were preferred because they were seen as less likely to move or break. This was especially interesting in this research as triangular shapes played a significant role in the design chosen. The perception of stability and strength was also compelling to investigate through equilateral triangles as stability and strength were also used as objectives in the optimisation.

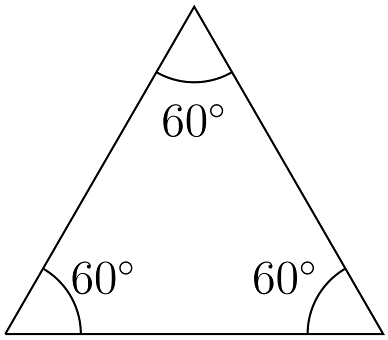


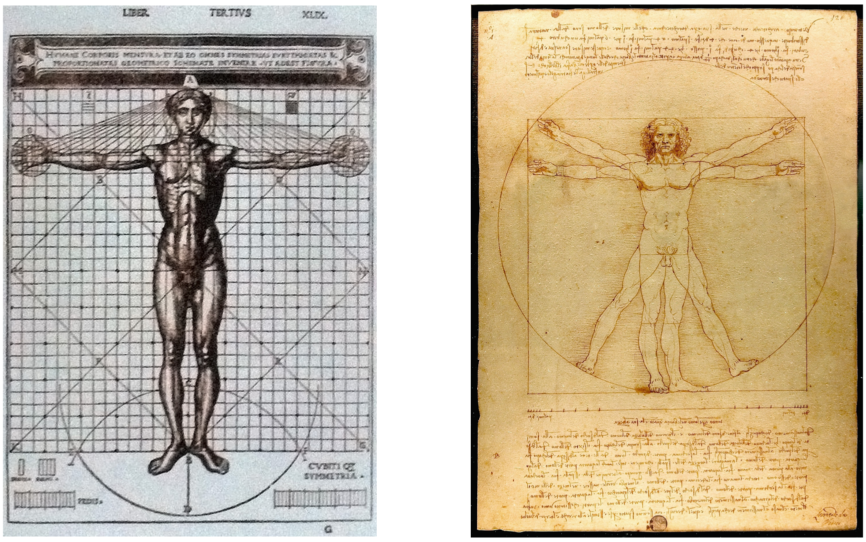
Figure 3.6: An equilateral triangle

3.3.4 Symmetry

Symmetry can give a sense of harmony and balance and can be found in everything from nature, mathematics and other natural sciences, as well as in art. It occurs in many fields within mathematics that are not related to geometry, like linear algebra, probability and calculus. The opposite of symmetry is asymmetry and this occurs when there is an absence of symmetry.

In geometry, an object is symmetrical if it constitutes two or more identical pieces that are organised in a way that does not alter its overall shape. Many types of symmetry occur in geometry. An object can for instance be symmetrical over a line and the two parts are mirrored images of each other. When an object is rotated about a fixed point without the shape of the object being changed, it has rotational symmetry. Scale symmetry occurs when an object does not change shape when expanded or contracted. Fractals are an example of scale symmetry and were explored further in section 3.3.5.

Humans and many animals are approximately symmetrical over a mirroring line through the middle of the body making the left and the right side almost symmetrical. Whilst the outer part of the body of the animal tends to be close to symmetrical, the inner part is often asymmetrical as the organs usually are not arranged in a symmetrical fashion. The famous drawing *The Vitruvian Man*, or *The proportions of the human body according to Vitruvius*, by Leonardo da Vinci, shows the symmetry and proportions of the ideal human body. Figure 3.7b shows this drawing and it is thought to be based on the proportions of the human body described by the architect Vitruvius in his work *De architectura* [23]. Da Vinci based the proportions of his drawing on measurements of male models. An illustration of the Vitru-



(a) Cesare Cesariano (1521) [1]   (b) Leonardo DaVinci (ca 1492) [17]

Figure 3.7: Two versions of the Vitruvian Man

vian Man by Cesare Cesariano from 1521 featured in the illustrated edition of *De architectura* can be seen in figure 3.7a.

In the paper *Symmetry as an aesthetic factor* [21], Harold Osborne wrote that too much or too obvious symmetry defeats its own purpose". Too apparent symmetry can be perceived as unnatural and manufactured even though perfect symmetry can be seen as an ideal. Asymmetry can break with what is expected and can therefore be perceived as interesting, but asymmetry to a too large degree can give the impression of imbalance. Thus the slight asymmetry seen in humans and animals alike is what makes them appear natural.

Osborne also states that when symmetry "is unobtrusively subordinated to other perceptual stimuli symmetry may enhance the overall aesthetic potentiality of a work; otherwise the aesthetic appeal is annulled" [21]. When symmetry is a part of the visual without being the main focus it can add to the overall appeal without taking away from it.

3.3.5 Fractal Geometry

Fractals are a type of symmetry found in mathematical structures that make them continue indefinitely where their pattern is repeated at different scales. When zooming in on the pattern the same shapes will appear infinitely making them extremely complex whilst still being easy to create. Fractals can make visually interesting and beautiful patterns. Patterns resembling fractals are widely found in nature, but as patterns in nature are not infinite these patterns only appear to be fractals. Examples of this can be found in snowflakes and crystals, but also in trees, leaves and other plants. Figure 3.8 shows how the Romanesco broccoli has a structure that resembles fractals.

Figure 3.8: Romanesco broccoli with a structure that resembles fractals [14]

A well-known example of fractals is the Mandelbrot set where one can zoom in indefinitely anywhere on the structure and find interesting and unexpected shapes and structures. Mandelbrot found that not all geometry can be described by Euclidean geometry and therefore suggested that there were other rules that had to be in place to describe “some grossly irregular and fragmented facets of nature” [19]. Using fractals Mandelbrot constructed interesting new shapes and it facilitated him to construct landscapes and maps of countries using geometric shapes that were completely artificial but appeared to be natural.

3.4 Measurements of Aesthetics

As the overall goal was to optimise the structure based on aesthetical measurability measurements for beauty had to be quantified and defined. In this thesis, 10 different measurements or objectives for aesthetics have been defined. These objectives were used for further analysis and discussion to determine what objectives were realistic to use in the optimisation process. Beauty is subjective, but the mathematical properties mentioned earlier in this chapter were used as a foundation for further discussion.

In *Oxford English Dictionary* 'aesthetic' refers to concern with beauty or the appreciation of beauty, but it can also mean that something is giving or is designed to give pleasure through beauty [22]. The origin is the Greek word aisthētikos, from aisthēta 'perceptible things', from aisthesthai 'perceive'. Thus the word aesthetic refers to what we as humans perceive as beautiful and the fact that we appreciate it because it is beautiful.

These are the measurements that were considered alongside the ability to endure weight in the multiobjective optimisation:

1. Deflection of the seat
2. The shortest total length of edges
3. The maximum length of edges longest edge
4. Golden rectangle for the whole structure
5. Golden triangles
6. Silver triangles
7. The compactness of triangles - Equilateral triangles
8. Stability - The centre of gravity
9. Symmetry over axes
10. Presence of planes

These goals were divided further into two groups: Either they were defined for the structure as a whole or they were defined for smaller parts that were added up to make the whole. The total length of edges and the stability of the structure were objectives defined for the whole structure. When the length of one of the edges was changed, the length of other edges was changed as well, and the stability of the structure was dependent on the length and angles of the edges in that exact composition. Objectives that assessed the triangles within the design evaluated the parts of the stool individually. Every triangle was evaluated and the fitnesses of the individual triangles were added up to provide an overall fitness. It could therefore be assumed that each triangle had to be quite a good approximation to the shape of the triangle optimised for. Suboptimal shapes of triangles could still occur as a part of a structure that had an overall good fitness if the other triangles had exceptionally good evaluations. For the fitness to be considered a good option the overall evaluation had to be favourable.

In this section, the objectives listed have been described and the possible advantages and disadvantages of each measurement have been discussed. Table 3.1 assesses what objectives were possible to combine in multiobjective optimisation for the generative design process. Only the upper part of the table was used to make it as intelligible as possible as the table was symmetrical.

	1	2	3	4	5	6	7	8	9	10
1		✓	✓	-	✓	✓	✓	✓	✓	✓
2			✓	-	✓	✓	✓	✓	✓	✓
3				-	✓	✓	✓	✓	✓	✓
4					-	-	-	-	-	-
5						✗	✗	✓	✓	✓
6							✗	✓	✓	✓
7								✓	✓	✓
8									✓	✓
9										✓
10										

Table 3.1: Possible combinations of measurements of optimisation

3.4.1 Deflection of the Seat

The deflection of the seat correlated with how much stress the truss could withstand and was integral for the function of the stool as it directly determined whether or not the stool was usable. Hence, this criterion was included in every optimisation.

3.4.2 Minimum Total Length of Edges

To ensure that the chair would not become wider than it was tall or for it to be challenging to use, it was necessary to limit the width of the frame truss. One option when limiting the width of the structure was to minimise the total length of edges. This objective was easily combined with most other objectives. When it was combined with objectives the impact depended on how much it was weighted in the overall evaluation of the design.

3.4.3 Minimising the Length of the Longest Edge

Another method used to limit the growth of the structure was to limit the length of the longest edge. This gave a similar effect to minimising the total length of the edges, but instead of minimising the length of every edge this approach rather ensured that no single edge was particularly long while still allowing for a wider range when it came to the length of the edges.

3.4.4 Golden Rectangle for the Whole Structure

When the mathematical measures of beauty were looked into, evaluating whether the structure conformed to the Golden ratio was the first objective that was considered. This objective was discarded early on in the process as this factor would only alter the relationship between the height and the width of the stool. For the truss to be considered to follow the Golden rectangle, the whole structure had to conform to the Golden rectangle as shown in figure 3.9. It became evident that this did not make a great change to the optimisation as it did not have the intended effect.

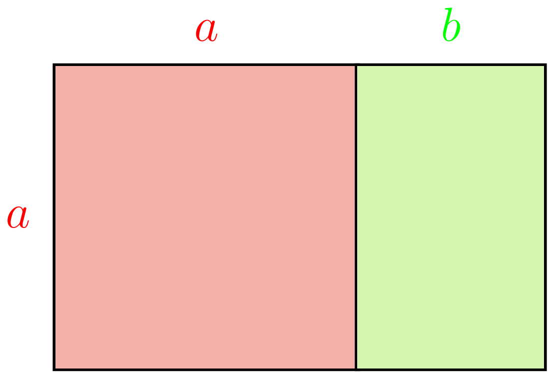


Figure 3.9: Figure showing a Golden Rectangle

3.4.5 The Golden Triangle

The Golden ratio can not only be used for rectangles but a similar concept in relation to the Golden ratio was applied to triangles resulting in the Golden triangle. An isosceles triangle, a triangle where at least two of the sides are of equal length, is a Golden triangle if the ratio between the shortest side, b, and the duplicated sides, a, follows the ratio $\frac{a}{b} \approx 1.618\ldots$. An example of the Golden triangle is found in figure 3.10.

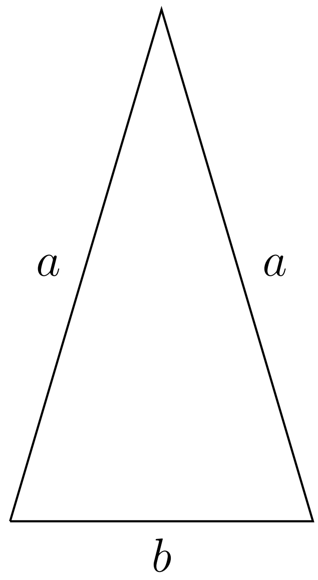


Figure 3.10: A triangle based on the Golden Ratio

3.4.6 The Silver Ratio Relating to Triangles

Similarly to the Golden triangle, an isosceles triangle relating to the Silver ratio can be derived. This triangle was referred to as the Silver triangle and a figure showing this can be found in figure 3.11. An isosceles triangle is a Silver triangle if the ratio between the shortest side, b, and the duplicated sides, a, follows the ratio $\frac{a}{b} \approx 2.414\ldots$.

3.4.7 Compactness of Triangles

The compactness of triangles was calculated by determining the closeness to equilateral triangles. Equilateral triangles have also been related to perceived strength. Therefore optimisation for this type of triangle was integral to the optimisation.

3.4.8 Stability

Stability in the structure was considered to be important to make it able to withstand stress. The stability of an object is determined by the centre of gravity and can be calculated for any object.

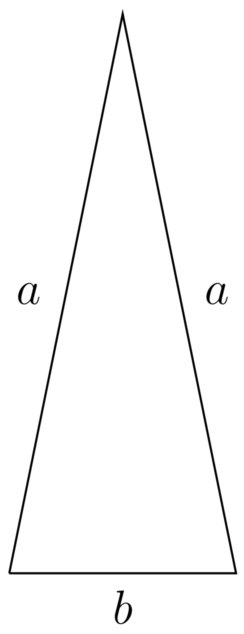


Figure 3.11: A triangle based on the Silver Ratio

3.4.9 Symmetry over Axes

Symmetrical or almost symmetrical forms are often found in nature and as the design of the original stool was chosen to have a mirrored symmetrical design both when it came to the placement of the beams and the joints, optimising for symmetry was seen as an intriguing objective for the optimisation. This objective could easily be combined with other objectives and could then give designs where symmetry enhanced the rest of the design.

3.4.10 Proximity to Forming a Plane

As the top and bottom layers of the stool were set to fixed positions, the only nodes that have the ability to move were the four nodes that constituted the middle layer. Thus the stool could contain a plane formed by these four nodes. The search for the presence of planes in the different layers of the stool, and the proximity of layers of the stool to forming planes, was thought to have the possibility of giving visually interesting designs. This could give a chair that had sections that had straight lines when seen from specific angles when the planes were looked at straight ahead, whilst the formation of a plane would not be evident from different angles. A hidden feature like this could give a focal point to the stool and be regarded as intriguing.

In geometry, a plane is a two-dimensional surface that is flat and extends indefinitely. A plane can be defined uniquely by three points in the plane that are not on a single line. When four points, A, B, C and D, in three-dimensional space, are defined, it is possible to determine whether they form a plane by utilising the method for defining a plane using three points and determining if the last point is in the plane as well. Using this approach it is also possible to determine how close the four points are to forming a plane. This can be done by calculating the distance between the plane defined by three of the four points and the last point. The method used for calculating how close the four points are to forming a plane is detailed further.

The general equation for a plane is determined by:

$$ax + by + cz + d = 0$$

Every point (x, y, z) in the plane meet this requirement. For points (x_1, y_1, z_1) that are not in the plane the shortest distance from the plane to the point is determined by the equation:

$$q = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

3.4.11 General Notions

Some of the objectives discussed are contradictions. This has been explored further in chapter 6 and tested to see what factors are not possible to combine. To determine this, a function was made to combine the objectives to find if good results were obtainable from different combinations of objectives. If a good evaluation was not achievable during optimisations combining two objectives, it was assumed that the objectives were incompatible and could thus not be used in the same optimisation.

Chapter 4

Optimisation Strategies

This chapter covers the frameworks and strategies of optimisation that were considered for the implementation of this thesis. MATLAB was chosen as a mathematical tool to both visualise and optimise mathematical problems during the multiobjective optimisation.

In evolutionary programming, common programming languages used are Python and MATLAB. Both have their advantages and disadvantages which have been discussed in the first section of this chapter. Here MATLAB was used, and the library genLib developed by Mats Høvin [13] was used as it was developed specifically to be a tool for working with generative design. In this chapter, different methods of solving optimisation problems in MATLAB were discussed. The Genetic algorithm and Simulated annealing were described separately. An approach using Weighted fitness optimisation was used to single out and focus on specific objectives for each optimisation and to weight them to suit the conditions of each optimisation.

4.1 Choice of Framework

Both MATLAB and Python are good options when it comes to the choice of programming language for multiobjective optimisation within generative design. They have most of the same functionality when it comes to applications concerning optimisation and visualisation. All methods used in this thesis would be possible to implement in both MATLAB and Python, and the methods used should therefore be possible to reproduce no matter what framework is chosen.

Python is a well-known open-source programming language with almost endless possibilities when it comes to what it is possible to achieve. It has a simple and concise syntax and is frequently used within evolutionary programming as well as machine learning. With a large community of users as well as a diverse selection of available frameworks and libraries targeting almost all possible areas in programming, it is a good first choice of programming language for both students and other programmers. Optimisation algorithms are easy to implement and use in Python.

MATLAB is a programming language based on the programming language C and it was designed to analyse data and create models and applications. It was specifically designed with engineers and scientists in mind and can be used for machine learning, signal processing, optimisation and modelling to name a few examples. MATLAB has evolved from being a program to easily manipulate matrices and can now be used for nearly all mathematical applications. With pop-up graphic windows that support both $2D$ and $3D$ views, it is great for visualisation. MATLAB also has a command window that can be used for executing programs or using an interactive shell and is a great tool when troubleshooting and testing out new ideas.

One of the most significant disadvantages of using MATLAB is its availability. MATLAB is only available through an annual or perpetual license with a fee. Many universities provide licenses for MATLAB for their students and their employees.

As the genLib library was developed specifically for working with generative design in MATLAB, this was the determining factor for using MATLAB. The genLib library was heavily utilised in the development.

4.2 Optimalisation in MATLAB

In MATLAB multiple algorithms for optimisation have already been implemented. The two algorithms that were considered were the Genetic algorithm and the Simulated annealing algorithm. For this application, both algorithms were deemed reliable as they gave similar results during testing. Simulated annealing is considered to be a computationally intensive algorithm. For optimising a small framework, this should not have a great impact on the runtime of the optimisation. For larger optimisation problems the difference between Simulated annealing and the Genetic algorithm would be more noticeable. Ultimately the Genetic algorithm was chosen.

4.2.1 Genetic Algorithm

The Genetic algorithm is used for optimisation with the objective of evolving toward a better solution using a process that imitates biological evolution. The Genetic algorithm was developed by John Holland in the 1960s and

1970s and "is a model or abstraction of biological evolution based on Charles Darwin's theory of natural selection" [28]. It can be used to search for global minima or maxima for highly nonlinear problems.

To initialise the optimisation, a randomly selected population of solutions is selected. The members of the population are also called individuals. Each individual in a generation is evaluated and assigned a fitness accordingly. Individuals are selected at random to use as parents to produce the children used for the next generation of individuals. Combining this with mutation and crossover among the members of the population, the genetic algorithm is more likely to avoid local minima or maxima. The population will over time evolve toward an optimal solution for the problem. The greatest advantage of the Genetic algorithm is the ability to do a search that explores many different directions simultaneously as the children act as individual entities [28] and covers a larger search field. This makes it possible to search larger areas of the fitness landscape. However, weaknesses of this algorithm are the scalar way individual performance is rewarded [7] and how it tends to be time-consuming.

4.2.2 Simulated Annealing

The Simulated annealing algorithm can solve unconstrained and bound-constrained optimisation problems. The algorithm simulates the annealing process that happens in metals when heated and cooled to alter their physical properties, and the simulated annealing process emulates the cooling process of metals where the temperature is controlled carefully [28]. This method works similarly to the genetic algorithm in the way the current solution is used to find the next step. Unlike the Genetic algorithm, simulated annealing only utilises one solution in each step of the optimisation. It also accepts worse solutions than the current for the next step with a certain probability. This is done to prevent the algorithm from getting trapped in a local minima or maxima.

Chapter 5

Implementation

In this chapter, the implementation of each measurement of evaluation used for the optimisation of the frame truss design presented in section 3.2 was described. For each algorithm used examples that demonstrated the effect they had when applied to the frame truss were shown. Each objective was implemented alongside the ability to endure weight as well as the physical constraints described. As can be seen, not all the measurements that initially seem like a good idea were so in reality. Others performed differently in practice than how they were thought to work.

5.1 Basic Criteria

The intended outcome of this analysis was a stool that would endure stress and was possible to sit on comfortably. The criteria in regards to the design of the stool were defined in section 3.2. Those constraints defined the practical point of view concerning how the stool looked and was used in the implementation. The criteria laid down ensured a usable chair.

All measurements presented are in metres. The stool was 0.2 metres tall and the nodes of each layer were 0.1 metres away from the closest nodes. The objectives listed were taken into consideration alongside the ability to endure weight in the form of deflection in the fitness evaluations. The impact of each objective on the evaluations was exemplified with one or more optimisations. All factors in the Weighted fitness function apart from the ones that were relevant to the optimisation were set to zero.

The Weighted fitness function of the optimisation problem was given as:

$$
\begin{aligned}
g = \sum_{i=1}^{n} \omega_i f_i &= \omega_1 \cdot \text{deflection} + \omega_2 \cdot \text{sumEdge} + \omega_3 \cdot \text{maxEdge} \\
&+ \omega_4 \cdot \text{goldenTri} + \omega_5 \cdot \text{silverTri} + \omega_6 \cdot \text{equiTri} + \omega_7 \cdot \text{chooseTri} \\
&+ \omega_8 \cdot \text{centreofMass} + \omega_9 \cdot \text{symmetry} + \omega_{10} \cdot \text{closePlane}
\end{aligned}
\tag{5.1}
$$

The names of the objectives used for the Weighted fitness function and the descriptions of each objective used in the optimisation:

- Deflection: The sum of the movement of each node of the top layer in all directions. The absolute values were used to account for the possibility of one side of the seat moving up when stress was applied.
- sumEdge: The sum of the length of all edges in metres. Each length was squared and as each edge was < 1 metre this factor did not become too significant and outweigh the other objectives.
- maxEdge: The length of the longest edge of the frame truss in metres.
- goldenTri: Each triangle was evaluated how far it was from forming a Golden triangle, and the differences were summed up. This was represented in metres.
- silverTri: A similar evaluation to the evaluation of goldenTri, but for Silver triangles.
- equiTri: A similar evaluation to the evaluation of goldenTri and silver-Tri, but for equilateral triangles.
- chooseTri: Each triangle was evaluated separately and compared to the type of triangle from the three above it resembled the most. The differences in metres were summed up.
- centreofMass: The stability of the structure was measured by finding how far the centre of mass was from the middle of the seat. This was measured in metres.
- symmetry: Specific nodes of the frame truss were compared in relation to how far they were from being mirrored symmetries over an imagined plane. How far the points were from being symmetrical was evaluated for each direction and summed up. This was measured in metres.

- closePlane: How close the nodes of the middle layer were to form a plane. The plane containing three of the four nodes was found and the distance in metres to the last node was found.

The evaluation of each of the objectives for the original stool was presented in table 5.1. The objectives of stability, symmetry and closeness to forming a plane were 0 in the original design as it was designed to contain mirrored symmetry and all four nodes of the middle layer formed a plane.

Deflection	0	equiTri	0.9941
sumEdge	0.3600	chooseTri	0.9941
maxEdge	0.1414	centreofMass	0
goldenTri	2.9429	symmetry	0
silverTri	12.4971	closePlane	0

Table 5.1: Evaluations of the original stool design

5.2 Handling Physical Constraints

During the optimisation of the practical problem, physical constraints that would not have been a problem in a purely theoretical case needed to be considered. Therefore cases that could alter the physical properties fo the truss and be a problem when building or printing the stool had to be considered. In the implementation of the stool, the two physical constraints considered were:

- A minimum angle between edges that shared a node
- A minimum distance between edges that did not share a node

When the constraints were not met a penalty was added to the evaluation of the stool. This gave designs that did not meet these two restrictions a worse overall evaluation than they would get otherwise, and thus they were less likely to be picked than designs that did. Both of these constraints were used in all optimisations.

5.2.1 The Angle Between Edges That Share a Node

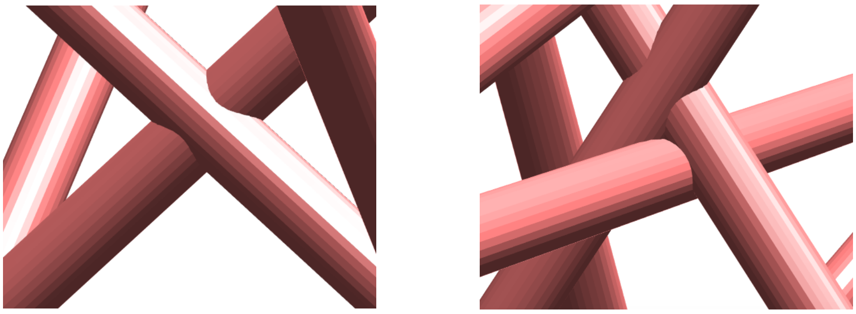
When the angle between two edges that share a node was very small, edges would either partially or fully overlap. A too-small angle between two edges that shared a node could, therefore, make the corner impossible to construct. To counteract this problem the optimisation was discouraged from choosing these options. This was done by specifying a limit of 15° for how small the angle between edges that shared a node could be. Angles smaller than the limit got a worse evaluation than structures that did not have angles lower than the limit. Instead of a hard limit where structures with angles smaller than the limit were discarded, a soft limit was used. A penalty that grew exponentially based on how small the angle was compared to the limit was used. This penalty was then added to the evaluation of the structure. As the total evaluation of the stool would be worse, the chair would be less likely to be favoured and thus less likely to be chosen.

5.2.2 The Distance Between Edges That Do Not Share a Node

When edges that did not share a node were too close to one another problems considering construction would occur. Edges would collide or cross. There were instances where edges barely intersected, fully collided or scenarios where multiple instances of colliding edges happened simultaneously. Two instances of intersection edges can be seen in figure 5.1. These scenarios were seen as unfavourable to construct as the frame truss could become challenging or impossible without altering the design of the edges.

One way to solve the problem of crossing edges was by cutting into the edges before assembling. The edges would then slot into place like a puzzle. In this case, the calculation of the strength of the stool would be incorrect as the algorithm calculated the stress applied to the edges according to which edges and nodes were initialised to be connected. The edges would gain support by being connected to intersecting edges and thus gain support that was not considered in the evaluation. The calculated strength of the chair, as well as the calculated deflection of the seat, would therefore be invalid.

The problem of colliding edges could also be solved by placing a new node at the intersection point of two edges and dividing the two previous edges into four new ones. Sebastian Tangen Olsen proposed a method for how to solve this in his thesis, *Optimizing a Quadcopter Frame Prototype With a Novel Generative Design Framework* [20]. This method was, however, not used in this case as adding new nodes in the points of intersection would add more complexity to the truss that would need further analysis. The main focus of this work was the analysis of triangular shapes, and placing new nodes at the intersection point of two edges would not necessarily result in the desired triangular shapes. Adding this extra step to the analysis would



(a) Two edges that intersect (b) Multiple edges intersecting

Figure 5.1: Intersection of edges

also add more complexity to the truss design, and the purpose of the choice of design was to make it as fundamental as possible. Adding new nodes at the point of intersection was thus not considered to be a viable option in this instance.

Instead, a limit for how close the edges that did not share a node were allowed to be from one another was set. This limit was a soft limit similar to the one used for the angle between edges that shared a node as shown in section 5.2.1. A penalty was added to the evaluation that grew exponentially for distances shorter than the smallest desired distance. The radius of the edges was set to 3 mm for the truss. To discourage the choice of designs with overlapping edges the desired minimum distance from the middle of each beam was set to 10 millimetres.

5.3 Structural Constraints

In this section, the objectives relating to the structural constraints of the stool were outlined. These objectives were related to the built of the stool and were related to the physical constraints of the chair that were described in section 5.2. Where physical constraints considered the aspects that make the stool impossible or challenging to build, the structural objectives considered the aspects that made the stool impractical or impossible to use for its intended purpose. This both included the ability to support weight and for the size to not expand outside the bounds that made it possible to sit on. They consisted of:

- Deflection of the seat
- Shortest total length of edges

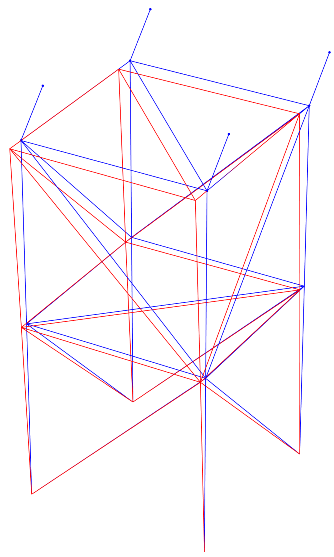


Figure 5.2: Deflection of the seat

- Maximum length of edges

5.3.1 Deflection of the Seat

To find the durability of the stool or how much weight it could withstand, the deflection of the seat was calculated using the deflection matrix. This matrix showed the displacement of each node, and it could be used to find the displacement of specific nodes. As the seat constituted of the four nodes that made up the top layer these nodes were the only ones used for this evaluation. The weight was also applied to these four nodes. Figure 5.2 shows the original truss design as well as the vectors of the forces applied to the stool in blue and the same truss with deflection in red. The displacements of the four nodes that constituted the seat were summed up to give an evaluation of the displacement of the seat. The stools with less deflection of the seat were favoured over stools with more deflection.

From experiments where the deflection of the seat was used for the evaluation, the nodes were found to move to make the edges parallel to the applied forces to counteract the stress applied. An example of this was presented in figure 5.3.

Figure 5.4 shows three examples of stools that were optimised for weight placed on all four nodes that constituted the seat. This resulted in stools of vastly different designs, even though the designs were only optimised for weight. One common variety of resulting designs from optimisations was

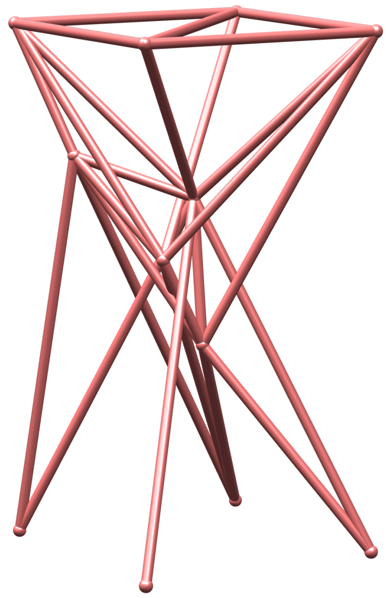


Figure 5.3: Force placed at an angle

Figure	5.4a	5.4b	5.4c
20N right down	0	0	0
Force applied at an angle	$4.8173 \cdot 10^{-7}$	$1.1662 \cdot 10^{-6}$	$5.9195 \cdot 10^{-7}$
Force applied at an angle	$4.5833 \cdot 10^{-7}$	$1.1541 \cdot 10^{-6}$	$1.1541 \cdot 10^{-6}$

Table 5.2: Evaluations of deflection of the seat for the optimised stools

excessively wide stools that were too wide for comfortable use. Two examples of wide stools are pictured in figure 5.4b and 5.4c. Other optimisations resulted in designs that appeared flat when observed from one direction whilst appearing to be wide when observed from a different direction. An example of this can be found in figure 5.4a. Table 5.2 shows the deflection for each of the stools shown in figure 5.4. There was no calculated deflection of the seat when the weight was placed straight down on all three stools, but when the weight was placed at an angle on the seat, the narrow stool shown in figure 5.4a gave somewhat better results for deflection than the wider stools shown in figure 5.4b and 5.4c.

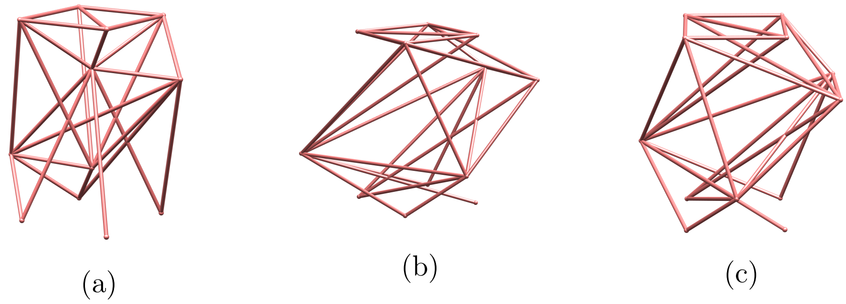


Figure 5.4: Three examples showing optimisation for deflection

5.3.2 Minimum Total Length of Edges

It became evident that one of the ways the algorithm would stabilise the structure and make it withstand more weight was to navigate towards wider structures. The structure would either expand in all directions or two corners on opposite sides would expand outwards whilst the two other corners stayed close to the middle of the structure. This would make the centre of gravity more centred in the stool thus making it more stable. Even with the positive effects of a wider stool, this was not an ideal choice for the usability of the stool.

To counteract for the chair from expanding and becoming too wide, one of the objectives was to limit the length of the edges of the structure. This was done by calculating the squares of the length of the edges and summing up the values. The value was added to the evaluation. Along with the deflection of the structure, this factor could be used to evaluate the performance of the chair. When used in combination with other objectives, the shortest length possible might not be attained, but it would ensure a chair that would not grow to become excessively wide. Even when only the total length of the edges and the deflection were used, intriguing shapes emerged as shown in figure 5.5.

5.3.3 Maximum Length of Edges

A different method used to discourage the choice of stools with wider structures was proposed. This method only evaluated the length of the longest edge, ignoring the length of the other edges. Two vastly different designs can be found in figure 5.6 that illustrate the effect this approach could have on the overall structure. Figure 5.6a resembled the result from the optimisation for the total length of edges where the nodes of the middle layer gathered in the middle of the figure. Nevertheless, this approach tended to result in

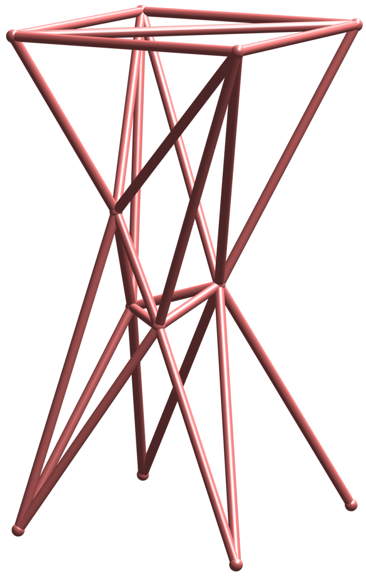


Figure 5.5: Stool optimised for minimising the total length of edges

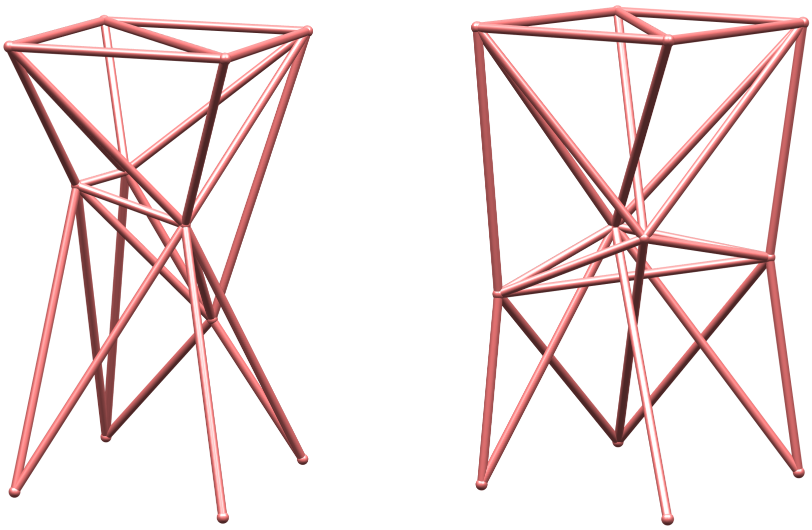
designs where no edge was extremely short. The result found in figure 5.6b shows a stool where all the edges ended up being more or less the same length while the stool still stayed narrow.

5.4 Aesthetical Objectives

As the framework of the stool primarily consisted of triangular shapes, objectives assessing the three types of triangular shapes discussed previously were used in optimisation:

- Golden triangle
- Silver triangle
- Equilateral triangle

A strategy that combined the optimisation of these three types of triangles and optimised each triangle to favour the shape it most resembled was also proposed.



(a) (b)

Figure 5.6: Two stools optimised for minimising the length of the longest edges

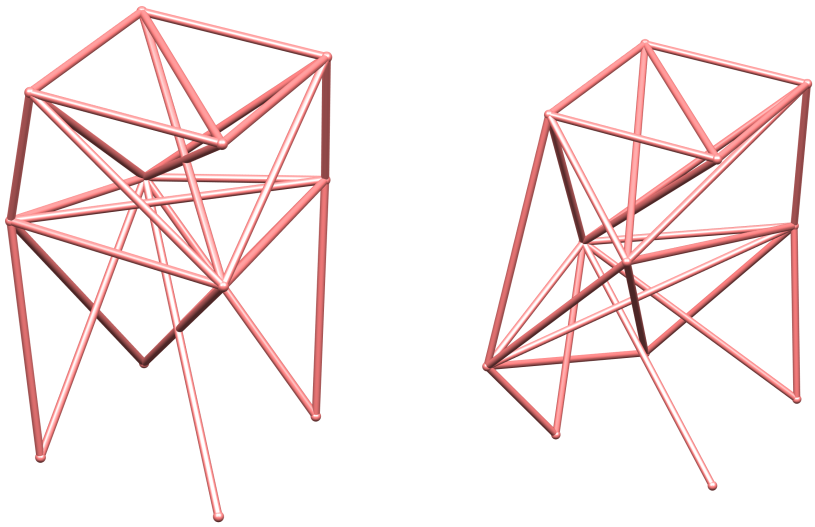
The other objectives used for aesthetical optimisation were:

- Stability
- Symmetry over axes
- Presence of planes

5.4.1 Golden Triangle

To optimise for the prevalence of Golden triangles in the stool, each triangle that was visible from the outside was evaluated and an overall evaluation for the whole design was made by summing up the individual evaluations. The method used to determine how close each triangle was to being a Golden triangle can be found in Algorithm 1. For each triangle, the length of the edges was found and sorted according to size. The difference between the ratio of the longest and the shortest edge compared to $\phi = 1.6180\ldots$, was added to the difference between the length of the longest edge compared to the length of the second-longest edge. The smaller this number was for a singular triangle, the closer it was to forming the shape of a Golden triangle.

Two resulting stools from the optimisation for the prevalence of Golden triangles can be found in figure 5.7. Interestingly, figure 5.7a had edges two



(a) (b)

Figure 5.7: Two stools optimised for Golden triangles

Figure	5.7a	5.7b	5.8
Deflection	$4.5348 \cdot 10^{-7}$	$4.3964 \cdot 10^{-7}$	$4.0030 \cdot 10^{-7}$
goldenTri	0.4838	0.6934	0.5939

Table 5.3: Evaluations of the stools presented in figure 5.7

instances of colliding edges whilst figure 5.7b did not have this problem. This meant that the significance of the evaluation of Golden triangles was deemed to be greater than the evaluation for edges that did not collide for the framework in figure 5.7a. To counteract this, either the evaluation of the prevalence of Golden triangles could be weighted to have less impact on the total evaluation, or the evaluation of the edges not colliding could be weighted to have a larger impact on the total evaluation.

The evaluation of the deflection of the seat and the prevalence of Golden triangles for the optimised stool designs from figure 5.7 along with evaluations of a stool that weighted the objective for Golden triangles with $\omega = 10^{-6}$ can be found in table 5.3. Figure 5.7a had a better evaluation for Golden triangles than figure 5.7b, whilst figure 5.7b had a better evaluation for the deflection of the seat. The stool in the figure where the Golden ratio was weighted to become less significant, figure 5.8, had a better evaluation for deflection of the seat, but an evaluation of Golden triangles that was between the evaluations

Algorithm 1 How close the triangles were to forming Golden triangles

function GOLDENTRIANGLES
  for every triangle with corners (a, b, c) in the stool **do**
    $\vec{ab} \leftarrow$ the vector from a to b;
    $\vec{bc} \leftarrow$ the vector from b to c;
    $\vec{ac} \leftarrow$ the vector from a to c;
    $(a1, b1, c1) \leftarrow$ vectors $\vec{ab}$, $\vec{bc}$ and $\vec{ac}$ sorted from smallest to largest;
    offset $\leftarrow$ abs$(c1 - b1)$ + abs$(\phi - c1/a1)$;
    sum_offset $\leftarrow$ sum_offset + offset;
  end for
  return sum_offset;
end function

for this factor for the two other stools. This shows how weighting a factor in the optimisation differently could result in designs that still had a good overall evaluation.

5.4.2 Silver Triangle

The method used to find how close each triangle was to forming a Silver triangle was similar to the one used to find how close each triangle was to forming a Golden triangle. However, instead of comparing the ratio between the length of the longest edge and the length of the shortest edge to ϕ it was compared to $\rho = 2.414\ldots$. This implementation has been mapped out in Algorithm 2.

The two stools presented in figure 5.9 show two resulting designs from the optimisation for Silver triangles. The two frameworks resembled one another somewhat as they both had a middle layer node situated far down and the node diagonally across from it was situated far up on the opposite side of the stool, making the middle layer of both stools appear tilted. The edges of figure 5.9b collided in two areas, but figure 5.9a did not have any problem with colliding edges. Similarly to the stool presented in figure 5.7a, the search for Silver triangles had more of an impact on the optimisation for figure 5.9b than the objective of avoiding collision of edges.

Table 5.4 shows the evaluation of the deflection of the seat as well as the prevalence of Silver triangles along with the evaluations for the design presented in figure 5.10 where the search for triangles resembling Silver triangles was weighted with $\omega = 10^{-6}$ and had a design similar to the stool shown in figure 5.9a. The stool presented in figure 5.9a had a better evaluation for the deflection of the seat than the stool in figure 5.9b, but the stool in figure

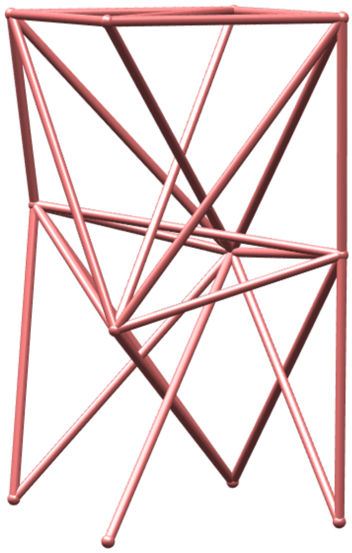
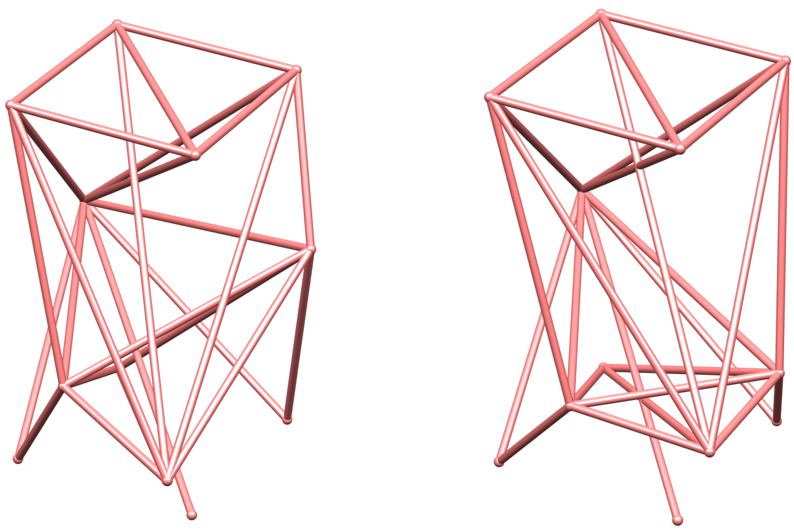


Figure 5.8: A stool where the evaluation for Golden triangles was weighted with $\omega = 10^{-6}$



(a) (b)

Figure 5.9: Two stools optimised for Silver triangles

Algorithm 2 How close the triangles were to forming Silver triangles

function SILVERTRIANGLES
    for every triangle with corners (a, b, c) in the stool **do**
        $\vec{ab} \leftarrow$ the vector from a to b;
        $\vec{bc} \leftarrow$ the vector from b to c;
        $\vec{ac} \leftarrow$ the vector from a to c;
        $(a1, b1, c1) \leftarrow$ vectors $\vec{ab}$, $\vec{bc}$ and $\vec{ac}$ sorted from smallest to largest;
        offset $\leftarrow$ abs($c1 - b1$) + abs($\sigma - c1/a1$);
        sum_offset $\leftarrow$ sum_offset + offset;
    end for
    return sum_offset;
end function

Figure	5.9a	5.9b	5.10
Deflection	$7.5123 \cdot 10^{-7}$	$8.3023 \cdot 10^{-7}$	0
silverTri	3.5197	2.9598	3.2288

Table 5.4: Evaluations of the stools presented in figure 5.9

5.9b had a better score for the prevalence of Silver triangles. However, the stool in figure 5.9b had colliding edges while the stools from figure 5.9a and 5.10 did not.

Comparing the two unweighted stool designs to the design where the Silver triangle function was weighted with $\omega = 10^{-6}$, the weighted stool design had an evaluation for Silver triangles that was between the evaluations for the unweighted designs. The weighted stool had no deflection of the seat and was therefore evaluated to be a better design than the stool in figure 5.9a when only considering these two objectives. Interestingly, the design where the evaluation of Silver triangles was weighted with such a small factor as $\omega = 10^{-6}$ had the best overall evaluation. Neither optimisation gained good evaluations for Silver triangles, and it can be assumed that the framework was not ideal for the optimisation of Silver triangles.

5.4.3 Equilateral Triangle - Compactness of Triangles

The search for compact triangles, or triangles with the largest possible triangle compared to the length of the edges, was done by searching for triangles that closely resembled equilateral triangles. In an equilateral triangle, all three edges are of the same length, and how close a triangle was to forming

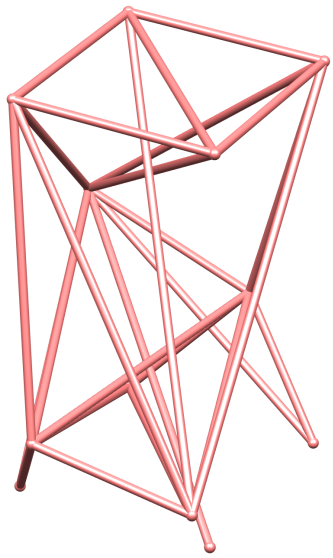
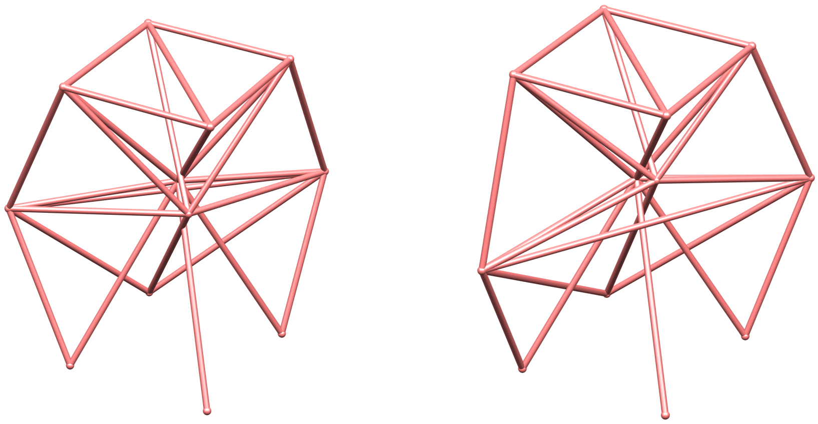


Figure 5.10: A stool where the evaluation for Silver triangles was weighted with $\omega = 10^{-6}$

an equilateral triangle was calculated by comparing the length of each of the three edges. The differences were summed up to give an evaluation of each triangle, and these evaluations were summed up to give an evaluation of the framework as a whole. The method used can be found in Algorithm 3.

Two examples of stools generated with the purpose of obtaining equilateral triangles can be found in figure 5.11, and the evaluation of the stools can be found in table 5.5. A stool where the compactness of triangles was weighted with $\omega = 10^{-6}$ can be found in figure 5.12 and was also used in the comparison. The same pattern as seen in sections 5.4.1 and 5.4.2 appeared when optimising for compactness of triangles as well. The stool presented in figure 5.11a had a better evaluation for compactness of triangles than the stool in figure 5.11b, but this stool also had edges that collided in the same way as we saw for figure 5.7a and 5.9b. This stool also had the best evaluation for deflection of the seat. To avoid this problem, a minimally worse evaluation had to be tolerated. When compared to the stool where the compactness of triangles was weighted by $\omega = 10^{-6}$, the evaluation of the compactness of triangles was worse, but this stool had no deflection of the seat.

In an ideal stool optimised for compactness for triangles, all edges would be of the exact same length making all triangles have the exact same size as well. Even though, none of the stools compared in table 5.5 consisted



(a) (b)

Figure 5.11: Two stools optimised for equilateral triangles

Algorithm 3 How close the triangles were to forming equilateral triangles

function EQUILATERALTRIANGLES
  for every triangle with corners (a, b, c) in the stool **do**
    $\vec{ab} \leftarrow$ the vector from a to b;
    $\vec{bc} \leftarrow$ the vector from b to c;
    $\vec{ac} \leftarrow$ the vector from a to c;
    offset $\leftarrow$ abs($\vec{ab} - \vec{bc}$) + abs($\vec{bc} - \vec{ac}$) + abs($\vec{ab} - \vec{ac}$);
    sum_offset $\leftarrow$ sum_offset + offset;
  end for
  return sum_offset;
end function

Figure	5.11a	5.11b	5.12
Deflection	$3.9704 \cdot 10^{-7}$	$4.1322 \cdot 10^{-7}$	0
equiTri	0.4409	0.5079	0.5304

Table 5.5: Evaluations of the stools presented in figure 5.11

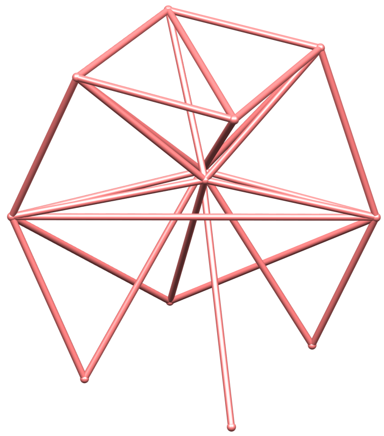
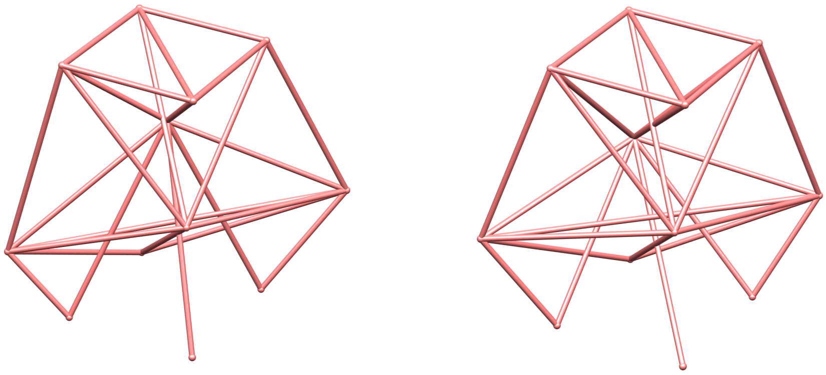


Figure 5.12: A stool where the evaluation for equilateral triangles was weighted with $\omega = 10^{-6}$.

of only triangles that were of the exact same size all of them came close. They had all had total evaluations > 0.55 for how close the triangles were to forming equilateral triangles, meaning all triangles in the three examples closely resembled equilateral triangles.

5.4.4 Combined Search for Triangles

As the search of the three different types of triangles would conflicd when used in the same optimisation the next step was to find a way to combine them in one algorithm. The three objectives were contradictory as they searched for completely different types of triangles, and a good evaluation for one type of triangle would give a bad evaluation for the other types. A method that determined which type of triangle each triangle in the design resembled the most was thus proposed. The search method was based on the methods proposed in sections 5.4.1, 5.4.2 and 5.4.3. The smallest evaluation was chosen for each triangle as this coincided with the type of triangle that particular triangle resembled the most. The evaluations of each triangle in the desing were then added up. The method used for the combined search



(a) (b)

Figure 5.13: Two stools optimised with a combined search for choice triangles with colliding edges

Figure	5.13a	5.13b	5.14
Deflection	0	0	0
chooseTri	0.2593	0.2229	0.3007
goldenTri	3.9638	3.4657	4.5252
silverTri	11.9254	13.0199	14.0793
equiTri	0.9529	0.8124	0.5899
Total	17.1014	17.5209	19.4951

Table 5.6: Evaluations of the stools presented in figure 5.13

for triangles can be found in Algorithm 4.

This search more often than not gave resulting stools with colliding edges, and two such examples can be found in figure 5.13. A stool without colliding edges can be found in figure 5.14. Table 5.6 shows the evaluations of each of the stools and shows good results for the combined search for triangles for all designs, but the stool without crossing edges had the worst evaluation.

An evaluation was done for each type of triangle for the whole designs to get an idea of what triangles this method gravitated towards. Table 5.6 shows how the combined search favoured the shape of equilateral triangles and moved away from Silver triangles. As the method for equilateral triangles differs from those used for Golden and Silver triangles, the numbers cannot be compared directly without further analysis.

Algorithm 4 How close the triangles were to forming the triangle each resembled the most

function CHOOSEBESTTRIANGLE
  for every triangle with corners (a, b, c) in the stool **do**
    $\vec{ab} \leftarrow$ the vector from a to b;
    $\vec{bc} \leftarrow$ the vector from b to c;
    $\vec{ac} \leftarrow$ the vector from a to c;
    $(a1, b1, c1) \leftarrow$ vectors $\vec{ab}$, $\vec{bc}$ and $\vec{ac}$ sorted from smallest to largest;
    offset_goldenTri $\leftarrow$ abs($c1 - b1$) + abs($\phi - c1/a1$);
    offset_silverTri $\leftarrow$ abs($c1 - b1$) + abs($\sigma - c1/a1$);
    offset_equiTri $\leftarrow$ abs($\vec{ab} - \vec{bc}$) + abs($\vec{bc} - \vec{ac}$) + abs($\vec{ab} - \vec{ac}$);
    offset $\leftarrow$ the smallest offset of the three above;
    sum_offset $\leftarrow$ sum_offset + offset;
  end for
  return sum_offset;
end function

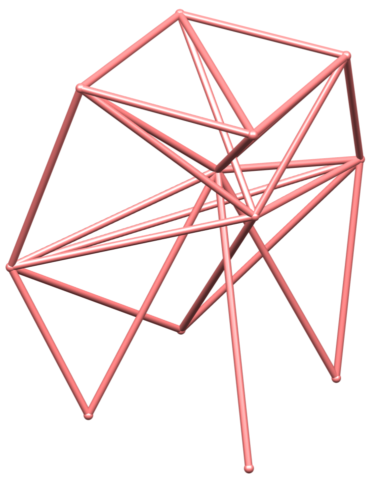
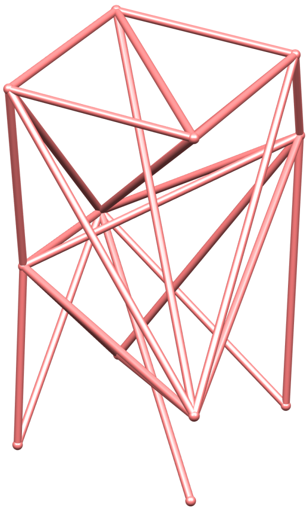


Figure 5.14: A stool for optimised for a combined search for triangles with no colliding edges.


(a)

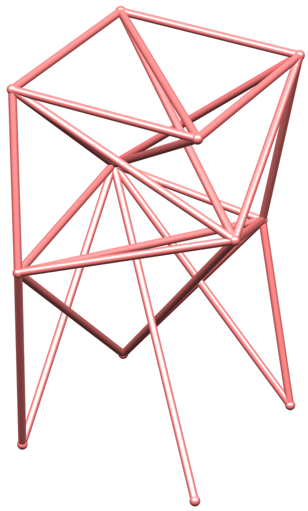

(b)

Figure 5.15: Two stools optimised for stability

5.4.5 Stability

The stability of the stool can be seen as a structural concern, but it is also related to the visual aspect of the stool as the appearance of the stool relates to how the stool is perceived. As discussed in section 3.4.8, the stability of the stool was determined by the centre of gravity. A stable stool was obtained by the centre of gravity being placed close to the middle of the stool when looking straight down at the seat. This was calculated by comparing the position of the middle point of the middle layer to the position of the middle of the top layer. For the centre of gravity to be situated in the lower part of the stool, the largest part of the mass has to be at the bottom of the structure. To obtain this, the aim was for the middle point of the middle layer to be placed in the upper half of the frame truss. Therefore, a penalty that grew exponentially was used when the middle of the middle layer was situated in the lower half of the frame truss. To make sure that the middle layer did not move above or below the top or bottom layers of the stool, these variations got an unfavourably high evaluation making sure these options would not be favoured. The algorithm used was presented in Algorithm 5.

The two stools obtained from optimisation were presented in figure 5.15 and their evaluation can be found in table 5.7. The stool presented in figure 5.15a had a better evaluation for stability than the stool presented in figure 5.15b. From this evaluation alone the stool found in figure 5.15a was evalu-

Algorithm 5 The stability of the frame truss

```
function STABILITY
    M ← centre of the middle layer;
    T ← centre of the top layer;
    C ← centre of the structure;
    B ← centre of the bottom layer;
    centretoMiddleLayer ← distance from C to M;
    if M is between T and C then
        stability ← centretoMiddleLayer;
    else if M is between B and C then
        factor ← 1 + height difference between M and C;
        stability ← centretoMiddleLayer ∗ factor;
    else if M is below B or above T then
        stability ← 10^21;
    end if
    return stability;
end function
```

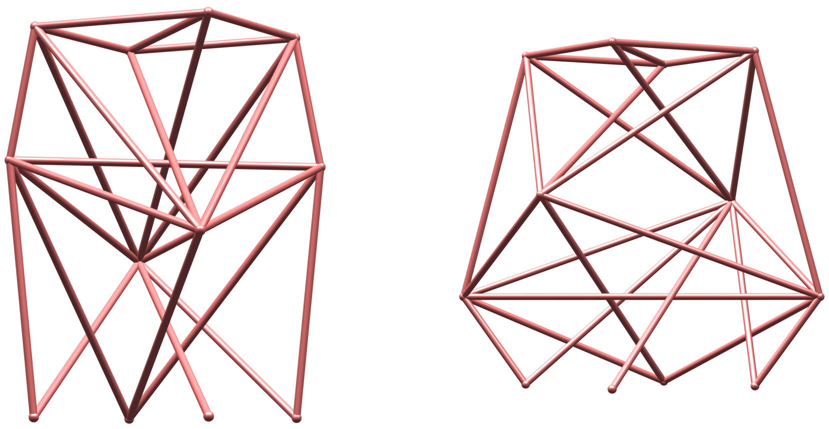
Figure	5.15a	5.15b
Deflection	0	0
Stability	$1.5482 \cdot 10^{-7}$	$2.4321 \cdot 10^{-7}$

Table 5.7: Evaluations of the stools presented in figure 5.15

ated to be the best of the two, but both these two stools were deemed to be stable. To what degree was not as important. The primary purpose with the optimisation for stability was to make sure the stool would not feel unstable when used and to make sure that the stool would not fall over. This measurement for optimisation was primarily meant to be used in combinations with other measurements and not separately.

5.4.6 Symmetry Over Axes

The importance of symmetry when quantifying beauty was discussed in section 3.3.4. The original design of the frame truss used for the optimisation contained mirrored symmetry as discussed in section 3.2, and a method for exploring the symmetry or near symmetry of the framework was proposed. Algorithm 6 shows the method used to quantify how close the stool was to symmetrical. The symmetry was measured from the same diagonal as



(a) (b)

Figure 5.16: Two stools optimised for symmetry over axes

the edges and nodes of the framework were mirrored symmetries over, and symmetry over this imaginary line would ensure symmetry for the whole framework. As only the nodes of the middle layer were allowed to move, the symmetry was only measured for these four nodes. The placement of each node was measured from the middle of the stool, and the placement was compared to the placement of the node diagonally across the stool. The differences in position in each direction were summed up to make an evaluation of how close the stool was to symmetrical.

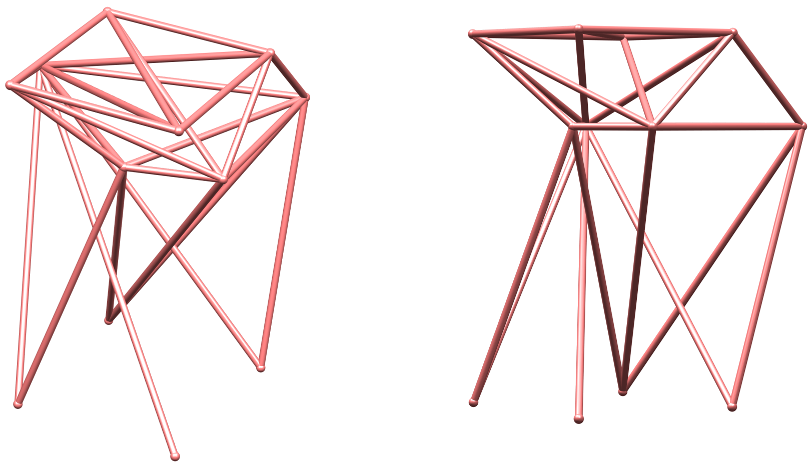
During optimisation for symmetry over the axes, vastly different designs appeared and two of these are found in figure 5.16. A different viewpoint than what has been used previously in this chapter was chosen to highlight the symmetry of the framework and how dissimilar the two designs were.

5.4.7 Presence of Planes

The method put forwards in section 3.4.10 was used to calculate the presence of planes contained in the frame truss of the stool. The top and bottom layers already formed planes, so this method was used to make the middle layer form a plane as well. The equation for a plane formed by three of the nodes that constitute the middle layer was found, and the distance from the plane to the last node of the middle layer was calculated. This was done for all four nodes of the middle layer, and the smallest distance to the plane gave the evaluation of how close the four nodes were to forming a plane. The algorithm used can be found in Algorithm 7.

Algorithm 6 How far the frame truss is from being symmetrical

function SYMMETRYOVERMIDDLE
  for two points A and B opposite from middle M **do**
    $(x_A, y_A, z_A) \leftarrow$ the vector from point M to point A;
    $(x_B, y_B, z_B) \leftarrow$ the vector from point M to point B;
    $x \leftarrow$ difference between x_A and x_B;
    $y \leftarrow$ difference between y_A and y_B;
    $z \leftarrow$ difference between z_A and z_B;
    symmetry $\leftarrow x + y + z$;
  end for
  return symmetryOverMiddle $\leftarrow$ symmetryOverMiddle + symmetry;
end function

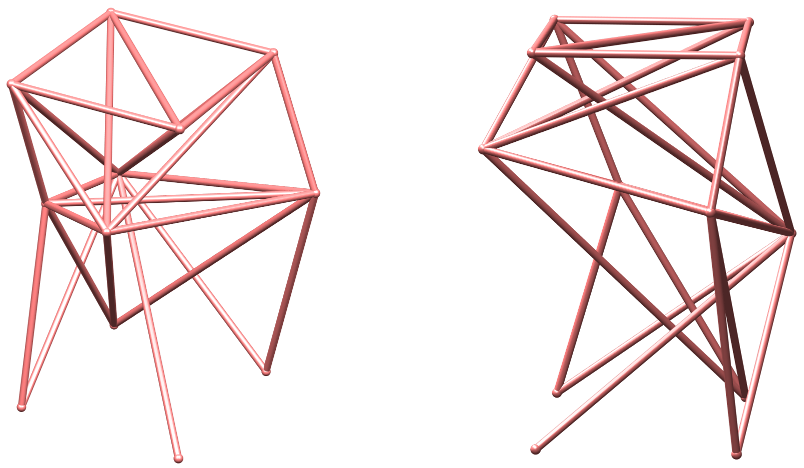


(a) (b)

Figure 5.17: Two points of view of a stool optimised for the presence of planes

Algorithm 7 Using the equation for the distance to the plane to find the distance from a plane to a point

function DISTANCEFROMPLANE
  $\vec{AB} \leftarrow$ the vector from points A to B in the plane;
  $\vec{AC} \leftarrow$ the vector from points A to C in the plane;
  $[a, b, c] \leftarrow$ crossproduct$(\vec{AB}, \vec{AC})$;
  d $\leftarrow -(a \cdot x + b \cdot y + c \cdot z)$;
  distancefromPlane $\leftarrow \mid (ax_1 + by_1 + cz_1 + d)/(\sqrt{a^2 + b^2 + c^2}) \mid$;
  return distancefromPlane;
end function



(a) (b)

Figure 5.18: Two points of view of a stool optimised for the presence of planes

This optimisation strategy was intended to be used in combination with other optimisation strategies, along with the strategies used for stability and symmetry, to obtain more interesting designs. Two stools optimised using this approach can be found in figure 5.17 and figure 5.18. Both designs were shown from two different angles to illustrate the effect of a plane and how this feature could be integrated into a multiobjective optimisation. Other than this, the two designs were vastly different.

5.4.8 Dialling Function

To combine the objectives of the optimisation and control the impact each had on the overall evaluation of the design a Weighted fitness function was used. For the implementation, a function to dial the evaluation of each objective was proposed. This was done using a function that required variables corresponding to the importance of each evaluation for that specific optimisation. For each optimisation, weights were set to determine how much each separate evaluation contributed to the total. All evaluations were weighted appropriately, and the sum of the weighted evaluations gave the total evaluation. This function was essential in the experiments presented in chapter 6.

Chapter 6

Experiments & Results

In the following experiments, the multiobjective optimisation used in a generative design process was expanded and new combinations were explored. The objectives were discussed and assessed individually in chapter 5. All possible combinations of two objectives were mapped out in table 3.1 in chapter 3, and these combinations were the basis for further exploration.

The experiments were organised by the type of triangle they correlated to, and the experiments for a combined search of triangles were put as a separate section. There were multiple interesting ways to organise the experiments, but this method was chosen to investigate the impact of the search for triangular shapes fully. All of these optimisations took the deflection of the seat into account as this is the main purpose of a stool as well as the constraints of a minimum angle between edges that shared a node and a minimum distance between edges that did not share a node.

Through the optimisation process, the objectives were weighted using the Weighted fitness function described in chapter 5. The analysis of each combination explored the impact each objective had on the combined optimisation. If a good evaluation of one of the objectives was not attainable during a multiobjective optimisation, it was presumed that the constraints were incompatible and could thus not obtain favourable results when combined in the same optimisation. The evaluations can be compared to the evaluations of the original truss design presented in table 5.1.

These experiments and the combinations explored were not extensive as the possible combinations were too numerous for it to be feasible to explore them all. Rather, it was a selection of combinations to get an idea of what was achievable in the proposed generative design process. Only a selection of the $3D$ figures showing the stools produced are shown, and the chosen selection was of stools that had features that were commented specifically. The names of the objectives in the implementation were the same as in chapter 5.

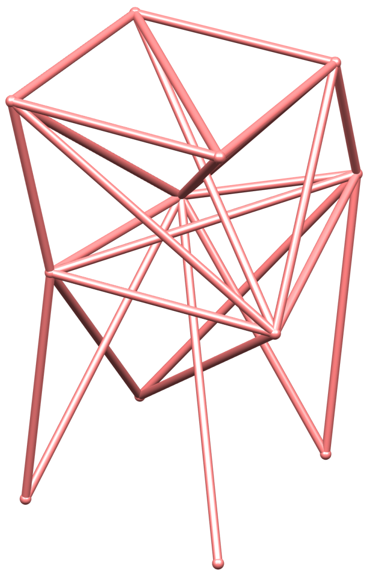


Figure 6.1: Stool optimised for Golden triangles and stability

6.1 Experiments Based on the Search for Golden Triangles

The search for the prevalence of Golden triangles was combined with the three goals of aesthetical optimisation not related to triangles. These optimisations were run separately as well as combined with the objectives of minimising the total length of edges and the length of the longest edge.

6.1.1 Golden Triangles & Stability

From the results in table 6.1, it was found that the stool that also optimised the maximum length of the longes edge gave the best evaluation for the optimisation of shapes resembling Golden triangles. The stability was evaluated to be approximately the same for all optimisations. The total length of edges was similar in each design although the optimisation that included the total length of edges interestingly gave the worst evaluation for the total length of edges. The optimised design using no restrictions of length was presented in figure 6.1.

	No edge restriction (6.1)	sumEdge	maxE
goldenTri	0.4514	0.5901	0.3493
Stability	0.0285	0.0208	0.0222
Total length of edges	0.4365	0.4646	0.4415
Max length of edge	0.1736	0.2140	0.1751

Table 6.1: Evaluations for Golden triangles and stability

	No edge restriction	sumEdge	maxE
goldenTri	0.3893	0.5236	0.3900
Symmetry	0.0953	0.0868	0.0728
Total length of edges	0.5201	0.4083	0.4686
Max length of edge	0.2149	0.1618	0.1923

Table 6.2: Evaluations for Golden triangles and symmetry

6.1.2 Golden Triangles & Symmetry

From table 6.2 it was found that the optimisation that did not include any limitations for the length of the edges and the optimisation that also considered the maximum length of the edges gave the best evaluations for Golden triangles. All three designs gave similar evaluations for symmetry.

6.1.3 Golden Triangles & Planes

The optimisation that also optimised for the total length of the edges gave the best evaluation for triangles resembling Golden triangles according to table 6.3. However, the search not considering the length of the edges gave an evaluation for closeness to planes that was considerably better than the other two optimisations presented in the table.

	No edge restriction	sumEdge	maxE
goldenTri	0.4876	0.3517	0.4429
Planes	0.0089	0.0394	0.0169
Total length of edges	0.3951	0.5428	0.4061
Max length of edge	0.1617	0.2649	0.1618

Table 6.3: Evaluations for Golden triangles and planes

	No edge restriction	sumEdge	maxE
silverTri	3.1136	3.1615	2.7157
Stability	0.0043	0.0186	0.0337
Total length of edges	0.6397	0.6101	0.7058
Max length of edge	0.2414	0.2414	0.2414

Table 6.4: Evaluations for Silver triangles and stability

6.2 Experiments Based on the Search for Silver Triangles

As seen in section 5.4.2, the evaluation for Silver triangles generally provided evaluations that were considerably worse than the evaluations for Golden and equilateral triangles and it could be assumed that the framework was not optimal for the optimisation for Silver triangles. The evaluations for Silver triangles were consistently larger than the other evaluations for the same design. In some cases, the evaluations would become more than a hundred times more significant.

The combinations of objectives were then optimised without any limitations when it came to the length of the edges, combined with the total length of edges, and when combined with the maximum length of the longest edge. Interestingly, all optimisations for these searches resulted in the same evaluation for the length of the longest edge, 0.2414 metres. Whether this was a coincidence or not was uncertain.

6.2.1 Silver Triangles & Stability

The best evaluation for the stability of the stools optimised with regard to Silver triangles was the optimisation that did not consider the length of the edges as shown in table 6.4. The best evaluation for Silver triangles was achieved with the optimisation also considering the maximum length of the edges, even though this evaluation was about eighty times as significant as the evaluation for stability for the same design.

6.2.2 Silver Triangles & Symmetry

All three optimisations concerning the search for triangles resembling Silver triangles combined with symmetry resulted in designs that all had almost the same evaluations as can be seen in table 6.5. This was interesting as

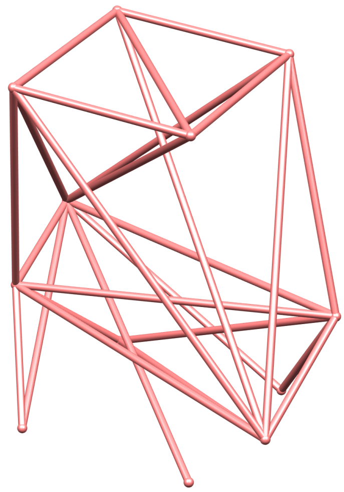


Figure 6.2: Stool optimised for Silver triangles and symmetry

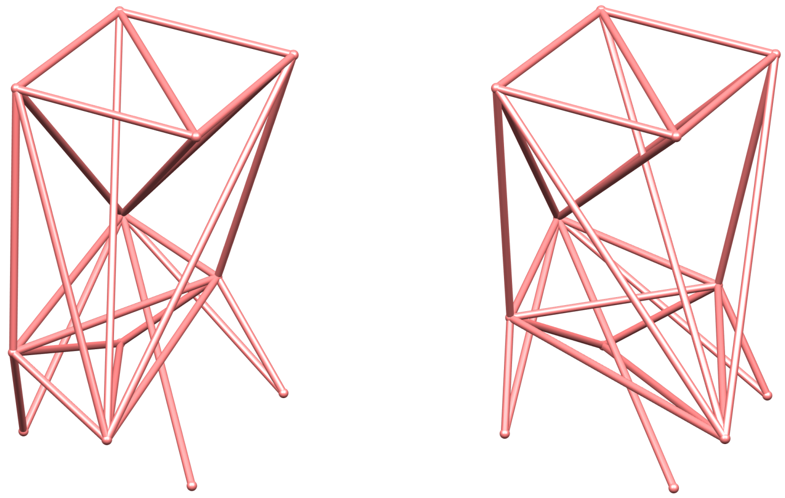
	No edge restriction (6.2)	sumEdge (6.3a)	maxE (6.3b)
silverTri	3.2319	3.0872	3.7314
Symmetry	0.0821	0.1181	0.1105
Total length of edges	0.6965	0.6166	0.6291
Max length of edge	0.2414	0.2414	0.2414

Table 6.5: Evaluations for Silver triangles and symmetry

the three stools looked vastly different as seen in figures 6.2 and 6.3. When analysed manually, the three examples did present a degree of symmetry, but neither presented perfect mirrored symmetry.

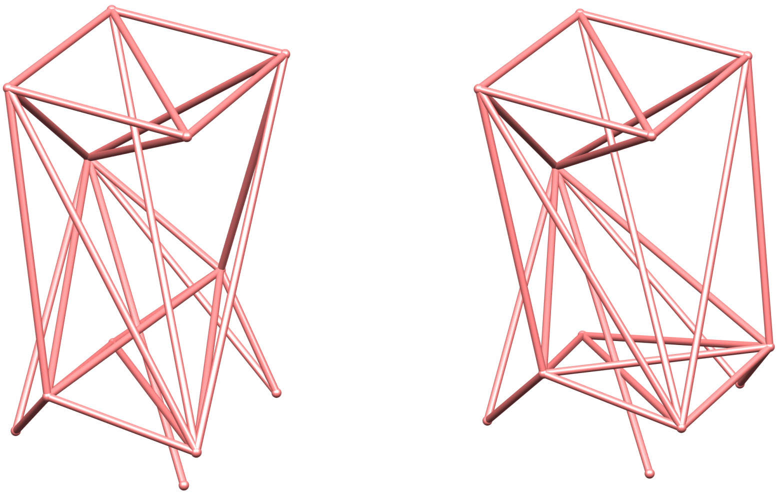
6.2.3 Silver Triangles & Planes

The combined search for the presence of Silver triangles and planes in the stool gave excellent evaluations for planes as seen in table 6.6. All designs had planes that were clearly visible when manually analysed. The stool that produced the best evaluation for the presence of planes was presented in figure 6.4b. This stool had a slight collision of edges as can be seen on the bottom edges of the stool, meaning that the objective of preventing overlapping edges was not given enough importance in the multiobjective optimisation to avoid collisions. The stool presented in figure 6.4a gave the worst evaluation for planes of the three designs presented in table 6.6, but it was the only design



(a) (b)

Figure 6.3: Two stools optimised for Silver triangles and symmetry



(a) (b)

Figure 6.4: Two stools optimised for Silver triangles and planes

	No edge restriction (6.4a)	sumEdge	maxE (6.4b)
silverTri	3.1950	2.8176	2.9030
Planes	0.0100	0.0036	0.0021
Total length of edges	0.6985	0.6823	0.6835
Max length of edge	0.2414	0.2414	0.2414

Table 6.6: Evaluations for Silver triangles and planes

that did not have colliding edges. During manual analysis, this was deemed to be the most interesting design due to the way the plane slanted.

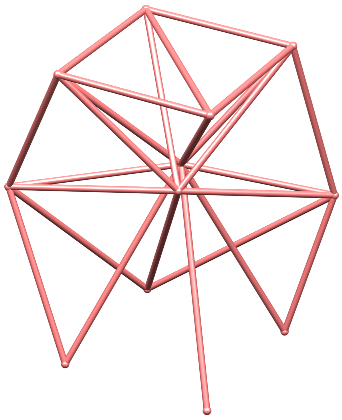
6.3 Experiments Based on the Search for Equilateral Triangles

The search for stools containing triangles resembling equilateral triangles was similarly combined with the three other aesthetical objectives not related to triangles. These searches were also combined with the goals of minimising the total length of the edges, and with the maximum length of the edges. These optimisations did not end up resulting in stools that were extremely wide in any direction. Designs produced tended to bulge out from the middle on two opposite sides. This was enough to be noticeable, but not enough to make the stool unusable. Examples of this can be found in figure 6.5.

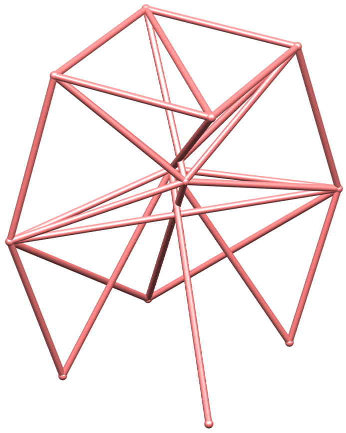
6.3.1 Equilateral Triangles & Stability

The evaluations of the three designs produced from the optimisation considering equilateral triangles and stability were presented in table 6.7. All optimisation strategies produced stable stools that had good evaluations for all objectives considered. The stool where the length of the longest edge was optimised produced the best evaluation for stability. This stool did not have the best evaluation for the length of the longest edge, but the evaluation of the stability for this stool was almost 50 times better than the evaluation of stability for the two other stools.

All the three designs produced looked similar, but the optimisation strategy where the total length of the edges was optimised gave the only design with no colliding edges. This design was presented in figure 6.5a. The design with the best evaluation for stability was presented in figure 6.5b.



(a)



(b)

Figure 6.5: Two stools optimised for equilateral triangles and stability

	No edge restriction	sumEdge (6.5a)	maxE (6.5b)
equiTri	0.4404	0.4810	0.4795
Stability	0.0166	0.0113	$2.2024 \cdot 10^{-4}$
Total length of edges	0.4115	0.3929	0.3928
Max length of edge	0.2122	0.1989	0.2013

Table 6.7: Evaluations for equilateral triangles and stability

	No edge restriction (6.6)	sumEdge	maxE
equiTri	0.4826	0.4825	0.4910
Symmetry	$2.0987 \cdot 10^{-4}$	$9.0759 \cdot 10^{-4}$	$3.0997 \cdot 10^{-5}$
Total length of edges	0.3922	0.3943	0.4013
Max length of edge	0.1996	0.2016	0.2059

Table 6.8: Evaluations for equilateral triangles and symmetry

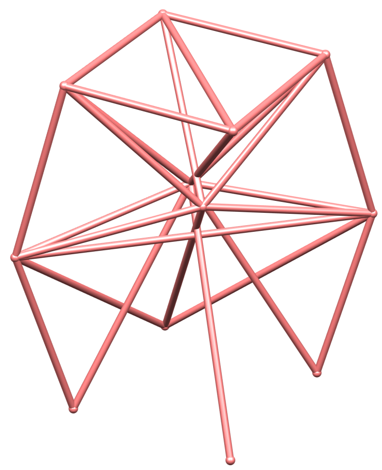
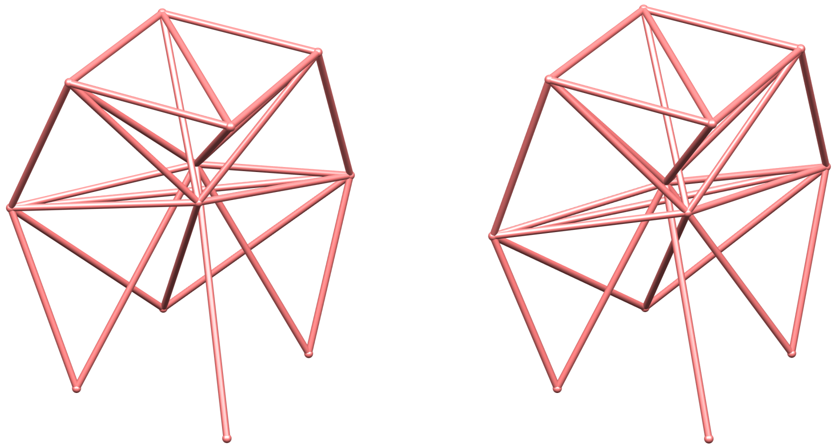


Figure 6.6: Stool optimised for equilateral triangles and symmetry

6.3.2 Equilateral Triangles & Symmetry

The search for equilateral triangles combined with symmetry gave very similar designs for the three optimisations. This could be due to the restrictions provided by the combination of the search for equilateral triangles and the desire for symmetry. All the three evaluations represented in table 6.8 show evaluations for symmetry, resulting in designs that were almost perfectly symmetrical. All three designs had multiple instances of colliding edges and this could be the result of the choice of truss design for the foundation along with the desire for symmetry. The best result for symmetry was obtained in the optimisation without any restriction to the length of the edges. This design was also the one with the most overlap of colliding edges and was presented in figure 6.6.



(a) (b)

Figure 6.7: Two stools optimised for equilateral triangles and planes

	No edge restriction	sumEdge (6.7a)	maxE (6.7b)
equiTri	0.5133	0.4602	0.4482
Planes	0.0265	0.0185	0.0143
Total length of edges	0.4208	0.3977	0.3968
Max length of edge	0.2221	0.2012	0.2026

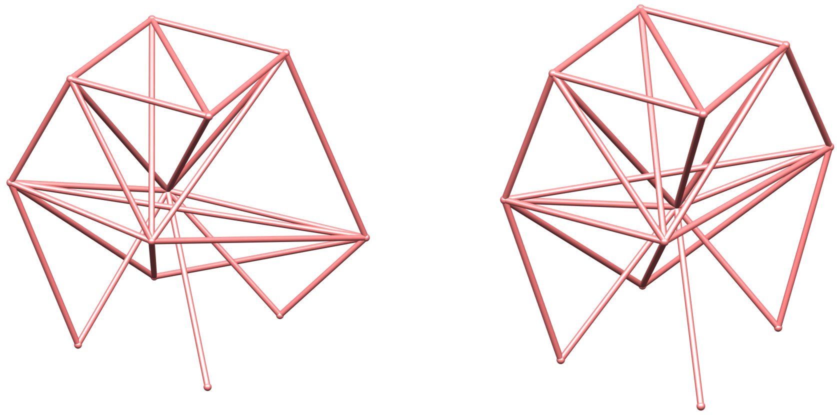
Table 6.9: Evaluations for equilateral triangles and planes

6.3.3 Equilateral Triangles & Planes

Table 6.9 shows the evaluations for the stool optimised for the presence of planes along with equilateral triangles. The searches that included restriction of the length of the edges gave somewhat better evaluations for both the search for equilateral triangles and the search for planes. The two resulting designs can be found in figure 6.7. Neither design contained any colliding edges.

6.4 Experiments Based on the Combined Search for Triangular Shapes

The optimisations using a combined search for triangles as described in section 5.4.4, produced vastly different designs. Here each triangle was optimised to get more similar to the triangular shape it resembled the most.



(a)

(b)

Figure 6.8: Two stools optimised for a combined search of triangles and stability

The searches including other objectives were described and discussed in this section.

In general, the combined search for triangles gave good evaluations during the multiobjective optimisation. This could be because this search method allowed for a larger variety when it came to construction and therefore the other objectives could be optimised considerably while still having a good evaluation for triangles.

6.4.1 Combined Search for Triangles & Stability

From the evaluations in table 6.10 it could be concluded that the optimisation that did not restrict the length of the edges gave the best evaluation for the search of triangles. Even though this stool had no limitations considering the length of the edges, the resulting stool was not excessively wide. The stool also optimising for the maximum length of the edges gave the best evaluation for stability. These two stools were presented in figure 6.8.

6.4.2 Combined Search for Triangles & Symmetry

From table 6.11 it was discovered that the optimisation that also minimised the total length of the edges gave the best evaluation for symmetry, whilst the optimisation that also minimised the length of the longest edge gave the best evaluation for the combined search for triangles. As seen previously, the designs where symmetry was used in the optimisation tended to have edges

	No edge restriction (6.8a)	sumEdge	maxE (6.8b)
Combined triangles	0.2940	0.3606	0.3875
Stability	0.0060	0.0372	0.0022
Total length of edges	0.5095	0.3761	0.4698
Max length of edge	0.2429	0.1800	0.2094

Table 6.10: Evaluations for a combined search of triangles and stability

	No edge restriction	sumEdge	maxE
Combined triangles	0.2636	0.2475	0.1991
Symmetry	0.0269	0.0091	0.0448
Total length of edges	0.3900	0.3810	0.3800
Max length of edge	0.1889	0.1850	0.1821

Table 6.11: Evaluations for a combined search of triangles and symmetry

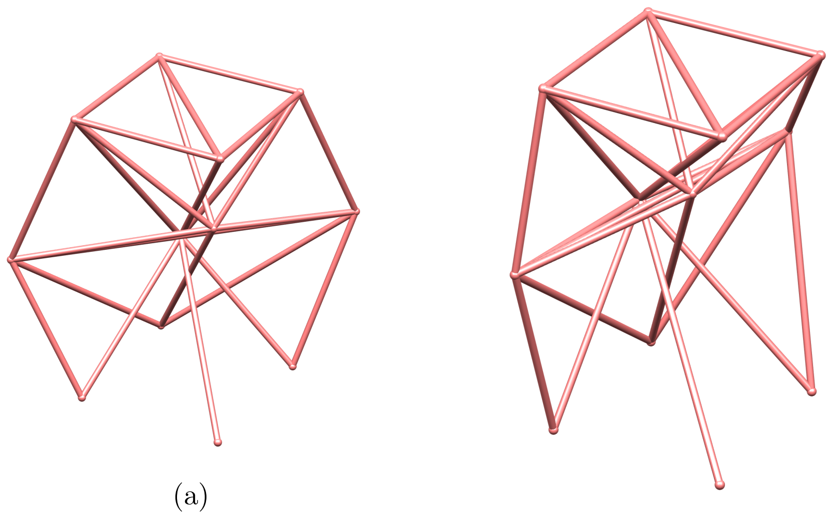
that collided. All three designs obtained from these optimisations had edges that collided, although neither had edges that crossed fully. This meant that the designs did not contain perfect mirrored symmetry.

6.4.3 Combined Search for Triangles & Planes

For the optimisations where the combined search for triangles was combined with the search for planes, the evaluations were presented in table 6.12. All three gave good approximations for the presence of planes. The optimisation considering the length of the longest edge of the stool gave an almost perfect approximation to a plane and the stool was presented in figure 6.9b. Another very good approximation of a plane was found in the optimisation that did not restrict the length of the edges and this stool was presented in figure 6.9a.

6.5 Miscellaneous Combinations of Objectives

From the observations from the multiobjective optimisation process, a couple of further optimisations were done by using different combinations of objectives. A set of optimisations exploring various combinations were executed and presented in this section. Some of the combinations were more favourable than others.



(a)

(b)

Figure 6.9: Two stools optimised for a combined search of triangles and planes

	No edge restriction (6.9a)	sumEdge	maxE (6.9b)
Deflection	0	0	0
Combined triangles	0.3664	0.2668	0.2420
Planes	0.0014	0.0223	$1.4467 \cdot 10^{-6}$
Total length of edges	0.4349	0.3950	0.4024
Max length of edge	0.2223	0.1954	0.1785

Table 6.12: Evaluations for a combined search of triangles and planes

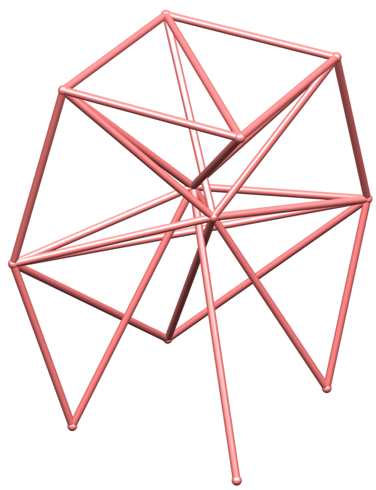


Figure 6.10: Stool optimised for equilateral triangles, symmetry and planes

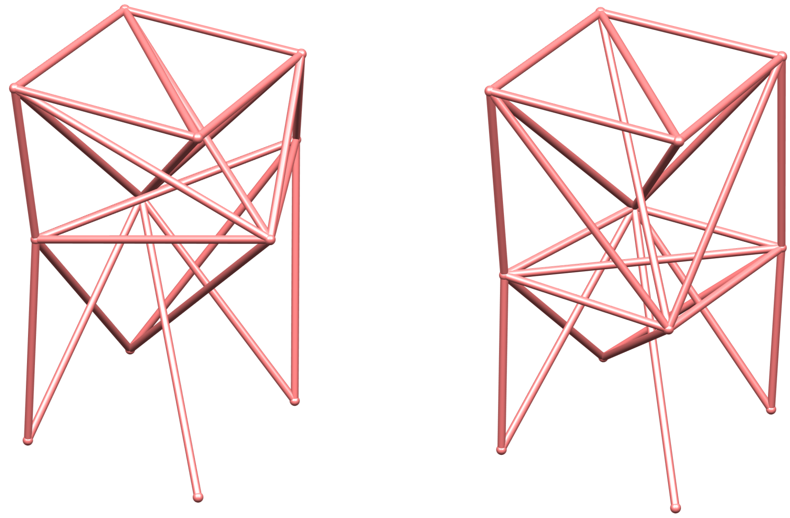
Some combinations, whilst not conflicting were deemed to result in designs that were unfavourable in one way or another. This could be because the combination would result in a design that was seen as uninteresting or where the combination of goals would diminish the individual features. It was found that symmetry was not easily combined with the objectives related to the optimisation of triangles and this objective was thus not used in further experiments.

6.5.1 Combinations Including Symmetry & Planes

The best evaluations for triangles were found when optimising for equilateral and Golden triangles. Stability gave interesting results that made the design appear balanced, and the optimisation for the presence of planes provided an area of visual interest and added to the attraction of the design.

When the optimisation of stability and planes was combined with the search for equilateral triangles, the constraint of the distance between edges that did not share a node had to be weighted much higher than previously in the attempt to avoid collision of edges. The results tended to look similar to the previous designs where the search for equilateral triangles was present as can be seen in figure 6.10. The objective for stability gave a similar effect to what was observed in designs optimised for symmetry.

Optimisation of stability and planes combined with the search for Golden triangles tended to provide more varied and interesting results as can be seen in figure 6.11. The constraint for the distance between edges that did not



(a) (b)

Figure 6.11: Two stools optimised for Golden triangles, symmetry and planes

share a node had to be weighted higher to avoid collision in these optimisations as well.

6.5.2 Symmetry Over Axes & Presence of Planes

The combination of symmetry with the presence of planes in the design was one that was deemed to be uninteresting. As the symmetry of the stool was found by comparing the placement of the nodes to the placement of the node diagonally across the plane this combination would result in a stool where the middle layer would be parallel to both the bottom and top layer similarly to the original design of the stool. As this would defeat the intended purposes for both optimisation strategies, this combination was not explored.

6.6 Ability to Assemble

Models of the frame truss design could be $3D$ printed or constructed using carbon steel rods. $3D$ printed models could be printed directly, either with support that was created manually or by using software that automatically provided this. The hollow rods made of carbon fibres were light but retained an incredible strength. The stool was built by connecting carbon fibre rods cut to the right dimension using generated corner pieces that were $3D$ printed. For easy assembly, the carbon steel rods and the corner pieces had to be possible to piece together and the corner pieces had to be distinguishable from each other. The length of the carbon steel pipes also had to be of a minimum length for two adjacent corners not to merge together and form one corner. Angles between pipes also had to be large enough for the pipes to not overlap. This was ensured by implementing the physical constraints explained in section 5.2.

Chapter 7

Discussion

In this chapter, the results from the implementation in chapter 5 and the experiments in chapter 6 were discussed. Methods for improvements were proposed along with suggestions for better choices of frame truss for specific objectives. The multiobjective optimisation both provided interesting results and results that were not deemed to be as favourable.

7.1 Discussion on the Objectives for Triangles

The search for triangles resembling equilateral and Golden triangles generally resulted in the best evaluations in regard to the optimisation of triangular shapes although equilateral triangles, in general, gave the best evaluations. When optimising for triangles resembling Silver triangles poor evaluations were achieved even when this was the only objective used for the optimisation. It could be concluded that the chosen foundation for the frame truss design was not optimal for the search for Silver triangles.

As the Silver triangle is a slimmer triangle than both the equilateral and Golden triangles, it was proposed that a truss design that facilitated more for this would provide better evaluations for the optimisation of triangles resembling Silver triangles. A taller and slimmer design was suggested as an option as well as a more detailed and complex design where the frame truss could be altered further to accommodate for slimmer triangles. Other designs might give better evaluations for this specific objective, and even though it was not a good choice for the frame truss design presented it should not be discarded as a viable objective for optimisation.

Through the combined search for triangles, where each triangle was evaluated and optimised to look more like the type of triangle it resembled the most, it was found that this method gravitated toward triangles resembling equilateral triangles. The evaluations for triangles resembling equilateral triangles were significantly better than the evaluations for Golden and Silver triangles. Of the three types of triangles, equilateral and Golden triangles resembled each other the most whilst equilateral and Silver triangles resemble each other the least. As it seems like the equilateral triangle was the most ideal shape of a triangle for this truss design, it was less likely for the majority of the triangles to converge towards triangles resembling Silver triangles than equilateral and Golden triangles. It could be assumed that equilateral triangles were the most optimal shape of triangle for the foundation chosen.

7.2 Discussion on the Structural objectives

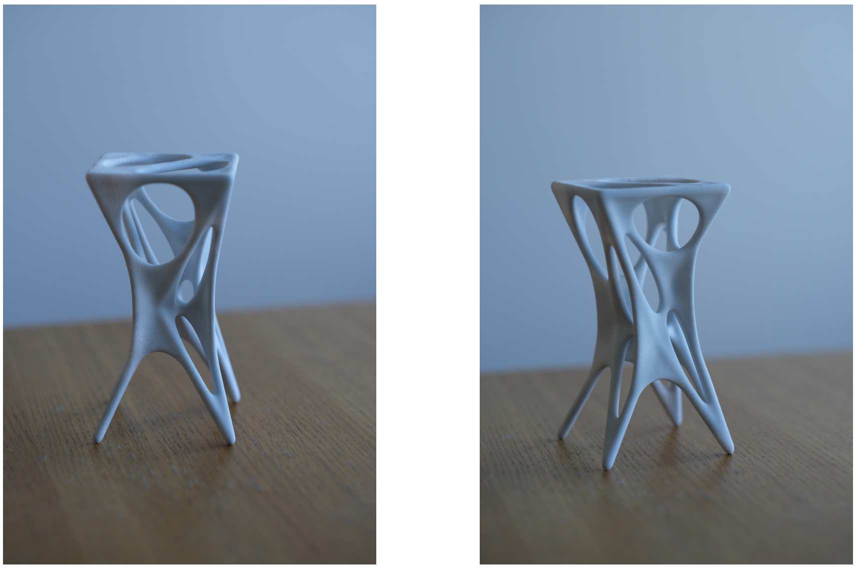
When symmetry was combined with other objectives, especially objectives for triangular shapes, issues with colliding edges often emerged. As the objectives for triangles tended to result in structures where two of the nodes of the middle layer switched sides, good symmetry resulted in collisions of the edges. To ensure that the edges did not collide, a worse evaluation for symmetry had to be tolerated. This gave a design that did not contain perfect mirrored symmetry but rather approximate symmetry. As this also mirrored nature better and gave a more lifelike and visually interesting result.

Stability could be combined with most other objectives and gave stools that were perceived as balanced. This gave a similar harmony to what well-applied symmetry or approximate symmetry provided. For the purpose of the optimisation of this frame truss stability could be used for a similar effect to what was provided by symmetry while not having the disadvantage of resulting in colliding edges as often.

The optimisation for planes gave interesting structures both when used alone and when combined with other objectives. The effect was only evident when a good evaluation was found. Otherwise, the design did not look like it was optimised for the presence of planes at all. The combination of symmetry and presence of planes was assumed to give uninteresting results as perfect evaluations of the two would give planes that were parallel to the seat. When applied correctly, the objective for the presence of planes was very effective and provided beautiful results.

7.3 Models of Stools

Models of the stools were made by $3D$ printing the whole structure as well as building with carbon steel rods that were assembled using $3D$ printed corners



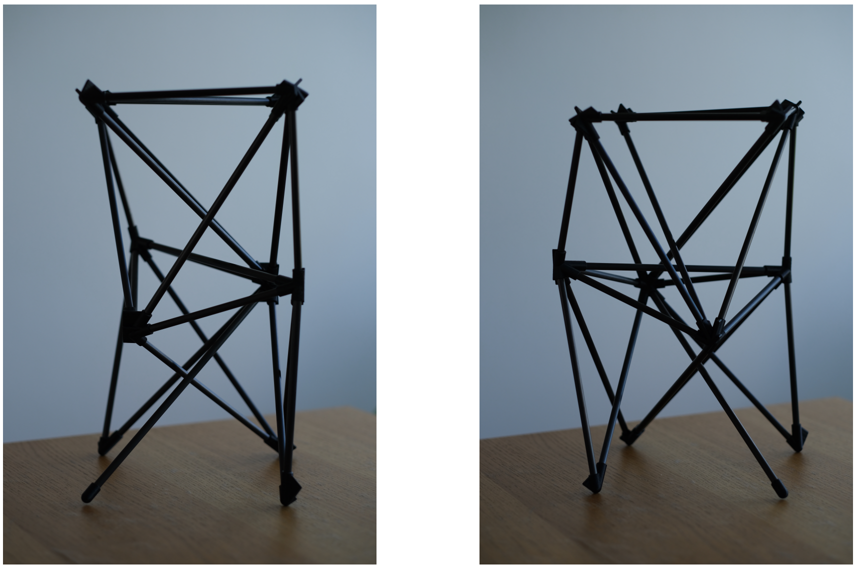
(a) (b)

Figure 7.1: A $3D$ printed stool from two points of view

as the joints. Both methods were viable, but they gave very different looks and could also be used for different applications.

When making a model for $3D$ printing liberties could be taken when it came to the transitions between connecting edges. The natural looking design as seen in figure 7.1 was achieved using splines to soften the transition between the edges. For fully $3D$ printed designs, the size of the stool was the main limitation as $3D$ printers usually do not have the ability to print large designs and a full-size stool would be too large to print for most $3D$ printers. The stool in the example was 20 cm tall. A stool of a large enough size to be used by an adult person would require a printer that could accommodate for this or it would have to be printed in multiple pieces that were glued together. Neither option would be ideal.

Constructing the stool from carbon steel rods did not present the same limitations when it came to size as carbon steel rods can be bought in almost any dimension. Figure 7.2 shows a stool made carbon steel rods. The design of these stools resembled the frame trusses presented throughout this thesis more closely. The joints were $3D$ printed, but the size limitation of the printer was not an issue here as these corners were small. The stool presented was 40 cm tall, but a larger stool could easily be built by scaling the design up or down.



(a) (b)

Figure 7.2: A stool made from carbon steel rods from two points of view

Chapter 8

Conclusion

In this chapter, a conclusion in regards to the generative design process utilising a frame truss design of a stool was met. The research goals from chapter 1 were reevaluated and conclusions to the generative design process utilising multiobjective optimisation were drawn.

8.1 Reevaluation of the Research Goals

The goal of the research was presented in section 1.2 in chapter 1. To conclude whether the main goal of the thesis was attainable, four secondary goals were presented and analysed.

In chapter 3, the objectives that were possible to investigate were determined and their conceived effect was analysed. Not all objectives proposed were possible to apply to this optimisation problem, but they might be possible to utilise in other generative design problems where aesthetics is a part of the optimisation. Table 3.1 assessed what objectives were possible to combine and also what objectives were not possible to combine.

A frame truss design that depicted a stool was used in the generative design process and the effect each of the objectives had when applied to the truss design was explored in chapter 5. Examples of the impact of each objective were presented by both evaluations of the truss and figures showing the resulting frame truss designs. In chapter 6 experiments where different combinations of objectives of optimisation were applied to the frame truss were conducted, and conclusions were drawn in regards to what combinations resulted in interesting designs and not. Not all combinations resulted in interesting designs even though they were theoretically possible to combine.

As all the secondary goals were attainable during the generative design process, it could be concluded that it was possible to attain visually interesting $3D$ designs when simultaneously optimising for practical and aesthetical objectives.

8.2 Future Work

As it was concluded that it was attainable to produce visually interesting $3D$ designs when the aesthetical objectives presented were combined with structural objectives in a practical optimisation problem. Of the measurements of aesthetics mapped out in chapter 3, only those pertaining to triangles like the Golden and Silver ratios were explored along with symmetry and the presence of planes. This could be expanded to involve additional and more complex geometrical shapes and other mathematical principles.

To discover more aspects, a method where the edges were allowed to reorganise for the option of a more beneficial organisation would be interesting to look into. This could facilitate the multiobjective optimisation to discover other geometrical shapes than just triangles. Different shapes to the seat and different numbers of legs could also result interesting designs.

For a larger structure with more beams and joints, the concepts of fractals and spiral geometry, like Fibonacci spirals and logarithmic spirals, might be appropriate to apply. Because of the nature of fractal and spiral geometry, this would require quite a large and detailed structure to work.

The aspect of stability was evaluated, but this resulted in figures where the mass was more or less evenly distributed. Seeming instability where designs that look like they would be unstable actually are stable is an aspect that could have been interesting to explore further. By breaking the rules set previously, more interesting aspects of optimisation could be discovered.

A deeper analysis of the methods used for calculating how closely a triangle resembled one of the given triangles could be done. The methods used for comparison of Golden and Silver triangles resembled one another, but the method for comparison of equilateral triangles was quite different. By analysing the results further these methods could be altered to correlate better and for the evaluations to be compared directly. Future research could also look into more and improved methods of optimisation that could be applied to larger and more complex trusses.

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