

Present biased compared to time consistent preferences

How is an agents' decision-making in a public good game affected?

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Thesis Submitted for the Degree of
Master in Economic Theory and Econometrics

30 credits

The Department of Economics

The Faculty of Social Science

THE UNIVERSITY OF OSLO

May, 2022

Keywords: Climate change, Stochastic game, Cooperation, Quasi-hyperbolic discounting, green technology investment.

Abstract

This paper will analyze a specific public good game referred to as *the climate game* where the public good is the quality of the climate. In our climate game have a number of N agents who participate in a game with repeating rounds over a finite time horizon. We assume every agent i will benefit from having a sustainable climate. Furthermore, we assume every agent emits greenhouse gases (GHG) from their performed production in each time period of the game. Over time, the GHG emissions will accumulate in the atmosphere, destabilizing the climate making everyone worse off. On the other hand, if the agents collectively invest in abatement technology, the total emission level will decrease which would be welfare improving.

The results of the game are based upon the strategy of agent i in each time period of the game. Where their given strategy profile will determine the agent's decision-making regarding their investment level in abatement technology. We will analyze three different circumstances of the game referred to as the first-best allocation, the business as usual result, and a self-enforcing agreement. The analysis will focus on how the agent's strategy profile is affected by including the assumption of agent i having time-inconsistent preferences, also referred to as present-biased preferences. Specifically investigating how the performed investment level during the game differs when the agents have time-inconsistent compared to time-consistent preferences. We will further analyze if having present-biased agents affects the total welfare by comparing the emission levels given the performed investment levels.

The results of the performed analysis show that the assumption of time-inconsistent preferences leads to a lower investment level and consequentially a higher emission level compared to if the agents have time-consistent preferences. Making the agents relatively worse off when having present-biased preferences. This result holds for both the first-best allocation and the business as usual result. The self-enforcing agreement results show that having present-biased preferences can actually be welfare improving compared to when having time-consistent preferences. This outcome is constrained on the assumption that the present-biased agents are patient and sophisticated.

Acknowledgments

I want to provide special gratitude to my supervisors, Katinka K. Holtmark and Geir B. Asheim, for their guidance and academic lead throughout writing my dissertation. The research question started as an exciting thought experiment inspired by my interest in Resource and Behavioral Economics. Recognition has to be made for their contribution and constructive feedback leading the way to develop an analytical approach providing the means to further investigate the question I was eager to take on. Furthermore, I want to emphasize that the presented model framework is solely inspired by Bård Harstad's model "Dynamic Games and Environmental Agreements" from taking his course in Environmental Economics. At last, I want to dedicate this thesis to my right hand, Arslan Ejaz Ahmed, who has kept me motivated and assured throughout the process of writing my master thesis.

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1. Preface

This paper will present a model framework based on a public good game where the analysis seeks to investigate the effect of including specific behavioral features. In the public good game, there are a number of N “players”, where every player, i , has an endowment as well as benefitting from a given public good. The quality of the public good depreciates over time if not invested in by the players. The classical framework of the game assumes every player i is completely rational with time-consistent preferences and is entirely selfish when maximizing their utility. Given these assumptions every player i 's dominant strategy is acting as a free rider. When acting as a free rider, the player can continue to benefit from the public good, given there is a group of other players who are willing to invest in the public good. Consequentially, every rational player prefers to act as a free rider instead of paying the cost of investing in the public good if there are no consequences to doing so. Therefore, it is interesting to analyze which circumstances can incentivize the players to cooperate and invest in the public good which would be welfare improving.

This paper discusses how using the classical approach of assuming the participating players in a public good game have time-consistent preferences has its limitations.¹ To build on the literature, the model framework, therefore, includes behavioral theory characteristics. The players participating in the climate game at hand have time-inconsistent preferences. Leading to the research question: Present-biased compared to time consistent preferences; how is an agent's decision-making in a public good game affected? In our model framework the public good is the quality of the climate and will therefore be referred to as the climate game. In the climate game, there are a number of N participating players who each are referred to as country i . The strategy profile of country i is given by their maximized present utility given their present discounted future utility. The countries are first described in the model framework using the classical approach. Where their present discounted future utility is obtained by using an exponential discount function, where the given structure of the discount function represents time-consistent preferences. Followed by describing the countries time preferences as present biased by using a quasi-hyperbolic discount function. Where the given

¹ Laibson, “Life cycle consumption and hyperbolic discount function.”

structure of the quasi-hyperbolic discount functions includes the assumption of present biasedness.

The approach is motivated by being able to analyze the effect of not having countries with time consistent preferences when it comes to their decision-making regarding their allocation of resources under the climate game at hand. Furthermore, it is interesting to analyze whether including present-biased preferences will have a diverse effect on the player's strategy profile under the different analyzed circumstances of the game. Firstly, we will analyze the first-best allocation of resources where the decision is made from a social planner's perspective. The first best allocation is thus the socially optimal outcome, where every participating country invests fully in the public good. Secondly, we will analyze the business as a usual result where every country is entirely selfish and does not take into account the climate cost of other countries' emission levels. Therefore, each country will only take into account the climate cost of their own emission level when maximizing their utility. The last circumstance of the game we will analyze is a self-enforcing agreement where a share of the countries cooperate and collectively agrees to invest in the abatement technology. Under the self-enforcing agreement we assume the rest of the non-cooperating countries will act as business as usual.

2. Theoretical Groundwork

This chapter will discuss the relevance and the theoretical aspects behind the model framework of the climate game at hand. As mentioned in the introduction a welfare improving outcome in the climate game is achieved if players decrease their emission level collectively. For an agreement between the players to occur without any external force, it has to be self-enforcing. A self-enforcing agreement implies that the players willing to cooperate decide upon an agreement amongst themselves, where no one else except them can implement the terms of the agreement. The agreement will be held as long as the terms are not breached and each of the cooperating players benefits from it.²

We can better understand what a self-enforcing agreement is by discussing a similar example from the real-world economy referred to as an international environmental agreement (IEA). Therefore, this chapter will present the details regarding the Paris agreement, discussing the strength and weaknesses behind such an IEA. The discussion offers an insight as to why it is interesting to theoretically analyze the circumstance of a self-enforcing agreement and how behavioral characteristics of the cooperating players can affect the outcome of such an agreement.

2.1 Discussion on IEA's

The latest achieved international environmental agreement is the treaty of the Paris agreement, which is a legally binding international treaty on climate change. Established during the COP 21 in Paris 12th of December 2015. The agreement entered into force by November 2016, signed by 196 countries.³ The goal of the Paris agreement is to limit the total climate damage and not exceed a rise in the global average temperature above 2 degrees Celsius compared to the pre-industrial time's temperature levels. To achieve the agreement's goals, the signatories seek to develop technologies and environmentally friendly solutions substituting high pollutants in order to have a climate-neutral global society within the year 2050. To achieve the long-term goals, the Paris agreement has encouraged the signees to design and deliver their long-term GHG emission development strategies (LT-LEDS) by 2022.

² Telser, "A Theory of self-enforcing Agreements."

³ UNFCCC, The Paris agreement.

Furthermore, in 2024 the enrollment of an enhanced transparency framework (EFT) will begin, where each cooperating country has to report its contributions in order to become climate neutral.⁴

There are thus expectations set to the signees, where the cooperating countries are assumed to be held accountable for their performed contributions, such as their level of commitment to their performed LT-LEDS. Even though economists have argued that an IEA such as the Paris agreement is not necessarily a strong enough legally binding contract to achieve the set long-term goals. Economists typically argue that the expectations of the cooperating countries are unrealistic.⁵ To hold the signees accountable, there needs to be a more potent force of liability demonstrated by organs such as direct civil society engagement, internal government process, and other national and international institutions has to be involved. Where such forces would instigate a more substantial total consequence for cooperating countries who does not hold up their end of the bargain. Research also suggests that one vital factor in achieving the long-term goals is for the domestic government of each cooperating country to be held accountable internally.⁶ If the national institutions and the population prefer that their country become climate neutral. Then there is a likelier chance for the cooperating country to follow through with their commitments.⁷ Therefore, the force of accountability should not only be external but internal as well. Concluding remarks; both the terms of the agreement as well as the behavioral characteristics of the participating countries are important factors affecting the outcome of an IEA.

2.2 Game theory

This chapter seeks to present an insight into the behavioral game theory used to construct the model framework of the climate game. The model framework is a stochastic game with repeating rounds over a discrete-time horizon. This chapter will thus present an introduction

⁴ UNFCCC, The Paris agreement.

⁵ Vinkhuyzen, Groff, Tamas, Dahl, "Entry into force and then?"

⁶ Vinkhuyzen, Groff, Tamas, Dahl, "Entry into force and then?"

⁷ Vinkhuyzen, Groff, Tamas, Dahl, "Entry into force and then?"

on the theory of a stochastic game. Including the mechanisms behind the strategy profile of the players throughout the timespan of the game. Furthermore, in a stochastic game when analyzing the player's present payoff in a game with repeating rounds, the player's strategy is given by his maximized present payoff. Where his present payoff will be determined by the present discounted value of his future payoff as well as his present utility. Therefore, the discount factor is of importance when analyzing the player's decision-making in each time period of the game. Leading us to the topic of the last sub-chapter of this chapter where we present the two different discount functions used in the climate game.

2.2.1 Stochastic Game Theory

Shapley (1953) published an insightful paper discussing the mathematical mechanisms behind stochastic games.⁸ The author states the preliminaries of a stochastic game which consists of a number of players $i = 1, 2, \dots, N$ who move from one state to the next. Each state is determined by the decision made by the participating players of the game, based on the assumption that there is a finite amount of possible states, and a finite number of possible choices referred to as strategies player i can perform in each state of the game. On the other hand, the time horizon of the game is not necessarily bound to its extent. A stochastic game is thus starting at a specified initial state, determining the starting position of the game, referred to as Γ^k , where the parameter k is denoted as a given state. We can therefore refer to a stochastic game as a collection of:

$$\Gamma = \{\Gamma^k | k = 1, 2, \dots, N\}$$

The specific strategies of a stochastic game are referred to as stationary strategies, where the agents will face the same probability for their actions during each time a given state is reached, independently of how the agent reached the given state. Furthermore, the payoff function for each player, g_i , is dependent on the given state, and the strategy profile. When we have repeating rounds of the game the present discounted value of the payoff function is dependent on the discount factor, δ , ($0 < \delta \leq 1$). We thus have the following formula for the discounted payoff function for each player i in time period t :

⁸ Shapley, "Stochastic Games."

$$\delta \sum_{\tau=t+1}^T (1 - \delta)^{\tau-t} g_{i,\tau}$$

Where t is the present time period and τ is the next time period given t , and the total timespan of the stochastic game is given by T .

Shapley found a noteworthy result for each state of the game, stating there exists a unique solution when there is a linear transformation between the states. For any stationary strategy that is optimal in each state will be an optimal pure strategy for all states given the circumstances of the initiated game. Furthermore, the obtained optimal pure strategy might not be optimal for another set of stochastic game and is thus not universal. Therefore, the set of optimal strategies for a stochastic game Γ is closed and convex.⁹

2.3 Discount value

One central aspect of the analysis is distinguishing between how assuming for time consistent preferences versus time-inconsistent preferences affects the strategy profile of the players. This sub-chapter will give a theoretical insight into the discount factors used in the model framework. Discussing how exponential discounting can be used to present time-consistent preferences. While, quasi-hyperbolic discounting is argued to be a better fit for analyzing time-inconsistent preferences. At last, we will discuss the fitness of exponential and quasi-hyperbolic discounting when assuming for behavioral characteristics such as present-biased preferences.

2.3.1 Exponential discount value

The exponential discount factor, δ , represents the long-term discount factor of utility. The discount factor is constant over time, leading the agent to value his present and future utility equally over the time span of the consumption period. We denote the present time period as

⁹ Shapley, "Stochastic Games."

t and the next time period is denoted by $\tau = t + 1$. We can therefore use the following expression to describe an agent's continuation value in discrete time:

$$U_{i,t} = u_{i,t} + \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau}^{10}$$

Where $u_{i,t}$ is agent i 's utility in time period t , $u_{i,\tau}$ is i 's utility in the next time period, $\tau = t + 1$, where the total time span is denoted by T and the discount factor is given by: $0 < \delta \leq 1$.

The continuation value is determined by the utility in the present time period as well as the discounted value of the agents summarized utility over the given time horizon, T . As we can see from the continuation value, the discount factor, δ , determines how the agent weighs his future utility in the present time period, t . Because the discount factor is constant over the given timespan. The agent will have time consistent preferences when maximizing his utility over the given consumption period. Furthermore, the formula for the exponential discount factor, δ , and discount rate, denoted as ρ , is given respectively by the following expressions:

$$\delta = \frac{1}{1 + \rho} \quad , \quad \rho = \frac{1}{\delta} - 1$$

Because the discount factor is constant over time the agent will value his present utility equally as his future utility and does not have any present-biased preferences when maximizing his utility over the given time horizon. If the discount factor is approaching one, i.e. the discount rate is low, the agent is patient when it comes to allocating his resources over time. If on the other hand the discount factor is approaching zero, we have the case of a high discount rate leading the agent to be very impatient in his decision making. Often in economic literature the agents are assumed to be patient, mimicking the characteristics of a rational agent.^{11, 12} We will assume the agents are patient throughout the analysis, thus we have a low discount rate, ρ .

¹⁰ Samuelson, "A note on measurement of utility."

¹¹ O'Donoghue, T, Rabin, M, "Doing it now or later."

¹² Strotz, "Myopia and inconsistency in dynamic utility maximization."

2.3.2 Quasi-hyperbolic discount value

The quasi-hyperbolic discount functions include both a long-term discount parameter as well as a present-biased discount parameter. Where the parameter δ represents the long-term discounting of utility, while the parameter β represents the present-biased discounting of utility. If β equals one, then the agent would have the same preferences as when using the exponential discount function. On the other hand, if β is strictly less than one then the agent will have present-biased preferences. In the analysis we will assume $\beta < 1$ in order to take into account the consequences of having strictly time-inconsistent preferences. The continuation value of the agent using quasi-hyperbolic discounting in discrete is given by the following expression:

$$U_{i,t} = u_{i,t} + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau}^{13}$$

Where $u_{i,t}$ is agent i 's utility in time period t , $u_{i,\tau}$ is i 's utility in the next time period, $\tau = t + 1$, where the total time span is denoted by T and the discount factors is given by: $0 < \delta \leq 1$ and $0 < \beta < 1$.

We can formulate the expression for long-term discount factor, δ , and discount rate denoted as ρ , given respectively by the following:

$$\delta = \frac{1}{1 + \rho} \quad , \quad \rho = \frac{1 - \delta}{\delta}$$

From the continuation value, we can see that the agent will value his present utility as well as the discounted sum of his future utility over the total time-horizon, T . Because the present-biased parameter, β , is less than one the agent will value his future utility less than his present utility when deciding on his consumption in the present time period. As a consequence, the agent will not be indifferent between decisions regarding facing costs and benefits in the present or future time periods. The agent will weigh the burden of costs higher

¹³ Elster, "Ulysses and Sirens."

than the benefits as the decision comes closer in time, leading the agent to preferring benefits in the present time period and costs in the future time periods. Consequentially, when the next time period emerges the same trade-offs between cost and benefit will take place, thus leading the agent to procrastinate bearing the costs.¹⁴

The characteristics of the agent on the other hand can differ when having present-biased preferences. Where we have two distinctive behavioral traits; an agent can be assumed to be sophisticated or naïve. If sophisticated, the agent will be aware of his present-biased preferences, leading to the incentive of holding himself accountable for his future-self's actions. Thus, when being a sophisticate, the agent will prefer to pre-commit to his future self and choose a consumption plan he will be able to follow as well as smooth his utility level over the given time span. On the other hand, if the agent is naïve, he will not be aware of his present-biased preferences. Therefore, a naïveté when deciding on his consumption plan, he will typically overestimate his future self. Believing he will be able to discipline himself in the following time period and utilize his present benefits and procrastinate the costs of doing so. Consequentially a naïve agent will not be able to hold himself accountable in head of time for his future-self's actions, making him potentially worse off in the end. In the analysis, we will assume the participating countries in the game are all patient and sophisticated when using quasi-hyperbolic discounting for tractability reasons between the two discount factors and the assumptions they follow.^{15, 16}

2.3.3 Comparing the fitness of the two discount values

In behavioral economics, it is argued using exponential discounting is not necessarily a strong theoretical approach for describing a human's utility preferences over time. When discussing consumer behavior Strotz (1956) argued every individual is a separate "self" in each time period, t , seeking to maximize his present utility. Because the exponential discount function solely consists of the long-term discount factor it does not capture the mechanisms behind an agent's present-biased preferences. Strotz argued one of the circumstances where exponential discounting can be a good fit is if the agent is aware of his intertemporal

¹⁴ O'Donoghue, Rabin, "Doing it now or later."

¹⁵ O'Donoghue, Rabin, "Doing it now or later."

¹⁶ Strotz, "Myopia and inconsistency in dynamic utility maximization."

preferences and seeks to pre-commit to his future self by acting disciplined, then an agent can be able to act as if he has constant time preferences. Where upbringing and the rules of society can help form the consumer-mind into minimizing their intertemporal preferences.¹⁷

O'Donoghue and Rabin (1999) discussed using quasi-hyperbolic discounting in order to explain human's tendency to procrastinate tasks, especially when faced with immediate costs and future benefit is a better fit than using exponential discounting. The authors based their argument on the fact that the discount rate and the weight on present biasedness is at a reasonable level when using quasi-hyperbolic discounting in contrast to be able to demonstrate the same behavioral characteristics using exponential discounting. Furthermore, O'Donoghue and Rabin argued using a discount factor representing the human behavior of intertemporal preferences and especially by including the characteristics of being sophisticated or naive has its economic analytical benefits when it comes to understanding the complexity of welfare problems.¹⁸

Laibson (1992) also argues based upon studies published discussing the human behavior that quasi-hyperbolic discounting is a good theoretical approach reflecting human's time preferences.¹⁹ The author also discusses the same dilemmas of human consumer behavior as Strotz and O'Donoghue & Rabin regarding the tendency of acting as separate "self" in each present time period. Where human's typically has a higher expectancy of their self the further away the time of doing the task is. Consequentially, as the time of doing the task approaches human's will procrastinate or ending up defaulting and not follow through with their plan when the time of doing the task is in the present time period. There are many different scenarios through life were humans face themselves in this dilemma. Laibson discusses how intertemporal preferences affect the ability to save for pensions, invariance of patience through the lifespan of the consumer and other scenarios a consumer face through his lifetime. Laibson points out hyperbolic discounting has been used in the psychological literature in order to explain human behavior. His concluding remarks states using hyperbolic

¹⁷ Strotz, "Myopia and inconsistency in dynamic utility maximization."

¹⁸ O'Donoghue, T, Rabin, M, "Doing it now or later."

¹⁹ Laibson, D, «Life-cycle consumption and hyperbolic discount function."

discounting, especially quasi-hyperbolic discounting offers an approach answering a comprehensive range of questions regarding consumer behavior.²⁰

In the model framework we will use exponential discounting when assuming the countries have time-consistent preferences and use quasi-hyperbolic discounting when assuming the countries have present-biased preferences.

2.4 Game structure

The model framework of the climate game is based upon the theory and structure behind the public good game. In order to give a further insight behind the foundations of the presented model framework this sub-chapter thus seeks to give a theoretical explanation of the public good game. Including a discussion on the different behavioral aspects of the game based upon laboratory experiments and theoretical regression analyzes published in the literature.

2.4.1 The public good game

The public good game discusses the dilemma of having free riders when the optimal outcome is achieving full cooperation between the participating agents. Let us assume there are a number of n participating agents, $i = \{1, \dots, n\}$, who are endowed with an income denoted, y , where each player i benefits of a given public good. Each player is assumed to be selfish and therefore only care about their own monetary payoff. Furthermore, each player will decide simultaneously on their contribution to maintain the public good. We therefore have contribution to the public good performed by player i is denoted as $g_i \in [0, y]$. The monetary payoff of each player is given by:

$$x_i(g_1, \dots, g_n) = y - g_i + \alpha \sum_{j=1}^n g_j$$

Where alpha is a parameter with value; $1/n < \alpha < 1$, denoting the constant marginal return to the public good. The dominant strategy of a completely selfish player is to choose $g_i = 0$, i.e. act as a free rider, because investing in the public good leads to a monetary loss of $1 - \alpha$.

²⁰ Laibson, D, «Life-cycle consumption and hyperbolic discount function.»

On the other hand, the aggregate monetary payoff is maximized if each player in the game chooses to fully invest in the public good, $g_i = y$.²¹ In order for the maximized payoff to be realized every player simultaneously has to maximize their effort to contribute to the public good by offering their entire endowment. This scenario seems unlikely without knowing every other player's preferences and having inequity aversion. If we assume each player prefer his own payoff only, then we will have the outcome of no cooperation.

Laboratory experiments performed by Chaudhuri (2010) studying the willingness to cooperate by contributing to the public good shows a high level of heterogeneity between the strategies of the players.²² Where typically the representative players will either act as a complete free rider or have the tendency of contributing a high share of their endowment. Furthermore, the willingness to contribute to the public good typically declines over the timespan of the game, given we have repeating rounds. If on the other hand a sanctioning system is introduced, the players incentive to repeatedly contribute to the public good increase in order to prevent being charged with a fee for acting as a free rider. Instead of introducing a negative consequence for not contributing, the opportunity of communicating with the other players before the game can have a positive effect on cooperation as well.²³

Fehr and Schmidt (1999) published a paper based upon a theoretical regression analysis of the agent's different strategies under various scenarios affecting the incentives of cooperation under the public good game. They as well found the dominant strategy of player i is to not invest in the public good at all, assuming there are no consequences of not cooperating and the agent is completely selfish in his preferences. If there is a punishment for not contributing to the public good, then the authors found a significant result were up to 80% of the participating agents prefer to fully cooperate in order to prevent being punished.²⁴

²¹ Fehr, Schmidt, "A theory of fairness, competition and cooperation."

²² Chaudhuri, "Sustaining cooperation in laboratory public good game experiments."

²³ Chaudhuri, "Sustaining cooperation in laboratory public good game experiments."

²⁴ Fehr, Schmidt, "A theory of fairness, competition and cooperation."

Where the contributors charged a higher fee the lower the investment in the public good performed by the players were, in each time period of the game. Consequentially, there are no gains from acting as a free rider. Under such a scheme the authors obtained a sub-game perfect equilibrium where deviating from the agreement of cooperation is not beneficial, hence not a dominant strategy. Furthermore, one of their propositions presents an equilibrium based on a sub-set of conjectures where full cooperation is substantial throughout the game. Based on the assumption of having a group of players who enforce an agreement to collectively cooperate and invest in the public good.²⁵

2.5 Theory behind self-enforcing IEA's

The end result of the analysis presents a theoretical result of a self-enforcing agreement. In order to give an insight on the theoretical groundwork of a self-enforcing agreement this sub-chapter will discuss relevant published papers in the literature discussing the matter.

2.5.1 Self-enforcing IEA's

Barrett (1994) published an influential paper analyzing how under his constructed climate game the participating players can achieve a self-enforcing international environmental agreement.²⁶ The paper discusses how a self-enforcing agreement can be achieved by a given number of committed cooperating countries under a set of stated assumptions. Where the challenge threatening the welfare improving outcome is the fact that every country would earn a higher return if acting as a free rider given the remaining countries act committed to the agreement. Barret constructs a model with linear marginal abatement costs and benefits. Where the first best equilibrium, i.e. full cooperation between all countries can be obtained when each country's marginal cost of abatement is equal to the global benefit of abatement. Given the assumption of each country taking into account the emission level performed by other countries as well as their own. Each country would benefit under full cooperation, at the same time no country has the incentive to cooperate unilaterally. When discussing the circumstances of a self-enforced agreement Barret argues the agreement will not be able to

²⁵ Fehr, Schmidt, "A theory of fairness, competition and cooperation."

²⁶ Barrett, "Self-enforcing International Environmental Agreements."

achieve a sustainable abatement level when the number of cooperating countries is large. Furthermore, based upon his performed regression analyzes he proposes a self-enforcing agreement given its design might be substantial when the number of signatories are between two and three countries. Consequentially, an agreement consisting of up to three countries will not be a great enough force of cooperators given the number of remaining countries acting as free riders is high in order to achieve an efficient reduction in the total emission levels.²⁷

Harstad (2012) discusses self-enforcing IEA's by analyzing a dynamic game consisting of participating countries who can cooperate by investing in abatement technology. Where an increased investment level in abatement technology leads to a reduction in the country's total emissions level. Additionally, the design of the agreement is assumed to be a legally binding, preventing the signatory countries to be free from deviating without some sort of consequence. Furthermore, the framework assumes the pollution stock and the technology stock accumulate over time. Given the assumption of each country having the ability to commit to future actions regarding their level of investment and thus emission levels, the first best result is feasible. Harstad further argues that in reality the ability for countries to pre-commit to future actions in the long run is not always realistic.

Furthermore, based upon the performed analysis committing to a short-term agreement can actually make the participating countries worse off than if acting as business as usual. The author argues this is because under a short-term agreement the participating countries actually ends up investing less in abatement technology than under the business as usual outcome. The reason as to why is because there occurs a hold-up problem in investment, when the agreement is set for a short time-horizon, discouraging further investments. On the other hand, a long-term agreement can be substantial as well as beneficial if under such an agreement the investment level in abatement technology amongst the signatories increases and the technology depreciation rate of the technology stock is low.²⁸

²⁷ Barrett, "Self-enforcing International Environmental Agreements."

²⁸ Harstad, "Dynamic games and environmental agreements."

3. Framework

This chapter present the model framework of the climate game. The theory used in order to construct the presented climate game is based upon the theoretical aspects discussed in chapter 2. The assumptions stated in the model is the foundation for the later performed analysis.

In the model framework we assume no risk or uncertainty is included in the game. The players have symmetric information throughout the time span of the game. We also assume the players are homogenous in their time preferences, where each player is assumed to be patient when both using exponential- and quasi-hyperbolic discounting. Consequentially, when analyzing the effect of time-inconsistent preferences every country has the same level of present-biasedness, represented by the value of β in the quasi-hyperbolic discount function. Additionally, when the player's has present-biased preferences they are assumed to have the characteristics of a sophisticate. In the last sub-chapter of chapter 3, the mathematical groundworks simplifying the model framework is presented. Giving insight to the equations summarizing the mechanisms behind the model.

3.1 The climate game

Our *climate game* is a stochastic game with repeating rounds over a discrete time horizon, where the total time periods of the game is three, $T = 3$. The time span of the game is thus finite where one time period t consists of an investment stage and a pollution stage. There are number of N representative players i who we refer to as countries, where $i = 1, \dots, N$. We define the strategy of country i by its objective function, where a country's objective is to maximize their present utility given their present discounted value of their future utility. We will present the country's present utility and objective function in detail below.

The participating players will have the possibility to cooperate by investing in abatement technology. The performed investment level in abatement technology will reduce country i 's emission level which contributes to maintaining the public good of a sustainable climate. When the countries enter the game, each country has the opportunity to cooperate in both the first and second time period. More specifically, this entails that a country who decides to

not invest in the first time period, $t = 1$, has the opportunity to do so in the next time period, $t = 2$. In the last time period, $t = 3$, there will be no investment stage and therefore no opportunity to invest in the abatement technology. The utility function of country i in each time period t is given by the following expression:

$$u_{i,t} = B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t})$$

Let $B_i(y_{i,t})$ denote country i 's benefit function from their consumption of energy, where the variable $y_{i,t}$ denotes i 's energy production level in time period t . The variable $g_{i,t}$ denotes country i 's emission level in time period t , where the climate cost function from i 's emission level is given by $C(g_{i,t})$. Furthermore, let $r_{i,t}$ denote country i 's investment level in abatement technology in time period t , where the cost function of investment is given by $K(r_{i,t})$.

We have that the associated benefit function of producing energy for country i in time period t is given by:

$$B_i(y_{i,t}) = -\frac{b}{2}(\bar{y}_{i,t} - y_{i,t})^2$$

The benefit function is concave and increasing in $y_{i,t}$ up to its *bliss* point given by the variable denoted $\bar{y}_{i,t}$, where the parameter, $b > 0$ measures the importance of energy production for country i . The *bliss* point represents the ideal energy level if there were no concern regarding emission levels. Each country i can vary between their own individual *bliss* point.²⁹

The associated cost function of investment in time period t for country i is given by:

$$K(r_{i,t}) = \frac{k(r_{i,t})^2}{2}$$

The parameter k denotes country i 's marginal cost of investing in abatement technology, $r_{i,t}$, in time period t .

²⁹ Harstad, B, "Dynamic Games and Environmental agreements."

Pollution leads to a negative externality affecting every country and is therefore referred to as a public bad. We let the parameter g_t denote the total emission rate in time period t , generated from the total energy production from fossil fuel. The total climate cost of emissions is given by the linear cost function:

$$C(g_t) = d \sum_{i=1}^n g_{i,t}$$

Where the parameter $d > 0$ measures the marginal cost of emissions. We will denote $g_{i,t}$ as the emission level performed by country i in period t . The linear cost function of country i 's emission level is thus given by the following:

$$c(g_{i,t}) = dg_{i,t}$$

Country i obtains benefit from consuming energy from fossil fuel production, at the same time as the units of energy produced from fossil fuel contributes to the public bad. The countries have on the other hand the opportunity to produce renewable energy from their abatement technology. Renewable energy is a substitute for the energy production from fossil fuel use where an increase in renewable energy use will reduce country i 's emission level. The level of renewable energy produced by country i is determined by their stock of abatement technology. Let the variable $R_{i,t}$ measure the abatement technology stock country i can use to produce renewable energy in period t . In total we have that the amount of energy country i can produce in time period t is given by the equation:

$$y_{i,t} = g_{i,t} + R_{i,t}$$

Let's make the assumption that the abatement technology stock, $R_{i,t}$, measures the quantity country i can reduce of its potential emissions in time period t , where the cost of doing so is zero. We can therefore reformulate the energy production function into an expression for the actual emission level of country i :

$$g_{i,t} = y_{i,t} - R_{i,t}$$

The technology stock, $R_{i,t}$ evolves in a natural manner. Where the technology's depreciating rate is expected to decline at the rate of $1 - q_R \in [0,1]$ in each time period, t . Furthermore, as mentioned above the variable, $r_{i,t}$, measures country i 's investment in abatement

technology in time period, t . In total we can describe the abatement technology stock of country i in time period t by the following equation:

$$R_{i,t} = q_R R_{i,t-1} + r_{i,t}.$$

The technology stock in the present time period is thus given by the remaining technology stock from the previous time period, $R_{i,t-1}$, where the state of the technology stock is given by the depreciation rate, q_R . At last the present level of technology stock will also be determined by the investment level in abatement technology in the present time period, $r_{i,t}$.

Both the investment stage and the pollution stage alternate over time. One time period t is defined by consists of an investment stage and a pollution stage. The information regarding each country's pollution and investment level is symmetric between all representative countries at all stages.

We will now present country i 's objective function when using exponential discounting in discrete time by the following expression:

$$U_{i,t} = u_{i,t} + \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau} \quad ^{30}$$

Where $u_{i,t}$ is country i 's utility in time period t and the utility function of the next time period given the present time period t is denoted $u_{i,\tau}$, where $\tau = t + 1$ and the total time span is given by $T = 3$.

The objective function can also be referred to as the continuation value of country i , as we can see from the expression the country's objective is to maximize its present utility. Furthermore, the countries present utility preference is determined by the present discounted value of the summarized utility over the remaining time span of the game. Under exponential discounting country i 's preferences are time consistent where the discount factor is given by $0 < \delta \leq 1$. The continuation value is country i 's strategy profile in time period t and will determine the country's decision-making regarding their performed energy

³⁰ Samuelson, "A note on Measurement of utility."

production level and investment level. We assume the countries are patient in their time preferences and homogenous with the same discount factor throughout the time horizon of the game.

When using quasi-hyperbolic discounting we have a specific form of present biasedness representing the countries' time-inconsistent preferences. Under this scenario, country i 's objective function will be given by the following expression:

$$U_{i,t} = u_{i,t} + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau}$$

Where $u_{i,t}$ is country i 's utility in time period t , the utility function of the next time period given the present time period t is denoted $u_{i,\tau}$, where $\tau = t + 1$ and the total time span is given by $T = 3$.

We have that β , ($0 < \beta < 1$), is the discount parameter representing the country's present-biased preferences and the discount factor $0 < \delta \leq 1$ represent the country's long-term preference.³¹ The value of the parameter, β , is beneficial for our analysis because the discount structure of the utility function under quasi-hyperbolic discounting still resembles the qualitative property and remain the analytical tractability of the exponential discounting approach. We assume the countries are homogenous and thus have the same value of β in each time period of the game. Meaning the countries has the same level of present biasedness, furthermore we assume the countries are patient and sophisticated. The quasi-hyperbolic discount function is used under a discrete-time horizon and therefore takes on the following structure over the timespan of the game:³²

$$\{1, \beta\delta^1, \beta\delta^2\}$$

³¹ Elster, "Ulysses and Sirens".

³² Laibson, D, «Life-cycle consumption and hyperbolic discount function.»

The strategy of the player given by their continuation value in time period t , will determine their performed energy production and investment level. Their performed decision regarding the latter determines the total emission level.

The analysis will derive the result of the first best allocation (FB), the business as usual result (BAU) and a self-enforcing agreement. The results will be based on the country's strategy profile, i.e. their continuation value in each time period of the game. I will first present the results obtained from deriving the continuation value using exponential discounting. Where given the assumptions stated in the exponential discount function the countries have time-consistent preferences. Secondly, I will present the results obtained from deriving the continuation value using quasi-hyperbolic discounting. Where given the assumptions stated in the quasi-hyperbolic discount function the countries has present-biased preferences. The thought of the approach is being able to analyze the impact of including the assumption of time-inconsistent preferences. More specifically, the analysis will focus on how the investment levels differ when the countries have time-consistent preferences and when they have present-biased preferences. Investigating if including the assumption of present biasedness affects the performed investment level which will consequentially affect the emission level and thus the welfare of the countries. Furthermore, it is interesting to investigate if the effect of including present-biased preferences has a different impact on welfare under the FB, BAU and self-enforcing agreement.

3.2 Groundworks for the result

Before deriving the results of the analysis, it is beneficial to specify the notation from the presented model framework. This sub-chapter will present the specific equations used when deriving the result of the first best allocation, business as usual result and the self-enforcing agreement in some more detail.

3.2.1 Specified notation

As we know, the total energy production of country i in each time period t , is given by the following equation:

$$\textit{Total energy prodcutio}n: y_{i,t} = g_{i,t} + R_{i,t}$$

Furthermore, we presented the energy production bliss point of country i , denoted as $\bar{y}_{i,t}$. In order to for the country's to have the same standpoint regarding energy production level in each time period of the game we will reformulate the expression for energy production.

Where we have that the bliss point of country i remains $\bar{y}_{i,t}$, but the total bliss point of every country is denoted as \bar{y}_t . In total we therefore have the following expression for the energy production level of country i in time period t , denoted as $\tilde{y}_{i,t}$:

$$\text{Energy production: } \tilde{y}_{i,t} = y_{i,t} + (\bar{y}_t - \bar{y}_{i,t}) \rightarrow \text{Where } \bar{y}_t \equiv \sum_{i \in N} \bar{y}_i / n$$

The total cost of fossil fuel emission in time period t is given by the following expression:

$$\text{Total cost of emissions: } C(g_t) = d \sum_{i \in N}^n g_{i,t}, \text{ where } g_{i,t} = y_{i,t} - R_{i,t}$$

Where the fossil fuel emission of country i in time period t , $g_{i,t}$, is given by country i 's energy production level, $y_{i,t}$ subtracted from their present abatement technology stock, $R_{i,t}$.

We let $R_{i,t}$ denote country i 's technology stock in the present time period, and the total abatement technology stock amongst all the participating countries in the present time period is denoted R_t . The present technology stock is given by the technology stock from the previous time period where the depreciation rate determines the present quality of the stock. Furthermore, the present investment level in abatement technology, $r_{i,t}$, depicts the present and future level of the technology stock. To summarize, we have that the total technology stock is given by the following:

$$\text{Technology stock: } R_t \equiv \sum_{i \in N} R_{i,t} = q_R R_{t-1} + \sum_{i \in N} r_{i,t}$$

The investment level of each country i in the present time period t is thus given by the following reformulation of the technology stock equation:

$$\text{Investment level of player } i: r_{i,t} = R_{i,t} - q_R R_{t-1} + \sum_{j \in N/i} r_{j,t}$$

The cost function of investment in abatement technology in the present time period is given by the following quadratic function. Let the parameter k denote the marginal cost of investment.

$$\text{Investments cost for abatement technology: } K(r_{i,t}) = \frac{k(r_{i,t})^2}{2}$$

The benefit country i receive from consuming the given energy production level is given by the following quadratic function. Let the parameter b denote the marginal benefit of consuming energy, where \bar{y}_t is the total bliss point of energy production for every country and let $\tilde{y}_{i,t}$ represent the homogenous decision-making regarding energy production level:

Benefit from consumption of energy is given by:

$$B_i(y_{i,t}) = -\frac{b}{2}(\bar{y}_t - \tilde{y}_{i,t})^2$$

$B(\cdot)$: increasing and concave in $y_{i,t}$

With abatement technology we have that the benefit of energy production is strictly positive, $B_i(y_{i,t}) < 0$. Furthermore, we have that the cost of investing in abatement technology yields zero.

In total we can now reformulate the utility function into a more specific expression. As we know the utility function of country i in the present time period is given by the following expression:

$$u_{i,t} = B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t})$$

If we now insert the stated functions for the benefit of energy production, cost of emission level and cost function of investment in abatement technology we can reformulate the utility function into the following expression:

$$u_{i,t} = -\frac{b}{2}(\bar{y}_t - \tilde{y}_{i,t})^2 - dg_{i,t} - \frac{k(r_{i,t})^2}{2}$$

We are thus able to eliminate each country's individual bliss point and abatement technology stock when stating i 's utility function. The benefit of this is to obtain countries who are

symmetric in their decision making regarding total energy production, represented by the expression for $\tilde{y}_{i,t}$. Furthermore, each country will thus be symmetric in their decision regarding investment in abatement technology in each time period t . Meaning that each country have the same standpoint independently from their heterogeneity in their bliss point of energy production and initial technology stock. We make the assumption that the country's strategies are not contingent on the possible technology differences. Therefore, we can state that a country's objective is only dependent on their investment in abatement technology. We refer to the continuation value as the maximized value of a country's utility function in the present time period of the game, given the sum of the present discounted value of the country's future utility. This will be the country's strategy in time period of the game.

This simplifies the analysis and leads us to obtaining a stationary strategy in each time period of the game based on the country's present continuation value.³³ The stationary strategies of the country i will differ when deriving the result of the FB, BAU and self-enforcing agreement. As mentioned, when presenting the stochastic game theory, a player's obtained stationary strategy in a stochastic game with repeating rounds is optimal given the initial state of the game. When deriving the first best allocation, the business as usual result and the self-enforcing agreement the initial state will differ, this is further specified when presenting each result individually in chapter 4. In concluding remarks, the representative country's optimal set of strategies throughout the game will thus differentiate when analyzing the three different scenarios of the game. But in each of the given initial states of the game country i will have the same set of strategy profile throughout the timespan of the game.

³³ Harstad, B, "Dynamic Games and Environmental agreements."

4. Results

The full mathematical approach used in order to obtain the results of the analysis are presented in the appendix. The appendix includes mathematical insight behind the presented result of the energy production level, the investment level and the emission level for the three analyzed outcomes of the game. In the results we first present the first best allocation from both using exponential discounting and quasi-hyperbolic discounting. Secondly, we present the business as usual result obtained from first using exponential discounting followed by using quasi-hyperbolic discounting. At last we present the self-enforcing agreement when using exponential discounting compared to quasi-hyperbolic discounting. The presented order of the results is given in the same chronologically order in the appendix.

4.1 First Best allocation

In order to obtain the first best allocation each participating country of the game has to cooperate and collectively fully invest in the abatement technology throughout the timespan of the game. Therefore, in order to achieve the first best allocation, we assume all countries are willing to enter the game as cooperators and invest in the abatement technology at both investment stages of the game. Where the investment level performed is referred to as the “first best” investment level, i.e. the optimal investment level. One main take away from the first best allocation is that every participating country takes into account the total environmental cost of emission when maximizing their utility, as opposed to a country’s business as usual objective where each country only takes into account their own emission cost.

4.1.1 First Best result – Exponential Discounting

The continuation value under the first best result is given by the summarized utility of all countries in the present time period as well as the summarized value of the discounted future utility of all countries over the given time span of the game. In total we have the following expression for the continuation value when using exponential discounting:

$$U_t^{FB} = \sum_{i=1}^n u_{i,t} + \sum_{\tau=t+1}^3 \delta^{\tau-t} \sum_{i=1}^n u_{i,\tau}$$

The stated continuation value is the collective strategy of the countries under the first best allocation. Where the exponential discount factor is given by $0 < \delta \leq 1$. Where the utility function of the representative countries i is given by the following:

$$\text{Utility function: } u_{i,t} = -\frac{b}{2}(\bar{y}_t - \tilde{y}_{i,t})^2 - C(g_t) - K(r_{i,t})$$

$$\text{s. t. } C(g_t) = d \sum_{i \in N}^n g_{i,t}, \text{ where } g_{i,t} = y_{i,t} - R_{i,t}, K(r_{i,t}) = \frac{k(r_{i,t})^2}{2}$$

By deriving the first order conditions of the given continuation value with respect to the energy production level and the investment level we at last obtain the first best investment and emission level. The details behind the presented results is given in the appendix, chapter 7, sub-chapter first best allocation – exponential discounting.

The first best investment level is given by following expression:

$$r_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - q_R R_{t-1} - \left[\frac{b}{nk(1 + \delta) + b} \right]$$

The FB investment level standpoint in time period t , is given by country i 's energy production bliss point. Furthermore, the technology stock remaining from the previous time period is negatively correlated with the optimal present investment level. This is because if the remaining technology stock is large, then the necessary present investment level is relatively smaller in order to remain the optimal technology stock over the timespan of the game. At last, the FB investment level is determined by the discounted value of the relative payoff from investment. Where the cost of investment has a negative effect on the investment level, while the benefit of energy production can have both a positive and negative effect. If the benefit of energy production increases this can have a distortion effect leading to energy production from fossil fuel usage becoming more attractive. At the same time as it makes energy production from abatement technology attractive as well. Therefore, if the marginal benefit of energy production increases then this can lead to the optimal emission level to both increase and decrease, the total effect depends on the relative increase in fossil fuel or abatement technology usage.

The performed investment level in abatement technology in time period t determines the level of emission in time period t as well. Where the FB emission level is positively correlated with the total emission from energy production from fossil fuel use. The abatement technology stock on the other hand will lead the emission level to decrease. The first best emission level, $g_{i,t}^{FB}$, is thus given by the obtained technology stock in each time period of the game; $R_{i,t} = \{R_{i,1}, R_{i,2}, R_{i,3}\}$

$$g_{i,t}^{FB}(R_{i,t}) = [\bar{y}_{i,1} - (R_{i,1})] + \delta[\bar{y}_{i,2} - (R_{i,2})] + \delta[\bar{y}_{i,3} - (R_{i,3})] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right]$$

As we can see from the expression of the FB emission level, country i 's optimal emission level in the present time period is given by i 's energy production bliss point where the obtained technology stock will decrease the performed emission level. One also has to take into account the present discounted value of both stated variables; the bliss point $\bar{y}_{i,t}$ and the technology stock $R_{i,t}$. Furthermore, the discounted relative climate cost of emission is negatively correlated with the FB emission level. If the obtained technology stock is large due to a high investment level, then the emission level will be low. The present discounted value of the ratio between marginal cost of emissions and the marginal benefit of energy production will also have an indirect effect on the emission level. If the marginal cost of emissions increase, then the FB emission level goes down. If on the other hand the benefit of energy production goes up, then this can lead to the emission level going up as well. If an increase in marginal benefit of energy production leads to a higher level of fossil fuel energy production.

4.1.4 First best result - Quasi-hyperbolic discounting

The continuation value for all countries under the first best result when using quasi-hyperbolic discounting is given by the utility in the present time period as well as the summarized discounted value of all country's future utility. We denote the continuation value when using quasi-hyperbolic discounting as \hat{U} . In total the continuation value under the first best allocation is given by the following:

$$\hat{U}_t^{FB} = \sum_{i=1}^n u_{i,t} + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} \sum_{i=1}^n u_{i,\tau}$$

The stated continuation value is the countries collectively strategy under the first best allocation. Where the utility function of each country i in time period t is given by the following:

$$u_{i,t} = B_i(y_{i,t}) - C(g_t) - K(r_{i,t})$$

$$\text{s. t. } C(g_t) = d \sum_{i \in N} g_{i,t}, \text{ where } g_{i,t} = y_{i,t} - R_{i,t}, K(r_{i,t}) = \frac{k(r_{i,t})^2}{2}$$

As we know from chapter 2, the present-biased preference of the representative countries is represented by the parameter $\beta < 1$. Furthermore, the quasi-hyperbolic discount value in discrete time takes on the following interval: $\{1, \beta\delta^1, \beta\delta^2\}$.³⁴ Meaning the discount factor in the present time period t is given 1, i.e. we do not discount the present time period. The next time period, $t+1$, is then discounted by $\beta\delta^1$ and the time period $t+2$ is discounted by $\beta\delta^2$. We will further investigate the effect of including the assumption of present-biasedness when it comes to the performed investment and emission level in the next sub-chapter of the first best result.

The approach in order to obtain the first best result is symmetrically for both discounting approaches. We therefore have the following result for the first best investment level in each time period t , given the quasi-hyperbolic discount function:

$$\hat{r}_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - q_R R_{t-1} - \frac{b}{nk(1 + \beta\delta) + b}$$

The first best result for optimal investment given the emission level is dependent on the bliss point of energy production and the present value of the technology stock from the previous time period. Furthermore, the investment level is determined by the discounted value of the relative benefit and cost of investment. As we can see from the expression the FB investment level in the present time period will decrease if the technology stock from the previous time period is relatively high. This can be the case if the depreciation rate of the technology stock is low or the investment level in the previous time period was high. Both scenarios lead to less

³⁴ Laibson, D, "Life-cycle consumption and hyperbolic discount function."

pressure on the present investment level in order to maintain the optimal emission level. Furthermore, the relative cost and benefit from investment will have some distortion effects. If the marginal cost of investment increases, then the investment level goes down. Additionally, if the marginal benefit of energy production increases this can both lead to a higher investment level in abatement technology or it can lead to fossil fuel energy production becoming more attractive. The total effect of an increased marginal benefit of energy production depends on which of the two direct effects is relatively higher than the other.

The first best emission level in time period t is given by the present emission level from fossil fuel energy production and the present discounted value of the next emission stages of the game. Furthermore, the performed FB emission level is determined by the relative marginal cost and benefit of emissions. The marginal cost of emission is negatively correlated with the emission level, if parameter d increases then the emission level goes down. The first best emission level is given by the obtained technology stock in each time period of the game; $R_{i,t} = \{R_{i,1}, R_{i,2}, R_{i,3}\}$

$$\hat{g}_{i,t}^{FB}(R_{i,t}) = [\bar{y}_{i,1} - (R_{i,1})] + \delta[\bar{y}_{i,2} - (R_{i,2})] + \delta[\bar{y}_{i,3} - (R_{i,3})] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right]$$

The technology stock in abatement technology will determine the first best emission level, where the higher level of technology stock the lower the emission level. We have the same dynamics behind the first best emission level result when using quasi-hyperbolic discounting as presented when using exponential discounting. The only parameter differentiating the two results is the discount factor. We will therefore discuss the impact of including present-biased preferences in the next sub-chapter.

4.1.5 First Best Allocation – Exponential VS Quasi-hyperbolic Discounting

One can discuss the benefit of taking into account time inconsistent preferences since the motivation behind the first best result is to obtain the *optimal* allocation of resources. On the other hand, for the sake of tractability in our analysis we will analyze all results using time consistent as well as time inconsistent preferences.

The interesting difference between the results when using both the exponential and quasi-hyperbolic discount function is the difference between the weight on benefit of energy consumption and the importance of minimizing the climate damage from fossil fuel production. Taking time-inconsistent preferences into account changes the first best result compared to the result obtained by assuming for time-consistent preferences. The result obtained from using exponential discounting the preference regarding investment level is weighted the same throughout the game. Whereas the first best result obtained when using quasi-hyperbolic discounting the social planner weighs energy consumption in the present time period greater than the cost of emission level in the next time period. Leading to procrastination of investment level in abatement technology.

One important question still remains; will the total first best investment levels under exponential discounting differ compared to the result obtained under quasi-hyperbolic discounting? We will assume that all other variables are the same under both approaches except from the discount factors. Furthermore, we will assume the value of the long-term discount parameters is greater than the present-biased parameter; $\delta > \beta$ ³⁵ where δ approaches 1. The first best investment level obtained by exponential and quasi-hyperbolic discounting is given respectively:

$$r_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - \frac{q_R}{n} R_{t-1} - \left[\frac{b}{nk(1 + \delta) + b} \right]$$

$$\hat{r}_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - \frac{q_R}{n} R_{t-1} - \left[\frac{b}{nk(1 + \beta\delta) + b} \right]$$

Because the quasi-hyperbolic discount factor is lower compared to the exponential discount factor for all time periods of the game, the performed investment level in each time period will be relatively lower as a consequence when the countries have present-biased preferences. Thus, we have that the optimal investment level when the representative countries have time consistent preferences is higher compared to when having present-biased preferences. In total we have that the first best investment level under exponential

³⁵ Laibson, D, "Life-cycle consumption and hyperbolic discount function."

discounting is greater than the first best investment level obtained under quasi-hyperbolic discounting:

$$r_{i,t}^{FB}(g_t) > \hat{r}_{i,t}^{FB}(g_t)$$

Where $r_{i,t}^{FB}(g_t)$ is the first best emission level of country i in time period t given the emission stock when using exponential discounting and $\hat{r}_{i,t}^{FB}(g_t)$ is the result obtained when using quasi-hyperbolic discounting.

As we have stated in the first best results the total emission levels is determined by the investment level. The results show that the total emission level under quasi-hyperbolic discounting is higher compared the total emission level when using exponential discounting. This is a direct effect of the inequality between the performed investment level when countries have time-consistent preferences compared to present-biased preferences. We therefore have the following inequality regarding the FB emission level given the abatement technology stock when comparing present-biased preferences to time consistent preferences.

$$\hat{g}_{i,t}^{FB}(R_{i,t}) > g_{i,t}^{FB}(R_{i,t})$$

Where $\hat{g}_{i,t}^{FB}(R_{i,t})$ is the FB emission level when the countries have present biased preferences and $g_{i,t}^{FB}(R_{i,t})$ is the emission level when the countries have time consistent preferences.

Consequently, we can see that the FB emission levels is higher when the countries have intertemporal preferences due to their first best investment level being lower when having present-biased preferences. Because the first best investment level is higher under exponential discounting we have as a direct effect that the total emission level will be lower when not taking into account present-biased preferences when deriving the first best allocation of resources. The first best result can thus be described as more socially desirable when the countries does not have time-inconsistent preferences.

4.2 Business as usual result

Under the business as usual equilibrium the participating countries does not take into account the environmental cost caused by other countries fossil fuel production when deciding on their optimal energy consumption. Therefore, under such a scenario each country i decision on energy production is based solely on their own emission cost. In total, marginal benefit of energy consumption equals country i 's marginal environmental cost from fossil fuel production. Thus, we have the following equilibrium under the BAU result:

$$B'_i(y_{i,t}) = C'(g_{i,t})$$

One time period t consists firstly of an investment stage followed by an emission stage. We have stated that the continuation value $U_{i,t}(g_{i,t}, R_{i,t})$ is the value of a sub-game starting at the investment stage. Furthermore, let us define country i 's continuation value before the emission stage by $V_{i,t}(g_{i,t}, R_{i,t})$. We can refer $V_{i,t}$ as country i 's temporary continuation value in each time period t between the two stages. Each country i takes the other countries j as given in their decision making on technology investment, $r_{i,t}$, where $j \neq i$. Therefore, we have that deciding on $r_{i,t}$ is equivalent to deciding on $R_{i,t}$.³⁶ We can express R_t by the following equation:

$$R_t = \sum_{i \in N} R_{i,t} = q_R R_{t-1} + \sum_{j \in N/i} r_{j,t} + r_{i,t}$$

Furthermore, we can reformulate the technology stock equation and express $r_{i,t}$ by:

$$r_{i,t} = R_{i,t} - q_R R_{i,t-1}$$

We thus have country i 's maximization problem based upon their temporary continuation value, i.e. country i 's strategy profile is given by:

$$\begin{aligned} & \text{Max}_{r_{i,t}} V_{i,t}(g_{i,t}, R_{i,t}) - K(r_{i,t}) \\ & = B_i(y_{i,t}) - d[y_{i,t} - (q_R R_{i,t-1} + r_{i,t})] - \frac{k(r_{i,t})^2}{2} \end{aligned}$$

³⁶ Harstad, B, "Dynamic Games and Environmental Agreements."

$$\frac{\partial V_{i,t}}{\partial R_{i,t-1}} = dq_r + kq_r = 0 \rightarrow q_r(d + k)$$

We see from the F.O.C that $R_{i,t}$ is independent from the previous time period technology stock $R_{i,t-1}$. The interpretation from this result is that the level of $R_{i,t-1}$ is payoff irrelevant and that the equilibrium level of $R_{i,t}$ is independent from the initial technology stock. This means that if all countries invest the same amount in the abatement technology even though a country has a marginally larger $R_{i,t-1}$ does not determine the level of $R_{i,t}$. What does have a direct effect on the investment level of country i performed in time period t is the depreciation rate of the technology stock. We thus have that $r_{i,t}$ will decline by q_r , where $1 - q_r \in [0,1]$. Furthermore, given the temporal continuation value country i 's first order condition with respect to their total energy production in the given time period t , is given by:

$$\begin{aligned} & \text{Max}_{y_{i,t}} V_{i,t}(g_{i,t}, R_{i,t}) - K(r_{i,t}) \\ & = -\frac{b}{2}(\bar{y}_t - \tilde{y}_{i,t})^2 - d[y_{i,t} - (q_r R_{i,t-1} + r_{i,t})] - \frac{k(r_{i,t})^2}{2} \\ & \frac{\partial V_{i,t}}{\partial y_{i,t}} = -b(\bar{y}_t - \tilde{y}_{i,t}) - d = 0 \rightarrow -b(\bar{y}_t - \tilde{y}_{i,t}) = d \\ & \quad -b\bar{y}_t + b\tilde{y}_{i,t} = d \\ & b\tilde{y}_{i,t} = d + b\bar{y}_t \rightarrow \tilde{y}_{i,t} = \frac{d}{b} + \bar{y}_t \end{aligned}$$

We see from the first order condition the representative countries are homogenous in their standpoint regarding energy production. We have that each country's total energy production level is decided based upon the ratio of the marginal cost of emissions from fossil fuel production and the marginal benefit of energy production given the total energy production bliss point, denoted as \bar{y}_t .

4.2.1 Business as usual result – Exponential Discounting

The continuation value of the representative country when using exponential discounting is given by the following expression:

$$U_{i,t} = u_{i,t} + \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau}$$

Where the discount factor is given by $\delta \in [0,1]$ ³⁷

The objective function of country i in time period t is given by:

$$u_{i,t} = B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t})$$

The backward induction approach is sufficient in order to determine each participant optimal strategy under the BAU equilibrium. Thus, we start discounting country i 's objective function at the last time period moving forward towards the first time period of the game. The incentive behind the approach is to at last obtain each country's individual continuation value determining their decision regarding investment and emission level in each stage of the game.

Time period 3

At time period three there is no investment stage, implicitly leading to $r_{i,3}$ being equal to zero. We therefore have the following objective function for the representative countries:

$$U_{i,3}^{BAU} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - C(g_{i,t})$$

$$s. t. g_{i,3} = y_{i,3} + R_{i,3}, R_{i,3} = q_R R_{i,2}$$

Based upon the derived first order conditions of the maximization problem we obtain the following result for the technology stock and emission level at the last stage of the game:

$$\text{Technology stock: } R_{i,3}^{BAU} = q_R R_{i,2}^{BAU} \rightarrow R_{i,3}^{BAU} = dq_R^2$$

$$\text{Emission level: } g_{i,3}^{BAU}(R_{i,3}^{BAU}) = \bar{y}_{i,3} - [q_R R_{i,2}^{BAU}] - \frac{d}{b}$$

Time period 2

In the second time period of the game there is an investment stage, leading to the assumption of $r_{i,t} > 0$. We now have to take into account country i 's private cost of investments, k , and insert the cost function for investment in abatement technology, $K(r_{i,t})$. We thus have the following objective function for the representative countries:

³⁷ O'Donoghue, T, Rabin, M, "Doing it now or later."

$$U_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - C(g_{i,2}) - \frac{k(r_{i,2})^2}{2} + \delta \bar{U}_{i,3}^{BAU}$$

$$s. t. g_{i,2} = y_{i,2} + R_{i,2} \quad R_{i,2} = q_R R_{i,1} + r_{i,2}$$

Based upon the derived first order conditions of the maximization problem including the investment stage we obtain the following result for the technology stock, emission level and investment level in the second time period of the game given respectively:

$$\text{Technology stock: } R_{i,2}^{BAU} = q_R R_{i,1}^{BAU} \rightarrow R_{i,2}^{BAU} = q_R [dq_R + \delta dq_R^2]$$

$$\text{Emission level: } g_{i,2}^{BAU}(R_{i,2}^{BAU}) = \bar{y}_{i,2} - [q_R R_{i,1}^{BAU}] - \frac{d}{b}$$

$$\text{Investment level: } r_{i,2}^{BAU} = [\bar{y}_{i,2} - [q_R R_{i,t-1}^{BAU}] - g_{i,2}] - \frac{d}{b} + \frac{\delta dq_R}{bk}$$

Time period 1

In time period 1 the first investment stage where country i has the option to invest in abatement technology takes place. We thus have the following objective function for the representative countries:

$$U_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - C(g_{i,1}) - \frac{k(r_{i,1})^2}{2} + \delta \bar{U}_{i,2}^{BAU} + \delta \bar{U}_{i,3}^{BAU}$$

$$s. t. g_{i,1} = y_{i,1} + R_{i,1} \quad R_{i,1} = q_R R_0 + r_{i,1}$$

From the derived first order conditions from the presented maximization problem we obtain the following result for the technology stock, emission level and investment level given respectively for the first time period of the game:

$$\text{Technology stock: } R_{i,1}^{BAU} = q_R R_{i,0}^* \rightarrow R_{i,1}^{BAU} = q_R [dq_R + \delta dq_R^2 + \delta^2 dq_R^3 + \delta dq_R^3]$$

$$\text{Emission level: } g_{i,1}^{BAU}(R_{i,1}^{BAU}) = \bar{y}_{i,1} - [q_R R_{i,0}^{BAU}] - \frac{d}{b}$$

$$\text{Investment level: } r_{i,1}^{BAU} = [\bar{y}_{i,1} - [q_R R_{i,0}^{BAU}] - g_{i,1}] - \frac{d}{b} + \frac{\delta [dq_R + \delta dq_R^2] + \delta dq_R^2}{bk}$$

4.2.2 Business as usual result – Quasi-Hyperbolic Discounting

The countries continuation value and utility function in time period t , using quasi-hyperbolic discounting is given respectively:

$$U_{i,t} = \left[u_{i,t} + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau} \right]$$

$$u_{i,t} = B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t})$$

Where the present-biased parameter is given by $0 < \beta < 1$ and δ is the long-term discount factor. Last remark, we have that the quasi-hyperbolic discount function in discrete time can be described by; $\{1, \beta\delta^1, \beta\delta^2\}$.³⁸

Time period 3

In the last time period of the game there is no investment stage, we therefore have that $r_{i,3}$ yields zero. Thus, we have the following objective for the representative countries:

$$\hat{U}_{i,3}^{BAU} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - C(g_{i,t})$$

$$s. t. g_{i,3} = y_{i,3} + R_{i,3}, R_{i,3} = q_R R_{i,2}$$

Based upon the first order conditions from the presented maximization problem we obtain the following result for the technology stock and investment level in the last stage of the game given respectively:

$$\text{Technology stock: } \hat{R}_{i,3}^{BAU} = q_R R_{i,2}^{BAU} \rightarrow \hat{R}_{i,3}^{BAU} = dq_R^2$$

$$\text{Emission level: } \hat{g}_{i,3}^{BAU}(R_{i,3}^{BAU}) = \bar{y}_{i,3} - [q_R R_{i,2}^{BAU}] - \frac{d}{b}$$

Time period 2

In the second time period there is an investment stage, we therefore assume that $r_{i,2} > 0$. Therefore, we have to take into account country i 's private cost of investment, k , as well. We thus have the following objective function for the representative countries:

$$\hat{U}_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - C(g_{i,2}) - \frac{k(r_{i,2})^2}{2} + \beta\delta\hat{U}_{i,3}^{BAU}$$

³⁸ Laibson, D, «Life-cycle consumption and hyperbolic discount function.»

$$s. t. g_{i,2} = y_{i,2} + R_{i,2} \quad R_{i,2} = q_R R_{i,1} + r_{i,2}$$

Based upon the first order conditions derived from the presented maximization problem we obtain the following result for the technology stock, emission and investment level in the second time period of the game given respectively:

$$\text{Technology stock: } \hat{R}_{i,2}^{BAU} = q_R R_{i,1}^{BAU} \rightarrow \hat{R}_{i,2}^{BAU} = q_R [dq_R + \beta \delta dq_R^2]$$

$$\text{Emission level: } \hat{g}_{i,2}^{BAU}(\hat{R}_{i,2}^{BAU}) = \bar{y}_{i,2} - [q_R R_{i,1}^{BAU}] - \frac{d}{b}$$

$$\text{Investment level: } \hat{r}_{i,2}^{BAU} = \left[\bar{y}_{i,2} - [q_R R_{i,1}^{BAU}] - g_{i,2} \right] - \frac{d}{b} + \frac{\beta \delta dq_R}{bk}$$

Time period 1

In time period 1 the first investment stage takes place, implicitly meaning that this is the first time period where country i has the option to invest in the abatement technology in the time span of the game. We thus have the following objective function for the representative countries:

$$\bar{U}_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - C(g_{i,1}) - \frac{k(r_{i,1})^2}{2} + \beta \delta \bar{U}_{i,2}^{BAU} + \beta \delta \bar{U}_{i,3}^{BAU}$$

$$s. t. g_{i,1} = y_{i,1} + R_{i,1} \quad R_{i,1} = q_R R_0 + r_{i,1}$$

Based upon the first order conditions derived from the presented maximization problem we obtain the following result for the technology stock, emission and investment level in the first time period of the game given respectively:

$$\text{Technology stock: } \hat{R}_{i,1}^{BAU} = q_R R_{i,0}^* \rightarrow \hat{R}_{i,1}^{BAU} = q_R [dq_R + \beta \delta dq_R^2 + \beta \delta^2 dq_R^3 + \beta \delta dq_R^3]$$

$$\text{Emission level: } \hat{g}_{i,1}^{BAU}(\hat{R}_{i,1}^{BAU}) = \bar{y}_{i,1} - [q_R R_{i,0}^*] - \frac{d}{b}$$

$$\text{Investment level: } \hat{r}_{i,1}^{BAU} = \left[\bar{y}_{i,1} - [q_R R_{i,0}^{BAU}] - g_{i,1} \right] - \frac{d}{b} + \frac{\beta \delta [dq_R + \beta \delta dq_R^2] + \beta \delta dq_R^2}{bk}$$

4.2.3 Comparing the two Business as Usual Results

The interesting insight from the business as usual result is the difference between the investment and emission level when the representative countries have intertemporal preferences contra time consistent preferences. Let us assume that all other variables are the same except from the discount parameters. When comparing the business as usual investment level, the first order condition with respect to the exponential discount parameter compared to the quasi-hyperbolic discount parameter gives the following inequality:

$$\frac{\partial r_{i,t}^{BAU}}{\partial \delta} > \frac{\partial \hat{r}_{i,t}^{BAU}}{\partial \beta \delta}$$

The inequality is a direct effect from the fact that the exponential discount factor, δ , is strictly greater than the quasi-hyperbolic discount factor $\beta \delta$.

The first order conditions show when a country has intertemporal time preferences, the investment level is relatively lower than the investment level when having time consistent preferences. As a consequence, the emission level will be greater when a country has time inconsistent preferences. Thus, we have the following result from the first order conditions of the business as usual emission levels with respect to the two discount rates:

$$\frac{\partial g_{i,t}^{BAU}}{\partial \delta} < \frac{\partial \hat{g}_{i,t}^{BAU}}{\partial \beta \delta}$$

Where $g_{i,t}^{BAU}$ denoted the emission level when the country i has time consistent preferences and $\hat{g}_{i,t}^{BAU}$ denotes the country i 's emission level when having present-biased preferences.

Concluding remarks; there is an impact in the decision-making of the representative countries when comparing the two type of time preferences. The discount rate when using quasi-hyperbolic discounting is lower in each time period of the game compared to the discount factor when using exponential discounting. Leading to the representative country when having present-biased preferences to value their present benefit over their future benefit.

The representative countries will thus invest less in abatement technology when we include the assumption of time-inconsistent preferences compared to when assuming for time-inconsistent preferences. As a consequence, the business as usual emission level when the countries have present-biased preferences will be relatively higher. It is important to note that this inequality in emission levels is as a direct effect from the structure of the two discount functions. The long-term discount factor, δ , in both discount functions remains the same. While the present-biased preference represented by the discount parameter, $\beta < \delta$, leads to the discount factor under the quasi-hyperbolic approach being relative lower in each present time period compared to the exponential discount factor. Consequentially leading to country i weighing present benefit being higher than future payoff and preferring to procrastinate the cost of investment when having present-biased preferences.

4.2.4 Business-as-usual VS First Best Result

The overall investment level under the business as usual result is lower than the first best investment level. Therefore, we have the following consequence of the emission level under business as usual being greater than the first best emission level. The main reason behind this is because under the business as usual result, regardless of having intertemporal time preferences or not, the representative countries do not take into account other country's emission level when performing their own decision-making regarding energy production and investment level in abatement technology. We thus have the following inequality between the objective function under the first best result and the business as usual result:

$$d \sum_{i=1}^n g_{i,t} > dg_{i,t} \rightarrow \frac{d}{b} < \frac{\sum_{i=1}^n d}{b}$$

As we saw under both the FB and BAU result, the countries strategy is to maximize their continuation value in the present time period. Furthermore, country i 's continuation value under the FB condition takes into account the total emission cost. While country i 's continuation value under the BAU condition takes into account i 's private cost of emission. Consequentially, the investment level under the FB result, independently of i 's time preference is higher than the investment level under the BAU result. Furthermore, because i 's

investment level under the BAU result is relatively lower the business as usual emission level of country i will be greater than the FB emission level. This holds for all investment and emission stages throughout the time span of the game. Therefore, we have the following inequality between the BAU and FB investment levels:

$$r_{i,t}^{BAU} < r_{i,t}^{FB}$$

Consequentially, leading to the BAU emission level being greater than the FB emission level:

$$g_{i,t}^{BAU} > g_{i,t}^{FB}$$

We can thus conclude all countries i would be better off under the first best allocation, when there is more than one country participating in the game. This is due to the emission level from fossil fuel production will be at a lower level due to the relatively higher investment level under the first best allocation of resources. The statement holds independently of the time preference of the representative countries. Even though the first best allocation is welfare improving, under the business as usual conditions no country would choose the first best investment level unilaterally.³⁹

4.3 Self-enforcing agreement

The self-enforcing agreement result is based upon previously stated assumptions. Where the investment level in abatement technology is independent of the representative countries' initial technology stock. Additionally, as mentioned in chapter 3, sub-chapter *groundwork for the result*; we have homogenous countries where the discount function is the only condition differentiating their decision-making regarding their investment level and energy production level.

We can define a self-enforcing agreement as an agreement between a share of the participating countries in the game, referred to as cooperators. Where the group of cooperators comes together and agrees upon the terms of the agreement collectively. In

³⁹ Barrett, S, "Self-enforcing International Environmental Agreements."

order for the agreement to be self-enforcing the terms of the agreement has to be beneficial for each cooperating country. Furthermore, in order for the agreement to be sustained each cooperator has to keep up their end of the bargain and follow through with the terms of agreement. Each cooperating country will be willing to do so as long as it is beneficial for them. We will make the assumption that the terms of the agreement will include an agreed upon level of investment in abatement technology which each cooperator has to perform during the timespan of the game. We will refer to the investment level performed by the cooperating countries as the optimal level given the circumstances of the agreement, leading to the following notation regarding the cooperating countries performances during the self-enforcing agreement.

Investment level performed by the cooperating countries: $r_{i,t}^{OPT}$

Emission level performed by the cooperating countries: $g_{i,t}^{OPT}$

The non-cooperating countries on the other hand will act as business as usual, therefore we will use the notation BAU when referring to the non-cooperator's performance under the self-enforcing agreement:

Investment level performed by the non-cooperating countries: $r_{i,t}^{BAU}$

Emission level performed by the non-cooperating countries: $g_{i,t}^{BAU}$

In total we therefore have a share m of the total countries N who chooses to cooperate, leaving $(1 - m)N$ countries acting as non-cooperating countries. Let $R_{n,t}$ denote the investment level in abatement technology performed by the non-cooperating countries. Where the non-cooperating countries individual investment level can be described by the following expression:

$$R_{n,t} = kr_{i,t} - b(\bar{y}_t - (g_{i,t}^{BAU} + R_{i,t}))$$

Due to all countries being homogenous, the non-cooperating countries total investment level can be described as:

$$R_{n,t} = (1 - m)N[-b(\bar{y}_t - (g_t^{BAU} + R_t)) - kr_t]$$

In total, the reaction function for the non-cooperating countries under the agreement can be expressed by the following expression:

$$R_{n,t}(m, A_{c,t}) = (1 - m) \left[\frac{-b(\bar{y}_t - (g_{i,t}^{BAU} + R_{i,t}) - kr_{i,t})}{(k/b + 1 - m)} \right]$$

Cooperating countries are assumed to choose their total investment level maximizing their combined net-benefits. The cooperating countries individual investment level can thus be described by the following expression:

$$R_{c,t} = -b(\bar{y}_t - (g_{i,t}^{OPT} + R_{i,t}) - kr_{i,t})$$

Each individual cooperating country's investment level is identical. Therefore, the cooperating countries total investment level can be described as:

$$R_{c,t} = (m)N[-b(\bar{y}_t - (g_t^{OPT} + R_t) - kr_t)]$$

Furthermore, we have the following expression describing the total investment level in abatement technology performed by the cooperating countries, given the share of cooperators:

$$R_{c,t}^*(m) = \frac{(kr_t - b(\bar{y}_t - (g_t^{OPT} + R_t)))m^2N(k/b)}{[(k/b) + 1 - m]^2 + m^2N(k/b)}$$

We use the same approach to obtain the investment level performed by the non-cooperating countries given the number of cooperators:

$$R_{n,t}^*(m) = \frac{(kr_t - b(\bar{y}_t - (g_t^{BAU} + R_t)))(1 - m)((k/b) + 1 - m)}{[(k/b) + 1 - m]^2 + m^2N(k/b)}$$

As we know country i will initially decide its strategy of acting as a cooperator or not before entering the game. Because there is no investment stage at the last time period of the game, the agent will invest in abatement technology during the first and or second time period of the game. Where the groundworks stated below has to hold for the whole timespan of the game in order to achieve a self-enforced agreement. (See appendix for the derived result)

4.3.1 Groundworks for the self-enforcing agreement

In order for the self-enforcing agreement to hold the net-benefit of acting as a cooperator has to exceed the net-benefit of deviating and acting as business as usual. In order to determine if country i will commit to acting as a cooperator or not is thus based on the relative net-benefit of changing position under the achieved agreement. Let π_n and π_c denote the net-benefit of non-cooperating and cooperating countries, given respectively. The following inequalities has to hold in order for the agreement to hold during each time period of the game.

First, the net-benefit of acting as a non-cooperative country given the share of non-cooperator's has to less or equal to the net-benefit of acting as a cooperator country:

$$\pi_{n,t}(m - 1/N) \leq \pi_{c,t}(m)$$

Secondly, the net-benefit of acting as a non-cooperative country given the share of cooperating countries has to be greater or equal to the net-benefit of acting as a cooperating country:

$$\pi_{n,t}(m) \geq \pi_{c,t}(m + 1/N)$$

We will now present the net-benefit of the cooperating and non-cooperating countries when using exponential discounting, i.e. the countries does not have present-biased preferences. Followed by the net-benefit of the cooperating and non-cooperating countries when using quasi-hyperbolic discounting, where we include the assumption of the countries having present-biased preferences.

The net-benefit of acting as a non-cooperating country when using exponential discounting is given by the following expression:

$$\pi_{n,t} = \left(\left[-\frac{b}{2} \left((g_t^{BAU} + R_t) - \bar{y}_2 \right) \right]^2 - dng_t^{BAU} - d(n-1)g_t^{OPT} - knr_t^{BAU} - k(n-1)r_t^{OPT} \right) + \sum_{\tau=t+1}^3 \delta^{\tau-t} \pi_{n,\tau}$$

The net-benefit of acting as a cooperating country when using exponential discounting is given by the following expression:

$$\pi_{c,t} = \left(\left[-\frac{b}{2} \left((g_t^{OPT} + R_t) - \bar{y}_2 \right) \right]^2 - dng_t^{OPT} - knr_t^{OPT} \right) + \sum_{\tau=t+1}^3 \delta^{\tau-t} \pi_{c,\tau}$$

Secondly, the same inequality has to be hold in order for the agreement to hold satisfactorily for each time period of the game when using quasi-hyperbolic discounting. Where the net-

benefit of acting as a non-cooperating country with present biased preferences is given by the following expression:

$$\pi_{n,t} = \left(\left[-\frac{b}{2} \left((g_t^{BAU} + R_t) - \bar{y}_2 \right) \right]^2 - dn g_t^{BAU} - d(n-1) g_t^{OPT} - knr_t^{BAU} - k(n-1)r_t^{OPT} \right) + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} \pi_{n,\tau}$$

The net-benefit of acting as a cooperating country when using quasi-hyperbolic discounting is given by the following expression:

$$\pi_{c,t} = \left(\left[-\frac{b}{2} \left((g_t^{OPT} + R_t) - \bar{y}_2 \right) \right]^2 - dn g_t^{OPT} - knr_t^{OPT} \right) + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} \pi_{c,\tau}$$

Given the inequalities and net-benefits of acting as a cooperator and non-cooperator stated above there are still possible actions during the game for the countries to consider under the self-enforcing agreement. More specifically the number of countries, m , who decides to cooperate can decide between deviating from the agreement or stay committed throughout the timespan of the game. The cooperating country stays committed to the agreement through performing the agreed upon investment level in the abatement technology. If the cooperating country does not hold up his commitment, then the terms of agreement will be breached reducing the net-benefit of the remaining cooperating countries. Furthermore, the non-cooperating countries can decide between entering the agreement or continue to act as business as usual. agreement.

Let us assume a cooperating country considers deviating from the self-enforcing agreement. Then the deviating cooperator's cost of investing in abatement technology in addition to their abatement level will be reduced as a direct effect. This is due to the investment level as a cooperating country is higher than the investment level when acting as business as usual. When deviating, consequently the terms of the agreement will be destabilized decreasing the net-benefit of staying as a committed cooperating country. The reduction in total net-benefits of the agreement will exceed the reduction in costs of investment country i faces. In total the benefit of staying committed exceeds the benefit of deviating. Therefore, the cooperating countries will not benefit from deviating given the stated conditions of the self-enforcing agreement. The same type of argument holds for the non-cooperating countries, who will not benefit from entering the agreement. If an initial non-cooperating country decides to enter

the agreement, they will immediately face an increased investment cost. Additionally, the cost of investment will exceed the net-benefit of them entering the agreement. Thus, the non-cooperating countries will not benefit from entering the agreement once the self-enforcing agreement is set.⁴⁰ This argument holds independently of the countries net-benefit is obtained using exponential or quasi-hyperbolic discounting.

4.3.2 Self-enforcing agreement – Exponential VS Quasi-hyperbolic discounting

This sub-chapter will present the results of the investment and emission level under the presented self-enforcing agreement. The full mathematical approach of obtaining the results is given in the appendix. Where the net-benefit of acting as a cooperator is the cooperating countries strategy profile under the agreement and the net-benefit of acting as a non-cooperating country is the non-cooperating countries strategy profile under the agreement. We will not present the outcome of the agreement in each time period individually, but discuss the outcome of the agreement in the given time period t . Furthermore, we will investigate how the investment level of the cooperating countries is affected when including the assumption of present-biased preferences and thus compare the results of when using exponential discounting compared to when using quasi-hyperbolic discounting. At last we will also compare the total emission level under the self-enforcing agreement obtained under the two discount functions.

4.3.3 The results of the self-enforcing agreement

The investment level performed by the cooperating countries, $R_{c,t}^*$, is dependent on the total number of participants, m , and the total emission level performed by the participating countries in the given time period t , $g_t^{OPT^*}$. We thus obtain the following result for the cooperating country's investment level with time consistent preferences in time period t under the self-enforcing agreement:

$$R_{c,t}^*(m, g_t^{OPT^*}) =$$

⁴⁰ Barrett, S, «Self-enforcing International Environmental agreements.»

$$\frac{\left(kr_t - b\left(\bar{y}_t - ((g_1^{OPT^*}) + R_t)\right)\right) m^2 N(k/b)}{\left[\left((k/b) + 1 - m\right)^2 + m^2 N(k/b)\right]}$$

The investment level performed by the non-cooperating countries, $R_{n,t}^*$, under the self-enforced agreement is dependent on the share of participating countries, m , and the total emission level in time period t performed by the non-cooperating countries, g_t^{BAU} . We obtain the following result for the non-cooperating country's investment level with time consistent preferences under the self-enforcing agreement:

$$R_{n,t}^*(m, g_t^{BAU}) = \frac{\left(kr_t - b\left(\bar{y}_t - ((g_t^{BAU}) + R_t)\right)\right) (1-m)\left((k/b) + 1 - m\right)}{\left[\left((k/b) + 1 - m\right)^2 + m^2 N(k/b)\right]}$$

Secondly, the investment level performed by the cooperating and non-cooperating countries with present-biased preferences under the self-enforced agreement is based upon the same dependent variables. Where we refer to the cooperating countries investment and emission level with present-biased preferences in time period t as $\hat{R}_{c,t}^*$ and $\hat{g}_t^{OPT^*}$ given respectively. The investment level and the emission level performed by the non-cooperating countries with time-inconsistent preferences in time period t is referred to as $\hat{R}_{n,t}^*$ and \hat{g}_1^{BAU} given respectively. In total we obtain the following result for the cooperating countries investment level in time period t when including time-inconsistent preferences:

$$\hat{R}_{c,t}^*(m, \hat{g}_t^{OPT^*}) = \frac{\left(kr_t - b\left(\bar{y}_t - ((\hat{g}_1^{OPT^*}) + R_t)\right)\right) m^2 N(k/b)}{\left[\left((k/b) + 1 - m\right)^2 + m^2 N(k/b)\right]}$$

At last we obtain the following result for the investment level performed by the non-cooperating countries in time period t when including time-inconsistent preferences:

$$\hat{R}_{n,t}^*(m, \hat{g}_1^{BAU}) = \frac{\left(kr_t - b\left(\bar{y}_t - ((\hat{g}_1^{BAU}) + R_t)\right)\right) (1-m)\left((k/b) + 1 - m\right)}{\left[\left((k/b) + 1 - m\right)^2 + m^2 N(k/b)\right]}$$

4.3.4 Discussing the optimal self-enforced agreement

The outcome of the performed investment level of the cooperating countries under the self-enforced agreement is as we have seen dependent on the number of countries acting as cooperators in addition to the level of emission from the cooperating countries. Furthermore, the environmental cost from fossil fuel use and the net-benefit of investing in abatement technology is of importance. As we know, we have a number of N countries participating in the game, whereas the share of cooperating and non-cooperating countries is uncertain. We are thus not able to analyze the exact outcome of the self-enforcing agreement. Even though, by deriving the first order conditions of the net-benefit of the cooperating countries with respect to the investment level, the emission level and the number of countries we are able to gain some insight behind the dynamics behind the self-enforcing agreement. The mathematical details behind the results are presented in the appendix.

The results show that if the quantity of cooperating countries increase the emission level performed by the cooperating countries will decrease. We can see this result from the obtained the first order conditions of the cooperating country's emission level under the self-enforced agreement, $g_t^{OPT^*}$, with respect to the number of countries, n . When using both exponential and quasi-hyperbolic discount factors we obtain the following inequality:

$$\frac{\partial g_t^{OPT^*}}{\partial n} < 0, \frac{\partial \hat{g}_t^{OPT^*}}{\partial n} < 0$$

As we can see, the argument holds regardless of the time preference of the cooperating countries. Consequently, there will be a lower bound of the number of cooperating countries for the self-enforced agreement to be sufficient under the stated conditions. Where a satisfactory agreement would consist of a high enough number of cooperators in order to achieve the lowest feasible maximum level of fossil fuel usage.

The first best emission level under the self-enforced agreement is negatively correlated with the climate cost from fossil fuel production. Where the higher the climate damage from fossil fuel usage is, the lower will the maximum emission level performed by the cooperating countries be. We therefore have the following result from the first order conditions of $g_t^{OPT^*}$ with respect to the marginal climate cost, d :

$$\frac{\partial g_t^{OPT*}}{\partial d} < 0, \frac{\partial \hat{g}_t^{OPT*}}{\partial d} < 0$$

The result also holds for both exponential and quasi-hyperbolic discounting. Let us now compare the effect between having cooperating countries with consistent and inconsistent time preferences. The result shows the more patient the cooperating countries are the lower will the maximum level of emission be under the self-enforced agreement. Having a lower discount rate represents being more patient, we thus have a negative correlation between the discount factor and the first best emission level performed by the cooperating countries. This can be observed by the first order conditions of the FB emission level with respect to the discount factors:

$$\frac{\partial g_t^{OPT*}}{\partial \delta} < 0, \frac{\partial \hat{g}_t^{OPT*}}{\partial \beta\delta} < 0$$

One interesting observation is the difference in the first order conditions of the optimal emission level when comparing the two different discount factors. The F.O.C show when cooperating countries are patient, i.e. the discount factor is high, the first best emission level will be lower under the quasi-hyperbolic discount factor compared to the exponential discount factor if we assume all other parameters has identical values. We therefore obtain the following relationship between the first order conditions of the FB emission level with respect to the discount factors:

$$\frac{\partial \hat{g}_t^{OPT*}}{\partial \beta\delta} \leq \frac{\partial g_t^{OPT*}}{\partial \delta} < 0$$

One explanation as to why the emission levels can differ is the underlying mechanisms of having intertemporal preferences compared to having time consistent preferences. One essential aspect to take into consideration is thus the difference between the behavioral characteristics behind the two distinctive type of time preferences. A cooperator with intertemporal preferences is assumed to be both patient and sophisticated, while a cooperator with consistent time preferences is assumed to be patient with no additional behavioral characteristics assumed for. Furthermore, due to the stated inequalities for the

agreement to hold, a cooperator is inevitably faced with a cost benefit schedule from investing in the abatement technology.

As a consequence, under the scenario of having sophisticated cooperators, each country will prefer to invest in the first investment stage of the game, instead of procrastinating to the second investment stage. The decision is not dependent on performing the investment during the first investment stage being strictly more beneficial than investing in the second. When having the behavioral trait of a sophisticate the country will simply prefer not to procrastinate when faced with a decision between when to perform a task with an immediate cost. Even though the country will face immediate costs and future benefit of doing so. This statement holds true when the costly task has to be performed within a given timespan.⁴¹ Additionally, the sophisticated cooperators might choose to invest in the second time period as well if the climate damage or quantity of cooperating countries is at a high enough level.

When the cooperating countries have time-consistent preferences on the other hand, they will be more prone to perform the investment in the second time period of the game. Where in the second investment stage of the game the choice stands between performing the investment or deviate from the agreement, which would make them relatively worse off. Last remark; the inequality in total emission level under the self-enforced agreement when comparing the two discount factors is also dependent on the total investment level performed during the timespan of the game and the quality of the abatement technology. We have not performed any further analysis regarding the effect of a relatively change in the level of these two variables when it comes to comparing the cooperating countries decision-making with and without present-biased preferences.

⁴¹ O'Donoghue, T, Rabin, M, "Doing it now or later."

5. Discussion

As we can see from the derived results of the first best allocation, business as usual result and the self-enforced agreement, using quasi-hyperbolic discount has an effect on the representative countries decision making as opposed to when using exponential discounting. Countries with intertemporal preferences leads to the decisionmaker not acting completely rational when deciding on energy consumption level and investment level in abatement technology. When having present-biased preferences the countries prefer to utilize present benefit and weighs present costs over present benefit. Thus, leading to the preference of maintain their energy consumption level in the present time period and wait to the future time period to perform investment in green technology. The analysis consists of a sub-set of assumptions determining the derived results and as a counter affect there are some limitations to them as well.

The representative countries are assumed to be sophisticated and patient which is not necessarily true behavioral characteristics for countries in reality. Therefore, as a consequence the energy production and investment level obtained under the FB, BAU and self-enforced agreement results holds under the behavioral conditions but are not representative if countries for example have other behavioral characteristics such as impatient or naïve instead. One interesting observation on the other hand, is given the stated assumptions of the representative countries behavioral characteristics, how intertemporal preferences has affected the results of the game compared to when the representative countries have time consistent preferences. The social planner's choice under the FB result as well as the business as usual result show how countries when having time inconsistent preferences perform an overall lower level of investment leading to a higher emission level compared to when the countries have time consistent preferences.

Under the self-enforced agreement, the investment level of the cooperators show how being a sophisticate can actually be welfare improving under the stated conditions of the agreement compared to when the cooperators are completely rational. It is important to note were it not for the cost-benefit schedule the countries face as cooperators and the features of being a sophisticate the result would not necessarily hold. As an example, under

the BAU result, the countries are also assumed to be sophisticated but are not faced with the same conditions regarding their cost-benefit schedule. Because non-cooperators have the opportunity to not invest at all without any consequences of reducing their individual benefits. Thus, leading to the main observation of how quasi-hyperbolic discounting represents the tendency of agents not being able to commit to allocating their resources in order to maximize their continuation value throughout the game. The reason as to why is because country i will typically weigh present cost higher than future benefit when having intertemporal preferences. Therefore, when faced with the decision of bearing the cost of investment in the present time period offsets the benefit of abatement technology in the future time period. Leading to a general lower emission level and consequentially higher emission level when taking into account intertemporal preferences.

The decision of analyzing a game of three repeating rounds was made in order to enable the analysis to primarily focus on the effect of performing the analysis using the two distinctive discount factors. Inevitably, there are some weaknesses of analyzing a climate game of repeating rounds under a discrete timespan instead of having a dynamic game with an infinite time horizon. As stated in the results the cooperating countries with intertemporal preferences prefer to invest during the first investment stage of the game under the self-enforced agreement.⁴² If on the other hand, the game was dynamic this argument would not necessarily hold. This is because under a dynamic game the cost and benefit schedule as a cooperator would be different. Potentially changing the preference of the sophisticated cooperator, dependent on the design of the schedule.

The technology stock which is dependent on the previous investment level as well as the depreciation rate would also have additional affects on the results if we were analyzing a game with infinite time periods. Because the countries partaking in the game only have two time periods to make a decision to invest or not the incentive to perform the investment can be limited. Especially because the game is finished in the next time period after the last investment stage, excluding the countries of experiencing the long-term effect of investing in

⁴² O'Donoghue, T, Rabin, M, "Doing it now or later."

abatement technology. Furthermore, the accumulation of abatement technology in order to achieve a steady state of low emission levels in the long run is dependent on repeating investment stages over a longer time horizon. Therefore, having a time limit of two investment stages can have some distortions affects in the participating countries incentives to invest during the game. Bringing back to the argument of analyzing a dynamic game with an accumulating investment stock including intertemporal preferences would be interesting for future research.

Furthermore, we have made a set of conditions in order for the self-enforced agreement to hold. Further research should perform regression analyzes simulating the specific dynamics behind such a self-enforced agreement. More specifically, how the value of climate cost, net-benefit of investment and the specific quantity of cooperating countries affect the conditions of the attained agreement. Such regression analyzes would enable insight into what specific parameter-values could an optimal first best emission level be achieved by the cooperating countries. Additionally, show how many cooperators are acquired for achieving a sustainable agreement under the given circumstances.

More interestingly in the highlight of the research question, how would including behavioral characteristics such as impatience and naiveté affect the agreement? Further research should analyze the consequence of cooperating countries under the self-enforced agreement being impatient instead of patient and naïve instead of sophisticated. Including these aspects offers an insight on how these assumptions affects the representative countries decisions regarding acting committed to the agreement and the performed investment level under the agreement. Such research could be interesting contributing to the literature on behavioral game theory, especially when analyzing the *climate game* and *collective-risk game* and cooperation. Especially since these are all characteristics describing observed human behavior when it comes to present-biased time preferences and agent's ability of commitment to future self.^{43, 44}

⁴³ Wang, M. Rieger, M, O. Hens, T, «How time preferences differ.»

⁴⁴ O'Donoghue, T, Rabin, M, "Doing it now or later."

All these analytical aspects also contribute to stress the relevancy of including behavioral theoretical characteristics when describing the representative agents in Economic theory in general. Economists such as Strotz argued using exponential discounting is not necessarily a representative approach for describing consumption behavior. His statement was based upon human behavioral studies arguing humans are not always able to stay true to their pre-commitments of future decisions.⁴⁵ Laibson as well argued quasi-hyperbolic discounting has a better fit in order to explain human behavior when analyzing consumer decisions-making compared to exponential discounting. Meaning including present-biased preferences has its analytical benefits as opposed to assuming time consistent preferences. More previous research in the economic field as well has drawn the same type of conclusions when it comes to using a representative discount rate describing human behavior.^{46, 47} The common reasoning behind these stated arguments are based upon humans behaving time-inconsistent, typically preferring present benefits and weighs present costs over future benefit, leading to procrastination of actions with immediate costs. As well as uncertainty and risk make it difficult to stay committed to agreements made previous in time.

Last remark; in the presented framework we have a linear costs and benefits schedule. Additionally, we do not take into account risks or the fact that the GHG level in the atmosphere accumulate over time. Therefore, there are some important real-life aspects of the climate change problem not included in the presented framework. Economists and scientists working for organizations such as the IPCC has stated that real world problem such as climate change is complex and consists of non-linear interactions and uncertainties.⁴⁸ Thus, the results discussed in this paper is not representative for the real-world complexity of climate change. On the other hand, the dissertation offers a theoretical discussion on the benefits of including behavioral theory when stating the characteristics for the participating countries in the climate game in order to take into account non-rational aspects of performed decision making, reflecting the real-life human behavior.

⁴⁵ Strotz, R, H, "Myopia and Inconsistency in dynamic utility maximization."

⁴⁶ Harrison, M, "Valuing the Future."

⁴⁷ Gowdy, J, Roy, L, Rosser, B, J, "The evolution of hyperbolic discounting."

⁴⁸ Pittock, A, B, "What we know and don't know about climate change."

6. Concluding Remarks

The dissertation seeks to discuss the research question; how is an agent's decision-making in a public good game affected when comparing present biased with time-consistent preferences? Based upon the presented analysis accommodating the stated assumptions we can appreciate there is a difference between the decisions made by the representative countries with intertemporal preferences compared to if their preferences are time consistent. One main reason behind the observed dissimilarity in performed investment level is due to the distinction between the discount rates. Under intertemporal preferences country i has a declining discount rate over the time span of the game, leading to *present-biased preferences*. Therefore, country i will prefer present benefit from energy consumption over future benefit from abatement technology, because immediate costs are weighed heavier than future benefit in the present time period. Consequentially, when all participating countries are assumed to have time inconsistent preferences the total level of investment in abatement technology will be at a relative lower level leading to an overall higher level of emissions from fossil fuel use. This was found to be true under both the first best allocation and business as usual result.

One interesting observation is how being a sophisticated agent can actually be welfare improving under the set of assumptions of the self-enforced agreement. Under the agreement, when entering the game as a cooperator there is no incentive to deviate because the decreased private cost country i face from doing so does not exceed the benefit of maintaining the agreement. Essentially leading the sophisticated cooperators with a cost-benefit schedule over a time-horizon of three time periods, where the investment has to be performed during the first and or second time period. If not, the cooperator automatically acts as business as usual, i.e. deviates from the agreement. Therefore, when a sophisticated cooperator is faced with a cost schedule with immediate costs and future benefits country i will thus prefer to invest in the abatement technology during the first investment stage.⁴⁹

⁴⁹ O'Donoghue, T, Rabin, M, "Doing it now or later".

The main incentive behind discussing the presented research question is to stress the limitations behind using exponential discounting, especially when developing such relevant research in the literature regarding *the climate game*. It appears to be a gap in the literature between focusing on the issues regarding cooperation and the behavioral assumptions of the representative players in the game. One aspect the literature is highly concentrated on is how to achieve a self-enforced IEA. Where the goal of such an agreement is to reduce the total emission rate by incentivizing each country to cooperate by investing in the public good. The reason behind this decentralized focus can be due to the reality behind the dilemma of the public good game such as the climate game. Society is under pressure regarding achieving sustainable solutions in order to prevent excessive climate change in the future.^{50,51}

Even though, there should be attention paid to the disadvantages of the classical approach by describing the agent's time preferences as time consistent in the behavioral game theory literature, especially when analyzing the climate game. OECD working papers (2012) points out assuming rational behavior is a poor approach to design efficient environmental policies. Where the authors argue behavioral economics might have a crucial role in order to design effective environmental policies. *Environmental policy might well be more cost-effective if we transform our rational choice models to include bounded rationality, bounded self-interest and bounded willpower.*⁵² Even though it is quite the norm to assume complete rationality and thus using exponential discounting when designing the framework of the climate game. In order to obtain further relevant research regarding the general dilemma behind the climate game, including behavioral characteristics by assuming the representative players have present-biased preferences can contribute into achieving economic theoretical research more representative for the real-world dilemma of climate change.

⁵⁰ IPCC, Climate change rapport 2013.

⁵¹ UNFCCC, UN Climate panel rapport 2019.

⁵² Prof Shogren, J. "Behavioral Economics and Environmental Incentives."

7. Appendix

7.1 The optimal level of energy production – The Envelope Theorem

We use notation $y_{i,t}^*$ for optimal energy production of country i which is given by variables $g_{i,t}$ and $R_{i,t}$. We therefore have that $y_{i,t}(g_{i,t}, R_{i,t})$ yields optimal level of energy production. In order to obtain $y_{i,t}^*$ we use the Envelope Theorem as shown below.

$$y_{i,t} = (g_{i,t} + R_{i,t}), C(g_{i,t}) = d(y_{i,t} - R_{i,t})$$

$$U_t = \max_{g_{i,t}, y_{i,t}} (g_{i,t}, y_{i,t})$$

$$\frac{\partial u_{i,t}}{\partial g_{i,t}} = 0 \rightarrow g_{i,t}^*(y_{i,t})$$

$$U_t(y_{i,t}) = u(g_{i,t}^*(y_{i,t}), y_{i,t})$$

$$U'_t(y_{i,t}) = \frac{\partial u_t(g_{i,t}^*(y_{i,t}), y_{i,t})}{\partial y_{i,t}} = \frac{\partial u_{i,t}}{\partial g_{i,t}} * g'_{i,t}(y_{i,t}) + \frac{\partial u_{i,t}}{\partial y_{i,t}}$$

$$\text{We know that } \frac{\partial u_{i,t}}{\partial g_{i,t}} = 0$$

$$U'_t(y_{i,t}) = \frac{\partial u_{i,t}}{\partial y_{i,t}} \rightarrow \text{We refer to this point as } y_{i,t}^* \text{ throughout}$$

The first order condition shows that $y_{i,t}^*$ is directly independent of $g_{i,t}$ and $R_{i,t}$.

Interpretation: Country i 's marginal benefit of total energy production given the emission rate and renewable energy stock in the third time period equals the marginal cost of emissions.

Secondly, we maximize the objective function w.r.t $y_{i,t}$ taking $y_{i,t}^*$ as given when obtaining the equilibria result of FB, BAU and optimal self-enforced agreement.

7.2 Deriving the First Best result

7.2.1 FB Result – Exponential Discounting

$$U_t^{FB} = \sum_{i=1}^n u_t + \sum_{\tau=t+1}^3 \delta^{\tau-t} \sum_{i=1}^n u_\tau$$

$$u_t = -\frac{b}{2} (\bar{y}_t - \tilde{y}_{i,t})^2 - C(g_t) - K(r_{i,t})$$

$$U_t^{FB} = \left[-\frac{b}{2}(\bar{y}_1 - \tilde{y}_{i,1})^2 - d \sum_{i=1}^n g_{i,1} - \sum_{i=1}^n \frac{(kr_{i,1})^2}{2} \right] + \delta \left[-\frac{b}{2}(\bar{y}_2 - \tilde{y}_{i,2})^2 - d \sum_{i=1}^n g_{i,2} - \sum_{i=1}^n \frac{(kr_{i,2})^2}{2} \right] \\ + \delta \left[-\frac{b}{2}(\bar{y}_3 - \tilde{y}_{i,3})^2 - d \sum_{i=1}^n g_{i,3} - \sum_{i=1}^n \frac{(kr_{i,3})^2}{2} \right]$$

$$S. t. y_{i,t} = g_{i,t} + R_{i,t}, \text{ where } R_{i,t} = \frac{q_R}{n} R_{t-1} + r_{i,t}$$

$$U_t^{FB} = \left[-\frac{b}{2}(\bar{y}_1 - \tilde{y}_{i,1})^2 - d \sum_{i=1}^n (y_{i,1} - R_{i,1}) - \sum_{i=1}^n \frac{(kr_{i,1})^2}{2} \right] \\ + \delta \left[-\frac{b}{2}(\bar{y}_2 - \tilde{y}_{i,2})^2 - d \sum_{i=1}^n (y_{i,2} - R_{i,2}) - \sum_{i=1}^n \frac{(kr_{i,2})^2}{2} \right] \\ + \delta \left[-\frac{b}{2}(\bar{y}_3 - \tilde{y}_{i,3})^2 - d \sum_{i=1}^n (y_{i,3} - R_{i,3}) - \sum_{i=1}^n \frac{(kr_{i,3})^2}{2} \right]$$

$$\frac{\partial U_t^{FB}}{\partial y_{1,2,3}} = (-b(\bar{y}_1 - \tilde{y}_{i,1}) - dn) + \delta(-b(\bar{y}_2 - \tilde{y}_{i,2}) - dn) + \delta(-b(\bar{y}_3 - \tilde{y}_{i,3}) - dn) = 0$$

$$\rightarrow \tilde{y}_{i,t} = y_{i,t} + (\bar{y}_t - \bar{y}_{i,t}) \rightarrow -b(\bar{y}_t - \tilde{y}_{i,t}) = b(y_{i,t} - \bar{y}_{i,t})$$

$$\frac{\partial U_t^{FB}}{\partial y_{1,2,3}} = b(y_{i,1} - \bar{y}_{i,1}) - dn + \delta[b(y_{i,2} - \bar{y}_{i,2}) - dn] + \delta[b(y_{i,3} - \bar{y}_{i,3}) - dn]$$

$$= b[(g_{i,1} + R_{i,1}) - \bar{y}_{i,1}] - dn + \delta[b((g_{i,1} + R_{i,1}) - \bar{y}_{i,2}) - dn] + \delta[b((g_{i,3} + R_{i,3}) - \bar{y}_{i,2}) - dn]$$

$$-dn - \delta dn - \delta dn = b[\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] + \delta b[\bar{y}_{i,2} - (g_{i,2} + R_{i,2})] + \delta b[\bar{y}_{i,3} - (g_{i,3} + R_{i,3})]$$

$$-\frac{dn(1+2\delta)}{b} = [\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] + \delta[\bar{y}_{i,2} - (g_{i,2} + R_{i,2})] + \delta[\bar{y}_{i,3} - (g_{i,3} + R_{i,3})]$$

$$-\frac{dn(1+2\delta)}{b} = -g_{i,1} + [\bar{y}_{i,1} - (R_{i,1})] - \delta g_{i,2} + \delta[\bar{y}_{i,2} - (R_{i,2})] - \delta g_{i,3} + \delta[\bar{y}_{i,3} - (R_{i,3})]$$

$$-\frac{dn(1+2\delta)}{b} + g_{i,1} + \delta g_{i,2} + \delta g_{i,3} = [\bar{y}_{i,1} - (R_{i,1})] + \delta[\bar{y}_{i,2} - (R_{i,2})] + \delta[\bar{y}_{i,3} - (R_{i,3})]$$

$$g_{i,t}^*(R_t) = [\bar{y}_{i,1} - (\frac{q_R}{n} R_0 + r_{i,1})] + \delta [\bar{y}_{i,2} - (\frac{q_R}{n} R_1 + r_{i,2})] + \delta [\bar{y}_{i,3} - (\frac{q_R}{n} R_2 + r_{i,3})] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right]$$

$$\frac{\partial U_t^{FB}}{\partial r_{1,2,3}} = dn - nkr_{i,1} + \delta(dn - nkr_{i,2}) + \delta dn = 0$$

→

$$-nkr_{i,1} - \delta nkr_{i,2} = -dn - \delta dn - \delta dn = b(\bar{y}_1 - \tilde{y}_{i,1}) + \delta b(\bar{y}_2 - \tilde{y}_{i,2}) + \delta b(\bar{y}_3 - \tilde{y}_{i,3})$$

$$-nkr_{i,1} - \delta nkr_{i,2} = b(\bar{y}_{i,1} - (g_{i,1} + R_{i,1})) + \delta b(\bar{y}_{i,2} - (g_{i,2} + R_{i,2})) + \delta b(\bar{y}_{i,3} - (g_{i,3} + R_{i,3}))$$

$$r_{i,t}^*(g_t) = (\bar{y}_{i,1} - (g_{i,1} + R_{i,1})) + \delta (\bar{y}_{i,2} - (g_{i,2} + R_{i,2})) + \delta (\bar{y}_{i,3} - (g_{i,3} + R_{i,3})) \left[-\frac{b}{nk(1 + \delta) + b} \right]$$

The first best result for optimal investment is thus given by:

$$r_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - \frac{q_R}{n} R_{t-1} - \frac{b}{nk(1 + \delta) + b}$$

The first best emission level is given by the technology stock, $R_{i,t} = \{R_{i,1}, R_{i,2}, R_{i,3}\}$

$$g_{i,t}^{FB}(R_{i,t}) = [\bar{y}_{i,1} - (R_{i,1})] + \delta [\bar{y}_{i,2} - (R_{i,2})] + \delta [\bar{y}_{i,3} - (R_{i,3})] \left[-\frac{dn(1 + 2\delta)}{b\delta^2} \right]$$

7.2.2 FB Result – Quasi Hyperbolic Discounting

$$\hat{U}_t^{FB} = \sum_{i=1}^n u_t + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} \sum_{i=1}^n u_\tau$$

$$u_{i,t} = B_i(y_{i,t}) - C(g_t) - K(r_{i,t})$$

$$\begin{aligned} \hat{U}_t^{FB} = & \left[-\frac{b}{2} (\bar{y}_1 - \tilde{y}_{i,1})^2 - d \sum_{i=1}^n g_{i,1} - \sum_{i=1}^n \frac{(kr_{i,1})^2}{2} \right] \\ & + \beta \delta \left[-\frac{b}{2} (\bar{y}_2 - \tilde{y}_{i,2})^2 - d \sum_{i=1}^n g_{i,2} - \sum_{i=1}^n \frac{(kr_{i,2})^2}{2} \right] \\ & + \beta \delta \left[-\frac{b}{2} (\bar{y}_3 - \tilde{y}_{i,3})^2 - d \sum_{i=1}^n g_{i,3} - \sum_{i=1}^n \frac{(kr_{i,3})^2}{2} \right] \end{aligned}$$

$$S. t. y_{i,t} = g_{i,t} + R_{i,t}, \text{ where } R_{i,t} = \frac{q_R}{n} R_{t-1} + r_{i,t}$$

$$\begin{aligned} \hat{U}_t^{FB} = & \left[-\frac{b}{2} (\bar{y}_1 - \tilde{y}_{i,1})^2 - d \sum_{i=1}^n (y_{i,1} - R_{i,1}) - \sum_{i=1}^n \frac{(kr_{i,1})^2}{2} \right] \\ & + \beta \delta \left[-\frac{b}{2} (\bar{y}_2 - \tilde{y}_{i,2})^2 - d \sum_{i=1}^n (y_{i,2} - R_{i,2}) - \sum_{i=1}^n \frac{(kr_{i,2})^2}{2} \right] \\ & + \beta \delta \left[-\frac{b}{2} (\bar{y}_3 - \tilde{y}_{i,3})^2 - d \sum_{i=1}^n (y_{i,3} - R_{i,3}) - \sum_{i=1}^n \frac{(kr_{i,3})^2}{2} \right] \end{aligned}$$

$$\frac{\partial \hat{U}_t^{FB}}{\partial y_{1,2,3}} = (-b(\bar{y}_1 - \tilde{y}_{i,1}) - dn) + \beta \delta (-b(\bar{y}_2 - \tilde{y}_{i,2}) - dn) + \beta \delta (-b(\bar{y}_3 - \tilde{y}_{i,3}) - dn) = 0$$

$$\rightarrow \tilde{y}_{i,t} = y_{i,t} + (\bar{y}_t - \bar{y}_{i,t}) \rightarrow -b(\bar{y}_t - \tilde{y}_{i,t}) = b(y_{i,t} - \bar{y}_{i,t})$$

$$\begin{aligned}
\frac{\partial \widehat{U}_t^{FB}}{\partial y_{1,2,3}} &= b(y_{i,1} - \bar{y}_{i,1}) - dn + \beta\delta[b(y_{i,2} - \bar{y}_{i,2}) - dn] + \beta\delta[b(y_{i,3} - \bar{y}_{i,3}) - dn] \\
&= b[(g_{i,1} + R_{i,1}) - \bar{y}_{i,1}] - dn + \beta\delta[b((g_{i,1} + R_{i,1}) - \bar{y}_{i,2}) - dn] + \beta\delta[b((g_{i,3} + R_{i,3}) - \bar{y}_{i,2}) - dn] \\
&\quad - dn - \beta\delta dn - \beta\delta dn = b[\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] + \beta\delta b[\bar{y}_{i,2} - (g_{i,2} + R_{i,2})] + \beta\delta b[\bar{y}_{i,3} - (g_{i,3} + R_{i,3})] \\
&\quad - \frac{dn(1 + 2\beta\delta)}{b} = [\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] + \delta\beta[\bar{y}_{i,2} - (g_{i,2} + R_{i,2})] + \delta\beta[\bar{y}_{i,3} - (g_{i,3} + R_{i,3})] \\
&\quad - \frac{dn(1 + 2\delta)}{b} = -g_{i,1} + [\bar{y}_{i,1} - (R_{i,1})] - \delta\beta g_{i,2} + \delta[\bar{y}_{i,2} - (R_{i,2})] - \delta\beta g_{i,3} + \delta[\bar{y}_{i,3} - (R_{i,3})] \\
&\quad - \frac{dn(1 + 2\beta\delta)}{b} + g_{i,1} + \beta\delta g_{i,2} + \beta\delta g_{i,3} = [\bar{y}_{i,1} - (R_{i,1})] + \beta\delta[\bar{y}_{i,2} - (R_{i,2})] + \beta\delta[\bar{y}_{i,3} - (R_{i,3})] \\
\hat{g}_{i,t}^*(R_t) &= [\bar{y}_{i,1} - (\frac{q_R}{n}R_0 + r_{i,1})] + \beta\delta[\bar{y}_{i,2} - (\frac{q_R}{n}R_1 + r_{i,2})] \\
&\quad + \beta\delta[\bar{y}_{i,3} - (\frac{q_R}{n}R_2 + r_{i,3})] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right]
\end{aligned}$$

$$\frac{\partial \widehat{U}_t^{FB}}{\partial r_{1,2,3}} = dn - nkr_{i,1} + \beta\delta(dn - nkr_{i,2}) + \beta\delta dn = 0$$

→

$$\begin{aligned}
-nkr_{i,1} - \beta\delta nkr_{i,2} &= -dn - \beta\delta dn - \beta\delta dn = b(\bar{y}_1 - \tilde{y}_{i,1}) + \beta\delta b(\bar{y}_2 - \tilde{y}_{i,2}) + \beta\delta b(\bar{y}_3 - \tilde{y}_{i,3}) \\
-nkr_{i,1} - \beta\delta nkr_{i,2} &= b(\bar{y}_{i,1} - (g_{i,1} + R_{i,1})) + \beta\delta b(\bar{y}_{i,2} - (g_{i,2} + R_{i,2})) + \beta\delta b(\bar{y}_{i,3} - (g_{i,3} + R_{i,3})) \\
r_{i,t}^*(g_t) &= (\bar{y}_{i,1} - (g_{i,1} + R_{i,1})) + \beta\delta(\bar{y}_{i,2} - (g_{i,2} + R_{i,2})) \\
&\quad + \beta\delta(\bar{y}_{i,3} - (g_{i,3} + R_{i,3})) \left[-\frac{b}{nk(1 + \beta\delta) + b} \right]
\end{aligned}$$

The first best result for optimal investment is thus given by:

$$\hat{r}_{i,t}^{FB}(g_t) = \bar{y}_{i,t} - \frac{q_R}{n}R_{t-1} - \frac{b}{nk(1 + \beta\delta) + b}$$

The first best emission level is given by the technology stock, $R_{i,t} = \{R_{i,1}, R_{i,2}, R_{i,3}\}$

$$\hat{g}_{i,t}^{FB}(R_{i,t}) = [\bar{y}_{i,1} - (R_{i,1})] + \delta[\bar{y}_{i,2} - (R_{i,2})] + \delta[\bar{y}_{i,3} - (R_{i,3})] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right]$$

7.3. Business as usual result – Exponential Discounting

Obtaining the intertemporal result for $y_{i,t}^{BAU}$

$$\tilde{y}_{i,t} = y_{i,t} + (\bar{y}_t - \bar{y}_{i,t})$$

$$B_i(y_{i,t}) = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2}$$

$$U_{i,t} = \left[u_{i,t} + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau} \right]$$

$$u_{i,t} = B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t})$$

$$U_{i,t} = \left[\frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d[y_{i,t} - (q_r R_{t-1} + r_{i,t})] - \frac{k(r_{i,t})^2}{2} + \delta^\tau \sum_{\tau=t+1}^3 u_{i,\tau} \right]$$

$$\frac{\partial U_{i,t}}{\partial y_{i,t}} = \frac{-2b(\bar{y}_t - \tilde{y}_{i,t})}{2} - d = 0$$

$$\rightarrow -b(\bar{y}_t - \tilde{y}_{i,t}) = d$$

$$-b\bar{y}_t + b\tilde{y}_{i,t} = d$$

$$b\tilde{y}_{i,t} = d + b\bar{y}_t$$

$$\tilde{y}_{i,t} = \frac{d + b\bar{y}_t}{b} \rightarrow \tilde{y}_{i,t} = \frac{d}{b} + \bar{y}_t$$

Deciding on $r_{i,t}$ for country i is equivalent to deciding on the technology stock $R_{i,t}$.

$$R_t = \sum_{i \in N} R_{i,t} = q_R R_{t-1} + \sum_{j \in \frac{n}{T}} r_{j,t} + r_{i,t} \rightarrow r_{i,t} = R_{i,t} - \frac{q_R}{n} R_{t-1}$$

We insert the equation for $r_{i,t}$ into the temporary continuation value function for the representative country:

$$V_{i,t} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d[y_{i,t} - (q_r R_{t-1} + r_{i,t})] + \delta U_{i,t}(g_{i,t}, R_{i,t})$$

$$V_{i,t} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d \left[y_{i,t} - \left(q_r R_{t-1} + \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right) \right] + \delta \left[B_i(y_{i,t}) - d \left[y_{i,t} - \left(q_r R_{t-1} + \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right) \right] - \frac{\left(k \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right)^2}{2} \right]$$

$$\frac{\partial V_{i,t}}{\partial R_{i,t}} = d + \delta(d - k)$$

$$\text{Max}_{R_{i,t}} V_{i,t}(g_{i,t}, R_{i,t}) - k r_{i,t}$$

$$= B_i(y_{i,t}) - d \left[y_{i,t} - (q_r R_{t-1} + (R_{i,t} - \frac{q_R}{n} R_{t-1})) \right] - \frac{\left(k \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right)^2}{2}$$

$$\frac{\partial V_{i,t}}{\partial R_{t-1}} = dq_R - d \frac{q_R}{n} + k \frac{q_R}{n} R_{t-1} = 0 \rightarrow dq_r \left(1 + \frac{1}{n} \right) + q_R (k/n) R_{t-1}$$

$$r_{i,t}^{BAU} = R_t^{BAU} - q_R R_{t-1}^{BAU}$$

$$r_{i,t}^{BAU} = d + \delta(d - k) - \left[q_r \left(1 + \frac{1}{n} \right) + q_R (k/n) R_{t-1} \right]$$

7.3.1 Deriving the BAU result for each time period – Exponential Discounting

Time period 3

No investment stage at the last time period of the game.

$$U_{i,3}^{BAU} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - C(g_{i,t}), \quad s. t. g_{i,3} = y_{i,3} - R_{i,3}, R_{i,3} = q_R R_{i,2}$$

$$U_{i,3}^{BAU} = \frac{-b(\bar{y}_3 - \tilde{y}_{i,3})^2}{2} - d[y_{i,3} - (q_R R_{i,2})]$$

$$\frac{\partial U_{i,3}^{BAU}}{\partial y_{i,3}} = -b(\bar{y}_3 - \tilde{y}_{i,3}) + d = 0$$

$$-b\left(\bar{y}_3 - \left(y_{i,3} + (\bar{y}_3 - \bar{y}_{i,3})\right)\right) = -d$$

$$b(y_{i,3} - \bar{y}_{i,3}) = -d$$

$$b((g_{i,3} + R_{i,3}) - \bar{y}_{i,3}) = -d$$

$$g_{i,3}^* = \bar{y}_{i,3} - (R_{i,3}) - \frac{d}{b}$$

$$\frac{\partial U_{i,3}^{BAU}}{\partial R_{i,2}} = dq_R$$

$$\bar{U}_{i,3}^{BAU} = dq_R R_{i,2}, R_{i,2}^* = dq_R$$

$$R_{i,3}^{BAU} = q_R R_{i,2}^{BAU} \rightarrow R_{i,3}^{BAU} = dq_R^2$$

$$g_{i,3}^{BAU}(R_{i,3}^{BAU}) = \bar{y}_{i,3} - [dq_R^2] - \frac{d}{b}$$

Time period 2.

$$U_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - C(g_{i,2}) - \frac{k(r_{i,2})^2}{2} + \delta \bar{U}_{i,3}^{BAU}, \quad s. t. g_{i,2} = y_{i,2} + R_{i,2}, R_{i,2} = q_R R_{i,1} + r_{i,2}$$

$$U_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - d[y_{i,2} - (q_R R_{i,1} + r_{i,2})] - \frac{k(r_{i,2})^2}{2} + \delta(R_{i,2} dq_R)$$

$$U_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - d[y_{i,2} - (q_R R_1 + r_{i,2})] - \frac{k(r_{i,2})^2}{2} + \delta((q_R R_1 + r_{i,2})dq_R)$$

$$\frac{\partial U_{i,2}^{BAU}}{\partial y_{i,2}} = -b(\bar{y}_2 - \tilde{y}_{i,2}) + d = 0$$

$$= -b(\bar{y}_2 - (y_{i,2} + (\bar{y}_2 - \bar{y}_{i,2}))) = -d$$

$$b(y_{i,2} - \bar{y}_{i,2}) = -d \rightarrow b((g_{i,2} + R_{i,2}) - \bar{y}_{i,2}) = -d$$

$$g_{i,2}^* = \bar{y}_{i,2} - (R_{i,2}) - \frac{d}{b}$$

$$\frac{\partial U_{i,2}^{BAU}}{\partial R_{i,1}} = dq_R + \delta dq_R^2, R_{i,1}^* = [dq_R + \delta dq_R^2]$$

$$\bar{U}_{i,2}^{BAU} = R_{i,1} [dq_R + \delta dq_R^2]$$

$$\text{Technology stock: } R_{i,2}^{BAU} = q_R R_{i,1}^{BAU} \rightarrow R_{i,2}^{BAU} = q_R [dq_R + \delta dq_R^2]$$

$$\text{BAU emission level: } g_{i,2}^{BAU}(R_{i,2}^{BAU}) = \bar{y}_{i,3} - [q_R [dq_R + \delta dq_R^2]] - \frac{d}{b}$$

$$\frac{\partial U_{i,2}^{BAU}}{\partial r_{i,2}} = d - kr_{i,2} + \delta dq_R = 0$$

$$-kr_{i,2} + \delta dq_R = -d = b((g_{i,2} + R_{i,2}) - \bar{y}_{i,2})$$

$$kr_{i,2} = -d - b((g_{i,2} + R_{i,2}) - \bar{y}_{i,2}) - \delta dq_R$$

$$r_{i,2}^{BAU} = [\bar{y}_{i,2} - [R_{i,2}^{BAU}] - g_{i,2}] - \frac{d}{b} - \frac{\delta dq_R}{b} \left[-\frac{1}{k} \right]$$

$$\text{BAU investment level: } r_{i,2}^{BAU} = [\bar{y}_{i,2} - [q_R R_{i,t-1}^{BAU}] - g_{i,2}] - \frac{d}{b} + \frac{\delta dq_R}{bk}$$

Time period 1

$$U_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - C(g_{i,1}) - \frac{k(r_{i,1})^2}{2} + \delta \bar{U}_{i,2}^{BAU} + \delta \bar{U}_{i,3}^{BAU},$$

$$s. t. g_{i,1} = y_{i,1} + R_{i,1} \quad R_{i,1} = q_R R_0 + r_{i,1}$$

$$U_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - d[y_{i,1} - (q_R R_0 + r_{i,2})] - \frac{k(r_{i,1})^2}{2} + \delta [R_{i,1}[dq_R + \delta dq_R^2]] + \delta [R_{i,2}dq_R]$$

$$U_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - d[y_{i,1} - (q_R R_0 + r_{i,1})] - \frac{k(r_{i,1})^2}{2} + \delta [q_R R_0 + r_{i,1}[dq_R + \delta dq_R^2]] + \delta [(q_R(q_R R_0 + r_{i,1}) + r_{i,2})dq_R]$$

$$\frac{\partial U_{i,2}^{BAU}}{\partial y_{i,1}} = -b(\bar{y}_1 - \tilde{y}_{i,1}) + d = 0$$

$$= -b(\bar{y}_1 - (y_{i,1} + (\bar{y}_1 - \bar{y}_{i,1}))) = -d$$

$$b(y_{i,1} - \bar{y}_{i,1}) = -d \rightarrow b((g_{i,1} + R_{i,1}) - \bar{y}_{i,1}) = -d$$

$$g_{i,1}^* = \bar{y}_{i,1} - (R_{i,1}) - \frac{d}{b}$$

$$\frac{\partial U_{i,1}^{BAU}}{\partial R_{i,0}} = dq_R + \delta dq_R^2 + \delta^2 dq_R^3 + \delta dq_R^3$$

$$\bar{U}_{i,1}^{BAU} = R_{i,0}[dq_R + \delta dq_R^2 + \delta^2 dq_R^3 + \delta dq_R^3]$$

$$R_{i,0}^* = [dq_R + \delta dq_R^2 + \delta^2 dq_R^3 + \delta dq_R^3]$$

$$\text{Technology stock: } R_{i,1}^{BAU} = q_R R_{i,0}^* \rightarrow R_{i,1}^{BAU} = q_R [dq_R + \delta dq_R^2 + \delta^2 dq_R^3 + \delta dq_R^3]$$

$$\text{BAU emission level: } g_{i,1}^{BAU}(R_{i,1}^{BAU}) = \bar{y}_{i,3} - [R_{i,1}^{BAU}] - \frac{d}{b}$$

$$\frac{\partial U_{i,1}^{BAU}}{\partial r_{i,1}} = d - kr_{i,1} + \delta[dq_R + \delta dq_R^2] + \delta dq_R^2 = 0$$

$$-kr_{i,1} + \delta[dq_R + \delta dq_R^2] + \delta dq_R^2 = -d = b((g_{i,1} + R_{i,1}) - \bar{y}_{i,1})$$

$$-kr_{i,1} = -b((g_{i,1} + R_{i,1}) - \bar{y}_{i,1}) - d - \delta[dq_R + \delta dq_R^2] + \delta dq_R^2$$

$$r_{i,1} = [\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] - \frac{d}{b} + \frac{\delta[dq_R + \delta dq_R^2] + \delta dq_R^2}{b} \left[-\frac{1}{k} \right]$$

$$r_{i,1}^{BAU} = [\bar{y}_{i,1} - [R_{i,1}^{BAU}] - g_{i,1}] - \frac{d}{b} + \frac{\delta[dq_R + \delta dq_R^2] + \delta dq_R^2}{bk}$$

$$\text{BAU investment level: } r_{i,1}^{BAU} = [\bar{y}_{i,1} - [q_R R_{i,0}^{BAU}] - g_{i,1}] - \frac{d}{b} + \frac{\delta[dq_R + \delta dq_R^2] + \delta dq_R^2}{bk}$$

7.3.2 BAU result – Quasi-Hyperbolic Discounting

Obtaining the intertemporal result for $y_{i,t}^{BAU}$

$$\begin{aligned}\tilde{y}_{i,t} &= y_{i,t} + (\bar{y}_t - \bar{y}_{i,t}), B_i(y_{i,t}) = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} \\ U_{i,t} &= \left[u_{i,t} + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} u_{i,\tau} \right] \\ u_{i,t} &= B_i(y_{i,t}) - C(g_{i,t}) - K(r_{i,t}) \\ U_{i,t} &= \left[\frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d[y_{i,t} - (q_r R_{t-1} + r_{i,t})] - \frac{k(r_{i,t})^2}{2} + \beta \sum_{\tau=t+1}^3 \delta^{\tau-t} u_{i,\tau} \right] \\ \frac{\partial U_{i,t}}{\partial y_{i,t}} &= \frac{-2b(\bar{y}_t - \tilde{y}_{i,t})}{2} - d = 0 \\ &\rightarrow -b(\bar{y}_t - \tilde{y}_{i,t}) = d \\ -b\bar{y}_t + b\tilde{y}_{i,t} &= d \rightarrow b\tilde{y}_{i,t} = d + b\bar{y}_t \\ \tilde{y}_{i,t} &= \frac{d + b\bar{y}_t}{b} \rightarrow \tilde{y}_{i,t} = \frac{d}{b} + \bar{y}_t\end{aligned}$$

Deciding on $r_{i,t}$ for country i is equivalent to deciding on the technology stock $R_{i,t}$.

$$R_t = \sum_{i \in N} R_{i,t} = q_R R_{t-1} + \sum_{j \in \frac{N}{I}} r_{j,t} + r_{i,t}, r_{i,t} = R_{i,t} - \frac{q_R}{n} R_{t-1}$$

Inserting $r_{i,t}$ into the temporary continuation value for country i .

$$\begin{aligned}V_{i,t} &= \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d[y_{i,t} - (q_r R_{t-1} + r_{i,t})] + \beta \delta U_{i,t}(g_{i,t}, R_{i,t}) \\ V_{i,t} &= \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - d \left[y_{i,t} - \left(q_r R_{t-1} + \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right) \right] \\ &\quad + \beta \delta \left[B_i(y_{i,t}) - d \left[y_{i,t} - \left(q_r R_{t-1} + \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right) \right] - \frac{k \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right)^2}{2} \right] \\ \frac{\partial V_{i,t}}{\partial R_{i,t}} &= d + \beta \delta (d - k) \\ \text{Max}_{R_{i,t}} V_{i,t}(g_{i,t}, R_{i,t}) &- k r_{i,t} \\ &= B_i(y_{i,t}) - d \left[y_{i,t} - \left(q_r R_{t-1} + \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right) \right) \right] - \frac{k \left(R_{i,t} - \frac{q_R}{n} R_{t-1} \right)^2}{2}\end{aligned}$$

$$\frac{\partial V_{i,t}}{\partial R_{t-1}} = dq_r - dq_r/n + kq_r/nR_{t-1} = 0 \rightarrow dq_r \left(1 + \frac{1}{n}\right) + q_r(k/n) R_{t-1}$$

$$r_{i,t}^{BAU} = R_t^{BAU} - q_R R_{t-1}^{BAU}$$

$$r_{i,t}^{BAU} = d + \beta\delta(d - k) - \left[dq_r \left(1 + \frac{1}{n}\right) + q_r(k/n) R_{t-1} \right]$$

7.3.3 Deriving the BAU result for each time period t – Quasi Hyperbolic Discounting

Time period 3

No investment stage in the last time period of the game, thus we have that $r_{i,3} = 0$ yielding the following maximization problem for the representative countries:

$$\widehat{U}_{i,3}^{BAU} = \frac{-b(\bar{y}_t - \tilde{y}_{i,t})^2}{2} - C(g_{i,t}), \quad s. t. g_{i,3} = y_{i,3} + R_{i,3}, R_{i,3} = q_R R_{i,2}$$

$$\widehat{U}_{i,3}^{BAU} = \frac{-b(\bar{y}_3 - \tilde{y}_{i,3})^2}{2} - d[y_{i,3} - (q_R R_{i,2})]$$

$$\frac{\partial \widehat{U}_{i,3}^{BAU}}{\partial y_{i,3}} = -b(\bar{y}_3 - \tilde{y}_{i,3}) + d = 0$$

$$-b\left(\bar{y}_3 - \left(y_{i,3} + (\bar{y}_3 - \bar{y}_{i,3})\right)\right) = -d$$

$$b(y_{i,3} - \bar{y}_{i,3}) = -d$$

$$b((g_{i,3} + R_{i,3}) - \bar{y}_{i,3}) = -d$$

$$g_{i,3}^* = \bar{y}_{i,3} - (R_{i,3}) - \frac{d}{b}$$

$$\frac{\partial \widehat{U}_{i,3}^{BAU}}{\partial R_{i,2}} = dq_R$$

$$\check{U}_{i,3}^{BAU} = R_{i,2} dq_R, R_{i,2}^* = dq_R$$

$$\text{Technology stock: } \widehat{R}_{i,3}^{BAU} = q_R R_{i,2}^{BAU} \rightarrow \widehat{R}_{i,3}^{BAU} = dq_R^2$$

$$\text{BAU emission level: } \widehat{g}_{i,3}^{BAU}(R_{i,3}^{BAU}) = \bar{y}_{i,3} - [dq_R^2] - \frac{d}{b}$$

Time period 2

$$\widehat{U}_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - C(g_{i,2}) - \frac{k(r_{i,2})^2}{2} + \beta\delta \check{U}_{i,3}^{BAU}$$

$$s. t. g_{i,2} = y_{i,2} + R_{i,2} \quad R_{i,2} = q_R R_{i,1} + r_{i,2}$$

$$\hat{U}_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - d[y_{i,2} - (q_R R_{i,1} + r_{i,2})] - \frac{k(r_{i,2})^2}{2} + \beta\delta(R_{i,2}dq_R)$$

$$\hat{U}_{i,2}^{BAU} = \frac{-b(\bar{y}_2 - \tilde{y}_{i,2})^2}{2} - d[y_{i,2} - (q_R R_1 + r_{i,2})] - \frac{k(r_{i,2})^2}{2} + \beta\delta((q_R R_1 + r_{i,2})dq_R)$$

$$\frac{\partial \hat{U}_{i,2}^{BAU}}{\partial y_{i,2}} = -b(\bar{y}_2 - \tilde{y}_{i,2}) + d = 0$$

$$= -b(\bar{y}_2 - (y_{i,2} + (\bar{y}_2 - \bar{y}_{i,2}))) = -d$$

$$b(y_{i,2} - \bar{y}_{i,2}) = -d$$

$$b((g_{i,2} + R_{i,2}) - \bar{y}_{i,2}) = -d$$

$$g_{i,2}^* = \bar{y}_{i,2} - (R_{i,2}) - \frac{d}{b}$$

$$\frac{\partial \hat{U}_{i,2}^{BAU}}{\partial R_{i,1}} = dq_R + \beta\delta dq_R^2, \tilde{U}_{i,2}^{BAU} = R_{i,1}[dq_R + \beta\delta dq_R^2]$$

$$R_{i,1}^* = [dq_R + \beta\delta dq_R^2]$$

$$\text{Technology stock: } \hat{R}_{i,2}^{BAU} = q_R R_{i,1}^{BAU} \rightarrow \hat{R}_{i,2}^{BAU} = q_R [dq_R + \beta\delta dq_R^2]$$

$$\text{BAU emission level: } \hat{g}_{i,2}^{BAU}(\hat{R}_{i,2}^{BAU}) = \bar{y}_{i,3} - (q_R [dq_R + \beta\delta dq_R^2]) - \frac{d}{b}$$

$$\frac{\partial \hat{U}_{i,2}^{BAU}}{\partial r_{i,2}} = d - kr_{i,2} + \beta\delta dq_R = 0$$

$$-kr_{i,2} + \beta\delta dq_R = -d = b((g_{i,2} + R_{i,2}) - \bar{y}_{i,2})$$

$$-kr_{i,2} = b(\bar{y}_{i,2} - (g_{i,2} + R_{i,2})) - d - \beta\delta dq_R$$

$$\text{BAU investment level: } \hat{r}_{i,2}^{BAU} = [\bar{y}_{i,2} - [\hat{R}_{i,2}^{BAU}] - g_{i,2}] - \frac{d}{b} + \frac{\beta\delta dq_R}{bk}$$

Time period 1

$$\hat{U}_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - C(g_{i,1}) - \frac{k(r_{i,1})^2}{2} + \beta\delta\tilde{U}_{i,2}^{BAU} + \beta\delta\tilde{U}_{i,3}^{BAU},$$

s. t. $g_{i,1} = y_{i,1} + R_{i,1}$ $R_{i,1} = q_R R_0 + r_{i,1}$

$$\hat{U}_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - d[y_{i,1} - (q_R R_0 + r_{i,1})] - \frac{k(r_{i,1})^2}{2} + \beta\delta [R_{i,1}[dq_R + \delta dq_R^2]] + \beta\delta [R_{i,2}dq_R]$$

$$\hat{U}_{i,1}^{BAU} = \frac{-b(\bar{y}_1 - \tilde{y}_{i,1})^2}{2} - d[y_{i,1} - (q_R R_0 + r_{i,1})] - \frac{k(r_{i,1})^2}{2} + \beta\delta [q_R R_0 + r_{i,1}[dq_R + \delta dq_R^2]] + \beta\delta [(q_R(q_R R_0 + r_{i,1}) + r_{i,2})dq_R]$$

$$\begin{aligned} \frac{\partial \hat{U}_{i,2}^{BAU}}{\partial y_{i,1}} &= -b(\bar{y}_1 - \tilde{y}_{i,1}) + d = 0 \\ &= -b\left(\bar{y}_1 - (y_{i,1} + (\bar{y}_1 - \bar{y}_{i,1}))\right) = -d \\ b(y_{i,1} - \bar{y}_{i,1}) &= -d \rightarrow b((g_{i,1} + R_{i,1}) - \bar{y}_{i,1}) = -d \end{aligned}$$

$$\begin{aligned} g_{i,1}^* &= \bar{y}_{i,1} - (R_{i,1}) - \frac{d}{b} \\ \frac{\partial \hat{U}_{i,1}^{BAU}}{\partial R_{i,0}} &= dq_R + \beta\delta dq_R^2 + \beta\delta^2 dq_R^3 + \beta\delta dq_R^3 \end{aligned}$$

$$\tilde{U}_{i,1}^{BAU} = R_{i,0}[dq_R + \beta\delta dq_R^2 + \beta\delta^2 dq_R^3 + \beta\delta dq_R^3]$$

$$R_{i,0}^* = [dq_R + \beta\delta dq_R^2 + \beta\delta^2 dq_R^3 + \beta\delta dq_R^3]$$

Technology stock: $\hat{R}_{i,1}^{BAU} = q_R R_{i,0}^* \rightarrow \hat{R}_{i,1}^{BAU} = q_R [dq_R + \beta\delta dq_R^2 + \beta\delta^2 dq_R^3 + \beta\delta dq_R^3]$

BAU emission level: $\hat{g}_{i,1}^{BAU}(\hat{R}_{i,1}^{BAU}) = \bar{y}_{i,3} - [q_R [dq_R + \beta\delta dq_R^2 + \beta\delta^2 dq_R^3 + \beta\delta dq_R^3]] - \frac{d}{b}$

$$\frac{\partial \hat{U}_{i,1}^{BAU}}{\partial r_{i,1}} = d - kr_{i,1} + \beta\delta [dq_R + \delta dq_R^2] + \beta\delta dq_R^2 = 0$$

$$-kr_{i,1} + \beta\delta [dq_R + \delta dq_R^2] + \beta\delta dq_R^2 = -d = b((g_{i,1} + R_{i,1}) - \bar{y}_{i,1})$$

$$b(\bar{y}_{i,1} - (g_{i,1} + R_{i,1})) - d - \beta\delta [dq_R + \delta dq_R^2] + \beta\delta dq_R^2 = kr_{i,1}$$

$$r_{i,1} = [\bar{y}_{i,1} - (g_{i,1} + R_{i,1})] - \frac{d}{b} - \frac{\beta\delta [dq_R + \delta dq_R^2] + \beta\delta dq_R^2}{b} \left[-\frac{1}{k} \right]$$

BAU investment level: $\hat{r}_{i,1}^{BAU} = [\bar{y}_{i,1} - [\hat{R}_{i,1}^{BAU}] - g_{i,1}] - \frac{d}{b} - \frac{\beta\delta [dq_R + \delta dq_R^2] + \beta\delta dq_R^2}{bk}$

7.4 Deriving the result of the self-enforcing agreement

7.4.1 Self-enforcing agreement - Exponential discounting

The objective function and utility function of country i under the self-enforcing agreement is given respectively by the following equations when using exponential discounting:

$$U_{i,t} = u_{i,t} + \sum_{\tau=t}^T \delta^{\tau-t} u_{i,\tau}$$

$$u_{i,t} = B_i(y_{i,t}) - C(g_t) - kr_{i,t}$$

$$s. t. g_{i,t}^{BAU}, r_{i,t}^{BAU}, g_{i,t}^{OPT}, r_{i,t}^{OPT} \text{ for } t = 1, 2, 3.$$

Time period 3

In order for country i to be willing to cooperate during the third time period of the game the following inequality has to hold:

$$U_3^{OPT} = \left\{ \left[-\frac{b}{2} \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \right\} \geq \left\{ \left[-\frac{b}{2} \left((g_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{BAU} - d(n-1) g_3^{OPT} \right\}$$

We define the left-hand side (LHS) as the cooperating country's strategy in the given time period, which will be followed as long as the net-benefit of doing so is equal to or greater than the net-benefit of deviating and acting as a non-cooperator. In order to analyze the outcome of the self-enforcing agreement regarding the performed investment and emission level of the cooperating countries we derive the first order condition of the LHS with respect to the given variables. Because there is no investment stage in the last time period of the game, we only find the F.O.C of the LHS with respect to the emission level when deriving the result for time period 3.

$$\begin{aligned} LHS_3 &= \left[-\frac{b}{2} \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \\ \frac{\partial LHS_3}{\partial g_3^{OPT}} &= -b \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) - dn = 0 \\ -b \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] + R_3 - \bar{y}_3 &= dn \\ -b \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] &= dn \end{aligned}$$

→ Solving for g_3^{OPT} yielding the minimum emission level performed by the cooperators in time period 3:

$$FB \text{ emission level under the agreement: } g_3^{OPT*} = -b \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] - dn$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial g_3^{OPT*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial g_3^{OPT*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial g_3^{OPT*}}{\partial \delta} < 0$$

Time period 2

In time period 2 of the game the second investment stage takes place, the following inequality has to hold in order for the cooperating countries to hold their commitment to the self-enforcing agreement:

$$\begin{aligned} U_2^{OPT} &= \left(\left[-\frac{b}{2} \left((g_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{OPT} - knr_2^{OPT} \right) \\ &\quad + \delta \left(\left[-\frac{b}{2} \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \right) \\ &\geq \\ &\left(\left[-\frac{b}{2} \left((g_2^{BAU} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{BAU} - d(n-1)g_2^{OPT} - knr_2^{BAU} - k(n-1)r_2^{OPT} \right) \\ &\quad + \delta \left(\left[-\frac{b}{2} \left((g_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{BAU} - d(n-1)g_3^{OPT} \right) \end{aligned}$$

We define the left-hand side (LHS) as the strategy of the cooperating country, which will be followed as long as the net-benefit of doing so is greater or equal to the net-benefit of acting as a non-cooperator. In order to analyze the investment and emission level performed by the cooperating countries we derive the first order condition of the LHS with respect to the emission level and the investment level.

$$\begin{aligned}
LHS_2 &= \left(\left[-\frac{b}{2} \left((g_2^{BAU} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{OPT} - knr_2^{OPT} \right) \\
&\quad + \delta \left(\left[-\frac{b}{2} \left((g_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \right) \\
\frac{\partial LHS_2}{\partial g_2^{FB}} &= -b \left((g_2^{BAU} + R_2) - \bar{y}_2 \right) - dn = 0 \\
\frac{\partial LHS_2}{\partial g_2^{OPT}} &= -b \left[\bar{y}_2 - (R_2) \right] + \delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] + R_2 - \bar{y}_2 - dn = 0
\end{aligned}$$

→ Solving for g_2^{OPT} yielding the minimum emission level performed by the cooperators in time period 2:

$$FB \text{ emission level under the agreement: } g_2^{FB*} = -b \left[\delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] \right] - dn$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial g_2^{FB*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial g_2^{FB*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial g_2^{FB*}}{\partial \delta} < 0$$

Time period 1

In time period 1 of the game the first investment stage takes place, where the following inequality has to hold in order for the cooperating countries to act committed to the self-enforcing agreement:

$$\begin{aligned}
U_1^{OPT} &= \left(\left[-\frac{b}{2} \left((g_1^{OPT} + R_1) - \bar{y}_1 \right) \right]^2 - dn g_1^{OPT} - knr_1^{OPT} \right) \\
&\quad + \delta \left(\left[-\frac{b}{2} \left((g_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{OPT} - knr_2^{OPT} \right) \\
&\quad + \delta \left(\left[-\frac{b}{2} \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \right)
\end{aligned}$$

$$\begin{aligned}
& \geq \\
& \left(\left[-\frac{b}{2} \left((g_1^{BAU} + R_1) - \bar{y}_1 \right) \right]^2 - dn g_1^{BAU} - d(n-1) g_1^{OPT} - knr_1^{BAU} - k(n-1) r_1^{OPT} \right) \\
& + \delta \left(\left[-\frac{b}{2} \left((g_2^{BAU} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{BAU} - d(n-1) g_2^{OPT} - knr_2^{BAU} - k(n-1) r_2^{OPT} \right) \\
& + \delta \left(\left[-\frac{b}{2} \left((g_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{BAU} - d(n-1) g_3^{OPT} \right) \\
& LHS_1 = \left(\left[-\frac{b}{2} \left((g_1^{OPT} + R_1) - \bar{y}_1 \right) \right]^2 - dn g_1^{OPT} - knr_1^{OPT} \right) \\
& + \delta \left(\left[-\frac{b}{2} \left((g_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn g_2^{OPT} - knr_2^{OPT} \right) \\
& + \delta \left(\left[-\frac{b}{2} \left((g_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn g_3^{OPT} \right) \\
& \frac{\partial LHS_1}{\partial g_1^{OPT}} = -b \left((g_1^{OPT} + R_1) - \bar{y}_1 \right) - dn = 0 \\
& = -b \left[\bar{y}_1 - (R_1) \right] + \delta \left[\bar{y}_2 - (R_2) \right] + \delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] + R_1 - \bar{y}_1 - dn = 0
\end{aligned}$$

→ Solving for g_1^{OPT} yielding the minimum emission level performed by the cooperators in time period 1:

$$\begin{aligned}
& \text{FB emission level under the agreement: } g_1^{OPT*} \\
& = -b \left[\delta \left[\bar{y}_2 - (R_2) \right] + \delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\delta)}{b\delta^2} \right] \right] - dn
\end{aligned}$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial g_1^{OPT*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial g_1^{OPT*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial g_1^{OPT*}}{\partial \delta} < 0$$

7.4.2 Self-enforcing agreement - Quasi-hyperbolic discounting

The objective function and utility function of country i under the self-enforcing agreement is given respectively by the following equations when using quasi-hyperbolic discounting:

$$\begin{aligned}\widehat{U}_{i,t} &= \left[u_{i,t} + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u_{i,\tau} \right] \\ u_{i,t} &= B_i(y_{i,t}) - C(g_t) - kr_{i,t} \\ \text{s. t } &\widehat{g}_{i,t}^{BAU}, \widehat{r}_{i,t}^{BAU}, \widehat{g}_{i,t}^{OPT}, \widehat{r}_{i,t}^{OPT} \text{ for } t = 1, 2, 3.\end{aligned}$$

The parameters take on the value $0 < \beta < 1$ and $0 < \delta \leq 1$, furthermore, the quasi-hyperbolic discount function in discrete time can be described by the following structure: $\{1, \beta\delta^1, \beta\delta^2\}$.⁵³

Time period 3

In the last time period, there is no investment stage, even though the following inequality has to hold in order for country i to be willing to cooperate during the last time period of the game:

$$\begin{aligned}\widehat{U}_3^{OPT} &= \left\{ \left[-\frac{b}{2} \left((\widehat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - d n \widehat{g}_3^{OPT} \right\} \geq \\ &\left\{ \left[-\frac{b}{2} \left((\widehat{g}_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - d n \widehat{g}_3^{BAU} - d(n-1) \widehat{g}_3^{OPT} \right\}\end{aligned}$$

We define the left-hand side of the inequality as the cooperating country's strategy under the self-enforcing agreement in time period 3, which will be followed as long as the net-benefit of doing so is greater than or equal to the net-benefit of acting as a non-cooperator.

$$\widehat{LHS}_3 = \left\{ \left[-\frac{b}{2} \left((\widehat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - d n \widehat{g}_3^{OPT} \right\}$$

We take the first order condition of the LHS with respect to the emission level of the cooperating countries in order to find the performed emission level under the self-enforcing agreement.

⁵³ Laibson, "Life-cycle consumption and hyperbolic discount function."

$$\begin{aligned}\frac{\partial LHS_3}{\partial \hat{g}_3^{OPT}} &= -b \left((\hat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) - dn = 0 \\ &= -b \left[[\bar{y}_3 - (R_3)] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right] + R_3 - \bar{y}_3 \right] - dn = 0\end{aligned}$$

→ Solving for \hat{g}_3^{OPT} yield the minimal emission level performed by the cooperators in time period 3:

$$FB \text{ emission level under the agreement: } \hat{g}_3^{OPT*} = -b \left[\left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right] \right] - dn$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_3^{OPT*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_3^{OPT*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial \hat{g}_3^{OPT*}}{\partial \beta\delta} < 0$$

Time period 2

In time period 2 of the game the second investment stage takes place, where the following inequality has to hold in order for the cooperating countries to act committed to the self-enforcing agreement:

$$\begin{aligned}U_2^{OPT} &= \left(\left[-\frac{b}{2} \left((\hat{g}_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{OPT} - kn\hat{r}_2^{OPT} \right) \\ &\quad + \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{OPT} \right) \\ &\geq \\ &\left(\left[-\frac{b}{2} \left((\hat{g}_2^{BAU} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{BAU} - d(n-1)\hat{g}_2^{OPT} - kn\hat{r}_2^{BAU} - k(n-1)\hat{r}_2^{OPT} \right) \\ &\quad + \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{BAU} - d(n-1)\hat{g}_3^{OPT} \right)\end{aligned}$$

Where we define the left-hand side as the strategy of the cooperating country in time period 2. The strategy will be followed as long as the net-benefit of doing so is greater than or equal to the net-benefit of acting as a non-cooperator.

$$\begin{aligned} \widehat{LHS}_2 = & \left(\left[-\frac{b}{2} \left((\hat{g}_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{OPT} - kn\hat{r}_2^{OPT} \right) \\ & + \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{OPT} \right) \end{aligned}$$

We now derive the first order condition of the LHS with respect to the emission level of the cooperating countries in order to analyze the outcome of the self-enforcing agreement.

$$\begin{aligned} \frac{\partial \widehat{LHS}_2}{\partial \hat{g}_2^{OPT}} &= -b \left((\hat{g}_2^{OPT} + R_2) - \bar{y}_2 \right) - dn = 0 \\ &= -b \left[[\bar{y}_2 - (R_2)] + \delta[\bar{y}_3 - (R_3)] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right] + R_2 - \bar{y}_2 \right] - dn = 0 \end{aligned}$$

→ Solving for \hat{g}_2^{OPT} yield the minimal emission level performed by the cooperators in time period 2:

$$FB \text{ emission level under the agreement: } \hat{g}_2^{OPT*} = -b \left[\delta[\bar{y}_3 - (R_3)] \left[-\frac{dn(1 + 2\beta\delta)}{b\beta\delta^2} \right] \right] - dn$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_2^{OPT*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_2^{OPT*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial \hat{g}_2^{OPT*}}{\partial \beta\delta} < 0$$

Time period 1

In time period 1 of the game the first investment stage takes place, where the following inequality has to hold in order for the cooperating countries to act committed to the self-enforcing agreement.

$$\begin{aligned}
U_1^{OPT} &= \left(\left[-\frac{b}{2} \left((\hat{g}_1^{OPT} + R_1) - \bar{y}_1 \right) \right]^2 - dn\hat{g}_1^{OPT} - kn\hat{r}_1^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{OPT} - kn\hat{r}_2^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{OPT} \right) \\
&\geq \\
&\left(\left[-\frac{b}{2} \left((\hat{g}_1^{BAU} + R_1) - \bar{y}_1 \right) \right]^2 - dn\hat{g}_1^{BAU} - d(n-1)\hat{g}_1^{OPT} - kn\hat{r}_1^{BAU} - k(n-1)\hat{r}_1^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_2^{BAU} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{BAU} - d(n-1)\hat{g}_2^{OPT} - kn\hat{r}_2^{BAU} - k(n-1)\hat{r}_2^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{BAU} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{BAU} - d(n-1)\hat{g}_3^{OPT} \right)
\end{aligned}$$

We define the left-hand side (LHS) as the strategy of the cooperating countries in time period 1, which will be followed as long as the net-benefit of doing so exceeds or is equal to the net-benefit of deviating and acting as a non-cooperator.

$$\begin{aligned}
\widehat{LHS}_1 &= \left(\left[-\frac{b}{2} \left((\hat{g}_{i,1}^{OPT} + R_{i,1}) - \bar{y}_1 \right) \right]^2 - dn\hat{g}_1^{OPT} - kn\hat{r}_1^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_2^{OPT} + R_2) - \bar{y}_2 \right) \right]^2 - dn\hat{g}_2^{OPT} - kn\hat{r}_2^{OPT} \right) \\
&+ \beta\delta \left(\left[-\frac{b}{2} \left((\hat{g}_3^{OPT} + R_3) - \bar{y}_3 \right) \right]^2 - dn\hat{g}_3^{OPT} \right)
\end{aligned}$$

We derive the first order conditions of LHS with respect to the emission level performed by the cooperating countries in order to analyze the outcome of the self-enforcing agreement:

$$\begin{aligned}
\frac{\partial \widehat{LHS}_1}{\partial \hat{g}_1^{OPT}} &= -b \left((\hat{g}_{i,1}^{OPT} + R_{i,1}) - \bar{y}_1 \right) - dn = 0 \\
&= -b \left[\bar{y}_1 - (R_1) \right] + \delta \left[\bar{y}_2 - (R_2) \right] + \delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\beta\delta)}{b\beta\delta^2} \right] + R_{i,1} - \bar{y}_1 - dn = 0
\end{aligned}$$

→ Solving for \hat{g}_1^{OPT} yield the minimal emission level performed by the cooperators in time period 1:

$$\begin{aligned}
&\text{FB emission level under the agreement: } \hat{g}_1^{OPT*} \\
&= -b \left[\delta \left[\bar{y}_2 - (R_2) \right] + \delta \left[\bar{y}_3 - (R_3) \right] \left[-\frac{dn(1+2\beta\delta)}{b\beta\delta^2} \right] \right] - dn
\end{aligned}$$

The more cooperators the smaller is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_1^{OPT*}}{\partial n} > 0$$

The higher the marginal cost of emissions the lower is the emission level performed under the agreement:

$$\frac{\partial \hat{g}_1^{OPT*}}{\partial d} > 0$$

The more patient the cooperating countries are the smaller is the performed emission level performed under the agreement:

$$\frac{\partial \hat{g}_1^{OPT*}}{\partial \beta\delta} < 0$$

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