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**Stochastic Event
Simulation of Oil
Recovery Projects**

Master thesis

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Abstract

Risk handling of oil recovery projects has become more important as the biggest and least complicated oil fields have been exploited. As exploitation of smaller or more complex oil fields are not as obviously profitable, the need for describing the risk of the recovery projects have become more apparent. In this thesis the uncertainties of the reservoir, expenses, oil prices and more are combined to produce joint risk profiles. The uncertainties may be studied at several levels. Production profile, income profile, expenses profiles.

An application that produces distributions for net present value, internal rate of return, capital efficiency, and profiles for cash flow, oil recovery, oil prices, income and various expenses, has been developed. The input needed for the simulations are stochastic variables that are available at the early stages of an oil recovery project. Defining distributions for each stochastic variable, you are able to study the effect that various variables have on the results. Changing exploitation plan from a semi-submersible rig to a drilling ship, greatly decreases capital expenses and abandonment expenses. In turn, the maximum processing capacity drops and field operation time increases, reducing the net present value.

Preface

When I signed up to the subject mat-stk2010, *Project in Finance, Insurance and Risks*, spring semester of 2004, little did I know about the fact that I would end up writing about oil production, and certainly not that it would become the basis of my masters thesis. By being an informatics student, my tutor in the subject, Arne Bang Huseby, and later, my master thesis tutor, assigned a project to me where I could take advantage of my programming skills. The task was to implement discrete events to simulate production profiles in oil fields. Since then, the work, and the application made, have been totally rewritten and extended, featuring both income and expenses in addition to an extended model for the oil recovery.

I am a student of Teknisk Programvare, Profesjonsstudiet, at the Institute of Informatics, University of Oslo. I began my studies August 2001 and are due to finish my masters degree in December 2005, half a year prior to standard. The deadline of this thesis is 1. November 2005.

I wish to thank my tutors, Hans Petter Langtangen and Arne Bang Huseby. Thanks to Eivind Damsleth, for helping me in getting a relevant summer job in Norsk Hydro ASA. Specially thanks to Thore Thuestad and Gudmund Kleiven in Norsk Hydro Oil&Energy for welcoming me, and for all the very valuable input I got. I felt respected and the information and suggestions I got holds a big part of this thesis. Explicitly, that is the STOIP, RF, RCI, Expenses and Maximum Processing Capacity. Also thanks to Eirik A.Berg, Hydro, helping me making a formula for distributing total reserves recoverable to the oil wells total reserves recoverable.

At last, I wish to issue special thanks to my tutor, Arne Bang Huseby, for picking this project for me and inspiring me with your enthusiasm. I have appreciated your always quick responses and thoroughly feedback.

A resource page for the thesis may be found at <http://master.dontsay.no>. Contact information is located at my personal home page <http://dontsay.no>. The program should be available as a runnable program on the resource page as of 1.Nov 2005, using Java web start(javaws). The application is written in Jython, Java and Fortran 77 (Duplicate java code has been written as well, for web compatibility).

Contents

1	Introduction	9
1.1	What is the Purpose of Stochastic Event Simulation of Oil Recovery Projects?	10
1.2	What are the Goals of Stochastic Event Simulation of Oil Recovery Projects?	10
1.2.1	Should the oil field be exploited?	10
1.2.2	What field exploitation plan should be chosen?	11
1.3	What partial goals do we have?	11
1.4	How should this document be read?	12
2	Stochastic Simulation	15
2.1	How are the distributions and profiles built from the stochastic variables?	15
2.2	What are discrete events?	17
2.2.1	Why choose discrete event simulation over functional simulation?	17
2.3	What information does stochastic event simulations give us?	17
2.4	Why represent the uncertainties with percentiles?	18
3	Production Profile	19
3.1	How is this chapter set up?	19
3.2	What stochastic variables do we have?	19
3.3	How are the discrete events implemented?	21
3.3.1	What would an application of discrete events in the stochastic simulation of oil production look like?	23
3.4	How is the oil well model defined?	25
3.4.1	How much oil can an Oil Well recover in a given time period?	25

3.4.2	What are the recoverable reserves of an oil well?	25
3.4.2.1	What is STOIIP?	26
3.4.2.2	What is RCI?	26
3.4.2.3	What is RF?	27
3.4.2.4	How can the total reserves recoverable be distributed onto the oil wells?	27
3.4.2.5	When should STOIIP and RF be preferred over setting distributions for each oil well respectively?	27
3.4.3	When are the oil wells phased in?	27
3.5	What is the oil field model?	28
3.5.1	What is the recoverable reserves of the oil field?	29
3.5.2	What is the Oil Field Production Rate?	29
3.5.2.1	How does a change in the production state occur?	29
3.5.2.2	What production rate do we have off plateau after insertion of the oil well?	30
3.5.3	What is maximum processing capacity?	31
3.5.3.1	How can we determine the state and production rate after phasing in a new oil well at below maximum capacity?	32
3.6	What does the simulated production profiles look like?	32
3.6.1	What can be said about the spread of the production profile simulations?	34
4	Income	35
4.1	How are the oil prices developed?	35
4.2	How is the income calculated?	36
5	Expenses	37
5.1	What expenses do we have?	37
5.2	How are Expenses related to the Production?	38
5.3	How are capital expenses implemented?	39
5.4	How are abandonment expenses implemented?	40
5.5	How are drilling expenses implemented?	40
5.6	How are operating expenses implemented?	40
5.7	How are tariff expenses implemented?	41
5.8	What are the Taxation Expenses?	41

6	Finance	43
6.1	How is the cash flow represented?	43
6.2	How is the net present value represented?	44
6.3	What is capital efficiency?	45
6.4	What is internal rate of return?	45
7	Concluding remarks	47
7.1	Notes for the reader	47
7.2	Known errors in the application	47
7.3	Future Development	48
A	Deductions for the Production Profile Calculations	51
A.1	What is the oil field production rate excluding the maximum processing capacity?	51
A.2	What is the oil field production rate with a maximum process- ing capacity cap?	58
B	ProductionProfiler - The Application	65
C	A Study Of Recovery Factor And Reservoir Complexity In- dex	69
D	Oil Recovery Project - Polar Star	71

Chapter 1

Introduction

Pessimism in estimations prior to the boom in the oil prices, caused many oil companies to calculate estimated income using fixed oil prices of \$16 per barrel, fearing \$10 per barrel. A consequence of this was that many oil fields were discarded, fields that in reality would have been very profitable. This is but one example. The anxiety factor for failure also caused the probability distributions for reserves and the like to be given lower estimates than they probably should have. In sum, this form of risk aversity obviously discarded a lot of projects. As there are few big and easy oil fields known left, exploitation of smaller or more complex oil fields have become more important. They are not as obviously profitable and the need for describing the risk of the projects is vital for hedging purposes.

Most of what I have learned I have been taught in person. References to articles in this text are near to absent. Before my work at Norsk Hydro ASA, I had not read any related articles. At Hydro I there were many, but as I signed a confidentiality agreement there are no documents to refer to. Most of the material in this text, I had prior to the Hydro job. The material that i did not have prior is not considered confidential and was granted use by my tutors in Hydro. Gas, associated gas and CO_2 are not covered by this text, and as income potential it is totally disregarded. Expenses associated to these are supposed covered in the expenses model.

1.1 What is the Purpose of Stochastic Event Simulation of Oil Recovery Projects?

The main purpose is to provide a fast and sufficient financial basis for the decision makers to answer these two fundamental questions:

1. Should the oil field be exploited?
2. What field exploitation plan should be chosen?

1.2 What are the Goals of Stochastic Event Simulation of Oil Recovery Projects?

The main goals are to create the elements of the financial basis for the decision makers. These are the

- Net Present Value, NPV (6.2)
- Cash Flow (6.1)
- Internal Rate of Return, IRR (6.4)
- Capital Efficiency, Capeff (6.3)

We want the expected values of them and their associated uncertainties. These elements are vital tools for the decision makers, representing the risk and potentials of the investment project. Using stochastic simulation is a good way to find the risk, represented as probability distributions of uncertainty. Reaching these main goals, we have what is needed to answer the questions of the purpose.

1.2.1 Should the oil field be exploited?

Given the estimations and the uncertainty descriptions of the financial basis, the risk aversity of the decision makers will determine whether or not to integrate this investment project in their portfolio. The rate of return of money invested in an oil project is important in a risk perspective, demanding a relatively high value, normally well above 15%, as the expenses of the oil exploitation is very hard to predict, and the potential hard to estimate.

1.2.2 What field exploitation plan should be chosen?

There exists several different oil field exploitation possibilities. All oil fields are different and the exploitation plan must be customized for each one. However, there are usually more than one solution for the different elements, for example:

- Jack-up or semi-submersible oil rig, platform or drilling ships?
- Ships or pipes for the oil transportation?
- Unmanned or manned platforms?

Simulating using the input data of the different field development plans, respectively, we get different results, which may be compared and weighted against each other. Equivalently, stochastic input variables may be altered to study the effect the variable has on the profitability. For example, suppose that if we used a new method for drilling the oil wells, we could reduce the drilling expenses and/or the drilling time. Alternatively, maybe the new method is more expensive, but reduces the number of oil wells needed. The aim of the development of the stochastic event simulation application is for the user to be able to study this, by altering the input variables.

1.3 What partial goals do we have?

It is a long way to go to reach the main goals. To reach them, we need to find the:

- Production Profile (Chapter 3 and Appendix A)
 - Recoverable Reserves (3.4.2)
 - Oil Well, and Oil Field, Oil Recovery Speed (3.4 -3.5).
- Income (Chapter 4)
 - Oil Prices (4.1)
- Expenses (Chapter 5)

- Capital Expenses, CAPEX (5.3)
- Abandonment Expenses, ABDEX (5.4)
- Operating Expenses, OPEX (5.6)
- Drilling Expenses, DRILLEX (5.5)
- Tariff Expenses, TAREX (5.7)
- Tax Expenses, TAXEX (5.3)

The idea is to use the uncertainty in the low level variables, such as total reserves, recovery factor, recovery rate of oil wells, processing rate of the rig and so on, to build the joint risk profiles. Using stochastic variables and discrete events (3.3) for the oil production and repeating the simulations many times, we obtain production profile. This profile and its distribution can we study the uncertainties of. (2.4)

By splitting up variables that are the sources of the greatest uncertainties, into their lower level parts, their joint distribution will more accurately describe the uncertainty of the variable. For example, the production in an oil rig is determined by the production of the oil wells and the rigs maximum processing or transport capacity, amongst others.

There are other elements of an oil field exploitation project as well. Gas, associated gas and CO_2 are not considered here, the thesis is limited to discussing the oil recovery. The expenses associated with the recovery, as gas injection etc. are supposed covered by the expenses model, especially the operating expenses.

1.4 How should this document be read?

The target reader of this document is an academic or professional person within the area of stochastic simulation or specialist on early stage oil recovery projects. It is also written to serve as basis for an eventual successor wishing to further develop the various models and/or the application [Production Profiler].

The requirements for an oil project suitable for this text, is that it is at the very early stages of planning and development. That is, an oil field is supposed revealed, and reservoir engineers to have given their estimates about reserves, field complexity etc. It is to be determined whether or not to further invest in the oil field. This is a stage referred to by some as

“tech-øk” in the oil industry. The “tech-øk” method has procedures highly dominated by expectations and thumb rules, which works great, but does not give a good description of the uncertainties. This document presents a very different procedure, focusing on uncertainty representations of the variables and joining them.

My thesis treats the issue of implementing stochastic event simulations to form oil production profiles, oil prices, income, expenses and more. The application provides graphical layout and analysis of these [Production Profiler]. Appendix B describes the application in short and an example input file for it is given in Appendix D.

I will start generally with stochastic event simulations (Chapter 2), then introduce the model for building the production model (Chapter 3). The calculations for the results presented there may be found in Appendix A. Having explained the production profile, I will introduce a model for oil prices and income (Chapter 4), then the expenses model (Chapter 5) and finally, I will introduce the finance model (Chapter 6).

Chapter 2

Stochastic Simulation

2.1 How are the distributions and profiles built from the stochastic variables?

In figure 2.1 I intend to illustrate the path of the stochastic in the simulations, combining to joint distributions and/or profiles. The bottom levels in the illustration, are the stochastic variables defined as input variables. When the simulations start we draw from these. The algorithms decide the outcomes and builds the distributions and profiles for each simulation. As the income side and the expenses side of the simulations are realized, they combine to the four desired results: the NPV, Capeff and IRR probability distributions and the cash flow profile.

The stochastic variables discussed in this paper are all represented in the diagram. The diagram is not all complete, it is huge, so duration and start time variables for the expenses and the CAPEX, TAXEX and TAREX are only mentioned in text. They are constructed in the same manner as the others, as you may expect. The stochastic variables of the expenses are covered in chapter 5 and the realization of the production profile is deeply covered in chapter 3 and appendix A. A fictive oil recovery project “Polar Star” (Appendix D) , which are the source of the graphs, are given as an example of a possible input file for the various stochastic variables.

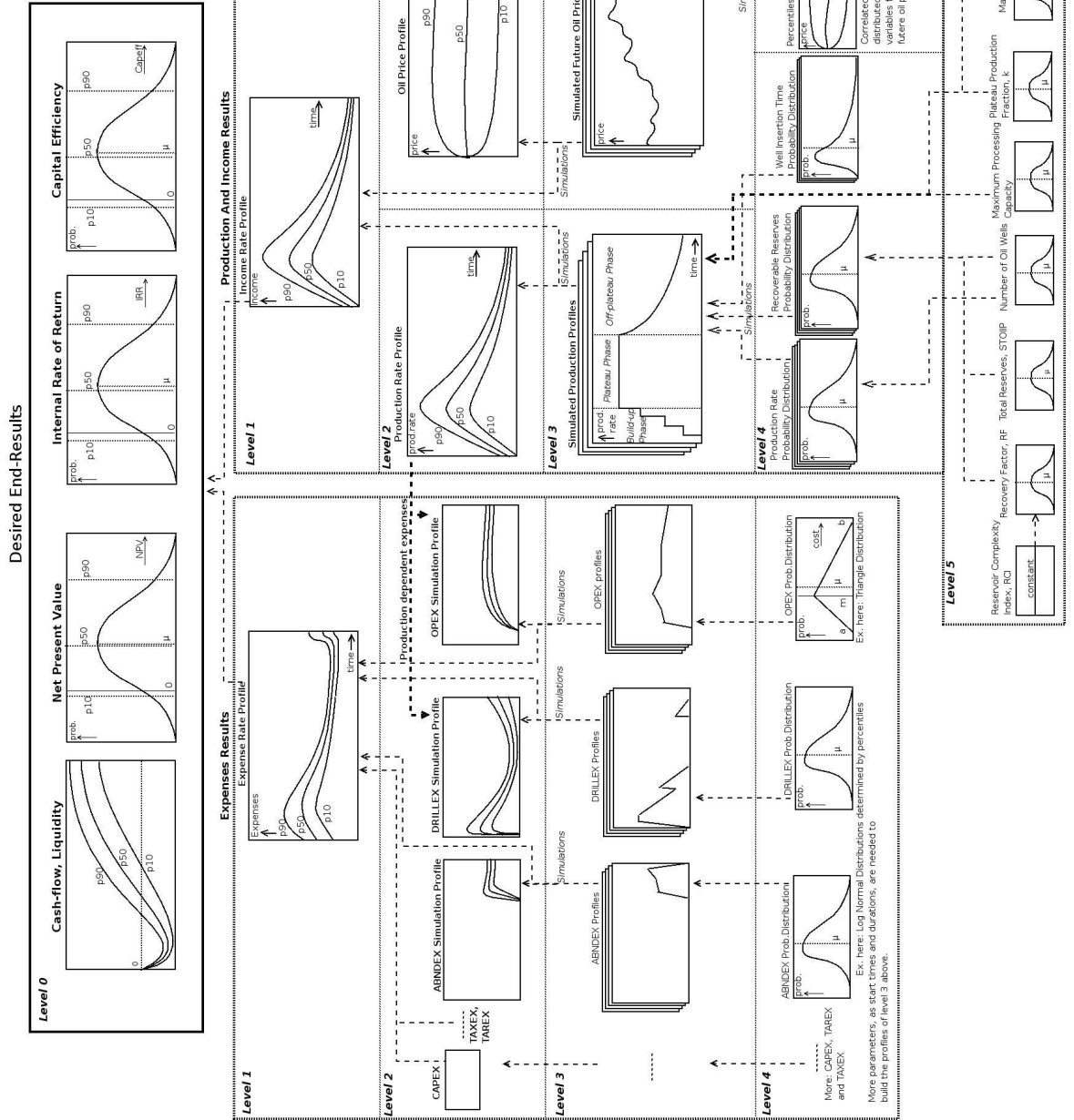


Figure 2.1: Simulations from stochastic variables, building distributions and profiles.

2.2 What are discrete events?

Discrete events are events set to happen at given time. Many such events have already been mentioned, such as insertion of an oil well. The events need not be physical ones as this one, but can be others, as change in the state of production, or abandonment event of the oil field due to high costs and low income. Events may cause other events to occur. For example, insertion of an oil well is an event and creates a plateau end event, or causes the time of the plateau end event to change (3.5.2).

2.2.1 Why choose discrete event simulation over functional simulation?

Functional simulation is faster than discrete event simulation, but sacrifices flexibility. In functional simulation you need to define a finite set of possible scenarios, which in our case is indeterminable. The number of events cannot be determined in advance as events may or may not create additional events, and their order may vary or change. These are essential properties of the model, and cannot be satisfactorily implemented with functional simulation.¹

2.3 What information does stochastic event simulations give us?

For each simulation, we end up with a, supposedly, possible realization of an oil recovery project. Repeating this, we will get more and different realizations, and with the increasing number of simulations, a pattern will present itself. This gives us the ability to describe the likelihoods of success of the project. By varying the probability distributions of the stochastic input variables the simulations will provide different likelihoods.

¹Some of the expenses are subject to functional simulation, as they are supposed known ahead or that the irregularities are represented in the uncertainty.

2.4 Why represent the uncertainties with percentiles?

Studying the results from the simulations with the use of percentiles is valuable as it gives a good impression of the spread and density of the realizations in a profile. By choosing many percentiles, combined with a high number of simulations, the shape will quickly reveal itself. Other issues, as mentioned in section 1.2.2, the changing of input values, will reflect itself in the spread or value of the percentiles. For example, changing the uncertainty and expectation of the production potentials or recovery potentials of the Oil Wells, will have a high impact on the distribution. This property should then serve as a useful tool in identifying the variables with the greatest impact on the overall uncertainties.

Chapter 3

Production Profile

3.1 How is this chapter set up?

The event simulation of the production profiles are very complex. This chapter is written top-down, discussing the overall issues first, getting more and more specific for each section through 3.2 to 3.5. We start by declaring the stochastic variables in the oil recovery simulations, applying discrete events, then discuss the use of them in the Oil Well Model. At the end we put it all together in the Oil Field Model. The chapter is limited to explaining the model and the states of the production, based on results of Appendix A. This appendix contains all the necessary calculations, based on this chapter, presenting the results in a *claim - proof* manner.

The idea behind the top-down approach of this chapter is to write each section so that they are only dependent on the theory of the above layers (figure 3.1). This way, it makes it easier to find and study the areas of the readers interest at later points, and the reader may himself choose how deep into the details he want to go, following the links to appendix A.

3.2 What stochastic variables do we have?

The number of stochastic variables of the production are only limited by the desired complexity of the model. The most important probability distributions for these, are the constant, uniform, triangle, normal, lognormal and exponential distributions, all implemented in the application as valid representations for uncertainty of each the stochastic input variable, respectively.

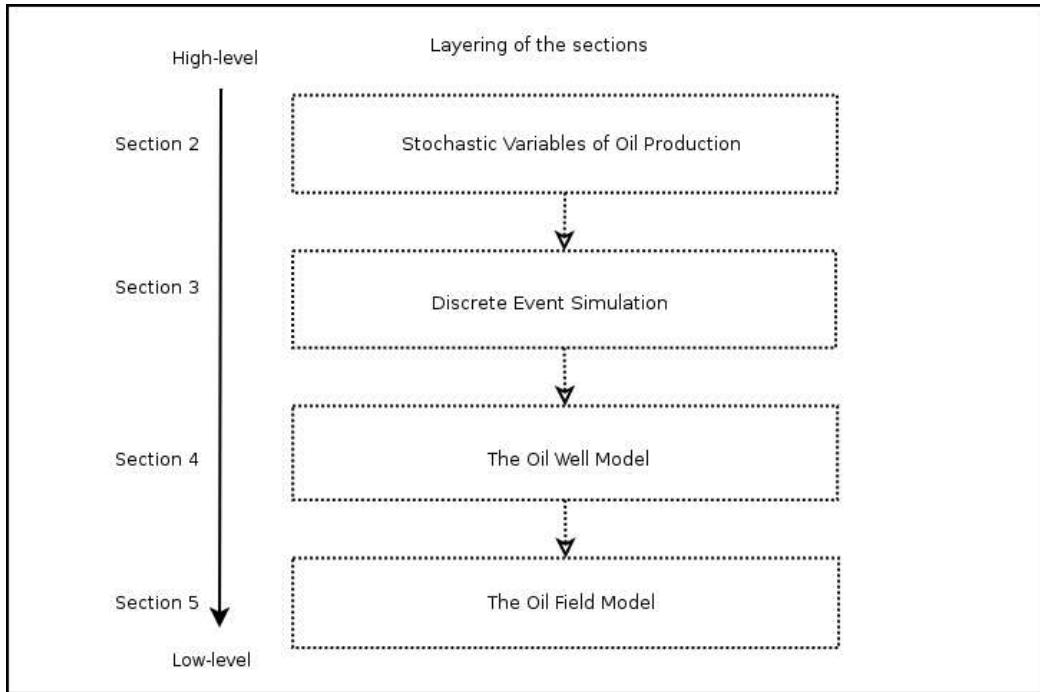


Figure 3.1: Progress chart. Layering of this chapter.

The main objects of my simulations are the *Oil Wells*. Combined with an oil rig and transportation utilities they form the *Oil Field*. The chosen complexity of my model uses the stochastic variables listed below.

Production Rate Defined as the amount of oil recovered at each discrete time step(day, month etc.) for each respective Oil Well (3.4.1, 3.5.2).

Recoverable Reserves Reserves recoverable in the oil field (3.5.1) or, alternatively, for each oil well respectively(3.4.2). This distribution may be constructed from two other stochastic variables:

Total Reserves The total reserves in the oil field (3.4.2.1).

Recovery Factor The fraction of total reserves that are recoverable (3.4.2.3).

Phase In Time The insertion time of an Oil Well, also referred to as a *phase in*, is defined to be the time of the first oil recovered in that oil

well (3.4.3). A widely used distribution for start times, duration times and the like, is the exponential distribution.

Maximum Processing Capacity Defined to be the value of the “bottle neck” related to the oil recovery. This is usually limited to the maximum processable oil recovery speed of the oil rig. It may also be that it is the transportation capacity that is the limiting factor.

Plateau Production The sum of the maximum production rate potentials of the all the oil wells, respectively.

Plateau Production Fraction Defined as the portion of oil that may be recovered at the oil wells full production capacity, before pressure and flow drops. The declination in production rate is defined as an exponential function.

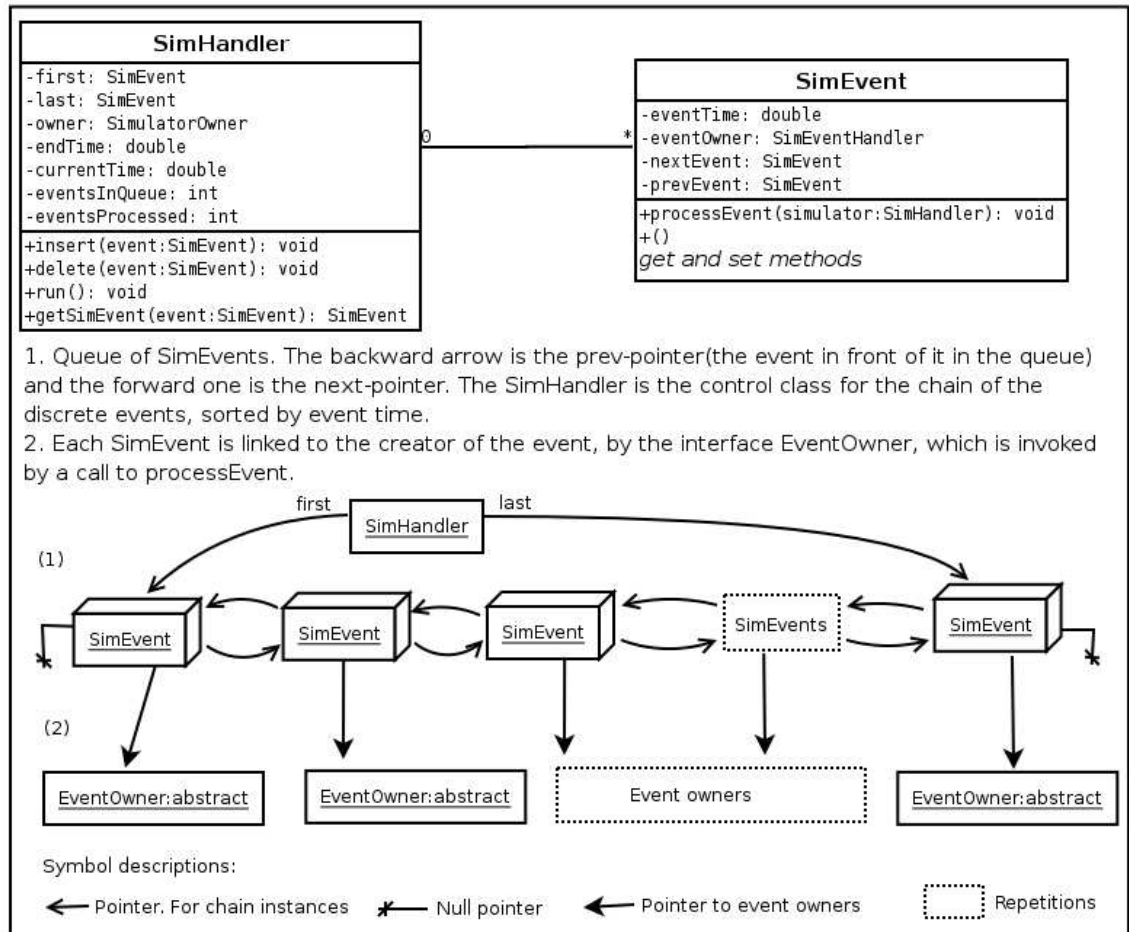
Production State There are defined two states for the oil field, *on plateau* and *off plateau*.

3.3 How are the discrete events implemented?

Discrete events in a simulation program may be implemented as a linked list or a chain of pointers, sorted by the event time. When an event is created, it is inserted into the chain/linked list at the position indicated by its event time. The simulator class then traverses the list by processing event by event, allowing the events to impose the creation of new events. These will then be put into the event queue according to the event time, as normal. The simulation ends when the queue of events is empty or when a defined end-time is passed or an event causes it all to end (loss/no profit). An illustration of discrete events is given in figure 3.2.

In an object oriented programming language like java, you may implement discrete events using a control class, the list handler (here: SimHandler), and an event class (SimEvent), as shown in the figure. The program instantiates the SimHandler and initializes it by passing SimEvents to it. When the simulation starts, the SimHandler processes the first event, then the next. Each SimEvent contains a pointer to its owner, which contains the implementation of the processing algorithm of that event, triggered by the SimHandler. For example, the events it processes may be:

Figure 3.2: Discrete Events. UML control class and model class, and an illustration of the chain structure.



- Production start of an oil well.
- End of plateau production.
- Malfunction events
- Abandonment event.

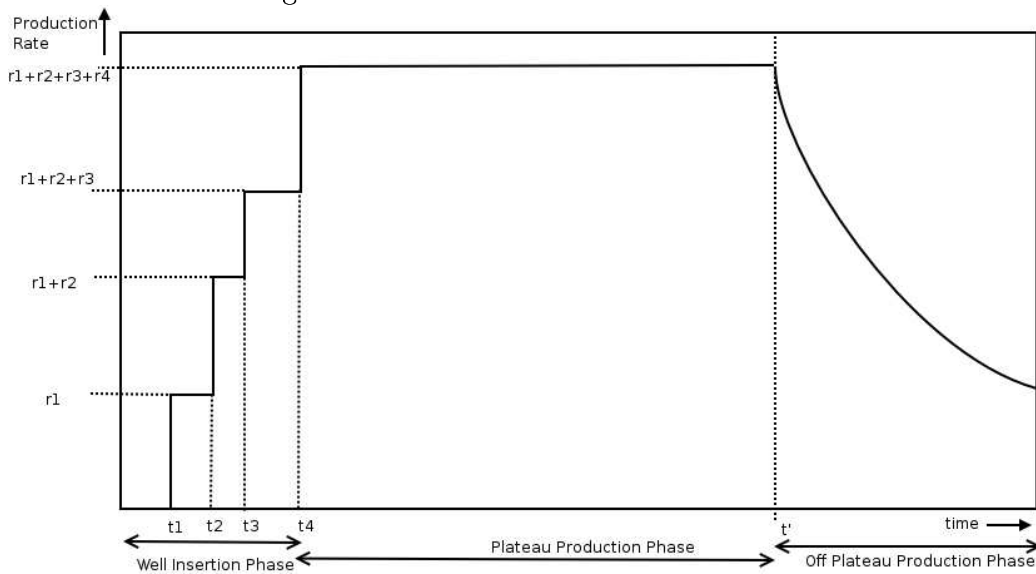
A standard chain of discrete events may look like: 'well insertion', 'well insertion', ..., 'well insertion', 'plateau production end', 'end of production'.

This is the most basic chain, but we can get many different scenarios, depending on the complexity of the model and the number of distinctive types of events. For example, if malfunctions are taken into account, we might get events which would cause a production halt of a well. This would then imply creating a 'well insertion' event at a later time determined by the repair period.

3.3.1 What would an application of discrete events in the stochastic simulation of oil production look like?

Depending on the amount of reserves of oil reached, when inserting new oil wells, we may jump on and off plateau production. The algorithms and mathematical calculations will be discussed in section A.1 and A.2. Below, figure 3.3, is a simple example of a possible production scenario¹.

Figure 3.3: Standard Production Profile



From the graph, we recognize the shape of a simple production profile of

¹The values in the examples, figure 3.3 and 3.4, are based on both the Oil Field Model (3.5). and the Oil Well Model (3.4) which are described later. They are the outcome of stochastic variables which are given here to illustrate the use of discrete events.

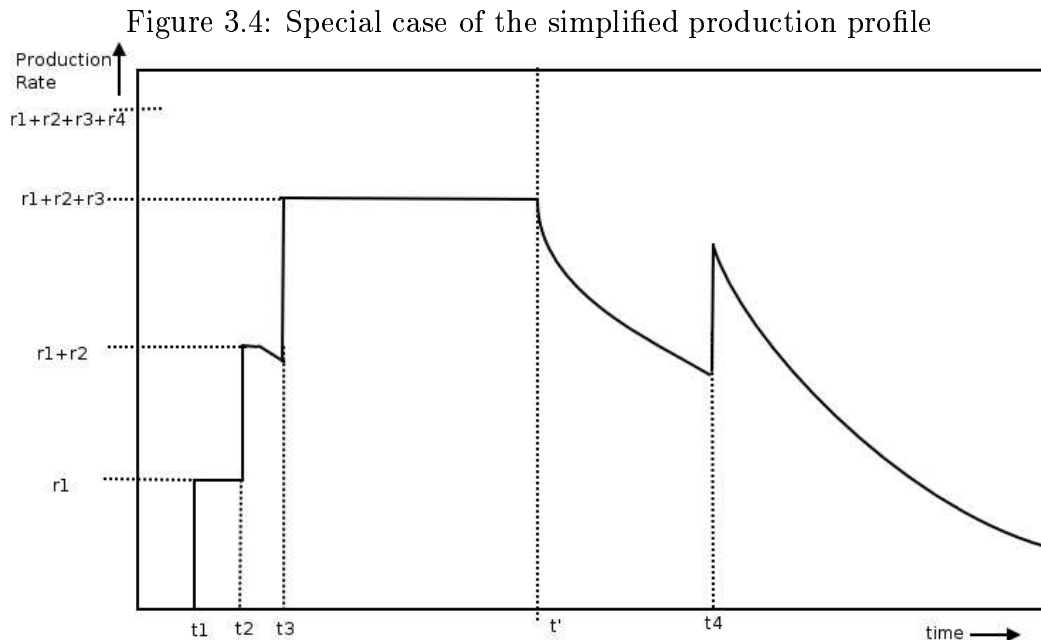
three parts, familiar to oil industry engineers [Arne B. Huseby].

Part 1 (Well insertions) In the *build up* period, four wells are phased in at the times t_1 , t_2 , t_3 and t_4 , respectively.

Part 2 (Plateau production) Together, the oil wells produce at plateau production until time t' .

Part 3 (Off-plateau) As the pressure of the oil field drops, we have an exponentially declining production. Called the *run down* phase.

In figure 3.4, we have a bit more advanced production profile. Before the insertion of well 3, between time t_2 and t_3 , we see that the production goes off plateau production for a short period of time, until well 3 is inserted and stabilizes the total production. We then produce on plateau until the pressure drops at time t' , and production goes off plateau. The similar situation occurs at insertion of well 4, at time t_4 , but this time, too much oil has been recovered to keep up pressure².



²Note that the duration or lengths of the plateau production is somewhat extended. This is the case in these examples and the coming ones, for illustration purposes.

The objective of the above example is to show an application of discrete events and introduce the concept of an oil field production profile. Given that there also would appear malfunctions, we would spot drops and peaks in the production profile. Enabling us to expand the model in such a manner is one of the strengths of discrete events, making future expansions relatively easy to implement.

3.4 How is the oil well model defined?

We are now close to the border of what is known definitively and what best can be characterized as sensible solutions and intuitive models. From this point on, I rely on empirical models, models given to me by my tutor, Huseby, and from professionals at Norsk Hydro ASA; Thuestad, Kleiven, Berg and Rasmussen. All implementations, algorithms and calculations have been done by me and I have derived models from the models given.

3.4.1 How much oil can an Oil Well recover in a given time period?

As default, the time period is set to daily, but others are allowed. The question then is how much oil can be recovered by the oil well every day. This is a stochastic variable of usually a high uncertainty. Depending on the information available you may specify distributions for each oil well separately or simply draw the values from an identical distribution for them all. The latter is the one based on the least amount of information, as is the case in the early stages of a project realization. One possible distribution for daily production, could be a lognormal distribution with 90%, 50%, 10% percentiles, for example $p_{10} = 1200Sm^3$, low, $p_{50} = 2000Sm^3$, medium, and $p_{90} = 2500Sm^3$, high. Note, $1Sm^3 \approx 6.3$ barrels.³

3.4.2 What are the recoverable reserves of an oil well?

The area of which an oil well may recover oil, the reach, is limited, and all oil reserves in the field may not be recovered. The amount recoverable depends on several factors, fluid flow, water, gas, porosity, amongst others.

³In some companies in the oil industry, as in Norsk Hydro ASA, the meaning of the percentiles is reversed. That is, the 90% percentile, p_{90} , is the low value, and p_{10} high.

In the application developed, two alternatives are given for representing the recoverable reserves for the oil wells:

Alternative 1 Fit distributions for recoverable oil for each oil well, then draw from these. The sum of this will then become the value for total recoverable oil. For example, if this volume is assumed lognormal, experts will estimate 90%-, 50%-, and 10% percentiles for it, alternatively, setting μ and σ directly.

Alternative 2 Find the total recoverable reserves first, then distribute it onto the oil wells. This amount may be found by drawing from two distinctive distributions, STOIP (3.4.2.1) and RF (3.4.2.3). The main difference from alternative 1, other than being built from two probability distributions, is that we this time starts with total oil recoverable and then divide it between the oil wells, not the other way around.

Alternative 1 is trivial, below we will exploit alternative 2, a lot more complex solution. Alternative 2 is a very good example of how increasing the complexity creates a distribution more representative for the uncertainty. We split a stochastic variable into its building blocks and construct its distribution from these. This increases accuracy of the estimates, but contributes to the complexity and hence increases simulation time.

3.4.2.1 What is STOIP?

Reads *Stock Tank Oil Initially in Place* and is a probability distribution based on several stochastic variables; sand, water, gas, dimensions etc. It describes the total amount of reserves in the oil field. This distribution is given by reservoir engineers at an early stage if the oil project.

3.4.2.2 What is RCI?

The complexity of a reservoir is an important indicator of how much of the oil that actually may be recovered. Some oil companies then created a routine for describing the complexity as an index ranging from 0 to 1. The index they called RCI and reads Reservoir Complexity Index [Ressursrapporten].

3.4.2.3 What is RF?

The *Recovery Factor*, RF, describes the fraction or percentage of total reserves which may be recovered. RF has a linear relation to RCI (3.4.2.2), normally distributed. In other words, we have a conditional distribution where knowing the RCI yields a normal distribution for RF.⁴ A study of this relationship has been done and is summarized in Appendix C.

3.4.2.4 How can the total reserves recoverable be distributed onto the oil wells?

Given the number of oil wells to be inserted, and distributions for their reach, we may scale the outcomes of the stochastic values so that they sum to the value of the total reserves recoverable (drawn from STOIIP and RF|RCI). In other words, given NW (number of oil wells), recoverable reserves, $R = STOIIP \cdot RF$, and distributions V_1, V_2, \dots, V_{NW} for the oil recoverable for each well, we scale by

$$\Delta = \frac{\sum_{i=1}^{NW} V_i}{R} \quad (3.1)$$

to end up with $\Delta \cdot V_i$ oil recoverable for oil well i [Eirik A. Berg].

3.4.2.5 When should STOIIP and RF be preferred over setting distributions for each oil well respectively?

The answer to this depends on the information available. If the geographical positions of the oil wells are known, alternative 1 should be chosen, because it is only under such circumstances that analysis of the reach of each respective oil well may be performed. However, this information is a very expensive, arriving at a late stage of the oil project development, requiring extensive analysis. At earlier stages, alternative 2 would be preferred one. Both STOIIP and RCI are given at an early stage.

3.4.3 When are the oil wells phased in?

The insertion times of the oil wells cannot be definitively determined in advance, although plans might have been laid. Again, depending on the information available, we may set insertion time distributions for each well

⁴Based on historic RF-RCI data, knowing the RCI will yield a normal distribution for the RF within a standard deviation of approximately 0.08 (8%), from the linear mean.

separately. If insufficient information is available to lay such plans, as usually is the case of alternative 2 (3.4.2), a more indeterminative procedure should be chosen. If so, estimate with:

TOFO Time Of First Oil The stochastic phase in time of the very first oil well.

Phase in interval The stochastic intervals between the phase in times of the oil wells, after the first one has been phased in.

3.5 What is the oil field model?

The Oil Field model is the complete model for the production. It manages the oil wells and consists of several other time dependent and time independent stochastic variables. They are defined in section 3.2 and repeated here:

- Recoverable Volume of Oil, V
- Plateau Production Rate, r_p
- Plateau Production Fraction, k
- Maximum Processing Capacity, r_m

Some other variables needed for the representations are:

Oil Production Rate, r The production rate at a given time is a function of time and pressure. On plateau production it equals r_m or r_p and at off plateau production it is represented by the function $f(t)$ (definition A.7).

In section 3.3.1, figures 3.3 and 3.4, we saw examples of the behavior of production rate in an oil field, excluding the maximum processing capacity. Figure 3.4 is a good example of a complex production profile, involving production rate, recoverable volume and plateau production. Some elementary properties of the oil field model is:

- *The oil field is treated as one.* It is the oil wells production rate and oil recoverable together that determines the state, *off* or *on* plateau. That is, one may calculate the time of which we cumulatively will have recovered all the oil recoverable on plateau.

- As a consequence of the above point, when a new oil well is phased in, total production rate and oil recoverable will increase. The state of the field, will be determined as if all the oil wells simultaneously started production, recovering the amount having been recovered. More of this in section 3.5.2.
- Off plateau production rate is defined to be an exponential function [Arne B. Huseby].

3.5.1 What is the recoverable reserves of the oil field?

The recoverable reserves of the oil field is set by the sum of the recoverable reserves of all the oil wells, as described in section 3.4.2. It may be directly deduced from 3.4.2.1, 3.4.2.3 and 3.4.2.2, if these models are used, as this yields the total recoverable reserves in the oil field.

3.5.2 What is the Oil Field Production Rate?

If the question was what the oil field plateau production rate was, the answer would be as trivial as the one above; the sum of the oil wells plateau production rates. But as we have other factors such as production states, the answer is much more complicated. As an additional constraint, we have the *Maximum Oil Processable* (3.5), a variable setting an upper limit to the oil recovery.

In the following, the maximum processing capacity will be ignored. The focus will be on trying to explain the behaviour of the production rate of the oil field, with respect to its oil wells plateau production rates and recoverable oil (A.1). The limit will be reintroduced in section 3.5.3, adding another dimension to the complexity of the model (A.2).

3.5.2.1 How does a change in the production state occur?

The production state represents the pressure and flow, amongst others, in the field. If V is the total volume of reserves recoverable of the oil wells, kV may be recovered *on plateau* [Arne B. Huseby]. When kV oil has been recovered, the oil wells may no longer recover oil at max potential, and the remaining $(1-k)V$ oil is recovered at an exponentially declining speed. Mathematically, we define that at time t after the plateau end time, $t_{plateau}$, the production is

$$f(t) = r_p e^{-\lambda(t-t_{\text{plateau}})}$$

where λ is the constant ensuring that the remaining $(1 - k)V$ oil will be recovered as $t \rightarrow \infty$ (A.1).

A change in the production state may also occur when phasing in a new well, as this adds to the amount of oil recoverable, V . If the amount of oil recovered up to this point, ω , is smaller than the new value of kV , production will continue *on plateau*, else it will continue *off plateau*.

3.5.2.2 What production rate do we have off plateau after insertion of the oil well?

Figure 3.5 illustrates the situation of phasing in an oil well at off plateau production. The state of the oil field is shown by the top graph, called *Fixed Production Profile*. The dash-dotted curve starting from the point $(u, f_f(u))$ shows the future development of the production rate given the absence of more events.

In the realization of the production profile currently being simulated (bottom graph figure 3.5), we apply the results from the fixed production profile. The curve marked (2), is identical for both graphs with regard to the production function. After finding the function $f_f(u)$, the challenge is mapping it to the time domain of our “real” production profile. In the real production profile, we find ourselves at t_w (t_{well}) in the figure, having recovered ω oil. The production rate at this time, $f_f(u) = f(t_w)$, (eq. A.16) is

$$f(t_w) = r_p \left(1 - \frac{\omega - kV}{(1 - k)V}\right)$$

hence (claim A.4), at time $t > t_w$ the production rate is

$$f(t) = r_p \left(1 - \frac{\omega - kV}{(1 - k)V}\right) e^{-\lambda(t-t_w)} \quad (3.2)$$

Studying this result we note that the more oil having been recovered, ω , the lesser the production rate will be at well insertion time, t_w , as one might expect.

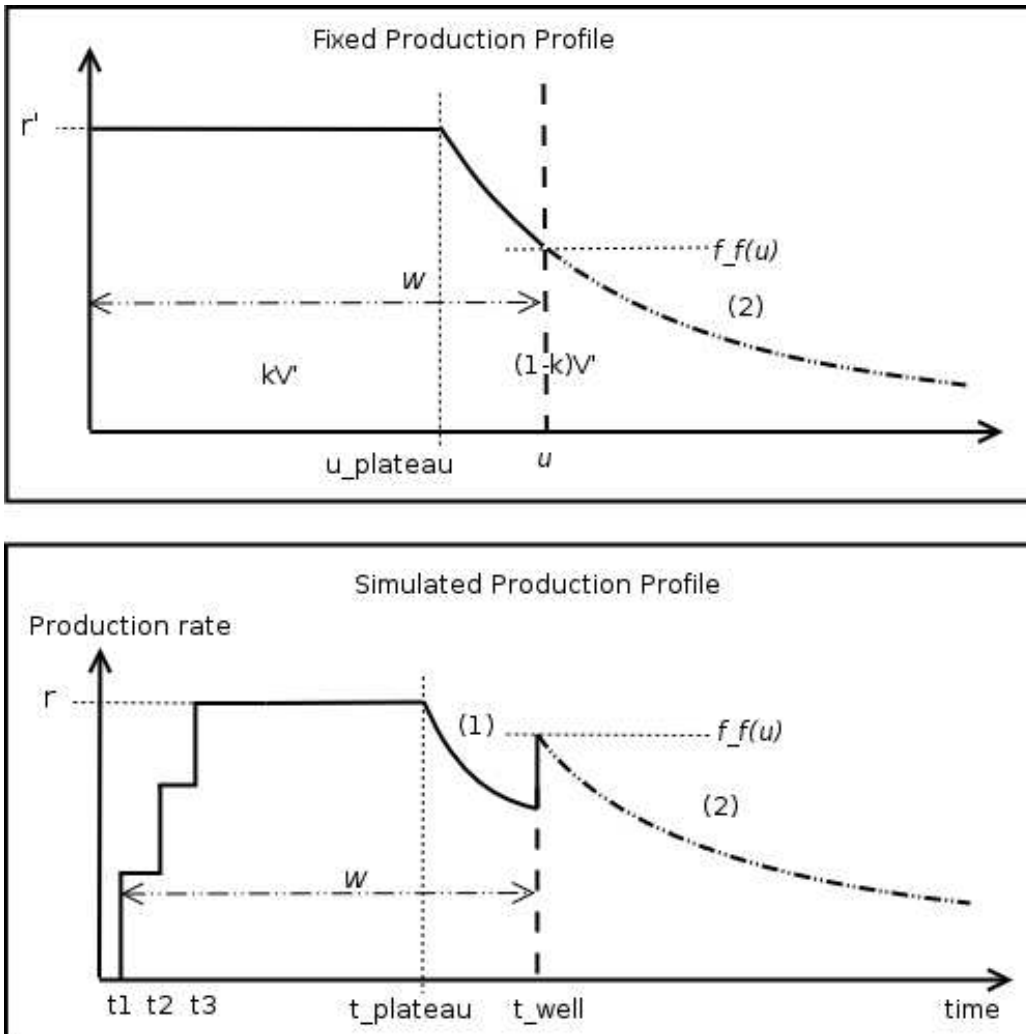


Figure 3.5: Determination of the off-plateau production rate

3.5.3 What is maximum processing capacity?

The processing plant of the oil field, an oil rig, ship or other, might not have the capacity to process all of the oil delivered by the oil wells. Extending the definition in section 3.2, the *maximum processing capacity*, r_m , is the *minimum* of the maximum transport capacity from the oil rigs to its purchaser (refinery), and the maximum oil processing capacity of the oil rig itself. This extra feature will cause the production profile to have an extended “plateau

production” curve, due to the reduced recovery speed. Note that this does only apply when $r_p > r_m$. The plateau end time is subject to the same rules as above, as it depends on the amount of oil being recovered, ω , but for convenience purposes we will redefine it.

Visible Plateau End The time of which the drop in production drops below the maximum processing capacity, r_m , see figure A.4.

Redefining the plateau end, $t_{plateau}$, to be the visible plateau end, the production rate function for $t > t_{plateau}$ becomes:

$$f(t) = r_m e^{-\lambda_m(t-t_{plateau})} \quad (3.3)$$

λ_m determined as before.

3.5.3.1 How can we determine the state and production rate after phasing in a new oil well at below maximum capacity?

As before, when phasing in a new oil well, depending on the oil recovered, ω , the production rate may be larger or less than the maximum processing capacity. If larger, production continues on this plateau and the visible plateau end time, $t_{plateau}$, needs recalculation. If production is less, we need to find the correct exponential function for the production (claim A.7). First, one must identify the production rate at the well insertion time, t_w , which is

$$f(t_w) = (r_m - \lambda_m(\omega - kV - V_m)) \quad (3.4)$$

where V_m is the amount of oil recovered in the time between the off plateau production and down to the visible off plateau production (claim A.5). Thus, the below maximum capacity production rate at time t after phasing in the well at time t_w is

$$f(t) = (r_m - \lambda_m(\omega - kV - V_m)) e^{-\lambda(t-t_w)} \quad (3.5)$$

3.6 What does the simulated production profiles look like?

Illustrations of the realisation of single simulations was given in 3.3.1, figure 3.3 and 3.4. Figure 3.6 shows the result from 5000 simulations from the fictive oil recovery project “Polar Star”.

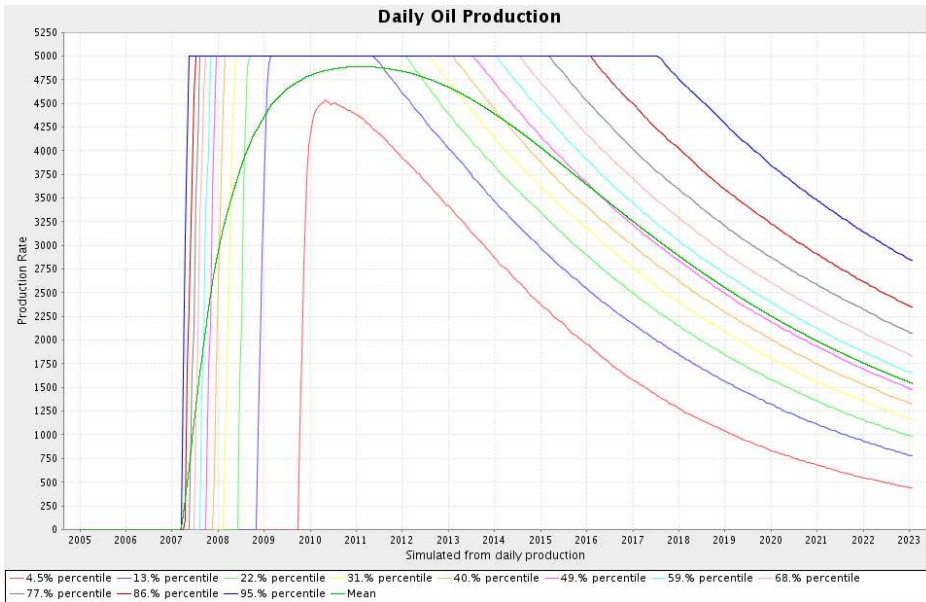


Figure 3.6: 5000 simulations from “Polar Star”. Plotting 11 percentiles and the mean daily oil production.

From the graph, we see that the uncertainties are greatest in the “run down” phase of project. An early TOFO normally implies an early drop, relative to the amount of oil recoverable. The amount of oil recoverable and the plateau production rate determines the length on maximum production capacity, supposing plateau production is above that level.

The curve of the profile is correlated, so the probability intervals of the percentile curves only applies when nothing is known about the development of the profile up to the time in question. ⁵Plotting of these results are further discussed under Future Development, chapter 7.3.

⁵Knowing a value in a point, however, gives a reasonable indicator of how the profile is curved, consequently, the 68% probability interval of the other values in that profile would look very different from the one calculated from the collection of the simulations(no states known).

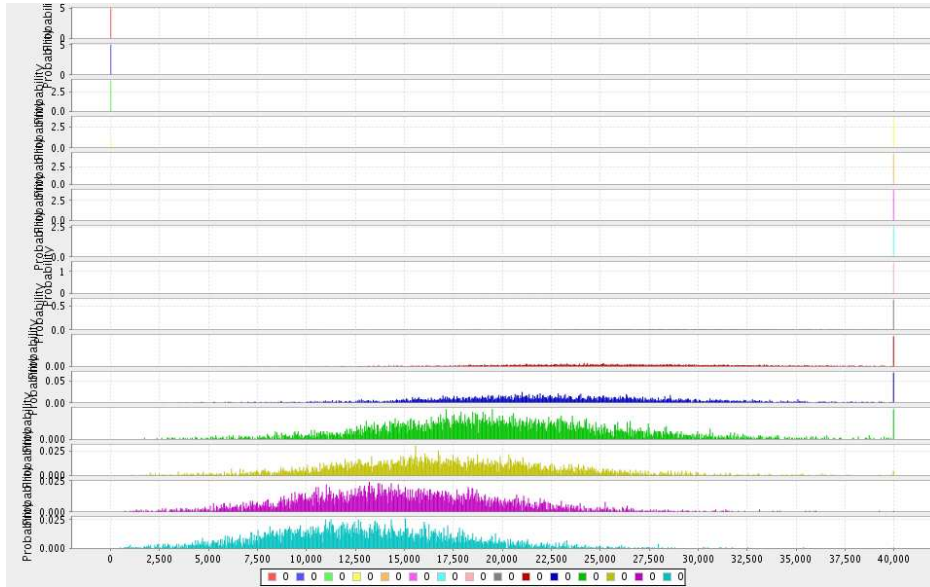


Figure 3.7: Density plots of 15 time points with equal spacing between them. From the production simulation of “Polar Star”.

3.6.1 What can be said about the spread of the production profile simulations?

As a simple additional feature of the application, one can study the density distributions of the vertical spread of simulations by extracting x time points with equal length y in between them.

In figure 3.7, there are plotted vertical density for 15 time points from beginning to start with accordingly equal spacing. It is the density representations equivalent to the production profile of figure 3.6 (Rotate it 90° and the it looks more like the production profile). For all the models, it is an objective for these curves to have the highest peak and narrower spread. Long tails are also not to good, but they may normally be ignored if they are not very dense, that is, highly improbable.

Chapter 4

Income

4.1 How are the oil prices developed?

There are several models for developing oil prices. (Partial) Geometric Brownian Motion is one such method, often used in finance for stock options and bonds. Another, simpler, method is based on future estimations of the oil prices, using 10%-, 50%- and 90%- percentiles . The percentiles are given for a number of future dates and the dates missing are autofilled with parabolic curves estimated from the neighbouring values. With the complete list of percentiles for the oil production, we may start to simulate oil prices. Using a model where the oil prices are normally distributed $X_i \sim N(\mu_i, \sigma_i^2)$, determined by the percentiles, and correlated with correlation, ρ , we may develop the oil prices as follows:

$$Y_i = \rho Y_{i-1} + \sqrt{(1 - \rho^2)} X_i, \quad i = 1, \dots, N$$

where Y_i is the oil price at time i . This is repeated for each simulation (each realization), yielding very different oil prices, relative to the correlation ρ . The oil prices are subject to high uncertainty, and may greatly affect the final outcome of the income profile distributions, as can be seen in figure 4.1. The uncertainties of the production profile (figure 3.6) has been greatly scaled. It is obvious that the price model selected (percentiles and correlation of “Polar Star”) is too oscillating, and should be somehow controlled, either by increasing the correlation or by narrowing the percentile levels. But then again, in reality the oil prices are truly indeterminative.

4.2 How is the income calculated?

The (not discounted) income is easy to calculate once we have the oil recovery at each discrete time and the the oil price (4.1) at these same times. Just multiply the production with the oil prices. As we look at the oil field in particular, we may suppose there are no other income possibilities other than producing and selling oil.



Figure 4.1: Daily cumulated income profile from 5000 simulations of “Polar Star”.

Chapter 5

Expenses

5.1 What expenses do we have?

We define all expenses to be covered in one of this categories:

CAPEX Capital Expenses. Initial Expenses, as planning, purchase of oil rig, office buildings, exploration, and other equipment necessary to exploit the field.

OPEX Operating Expenses. Expenses related to equipment, maintenance of these, gas injection and continuous production.

TAREX Tariff Expenses. People related, salary and such.

DRILLEX Drilling Expenses. Expenses related to the drilling of oil wells.

ABDEX Abandonment Expenses. The cost of getting rid of the equipment when the exploitation of the field is finished.

TAXEX Tax Expenses. Special tax regulations exists for oil recovery projects.

The stochastic simulations applied for these expenses are based on functional simulation, only the DRILLEX is based on event simulation. That is, for all the expenses except for the DRILLEX, it is assumed that all expenses are known prior and covered by the probability distributions defined for them. It is allowed to provide several expenses for each category, enabling the possibility of dividing the expenses into their different elements. Some of the expenses are production dependent, this will be covered below (5.2).

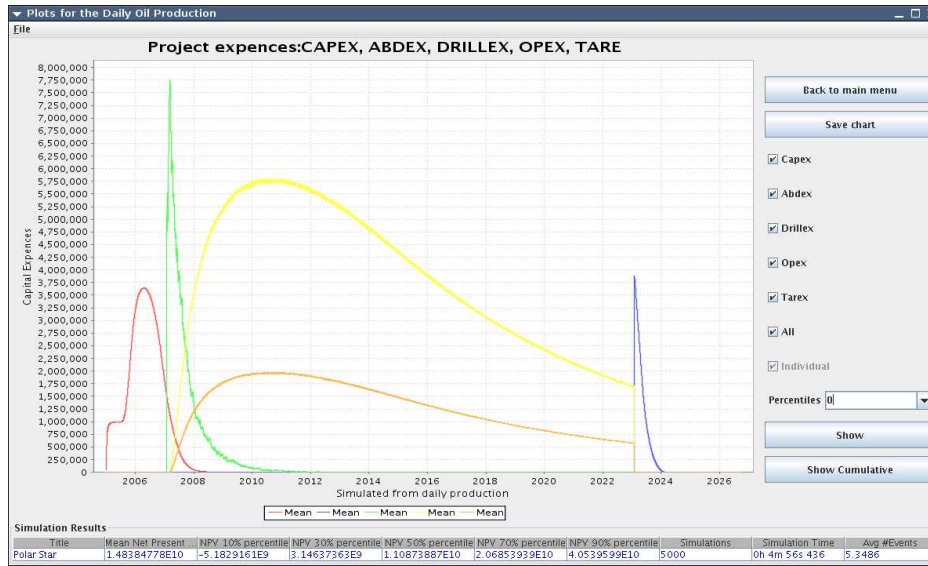


Figure 5.1: Daily expenses plot of “Polar Star”. Screen shot [Production Profiler]. Mean of all plotted together, individually. From the left by appearance at 50M; CAPEX, DRILLEX, OPEX, TAREX and ABDEX.

The models used are simple, they exist mainly for enabling us to easily represent the costs of the project, so that NPV etc. may be shown. The cost of each task for the production independent expenses and the DRILLEX, is divided on the task duration and set constant over the duration.

5.2 How are Expenses related to the Production?

Some expenses are directly linked to the speed of oil recovery. These are the OPEX and TAREX. The DRILLEX is indirectly dependent, as it is triggered by the drilling of oil wells. From figure 5.1, you see that the operating expenses and the tariff expenses begins at the same time as the production profiles did, inheriting its shape, and the DRILLEX grows also grows rapidly at the beginning of the production, as one would expect. The CAPEX and ABDEX are the two curves starting prior and posterior to the production, respectively.

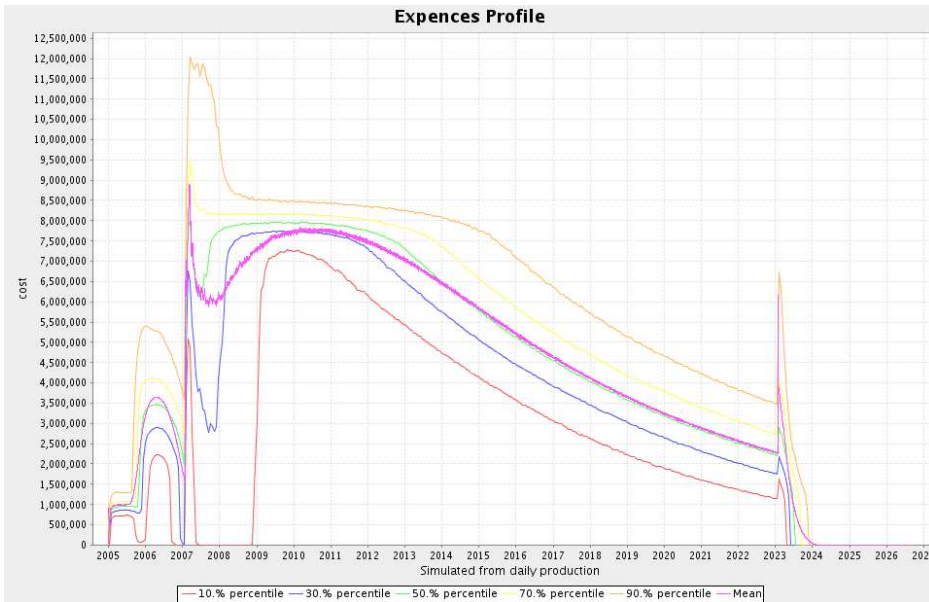


Figure 5.2: Daily expenses plot. All expenses summed at each time period.

When the sum of all the expenses plotted together, with five percentiles, (figure 5.2), it all feels like a mess. But the expenses presents themselves quite clearly. It starts with the CAPEX, the first and second peak, then the drilling starts, yielding the highest peak. Moments after the DRILLEX, the OPEX and TAREX runs. They cannot be identified independently, running together until the abandonment expenses starts, resulting in the last peak.

5.3 How are capital expenses implemented?

Expenses related to planning and development of the oil recovery project up to the start of the oil recovery, excluding the drilling of the oil wells, including the exploration, are defined to belong to this category. The model defined for the application uses three distributions for each expense defined:

cost The total cost distribution of the task

start time The start time distribution for the task

duration The duration distribution of the task

As many CAPEX expenses as you want may be given as input variables for the application.

5.4 How are abandonment expenses implemented?

The start time of the abandonment expenses is defined to equal the time of production stop (chapter 7.3) of the oil field. The stochastic variables thus are:

cost The total cost distribution of the task

duration The duration distribution of the task

As above, several ABDEX expenses may be given as input.

5.5 How are drilling expenses implemented?

The DRILLEX are the expenses that are closest to event simulation in this model. It is triggered and drawn for each oil well that is phased in. The start time of the drilling is determined by the duration of the task. The task is supposed ended when the oil well is phased in. For each oil well, we have the stochastic variables;

cost The total cost distribution of the task

duration The duration distribution of the task

A single DRILLEX distribution is given for each oil well, or one defined for all.

5.6 How are operating expenses implemented?

The operating expenses are expenses directly linked to the oil recovery rate. This includes maintenance, transport, gas injection and such. The stochastic variables are drawn for each time period, history independent (no correlation):

cost per unit of oil The operating cost to produce one unit of oil, usually in barrels or Sm^3 . This value is normally within the interval of \$2 – \$4 per barrel.

5.7 How are tariff expenses implemented?

Also dependent on the production are the tariff expenses. As the other expenses, this is a simple model for representing it, and it is supposed to include salaries, the DD&A (depreciation, depletion and amortization), tax other than income and others not covered by the other expenses.

cost per unit of oil Tariff expenses per unit of oil. This value is normally within the interval of \$1 – \$2 per barrel, drawn for each time point, history independent.

5.8 What are the Taxation Expenses?

There are special tax rules given for oil companies exploiting oil in Norway. These are also stochastic variables, as they are subject to change with changing powers in the government. The taxation expenses are not implemented in the application and will only be mentioned in short here. Tax rates for installations on NCS (Norwegian Continental Shelf) are high, but also have some favorable tax rules for tax deduction. The taxes are

- 28% corporate tax + 50% special petroleum tax which yields a 78% marginal tax.

The tax deductions are highly favourable to investment. Norsk Hydro ASA has estimated that for 1 NOK invested, 93 øre are paid back through tax deductions. Some special tax deduction rules are:

- CAPEX are tax deductible by depreciation, with special rules applied:
 - Offshore ordinary CAPEX as fixed installations on NCS are depreciated over 6 years in a straight line starting in the investment year. In addition it is an uplift against special tax base (50%) of 5% annually over the same 6 years.
 - Offshore exploration CAPEX are 100% deductible the first year.
 - Other investments as office buildings, ships are depreciated according to standard rules.

- OPEX, TAREX and DRILLEX are charged as expenses (100% deduction)
- ABDEX are also given special rules, being able to deduct some of the expenses on the budget years prior to the event.

Chapter 6

Finance

The subjects discussed in this chapter provides the basis for the financial decision makings of whether or not to invest in the oil recovery project, or what exploitation solution that should be used. Some variables needed to be defined prior to introducing them is:

- r_n is the the global discount rate, for year n .
- $income_n$ and $cost_n$ is the income and expenses in year n , respectively.
- N is the lifetime of the project in years.

In the above examples, it has been used a daily time basis. Either by summing the values to years or by dividing the the global discount rate by the number of days, we can calculate with the daily values. The latter being used in the application.

6.1 How is the cash flow represented?

The cash flow is straight forward to calculate. The cash flow at year n is

$$cashflow_n = income_n - cost_n$$

and the discounted cash flow

$$dcashflow_n = \frac{income_n - cost_n}{(1 + r_n)^n}$$

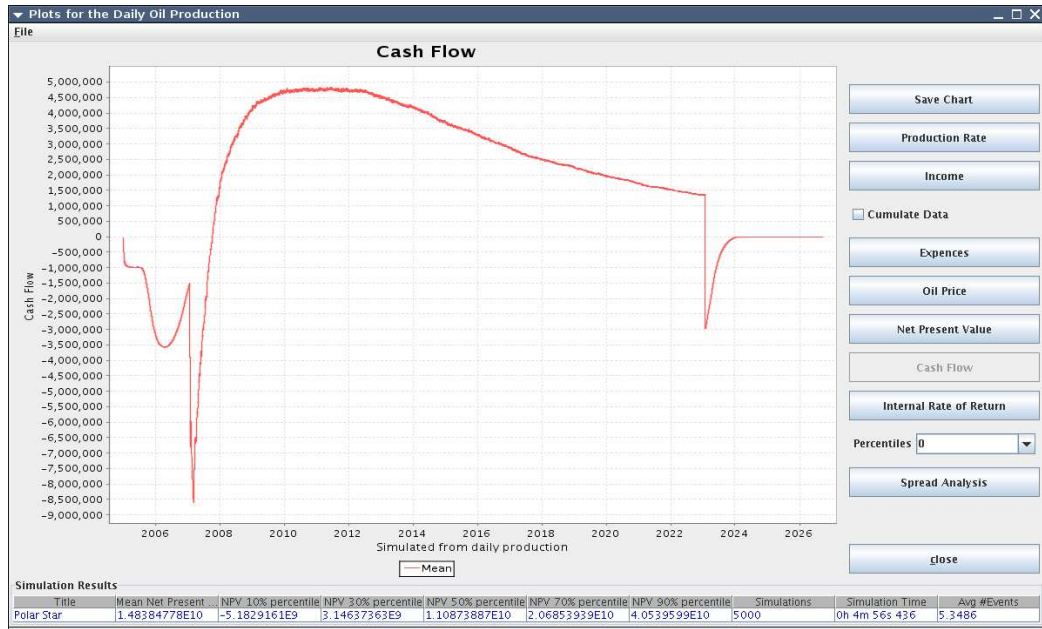


Figure 6.1: Cumulative cash flow of “Polar Star” screen-shot from the application[Production Profiler]. Plot of the mean.

The figure of the cash flow only plots the mean. This is chosen for the purpose of acknowledging the logical form of the curve, starting and ending with expenses, earning in between due to oil recovery.

6.2 How is the net present value represented?

The net present value equals the cumulated discounted cash flow

$$npv = \sum_{n=0}^N dcashflow_n = \sum_{n=0}^N \frac{income_n - cost_n}{(1 + r_n)^n}$$

The figure (6.2) shows a density plot of the outcomes from the 5000 simulations of the “Polar Star”. From the cash flow screen shot, figure 6.1, it says the 10% percentile of the NPV is –5 billion NOK. That is, from the simulations, it says that it is a 10% chance of losing 5 billion NOK or more (\$1≈NOK6.4). On average they earned 15 billion, even though 50% of the outcomes generated 11.1 billion or less.

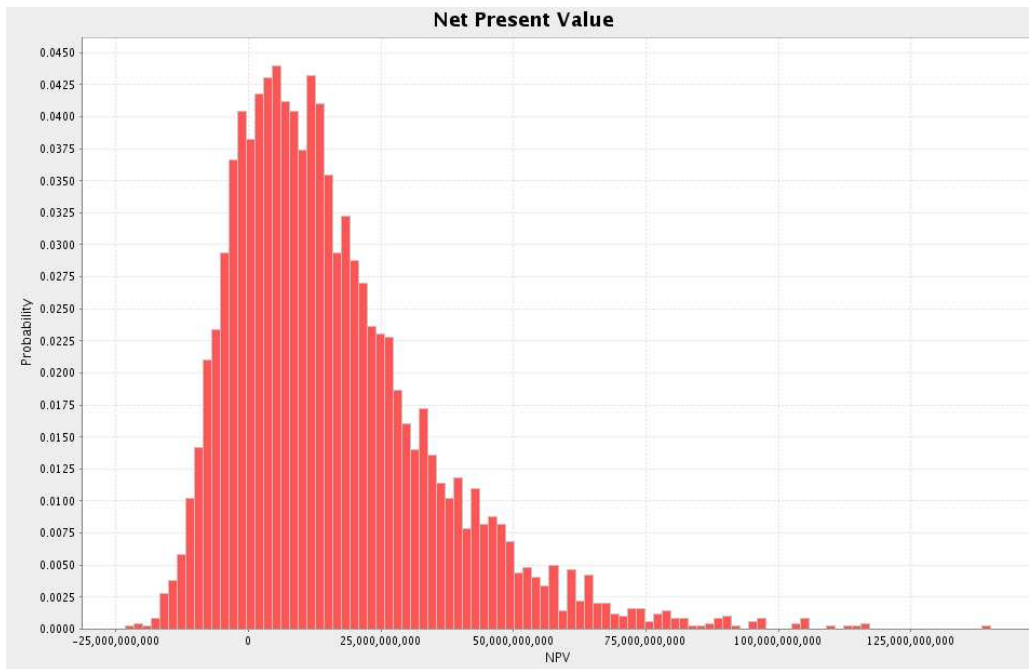


Figure 6.2: Net Present Value probability density distribution of “Polar Star”

6.3 What is capital efficiency?

The informal ratio of output divided by capital expenditure. The larger the ratio, the better the capital efficiency. Straight forward to calculate, as we already have got the net present value and the expenses. The formula is:

$$capeff = \frac{npv}{\sum_{n=0}^N \frac{cost_n}{(1+r_n)^n}}$$

6.4 What is internal rate of return?

Essentially, this is the return that a company would earn if they expanded or invested in themselves, rather than investing that money abroad. I have seen definitions using both the discounted and not discounted cash flows. I choose not to discount the values, as risk free interests as government bonds are not discounted either. The definition of IRR then becomes; The interest rate that makes the net value of all cash flow equal zero.

$$\sum_{n=0}^N \frac{income_n - cost_n}{(1 + irr_n)^n} = 0$$

In general there is no closed-form solution for IRR. One must find it iteratively. In other words, pick a value for IRR. Plug it into the NPV calculation. See how close to zero the NPV is. Based on that, pick a different IRR value and repeat until the NPV is as close to zero as you care.

Chapter 7

Concluding remarks

7.1 Notes for the reader

As this paper goes into print five days ahead of the final delivery date 1 November 2005, there may be a few changes in the final version of the application. The changes may be noticed when comparing with the screen shots shown in this text, but most will essentially be the same.

7.2 Known errors in the application

The program easily fails, as it is very sensible on input variables. If there are any errors in the XML structure of a document, it will not even load into the window and must be edited externally. If some variables are missing, misspelled, given wrong attribute types or in any other way not of the form it is programmed to read, it will not give any sensible error message.

Choosing to many simulations in combination with a high number of points for storing the simulations, the memory usage easily becomes to big. Even though all numbers have been changed from doubles to *floats*, the storage need for 10000 simulations, storing 500 points is $10000 * 500 * 4 \approx 20MB$ for each profile. The program stores 10 such profiles (6 expenses + oil prices + income + cash flow + production). A large number of simulations and storage points may use some time when creating the time series for the percentiles, although this will go fairly fast for pretty high numbers.

7.3 Future Development

As illustrated in figure 2.1, the simulation model is layer based. It is made that way so that a part can be removed or modified without forcing changes in exterior code. However, if event based expenses are wanted, most of the expenses classes will have to be changed. All other parts would remain unaffected. Below, I list some aspects that could and should be developed, with a short discussing of each:

Condensate and associated gas In many oil fields there are a lot of gas in addition to oil. This is also profitable and many fields extract both. Extending the model to cover this will impose huge and time consuming changes. Different types of oil wells, production states, gas prices and much more.

CO₂ Tariffs, and income or subsidiaries by pumping the *CO₂* back into the ground.

TOFO triggered by CAPEX Should be fairly simple. As of now, the TOFO is drawn as stochastic variable defined at input, unaffected by the CAPEX. In general, if CAPEX is not done, production cannot have started. This implies that the CAPEX must be able to create a TOFO event. The TOFO event already triggers the first DRILLEX, the production, OPEX and TAREX start.

ABDEX start Equivalent to **production stop**, this value is now a fixed value, set at input. As for most of the topics in this section, time and functionality is the reason for it not yet being implemented. A rule for the production stop could be that the first year after phasing in all the wells, that field is losing money, one cuts the production at the time it started losing money for longer than a week. We may suppose that we are able to backtrack like this, as we in reality would have been able to pretty accurately estimate the best time of abandonment the field.

percentile representation in the plots Yes, this is already implemented. But it only takes the percentiles of each time point independently of the other times. So any realization will probably never walk the path of the 10% percentile, or the 90%. It would be interesting to find other means of characterizing the simulations, in order to sort them.

The first obvious possibility is to start with the NPV, displaying the various profiles, income, production, expenses, etc. for the realization yielding the 10% percentile of the npv. Problems with this one is that the production might have been very good, but the oil prices horrible. As yet another alternative, using a formula for recovery speed, cumulate the production and look at the simulations half way through, take the percentiles, then plot the realizations of the percentiles.

TAXEX Implementing the taxes model.

discounting values Creating a model for the global discount rate.

event based expenses Implementing expenses as events.

oil prices Partial geometric Brownian motion, or other models. Give the user more freedom of choice of oil prices emulator. Already a bottle neck in the simulations, this should be improved.

malfunction events Enabling malfunction events, causing production stop in an oil well, or in the whole field. Maybe the malfunction only causes a reduced plateau production. Create models for calculating the repair cost and repair time versus the expected added income if repaired.

reinforcement of oil wells New technology, cost effective solutions dependent on the oil prices, may increase the tail production of an oil wells. Such events should be one of the first to integrated if extending the models.

error handling Error handling of the application. Instructive messages to the user when an error occurs. Allowing more ways of representing the variables and the XML file. Now it has a very strict rule set that easily fails without giving instructive feedback.

input The way of giving input variables could be improved significantly. However, not very cost effective, a drag and drop system, clicking and choosing the variables, and selecting distributions for them, would be a nice feature.

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Appendix A

Deductions for the Production Profile Calculations

This chapter presents the mathematical formulas for calculating the production rates as events occur, or as the state of the field changes. It is based on the oil field model of this text. Divided in two sections, I will start to present the calculations of the production rate without the maximum processing capacity represented. At the end, it will be reintroduced, using the result from the first section. As I have close to no references, all my “propositions” are presented as claims, deducted from the definitions, claims and models defined prior.

A.1 What is the oil field production rate excluding the maximum processing capacity?

Definition A.1. *The cumulative amount of oil recoverable for the oil wells in an oil field is V .*

Definition A.2. *There are two possible states of the production, plateau production and off plateau production.*

Definition A.3. *Plateau production, r_p , is the sum of the maximum recovery speed potentials of oil wells.*

Definition A.4. *The amount of oil recoverable for the oil wells at plateau production is kV , $0 < k < 1$.*

Definition A.5. ω is the amount of oil recovered up to a given time t .

Definition A.6. The plateau end time is the time of which $\omega = kV$ oil has been recovered.

Claim A.1. Suppose that at time t , the production is at plateau production. Then the expected time of the plateau end is

$$t_{\text{plateau}} = t + \frac{kV - \omega}{r_p} \quad (\text{A.1})$$

Deduction. This follows directly from definitions A.3-A.6, as there are $kV - \omega$ oil left to recover at plateau production r_p . \square

Definition A.7. The off plateau production rate is an exponential function

$$f(t) = r_p e^{-\lambda(t-t_{\text{plateau}})} \quad (\text{A.2})$$

Claim A.2. The exponential constant, λ , is

$$\lambda = \frac{r_p}{(1-k)V} \quad (\text{A.3})$$

Deduction. As kV oil had been recovered at t_{plateau} there are $(1-k)V$ recoverable oil left. This must be recover at off plateau production. Thus

$$\int_{t_{\text{plateau}}}^{\infty} r_p e^{-\lambda(t-t_{\text{plateau}})} dt = (1-k)V \quad (\text{A.4})$$

$$\left[-\frac{r_p}{\lambda} e^{-\lambda(t-t_{\text{plateau}})} \right]_{t_{\text{plateau}}}^{\infty} = (1-k)V \quad (\text{A.5})$$

$$\frac{r_p}{\lambda} = (1-k)V \quad (\text{A.6})$$

$$\lambda = \frac{r_p}{(1-k)V} \quad (\text{A.7})$$

\square

As an additional, and useful, deduction of the above we may use the discrete event times at off plateau production as pivots, that is as the t_{plateau} values used above. More accurately:

Claim A.3. *If production rate at time $t_0 \geq t_{plateau}$ was r_0 , then at time $t \geq t_0$ the production rate is:*

$$f(t) = r_0 e^{-\lambda_0(t-t_0)} \quad (\text{A.8})$$

where $\lambda_0 = \frac{r_0}{V-\omega}$,

Deduction. Follows from the deduction above and definition A.7. \square

A phase in of an oil well cause a change in the oil field state. It is a complicated process and the production may or may not continue on plateau production, even if it was currently at plateau production.

Definition A.8. *When an oil well is phased in, the state of the oil field is altered. The recoverable reserves and production rate of the oil well are added to the totals, V and r_p respectively.*

Definition A.8 is a direct result of the fundamental property of the oil field model, that we treat it as a whole, that is, the oil wells all work together as one.

Claim A.4. *If $\omega < kV$ after phasing in a new oil well at time t_w , the production rate is r_p and the estimated plateau end time is*

$$t_{plateau} = t_w + \frac{kV - \omega}{r_p} \quad (\text{A.9})$$

If $\omega \geq kV$, then the production rate is

$$f(t) = r_p \left(1 - \frac{\omega - kV}{(1 - k)V}\right) e^{-\lambda(t-t_w)} \quad (\text{A.10})$$

Deduction. Both cases are shown in figure A.1. Either we are on plateau, or off¹. Case 1, $\omega < kV$, ($\omega = w_case\ 1$) follows from definitions A.7 and A.8. If $\omega = w_case\ 1$, is the situation,, and in the opposite is the case at $\omega = w_case\ 2$.

Case 2 is more complex, the amount of oil recovered is greater than what can be recovered on plateau. The answer however , results in a relatively

¹Note that this is *not* our production profile, it is an illustration of showing what state the field would be at after recovering ω oil with the n oil wells in production. In other words, it is being used for determining the production rate in the oil field at the time of production start of a new well.

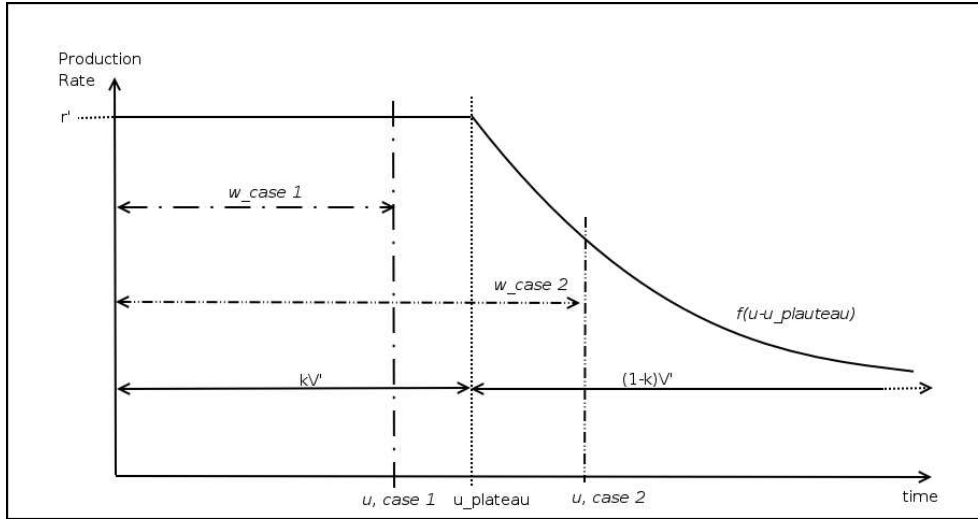


Figure A.1: State and production rate after insertion of a new Oil Well

pretty answer (eq. A.10). Below, I will present a stepwise procedure to find this production rate. The first three steps are repetitions of definitions A.5 and A.8, and by progressively solving these 9 ordered problems, we will find the production rate.

1. Set $r_p = r_p' + r_w$, where r_p' was the oil fields plateau production rate before t_w .
2. Set $V = V' + V_w$, where V' was the oil fields recoverable reserves before t_w .
3. Set ω as the recovered oil up to t_w
4. Find $u_{plateau}$
5. Find λ_f
6. Find u
7. Find $f_f(u)$
8. Find λ
9. Find $f(t)$

Figure A.2 illustrates the situation of our case (case 2 in figure A.1). The state of the oil field is shown by the top graph, called *Fixed Production Profile*, the profile we would have had if we started production in all the oil wells at the same time. The dash-dotted curve marked (2), starting from the

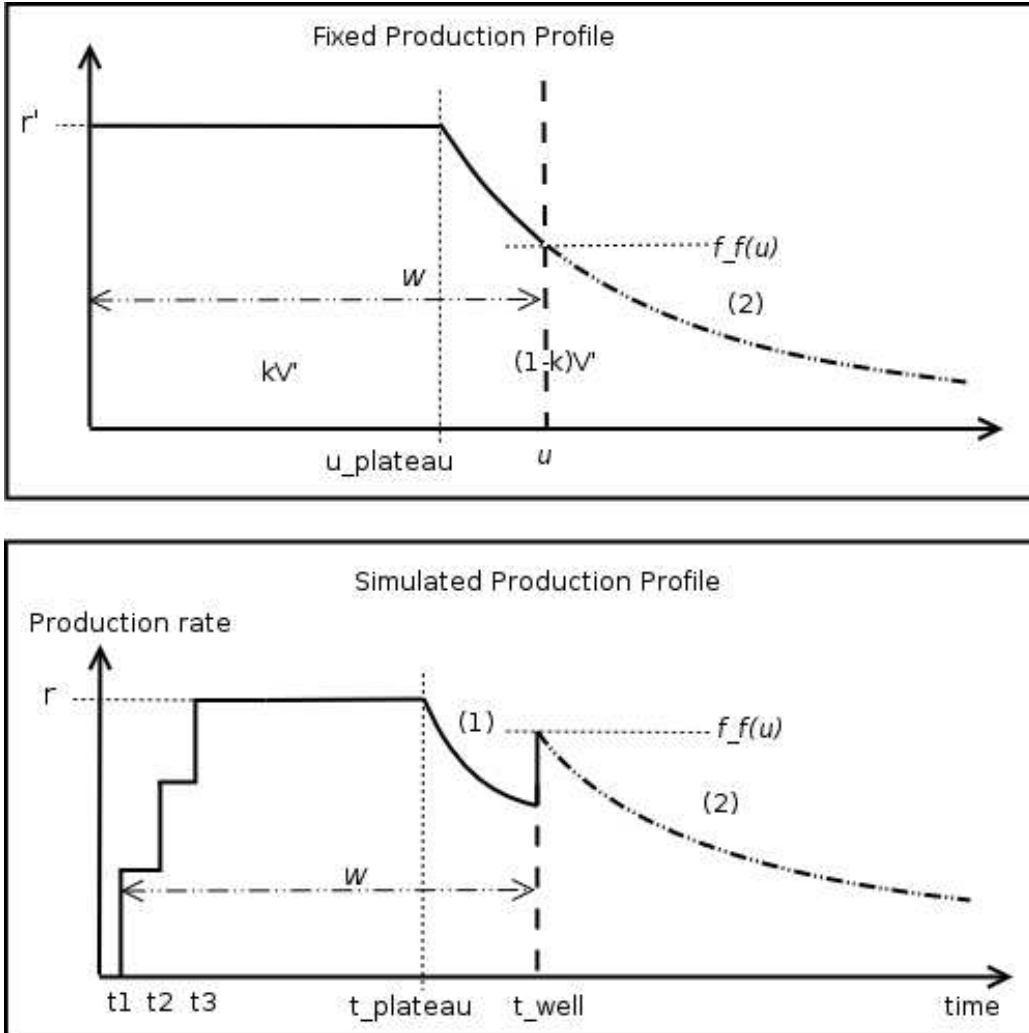


Figure A.2: Determination of the off plateau production rate

point $(u, f_f(u))$, is identical for both graphs with regard to the production function. It shows the future development of the production rate, given the absence of more events. In the realization of the production profile currently

being simulated (bottom graph figure A.2), we apply the results from the fixed production profile. The curve , After finding the function $f_f(u)$, the challenge is mapping it to the time domain of our “real” production profile. In the real production profile, we find ourselves at t_w (t_well) in the figure. Using this model, we may solve the problems, continuing from step 4.

Step 4 Find $u_{plateau}$

The *fixed production* plateau end time is

$$u_{plateau} = \frac{kV}{r_p} \quad (A.11)$$

that is, the time of the plateau end if all the oil wells had been at plateau production from time 0, recovering ω oil.

Step 5 Calculating the lambda constant for the fixed profile, λ_f .

The off plateau production rate is $f_f(u) = r_p e^{-\lambda_f(u-u_{plateau})}$. As kV oil has been recovered at plateau production, $(1-k)V$ will be recovered off plateau. Hence, as in claim A.2,

$$\lambda_f = \frac{r_p}{(1-k)V} \quad (A.12)$$

Step 6 Finding the current state time, u , of the fixed profile.

As you may see in the top graph, figure A.2, ω must equal the volume recovered at plateau production, kV , plus the volume produced off plateau up to time u . This produces the equation

$$kV + \int_{u_{plateau}}^u r_p e^{-\lambda_f(t-u_{plateau})} dt = \omega \quad (A.13)$$

where u is the only unknown. Solving for u

$$\begin{aligned} \frac{r_p}{\lambda_f} (1 - e^{-\lambda_f(u-u_{plateau})}) &= \omega - kV \\ e^{-\lambda_f(u-u_{plateau})} &= 1 - \frac{\lambda_f}{r_p}(\omega - kV) \\ -\lambda_f(u - u_{plateau}) &= \ln\left(1 - \frac{\lambda_f}{r_p}(\omega - kV)\right) \end{aligned}$$

yields

$$u = u_{plateau} - \frac{\ln\left(1 - \frac{\lambda_f}{r_p}(\omega - kV)\right)}{\lambda_f} \quad (A.14)$$

Step 7 Finding $f_f(u)$.

Knowing u (A.14), u_{plateau} (A.11) and λ_f (A.12), we may find the production rate at time u , that is, the exact value for the production rate at the oil well phase in time, is

$$f_f(u) = r_p e^{-\lambda_f(u - u_{\text{plateau}})} \quad (\text{A.15})$$

substituting for u , yields

$$\begin{aligned} f_f(u) &= r_p e^{-\lambda_f \left(u_{\text{plateau}} - \frac{\ln\left(1 - \frac{\lambda_f}{r_p}(\omega - kV)\right)}{\lambda_f} - u_{\text{plateau}} \right)} \\ &= r_p e^{\ln\left(1 - \frac{\lambda_f}{r_p}(\omega - kV)\right)} \\ &= r_p - \lambda_f(\omega - kV) \end{aligned} \quad (\text{A.16})$$

Step 8 Finding λ , of $f(t)$.

The remaining oil recoverable after the oil well phase in time t_w , is $V - \omega$. Similar to claim 5, we find

$$\int_{t_w}^{\infty} f_f(u) e^{-\lambda(t - t_w)} dt = V - \omega \quad (\text{A.17})$$

$$\lambda = \frac{f_f(u)}{V - \omega} \quad (\text{A.18})$$

Step 9 Finding $f(t)$, the off plateau production rate function of our time domain.

The production rate at time t_w is

$$f(t_w) = f_f(u) \quad (\text{A.19})$$

At time $t \geq t_w$, the production rate is $f(t) = f(t_w) e^{-\lambda(t - t_w)}$ or, equivalently:

$$f(t) = f_f(u) e^{-\lambda(t - t_w)} \quad (\text{A.20})$$

Substituting for $f_f(u)$ (A.16) gives

$$f(t) = (r_p - \lambda_f(\omega - kV)) e^{-\lambda(t - t_w)} \quad (\text{A.21})$$

substituting for λ_f (A.12) yields

$$f(t) = r_p \left(1 - \frac{\omega - kV}{(1 - k)V}\right) e^{-\lambda(t - t_w)} \quad (\text{A.22})$$

□

Remark: Setting $r_0 = r_p(1 - \frac{\omega - kV}{(1-k)V})$, $\lambda_0 = \lambda$ and $t_0 = t_w$, as in claim A.3, it is easy to reuse these values for the next event, as the production rate of that event time, $t_e > t_0$ (or more accurately, the moment just before the event action), is

$$f(t) = r_0 e^{-\lambda_0(t_e - t_0)} \quad (\text{A.23})$$

A.2 What is the oil field production rate with a maximum processing capacity cap?

Definition A.9. *The production rate may never be greater than the maximum processing capacity, r_m .*

Definition A.10. *The visible plateau end is the point in time of which the oil production rate drops below the maximum processing capacity.*

Definition A.11. *The visible plateau end is t_{plateau} .²*

The plateau end will be triggered as before, when the amount kV of oil has been recovered (A.1), but its effect is not be *visible* until the off plateau production is less than r_m .

Claim A.5. *The amount of oil recovered from the start of the off plateau plateau production, up to the intersection point of the maximum processing capacity and the off plateau production, is*

$$V_m = (1 - k)V \left(\frac{r_p - r_m}{r_p} \right)$$

Deduction. As shown in figure A.3, V_m (V_m) is area under the graph from z_0 to z_1 in (A) of the figure.

To find V_m , we need to locate the time z_1 . This is the time of where the production level drops noticeably. The z_0 will disappear and is not needed. The z_1 is the intersection time of the off plateau production and r_m .

$$r_p e^{-\lambda_f(z_1 - z_0)} = r_m \quad (\text{A.24})$$

solving for z_1 , yields

$$z_1 = -\frac{\ln \frac{r_m}{r_p}}{\lambda_f} + z_0 \quad (\text{A.25})$$

²This is a redefinition of t_{plateau} , hence its calculation is also changed.

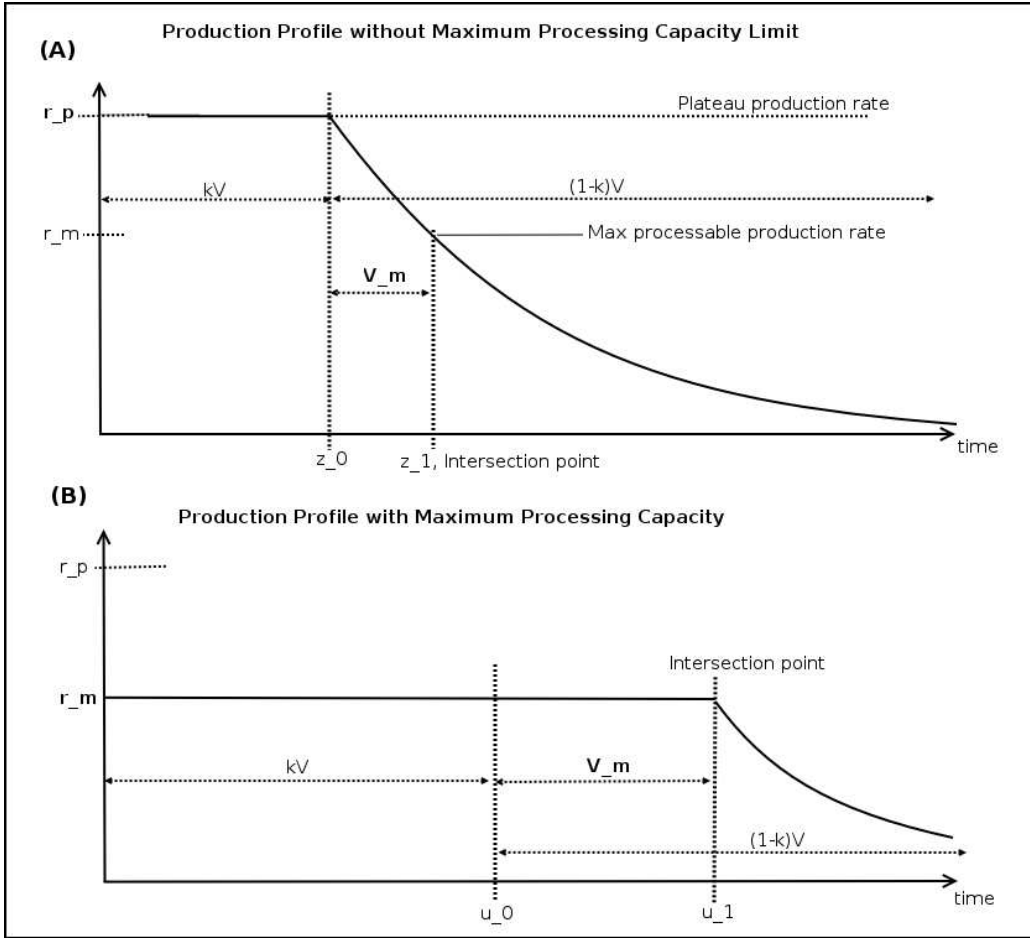


Figure A.3: Time of visible off-plateau rate

The area under the graph (A) from z_0 to z_1 is

$$V_m = \int_{z_0}^{z_1} r_p e^{-\lambda_f(u-z_0)} du \quad (\text{A.26})$$

$$V_m = \frac{r_p}{\lambda_f} (1 - e^{-\lambda_f(z_1-z_0)}) \quad (\text{A.27})$$

substituting for z_1 (eq. A.25), yields

$$V_m = \frac{r_p}{\lambda_f} (1 - e^{-\lambda_f(-\frac{\ln \frac{r_m}{r_p}}{\lambda_f} + z_0 - z_0)}) \quad (\text{A.28})$$

$$V_m = \frac{r_p}{\lambda_f} \left(1 - \frac{r_m}{r_p}\right) \quad (\text{A.29})$$

further substituting for λ_f (eq. A.12) yields

$$V_m = (1 - k)V \left(\frac{r_p - r_m}{r_p}\right) \quad (\text{A.30})$$

□

Having found a formula for the amount of oil to be recovered before the production rate drop will be noticeable, we may find the production rate after this level. Note that the profiles shown here correspond to a *fixed production profile* as in figure A.2. Below, I will present the formula for finding the production rate at time $u > u_1$, where u_1 is the visible plateau end time. Following this, a formula for finding the production rate in the realized production profile will be presented, similar to that of the bottom graph of figure A.2.

Claim A.6. *The production rate after the visible plateau end, starting all the oil wells simultaneously, is*

$$f_m(u) = r_m e^{-\lambda_m(u-u_1)}$$

where $\lambda_m = \frac{r_m}{kV+V_m}$ and u_1 the visible plateau end time.

Deduction. The areas under the graphs (A) and (B) for the V_m are equal:

$$V_m = \int_{z_0}^{z_1} r_p e^{-\lambda_f(u-z_0)} du = \int_{u_0}^{u_1} r_m du \quad (\text{A.31})$$

$$(1 - k)V \left(\frac{r_p - r_m}{r_p}\right) = r_m(u_1 - u_0) \quad (\text{A.32})$$

$$u_1 = (1 - k)V \frac{r_p - r_m}{r_p r_m} + u_0 \quad (\text{A.33})$$

Finally, substituting for u_0 (eq. A.1) yields

$$u_1 = (1 - k)V \frac{r_p - r_m}{r_p r_m} + \frac{kV}{r_m} \quad (\text{A.34})$$

This is the time when the production rate goes below max processing capacity, having recovered $\omega = kV + V_m$ oil. Following claim A.3, the production rate at a given time $u > u_1$, is

$$f_m(u) = r_m e^{-\lambda_m(u-u_1)} \quad (\text{A.35})$$

where $\lambda_m = \frac{r_m}{(1-k)V - V_m}$. □

Claim A.7. *Suppose $r_p > r_m$, the production rate after phasing in a new oil well is*

$$g_f(u) = \begin{cases} r_m & , \quad u < u_{\text{plateau}} \\ f_m(u - u_{\text{plateau}}) & , \quad u \geq u_{\text{plateau}} \end{cases}$$

where $u_{\text{plateau}} = u_1$ is the visible plateau end time and g_f and f_m functions of the fixed production profile, all the oil wells started simultaneously.

Deduction. This follows directly from the definition of the visible plateau end and claim A.6. □

As in section A.1, we find the state by comparing with the oil recovered ω . If $\omega < kV + V_m$ then $u < u_{\text{plateau}}$, conversely, if $\omega \geq kV + V_m$ then $u \geq u_{\text{plateau}}$ and we are at below maximum capacity production. In the first case, the production continues at r_m , and the visible plateau end time is

$$t_{\text{plateau}} = t + \frac{kv + V_m - \omega}{r_m} \quad (\text{A.36})$$

In the latter case, production will continue below r_m .

Claim A.8. *When phasing in a new oil well at time t_w , yielding a production rate below the maximum processing capacity, the production rate at a time $t > t_w$ is*

$$f(t) = (r_m - \lambda_m(\omega - kV - V_m)) e^{-\lambda(t-t_w)}$$

Deduction. Figure A.4 is an extension of figure A.2, where the production state time is beyond the intersection time discussed above.

The dashed curves after the well insertion at time t_{new} , u_{new} and z_{new} in the figure, are the deterministic exponential curves given by the state at u_{new} . What we have from earlier is the production function $f_m(u)$ (eq. A.35), that is, the production rate function of the middle graph (figure A.4) with

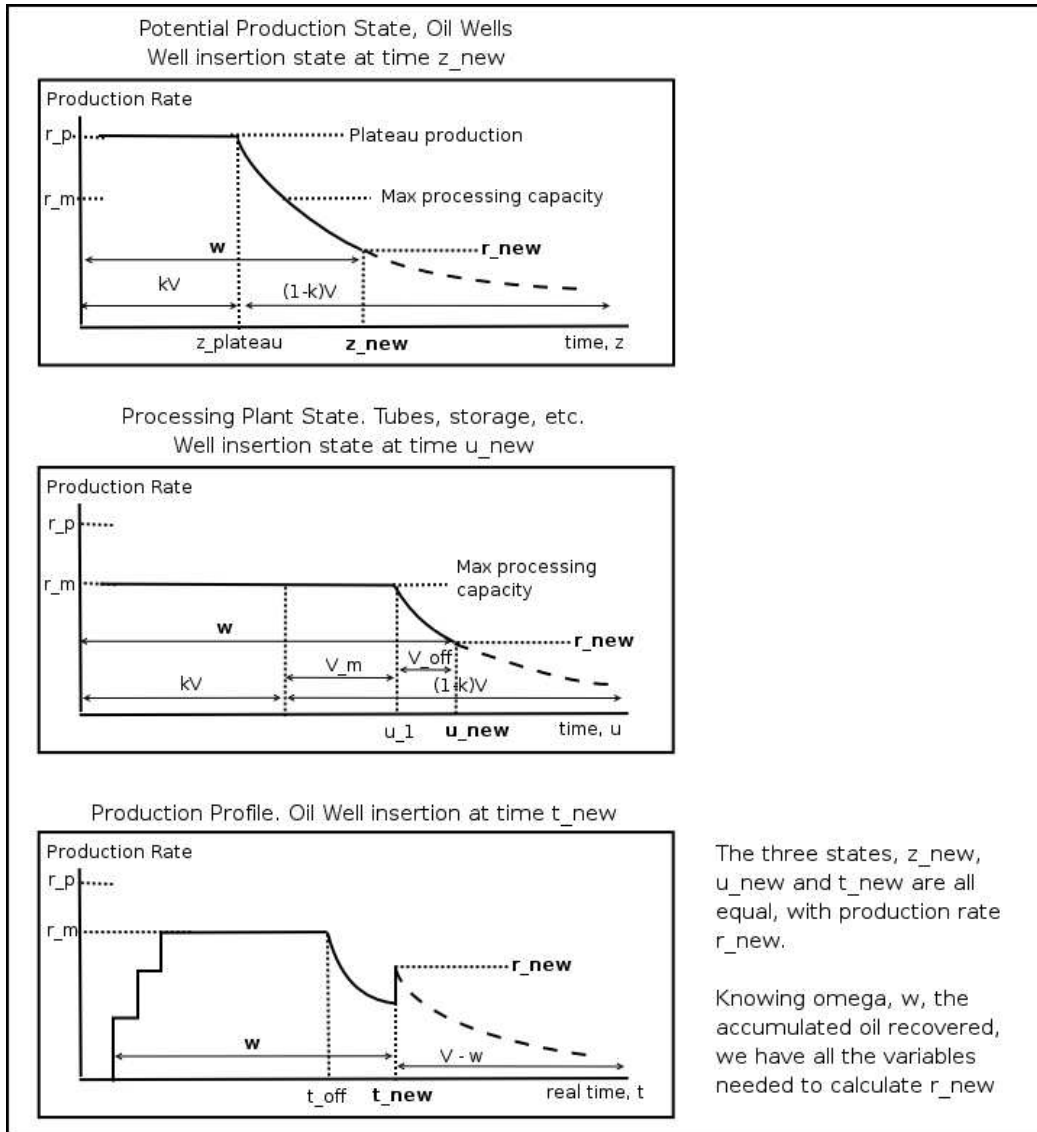


Figure A.4: Well insertion on visible off-plateau production

respect to u_1 . As in the previous section, we need to map from the fixed profile to the time domain of the production profile being realized. That is,

$$f_m(u) = r_m e^{-\lambda_m(u-u_0)} \rightarrow r_{new} e^{-\lambda(t-t_{new})} = f(t) \quad (A.37)$$

ω oil is recovered at time $u_{new} > u_0$. Finding $u = u_{new}$ will give us the

new production rate, $f_m(u_{new})$, at t_{new} (t_{new}). Hence, the production rate at $t \geq t_{new}$ is

$$f(t) = f_m(u_{new})e^{-\lambda(t-t_{new})} \quad (\text{A.38})$$

where λ is given by

$$V - \omega = \int_{t_{new}}^{\infty} f_m(u_{new})e^{-\lambda(t-t_{new})} dt \quad (\text{A.39})$$

$$\lambda = \frac{f_m(u_{new})}{V - \omega} \quad (\text{A.40})$$

Then, from equation A.35 we get

$$f(t) = (r_m e^{-\lambda_m(u_{new}-u_1)}) e^{-\lambda(t-t_{new})} \quad (\text{A.41})$$

The state time u_{new} is determined by the amount of oil recovered off visible plateau:

$$V_{off} = \omega - (kV + V_m) = \int_{u_0}^{u_{new}} r_m e^{-\lambda_m(u-u_1)} du \quad (\text{A.42})$$

$$\omega - kV - V_m = \frac{r_m}{\lambda_m} (1 - e^{-\lambda_m(u_{new}-u_1)}) \quad (\text{A.43})$$

$$u_{new} = \frac{\ln\left(1 - \frac{\lambda_m}{r_m}(\omega - kV - V_m)\right)}{-\lambda_m} + u_1 \quad (\text{A.44})$$

With no unknowns in the current production rate left, we find that:

$$\begin{aligned} f(t) &= f_m(u_{new})e^{-\lambda(t-t_{new})} \\ &= r_m e^{-\lambda_m(u_{new}-u_0)} e^{-\lambda(t-t_{new})} \\ &= r_m e^{-\lambda_m\left(\frac{\ln\left(1 - \frac{\lambda_m}{r_m}(\omega - kV - V_m)\right)}{\lambda_m}\right)} e^{-\lambda(t-t_{new})} \\ &= (r_m - \lambda_m(\omega - kV - V_m)) e^{-\lambda(t-t_{new})} \end{aligned} \quad (\text{A.45})$$

This result is very similar to that of equation (A.21), as one might suspect. \square

Appendix B

ProductionProfiler - The Application

The application has been written in java and jython. The oil prices was once implemented in fortran 77 for speed purposes. Later, this has also been written in java, experiencing little differences in run time. An advantage of having it all in java and jython is that it all compiles to java byte code. It may be run as a web application, as is what I have done. The application may be found as a runnable on my web page <http://master.dontsay.no>. It runs with java web start for j2EE5.0+. Figure B.1 shows a screen shot of the main window of the application. The screen shots shown earlier are of the results window that pops up when the simulations are complete.

As big changes have continuously been made, I have not been able to keep up and adapt the tab panels located at the left. Therefore, I have disabled them. The “Tree View” button will display the XML file similar to a directory structure, but as this editor builds on the same class structure as the tab panels, it is not very useful for editing purposes.

Choosing Options -> Analyse RF-RCI data you may study the reservoir data file of RF and RCI values used in determining the distribution for RF. The Help contains more information about the application and its use, of needed.

An overview of how the program is built is given in figure B.2. It presents a sort of MVC model, listing the packages of the program. The package names should be fairly self explanatory. As the application as this point consist of about 80 code files, hence even more classes, and about 12000 lines of code, a class diagram would be too big. The discrete event model, package

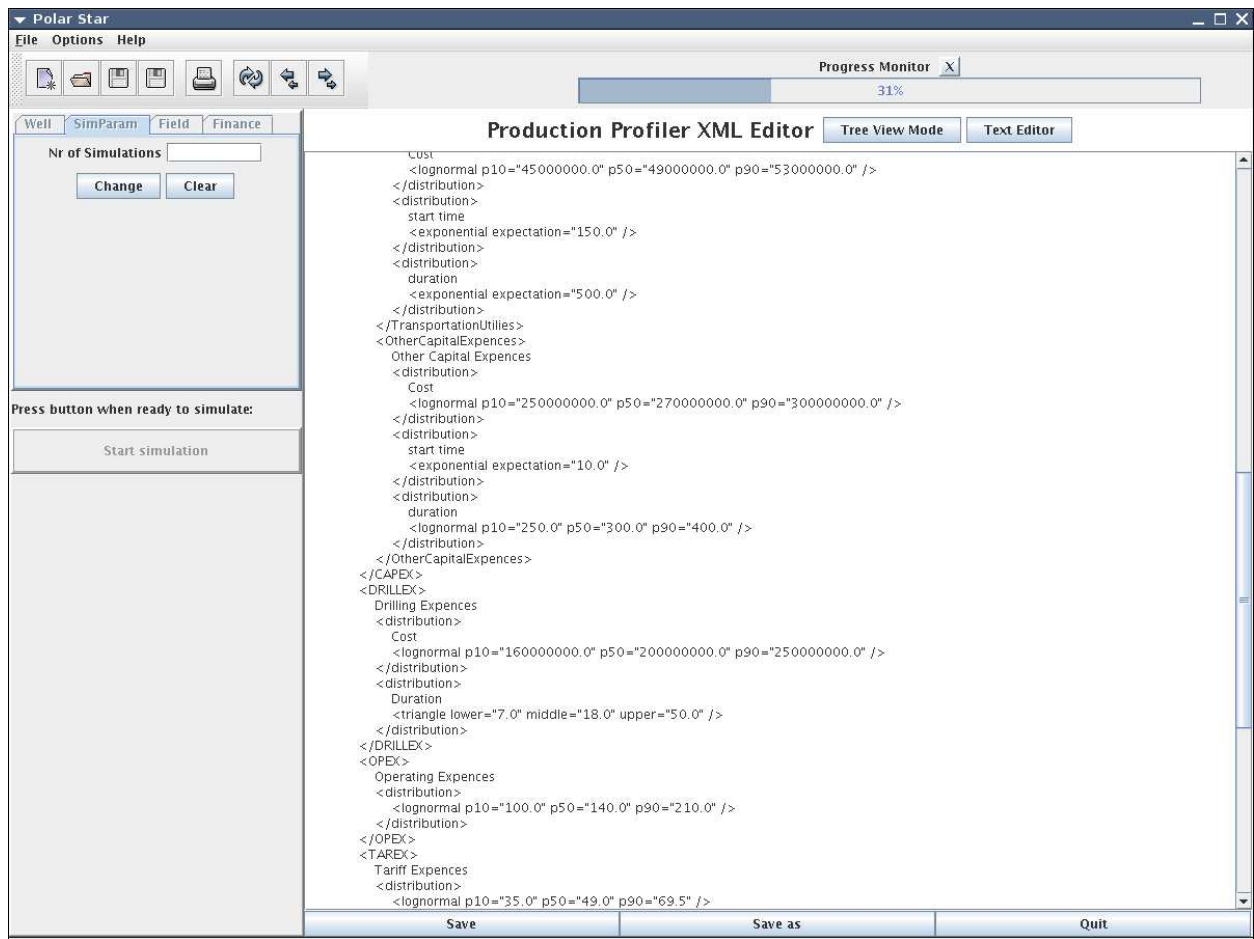


Figure B.1: Screen shot of the main screen of the application, having finish 31% of the simulations.

Simulation, was illustrated in figure 3.2, and with this diagram intend to illustrate the hierarchy of the classes.

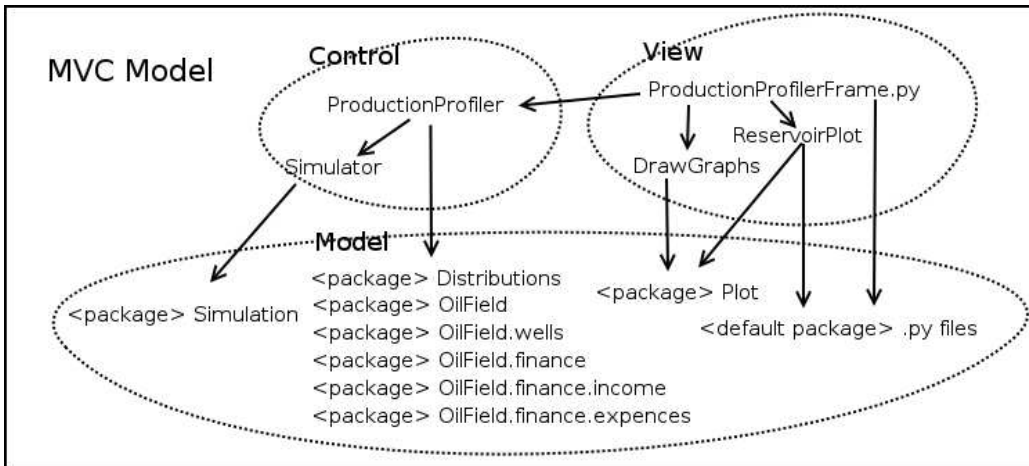


Figure B.2: MVC model for the ProductionProfiler application

Appendix C

A Study Of Recovery Factor And Reservoir Complexity Index

From a report from the Norwegian Oil Directorate [Ressursrapporten] summarized several results and studies done for the oil industry. It indicated a relation between the Recovery Factor and the Reservoir Complexity Index, plotting them together, based on observed data. Some observations I put into a file and plotted the values. The result is shown in figure C.1. Using linear regression, producing a linear mean from the top left to the bottom right, I could study the distribution of the points around the linear mean. Collecting the vertical distances of the points from the linear mean, I could calculate the mean and the variance of the vertical distribution along it. Note that there should be more data points and the source of these points are not very accurate. The application supports giving a distribution directly for the recovery factor, instead of feeding a reservoir data file of the RF and RCI data.

Using the mean and variance calculated from the observed data vertical distribution along the linear mean, I plotted the cumulative distribution function for the normal distribution. Adding the observed data points, we see that the normal distribution is not a bad estimate. But the uniform distribution would not have been a worse estimate either.

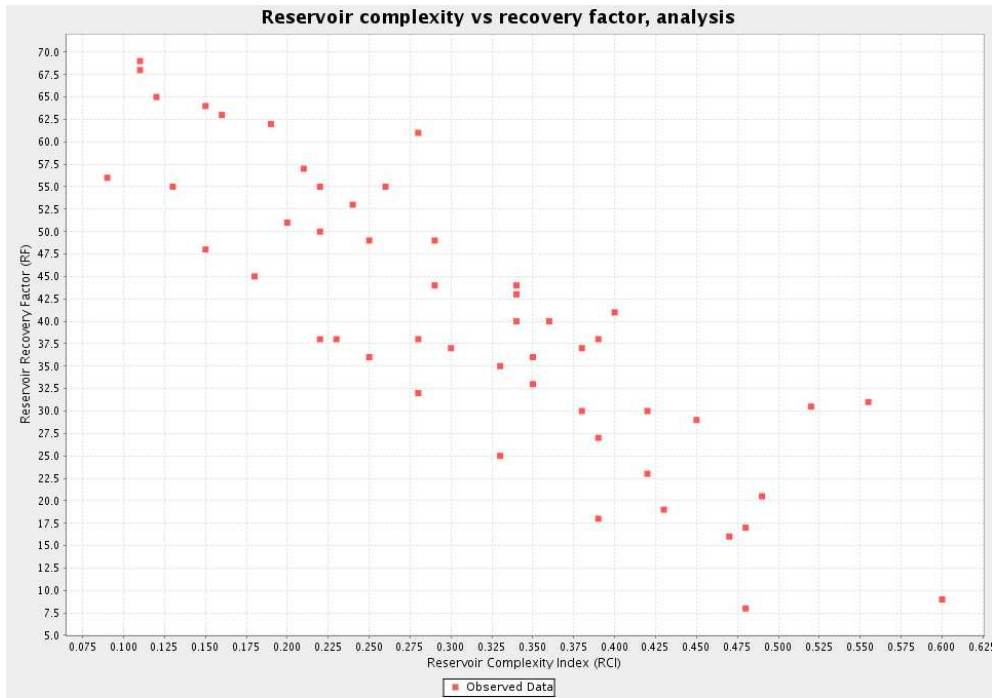


Figure C.1: Scatter plot of RF versus RCI

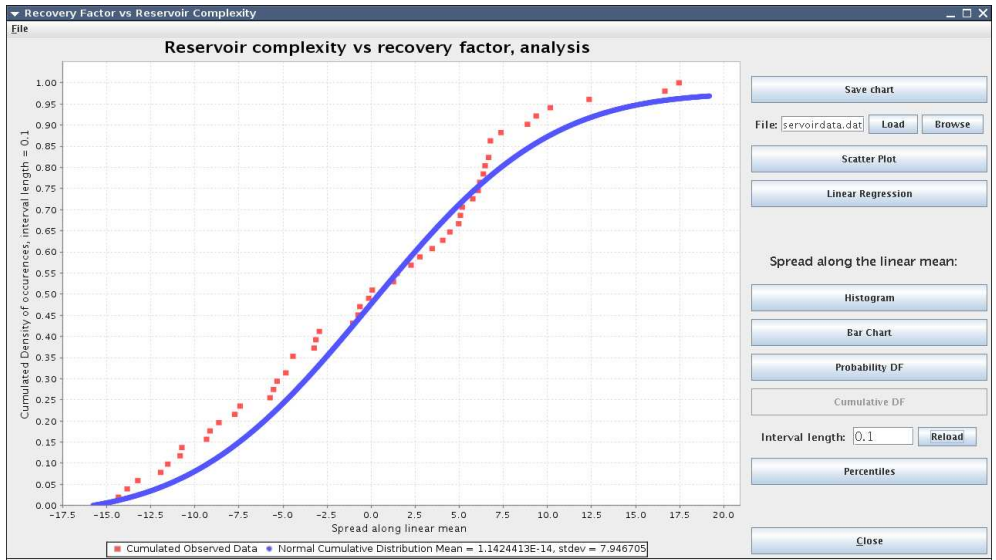


Figure C.2: Normal CDF from the data mean and variance versus the cumulative density of the observed data.

Appendix D

Oil Recovery Project - Polar Star

This is the fictive oil recovery project used for illustrating an input file in correct XML format. It should be pretty self explanatory, but further descriptions exists within the application. polarstar.xml:

```
<?xml version="1.0"?>
<ProductionProfiler>
  <Description>Example xml data for the Production Profiler program.
  Last updated 114 oct. 2005</Description>
  <name>Polar Star</name>
  <Simulation nrOfSimulations="5000">
    <time year="2005" month="01" date="01" />
    <percentiles p10="0.1" p30="0.3" p50="0.5" p70="0.7" p90="0.9" />
  </Simulation>
  <well>
    <nrOfWells distribution="triangle" lower="3" middle="4" upper="6" />
    <production distribution="lognormal" p10="10600.0" p50="13000.0" p90="16000.0" />
    <reserves>
      <STOIIP distribution="normal" mean="400000000.0" stdev="60000000.0"/>
      <RCI>0.25</RCI>
      <RF datafile="http://folk.uio.no/omamunds/master/reservoirdata.dat">Build
the rf distribution from the historic data</RF>
      <OilWellRecovery distribution="lognormal" p10="1.0" p50="1.5" p90="1.7">
oilwell recoverable reserves. Relative. Will be scaled to sum to the stoiip*(rf/rci)
      </OilWellRecovery>
    </reserves>
    <time> Time data
      <TOFO distribution="exponential" expectation="300" lower="800">Time Of
First Oil</TOFO>
      <interval distribution="exponential" expectation="30"> Well insertions
intervals</interval>
    </time>
  </well>
  <Field_and_grid>
    <PlateauFraction distribution="triangle" lower="0.03" middle="0.08"
upper="0.12"> Fraction of production recoverable on plateau </PlateauFraction>
    <OilRig>
      <processing_capacity distribution="constant" value="40000">Processing Plant
```

```

max processing capacity</processing_capacity>
</OilRig>
  <grid stepsize="1.0" storagepoints="400" />
  <time format="dd" start="0" end="6600">Daily</time>
  <comment>time formats should be "dd","mm" or "yyyy"</comment>
</Field_and_grid>
<Finance>
  <Expences>
    <CAPEX> Capital Expences
      <comment>Several abandonment expences may be defined</comment>
      <OilRigExpences>Rig Cost
        <distribution>Cost
          <lognormal p10="1200000000.0" p50="1500000000.0" p90="2000000000.0" />
        </distribution>
        <distribution>start time
          <lognormal p10="250.0" p50="300.0" p90="400.0" />
        </distribution>
        <distribution>duration
          <lognormal p10="300.0" p50="500.0" p90="600.0" />
        </distribution>
      </OilRigExpences>
      <TransportationUtilies> Transportation Expences
        <distribution> Cost
          <lognormal p10="45000000.0" p50="49000000.0" p90="53000000.0"/>
        </distribution>
        <distribution> start time
          <exponential expectation="150.0" />
        </distribution>
        <distribution> duration
          <exponential expectation="500.0" />
        </distribution>
      </TransportationUtilies>
      <OtherCapitalExpences> Other Capital Expences
        <distribution> Cost
          <lognormal p10="250000000.0" p50="270000000.0" p90="300000000.0"/>
        </distribution>
        <distribution> start time
          <exponential expectation="10.0" />
        </distribution>
        <distribution> duration
          <lognormal p10="250.0" p50="300.0" p90="400.0" />
        </distribution>
      </OtherCapitalExpences>
    </CAPEX>
    <DRILLEX> Drilling Expences
      <distribution> Cost
        <lognormal p10="160000000.0" p50="200000000.0" p90="250000000.0"/>
      </distribution>
      <distribution> Duration
        <triangle lower="6.0" middle="15.0" upper="30.0" />
      </distribution>
    </DRILLEX>
    <OPEX> Operating Expences
      <distribution>
        <lognormal p10="15.0" p50="20.0" p90="30.0" />
      </distribution>
    </OPEX>
  </Expences>
</Finance>

```

```

<TAREX> Tariff Expences
  <distribution>
    <lognormal p10="5.0" p50="7.0" p90="9.5" />
  </distribution>
</TAREX>
<ABDEX> Abandonment Expences
  <comment>Several abandonment expences may be defined</comment>
  <AbandonmentExpences>
    <distribution> Cost
      <lognormal p10="390000000.0" p50="420000000.0" p90="490000000.0"/>
    </distribution>
    <distribution> duration
      <triangle lower="1.0" middle="100.0" upper="400.0" />
    </distribution>
  </AbandonmentExpences>
</ABDEX>
<TAXEX>Tax</TAXEX>
</Expences>
<Oilprices> Future estimate percentiles
  <percentiles corrolation="0.9995">
    <p10 dd0="300" dd40="240" dd100="200" dd200="180" dd500="175"
dd1000="170" dd2000="155" dd3000="150" dd4000="145" dd5000="140" dd6000="140" />
    <p50 dd0="300" dd40="310" dd100="330" dd200="350" dd500="320"
dd1000="290" dd2000="270" dd3000="260" dd4000="250" dd5000="245" dd6000="240" />
    <p90 dd0="300" dd40="360" dd100="400" dd200="450" dd500="500"
dd1000="550" dd2000="600" dd3000="650" dd4000="690" dd5000="700" dd6000="710" />
  </percentiles>
</Oilprices>
</Finance>
</ProductionProfiler>

```