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Poisson vs. Long-Tailed Internet traffic

Master thesis

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26th September 2005

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Preface

This project is submitted in partial fulfillment of the Network and System Administration master program, a collaboration between Oslo College University and University in Oslo.

It is a result of continuing and intensive work in 6 weeks under the tutor of Professor Mark Burgess.

Le Anh Dung,
26 September 2005

Acknowledgement

Many thanks to my supervisor, professor Mark Burgess, for his tutor and encouragement through this project.

Many thanks to my wife, Anh Ngoc Tran, for being so patient with me for the last 10 years, constantly supporting my desire to study, from Norwegian Course to Adult HighSchool, from the disappointed years at Domus Medica to these recent years at Oslo College University. Like a widow, she spent most weekends, holidays together with our 2 children in my absence …
Without her, this thesis would not be finished.

Abstract

In this thesis, we reexamine the long discussion on which model is suitable for studying Internet traffic: Poisson or Long-tailed Internet traffic.

Poisson model, adapted from telephone network, has been used since the beginning of World Wide Web, while long-tailed distribution gradually takes over with believable evidence.

Instead of using Superposition of Point Processes to explain why traffic that is not Poisson tends towards Poisson traffic as the load increases, as it is recently claimed in [3], we try to approach this result by another simpler way.

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1. Introduction

“Poisson or not Poisson, that is the question”.

For nearly a quarter of a century, researchers have been looking for a stochastic process which could be used as an accurate and simple model for network traffic. Numerous professional literatures have written on traffic patterns in the Internet.

Since the 1970s, inherited from the voice telephone networks, the simple Poisson models are used.

Poisson models have also remained valid for modeling purposes for technology for at least fifty years. They are mathematically attractive, often amenable to elegant analysis, leading to closed-form expressions.

In nineties, a new wave of reports began to question the valid of Poisson models. Detailed studies of Internet network traffic show that the packet arrival process is not Poisson, even though the session arrival process is Poisson. Wide area traffic is much burstier than Poisson models predict [1], [2].

Queueing behaviour can be much more variable than predicted by a Poisson model and Internet traffic is rather Long-tailed distribution than Poisson.

Recently, in the beginning of this century, a new research [3] makes the discussion more interesting again : Internet traffic is really not Poisson, but tends towards Poisson and independent as the load increases.

In this thesis, we consider Poisson Internet traffic versus Long-tailed Internet traffic and try to find out how Poisson distribution arise as a limiting process by the convolution of several arrival processes.

2. Background

One of the most important concepts in probability is the idea of a probablility distribution.

There are discrete distribution and continuous distribution.

Poisson and binomial distribution are discrete, while continuous distribution consists of : Normal, Exponential, Power, or Cauchy distribution …

Example :

The binomial distribution is applicable whenever a series of trials is made, each trial has only two possible outcomes, either success with probability p or failure with probability (1-p). The number of successes which is observed may be any integer between 0 and n.

The probability of getting q successes out of n :

$$f(q) = {}^{n}C_{q}\, p^{q}\, (1 - p)^{n-q} \qquad (q = 0, 1, 2 \ldots n)$$

Where

$$ {}^{n}C_{q} = \binom{n}{q} = \frac{n!}{(n-q)!\, q!}$$

n! is the factorial of n, with n is an integer.

We shall use this as a basic for our proof later in Chapter §5.4

2.1 Poisson distribution

”*Life is good for only two things,*
discovering mathematics and teaching mathematics”
Simeon Denis Poisson

In statistical and probability theory, the Poisson distribution is a discrete probability distribution discovered by Simeon Denis Poisson (1781 - 1840), an eminent French mathematician and physicist, an academic administrator.

One of Poisson's many interests was the application of probability to the law, and a limit theorem for the binomial distribution was published in 1837 in his ' Recherches sur la Probabilité de Judgements'. Although initially viewed as a little more than a welcome approximation for hard-to-compute binomial probabilities, this particular result was destined for bigger things : it was the analytical seed out of which grew what is now one of the most important of all probability models, the Poisson distribution [6].

The Poisson distribution is used to model the a number of discrete events (for example : called arrivals) occurring during a time interval of given length.

The probability that there are exactly x arrivals (x = 0,1,2,3…)

$$p(x, \lambda) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Where :

e is the base of the natural logarithm e = 2.71828…

x! is the factorial of x, with x is an integer.

λ is the shape parameter, a positive real number, which indicate the average number of arrivals that occur during the given interval.

These probabilities are non-negative and sum to one.

With the help of Gamma function , defined by :
$x! = \Gamma(x+1)$

The above Poisson discrete (x : integer) can be written under continuous distribution as follows :

$$p(x, \lambda) = \frac{e^{-\lambda} \lambda^{x}}{\Gamma(x+1)} \quad , \quad \text{for alle } x$$

The formula for the Poisson cumulative probability function is :

$$F(x, \lambda) = \sum_{i=0}^{x} \frac{e^{-\lambda} \lambda^{i}}{i!}$$

A fundamental property of the Poisson distribution is that the variance of the distribution equals the mean. The standard deviation is equal to the square root of the mean.

The Poisson distribution has two main applications : firstly for describing the number of arrivals which occur in a certain time interval, and secondly as a useful approximation to the binomial distribution when the binomial parameter p is small [7].

The sum of two independent Poisson is still Poisson distributed :
If N and M are two independent random variables, both following a Poisson distribution with parameters λ and μ, respectively, then N + M follows a Poisson distribution with parameter $\lambda + \mu$ [18].

2.2 The Poisson process

The Poisson process is a model for series of events occurring from time to time at random.

In order to understand the Poisson process, we shall first find out what counting process mean.

A counting process is a function N_t that counts the total number of occurrence of certain 'events' up to time t. By 'up to time t' we mean 'before and at time t', i.e., if an event occurs at time t, it is counted immediately. Graphical describing of this function is called a sample path of that counting process.

Definition :

A counting process is a continuous time discrete valued process (N_t) such that :

- $N_0 = 0$
- (N_t) is continuous from the right and has limits from the left with probability one.
- At each point of discontinuity, the sample path jumps up exactly by size one.

Define $T_n := S_n - S_{n-1}$, $n = 1,2,\ldots$ by the inter-arrival time between the $(n-1)^{th}$ arrival time and the n^{th} arrival time.

<u>Definition of Poisson process in terms of inter-arrival time:</u>

A counting process (N_t) that counts events with i.i.d. (independent identical distributed) exponential inter-arrival times with λ is called a Poisson process with rate λ .

N_t follows a Poisson distribution with mean λt. that is

$$P(N(t) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad \text{for } k = 0, 1, 2..$$

For any $t \geq 0, s > 0$

- $N_{t+s} - N_t$ follows the same distribution as N_s.
- $N_{t+s} - N_t$ is independent of N_t.

In other words, for a Poisson process it does not matter when we start the counting, what matters is merely how long we count. This is the memoryless property of exponential distributions.

The Poisson process has independent increment property (and is therefore a Markov process) which means any two increments involving disjoint (non-overlapping) intervals are independent :

If $s_1 < t_1 < s_2 < t_2$ then the two increments $N(t_1) - N(s_1)$ and $N(t_2) - N(s_2)$ are independent.

The Poisson process is regenerative, that is each deterministic time and at each arrival time, the process "starts over" independently from the past.

One can also split a Poisson process into m independent Poisson processes [18].

2.3 Long-tailed distribution

According to [2], a distribution is long-tail if :

$$\mathbf{P[X > x] \sim x^{-\alpha}\text{ , as } x \to \infty,\ 0 \le \alpha}$$

This means that regardless of the distribution for small values of the random variable, if the asymptotic shape of the distribution is hyperbolic, it is long-tailed. For a long-tail distribution, the tails declines to zero very slowly.
The simple long-tailed distributions are Cauchy distribution, Pareto distribution…

If $\alpha \le 2$, then the long-tailed distribution has infinite variance.

If $\alpha \le 1$, then the long-tailed distribution has infinite mean.

A cumulative distribution function, F(x), is said to have a long tail if there exist positive constant β that

$$\mathbf{1 - F(x) \sim cx^{-\beta}\text{ as } x \to \infty}$$

c is a positive finite constant that does not depend on x
β is the tail index in the interval (0,2)

This property is, for example, satisfied by the well-known family of “Pareto distributions”, originally introduced for modeling the distribution of income within a population.

The characteristic of long-tailed distributions is that the log-log plot of the tail of a long-tailed distribution is approximately linear over many orders of magnitude (on contrast that of an exponential distribution is linear) [8].

3. Review of Related Research

“Teletraffic theory” refers to all mathematical modeling, statistical interference, queueing and performance in voice telephon network.
Later it was extended to data networks as Internet too, and Internet engineering: the design, management, control, and operations of the global Internet, would become part of teletraffic theory.

The call arrivals at links in the voice telephone network are presumed Poisson process, which means that the call arrivals are mutually independent and the call interarrival times are all exponentially distributed with one and the same parameter λ [4].

Voice traffic has the property that it is relatively homogeneous and predictable, and from a signaling perspective, spans long time scales.
Consequently, voice networks have been engineered in a circuit-switching fashion.

In traditional statistical theory, arrival events are always assumed to follow the pattern of a Poisson arrival process, that is, as a stream of random, independent arrivals. This assumption is made because it is simple and has a special analytical properties [16].

But the advent of faxes in the 1980s and the drastic explosion of the Web make great changes in teletraffic modeling. The key change is that telephone calls used for fax transmission and Internet access have statistical characteristics dramatically different from a typical voice call. They tend to be significantly longer and much more variable in their duration than a voice call, and their numbers have recentlty increased dramatically, especially in terms of Internet.

In contrast to voice traffic, data traffic is much more variable, with individual connections ranging from extremely short to extremely long and from extremely low-rate to extremely high-rate. It does not come at a steady rate, but instead in starts and fits with lulls in between. We call this characteristic “bursty”; that is, if one event arrives, several tend to arrive in a cluster. These properties have led to a design for data networks in which each individual data ‘packet’ or datagram transmitted over the network is forwarded through the network independently of previous packets that may have been transmitted by the same connection.

And the Internet engineering community has come to consider teletraffic theory as irrelevant and detrimental to the development of the Internet.

Detailed studies of Internet network traffic show that the packet arrival process is not Poisson in general. That is, the inter-arrival times between packets are not exponentially distributed, nor are they independent. The queueing behaviour can be much more variable than predicted by a Poisson model.

Internet network traffic analysis studies later show that the arrival times of data packets within a stream have a long-tailed distribution, often modeled as a Pareto distribution in the asymptotic limit [1],[2].

Is there any meaning in this discovering?

It is an important consideration to designers of routers and switching hardware. It implies that a fundamental change in the nature of network traffic has taken place. A partial explanation of this behaviour is that packet arrival times consist not only of Poisson random processes for session arrivals but also of internal correlations within a session. Thus, it is important to distinguish between measurements of packet traffic and measurements of numbers of sockets or TCP sessions. The long tailed distribution exhibited by Pareto tails is often indicative of clustered behaviour. If one event arrives, several tend to arrive in a cluster or a burst [16].

This non-Poisson structure is believed to be due in part to the protocols used for data transmission.

Although the packet arrival process is not Poisson, there is strong evidence that the session arrival process is Poisson. That is, human Internet users seem to operate independently at random when initiating access to certain Internet resources. Remote-login with TELNET connection, file transfer with FTP, for example, can be well-modeled with a Poisson process, with fixed hourly rates [1].

Recent researches [3],[5] further conclude that, yes, the arrival process is really not Poisson, but it tends toward Poisson as the load increases.

Extensive empirical and theoretical studies of packet traffic demonstrate that the number of active connections has a dramatic effect on traffic characteristics. The packet traffic on a link can be modeled as a marked point process. The arrival times of the process are the arrival times of the packets on the link. The marks of the process are the packet sizes.
As the load increases, the laws of superposition of marked point processes push the arrivals toward Poisson.

(Point process theory and the superposition of Point Processes are advanced statistical themes we try to have a bird's eye view in Chapter 5. At the end of this chapter, §5.4, we also try to approach this idea in another simpler way).

Once the connection load is sufficiently large, the network begins pushing back on the attraction to Poisson by causing queueing on the link-input router. But if the link speed is high enough, the traffic can get quite close to Poisson before the push-back begins in force [3].

Article [8] concluded that even though there is some evidence that bursts sizes for ftp and HTTP file transfers are long-tailed, but file sizes, transfer times, and interarrival times are not sufficiently long-tailed.

Detailed researches do not stop at long-tail Internet traffic but go further, focusing on the tail behaviour, but the debate on the nature of the tail is of little practical interest or consequence, and there is never sufficient data to support any analytical form summarizing the tail behavior and therefore any summary could be misleading and dangerous [11]

On the other hand, it is also believed that long-tail distributions are poor models for average waiting time, queue length and their variance, etc…[12]

4. The Power Law

Researchers focused their efforts on detecting power laws, and sure enough, power laws were discovered for web files sizes, web site connectivities, and the router connection degrees [11].

Why are people focusing on power law?

In the past few decades scientists have recognized that on occasion nature generates quantities that follow a power distribution instead of Poisson.

Power law distribution does not have a peak, and its histogram is continuously decreasing curve, implying that many small events coexist with a few large events.

There is an important qualitative difference between a power law and a Poisson curve when it comes to the tail of the distribution. Poisson curves have an exponentially decaying tail, which is much faster decrease than that displayed by a power law. A power law decays far more slowly, allowing for 'rare events'.

Power law is characterized by a unique exponent. In networks, power law describes the degree distribution; the exponent is often called the degree exponent. Measurements indicates that the distribution of incoming links on Webpages followed a power law with a unique and well-defined degree exponent close to two [19].

This implies that the number of Webpages with exactly k incoming links, denoted by N(k), follows :

$$\mathbf{N\,(k) \sim k^{-\gamma}}$$

Where γ ~ 2 is the degree exponent.

With outgoing links, the degree exponent γ is slightly larger : 2,5.

In 1999, an analysis of the Internet topology by Faloutsos et.al [23] suggested that the distribution of node degrees of the Internet decays as a power law with γ= 2,22.

5. Method

5.1 Point Process Theory

Point Process Theory is a branch of Applied Mathematics that has been developed only in the last three decades and deals with events that occur at isolated points in time or space.

A Point Process is a model for describing the random numbers of occurrences of a certain event in time intervals or of the numbers of points in regions of a space. It is pictured best as a collection of random points.

Some of examples are times at which items enter or leave a certain place such as data packets entering a computer, telephone calls arriving to a switching center, times of births, police emergencies or earthquakes, etc [22].

A useful mathematical characterization of a point process is in terms of the inter-arrival $[t, t + \Delta t)$.
Alternatively, a point process can be characterized in terms of the associated counting process $N_X(t)$, formally defined as the number of arrivals in the interval $[0,t]$.

Because of the simplicity of the Poisson process, it traditionally plays a central role in point process studies.

5.2 Renewal theory

Poisson processes are members of a broader class of point processes known as renewal processes.

An ordinary renewal process is a point process in which the interarrivals are independent, and identically distributed according to some common probability density function [17].

A counting process $\{ N_t , t \geq 0\}$ is a renewal process if for each n, the inter-arrival time X_n [the time between the $(n-1)^{th}$ and the n^{th} arrivals] and $\{ X_n , n \geq 1\}$ are independent with the same distribution.

The time of the n^{th} arrival is

$$S_n = \sum_{i=1}^{n} X_i \,,\; n \geq 1 \,,$$

with $S_0 = 0$

We can write $N(t) = \max \{n : S_n \leq t\}$

A Point process N on R_+ with points at $T_1 < T_2 < \ldots$, is a renewal process if the interpoint distances $T_n - T_{n-1}$, n= 1, 2, … are independent and identically distributed.

A renewal process is regenerative, that is, at each arrival time, it starts over independently from the past. However, a renewal process does not necessarily hold the independent increment property, actually. It often fails to be Markov. It is because that the distribution usually is not memoryless.

Renewal processes are used to represent times of system failures, demands for services or information, emergencies, setups on a machines, etc. They are also used to model times at which complex systems 'regenerate'.

5.3 Limit theorem for Superpositions of Poissson and other distributions.

A Point Process is pictured best as a collection of random points; the superposition of two point processes is then obtained by taking the union of the points of both.

The simplest and best known result is that the superposition of two independent Poisson processes is again a Poisson process. That is, the class of Poisson processes is invariant under the superposition of its independent members.

In the case of modeling exogenous arrival processes consisting of many component processes, one may invoke the convergence of the superposition to the Poisson process. As the number of independent and relatively sparse component processes tends to infinity, the superposition asymptotically behaves like a Poisson process [18].

Precise proofs of the above conclusions are clearly written in [15].

An essential intuitive explaining is that, in a limit, the number of processes being superposed increases while each individual component contributes less and less so as to keep the intensity of the superposition process constant. This way, no component can contribute more than one point to the superpostition in any finite interval [0,t].

Suppose, there are n_t points in the interval [0,t]; then each belongs to a different component process; the positions of these n_t points are independent of each other by the independence of the components; and each position has the uniform distribution over [0,t] by the stationarity of the component it belongs to. The number n_t must be so that n_t /t is the intensity of the superposition as $t \rightarrow \infty$. This is precisely the description of the Poisson process.

5.4 Poisson distribution as a limit

The prominence of Poisson processes stems from the fact that they arise as limits of sums or superposition of uniformly sparce point process. They also arise as limits of processes of rare events [22].

An actual point process can often be viewed as a superposition of points from many source or as a process of very rare events, and so a Poisson model may be appropriate.

Another important property of a Poisson process is that it is Markovian. If N is a stationary Poisson process on R_+, then $\{N_t : t \in R_+\}$ has independent increments and hence is a Markov process. In general, a Poisson process on any space is a Markov random field. Poisson processes are also building blocks for a variety of other stochastic processes as well as point processes.

Below we try to understand how Poisson distribution can arise as a limiting process by the convolution of several arrival processes.

Consider an observational process in which either an event is registered or not i.e. we make a stream of observations i= 1, 2, 3, …, n, whose outcome is either true or false (such a variable is called a Bernouilli variable).

After n observations, suppose there are q "true" or positive observations. The distribution of probabilities of obtaining q positives in n observations is

$$f(q) = {}^{n}C_{q}\, p^{q}\, (1 - p)^{n-q} \qquad (1)$$

Where p : probability of obtaining a positive result on each independent observation.

By definition :

$${}^{n}C_{q} = \binom{n}{q} = \frac{n!}{(n-q)!\,q!}$$

Now consider this binomial distribution in the limit of small p and large n.
A binomial distribution has mean value $\lambda = np = \langle q \rangle$.

We interprete (1) as :

$$\begin{aligned} \text{Prob}(q = k) &= {}^{n}C_{k}\, p^{k}\, (1 - p)^{n-k} \\ &= \frac{n!}{(n-k)!\,k!} \cdot p^{k}\, (1 - p)^{n-k} \qquad (2) \\ &\leq \frac{n^{k}}{k!} \cdot p^{k}\, (1 - p)^{n-k} \end{aligned}$$

Because we knew from theory :

$${}^{n}C_{r} \leq \frac{n^{k}}{k!}$$

Suppose we assume that $np = \lambda$ = constant, but $n \to \infty$, $p \to 0$

$$\text{Prob}(q = k) \leq \frac{n^{k}}{k!} \cdot p^{k}\, (1 - p)^{n-k} \to \frac{(np)^{k}}{k!}\, (1 - p)^{n-k} \to \frac{\lambda^{k}}{k!}\, e^{-\lambda}$$

Thus :

$$\boxed{\lim_{\substack{n\to\infty \\ p\to 0}} \text{Prob}(q = k) \to \frac{\lambda^{k}}{k!}\, e^{-\lambda} \qquad (3)}$$

This is exactly the Poisson distribution previous defined in chapter §2.1.

We can verify that:

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} = 1$$

Since :

$$e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$$

(series expansion for exponential function)

Thus, based purely on a model of rare events measured over long times, or many samples, we expect that the probabilities are approximate the Poisson limit, or converge to Poisson distribution.

$$\boxed{\text{Prob}(q=k) \leq \text{Poisson}(q=k) \qquad (4)}$$

On purely general grounds we expect a true/false process, a counting process to have a Poisson limit for $n\rightarrow\infty$.

How big does n have to be?

In research [13], the authors note that there are asymptotes like power laws in discrete processes that tend to Poisson.
They suggest that one needs $n = 10^{10}$ to see Poisson behaviour.

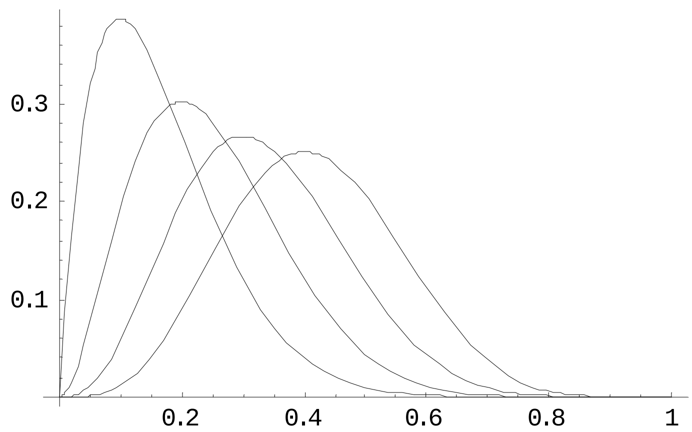
What does the distribution look like for finite n?

We try to plot the function (2) above for various n value, using Mathematica, for example :

$$\text{Prob}[n\_,\ k\_][p\_] := \frac{n!}{(n-k)!\,k!} p^k (1-p)^{n-k};$$

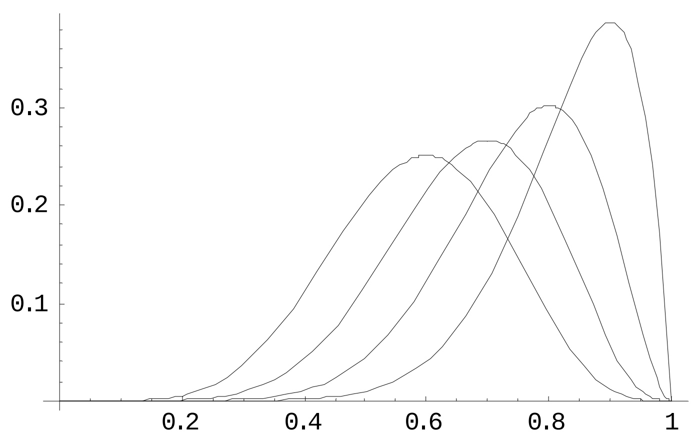
Plot[Evaluate[Table[Prob[10,i][p],{i,1,4}]],{p,0,1}, PlotRange →All]

- n= 10 with k=1, k=2, k=3, k=4 following this order from left to right :

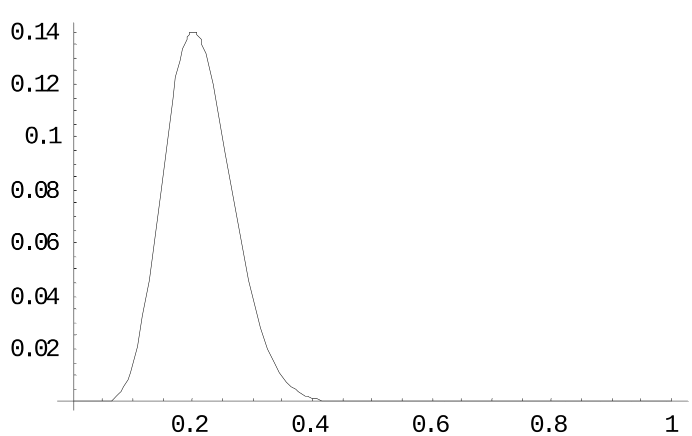


Using the above command with different parameters, we have :

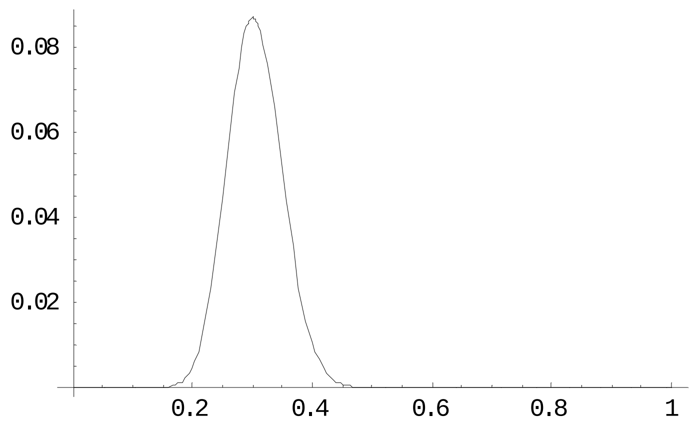
- n= 10 with k=6, k=7, k=8, k=9 following this order from left to right :



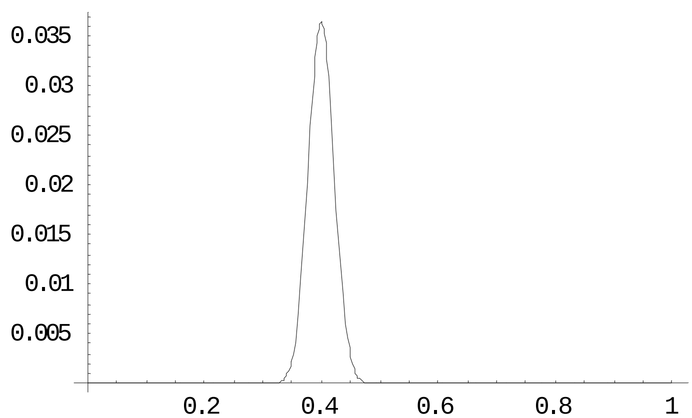
- n= 50 with k=10: (the maximum value now reduces to 0.14 while the maximum value when n =10 above is 0.39)



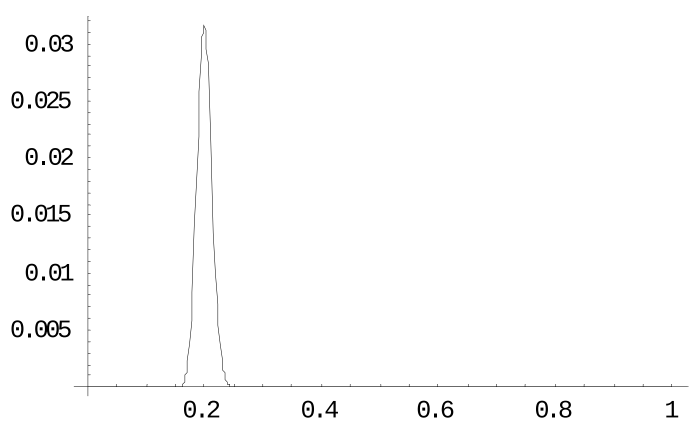
- n= 100 with k=30:



- n= 500 with k=200:

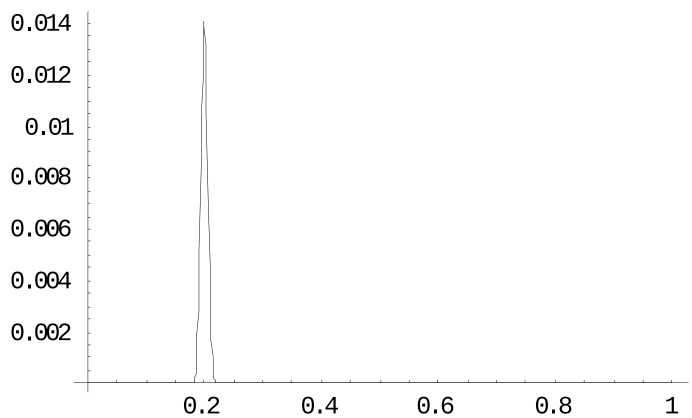


- n= 1000 with k=200:

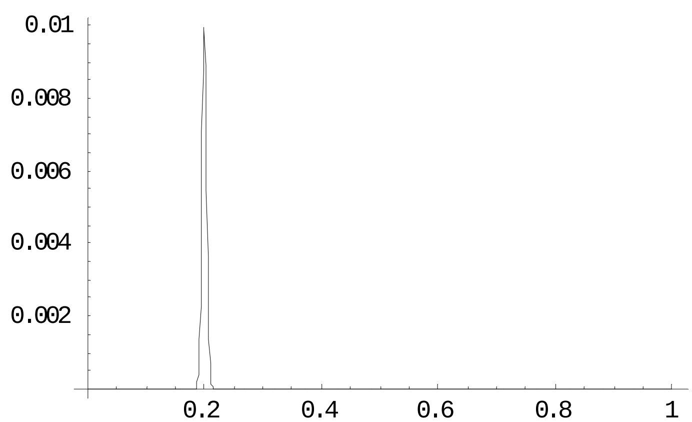


Comparing the graphs above, we see that the peaks are lowered down while the distances between the two zero points shortened.

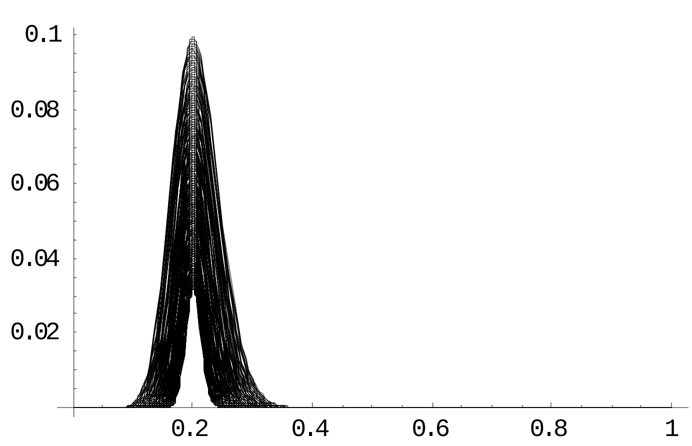
n= 5000 with k=1000:



- n= 10000 with k=2000:



The following figure show equation (2) for different n, from n=100 (outmost) to n=1000 (innermost) with the same value k/n=0.2 in the same graph :



Our remark is that the greater n is, the lower is the peak and the faster decreases the tail of the graph to zero. Those are also the signs of Poisson distribution we discussed in Chapter 4.

We try to find out how many observations are made before coming to the conclusion among the researches we have read. For example :

- 72000 observations of the number of Telnet originator packets arriving, 1.7 million arriving packets and then 2.4 million arriving packets, and in another experiment, there are 1.3 million packets, and another : 85000 packets [1].

- Under the research [2], 24 data sets are used to analysis the Ethernet traffic measurements, some of which come to nearly 28 million packets arrivals.

- The result in [5] is an emprical study based only on 3026 packets traces.

- In [8], the researcher uses a data sets of 4410851 interarrival times in 455992 connections, and then 739005 connections.

- The authors of [11] generated and running through the server about 500000 arrivals.

The above numbers show that the observations are far below 10^{10} (to see Poisson behaviour). This explains why their conclusions are another distribution rather than tend to Poisson.

Why it is so difficult to study the Internet traffic with a general model?

One of the reasons making the Internet exceedingly hard to characterize is that Internet changes in drastic ways over time, as an immense moving target.

Another problem is the heterogeneity of the Internet in topology, in link properties, and in protocols: Internet topology is also constantly changing in different entities, dynamic routing can make routes through network which are asymmetric change on short time scales, and the widely used TCP protocol has undergone major evolutionary changes in different implementations [20].

The Internet's technical and administrative diversity, sustained growth over time, and immense variations over time present a tremendous difficulties for attempts to simulate the Internet with a goal of obtaining "general" results.

6. Conclusion

We have reviewed the Internet traffic with two main models : Poisson versus Long-Tailed. The simple Poisson model does not capture all the statistical properties of packet traffic, but can describe the session arrival process, aggregating Web traffic.

The switch from Poisson to Long-Tailed thinking in Internet traffic research has had a major impact on our understanding of actual Internet traffic.

The recent research on Internet traffic tending toward Poisson, as the load increases, can be theoretically proved only, because it is difficult to accomplish the experiment with 10^{10} observations to verify this point of view.

In our opinion, looking for a stochastic process which could be used as an accurate and simple general model for Internet traffic seems like a vain effort because of the explosive growth of Internet over time, with immense variations.

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