Coastal Engineering 172 (2022) 104077

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

A truncated, translated Weibull distribution for shallow water sea states

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ARTICLE INFO

Keywords:
Ocean waves
Marine structures
Shallow water
Significant wave height
Coastal environments
Statistical modelling
Probabilistic wave models

ABSTRACT

Reliable probabilistic information of the long-term ocean wave climate is important in many coastal and ocean engineering applications such as the design of marine and coastal structures. There is therefore a need to establish accurate models for the long-term distribution of relevant metocean variables. One of the most important wave parameters in marine design is the significant wave height, which is a measure of the severity of the sea state, and wave loads are typically described in terms of this and other variables. Hence, a lot of effort has been put on the proper probabilistic modelling of significant wave height. However, probability distributions proposed to model significant wave height have mostly focused on deep water conditions and less efforts have been put on shallow water conditions. This paper proposes a truncated, translated Weibull distribution for the long-time distribution of significant wave height in shallow waters and demonstrates that the model fit shallow water data quite well. In particular, the proposed model describes shallow-water wave data better than the commonly used 3-parameter Weibull distribution, which is often the preferred distribution for deep water locations.

1. Introduction and background

Reliable probabilistic information about the long-term wave climate is essential for the design and operation of marine and coastal structures and other ocean engineering applications (Malliouri et al., 2021a; Fazeres-Ferradosa et al., 2018a). Such information can be found in wave data collected from in-situ measurements, satellite observations or numerical wave models, but recorded time series are typically supplemented by statistical models or probability distributions fitted to the data. Hence, there is a need for appropriate distribution functions to use for the long-term distribution of relevant metocean variables.

Significant wave height is one of the most important wave parameters in marine and coastal design, and it is a measure of the severity of the sea state. One classical definition of significant wave height, H_S, is that it is the mean of the highest one-third of the individual waves in a sea state; alternative definitions are that it is four times the standard deviation of the surface elevation or four times the square root of the zeroth-order moment of the wave spectrum (Holthuijsen, 2007). Typically, the short-term distribution of individual wave heights in a sea state is parametrized in terms of significant wave height and other sea state parameters, and the sea state distributions are needed for a long-term description of wave crests and heights (Mackay and Johanning, 2018; Bulgakov et al., 2018). Wave loads are typically described in terms of significant wave height making this a very important variable in structural reliability assessment and design of coastal structures. Often, the joint distribution of several metocean variables are needed, and the zero up-crossing wave period, mean wind speed and mean wind-and wave directions are other relevant variables. For coastal hazards, also the joint distribution of waves and water levels may be relevant (Hawkes et al., 2002). The environmental contour method is often used in structural reliability analysis based on the joint long-term distributions of relevant metocean variables (Winterstein et al., 1993; Haver and Winterstein, 2009). An alternative approach to combine long-term distributions of sea states with short-term distributions of structural response is recently proposed in Gramstad et al. (2020).

Due to its importance in coastal and ocean engineering, a lot of efforts have been made to find appropriate, parametric models for the long-term distribution of significant wave height. The log-normal distribution and the Weibull distribution, in its 2- or 3-parameter forms, have been widely used to model significant wave height (Jasper, 1956; Battjes, 1972; Mathisen and Bitner-Gregersen, 1990), and also hybrids of these models have been suggested (Haver, 1985). However, there are large model uncertainties in fitting a parametric model to significant wave height data, a number of other probability distributions have been explored (Ferreira and Guedes Soares, 2000). The generalized gamma distribution have been proposed for modelling significant wave

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https://doi.org/10.1016/j.coastaleng.2021.104077
Received 15 January 2021; Received in revised form 27 November 2021; Accepted 5 December 2021

height (Ochi, 1998) and various beta and gamma-distributions were fitted to significant wave height data in Ferreira and Guedes Soares (1999) and Guedes Soares and Scotto (2001). The Rayleigh distribution was compared to the Weibull distribution in Shariff and Hafezi (2012). Other parametric models used for significant wave height include the exponential distribution, the Gaussian distribution, the skewness-corrected log-normal distribution, the logistic distribution, and also non-parametric approaches such as kernel-based models have been proposed (Athanassoulis and Belibassakis, 2002). Recently, an exponentiated Weibull distribution was proposed in Haselsteiner and Thoben (2020). A maximum entropy distribution is proposed in Dong et al. (2013), and various more elaborated models include Bayesian models (Scotto and Guedes Soares, 2007), mixed models (Li et al., 2016), fractal-based models (Liu et al., 2019), time-series models (Athanassoulis and Stefanakos, 1995; Scotto and Guedes Soares, 2000), spatial models (Altunkaynak, 2005) and spatio-temporal models (Baxevani et al., 2009; Vanem, 2013) to describe the long-term distribution of significant wave height. Different extreme value models have also been widely used to model extreme wave conditions (Jonathan and Ewans, 2013). Several candidate models for significant wave height are examined in Soukissian and Takvor (2021), and compared to a model not previously applied to ocean data, i.e., the extended generalized inverse Gaussian distribution. Results indicate that the proposed new distribution outperforms the other candidate models for several, but not all datasets that were analysed. Nevertheless, the 3-parameter Weibull distribution is recommended in DNV's recommended practice on environmental loads (DNV, 2021) and remains a very common approach for describing the distribution of significant wave height in deep waters.

It is well known that restricted water depths influence the distribution of wave heights, e.g. due to depth-induced wave breaking. Moreover, coastal waters may be fetch-limited, which would also prevent generation of very large wind generated waves. The attenuation of the average wave climate towards the coast were observed and quantified from radar altimetry data in Passaro et al. (2021). Therefore, different models are required to describe shallow water waves compared to deep ocean waves (Battjes and Groenendijk, 2000; Méndez and Castanedo, 2007; Wu et al., 2016). Spectral wave models are often used to simulate wave characteristics, and several wave models have also been used for shallow waters, see e.g. Rusu et al. (2008) and Fonseca et al. (2017), and a framework for downscaling wave reanalyses to coastal areas combining dynamical and statistical downscaling is presented in Camus et al. (2013). Another approach to describe shallow water sea states is based on the empirical relationship between deep sea characteristics and nearshore characteristics, and regression-type statistical models can be established using for example neural networks or other regression models, see e.g. Kalra et al. (2005) and Browne et al. (2007). Regression models for coastal sea states with wind data or other meteorological data as input has also been suggested in e.g. Shamshirband et al. (2020) and Casas-Prat et al. (2014).

There has recently been much interest in the statistics of shallow water waves and several studies have been reported. However, there seem to have been more focus on the statistics of individual wave height compared to that of shallow water sea states. A recent study based on deep waters measurements presented in Kvingedal et al. (2018) suggests that the Forristall distributions for individual wave and crest heights generally fits the deep-water data well, but that it is less accurate in steeper sea states corresponding to high wind speeds. An analysis of laboratory data presented in Karmpadakis et al. (2019) and Zhang et al. (2019) confirms systematic departures from the deep ocean distributions. It is suggested in Zhang et al. (2019) to use a generalized Boccotti distribution (Alkhalidi and Tayfun, 2013) for shallow water waves. The fact that the steepness and asymmetries of extreme waves increase with shallower water depths is found by Chen et al. (2018), which proposes an empirical parametrization of wave steepness and asymmetries in nearshore environments. The effect of bottom topography on the distribution of wave heights in shallow waters is investigated in Bolles et al. (2019), Majda and Qi (2019), and Majda et al. (2019) presents a statistical dynamical model that accounts for the strong positive skewness that are observed downstream of an abrupt depth change. Subspace analysis is applied for the exceedance probability of shallow-water waves in a sea state in Šehić et al. (2021).

The study presented in Malliouri et al. (2019) aims at obtaining a more accurate description of the long-term wave climate in shallow waters by combining short- and long-term statistics in deep waters. First, the joint long-term statistics of wave height and period in deep waters are found by combining the conditional short-term joint distribution of these parameters with the long-term joint distribution of sea states parameters significant wave height and mean zero-crossing wave period. Then, the joint distribution in shallow waters is estimated by considering wave transformation of each individual wave as waves propagate from the open sea towards shallower waters. They find that wave statistics in shallower water differ from those in deeper waters, but ends up with the same parametric family for intermediate waters as in deep water for the long-term distribution of sea states, i.e. Weibull or Gamma distributions for the significant wave height and conditional lognormal for mean zero-crossing period. However, the distributional parameters change. This statistical framework is applied to reliability analysis of coastal structures in Mallouri et al. (2021b). A regional frequency analysis of extreme sea states in coastal areas is reported in Lucas et al. (2017).

Notwithstanding the huge amount of literature focusing on the probabilistic modelling of significant wave height, and the increasing interest in the distribution of individual wave heights in shallow water, there has been considerable less efforts on statistical modelling of the long-term distribution of sea-state parameters such as significant wave height especially for shallow water conditions. However, it cannot be expected that probabilistic models used for sea states in deep water conditions are equally suited for coastal waters (Bitner-Gregersen, 2018). Hence, this paper proposes a truncated, translated Weibull distribution for significant wave height in shallow waters. A case study is shown where the proposed distribution function is fitted to a set of data from a shallow water location and compared to the standard 3-parameter Weibull distribution. Results indicate that the truncated Weibull distribution yields better fit to the data than the un-truncated one. The effect of marginal distribution of H_S on the joint distribution of significant wave height and zero up-crossing wave period is illustrated by environmental contours, again demonstrating an improved fit to the data with the truncated model.

The remainder of this paper is as follows. First, an introduction to the truncated, translated Weibull distribution is given in Section 2. Then, a description of the data used in this study is presented in Section 3 and the case study is presented in Section 4. Finally, a summary and general conclusions is given in Section 5.

2. A truncated, translated Weibull distribution

The 3-parameter, translated Weibull distribution is often used for modelling significant wave height. It has probability density function

$$f_X(x) = \frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}}, \quad x \geq \gamma \tag{1}$$

and cumulative distribution function

$$F_X(x) = 1 - e^{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}}, \quad x \geq \gamma, \tag{2}$$

where α is the scale parameter, β is the shape parameter and γ is the location parameter. Fitting such distributions to data amounts to estimating these parameters, and several techniques exist including maximum likelihood, method of moments, least squares and minimization of various goodness-of-fit measures.

The 3-parameter Weibull distribution has a lower bound of the support dictated by the location parameter γ, with $f_X(x) = 0 \; \forall x < \gamma$, and $\gamma = 0$ reduces to the 2-parameter Weibull distribution with support on $[0, \infty)$. Hence, this distribution has no upper bound.

Waves in shallow water, on the other hand, are bounded by the effect of the water depth and wave heights in shallow waters will have an upper limit. These upper bounds will not be captured by the 2- or 3-parameter Weibull distributions. Hence, in this paper it is proposed to rather use a 4-parameter Weibull distribution where the upper tail is truncated by an additional parameter. As far as the authors know, this distribution has not yet been applied to significant wave heights, but is reported to be useful for describing wind speeds in Savenkov (2009) and Kantar and Usta (2015). However, the distribution in Savenkov (2009) is a left-truncated Weibull distribution, and the one proposed in Kantar and Usta (2015) is a truncated 2-parameter distribution. Doubly-truncated versions of the Weibull distribution also exist (Zhang and Xie, 2011). In this paper, however, a truncated and translated Weibull distribution is proposed, corresponding to an upper-truncated 3-parameter Weibull distribution.

The upper-truncated, translated Weibull distribution is defined on the interval $\gamma \leq x \leq \tau$, where τ is the truncation point, and has cumulative distribution function

$$G_X(x) = \frac{F_X(x)}{F_X(\tau)} = \frac{1 - e^{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}}}{1 - e^{-\left(\frac{\tau-\gamma}{\alpha}\right)^{\beta}}}, \quad 0 \leq x \leq \tau. \tag{3}$$

The probability density function is simply the derivative of this, hence the truncated, translated Weibull distribution has pdf

$$g_X(x) = \frac{f_X(x)}{F_X(\tau)} = \frac{\frac{\beta}{\alpha}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}}}{1 - e^{-\left(\frac{\tau-\gamma}{\alpha}\right)^{\beta}}}, \quad 0 \leq x \leq \tau. \tag{4}$$

Often, the truncation point τ is assumed known, but in this paper this is regarded as an additional model parameter that is estimated from the data.

Model parameters are estimated in this study by maximum likelihood, as well as maximum goodness-of-fit. For a discussion on estimation methods for the truncated Weibull distribution, reference is made to Mittal and Dahiya (1989).

3. Data description

In this study, data of significant wave height (H_S) and zero up-crossing wave period (T_Z) from a shallow water location are analysed. The data are extracted from hindcast data over an area in the Danish sector of the North Sea with water depths ranging from 10 to 20 m. The exact location of the data being analysed in this paper is 55.725°N and 7.750°E. The temporal resolution is 1 h and data from 124 months from January 1. 2003 to May 1. 2013 are included, corresponding to a total of 90 553 joint observations of H_S and T_Z. The water depth at this particular location is reported to be 18 m. The same data was used in Fazeres-Ferradosa et al. (2018a) and Fazeres-Ferradosa et al. (2018b) which give a more detailed description including a table of descriptive statistics.

A scatterplot of the data is shown in Fig. 1. Note in particular the peculiar characteristics of the upper significant wave heights, which presumably is an effect of the limited water depth. This feature makes it difficult to obtain a good fit using the standard 2- or 3-parameter Weibull distributions, but it will be demonstrated that reasonable fits can be obtained using the truncated distribution.

4. Analysis and results

4.1. Univariate distribution of significant wave height in shallow waters

First, the commonly used 3-parameter Weibull distribution is fitted to the significant wave height data using standard maximum likelihood (MLE) and maximized goodness of fit according to the 2nd order Anderson–Darling statistic (A2D) and the Cramer–von Mises statistic (CvM), respectively. The estimated pdfs and cdfs are shown in Fig. 2 together with the empirical ones. At first sight, the plots indicate a reasonable fit to the data. However, focusing on the upper tail it appears that there is an upper bound in the empirical distribution that is not captured by the 3-parameter Weibull distribution, as shown in the close-up plots in Fig. 3. The horizontal dashed lines in the plot of the CDFs in Fig. 3 corresponds to return values for return periods of 10, 25, 50 and 100 years, respectively.

Even though the various Weibull-fits look reasonable overall, it is cleat that the upper tails are not well captured and that the model needs to be improved. Hence, additional endpoint-parameter is introduced and the truncated 3-parameter Weibull distribution is fitted to the data. It is noted that the endpoint is often assumed known, and may typically be taken as the highest valued observation. However, in this study, this is assumed to be an unknown parameter that is fitted by maximum likelihood. However, in order to avoid unreasonable estimates for this parameter, an upper bound is used in the optimization algorithms. In this paper, an upper bound is assumed to be $0.6 \times d$ where d denotes the water depth. It is noted that this value is much higher than the highest observed significant wave height in the data, and corresponds to an upper bound on significant wave height of 10.8 m for this location.

The results of fitting the truncated 3-parameter Weibull distribution to these data are shown in Figs. 4–5 and compared to the initial fits using the un-truncated distribution. Only maximum likelihood fitting is used for the truncated distribution, and it is obvious from Fig. 4 that the truncated distribution is nearly identical to the un-truncated one based on the same fitting technique. However, when zooming in on the tail (Fig. 5), it is observed that the truncated distribution captures the upper bound of the empirical distribution, which is an effect of the restricted water depth. It is also noted that the estimated endpoint parameter is just slightly higher than the maximum H_S observed in the data. The estimated model parameters for the different models are presented in Table 1.

It is observed that the parameters of the estimated truncated Weibull distribution are nearly identical to the ones estimated for the un-truncated version, using the same fitting technique (MLE). Hence, the main body of the fitted distribution is hardly affected by the truncation. In order to aid model selection, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are used. These are defined as follows

$$\begin{aligned} AIC &= 2k - 2\ln(\hat{L}) \\ BIC &= k\ln(n) - 2\ln(\hat{L}), \end{aligned} \tag{5}$$

where k is the number of model parameters, n the sample size and $\hat{L}$ is the maximized likelihood. The model with lowest value of AIC/BIC is preferable according to these criteria. The AIC and BIC values for the model alternatives fitted by MLE are included in Table 1. Since AIC/BIC are based on the maximized likelihood, it would not be fair to compare AIC and BIC for the models fitted by goodness-of-fit, so these values are not reported. Indeed, both these models have a higher threshold than the minimum value in the data, and would hence get zero likelihood due to the poor fit of the lower part of the distribution. Notwithstanding, both AIC and BIC are lower for the truncated model, confirming that this gives better fit to the data.

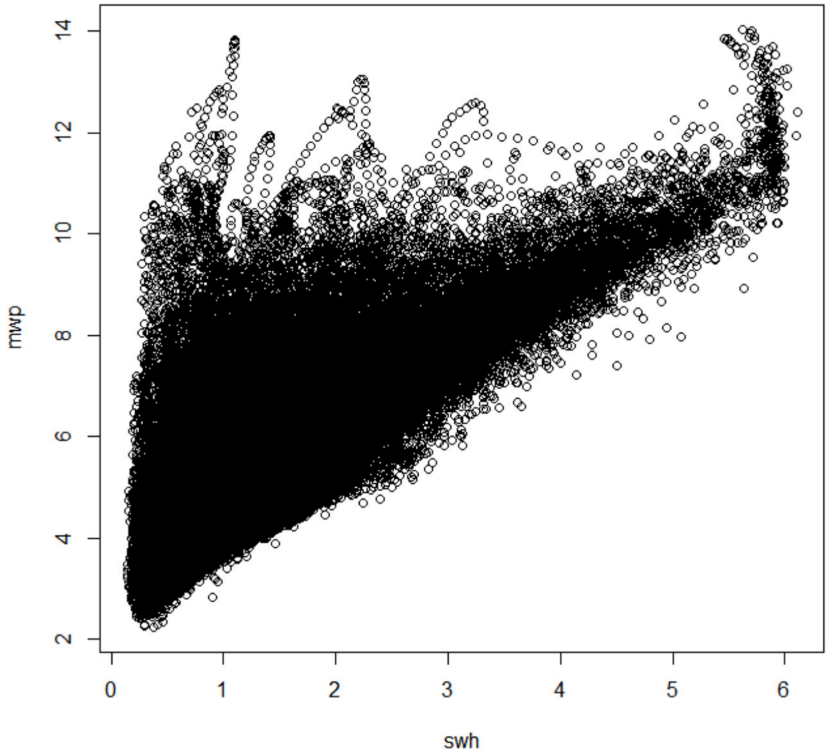

Fig. 1. Scatterplot of the data of significant wave height and mean wave period.

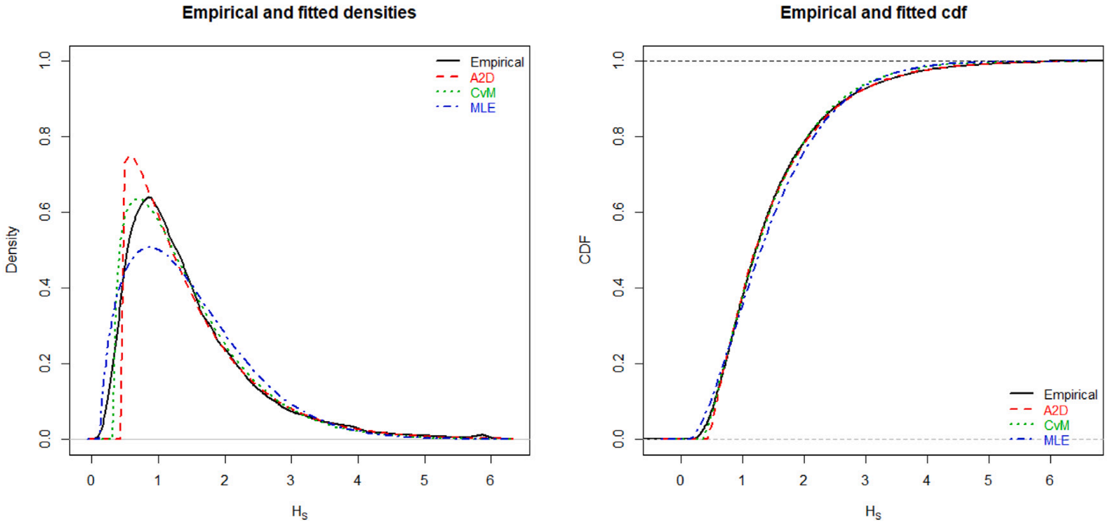

Fig. 2. Estimated density functions (left) and cumulative distribution functions (right) for the significant wave height data; 3-parameter Weibull.

Table 1
Estimated model parameters for the 3-parameter Weibull distributions and the truncated distribution.

	Shape (β)	Scale (α)	Location (γ)	Endpoint (τ)	AIC	BIC
3p Weibull; MLE	1.52098	1.47108	0.13986		208 135	208 163
3p Weibull; A2D	1.10667	1.06753	0.43788			
3p Weibull; CvM	1.25974	1.16200	0.33575			
Truncated 3p Weibull; MLE	1.52194	1.47294	0.13985	6.11001	208 108	208 145

4.2. Joint distribution of significant wave height and wave period

In many engineering applications, the joint distribution of several metocean variables are needed and this study considers the joint distribution of significant wave height and zero up-crossing wave period. The effect of truncating the 3-parameter Weibull distribution on the joint distribution will be illustrated by calculating environmental contours.

A conditional model is assumed for modelling the joint distribution of significant wave height and wave period, where the marginal distribution of H_S is combined with a conditional log-normal distribution for T_Z, conditioned on the value of H_S (Mathisen and Bitner-Gregersen,

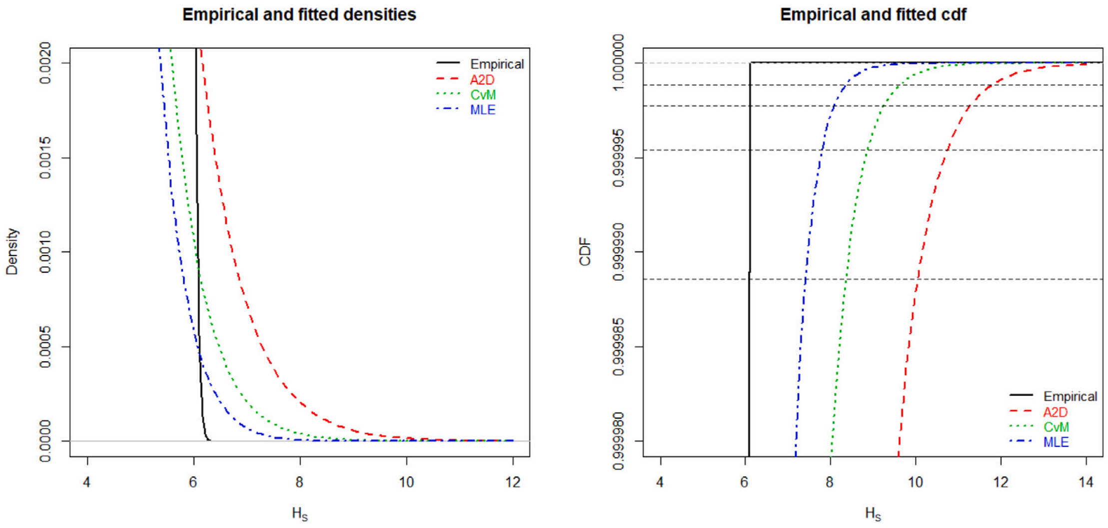

Fig. 3. Close-up of upper tail of estimated density functions (left) and cumulative distribution functions (right) for the significant wave height data.

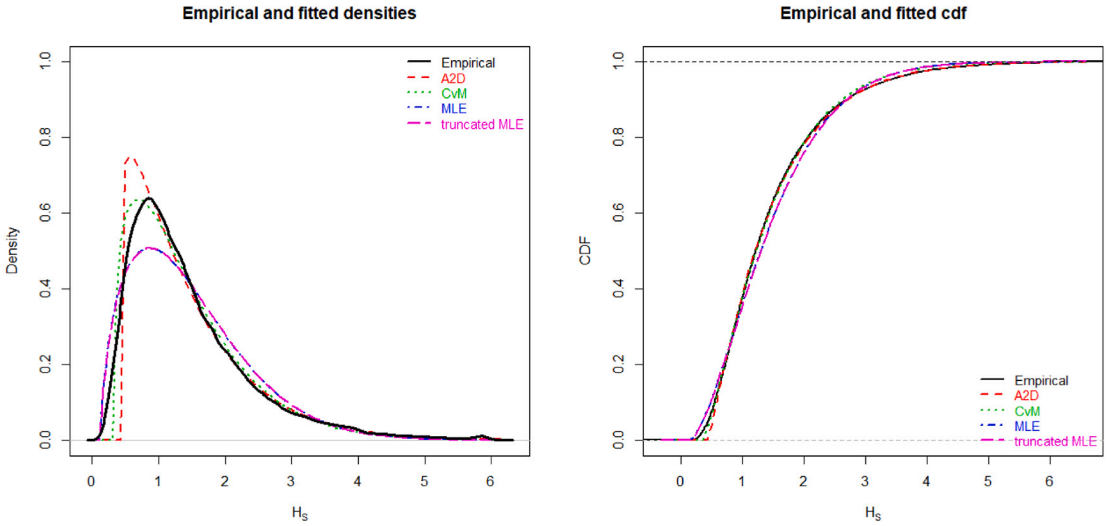

Fig. 4. Estimated density functions (left) and cumulative distribution functions (right) for the significant wave height data; truncated 3-parameter Weibull.

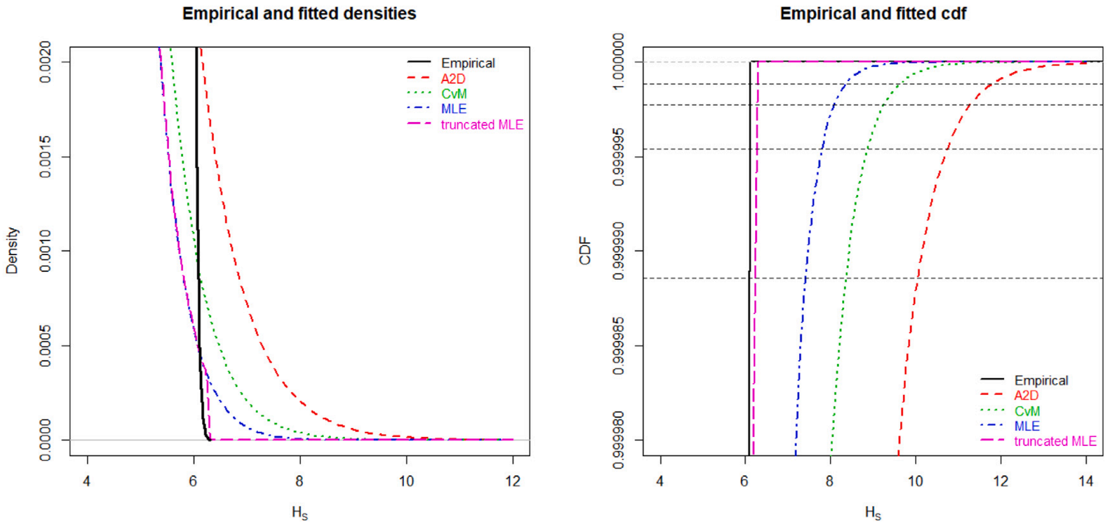

Fig. 5. Close-up of upper tail of estimated density functions (left) and cumulative distribution functions (right) for the significant wave height data.

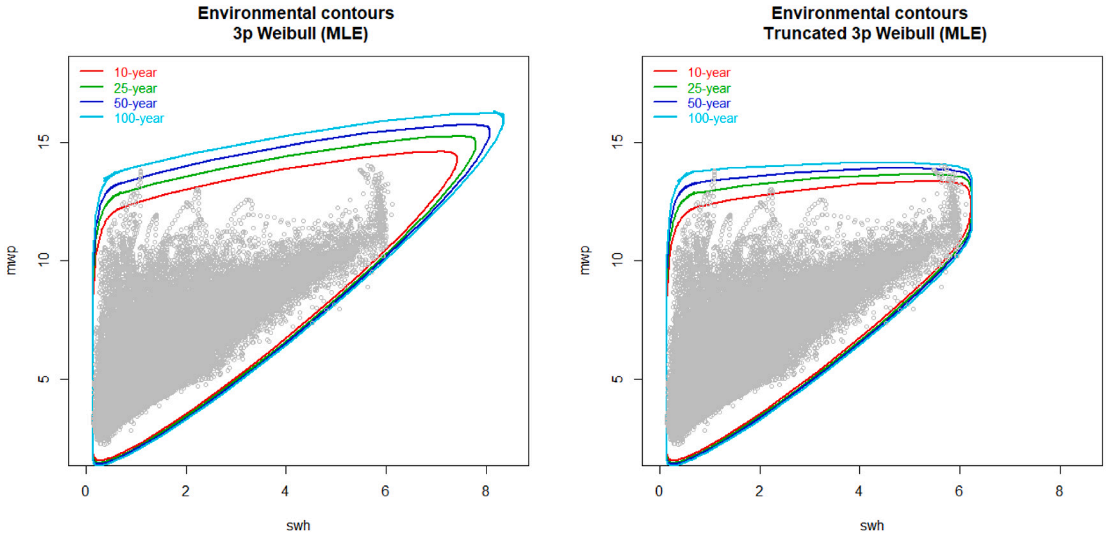

Fig. 6. Environmental contours for joint distribution of H_S and T_Z assuming a marginal 3-parameter Weibull (left) and a truncated Weibull (right).

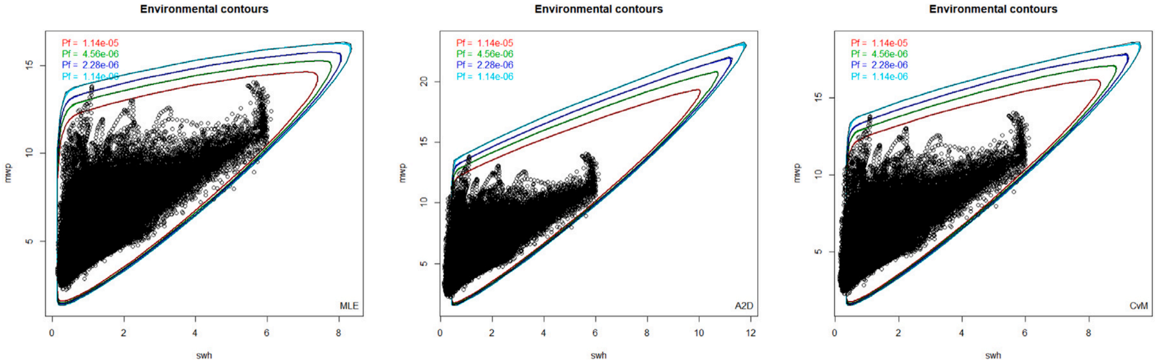

Fig. 7. Environmental contours for joint distribution of H_S and T_Z assuming a marginal 3-parameter Weibull with different fitting methods; MLE (left), A2D (middle) and CvM (right).

1990). The conditional model for T_Z is fitted by binning data and fitting a parametric model by least squares, see e.g. Vanem (2016) for details. It is noted that the conditional model for T_Z will obtain the same parameters regardless of the marginal distribution for H_S, but obviously different marginal distributions will give different joint distributions.

Having estimated the joint distribution, environmental contours are calculated using the direct sampling approach outlined in Huseby et al. (2013, 2015), using the tail sampling scheme suggested in Vanem (2018). The resulting environmental contours are shown in Fig. 6 for initial 3-parameter Weibull distribution and the truncated Weibull distribution, both fitted by MLE. Scatter plots of the data are included in the plots. The 10-, 25-, 50- and 100-year contours are shown, and it is clearly seen that the upper tail of the initial H_S distribution gives too high and unrealistic sea states in this shallow water location. However, this is remedied by the truncated Weibull distributions. As can be clearly seen from Fig. 6, no overly unrealistic sea states are contained within the modified contours.

It is noted that the alternative fitting methods, based on goodness-of-fit and with a particular focus on the upper tail of the distribution yields even more unrealistic upper tails and even more unrealistic sea states, as can be seen in Fig. 7 where contours for the alternative fitting methods for the 3-parameter Weibull distribution are shown.

5. Summary and conclusions

This paper has proposed a truncated, translated Weibull distribution for modelling the long-term distribution of significant wave height in shallow waters. The probabilistic model is almost identical to the 3-parameter Weibull distribution commonly used for modelling significant wave height at deep sea locations, but provide a means for truncating the upper part of the distribution to avoid predicting sea states that would be unrealistic in shallow waters.

The proposed model is fitted to a shallow-water dataset with a distinctive upper bound on observed significant wave height and it is showed that it yields improved fit to the data compared to the commonly used 3-parameter Weibull distribution. Moreover, statistical model selection criteria confirm that the truncated model is an improvement. The effect of truncating the marginal distribution of H_S on the joint distribution of H_S and T_Z is also shown, by way of environmental contour lines. This illustrates a big difference and suggests that the standard distribution functions that are known to perform well for deep-sea locations may not be used in shallow waters, where limited water depth restricts large wave heights. Thus, it is suggested that the truncated version of the 3-parameter Weibull distribution can be a useful model for long-term metocean description in shallow waters that can be utilized in design and analysis of coastal structures.

CRediT authorship contribution statement

Erik Vanem: Idea, Conceptualization, Methodology, Analysis, Writing of paper. **Tiago Fazeres-Ferradosa:** Conceptualization, Methodology, Validation, Review and editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work has partly been carried out within the HIPERWIND project, supported by EU H2020, grant agreement number 101006689.

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