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Ground and Grain

Peter Fritz[1,2]

[1]Dianoia Institute of Philosophy, Australian Catholic University, Melbourne, Australia

[2]Department of Philosophy, Classics, History of Art and Ideas, University of Oslo, Oslo, Norway

Correspondence
Peter Fritz, Dianoia Institute of Philosophy, Australian Catholic University, Melbourne, Australia.
Email: peter.fritz@acu.edu.au

Abstract

Current views of metaphysical ground suggest that a true conjunction is immediately grounded in its conjuncts, and only its conjuncts. Similar principles are suggested for disjunction and universal quantification. Here, it is shown that these principles are jointly inconsistent: They require that there is a distinct truth for any plurality of truths. By a variant of Cantor's Theorem, such a fine-grained individuation of truths is inconsistent. This shows that the notion of grounding is either not in good standing, or that natural assumptions about it need to be revised.

1 | INTRODUCTION

The facts we encounter in our lives are not completely chaotic. From the fact that this is red, we can infer the fact that something is red. We draw on such connections in providing explanations: something is red because this is red. Such an explanation may be used merely to record how we came to know the explanandum – in this case, the fact that something is red. But it seems that the connection between something being red and this being red on which the explanation draws is not itself dependent on our or any other agent's cognitive attitudes to these facts. Doesn't this being red *make it the case* that something is red, in a way which is entirely independent of us?

Clearly, there is a connection between this being red and something being red that is independent of us: if this is red, then something is red. This has nothing to do with us. And it is no accident: necessarily, if this red, then something is red. But some metaphysicists have recently argued that there is an important sense in which this being red makes it the case that something is red which is not captured by such material and modal connections. In the relevant sense, $2 + 2 = 4$ is taken to make it the case that $2 + 2 = 4$ or Socrates was a philosopher, but not *vice versa*: that $2 + 2 = 4$ or Socrates was a philosopher is not taken to make it the case that $2 + 2 = 4$. Assuming that it is necessary that $2 + 2 = 4$, the two claims are necessarily materially equivalent, so there seems to be no way of capturing their relevant asymmetric relationship using merely modal vocabulary. These metaphysicists therefore propose to introduce a new term – *grounding* – to express this notion of dependence which draws such fine distinctions among facts (see Correia and Schnieder, 2012a, pp. 13–14; Fine, 2012, p. 36).

Accordingly, many proponents of grounding hold that this being red grounds that something is red. And this is generally not considered to be an exception: if something is some way, then it being that way is taken to ground that something is that way. Likewise, it is often held that disjunctions are always grounded in their true disjuncts, and true conjunctions in their conjuncts. But it was noted early on (Fine, 2010) that such general principles lead to violations of the widely endorsed principle of the irreflexivity of ground, according to which no fact grounds itself. Few proponents of grounding have taken a position on how to respond to these problems, but most who have done so (such as Correia, 2014, Woods, 2018) accept the conclusion that some facts ground themselves.

Here, it will be shown that problems also arise without the assumption of irreflexivity. The discussion by Fine and others sketched so far concerns a relation of *mediate* ground, according to which the grounds of a fact include the grounds of its grounds. But grounding is also said to come in an *immediate* variant, which requires an unmediated connection between a grounded fact and its grounds. It turns out that in the case of immediate ground, natural principles – concerning the grounds of quantified truths, conjunctive truths and disjunctive truths – lead to inconsistency by themselves, without assuming the irreflexivity of immediate ground. This is the main result to be established in this article. The result can be traced to the constraints which principles of immediate ground impose on the individuation of facts: They require the individuation of certain facts to be so fine-grained that for each plurality of facts, there is a unique corresponding fact. By a well-known result of Russell (1903, Appendix B) and Myhill (1958), such a fine-grained individuation of facts is logically inconsistent. The result can be seen as an application, or variant, of Cantor's Theorem.

The inconsistency result just described is presented in section 2. This section follows a substantial part of the literature in formalizing grounding relations using sentential operators. Correspondingly, it uses propositional quantifiers to regiment quantification over the relata of ground. Section 3 shows that this choice is immaterial for the inconsistency result: with minor modifications, it goes through as well if grounding relations are formalized using relation symbols of first-order logic, and first-order quantifiers are used to quantify over the relata of ground. Those who are happy with formalizing ground using sentential operators may safely skip section 3. Sections 2 and 3 employ formal languages in stating the relevant principles of ground, but otherwise keep technicalities to a minimum.

The results established here show that rejecting irreflexivity is not an adequate response to the problems of ground. This is not all they bring out: What leads to inconsistency is the requirement for ground to draw very fine distinctions, even between facts expressed by certain logically equivalent sentences. But this was one of the very features which motivated the introduction of ground as a new theoretical primitive, in addition to the less discerning dependency relations which can be formulated using familiar metaphysical vocabulary, such as modal terms. The problem developed here therefore not only poses a challenge to any fuller development of a theory of ground, but calls into question one of the very motivations for theorizing in terms of ground.

Section 4 canvasses a range of responses to the inconsistency results which retain the problematic principles of immediate ground. This means either rejecting some of the logical principles used in the derivation of a contradiction, or reformulating some of these principles. It will be argued that none of these options is promising. Section 5 therefore considers the options for rejecting some of the principles of immediate ground. This can take the form of giving up the notion of immediate grounding, giving up the idea of logical grounding, giving up specific principles, or giving up the ideology of grounding altogether. None of these options is easily taken by the grounding theorist. The inconsistency arguments to be developed here therefore pose a significant challenge to proponents of grounding. In section 6, I conclude by noting that my personal inclinations lie with the last option, of rejecting the notion of metaphysical grounding altogether.

2 | THE INCONSISTENCY OF IMMEDIATE GROUND

The inconsistency will first be demonstrated using sentential grounding connectives and propositional quantifiers.

2.1 | The language of ground

Fine (2012, pp. 48–54) draws a number of distinctions between different notions of ground, introducing terminology which has become widely used. For present purposes, two distinctions are important: the distinction between *partial* and *full* ground, and the distinction between *mediate* and *immediate* ground. The idea behind the first is this: while partial grounds need only contribute to make what is grounded the case, full grounds must also collectively suffice to make what is grounded the case. Fine illustrates this schematically with the grounding of conjunctive facts: while p is a partial but not a full ground of $p \wedge q$ (assuming p and q are distinct), p and q together fully ground $p \wedge q$. Consequently, while partial ground is a relation in which one fact stands to another, full ground is a relation in which many facts stand, together, to one fact.

Fine's second distinction, between mediate and immediate ground, is orthogonal to the distinction between partial and full ground, so that one must distinguish between mediate partial, mediate full, immediate partial, and immediate full ground. The mediate/immediate distinction concerns the question whether grounds immediately make the grounded the case, or whether they do so mediately – by making mediate facts the case, which themselves make the grounded the case. Fine also illustrates this distinction with the case of conjunctive facts, considering partial grounds: p immediately grounds $p \wedge q$ on account of being a conjunct, but it only mediately grounds $(p \wedge q) \wedge r$ (via $p \wedge q$) on account of being a conjunct of a conjunct. In this article, we will mainly be concerned with *immediate partial* ground, so unless noted otherwise, ground is immediate partial ground.

In order to investigate theories of grounding systematically, grounding theorists often use formal languages, and doing so will be useful here as well. A common option is to take the language of propositional logic, the formulas of which are built up from proposition letters $p, q, \ldots$ using Boolean connectives $\neg, \wedge, \vee, \rightarrow$ and $\leftrightarrow$, and to add a sentential operator $\prec$ for (immediate partial) ground. This language allows one to make schematic generalizations such as the following, stating schematically that ground is irreflexive:

$$\varphi \nprec \varphi$$

(This is meant to abbreviate $\neg(\varphi \prec \varphi)$, a convention which will be used for $\prec$ and similar operators in the following. Lowercase Greek letters will be used in schemas to indicate occurrences of formulas.)

However, not all general claims grounding theorists engage with are of this straightforward universal form. Consider the claim that some fact has no grounds, a claim discussed by, e.g., Rosen (2010, p. 116) and Fine (2012, p. 47). Such a quantificational claim cannot be formulated schematically; rather, it requires some way of quantifying into the position of arguments of the grounding connective. One option, discussed in Correia and Schnieder (2012a, pp. 11–12) and Krämer (2013), is to conceive of proposition letters as variables, and introduce quantifiers $\exists$ and $\forall$ binding them. To a first approximation, such quantifiers may informally be read as quantifying over propositions. One may then take facts simply to be true propositions – truths – and so state the principle as follows:

$$\exists p(p \wedge \forall q(q \nless p))$$

This is the option we will adopt first. An alternative is to start with first-order logic, and regiment ground using a binary predicate. This alternative allows one to use standard first-order quantifiers to make general claims about grounding. It will be considered in section 3, along with a hybrid approach which combines a sentential grounding operator with first-order quantifiers. In addition to propositional variables p, q, …, we will also use propositional constants a, b, … to stand for specific propositions.

Throughout the following, it will be assumed that facts are true propositions, so from now on, we will speak only of *truths*. This identification is not uncontroversial. E.g., Fine (1982, pp. 46–49) argues against identifying facts with true propositions, and Fine (2012, p. 43) explicitly takes the relata of ground to be facts rather than true propositions. But as Fine (1982) also argues, it is plausible that on the relevant conception of facts, true propositions and facts stand in a one-to-one correspondence. For present purposes, the identification of facts with true propositions is therefore harmless: any logical limitation on the fineness of grain of propositions translates straightforwardly into a limitation on the fineness of grain of facts.

Recall that Fine's conjunction example shows not only that partial and full ground are distinct relations, but also that they are fundamentally different kinds of relations: whereas a single truth partially grounds another, it is in general many truths which together fully ground one truth. Thus full ground cannot be formalized using a binary sentential connective. Instead, grounding theorists have used connectives which can be flanked by a comma-separated sequence of formulas (possibly infinite) on the left, and a single formula on the right, e.g., in Fine (2012). If we added such a connective, we could schematically state the claim that true conjunctions are fully grounded in their conjuncts as follows:

$$\varphi \wedge \psi \rightarrow (\varphi, \psi < (\varphi \wedge \psi))$$

The notation is, however, limiting when more demanding quantificational claims are considered, such as the claim that some truth has no full grounds. Propositional quantifiers provide the resources to make some related claims, such as instances of the following schema:

$$\exists p(p \wedge \forall q_1 \ldots \forall q_n(q_1, \ldots, q_n \nless p))$$

But no instance of this schema, nor all of them taken together, expresses the intended claim, since they do not even rule out that every truth is either fully grounded in one proposition, or fully grounded in two propositions:

$$\forall p(p \rightarrow (\exists q_1(q_1 < p) \vee \exists q_1 \exists q_2(q_1, q_2 < p)))$$

What is needed is a way of quantifying into the first argument-place of $<$, and so binding a variable which takes the place of a whole sequence of formulas, rather than the place of an individual formula.

How should we conceive of such quantification? A natural idea, which is suggested by the formulations used by authors such as Correia and Schnieder (2012a, p. 12) and Fine (2012, p. 46), is that $<$ ought to take a *plural* term as its first argument, along the lines of plural logic pioneered by Boolos (1984). To a first approximation, the relevant form of plural quantification can be paraphrased using formulations such as “some propositions are such that”. Instead of using comma-separated sequences of formulas, we therefore introduce a new kind of *plural* propositional variables pp, qq, …, which can be bound by quantifiers and used as the first argument of $<$. A connective is also needed to formalize “is one of”. Usually the symbol $\prec$ is used for this, but doing so would be confusing here, since $\prec$ is already used for partial ground. Instead, we will use the symbol ε, and so write $p\varepsilon qq$ to state that p is one of qq.

In English, talk of “some things” carries an existential commitment in the sense that it can be paraphrased by saying “one or more things” (or possibly “two or more things”). But for many applications of plural quantifiers, requiring pluralities to be non-empty is undesirable, and this is also the case in the context of grounding. E.g., Fine (2012, p. 48) distinguishes between a truth being “ungrounded” in the sense that there are no truths which ground it, and a truth being “zero-grounded” in the sense that the empty plurality grounds it. The distinction is straightforward to state in the formal language if the range of plural propositional quantifiers includes such an empty plurality – propositions such that no proposition is one of them. This will from now on be assumed, even if it means that plural talk here departs from ordinary English usage.

The final resource needed is identity. For example, if partial grounding is irreflexive, then from p partially grounding q we can conclude that p is distinct from q. Using $=$ as a binary sentential operator, we can state this as follows:

$$\forall p \forall q((p \prec q) \rightarrow (p \neq q))$$

As this example shows, formulas with propositional variables tend to require lots of parentheses in order to avoid structural ambiguities. For brevity, we therefore adopt and extend standard conventions on omitting parentheses, stipulating that unary operators like negation and quantifiers bind strongest, after which come the grounding, identity, and “one of” connectives, which are followed by conjunction and disjunction, with implication and bi-implication coming last. E.g., the last formula may thus be written as:

$$\forall p \forall q(p \prec q \rightarrow p \neq q)$$

Summing up, the language to be used in this section provides propositional variables and constants, plural propositional variables, Boolean operators, quantifiers, the grounding connectives $\prec$ and $<$, a connective ε for “is one of”, and a propositional identity operator.

2.2 | Principles of ground

With the language of ground in place, the principles to be shown inconsistent can be stated. They are principles concerning the immediate partial grounds of conjunctions, disjunctions and universal truths, as well as a non-triviality principle. There is little explicit literature on such formal principles of immediate ground. Furthermore, as Fine (2012, p. 51) notes, it is not clear that immediate grounding can be defined in terms of mediate grounding, so principles of immediate grounding cannot simply be derived from principles of mediate grounding. (However, see section 5.1 for further discussion of this issue.) The principles used here have therefore not played a major *explicit* role in motivating and pinning down the notion of ground. However, all of the principles used here are strongly motivated by discussions of analogous principles of mediate ground (e.g., Fine, 2012, pp. 58–63; Correia, 2014), and general discussions of immediate ground (especially Fine, 2012, pp. 50–51). Furthermore, it will be argued in this section that these principles have been covertly appealed to in order to introduce certain notions of ground, to illustrate them, and to motivate various theses about them. In this sense, they can be considered to be paradigmatic. In any case in which there is any controversy about the relevant principle, a weak variant will be used which remains neutral on the debated question. Recall that the principles to be stated concern only *immediate partial* ground; this qualification will mostly be left tacit.

The case of conjunctions is straightforward: the grounds of a true conjunction are just the conjuncts. For readability, this will be split up into two formal principles, one stating that any true conjunction is grounded in both conjuncts, and one stating that grounds of a conjunction must be conjuncts:

($\wedge$1) $\forall p \forall q\,(p \wedge q \rightarrow p \prec (p \wedge q) \wedge q \prec (p \wedge q))$
($\wedge$2) $\forall p \forall q \forall r\,(r \prec (p \wedge q) \rightarrow r = p \vee r = q)$

As noted above, Fine (2012, p. 50) uses the grounding of conjunctions in their conjuncts as a way to introduce and motivate the distinction between full and partial ground. In fact, already the very first example of grounding in Fine (2012, p. 37) is a case of a conjunct grounding a conjunction. In this sense, these logical connections between conjunction and grounding are paradigmatic. To be clear, the examples just mentioned concern mediate rather than immediate grounding. But ($\wedge$1) and ($\wedge$2) are similarly paradigmatic, as Fine (2012, p. 50) implicitly relies on them to motivate the distinction between mediate and immediate ground: Recall that he notes that while p is an immediate ground of $p \wedge q$, it is only a mediate ground of $(p \wedge q) \wedge r$, as in the latter case, the grounding must be seen to be mediated through $p \wedge q$. (Here, p is presumably taken to be distinct from r.) It is natural to assume that what makes p an immediate ground of $p \wedge q$ and what keeps it from being an immediate ground of $(p \wedge q) \wedge r$ is that it is a conjunct of the former, but only a conjunct of a conjunct of the latter. Thus, what is implicit is the assumption that the immediate grounds of a true conjunction are just the conjuncts, as ($\wedge$1) and ($\wedge$2) state. And although Fine does not explicitly present any systematic principles of immediate grounding, such an explicit endorsement of ($\wedge$1) and ($\wedge$2) can be found in Litland (2018, p. 63); indeed, Litland holds that these principles jointly encapsulate the essence of conjunction.

It would be natural to endorse principles for disjunctions analogous to ($\wedge$1) and ($\wedge$2), but this is not entirely uncontroversial: It seems clear enough that only disjuncts should be grounds of a disjunction, and that a disjunction with exactly one true disjunct should be grounded in it. But (Fine, 2010, p. 108) questions whether a disjunction with two true disjuncts must have both of them as grounds. In order to remain neutral on this issue, only weak principles for disjunctions will be used, according to which every true disjunction has at least one disjunct as a ground, and every ground of a disjunction must be a disjunct:

($\vee$1) $\forall p \forall q\,(p \vee q \rightarrow p \prec (p \vee q) \vee q \prec (p \vee q))$
($\vee$2) $\forall p \forall q \forall r\,(r \prec (p \vee q) \rightarrow r = p \vee r = q)$

As in the conjunctive case, principles of disjunctive grounding also figure in arguments which introduce and motivate the various notions of ground. For example, Fine (2012, p. 40) uses the idea that a truth r grounds a disjunction $r \vee r$ in an argument concerning the relationship between different notions of modality and explanatory relations. Again, in this and similar examples, mediate notions of ground are appealed to. But Fine (2012, p. 51) also implicitly draws on ($\vee$1) and ($\vee$2) in elaborating the distinction between mediate and immediate ground: He notes that $p \vee (p \vee p)$ is grounded in p both mediately and immediately, noting explicitly that p is an immediate ground due to its being a disjunct. Analogous to the conjunctive case, it is natural to assume that the underlying principle is that disjunctions are immediately grounded in just their true disjuncts. This is captured by ($\vee$1) and ($\vee$2) in a way which is neutral on the debated case in which both disjuncts are true.

The next two principles concern universal quantification. A natural idea is that the grounds of a true universal claim should be just its instances. If one thinks of universal claims as big conjunctions, then this is suggested by the principle for conjunction: if a truth $\forall xFx$ is a big conjunction whose conjuncts comprise the truths Fx for all x, then the grounds of $\forall xFx$ should be just those truths. But there is a familiar worry for this picture of quantification: such a big conjunction intuitively doesn't rule out the existence of further things which are not F. Correspondingly, Fine (2012, pp. 60–62) considers whether the grounds of a true universal statement should also include, in addition to its instances, a truth stating "that's all", and discusses some difficulties in formulating it. Since it is unclear whether such an additional ground is needed, and if so, how to formulate it, we will sidestep the issue by allowing one exception to the claim that universal claims may only have instances as grounds. So, let the propositional constant $\mathtt{t}$ stand for the "that's all" claim if there is such a truth, and an arbitrary proposition if not – say $2 + 2 = 4$. The two principles for universal quantification to be used therefore say that all instances of a true universal claim are among its grounds, and that $\mathtt{t}$ is the only possible exception to the principle that the grounds of a universal claim are instances. Schematically, these can be formulated as follows:

($\forall$1) $\forall p\varphi \to \forall p(\varphi \prec \forall p\varphi)$
($\forall$2) $\forall q(q \prec \forall p\varphi \wedge \forall p(q \neq \varphi) \to q = \mathtt{t})$

The schematicity of these principles requires some clarifications. First, φ may be any formula in which p but not q occurs freely. In particular, this rules out cases of vacuous quantification, as they may be controversial and won't be needed here. Second, φ may contain variables other than p freely; these are considered as "parameters" bound by a tacit initial universal quantifier prefix. Thus an instance of one of the two principles for a formula φ with free variables $p_1, \ldots, p_n, qq_1, \ldots, qq_m$ distinct from p is of the form $\forall p_1 \ldots \forall p_n \forall qq_1 \ldots \forall qq_m(\ldots)$. Note also that ($\forall$2) does not say that $\mathtt{t}$ is an exception to the principle that only instances are grounds – it only says that *if* a proposition is an exception, it must be $\mathtt{t}$. The principle is thus neutral on whether there is the controversial "that's all" proposition, and if so, whether it is a ground of any universal truths.

Quantificational examples of grounding are less often appealed to in order to motivate and introduce various notions of ground. This is unsurprising, as they are more complicated to state than principles involving only sentential operators, such as the conjunctive and disjunctive grounding principles used above. But there is no reason to think that quantificational principles of grounding are less central to the conception of grounding and its relation to logical connectives. In fact, in discussing logical principles of ground, Fine (2012, pp. 58–67) discusses universal and existential quantification at length, immediately after discussing conjunction and disjunction, and before finally considering negation. There, the connection between universal quantification and (possibly infinite) conjunction is developed at length, which provides a firm basis for extrapolating from ($\wedge$1) and ($\wedge$2) to ($\forall$1) and ($\forall$2), as done here. Furthermore, Schnieder (2020, p. 115–118), assumes and defends principles of immediate grounding for (higher-order) existential quantification corresponding to strengthenings of ($\forall$1) and ($\forall$2), according to which the immediate grounds of an existential truth are its true instances. Indeed, like Litland, Schnieder considers the principles of immediate grounding to constitute the essence of the relevant logical connective.

The final principle is a non-triviality principle, ruling out that all truths ground each other:

(*nt*) $\neg\forall p\forall q(p \wedge q \to p \prec q)$

Clearly, if grounding is to play the role its proponents envisage, it cannot be trivial for truths to stand in this relation, in the sense that every truth grounds every truth.

2.3 | Arbitrary conjunctions

The seven labeled principles stated in the previous section are inconsistent. The axioms and rules required for the derivation of the inconsistency are the principles of classical truth-functional reasoning, elementary principles of identity and quantification for propositional quantifiers (as, e.g., formulated in Fine, 1970), elementary principles of quantification for plural propositional quantifiers, and the standard plural comprehension principle for plural propositional quantifiers according to which for every (instantiated) condition φ, there are the propositions satisfying φ. Although the proof system is straightforward, the proof of the inconsistency is somewhat complex. To understand its structure, it is helpful to consider first a variant argument which uses more controversial assumptions about grounding, but which is formally simpler. This section presents this variant argument; the next returns to the inconsistency of the seven principles just stated.

The variant argument uses a device of arbitrary conjunction, which allows us to turn any plural propositional variable pp into a conjunctive formula $\bigwedge pp$ – the conjunction of pp. To state the argument, two abbreviations will be useful: let Tpp abbreviate the claim that all pp are true, and let $\overline{\forall}$ be a restriction of the plural propositional quantifier to truths:

$$Tpp := \forall p(p \mathrel{\varepsilon} pp \rightarrow p)$$
$$\overline{\forall} pp\varphi := \forall pp(Tpp \rightarrow \varphi)$$

Only a single principle of ground is needed, which corresponds to the pair of principles for binary conjunctions ($\wedge$ 1) and ($\wedge$ 2) above: the grounds of a conjunction $\bigwedge pp$ of some truths pp are just pp. In symbols:

$$(\textstyle\bigwedge\prec)\ \overline{\forall} pp\forall p(p \prec \bigwedge pp \leftrightarrow p \mathrel{\varepsilon} pp)$$

The only further principle needed is a principle of truth-functionality of arbitrary conjunction, according to which the conjunction of pp is true just in case pp are true:

$$(\textstyle\bigwedge T)\ \forall pp(\bigwedge pp \leftrightarrow Tpp)$$

It is not straightforward whether the inconsistency of these two principles poses a problem for ground, since there are reasons for questioning ($\bigwedge\prec$), to which we return shortly. But assuming for the sake of argument that ($\bigwedge\prec$) is true, how does the inconsistency arise in this case? Let $pp \equiv qq$ abbreviate the claim that pp are qq, in the sense that every proposition is one of pp just in case it is one of qq:

$$pp \equiv qq := \forall p(p \mathrel{\varepsilon} pp \leftrightarrow p \mathrel{\varepsilon} qq)$$

If pp and qq are truths such that $pp \not\equiv qq$, then there is some p which is one of either pp or qq, but not both. By ($\bigwedge\prec$), p is therefore a ground of either $\bigwedge pp$ or $\bigwedge qq$, but not both. Hence $\bigwedge pp \neq \bigwedge qq$. Thus $pp \not\equiv qq$ only if $\bigwedge pp \neq \bigwedge qq$, or, contraposing:

$$(S\textstyle\bigwedge)\ \overline{\forall} pp\overline{\forall} qq(\bigwedge pp = \bigwedge qq \rightarrow pp \equiv qq)$$

Furthermore, ($\bigwedge T$) entails that conjunctions of truths are true:

$$(T\bigwedge)\ \overline{\forall} pp \bigwedge pp$$

And these two consequences of ($\bigwedge\prec$) and ($\bigwedge T$) are inconsistent by a familiar argument. Informally, it can be understood as an application of a plural version of Cantor's theorem: ($S\bigwedge$) requires there to be a distinct proposition $\bigwedge pp$ for any truths pp, which by ($T\bigwedge$) must be a truth itself. Thus there is a distinct truth $\bigwedge pp$ for every plurality of truths pp. But this is impossible by a plural version of Cantor's theorem, which shows that in the relevant sense, there are more pluralities of truths than truths.

The argument given here is informal, but it is routine to turn it into a formal derivation. In fact, the derivation showing that ($S\bigwedge$) and ($T\bigwedge$) are jointly inconsistent is merely a minor variant of a deductive argument which was already given by Russell (1903, Appendix B), and discussed widely since then. The argument is sometimes named after both Russell and Myhill, since it was rediscovered, apparently independently, by Myhill (1958). A more detailed presentation of this argument in the present setting of plural propositional quantification can be found in Fritz (forthcoming b).

Uzquiano (2015), Dorr (2016) and Goodman (2017) understand such Cantorian or Russellian arguments to impose limitations on the fineness of grain of propositions: the individuation of propositions cannot be so fine-grained as to enforce the existence of a distinct proposition for every plurality of propositions. Applying this to truths, we can trace the inconsistency of the principle of grounding for arbitrary conjunctions to such a limitation on the fineness of grain of truths: the grounding principle requires a degree of fineness of grain of truths which is logically inconsistent.

Is ($\bigwedge\prec$) true? If $\bigwedge$ is of the same standing as $\wedge$, then it is hard to see how someone could endorse ($\wedge$1) and ($\wedge$2) without also endorsing ($\bigwedge\prec$). But one might hold that $\bigwedge$ must be understood quantificationally, with statements of the form $\bigwedge pp$ serving to abbreviate more complex statements involving quantifiers whose grounds they inherit. In such a case, the status of ($\bigwedge\prec$) depends on the particular quantificational statements which $\bigwedge$ serves to abbreviate.

One option is that $\bigwedge$ functions just like T to abbreviate quantificational claims of the form $\forall p(p\ \varepsilon\ pp \to p)$. In this case, the grounds of $\bigwedge pp$ will include, by ($\forall$1), the conditional propositions $p\ \varepsilon\ pp \to p$ for all propositions p, rather than pp. Since standard accounts of the interaction between ground and logical connectives typically do not treat conditionals (see, e.g., Fine, 2012; Correia, 2014), it is not clear whether this option leads to any problems.

Another option is that $\bigwedge$ serves to abbreviate restricted quantificational statements using a primitive restricted quantifier $\forall p(\varphi : \psi)$ rather than the construction $\forall p(\varphi \to \psi)$. As suggested by remarks of Fine (2012, p. 59, fn. 17; 2017b, p. 568), it is plausible that such a primitive restricted quantifier is governed by different grounding principles. Setting a potentially needed "that's all" truth aside for simplicity, the natural idea is that if $\forall p(\varphi : \psi)$ is true, then its grounds are the instances ψ for the p such that φ. I.e.:

$$(\forall{:})\ \forall p(\varphi : \psi) \to \forall q(q \prec \forall p(\varphi : \psi) \leftrightarrow \exists p(\varphi \wedge q = \psi))$$

Since $\bigwedge pp$ is now taken to abbreviate a statement of the form $\forall p(p\ \varepsilon\ pp : p)$, consider the instance of ($\forall$:) for φ being $p\ \varepsilon\ pp$ and ψ being p:

$$\forall pp(\forall p(p\,\varepsilon\, pp : p) \to \forall q(q \prec \forall p(p\,\varepsilon\, pp : p) \leftrightarrow \exists p(p\,\varepsilon\, pp \wedge q = p)))$$

Assuming that $\forall p(\varphi : \psi)$ is at least materially equivalent to $\forall p(\varphi \to \psi)$, this can be simplified to:

$$\overline{\forall} pp \forall q(q \prec \forall p(p\ \varepsilon\ pp : p) \leftrightarrow q\ \varepsilon\ pp)$$

And this is just an alphabetic variant of ($\bigwedge\prec$) with $\bigwedge pp$ replaced by $\forall p(p \mathrel{\varepsilon} pp : p)$. Thus an inconsistency arises from arbitrary conjunction if it is treated along present lines using a primitively restricted quantifier governed by ($\forall$:), and *a fortiori* more directly from any such a restricted quantifier.

The plausibility of ($\bigwedge\prec$) therefore depends on matters of arbitrary conjunction and restricted quantification which are more controversial than the earlier seven principles. The main argumentative weight of this paper will therefore rest on the inconsistency result arising from the latter, to which we now return.

2.4 | The inconsistency result

The argument for the inconsistency of the original seven principles of section 2.2 has the same general form as the one just given: An open formula with a plural propositional parameter is specified which according to the grounding principles expresses a distinct truth for any truths (used to interpret the parameter). To state this more formally, we adopt the common convention of writing, e.g., $\psi(pp)$ for the result of replacing a contextually salient free variable in ψ by pp. In order to avoid unintended variable binding, it will be assumed that in such cases, pp itself does not occur in ψ. It will be shown that there is a formula ψ with a free plural propositional variable for which the following statements are derivable from the principles of ground:

($S\psi$) $\overline{\forall} pp \overline{\forall} qq(\psi(pp) = \psi(qq) \rightarrow pp \equiv qq)$
($T\psi$) $\overline{\forall} pp \psi(pp)$

As before, these are jointly inconsistent by a version of the Russell-Myhill argument restricted to truths: according to ($S\psi$) and ($T\psi$), there is a distinct truth $\psi(pp)$ for any truths pp, which is impossible since there are more pluralities of truths than truths. Providing a formal derivation of the joint inconsistency of ($S\psi$) and ($T\psi$), for any given formula ψ, along with a rigorous statement of the proof system will have to wait for another occasion. But it is worth nothing that this is the only part of the argument which relies on the principle of plural comprehension. The deduction of ($S\psi$) and ($T\psi$) from the seven principles of ground can be regimented in a weaker calculus which omits this principle. The following presents the reasoning informally; since providing a formal regimentation is routine and not illuminating, it is omitted.

A few tools are needed to specify the formula ψ for the present argument. First we show that there are two truths: There is of course some truth, such as $\forall q(q \rightarrow q)$, and so by ($\wedge 1$), $\forall q(q \rightarrow q) \prec (\forall q(q \rightarrow q) \wedge \forall q(q \rightarrow q))$. By (*nti*), there are truths q and r such that $q \nprec r$. So these cannot all be the same truth, hence there are at least two truths. We may therefore work with two propositional constants a and b and assume $\mathsf{a} \wedge \mathsf{b} \wedge \mathsf{a} \neq \mathsf{b}$. The use of these constants is not necessary – we could use variables bound by suitably restricted universal quantifiers instead, but using a and b simplifies the presentation.

Next, we adopt, for brevity, the use of $\cdot^2$ to indicate self-conjunction:

$$\varphi^2 := \varphi \wedge \varphi$$

Further, we define two abbreviations, which can be thought of as defining variants of conjunction and disjunction:

$$\varphi \mathbin{\hat{\wedge}} \psi := ((\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2) \wedge (\mathsf{a}^2 \wedge \psi^2)$$
$$\varphi \mathbin{\hat{\vee}} \psi := ((\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2) \vee (\mathsf{a}^2 \wedge \psi^2)$$

Since a and b are true, these are truth-functionally equivalent to conjunction and disjunction, respectively, in the following sense:

$$\varphi \mathbin{\hat{\wedge}} \psi \leftrightarrow \varphi \wedge \psi$$
$$\varphi \mathbin{\hat{\vee}} \psi \leftrightarrow \varphi \vee \psi$$

As before, such schematic principles allow parameters bound by tacit universal quantifiers. $\hat{\wedge}$ and $\hat{\vee}$ are useful since they allow us to recover, under certain assumptions, information about the constituents of formulas constructed with them. In the case of $\hat{\wedge}$, we can recover the "conjuncts" when they are true:

$$(\hat{\wedge})\ \varphi \wedge \psi \wedge (\varphi \mathbin{\hat{\wedge}} \psi) = (\chi \mathbin{\hat{\wedge}} \vartheta) \rightarrow \varphi = \chi \wedge \psi = \vartheta$$

In the case of $\hat{\vee}$, we can recover the true "disjunct" when exactly one disjunct is true:

$$(\hat{\vee})\ (\varphi \mathbin{\hat{\vee}} \psi) = (\chi \mathbin{\hat{\vee}} \vartheta) \rightarrow (\varphi \wedge \neg\psi \rightarrow \varphi = \chi) \wedge (\psi \wedge \neg\varphi \rightarrow \psi = \vartheta)$$

These principles follow from the grounding principles for conjunction and disjunction. Consider $(\hat{\wedge})$: Let φ and ψ be truths such that $\varphi \mathbin{\hat{\wedge}} \psi$ is $\chi \mathbin{\hat{\wedge}} \vartheta$. By the grounding principles for conjunction, $\varphi \mathbin{\hat{\wedge}} \psi$ has a ground $(\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2$, which is therefore also a ground of $\chi \mathbin{\hat{\wedge}} \vartheta$. The grounds of the latter are $(\mathsf{a} \wedge \mathsf{b}) \wedge \chi^2$ and $\mathsf{a}^2 \wedge \vartheta^2$. Thus $(\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2$ must be one of these. It cannot be $\mathsf{a}^2 \wedge \vartheta^2$, since $(\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2$ has a ground (namely $\mathsf{a} \wedge \mathsf{b}$) with two distinct grounds (a and b), whereas the grounds of $\mathsf{a}^2 \wedge \vartheta^2$ (a^2, ϑ^2) all have only one ground (a/ϑ). Therefore $(\mathsf{a} \wedge \mathsf{b}) \wedge \varphi^2$ is $(\mathsf{a} \wedge \mathsf{b}) \wedge \chi^2$. Both have one ground (namely $\mathsf{a} \wedge \mathsf{b}$) with two grounds ($\mathsf{a}$ and b), as well as one ground (φ^2/χ^2) with one ground (φ/χ). Thus the grounds with one ground must be the same, i.e., $\varphi^2 = \chi^2$. Finally, their grounds must be the same, whence $\varphi = \chi$.

By similar reasoning, one can show that for truths φ and ψ, $\varphi \mathbin{\hat{\wedge}} \psi$ being $\chi \mathbin{\hat{\wedge}} \vartheta$ entails that $\psi = \vartheta$. Likewise, one can obtain $(\hat{\vee})$ using the fact that the grounding principles for disjunction entail that a disjunct with exactly one true disjunct has it as its sole ground. Since the arguments are straightforward and follow the pattern of the case of $(\hat{\wedge})$, the details are omitted.

We can now define ψ, via φ, as follows:

$$\varphi := (r \mathbin{\hat{\wedge}} (r \mathbin{\varepsilon} rr \mathbin{\hat{\vee}} \neg(r \mathbin{\varepsilon} rr))) \mathbin{\hat{\vee}} \neg r$$
$$\psi := \forall p \varphi(p)$$

It remains to establish $(S\psi)$ and $(T\psi)$. $(T\psi)$ – i.e., $\overline{\forall} pp \forall p \varphi(p, pp)$ – follows from the truth-functional behaviour of $\hat{\wedge}$ and $\hat{\vee}$ noted above. For $(S\psi)$, we first establish as a lemma the following claim:

$$(L\varphi)\ \overline{\forall} pp \overline{\forall} qq \forall p \forall q (p \wedge \varphi(p, pp) = \varphi(q, qq) \rightarrow (p \mathbin{\varepsilon} pp \leftrightarrow p \mathbin{\varepsilon} qq))$$

Note that for both uses of $\hat{\vee}$ in φ, truth-functional reasoning shows that exactly one "disjunct" is true. Consider any truths pp and qq, truth p, and proposition q such that $\varphi(p, pp)$ is $\varphi(q, qq)$. Then by $(\hat{\vee})$, $p \mathbin{\hat{\wedge}} (p \mathbin{\varepsilon} pp \mathbin{\hat{\vee}} \neg(p \mathbin{\varepsilon} pp))$ is $q \mathbin{\hat{\wedge}} (q \mathbin{\varepsilon} qq \mathbin{\hat{\vee}} \neg(q \mathbin{\varepsilon} qq))$. Since these are true, it follows with $(\hat{\wedge})$ that p is q. Likewise, it follows that $p \mathbin{\varepsilon} pp \mathbin{\hat{\vee}} \neg(p \mathbin{\varepsilon} pp)$ is $q \mathbin{\varepsilon} qq \mathbin{\hat{\vee}} \neg(q \mathbin{\varepsilon} qq)$, which – since p is q – is $p \mathbin{\varepsilon} qq \mathbin{\hat{\vee}} \neg(p \mathbin{\varepsilon} qq)$. If $p \varepsilon pp$ or $p \varepsilon qq$, then by $(\hat{\vee})$, $p \mathbin{\varepsilon} pp$ is $p \varepsilon qq$, and so $p \varepsilon pp$ if and only if $p \varepsilon qq$. Thus $p \varepsilon pp$ if and only if $p \varepsilon qq$ must hold unconditionally. This establishes $(L\varphi)$.

We turn to ($S\psi$): Consider any truths pp and qq such that $pp \not\equiv qq$. Without loss of generality, we may assume that there is a p among pp which is not one of qq. Since p is among pp, it is true. Assume for contradiction that $\psi(pp)$ is $\psi(qq)$.

By ($\forall$1), $\varphi(p, pp)$ grounds $\psi(pp)$, and thus also $\psi(qq)$. But $\varphi(p, pp)$ cannot be $\varphi(q, qq)$ for any q: if it were, then by ($L\varphi$), it would follow that p is one of qq, which is false. So by ($\forall$2), $\varphi(p, pp)$ must be $\mathtt{t}$.

By ($\forall$1), $\varphi(p, qq)$ grounds $\psi(qq)$, and thus also $\psi(pp)$. But $\varphi(p, qq)$ cannot be $\varphi(q, pp)$ for any q: if it were, then by ($L\varphi$), it would follow that p is one of qq, which is false. So by ($\forall$2), $\varphi(p, qq)$ must be $\mathtt{t}$.

Hence $\varphi(p, pp)$ and $\varphi(p, qq)$ are both $\mathtt{t}$, and so are identical. But then by ($L\varphi$), p is one of qq, contradicting our assumption. Thus $\psi(pp)$ is distinct from $\psi(qq)$. So if $\psi(pp) = \psi(qq)$, then $pp \equiv qq$, as claimed by ($S\psi$).

This concludes the derivation of ($S\psi$) and ($T\psi$), and so the derivation of an inconsistency. Similar to the case of arbitrary conjunction, the seven paradigmatic principles of ground set out in section 2.2 require a degree of fineness of grain of truths which is logically inconsistent, demanding a distinct truth $\psi(pp)$ for any truths pp.

From the details of this argument, one can also see that it does not matter that it was the same relation of grounding which related true conjunctions to their conjuncts, true disjunctions to their disjuncts, and true universal claims to their instances. This means that the argument shows more generally that any naive treatment of the relations of being a conjunct, disjunct or instance among *propositions* – as opposed to sentences – leads to inconsistency. In fact, since in this case, we need not restrict ourselves to truths, we can simplify the argument, and consider only conjuncts and instances of universal generalizations. That is, we can show that it is inconsistent to hold both that for any propositions p, q and r, p is a conjunct of $q \wedge r$ if and only if p is q or r, and – schematically, for any formula φ – that for any proposition p, p is an instance of $\forall q\varphi$ if and only if p is φ, for some q. It can also be shown that this inconsistency essentially arises out of combining these two principles, as each of them is consistent on its own. I do so in Fritz (forthcoming b).

3 | PROPOSITIONS AS INDIVIDUALS

Many grounding theorists do not use propositional quantifiers, but instead work with first-order quantifiers. In this section, it is shown that the inconsistency also arises in such a setting. The argument is very similar, but requires some adjustments which are not completely mechanical. The first-order variant will therefore be discussed in some detail. Those who are happy with the formalization given above may safely skip this section.

3.1 | The first-order language of ground

We start with a standard first-order language, with formulas built up from individual variables and constants, function symbols, and relation symbols, using Boolean operators, an identity symbol, and existential and universal quantifiers. In order to theorize about the relata of ground, many grounding theorists (e.g., Fine, 2010, Rosen, 2010, Audi, 2012) add a term-forming operator which can be applied to a formula φ to yield an individual term $[\varphi]$. We can think of such a term as denoting *the proposition that* φ. Note that in order to be able to specify propositions using parameters, any occurrence of a variable free in φ must also be free in $[\varphi]$.

There are two ways in which grounding connectives are treated in such a first-order setting: some, like Fine (2010) and Litland (2015), use sentential operators, and while others, like Rosen (2010) and

Audi (2012), use binary relation symbols. The former can be thought of as a hybrid approach, since the variables bound by quantifiers do not match, syntactically, the arguments of the grounding connective. This makes it difficult to state many principles about grounding in the intended generality. For example, let ψ be a formula expressing some ungrounded proposition. That this proposition has no grounds is a universal statement, but if $\prec$ is a sentential operator, then one cannot formulate it as $\forall x(x \not\prec \psi)$, since this is then not a well-formed formula. Fine therefore employs a truth predicate T, with which one can state $\forall x(Tx \not\prec \psi)$. But this doesn't capture the intended claim in sufficient generality, since it does not rule out, for any given formula φ, that $\varphi \prec \psi$. To see why not, note that we can of course instantiate the universal claim using $[\varphi]$, and so obtain $T[\varphi] \not\prec \psi$. But there is no obvious way to conclude $\varphi \not\prec \psi$ from this, since Fine (2010, p. 106) explicitly denies that $T[\varphi]$ and φ are intersubstitutable in grounding contexts. We therefore adopt the second option here, and add the grounding connective $\prec$ to the language as a binary relation symbol, writing, e.g., $x \prec y$ for the claim that x grounds y. With this, one can straightforwardly express the claim that $[\varphi]$ has no grounds as $\forall x(x \not\prec [\varphi])$, from which $[\psi] \not\prec [\varphi]$ follows by universal instantiation.

A truth predicate is not only needed on the hybrid approach, but also on the uniform first-order approach used here. To illustrate this, consider the factivity of ground. Without a truth predicate, one can state schematically: $[\varphi] \prec [\psi] \rightarrow \varphi \wedge \psi$. But there is a natural sense in which this doesn't express the idea of factivity in the intended generality, as there may be propositions not expressed by any formula. Using a truth predicate T, the principle can be stated as $\forall x \forall y(x \prec y \rightarrow Tx \wedge Ty)$. Of course, to obtain the instances of the schematic principle from the quantified one, the truth of $[\varphi]$ must entail φ. We therefore include a unary predicate T expressing truth, and assume – with Fine (1980, p. 191) – that it satisfies the natural schema $T[\varphi] \leftrightarrow \varphi$.

It is important to note that the need for such a truth predicate does not by itself pose any problem for ground. In particular, there is no danger of inconsistency arising from the schema $T[\varphi] \leftrightarrow \varphi$, since $[\varphi]$ is *the proposition that* φ rather than the sentence $\ulcorner \varphi \urcorner$. (See Schwarz (2013) for a detailed discussion of this general point.) In fact, a truth predicate satisfying the schema is plausibly *definable* using the grounding relation as *grounding something*: The schema $\exists x([\varphi] \prec x) \leftrightarrow \varphi$ can be derived from the following principles, capturing the idea that true conjunctions are grounded in their conjuncts, and the idea that grounding is factive in the sense that $[\varphi]$ can only stand in the grounding relation if φ:

$$\varphi \wedge \psi \rightarrow [\varphi] \prec [\varphi \wedge \psi] \wedge [\psi] \prec [\varphi \wedge \psi]$$
$$\forall x([\varphi] \prec x \vee x \prec [\varphi] \rightarrow \varphi)$$

If φ, then by the first principle, $[\varphi] \prec [\varphi \wedge \varphi]$, hence $[\varphi]$ grounds something. And if $[\varphi]$ grounds something, then by the second principle, φ. (A similar observation is made by Korbmacher (2015).) Both of these principles are easily shown to be consistent with a strong background theory like true arithmetic, alongside the schema $T[\varphi] \leftrightarrow \varphi$ and the claim $\forall y(Ty \leftrightarrow \exists x(y \prec x))$: Consider a standard first-order language of arithmetic, and expand it with $[\cdot]$, T and $\prec$. Interpret it on the standard model of the natural numbers as usual, adding the following evaluation clauses for the additional connectives: relative to a given variable assignment, $[\varphi]$ is 1 if φ is true, and 0 otherwise, T is interpreted as the singleton of 1, and $\prec$ is interpreted as the relation which relates 1 to itself and nothing else. There is thus no reason to worry about the consistency of $T[\varphi] \leftrightarrow \varphi$, unless one thinks of the relata of ground as sentences – which is implausible if ground is to be a central metaphysical relation. (*Pace* Korbmacher (2018a,b); we return to this point in section 4.2.)

Finally, we again need to include plural quantifiers to quantify into the first argument of a connective of full ground. Thus we include the standard resources of plural logic, consisting of plural variables xx, yy, …, which may be bound by quantifiers and used in *is one of* statements of the form $x \varepsilon xx$.

3.2 | First-order principles of ground

Using the resources just outlined, principles corresponding to those in section 2.2 can be formulated as follows:

$(\wedge 1')\ \varphi \wedge \psi \rightarrow [\varphi] \prec [\varphi \wedge \psi] \wedge [\psi] \prec [\varphi \wedge \psi]$
$(\wedge 2')\ \forall x(x \prec [\varphi \wedge \psi] \rightarrow x = [\varphi] \vee x = [\psi])$
$(\vee 1')\ \varphi \vee \psi \rightarrow [\varphi] \prec [\varphi \vee \psi] \vee [\psi] \prec [\varphi \vee \psi]$
$(\vee 2')\ \forall x(x \prec [\varphi \vee \psi] \rightarrow x = [\varphi] \vee x = [\psi])$
$(\forall 1')\ \forall x\varphi \rightarrow \forall x([\varphi] \prec [\forall x\varphi])$
$(\forall 2')\ \forall y(y \prec [\forall x\varphi] \wedge \forall x(y \neq [\varphi]) \rightarrow y = \mathsf{t})$
$(nt')\ \neg \forall x \forall y(Tx \wedge Ty \rightarrow x \prec y)$

Of course, t is here treated as an individual constant.

Note that we must now distinguish between a statement and the statement that the proposition it expresses is true (between φ and $T[\varphi]$), and between a proposition and the proposition that it is true (between x and $[Tx]$). We will require some additional principles concerning the systematic connections which obtain between them. The first required principle is the truth schema already discussed:

$(T0)\ T[\varphi] \leftrightarrow \varphi$

Concerning truths and the truths ascribing truth to them, grounding theorists generally endorse that a truth $[Tx]$ is mediately grounded in x (Fine, 2010, Schnieder, 2011, Correia and Schnieder, 2012a). The natural corresponding principles for immediate ground states that a truth x is an immediate ground of $[Tx]$, and the only such ground:

$(T1)\ \forall x(Tx \rightarrow x \prec [Tx])$
$(T2)\ \forall x \forall y(y \prec [Tx] \rightarrow y = x)$

3.3 | The first-order inconsistency result

The ten principles just stated are inconsistent. Similar to before, only classical truth-functional reasoning, elementary principles of identity and quantification for first-order and plural quantifiers, and plural comprehension are required.

As before, we argue that there are distinct truths, which we call a and b: By $(\wedge 1')$, $[\forall x(x = x)]$ grounds $[\forall x(x = x) \wedge \forall x(x = x)]$, which according to $(T0)$ must be truths. By (nt'), there are truths x and y such that x does not ground y, so there are at least two truths a and b (using the symbols a and b now as individual constants). We continue to use $\cdot^2$ to indicate self-conjunction, and define $\hat{\wedge}$ and $\hat{\vee}$ almost as before, apart from using $T\mathsf{a}$ and $T\mathsf{b}$ instead of a and b. Since a and b are truths, $\hat{\wedge}$ and $\hat{\vee}$ are still truth-functionally equivalent to conjunction and disjunction. To show that they also still allow a certain amount of constituent recovery, it is useful to establish as a lemma that a truth x can only be distinct from y if their truth-ascriptions are distinct:

$\forall x \forall y(Tx \wedge [Tx] = [Ty] \rightarrow x = y)$

The argument is simple: If Tx, the by $(T1)$, x grounds $[Tx]$. If additionally $[Tx] = [Ty]$, then x grounds $[Ty]$. By $(T2)$, only y grounds $[Ty]$, so $x = y$.

As $T\mathsf{a}$ and $\mathsf{a} \neq \mathsf{b}$, it follows that $[T\mathsf{a}] \neq [T\mathsf{b}]$. With this, the arguments for the following lemmas go through as before:

$$(\hat{\wedge}')\ \varphi \wedge \psi \wedge [\varphi \hat{\wedge} \psi] = [\chi \hat{\wedge} \vartheta] \rightarrow [\varphi] = [\chi] \wedge [\psi] = [\vartheta]$$
$$(\hat{\vee}')\ [\varphi \hat{\vee} \psi] = [\chi \hat{\vee} \vartheta] \rightarrow (\varphi \wedge \neg\psi \rightarrow [\varphi] = [\chi]) \wedge (\psi \wedge \neg\varphi \rightarrow [\psi] = [\vartheta])$$

Analogous to before, the central formula ψ is defined via φ:

$$\varphi := (Tz \hat{\wedge} (z \,\varepsilon\, zz \hat{\vee} \neg(z \,\varepsilon\, zz))) \hat{\vee} \neg Tz$$
$$\psi := \forall x \varphi(x)$$

Again, let $\overline{\forall}$ abbreviate plural quantification restricted to truths, and $\equiv$ abbreviate *these* being *those*:

$$\overline{\forall} xx\varphi := \forall xx(\forall x(x \,\varepsilon\, xx \rightarrow Tx) \rightarrow \varphi)$$
$$xx \equiv yy := \forall x(x \,\varepsilon\, xx \leftrightarrow x \,\varepsilon\, yy)$$

It is again useful to establish a lemma, which now states:

$$(L\varphi')\ \overline{\forall} xx \overline{\forall} yy \forall x \forall y(Tx \wedge [\varphi(x, xx)] = [\varphi(y, yy)] \rightarrow (x \,\varepsilon\, xx \leftrightarrow x \,\varepsilon\, yy))$$

Note that if $[\varphi] = [\psi]$ then $T[\varphi] \leftrightarrow T[\psi]$, so by $(T0)$, $\varphi \leftrightarrow \psi$; hence, in general:

$$[\varphi] = [\psi] \rightarrow (\varphi \leftrightarrow \psi)$$

With this, the argument for $(L\varphi')$ is effectively as before, as is the argument from $(L\varphi')$ to the two principles which state that there is a distinct truth $[\psi(xx)]$ for any truths xx:

$$(S\psi')\ \overline{\forall} xx \overline{\forall} yy([\psi(xx)] = [\psi(yy)] \rightarrow xx \equiv yy)$$
$$(T\psi')\ \overline{\forall} xx T[\psi(xx)]$$

As before, these are jointly inconsistent by plural Cantorian reasoning.

4 | RETAINING THE PRINCIPLES OF IMMEDIATE LOGICAL GROUND

How should we respond to the inconsistency results established here? The possible responses can roughly be divided into two kinds: On the first kind of response, the principles just argued to be inconsistent are retained, either in their present form or in a reformulated one. On the second kind of response, some or all of these principles are given up, without being replaced by a similar principle capturing the same idea. This section considers responses of the first kind. The discussions of these options will be largely independent, so the subsections of this section may be read selectively and out of order. I will argue that all of these responses are unpromising, and that therefore, a response of the second kind must be given. Thus, some of the principles of immediate logical ground discussed here must be given up, without being replaced by any similar principle capturing the same idea. Readers who are already convinced of this conclusion may safely skip to the next section.

4.1 | Rejecting the logic

If all of the problematic principles are to be retained in their present form, then some of the logical principles used to derive their inconsistency must be rejected. The most controversial logical principle is probably plural comprehension, the schematic claim that there are some propositions *pp* which are just the q such that $\varphi(q)$, whatever (instantiated) condition φ might be. Although it strikes many as overwhelmingly plausible, independent grounds for rejecting it are discussed by Florio and Linnebo (2021). Somewhat similarly, one might also consider weakening the classical quantificational principles by imposing the restrictions of free logic. This would make it possible to deny that there is such a truth as φ (or $[\varphi]$), even if φ is the case. In the first-order formulation of section 3, one might also reject the broadly logical principle ($T0$) on the basis that it is inconsistent with a structured conception of propositions, as discussed by Schwarz (2013) and Whittle (2017). Finally, one may reject the laws of classical propositional logic, e.g., in order to pursue a naive property theory such as Field (2004).

There are many avenues along these lines. A particularly natural one is a potentialist position, following ideas of, e.g., Fine (2005) and Linnebo (2010). I explore this in some more detail in Fritz (forthcoming a), and point out some problems arising specifically in connection with ground. Here, I will limit myself to a couple of general reasons against rejecting logical principles in response to the problems of ground, and to the details of one way of weakening logical principles suggested by recent work on the Russell-Myhill argument.

The first reason is that rejecting these classical inferences is highly disruptive, as we tacitly rely on them in our informal theorizing, and the relevant restrictions are often very difficult to isolate, as argued by Williamson (2017). Any such rejection should therefore be considered as very costly.

The second reason concerns the status of the language in which the inconsistency is derived. Natural languages exhibit a variety of complex features such as vagueness, which many take to lead to violations of classical logic in some form or another. Often, such violations can be understood as arising from complexities in the relationship between expressions and the features of reality they express, such as an expression not having a unique denotation. E.g., a vague term may be considered to have many admissible precise denotations, or a description like “the present king of France” may be considered to have no denotation. But the languages used here are very different: They are formal languages whose syntax is defined in a mathematically rigorous ways, and whose interpretation is explicitly stipulated. Furthermore, these languages are tailored to metaphysical investigation, not unlike the *Begriffsschrift* which Frege (1879) tailored to his purposes in the philosophy of mathematics. The terms of such languages ought to relate to reality in a straightforward way, with a symbol like $\prec$ uniquely expressing the intended grounding relation. The idea that such a language should exhibit failures of classical logic is thus a much more radical proposal than the idea that, e.g., vagueness leads to some form of non-classicality in natural languages. Yet, even in the latter cases, many consider it too costly to give up classical logic, such as Williamson (1994). Since the cost of non-classicality is even higher in cases like the classical inconsistency of grounding principles, it is even more compelling to reject the notion of ground instead of weakening classical logic.

Despite these general reasons to be skeptical about revising logical principles in the face of problems with ground, the remainder of this section considers one such revision. According to it, plural comprehension restricted to so-called predicative instances. Plural comprehension may be formulated as the following axiom schema:

$\exists pp \forall q(q \, \varepsilon \, pp \leftrightarrow \varphi)$ (pp not free in φ)

An instance of this is *predicative* if φ contains no plural quantifiers or free plural variables. It turns out that with this kind of restriction, the Russell-Myhill argument can be blocked, as worked out by Walsh (2016). The inconsistency arguments discussed here rely on a variant of the Russell-Myhill argument restricted to truth, and the standard way of deriving an inconsistency from instances of the schematic principles ($S\psi$) and ($T\psi$) stated in section 2.4 involves an instance of impredicative plural comprehension. So it is *prima facie* plausible that the present inconsistency arguments are also blocked by restricting plural comprehension to predicative instances.

This, however, is not the case. As discussed recently by Uzquiano (2019), certain types of Cantorian arguments can be carried out using only predicative plural comprehension, and such arguments turn out to be possible in the present case as well. The matter is again most easily illustrated with the variant using arbitrary conjunction presented in section 2.3. One can give a variant argument for the inconsistency of the grounding principle for arbitrary conjunction which only relies on a predicative instance of plural comprehension, according to which there are the non-self-grounding truths rr:

$$\forall p(p \, \varepsilon \, rr \leftrightarrow p \wedge p \nprec p).$$

The relevant instance of plural comprehension is in fact not just predicative; it involves no quantifiers or free variables of any kind.

Recall the basic grounding principle for arbitrary conjunctions ($\bigwedge\prec$), according to which a conjunction of truths is (immediately) grounded in just its conjuncts. Considering the question whether $\bigwedge rr$ (the conjunction of the non-self-grounding truths) grounds itself. First, by choice of rr, all $p \varepsilon rr$ are true, so with the principle ($\bigwedge T$) capturing the truth-functionality of $\bigwedge$, it follows that $\bigwedge rr$ is also true. By ($\bigwedge\prec$), $\bigwedge rr$ grounds $\bigwedge rr$ just in case $\bigwedge rr$ is one of rr:

$$\bigwedge rr \prec \bigwedge rr \leftrightarrow \bigwedge rr \, \varepsilon \, rr.$$

And by choice of rr, $\bigwedge rr$ is one of rr just in case $\bigwedge rr$ is true and does not ground itself. Since $\bigwedge rr$ is true, $\bigwedge rr$ is one of rr just in case $\bigwedge rr$ does not ground itself:

$$\bigwedge rr \, \varepsilon \, rr \leftrightarrow \bigwedge rr \nprec \bigwedge rr.$$

And these two conclusions are straightforwardly inconsistent.

As above, this type of argument can be generalized. Assume that $\varphi(p, q)$ and $\psi(pp)$ are formulas such that the following hold:

($S'\varphi, \psi$) $\overline{\forall} pp \forall p(p \rightarrow (\varphi(p, \psi(pp)) \leftrightarrow p \, \varepsilon \, pp))$
($T\psi$) $\overline{\forall} pp \psi(pp)$

($S'\varphi, \psi$) effectively states that for any truths pp, pp can be recovered from $\psi(pp)$ via the relation expressed by $\varphi(p, q)$. ($T\psi$) states, as before, that $\psi(pp)$ is true, for any truths pp. Thus ($S'\varphi, \psi$) and ($T\psi$) also formulate some version of the claim that for any truths pp, there is a unique corresponding truth $\psi(pp)$. But now we can derive a contradiction using just the instance of plural comprehension according to which there are the truths rr which don't stand in the relation expressed by $\varphi(p, q)$ to themselves:

$$\forall q(q \, \varepsilon \, rr \leftrightarrow q \wedge \neg\varphi(q, q))$$

The existence of such rr is guaranteed by predicative plural comprehension as long as $\varphi(q, q)$ does not involve plural resources. The derivation of a contradiction is simple: Since rr are all true, $\psi(rr)$ follows by $(T\psi)$. So $\psi(rr)\varepsilon rr$ just in case $\varphi(\psi(rr), \psi(rr))$. And with $(S'\varphi, \psi)$, it follows that $\varphi(\psi(rr), \psi(rr))$ just in case $\psi(rr)\varepsilon rr$. These conclusions are inconsistent. The case of arbitrary conjunction is obtained by letting $\varphi(p, q)$ be $p \prec q$, and $\psi(pp)$ be $\bigwedge pp$. (The use of a binary relation in $(S'\varphi, \psi)$ is modeled on a version of Cantor's theorem in Bernays (1942); see Uzquiano (2019, p. 212) for further discussion related to predicative comprehension.)

What about the seven principles of section 2.2? We can give a predicative argument if (∀2) is strengthened to say that universal statements are only grounded in their instances (thus dropping the potential exception of a "that's all" truth):

$(\forall 2_s)$ $\forall q(q \prec \forall p\varphi \rightarrow \exists p(q = \varphi))$

We can then use φ and ψ defined as follows:

$$\varphi(p, q) := \exists r(r \wedge ((p \mathbin{\hat{\vee}} \neg p) \mathbin{\hat{\wedge}} (r \mathbin{\hat{\vee}} \neg r)) \prec q)$$
$$\psi(pp) := \forall p((p \mathbin{\hat{\vee}} \neg p) \mathbin{\hat{\wedge}} (p \,\varepsilon\, pp \mathbin{\hat{\vee}} \neg(p \,\varepsilon\, pp)))$$

With (∀2) strengthened to $(\forall 2_s)$, arguments similar to those in section 2.4 establish $(S'\varphi, \psi)$ and $(T\psi)$. The required instance of plural comprehension uses the clause $q \wedge \neg\varphi(q, q)$, which is predicative. So with $(\forall 2_s)$, the seven principles of section 2.2 can be shown to be inconsistent without invoking impredicative plural comprehension.

It remains to be settled whether we can also give a predicative inconsistency argument for the original seven principles, with (∀2) instead of $(\forall 2_s)$. But what we have seen here suffices to show that not every way of blocking the Russell-Myhill argument also blocks the inconsistency arguments for ground. In particular, it is far from clear that restricting plural comprehension to predicative instances suffices to circumvent the inconsistency. And of course, even it this turns out to be the case, one still needs to give a compelling philosophical motivation for rejecting impredicative plural comprehension, which is by no means easy.

4.2 | Reformulating principles

If the logical principles used here are to be retained, then some of the problematic principles of immediate ground must be given up. A conservative approach is to merely reformulate these problematic principles, in an attempt to save the intuitions which motivate them. Such a reformulation can be motivated by arguing that the very languages employed here are inadequate. As Correia and Schnieder (2012a, pp. 10–12) note, the two languages used in sections 2 and 3 are the most widely used ones. But there are alternatives. Korbmacher (2018a,b), for example, adopts a third option, arguing that the relata of ground should be taken to be sentences; and a similar approach is taken in Poggiolesi (2016, 2018). As in section 3, the appropriate language is accordingly a first-order language with binary relation symbols formalizing notions of partial ground, albeit without a propositional abstraction operator. Instead, sentences are taken to be coded using natural numbers, so that the theory of ground can be carried out using techniques familiar from the literature on formal theories of (sentential) truth. The main argument given by Korbmacher (2018a, pp. 163–166) for this choice of language is technical convenience: it allows him to draw on a rich and well-developed body of logical theory, including the proof theory and model theory of first-order logic, as well as formal theories of truth.

However, as Korbmacher (2018a, pp. 189–190) acknowledges, his choice of language also comes with severe formal limitations, since there is no natural way of extending the coding of sentences using numbers to pluralities of sentences, and so no natural way to treat the notion of full ground. Even from just a formal perspective, it is thus doubtful that the proposed language is to be preferred. More importantly, the idea that ground relates sentences is simply implausible from a philosophical perspective. Ground is meant to be a matter of metaphysics, and so a matter of how the world is, not how we represent it. Such a perspective is sometimes endorsed explicitly, e.g., by Audi (2012, p. 691), and sometimes implicitly, e.g., by adopting *Metaphysical Grounding* as the title for the volume edited by Correia and Schnieder (2012b). Consequently, ground should relate the features of reality we express with our sentences, not the sentences themselves.

Those who want to avoid the inconsistency of ground by adopting a different language thus face a difficult challenge: They need to produce a convincing way of regimenting talk of ground such that the relata of ground are not implausibly representational like sentences. Furthermore, this regimentation needs to admit a treatment of full ground and so of plural quantification over the relata of ground. Finally, adopting this language must somehow allow paradigmatic principles of logical ground to escape the constraints of Cantor's theorem, which rules out distinguishing a truth for every plurality of truth. I can see two options which have some initial promise in satisfying these desiderata, which will be discussed in the next two subsections.

4.3 | Reformulating quantificational grounds

The first option reformulates the principles of immediate grounds of universal truths. The relevant reformulation can be motivated independently. To do so, note that principles (∀1) and (∀2) are schematic, in opposition to the principles for conjunction and disjunction. They may thus be less general than we would like them to be, being limited to the universal claims which can be formulated in the relevant language. Using some tools of higher-order logic, this limitation can be overcome.

For any propositional variable p and formula φ, let $\lambda p\varphi$ be an expression which can be used in the position of a sentential operator. To a first approximation, $\lambda p\varphi$ can be read as expressing the property of being a proposition p such that φ; consequently, $(\lambda p\varphi)\psi$ attributes to ψ being a p such that φ. Next, add to the language of section 2 variables X, Y, Z, … which can take the place of sentential operators, and allow them to be bound by existential and universal quantifiers. To a first approximation, we can read $\forall X\varphi$ as stating that every property of propositions X is such that φ. Following ideas of Frege (1884), we can now take propositional quantifiers to express properties of properties of propositions, and so understand them as expressions which take sentential operators as arguments. We will use the symbols E and A for this purpose. $\mathsf{E}X$ can thus be read as the claim that there is an X, or that X is instantiated, and $\mathsf{A}X$ as the claim that all propositions are X, or that X is universal. Consequently, $\mathsf{E}\lambda p\varphi$ states that some p is such that φ, and $\mathsf{A}\lambda p\varphi$ that every p is such that φ. The variable-binding $\exists$ and $\forall$ can now be read as abbreviating the strings $\mathsf{E}\lambda$ and $\mathsf{A}\lambda$.

In such a higher-order variant of the language of section 2, the schematicity of (∀1) and (∀2) can be overcome, as we can use an object-language variable X bound by a universal quantifier instead of a schematic meta-variable φ:

($\forall 1^*$) $\forall X(\mathsf{A}X \to \forall p(Xp \prec \mathsf{A}X))$
($\forall 2^*$) $\forall X \forall q(q \prec \mathsf{A}X \land \forall p(q \neq Xp) \to q = \mathsf{t})$

These two principle are also natural ways of formulating the informal idea that – setting t aside for the moment for simplicity – the grounds of a universal claim $\mathsf{A}X$ are just the instances, i.e., the propositions Xp, for any p. In fact, they do so in a more elegant and possibly more general way, since they are not schematic.

The proof of the inconsistency no longer goes through without additional assumptions when ($\forall$1) and ($\forall$2) are replaced by ($\forall 1^*$) and ($\forall 2^*$). To see why, consider the grounds of $\forall p\varphi(p,pp)$, where φ is the formula defined in section 2.4. Since this is the formula $\mathsf{A}\lambda p\varphi(p, pp)$, the instances which ground it are no longer $\varphi(q, pp)$, for any q, but $(\lambda p\varphi(p, pp))q$, for any q. The presence of λ in these instances blocks the application of the principles of grounds of conjunctions and disjunctions in the argument for lemma ($L\varphi$).

This is of course not the end of the matter. The reason why the argument no longer goes through is that we have made no assumptions about the relationship between pairs of formulas of the form $(\lambda p\varphi(p))\psi$ and $\varphi(\psi)$. But it is clear that the two are tightly related. Consider for simplicity corresponding instances $(\lambda pXp)q$ and Xq of these schemas. Roughly, the first states that q has the property of being a p such that Xp, whereas the second states that q is X (or has property X). Assuming that these are both true, there are two natural positions to take about their relationship in the present context: According to the first, the two formulas express the same truth, so that $(\lambda pXp)q = Xq$. According to the second, Xq is the unique immediate ground of $(\lambda pXp)q$. (The first corresponds to a principle widely appealed to in higher-order logic under the label *β-conversion*; the second is suggested by Fine (2012, pp. 67–71).)

Whichever of these two position is chosen, if it is endorsed for all pairs of formulas $(\lambda p\varphi(p))\psi$ and $\varphi(\psi)$, the inconsistency result is reinstated. This is immediate in the first case of endorsing all instances of $(\lambda p\varphi(p))\psi = \varphi(\psi)$. In the second case, it is easy to see that the inconsistency argument can be patched up using the following schema:

$$(\lambda r\varphi(r))p = (\lambda r\psi(r))q \to \varphi(p) = \psi(q)$$

And this follows from the schematic assumption that $\varphi(p)$ is the unique immediate ground of $(\lambda q\varphi(q))p$.

For the response explored here to be viable, a third position on the relationship between $(\lambda p\varphi(p))\psi$ and $\varphi(\psi)$ is required, and it is not clear what this could be. Although the current literature on ground does not suggest any such third position, there is also no consensus on whether one of the two positions discussed here is correct. In Fritz (2020), I explore in a little more detail what such a third option might look like. There, I also show that adopting quantified instead of schematic principles of grounds of quantified statements provides a novel solution to the known problems of ground of Fine (2010), and one which does not require rejecting the irreflexivity of ground. It is therefore worth pursuing this option further. But at present, no position worked out in any detail – let alone argued for – is available which would show how to solve the problems of ground using quantified rather than schematic principles of quantificational ground.

A final limitation of the present proposal concerns the variant inconsistency argument using arbitrary conjunctions discussed in section 2.3. Adopting ($\forall 1^*$) and ($\forall 2^*$) instead of ($\forall$1) and ($\forall$2) has no influence on the grounds of arbitrary conjunctions if $\bigwedge$ is understood as a primitive connective – it provides no reason to deny that the immediate grounds of a conjunction $\bigwedge pp$ of truths pp are just those truths. One must therefore hold that $\bigwedge$ can only be understood as an abbreviating device along the lines discussed above, and therefore does not have the grounds one would naively expect it to have. It is not clear how big of a cost this is.

4.4 | Grounding as a relation among *t*-complexes

The second option for reformulating principles of immediate ground uses a language which takes the extension of the propositionally quantified language of section 4.3 to its logical conclusion: it provides not only variables for propositions and properties of propositions, but also variables for properties of properties of propositions, and so on, and generalizes talk of unary properties to relations of arbitrary finite arity. We thus work in a standard relational type theory. Since we don't need to talk about individuals, the hierarchy of types can be recursively generated by the single rule which states that if $t_1, \ldots, t_n$ are types (n being any natural number), then the sequence $\langle t_1, \ldots, t_n \rangle$ is a type as well, namely the type of a relational expression A taking n arguments $b_1, \ldots, b_n$, of types $t_1, \ldots, t_n$, respectively, to produce an expression $Ab_1 \ldots b_n$ of type $\langle\rangle$. Type $\langle\rangle$ is generated by the case $n = 0$ of the type formation rule, and is the type of formulas – expressions requiring no arguments to form a formula. Having generated atomic formulas from a stock of variables and constants of all types in the way just indicated, we allow, as before, the formation of complex formulas using Boolean operators, and quantifiers binding arbitrary variables.

Relationally typed languages like this go back to Myhill (1958) and Orey (1959), and have played an important role in formal semantics following Gallin (1975); Williamson (2013) makes a case for their usefulness in metaphysics. The language just sketched does not include plural quantifiers, but the constraints of Cantor's Theorem, via the Russell-Myhill result, still affect it: Using the principles of classical higher-order logic, we can prove that there cannot be a distinct proposition p for every property of propositions X (where now, p and X are variables of types $\langle\rangle$ and $\langle\langle\rangle\rangle$, respectively). As before, this shows that propositions are not structured, in the sense that we cannot in general recover, from a proposition Xp, the property X and proposition p as its unique constituents. We can, however, identify other entities using the higher-order language which encode, in more or less straightforward ways, X and p, as long as we go up sufficiently high in the type hierarchy. A simple example is the binary relation R among properties of propositions and propositions which relates X to p, and nothing else. Syntactically, R is thus an expression of type $\langle\langle\langle\rangle\rangle, \langle\rangle\rangle$. So while Xp is not itself structured, there is a proxy of type $\langle\langle\langle\rangle\rangle, \langle\rangle\rangle$ for the elusive structured proposition Xp.

This can be generalized. We can associate with every formula φ a higher-order relation, the type of which corresponds to the syntactic structure of φ, and which encodes all of the entities expressed by atomic constituents of φ. This is developed in more detail in Fritz (2021). There, it is also shown how to formulate the notion of such a higher-order relation of a type t corresponding to the syntactic structure of a formula independently of any formula with which it might be associated. Such relations are there called *t-complexes*. This provides us with a general notion of a proxy for a structured proposition of a certain syntactic structure. The resulting construction is in many ways familiar, as it can be seen as an instance of the proposal to assign to sentences trees corresponding to their syntactic structure, labeled by some kind of semantic value of their atomic constituents, as in, e.g., Lewis (1970), Cresswell (1985, 2002), Salmon (1986) and Soames (1987), except that the construction of trees is here carried out in type theory rather than the more common construction in set theory. This allows for an especially simple although rigorous theory, and consequently an easy consistency argument.

t-complexes may play various of the roles structured propositions have been thought to play. Could they be the relata of ground? Endorsing this means rejecting the language in which the inconsistency result of section 2 is formulated, since it requires that grounding relations are not relations among propositions. Instead, it requires that each of the grounding relations discussed so far divides further into an array of relations of different types, relating entities of various types corresponding to syntactic structures of formulas.

To illustrate this, consider two conjunctions of different syntactic structure, such as $p \wedge p$ and $p \wedge (p \wedge p)$, for a sentential constant p. Both are grounded in p. But the t-complexes associated with the two conjunctions are not just distinct, but higher-order relations of different types t_1 and t_2, corresponding to the syntactic structures of the two formulas. Consequently, one stands in a grounding relation of type $\langle\langle\rangle, t_1\rangle$, and the other in a grounding relation of type $\langle\langle\rangle, t_2\rangle$, to p. Grounding relations must thus be further indexed by types.

Schematically, we can still formulate general grounding principles for, e.g., conjunctions, corresponding to those used in the inconsistency result. Does the argument for the inconsistency still arise? Note first that the present higher-order language does not include plural propositional quantifiers. But this is no real limitation, since we may assume that for every plurality of propositions, there is a unique corresponding property of propositions – the property of being one of them – which can thus go proxy for the plurality. Alternatively, the construction of t-complexes could also be carried out, with minor adjustments, in a language with primitive plural quantifiers. By the construction of t-complexes, constituents can be recovered uniquely. There is thus no problem if grounding principles entail that the t-complex associated with $\psi(pp)$ can only be the t-complex associated with $\psi(qq)$ if pp are qq, since this consequence already follows more directly from the construction of t-complexes. All that is required is the consistency of the construction of t-complexes itself, which is demonstrated model-theoretically in Fritz (2021).

How do t-complexes escape the conclusion of the Russell-Myhill argument? How can it be that for every property X of t-complexes, there is a unique corresponding t-complex? The answer lies in the type parameter t. Let t_1 be any type corresponding to the syntactic structure of a formula, and X a property of t_1-complexes. Then the variable X is of type $\langle t_1\rangle$. We can use it in a formula, such as Xy, where y is a variable of type t_1. What is associated with this formula is not a t_1-complex, but a proxy of a higher-order relational type reflecting the syntactic structure of Xy: a $\langle\langle t_1\rangle, t_1\rangle$-complex. No Cantorian argument rules out the existence of a distinct relation of type $\langle\langle t_1\rangle, t_1\rangle$ for every property of type $\langle t_1\rangle$.

The distribution of t-complexes across different types makes their construction possible. But it also imposes severe limitations on what can be said about t-complexes in the higher-order language. Consider the claim which was used above to motivate the introduction of quantifiers: the claim that some truth has no grounds. Since all variables have one specific type, there is no straightforward way of stating that some t_1-complex of some type t_1 has no t_2-complex of any type t_2 as its ground. This is an instance of general limitations of higher-order languages, and there is no obvious answer what to make of them. A similar issue arises in the formulation of the notion of full ground: since the full grounds of a t-complex may be of different types – take, e.g., the case of a conjunction with conjuncts of different syntactic structure – it is not clear what the type of the entity which fully grounds it should be. I explore these issues in more detail in Fritz (2021), and tentatively sketch some possible responses. There, I also show that taking the relata of ground to be t-complexes solves the problems of Fine (2010) and others as well, without requiring rejecting the irreflexivity of ground. Whether the proposal could be worked out to a satisfactory theory of ground despite the limitations noted here is left for further investigation.

4.5 | A Metaphysicists' First Amendment?

I have argued that the prospects of retaining the problematic principles of immediate ground – either in their present form, by rejecting some logical principles, or in some reformulation – are unpromising. There is one final response to the inconsistency which retains the problematic principles: the option to ignore it. Faced with a version of the inconsistency of naive set comprehension, Landman (2000, p. 79) invokes what he calls the *Semanticists' First Amendment*:

> "The right to solve Russell's Paradox some other time shall not be restricted."

What about a grounding theorist who invokes a similar *Metaphysicists' First Amendment* and simply ignores the problem? Can't they just work with structured propositions, and leave it to the logicians to figure out how to get them?

Clearly, any metaphysicist unwilling to give up classical logic should not endorse a classically inconsistent theory. Furthermore, we have every reason to expect that ignoring the issue will stymie research on grounding. It is helpful to consider the analogous case of set theory in mathematics. A grounding theorist may want to argue as follows: "A mathematician working in, e.g., real analysis, who considers the reals and functions on the reals to be certain sets, or a theoretical physicist applying results in this area to concrete phenomena, doesn't have to worry about abstract set-theoretic questions concerning the paradoxes of set theory, such as Russell's paradox, since they are only concerned with such a limited and well-behaved domain. Likewise, grounding theorists don't have to worry about paradoxes of propositions, since they are only concerned with a very simple metaphysical relation of ground." The analogy seems a bit loose, but even if the two cases are relevantly similar, the situation in the case of set theory simply isn't as this fictional grounding theorist describes it: Standard work on reals and functions on reals involves heavy use of the axiom of choice, as noted, e.g., by Bruckner et al. (1997, p. 12), in keeping with the fact that it is one of the axioms of the most widely used set theory, ZFC. (For more on the importance of the axiom of choice, see Moore, 1982.) But the inclusion of the axiom of choice is specific to ZFC, which is only one way of developing a theory of sets which is not affected by Russell's theorem of the inconsistency of naive set comprehension. An example of a different theory is NF, proposed by Quine (1937), which Specker (1953) showed to be inconsistent with the axiom of choice. The fact that ZFC became the *de facto* standard for reasoning about sets in the middle of the 20th century has deeply shaped contemporary mathematics. It is hard to imagine how mathematics could have progressed as much as it actually did if the response to Russell's result concerning naive set comprehension in the early 20th century had been to perpetually invoke the right to solve it some other time and continue to work with naive set comprehension. Likewise, grounding is unlikely to live up to its promise if a systematic treatment of its relata is indefinitely deferred. As Deutsch (2008) urges in the case of the philosophy of language: Russell's limitative result concerning the individuation of propositions has been known for over a century – there is no excuse anymore for naively working with "structured propositions".

5 | REJECTING THE PRINCIPLES OF IMMEDIATE LOGICAL GROUND

The previous section argued that retaining the problematic principles of immediate logical ground is unpromising, even if these principles are reformulated in various ways. Some of these principles therefore have to simply be given up. This in turn can take various forms, which this section discusses.

Again, the discussions of these options will be largely independent, so that subsections may be read selectively and out of order.

5.1 | Rejecting immediate grounding

The first option is to give up the the problematic principles wholesale, by giving up the notion of immediate grounding entirely. According to this response, the only legitimate varieties of ground are the mediate ones.

Is this a plausible response? First, note that it is a costly one. Although grounding theorists rarely focus on immediate ground – with some exceptions, like Poggiolesi (2016, 2018) and Werner (2020) – many grounding theorists consider the notion of immediate ground to be important. Fine (2012, p. 51), for example, writes: “It is the notion of immediate ground that provides us with our sense of a ground-theoretic hierarchy”. And Correia and Schnieder (2012a, section 2.3) note that immediate ground already appears in the writings of Bernard Bolzano, who is arguably the most important precursor to the modern debate about grounding. (A more in-depth discussion of Bolzano’s theory of ground, including the immediate variant, can be found in Tatzel (2002).) A version of the present inconsistency argument can also be formulated for the grounding relation called “pre-ground” by Litland (2020), a precursor of which occurs in Litland (2015) under the label “putative ground”. In Litland’s discussion, pre-ground arguably emerges as more fundamental than more standard notions of ground. Furthermore, pre-ground is not taken to be irreflexive, so that the problems of Fine (2010) do not apply to it. The present argument therefore appears to the be most important logical challenge for this conception of grounding.

Second, the response of giving up immediate notions of ground depends on them being conceptually separable from notions of mediate ground. Whether this is so is not a straightforward matter. Clearly, such a separability requires that immediate ground cannot be defined in terms of mediate ground. Fine (2012, p. 51) maintains that no such definition is available, but the only argument he gives is an argument against one specific proposed definition, according to which p immediately grounds q just in case p mediately grounds q and there is no r such that p mediately grounds r and r mediately grounds q. By taking into account full grounds, various alternative proposals suggest themselves. E.g., let *pp support qq* just in case every q among qq which is not one of pp is fully grounded in some of pp. One might then propose that pp are immediate full grounds of qq just in case they are supported by every plurality which mediately fully grounds qq, and that p is an immediate partial ground of q if it is among some immediate full grounds of q. Evaluating this proposal is beyond the scope of this article, but the possibility of such definitions show that we have no conclusive reason to think that the notion of immediate ground is an extraneous addition to a conceptually independent notion of mediate ground, instead of considering it part and parcel of our overall intuitive conception of ground to which grounding theorists appeal.

The latter viewpoint is further supported when we consider certain general features of mediate ground, which can be given a natural unifying explanation in terms of immediate ground. Take the case of conjunction. The mediate grounds of a true conjunction $p \wedge q$ include the conjuncts p and q. But in contrast to the immediate case, mediate grounds need not be conjuncts: by the transitivity of mediate ground, any mediate ground of p or q is a mediate ground of $p \wedge q$ as well. It is natural to think that there should be no further mediate grounds of $p \wedge q$. Thus, the mediate grounds of a true conjunction $p \wedge q$ would comprise exactly p, q, the mediate grounds of p and the mediate grounds of q. The same kind of reasoning applies to other cases like disjunctive and quantificational truths. E.g., assuming for simplicity that all true disjuncts of a true disjunction are among its mediate grounds, the mediate grounds of a true disjunction should be its true disjuncts and their mediate grounds. There is clearly a pattern here. It can be be brought out using immediate ground if one assumes with Fine (2012, p. 51) that the relation of mediate ground is the transitive closure of the relation of immediate ground. The necessary and sufficient conditions of mediate grounds just stated then follow from the corresponding necessary and sufficient conditions of immediate ground. E.g., from the claim that the immediate grounds of a true conjunction are just the conjuncts, it follows that the mediate grounds of a true conjunction are just the conjuncts and their mediate grounds. (The account of necessary conditions on mediate grounds sketched in this paragraph may be in conflict with the proposal of Fine (2012, p. 63). But for the same reasons, Fine's own account of the relationship between mediate and immediate ground just mentioned may be in conflict with this proposal as well.)

5.2 | Rejecting logical grounding

A second way of giving up the problematic principles wholesale is by giving up the idea of logical grounding, i.e., the idea that the truth-functional behaviour of conjunction, disjunction and quantification is reflected in grounding relations along the lines of the principles of section 2.2. Instead, one might claim that the paradigmatic cases of grounding are cases such as the following (adapted from Correia and Schnieder, 2012a, p. 1):

Mental facts are grounded in neurophysiological facts.
Normative facts are grounded in natural facts.

Some grounding theorists, like Correia (2014) and Fine (2017a, section 6), distinguish between different notions of ground, including a logical notion of the kind discussed here, and a metaphysical notion which is less discerning when it comes to logical connectives. In such a setting, the present response would amount to rejecting the logical notion of ground, while retaining the metaphysical notion.

As far as the current literature of ground is concerned, such a response is highly revisionary, and would require abandoning a significant portion of the grounding research program. E.g., Fine (2012) almost exclusively uses the kind of logical grounding relations which would have to be abandoned to illustrate the various notions and distinctions concerning ground which he introduces. On this response, the relations of ground would have to behave radically differently from how they are widely assumed to behave, and the motivations for various notions and distinctions would have to be revisited.

Furthermore, abandoning the idea of logical grounding calls into question the idea that grounding draws hyperintensional distinctions, i.e., distinctions between necessarily equivalent truths. To see why, we need to step back from the particular concerns of ground, and consider grain more generally.

There are two standard pictures of the individuation of truths, and propositions more generally: On the *modal* picture, propositions are identical just in case they are necessarily equivalent. On the *structured* picture, propositions are identical just in case they have the same structure, and are constituted of the same entities in corresponding positions. The structured picture is incompatible with the standard principles of logic used here, for very much the same reason the principles of immediate ground turned out to be inconsistent: structure requires that propositions $\forall p(p \; \varepsilon \; pp \rightarrow p)$ and $\forall p(p \varepsilon qq \rightarrow p)$ are identical only if pp are qq, which is impossible – by Cantorian reasoning, there cannot be a proposition for every plurality of propositions. (There is of course much more to be said about the two pictures, and the inconsistency of structure, but going into further detail would lead us too far from matters of ground. Some relevant discussion can be found in Dorr (2016) and Goodman (2017).)

We may thus take the modal individuation of propositions as the default position. This is incompatible with logical grounding, which we can now easily see: Working in the language with propositional quantifiers, it was shown above that the principles of logical ground entail that there are distinct truths a and b, which are, respectively, the unique immediate grounds of $a \vee a$ and $b \vee b$. Since these disjunctions are necessary, they are identified by the modal individuation of propositions, contradicting the claim that they have different grounds. This motivates introducing a new metaphysical primitive of grounding, and rejecting the modal individuation of propositions – ideally replacing the modal individuation of propositions with an alternative in terms of the new primitive notion of ground. But now consider the case in which the principles of logical ground are rejected. Does the same motivation still apply? One might think so, since many proponents of grounding support their claim that grounding draws distinctions between necessarily equivalent propositions using examples which do not involve logical grounding. E.g., Fine (2012, p. 45) adapts the well-known example from his work on essence (1994), and proposes that Socrates existing grounds {Socrates} existing, but not *vice versa*, even though necessarily, Socrates exists just in case {Socrates} exists. However, I will argue that in any such case, we should take the asymmetric relationship between the relevant truths to show that they are not necessarily equivalent, instead of revising the modal individuation of propositions.

Let *Booleanism* be the view that any two sentences which are equivalent by classical propositional logic express the same proposition. E.g., according to this view, $p \vee p$ is $q \vee q$ for all p and q. Since tautologies are necessary, Booleanism follows from the modal individuation of propositions, and so may be counted as being part of the default position. Let *top* be the proposition expressed by some sentence which is a tautology of classical propositional logic; by Booleanism, all such sentences express the same proposition, so it does not matter which one we choose. Thinking of modalities as properties of propositions, consider *being top*, the modality of being identical to the top proposition. This modality displays the standard logical features of a notion of necessity, as noted already by Cresswell and (1965) and Suszko (1971). And being necessary – in the sense operative in the modal individuation of propositions – must at least be materially equivalent to being top: p is necessary just in case it is necessary that p iff top, which is the case if and only if p is top.

Bacon (2018) shows with natural auxiliary assumptions that *being top* is the unique strongest notion of necessity. In my opinion, this means that *being top* is metaphysical necessity. If there is any place in the literature in which the notion of metaphysical necessity was introduced, it is *Naming and Necessity*, and the only thing Kripke (1980 [1972]) says about which notion of necessity he is concerned with is that it is necessity "in the highest degree" (p. 99). Bacon (2018, p. 743) disagrees, but for present purposes, the issue is not important. Whether *being top* is metaphysical necessity or not, there are few things we know about what is top and what isn't. Plausibly, we can use the argument of Kripke (1971) to argue that true identities are top. But we have little reason to take anything else for granted, including many of the essentialist judgements of Kripke (1980 [1972]). Even if it strikes one intuitively as plausible that one could not have been a fried egg, this may well merely reflect correct judgements about everyday notions of necessity, rather than any reliable answer to the question whether the proposition that you are not a fried egg is top.

Let us return once again to grounding, taking the modal individuation of propositions as the default position, and considering little as settled concerning the relevant notion of necessity. What if we become convinced that there is an asymmetric metaphysical dependence between Socrates existing and {Socrates} existing? If these truths are to be distinguished, then they must either not be necessarily equivalent or the modal individuation of propositions must be given up. In this case, it seems far more plausible to take this to show that Socrates existing is not necessarily equivalent to {Socrates} existing, since this only requires revising some particular judgements about the necessity of impure set formation. In contrast, rejecting the modal individuation of propositions is theoretically more costly, as it requires rejecting a general principle of the individuation of propositions without supplying any alternative proposal. The example therefore does not motivate the need for a new metaphysical primitive which draws distinctions between necessarily equivalent propositions.

As noted by Duncan et al. (2017), grounding is often said to be hyperintensional. We have seen that without the idea of logical grounding, the case for hyperintensionality is significantly limited. This is a substantial cost for the present response, for two reasons: First, the claimed hyperintensionality of grounding is one of the motivations for introducing it as a new primitive metaphysical notion. Second, the existence of a hyperintensional relation among propositions entails that necessarily equivalent propositions need not be identical, a conclusion with pervasive ramifications in metaphysics. Giving up the logical grounding principles appealed to in the inconsistency arguments therefore not only requires substantial revisions to the current understanding of ground, but both undermines one of the motivations for introducing such a new primitive, and eliminates one of its most interesting features.

5.3 | Rejecting specific principles

Those who wish to retain both an immediate notion of ground and the idea that grounding tracks truth functions will have to reject some of the specific principles.

In the case of the first-order formulation of section 3, one might object to the principles of truth grounding ($T1$) and ($T2$) on the basis of a kind of deflationary theory of truth. One might hold that attributing truth to a truth just amounts to expressing that truth, so that $[Tx]$ should be x when Tx. In fact, this is does not help: the argument for inconsistency only uses ($T1$) and ($T2$) to derive that Tx and $[Tx] = [Ty]$ only if $x = y$, which also follows straightforwardly from the proposed identification of $[Tx]$ with x.

In the case of the propositionally quantified formulation of section 3, the only option is to reject some conjunctive, disjunctive, and universally quantified principles of immediate logical ground. Rosen (2010, section 8) presents some possible alternative grounds for universal truths, considering the possibility of certain universal truths being grounded in truths about essences or laws. However, Rosen (2010, section 6) does endorse the principle that existential truths are grounded in their true instances, and it is easy to see that the above inconsistency arguments go through as well if the principles governing universal quantifiers are replaced by the corresponding principles for existential quantifiers. E.g., the principles corresponding to (∀1) and (∀2) are:

(∃1) $\forall p(\varphi \to \varphi \prec \exists p\varphi)$
(∃2) $\forall q(q \prec \exists p\varphi \land \forall p(q \neq \varphi) \to q = \mathsf{t})$

In fact, such principles of grounds for existential truths are controversial as well. E.g., Fine (2010, p. 108) considers whether every true instance of an existential claim must count as a ground, or whether it suffices to require every true existential claim to be grounded in some instance. In the second case, (∃1) would have to be replaced by a weaker principle, corresponding to the weak principle ∨1 for disjunction used above:

$$\exists p\varphi \to \exists p(\varphi \prec \exists p\varphi)$$

This may well restore consistency. But the resulting theory of grounds of quantified truths is excessively weak: no systematic principles of grounds of universal truths are endorsed, and only very weak restrictions are imposed on the grounds of existential truths.

Similarly, it is not easy to see how the conjunctive and disjunctive principles (∧ 1), (∧ 2), (∨1) and (∨2) could be weakened without rendering the resulting theory unsatisfyingly incomplete, and without making apparently arbitrary choices about which of our intuitions concerning grounding are to be respected. Note in particular that it is implausible to appeal to model theories of ground based on state spaces, such as the one developed by Fine (2012), which support weak principles of interaction between grounding and logical connectives, in order to identify the acceptable principles of logical grounding. Doing so may be acceptable to someone who assumes that metaphysics should operate with a primitive notion of a state of affairs, in terms of which various derivative notions can be defined, some of which may be labeled using the ideology of grounding. But much of the grounding literature appears to adopt a different standpoint, namely one on which the central new metaphysical primitives are those of ground, rather than that of a state of affairs. This is suggested by the fact that it is normally intuitive judgements about grounding claims which are appealed to, rather than intuitive judgements about claims concerning states of affairs. On this latter standpoint, the adequacy of a set-theoretic model theory of state spaces therefore has to be judged on the basis of whether it supports the intuitive grounding principles. Conversely, motivating principles of grounding using any kind of model theory would thus be putting the cart before the horse.

5.4 | Rejecting grounding

Those who are already committed to grounding in metaphysics will no doubt take the inconsistency results of this paper merely to uncover some limitations on the correct principles of grounding. But those not already so committed may well consider the results to contribute to doubts about the new theoretical primitives which grounding theorists aim to introduce into metaphysics. A final response to the inconsistency results which must be taken seriously is therefore the option to reject grounding altogether.

To see that this response need not be seen as an over-reaction, recall that the ideology of grounding was only introduced recently as a new primitive theoretical notion in metaphysics. It is a substantial claim that this introduction was successful: it requires that our ordinary talk involving phrases such as "because" or "in virtue of", alongside other stipulations such as paradigmatic examples and disambiguations (e.g., distinguishing between partial versus full ground), suffice to pin down a unique notion of grounding. Even independently of any formal arguments against ground, one may doubt, as Daly (2012), Wilson (2014) and Koslicki (2015) do, that the introduction of this new theoretical primitive succeeds. Whether or not we are initially sympathetic to the notion of grounding, the fact that basic principles – along the lines of paradigmatic instances meant to pin down the notion under consideration – lead to inconsistency is a serious reason to question whether the notion is in fact in good standing. This is explicitly acknowledged by Rosen (2010, p. 114), who writes:

> "I begin with the working hypothesis that there is a single salient form of metaphysical dependence to which the idioms we have been invoking all refer. The plan is to begin to lay out the principles that govern this relation and its interaction with other important philosophical notions. If the notion is confused or incoherent, we should get some inkling of this as we proceed."

The above inconsistency results provide such an inkling of the incoherence of ground. In fact, these results are not the first indication that the notion of grounding is – in Rosen's terminology – confused or incoherent: one may already take the inconsistency results of Fine (2010), and the variant of Krämer (2013), to provide an inkling of this.

Such a skeptical attitude towards grounding is further supported by the fact that in roughly a decade of intense work on grounding in metaphysics, no models of ground have been constructed which have found any significant endorsement by grounding theorists themselves. This even applies to notions of mediate ground. E.g., Fine (2012, pp. 71–74) proposes a model theory for ground in which a space of states of affairs is taken as basic, and in which propositions can be identified with sets of verifying states or pairs of sets of verifying and falsifying states. He has subsequently developed this in much more detail and used it in a variety of other applications (for a recent survey, see Fine, 2017b). However, such models fail to distinguish between truths p and $p \wedge p$, as Fine (2012, p. 74, fn. 22) notes himself. By construction (see Fine, 2012, p. 72, clause (iv)), strict partial ground is irreflexive in these models, so they even fail to validate the basic idea that the strict partial grounds of a true conjunction include its conjuncts. Other proposals are highly restricted, such as the models of Krämer (2018) which do not treat quantifiers. (Some grounding theorists, such as Audi (2012, p. 686), endorse a structured view of propositions. Models for such views have been developed, e.g., by Bealer (1982), but these involve highly controversial restrictions needed to avoid inconsistency via the Russell-Myhill argument, some of which were discussed in section 4.1.)

The formal inconsistency of immediate ground combined with the inconsistency results of Fine (2010) and the lack of success in developing models of even mediate ground casts doubt on the coherence of the notion of ground envisaged by its proponents. We must therefore take seriously the possibility that the introduction of talk of ground in metaphysics failed. Such a failure can take a number of forms, corresponding to various semantic and metasemantic defects which words can display. For example, it might simply be that “ground” and similar terms as used in contemporary metaphysics are meaningless. Or it might be that “ground” is highly ambiguous, even after standard clarifications have been made, such as distinguishing between mediate and immediate ground. Or it might be that “ground” picks out a single relation, but that this relation is one among a continuum of relations none of which plays a distinguished role in metaphysics. For the purposes of metaphysics, the most important question is just whether “ground” suffers from any of these defects, not which particular defect it displays, if any.

It is worth re-iterating that skepticism about ground does not entail skepticism about metaphysical dependence relations in general: For example, the problems for ground developed here are known not to arise for modal ideology. This can be demonstrated model-theoretically using the well-known possible worlds models for modal logic, including higher-order logic (Gallin, 1975). Therefore, no doubt is cast on, e.g., modal dependence relations in metaphysics. For this reason, it is clear that there is *some* way of making sense of talk of ground, and *some* way of revising paradigmatic principles like those of section 2.2 to arrive at a coherent picture: we can always understand ground to amount to modal dependence. But such a defense of the new paradigm of grounding-based metaphysics achieves at most a pyrrhic victory. This new paradigm crucially involves the claim that grounding should be adopted as a new primitive metaphysical notion which draws distinctions which go beyond what can be drawn in modal terms. This is called into question by the inconsistency results developed here.

6 | CONCLUSION

Paradigmatic principles of immediate logical ground lead to inconsistency. A wide range of ways of retaining these principles – potentially in some reformulated form – were surveyed here, but were found to be implausible. Some of the more promising ones are further developed in Fritz (2020, 2021, forthcoming a). They were seen to face their own difficulties, although room was left for the hope that developing them fuller would lead to a more satisfactory theory. Nevertheless, at the current stage of inquiry, no response to the inconsistency is more compelling than the conclusion that some of the problematic principles simply have to be given up.

Giving up some of these principles can take a number of forms. One option is to reject them wholesale, either by giving up the notion of *immediate* ground, or by giving up the idea of *logical* ground (the idea that grounding relations track the truth-functional behaviour of logical connectives). In the previous section, it was argued that both of these options come with substantial costs. In particular, if the idea of logical ground is given up, then the claim that grounding leads to distinctions between necessarily equivalent propositions loses much of its appeal. Two further options remain: One is to provide an account of the interaction of immediate grounding with logical connectives which is both non-trivial and avoids the present inconsistency results. Developing such an account poses a substantial challenge to the advocate of grounding. The final option is to reject altogether the recent proposal of introducing notions of metaphysical ground into philosophy.

Among all of these possible responses to the inconsistency results presented here, my own inclinations lie with the last one: It seems most plausible to me that the introduction of the new metaphysical vocabulary of ground simply failed. Paradigmatic principles of ground require a degree of fineness of grain in the individuation of truths which is logically inconsistent. There are therefore no relations satisfying the paradigmatic principles of ground, and so no relations which grounding theorists talk about when they use the ideology of ground.

If this is right, then we should not claim that this being red grounds something being red. Nevertheless, there are metaphysical dependence relations which relate such truths: this is only red if something is red, and necessarily so. As was shown here, such relations of metaphysical dependence cannot obtain between conjunctive truths and their conjuncts, disjunctive truths and their disjuncts, and universal truths and their instances along the lines grounding theorists imagine. But it is important to note that this negative conclusion concerns only metaphysical relations among truths. There may well be important asymmetric relations between conjunctive *sentences* and their conjuncts, disjunctive *sentences* and their disjuncts, and universal *sentences* and their instances along these lines. And such relations may well play an important explanatory role when we are concerned with an agent's attitude, for example in explaining how the agent came to know that something is red – by knowing that this is red. But these are then relations tracking features of our representation of reality, not of reality itself. On pain of inconsistency, reality is not so fine-grained as to admit relations of *metaphysical* ground.[1]

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