Coalitional Game Theory for Distributed Cooperation in Next Generation Wireless Networks

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Preface

This dissertation has been submitted to the Faculty of Mathematics and Natural Sciences at the University of Oslo in partial fulfillment of the requirements for the degree *Philosophiae Doctor* (Ph.D.). The studies were carried out over a period of three years, from August 2007 to June 2010. During the first and third year the work was mainly done at UNIK- University Graduate Center, Kjeller, Norway, while in the second year my workplace has been at the Coordinate Science Laboratory at the University of Illinois at Urbana-Champaign, USA. The research was funded by the Research Council of Norway through the project 183311/S10 entitled "Mobile-to-Mobile Communication Systems – M2M". My supervisors have been Prof. Are Hjørungnes, Prof. Zhu Han, Prof. Merouane Debbah, and Prof. Nils Henrik Risebro.

The symbol usage may vary from one paper to another as the papers included in this dissertation are *not* published/submitted/revised at the same time.

Dedication

This dissertation is dedicated to my late father Emile.

Acknowledgments

I am ever grateful to the almighty God for his blessings and for having made this work possible.

I would like to acknowledge many people and collaborators who helped me during the course of this work. First and foremost, I am heartily thankful to my advisor, Prof. Are Hjørungnes, whose guidance and support from the initial to the final stage ensured the success of this work. I sincerely thank him for giving me the opportunity to be part of his research group and for his persistent support. Further, I owe my deepest gratitude to Prof. Zhu Han for his tremendous efforts and his invaluable assistance throughout the work done in this thesis. His truly scientific intuition has made him a constant oasis of ideas and passions in engineering, which helped exceptionally inspire and enrich my growth as a student and a researcher. Without his guidance, broad technical expertise, and constant encouragement this dissertation would not have been possible. I gratefully acknowledge Prof. Merouane Debbah for his constant encouragement, his constructive suggestions, and helpful comments on my thesis work. Prof. Merouane Debbah is a great motivator and inspirer and I have immensely benefited from his energy, vision, and knowledge.

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Words fail me to express my appreciation to my wife Mary whose dedication, love, and persistent confidence in me, has taken the load off my shoulders. I owe her for being my source of strength without which this thesis would never have started much less finished.

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Walid Saad
Oslo, May 31, 2010

Abstract

Next generation wireless networks are bound to go beyond the classical point-to-point or point-to-multipoint communication paradigms of traditional networks such as cellular networks. For instance, next generation networks will witness a highly complex and dynamic environment whereby the nodes can interact and cooperate for improving their performance. In this context, cooperation has emerged as a novel communication paradigm that can yield tremendous performance gains from the physical layer all the way up to the application layer.

Consequently, a significant amount of research efforts has been dedicated to studying cooperation in wireless networks. The main research in this area has focused on examining the performance gains that cooperation can entail, in different scenarios. For example, on one hand, it was demonstrated that, by cooperating, a group of single-antenna nodes can exploit the highly acclaimed performance gains of multiple-input-multiple-output (MIMO) systems in terms of throughput and bit error rate. On the other hand, it has been shown that, by forwarding each others packets, the wireless nodes can increase their throughput and improve the connectivity of the network. However, most of the cooperative systems proposed so far are based on ideal cooperation, e.g., with no cost, and are mostly concerned with studying the benefits from cooperation, while giving little attention to the impact of cooperation on the network's structure and the users' behavior. Hence, there is a need for well-designed cooperative algorithms that can reap the numerous gains from cooperation while taking into account the costs for cooperation as well as its impact on the overall network structure and dynamics.

Designing such efficient cooperation algorithms faces numerous challenges. First, cooperation entails various costs, such as power, that can limit its benefits or even impair the users' performance. Second, wireless network users tend to be selfish in nature. Therefore, deriving a fair and practical cooperation algorithm where the decision to cooperate does not degrade the performance of any of the cooperating users is a challenging

task. Moreover, it is highly desirable that the nodes adopt distributed cooperative strategies with little or no reliance on centralized entities such as base stations. Thus, deriving distributed and fair cooperative strategies is highly challenging and desirable in practice.

In this regard, *coalitional game theory* serves as a highly suited mathematical tool for modeling cooperation in wireless networks. Most of the current research in the field is restricted to applying standard coalitional game models and techniques to study very limited aspects of ideal cooperation in networks. In this dissertation, first, we introduce a novel application-oriented classification of coalitional games by grouping the sparse literature into three distinct classes of games: Canonical coalitional games, coalition formation games, and coalitional graph games. By doing so, we provide a unified treatment of coalitional game theory tailored to the demands of communications and network engineers and which fills an important void in current wireless literature.

Further, we leverage the use of coalitional game theory to design novel cooperation models and distributed algorithms that take into account the tradeoff between the gains from cooperation and the costs for cooperation, for various next generation wireless networks. In particular, we devise coalitional game based models and algorithms for studying distributed cooperation in the following areas: (i)- Distributed formation of virtual MIMO systems, (ii)- Distributed collaborative spectrum sensing in cognitive radio networks, (iii)- Joint sensing and access in cognitive radio networks, (iv)- Distributed task allocation in multi-agent wireless systems, (v)- Distributed cooperation for physical layer security improvement, and (vi)- Network formation in wireless multi-hop networks.

In every area, we propose suited cooperation models that can capture the different benefit-cost tradeoffs that exist in these networks. Using novel concept from coalitional game theory, such as merge-and-split rules, hedonic games, network formation games, and coalition formation in partition form, we devise distributed cooperation algorithms that allow the nodes to self-organize into a stable network partition and adapt this structure to environmental changes such as periodic mobility. We show different properties of the resulting coalitional structures and we study their characteristics. Finally, using thorough simulations and numerical results, we investigate various aspects of the proposed algorithms and we analyze their performance.

List of Publications

This dissertation is based on the following seven papers, referred to in the text by letters (A-G).

A. W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, “Coalitional Game Theory for Communication Networks,” *IEEE Signal Processing Magazine, Special Issue on Game Theory for Signal Processing and Communication*, volume 26, issue 5, pages 77-97, September 2009.

B. W. Saad, Z. Han, M. Debbah, and A. Hjørungnes, "A Distributed Coalition Formation Framework for Fair User Cooperation in Wireless Networks," *IEEE Transactions on Wireless Communications*, volume 8, issue 9, pages 4580-4593, September 2009.

C. W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, “Coalitional Games for Distributed Collaborative Spectrum Sensing in Cognitive Radio Networks,” in Proceedings of *the IEEE International Conference on Computer Communications (INFOCOM)*, pages 2114 - 2122, Rio de Janeiro, Brazil, April 2009.

D. W. Saad, Z. Han, R. Zheng, A. Hjørungnes, and T. Başar, “Coalitional Games in Partition Form for Joint Spectrum Sensing and Access in Cognitive Radio Networks,” submitted to *the IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Advances on Cognitive Radio Networking and Communications*, December 2009.

E. W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, “Hedonic Coalition Formation for Distributed Task Allocation among Wireless Agents,” conditionally accepted for publication in *IEEE Transactions on Mobile Computing* (subject to reviewers’ and editor’s final approval of the revised manuscript submitted 03-02-2010).

F. W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Distributed Coalition Formation Games for Secure Wireless Transmission" submitted to *ACM/Springer Journal on Mobile Networks and Applications*, October 2009.

G. W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Network Formation Games among the Relay Stations in Next Generation Wireless Networks," submitted to *IEEE Transactions on Communications*, January 2010.

The list of other related publications during my PhD studies are given in Section 8 of Chapter I.

Contents

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Abbreviations

3GPP	Third generation partnership project
AP	Access point
AWGN	Additive white Gaussian noise
BER	Bit error rate
BPSK	Binary phase shift keying
BS	Base station
BW	Band-width
CDMA	Code division multiple access
CSS	Collaborative spectrum sensing
dB	Decibel
D&F/DF	Decode-and-forward
FDD	Frequency division duplexing
Hz	Hertz
i.i.d.	Independent and identically distributed
IEEE	Institute of Electrical and Electronics Engineers
LAN	Local area network
LTE	Long term evolution
MAC	Medium access control
MIMO	Multiple-input multiple-output
MRC	Maximum ratio combining
NTU	Non transferable utility
PU	Primary user
QoS	Quality of service
QOSTBC	Quasi-orthogonal space-time block code
RS	Relay station
SER	Symbol error rate
SISO	Single-input single-output
SNR	Signal-to-noise ratio
SINR	Signal-to-interference-plus-noise ratio
SR	Selective relaying

SU	Secondary user
TU	Transferable utility
TDMA	Time division multiple access
ZF	Zero forcing

Part I

Introduction

Introduction

In recent years, the nature of the mobile communications market witnessed a radical change whereby the need for new resource demanding wireless services has drastically increased. Meeting this demand is quite challenging since wireless networks are subject to several constraints such as complex and harsh fading channels, a scarce usable radio spectrum, limitations on the power and size of hand-held terminals, among others. For instance, classical cellular-like network architectures are unable to efficiently overcome these challenges and cope with the stringent quality of service requirements of emerging services. Thus, there is a need for advanced communication techniques for allowing next generation wireless networks to deliver resource demanding services to their subscribers.

In this context, cooperation among wireless devices has recently emerged as a novel communications paradigm that can yield tremendous performance gains and enable the efficient delivery of next generation wireless services. It has been shown that cooperation can yield an enhanced quality of service in terms of throughput improvement, bit error rate reduction, or energy savings, in many scenarios. Nonetheless, cooperation comes at a cost which can hinder these gains and renders the implementation aspects of cooperative networks quite challenging.

In this part of the dissertation, first, we motivate the concept of cooperation in wireless communication networks. Then, we introduce the analytical framework of coalitional game theory. In this regards, we provide an engineering-oriented description of coalitional game theory and we discuss its potential application in wireless networks. Further, we discuss the basics and challenges of implementing cooperation in the following areas: (i)- Cooperation for virtual MIMO formation, (ii)- Cooperation in cognitive radio networks, (iii)- Distributed task allocation in multi-agent wireless systems, (iv)- Cooperation for physical layer security improvement, and (vii)- Cooperation in next generation wireless multi-hop networks.

Following this, we discuss in details the main contributions of the included papers as well as the key suggestions for future work.

1 Cooperation in Wireless Networks

The wireless industry is recently experiencing an unprecedented growth in the number of services, subscribers, and applications that must be supported by next generation mobile communication systems. The growing diffusion of new services such as mobile television and multimedia communications emphasizes the need of advanced wireless networking paradigms that can fundamentally increase the system performance. In this context, cooperation between devices in wireless networks has been identified as one of the key technology enablers required to facilitate next generation wireless communication systems. Much of the research of the past decade used to assume a point-to-point communication between a terminal and a centralized access point or a point-to-multipoint communication between the central entity and the terminals. However, as per the famous saying "The real egoistic behavior is to cooperate!" (K. Edwin), for reasons of self-interest, the wireless users may engage in cooperative behavior, resulting in an improved overall network performance. Further, cooperation among the devices can coexist with a centralized infrastructure, e.g., in a cellular network, but is also of interest in ad hoc autonomous networks.

The simplest example of cooperation in wireless networks is the relay channel, first introduced in [1, 2]. Relaying implies that a given wireless helper-node can assist other nodes in transmitting their data to their destination. By doing so, it has been shown that the network and the nodes can witness performance improvements at different levels such as coverage extension, bit error rate improvement, increased throughput, among others [3]. Further, a number of single-antenna nodes can cooperate in order to exploit the advantages of multiple-input-multiple-output (MIMO) systems without the need for multiple antennas physically present on the devices [4–6]. Beyond the physical layer, cooperative schemes have also been studied at the upper layers such as the MAC and the network layer. For example, by cooperating at the MAC layer, the access points in a wireless LAN can achieve higher throughput and a reduced interference [7, 8]. Also, it has been shown that, by cooperating in packet forwarding, the nodes in an ad hoc wireless network can improve the connectivity of the network through adequate cooperation decisions [9].

Thus, cooperation has emerged as a novel networking paradigm that will yield significant performance advantages in next generation networks. In fact, due to the advantage of cooperation, numerous aspects of cooperative communication are making their way into wireless standards such as 3GPP's long term evolution advanced (LTE-Advanced) [10], or the forthcoming IEEE 802.16j WiMAX standard [11]. For studying cooperation in next generation wireless networks, numerous questions need to be answered such as "what are the gains and costs from cooperation?" or "who should cooperate with whom and why?". Thus, in order to efficiently answer these questions, one must be able to characterize and overcome numerous challenges such as (but not limited to): (i)- Modeling the benefit and cost tradeoffs that exist in cooperation, (ii)- Providing fair rules for cooperation, and (iii)- Designing distributed approaches for cooperation among others.

Therefore, there is a strong need for an analytical framework that can appropriately capture these challenges of cooperation and which can provide guidelines for deploying cooperative nodes in next generation networks. Hence, in the next section, we provide an introduction to coalitional game theory, which is a suited framework for modeling cooperative behavior.

2 Coalitional Game Theory

2.1 Motivation

Game theory is a formal analytical framework with a set of mathematical tools to study the complex interactions among independent rational players. Throughout the past decades, game theory has made a revolutionary impact on a wide number of disciplines ranging from economics, politics, philosophy, or even psychology [12]. The emergence of large-scale distributed wireless networks, as well as the recent interest in mobile flexible network where the nodes are autonomous decision makers has brought to surface many interesting game theoretic problems that arise from the competitive and cooperative interplay of the different wireless entities. Further, the need for self-organizing, self-configuring, and self-optimizing networks eventually led to the use of many game theoretic concepts in wireless communication networks [13, 14].

In a game theoretic framework, one can distinguish between two main categories: Non-cooperative [15, 16] and cooperative game theory [12, 17].

While non-cooperative game theory mainly deals with modeling competitive behavior, cooperative game theory is dedicated to the study of cooperation among a number of players. Cooperative game theory mainly includes two branches: Nash bargaining and coalitional game theory. In this dissertation, we restrict our attention to the latter, although the former can also be quite useful in different scenarios (the interested reader is referred to [12] for details on Nash bargaining solutions).

Coalitional game theory mainly deals with the formation of cooperative groups, i.e., coalitions, that allow the cooperating player to strengthen their positions in a given game. In consequence, the recent interest in the cooperative paradigm in wireless networks, implied that the adoption of coalitional games-based approaches is quite natural. In this context, coalitional games prove to be a very powerful tool for designing fair, robust, practical, and efficient cooperation strategies in communication networks. However, most of the research in the wireless community has been focusing on either non-cooperative games [13, 18–20] or on applying standard coalitional game theory models and techniques to study very limited aspects of cooperation in networks such as stability under ideal cooperation or fairness. This is mainly due to the sparse literature that tackles coalitional games as most pioneering game theoretical references such as [12, 15–17] focus on non-cooperative games.

For this purpose, it is of strong interest to: (i)- Provide an engineering-oriented introduction to coalitional game theory that can unify the various aspects of coalitional games and (ii)- Propose advanced applications for coalitional games in wireless networks. This dissertation will tackle both points thoroughly as will be seen in the next sections. First, in the following subsection, we describe the main concepts of coalitional games, and we discuss an engineering-oriented classification based on the work done in Paper A.

2.2 Basic Concepts

The main two components of a coalitional game are the players set, normally denoted by $\mathcal{N} = \{1, \ldots, N\}$, and the coalition *value*. The set $\mathcal{N}$ represents the players that interact in order to form cooperative groups, i.e., coalitions, in order to improve their position in the game. Thus, a coalition $S \subseteq \mathcal{N}$ represents an agreement between the members of S to act as a single entity in a given game. Forming coalitions or alliances is pervasive in many disciplines such as politics, economics, and, more recently, wireless networks. Further, the coalition value, usually denoted as v, quantifies the worth or *utility* of a coalition in a game. Depending on the definition of the value, a coalitional game can have different properties. Nonetheless, independent of the definition of the value, any coalitional game can be uniquely defined by the pair $(\mathcal{N}, v)$ [1].

In general, a coalition value can be in three different forms:

- Characteristic form.
- Partition form.
- Graph form.

The characteristic form is the most common in game theory literature, and it was first introduced by Von Neuman and Morgenstern [21]. In *characteristic form*, the worth or utility of any coalition $S \subseteq \mathcal{N}$ is *independent* of the coalitions/structure formed among the players outside S, i.e., the players in $\mathcal{N} \setminus S$. Thus, a value of a coalition S in characteristic form depends *solely* on the members of that coalition. In contrast, a game is in *partition form* if, for any coalition $S \subseteq \mathcal{N}$, the coalitional value depends both on the members of S as well as on the coalitions formed by the members in in $\mathcal{N} \setminus S$. The concept of the partition form was introduced by Thrall and Lucas [22] with the characteristic form as a particular case. Further, in many coalitional games, the connection between the players in the coalition, i.e., "who is connected to whom", strongly impacts the value of the game. In such coalitional games, the interconnection between the players is usually captured through a graph structure. Subsequently, for modeling such coalitional games, the value is considered in *graph form*, i.e., for each graph structure, a different utility can be yielded. The idea of capturing the interconnection graph within coalitions started with the pioneering work of Myerson [23]. In [23], Myerson started with a game in characteristic form, and layered on top of that a network structure, represented by a graph that indicated which players can communicate. Consequently, the value function became dependent on the communication graph which led to the idea of a game in graph form (this work was further generalized in [24]).

In any coalitional game (independent of its form), it is always important to distinguish between two entities: the value of a *coalition* and the payoff received by a *player*. The value of a coalition represents the amount of

[1]Since, for a given players' set $\mathcal{N}$, the value v completely describe the coalitional game, some references refer to the value in a coalitional game as *the game*.

utility that a coalition, as a whole, can obtain. In contrast, the payoff of a player, represents the amount of utility that a player, member of a certain coalition, will obtain. For instance, depending on how the value is mapped into payoffs, the coalitional game can be either with *transferable utility (TU)* or with *non-transferable utility (NTU)*. A TU game implies that the total utility received by any coalition $S \subseteq \mathcal{N}$ can be apportioned in any manner between the members of S. A prominent example of TU type games is when the value represents an amount of money, which can be distributed in any way between coalition members. In particular, when considering a TU game in characteristic form, the value is a function[2] over the real line defined as $v : 2^{\mathcal{N}} \to \mathbb{R}$. In such a setting, for every coalition, the value function associates a real number which represents the overall utility or worth of this coalition. Further, due to the TU property, this real number can be divided in any manner (e.g., using some fairness rule), in order to obtain each player's payoff from the value received by any coalition S. The amount of utility that a player $i \in S$ receives from the division of $v(S)$ constitutes the user's *payoff* and is denoted by x_i hereafter. The vector $\boldsymbol{x} \in \mathbb{R}^S$ with each element x_i being the payoff of user $i \in S$ constitutes a *payoff allocation*.

Although TU models are quite popular and useful, many scenarios exist where the coalition value cannot be assigned a single real number or rigid restrictions exist on the division of the utility. These games are known as coalitional games with non-transferable utility (NTU) and were first derived using non-cooperative strategic games as a basis [12]. The idea is that, in a NTU game, the payoff that each user in a coalition S receives is dependent on the joint actions that the players of coalition S select[3]. Hence, in a NTU game, the value of a coalition S is no longer a function over the real line but a set of payoff vectors. For example, in an NTU game in characteristic form, the value of a coalition S would be given by the set $v(S) \subseteq \mathbb{R}^S$, where each element x_i of a vector $\boldsymbol{x} \in v(S)$ represents a payoff that user $i \in S$ can obtain when acting within coalition S given a certain strategy. Moreover, given this definition, a TU game can be seen as a particular case of the NTU framework [12].

In general, the most well studied aspect of coalitional game theory is that pertaining to games in *characteristic form with TU or NTU* which are widely spread in game theory literature. Different properties and solution concepts can be defined for these games, as will be also seen in the re-

[2]In these games, the value is commonly known as the *characteristic function*.

[3]The action space depends on the underlying non-cooperative game.

mainder of this section. In particular, given a game in characteristic form with TU, we define the following property:

Definition 1 *A coalitional game* $(\mathcal{N}, v)$*, in characteristic form with transferable utility, is said to be superadditive if for any two disjoint coalitions* $S_1, S_2 \subset \mathcal{N},\ S_1 \cup S_2 = \emptyset$*,* $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$*.*

Superadditivity implies that, given any two disjoint coalitions S_1 and S_2, if coalition $S_1 \cup S_2$ forms, then it can give its members any allocations they can achieve when acting in S_1 and S_2 separately. In other words, a game is superadditive, if cooperation, i.e., the formation of a large coalition out of disjoint coalitions, guarantees at least the value that is obtained by the disjoint coalitions separately. The rationale behind the superadditivity property is that within a coalition, the players can always revert back to their non-cooperative behavior and, thus, achieving their non-cooperative payoffs. Consequently, in a superadditive game, cooperation is always beneficial. Note that, an analogous definition of a superadditive game also exists in the NTU framework [12].

For superadditive games, it is to the joint benefit of the players to always form the *grand coalition* $\mathcal{N}$, i.e, the coalitions of all the users in $\mathcal{N}$, since the payoff received from $v(\mathcal{N})$ is at least as large as the amount received by the players in any disjoint set of coalitions they could form. As a result, determining whether a game is superadditive or not strongly impacts the approach that must be used to solve the game.

Having laid out the basic concepts of coalitional game theory, in the next subsection, using these properties and concepts, we provide a novel engineering-oriented classification of coalitional game theory.

2.3 A Novel Classification of Coalitional Game Theory

For better identifying the potential wireless and communications applications of coalitional game theory, we propose a novel classification, as per Paper A, which allows to group various types of games under one class based on several game properties. We group coalitional games into three distinct classes

1. **Class I:** Canonical coalitional games.
2. **Class II:** Coalition formation games.
3. **Class III:** Coalitional graph games.

The key features of these classes are summarized in Fig. 1.1.

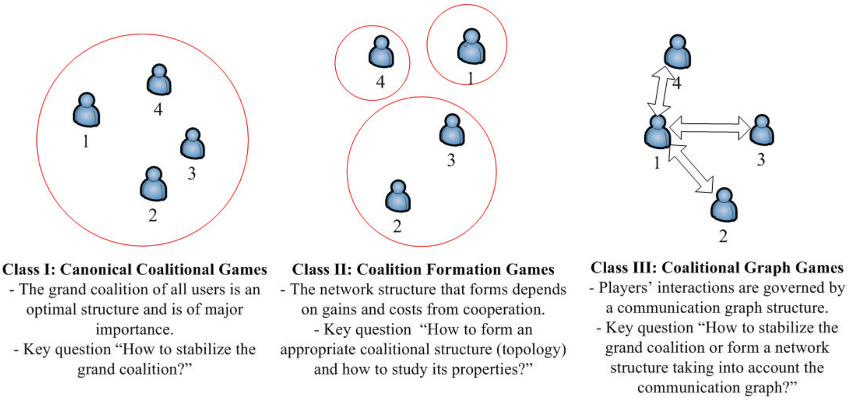

Figure 1.1: An illustration of the proposed classification of coalitional games.

2.3.1 Canonical Coalitional Games

Canonical coalitional games refer to the most classical and well studied type of coalitional game theory. In this regards, the canonical coalitional game class can be used to model problems where:

1. The value is in characteristic form (or can be mapped to the characteristic form through some assumptions).

2. The game is superadditive. As a result, cooperation is always beneficial, i.e., including more players in a coalition does not decrease its value.

3. There is a need to study how payoffs can be allocated in a fair manner that stabilizes the grand coalition, i.e., coalition of all players.

Consequently, due to these properties, in canonical games[4], the main emphasis is on studying the properties of the grand coalition such as stability and fairness. One core objective is to propose a payoff allocation which guarantees that no group of players have an incentive to leave the grand coalition. Hence, the stability of the grand coalition under a certain *payoff division* is a critical objective of canonical games. In addition, assessing

[4]In this dissertation, we use the term canonical games to refer to canonical coalitional games.

the gains that the grand coalition can achieve as well as the fairness criteria that must be used for distributing these gains is another key objective for this class of games.

For solving canonical coalitional games, game theory literature presents a broad range of concepts [12, 17]. Most of these solutions fully exploit the properties of canonical games as well as the key objectives that we identified. The main solution concepts used for solving a canonical game are: The core, the Shapley value, and the nucleolus.

The Core

The core is the most used solution concept for coalitional games in general and for canonical games in particular. In fact, most existing game theoretical literature considers the core as the most important concept for solving a canonical coalitional game.

In essence, the core of a canonical coalitional game is simply the set of payoff allocations which guarantees that no group of players has an incentive to leave the grand coalition $\mathcal{N}$ in order to form another coalition $S \subset \mathcal{N}$. The idea is, thus, to provide a payoff allocation that stabilize the grand coalition (recall that, in a canonical game, the grand coalition generates the most utility due to superadditivity). This main concept of the core applies to both TU and NTU games, although the mathematical definition of the core slightly differs between the two types.

For a TU game, prior to mathematically specifying the core, we first define the following concepts:

Definition 2 *Consider a coalitional game in characteristic form with TU defined by the pair $(\mathcal{N}, v)$. Given the grand coalition $\mathcal{N}$, a payoff vector $\boldsymbol{x} \in \mathbb{R}^N$ ($N = |\mathcal{N}|$) for dividing $v(\mathcal{N})$ is said to be* group rational *if $\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N})$, where x_i is the ith component of $\boldsymbol{x}$. A payoff vector $\boldsymbol{x}$ is said to be* individually rational *if every player can obtain a benefit no less than acting alone, i.e. $x_i \geq v(\{i\}), \forall\, i$. An* imputation *is a payoff vector satisfying the above two conditions.*

Consequently, *the core* of a coalitional game $(\mathcal{N}, v)$ in characteristic form with TU is defined as

$$\mathcal{C}_{\text{TU}} = \left\{ \boldsymbol{x} : \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) \text{ and } \sum_{i \in S} x_i \geq v(S)\ \forall\, S \subseteq \mathcal{N} \right\}. \tag{1.1}$$

In other words, the core is the set of imputations where no coalition $S \subseteq \mathcal{N}$ has an incentive to reject the proposed payoff allocation, deviate from the grand coalition, and form coalition S instead. The core guarantees that these deviations do not occur through the fact that any payoff allocation $\boldsymbol{x}$ that is in the core guarantees at least an amount of utility equal to $v(S)$ for every $S \subset \mathcal{N}$. Note that, for an NTU game, an analogous definition of the core can be used (see [12] or Paper A).

As a result, whenever one is able to find a payoff allocation that lies in the core, then the grand coalition is a stable and optimal solution for the coalitional game. This highlights clearly the attractiveness of the core as a solution concept. However, the core of a coalitional game suffers from several drawbacks. On one hand, in many games, the core is empty, and, thus, there does not exist an allocation that can stabilize the grand coalition. On the other hand, even when it exists, the core might be a huge set and selecting a fair allocation out of this set is a big challenge. Nonetheless, there exists numerous classes of canonical coalitional games where the core is non-empty and where interesting properties can be derived. Moreover, for a given game, several methods can be used to determine the existence of the core as well as the allocations that lie in the core. A more detailed exposition of these properties and methods is found in Paper A.

The Shapley Value

Due to the various drawbacks of the core previously mentioned, Shapley provided an axiomatic approach for associating with every coalitional game $(\mathcal{N}, v)$ a *unique* payoff vector known as the *value* of the game (which is quite different from the value of a coalition). Shapley showed [12] that there exists a unique mapping, *the Shapley value* $\phi(v)$, from the space of all coalitional games to $\mathbb{R}^{\mathcal{N}}$, that satisfies the following four axioms[5] (ϕ_i is the payoff given to player i by the Shapley value ϕ):

1. *Efficiency Axiom*: $\sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N})$.
2. *Symmetry Axiom*: If player i and player j are such that $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition S not containing player i and player j, then $\phi_i(v) = \phi_j(v)$.
3. *Dummy Axiom*: If player i is such that $v(S) = v(S \cup \{i\})$ for every coalition S not containing i, then $\phi_i(v) = 0$.

[5] In some references, the Shapley axioms are compressed into three by combining the dummy and efficiency axioms.

4. *Additivity Axiom*: If u and v are characteristic functions, then $\phi(u+v) = \phi(v+u) = \phi(u) + \phi(v)$.

Thus, for every game $(\mathcal{N}, v)$, the Shapley value ϕ assigns a unique payoff allocation in $\mathbb{R}^{\mathcal{N}}$ which satisfies the four axioms. The efficiency axiom is in fact group rationality. The symmetry axiom implies that, when two players have the same contribution in a coalition, their assigned payoffs must be equal. The dummy axiom assigns no payoff to players that do not improve the value of any coalition. Finally, the additivity axiom links the value of different games u and v, and asserts that ϕ is a unique mapping over the space of all coalitional games.

The Shapley value also has an alternative interpretation which takes into account the order in which the players join the grand coalition $\mathcal{N}$. In the event where the players join the grand coalition in a *random* order, the payoff allotted by the Shapley value to a player $i \in \mathcal{N}$ is the expected marginal contribution of player i when it joins the grand coalition. The basis of this interpretation is that, given any canonical TU game $(\mathcal{N}, v)$, for every player $i \in \mathcal{N}$ the Shapley value $\phi(v)$ assigns the payoff $\phi_i(v)$ given by

$$\phi_i(v) = \sum_{S \subseteq \mathcal{N} \setminus \{i\}} \frac{|S|!(N - |S| - 1)!}{N!} [v(S \cup \{i\}) - v(S)]. \tag{1.2}$$

In (1.2), it is clearly seen that the marginal contribution of every player i in a coalition S is $v(S \cup \{i\}) - v(S)$. The weight that is used in front of $v(S \cup \{i\}) - v(S)$ is the probability that player i faces the coalition S when entering in a random order, i.e., the players in front of i are the ones already in S. In this context, there are $|S|!$ ways of positioning the players of S at the start of an ordering, and $(N - |S| - 1)!$ ways of positioning the remaining players except i at the end of an ordering. The probability that such an ordering occurs (when all orderings are equally likely) is therefore $\frac{|S|!(N-|S|-1)!}{N!}$, consequently, the resulting payoff $\phi_i(v)$ is the expected marginal contribution, under random-order joining of the players for forming the grand coalition.

The computation of the Shapley value is generally done using (1.2); however, in games with a large number of players the computational complexity of the Shapley value grows significantly. For computing the Shapley value in reasonable time, several analytical techniques have been proposed such as multi-linear extensions [17], and sampling methods for simple games [25], among others. The Shapley value was essentially defined for TU games; however, extensions to NTU games exist as seen in [12].

The Nucleolus

In addition to the core and the Shapley value, an interesting solution concept for canonical games is *the nucleolus*[6] [17]. For a given $(\mathcal{N}, v)$ canonical coalitional game, the nucleolus provides an allocation that minimizes the dissatisfaction of the players from a given allocation. For a coalition S, the measure of dissatisfaction from an allocation $\boldsymbol{x} \in \mathbb{R}^{\mathcal{N}}$ is defined as the *excess* $e(\boldsymbol{x}, S) = v(S) - \sum_{j \in S} x_j$. Clearly, an allocation $\boldsymbol{x}$ which can ensure that all excesses (or dissatisfactions) are minimized is of particular interest as a solution[7] and hence, constitutes the main motivation behind the concept of the nucleolus. Let $\boldsymbol{O}(\boldsymbol{x})$ be the vector of all excesses in a canonical game $(\mathcal{N}, v)$ arranged in non-increasing order (except the excess of the grand coalition $\mathcal{N}$). A vector $\boldsymbol{y} = (y_1, \dots, y_k)$ is said to be lexographically less than a vector $\boldsymbol{z} = (z_1, \dots, z_k)$ (denoted by $\mathbf{y} \prec_{\text{lex}} \mathbf{z}$) if $\exists l \in \{1, \dots, k\}$ where $y_1 = z_1, y_2 = z_2, \dots, y_{l-1} = z_{l-1}, y_l < z_l$. An *imputation* $\boldsymbol{x}$ is a *nucleolus* if for every other imputation $\boldsymbol{\delta}$, $\boldsymbol{O}(\boldsymbol{x}) \prec_{\text{lex}} \boldsymbol{O}(\boldsymbol{\delta})$. Hence, the nucleolus is the imputation $\boldsymbol{x}$ which *minimizes the excesses* in a non-increasing order. The nucleolus of a canonical coalitional game exists and is unique. The nucleolus is group and individually rational (since it is an imputation), and satisfies the symmetry and dummy axioms of Shapley. If the core is not empty, the nucleolus is in the core. Moreover, the nucleolus lies in the *kernel* of the game, which is the set of all allocations $\boldsymbol{x}$ such that

$$\max_{S \subseteq \mathcal{N} \setminus \{j\}, i \in S} e(\boldsymbol{x}, S) = \max_{G \subseteq \mathcal{N} \setminus \{i\}, j \in G} e(\boldsymbol{x}, G). \tag{1.3}$$

The kernel states that if players i and j are in the same coalition, then the highest excess that i can make in a coalition without j is equal to the highest excess that j can make in a coalition without i. As the nucleolus lies in the kernel, it also verifies this property. Thus, the nucleolus is the best allocation under a min-max criterion. The process for computing the nucleolus is more complex than the Shapley value, and is described as follows:

First, we start by finding the imputations that distribute the worth of the grand coalition in such a way that the maximum excess (dissatisfaction) is minimized. In the event where this minimization has a unique solution, this solution is the nucleolus. Otherwise, we search for the im-

[6]The nucleolus is only applicable to games in characteristic form with TU [17].

[7]In particular, an imputation x lies in the core of (N, v), if and only if all its excesses are negative or zero.

putations which minimize the second largest excess. The procedure is repeated for all subsequent excesses, until finding a unique solution which would be the nucleolus. These sequential minimizations are solved using linear programming techniques such as the simplex method [26].

Thus, the nucleolus is an interesting concept, since it combines a number of fairness criteria with stability. However, the communications applications that utilized the nucleolus are still few, with one example being [9], where it was used for allocating the utilities in the modeled game. The main drawback of the nucleolus is its computational complexity in some games. However, with appropriate models, the nucleolus can be an optimal and fair solution to many applications.

Canonical Coalitional Games in Wireless Networks

By closely inspecting the characteristics of canonical coalitional games, one can see that the main interest of these games is focused on the ideas of stability and fairness. In wireless networks, the main usage model of canonical coalitional games is to study the limits of cooperation, as well as the possibility of maintaining cooperation, in a setting where no cost for cooperation exists. In fact, when dealing with a wireless problem where cooperation is ideal and the grand coalition forms, solution concepts such as the core or the nucleolus can be of utter importance in order to assess the stability of this grand coalition as well as whether any fair solution can be obtained for maintaining cooperation among the users.

Numerous examples applying canonical coalitional games already exist in the literature. For instance, the work in [27, 28] focused on devising a cooperative model for rate improvement through ideal *receivers* cooperation. Using canonical games and the concept of the core, the authors in [27] showed that, for the receiver coalition game in a Gaussian interference channel, a stable grand coalition of all users can be formed if no cost for cooperation is accounted for. In addition, using canonical games, the fair allocation of rate for cooperating users in an interference channel was studied in [29] for the transmitters. Under some assumptions on the users' behavior, the authors showed that a unique rate allocation exists verifying certain well defined fairness axioms from canonical coalitional games. At the network layer, the authors in [9] present a canonical coalitional game model for solving an inherent problem in packet forwarding networks known as *the curse of the boundary nodes*. In this work, the nodes at the boundary of the network would cooperate with those in the backbone in order to forward their packets, and they reward these nodes by a reduced power consumption. Using the concepts of the core, the Shapley value and the nucleolus, it was shown that a stable grand coalition can exist.

In a nutshell, canonical coalitional game are a solid tool for studying cooperation, fairness, and stability in a variety of application. In particular, applying these concepts in wireless networks can provide useful and interesting insights on cooperative protocols.

2.3.2 Coalition Formation Games

In many cooperative scenarios, superadditivity can be quite a restrictive concepts. For instance, it is quite natural to consider that any cooperation is accompanied with an inherent cost that can limit the benefits of this cooperation. In consequence, the formation of a grand coalition is seldom guaranteed. In such cases, canonical coalitional games are not suited for modeling the cooperative behavior of the players. In this regards, coalition formation games encompass coalitional games where, unlike the canonical class, *network structure* and *cost* for cooperation play a major role. The characteristics of a coalition formation game can be summarized as follows:

1. Forming a coalition brings gains to its members, but the gains are limited by a *cost* for forming the coalition. Thus, the game is non-superadditive and the formation of a grand coalition is not guaranteed.

2. The game can be in characteristic or partition form.

3. The objective is to study the *network coalitional structure*, i.e., answering questions like which coalitions will form, what is the optimal coalition size and how can we assess the structure's characteristics, and so on.

In many problems, forming a coalition requires a negotiation process or an information exchange process which can incur a cost, thus, reducing the gains from forming the coalition. Therefore, in such scenarios, coalition formation games prove to be quite a solid tool. In contrast to canonical games, where formal rules and analytical concepts exist, solving a coalition formation game, is more difficult, and application-specific. In any coalition formation game, the following definition is useful:

Definition 3 *Given a coalition formation game among a set of players $\mathcal{N}$, a* collection *of coalitions, denoted by $\mathcal{S}$, is defined as the set $\mathcal{S} = \{S_1, \ldots, S_l\}$ of mutually disjoint non-empty coalitions $S_i \subset \mathcal{N}$. In other words, a collection is any arbitrary group of disjoint coalitions S_i of $\mathcal{N}$ not necessarily spanning all players of $\mathcal{N}$. If the collection spans* all *the players of $\mathcal{N}$; that is $\bigcup_{j=1}^{l} S_j = \mathcal{N}$, the collection is a* partition *of $\mathcal{N}$ or a* coalitional structure.

In the presence of a coalitional structure, the solution concepts discussed in the previous subsection need substantial changes in their definition for applying them in a coalition formation setting. Even by changing the definition, finding these solutions is by no means straightforward and can be cumbersome. In [30], it was shown that, in the presence of a coalitional structure, the core and the nucleolus, as defined in canonical coalitional games, are inapplicable and an alternative definition is provided instead. In contrast, by a slight modification of its definition, the Shapley value can be found by computing the Shapley value over each coalition present in the partition [30]. Hence, finding optimal coalitional structure and characterizing their formation is quite a challenging process, and, unlike canonical coalitional games, no unified or formal solution concepts exist. In fact, a majority of the literature dealing with coalition formation games, such as [31–34] or others, usually re-defines the solution concepts or presents alternatives that are specific to the game being studied.

For coalition formation games, the most important aspect is the formation of the coalitions, i.e., answering the question of "how to form a coalitional structure that is suitable to the studied game". In practice, coalition formation entails finding a coalitional structure which either maximizes the total utility (social welfare) if the game is TU, or finding a structure with Pareto optimal payoff distribution for the players if the game is NTU. For achieving such a goal, a *centralized* approach can be used; however, such an approach is generally NP-complete [31–34]. The reason is that, finding an optimal partition in a general case, requires iterating over all the partitions of the player set $\mathcal{N}$. The number of partitions of a set $\mathcal{N}$ grows exponentially with the number of players in $\mathcal{N}$ and is given by a value known as the Bell number [31]. For example, for a game where $\mathcal{N}$ has only 10 elements, the number of partitions that a centralized approach must go through is 115975 (computed through the Bell number). Hence, finding an optimal partition from a centralized approach is, in general, computationally complex and impractical. In some cases, it may be possible to explore the properties of the game, notably of the value v, for reducing the centralized complexity. Nonetheless, in many practical applications, it is desirable that the coalition formation process takes place in a distributed manner, whereby the players have an autonomy on the decision as to whether or not they join a coalition. In fact, the complexity of the centralized approach as well as the need for distributed solutions have sparked a huge growth in the coalition formation literature that aims to find low complexity and distributed algorithms for forming coalitions [31–34].

The approaches used for distributed coalition formation are quite varied and range from heuristic approaches [31], Markov chain-based methods [32], to set theory based methods [33] as well as approaches that use bargaining theory or other negotiation techniques from economics [34]. Clearly, constructing coalition formation algorithms is application-specific, however, some work, such as [33] provides generic rules that can be used to derive coalition formation algorithms in different scenarios. In this regards, the work in [33] does not provide an algorithm for coalition formation, but it presents a framework that can be tailored for developing such an algorithm. The main ingredients presented in [33] that are presented in [33] are the following:

1. Well-defined orders suitable to compare *collections* of coalitions.

2. Two simple operations for forming or breaking coalitions.

3. Stability notions that can be suited in a coalition formation context.

By using the guidelines in [33], one can devise different coalition formation algorithms. Moreover, many of the algorithms in the literature can also be tailored to new applications through adequate modifications.

Further, we note that coalition formation approaches can be either fully reversible, partially reversible or irreversible [32]. An irreversible coalition formation approach implies that, whenever a coalition forms, its members are not allowed to leave it. In contrast, in a fully reversible approach, the players can join and leave coalitions with no restrictions. On one hand, a fully reversible approach is practical and flexible, however, deriving such an approach can be complicated. On the other hand, although irreversible approaches are easy to construct, their practicality is limited as the players are bound to remain in a coalition they join with no possibility of breaking the agreement. For this purpose, partially reversible approaches have been recently sought as they provide a balance between practicality and complexity. In partially reversible coalition formation approaches, once the players form a coalition, they can break that coalition under certain conditions. Under different applications, one can carefully select the most practical and suited approach.

In summary, coalition formation games are diverse, and, in addition to the previously mentioned approaches, numerous schemes and rules exist. For example, a type of coalition formation games, known as *hedonic coalition formation* games has been widely studied in game theory. Hedonic games are interesting since they allow the formation of coalitions (whether dynamic or static) based on the individual preferences of the players. Further, hedonic games admit different stability concepts that are extensions to well known concepts such as the core or the Nash equilibrium used in a coalition formation setting [35]. In this regard, hedonic games constitute a very useful analytical framework which has a very strong potential to be adopted in modeling problems in wireless and communication networks (only few contributions such as [36] used this framework in a communication/wireless model). Further, a multitude of algorithms and concepts pertaining to coalition formation games can be found in [31–34] and many others.

Coalition Formation Games in Wireless Networks

While canonical coalitional games have had several applications in wireless networks, surprisingly, coalition formation games applications are still scarce (e.g., in [37, 38]). This is mainly due to the fact that, unlike canonical coalitional games, no unified reference or formal rules exist for solving coalition formation games. In addition, most existing tutorial or references on coalitional game theory mainly focus on canonical coalitional games, with little mention of coalition formation.

However, one can see that coalition formation games have a huge potential of applications within wireless networks. For instance, in a wireless or communications environment, cooperation always entails costs such as energy, power, time, or others. In most wireless problems, cooperating, i.e., forming a coalition, is preceded by a negotiation process or an information exchange process which incurs costs that can significantly reduce the gains from forming a coalition. Hence, in these scenarios, canonical coalitional games are inapplicable and one must revert to formulating a coalition formation approach. In addition, next generation wireless networks are large-scale, heterogeneous, and characterized by a dynamically varying environment. In such a setting, it is restrictive to assume that a grand coalition would form and it is imperative to study how the network structure would be affected by the presence of cooperative nodes. Further, with the recent interest in cooperation as well as the need for next generation wireless users to learn and adapt to their environment (changes in topology, technologies, service demands, application context, etc), coalition formation game models are bound to be ubiquitous in future wireless communication networks. In brief, any problem involving the study of cooperative wireless nodes behavior when a cost is present, is a candidate for modeling using coalition formation games. Thus, the potential applications of coalition formation games in wireless networks are numerous and diverse.

In Section 8 of this dissertation, we provide numerous coalition formation models, algorithms and applications suited for wireless and communication networks.

2.3.3 Coalitional Graph Games

In canonical and coalition formation games, the utility or value of a coalition does *not* depend on how the players are interconnected within the coalition. In this sense, every coalition is simply seen as a subset of the players set $\mathcal{N}$. However, in many scenarios, the communication or interconnection between the players of the game have a major impact on the outcome and the characteristics of this game [23, 39]. This interconnection can be captured by a graph representing the connectivity of the players among each other, i.e., which player communicates with which ones inside each and every coalition. In general, the main properties that distinguish a coalitional graph game are as follows:

1. The coalitional game is in graph form, and can be TU or NTU. However, the value of a coalition may depend on the external network structure.

2. The *interconnection* between the players within each coalition, i.e., who is connected to whom, strongly impacts the characteristics and outcome of the game.

3. The main objectives are to derive low complexity distributed algorithms for players that wish to build a network *graph* (directed or undirected) as well as to study the properties (stability, efficiency, etc) of the formed network graph.

4. In games where there is a *hierarchy* that governs the interactions among the players, coalitional graph games are a suitable tool.

Typically, in a coalitional graph game, there are two objectives. The first and most important objective, is to provide low complexity algorithms for building a network graph to connect the players. A second objective is to study the properties and stability of the formed network graph. In some scenarios, the network graph is given, and hence analyzing its stability and efficiency is the only goal of the game.

The idea of coalitional graph games mainly started with the work done by Myerson [23], through the graph function form for TU games. In this work, starting with a TU canonical coalitional game $(\mathcal{N}, v)$ and given an undirected graph G that interconnects the players in the game, Myerson defined a fair solution, later known as the *Myerson value*. Based on this work, the family of coalitional graph games evolved significantly with many different approaches. One prominent branch of these games is the so called *network formation games*.

In a network formation game, the main objective in these games is to study the interactions among a group of players that wish to form a graph. Network formation games can be thought of as a hybrid between coalitional graph games and non-cooperative games. The reason is that, for forming the network, non-cooperative game theory plays an important role. In any network formation game two objectives are of interest: (i)- Forming a network graph and (ii)- Studying the properties and stability of this graph, through concepts analogous to those used in canonical coalitional games. For forming the graph, a broad range of approaches exist, and are grouped into two types: *Myopic* and *far sighted* [8]. The main difference between these two types is that, in myopic approaches, the players play their strategies given the current state of the network, while in far sighted algorithms, the players adapt their strategy by learning, and predicting future strategies of the other players. For both approaches, well-known concepts from non-cooperative game theory can be used. The most popular of such approaches is to consider the network formation as a non-zero sum non-cooperative game, where the players' strategies are to select one or more links to form or break. One approach to solve the game is to play *myopic best response dynamics* whereby each player selects the strategy, i.e., the link(s) to form or break, that maximizes its utility. Under certain conditions on the utilities, the best response dynamics converge to a Nash equilibrium, which constitutes a Nash network. These approaches are widespread in network formation games [41–43], and also,

[8] These approaches are sometimes referred to as dynamics of network formation (see [40]).

several refinements to the Nash equilibrium suitable for network formation are used [41–43]. The main drawback of aiming for a Nash network is that, in many network formation games, the Nash networks are trivial graphs such as the empty graph or can be inefficient. For these reasons, a new type of network formation games has been developed, which utilizes new concepts for stability such as *pairwise stability* and *coalitional stability* [40]. The basic idea is to present stability notions that depend on deviations by a group of players instead of the unilateral deviations allowed by the Nash equilibrium. Independent of the stability concept, a key design issue in network formation games is the tradeoff between stability and efficiency. It is desirable to devise algorithms for forming stable networks that can also be efficient in terms of payoff distribution or total social welfare. Several approaches for devising such algorithms exist, notably using stochastic processes, graph theoretical techniques or non-cooperative games. A comprehensive survey of such algorithms can be found in [40].

Beyond network formation games, other approaches, which are closely tied to canonical games can be proposed for solving a coalitional graph game. For example, the work in [39], proposes to formulate a canonical game-like model for an NTU game, whereby the graph structure is taken into account. In this work, the authors propose an extension to the core called the *balanced core* which takes into account the graph structure. Further, under certain conditions, analogous to the balanced conditions of canonical games, the authors in [39] show that this balanced core is non-empty. Hence, coalitional graph games constitute a rich and diverse framework which can have interesting application scenarios.

Coalitional Graph Games in Wireless Networks

The presence of a network graph is prevalent in wireless applications. For example, next generation networks will be characterized by multi-hop communication which imposes a tree architecture on the network. Moreover, in any routing application, different graph structures can inter-connect the wireless nodes. For designing, understanding, and analyzing such graphs, coalitional graph games are the accurate tool. Through the various concepts pertaining to network formation, stability, fairness, or others, one can model a diversity of problems.

In existing work, network formation games have been applied to study many aspects of routing in wireless and communication networks. In [44], a stochastic approach for network formation is provided. In the proposed model, a network of nodes that are interested in forming a graph for routing traffic among themselves is considered with each node aiming at minimizing its cost function which reflects the various costs that routing traffic can incur (routing cost, link maintenance cost, disconnection cost, etc.). Using a myopic best respone algorithms, the authors in [44] show that their proposed algorithm converges to a stable and Pareto efficient network. The usage of network formation games in routing applications is not solely restricted to forming the network, but also for studying properties of an existing network. For instance, in [45], the authors study the stability and the flow of the traffic in a given directed graph. For determining the network flows, the work [45] uses non-cooperative game theory while taking into account the stability of the network graph.

The applications of coalitional graph games are by no means limited to routing problems. The main future potential of using this class of games lies in problems beyond network routing. For instance, coalitional graph games are suitable tools to analyze several problems in next generation networks pertaining to information trust management in wireless networks, multi-hop cognitive radio, relay selection in cooperative communications, intrusion detection, peer-to-peer data transfer, multi-hop relaying, packet forwarding in sensor networks, and many others. Certainly, this rich framework is bound to be used thoroughly in the design of many aspects of future communication networks with a broad range of applications.

In Section 8 of this introduction, we discuss how network formation games can be used for studying the multi-hop network architecture in next generation wireless networks.

2.4 Summary

Coalitional game theory presents a rich framework that can be used to model various aspects of cooperative behavior in next generation wireless networks. On one hand, for ideal cooperation, one can utilize the various solution concepts of canonical coalitional games for studying the stability and fairness of allocating utilities when all the users in the network cooperate. Although, in this dissertation, the applications of canonical coalitional games were not explored, however, their applications are numerous and they are quite a suited analytical for studying the limits of cooperation and the feasibility of providing incentives for the wireless users to maintain a cooperative behavior, when cooperation is ideal.

On the other hand, whenever there exists a benefit-cost tradeoff for cooperation, one can revert to a class of coalitional games, known as coalition formation games, for deriving models and algorithms that can help in analyzing the cooperating groups that will emerge in a given wireless network. For instance, coalition formation games can model a variety of problems related to distributed cooperation in next generation wireless networks. Further, for analyzing routing problems, network structure formation, and graph interconnection, coalitional graph game provide several algorithms and solutions, notably through the framework of network formation games. In this context, one can design efficient and robust strategies for forming the network structure that will govern the architecture of wireless systems.

Hence, using different concepts from coalitional game theory, one can study different problems in next generation networks such as (but not limited to): (i)- Distributed formation of virtual MIMO systems, (ii)- Distributed collaborative spectrum sensing in cognitive radio networks, (iii)- Joint sensing and access in cognitive radio networks, (iv)- Distributed task allocation in multi-agent wireless systems, (v)- Distributed cooperation for physical layer security improvement, and (vi)- Network formation in wireless multi-hop networks.

In the next sections, we introduce the basic challenges in each one of these areas, and, then, we present the different contributions of this dissertation in these fields.

3 Virtual MIMO Systems

In this section, we first introduce the basics of multiple antenna systems, and, then, we identify the basic challenges of using cooperation for forming virtual multiple antenna systems.

3.1 Basics of a MIMO System

Next generation wireless communication systems are bound to provide higher data rates and an improved quality of service, in order to support several resource demanding applications such as video transmission or mobile TV. In addition to conventional methods, such as introducing higher modulation types or providing larger bandwidths, improving the performance of wireless systems can be achieved by using multiple antenna systems (multiple-input multiple-output - MIMO).

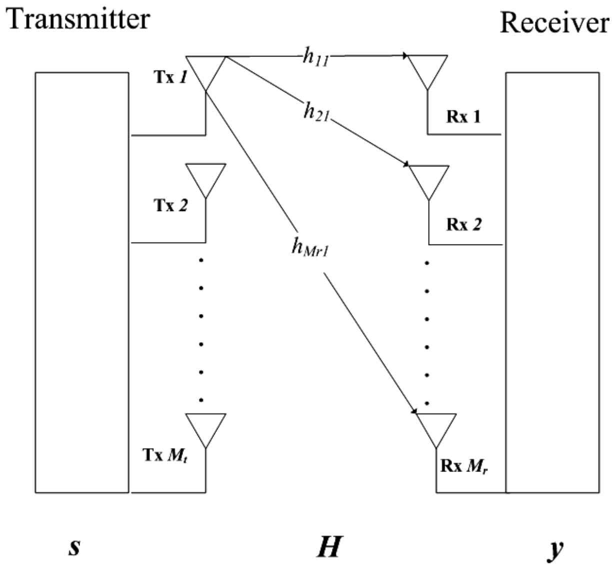

Figure 1.2: Block diagram of a typical MIMO system.

Following the pioneering work in [46, 47], communications using MIMO links has emerged as one of the most significant breakthroughs in modern communication systems. It has been show that, by deploying multiple antennas at the transmitters or the receivers of a wireless system, significant performance gains, in terms of improved throughput, improved bit error rate, or others can be achieved [46–49]. The key feature of MIMO systems is the ability to turn multipath propagation, a traditional impairment of the wireless channel, into a performance benefit. For example, MIMO systems can take advantage of random fading and, when possible, multipath delay spread for increasing data rates.

A typical MIMO system consists of M_t transmitters and M_r receivers that are communicating over a wireless channel. Let $\boldsymbol{s} \in \mathbb{C}^{M_t \times 1}$ denote the $M_t \times 1$ transmitted signal vector (each element is the signal transmitted from the corresponding antenna), $\boldsymbol{y} \in \mathbb{C}^{M_r \times 1}$ denote the $M_r \times 1$ received signal vector, and $\boldsymbol{H}$ denote the $M_r \times M_t$ channel matrix (each element h_{ij} of the matrix represents the channel between transmitter j and receiver i). A representation of this model is shown in Figure 1.2. The MIMO system model shown in Figure 1.2 can be written analytically as [49, p. 149]

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \tag{1.4}$$

where $\boldsymbol{n}$ is the additive noise vector with its elements typically considered as independent, circularly symmetric complex Gaussian random variables, $\boldsymbol{n} \sim \mathcal{CN}(\boldsymbol{0}_{M_r \times 1}, \sigma^2 \boldsymbol{I}_{M_r})$ with zero mean and covariance matrix $\mathrm{E}[\boldsymbol{n}\boldsymbol{n}^\dagger] = \sigma^2 \boldsymbol{I}_{M_r}$ ($\boldsymbol{n}^\dagger$ is the conjugate transpose of $\boldsymbol{n}$). In recent years, more advanced MIMO techniques have been studied, by combining techniques such as precoding, space-time codes, among others [48].

Information theoretic investigations in the past few years [46–49] have shown that very high capacity gains can be obtained from deploying MIMO systems. For example, when a block fading channel is considered, Gaussian signaling is used, and the transmitter is constrained in its total power, i.e., $\mathrm{E}\left[\boldsymbol{s}^\dagger \boldsymbol{s}\right] \leq \tilde{P}$, the capacity of the MIMO system can be given by [46]

$$C = \max_{\boldsymbol{Q}} I(\boldsymbol{s}; \boldsymbol{y}) = \max_{\boldsymbol{Q}} (\log \det(\boldsymbol{I}_{M_r} + \boldsymbol{H} \cdot \boldsymbol{Q} \cdot \boldsymbol{H}^\dagger)), \tag{1.5}$$

$$\text{s.t. } \operatorname{tr}[\boldsymbol{Q}] \leq \tilde{P},$$

where $I(\boldsymbol{s}; \boldsymbol{y})$ is the mutual information between $\boldsymbol{s}$ and $\boldsymbol{y}$ and $\boldsymbol{Q} = \mathrm{E}\left[\boldsymbol{s} \cdot \boldsymbol{s}^\dagger\right]$ is the covariance of $\boldsymbol{s}$. When the channel matrix $\boldsymbol{H}$ is perfectly known at the transmitter and the receiver, the maximizing input signal covariance is given by [46, 48]

$$\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^\dagger, \tag{1.6}$$

where $\boldsymbol{V}$ is the $M_t \times M_t$ unitary matrix given by the singular value decomposition of $\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\dagger$ and $\boldsymbol{D}$ is an $M_t \times M_t$ diagonal matrix given by $\boldsymbol{D} = \operatorname{diag}(D_1, \ldots, D_K, 0, \ldots, 0)$ where $K \leq \min(M_r, M_t)$ is the number of positive singular values of the channel $\boldsymbol{H}$ (eigenmodes) and each D_i given by

$$D_i = (\mu - \lambda_i^{-1})^+, \tag{1.7}$$

where $a^+ \triangleq \max(a, 0)$ and μ is determined by water-filling to satisfy the power constraint $\operatorname{tr}[\boldsymbol{Q}] = \operatorname{tr}[\boldsymbol{D}] = \sum_i D_i = \tilde{P}$, and λ_i is the ith eigenvalue of $\boldsymbol{H}^\dagger\boldsymbol{H}$. Using [46], the resulting capacity for the MIMO system is

$$C = \sum_{i=1}^{K} (\log(\mu \lambda_i))^+. \tag{1.8}$$

As a result, it is demonstrated in [46–48] that, even with a block fading channel, the capacity in (1.8) can be significantly better than a traditional single-antenna system. These gains can vary depending on different aspects such as channel fading, number of antennas, and so on. These widely acknowledged advantages of MIMO systems, led to the standardization of MIMO in many wireless systems [10, 11, 48].

Nonetheless, in order to reap the benefits of MIMO systems, numerous practical challenges arise. Most importantly, the feasibility of deploying multiple antennas on small devices, such as mobile phones, is questionable. For this purpose, alternative techniques for benefiting from MIMO gains, such as cooperation, need to be investigated. In the next subsection, we describe how cooperation can be used for providing MIMO gains to single-antenna users in wireless networks.

3.2 Virtual MIMO through Cooperation

While the information theoretic studies corroborated the gains from MIMO systems, the possibility of exploiting these gains in practice remains a big challenge. In particular, in ad-hoc or distributed large scale wireless networks, nodes are often constrained in hardware complexity and size, which makes implementing multiple antenna systems highly impractical for many applications. For this purpose, an alternative approach for exploiting the MIMO gains, through nodes cooperation, needs to be sought.

In fact, an important application for cooperation in next generation wireless networks is the formation of *virtual MIMO systems* through cooperation among single antenna devices. In this context, a number of single antenna devices can form virtual multiple antenna transmitters or receivers through cooperation, consequently, benefiting from the advantages of MIMO systems without the extra burden of having multiple antennas physically present on each transmitter or receiver. Thus, the basic idea of virtual MIMO is to rely on cooperation among mobile devices for benefiting from the widely acclaimed performance gains of MIMO systems. In Figure 1.3, we show an illustrative example of cooperation for virtual MIMO formation in a wireless network.

Similar to their single user MIMO counterparts discussed in the previous subsection, an intensive amount of research has been dedicated to the information theoretic studies of virtual MIMO systems. For instance, the authors in [50] showed the interesting gains in terms of outage capacity resulting from the cooperation of two single antenna devices that are transmitting to a far away receiver in a Rayleigh fading channel. Further,

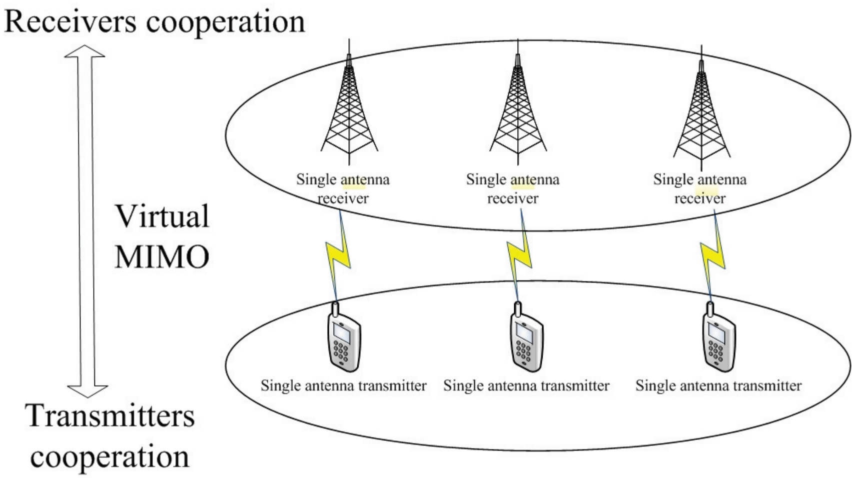

Figure 1.3: An illustrative example of a virtual MIMO system.

the work in [4, 5] considered cooperation among multiple single antenna transmitters as well as receivers in a broadcast channel. Different cooperative scenarios were, thus, studied and the results showed the benefits of cooperation from a sum-rate perspective. It is important to also note that virtual MIMO gains are not only limited to rate gains. For example, forming virtual MIMO clusters in sensor networks can yield gains in terms of energy conservation [6].

Using a canonical coalitional game the work in [27, 28] studied fairness and cooperation gains in virtual MIMO systems. The model considered in [28] consists of a set of transmitter-receiver pairs, in a Gaussian interference channel. The authors study the cooperation between the receivers under two coalitional game models: A TU model where the receivers communicate through noise-free channels and jointly decode the received signals, and an NTU model where the receivers cooperate by forming a linear multiuser detector. Further, the authors study the transmitters cooperation problem under perfect cooperation and partial decode and forward cooperation, while considering that the receivers have formed the grand coalition. The main interest was to study the properties of the grand coalitions for the receivers and the transmitters. In the joint decoding game, it is shown that the game is superadditive and that the network can be seen as a single-input multiple-output (SIMO) MAC channel (when the transmitters do not cooperate). For this game, the authors in [27] show that the core is non-empty and it contains *all the imputations* which lie on the SIMO-MAC capacity region. Further, it is proven that the Nash bargaining solution, and in particular, a proportional fair rate allocation lie in the core, and, hence, constitute suitable fair and stable allocations. For the linear multiuser detector game, the model is similar to the joint decoding game, with one major difference: Instead of jointly decoding the received signals, the receivers form linear multiuser detectors (MUD). The MUD coalitional game is inherently different from the joint decoding game since, in a MUD, the SINR ratio achieved by a user i in coalition S cannot be shared with the other users, and hence the game becomes an NTU game with the SINR representing the payoff of each player. In this NTU setting, the value $v(S)$ of a coalition S becomes the set of SINR vectors that a coalition S can achieve. For this NTU game, the grand coalition is proven to be stable and sum-rate maximizing at high SINR regime using limiting conditions on the SINR expression.

Further, the authors consider the transmitters cooperation along with the receivers cooperation. In this case, the interference channel is mapped unto a virtual MIMO MAC channel. For maintaining a characteristic form, the authors consider a utility that captures the sum-rate under worst case interference. Using this and other assumptions, the authors show that in general the game has an empty core. Further, through [28, Th. 19], it is shown that the grand coalition is the optimal partition, from a total utility point of view. The authors conjecture that in some cases, the core can also be non-empty depending on the power and channel gains. However, no existence results for the core are provided in this game. Finally, the authors in [28] provide a discussion on the grand coalition and its feasibility when the transmitters employ a partial decode and forward cooperation. In summary, the work in [27, 28] provides valuable insights and results pertaining to fairness and to the cooperation gains when performing virtual MIMO systems. However, this work does not consider any cost for virtual MIMO formation (whether it be at the transmitters or receivers side) nor does it propose any strategies for forming coalitions.

Although the gains from virtual MIMO are quite well studied and established, implementing distributed cooperation algorithms that allow to exploit these advantages in a practical wireless network is challenging and desirable. In this regards, it is of interest to study a model for distributed virtual MIMO formation which accounts for both benefits and costs for cooperation. In particular, the key issues that need to be tackled in such a scenario are (among many others)

1. What are the benefits and costs from cooperation?
2. Given the benefit-cost tradeoff, which groups of users must cooperate?
3. How can this cooperation be performed in a distributed manner?
4. How does the cooperative behavior of the users affect the network structure?
5. Can the network structure adapt to environmental changes such as slow mobility?

In order to answer these questions, and deploy distributed cooperation for virtual MIMO formation in next generation wireless networks, one needs to tackle and overcome many challenges. In this dissertation, we study the problem of virtual MIMO formation among the transmitters in the uplink of a wireless network using the analytical framework of coalition formation games. The main contributions of this work are summarized in Section 8 and the details are found in Paper B.

4 Spectrum Sensing in Cognitive Radio Networks

In this section, we introduce the basic concepts of cognitive radio networks and, then, we identify the key challenges for spectrum sensing as well as the design issues of performing joint spectrum sensing and access in cognitive networks.

4.1 Basics of Cognitive Radio Networks

With the recent growth in wireless services, the demand for the radio spectrum has significantly increased. As the demand for wireless services becomes more and more ubiquitous, the wireless devices must find a way to transmit within extremely constrained radio resources. In fact, the spectrum resources are scarce and most of them have been already licensed to existing operators. Numerous studies, such as those done by the Federal Communications Commission (FCC) in the United States, have shown that the licensed spectrum remains unoccupied for large periods of time [51]. In general, a large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum ranges from 15% to 85% with a high variance in time [51]. As a result, under the current

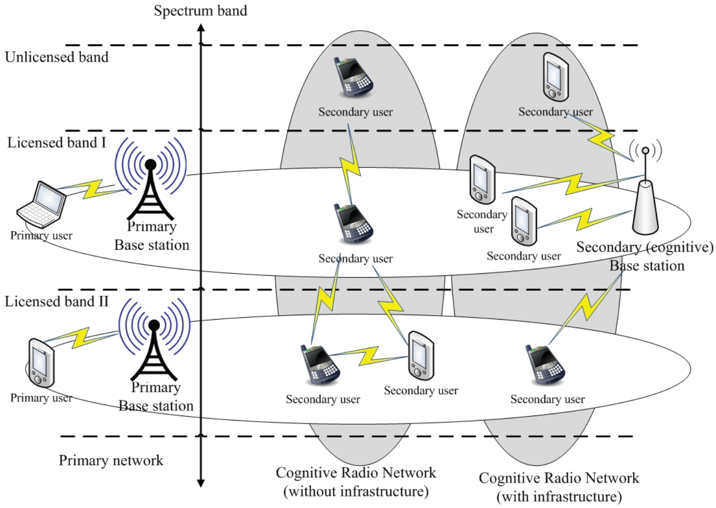

Figure 1.4: Illustration of a cognitive radio network.

fixed spectrum assignment policy, the utilization of the radio resources is quite inefficient. This limited availability and inefficiency of the spectrum usage necessitates a new communication paradigm to exploit the existing wireless spectrum opportunistically. This new networking paradigm is referred to as *cognitive radio networks* [19, 52–54].

In cognitive radio networks, the wireless devices can change and tune their transmission or reception parameters in order to achieve efficient wireless communication without interfering with the licensed users. For performing this parameter adaptation, cognitive devices can actively monitor different several external and internal radio parameters, such as radio frequency spectra, user behavior, and network states. By sensing and monitoring the available spectrum, unlicensed cognitive radio users, or secondary users (SUs), can intelligently adapt to the most suitable available communication links in the licensed bands, and, hence, by exploiting the *spectrum holes*, they are able to share the spectrum with the licensed primary users (PUs), operating whenever the PUs are idle.

The deployment of cognitive radio technology can bring a variety of benefits for the different entities of the wireless networks. For a regulator, cognitive radios can significantly increase spectrum availability for new and existing applications. For a license holder, cognitive radios can reduce the complexity of frequency planning, increase system capacity, and reduce interference. For equipment manufacturers, cognitive radios can increase demands for wireless devices. Finally, for an individual user, cognitive radios can bring more capacity per user, enhance inter-operability and bandwidth-on-demand, and provide ubiquitous mobility with a single user device across disparate spectrum-access environments. Due to the advantages of the cognitive radio networking paradigm, several of its aspects have recently made their way into different standards such as IEEE 802.11h (dynamic frequency selection and transmitter power control for WLAN sharing), IEEE P1900 (standards for advanced spectrum management), IEEE 802.22 (WRANs in unused TV bands), IEEE 802.15 task group 2 (coexistence of IEEE 802.11 and Bluetooth) [19]. An illustration of a cognitive radio network is shown in Figure 1.4.

Nonetheless, implementing practical cognitive radio networks faces numerous challenges at different levels such as: Spectrum sensing, spectrum sharing or access, spectrum management, and spectrum mobility. Spectrum sensing mainly deals with the stage during which the SUs attempt to learn their environment prior to the spectrum access (or spectrum sharing) stage where the SUs actually transmit their data. Further, spectrum management deals with allocating the spectrum between the different operators as well as matching the available spectrum to the different users and operators requirements. Finally, spectrum mobility attempts to study how the cognitive users can maintain a seamless connection while frequently changing over to better frequency bands.

These various aspects of cognitive radio have been exhaustively treated in the literature, such as in [19, 52–54] and the references therein. In this dissertation, we limit our attention to spectrum sensing, as well as the possibility of performing, jointly, spectrum sensing and access.

4.2 Spectrum Sensing

One of the major challenges of cognitive radio networks is the development of efficient spectrum sensing techniques for the SUs. Spectrum sensing refers to the phase during which the SUs must sense the radio frequencies in order to make a decision on whether to transmit or not, depending on the state of the PUs. The main objective of spectrum sensing is the design of high quality spectrum sensing devices and algorithms for exchanging spectrum sensing data between nodes to (i)- Reliably detect spectral holes for use by the cognitive radio devices and (ii)- Reliably detect when the primary transmitter becomes active. In order to achieve those goals, cross-layer design problems need to be addressed by exploiting advanced digital signal processing techniques, introducing efficient detection and estimation approaches as well as exploiting users' cooperation.

By using local measurements and local observations, a secondary user can detect the transmitted signal from a PU. The model for signal detection at time t can be described as follows [19, 54]:

$$y(t) = \begin{cases} n(t), & H_0, \\ h \cdot s(t) + n(t), & H_1, \end{cases} \tag{1.9}$$

where $y(t)$ is the received signal at an SU, $s(t)$ is the transmitted signal of the licensed PU, $n(t)$ is the additive white Gaussian noise (AWGN), and h is the channel gain. In (1.9), H_0 and H_1 represent, respectively, the hypotheses of having and not having a signal from a licensed PU in the target frequency band. Consequently, the spectrum sensing phase boils down to a decision between two hypotheses H_0 or H_1, depending on the received signal at the SU. In order to detect the signal of the PU, different methods can be used such as

1. *Matched Filter Detection*: Matched filter detection is generally used to detect a signal by comparison between a known signal (i.e., template) and the input signal. It is well known that the optimal method for signal detection is through a matched filter [49], since it maximizes the received signal-to-noise ratio. In addition, by using a matched filter detector, the detection of the PU signal can take a small amount of time [54] which is one of the main advantages of matched filter detection. However, utilizing a matched filter in spectrum sensing requires demodulation of a PU signal which implies that the cognitive radio must have a priori knowledge of different PHY and MAC characteristics of the PU signal such as pulse shaping, packet format, and so on. Further, in the event where this information is not available or is incorrect, the performance of spectrum sensing degrades significantly [19, 54]. As a result, matched filter detection is mainly useful whenever the PU can convey some information on its signal using some sort of pilot channel, preambles, spreading codes, or other techniques that can help the SUs to construct an estimate of the signal.

2. *Energy Detection*: While the matched filter approach requires coherent detection, a more simplified filtering approach is to perform non-

coherent detection through energy detection. Whenever the information on the PU signal is unavailable at the SUs, energy detection can be quite a useful approach [19, 54]. For energy detection, the output signal from a bandpass filter is squared and integrated over the observation interval. Subsequently, a decision algorithm compares the integrator output with a threshold to decide whether a licensed user exists or not [19, 54]. Basically, for energy detection, the performance deteriorates as the received SNR from the PU signal decreases. Energy detection has been widely adopted in many spectrum sensing scenarios [19]. Despite its practicality and appeal, energy detection suffers from three main drawbacks. First, it is susceptible to the uncertainty of noise. Second, energy detection can only detect the presence of the signal without being able to differentiate the type of the signal. As a result, energy detection can confuse signals resulting, for example, from other SUs with the PU signal. In addition, an energy detectors do not work for spread spectrum signals, for which more sophisticated signal processing algorithms need to be devised.

3. *Cyclostationary Feature Detection*: The transmitted signal from a licensed PU generally possesses a period pattern. Such signals are commonly referred to as cyclostationary. By using this period pattern one can detect the presence of a licensed PU [54]. A signal is cyclostationary (in the wide sense) if the autocorrelation is a periodic function. With this periodic pattern, the transmitted signal from a licensed PU can be distinguished from noise which is a wide-sense stationary signal without correlation. In general, cyclostationary detection can provide a more accurate sensing result and it is robust to variation in the noise power. However, these advantages come at the expense of a higher complexity for implementation and the need for long observation times. Different aspects of cyclostationary detectors are found in [19, 54].

In addition to these methods, recent work has also investigated the use of advanced detection techniques, such as wavelet detectors [19], for performing spectrum sensing. Moreover, one can also integrate, in a single secondary system, different detection methods. For example, energy detection can be used to perform a fast scan of a wide range of spectrum bands. Subsequently, the results from energy detection can be used to eliminate the spectrum bands with high energy densities (e.g., due to the transmission of PUs). Then, feature detection can be applied to a few candidate bands with low energy densities to search for a unique feature of signals pertaining to PUs.

For measuring the performance of spectrum sensing, three key metrics are explored: The probability of correct detection, the probability of miss and the probability of false alarm. The probability of correct detection is defined as $P_d = \text{Prob}\{\text{decision} = H_1|H_1\}$, which is the probability of correctly detecting the transmission of the PU when this PU is active. Subsequently, the probability of miss is defined as $P_m = \text{Prob}\{\text{decision} = H_0|H_1\}$ which is the probability of not detecting the PUs transmission while this PU is active, i.e., $P_m = 1 - P_d$. Finally, the probability of false alarm is defined as $P_f = \text{Prob}\{\text{decision} = H_1|H_0\}$ which is the probability of deciding that the PU is transmitting while the PU is, in fact, idle.

The expressions for computing the different probabilities depend largely on the detection method being employed as well as on the channel conditions between the PUs and the SUs. As an example, when one considers energy detection, the probability of false alarm can be given by

$$P_f = \frac{\Gamma(m, \frac{\lambda}{2})}{\Gamma(m)}, \tag{1.10}$$

where m is the time bandwidth product for energy detection, λ is the energy detection threshold, $\Gamma(\cdot,\cdot)$ is the incomplete gamma function, and $\Gamma(\cdot)$ is the gamma function. Furthermore, for SUs using energy detectors in a Rayleigh fading environment, the average probability of detection can be given by

$$P_d = e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{m-1} \left[e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})}\right)^n \right], \tag{1.11}$$

where $\bar{\gamma}$ is the average received SNR of the PU signal.

The performance of spectrum sensing is significantly affected by the degradation of the PU signal due to path loss or shadowing. For example, energy or feature detection might be quite affected by a low received SNR from PU signal, due to fading for example. Added to the issue of low SNR is the hidden terminal problem that arises because of shadowing. SUs may be shadowed away from the PU's transmitter but there may be primary receivers close to the SUs that are not shadowed from the PU transmitter. Thus, if the SU transmits, it may interfere with the primary receiver's reception. Consequently, advanced methods for improving spectrum sensing

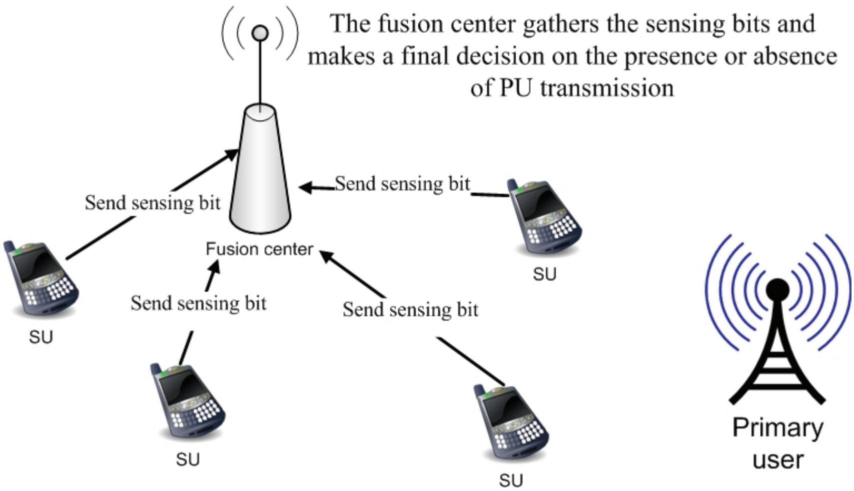

Figure 1.5: An illustration of collaborative spectrum sensing in cognitive networks.

are being sought. In particular, it has been shown that, through cooperation among the SUs, i.e., collaborative spectrum sensing (CSS), the effects of this hidden terminal problem can be reduced and the probability of detecting the PU can be improved [55–61].

The main idea of CSS is mainly composed of two steps. In the first step, each SU perform its individual detection for spectrum sensing. Then, the SUs would send their sensing bits to a fusion center which, using adequate decision fusion rules, can combined the bits from the different SUs and make a better decision on the presence or absence of the PU. An illustration of a typical CSS approach is shown in Figure 1.5.

The interest in CSS has grown significantly in the past few years. Existing literature has, in fact, studied thoroughly the performance of CSS in cognitive radio networks. For instance, in [55], the SUs perform CSS by sharing their sensing decisions through a centralized fusion center which combines the SUs' sensing bits using the OR-rule for data fusion. A similar approach is used in [56] using different decision-combining methods. In [57], it is shown that, in CSS, soft decisions can have an almost comparable performance with hard decisions while reducing complexity. The authors in [59] propose an evolutionary game model for CSS in order to inspect the strategies of the SUs and their contribution to the sensing process. The effect of the sensing time on the access performance of the SUs in a cognitive network is analyzed in [62]. For improving the performance of CSS, spatial diversity techniques are presented in [58] as a means for combatting the error probability due to fading on the reporting channel between the SUs and the central fusion center. Other interesting performance aspects of CSS are studied in [60, 61, 63–65].

Existing literature mainly focused on the performance assessment of CSS in the presence of a centralized fusion center that combines *all* the SUs bits in the network. In practice, the SUs can be at different locations in the network, and, thus, prefer to form nearby groups for CSS without relying on a centralized entity. Moreover, the SUs can belong to different service providers and need to interact with each other for CSS, instead of relaying their bits to a centralized fusion center (which may not even exist in an ad hoc network of SUs). In addition, a centralized approach leads to a significant overhead and increased complexity, notably in large networks. Further, as the number of collaborating SUs increase, the improvement in the probability of detection is accompanied by an increase in the false alarm probability. As a result, given this probability of detection-false alarm tradeoff each SU may only be willing to share their sensing bits with a selected subset. In summary, there is a need for devising models and algorithms that allow the SUs to autonomously interact for performing collaborative spectrum sensing, in a distributed manner, with no need for centralized fusion centers.

For this purpose, in this dissertation, using a coalition formation game formulation, we study the problem of distributed cooperation among the SUs in a cognitive network that seek to improve their sensing performance through CSS. The main contributions and motivations of this work are summarized in Section 8 and the details are found in Paper C.

4.3 Tradeoff between Spectrum Sensing and Spectrum Access

In general, for performing dynamic spectrum access in cognitive radio networks, the SUs must be able to perform efficient spectrum sensing and access. While spectrum sensing is the first step in dynamic spectrum access, the next step is the actual access of the channel, in the spectrum access phase. In the spectrum sensing phase, the SUs are required to decided on whether or not PUs are active as well as to discover which channels are available. In the spectrum access phase, the SUs engage in deciding on how to access the spectrum, which channel to select, which power level to use, what kind of MAC protocol to support, and so on.

As mentioned in the previous section, the technical challenges of spectrum sensing has been widely explored in the literature. Similarly, spectrum access has also received an increased attention [62, 66–76]. For instance, using a partially observable Markov decision process (POMDP), the authors in [66] devised a MAC protocol for spectrum access that maximizes the expected total number of transmitted bits over a certain duration under the constraint that the collision probability with the PU should be maintained below a target level. Further, in [62], the authors propose a dynamic programming approach to maximize their channel access time, given a penalty factor conceded when a collision with the PU occurs. In [67], a channel selection scheme based on stochastic control was derived. The work in [68] proposes a novel multiple access scheme that takes into account the physical layer transmission in cognitive networks. In [69], the authors model the spectrum access problem as a non-cooperative game, and propose learning algorithms to find the correlated equilibria of the game. Non-cooperative solutions for dynamic spectrum access are also proposed in [70] while taking into account changes in the SUs' environment such as the arrival of new PUs, among others. Additional challenges of spectrum access are tackled in [71–76].

One important aspect of dynamic spectrum access is to inspect the effect of the spectrum sensing phase on the spectrum access phase. In fact, cognitive radio networks exhibit an inherent tradeoff between exploration and exploitation. On one hand, spectrum exploration through sensing is the process by which the cognitive users tend to probe more channels to discover better channel opportunities. On the other hand, exploiting the spectrum refers to the immediate benefit gained from accessing the channel with the estimated highest reward or performance. While the SUs have an incentive to explore the spectrum in the hope of finding their best transmission opportunity, they are also bound to cease the transmission opportunity as soon as possible before the PU occupies the channel. Further, the time spent on exploring the spectrum can significantly affect the performance of data transmission during the spectrum access phase.

In fact, this tradeoff between exploration (spectrum sensing) and exploitation (spectrum access) in cognitive networks arises from different aspects. Notably, it has been established [62] that, in practice, the sensing time of cognitive radio networks is *large* and can significantly affect the access performance of the SUs. Thus, although each SU has an incentive to sense as many PU channels as possible for locating access opportunities, this spectrum exploration may come at the expense of a smaller transmission time, and, hence, a possibly smaller effective capacity for data transmission. In summary, there exists an interesting tradeoff between exploring and exploiting the spectrum in cognitive networks that needs to be addressed. While this aspect has been addressed in some existing work such as [66] or [77], most of this work is focused on non-cooperative learning approaches for tackling the problem. As a result, it is of interest to study how *cooperation* can be used to allows the SUs to improve their performance while jointly considering the *spectrum sensing* and *spectrum access* phases.

In this dissertation, we study and analyze the use of cooperation for performing joint spectrum sensing and access in cognitive radio networks using the analytical framework of coalition formation games. The main contributions of this work are summarized in Section 8 and the details are found in Paper D.

5 Multi-agent Systems

In this section, first, we provide an overview on multi-agent systems and its disciplines, then we discuss the future of applying multi-agent concepts in the context of wireless communication networks.

5.1 Overview

In many disciplines such as computer science or robotics, the concept of an *agent* is ubiquitous. The birth of the term "agent" has its roots in computer science, whereby an agent is, roughly, defined as an autonomous computer program[9]. The notion of an agent is quite difficult to define. Although numerous papers on the subject of agents and multi-agent systems have been written, a tremendous number of definitions exist. In essence, an agent is an entity that has the capabilities of an intelligent person or human being. Due to this characteristic, being able to find a unified definition of an agent is quite tough.

Although the definition can vary from one discipline to the other, in general, the main characteristics of an agent are its proactive and intelligent ability to sense its environment, interact with it, and take autonomous decisions. In some sense, the role of an agent is to mimic human behavior in

[9]This definition refers to the concept of computational agents in computer science.

a given technical problem whether it be, for example, in computer science, robotics or control systems. Thus, a multi-agent system is a system composed of multiple interacting intelligent agents that can interact, collaborate, and act together in order to solve different problems. For example, multi-agent systems can be used to solve problems in online trading, software engineering, disaster response, military applications, and modeling social structures [78].

The main challenge in designing multi-agent systems is to be able to allow the agents to somehow simulate the way humans act in their environment, interact with one another, cooperatively solve problems or act on behalf of others, solve more and more complex problems by distributing tasks or enhance their problem solving performances by competition. Clearly, the use of agents and multi-agent systems will be one of the landmark technology in many disciplines in years to come, as it will bring extra conceptual power, new methods and techniques, and advanced design approaches. Consequently, this will essentially broaden the spectrum of applications and expand it beyond the computer world into disciplines such as wireless networks or communications theory.

Independent from its application, a general problem that is of strong interest in multi-agent systems, is the distribution of tasks among the different agents. For instance, it is of importance to study how, a number of agents, can autonomously and intelligently allocate different tasks among each others using cooperative as well as non-cooperative approaches [78]. In a software system, the tasks can represent, for example, threads or programs that need to be executed. In a control system, the tasks can be points in time or space that the agents are required to attend to. For example, in [79, 80], the problem of enabling a number of vehicle-agents to move to randomly generated tasks is studied in a non-cooperative approach, while in [81, 82], the problem of task allocation in a software system is studied using a heuristic coalition formation approach. Additional approaches for agents task allocation in robotics and artificial intelligence are found in [83–87].

In a nutshell, the use of agents in future technology applications will be pervasive and centric. In consequence, there is a need to better understand the different behaviors, strategies, and usage models that these agents can have in a multitude of applications.

5.2 Deployment of Agents in Wireless Networks

Future wireless networks will present a highly complex and dynamic environment characterized by a large number of heterogeneous information sources, and a variety of distributed network nodes. This is mainly due to the recent emergence of large-scale, distributed, and heterogeneous communication systems which are continuously increasing in size, traffic, applications, services, etc. For maintaining a satisfactory operation of such networks, there is a constant need for dynamically optimizing their performance, monitoring their operation and reconfiguring their topology. For doing so, different autonomous nodes, that can be thought of as “agents”, will be deployed in future wireless networks in order to service these networks at different levels such as data collection, monitoring, optimization, management, maintenance, among others [14, 19, 88–93]. These nodes belong to the authority maintaining the network, and must be able to survey large scale networks, and perform very specific tasks at different points in time, in a distributed and autonomous manner, with very little reliance on any centralized authority [14, 19, 88, 91–93].

Although most approaches pertaining to multi-agent systems and multi-agent task allocation are oriented to robotics, control, software, or even military applications as seen in the previous subsection, it is, thus, important to leverage these problems to applications in wireless and communication networks. Some existing work has already studied the role of agents in wireless networks, although the concept was rather implicit. One prominent application of agents in wireless networks is the deployment of unmanned aerial vehicles (UAVs). For instance, in [94], the authors study how a number of UAVs, acting as agents, can self-deploy to improve the connectivity of a wireless ad hoc network. In this work, the main focus is on the optimal locations of the UAVs. In [95], a 2-level hierarchical network structure is proposed, using UAVs as an embedded mobile backbone, for improving the routing performance in ad hoc wireless networks such as military networks. This idea of hierarchical routing using UAVs is further investigated in [96]. Further, in [97], a novel MAC protocol suited for communication between ground nodes and UAVs is proposed.

In addition to using UAVs as agents in wireless networks, there has been also a recent emergence of self-deploying mobile nodes such as mobile relay stations or mobile base stations. For instance, in [91], the concept of autonomous mobile base stations is studied for improving the connectivity of vehicular ad hoc networks, in roads where the traffic flow and vehicles' speed hinders this connectivity. Further, in [93], the concept of mobile base stations is used for improving the network lifetime of wireless sensor networks. The idea of agents can also be mapped to the concept of message ferrying, whereby a special node, called a message ferry, facilitates the connectivity in a mobile ad hoc network where the nodes are sparsely deployed. The message ferry can be easily seen as an agent as it has autonomy and intelligence to self-deploy and interact with the nodes to improve connectivity. Different performance analysis of message ferrying in ad hoc networks is studied in [92, 98] and the references therein.

In a nutshell, the deployment of agents in next generation wireless networks is imminent, as many of the current research has directly or indirectly investigated the use of such nodes such as UAVs mobile base stations, or message ferries among others. One fundamental problem in this regards that remains relatively unexplored is to study the task allocation among agents in the context of wireless networks. While this problem has been studied in other disciplines as mentioned in the previous subsection, most of these existing models are unsuitable for task allocation problems in the context of wireless networks due to various reasons such as: (i)- The task allocation problems studied in the existing papers are mainly tailored for military operations, computer systems, or software engineering and, thus, cannot be readily applied in models pertaining to wireless networks, (ii)- The tasks are generally considered as static abstract entities with very simple characteristics and no intelligence (e.g. the tasks are just points in a plane) which is a major limitation, and (iii)- The existing models do not consider any aspects of wireless communication networks such as the characteristics of the wireless channel, the presence of data traffic, the need for wireless data transmission, or other wireless-specific specifications.

In this dissertation, we introduce a novel model and we provide an algorithm that allow a number of wireless agents to autonomously share a group of arbitrarily located tasks among each other. The main contributions of this work are summarized in Section 8 and the details are found in Paper E.

6 Physical Layer Security

In this section, first we discuss the main concepts of physical layer security, then we present how cooperation can be used to improve the security of wireless transmission.

6.1 Basics of Physical Layer Security

Due to the broadcast nature of the wireless channel, any unauthorized receiver, i.e., eavesdropper, located within transmission range is capable of observing the signals being communicated between the legitimate transmitters. Moreover, the malicious node has the freedom to combine its own observations with those of neighboring eavesdroppers for example, thus improving its reception by means of cooperative inference. Although much has been achieved in terms of securing the higher layers of the classical protocol stack, protecting the physical layer of wireless networks from one or multiple eavesdroppers remains a challenging task. In fact, with the emergence of large-scale heterogeneous wireless networks with little infrastructure, applying higher-layer techniques such as encryption can be quite complex and difficult. For this purpose, the implementation of information-theoretically secure communications means over the wireless channel has been receiving a recently increased attention. The main idea is to exploit the wireless channel physical layer characteristics such as fading or noise for improving the reliability of wireless transmission. This reliability is quantified by the rate of secret information sent from a wireless node to its destination in the presence of eavesdroppers, i.e., the so called *secrecy rate*. The maximal achievable secrecy rate is referred to as the *secrecy capacity*.

The idea of implementing *physical layer security* over noisy channels, which builds on the notion of *perfect secrecy* established by Shannon in [99], has its foundation in the work done in [100]. For instance, in [100], Wyner introduced the wiretap channel to model the degraded broadcast channel where the eavesdropper observes a degraded version of the receiver's signal. In his model, the confidentiality is measured by the equivocation rate, i.e., the mutual information between the confidential message and the eavesdropper's observation. For the discrete memoryless degraded wiretap channel, Wyner characterized the capacity-equivocation region and showed that a non-zero secrecy rate can be achieved [100]. The most important operating point on the capacity-equivocation region is the secrecy capacity, i.e., the largest reliable communication rate such that the eavesdropper obtains no information about the confidential message (the equivocation rate is as large as the message rate). The secrecy capacity of the Gaussian wiretap channel was given in [101]. Csizar and Korner considered a more general wiretap channel in which a common message for both receivers is sent in addition to the confidential message [102].

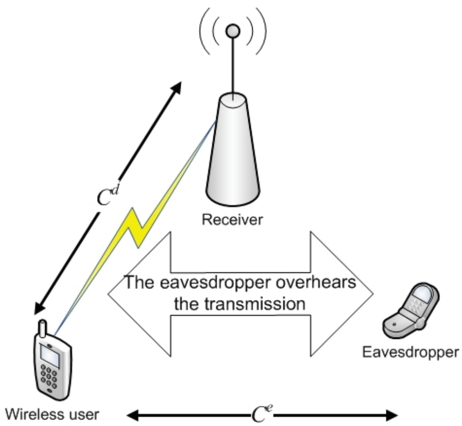

Figure 1.6: Illustration of a basic wireless transmission model in the presence of an eavesdropper.

Recently, there has been considerable efforts devoted to generalizing these results into the wireless channel and multi-user scenarios [103–110]. In [103], the secrecy capacity of the ergodic slow fading with perfect channel state information at the transmitter (CSIT) was characterized and the power/rate allocation under partial CSIT (the knowledge on the channel of the intended receiver only) was derived. The secrecy capacity of the parallel fading channels was given [104, 105] where [105] considered the model in [102] with a common message. The feasibility of traditional physical layer security approaches based on single antenna systems is hampered by channel conditions: If the channel between the source and the destination is worse than the channel between the source and an eavesdropper, the secrecy capacity is typical zero [100, 105]. For overcoming this limitation, in [106, 108, 109], the use of multiple antennas for improving the secrecy capacity was investigated. In summary, physical layer security presents an interesting and challenging field which is currently ongoing a significant growth, namely in the wireless community.

As previously mentioned, the main performance metric of interest in physical layer security problems is the concept of secrecy capacity (as well as the secrecy rate). The most basic model of transmission in the presence of an eavesdropper is shown in Figure 1.6. In this figure, C^d denotes the Shannon rate from the transmitter to its receiver while C^e represents the

Shannon rate at the eavesdropper. Given this basic model, the secrecy capacity C^s of the transmitter can be given by

$$C^s = \left(C^d - C^e\right)^+. \tag{1.12}$$

The fact that (1.12) represents the secrecy capacity and is achievable has been shown in [108, 109]. In the presence of multiple eavesdroppers, characterizing the secrecy capacity can be challenging [106–110]. However, it has been shown that, by the use of Gaussian inputs, an achievable secrecy rate R^s, in the presence of $K > 1$ eavesdroppers, can be given by [111]

$$R^s = \left(C^d - \max_{1 \le k \le K} C_k^e\right)^+, \tag{1.13}$$

where C^d represents the Shannon rate of the transmitter and C_k^e is the Shannon rate at eavesdropper k.

As demonstrated in [106, 108, 109], the use of multiple antennas can significantly improve the secrecy rate both in the single eavesdropper case of (1.12) as well as the multiple eavesdroppers case in (1.13). However, as thoroughly discussed in Section 3, due to cost and size limitations, multiple antennas may not be available at the wireless nodes and, under such scenarios, cooperation is an effective way to enable single-antenna nodes to enjoy the benefits of multiple-antenna systems. In the next subsection, we discuss how different cooperation techniques can be used for improving the physical layer security of wireless transmission.

6.2 Cooperation for Improving Physical Layer Security

As mentioned in the previous subsection, in many scenarios, the wireless channel conditions can lead to a zero secrecy capacity for some users. In this case, the users need to perform advanced communications techniques to improve their secrecy capacity. It has been shown that the use of multiple antennas can improve the secrecy capacity and overcome some of the challenges of the wireless channel [106, 108, 109].

However, due to hardware limitation as well as costs, physically implementing multiple antennas on wireless devices might not always be feasible. As an alternative, a number of single antenna can cooperate in order to improve their secrecy capacity. For instance, consider a source node equipped with a single antenna seeking to cooperate a number of single antenna relay nodes in its vicinity in order to transmit its data to

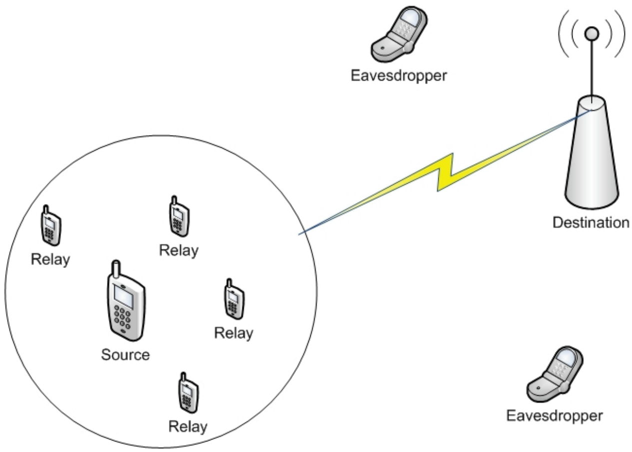

Figure 1.7: Illustration of a cooperation for wireless transmission in the presence of eavesdroppers.

a far away destination in the presence of one or more eavesdroppers. An illustration of the model is given in Figure 1.7.

Different other cooperative schemes for improving the transmission in the presence of eavesdroppers have been proposed in [107, 110, 112–117]. These approaches consider different roles for the relay such as helping the source, or the eavesdropper, or both. Techniques such as jamming the eavesdropper or nulling the signal at the eavesdropper are used to improve the secrecy rate of the users in different scenarios. Most of this work is mainly focused on performance assessment, analysis of the secrecy rate, as well as the rate-achieving relaying strategy.

In order to illustrate how cooperation can improve the secrecy rate of a source node such as in in Figure 1.7, one approach, as described in [107, 110], is to allow the source and the relays to adopt a cooperative protocol composed of two stages

- In the first stage, the source transmits its signal locally to the trusted relays in its vicinity.

- In the second stage, the source and the relays transmit, cooperatively, the signal to the destination using a well suited cooperation protocol.

Thus, the two-stage algorithm consists of an information exchange stage and a transmission stage. In the transmission stage, various well known techniques for relaying can be used such as decode-and-forward or amplify-and-forward. Using decode-and-forward and assuming that the signal is transmitted in the first stage with enough power to allow the relays to correctly decode it, the source as well as each relay (after decoding the message) transmit a weighed signal of the original decoded message. In contrast, using amplify-and-forward, in the second stage, the source transmits a weighed version of its signal while the relays transmit a weighed version of the noisy signal received during the first stage.

The weights used in the second stage can be optimized so that the secrecy rate of the source node is improved. Hereafter, it is assumed, as is often the case in current physical layer security literature [107], that the source and the relays have complete knowledge of the channels to the destination and the eavesdroppers. The assumption that the users have knowledge of the eavesdroppers channel is commonly used in most physical layer security related literature (see [107, 110, 118] and references therein), and as explained in [118] this channel information can be obtained by the users through a constant monitoring of the behavior of the eavesdroppers. Alternatively, the eavesdroppers can be considered as location in space where the source *suspects* the presence of a malicious node.

For the case of a single eavesdropper, the optimal weights can be found as follows. By considering the decode-and-forward case, given a total of $N-1$ relays and a single eavesdropper, we let $\boldsymbol{h} = [h_1, \ldots, h_N]^\dagger$, $\boldsymbol{g}_k = [g_{1,k}, \ldots, g_{N,k}]^\dagger$, and $\boldsymbol{w} = [w_1, \ldots, w_N]^\dagger$ be the the $N \times 1$ vectors representing, respectively, the channels between the nodes (source and relays) and the destination, the channels between the nodes (source and relays) and the eavesdropper k channels, and the signal weights (note that, the first element of each vector corresponds to the source). In this case, the secrecy capacity of the source, as defined in (1.12), can be written as [107, Eq. (6)]

$$C^s = \frac{1}{2}\log_2\left(\frac{\sigma^2 + \boldsymbol{w}^\dagger \boldsymbol{R}_h \boldsymbol{w}}{\sigma^2 + \boldsymbol{w}^\dagger \boldsymbol{R}_g^k \boldsymbol{w}}\right), \tag{1.14}$$

where the scalar factor $\frac{1}{2}$ is due to the fact that the algorithm requires two stage for cooperation, σ^2 is the variance of the Gaussian noise, $\boldsymbol{R}_h = \boldsymbol{h}\boldsymbol{h}^\dagger$, and $\boldsymbol{R}_g^k = \boldsymbol{g}_k \boldsymbol{g}_k^\dagger$.

For the case of one eavesdropper, assuming that the total power that the source and it relays can use to transmit is $\tilde{P}$, and considering that the power used in the first stage of cooperative transmission is negligible, the problem of maximizing the secrecy capacity can be written as

$$\max_{\boldsymbol{w}} \frac{\sigma^2 + \boldsymbol{w}^\dagger \boldsymbol{R}_h \boldsymbol{w}}{\sigma^2 + \boldsymbol{w}^\dagger \boldsymbol{R}_g^k \boldsymbol{w}}, \tag{1.15}$$

$$\text{s.t. } \boldsymbol{w}^\dagger \boldsymbol{w} = \tilde{P}.$$

The solution of this optimization problem, as found in [107, 108, 119], is the scaled eigenvector corresponding to the largest eigenvalue of the symmetric matrix $(\tilde{\boldsymbol{R}}_g^k)^{-1}\tilde{\boldsymbol{R}}_h$ where $\tilde{\boldsymbol{R}}_g^k \triangleq \frac{\sigma^2}{\tilde{P}}\boldsymbol{I}_N + \boldsymbol{R}_g^k$ and $\tilde{\boldsymbol{R}}_h \triangleq \frac{\sigma^2}{\tilde{P}}\boldsymbol{I}_N + \boldsymbol{R}_h$.

Unlike the single eavesdropper case, whenever there are $K > 1$ eavesdroppers in the network, finding an optimal weight can be quite difficult to find [107]. Alternatively, one approach is to weigh the signal in a way to *completely null out the signal at all the eavesdroppers*. By doing so, the secrecy rate of the source is certainly improved (although not maximized). In this case, it is shown that, cooperatively and while nulling the signal at the eavesdroppers', the source's secrecy rate, as per (1.13) would become [107, Eq. (14)]

$$R^s = \frac{1}{2}\log_2(1 + \frac{(\boldsymbol{w}^*)^\dagger \boldsymbol{R}_h \boldsymbol{w}^*}{\sigma^2}), \tag{1.16}$$

where $\boldsymbol{w}^*$ is the weight vector that maximizes the secrecy rate while nulling the signal at the eavesdropper and is given in [107, Eq.(20)] by

$$\boldsymbol{w}^* = \beta \boldsymbol{G}^\dagger (\boldsymbol{G}\boldsymbol{G}_S^\dagger)^{-1}\boldsymbol{e} \tag{1.17}$$

with $\boldsymbol{G} = [\boldsymbol{h}, \boldsymbol{g}_1, \ldots, \boldsymbol{g}_K]^\dagger$ a $(K+1) \times N$ matrix, $\beta = \sqrt{\frac{\tilde{P}}{\boldsymbol{e}^\dagger(\boldsymbol{G}\boldsymbol{G}^\dagger)^{-1}\boldsymbol{e}}}$ a scalar and $\boldsymbol{e} = [1, \boldsymbol{0}_{1\times K}]^\dagger$ a $(K+1)\times 1$ vector.

It is shown in [107] that, for a source node having a number of relays in its vicinity, the secrecy rate as per (1.16) is improved significantly with respect to the non-cooperative case, in the presence of multiple eavesdroppers. Similar analysis can also be found in [110], for the amplify-and-forward case. It is also shown [107, 110] that, for a given cluster of nearby relays and a single source the decode-and-forward case performs better than the amplify-and-forward case but at the expense of more complexity.

In brief, existing work has established that, by cooperation, gains in terms of secrecy rate can be achieved when wireless transmission occurs in the presence of eavesdroppers. However, as previously mentioned, most of the existing work in [107, 110, 112–117] focuses on information theoretic analysis of the secrecy rate, at a link level, with no cost for cooperation.

Although the main concepts behind physical layer security are still largely theoretical, it is of interest to study how the gains, in terms of secrecy rate, stemming from cooperation can be achieved in a practical network. In particular, it is important to study: (i)- What kind of tradeoff exists between the cooperation gains and the information exchange costs, (ii)- What kind of cooperative strategies the users can adopt to improve their secrecy rate, and (iii)- The impact of physical layer security cooperation on the network structure. In this dissertation, we attempt to answer these questions by studying and analyzing the use of cooperation for physical layer security improvement using the analytical framework of coalition formation games. The main contributions of this work are summarized in Section 8 of this introduction and the details are found in Paper F.

7 Multi-hop Architectures in Next Generation Networks

In this section, first, we briefly introduce the main ideas behind multi-hop communication, and, then, we discuss how the deployment of relay stations in next generation wireless networks impacts the network architecture.

7.1 Introduction

The concept of multi-hop wireless networks dates back to the 1970s at the time of the DARPA Packet Radio network [120]. The main idea is that, whenever a node needs to send its data to a far destination, it can send its data to neighboring node, which, in turn, send the data to other neighbors, until the arrival of the data at the destination. Although the design of multi-hop networks languished in the late 1980s partially due to the lack of low cost CPUs and memory for performing such multi-hop routing, it has been rekindled recently with the latest advances in device technology.

Multi-hop architectures are ubiquitous in both ad hoc wireless networks and infrastructure-based wireless networks (e.g., cellular networks). For instance, ad hoc wireless multi-hop networks can be defined as communication networks that consist entirely of wireless nodes, placed together in an ad hoc manner, i.e., with minimal prior planning. All the nodes possess routing capabilities, and can forward data packets for other nodes in multi-hop fashion. Nodes can enter or leave the network at any time, and may be mobile, so that the network topology continuously experiences alterations during deployment. The challenges of ad hoc wireless networks have been thoroughly tackled in [121–125] and references therein. Further, multi-hop communication can also take place among the different mobile users in a network with infrastructure, e.g., a cellular network as studied in [126–129] and the references therein.

Regardless of the network type, multi-hop communication can span the different networking layer. At the network and transport layer, the main interest is to study the efficient delivery of the packets through multi-hop communication. Another important aspect at those layers is to study routing protocols that can efficiently operate under multi-hop communication. At the physical and MAC layers, the recent interest in cooperative transmission has also led to the emergence of important challenges for multi-hop communication. For instance, the deployment of one or more relays that can decode a signal and re-transmit it, cooperatively with a source node, naturally leads to multi-hop communication. As the presence of relay nodes, that can act as helper nodes for other source nodes[10], becomes more and more pervasive in future wireless networks, multi-hop communication is bound to become a pillar of next generation wireless networks.

7.2 Relay Station Nodes in Next Generation Networks

It has been demonstrated that by deploying one or multiple relays that can perform cooperative transmission in a wireless network [130–132] a significant performance improvement can be witnessed in terms of throughput, bit error rate, capacity, or other metrics. Consequently, due to this performance gain that cooperation can yield in a wireless network, recently, the incorporation of relaying into next generation wireless networks has been proposed. In this context, the deployment of relay station (RS) nodes, dedicated for cooperative communications, is a key challenge in next generation networks such as 3GPP's long term evolution advanced (LTE-Advanced) [10] or the forthcoming IEEE 802.16j WiMAX standard [11].

The deployment of RSs can have in general two usage models. First,

[10] A relay node can be either a dedicated node or another independent wireless node which has an interest to relay data for one of its partner.

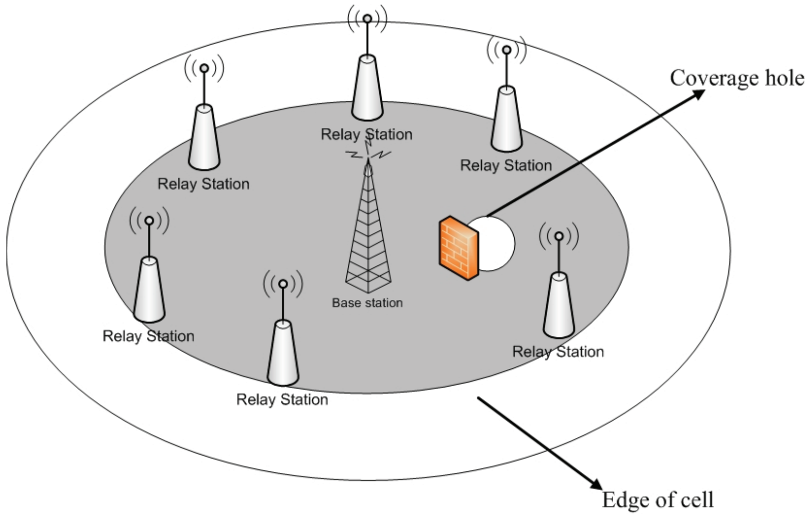

Figure 1.8: A number of RSs deployed to improve cell coverage.

RSs can be deployed in order to improve the coverage range of a cell, as shown in Figure 1.8. In this case, the communication between the users and the base station occurs, in general, through one hop (one RS). Alternatively, RSs can be deployed in order to improve the capacity and performance of the users. For such a scenario, a number of small RS cells would appear in the network as shown in Figure 1.9, and multi-hop communication becomes essential as every node can transmit its data through one or more RSs to the base station. For this model, the use of advanced communication techniques such as cooperative transmission can be quite common.

Independent of the usage model, for an efficient deployment of RSs in next generation networks, several key technical challenges need to be addressed at both the uplink and downlink levels. Recent literature has, indeed, studied different aspects of the deployment of RSs, notably in the context of LTE-Advanced and 802.16j networks. For instance, in [133], the authors study the capacity gains and the resource utilization in a multi-hop LTE network in the presence of RSs. Further, the performance of different relaying strategies in an LTE-Advanced network is studied in [134]. Furthermore, the authors in [135] study the possibility of coverage extension in an LTE-Advanced system, through the use of relaying. In [136],

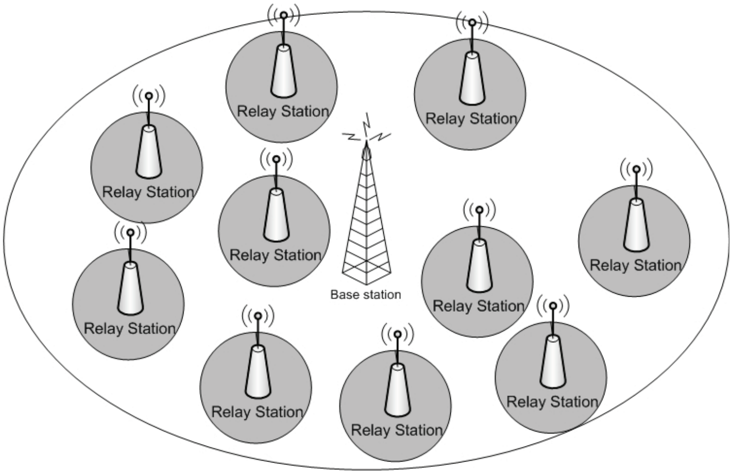

Figure 1.9: A number of RSs deployed to improve performance (e.g., capacity, bit error rate, etc.).

the communication possibilities between the RSs and the base station is studied and a need-basis algorithm for associating the RSs to their serving BS is proposed for LTE-Advanced networks. The possibilities for handover in an LTE network in the presence of RSs are analyzed in [137].

From an 802.16j perspective, the work in [138] studies the optimal placement of one RS in the downlink of 802.16j for the purpose of maximizing the total rate of transmission. In [139], the use of dual relaying is studied in the context of 802.16j networks with multiple RSs. Resource allocation and network planning techniques for 802.16j networks in the presence of RSs are proposed in [140]. Other aspects of RS deployment in next generation networks are also considered in [88, 141–144].

Clearly, the presence of RSs impacts the performance and network structure of next generation networks. While existing literature mostly focused on RS placement and performance assessment, one challenging area which remains relatively unexplored is the formation of the multi-hop architecture that can eventually connect a base station to the RSs in its coverage area. For both the uplink and the downlink, this architecture is certainly based on multi-hop communication and can include different structures such as trees, forests, or other multi-hop architectures. The network structure would depend on the communication techniques being used as well as on the performance metrics that the RSs and the wireless users are interested in optimizing.

In this dissertation, using network formation games, we study and analyze the formation of the multi-hop tree architecture in the uplink of a wireless network, whenever the nodes are interested in optimizing the tradeoff between improved bit error rate due to cooperative transmission and the delay incurred by multi-hop communication. The main contributions of this work are summarized in Section 8 of this introduction and the details are found in Paper G.

8 Contribution of the Included Papers

This dissertation consists of seven papers numbered by letters (A-G). In this section, we present a brief summary of these papers.

8.1 Paper A

W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional Game Theory for Communication Networks," *IEEE Signal Processing Magazine, Special Issue on Game Theory for Signal Processing and Communication*, volume 26, issue 5, pages 77-97, September 2009.

Paper A presents a tutorial on the applications of coalitional game theory within wireless and communication networks. In this paper, we focus on providing an application-oriented analysis of coalitional game theory that will enable wireless engineers to identify the suited tools for solving different problems in wireless and communication networks.

For doing so, we provide a novel classification of coalitional game theory, by compiling and dividing the sparse literature on the subject into three distinct classes: (i)- canonical coalitional games, (ii)- coalition formation games, and (iii)- coalitional graph games. First, we provide the main fundamental concepts that are applicable and common in all three classes. Then, for each class of coalitional games, we present the fundamental components, introduce the key properties, mathematical techniques, and solution concepts, and describe the methodologies for applying these games in several applications drawn from the state-of-the-art research in wireless and communication networks.

Canonical coalitional games describe the situation where cooperation, i.e., forming a coalition is always beneficial. The main issues to tackle in such games are the fairness and stability of the grand coalition, i.e., the coalition of all users. After thoroughly describing the main ingredients and solution concepts of canonical coalitional games, we present techniques and methods for finding each one of these solutions. Further, we analyze how canonical coalitional games can be used to solve different problems in wireless and communication networks such as rate allocation in a multiple access channel, as well as receivers and transmitters ideal cooperation, i.e., with no cost, in a wireless network.

Unlike canonical coalitional games, coalition formation games consider cooperation problems in the presence of both gains and costs from cooperation. This is quite a useful class of games since, in several problems, forming a coalition requires a negotiation process or an information exchange process which can incur a cost, thus, reducing the gains from forming the coalition. For coalition formation, we first describe the main properties of these games as well as how the canonical coalitional game solution concepts are affected by the presence of a coalitional structure. Then, we provide guidelines on developing coalition formation algorithms for practical applications. In addition, we analyze how coalition formation games can be applied to solve different applications in wireless networks such as transmitter cooperation with cost in a TDMA system as well as collaborative spectrum sensing in cognitive radio networks.

In both canonical and coalition formation games, the utility or value of a given coalition has *no* dependence on how the players inside (and outside) the coalition communicate. Nonetheless, in certain scenarios, the *underlying communication structure*, i.e., the graph that represents the connectivity between the players in a coalitional game can have a major impact on the utility and other characteristics of the game. In such scenarios, coalitional graph games constitute a strong tool for studying the graph structures that can form in a coalitional game based on the cooperative incentives of the various players. For these games, we present the main concepts for classifying a game as coalitional graph game, and we focus on an important subclass of these games, known as network formation games. We present the main building blocks of network formation games and we discuss how network formation algorithms can be built. Afterwards, we present how coalitional graph games can be applied within the context of wireless communications by applying these games for forming the network structure in an IEEE 802.16j network.

Further, for all three classes, we shed a light on their future potential as well as on possible future applications in next generation wireless networks.

In summary, Paper A fills an important void in current wireless literature by providing a unified engineering-oriented treatment of coalitional game theory. With the ongoing growth of the cooperation paradigm in wireless communications, the need for a tool such as coalitional game will grow incessantly, and, such a tutorial is of high interest. To the best of our knowledge, this paper constitutes the only such reference in existing literature which highlights the timeliness and significance of its contribution.

8.2 Paper B

W. Saad, Z. Han, M. Debbah, and A. Hjørungnes, "A Distributed Coalition Formation Framework for Fair User Cooperation in Wireless Networks," *IEEE Transactions on Wireless Communications*, volume 8, issue 9, pages 4580-4593, September 2009.

In this paper, we study the distributed formation of virtual MIMO systems through transmitters cooperation in the uplink of a TDMA wireless network. Given a number of single-antenna users seeking to transmit data in the uplink to a central base station having multiple antennas, we provide a cooperation model that takes into account both the gains from cooperation, in terms of an improved sum-rate, as well as the costs, in terms of the power used for information exchange.

The problem is modeled as a coalitional game with transferable utility where the players are the transmitters and the value is the sum-rate achieved by each coalition (over the slots owned by this coalition), which can be seen as a single-user virtual MIMO system, given the costs for information exchange. We show that the game is non-superadditive and that the core is, thus, empty. As a result the game is formulated as a coalition formation game with a transferable utility. We also discuss the effect of having various fairness rules for dividing the utility among each coalition's members such as: The egalitarian rule, the proportional fair rule, the Shapley value, and the nucleolus (applied at the level of each coalition).

Then, for forming coalitions, we derive a distributed coalition formation algorithm based on well-defined rules, referred to as the "merge" and "split" rules. The basic idea of the merge rule is that a group of coalitions would merge if at least one user improves its payoff without hurting any of the other involved user. The merge phase is seen as a binding agreement between the transmitters to act as a single coalition. In contrast, a coalition would break up into smaller coalitions, i.e., split, if, by doing so, at least one user improves its payoff without hurting any of the other involved user. Such a coalition formation algorithm is partially reversible.

We show how, using the proposed algorithm, the users can self-organize into independent disjoint coalitions. Further, we characterize the resulting coalitional structure using the concept of defection functions. We show that the proposed algorithm converges to a network partition that is $\mathbb{D}_{hp}$-stable, i.e., no user has an incentive to leave this partition through merge or split. Depending on the location of the users, we discuss how the proposed algorithm can also converge to a Pareto optimal (in terms of payoff distribution) $\mathbb{D}_c$-stable partition which is a unique outcome of any merge-and-split iteration.

Exhaustive simulations are performed in order to assess the different aspects of the proposed algorithm. Through simulations, we first show how the network can self-organize into disjoint coalitions, and we discuss how the network structure as well as the performance varies with the different fairness rules for payoff division. Then, we show how, through periodic runs of the merge-and-split algorithm, the transmitters can adapt the network structure to mobility as different coalitions form or split depending on the users' time varying positions. Simulation results also show that the proposed algorithm can improve the individual user's payoff up to 40.42% compared to the non-cooperative case, and, compared to a centralized optimal solution, the performance is only 1% below the optimal solution for a network having 20 users.

8.3 Paper C

W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional Games for Distributed Collaborative Spectrum Sensing in Cognitive Radio Networks," in Proceedings of *the IEEE International Conference on Computer Communications (INFOCOM)*, pages 2114 - 2122, Rio de Janeiro, Brazil, April 2009.

In Paper C, we apply the framework of coalition formation games for performing distributed collaborative spectrum sensing in a cognitive radio network with a single primary user. Unlike existing work which focused on the performance of centralized approaches for collaborative spectrum sensing, we propose a distributed approach and study the network interactions among the secondary users seeking to improve their sensing performance through cooperation.

We model the problem as a non-transferable utility coalitional game where the secondary users are the players. Each secondary user's payoff function captures the inherent collaborative spectrum tradeoff that exists between the gains, in terms of a reduced probability of miss and the costs, in terms of an increased false alarm. Then, we propose an algorithm for coalition formation, based on the merge and split rules, which captures the incentives of the secondary users for cooperating to minimize their probability of miss while maintaining a target false alarm level. Using the proposed algorithm, the secondary users are able to self-organize into disjoint independent coalitions while improving their sensing performance. Within each coalition, a secondary user, chosen as coalition head, acts as a fusion center which combines the bits from all the secondary users in its coalition and makes a final decision on the presence or absence of the primary user.

The proposed algorithm is composed of three phases: Local sensing phase where each individual secondary user computes its own local primary user detection bit based on the received primary user signal, adaptive coalition formation phase where merge-and-split occurs, and, once the network topology converges following merge-and-split, the last phase is the coalition sensing phase where the secondary users that belong to the same coalition report their local sensing bits to their local coalition head who makes the final decision. Further, we characterized the network structure resulting from the proposed algorithm, studied its stability and showed that a maximum number of SUs per coalition exists for the proposed utility model. This maximum is independent of the size of the network and mainly depends on the non-cooperative and target false alarm levels.

The performance and characteristics of the proposed algorithm are assessed through adequate simulations. Simulation results show that the proposed algorithm allows a reduction of up to 86.6% of the average missing probability per SU (probability of missing the detection of the PU) relative to the non-cooperative case, while maintaining a certain false alarm level below a target of 10%. The simulations also show how the algorithm can handle mobility through periodic decisions by the secondary users.

8.4 Paper D

W. Saad, Z. Han, R. Zheng, A. Hjørungnes, and T. Başar, "Coalitional Games in Partition Form for Joint Spectrum Sensing and Access in Cognitive Radio Networks," submitted to *the IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Advances on Cognitive Radio Networking and Communications*, December 2009.

In Paper D, we propose a novel cooperation protocol for allowing the secondary users in a multi-channel cognitive network to jointly improve their spectrum sensing and access performance. Due to the fact that: (i)- different cognitive users can have different views of the channels and (ii)- frequency-selective channels are different for different secondary users, it is beneficial for the secondary users to cooperate and share their channel knowledge in order to coordinate both their sensing.

From a sensing perspective, we propose a scheme through which the secondary users cooperate in order to share their channel knowledge, and, hence, improve their view of the spectrum, consequently, reducing their sensing time. From an access perspective, the proposed cooperation protocol allows the cognitive users to improve their access capacities by: (i)- Learning from their cooperating partners the existence of alternative channels with better conditions, (ii)- Reducing the interference among each other, and (iii)- Exploiting multiple channels simultaneously, when possible (in a non-cooperative approach, due to hardware limitation, the secondary user can only sense and access one channel at a time).

We model the problem as a coalitional game in *partition form*, and we propose an algorithm for coalition formation. To the best of our knowledge, this paper is among the first that applies the partition form in the design of wireless protocols and systems. Further, we propose a coalition formation algorithms that allows the secondary users to take distributed individual decisions to join or leave a coalition, while maximizing their utility which accounts for the average time needed to locate an unoccupied channel (spectrum sensing) and the average capacity achieved when transmitting the data (spectrum access). In this scheme, Thus, a secondary user can decide to move from its current coalition and join a new coalition while improving its payoff, given the approval and consent of the members of this new coalition.

We show that, using the proposed algorithm, the secondary users can self-organize into disjoint coalitions that constitute a Nash-stable network partition. Within every formed coalition, the secondary users act cooperatively by sharing their view of the spectrum, coordinating their sensing order, and distributing their powers over the seized channels whenever possible. Also, the proposed coalition formation algorithm allows the secondary users to adapt the topology to environmental changes such as the changes in the availability of the PU channels or the slow mobility of the secondary users.

We study and analyze the performance and characteristics of the proposed algorithm through extensive simulations. Simulation results show how the proposed algorithm allows the secondary users to self-organize while yielding a performance improvement, in terms of the average secondary user payoff, up to 77.25% relative to the non-cooperative case for a network with 20 secondary users. Further, simulation results also show how the algorithm can handle environmental changes such as slow mobility or a change in the traffic of the primary users.

8.5 Paper E

W. Saad, Z. Han, T. Başar, M. Debbah and A. Hjørungnes, "Hedonic Coalition Formation for Distributed Task Allocation among Wireless Agents," conditionally accepted for publication in *IEEE Transactions on Mobile Computing* (subject to reviewers' and editor's final approval of the revised manuscript submitted 03-02-2010).

In Paper E, we introduce a new model for deploying wireless agents in next generation networks. A "wireless agent" represents any node that can operate autonomously and can perform wireless transmission. Examples of wireless agents are unmanned aerial vehicles, mobile base stations, cognitive wireless devices, or self-deploying mobile relay stations, among others. Although the concept of agents has been ubiquitous in the context of software engineering, computer science, robotics or military applications, this paper attempts to study the use of agents within wireless networks. In this regards, we study the problem of spatial arrangement of wireless agents that need to service arbitrarily located tasks in a wireless communication network. The tasks are queues of packets generated by some source of data. These tasks can represent, for example, a group of mobile devices, such as sensors, video surveillance devices, or any other static or dynamic wireless nodes that have limited power and are unable to provide long-distance transmission These packets are collected by the agents and transmitted through a wireless channel to a central receiver. While servicing tasks, an agent can act as either a collector that extracts the data, and, thus, improves the throughput, or as a relay, that allows to improve the probability of successful transmission. Each agent and task receives a payoff which depends on the tradeoff between the net throughput and delay in transmitting the packets. The ultimate goal is to obtain spatial organization of agents corresponding to any given arrangement of tasks that can improve the average player (agent or task) payoff.

To address the above objective, we model the problem as a hedonic coalition formation game and we propose a distributed algorithm for forming coalitions. A hedonic coalition formation game is a coalition formation game where the payoff of a player depends solely on the members of its coalition and the coalitions are built using preference relations. The proposed algorithm leads to the formation of disjoint coalitions of players. In each coalition, the tasks belonging to that coalition are served in a cyclic order by the agents in the same coalition. For instance, each coalition is shown to be a polling system with an exhaustive polling strategy and deterministic non-zero switchover times (corresponding to the travel times of the agents) consisting of a number of collectors which act as a single server that moves continuously between the different tasks (queues) present in the coalition, gathering and transmitting the collected packets to a common receiver. We show that the set of coalitions resulting from the proposed algorithm form a Nash-stable coalitional structure. Further, we show that, for each coalition, a minimum number of collectors is needed.

The performance and characteristics of the proposed algorithm are analyzed through simulations. First, the simulations highlight how the agents and tasks can engage in the proposed algorithm for forming the network structure. In addition, we show how this structure can evolve over time when the environment is dynamic due to the arrival of new tasks, the removal of existing tasks, or the mobility of the tasks. Simulation results show that the proposed algorithm yields a performance improvement, in terms of average player (agent or task) payoff, of at least 30.26% (for a network of 5 agents with up to 25 tasks) relatively to a scheme that allocates nearby tasks equally among the agents.

In a nutshell, by combining concepts from wireless networks, queueing theory and novel concepts from coalitional game theory, this paper introduces a new model for task allocation among autonomous agents in communication networks which is well suited for many practical applications such as data collection, video surveillance in wireless networks, data transmission, autonomous relaying, operation of mobile base stations in vehicular ad hoc networks and mobile ad hoc networks (the so called *message ferry* operation), surveillance, autonomous deployment of unmanned air vehicles in military ad hoc networks, wireless monitoring of randomly located sites, or maintenance of failures in next generation wireless networks.

8.6 Paper F

W. Saad, Z. Han, T. Başar, M. Debbah and A. Hjørungnes, “Distributed Coalition Formation Games for Secure Wireless Transmission," submitted to *ACM/Springer Journal on Mobile Networks and Applications*, October 2009.

In this paper, we study network aspects of cooperation among wireless devices seeking to improve their physical layer security, in the presence of multiple eavesdroppers. While existing physical layer security literature answered the question “what are the link-level *secrecy rate* gains from cooperation?”, this paper seeks to answer the question of “how to achieve those gains in a practical decentralized wireless network and in the presence of a cost for information exchange?”. Note that the results derived in this paper build upon a preliminary version found in [145].

In this regards, given a TDMA network where a number of users are transmitting their data in the uplink with the presence of eavesdroppers, we study how the users can adopt decode-and-forward or amplify-and-forward cooperation strategies, in order to improve their secrecy rate. The problem is formulated as a non-transferable utility coalition formation games where the players are the users and the payoff of each player is a function of the cooperation gains, in terms of secrecy rate that this player achieves, as well as the costs, in terms of secrecy rate losses during information exchange. For forming coalitions, we devise a distributed algorithm, based on the rules of merge and split, that allows the users to form or break coalitions, depending on the achieved payoff and the consent of their cooperative partners. We show that, due to the cooperation costs, a number of disjoint coalitions will emerge in the network. We highlight the stability of the resulting coalitional structure using the defection function concept.

Simulation results show that, when adopting amplify-and-forward strategies, only users with highly favorable conditions can cooperate, and, thus few coalitions appear in the network. This is mainly due to the fact that the amplification of the noise resulting from beamforming using amplify-and-forward relaying hinders the gains from cooperation relative to the secrecy cost during the information exchange phase. As a result, amplify-and-forward cooperation has an average performance comparable to the non-cooperative case. In contrast, simulation results show that, by coalition formation using decode-and-forward, the average secrecy rate per user is increased of up to 25.3% and 24.4% relative to the non-cooperative and amplify-and-forward cases, respectively. Finally, the results briefly discuss how the users can self-organize and adapt the topology to mobility.

8.7 Paper G

W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Network Formation Games among the Relay Stations in Next Generation Wireless Networks," submitted to *IEEE Transactions on Communications*, January 2010.

This paper studies the problem of building the network tree structure that connects a base station to the relay stations in its coverage area in next generation wireless networks such as LTE-Advanced or WiMAX 802.16j. In this context, given a number of relay stations that need to transmit, through multi-hop communications to a base station, the data they receive from external mobile stations, we study how the relay stations can interact to select their next hop to eventually form the network's uplink tree structure. In this model, the mobile stations, considered as external data sources (queues with Poisson arrivals), connect to a serving relay station and deposit their packets. Subsequently, the relay stations transmit the data to the base station using cooperative transmission through the multi-hop decode-and-forward relaying channel considered in [132]. As a result, from the perspective of each relay station, there exists a tradeoff between the bit error rate reduction resulting from cooperative transmission as well as the delay incurred by multi-hop transmission.

Consequently, we formulate a network formation game among the relay stations where each relay station aims to optimize a cross-layer utility function that captures the gains from cooperative transmission, in terms of a reduced bit error rate and improved effective throughput, as well as the costs incurred by multi-hop transmission in terms of delay. The proposed utility is based on the queueing concept of power, which is a suited metric for evaluating the tradeoff between throughput and delay.

For building the tree structure, we propose a myopic best response-based network formation algorithm using which the relay stations engage in pairwise negotiations for selecting their next hop. In this algorithm, an given relay station i suggests to form a link with another relay station j, and, subsequently, relay station j has the opportunity to either accept or reject the offer. As a result of this pairwise negotiation process, each relay station ultimately selects the feasible strategy (i.e., strategy accepted by both initiating and accepting relay stations) that maximizes its utility. We show that, through the proposed algorithm, the relay stations are able to self-organize into a Nash network tree structure rooted at the serving base station. Moreover, we demonstrate how, by periodic runs of the algorithm, the relay stations can take autonomous decisions to adapt the network structure to environmental changes such as incoming traffic due to new mobile stations being deployed as well as mobility.

Through simulations, we show that the proposed network formation algorithm leads to a performance gain, in terms of average utility per mobile station, of at least 21.5% compared to the case of direct transmission with no relay station and up to 45.6% compared to a nearest neighbor algorithm.

Note that, base on this work, in [88, 144], we studied, respectively, the tree formation in an 802.16j network as well as for networks with Voice over IP services.

9 Summary of the Main Contributions of the Dissertation

The main contributions of this dissertation can be summarized as follows:

- Provided a novel classification of coalitional game theory suited for wireless and communications networks.

- Proposed a unified reference on coalitional game theory suited for engineering applications.

- Explored and analyzed the potential of applying coalitional game theory in a wide range of disciplines such as virtual MIMO, cognitive radio networks, physical layer security, autonomous agents, and multi-hop networks.

- Proposed and analyzed a coalitional game based model for distributed virtual MIMO formation which accounts for the tradeoff between the gains of cooperation, in terms of improved sum-rate, and the costs in terms of power for information exchange.
- Proposed and analyzed a distributed algorithm, based on coalition formation games, for virtual MIMO formation through cooperation among a number of single antenna transmitters in the uplink of a wireless network.
- Proposed and analyzed a novel coalition formation game model for distributed collaborative spectrum sensing among the secondary users in a cognitive radio network that captures the tradeoff between the cooperation improvement, in terms of reduced probability of miss and the costs in terms of an increased false alarm probability.
- Devised an algorithm for distributed coalition formation for improving the spectrum sensing performance of the secondary users in a cognitive radio network
- Proposed a cooperation model, using coalitional games in partition form, that allows the secondary users of a multi-channel cognitive network to share their sensing results and jointly optimize their spectrum sensing and spectrum access performance.
- Proposed and analyzed an algorithm, based on coalition formation games in partition form, that allows the secondary users in a multi-channel cognitive network to reduce their sensing time, learn from their cooperating partners the existence of alternative channels with better conditions, reduce the interference among each other, and exploit multiple channels simultaneously.
- Introduced a novel model for deployment of autonomous agents in wireless and communication networks that combines concepts from coalitional game theory, queueing theory, polling systems, and wireless networks.
- Proposed and analyzed a coalition formation algorithm, based on hedonic coalitional games, that enables a number of wireless agents to autonomously self-organize and distribute a number of arbitrarily located tasks among each others.

- Proposed and analyzed a cooperation model, based on coalitional game theory with non-transferable utility, that improves the security of wireless transmission in the presence of eavesdroppers.
- Proposed and analyzed a distributed algorithm, using coalition formation games, that allows the wireless users to cooperate while improving their secrecy rate, given the costs in terms of secrecy rate loss for information exchange prior to cooperation.
- Proposed and analyzed a network formation games model for building the uplink tree structure connecting the relay station nodes and their serving base station in next generation wireless networks such as LTE-Advanced or WiMAX 802.16j.
- Proposed and analyzed a distributed myopic network formation algorithm that allows the relay station to form the network structure among each others while maximizing cross-layer utility function that captures the gains from multi-hop cooperative transmission, in terms of reduced bit error rate, and the costs, in terms of delay incurred by multi-hop communication.
- For all algorithms, studied how coalition formation, through periodic decisions, allows to adapt the users' cooperative strategies to environmental changes such as slow mobility, arrival or departure of nodes, among others.
- For all approaches, analyzed and studied the properties, characteristics, and stability of the resulting network structures through adequate concepts from game theory.
- For all models and algorithms, performed extensive simulations that highlighted the key aspects of the proposed models as well as the performance and characteristics of the devised coalition formation or network formation algorithms.

10 Suggestions for Future Research and Extensions

In this section, we discuss potential future directions for the different approaches proposed in this dissertation.

First, for virtual MIMO formation, while this dissertation focused mainly on the uplink case and the transmitters cooperation, two important future extensions are: (i)- Studying the possibility of joint transmitters and receivers cooperation, in the presence of gains and costs for cooperation, and (ii)- Proposing coalition formation models for cooperation in the downlink of wireless networks (e.g., using dirty paper coding for example, or other techniques). Further, while this dissertation investigated virtual MIMO coalition formation in a multiple access TDMA channel, another important future extension of this work is to consider the transmitters cooperation in an interference channel. The main challenge in such an extension lies in the need for the framework of coalitional game theory in partition form for modeling the problem, due to the dependence of any defined utility on the external structure of the users (due to the dependence of the interference on the formed coalitions in that case).

For collaborative spectrum sensing, in this dissertation, it was assumed that all the secondary users are trusted users. One important future direction is to consider the case of coalition formation in the presence of malicious nodes. The presence of malicious nodes would strongly impact the cooperative strategies of the cognitive users, as they would be required to learn the trust value of each user prior to making a decision on whether to cooperate or not. In this regards, it would be of interest, in such a scenario, to combine the proposed coalition formation algorithm with a learning algorithm that can help in identifying malicious nodes. Further extensions of distributed collaborative spectrum sensing include studying these approaches between secondary base stations, as well as combining collaborative spectrum sensing through coalition formation with advanced signal processing techniques such as transmit beamforming or interference cancelation.

Further, in this dissertation, we proposed a coalitional game model in partition form for joint spectrum sensing and access. In the proposed model, for avoiding interference and distributing power over their seized channels, the secondary users employ a heuristic sorting algorithm and a basic social welfare maximizing optimization problem, respectively. For future work, it is of interest to study: (i)- an optimal sorting algorithm for interference avoidance within each coalition, and (ii)- advanced techniques for power allocation through concepts such as non-cooperative Nash game or Nash bargaining. Further, this dissertation did not consider the possibility of sensing errors, i.e., false alarm or probability of miss, in this model. For the future, it is of interest to modify the utility in order to account for the sensing errors. In addition, it would important to combine this work with distributed collaborative spectrum sensing, in order to further improve the sensing performance of the cognitive users.

For the deployment of wireless agents in communication networks, this dissertation introduced a generic model for task allocation among the agents. An important future direction for this work is to consider specific applications for the model, such as in ferry message passing applications, sensor networks, video surveillance, or next generation wireless systems. Further, in this dissertation, each coalition was modeled using basic concepts from polling systems. As polling systems are receiving more and more attention recently, it is of interest to improve the proposed utility and model by using advanced queueing techniques from polling systems. Another extension for this work is to consider advanced relaying techniques, e.g., cooperative transmission, for transmitting the data from the queue (tasks) to the central receiver.

In this dissertation, we considered a cooperation model for improving the physical layer security of the users which assumes that the users are aware of the locations/channels of the eavesdroppers. A very important future extension is to consider that the users are unaware of the eavesdroppers' location and are required to estimate these locations. For doing so, one direction is to combined some concepts from Bayesian learning with the proposed coalition formation algorithm. Further, in this dissertation, it was assumed that the eavesdroppers are always non-cooperative. However, similar to the users, the eavesdroppers can also engage in coalition formation, and collude to reduce the secrecy rate of the users and improve the damage that they incur on these users. Thus, as a future direction, it is of interest to: (i)- propose cooperative models among the eavesdroppers, and (ii)- propose coalition formation models that occur jointly at the users and eavesdroppers side. Finally, beyond transmit beamforming and relaying, other cooperative protocols can be accommodated for improving the secrecy rates of the users in a wireless network in the presence of eavesdroppers.

For multi-hop networks, this dissertation proposed a network formation game formulation that allows the relay stations to communicate with their serving base station using cooperative transmission while taking into account the delay incurred by multi-hop transmission. The proposed scheme considered myopic approaches for network formation where the relay stations seek to optimize their short-term performance without accounting for future evolution of the network. One important future extension in this regards is to consider far sighted network formation algorithms for uplink tree formation. Moreover, in this work, the mobile stations were considered as external entities and their choice of relay stations was considered as fixed (each mobile selects the relay station that maximizes its utility). For future work, it would be interesting to consider two jointly related games consisting of the mobile station assignment game and the network formation game among the relay stations. One approach to solve such a model can be sought in the framework of multi-leader multi-follower Stackleberg games where the leaders would be the mobile stations and the followers would be the relay stations. Furthermore, in game theory literature related to network formation games, there has been a recent interest in studying stability notions such as pairwise stability or strong stability. For this purpose, a future extension of this contribution in network formation games is to propose algorithm that can yield pairwise stable or strongly stable structures, which can be deemed more suitable than the Nash networks sought in this dissertation. Additional extensions of this work can also include considering jointly the uplink and downlink tree structures as well as allowing the relay stations to use mixed strategies.

Moreover, while this dissertation mainly focused on theoretical and algorithmic approaches to coalition formation, corroborated by analytical and numerical simulation results, an important and interesting future extension is to consider practical aspects of deploying these algorithms in real-life wireless networks. For doing so, the use of test beds or advanced simulators would be needed. Finally, beyond the applications of coalitional game theory presented in this dissertation, the framework can also be explored in numerous potential future applications such as vehicular networks, wireless sensor networks, advanced cognitive networks, social networks, peer-to-peer networks, queueing systems, and many others.

11 Journal and Conference Contributions during Ph.D. Studies

During the Ph.D. studies, the author has contributed to the following journals and conference publications:

List of Journal Publications during Ph.D. Studies:

- W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, “Coalitional Game Theory for Communication Networks: A Tutorial,” *IEEE Signal Processing Magazine, Special Issue on Game Theory for Signal*

Processing and Communication, vol. 26, no. 5, pp. 77-97, September 2009.

- W. Saad, Z. Han, M. Debbah, and A. Hjørungnes, "A Distributed Coalition Formation Framework for Fair User Cooperation in Wireless Networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 4580 - 4593, September 2009.
- W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Hedonic Coalition Formation for Distributed Task Allocation among Wireless Agents," conditionally accepted for publication in *IEEE Transactions on Mobile Computing* (subject to reviewers' and editor's final approval of the revised manuscript submitted 03-02-2010).
- W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Distributed Coalition Formation Games for Secure Wireless Transmission," submitted to *ACM/Springer Journal on Mobile Networks and Applications*, October 2009.
- W. Saad, Z. Han, R. Zheng, A. Hjørungnes, and T. Başar, "Coalitional Games in Partition Form for Joint Spectrum Sensing and Access in Cognitive Radio Networks," submitted to *the IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Advances on Cognitive Radio Networking and Communications*, December 2009.
- W. Saad, Z. Han, A. Hjørungnes, D. Niyato, and E. Hossain, "Coalition Formation Games for Distributed Roadside Units Cooperation in Vehicular Networks," submitted to *the IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Vehicular Communications and Networks*, December 2009.
- D. Niyato, P. Wang, E. Hossain, W. Saad, and A. Hjørungnes, "Exploiting Mobility Diversity in Wireless Access for Vehicular Telematics: A Game-Theoretic Approach," submitted to *the IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Vehicular Communications and Networks*, December 2009.
- W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Network Formation Games among the Relay Stations in Next Generation Wireless Networks," submitted to *IEEE Transactions on Communications*, January 2010.

List of Conference Publications during Ph.D. Studies:

- W. Saad, T. Alpcan, T. Başar, and A. Hjørungnes, "Coalitional Game Theory for Security Risk Management," in Proceedings of *the Fifth International Conference on Internet Monitoring and Protection (ICIMP)*, Barcelona, Spain, May 2010.

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- D. Niyato, P. Wang, W. Saad, and A. Hjørungnes, "Coalition Formation Games for Bandwidth Sharing in Vehicle-to-Roadside Communications," in Proceedings of *the IEEE Wireless Communications and Networking Conference (WCNC)*, Networking Symposium, Sydney, Australia, April 2010.

- W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Coalitional Games for Distributed Eavesdroppers Cooperation in Wireless Networks," in Proceedings of *the 3rd International Workshop on Game Theory in Communication Networks (Gamecomm)*, Pisa, Italy, October, 2009.

- W. Saad, A. Hjørungnes, Z. Han, and T. Başar, "Network Formation Games for Wireless Multi-hop Networks in the Presence of Eavesdroppers," invited paper, in Proceedings of *the 3rd International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, Aruba, December, 2009.

- W. Saad, Q. Zhu, T. Başar, Z. Han, and A. Hjørungnes, "Hierarchical Network Formation Games in the Uplink of Multi-hop Wireless Networks," in Proceedings of *the IEEE Global Communications Conference (GLOBECOM), Wireless Networking Symposium*, Hawaii, USA, December 2009.

- W. Saad, Z. Han, T. Başar, M. Debbah, and A. Hjørungnes, "Physical Layer Security: Coalitional Games for Distributed Cooperation," in Proceedings of *the 7th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt)*, Seoul, South Korea, June 2009. **Best Paper Award.**

- W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional Games for Distributed Collaborative Spectrum Sensing in Cognitive Radio Networks," in Proceedings of *the IEEE International Conference on Computer Communications (INFOCOM)*, Rio de Janeiro, Brazil, April 2009.
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Part II

Included Papers

Paper A

Coalitional Game Theory for Communication Networks

W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar

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Abstract

Game theoretical techniques have recently become prevalent in many engineering applications, notably in communications. With the emergence of cooperation as a new communication paradigm, and the need for self-organizing, decentralized, and autonomic networks, it has become imperative to seek suitable game theoretical tools that allow to analyze and study the behavior and interactions of the nodes in future communication networks. In this context, this tutorial introduces the concepts of cooperative game theory, namely coalitional games, and their potential applications in communication and wireless networks. For this purpose, we classify coalitional games into three categories: Canonical coalitional games, coalition formation games, and coalitional graph games. This new classification represents an application-oriented approach for understanding and analyzing coalitional games. For each class of coalitional games, we present the fundamental components, introduce the key properties, mathematical techniques, and solution concepts, and describe the methodologies for applying these games in several applications drawn from the state-of-the-art research in communications. In a nutshell, this article constitutes a unified treatment of coalitional game theory tailored to the demands of communications and network engineers.

1 Introduction and motivation

Game theory provides a formal analytical framework with a set of mathematical tools to study the complex interactions among rational players. Throughout the past decades, game theory has made revolutionary impact on a large number of disciplines ranging from engineering, economics, political science, philosophy, or even psychology [1]. In recent years, there has been a significant growth in research activities that use game theory for analyzing communication networks. This is mainly due to: (i)- The need for developing autonomous, distributed, and flexible mobile networks where the network devices can make independent and rational strategic decisions; and (ii)- the need for low complexity distributed algorithms that can efficiently represent competitive or collaborative scenarios between network entities.

In general, game theory can be divided into two branches: non-cooperative [2] and cooperative game theory [1, 3]. *Non-cooperative game theory* studies the strategic choices resulting from the interactions among *competing* players, where each player chooses its strategy independently for improving its own performance (utility) or reducing its losses (costs). For solving non-cooperative games, several concepts exist such as the celebrated Nash equilibrium [2]. The mainstream of existing research in communication networks focused on using non-cooperative games in various applications such as distributed resource allocation [4], congestion control [5], power control [6], and spectrum sharing in cognitive radio, among others. This need for non-cooperative games led to numerous tutorials and books outlining its concepts and usage in communication, e.g., [7], [8].

While non-cooperative game theory studies competitive scenarios, *cooperative game theory* provides analytical tools to study the behavior of rational players *when they cooperate*. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions [1], that can strengthen the players' positions in a game. In this tutorial, we restrict our attention to coalitional game theory albeit some other references can include other types of games, such as bargaining, under the umbrella of cooperative games. Coalitional games have also been widely explored in different disciplines such as economics or political science. Recently, cooperation has emerged as a new networking paradigm that has a dramatic effect of improving the performance from the physical

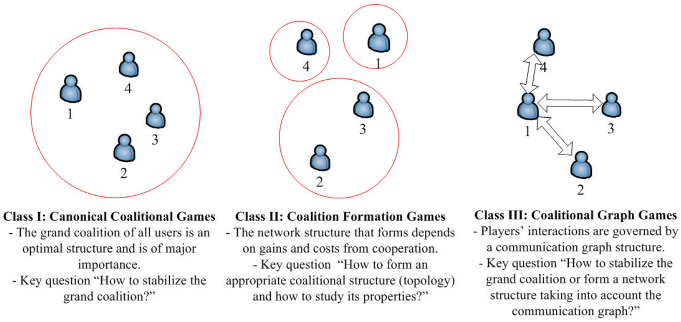


Fig. A.1: A novel classification of coalitional games.

layer [9], [10] up to the networking layers [4]. However, implementing cooperation in large scale communication networks faces several challenges such as adequate modeling, efficiency, complexity, and fairness, among others. Coalitional games prove to be a very powerful tool for designing fair, robust, practical, and efficient cooperation strategies in communication networks. Most of the current research in the field is restricted to applying standard coalitional game models and techniques to study very limited aspects of cooperation in networks. This is mainly due to the sparsity of the literature that tackles coalitional games. In fact, most pioneering game theoretical references, such as [1–3], focus on non-cooperative games; touching slightly on coalitional games within a few chapters.

In this article, we aim to provide a unified treatment of coalitional game theory oriented towards engineering applications. Thus, the goal is to gather the state-of-the-art research contributions, from game theory and communications, that address the major opportunities and challenges in applying coalitional games to the understanding and designing of modern communication systems, with emphasis on both new analytical techniques and novel application scenarios. With the incessant growth in research revolving around cooperation, self-organization and fairness in communication networks, this tutorial constitutes a comprehensive guide that enables to fully exploit the potential of coalitional game theory. The tutorial starts by laying out the main components of coalitional games in Section 2 while in the following sections it zooms in on an in-depth study of these games and their applications. Since the literature on coalitional games and their communication applications is sparse, we introduce a novel classification of coalitional games which allows grouping of various types of games under one class based on several game properties. Hence, we group coalitional games into three distinct classes:

1. **Class I:** Canonical (coalitional) games[11]
2. **Class II:** Coalition formation games
3. **Class III:** Coalitional graph games

This novel classification is intended to provide an application-oriented approach to coalitional games. The key features of these classes are summarized in Fig. A.1 and an in-depth study of each class is provided in Sections 3, 4, and 5.

2 Coalitional Game Theory: Preliminaries

In essence, coalitional games involve a set of players, denoted by $\mathcal{N} = \{1, \ldots, N\}$ who seek to form cooperative groups, i.e., coalitions, in order to strengthen their positions in the game. Any coalition $S \subseteq \mathcal{N}$ represents an agreement between the players in S to act as a single entity. The formation of coalitions or alliances is ubiquitous in many applications. For example, in political games, parties, or individuals can form coalitions for improving their voting power. In addition to the player set $\mathcal{N}$, the second fundamental concept of a coalitional game is the coalition *value*. Mainly, the coalition value, denoted by v, quantifies the worth of a coalition in a game. The definition of the coalition value determines the *form* and *type* of the game. Nonetheless, independent of the definition of the value, a coalitional game is uniquely defined by the pair $(\mathcal{N}, v)$. It must be noted that the value v is, in many instances, referred to as *the game*, since for every v a different game may be defined.

The most common form of a coalitional game is the *characteristic form*, whereby the value of a coalition S depends *solely* on the members of that coalition, with no dependence on how the players in $\mathcal{N} \setminus S$ are structured. The characteristic form was introduced, along with a category of coalitional games known as games with *transferable utility* (TU), by Von Neuman and Morgenstern [11]. The value of a game in characteristic form with TU is a function over the real line defined as $v : 2^{\mathcal{N}} \to \mathbb{R}$ (characteristic function). This characteristic function associates with every coalition $S \subseteq \mathcal{N}$ a real number quantifying the gains of S. The TU property implies that the total utility represented by this real number can be divided in any manner between the coalition members. The values in TU games are thought of as monetary values that the members in a coalition can distribute among themselves using an appropriate *fairness* rule (one such rule being an

[11] We will use the terminologies "canonical coalitional games" and "canonical games" interchangeably throughout this tutorial.

equal distribution of the utility). The amount of utility that a player $i \in S$ receives from the division of $v(S)$ constitutes the player's *payoff* and is denoted by x_i hereafter. The vector $\boldsymbol{x} \in \mathbb{R}^{|S|}$ ($| \cdot |$ represents the cardinality of a set) with each element x_i being the payoff of player $i \in S$ constitutes a *payoff allocation* (alternatively one can use the notation $\mathbb{R}^S$ for representing the set of all real valued functions over the set S, i.e., the set of payoff vectors achievable by the players in S). Although the TU characteristic function can model a broad range of games, many scenarios exist where the coalition value cannot be assigned a single real number, or rigid restrictions exist on the distribution of the utility. These games are known as *coalitional games with non-transferable utility (NTU)* and were first introduced by Aumann and Peleg using non-cooperative strategic games as a basis [1, 12]. In an NTU game, the payoff that each player in a coalition S receives is dependent on the joint actions that the players of coalition S select[12]. The value of a coalition S in an NTU game, $v(S)$, is no longer a function over the real line, but a set of payoff vectors, $v(S) \subseteq \mathbb{R}^{|S|}$, where each element x_i of a vector $\boldsymbol{x} \in v(S)$ represents a payoff that player $i \in S$ can obtain within coalition S given a certain strategy selected by i while being a member of S. Given this definition, a TU game can be seen as a particular case of the NTU framework [1]. Coalitional games in characteristic form with TU or NTU constitute one of the most important types of games, and their solutions are explored in detail in the following sections.

Recently, there has been an increasing interest in coalitional games where the value of a coalition depends on the partition of $\mathcal{N}$ that is in place at any time during the game. In such games, unlike the characteristic form, the value of a coalition S will have a strong dependence on how the players in $\mathcal{N} \setminus S$ are structured. For this purpose, Thrall and Lucas [13] introduced the concept of games in *partition form*. In these games, given a *coalitional structure* $\mathcal{B}$, defined as a *partition* of $\mathcal{N}$, i.e., a collection of coalitions $\mathcal{B} = \{B_1, \ldots, B_l\}$, such that $\forall\ i \neq j,\ B_i \cap B_j = \emptyset$, and $\cup_{i=1}^{l} B_i = \mathcal{N}$, the value of a coalition $S \in \mathcal{B}$ is defined as $v(S, \mathcal{B})$. This definition imposes a dependence on the coalitional structure when evaluating the value of S. Coalitional games in partition form are inherently complex to solve; however, the potential of these games is interesting and, thus, we will provide insights on these games in the following sections.

As an example on the difference between characteristic and partition forms, consider a 5-players game with $\mathcal{N} = \{1, 2, 3, 4, 5\}$ and let $S_1 = \{1, 2, 3\}$,

[12] The action space depends on the underlying non-cooperative game (see [12] for examples).

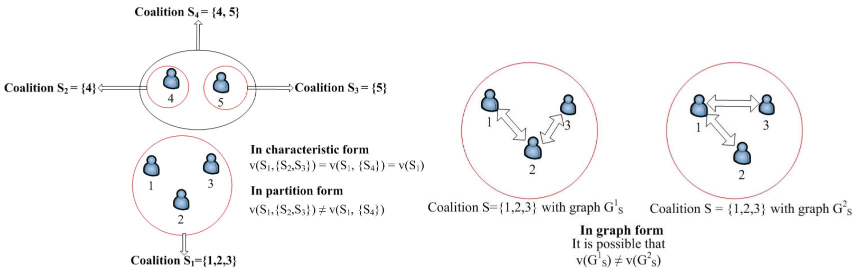


Fig. A.2: (a) Coalitional games in characteristic form vs. partition form. (b) Example of a coalitional game in graph form.

$S_2 = \{4\}$, $S_3 = \{5\}$, and $S_4 = \{4, 5\}$. Given two partitions $\mathcal{B}_1 = \{S_1, S_2, S_3\}$ and $\mathcal{B}_2 = \{S_1, S_4\}$ of $\mathcal{N}$, evaluating the value of coalition S_1 depends on the form of the game. If the game is in *characteristic form*, then $v(S_1, \mathcal{B}_1) = v(S_1, \mathcal{B}_2) = v(S_1)$ while in *partition form* $v(S_1, \mathcal{B}_1) \neq v(S_1, \mathcal{B}_2)$ (the value here can be either TU or NTU). The basic difference is that, unlike the characteristic form, the value of S_1 in partition form depends on whether players 4 and 5 cooperate or not. This is illustrated in Fig. A.2 (a).

In many coalitional games, the players are interconnected and communicate through pairwise links in a graph. In such scenarios, both the characteristic form and the partition form may be unsuitable since, in both forms, the value of a coalition S is independent of how the members of S are connected. For modeling the interconnection graphs, coalitional games in *graph form* were introduced by Myerson in [14] where *connected* graphs were mapped into coalitions. This work was generalized in [15] by making the value of each coalition $S \subseteq \mathcal{N}$ a function of the graph structure connecting the members of S. Hence, given a coalitional game $(\mathcal{N}, v)$ and a graph G_S (directed or undirected) with vertices the members of a coalition $S \subseteq \mathcal{N}$, the value of S in *graph form* is given by $v(G_S)$. For games in graph form, the value can also depend on the graph $G_{\mathcal{N} \setminus S}$ interconnecting the players in $\mathcal{N} \setminus S$. An example of a coalitional game in graph form is given in Fig. A.2 (b). In this figure, given two graphs $G_S^1 = \{(1, 2), (2, 3)\}$ and $G_S^2 = \{(1, 2), (1, 3)\}$ (a pair (i, j) is a link between two players i and j) defined over coalition $S = \{1, 2, 3\}$, a coalitional game in graph form could assign a different value for coalition S depending on the graph[13]. Hence, in graph form, it is possible that $v(G_S^1) \neq v(G_S^2)$, while in characteristic or

[13]In this example we considered an undirected graph and a single link between every pair of nodes. However, multiple links between pairs of nodes as well as directed graphs can also be considered within the graph form of coalitional games.

partition form, the presence of the graph does not affect the value. Having introduced the fundamental concepts for coalitional games, the rest of this tutorial provides an in-depth analysis of each class of games.

3 Class I: Canonical Coalitional Games

3.1 Main Properties of Canonical Coalitional Games

Under the class of canonical coalitional games, we group the most popular category of games in coalitional game theory. Hence, this class pertains to the coalitional games tools that have been widely understood, thoroughly formalized, and have clear solution concepts. For classifying a game as canonical, the main requirements are as follows:

1. The coalitional game is in *characteristic* form (TU or NTU).
2. Cooperation, i.e., the formation of large coalitions, is *never* detrimental to any of the involved players. Hence, in canonical games no group of players can do worse by cooperating, i.e., by joining a coalition, than by acting non-cooperatively. This pertains to the mathematical property of *superadditivity*.
3. The main objectives of a canonical game are: (i)- To study the properties and stability of the *grand coalition*, i.e., the coalition of all the players in the game, and (ii)- to study the gains resulting from cooperation with negligible or no cost, as well as the distribution of these gains in a *fair* manner to the players.

The first two conditions for classifying a game as canonical pertain to the mathematical properties of the game. First, any canonical game must be in characteristic form. Second, the canonical game must be superadditive, which is defined as

$$v(S_1 \cup S_2) \supset \{\boldsymbol{x} \in \mathbb{R}^{|S_1 \cup S_2|} | (x_i)_{i \in S_1} \in v(S_1), (x_j)_{j \in S_2} \in v(S_2)\}$$
$$\forall S_1 \subset \mathcal{N}, S_2 \subset \mathcal{N}, \text{ s.t. } S_1 \cap S_2 = \emptyset, \qquad \text{(A.1)}$$

where $\boldsymbol{x}$ is a payoff allocation for coalition $S_1 \cup S_2$. Superadditivity implies that, given any two disjoint coalitions S_1 and S_2, if coalition $S_1 \cup S_2$ forms, then it can give its members any allocations they can achieve when acting in S_1 and S_2 separately. The definition in (A.1) is used in an NTU case. For a TU game, superadditivity reduces to [1]

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \ \forall S_1 \subset \mathcal{N}, S_2 \subset \mathcal{N}, \text{ s.t. } S_1 \cap S_2 = \emptyset. \qquad \text{(A.2)}$$

From (A.2), the concept of a superadditive game is better grasped. Simply, a game is superadditive if cooperation, i.e., the formation of a large coalition out of disjoint coalitions, guarantees at least the value that is obtained by the disjoint coalitions separately. The rationale behind superadditivity is that, within a coalition, the players can always revert back to their non-cooperative behavior to obtain their non-cooperative payoffs. Thus, in a superadditive game, cooperation is always beneficial. Due to superadditvity in canonical games, it is to the joint benefit of the players to always form the *grand coalition* $\mathcal{N}$, i.e, the coalition of *all* the players, since the payoff received from $v(\mathcal{N})$ is at least as large as the amount received by the players in any disjoint set of coalitions they could form. The formation of the grand coalition in canonical games implies that the main emphasis is on studying the properties of this grand coalition. Two key aspects are of importance in canonical games: (i)- Finding a payoff allocation which guarantees that no group of players have an incentive to leave the grand coalition (having a *stable* grand coalition), and (ii)- assessing the gains that the grand coalition can achieve as well as the fairness criteria that must be used for distributing these gains (having a *fair* grand coalition). For solving canonical coalitional games, the literature presents a number of concepts [1, 3] that we will explore in detail in the following sections.

3.2 The Core as a Solution for Canonical Coalitional Games

3.2.1 Definition

The most renowned solution concept for coalitional games, and for games classified as canonical in particular, is *the core* [1, 3]. The core of a canonical game is directly related to the grand coalition's stability. In a canonical coalitional game $(\mathcal{N}, v)$, due to superadditvity, the players have an incentive to form the grand coalition $\mathcal{N}$. Thus, the core of a canonical game is the set of payoff allocations which guarantees that no group of players has an incentive to leave $\mathcal{N}$ in order to form another coalition $S \subset \mathcal{N}$. For a TU game, given the grand coalition $\mathcal{N}$, a payoff vector $\boldsymbol{x} \in \mathbb{R}^N$ ($N = |\mathcal{N}|$) for dividing $v(\mathcal{N})$ is *group rational* if $\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N})$. A payoff vector $\boldsymbol{x}$ is *individually rational* if every player can obtain a benefit no less than acting alone, i.e. $x_i \geq v(\{i\}), \forall\ i \in \mathcal{N}$. An *imputation* is a payoff vector satisfying the above two conditions. Having defined an imputation, *the core* is defined as

$$\mathcal{C}_{\text{TU}} = \left\{ \boldsymbol{x} : \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) \text{ and } \sum_{i \in S} x_i \geq v(S) \ \forall \ S \subseteq \mathcal{N} \right\}. \tag{A.3}$$

In other words, the core is the set of imputations where no coalition $S \subset \mathcal{N}$ has an incentive to reject the proposed payoff allocation, deviate from the grand coalition and form coalition S instead. The core guarantees that these deviations do not occur through the fact that any payoff allocation $\boldsymbol{x}$ that is in the core guarantees at least an amount of utility equal to $v(S)$ for every $S \subset \mathcal{N}$. Clearly, whenever one is able to find a payoff allocation that lies in the core, then the grand coalition is a stable and optimal solution for the coalitional game. For solving NTU games using the core, the value v of the NTU game is often assumed to satisfy the following, for any coalition S, [1]: (1)- The value $v(S)$ of any coalition S must be a closed and convex subset of $\mathbb{R}^{|S|}$, (2)- the value $v(S)$ must be *comprehensive*, i.e., if $\boldsymbol{x} \in v(S)$ and $\boldsymbol{y} \in \mathbb{R}^{|S|}$ are such that $\boldsymbol{y} \leq \boldsymbol{x}$, then $\boldsymbol{y} \in v(S)$, and (3)- the set $\{\boldsymbol{x} | \boldsymbol{x} \in v(S) \text{ and } x_i \geq z_i, \ \forall i \in S\}$ with $z_i = \max \{y_i | \boldsymbol{y} \in v(\{i\})\} < \infty \ \forall i \in \mathcal{N}$ must be a bounded subset of $\mathbb{R}^{|S|}$. The comprehensive property implies that if a certain payoff allocation $\boldsymbol{x}$ is achievable by the members of a coalition S, then, by changing their strategies, the members of S can achieve any allocation $\boldsymbol{y}$ where $\boldsymbol{y} \leq \boldsymbol{x}$. The last property implies that, for a coalition S, the set of vectors in $v(S)$ in which each player in S receives no less than the maximum that it can obtain non-cooperatively, i.e., z_i, is a bounded set. For a canonical NTU game $(\mathcal{N}, v)$ with v satisfying the above properties, the core is defined as

$$\mathcal{C}_{\text{NTU}} = \{\boldsymbol{x} \in v(\mathcal{N}) | \forall S, \nexists \boldsymbol{y} \in v(S), \text{ s.t. } y_i > x_i, \ \forall i \in S\}. \tag{A.4}$$

This definition for NTU also guarantees a stable grand coalition. The basic idea is that any payoff allocation in the core of an NTU game guarantees that no coalition S can leave the grand coalition and provide a better allocation *for all* of its members. The difference from the TU case is that, in the NTU core, the grand coalition's stability is acquired over the elements of the payoff vectors while in the TU game, it is acquired by the sum of the payoff vectors' elements.

3.2.2 Properties and Existence

The cores of TU or NTU canonical games are not always guaranteed to exist. In fact, in many games, the core is empty and hence, the grand coalition cannot be stabilized. In these cases, alternative solution concepts may be used, as we will see in the following sections. However, coalitional game theory provides several categories of games which fit under our canonical game class, where the core is guaranteed to be non-empty. Before surveying the existence results for the core, we provide a simple example of the core in a TU canonical game:

Example 1 Consider a majority voting TU game $(\mathcal{N}, v)$ where $\mathcal{N} = \{1, 2, 3\}$. The players, on their own, have no voting power, hence $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. Any 2-players coalition wins two thirds of the voting power, and hence, $v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = \frac{2}{3}$. The grand coalition wins the whole voting power, and thus $v(\{1,2,3\}) = 1$. Clearly, this game is superadditive and is in characteristic form and thus is classified as canonical. By (A.3), solving the following inequalities yields the core and shows what allocations stabilize the grand coalition.

$$x_1 + x_2 + x_3 = v(\{1,2,3\}) = 1,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,$$
$$x_1 + x_2 \geq v(\{1,2\}) = \frac{2}{3},\ x_1 + x_3 \geq v(\{1,3\}) = \frac{2}{3},\ x_2 + x_3 \geq v(\{2,3\}) = \frac{2}{3}.$$

By manipulating these inequalities, the core of this game is found to be the *unique* vector $\boldsymbol{x} = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$ which corresponds of an equal division of the total utility of the grand coalition among all three players.

In general, given a TU coalitional game $(\mathcal{N}, v)$ and an imputation $\boldsymbol{x} \in \mathbb{R}^N$, the core is found by a linear program (LP)

$$\min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} x_i,\ \text{s.t.}\ \sum_{i \in S} x_i \geq v(S),\ \forall S \subseteq \mathcal{N}. \tag{A.5}$$

The existence of the TU core is related to the feasibility of the LP in (A.5). In general, finding whether the core is non-empty through this LP, is NP-complete [16] due to the number of constraints growing exponentially with the number of players N (this is also true for NTU games, see [1, Ch. 9.7]). However, for determining the non-emptiness of the core as well as finding the allocations that lie in the core several techniques exist and are summarized in Table A.I.

Table A.I: Approaches for finding the core of a canonical coalitional game

Game theoretical and mathematical approaches	
	(T1) - A *graphical* approach can be used for finding the core of TU games with up to 3 players.
	(T2) - Using duality theory, a necessary and sufficient condition for the non-emptiness of the core exists through the *Bondareva-Shapley theorem* (Theorem 1) for TU and NTU [1, 3] .
	(T3) - A class of canonical games, known as *convex coalitional games* always has a non-empty core.
	(T4) - A necessary and sufficient condition for a non-empty core exists for a class of canonical games known as *simple games*, i.e., games where $v(S) \in \{0,1\},\ \forall S \subseteq \mathcal{N}$ and $v(\mathcal{N}) = 1$.
Application-specific approaches	
	(T5) - In several applications, it suffices to find whether payoff distributions that are of interest in a given game, e.g., fair distributions, lie in the core.
	(T6)- In many games, exploiting game-specific features such as the value's mathematical definition or the underlying nature and properties of the game model, helps finding the imputations that lie in the core.

The *first* technique in Table A.I deals with TU games with up to 3 players. In such games, the core can be found using an easy graphical approach. The main idea is to plot the constraints of (A.5) in the plane $\sum_{i=1}^{3} x_i = v(\{1,2,3\})$. By doing so, the region containing the core allocation can be easily identified. Several examples on the graphical techniques are found in [3] and the technique for solving them is straightforward. Although the graphical method can provide a lot of intuition into the core of a canonical game, its use is limited to TU games with up to 3 players.

The *second* technique in Table A.I utilizes the dual of the LP in (A.5) to show that the core is non-empty. The main result is given through the Bondareva-Shapley theorem [1, 3] which relies on the *balanced* property. A TU game is *balanced* if and only if the inequality [1]

$$\sum_{S \subseteq \mathcal{N}} \mu(S) v(S) \leq v(\mathcal{N}), \tag{A.6}$$

is satisfied for all non-negative weight collections $\mu = (\mu(S))_{S \subseteq \mathcal{N}}$ (μ is a collection of weights, i.e., numbers in $[0,1]$, associated with each coalition $S \subseteq \mathcal{N}$) which satisfy $\sum_{S \ni i} \mu(S) = 1,\ \forall i \in \mathcal{N}$; this set of non-negative weights is known as a balanced set. This notion of a balanced game is interpreted as follows. Each player $i \in \mathcal{N}$ possesses a single unit of time, which can be distributed between all the coalitions that i can be a member of. Every coalition $S \subseteq \mathcal{N}$ is active during a fraction of time $\mu(S)$ if all of its members are active during that time, and this coalition achieves a payoff of $\mu(S)v(S)$. In this context, the condition $\sum_{S \ni i} \mu(S) = 1,\ \forall i \in \mathcal{N}$ is simply a feasibility constraint on the players' time allocation, and the game is balanced if there is no feasible allocation of time which can yield a total payoff for the players that exceeds the value of the grand coalition $v(\mathcal{N})$. For NTU canonical games, two different definitions for balancedness exist (one of which is analogous to the TU case) and can be found in [1, 3]. The definitions for NTU accommodate the fact that the value v in an NTU game is a set and not a function. Subsequently, given a TU balanced canonical game, the following result holds [1, 3].

Theorem 1 *(Bondareva-Shapley) The core of a game is non-empty if and only if the game is balanced.* $\diamond$

For NTU, the Bondareva-Shapley theorem is also true however in that case the balanced condition presented is sufficient but not necessary as a second definition for a balanced game also exists (see [1, 3]). Therefore, in a given canonical game, one can always show that the core is non-empty by proving that the game is balanced through (A.6) for TU games or its counterparts for NTU [1, Ch. 9.7]. Proving the non-emptiness of the core through the balanced property is a popular approach and several examples on balanced games exist in the game theory literature [1, 3] as well as in the literature on communication networks [17, 18].

The *third* technique in Table A.I pertains to *convex* games. A TU canonical game is convex if

$$v(S_1) + v(S_2) \leq v(S_1 \cup S_2) + v(S_1 \cap S_2) \;\forall\; S_1, S_2 \subseteq \mathcal{N} \tag{A.7}$$

This convexity property implies that the value function, i.e., the game, is supermodular. Alternatively, a convex coalitional game is defined as any coalitional game that satisfies $v(S_1 \cup \{i\}) - v(S_1) \leq v(S_2 \cup \{i\}) - v(S_2)$, whenever $S_1 \subseteq S_2 \subseteq \mathcal{N} \setminus \{i\}$. This alternative definition implies that a game is convex if and only if for each player $i \in \mathcal{N}$ the marginal contribution of this player, i.e. the difference between the value of a coalition with and without this player, is nondecreasing with respect to set inclusion. The convexity property can also be extended to NTU in several ways, and the reader is referred to [3, Ch. 9.9] for more details. For both TU and NTU canonical games, a convex game is balanced and *has a non-empty* core, but the converse is not always true [3]. Thus, convex games constitute an important class of games where the core is non-empty. Examples of such games are ubiquitous in both game theory [1, 3] and communications [17].

The *fourth* technique pertains to *simple games* which are an interesting class of canonical games where the core can be shown to be non-empty. A simple game is a coalitional game where the value are either 0 or 1, i.e., $v(S) \in \{0, 1\}$, $\forall S \subseteq \mathcal{N}$ and the grand coalition has $v(\mathcal{N}) = 1$. These games model numerous scenarios, notably voting games. It is known that a simple game which contains at least one *veto* player $i \in \mathcal{N}$, i.e. a player i such that $v(\mathcal{N} \setminus \{i\}) = 0$ has a *non-empty core* [3]. Moreover, in such simple games, the core is fully characterized, and it consists of all non-negative payoff profiles $\boldsymbol{x} \in \mathbb{R}^N$ such that $x_i = 0$ for each player i that is a *non-veto* player, and $\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) = 1$

The first four techniques in Table A.I rely mainly on well-known game theoretical properties. In many practical scenarios, notably in wireless and communication networking applications, alternative techniques may be needed to find the allocations in the core. These alternatives are inherently application-specific, and depend on the nature of the defined game and the properties of the defined value function. One of these alternatives, the *fifth* technique in Table A.I, is to investigate whether well-known allocation rules yield vectors that lie in the core. In many communication applications (and even game theoretical settings), the objective is to assess whether certain well-defined types of fair allocations such as equal fairness or proportional fairness among others are in the core or not, without finding all the allocations that are in the core. In such games, showing the non-emptiness of the core is done by testing whether such well-known allocations lie in the core or not, using the intrinsic properties of the considered game and using (A.3) for TU games or (A.4) for NTU games. A simple example of such a technique is Example 1, where one can check the non-emptiness of the core by easily showing that the equal allocation lies in the core. In many canonical games, the nature of the defined value for the game can be explored for showing the non-emptiness of the core; this is done in many applications such as in [10] where information theoretical properties are used, in [19] where network properties are used, as well as in [18, 20] where the value is given as a convex optimization, and through duality, a set of allocations that lie in the core can be found. Hence, whenever techniques (T1)-(T4) are too complex or difficult to apply for solving a canonical game, as per the *sixth* technique in Table A.I, one can explore the properties of the considered game model such as in [10, 17–20].

In summary, the core is one of the most important solution concepts in coalitional games, notably in our canonical games class. It must be stressed that the existence of the core shows that the grand coalition $\mathcal{N}$ of a given $(\mathcal{N}, v)$ canonical coalitional game is stable, optimal (from a payoff perspective), and desirable.

3.3 The Shapley Value

As a solution concept, the core suffers from three main drawbacks: (i) - The core can be empty, (ii) - the core can be quite large, hence selecting a suitable core allocation can be difficult, and (iii)- in many scenarios, the allocations that lie in the core can be unfair to one or more players. These drawbacks motivated the search for a solution concept which can associate with every coalitional game $(\mathcal{N}, v)$ a *unique* payoff vector known as the *value* of the game (which is quite different from the value of a coalition). Shapley approached this problem axiomatically by defining a set of desirable properties and he characterized a unique mapping ϕ that satisfies these axioms, later known as the *Shapley value* [1]. The Shapley value was essentially defined for TU games; however, extensions to NTU games exist. In this tutorial, we restrict our attention to the Shapley value for TU canonical games, and refer the reader to [1, Ch. 9.9] for insights on how the Shapley value is extended to NTU games. Shapley provided four axioms[14] as follows (ϕ_i is the payoff given to player i by the Shapley value ϕ)

1. *Efficiency Axiom*: $\sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N})$.

2. *Symmetry Axiom*: If player i and player j are such that $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition S not containing player i and player j, then $\phi_i(v) = \phi_j(v)$.

3. *Dummy Axiom*: If player i is such that $v(S) = v(S \cup \{i\})$ for every coalition S not containing i, then $\phi_i(v) = 0$.

4. *Additivity Axiom*: If u and v are characteristic functions, then $\phi(u + v) = \phi(v + u) = \phi(u) + \phi(v)$.

Shapley showed that there exists a unique mapping, the Shapley value $\phi(v)$, from the space of all coalitional games to $\mathbb{R}^N$, that satisfies these axioms. Hence, for every game $(\mathcal{N}, v)$, the Shapley value ϕ assigns a unique payoff allocation in $\mathbb{R}^N$ which satisfies the four axioms. The efficiency axiom is in fact group rationality. The symmetry axiom implies that, when two players have the same contribution in a coalition, their assigned payoffs must be equal. The dummy axiom assigns no payoff to players that do not improve the value of any coalition. Finally, the additivity axiom

[14] In some references, the Shapley axioms are compressed into three by combining the dummy and efficiency axioms.

links the value of different games u and v and asserts that ϕ is a unique mapping over the space of all coalitional games.

The Shapley value also has an alternative interpretation which takes into account the order in which the players join the grand coalition $\mathcal{N}$. In the event where the players join the grand coalition in a *random* order, the payoff allotted by the Shapley value to a player $i \in \mathcal{N}$ is the expected marginal contribution of player i when it joins the grand coalition. The basis of this interpretation is that, given any canonical TU game $(\mathcal{N}, v)$, for every player $i \in \mathcal{N}$ the Shapley value $\phi(v)$ assigns the payoff $\phi_i(v)$ given by

$$\phi_i(v) = \sum_{S \subseteq \mathcal{N} \setminus \{i\}} \frac{|S|!(N - |S| - 1)!}{N!} [v(S \cup \{i\}) - v(S)]. \tag{A.8}$$

In (A.8), it is clearly seen that the marginal contribution of every player i in a coalition S is $v(S \cup \{i\}) - v(S)$. The weight that is used in front of $v(S \cup \{i\}) - v(S)$ is the probability that player i faces the coalition S when entering in a random order, i.e., the players in front of i are the ones already in S. In this context, there are $|S|!$ ways of positioning the players of S at the start of an ordering, and $(N - |S| - 1)!$ ways of positioning the remaining players except i at the end of an ordering. The probability that such an ordering occurs (when all orderings are equally probable) is therefore $\frac{|S|!(N-|S|-1)!}{N!}$, consequently, the resulting payoff $\phi_i(v)$ is the expected marginal contribution, under random-order joining of the players for forming the grand coalition.

In general, the Shapley value is unrelated to the core. However, in some applications, one can show that the Shapley value lies in the core. Such a result is of interest, since if such an allocation is found, it combines both the stability of the core as well as the axioms and fairness of the Shapley value. In this regard, an interesting result from game theory is that *for convex games the Shapley value lies in the core* [1, 3]. The Shapley value presents an interesting solution concept for canonical games, and has numerous applications in both game theory and communication networks. For instance, in coalitional voting simple games, the Shapley value of a player i represents its power in the game. In such games, the Shapley value is used as a power index (known as the Shapley-Shubik index), and it has a large number of applications in many game theoretical and political settings [3]. In communication networks, the Shapley value presents a suitable fairness criteria for allocating resources or data rates as in [9, 19, 21]. The computation of the Shapley value is generally done using (A.8); however, in games with a large number of players the computational complexity of the Shapley value grows significantly. For computing the Shapley value in reasonable time, several analytical techniques have been proposed such as multi-linear extensions [3], and sampling methods for simple games [22], among others.

3.4 The Nucleolus

Another prominent and interesting solution concept for canonical games is *the nucleolus* which was introduced mainly for TU games [3]. Extensions of the nucleolus for NTU games are not yet formalized in game theory, and hence this tutorial will only focus on the nucleolus for TU canonical games. The basic motivation behind the nucleolus is that, instead of applying a general fairness axiomatization for finding a unique payoff allocation, i.e., a value for the game, one can provide an allocation that minimizes the dissatisfaction of the players from the allocation they can receive in a given $(\mathcal{N}, v)$ game. For a coalition S, the measure of dissatisfaction from an allocation $\boldsymbol{x} \in \mathbb{R}^N$ is defined as the *excess* $e(\boldsymbol{x}, S) = v(S) - \sum_{j \in S} x_j$. Clearly, an allocation $\boldsymbol{x}$ which can ensure that all excesses (or dissatisfactions) are minimized is of particular interest as a solution[15] and hence, constitutes the main motivation behind the concept of the nucleolus. Let $\boldsymbol{O}(\boldsymbol{x})$ be the vector of all excesses in a canonical game $(\mathcal{N}, v)$ arranged in non-increasing order (except the excess of the grand coalition $\mathcal{N}$). A vector $\boldsymbol{y} = (y_1, \dots, y_k)$ is said to be lexographically less than a vector $\boldsymbol{z} = (z_1, \dots, z_k)$ (denoted by $\mathbf{y} \prec_{\text{lex}} \mathbf{z}$) if $\exists l \in \{1, \dots, k\}$ where $y_1 = z_1, y_2 = z_2, \dots, y_{l-1} = z_{l-1}, y_l < z_l$. An *imputation* $\boldsymbol{x}$ is a *nucleolus* if for every other imputation $\boldsymbol{\delta}$, $\boldsymbol{O}(\boldsymbol{x}) \prec_{\text{lex}} \boldsymbol{O}(\boldsymbol{\delta})$. Hence, the nucleolus is the imputation $\boldsymbol{x}$ which *minimizes the excesses* in a non-increasing order. The nucleolus of a canonical coalitional game exists and is unique. The nucleolus is group and individually rational (since it is an imputation), and satisfies the symmetry and dummy axioms of Shapley. If the core is not empty, the nucleolus is in the core. Moreover, the nucleolus lies in the *kernel* of the game, which is the set of all allocations $\boldsymbol{x}$ such that $\max_{S \subseteq \mathcal{N} \setminus \{j\}, i \in S} e(\boldsymbol{x}, S) = \max_{G \subseteq \mathcal{N} \setminus \{i\}, j \in G} e(\boldsymbol{x}, G)$. The kernel states that if players i and j are in the same coalition, then the highest excess that i can make in a coalition without j is equal to the highest excess that j can make in a coalition without i. As the nucleolus lies in the kernel, it also verifies this property. Thus, the nucleolus is the best allocation under a min-max criterion. The process for computing the nucleo-

[15]In particular, an imputation x lies in the core of (N, v), if and only if all its excesses are negative or zero.

lus is more complex than the Shapley value, and is described as follows. First, we start by finding the imputations that distribute the worth of the grand coalition in such a way that the maximum excess (dissatisfaction) is minimized. In the event where this minimization has a unique solution, this solution is the nucleolus. Otherwise, we search for the imputations which minimize the second largest excess. The procedure is repeated for all subsequent excesses, until finding a unique solution which would be the nucleolus. These sequential minimizations are solved using linear programming techniques such as the simplex method [23]. The applications of the nucleolus are numerous in game theory. One of the most prominent examples is the marriage contract problem which first appeared in the Babylonian Talmud (0-500 A.D).

Example 2 A man has three wives, and he is committed to a marriage contract that specifies that they should receive 100, 200 and 300 units respectively, after his death. This implies that, given a total amount of α units left after the man's death, the three wives can only claim 100, 200, and 300, respectively, out of the α units. If after the man dies, the amount of money left is not enough for this distribution, the Talmud recommends the following:

- If $\alpha = 100$ is available after the man dies, then each wife gets $\frac{100}{3}$.
- If $\alpha = 200$ is available after the man dies, wife 1 gets 50, and the other two get 75 each.
- If $\alpha = 300$ is available after the man dies, wife 1 gets 50, wife 2 gets 100 and wife 3 gets 150.

Note that the Talmud does not specify the allocation for other values of α but certainly, if $\alpha \geq 600$ each wife simply claims its full right. A key question that puzzled mathematicians and researchers in game theory was how this allocation was made and it turns out that the nucleolus is the answer. Let us model the game as a coalitional game $(\mathcal{N}, v)$ where $\mathcal{N}$ is the set of all three wives which constitute the players and v is the value defined for any coalition $S \subseteq \mathcal{N}$ as $v(S) = \max(0, \alpha - \sum_{i \in \mathcal{N} \setminus S} c_i)$, where $\alpha \in \{100, 200, 300\}$ is the total units left after the death of the man and c_i is the claim that wife i must obtain $(c_1 = 100, c_2 = 200, c_3 = 300)$. It then turns out that, with this formulation, the payoffs that were recommended by the Talmud coincide with the nucleolus of the game! This result highlights the importance of the nucleolus in allocating fair payoffs in a game.

In summary, the nucleolus is quite an interesting concept, since it combines a number of fairness criteria with stability. However, the communications applications that utilized the nucleolus are still few, with one example being [19], where it was used for allocating the utilities in the modeled game. The main drawback of the nucleolus is its computational complexity in some games. However, with appropriate models, the nucleolus can be an optimal and fair solution to many applications.

3.5 Applications of Canonical Coalitional Games

3.5.1 Rate allocation in a multiple access channel

An elegant and interesting use of canonical games within communication networks is presented in [9] for the study of rate allocation in multiple access channels (MAC). The model in [9] tackles the problem of how to fairly allocate the transmission rates between a number of users accessing a wireless Gaussian MAC channel. In this model, the users are bargaining for obtaining a fair allocation of the total transmission rate available. Every user, or group of users (coalition), that does not obtain a fair allocation of the rate can threaten to act on its own which can reduce the rate available for the remaining users. Consequently, the game is modeled as a coalitional game defined by $(\mathcal{N}, v)$ where $\mathcal{N} = \{1, \ldots, N\}$ is the set of players, i.e., the wireless network users that need to access the channel, and v is the maximum sum-rate that a coalition S can achieve. In order to have a characteristic function, [9] assumes that, when evaluating the value of a coalition $S \subset \mathcal{N}$, the users in $S^c = \mathcal{N} \setminus S$ known as jammers, cooperate in order to *jam* the transmission of the users in S. The jamming assumption is a neat way of maintaining the characteristic form of the game, and it was previously used in game theory for deriving a characteristic function from a strategic form non-cooperative game [1, 12]. Subsequently, when evaluating the sum-rate utility $v(S)$ of any coalition $S \subseteq \mathcal{N}$, the users in S^c form a single coalition to jam the transmission of S and hence, the coalitional structure of S^c is always pre-determined yielding a characteristic form. For a coalition S, the characteristic function in [9], $v(S)$, represents the capacity, i.e., the maximum sum-rate, that S achieves under the jamming assumption. Hence, $v(S)$ represents a rate that can be apportioned in an arbitrary manner between the players in S, and thus the game is a TU game. It is easily shown in [9] that the game is superadditive since the sum of sum-rates achieved by two disjoint coalitions is no less than the sum-rate achieved by the union of these two coalitions, since the jammer in both cases is the same (due to the assumption of a sin-

Table A.II: The main steps in solving the Gaussian MAC rate allocation canonical game as per [9]

1- The player set is the set $\mathcal{N}$ of users in a Gaussian MAC channel.
2- For a coalition $S \subseteq \mathcal{N}$, a superadditive value function in characteristic form with TU is defined as the maximum sum-rate (capacity) that S achieves under the assumption that the users in coalition $S^c = \mathcal{N} \setminus S$ attempt to jam the communication of S.
3- Through technique (T5) of Table A.I the core is shown to be non-empty and containing all imputations in the capacity region of the grand coalition.
4- The Shapley value is discussed as a fairness rule for rate-allocation, but is shown to be outside the core, hence, rendering the grand coalition unstable.
5- A new application-specific fairness rule, known as "envy-free" fairness, is shown to lie in the core and is presented as a solution to the rate-allocation game in Gaussian MAC.

gle coalition of jammers). Consequently, the problem lies in allocating the payoffs, i.e., the transmission rates, between the users in the grand coalition $\mathcal{N}$ which forms in the network. The grand coalition $\mathcal{N}$ has a capacity region $\mathcal{C} = \{\boldsymbol{R} \in \mathbb{R}^N | \sum_{i=1}^{N} R_i \leq C(\Gamma_S, \sigma^2),\ \forall S \subseteq \mathcal{N}\}$, where Γ_S captures the power constraints on the users in S, σ^2 is the Gaussian noise variance, and hence, $C(\Gamma_S, \sigma^2)$ is the maximum sum-rate (capacity) that coalition S can achieve. Based on these properties, the rate allocation game in [9] is clearly a *canonical coalitional game*, and the key question that [9] seeks to answer is "how to allocate the capacity of the grand coalition $v(\mathcal{N})$ among the users in a fair way that stabilizes $\mathcal{N}$". In answering this question, two main concepts from canonical games are used: The core and the Shapley value.

In this rate allocation game, it is shown that the core, which represents the set of rate allocations that stabilize the grand coalition, is *non-empty* using technique (T5) from Table A.I. By considering the *imputations* that lie in the capacity region $\mathcal{C}$, i.e., the rate vectors $\boldsymbol{R} \in \mathcal{C}$ such that $\sum_{i=1}^{N} R_i = C(\Gamma_{\mathcal{N}}, \sigma^2)$, it is shown that any such vector lies in the core. Therefore, the grand coalition $\mathcal{N}$ of the Gaussian MAC canonical game can be stabilized. However, the core of this game is *big* and contains a large number of rate vectors. Thus, the authors in [9] sought to answer the next question "how to select a single fair allocation which lies in the core?". For this purpose, the authors investigate the use of the Shapley value as a fair solution for rate allocation which accounts for the random-order of joining of the players in the grand coalition. In this setting, the Shapley value simply implies that no rate is left unallocated (efficiency axiom), dummy players receive no rate (dummy axiom), and the labeling of the players does not affect the rate that they receive (symmetry axiom). However, the authors show that: (i)- The fourth Shapley axiom (additivity) is not suitable for the proposed rate allocation game, and (ii)- the Shapley value does not lie in the core, and hence cannot stabilize the grand coalition. Based on these results for the Shapley value, the authors propose a new fairness criterion, named “envy-free” fairness. The envy-free fairness criterion relies on the first three axioms of Shapley (without the additivity axiom), and complements them with a fourth axiom, the *envy free allocation axiom* [9, Eq. (6)]. This axiom states that, given two players i and j, with power constraints $\Gamma_i > \Gamma_j$, an envy-free allocation ψ gives a payoff $\psi_j(v)$ for user j in the game $(\mathcal{N}, v)$, equal to the payoff $\psi_i(v^{i,j})$ of user i in the game $(\mathcal{N}, v^{i,j})$ where $v^{i,j}$ is the value of the game where user i utilizes a power $\Gamma_i = \Gamma_j$. Mathematically, this axiom implies that $\psi_j(v) = \psi_i(v^{i,j})$. With these axioms, it is shown that a unique allocation exists and this allocation lies in the core. Thus, the envy-free allocation is presented as a fair and suitable solution for the rate allocation game in [9]. Finally, the approach used for solving the rate allocation canonical coalitional game in [9] is summarized in Table A.II.

3.5.2 Canonical games for receivers and transmitters cooperation

In [10], canonical games are used for studying the cooperation possibilities between single antenna receivers and transmitters in an interference channel. The model considered in [10] consists of a set of transmitter-receiver pairs, in a Gaussian interference channel. The authors study the cooperation between the receivers under two coalitional game models: A TU model where the receivers communicate through noise-free channels and jointly decode the received signals, and an NTU model where the receivers cooperate by forming a linear multiuser detector (in this case the interference channel is reduced to a MAC channel). Further, the authors study the transmitters cooperation problem under perfect cooperation and partial decode and forward cooperation, while considering that the receivers have formed the grand coalition. Since all the considered games are canonical (as we will see later), the main interest is in studying the properties of the grand coalitions for the receivers and the transmitters.

For receiver cooperation using joint decoding, the coalitional game model is as follows: the player set $\mathcal{N}$ is the set of links (the players are the receivers of these links) and, assuming that the transmitters do *not* cooperate, the value $v(S)$ of a coalition $S \subseteq \mathcal{N}$ is the maximum sum-rate achieved by the links whose receivers belong to S. Under this model, one can easily see that the utility is transferable since it represents a sum-rate, hence the game is TU. The game is also in characteristic form, since, as the transmitters are considered non-cooperative, the sum-rate achieved when the receivers in S cooperate depends solely on the receivers in S while treating the signal from the links in $\mathcal{N} \setminus S$ as interference. In this game, the cooperation channels between the receivers are considered noiseless and hence, cooperation is always beneficial and the game is shown to be superadditive. Hence, under our proposed classification, this game is clearly a canonical game, and the interest is in studying the properties of the grand coalition of receivers. Under this cooperation scheme, the network can be seen as a single-input-multiple-output (SIMO) MAC channel, and the proposed coalitional game is shown to have a non-empty core which contains *all the imputations* which lie on the SIMO-MAC capacity region. The technique used for this proof is similar to the game in [9] which selects a particular set of rate vectors, those that are on the SIMO-MAC region, and shows that they lie in the core as per (T5) from Table A.I. The core of this game is very large, and for selecting fair allocations, it is proven in [10] that the Nash bargaining solution, and in particular, a proportional fair rate allocation lie in the core, and hence constitute suitable fair and stable allocations. For the second receiver cooperation game, the model is similar to the joint decoding game, with one major difference: Instead of jointly decoding the received signals, the receivers form linear multiuser detectors (MUD). The MUD coalitional game is inherently different from the joint decoding game since, in a MUD, the SINR ratio achieved by a user i in coalition S cannot be shared with the other users, and hence the game becomes an NTU game with the SINR representing the payoff of each player. In this NTU setting, the value $v(S)$ of a coalition S becomes the set of SINR vectors that a coalition S can achieve. For this NTU game, the grand coalition is proven to be stable and sum-rate maximizing at high SINR regime using limiting conditions on the SINR expression, hence technique (T6) in Table A.I.

For modeling the transmitters cooperation problem as a coalitional game the authors make two assumptions: (i)- The receivers jointly decode the signals, hence form a grand coalition, and (ii)- a jamming assumption similar to [9] is made for the purpose of maintaining the characteristic form. In the transmitters game, from the set of links $\mathcal{N}$, the transmitters are the players. When considering the transmitters cooperation along with the receivers cooperation, the interference channel is mapped unto a multiple-input-multiple-output (MIMO) MAC channel. For maintaining a characteristic form, the authors assumed, in a manner analogous to [9],

Table A.III: The main results for receivers and transmitters cooperation coalitional games as per [10]

1- The coalitional game between the receivers, where cooperation entails joint decoding of the received signal, is a canonical TU game which has a non-empty core. Hence, the grand coalition is the stable and sum-rate maximizing coalition.
2- The coalitional game between the receivers, where cooperation entails forming linear multiuser detectors, is a canonical NTU game which has a non-empty core. Hence, the grand coalition is the stable and sum-rate maximizing coalition.
3- For transmitters cooperation, under jamming assumption, the coalitional game is not, *superadditive* hence non-canonical. However, the grand coalition is shown to be the rate maximizing partition.
4- For transmitters cooperation under jamming assumption, no results for the existence of the core can be found due to mathematical intractability.

that whenever a coalition of transmitters S forms, the users in $S^c = \mathcal{N} \setminus S$ form one coalition and aim to jam the transmission of coalition S. Without this assumption, the maximum sum-rate that a coalition can obtain highly depends on how the users in S^c structure themselves, and hence requires a partition form that may be difficult to solve. With these assumptions, the value of a coalition S is defined as the maximum sum-rate achieved by S when the coalition S^c seeks to jam the transmission of S. Using this transmitters with jamming coalitional game, the authors show that in general the game has an empty core. This game is not totally canonical since it does not satisfy the superadditivity property. However, by proving through [10, Th. 19] that the grand coalition is the optimal partition, from a total utility point of view, the grand coalition becomes the main candidate partition for the core. The authors conjecture that in some cases, the core can also be non-empty depending on the power and channel gains. However, no existence results for the core are provided in this game. Finally, the authors in [10] provide a discussion on the grand coalition and its feasibility when the transmitters employ a partial decode and forward cooperation. The main results are summarized in Table A.III.

3.5.3 Other applications for canonical games and future directions

Canonical coalitional games cover a broad range of communication and networking applications and, indeed, most research activities in these areas utilize the tools that fall under the canonical coalitional games class. In addition to the previous examples, numerous applications used models that involve canonical games. For instance, in [19], canonical coalitional games are used to solve an inherent problem in packet forwarding ad hoc networks. In such networks, the users that are located in the center of the network, known as backbone nodes, have a mutual benefit to forward each others' packets. In contrast, users located at the boundary of the network, known as boundary nodes, are not helped by the backbone nodes due to the fact that the backbone nodes do not need the help of the boundary nodes at any time. Hence, in such a setting, the boundary nodes end up having no way of sending their packets to other nodes, and this is a problem known as the *curse of the boundary nodes*. In [19], a canonical coalitional game model is proposed between a player set $\mathcal{N}$ which includes all boundary nodes and a *single* backbone node. In this model, forming a coalition, entails the following benefits: (i)- By cooperating with a number of boundary nodes and using cooperative transmission, the backbone node can reduce its power consumption, and (ii)- in return, the backbone node agrees to forward the packets of the boundary nodes. For cooperative transmission, in a coalition S, the boundary nodes act as relays while the backbone node acts as a source. In this game, the *core* is shown to be non-empty using the property that any group of boundary nodes receive no utility if they break away from the grand coalition with the backbone node, this classifies as a (T6) technique from Table A.I. Further, the authors in [19] study the conditions under which a Shapley value and a nucleolus are suitable for modeling the game. By using a canonical game, the connectivity of the ad hoc network is significantly improved [19]. Beyond packet forwarding, many other applications such as in [17, 18, 21] utilize several of the techniques in Table A.I for studying the grand coalition in a variety of communications applications.

In summary, canonical games are an important tool for studying cooperation and fairness in communication networks, notably when cooperation is always beneficial. Future applications are numerous, such as studying cooperative transmission capacity gains, distributed cooperative source coding, cooperative relaying in cognitive radio and many other applications. In brief, whenever a cooperative scheme that yields significant gains at any layer is devised, one can utilize canonical coalitional games for assessing the stability of the grand coalition and identifying fairness criteria in allocating the gains that result from cooperation. Finally, it has to be noted that canonical games are not restricted to link-level analysis, but also extend to network-level studies as demonstrated in [18, 19].

4 Class II: Coalition Formation Games

4.1 Main Properties of Coalition Formation Games

Coalition formation games encompass coalitional games where, unlike the canonical class, *network structure* and *cost* for cooperation play a major role. Some of the main characteristics that make a game a coalition formation game are as follows:

1. The game is in either characteristic form or partition form (TU or NTU), and is generally not superadditive.

2. Forming a coalition brings gains to its members, but the gains are limited by a *cost* for forming the coalition, hence the grand coalition is seldom the optimal structure.

3. The objective is to study the *network coalitional structure*, i.e., answering questions like which coalitions will form, what is the optimal coalition size and how can we assess the structure's characteristics, and so on.

4. The coalitional game is subject to environmental changes such as a variation in the number of players, a change in the strength of each player or other factors which can affect the network's topology.

5. A coalitional structure is imposed by an external factor on the game (e.g., physical restrictions in the problem).

Unlike canonical games, a coalition formation game is generally not superadditive and can support the partition form model. Another important characteristic which classifies a game as a coalition formation game is the presence of a cost for forming coalitions. In canonical games, as well as in most of the literature, there is an implicit assumption that forming a coalition is always beneficial (e.g. through superadditivity). In many problems, forming a coalition requires a negotiation process or an information exchange process which can incur a cost, thus, reducing the gains from forming the coalition. In general, coalition formation games are of two types: *Static coalition formation games and dynamic coalition formation games*. In the former, an external factor imposes a certain coalitional structure, and the objective is to study this structure. The latter is a more rich framework. In dynamic coalition formation games, the main objectives are to analyze the formation of a coalitional structure, through players' interaction, as well as to study the properties of this structure and its adaptability to environmental variations or externalities. In contrast to canonical games, where formal rules and analytical concepts exist, solving a coalition formation game, notably dynamic coalition formation, is more difficult, and application-specific. The rest of this section is devoted to dissecting the key properties of coalition formation games.

4.2 Impact of a Coalitional Structure on Solution Concepts of Canonical Coalitional Games

In canonical games, the solution concepts defined, such as the core, the Shapley value and the nucleolus, assumed that the grand coalition would form due to the superadditivity property. The presence of a coalitional structure affects the definition and use of these concepts as was first pointed out by Aumann and Drèze in [24] for a static coalition formation game. In [24], a TU coalitional game is considered, in the presence of a static coalitional structure $\mathcal{B} = \{B_1, \dots, B_l\}$ (each B_i is a coalition), that is imposed by some external factor. Hence, [24] defines a coalitional game as the triplet $(\mathcal{N}, v, \mathcal{B})$ where v is a characteristic function. First, in the presence of $\mathcal{B}$, the concept of group rationality is substituted by *relative efficiency*. Given an allocation vector $\boldsymbol{x} \in \mathbb{R}^N$, relative efficiency implies that, for each coalition $B_k \in \mathcal{B}$, $\sum_{i \in B_k} x_i = v(B_k)$ [24]. Hence, for every present coalition B_k in $\mathcal{B}$, the total value available for coalition B_k is divided among its members unlike in canonical games where the value of the grand coalition $v(\mathcal{N})$ is distributed among all players. With regards to canonical solutions, we first turn our attention to the Shapley value. For the game $(\mathcal{N}, v, \mathcal{B})$, the previously defined Shapley axioms remain in place, except for the efficiency axiom which is replaced by a *relative efficiency* axiom. With this modified axiom, [24] shows that the Shapley value of $(\mathcal{N}, v, \mathcal{B})$, referred to as $\mathcal{B}$-value, has the *restriction property*. The restriction property implies that, for finding the $\mathcal{B}$-value, one can consider the *restricted* coalitional games $(B_k, v|B_k)$, $\forall B_k \in \mathcal{B}$ where $v|B_k$ is the value v of the original game $(\mathcal{N}, v, \mathcal{B})$, defined over player set (coalition) B_k. As a result, for finding the $\mathcal{B}$-value, we proceed in two steps, using the restriction property: (1)- Consider the games $(B_k, v|B_k), k = 1, \dots, l$ separately and for each such game $(B_k, v|B_k)$ find the Shapley value using the canonical definition (A.8), and (2)- the $\mathcal{B}$-value of the game is the $1 \times N$ vector $\boldsymbol{\phi}$ of payoffs constructed by combining the resulting allocations of each restricted game $(B_k, v|B_k)$.

In the presence of a coalitional structure $\mathcal{B}$, the canonical definitions of the core and the nucleolus are also mainly modified by replacing group rationality with relative efficiency. However, unlike the Shapley value, it is shown in [24] that the restriction property does not apply to the core, nor the nucleolus. This can be easily deduced from the fact that both the core and the nucleolus depend on *all* coalitions of $\mathcal{N}$. Hence, in the presence of $\mathcal{B}$, the core and the nucleolus depend on the values of coalitions $B_j \in \mathcal{B}$ *as well as* the values of coalitions that are not in $\mathcal{B}$, that is coalitions $S \subset \mathcal{N}, \nexists B_k \in \mathcal{B}$ s. t. $B_k = S$. Hence, the problem of finding the core and the nucleolus of $(\mathcal{N}, v, \mathcal{B})$ is more complex than for the Shapley value. In [24], an approach for finding these solutions for games where $v(\{i\}) = 0,\ \forall i \in \mathcal{N}$ is presented. The approach is based on finding a game equivalent to v by redefining the value, and hence, the core and nucleolus can be found for this equivalent game. For the detailed analysis, we refer the reader to [24, Th. 4 and Th. 5].

Even though the analysis in [24] is restricted to static coalition formation games with TU and in characteristic form, it shows that finding solutions for coalition formation games is not straightforward. The difficulty of such solutions increases whenever an NTU game, a partition form game, or a dynamic coalition formation game are considered, notably when the objective is to compute the solution in a distributed manner. For example, when considering a dynamic coalition formation game, one would need to evaluate the payoff allocations *jointly* with the formation of the coalitional structure, hence solution concepts become even more complex to find (although the restriction property of the Shapley value makes things easier). For this purpose, the literature dealing with coalition formation games, notably dynamic coalition formation such as [25–28] or others, usually re-defines the solution concepts or presents alternatives that are specific to the game being studied. Hence, unlike canonical games where formal solutions exist, the solution of a coalition formation game depends on the model and the objectives that are being considered.

4.3 Dynamic Coalition Formation Algorithms

In general, in a coalition formation game, the most important aspect is the formation of the coalitions in the game. In other words, one must answer the question of “how to form a coalitional structure that is suitable to the studied game”. In addition, the evolution of this structure is important, notably when changes to the game nature can occur due to external or internal factors (e.g., what happens to the coalition structure if one or more players leave the game). In many applications, coalition formation entails finding a coalitional structure which either maximizes the total utility (social welfare) if the game is TU, or finding a structure with Pareto optimal payoff distribution for the players if the game is NTU. For achieving such a goal, a *centralized* approach can be used; however, such an approach is generally NP-complete [25–28]. The reason is that, finding an optimal partition, requires iterating over all the partitions of the player set $\mathcal{N}$. The number of partitions of a set $\mathcal{N}$ grows exponentially with the number of players in $\mathcal{N}$ and is given by a value known as the Bell number [25]. For example, for a game where $\mathcal{N}$ has only 10 elements, the number of partitions that a centralized approach must go through is 115975 (easily computed through the Bell number). Hence, finding an optimal partition from a centralized approach is, in general, computationally complex and impractical. In some cases, it may be possible to explore the properties of the game, notably of the value v, for reducing the centralized complexity. Nonetheless, in many practical applications, it is desirable that the coalition formation process takes place in a distributed manner, whereby the players have an autonomy on the decision as to whether or not they join a coalition. In fact, the complexity of the centralized approach as well as the need for distributed solutions have sparked a huge growth in the coalition formation literature that aims to find low complexity and distributed algorithms for forming coalitions [25–28].

The approaches used for distributed coalition formation are quite varied and range from heuristic approaches [25], Markov chain-based methods [26], to set theory based methods [27] as well as approaches that use bargaining theory or other negotiation techniques from economics [28]. Although there are no general rules for distributed coalition formation, some work, such as [27] provides generic rules that can be used to derive application-specific coalition formation algorithms. Although [27] does not explicitly construct a coalition formation algorithm, the mathematical framework presented can be used to develop such algorithms. The main ingredients that are presented in [27] are three: (1)- Well-defined orders suitable to compare *collections* of coalitions, (2)- two simple rules for forming or breaking coalitions, and (3)- adequate notions for assessing the stability of a partition. For comparing collections of coalitions, a number of *orders* are defined in [27], two of which are of noticeable importance. The first order, known as the *utilitarian order*, states that, a group of players prefers to organize themselves into a collection $\mathcal{R} = \{R_1, \ldots, R_k\}$ instead of $\mathcal{S} = \{S_1, \ldots, S_l\}$, if the total social welfare achieved in $\mathcal{R}$ is strictly greater than in $\mathcal{S}$, i.e., $\sum_{i=1}^{k} v(R_i) > \sum_{i=1}^{l} v(S_i)$. This order is generally suitable for TU games. Another important order is the *Pareto order*, which bases the preference on the individual payoffs of the players rather than the coalition value. Given two allocations $\boldsymbol{x}$ and $\boldsymbol{y}$ that are allotted by $\mathcal{R}$ and $\mathcal{S}$, respectively, to the same players, $\mathcal{R}$ is preferred over $\mathcal{S}$ by Pareto order if at least one player improves in $\mathcal{R}$ without hurting the other players, i.e., $\boldsymbol{x} \geq \boldsymbol{y}$ with at least one element x_i of $\boldsymbol{x}$ such that $x_i > y_i$. The Pareto order is suitable for both TU and NTU games.

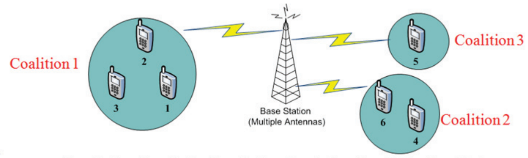
Using such orders, [27] presents two main rules for forming or breaking coalitions, referred to as *merge* and *split*. The basic idea behind the rules is that, given a set of players $\mathcal{N}$, any collection of disjoint coalitions $\{S_1, \ldots, S_l\}$, $S_i \subset \mathcal{N}$ can agree to *merge* into a single coalition $G = \cup_{i=1}^{l} S_i$, if this new coalition G is preferred by the players over the previous state depending on the selected comparison order. Similarly, a coalition S *splits* into smaller coalitions if the resulting collection $\{S_1, \ldots, S_l\}$ is preferred by the players over S. Independent of the selected order, any arbitrary sequence of these two rules is shown to converge to a final partition of $\mathcal{N}$ [27]. For assessing the stability of the final partition, the authors in [27] propose the concept of a *defection function*, which is a function that associates with every network partition, another partition, a group of other partitions, or a group of collections in $\mathcal{N}$. By defining various types of such a function, one can assess whether, in a given partition $\mathcal{T}$ of $\mathcal{N}$, there is an incentive for the players to deviate and form other partitions or collections. A first notion of stability, is a weak equilibrium-like stability, known as $\mathbb{D}_{hp}$ stability. A $\mathbb{D}_{hp}$-stable partition simply implies that, in this partition, no group of players has an interest in performing a merge or a split operation. This type of stability can be thought of as merge-and-split proofness of a partition, or a kind of equilibrium with respect to merge-and-split. The most important type of stability inspected in [27] is $\mathbb{D}_c$-stability. The existence of a $\mathbb{D}_c$-stable partition is not always guaranteed, and the two conditions needed for its existence can be found in [27]. However, when it exists, the $\mathbb{D}_c$-stable partition has numerous attractive properties. First and foremost, a $\mathbb{D}_c$-stable partition is a *unique* outcome of any arbitrary merge and split iteration. Hence, starting from any given partition, one would always reach the $\mathbb{D}_c$-stable partition by merge-and-split. Based on the selected order, the players prefer the $\mathbb{D}_c$-stable partition over *all other partitions*. On one hand, if the selected order is the utilitarian order, this implies that the $\mathbb{D}_c$-stable partition maximizes the social welfare (total utility), on the other hand, if the selected order is the Pareto order, the $\mathbb{D}_c$-stable partition has a Pareto optimal payoff distribution for the players. Finally, no group of players in a $\mathbb{D}_c$-stable partition have an incentive to leave this partition for forming any other collection in $\mathcal{N}$. Depending on the application being investigated, one can possibly define other suitable defection functions, as this concept is not limited to a particular problem.

Coalition formation games are diverse, and by no means limited to the concepts in [27]. For example, a type of coalition formation games, known as *hedonic coalition formation* games has been widely studied in game theory. Hedonic games are quite interesting since they allow the formation of coalitions (whether dynamic or static) based on the individual preferences of the players. In addition, these games admit different stability concepts that are extensions to well known concepts such as the core or the Nash equilibrium used in a coalition formation setting [29]. In this regard, hedonic games constitute a very useful analytical framework which has a very strong potential to be adopted in modeling problems in wireless and communication networks (only few contributions such as [30] used this framework in a communication/wireless model). Furthermore, beyond merge-and-split and hedonic games, dynamic coalition formation games encompass a multitude of algorithms and concepts such as in [25–28] and many others. Due to space limitations, this tutorial cannot provide an exhaustive survey of all such algorithms. Nonetheless, as will be seen in the following sections, many coalition formation algorithms and concepts can be tailored and adapted for communication applications.

4.4 Applications of Coalition Formation Games

4.4.1 Transmitter cooperation with cost in a TDMA system

The formation of virtual MIMO systems through distributed cooperation has received an increasing attention recently (see [10, 31] and the references therein). The problem involves a number of single antenna users which cooperate and share their antennas in order to benefit from spatial diversity or multiplexing, and hence form a virtual MIMO system. Most literature that studied the problem is either devoted to analyzing the link-level information theoretical gains from distributed cooperation, or focused on assessing the stability of the grand coalition, for cooperation with no cost, such as in the work of [10] previously described. However, there is a lack of literature which studies how a network of users can interact to form virtual MIMO systems, notably when there is a cost for cooperation. Hence, a study of the network topology and dynamics that result from the interaction of the users is needed and, for this purpose, coalition formation games are quite an appealing tool. These considerations motivated our work in [31] where we considered a network of single antenna trans-



Time slot 1	Time slot 2	Time slot 3	Time slot 4	Time slot 5	Time slot 6
User 1	User 2	User 3	User 4	User 5	User 6

Time slot 1	Time slot 2	Time slot 3	Time slot 4	Time slot 5	Time slot 6
Coal. 1	Coal. 1	Coal. 1	Coal. 2	Coal. 3	Coal. 2

Fig. A.3: The system model for the virtual MIMO formation game in [31].

mitters that send data in the uplink of a TDMA system to a receiver with multiple antennas. In a non-cooperative approach, each single antenna transmitter sends its data in an allotted slot. For improving their capacity, the transmitters can interact for forming coalitions, whereby each coalition S is seen as a single user MIMO that transmits in the slots that were previously held by the users of S. After cooperation, the TDMA system schedules one coalition per time slot. An illustration of the model is shown in Fig. A.3. To cooperate, the transmitters must exchange their data, and hence, this exchange of information incurs a cost in terms of power. The presence of this cost, as per [31], renders the game non-superadditive due to the fact that the information exchange incurs a cost in power which is increasing with the distances inside the coalition as well as the coalition size. For example, when two users are far away, information exchange can consume the total power, and the utility for cooperation is smaller than in the non-cooperative case. Similarly, adding more users to a coalition does not always yield an increase in the utility; for instance, a coalition consisting of a large number of users increases the cost for information exchange, and thus superadditivity cannot be guaranteed. As a consequence of this property, for the proposed game in [31] the grand coalition seldom forms[16] and the game is modeled as a dynamic coalition formation game between the transmitters (identified by the set $\mathcal{N}$) that seek to form cooperating coalitions. The dynamic aspect stems from the fact that many environmental changes, such as the mobility of the transmitters or the deployment of new users, may affect the coalitional structure that will form and any algorithm must be able to cope with these changes accurately.

[16]In this game, the grand coalition only forms in extremely favorable cases, such as when the network contains only two users and these users are very close by.

For the proposed game, the value function represents the sum-rate, or capacity, that the coalition can achieve, while taking into account the power cost. Due to the TDMA nature of the problem, a power constraint $\tilde{P}$ per time slot, and hence per coalition, is considered. Whenever a coalition forms, a fraction of $\tilde{P}$ is used for information exchange, hence constituting a cost for cooperation, while the remaining fraction will be used for the coalition to transmit its data, as a single user MIMO, to the receiver. For a coalition S, the fraction used for information exchange is the sum of the powers that each user $i \in S$ needs to transmit its data to the user $j \in S$ that is farthest from i; due to the broadcast nature of the wireless channel all other users in S can receive this data as well. This power cost scales with the number of users in the coalition, as well as the distance between these users. Hence, the sum-rate that a coalition can achieve is limited by the fraction of power spent for information exchange. For instance, if the power for information exchange for a coalition S is larger than $\tilde{P}$, then $v(S) = 0$. Otherwise, $v(S)$ represents the sum-rate achieved by the coalition using the remaining fraction of power. Clearly, the sum-rate is a transferable utility, and hence we deal with a TU game.

In this framework, a dynamic coalition formation algorithm based on the merge-and-split rules previously described can be built. In [31], for coalition formation, we start with a non-cooperative network, whereby each user discovers its neighbors starting with the closest, and attempts to merge based on the utilitarian order, i.e., if cooperating with a neighbor improves the total sum-rate that the involved users can achieve, then merging occurs (merge is done through pairwise interactions between a user or coalition and the users or coalitions in the vicinity). Further, if a formed coalition finds out that splitting into smaller coalitions improves the total utility achieved by its users, then a split occurs. Starting from the initial non-cooperative network, the coalition formation algorithm involves sequential merge and split rules. The network's coalition can autonomously decide on whether to perform a merge or split based on the utility evaluation. The convergence is guaranteed by virtue of the definition of merge-and-split. Further, if an optimal $\mathbb{D}_c$-stable partition exists, the proposed algorithm converges to it. The existence of the $\mathbb{D}_c$-stable partition in this model cannot always be guaranteed, as it depends on random locations of the users; however, the convergence to it, when it exists, is guaranteed.

The coalition formation algorithm proposed in [31] can handle any network size, as the implementation is inherently distributed, whereby each coalition (or user) can detect the strength of the other users' uplink signals (using techniques as in ad hoc routing), and discover the nearby candidate partners. Consequently, the distributed users can exchange the required information and then assess what kind of merge or split decisions they can make. The transmitters engage in merge-and-split periodically, and hence, adapt the topology to any environmental change, such as mobility or the joining/leaving of transmitters. In this regard, by adequate merge or split decisions, the topology is always dynamically changing, through individual and distributed decisions by the network's coalitions. As the proposed model is TU, several rules for dividing the coalition's value are used. These rules range from well-known fairness criteria such as the proportional fair division, to coalitional game-specific rules such as the Shapley value or the nucleolus. Due to the distributed nature of the problem, the nucleolus or the Shapley value are applied at the level of the coalitions that are forming or splitting. Hence, although for the Shapley value this allocation coincides with the Shapley value of the whole game as previously discussed, for the nucleolus, the resulting allocations lie in the nucleolus of the restricted games only. In this game, for any coalition $S \subseteq \mathcal{N}$ that forms through merge-and-split, the Shapley value presents a division of the payoff that takes into account the random order of joining of the transmitter in S when forming the coalition (this division is also efficient at the coalition level and treats the players symmetrically within S). In contrast, the division by the nucleolus at the level of every coalition $S \subseteq \mathcal{N}$ that forms through merge-and-split ensures that the dissatisfaction of any transmitter within S is minimized by minimizing the excesses inside S. Finally, although in [31] we used a utilitarian order, in extensions to the work [32], we reverted to the Pareto order, which allows every user of the coalition to assess the improvement to its own payoff during merge or split, instead of relying on the entire coalitional value. By doing so, the fairness criteria chosen impacts the network structure and hence, for every fairness type one can obtain a different topology.

4.4.2 Coalition formation for spectrum sensing in cognitive radio networks

In cognitive radio networks, the unlicensed secondary users (SU) are required to sense the environment in order to detect the presence of the licensed primary user (PU) and transmit during periods where the PU is inactive. Collaborative spectrum sensing (CSS) has been proposed for improving the sensing performance of the SUs, in terms of reducing the probability of missing the detection of the PU (probability of miss), and hence decreasing the interference on the PU. Even though CSS decreases the probability of miss, it also increases the false alarm probability, i.e., the probability of falsely detecting that the PU is transmitting. Hence, CSS presents an inherent tradeoff between reducing the probability of miss (reducing interference on the PU) and maintaining a good false alarm probability, which corresponds to a good spectrum utilization. In [33], we consider a network of SUs, that interact for improving their sensing performance, while taking into account the false alarm cost. For performing CSS, every group of SUs form a coalition, and within each coalition, an SU, selected as coalition head will gather the sensing bit from the coalition members. By using well-known decision fusion rules, the coalition head can decide on the presence or the absence of the PU. Using this CSS scheme, as shown in [33], each coalition reduces the probability of miss of its SUs. However, this reduction is accompanied by an increase in the false alarm probability. This tradeoff between the improvement of the probability of miss and the false alarm, impacts the coalitional structure that forms in the network.

Consequently, the CSS problem is modeled as a dynamic coalition formation game between the SUs ($\mathcal{N}$ is the set of SUs in this game). The utility $v(S)$ of each coalition S is a decreasing function of the probability of miss $Q_{m,S}$ within coalition S and a decreasing function of the false alarm probability $Q_{f,S}$. In the false alarm cost component, the proposed utility in [33, Eq. (8)] imposes a maximum tolerable false alarm probability, i.e., an upper bound constraint α on the false alarm, that cannot be exceeded by any SU. This utility represents probabilities, and hence, cannot be transferred arbitrarily between the SUs. Hence, the coalition formation game for CSS is an NTU game, whereby the payoff of an SU which is a member of any coalition S is given by $x_i = v(S),\ \forall i \in S$ and reflects the probabilities of miss and false alarm that any SU which is a member of S achieves [33, Property 1] (here, the NTU value is a singleton set). In this game, it is easily shown that the grand coalition *seldom* forms, due to the false alarm constraint α and the fact that the false alarm for a coalition increases with the coalition size and the distances between the coalition members [33, Property 3].

For this purpose, a coalition formation algorithm is needed. The algorithm proposed in [33] consists of three phases: In the first phase the SUs perform their local sensing, in the second phase the SUs engage in an

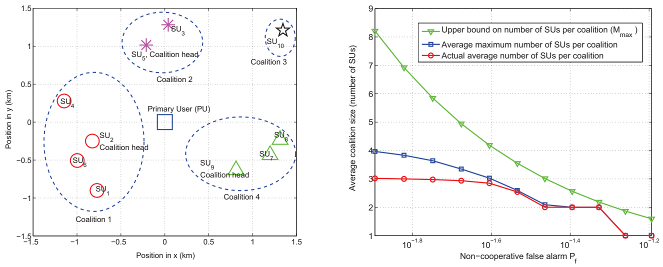


Fig. A.4: (a) Topology resulting from coalition formation in CSS for 10 SUs. (b) Maximum and average coalition size vs. non-cooperative false alarm P_f for the dynamic coalition formation game solution for a network of 30 SUs.

adaptive coalition formation algorithm based on the merge and split rules of Section 4.3, and in the third phase, once the coalitions have formed, each SU reports its sensing bit to the coalition head which makes a decision on whether or not the PU is present. Due to the NTU nature of the game, the adaptive coalition formation phase of the algorithm uses the Pareto order for performing merge or split operations. The merge and split decisions in the context of the CSS model can also be performed in a distributed manner by each coalition, or individual SU. The merge and split phase converges to the $\mathbb{D}_c$-stable partition which leads to a Pareto optimal payoff allocation, whenever this partition exists. Periodically, the formed coalitions engage in merge and split operations for adapting the topology to environmental changes such as the mobility of the SUs or the PU, or the deployment of more SUs. In Fig. A.4 (a), we show an example of a coalitional structure that the SUs form for CSS in a cognitive network of 10 SUs with a false alarm constraint of $\alpha = 0.1$. Clearly, the proposed algorithm allows the SUs to structure themselves into disjoint independent coalitions for the purpose of spectrum sensing. By forming such topologies, it is shown in [33] that the SUs can significantly improve their performance, in terms of probability of miss, reaching up to 86.6% per SU improvement relative to the non-cooperative sensing case for a network of 30 SUs, while maintaining the desired false alarm level of $\alpha = 0.1$. In addition to the performance improvement achieved by the proposed coalition formation algorithm in [33], an interesting upper bound on the coalition size is derived for the proposed utility. This upper bound is a function of only two quantities: The false alarm constraint α and the non-cooperative false alarm value P_f, i.e., the detection threshold. Hence, this upper bound does not depend on the location of the SUs in the network nor on the actual number of SUs in the network. Therefore, deploying more SUs or moving the SUs in the network for a fixed α and P_f does not increase the upper bound on coalition size. In Fig. A.4 (b), we show this upper bound in addition to the average and maximum achieved coalition size for a network of 30 SUs with a false alarm constraint of $\alpha = 0.1$. The coalition size variations are shown as a function of the non-cooperative false alarm P_f. The results in Fig. A.4 (b) show that, in general, the network topology is composed of a large number of small coalitions rather than a small number of large coalitions, even when P_f is small relative to α and the collaboration possibilities are high (a smaller P_f implies the cost for cooperation, in terms of false alarm increases more slowly with the coalition size). Also, when $P_f = \alpha = 0.1$, the network is non-cooperative, since cooperation would always violate the false alarm constraint α. In a nutshell, dynamic coalition formation provides novel collaboration strategies for SUs in a cognitive network which are seeking to improve their sensing performance, while maintaining a desired spectrum utilization (false alarm level). The framework of dynamic coalition formation games suitably models this problem, yields a significant performance improvement, and allows to characterize the network topology that will form.

4.4.3 Future applications of coalition formation games

Potential applications of coalition formation games in communication networks are numerous and diverse. Beyond the applications presented above, coalition formation games have already been applied in [34] to improve the physical layer security of wireless nodes through cooperation among the transmitters, while in [30] coalition formation among a number of autonomous agents, such as unmanned aerial vehicles, is studied in the context of data collection and transmission in wireless networks. Moreover, recently, there has been a significant increase of interest in designing autonomic communication systems. Autonomic systems are networks that are self-configuring, self-organizing, self-optimizing, and self-protecting. In such networks, the users should be able to learn and adapt to their environment (changes in topology, technologies, service demands, application context, etc), thus providing much needed flexibility and functional scalability. Coalition formation games present an adequate framework for the modeling and analysis of these self-organizing next generation communication networks. Hence, potential applications of coalition formation games encompass cooperative networks, wireless sensor networks, next generation IP networks, ad hoc self-configuring networks, and many others. In general, whenever there is a need for distributed algorithms for autonomic networks, coalition formation is a strong tool for modeling such problems. Also, any problem involving the study of cooperative wireless nodes behavior when a cost is present, is candidate for modeling using coalition formation games.

Finally, although the main applications we described in this tutorial required a characteristic form, coalition formation games in partition form are of major interest and can have potential applications in communication networks. For instance, in [10], the transmitter cooperation problem assumed that the players outside any coalition work as a single entity and jam the communication of this coalition. This assumption is made in order to have a characteristic form. For relaxing this assumption and taking into account the actual interference that affects a coalition, a coalitional game in partition form is needed. In the presence of a cooperative cost, this partition form game falls in the class of coalition formation games. Hence, coalition formation games in partition form are ripe for many future applications.

5 Class III: Coalitional Graph Games

5.1 Main Properties of Coalitional Graph Games

In canonical and coalition formation games, the utility or value of a coalition does *not* depend on how the players are interconnected within the coalition. However, it has been shown that, in certain scenarios, the *underlying communication structure* between the players in a coalitional game can have a major impact on the utility and other characteristics of the game [14, 35]. By the underlying communication structure, we mean the graph representing the connectivity of the players among each other, i.e., which player communicates with which one inside each and every coalition. We illustrated examples on such interconnections in Section 2 and Fig. A.2 (b). In general, the main properties that distinguish a coalitional graph game are as follows:

1. The coalitional game is in graph form, and can be TU or NTU. However, the value of a coalition may depend on the external network structure as explained in Section 2.
2. The *interconnection* between the players within each coalition, i.e., who is connected to whom, strongly impacts the characteristics and outcome of the game.

3. The main objective is to derive low complexity distributed algorithms for players that wish to build a network *graph* (directed or undirected) and not just coalitional groups as in coalition formation games (Class II). Another objective is to study the properties (stability, efficiency, etc) of the formed network graph.

In coalitional graph games, the main theme is the presence of a graph for communication between the players. Typically, there are two objectives for coalitional graph games. The first and most important objective, is to provide low complexity algorithms for building a network graph to connect the players. A second objective is to study the properties and stability of the formed network graph. In some scenarios, the network graph is given, and hence analyzing its stability and efficiency is the only goal of the game. The following sections provide an in-depth study of coalitional graph games.

5.2 Coalitional Graph Games and Network Formation Games

The idea of having a value dependent on a graph of communication between the players was first introduced by Myerson in [14], through the graph function for TU games. In this work, starting with a TU canonical coalitional game $(\mathcal{N}, v)$ and given an undirected graph G that interconnects the players in the game, Myerson attempts to find a fair solution. For this purpose, a new value function u, which depends on the graph, is defined. For evaluating the value u of a coalition S, this coalition is divided into smaller coalitions depending on the players that are connected through S. For example, given a 3-players coalition $S = \{1, 2, 3\}$ and a graph $G = \{(2, 3)\}$ (only players 2 and 3 are connected by a link in G), the value $u(S, G)$ is equal to $u(S, G) = v(\{2, 3\}) + v(\{1\})$, where v is the original value of the canonical game. Using the new value u, Myerson presents an axiomatic approach, similar to the Shapley value, for solving the game in graph function form. The work in [14] shows that, a fair solution of the canonical game $(\mathcal{N}, v)$ in the presence of a graph structure, is the Shapley value of the game $(\mathcal{N}, u)$ where u is the newly defined value. This solution is known as the *Myerson value*. The drawback of the approach in [14] is that the value u of a coalition depends only on the connected players in the coalition with no dependence on the structure, e.g., for both graphs G_S^1 and G_S^2 in Fig. A.2 (b), the values u are equal (although the payoffs received by the players in G_S^1 and G_S^2 through the Myerson value allocation would be different due to the different graphs).

Nevertheless, the work in [14] motivated future work, and in [15] the value was extended so as to depend on the graph structure, and not only on the connected components. By doing so, coalitional graph games became a richer framework, however, finding solutions became more complex. While in [14], the objective was to find a solution, *given a graph*, new research in the area sought algorithms for *forming the graph*. One prominent tool in this area is non-cooperative game theory which was extensively used for forming the network graph. For instance, in [1, Ch. 9.5], using the Myerson framework of [14], an extensive form game is proposed for forming the network graph. However, the extensive form approach is impractical in many situations, as it requires listing all possible links in the graph, which is a complex combinatorial problem. Nonetheless, a new breed of games started to appear following this work, and these games are known as *network formation games*. The main objective in these games is to study the interactions among a group of players that wish to form a graph. Although in some references these games are decoupled from coalitional game theory, we place these games under coalitional graph games due to several reasons: (i)- The basis of all network formation games is the work on coalitional graph games that started in [14], (ii)- network formation games share many objectives with coalitional graph games such as the presence of a value and an allocation rule, the need for stability among others, and (iii)- the solutions of network formation games are quite correlated with coalition formation games (in terms of forming the graph) and canonical games (in terms of having stable allocations).

Network formation games can be thought of as a hybrid between coalitional graph games and non-cooperative games. The reason is that, for forming the network, non-cooperative game theory plays a prominent role. Hence, in network formation games there is a need to form a network graph as well as to ensure the stability of this graph, through concepts analogous to those used in canonical coalitional games. For forming the graph, a broad range of approaches exist, and are grouped into two types: *myopic* and *far sighted* [17]. The main difference between these two types is that, in myopic approaches, the players play their strategies given the current state of the network, while in far sighted algorithms, the players adapt their strategy by learning, and predicting future strategies of the other players. For both approaches, well-known concepts from non-cooperative game theory can be used. The most popular of such approaches is to consider the network formation as a non-zero sum non-cooperative game,

[17]These approaches are sometimes referred to as dynamics of network formation (see [36]).

where the players' strategies are to select one or more links to form or break. One approach to solve the game is to play *myopic best response dynamics* whereby each player selects the strategy, i.e. the link(s) to form or break, that maximizes its utility. Under certain conditions on the utilities, the best response dynamics converge to a Nash equilibrium, which constitutes a Nash network. These approaches are widespread in network formation games [37–39], and also, several refinements to the Nash equilibrium suitable for network formation are used [37–39]. The main drawback of aiming for a Nash network is that, in many network formation games, the Nash networks are trivial graphs such as the empty graph or can be inefficient. For these reasons, a new type of network formation games has been developed, which utilizes new concepts for stability such as *pairwise stability* and *coalitional stability* [36]. The basic idea is to present stability notions that depend on deviations by a group of players instead of the unilateral deviations allowed by the Nash equilibrium. Independent of the stability concept, a key design issue in network formation games is the tradeoff between stability and efficiency. It is desirable to devise algorithms for forming stable networks that can also be efficient in terms of payoff distribution or total social welfare. Several approaches for devising such algorithms exist, notably using stochastic processes, graph theoretical techniques or non-cooperative games. For a comprehensive survey on such algorithms, we refer the reader to [36].

Finally, the Myerson value and network formation games are not the only approaches for solving coalitional graph games. Other approaches, which are closely tied to canonical games can be proposed. For example, the work in [35], proposes to formulate a canonical game-like model for an NTU game, whereby the graph structure is taken into account. In this work, the authors propose an extension to the core called the *balanced core* which takes into account the graph structure. Further, under certain conditions, analogous to the balanced conditions of canonical games, the authors in [35] show that this balanced core is non-empty. Hence, coalitional graph games constitute quite a rich and diverse framework, with a broad range of applications. In the rest of this section, we review sample applications from communication networks.

5.3 Applications of Coalitional Graph Games

5.3.1 Distributed uplink tree formation in IEEE 802.16j

The most recent WiMAX standard, the IEEE 802.16j, introduced a new node, the relay station (RS) for improving the network's capacity and coverage. The introduction of the RS impacts the network architecture of WiMAX networks as the mesh network is replaced by a tree architecture which connects the base station (BS) to its subordinate RSs. An efficient design of the tree topology is, thus, a challenging problem, notably because the RSs can be nomadic or mobile. The IEEE 802.16j standard does not provide any algorithm for the tree formation, however, it states that both distributed and centralized approaches may be used. For tackling the design of the tree topology in 802.16j networks from a distributed approach, coalitional graph games provide a suitable framework. In [40], we model the problem of the *uplink* tree formation in 802.16j using coalitional graph games, namely network formation games. In this model, the players are the RSs who interact for forming a *directed* uplink tree structure (directed towards the BS). Every RS i in the tree, acts as a source node, and transmits the packets that it receives from external mobile stations (MSs) to the BS, using multi-hop relaying. Hence, when RS i is transmitting its data to the BS, all the RSs that are parents of i in the tree relay the data of i using decode-and-forward relaying. Through multi-hop relaying, the probability of error is reduced, and consequently the packet success rate (PSR) achieved by a RS can be improved. Essentially, the value function in this game is NTU as each RS optimizes its own utility. The utility of a RS i is an increasing function of the effective number of packets received by the BS (effective throughput) while taking into account the PSR, as well as the number of packets received from other RS (the more a RS receives packet, the more it is rewarded by the network). The utility also reflects the cost of maintaining a link, hence, each RS i has a maximum number of links that it can support. As the number of links on a RS i increases, the rewards needed for *accepting* a link also increase, hence making it difficult for other RSs to form a link with i. The strategy of each RS is two-fold: (1)- Each RS can select another RS (or the BS) with whom to connect, and (2)- Each RS can choose to break a number of links that are connected to it. For forming a directed link (i, j) between RS i and RS j, the consent of RS j is needed. In other words, if RS i bids to connect to RS j, RS j can either accept this link as a new connection, accept this link by replacing one or more other links, or reject the link. Using this formulation, the network formation game is a non-cooperative non-zero sum game played between the RSs, with the previously defined strategies. Hence, the dynamics of network formation are performed using an algorithm consisting of two phases. In the first phase, the RSs are prioritized, and in the second phase, proceeding sequentially by priority, each RS is allowed to play its

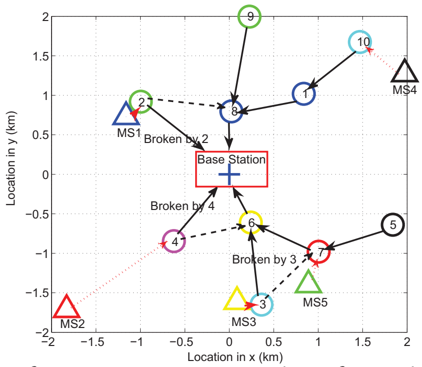


Fig. A.5: Example of an 802.16j tree topology formed using a distributed network formation game as per [40].

best response, i.e., the strategy that maximizes its utility. This algorithm is myopic, since the best response of a RS is played given the current state of the network graph. The end result is the formation of a Nash network tree structure that links the RSs to the BS. This tree structure is shown in [40] to yield an improvement in the overall PSR achieved by the MSs in the network, compared to a static star topology or a network with no relays. The proposed algorithm allows each RS to autonomously choose whether to cooperate or not, and hence, it can easily be implemented in a distributed manner.

In Fig. A.5, we show an example of a network topology formed by 10 RSs. In this figure, the solid arrows indicate the network topology that formed before the deployment of any MSs (in the presence of keep-alive packets only). The proposed network formation algorithm is, in fact, adaptive to environmental changes, such as the deployment of the external MSs as well as mobility of the RSs or MSs. Hence, in Fig. A.5, we can see how the RSs decide to break some of their link, replacing them with new links (in dashed arrows) hence adapting the topology, following the deployment of a number of MSs. In [41], the application of network formation games in 802.16j was extended and the algorithm was adapted to support the tradeoff between improving the effective throughput by relaying and the delay incurred by multi-hop transmission, for voice over IP services in particular. Future work can tackle various aspects of this problem using the tools of coalitional graph games. These aspects include devising a probabilistic approach to the network formation, or utilizing coalition graph games concepts such as the balanced core introduced in [35] among others.

5.3.2 Other applications and future potential

The presence of a network graph is ubiquitous in many wireless and communication applications. For designing, understanding, and analyzing such graphs, coalitional graph games are the accurate tool. Through the various concepts pertaining to network formation, stability, fairness, or others, one can model a diversity of problems. For instance, network formation games have been widely used in routing problems. For example, in [42], a stochastic approach for network formation is provided. In the proposed model, a network of nodes that are interested in forming a graph for routing traffic among themselves is considered. Each node in this model aims at minimizing its cost function which reflects the various costs that routing traffic can incur (routing cost, link maintenance cost, disconnection cost, etc.). For network formation, the work in [42] proposes a myopic dynamic best response algorithm. Each round of this algorithm begins by randomly selecting a pair of nodes i and j in the network. Once a random pair of nodes is selected, the algorithm proceeds in two steps. In the first step, if the link (i, j) is already formed in the network, node i is allowed to break this link, while in the second step node i is allowed to form a new link with a certain node k, if k accepts the formation of the link (i, k). In the model of [42], the benefit from forming a link (i, j) can be seen as some kind of cost sharing between nodes i and j. By using a stochastic process approach, the work in [42] shows that the proposed myopic algorithm always converges to a pairwise stable and efficient tree network. Under a certain condition on the cost function, the stable and efficient tree network is a simple star network. The efficiency is measured in terms of Pareto optimality of the utilities as the proposed game is inherently NTU. Although the network formation algorithm in [42] converges to a stable and efficient network, it suffers from a major drawback which is the slow convergence time, notably for large networks. The proposed algorithm is mainly implemented for undirected graphs but the authors provide sufficient insights on how this work can extend to directed graphs.

The usage of network formation games in routing applications is not solely restricted to forming the network, but also for studying properties of an existing network. For instance, in [43], the authors study the stability and the flow of the traffic in a given directed graph. For this purpose, several concepts from network formation games such as pairwise stability are used. In addition, the work in [43] generalizes the concept of pairwise stability making it more suitable for directed graphs. Finally, [43] uses non-cooperative game theory to determine the network flows at different nodes while taking into account the stability of the network graph. The applications of coalitional graph games are by no means limited to routing problems. The main future potential of using this class of games lies in problems beyond network routing. For instance, coalitional graph games are suitable tools to analyze problems pertaining to information trust management in wireless networks, multi-hop cognitive radio, relay selection in cooperative communications, intrusion detection, peer-to-peer data transfer, multi-hop relaying, packet forwarding in sensor networks, and many others. Certainly, this rich framework is bound to be used thoroughly in the design of many aspects of future communication networks.

6 Conclusions

In this tutorial, we provided a comprehensive overview of coalitional game theory, and its usage in wireless and communication networks. For this purpose, we introduced a novel classification of coalitional games by grouping the sparse literature into three distinct classes of games: canonical coalitional games, coalition formation games, and coalitional graph games. For each class, we explained in details the fundamental properties, discussed the main solution concepts, and provided an in-depth analysis of the methodologies and approaches for using these games in both game theory and communication applications. The presented applications have been carefully selected from a broad range of areas spanning a diverse number of research problems. The tutorial also sheds light on future opportunities for using the strong analytical tool of coalitional games in a number of applications. In a nutshell, this article fills a void in existing communications literature, by providing a novel tutorial on applying coalitional game theory in communication networks through comprehensive theory and technical details as well as through practical examples drawn from both game theory and communication applications.

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Paper B

A Distributed Coalition Formation Framework for Fair User Cooperation in Wireless Networks

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Abstract

Cooperation in wireless networks allows single antenna devices to improve their performance by forming virtual multiple antenna systems. However, performing a distributed and fair cooperation constitutes a major challenge. In this work, we model cooperation in wireless networks through a game theoretical algorithm derived from a novel concept from coalitional game theory. A simple and distributed merge-and-split algorithm is constructed to form coalition groups among single antenna devices and to allow them to maximize their utilities in terms of rate while accounting for the cost of cooperation in terms of power. The proposed algorithm enables the users to self-organize into independent disjoint coalitions and the resulting clustered network structure is characterized through novel stability notions. In addition, we prove the convergence of the algorithm and we investigate how the network structure changes when different fairness criteria are chosen for apportioning the coalition worth among its members. Simulation results show that the proposed algorithm can improve the individual user's payoff up to 40.42% as well as efficiently cope with the mobility of the distributed users.

1 Introduction

The wireless network performance can be improved through cooperation techniques. Cooperation allows wireless network users to benefit from various gains such as an increase in the achieved rate or an improvement in the bit error rate. Designing an efficient cooperation algorithm faces numerous challenges. First and foremost, cooperation entails various costs, such as power, that can limit its benefits or even impair the users' performance. Second, wireless network users tend to be selfish in nature. Therefore, deriving a fair and practical cooperation algorithm where the decision to cooperate does not degrade the performance of any of the cooperating users is a tedious task. Moreover, if a cooperation algorithm depends on a centralized entity in the network such as a base station (BS), an extra amount of communication overhead is required for information exchange among the users. Such a centralized scheme will heavily depend on the availability of resources at the centralized entity. Hence, there is a strong need to design a cooperation algorithm that can reduce this communication overhead by allowing the users to autonomously take the decision for cooperation without relying on a centralized intelligence. In summary, deriving a distributed and fair cooperative strategy is highly challenging but desirable in practice.

An important application for cooperation is the formation of virtual MIMO systems through cooperation among single antenna devices. In this context, a number of single-antenna devices can form virtual multiple antenna transmitters or receivers through cooperation, consequently, benefiting from the advantages of MIMO systems without the extra burden of having multiple antennas physically present on each transmitter or receiver. The information theoretical aspects of virtual MIMO systems were thoroughly exploited in [1], [2] and [3]. On one hand, the authors in [1] showed the gains in terms of outage capacity resulting from the cooperation of two single antenna devices that are transmitting to a far away receiver in a Rayleigh fading channel. On the other hand, the work in [2, 3] considered cooperation among multiple single antenna transmitters as well as receivers in a broadcast channel. Different cooperative scenarios were studied in the presence of a power cost for cooperation and the results showed the benefits of cooperation from a sum-rate perspective. Virtual MIMO gains are not only limited to rate gains. For example, forming virtual MIMO clusters in sensor networks can yield gains in terms of energy conservation [4].

Implementing distributed cooperation algorithms that allow the wireless network to reap these capacity or energy benefits requires an adequate analytical tool. In this regard, game theory provides a highly appealing mathematical tool for designing such distributed algorithms for cooperation or competition scenarios in wireless networks [5]. For instance, using coalitional game theory, the work in [6] and [7] focused on devising a distributed cooperative algorithm for rate improvement through *receivers* cooperation. The authors showed that for the receiver coalition game in a Gaussian interference channel and synchronous CDMA multiple access channel (MAC), a stable grand coalition of all users can be formed if no cost for cooperation is accounted for. Subsequently, two schemes were provided for dividing the payoffs, in terms of rate, among the users: the Nash Bargaining game solution and the proportional fair payoff division. Furthermore, using game theory, this fair allocation of rate for cooperating users in an interference channel was also studied in [8] for the transmitters. The authors in [8] assumed that the users in a Gaussian multiaccess channel will bargain for favorable rate allocation by threatening to cooperate and form coalitions of devices that will jam the channel. Based on this jamming assumption, the authors showed that a unique rate allocation exists verifying certain well defined fairness axioms from coalitional games.

Moreover, distributed cooperation through game theory is not restricted to the virtual MIMO problem but it is also of interest at higher layers such as the network and transport layers. Cooperation in routing protocols was tackled in [9] and [10] to reduce energy cost. The system derived in these papers encourages cooperation by rewarding service providers according to their contribution. Another aspect of cooperative networks, resource allocation, is discussed in [11]. Finally, cooperation in packet forwarding was studied in [12] and [13] using cooperative game theory, repeated game theory, and machine learning.

In summary, previous work on cooperation focused mainly on the information theoretical analysis of the cooperation gains, and characterized these gains in the presence of no cost, namely in the virtual MIMO problem [1–3, 6–8]. The main objectives of this paper are two fold: (1)- investigate the limitation on the cooperation gains in a virtual MIMO system, in the presence of a cost, and most importantly (2)- provide a distributed algorithm that models the users behavior when they interact in order to benefit from the widely established gains virtual MIMO gains (in the presence of a cost). In fact, while existing literature answered the question of “why to cooperate?”, we aim to answer questions such as when to cooperate and with whom to cooperate, notably when cooperation incurs a cost as well as a benefit. In this context, the main contributions of this paper are: (1)- to design a distributed game theoretical framework that enables single antenna *transmitters* to *autonomously* take decisions to cooperate and form virtual MIMO coalitions while accounting for the inherent benefit-cost trade off involved in this formation; (2) to study the topology and dynamics of a wireless network where the users seek cooperation through virtual MIMO, hence, assessing the possibility of achieving these gains in practice, and; (3)- to provide fair rules for performance improvement through cooperation. Thus, we construct a coalition formation algorithm based on well-defined and distributed merge-and-split rules from cooperative games suitable for tackling this transmitter cooperation problem. The convergence of this merge-and-split transmitter cooperation algorithm is discussed and the stability of the resulting coalition structure is characterized through suitable stability notions. Finally, various fairness criteria for allocating the extra benefits among coalition users are discussed and their effect on the network structure is analyzed. Simulation results show that our algorithm can improve the individual user’s payoff up to 40.42% as well as efficiently handle the users’ mobility.

The rest of this paper is organized as follows: Section 2 presents the transmitter cooperation system model. Section 3 presents the proposed game theoretic algorithm while Section 4 discusses fairness criteria for payoff division. Simulation results are analyzed in Section 5 and conclusions are drawn in Section 6.

2 System Model

In this section, we present the transmitter cooperation coalitional game model and discuss its properties.

2.1 Transmitter Cooperation Model

Consider a network having M_t single antenna transmitters (e.g. mobile users) sending data in the uplink to a fixed receiver, e.g., a BS, with M_r receive antennas (multiple access channel). Denote $N = \{1 \dots M_t\}$ as the set of all M_t users in the network, and let $S \subseteq N$ be a coalition consisting

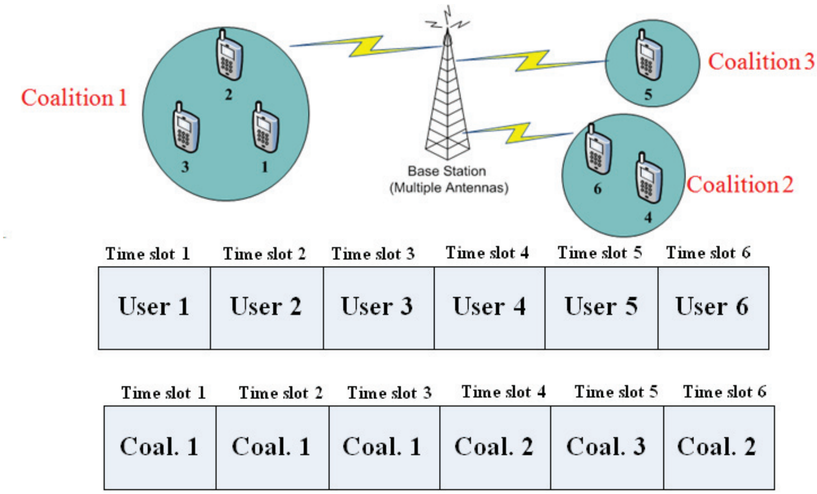

Fig. B.1: User cooperation example coalitions and TDMA transmission.

of $|S|$ users ($|\cdot|$ represents the cardinality of a set). We consider a TDMA transmission in the network thus, in a non-cooperative manner, the M_t users require a time scale of M_t slots to transmit as every user occupies one time slot. When cooperating, the single antenna transmitters form different disjoint coalitions (each coalition can be seen as a single user MIMO device) and they will subsequently transmit in a TDMA manner, which is one coalition per transmission. During the time scale M_t, each coalition is able to transmit within all the time slots *previously held by its users*. For a cooperating coalition S, we consider a block fading $M_r \times |S|$ channel matrix $\boldsymbol{H}_S$ with a path loss model between the users in S and the BS with each element of the matrix $h_{i,k} = e^{j\phi_{i,k}}\sqrt{\kappa/d_{i,k}^{\alpha}}$ with α the path loss exponent, κ the path loss constant, $\phi_{i,k}$ the phase of the signal from transmitter i to the BS receiver k, and $d_{i,k}$ the distance between transmitter i and the base BS's receiver k. An illustration of the model is shown in Fig. B.1 for $M_t = 6$.

As we are considering a TDMA system, we define a fixed transmit power constraint *per time slot*, i.e., a power constraint *per coalition* $\tilde{P}$ as in [2, 3]. This average power constraint is applied to *all* the transmitters that are part of the coalition active in the slot. In the non-cooperative scenario, this *same* power constraint per slot is simply the power constraint per individual user active in the slot. In fact, due to ergodicity, for each time slot, the average long term power constraint per individual user and the power constraint per slot (i.e. constraining all transmitters of a coalition active in a slot) are the same [2, 3]. In the considered TDMA system, each coalition transmits in a slot, hence, perceiving no interference from other coalitions during transmission. As a result, in a slot, the sum-rate capacity of the virtual MIMO formed by a coalition S, under a power constraint P_S with Gaussian signaling is given by [14]

$$C_S = \max_{\boldsymbol{Q}_S} I(\boldsymbol{x}_S; \boldsymbol{y}_S) = \max_{\boldsymbol{Q}_S} (\log\det(\boldsymbol{I}_{M_r} + \boldsymbol{H}_S \cdot \boldsymbol{Q}_S \cdot \boldsymbol{H}_S^{\dagger})) \tag{B.1}$$

$$\text{s.t. } \operatorname{tr}[\boldsymbol{Q}_S] \leq P_S,$$

where $\boldsymbol{x}_S$ and $\boldsymbol{y}_S$ are, respectively, the transmitted and received signal vectors of coalition S of size $|S| \times 1$ and $M_r \times 1$, $\boldsymbol{Q}_S = \mathrm{E}\,[\boldsymbol{x}_S \cdot \boldsymbol{x}_S^{\dagger}]$ is the covariance of $\boldsymbol{x}_S$ and $\boldsymbol{H}_S$ is the channel matrix with $\boldsymbol{H}_S^{\dagger}$ its conjugate transpose.

The considered channel matrix $\boldsymbol{H}_S$ is assumed perfectly known at the transmitter and receiver, thus, the maximizing input signal covariance is given by $\boldsymbol{Q}_S = \boldsymbol{V}_S \boldsymbol{D}_S \boldsymbol{V}_S^{\dagger}$ ([14, 15]) where $\boldsymbol{V}_S$ is the unitary matrix given by the singular value decomposition of $\boldsymbol{H}_S = \boldsymbol{U}_S \boldsymbol{\Sigma}_S \boldsymbol{V}_S^{\dagger}$ and $\boldsymbol{D}_S$ is an $|S| \times |S|$ diagonal matrix given by $\boldsymbol{D}_S = \operatorname{diag}(D_1, \ldots, D_K, 0, \ldots, 0)$ where $K \leq \min(M_r, |S|)$ is the number of positive singular values of the channel $\boldsymbol{H}_S$ (eigenmodes) and each D_i given by $D_i = (\mu - \lambda_i^{-1})^{+}$, where μ is determined by water-filling to satisfy the coalition power constraint $\operatorname{tr}[\boldsymbol{Q}_S] = \operatorname{tr}[\boldsymbol{D}_S] = \sum_i D_i = P_S$, and λ_i is the ith eigenvalue of $\boldsymbol{H}_S^{\dagger}\boldsymbol{H}_S$. Using [14], the resulting capacity, in a slot, for a coalition S is

$$C_S = \sum_{i=1}^{K} (\log(\mu\lambda_i))^{+}. \tag{B.2}$$

For forming the considered virtual MIMO coalitions and benefiting from the capacity gains, the users need to exchange their data information and their channel (user-BS) information. For this purpose, we will consider a cost for information exchange in terms of transmit power. This transmit power cost mainly models the data exchange penalty. As we consider block fading channels with a long coherence time, the additional power penalty for exchanging the user-BS channel information can be deemed as negligible relatively to the data exchange cost, since the considered channel varies slowly (for example, exchange of the channel information can be done only periodically). Consequently, the cost for information exchange is taken as the sum of the powers required by each user in a coalition S to broadcast to its corresponding farthest user inside S. Due to the broadcast nature of the wireless channel, once a coalition member broadcasts its information to the farthest user, all the other members can also receive this information simultaneously. The power needed for broadcast between a user $i \in S$ and its corresponding *farthest* user $\hat{i} \in S$ is

$$\bar{P}_{i,\hat{i}} = \frac{\nu_0 \cdot \sigma^2}{g_{i,\hat{i}}^2}, \tag{B.3}$$

where ν_0 is a target average SNR for information exchange, σ^2 is the noise variance and $g_{i,\hat{i}} = \sqrt{\kappa / d_{i,\hat{i}}^{\alpha}}$ is the path loss between users i and $\hat{i}$ with $d_{i,\hat{i}}$ the distance between users i and $\hat{i}$. In consequence, the total power cost for a coalition S having $|S|$ users is given by $\hat{P}_S$ as follows

$$\hat{P}_S = \sum_{i=1}^{|S|} \bar{P}_{i,\hat{i}}. \tag{B.4}$$

It is interesting to note that the defined cost depends on the location of the users and the size of the coalition; hence, a higher power cost is incurred whenever the distance between the users increases or the coalition size increases. Thus, the actual power constraint P_S per coalition S *with cost* is

$$P_S = (\tilde{P} - \hat{P}_S)^+, \tag{B.5}$$

where $\tilde{P}$ is the average power constraint per coalition (per slot), $\hat{P}_S$ the cooperation power cost given in (B.4) and $a^+ \triangleq \max(a, 0)$. In order to achieve the capacity in (B.2), within a slot, each user of a coalition S adjusts its power value based on the water-filling solution, taking into account the available power constraint P_S. Note that, since the power constraint $\tilde{P}$ applied over a coalition is the same as the maximum power constraint per individual user in the coalition, the water-filling solution always yields a power value per user that does not violate the user's available power after deducing the cost for cooperation in (B.3) from its individual long term power constraint.

The considered power cost does not take into account the interference for exchange of information between users and can be considered as a lower bound of the penalty incurred by cooperation. In addition to this power cost, a fraction of time may be required for the data exchange between the users prior to cooperation. However, due to the fact that the proposed power cost given in (B.4) depends on distance and coalition size, the formed coalitions will typically consist of small clusters of nearby close users (as will be verified through simulations), and thus the users can exchange information at high rates rendering the time for data exchange negligible relatively to the transmission time slot (typically, the distance between the users of a coalition and the BS is larger than the distance between the coalition users themselves). Furthermore, in practice, performing a cooperation for virtual MIMO formation can require a synchronization at the carrier frequency between the nodes, yielding some costs for practical implementation. In this work, we will not account for these carrier synchronization costs similar to existing virtual MIMO literature [1, 3, 4], [6, 7]. The coalition formation results derived in this paper could also be applied for other cost functions without loss of generality. For example, the cost of power can be replaced by a cost of bandwidth where one could quantify the use of an additional band for information exchange, orthogonal to the band of transmission.

Based on the defined capacity benefit and power cost, over the TDMA time scale of M_t, for every coalition $S \subseteq N$, we define the utility function (or *characteristic function* in coalitional game theory terms [16]) as

$$v(S) = \begin{cases} |S| \cdot C_S, & \text{if } P_S > 0; \\ 0, & \text{otherwise.} \end{cases} \tag{B.6}$$

where P_S is given by (B.5), C_S is given by (B.2) and $|S|$ is the number of users in S. This utility represents the total capacity achieved by coalition S during the time scale M_t while accounting for the cost through the power constraint. A coalition of $|S|$ users will transmit with capacity C_S during $|S|$ time slots; thus achieving a total capacity of $v(S)$ during the time scale M_t (e.g. in Fig. B.1 during $M_t = 6$ coalition 2 consisting of 2 users achieves C_2 in slot 4 and C_2 in slot 6; thus a total of $2 \cdot C_2$ during $M_t = 6$ slots). The second case in (B.6) implies that if the power for information exchange is larger than (or equal to) the available power constraint the coalition cannot be formed due to a utility of 0. The payoff of each user in a coalition S is computed by a fair division of the utility $v(S)$, through various criteria explained in Section 4. This individual user payoff denoted ϕ_i^v represents the total rate achieved by user i during the transmission time scale M_t.

Thus, we have a coalitional game (N,v) with a transferable utility (i.e. the coalition value can be arbitrarily divided among its users) and we seek a coalition structure that allows the users to maximize their utilities with cost.

2.2 Transmitter Cooperation Game Properties

For modeling cooperation in wireless networks, existing work mainly sought to prove that the grand coalition of all users can form and inspected its stability. In the proposed transmitter cooperation (N,v) coalitional game, due to the cooperation costs, the grand coalition will seldom form and, instead, disjoint coalitions will form in the network.

Definition 4 *A coalitional game (N, v) with transferable utility is said to be superadditive if for any two disjoint coalitions $S_1, S_2 \subset N$, $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$.*

Theorem 1 *The proposed transmitter (N,v) coalitional game with cost is non-superadditive.*

Proof: *Consider two disjoint coalitions $S_1 \subset N$ and $S_2 \subset N$ in the network with their corresponding utilities $v(S_1)$ and $v(S_2)$ when they do not cooperate with each other. Assume that the users of $S_1 \cup S_2$ are located far enough to yield a power cost per* (B.4) *$\hat{P}_{S_1 \cup S_1} \geq \tilde{P}$. In this case, by* (B.5) *$P_{S_1 \cup S_2} = 0$ yielding $v(S_1 \cup S_2) = 0 < v(S_1) + v(S_2)$* (B.6)*; hence the game is not superadditive.* □

Definition 5 *A payoff vector $\boldsymbol{\phi}^v = (\phi_1^v, \ldots, \phi_{M_t}^v)$ for dividing the value v of a coalition is said to be* group rational *or efficient if $\sum_{i=1}^{M_t} \phi_i^v = v(N)$. A payoff vector $\boldsymbol{\phi}^v$ is said to be* individually rational *if the player can obtain the benefit no less than acting alone, i.e. $\phi_i^v \geq v(\{i\}), \forall\, i$. An* imputation *is a payoff vector satisfying the above two conditions.*

Definition 6 *An imputation $\boldsymbol{\phi}^v$ is said to be unstable through a coalition S if $v(S) > \sum_{i \in S} \phi_i^v$, i.e., the players have incentive to form coalition S and reject the proposed $\boldsymbol{\phi}^v$. The set $\mathcal{C}$ of stable imputations is called the* core*, i.e.,*

$$\mathcal{C} = \left\{ \boldsymbol{\phi}^v : \sum_{i \in N} \phi_i^v = v(N) \text{ and } \sum_{i \in S} \phi_i^v \geq v(S)\ \forall\ S \subseteq N \right\}. \tag{B.7}$$

A non-empty core means that the players have an incentive to form the grand coalition.

Remark 1 *In general, the core of the (N,v) transmitter cooperation game with cost is empty.*

In the proposed model, the costs for cooperation for a coalition S increase as the number of users in a coalition increase as well as when the distance between the users increase hence affecting the topology. In particular, consider the grand coalition $\{N\}$ of all M_t users in the network. This coalition consists of a large number of users who are randomly located at different distances. Hence, the grand coalition $\{N\}$ will often have a value of $v(\{N\}) = 0$ due to the cooperation costs and several coalitions $S \subset N$ have an incentive to deviate from this grand coalition and form independent disjoint coalitions. Consequently, an imputation that lies in the core cannot be found, and, due to cost, the core of the the transmitter cooperation (N,v) game is generally empty. Briefly, as a result of the non-superadditivity of the game as well as the emptiness of the core, the grand coalition of all transmitters will not form. Instead, independent disjoint coalitions will form. Hence, in the next section, we will devise an algorithm for coalition formation that can characterize these disjoint coalitions.

3 Coalition Formation Algorithm

In this section, we construct a novel coalition formation algorithm and we prove its key properties.

3.1 Coalition Formation Concepts

Coalition formation has been a topic of high interest in game theory [17–21]. In [19], [20] and [21], an interesting approach for dynamic coalition formation is derived. The mathematical tools presented in [20] and [21] allow to build algorithms to dynamically form coalitions among players through two simple merge-and-split rules, which can be applied in a distributed manner and, thus, deemed suitable for wireless network games. Introducing this framework and applying it into wireless networks requires several concepts to be defined as follows.

Definition 7 *A* collection *of coalitions in the grand coalition N, denoted S, is defined as the set $S = \{S_1, \ldots, S_l\}$ of mutually disjoint coalitions S_i of N. In other words, a collection is any arbitrary group of disjoint coalitions S_i of N not necessarily spanning all players of N. If the collection spans all the* *players of* N*; that is* $\bigcup_{j=1}^{l} S_j = N$*, the collection is referred to as a* partition *of* N.

Definition 8 *A preference operator or* comparison relation $\triangleright$ *is defined for comparing two collections* $R = \{R_1, \ldots, R_l\}$ *and* $S = \{S_1, \ldots, S_p\}$ *that are partitions of the same subset* $A \subseteq N$ *(i.e. same players in* R *and* S*). Thus,* $R \triangleright S$ *implies that the way* R *partitions* A *is preferred to the way* S *partitions* A.

Various well known orders can be used as comparison relations [20], [21]. These orders are split into two categories: coalition value orders and individual value orders. Coalition value orders compare two collections (or partitions) using the value of the coalitions inside these collections such as in the utilitarian order where $R \triangleright S$ implies $\sum_{i=1}^{l} v(R_i) > \sum_{i=1}^{p} v(S_i)$. Individual value orders perform the comparison using the individual payoffs such as the Pareto order. For these orders, two collections R and S are seen as sets of individual payoffs of the same length L (number of players) resulting from a group rational division of the utilities of each coalition $R_i \in R$ and $S_i \in S$. For a collection $R = \{R_1, \ldots, R_l\}$, the payoff of a player j in a coalition $R_i \in R$ is denoted by $\phi_j^v(R)$; and $\sum_{i=1}^{l} v(R_i) = \sum_{j=1}^{L} \phi_j^v(R)$. The Pareto order is defined as

$$R \triangleright S \Longleftrightarrow \{\phi_j^v(R) \geq \phi_j^v(S) \ \forall\ j \in R, S\}$$
$$\text{with } \textit{at least one strict inequality } (>) \text{ for a player } k. \tag{B.8}$$

The Pareto order implies that a collection R is preferred over S, if at least one player is able to improve its payoff when the coalition structure changes from S to R without decreasing other players' payoffs.

3.2 Merge-and-Split Coalition Formation Algorithm

Using the coalition formation concepts prescribed in the previous section, a coalition formation algorithm for self organization in wireless networks can be generated. This algorithm will be based on simple rules of merge-and-split that allow to modify a partition T of N as follows [20]:

- **Merge Rule:** Merge any set of coalitions $\{S_1, \ldots, S_l\}$ where $\{\bigcup_{j=1}^{l} S_j\} \triangleright \{S_1, \ldots, S_l\}$, therefore, $\{S_1, \ldots, S_l\} \rightarrow \{\bigcup_{j=1}^{l} S_j\}$. (each S_i denotes a coalition).

- **Split Rule:** Split any coalition $\bigcup_{j=1}^{l} S_j$ where $\{S_1, \dots, S_l\} \triangleright \{\bigcup_{j=1}^{l} S_j\}$, thus, $\{\bigcup_{j=1}^{l} S_j\} \rightarrow \{S_1, \dots, S_l\}$. (each S_i denotes a coalition).

In brief, multiple coalitions will merge (split) if merging (splitting) yields a preferred collection based on a chosen $\triangleright$. In [20] and [21] it is shown that any arbitrary iteration of merge-and-split operations *terminates*, therefore, it will be suitable to devise a coalition formation algorithm by means of merge-and-split. In the transmitter cooperation game, the Pareto order given by (B.8) is highly appealing as a comparison relation $\triangleright$ for the merge-and-split rules. With the Pareto order, coalitions will merge only if at least one user can enhance its individual payoff through this merge without decreasing the other users' payoffs. Similarly, a coalition will split only if at least one user in that coalition is able to strictly improve its individual payoff through the split without hurting other users. A decision to merge or split is, thus, tied to the fact that all users must benefit from merge or split, thus, any merged (or split) form is reached only if it allows all involved users to maintain their payoffs with at least one user improving. In summary, the proposed algorithm is a coalition formation algorithm with partially reversible agreements [18], where the users sign a binding agreement to form a coalition through the merge operation (if *all* users are able to improve their individual payoffs from the previous state) and they can only split this coalition if splitting does not decrease the payoff of any coalition member (partial reversibility). Having partial reversibility through the split operation reduces the complexity of the coalition formation process relatively to a fully reversible process [18] but can impact the coalition stability as further discussed in Section 3.3.

For the proposed virtual MIMO formation game, the self-organizing algorithm consists of two phases: adaptive coalition formation and transmission. In the adaptive coalition formation phase, an iteration of sequential merge-and-split rules is performed until the iteration terminates yielding a final network partition composed of independent disjoint coalitions. In the transmission phase, the formed coalitions transmit in their corresponding slots in a TDMA manner. The transmission phase may occur several times prior to the repetition of the coalition formation phase, notably in low mobility environment where changes in the coalition structure due to mobility are seldom.

Although any arbitrary merge can be performed, we propose a distributed cost-based merge process allowing the coalitions (users) to perform a local search for partners. Consequently, the decision to merge with neighboring coalitions is taken based on the Pareto order proceeding from the partner that provides the lowest cost. In order for coalition S_1 to merge with another coalition S_2, the utility of the formed coalition through merge must be positive; that is $v(S_1 \cup S_2) > 0$ otherwise no benefits exist for the merge. Thus, based on the defined power cost (B.4) and utility (B.6); coalitions can only merge when the cost for cooperation is less than $\tilde{P}$. Otherwise, when the cost is greater than or equal $\tilde{P}$, through (B.6) the utility of the merged coalition will be 0 and there is no mutual benefit. Thus, using (B.4) the merge is possible (non zero utility) for S_1 with S_2 if $\hat{P}_{S_1 \cup S_2} < \tilde{P}$, that is $\sum_{i=1}^{|S_1 \cup S_2|} \bar{P}_{i,\hat{i}} < \tilde{P}$ which, by (B.3), yields

$$\sum_{i=1}^{|S_1 \cup S_2|} \frac{1}{d_{i,\hat{i}}^{\alpha}} < \frac{\tilde{P}}{\nu_0 \cdot \sigma^2 \cdot \kappa}. \tag{B.9}$$

Thus, a coalition will only attempt to merge with other coalitions where (B.9) can be verified.

Each stage of the proposed algorithm starts from an initial network partition $T = \{T_1, \ldots, T_l\}$ of N. In this partition, any random coalition (user) can start with the merge process. For implementation purposes, assume that the coalition $T_i \in T$ which has the highest utility in the initial partition T starts the merge by attempting to cooperate with the coalition yielding the lowest cost. On one hand, if merging occurs, a new coalition $\tilde{T}_i$ is formed and, in its turn, coalition $\tilde{T}_i$ will attempt to merge with the lowest cost partner. On the other hand, if T_i was unable to merge with the smallest cost coalition, it tries the next lowest cost partner, proceeding sequentially through the coalitions verifying (B.9). The search ends by a final merged coalition T_i^{final} composed of T_i and one or several of coalitions in its vicinity (or just T_i, if no merge occured). The algorithm is repeated for the remaining $T_i \in T$ until all the coalitions have made their local merge decisions, resulting in a final partition W. The coalitions in the resulting partition W are next subject to split operations, if any is possible. An iteration consisting of multiple successive merge-and-split operations is repeated until it terminates. Table B.I shows a summary of one stage of the proposed algorithm.

3.3 Partition Stability

The result of the proposed algorithm in Table B.I is a network partition composed of disjoint independent coalitions. The stability of this resulting network structure can be investigated with respect to a novel concept of

Table B.I: One stage of the proposed merge-and-split algorithm.

Initial State

The network is partitioned by $T = \{T_1, \ldots, T_k\}$ (At the beginning
of all time T = N = $\{1, \ldots, M_t\}$ with non-cooperative users).

Proposed Coalition Formation Algorithm

Phase I - Adaptive Coalition Formation:

In this phase, coalition formation using merge-and-split occurs.

repeat

a) Coalitions begin the local search merge operation in Section 3.2:

W = **Merge**(T).

b) Coalitions in W decide to split based on the Pareto order.

T = **Split**(W).

until merge-and-split iteration terminates.

Phase II - Virtual MIMO Transmission:

The coalitions transmit during the time scale M_t with 1 coalition per slot with each coalition occupying all the time slots previously held by its members.

The proposed algorithm is repeated periodically, enabling the users to autonomously self-organize and adapt the topology to environmental changes such as mobility.

defection function $\mathbb{D}$ [19], [20].

Definition 9 *A* defection *function* $\mathbb{D}$ *is a function which associates with each partition* T *of* N *a family (group) of collections in* N*. A partition* $T = \{T_1, \ldots, T_l\}$ *of* N *is* $\mathbb{D}$-stable *if no group of players is interested in leaving* T *when the players who wish to leave can only form the collections allowed by* $\mathbb{D}$*.*

Two important defection functions can be pinpointed [19], [20]. First, the $\mathbb{D}_{hp}(T)$ function (denoted $\mathbb{D}_{hp}$) which associates with each partition T of N the family of all partitions of N that can form by merging or splitting coalitions in T. This function allows any group of players to leave the partition T of N through merge-and-split operations to create another *partition* in N. Second, the $\mathbb{D}_c(T)$ function (denoted $\mathbb{D}_c$) which associates with each partition T of N the family of all collections in N. This function allows any group of players to leave the partition T of N through *any* operation and create an arbitrary *collection* in N. Two forms of stability stem from these definitions: $\mathbb{D}_{hp}$ stability and a stronger $\mathbb{D}_c$ stability. A partition T is $\mathbb{D}_{hp}$-stable, if no players in T are interested in leaving T through merge-and-split to form other partitions in N; while a partition T is $\mathbb{D}_c$-stable, if no players in T are interested in leaving T to form other collections in N.

The $\mathbb{D}$-stability of a partition depends on various properties of its coalitions. A partition $T = \{T_1, \dots, T_l\}$ is $\mathbb{D}_{hp}$-stable, if the following two conditions are met ([19], [20]): (i)- For each $i \in \{1, \dots, p\}$ and for each partition $\{R_1, \dots, R_p\}$ of T_i we have $\{R_1, \dots, R_p\} \ntriangleright T_i$, and (ii)- For each $S \subseteq \{1, \dots, l\}$ we have $\bigcup_{i \in S} T_i \ntriangleright \{T_i | i \in S\}$ ($\ntriangleright$ is the opposite rule of the preference operator $\triangleright$). By definition of $\mathbb{D}_{hp}$ stability, we have

Theorem 2 *Every partition resulting from our proposed merge-and-split algorithm is $\mathbb{D}_{hp}$-stable.*

Proof: *As every iteration of merge-and-split terminates, a resulting partition from such iterations cannot be subject to any further merge or split. Therefore, the players in a partition T resulting from sequential merge-and-split operations such as in the algorithm of Table* B.I *cannot leave this partition through merge or split. Assume $T = \{T_1, \dots, T_l\}$ is the partition resulting from the proposed merge-and-split algorithm. If for any $i \in \{1, \dots, l\}$ and for any partition $\{S_1, \dots, S_p\}$ of T_i we assume that $\{S_1, \dots, S_p\} \triangleright T_i$ then the partition T can still be modified through the application of the split rule on T_i contradicting with the fact that T resulted from a termination of the merge-and-split iteration; therefore $\{S_1, \dots, S_p\} \ntriangleright T_i$ (first $\mathbb{D}_{hp}$ stability condition verified). A similar reasoning is applicable in order to prove that T verifies the second condition; since otherwise a merge rule would still be applicable.*

□

One drawback of $\mathbb{D}_{hp}$-stable partitions is that coalitions within such partitions may be prone to deviations due to the partial reversibility of the merge-and-split algorithm. For instance, in a $\mathbb{D}_{hp}$-stable partition, once a group of users form a coalition S by Pareto order merge, some subset of S may be able to deviate from this coalition but is not allowed to do so unless the deviation does not decrease the payoff of the remaining users in S (split only by Pareto order). The rationale behind this is that, once an agreement is signed to form a coalition by Pareto order, the users can only deviate if they do not hurt the other coalition members. Coalitions exhibiting such internal deviation incentives are referred to as coalitions "prone to deviations". For the proposed algorithm, the number of such coalitions will generally be very small due to the usage of Pareto order for merge and the presence of cooperation costs which limit the possible deviations that are not captured by the split operation (generally, nearby users merge into a coalition and splitting occurs when they distance themselves due to mobility). Moreover, by imposing stringent fairness criteria for the payoff division as will be seen in Section 5, the number of such coalitions can be further reduced.

Due to the possibility of having coalitions prone to deviations in a $\mathbb{D}_{hp}$-stable partition, a stronger $\mathbb{D}_c$-stable partition can be sought by the proposed algorithm. For instance, the work in [20] showed that, if it exists, a $\mathbb{D}_c$-stable partition has the following properties:

1. If it exists, a $\mathbb{D}_c$-stable partition is the *unique* outcome of any *arbitrary* iteration of merge-and-split and is a unique $\mathbb{D}_{hp}$-stable partition.

2. A $\mathbb{D}_c$-stable partition T is a unique $\triangleright$-maximal partition, that is for all partitions $T' \neq T$ of N, $T \triangleright T'$. In the case where $\triangleright$ represents the Pareto order, this implies that the $\mathbb{D}_c$-stable partition T is the partition that presents a *Pareto optimal* payoff distribution for all the players.

3. A $\mathbb{D}_c$-stable partition does not contain any coalitions prone to deviations.

Clearly, a $\mathbb{D}_c$-stable partition is an optimal partition that the wireless network can seek as it provides a payoff distribution that is Pareto optimal with for all users with respect to any other network partition. In addition, this partition is a unique outcome of any arbitrary iteration of merge-and-split rules. However, the existence of a $\mathbb{D}_c$-stable partition is not always guaranteed [20]. The $\mathbb{D}_c$-stable partition $T = \{T_1, \ldots, T_l\}$ of the whole space N exists if a partition of N verifies two necessary and sufficient conditions [20]:

A) For each $i \in \{1, \ldots, l\}$ and each pair of disjoint *coalitions* A and B such that $\{A \cup B\} \subseteq T_i$ we have $\{A \cup B\} \triangleright \{A, B\}$ (referred to as cond. A) hereafter).

B) For the partition $T = \{T_1, \ldots, T_l\}$ a coalition $G \subset N$ formed of players belonging to different $T_i \in T$ is T-incompatible if for no $i \in \{1, \ldots, l\}$ we have $G \subset T_i$. $\mathbb{D}_c$-stability requires that for all T-incompatible coalitions

$\{G\}[T] \triangleright \{G\}$ where $\{G\}[T] = \{G \cap T_i \; \forall \, i \in \{1, \ldots, l\}\}$ is the projection of coalition G in partition T (referred to as cond. B) hereafter).

If no partition of N can satisfy these conditions, then no $\mathbb{D}_c$-stable partitions of N exists. Since the $\mathbb{D}_c$-stable partition is a unique outcome of any arbitrary merge-and-split iteration, we have

Lemma 1 *For the proposed (N, v) transmitter cooperation coalitional game, the merge-and-split algorithm of Table* B.I *converges to the optimal $\mathbb{D}_c$-stable partition, if such a partition exists. Otherwise, the proposed algorithm yields a final network partition that is $\mathbb{D}_{hp}$-stable.*

In the transmitter cooperation game, the existence of a $\mathbb{D}_c$-stable partition depends on various factors. For instance, cond. A) states that, for a $\mathbb{D}_c$-stable partition T, every coalition $T_i \in T$ must verify the Pareto order not only at the level of the whole coalition T_i but also at the level of all disjoint coalitions subsets of T_i. Verifying the Pareto order requires that the utility of every union of any two disjoint coalitions subsets of a coalition T_i must yield an extra utility over the disjoint case; that is $v(A \cup B) > v(A) + v(B) \; \forall \; A, B \subset T_i$. In an ideal case with no cost, as the number of transmit antennas is increased for a fixed power constraint, the overall system's diversity increases as the data passes through different channel values allowing, with adequate coding, the symbols to be recovered without error at a higher rate [15]. In such a case, since $A \cup B$ has a larger number of antennas than A and B, $\forall \; A, B \subset T_i$ and for each T_i we have $C_{A \cup B} > \max(C_A, C_B)$ and thus,

$$\begin{aligned} |A \cup B| \cdot C_{A\cup B} &> |A| \cdot \max(C_A, C_B) + |B| \cdot \max(C_A, C_B), \\ |A \cup B| \cdot C_{A\cup B} &> |A| \cdot C_A + |B| \cdot C_B \Leftrightarrow v(A \cup B) > v(A) + v(B), \end{aligned} \tag{B.10}$$

which is sufficient to verify cond. A) for $\mathbb{D}_c$-stability when adequate payoff divisions are done. However, due to the cost $C_{A\cup B}, C_A$ and C_B can have different power constraints and (B.10) may not be guaranteed $\forall A, B \subset T_i$. Guaranteeing this condition is directly dependent on the cooperation cost within the coalitions in T and, thus, on the users' location. In practical networks, verifying cond. A) for $\mathbb{D}_c$-stability depends on the users' random locations.

Cond. B) for the existence of a $\mathbb{D}_c$-stable partition T is that players formed from different $T_i \in T$ have no incentive to form a coalition G outside of T. In the transmitter cooperation game, cond. B is also dependent on the location of the coalitions $T_i \in T$; specifically on the distance between the users in different coalitions $T_i \in T$. Thus, cond. B) is verified whenever two users belonging to different coalitions in a partition T are separated by large distances. A sufficient condition for verifying this second requirement can be derived.

Theorem 3 *For a network partition $T = \{T_1, \dots, T_l\}$ resulting from the proposed algorithm; if the distance $d_{i,j}$ between* any *two users $i \in T_i$ and $j \in T_j$ with $T_i \neq T_j$ verifies $d_{i,j} > \left(\frac{\kappa \cdot \tilde{P}}{2 \cdot \nu_0 \cdot \sigma^2}\right)^{\frac{1}{\alpha}} = \hat{d}_0$ then the second condition for $\mathbb{D}_c$ stability, cond. B), is verified.*

Proof: *Since a $\mathbb{D}_c$-stable partition is a unique outcome of any merge-and-split iteration, we will consider the partition $T = \{T_1, \dots, T_l\}$ resulting from any merge-and-split iteration in order to show when cond. B) can be satisfied. A T-incompatible coalition is a coalition formed from users belonging to different $T_i \in T$. Consider the T-incompatible coalition $\{i, j\}$ that can potentially form between two users $i \in T_i$ and $j \in T_j$ with $T_i \neq T_j$. The total power cost for $\{i, j\}$ is given by* (B.3) *as $\hat{P}_{\{i,j\}} = \bar{P}_{i,j} + \bar{P}_{j,i} = 2 \cdot \bar{P}_{i,j}$. In the case where the total power cost is larger than the constraint, we have $\hat{P}_{\{i,j\}} \geq \tilde{P}$ and thus $\bar{P}_{i,j} \geq \frac{\tilde{P}}{2}$ which yields the required condition on distance $d_{i,j} \geq (\frac{\kappa \cdot \tilde{P}}{2 \cdot \nu_0 \cdot \sigma^2})^{\frac{1}{\alpha}} = \hat{d}_0$. We will have by* (B.6) *$v(\{i, j\}) = 0$, and, thus, $\phi_i^v(\{i, j\}) = \phi_j^v(\{i, j\}) = 0$. Or we have that, $\{i, j\}[T] = \{\{i, j\} \cap T_k \ \forall \ k \in \{1, \dots, l\}\} = \{\{i\}, \{j\}\}$, and, thus, $\phi_i^v(\{i, j\}[T]) = v(\{i\}) > \phi_i^v(\{i, j\}) = 0$ and $\phi_j^v(\{i, j\}[T]) = v(\{j\}) > \phi_j^v(\{i, j\}) = 0$. Consequently, $\{i, j\}[T] \triangleright \{i, j\}$ and cond. B) is verified for any T-incompatible coalition formed of 2 users. Moreover, when* any *two users $i \in T_i$ and $j \in T_j$ with $T_i \neq T_j$ are separated by $\hat{d}_0$, T-incompatible coalitions G with $|G| > 2$ will have a cost $\hat{P}_G > \hat{P}_{\{i,j\}} \geq \tilde{P}$ and thus $v(G) = 0$; yielding $G[T] \triangleright G$ for all T-incompatible coalitions G. Hence, when any two users in the network partition T resulting from merge-and-split are separated by a distance larger than $\hat{d}_0$, then cond. B) for $\mathbb{D}_c$ stability existence is verified.* □

In summary, the existence of the $\mathbb{D}_c$-stable partition is closely tied to the users' location; which is a random parameter in practical networks. A partition resulting from our algorithm will be either $\mathbb{D}_c$ or $\mathbb{D}_{hp}$-stable as per Lemma 1.

3.4 Distributed Implementation of Merge-and-Split

The proposed algorithm in Table B.I can be implemented in a distributed way. As the user can detect the strength of other users' uplink signals (through techniques similar to those used in the ad hoc routing discovery

[22]), nearby coalitions can be discovered and the local merge algorithm performed. Each coalition surveys neighboring coalitions satisfying (B.9) and attempts to merge based on the Pareto order. The users in a coalition need only to know the maximum distances with respect to the users in neighboring coalitions. Moreover, each formed coalition internally decides to split if its members find a split form by Pareto order. By using a control channel, the distributed users can exchange some channel information and then and then cooperate using our model (exchange data information if needed, form coalition then transmit). Signaling for this handshaking can be minimal.

The complexity of the algorithm in Table B.I lies in the complexity of the merge-and-split operations. For a given network structure, one run of the cost-based *merge* process detailed in Section 3.2 implies that each coalition will try to merge with other coalitions where (B.9) is verified. The most complex case for the merge occurs when the network partition consists of M_t non-cooperative users that are located closely to verify (B.9) but not close enough to merge. In such a scenario, every user attempts to merge with all the others; but the merge is unsuccessful due to high cost. The first user requires $M_t - 1$ attempts for merge, the second requires $M_t - 2$ attempts and so on. The total number of merge attempts will be $\sum_{i=1}^{M_t-1} i = \frac{M_t(M_t-1)}{2}$. In practice, the merge process requires a significantly *lower* number of attempts. For instance, after the first run of the algorithm, the initial M_t non-cooperative users will self-organize into larger coalitions. Subsequent runs of the algorithms will deal with a network composed of a number of coalitions that is much smaller than M_t; reducing the number of merge attempts per coalition. This complexity is further *reduced* by the fact that, due to the cost, a coalition does not need to attempt to merge with far away or large coalitions which violate (B.9). Finally, once a group of coalitions merges into a larger coalition, the number of merging possibilities for the remaining users will decrease. In summary, although in worst case scenarios the merge process requires around $\frac{M_t \cdot (M_t-1)}{2}$ attempts, in practice, the process is far less complex.

At first glance, the *split* rule can be seen as a complex operation. For instance, splitting can involve finding all the possible partitions of the set formed by the users in a coalition In set theory, the number of all possible partitions of a set, i.e., a coalition in our case, is given by a value known as the Bell number which grows exponentially with the number of elements in the set, i.e., the number of users in the coalition [23]. In practice, since the split operation is restricted to each formed coalition in the network, it will operate on relatively small sets. As per (B.4), the size of each coalition is limited by the increasing cost as well as by the users' locations, thus, the coalitions formed in the proposed game are relatively small. Consequently, the split complexity will be limited to finding all possible partitions for small sets which can be affordable in terms of complexity. This complexity is further reduced by the fact that a coalition is not required to search for all the split forms. Typically, a coalition does not need to go through all the possible partitions As soon as a coalition finds a split form verifying the Pareto order, the users in this coalition will split, and the search for further split forms is not required. Briefly, the practical aspects dictated by the wireless network such as the increasing cost, users' locations and sequential search will significantly diminish the split complexity. The reduction in the merge-and-split complexity will be further corroborated through simulations in Section 5.

4 Fairness Criteria for Payoff Division within Coalition

In this section, various fairness rules for dividing the utility $v(S)$ among the members of coalition S are inspected. A merge or a split operation by Pareto order directly depends on the fairness criterion selected for payoff division.

4.1 Egalitarian Fair (EF)

The most simple division method is to divide the *extra* utility *equally* among users. In other words, the utility of user i among the coalition S is

$$\phi_i = \frac{1}{|S|}\left(v(S) - \sum_{j\in S} v(\{j\})\right) + v(\{i\}), \tag{B.11}$$

where $v(\{i\})$ and $v(\{j\})$ are the non-cooperative payoffs of users i and j. EF does not imply dividing the whole utility equally but rather the *extra* benefits equally while conserving individual rationality.

4.2 Proportional Fairness (PF)

The EF is a very simple and strict fairness criterion. However, in practice, the user experiencing a good channel might not be willing to cooperate with a user under bad channel conditions, if the extra is divided equally. To account for the channel differences, we use a criterion named proportional fairness (PF) [24], in which the extra benefit is divided in weights according to the users' non-cooperative utilities. Thus,

$$\phi_i = w_i \left(v(S) - \sum_{j \in S} v(\{j\}) \right) + v(\{i\}), \tag{B.12}$$

where $\sum_{i \in S} w_i = 1$ and within the coalition $\frac{w_i}{w_j} = \frac{v(\{i\})}{v(\{j\})}$. Thus, within the coalition for PF, the users with good channel conditions deserve more extra benefits than the users with bad channel conditions.

4.3 Shapley Value Fairness (SV)

Additional fairness criteria can be used while dividing the worth of a coalition among its members. For instance, another measure of fairness is given using the Shapley value (SV) [16].

Definition 10 *A* Shapley value ϕ^v *is a function that assigns to each possible characteristic function* v *a vector of real numbers, i.e.,* $\boldsymbol{\phi}^v = (\phi_1^v, \phi_2^v, \ldots, \phi_{M_t}^v)$*, where* ϕ_i^v *represents the worth or value of user* i *in the game* (N, v)*. There are four Shapley axioms that* ϕ^v *must satisfy:*

1. Efficiency Axiom*:* $\sum_{i \in N} \phi_i^v = v(N)$.
2. Symmetry Axiom*: If user* i *and user* j *are s. t.* $v(S \cup \{i\}) = v(S \cup \{j\})$ *for every coalition* S *not containing user* i *and user* j*, then* $\phi_i^v = \phi_j^v$.
3. Dummy Axiom*: If user* i *is s. t.* $v(S) = v(S \cup \{i\})$ *for every coalition* S *not containing* i*, then* $\phi_i^v = 0$.
4. Additivity Axiom*: If* u *and* v *are characteristic functions, then* $\phi^{u+v} = \phi^{v+u} = \phi^u + \phi^v$.

It is shown [16], [25] that there exists a unique function ϕ *satisfying the Shapley axioms given by*

$$\phi_i^v = \sum_{S \subseteq N - \{i\}} \frac{(|S|)!(|N| - |S| - 1)!}{(|N|)!} [v(S \cup \{i\}) - v(S)]. \tag{B.13}$$

The SV provides a fair division which takes into account the random-ordered joining of the users in the coalition. Under the assumption of randomly-ordered joining, the Shapley function of each user is its expected marginal contribution when it joins the coalition [16].

In our transmitter cooperation game, we are interested in dividing payoffs using SV among the users in any formed coalition G by merge or split. Thus, the payoff division by SV occurs by applying (B.13) on each restricted game (G, v) in the structure. In [26], they proved that, in a game with coalitional structure, the SV of the whole game (N, v) is found by using the SV on the game restricted to each coalition G in the structure as in our case. For a non-superadditive game, the SV might not be individually rational, however, the proposed algorithm handles it with an appropriate merge-and-split decision by Pareto order.

4.4 Maximin Fairness Using the Nucleolus (NU)

A stricter fairness rule for payoff division is given using the nucleolus (NU). We introduce the concepts of *excess*, *kernel*, and *nucleolus* [16, 25, 27]. For a given characteristic function v, an allocation $\boldsymbol{x}$ is found such that, for each coalition S and its associated dissatisfaction, an optimal allocation is calculated to minimize the maximum dissatisfaction. The dissatisfaction is quantified as follows.

Definition 11 *The measure of dissatisfaction of an allocation ϕ^v for a coalition S is defined as the* excess $e(\boldsymbol{\phi}^v, S) = v(S) - \sum_{j \in S} \phi_j^v$. *A* kernel *of v is the set of all allocations ϕ^v such that*

$$\max_{S \subseteq N - \{j\}, i \in S} e(\boldsymbol{\phi}^v, S) = \max_{G \subseteq N - \{i\}, j \in G} e(\boldsymbol{\phi}^v, G). \tag{B.14}$$

The kernel states that if players i and j are in the same coalition, then the highest excess that i can make in a coalition without j is equal to the highest excess that j can make in a coalition without i.

Definition 12 *Define $\boldsymbol{O}(\phi^v)$ as the vector of all excesses in a game (N, v) arranged in non-increasing order (except the excess of the grand coalition $\{N\}$). A vector $\boldsymbol{y} = (y_1, \ldots, y_k)$ is said to be lexographically less than a vector $\boldsymbol{z} = (z_1, \ldots, z_k)$ (denoted by $\mathbf{y} \prec_{lex} \mathbf{z}$) if $\exists l \in \{1, \ldots, k\}$ where $y_1 = z_1, y_2 = z_2, \ldots, y_{l-1} = z_{l-1}, y_l < z_l$. A group rational allocation $\boldsymbol{\delta}^v$ is a* nucleolus *(or prenucleolus) if for every other ϕ^v, $\boldsymbol{O}(\boldsymbol{\delta}^v) \prec_{lex} \boldsymbol{O}(\phi^v)$. Hence, the nucleolus is the group rational allocation $\boldsymbol{\delta}^v$ which* minimizes the excesses *in a non-increasing order.*

The NU of a coalitional game exists and is unique. The NU is group rational, lies in the kernel of the game and satisfies the symmetry and dummy axioms. Moreover, if the core is not empty, the NU is in the core. In other words, the NU is the best allocation under a min-max criterion.

Our main interest is to use the NU to find the allocation $\boldsymbol{\phi}^v = (\phi_1^v, \ldots, \phi_{|G|}^v)$ for a coalition $G \subset N$ that will potentially form by merge (or split); that is the NU of the restricted game (G, v). Unlike the SV, the NU of a game (N, v) with a coalition structure T is not the same as the NU of the restricted games (G, v) with $G \in T$ [26]. When the NU is considered over the restricted game (G, v), it will not minimize the excesses pertaining to coalitions formed by a combination of players belonging to G and players outside G. A similar reasoning also applies to the kernel. However, finding the NU of the whole game (N, v) requires a centralized intelligence which can also find the excesses among the disjoint coalitions that are formed; which contradicts with the goal of deriving a distributed clustering algorithm. Thus, for distributed cooperation, it suffices to use the NU of the restricted game (G, v) as it allows a payoff allocation which minimizes excesses inside G while having all the fairness properties of the NU restricted over (G, v). If a game is non-superadditive the NU may not be individually rational; however, this will simply be handled by adequate merge or split decisions.

5 Simulation Results and Analysis

For simulations, a BS is placed at the origin with $M_r = 3$ equally spaced antennas. Without loss of generality, at the receiver, we consider antennas that are separated enough while $\phi_{i,k} = 0\ \forall i, k$. [18]. Users are randomly located within a square of $4\text{km} \times 4\text{km}$ around the BS. The propagation loss $\alpha = 3$ and the path loss constant $\kappa = 1$. The power constraint per slot is $\tilde{P} = 0.01$ W, the cost SNR target for information exchange is $\nu_0 = 10$ dB and noise level is -90 dBm. 10000 independent runs with randomly located users are conducted for different network sizes.

Fig. B.2 shows the resulting network's structure in terms of the average number of coalitions for different fairness and networks sizes. Cooperation organizes the network in clustered coalitions with the average number of users per coalition seen through Fig. B.2 by dividing the network size with

[18] This choice provides a lower bound on the performance gain of our algorithm (i.e. the gains mainly stemming from the transmitters cooperation which is the main objective of the paper); considering different phases certainly yields an additional multiplexing gain [14, 15] and it does not affect the analysis or the results hereafter.

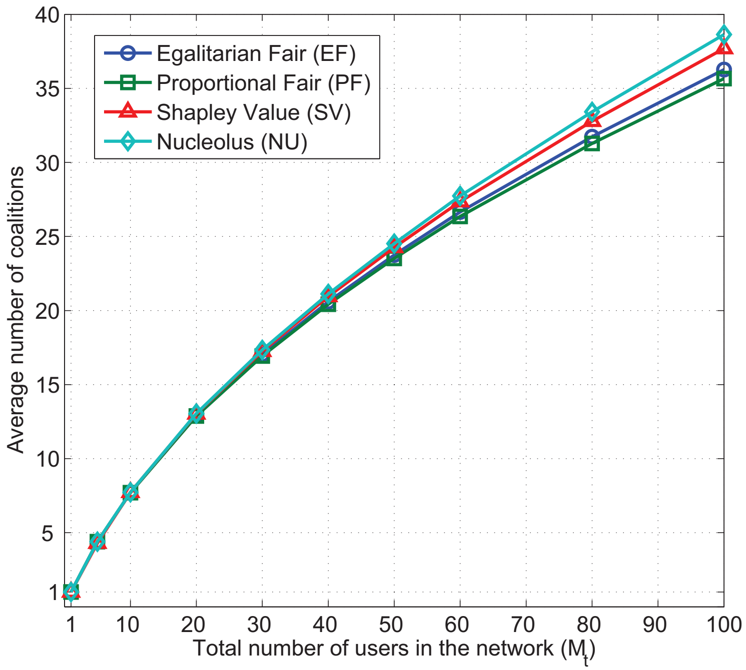

Fig. B.2: Resulting network structure shown through the average number of coalitions formed for different network sizes.

the number of coalitions. The number of coalitions and the average number of users per coalition increase with the network size due to the availability of more partners for forming coalitions. Moreover, as per Fig. B.2, in general, the network tends to self-organize into a large number of small coalitions rather than a small number of large coalitions. The PF division yields the largest average coalition size for all networks since it provides cooperation incentives to users with better non-cooperative channels by assigning them a larger weight. In contrast, the SV and NU division yield smaller coalitions due to additional fairness constraints imposed on the division (e.g. Shapley axioms and excess minimization for nucleolus). The results in this figure also provide us with an insight on the complexity of the merge-and-split operations as discussed in Section 3.4. For example,

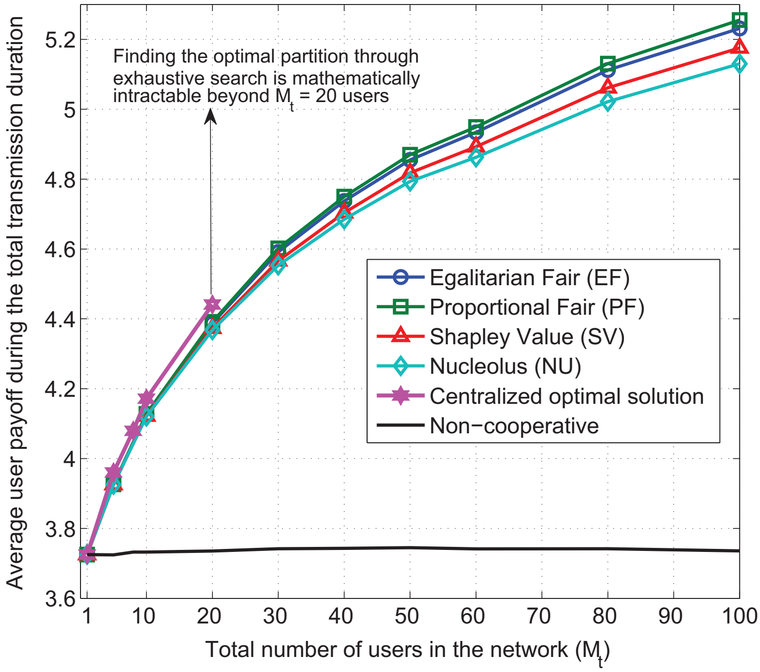

Fig. B.3: Cooperation gains in terms of the average individual user payoff achieved by the proposed scheme during the whole transmission duration compared to the non-cooperative case and the centralized optimal solution for different network sizes and different fairness criteria.

for the merge operation, we note that one run of the proposed algorithm transforms a network of $M_t = 100$ non-cooperative users into a network consisting of at most an average of 38.63 coalitions (NU case). Thus, the maximum number of merge attempts for future runs of the algorithm is reduced by a factor of almost 2.5. In addition, because the network self-organizes into a large number of small coalitions, the complexity of the split operation is generally affordable as it will be restricted to small coalitions. For instance, for a network as large as $M_t = 100$ users, the maximum average coalition size (over which a split may be applicable) is only 2.6 users for PF.

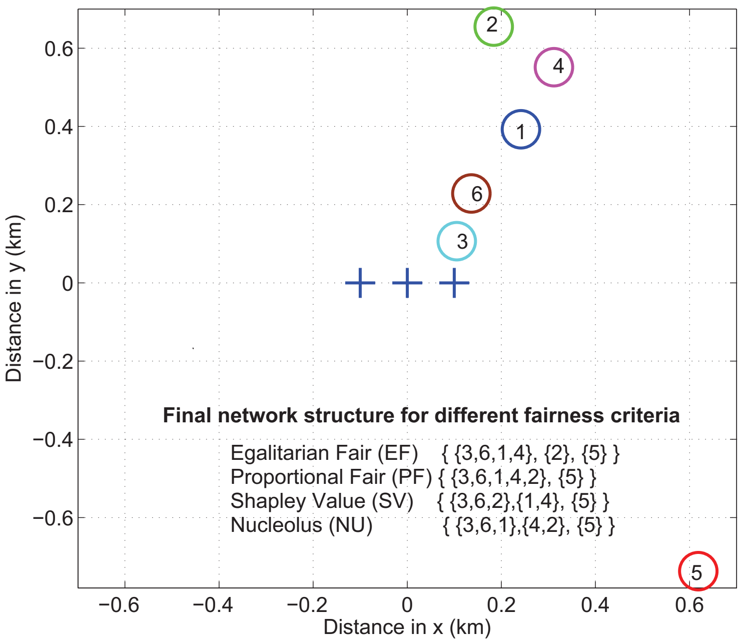

Fig. B.4: User positions and network structure for different fairness criteria.

In Fig. B.3, we show the average total individual user payoff (rate) improvement achieved during the whole transmission time scale as a function of the network size. We compare the performance of the proposed algorithm to that of the non-cooperative case as well as the optimal partition found by a centralized entity through exhaustive search. For the cooperative case, the average user's payoff increases with the number of users since the possibility of finding cooperating partners increases. In contrast, the non-cooperative approach presents an almost constant performance with different network sizes. Cooperation presents a significant advantage over the non-cooperative case in terms of average individual utility for all fairness types, and this advantage increases with the network size. The PF division presents the best performance, as it allows an improvement of up to 40.42% over the non-cooperative case at $M_t = 100$. This result also highlights the trade off between fairness and cooperation gains. For instance, while the PF presents an advantage in terms of payoff gain, since it allows larger coalitions to form (Fig. B.2), the SV and NU present lower gains but more fairness in allocating payoffs (Section 4). Furthermore, compared to the optimal solution, clearly the proposed merge and split algorithm achieve a highly comparable performance with a performance loss not exceeding 1% at $M_t = 20$ users. This clearly shows that, by using the proposed distributed merge-and-split algorithm, the network can achieve a performance that is very close to optimal. Note that, for more than 20 users, finding the optimal partition by exhaustive search is mathematically and computationally untractable.

Depending on the chosen fairness criteria for payoff division, the resulting network topology changes as the merge-and-split through Pareto order becomes different. For showing the fairness effect on the network structure, we show in Fig.B.4, for a random network of $M_t = 6$ users, the users' positions and the *final structure* for each fairness type. Moreover, the payoffs of relevant coalitions in this network are shown in Table B.II. The merge process starts with User 3 (best non-cooperative utility) which attempts to merge with User 6 (closest user). Forming coalition $\{3, 6\}$ allows both users to improve their payoff, for all fairness. The PF gives a different division than other fairness types as it assigns a higher weight to User 3 which has the best non-cooperative utility. Subsequently, for all fairness types except the SV, coalition $\{3, 6\}$ merges with User 1 to yield a 3-users coalition. For the SV, since User 6 cannot improve its payoff through this merge, coalition $\{3, 6, 1\}$ does not form. In this case, coalition $\{3, 6\}$ merges successfully with User 2 (the next lowest cost) as shown in Table B.II. Moreover, for the SV, coalition $\{3, 6, 2\}$ tries to merge with User 1; but the Pareto order cannot be verified and $\{3, 6, 2\}$ cannot merge any further. For the other fairness types, the newly formed coalition $\{3, 6, 1\}$ tries to merge with User 4. This merge is possible for EF and PF since all 4 users can improve their payoff. However, for the NU coalition $\{3, 6, 1, 4\}$ does not form since the payoff of User 6 would decrease by the merge. For EF, coalition $\{3, 6, 1, 4\}$ cannot merge with User 2 since the payoffs of Users 3, 6, 1 and 4 will degrade. As a result, for EF, this coalition cannot merge any further and the final network structure is $T = \{\{3, 6, 1, 4\}, \{2\}, \{5\}\}$. In contrast, for PF, coalition $\{3, 6, 1, 4\}$ is able to merge with User 2 but it can no longer merge with the remaining User 5. However, the grand coalition cannot form since the cost for cooperation would be 0.0939 W which is larger than the total power constraint $\tilde{P} = 0.01$ W yielding $v(\{3, 6, 1, 4, 2, 5\}) = 0$. For the

Table B.II: Dependence of the final network structure on the different fairness criteria for payoff division among the coalition users for network of Fig.B.4.

Non-Cooperative Payoffs					
$v(\{1\}) = 8.2827$, $v(\{2\}) = 6.5697$, $v(\{3\}) = 13.5481$, $v(\{4\}) = 6.9005$, $v(\{5\}) = 5.3854$, $v(\{6\}) = 10.6611$					
Individual Payoffs for Relevant Coalitions					
	Egalitarian Fair				
	User 1	User 2	User 3	User 4	User 6
$\{3,6\}$	-	-	19.1089	-	16.2219
$\{3,6,1\}$	15.4172	-	20.6826	-	17.7956
$\{3,6,2\}$	-	12.7609	19.7393	-	16.8523
$\{3,6,1,4\}$	15.6885	-	20.9539	14.3063	18.0669
$\{3,6,1,4,2\}$	15.1963	13.4833	20.4617	13.8141	17.5747
$\{1,4\}$	9.4233	-	-	8.0411	-
$\{4,2\}$	-	7.5588	-	7.8896	-
$\{3,6,2,1\}$	15.1604	13.4475	20.4258	-	17.5388
	Proportional Fair				
	User 1	User 2	User 3	User 4	User 6
$\{3,6\}$	-	-	19.7720	-	15.5587
$\{3,6,1\}$	13.7388	-	22.4726	-	17.6839
$\{3,6,2\}$	-	10.5342	21.7237	-	17.0946
$\{3,6,1,4\}$	14.5114	-	23.7364	12.0897	18.6783
$\{3,6,1,4,2\}$	14.5121	11.5108	23.7376	12.0904	18.6793
$\{1,4\}$	9.5271	-	-	7.9372	-
$\{4,2\}$	-	7.5345	-	7.9139	-
$\{3,6,2,1\}$	14.1162	11.1967	23.0900	-	18.1697
	Shapley				
	User 1	User 2	User 3	User 4	User 6
$\{3,6\}$	-	-	19.1089	-	16.2219
$\{3,6,1\}$	12.8630	-	25.1391	-	15.8931
$\{3,6,2\}$	-	10.1499	22.9161	-	16.2865
$\{3,6,1,4\}$	14.1229	-	26.8091	11.9101	16.1738
$\{3,6,1,4,2\}$	13.9112	10.9714	27.6011	11.8467	16.1999
$\{1,4\}$	9.4233	-	-	8.0411	-
$\{4,2\}$	-	7.5588	-	7.8896	-
$\{3,6,2,1\}$	13.7479	11.0347	25.7791	-	16.0108
	Nucleolus				
	User 1	User 2	User 3	User 4	User 6
$\{3,6\}$	-	-	19.1089	-	16.2219
$\{3,6,1\}$	13.4236	-	24.0180	-	16.4536
$\{3,6,2\}$	-	10.2958	22.6245	-	16.4323
$\{3,6,1,4\}$	13.1070	-	28.9640	11.0105	15.9342
$\{3,6,1,4,2\}$	12.7567	9.0421	32.0028	10.7246	16.0041
$\{1,4\}$	9.4233	-	-	8.0411	-
$\{4,2\}$	-	7.5588	-	7.8896	-
$\{3,6,2,1\}$	12.7514	9.6235	28.8135	-	15.3842

Table B.III: Payoffs for coalition $T_2 = \{7, 9, 10\}$ of Figure B.5 and its sub-coalitions (during $M_t = 10$ slots).

	Egalitarian Fair			**Proportional Fair**		
	User 7	User 9	User 10	User 7	User 9	User 10
$\{7\}$	3.1077	-	-	3.1077	-	-
$\{9\}$	-	2.7173	-	-	2.7173	-
$\{10\}$	-	-	2.7345	-	-	2.7345
$\{7, 9\}$	3.9829	3.5925	-	4.0416	3.5338	-
$\{7, 10\}$	4.0075	-	4.4406	4.0648	-	3.5779
$\{9, 10\}$	-	3.5996	3.6177	-	3.6206	3.5967
$T_2 = \{7, 9, 10\}$	4.5175	4.1452	4.1271	4.6431	4.0869	4.0598
	Shapley			**Nucleolus**		
	User 7	User 9	User 10	User 7	User 9	User 10
$\{7\}$	3.1077	-	-	3.1077	-	-
$\{9\}$	-	2.7173	-	-	2.7173	-
$\{10\}$	-	-	2.7345	-	-	2.7345
$\{7, 9\}$	3.9829	3.5925	-	3.9829	3.5925	-
$\{7, 10\}$	4.7863	-	4.4406	4.7863	-	4.4406
$\{9, 10\}$	-	3.5996	3.6177	-	3.5996	3.6177
$T_2 = \{7, 9, 10\}$	4.5210	4.1558	4.1131	4.5244	4.1663	4.0991

SV, User 1 will further merge with User 4 but coalition $\{1, 4\}$ cannot merge any further with User 5 since $v(\{1, 4, 5\}) = 0$. For the NU, User 4 merges with User 2 forming coalition $\{4, 2\}$ which can no longer merge with User 5 as $v(\{4, 2, 5\}) = 0$. Finally, for all 2-users coalitions the EF, SV and NU division coincide as they obey the same equations in this case [16], [25].

Furthermore, a network with $M_t = 10$ users is generated where the users are located in a way that a $\mathbb{D}_c$-stable partition exists. Fig. B.5 shows that the proposed algorithm converges to the final $\mathbb{D}_c$-stable network partition $T = \{T_1, \ldots, T_6\}$ (valid for all fairness criteria). Cond. A) for $\mathbb{D}_c$-stability is easily verified for coalitions consisting of at most 2 users since such coalitions do not form unless the Pareto order is internally verified (definition of the merge rule). For the 3-users coalition $T_2 = \{7, 9, 10\}$ Table B.III shows the payoffs of the different sub-coalitions for the various fairness types. Table B.III shows that the Pareto order is internally verified for T_2, that is $\forall A, B \subset T_2; \{A \cup B\} \triangleright \{A, B\}$ for all fairness cases. In addition, by inspecting Fig. B.5 it is clear that any two users belonging to T-incompatible coalitions are separated by a distance larger than the maximum distance, which is $\hat{d}_0 = 0.793$ km, computed using Theorem 4 for the simulation parameters. Thus, Theorem 3 is satisfied and cond. B) is verified. For example, for the T-incompatible coalition $G = \{4, 7\}$ equation (B.6) yields $v(G) = 0$ and thus $\phi_4^v(G) = \phi_7^v(G) = 0$ due

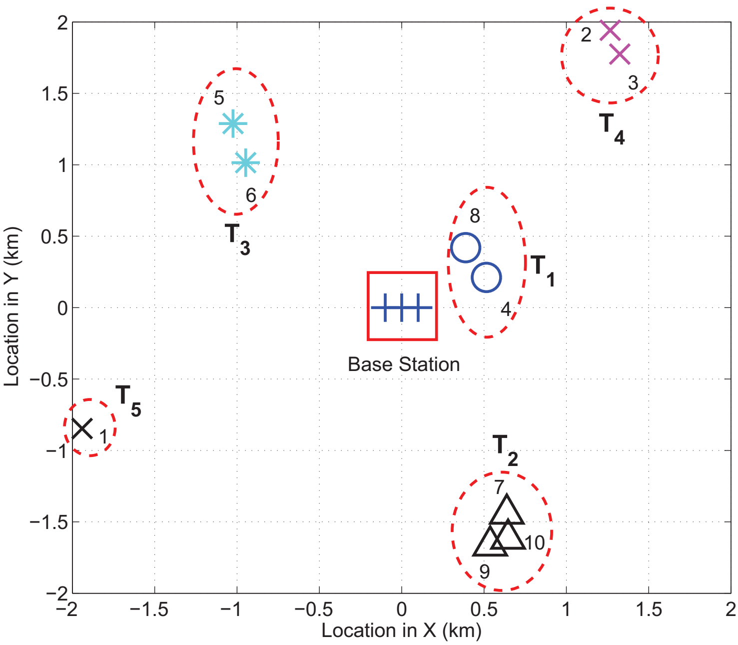

Fig. B.5: Convergence of the algorithm to a final $\mathbb{D}_c$-stable partition.

to the distance between Users 4 and 7. The projection of G in T is $G[T] = \{\{4,7\} \cap T_1, \{4,7\} \cap T_2, \{4,7\} \cap T_3, \{4,7\} \cap T_4, \{4,7\} \cap T_5, \{4,7\} \cap T_6\} = \{\{4\},\{7\}\}$. In $G[T]$, the payoffs of users 4 and 7 are respectively $\phi_4^v(G[T]) = v(\{4\}) = 7.6069$ and $\phi_7^v(G[T]) = v(\{7\}) = 3.1077$, by Pareto order $\phi_4^v(G[T]) > \phi_4^v(G)$ and $\phi_7^v(G[T]) > \phi_7^v(G)$, thus, $G[T] \triangleright G$.

In Fig. B.6, we plot the percentage of coalitions prone to deviations (averaged over random starting positions of the users) in the final network structure for different fairness criteria and different network sizes. First and foremost, this figure shows that the percentage of coalitions prone to deviation is generally small and does not exceed 10% for a relatively large network with $M_t = 100$ users, for the PF fairness criteria. This corroborates the fact that, by using Pareto order merge and split, the number of coalitions prone to deviations is generally small. Moreover, in this figure, we can see that for stringent fairness criteria such as the SV and the NU,

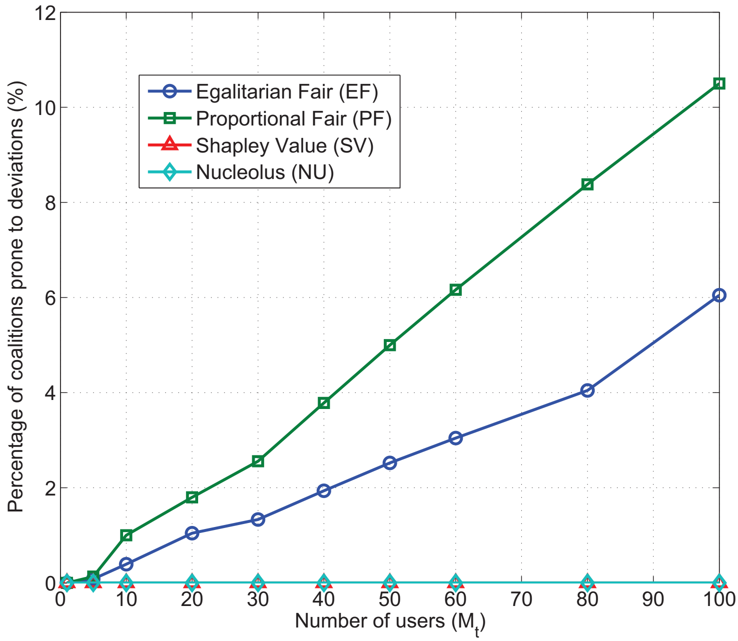

Fig. B.6: Average percentage of coalitions prone to deviations in the final network structure vs. number of users M_t for different fairness criteria.

the final network partitions contain no coalitions prone to deviation even if the partition is $\mathbb{D}_{hp}$-stable. This is mainly due to two reasons. On one hand, the SV and the NU have strict fairness requirements on the coalitions, hence yielding more cohesive and stable coalitions. On the other hand, for such fairness criteria, the size of the coalitions is generally small as previously shown. In contrast, the PF presents the largest percentage due to the large size of the coalitions that form for PF division as well as the fact that the PF can give incentives to deviate for coalition members that have a good non-cooperative utility. Similarly, the EF presents a percentage of coalitions prone to deviation, due to the coalition sizes that are generally smaller than PF, and to the fact that extra utility benefits are equally distributed among the users, giving less incentive for users to deviate than in the PF. In a nutshell, Fig. B.6 summarizes the possible loss of

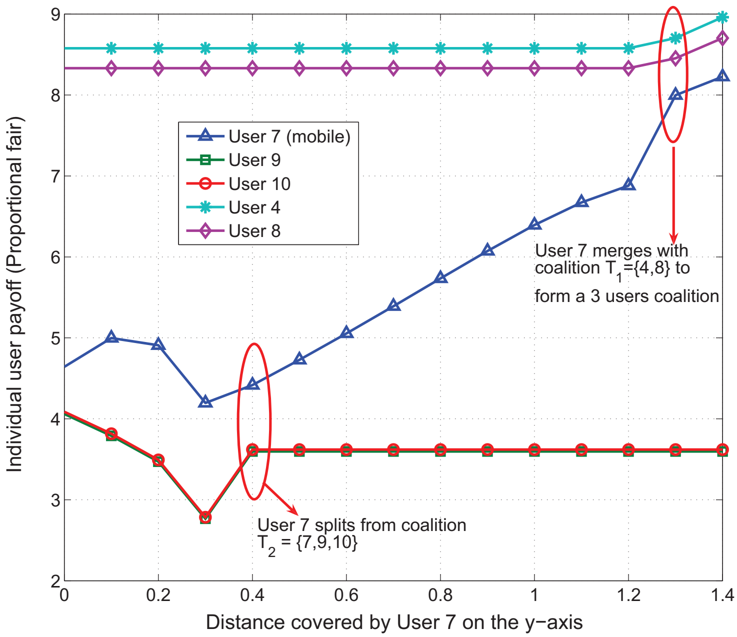

Fig. B.7: Adaptive coalition formation: coalition merging/splitting due to mobility of the users.

stability that can occur when the final network partitions are $\mathbb{D}_{hp}$-stable, while highlighting an interesting fairness-stability trade off.

In Fig. B.7, we show how the merge-and-split algorithm handles mobility. The network setup is the same as in Fig. B.5 with User 7 moving along the y-axis upwards for 1.4 km. The figure depicts the results for PF, as the other division types yield similar curves and are omitted for space limitation. When User 7 moves upwards, it becomes closer to the BS while the cost in coalition $\{7, 9, 10\}$ increases. As a result, its utility increases at first while the utilities of Users 9 and 10 decrease, since the PF division gives a higher weight to the user with the best non-cooperative utility, i.e., User 7. When User 7 covers 0.3 km, the cost in coalition $\{7, 9, 10\}$ increases significantly, and the utilities of all three users drop. Afterwards, at 0.4 km the splitting step occurs as User 7 splits from coalition $\{7, 9, 10\}$ and all three

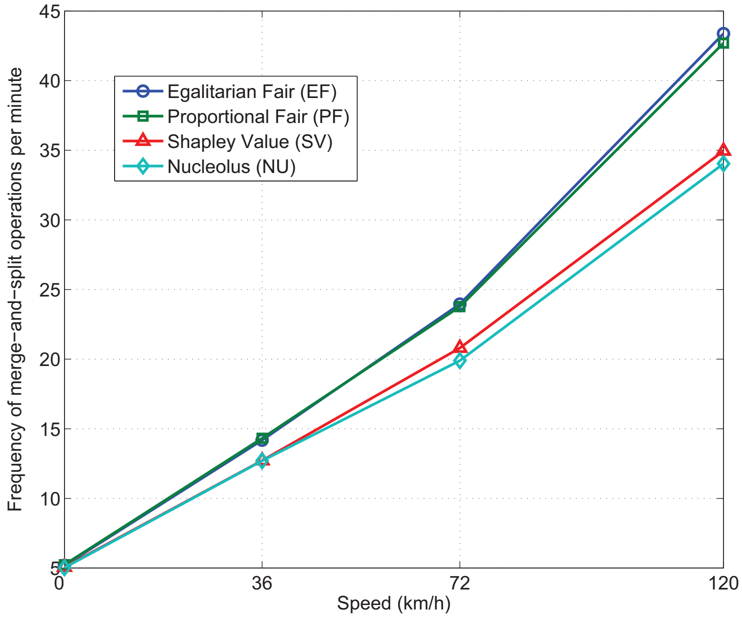

Fig. B.8: Frequency of merge-and-split operations per minute for different speeds in a mobile network of $M_t = 50$ users.

users improve their utilities by Pareto order. User 7 continues to improve its utility as it gets closer to the BS. Once User 7 moves about 1.3 km, it will be beneficial to Users 7, 4 and 8 to form a 3-user coalition. The merge algorithm allows User 7 to join the coalition of Users 4 and 8. Therefore, the payoffs of Users 4, 7 and 8 start improving significantly. These results show how merge-and-split algorithm operates in a wireless network.

The algorithm's performance is further investigated in a mobile network of $M_t = 50$ users (random walk mobility) for a period of 5 minutes. For $M_t = 50$, each TDMA transmission requires $50 \cdot \theta$ seconds with θ the slot duration, we let $\theta = 10$ ms. The results in terms of frequency of merge-and-split operations per minute are shown in Fig. B.8 for various speeds. As the speed increases, for all fairness types, the number of merge-and-split operations increases due to the changes in the network structure

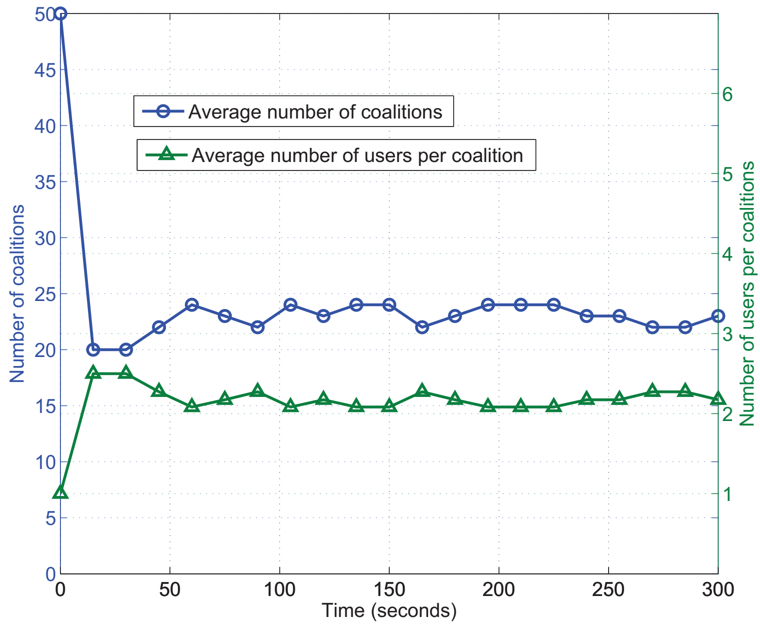

Fig. B.9: Network structure changes with time for $M_t = 50$ users, a constant speed of 120 km/h and a proportional fair division.

incurred by mobility. Fairness types that yield large coalitions, incur a higher frequency of merge-and-split since such coalitions require additional merge operations and are more prone to splitting due to mobility. In this regard, the PF and EF record the highest frequencies notably at high speeds. EF has the highest frequency since it yields coalitions with size comparable to PF, but these coalitions are more prone to split at high mobility since EF divides the extra benefit equally without accounting for the users' non-cooperative utilities like PF.

Fig. B.9 shows how the structure of a mobile network of $M_t = 50$ users evolves with time for PF (other fairness are omitted for space limitation), while the velocity of the users is constant equal to 120 km/h for a period of 5 minutes. As the users move, the structure of the network changes, with new coalitions forming and others splitting. The network starts with a non-

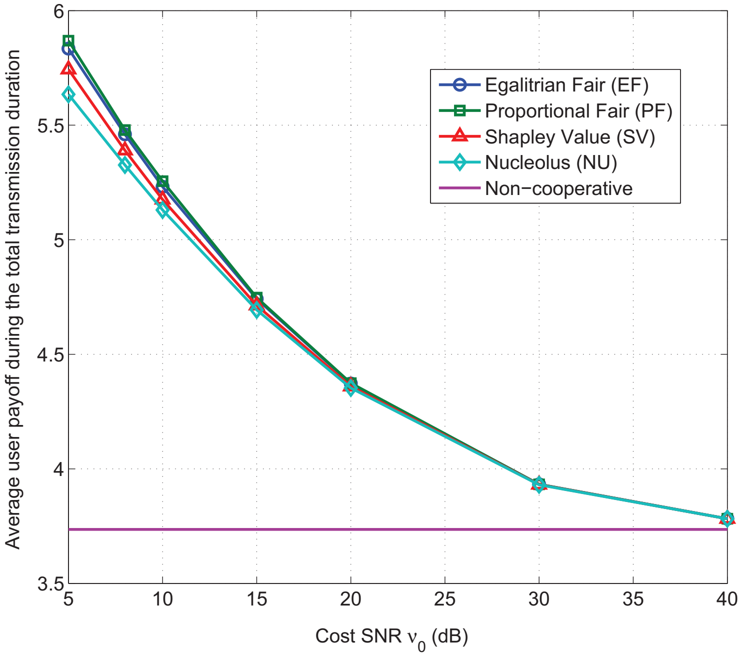

Fig. B.10: Average individual user payoff for different cost SNRs ν_0 for a network having $M_t = 100$ users for the cooperative and non-cooperative cases.

cooperative structure made up of 50 independent users. In the first step, the network self-organizes in 20 coalitions with an average of 2.5 users per coalition. With time the structure changes as new coalitions form or others split. After the 5 minutes have elapsed the final coalition structure is made up of 23 coalitions that is an average of 2.17 users per coalition. Finally, the behavior and performance of the network for different cost SNRs ν_0 is assessed. Fig. B.10 shows the average total user payoff achieved for different cost SNRs for $M_t = 100$ users. This figure shows that cooperation maintains utility gains at almost all costs; however, as the cost increases these gains decrease converging towards the non-cooperative gains at high cost since cooperation becomes difficult due to the cost. In fact, at $\nu_0 =$

40 dB all fairness types yield an average number of coalitions of around 96, i.e., almost every user acts non-cooperatively, hence the performance is close to that of the non-cooperative case.

6 Conclusions

In this paper, we constructed a novel game theoretical framework suitable for modeling cooperation among wireless network nodes. The framework is applied to the transmitter cooperation game with cost. Unlike existing literature which sought algorithms to form the grand coalition of transmitters, we inspected the possibility of forming disjoint independent coalitions using a novel algorithm from coalitional game theory. We proposed a simple and distributed merge-and-split algorithm for forming coalitions and benefiting from spatial gains. The proposed algorithm enables single antenna transmitters to cooperate and self-organize. Moreover, the algorithm can be implemented in a distributed way since the decision to merge or split is individually taken by each user based on the Pareto order, implying that at least one of the users is able to improve his payoff without hurting the others. The stability of the network partitions is studied through the novel concept of defection function $\mathbb{D}$. The proposed algorithm converges to a network partition that is $\mathbb{D}_{hp}$-stable, i.e., no user has an incentive to leave this partition through merge or split. Depending on the location of the users, the proposed algorithm can also converge to a Pareto optimal (in terms of payoff distribution) $\mathbb{D}_c$-stable partition which is a unique outcome of any merge-and-split iteration. The derived algorithm efficiently adapts to mobility as the coalitions form or split depending on the users' time varying positions. Simulation results show how the proposed algorithm allows the network to self-organize and improve the user payoff by 40.42%.

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Paper C

Coalitional Games for Distributed Collaborative Spectrum Sensing in Cognitive Radio Networks

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Abstract

Collaborative spectrum sensing among secondary users (SUs) in cognitive networks is shown to yield a significant performance improvement. However, there exists an inherent trade off between the gains in terms of probability of detection of the primary user (PU) and the costs in terms of false alarm probability. In this paper, we study the impact of this trade off on the topology and the dynamics of a network of SUs seeking to reduce the interference on the PU through collaborative sensing. Moreover, while existing literature mainly focused on centralized solutions for collaborative sensing, we propose distributed collaboration strategies through game theory. We model the problem as a non-transferable coalitional game, and propose a distributed algorithm for coalition formation through simple merge and split rules. Through the proposed algorithm, SUs can autonomously collaborate and self-organize into disjoint independent coalitions, while maximizing their detection probability taking into account the cooperation costs (in terms of false alarm). We study the stability of the resulting network structure, and show that a maximum number of SUs per formed coalition exists for the proposed utility model. Simulation results show that the proposed algorithm allows a reduction of up to 86.6% of the average missing probability per SU (probability of missing the detection of the PU) relative to the non-cooperative case, while maintaining a certain false alarm level. In addition, through simulations, we compare the performance of the proposed distributed solution with respect to an optimal centralized solution that minimizes the average missing probability per SU. Finally, the results also show how the proposed algorithm autonomously adapts the network topology to environmental changes such as mobility.

1 Introduction

In recent years, there has been an increasing growth in wireless services, yielding a huge demand on the radio spectrum. However, the spectrum resources are scarce and most of them have been already licensed to existing operators. Recent studies showed that the actual licensed spectrum remains unoccupied for large periods of time [1]. In order to efficiently exploit these spectrum holes, *cognitive radio* (CR) has been proposed [2]. By monitoring and adapting to the environment, CRs (secondary users) can share the spectrum with the licensed users (primary users), operating whenever the primary user (PU) is not using the spectrum. Implementing such flexible CRs faces several challenges [3]. For instance, CRs must constantly sense the spectrum in order to detect the presence of the PU and use the spectrum holes without causing harmful interference to the PU. Hence, efficient spectrum sensing constitutes a major challenge in cognitive networks.

For sensing the presence of the PU, the secondary users (SUs) must be able to detect the signal of the PU. Various kinds of detectors can be used for spectrum sensing such as matched filter detectors, energy detectors, cyclostationary detectors or wavelet detectors [4]. However, the performance of spectrum sensing is significantly affected by the degradation of the PU signal due to path loss or shadowing (hidden terminal). It has been shown that, through collaboration among SUs, the effects of this hidden terminal problem can be reduced and the probability of detecting the PU can be improved [5–7]. For instance, in [5] the SUs collaborate by sharing their sensing decisions through a centralized fusion center in the network. This centralized entity combines the sensing bits from the SUs using the OR-rule for data fusion and makes a final PU detection decision. A similar centralized approach is used in [6] using different decision-combining methods. The authors in [7] propose spatial diversity techniques for improving the performance of collaborative spectrum sensing by combatting the error probability due to fading on the reporting channel between the SUs and the central fusion center. Existing literature mainly focused on the performance assessment of collaborative spectrum sensing in the presence of a centralized fusion center. However, in practice, the SUs may belong to different service providers and they need to interact with each other for collaboration without relying on a centralized fusion center. Moreover, a centralized approach can lead to a significant overhead and increased complexity.

The main contribution of this paper is to devise distributed collaboration strategies for SUs in a cognitive network. Another major contribution is to study the impact on the network topology of the inherent trade off that exists between the collaborative spectrum sensing gains in terms of detection probability and the cooperation costs in terms of false alarm probability. This trade off can be pictured as a trade off between reducing the interference on the PU (increasing the detection probability) while maintaining a good spectrum utilization (reducing the false alarm probability). For distributed collaboration, we model the problem as a non-transferable coalitional game and we propose a distributed algorithm for coalition formation based on simple merge and split rules. Through the proposed algorithm, each SU autonomously decides to form or break a coalition for maximizing its utility in terms of detection probability while accounting for a false alarm cost. We show that, due to the cost for cooperation, independent disjoint coalitions will form in the network. We study the stability of the resulting coalition structure and show that a maximum coalition size exists for the proposed utility model. Through simulations, we assess the performance of the proposed algorithm relative to the non-cooperative case, we compare it with a centralized solution and we show how the proposed algorithm autonomously adapts the network topology to environmental changes such as mobility.

The rest of this paper is organized as follows: Section 2 presents the system model. In Section 3, we present the proposed coalitional game and prove different properties while in Section 4 we devise a distributed algorithm for coalition formation. Simulation results are presented and analyzed in Section 5. Finally, conclusions are drawn in Section 6.

2 System Model

Consider a cognitive network consisting of N transmit-receive pairs of SUs and a single PU. The SUs and the PU can be either stationary and mobile. Since the focus is on spectrum sensing, we are only interested in the transmitter part of each of the N SUs. The set of all SUs is denoted $\mathcal{N} = \{1, \ldots, N\}$. In a non-cooperative approach, each of the N SUs continuously senses the spectrum in order to detect the presence of the PU. For detecting the PU, we use energy detectors which are one of the main practical signal detectors in cognitive radio networks [5–7]. In such a non-cooperative setting, assuming Rayleigh fading, the detection probability and the false alarm probability of a SU i are, respectively, given by $P_{d,i}$ and $P_{f,i}$ [5, Eqs. (2), (4)]

$$P_{d,i} = e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n + \left(\frac{1+\bar{\gamma}_{i,PU}}{\bar{\gamma}_{i,PU}}\right)^{m-1} \times \left[e^{-\frac{\lambda}{2\left(1+\bar{\gamma}_{i,PU}\right)}} - e^{-\frac{\lambda}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} \left(\frac{\lambda\bar{\gamma}_{i,PU}}{2(1+\bar{\gamma}_{i,PU})}\right)^n \right], \tag{C.1}$$

$$P_{f,i} = P_f = \frac{\mathit{\Gamma}(m, \frac{\lambda}{2})}{\mathit{\Gamma}(m)}, \tag{C.2}$$

where m is the time bandwidth product, λ is the energy detection threshold assumed the same for all SUs without loss of generality as in [5–7], $\mathit{\Gamma}(\cdot,\cdot)$ is the incomplete gamma function and $\mathit{\Gamma}(\cdot)$ is the gamma function. Moreover, $\bar{\gamma}_i$ represents the average SNR of the received signal from the PU to SU given by $\bar{\gamma}_{i,\mathrm{PU}} = \frac{P_{\mathrm{PU}} h_{\mathrm{PU},i}}{\sigma^2}$ with P_{PU} the transmit power of the PU, σ^2 the Gaussian noise variance and $h_{\mathrm{PU},i} = \kappa / d^{\mu}_{\mathrm{PU},i}$ the path loss between the PU and SU i; κ being the path loss constant, μ the path loss exponent and $d_{\mathrm{PU},i}$ the distance between the PU and SU i. It is important to note that the non-cooperative false alarm probability expression depends *solely* on the detection threshold λ and does not depend on the SU's location; hence we dropped the subscript i in (C.2).

Moreover, an important metric that we will thoroughly use is the missing probability for a SU i, which is defined as the probability of missing the detection of a PU and given by [5]

$$P_{m,i} = 1 - P_{d,i}. \tag{C.3}$$

For instance, reducing the missing probability directly maps to increasing the probability of detection and, thus, reducing the interference on the PU. In order to minimize their missing probabilities, the SUs will interact for forming coalitions of collaborating SUs. Within each coalition $S \subseteq \mathcal{N} = \{1, \ldots, N\}$, a SU, selected as *coalition head*, collects the sensing bits from the coalition's SUs and acts as a fusion center in order to make a coalition-based decision on the presence or absence of the PU. This can be seen as having the centralized collaborative sensing of [5], [7] applied at the level of each coalition with the coalition head being the fusion center to which all the coalition members report. For combining the sensing bits and making

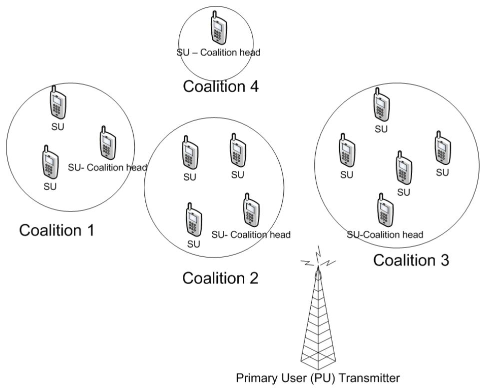

Fig. C.1: An illustrative example of coalition formation for collaborative spectrum sensing among SUs.

the final detection decision, the coalition head will use the decision fusion OR-rule. Within each coalition we take into account the probability of error due to the fading on the reporting channel between the SUs of a coalition and the coalition head [7]. Inside a coalition S, assuming BPSK modulation in Rayleigh fading environments, the probability of reporting error between a SU $i \in S$ and the coalition head $k \in S$ is given by [8]

$$P_{e,i,k} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_{i,k}}{1+\bar{\gamma}_{i,k}}}\right), \tag{C.4}$$

where $\bar{\gamma}_{i,k} = \frac{P_i h_{i,k}}{\sigma^2}$ is the average SNR for bit reporting between SU i and the coalition head k inside S with P_i the transmit power of SU i used for reporting the sensing bit to k and $h_{i,k} = \frac{\kappa}{d_{i,k}^{\mu}}$ the path loss between SU i and coalition head k. Any SU can be chosen as a coalition head within a coalition. However, for the remainder of this paper, we adopt the following convention without loss of generality.

Convention 1 *Within a coalition S, the SU $k \in S$ having the lowest non-cooperative missing probability $P_{m,k}$ is chosen as* coalition head. *Hence, the coalition head k of a coalition S is given by $k = \arg\min_{i \in S} P_{m,i}$ with $P_{m,i}$ given by (C.3).*

The motivation behind Convention 1 is that the SU having the lowest missing probability (best detection probability) within a coalition should not risk sending his local sensing bit over the fading reporting channel; and thus it will serve as a fusion center for the other SUs in the coalition. By collaborative sensing, the missing and false alarm probabilities of a coalition S having coalition head k are, respectively, given by [7]

$$Q_{m,S} = \prod_{i \in S} [P_{m,i}(1 - P_{e,i,k}) + (1 - P_{m,i})P_{e,i,k}], \tag{C.5}$$

$$Q_{f,S} = 1 - \prod_{i \in S} [(1 - P_f)(1 - P_{e,i,k}) + P_f P_{e,i,k}], \tag{C.6}$$

where P_f, $P_{m,i}$ and $P_{e,i,k}$ are respectively given by (C.2), (C.3) and (C.4) for a SU $i \in S$ and coalition head $k \in S$.

It is clear from (C.5) and (C.6) that as the number of SUs per coalition increases, the missing probability will decrease while the probability of false alarm will increase. This is a crucial trade off in collaborative spectrum sensing that can have a major impact on the collaboration strategies of each SU. Thus, our objective is to derive distributed strategies allowing the SUs to collaborate while accounting for this trade off. An example of the sought network structure is shown in Fig. C.1.

3 Collaborative Spectrum Sensing As Coalitional Game

In this section, we model the problem of collaborative spectrum sensing as a coalitional game. Then we prove and discuss its key properties.

3.1 Centralized Approach

A centralized approach can be used in order to find the optimal coalition structure, such as in Fig.C.1, that allows the SUs to maximize their benefits from collaborative spectrum sensing. For instance, we seek a centralized solution that minimizes the average missing probability (maximizes the average detection probability) per SU subject to a false alarm probability constraint per SU. In a centralized approach, we assume the existence of a centralized entity in the network that is able to gather information on the SUs such as their individual missing probabilities or their location. In brief, the centralized entity must be able to know all the required parameters for computing the probabilities in (C.5) and (C.6) in order to find the optimal structure. However, prior to deriving such an optimal centralized solution, the following property must be pinpointed within each coalition.

Property 1 *The missing and false alarm probabilities of any SU $i \in S$ are given by the missing and false alarm probabilities of the coalition S in (C.5) and (C.6), respectively.*

Proof: *Within each coalition S the SUs report their sensing bits to the coalition head. In its turn the coalition head of S combines the sensing bits using decision fusion and makes a final decision on the presence or absence of the PU. Thus, SUs belonging to a coalition S will transmit or not based on the final coalition head decision. Consequently, the missing and false alarm probabilities of any SU $i \in S$ are the missing and false alarm probabilities of the coalition S to which i belongs as given by in (C.5) and (C.6), respectively.* □

As a consequence of Property 1 the required false alarm probability constraint per SU directly maps to a false alarm probability constraint *per coalition*. Therefore, denoting $\mathcal{B}$ as the set of *all partitions* of $\mathcal{N}$, the centralized approach seeks to solve the following optimization problem

$$\min_{\mathcal{P}\in\mathcal{B}} \frac{\sum_{S\in\mathcal{P}} |S| \cdot Q_{m,S}}{N}, \qquad \text{(C.7)}$$

$$\text{s.t. } Q_{f,S} \leq \alpha \ \forall \ S \in \mathcal{P},$$

where $|\cdot|$ represents the cardinality of a set operator and S is a coalition belonging to the partition $\mathcal{P}$. Clearly, the centralized optimization problem seeks to find the optimal partition $\mathcal{P}^* \in \mathcal{B}$ that minimizes the average missing probability per SU, subject to a false alarm constraint per SU (coalition).

However, it is shown in [9] that finding the optimal coalition structure for solving an optimization problem such as in (C.7) is an NP-complete problem. This is mainly due to the fact that the number of possible coalition structures (partitions), given by the Bell number, grows exponentially with the number of SUs N [9]. Moreover, the complexity increases further due to the fact that the expressions of $Q_{m,S}$ and $Q_{f,S}$ given by (C.5) and (C.6) depend on the optimization parameter $\mathcal{P}$. For this purpose, deriving a distributed solution enabling the SUs to benefit from collaborative spectrum sensing with a low complexity is desirable. The above formulated centralized approach will be used as a benchmark for the distributed solution in the simulations, for reasonably small networks.

3.2 Game Formulation and Properties

For the purpose of deriving a distributed algorithm that can minimize the missing probability per SU, we refer to cooperative game theory [10] which provides a set of analytical tools suitable for such algorithms. For instance, the proposed collaborative sensing problem can be modeled as a $(\mathcal{N}, v)$ coalitional game [10] where $\mathcal{N}$ is the set of players (the SUs) and v is the utility function or value of a coalition.

The value $v(S)$ of a coalition $S \subseteq \mathcal{N}$ must capture the trade off between the probability of detection and the probability of false alarm. For this purpose, $v(S)$ must be an increasing function of the detection probability $Q_{d,S} = 1 - Q_{m,S}$ within coalition S and a decreasing function of the false alarm probability $Q_{f,S}$. A suitable utility function is given by

$$v(S) = Q_{d,S} - C(Q_{f,S}) = (1 - Q_{m,S}) - C(Q_{f,S}), \tag{C.8}$$

where $Q_{m,S}$ is the missing probability of coalition S given by (C.5) and $C(Q_{f,S})$ is a cost function of the false alarm probability within coalition S given by (C.6).

First of all, we provide the following definition from [10] and subsequently prove an interesting property pertaining to the proposed game model.

Definition 13 *A coalitional game $(\mathcal{N}, v)$ is said to have a* transferable *utility if the value $v(S)$ can be arbitrarily apportioned between the coalition's players. Otherwise, the coalitional game has a* non-transferable utility *and each player will have its own utility within coalition S.*

Property 2 *In the proposed collaborative sensing game, the utility of a coalition S is equal to the utility of each SU in the coalition, i.e., $v(S) = \phi_i(S)$, $\forall\, i \in S$, where $\phi_i(S)$ denotes the utility of SU i when i belongs to a coalition S. Consequently, the proposed $(\mathcal{N}, v)$ coalitional game model has a* non-transferable *utility.*

Proof: *The coalition value in the proposed game is given by (C.8) and is a function of $Q_{m,S}$ and $Q_{f,S}$. As per Property 1, the missing probabilities for each SU $i \in S$ are also given by $Q_{m,S}$ and $Q_{f,S}$ and, thus, the utility of each SU $i \in S$ is $\phi_i(S) = v(S)$. Hence, the coalition value $v(S)$ cannot be arbitrarily apportioned among the users of a coalition; and the proposed coalitional game has non-transferable utility.* □

In general, coalitional game based problems seek to characterize the properties and stability of the grand coalition of all players since it is generally assumed that the grand coalition maximizes the utilities of the players [10]. In our case, although collaborative spectrum sensing improves the detection probability for the SUs; the cost in terms of false alarm limits this gain. Therefore, for the proposed $(\mathcal{N}, v)$ coalitional game we have the following property.

Property 3 *For the proposed $(\mathcal{N}, v)$ coalitional game, the grand coalition of all the SUs does not always form due to the collaboration false alarm costs; thus disjoint independent coalitions will form in the network.*

Proof: *By inspecting $Q_{m,S}$ in (C.5) and through the results shown in [7] it is clear that as the number of SUs in a coalition increase $Q_{m,S}$ decreases and the performance in terms of detection probability improves. Hence, when no cost for collaboration exists, the grand coalition of all SUs is the optimal structure for maximizing the detection probability. However, when the number of SUs in a coalition S increases, it is shown in [7] through (C.5) that the false alarm probability increases. Therefore, for the proposed collaborative spectrum sensing model with cost for collaboration, the grand coalition of all SUs will, in general, not form due to the false alarm cost as taken into consideration in (C.8).* □

In a nutshell, we have a non-transferable $(\mathcal{N}, v)$ coalitional game and we seek to derive a distributed algorithm for forming coalitions among SUs. Before deriving such an algorithm, we will delve into the details of the cost function in (C.8).

3.3 Cost Function

Any well designed cost function $C(Q_{f,S})$ in (C.8) must satisfy several requirements needed for adequately modeling the false alarm cost. On one hand, $C(Q_{f,S})$ must be an increasing function of $Q_{f,S}$ with the increase slope becoming steeper as $Q_{f,S}$ increases. On the other hand, the cost function $C(Q_{f,S})$ must impose a maximum tolerable false alarm probability, i.e. an upper bound constraint on the false alarm, that cannot be exceeded by any SU in a manner similar to the centralized problem in (C.7) (due to Property 1, imposing a false alarm constraint on the coalition maps to a constraint per SU).

A well suited cost function satisfying the above requirements is the logarithmic barrier penalty function given by [11]

$$C(Q_{f,S}) = \begin{cases} -\alpha^2 \cdot \log\left(1 - \left(\frac{Q_{f,S}}{\alpha}\right)^2\right), & \text{if } Q_{f,S} < \alpha, \\ +\infty, & \text{if } Q_{f,S} \geq \alpha, \end{cases} \tag{C.9}$$

where $\log$ is the natural logarithm and α is a false alarm constraint per coalition (per SU). The cost function in (C.9) allows to incur a penalty which is increasing with the false alarm probability. Moreover, it imposes a maximum false alarm probability per SU. In addition, as the false alarm probability gets closer to α the cost for collaboration increases steeply, requiring a significant improvement in detection probability if the SUs wish to collaborate as per (C.8). Also, it is interesting to note that the proposed cost function depends on both distance and the number of SUs in the coalition, through the false alarm probability $Q_{f,S}$ (the distance lies within the probability of error). Hence, the cost for collaboration increases with the number of SUs in the coalition as well as when the distance between the coalition's SUs increases.

4 Distributed Coalition Formation Algorithm

In this section, we propose a distributed coalition formation algorithm and we discuss its main properties.

4.1 Coalition Formation Concepts

Coalition formation has been a topic of high interest in game theory [9, 12–14]. The goal is to find algorithms for characterizing the coalitional structures that form in a network where the grand coalition is not optimal. For instance, a generic framework for coalition formation is presented in [13–15] whereby coalitions form and break through two simple merge-and-split rules. This framework can be used to construct a distributed coalition formation algorithm for collaborative sensing, but first, we define the following concepts [13, 14].

Definition 14 *A* collection *of coalitions, denoted $\mathcal{S}$, is defined as the set $\mathcal{S} = \{S_1, \ldots, S_l\}$ of mutually disjoint coalitions $S_i \subset \mathcal{N}$. If the collection spans* all *the players of $\mathcal{N}$; that is $\bigcup_{j=1}^{l} S_j = \mathcal{N}$, the collection is a* partition *of $\mathcal{N}$.*

Definition 15 *A preference operator or* comparison relation $\triangleright$ *is defined for comparing two collections* $\mathcal{R} = \{R_1, \ldots, R_l\}$ *and* $\mathcal{S} = \{S_1, \ldots, S_m\}$ *that are partitions of the same subset* $\mathcal{A} \subseteq \mathcal{N}$ *(same players in* $\mathcal{R}$ *and* $\mathcal{S}$*). Thus,* $\mathcal{R} \triangleright \mathcal{S}$ *implies that the way* $\mathcal{R}$ *partitions* $\mathcal{A}$ *is preferred to the way* $\mathcal{S}$ *partitions* $\mathcal{A}$ *based on a criterion to be defined next.*

Various criteria (referred to as *orders*) can be used as comparison relations between collections or partitions [13], [14]. These orders are divided into two main categories: coalition value orders and individual value orders. Coalition value orders compare two collections (or partitions) using the value of the coalitions inside these collections such as in the utilitarian order where $\mathcal{R} \triangleright \mathcal{S}$ implies $\sum_{i=1}^{l} v(R_i) > \sum_{i=1}^{m} v(S_i)$. Individual value orders perform the comparison using the actual player utilities and not the coalition value. For such orders, two collections $\mathcal{R}$ and $\mathcal{S}$ are seen as sets of player utilities of the same length L (number of players). The players' utilities are either the payoffs after division of the value of the coalitions in a collection (transferable utility) or the actual utilities of the players belonging to the coalitions in a collection (non-transferable utility). Due to the non-transferable nature of the proposed $(\mathcal{N}, v)$ collaborative sensing game (Property 2), an individual value order must be used as a comparison relation $\triangleright$. An important example of individual value orders is the *Pareto order*. Denote for a collection $\mathcal{R} = \{R_1, \ldots, R_l\}$, the utility of a player j in a coalition $R_j \in \mathcal{R}$ by $\phi_j(\mathcal{R}) = \phi_j(R_j) = v(R_j)$ (as per Property 2); hence, the Pareto order is defined as follows

$$\mathcal{R} \triangleright \mathcal{S} \Longleftrightarrow \{\phi_j(\mathcal{R}) \geq \phi_j(\mathcal{S}) \;\forall\, j \in \mathcal{R}, \mathcal{S}\}, \qquad \text{(C.10)}$$

with *at least one strict inequality* ($>$) for a player k.

Due to the non-transferable nature of the proposed collaborative sensing model, the Pareto order is an adequate preference relation. Having defined the various concepts, we derive a distributed coalition formation algorithm in the next subsection.

4.2 Coalition Formation Algorithm

For autonomous coalition formation in cognitive radio networks, we propose a distributed algorithm based on two simple rules denoted as "merge" and "split" that allow to modify a partition $\mathcal{T}$ of the SUs set $\mathcal{N}$ as follows [13].

Definition 16 ***Merge Rule*** *- Merge any set of coalitions* $\{S_1, \ldots, S_l\}$ *where* $\{\bigcup_{j=1}^{l} S_j\} \triangleright \{S_1, \ldots, S_l\}$*, therefore,* $\{S_1, \ldots, S_l\} \rightarrow \{\bigcup_{j=1}^{l} S_j\}$*, (each* S_i *is a coalition in* $\mathcal{T}$*).*

Definition 17 ***Split Rule*** *- Split any coalition* $\bigcup_{j=1}^{l} S_j$ *where* $\{S_1, \ldots, S_l\} \triangleright \{\bigcup_{j=1}^{l} S_j\}$*, thus,* $\{\bigcup_{j=1}^{l} S_j\} \rightarrow \{S_1, \ldots, S_l\}$*, (each* S_i *is a coalition in* $\mathcal{T}$*).*

Using the above rules, multiple coalitions can merge into a larger coalition if merging yields a preferred collection based on the selected order $\triangleright$. Similarly, a coalition would split into smaller coalitions if splitting yields a preferred collection. When $\triangleright$ is the Pareto order, coalitions will merge (split) only if at least one SU is able to strictly improve its individual utility through this merge (split) without decreasing the other SUs' utilities. By using the merge-and-split rules combined with the Pareto order, a distributed coalition formation algorithm suited for collaborative spectrum sensing can be constructed. First and foremost, the appeal of forming coalitions using merge-and-split stems from the fact that it has been shown in [13] and [14] that any arbitrary iteration of merge-and-split operations *terminates*. Moreover, each merge or split decision can be taken in a distributed manner by each individual SU or by each already formed coalition. Subsequently, a merge-and-split coalition algorithm can adequately model the distributed interactions among the SUs of a cognitive network that are seeking to collaborate in the sensing process.

In consequence, for the proposed collaborative sensing game, we construct a coalition formation algorithm based on merge-and-split and divided into three phases: local sensing, adaptive coalition formation, and coalition sensing. In the local sensing phase, each individual SU computes its own local PU detection bit based on the received PU signal. In the adaptive coalition formation phase, the SUs (or existing coalitions of SUs) interact in order to assess whether to share their sensing results with nearby coalitions. For this purpose, an iteration of sequential merge-and-split rules occurs in the network, whereby each coalition decides to merge (or split) depending on the utility improvement that merging (or splitting) yields. In the final coalition sensing phase, once the network topology converges following merge-and-split, SUs that belong to the same coalition report their local sensing bits to their local coalition head. The coalition head subsequently uses decision fusion OR-rule to make a final decision on the presence or the absence of the PU. This decision is then reported by the coalition heads to all the SUs within their respective coalitions.

Table C.I: One round of the proposed collaborative sensing algorithm

Initial State

The network is partitioned by $\mathcal{T} = \{T_1, \ldots, T_k\}$ (At the beginning of all time $\mathcal{T}$ = $\mathcal{N}$ = $\{1, \ldots, N\}$ with non-cooperative SUs).

Three phases in each round of the coalition formation algorithm

Phase 1 - Local Sensing:

Each individual SU computes its local PU signal sensing bit.

Phase 2 - Adaptive Coalition Formation:

In this phase, coalition formation using merge-and-split occurs.

repeat

a) $\mathcal{F}$ = Merge($\mathcal{T}$); coalitions in $\mathcal{T}$ decide to merge based on the merge algorithm explained in Section 4.2.

b) $\mathcal{T}$ = Split($\mathcal{F}$); coalitions in $\mathcal{F}$ decide to split based on the Pareto order.

until merge-and-split terminates.

Phase 3 - Coalition Sensing:

a) Each SU reports its sensing bit to the coalition head.

b) The coalition head of each coalition makes a final decision on the absence or presence of he PU using decision fusion OR-rule.

c) The SUs in a coalition abide by the final decision made by the coalition head.

The above phases are repeated throughout the network operation. In Phase 2, through distributed and periodic merge-and-split decisions, the SUs can autonomously adapt the network topology to environmental changes such as mobility.

Each round of the three phases of the proposed algorithm starts from an initial network partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ of $\mathcal{N}$. During the adaptive coalition formation phase any random coalition (individual SU) can start with the merge process. For implementation purposes, assume that the coalition $T_i \in \mathcal{T}$ which has the highest utility in the initial partition $\mathcal{T}$ starts the merge by attempting to collaborate with a nearby coalition. On one hand, if merging occurs, a new coalition $\tilde{T}_i$ is formed and, in its turn, coalition

$\tilde{T}_i$ will attempt to merge with a nearby SU that can improve its utility. On the other hand, if T_i is unable to merge with the firstly discovered partner, it tries to find other coalitions that have a mutual benefit in merging. The search ends by a final merged coalition T_i^{final} composed of T_i and one or several of coalitions in its vicinity ($T_i^{\text{final}} = T_i$, if no merge occurred). The algorithm is repeated for the remaining $T_i \in \mathcal{T}$ until all the coalitions have made their merge decisions, resulting in a final partition $\mathcal{F}$. Following the merge process, the coalitions in the resulting partition $\mathcal{F}$ are next subject to split operations, if any is possible. An iteration consisting of multiple successive merge-and-split operations is repeated until it terminates. It must be stressed that the decisions to merge or split can be taken in a distributed way without relying on any centralized entity as each SU or coalition can make its own decision for merging or splitting. Table C.I summarizes one round of the proposed algorithm.

For handling environmental changes such as mobility or the joining/leaving of SUs, Phase 2 of the proposed algorithm in Table C.I is repeated periodically. In Phase 2, periodically, as time evolves and SUs (or the PU) move or join/leave, the SUs can autonomously self-organize and adapt the network's topology through new merge-and-split iterations with each coalition taking the decision to merge (or split) subject to satisfying the merge (or split) rule through Pareto order (C.10). In other words, every period of time θ the SUs assess the possibility of splitting into smaller coalitions or merging with new partners. The period θ is smaller in highly mobile environments to allow a more adequate adaptation of the topology. Similarly, every period θ, in the event where the current coalition head of a coalition has moved or is turned off, the coalition members may select a new coalition head if needed. The convergence of this merge-and-split adaptation to environmental changes is always guaranteed, since, by definition of the merge and split rules, any iteration of these rules certainly terminates.

For the proposed coalition formation algorithm, an upper bound on the maximum coalition size is imposed by the proposed utility and cost models in (C.8) and (C.9) as follows:

Theorem 1 *For the proposed collaborative sensing model, any coalition structure resulting from the distributed coalition formation algorithm will have coalitions limited in size to a maximum of* $M_{\max} = \frac{\log(1-\alpha)}{\log(1-P_f)}$ *SUs.*

Proof: *For forming coalitions, the proposed algorithm requires an improvement in the utility of the SUs through Pareto order. However, the benefit from collaboration is limited by the false alarm probability cost modeled* *by the barrier function (C.9). A minimum false alarm cost in a coalition S with coalition head $k \in S$ exists whenever the reporting channel is perfect, i.e., exhibiting no error, hence $P_{e,i,k} = 0 \; \forall i \in S$. In this perfect case, the false alarm probability in a perfect coalition S_p is given by*

$$Q_{f,S_p} = 1 - \prod_{i \in S_p} (1 - P_f) = 1 - (1 - P_f)^{|S_p|}, \tag{C.11}$$

where $|S_p|$ is the number of SUs in the perfect coalition S_p. A perfect coalition S_p where the reporting channels inside are perfect (i.e. SUs are grouped very close to each other) can accommodate the largest number of SUs relative to other coalitions. Hence, we can use this perfect coalition to find an upper bound on the maximum number of SUs per coalition. For instance, the log barrier function in (C.9) tends to infinity whenever the false alarm probability constraint per coalition is reached which implies an upper bound on the maximum number of SUs per coalition if $Q_{f,S_p} \geq \alpha$, yielding by (C.11)

$$|S_p| \leq \frac{\log(1-\alpha)}{\log(1-P_f)} = M_{\max} \,. \tag{C.12}$$

□

It is interesting to note that the maximum size of a coalition $M_{\max}$ depends mainly on two parameters: the false alarm constraint α and the non-cooperative false alarm P_f. For instance, larger false alarm constraints allow larger coalitions, as the maximum tolerable cost limit for collaboration is increased. Moreover, as the non-cooperative false alarm P_f decreases, the possibilities for collaboration are better since the increase of the false alarm due to coalition size becomes smaller as per (C.6). It must be noted that the dependence of $M_{\max}$ on P_f yields a direct dependence of $M_{\max}$ on the energy detection threshold λ as per (C.2). Finally, it is interesting to see that the upper bound on the coalition size does not depend on the location of the SUs in the network nor on the actual number of SUs in the network. Therefore, deploying more SUs or moving the SUs in the network for a fixed α and P_f does not increase the upper bound on coalition size.

4.3 Stability

The result of the proposed algorithm in Table C.I is a network partition composed of disjoint independent coalitions of SUs. The stability of this resulting network structure can be investigated using the concept of a defection function $\mathbb{D}$ [13].

Definition 18 *A* defection *function $\mathbb{D}$ is a function which associates with each partition $\mathcal{T}$ of $\mathcal{N}$ a group of collections in $\mathcal{N}$. A partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ of $\mathcal{N}$ is* $\mathbb{D}$-stable *if no group of players is interested in leaving $\mathcal{T}$ when the players who leave can only form the collections allowed by $\mathbb{D}$.*

Two important defection functions must be characterized [13–15]. First, the $\mathbb{D}_{hp}(\mathcal{T})$ function (denoted $\mathbb{D}_{hp}$) which associates with each partition $\mathcal{T}$ of $\mathcal{N}$ the group of all partitions of N that the players can form through merge-and-split operations applied to $\mathcal{T}$. This function allows any group of players to leave the partition $\mathcal{T}$ of $\mathcal{N}$ through merge-and-split operations to create another *partition* in $\mathcal{N}$. Second, the $\mathbb{D}_c(\mathcal{T})$ function (denoted $\mathbb{D}_c$) which associates with each partition $\mathcal{T}$ of $\mathcal{N}$ the family of all collections in $\mathcal{N}$. This function allows any group of players to leave the partition $\mathcal{T}$ of $\mathcal{N}$ through *any* operation and create an arbitrary *collection* in $\mathcal{N}$. Two forms of stability stem from these definitions: $\mathbb{D}_{hp}$ stability and a stronger $\mathbb{D}_c$ stability. A partition $\mathcal{T}$ is $\mathbb{D}_{hp}$-stable, if no players in $\mathcal{T}$ are interested in leaving $\mathcal{T}$ through merge-and-split to form other partitions in $\mathcal{N}$; while a partition $\mathcal{T}$ is $\mathbb{D}_c$-stable, if no players in $\mathcal{T}$ are interested in leaving $\mathcal{T}$ through *any* operation (not necessary merge or split) to form other collections in $\mathcal{N}$.

Characterizing any type of $\mathbb{D}$-stability for a partition depends on various properties of its coalitions. For instance, a partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ is $\mathbb{D}_{hp}$-stable if, for the partition $\mathcal{T}$, no coalition has an incentive to split or merge. As an immediate result of the definition of $\mathbb{D}_{hp}$-stability we have

Theorem 2 *Every partition resulting from our proposed coalition formation algorithm is $\mathbb{D}_{hp}$-stable.*

Briefly, a $\mathbb{D}_{hp}$-stable can be thought of as a state of equilibrium where no coalitions have an incentive to pursue coalition formation through merge or split. With regards to $\mathbb{D}_c$ stability, the work in [13–15] proved that a $\mathbb{D}_c$-stable partition has the following properties:

1. If it exists, a $\mathbb{D}_c$-stable partition is the *unique* outcome of any *arbitrary* iteration of merge-and-split and is a $\mathbb{D}_{hp}$-stable partition.

2. A $\mathbb{D}_c$-stable partition $\mathcal{T}$ is a unique $\triangleright$-maximal partition, that is for all partitions $\mathcal{T}' \neq \mathcal{T}$ of $\mathcal{N}$, $\mathcal{T} \triangleright \mathcal{T}'$. In the case where $\triangleright$ represents the Pareto order, this implies that the $\mathbb{D}_c$-stable partition $\mathcal{T}$ is the partition that presents a *Pareto optimal* utility distribution for all the players.

However, the existence of a $\mathbb{D}_c$-stable partition is not always guaranteed [13]. The $\mathbb{D}_c$-stable partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ of the whole space $\mathcal{N}$ exists if a partition of $\mathcal{N}$ that verifies the following two necessary and sufficient conditions exists [13]:

1. For each $i \in \{1, \ldots, l\}$ and each pair of disjoint *coalitions* S_1 and S_2 such that $\{S_1 \cup S_2\} \subseteq T_i$ we have $\{S_1 \cup S_2\} \rhd \{S_1, S_2\}$.

2. For the partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ a coalition $G \subset \mathcal{N}$ formed of players belonging to different $T_i \in \mathcal{T}$ is $\mathcal{T}$-incompatible if for no $i \in \{1, \ldots, l\}$ we have $G \subset T_i$. $\mathbb{D}_c$-stability requires that for all $\mathcal{T}$-incompatible coalitions $\{G\}[\mathcal{T}] \rhd \{G\}$ where $\{G\}[\mathcal{T}] = \{G \cap T_i \ \forall \ i \in \{1, \ldots, l\}\}$ is the projection of coalition G on $\mathcal{T}$.

If no partition of $\mathcal{N}$ can satisfy these conditions, then no $\mathbb{D}_c$-stable partitions of $\mathcal{N}$ exists. Nevertheless, we have

Lemma 1 *For the proposed $(\mathcal{N}, v)$ collaborative sensing coalitional game, the proposed algorithm of Table* C.I *converges to the optimal $\mathbb{D}_c$-stable partition, if such a partition exists. Otherwise, the proposed algorithm yields a final network partition that is $\mathbb{D}_{hp}$-stable.*

Proof: *The proof is an immediate consequence of Theorem 2 and the fact that the $\mathbb{D}_c$-stable partition is a unique outcome of any arbitrary merge-and-split iteration which is the case with any partition resulting from our algorithm.* □

Moreover, for the proposed game, the existence of the $\mathbb{D}_c$-stable partition cannot be always guaranteed. For instance, for verifying the first condition for existence of the $\mathbb{D}_c$-stable partition, the SUs belonging to partitions of each coalitions must verify the Pareto order through their utility given by (C.8). Similarly, for verifying the second condition of $\mathbb{D}_c$ stability, SUs belonging to all $\mathcal{T}$-incompatible coalitions in the network must verify the Pareto order. Consequently, finding a geometrical closed-form condition for the existence of such a partition is not feasible as it depends on the location of the SUs and the PU through the individual missing and false alarm probabilities in the utility expression (C.8). Hence, the existence of the $\mathbb{D}_c$-stable partition is closely tied to the location of the SUs and the PU which both can be random parameters in practical networks. However, the proposed algorithm will always guarantee convergence to this optimal $\mathbb{D}_c$-stable partition when it exists as stated in Lemma 1. Whenever a $\mathbb{D}_c$-stable partition does not exist, the coalition structure resulting from the

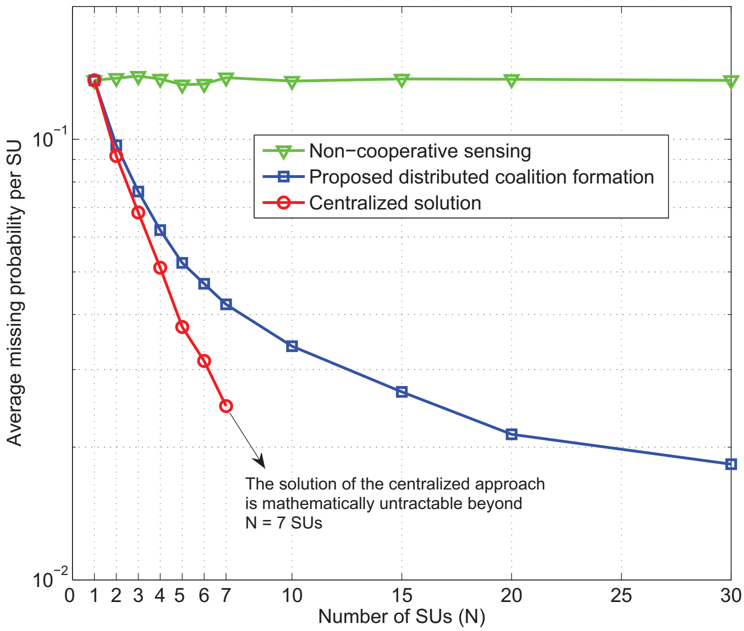

Fig. C.2: Average missing probabilities (average over locations of SUs and non-cooperative false alarm range $P_f \in (0, \alpha)$) vs. number of SUs.

proposed algorithm will be $\mathbb{D}_{hp}$-stable (no coalition or SU is able to merge or split any further).

5 Simulation Results and Analysis

For simulations, the following network is set up: The PU is placed at the origin of a 3 km $\times$3 km square with the SUs randomly deployed in the area around the PU. We set the time bandwidth product $m = 5$ [5–7], the PU transmit power $P_{\text{PU}} = 100$ mW, the SU transmit power for reporting the sensing bit $P_i = 10$ mW $\forall i \in \mathcal{N}$ and the noise level $\sigma^2 = -90$ dBm. For path loss, we set $\mu = 3$ and $\kappa = 1$. The maximum false alarm constraint is set to $\alpha = 0.1$, as recommended by the IEEE 802.22 standard [16].

In Figs. C.2 and C.3 we show, respectively, the average missing probabilities and the average false alarm probabilities achieved per SU for differ-

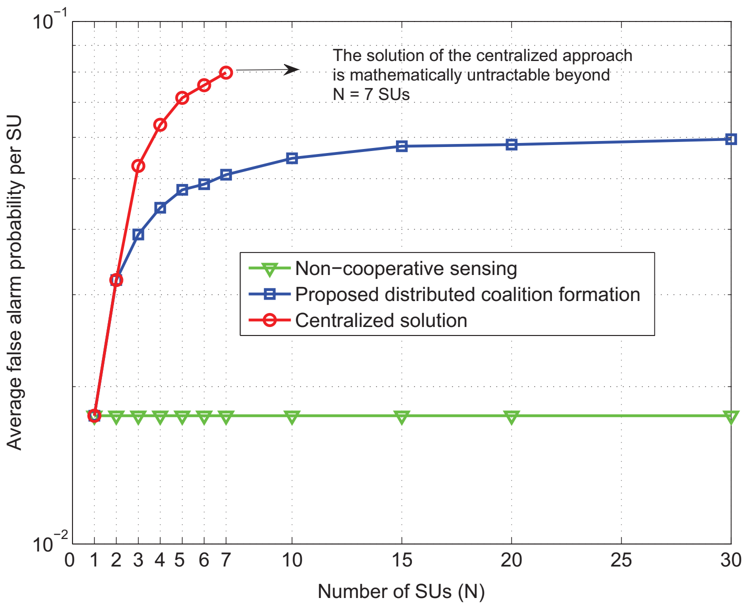

Fig. C.3: Average false alarm probabilities (average over locations of SUs and non-cooperative false alarm range $P_f \in (0, \alpha)$) vs. number of SUs.

ent network sizes. These probabilities are averaged over random locations of the SUs as well as a range of energy detection thresholds λ that do not violate the false alarm constraint; this in turn, maps into an average over the non-cooperative false alarm range $P_f \in (0, \alpha)$ (obviously, for $P_f > \alpha$ no cooperation is possible). In Fig. C.2, we show that the proposed algorithm yields a significant improvement in the average missing probability reaching up to 86.6% reduction (at $N = 30$) compared to the non-cooperative case. This advantage is increasing with the network size N. However, there exists a gap in the performance of the proposed algorithm and that of the optimal centralized solution. This gap stems mainly from the fact that the log barrier function used in the distributed algorithm (C.9) increases the cost drastically when the false alarm probability is in the vicinity of α. This increased cost makes it harder for coalitions with false alarm levels close to α to collaborate in the distributed approach as they require a large missing probability improvement to compensate the cost in their utility (C.8) so

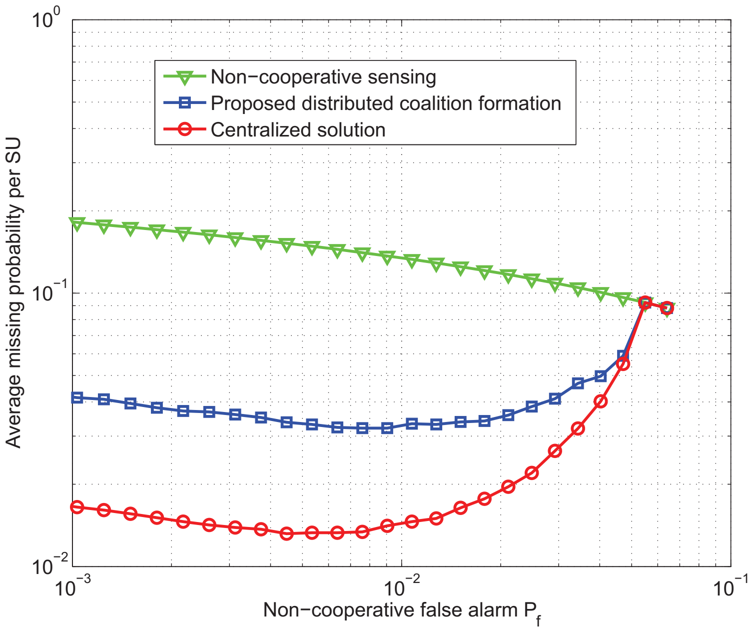

Fig. C.4: Average missing probabilities per SU vs. non-cooperative false alarm P_f (or energy detection threshold λ) for $N = 7$ SUs.

that a Pareto order merge or split becomes possible. However, albeit the proposed cost function yields a performance gap in terms of missing probability, it forces a false alarm for the distributed case smaller than that of the centralized solution as seen in Fig. C.3.

For instance, Fig. C.3 shows that the achieved average false alarm by the proposed distributed solution outperforms that of the centralized solution but is still outperformed by the non-cooperative case. Thus, while the centralized solution achieves a better missing probability; the proposed distributed algorithm compensates this performance gap through the average achieved false alarm. In summary, Figs. C.2 and C.3 clearly show the performance trade off that exists between the gains achieved by collaborative spectrum sensing in terms of average missing probability and the corresponding cost in terms of average false alarm probability.

In Fig. C.4, we show the average missing probabilities per SU for differ-

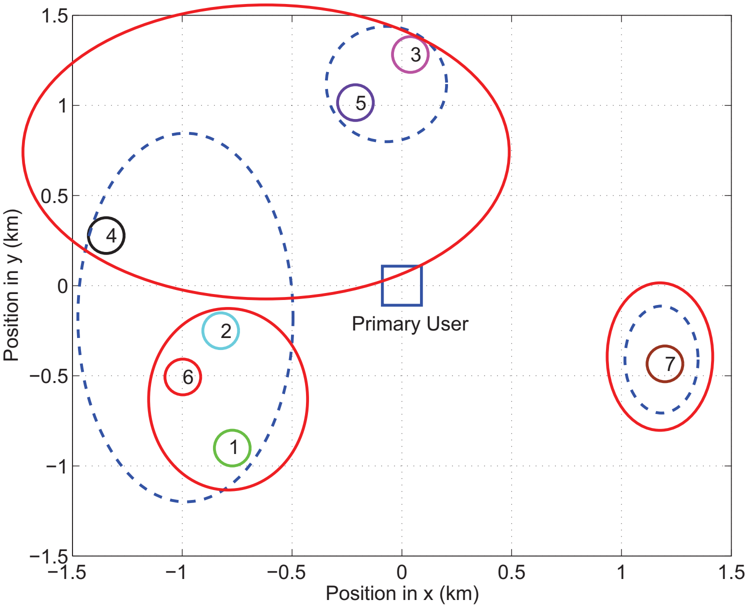

Fig. C.5: Final coalition structure from both distributed (dashed line) and centralized (solid line) collaborative spectrum sensing for $N = 7$ SUs.

ent energy detection thresholds λ expressed by the feasible range of non-cooperative false alarm probabilities $P_f \in (0, \alpha)$ for $N = 7$. In this figure, we show that as the non-cooperative P_f decreases the performance advantage of collaborative spectrum sensing for both the centralized and distributed solutions increases (except for very small P_f where the advantage in terms of missing probability reaches its maximum). The performance gap between centralized and distributed is once again compensated by a false alarm advantage for the distributed solution as already seen and explained in Fig. C.3 for $N = 7$. Finally, in this figure, it must be noted that as P_f approaches $\alpha = 0.1$, the advantage for collaborative spectrum sensing diminishes drastically as the network converges towards the non-cooperative case.

In Fig. C.5, we show a snapshot of the network structure resulting from the proposed distributed algorithm (dashed line) as well as the centralized

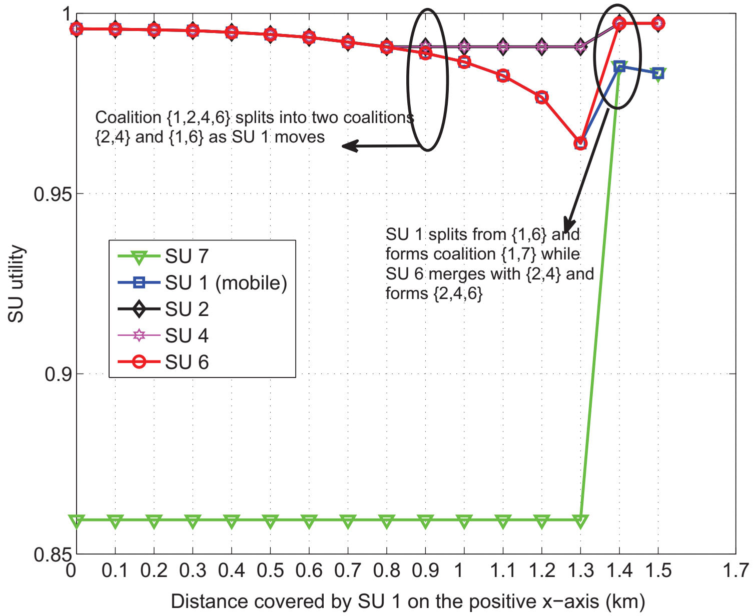

Fig. C.6: Self-adaptation of the network's topology to mobility through merge-and-split as SU 1 moves horizontally on the positive x-axis.

approach (solid line) for $N = 7$ randomly placed SUs and a non-cooperative false alarm $P_f = 0.01$. We notice that the structures resulting from both approaches are almost comparable, with nearby SUs forming collaborative coalitions for improving their missing probabilities. However, for the distributed solution, SU 4 is part of coalition $S_1 = \{1, 2, 4, 6\}$ while for the centralized approach SU 4 is part of coalition $\{3, 4, 5\}$. This difference in the network structure is due to the fact that, in the distributed case, SU 4 acts selfishly while aiming at improving its own utility. In fact, by merging with $\{3, 5\}$ SU 4 achieves a utility of $\phi_4(\{3, 5\}) = 0.9859$ with a missing probability of 0.0024 whereas by merging with $\{1, 2, 6\}$ SU 4 achieves a utility of $\phi_4(\{1, 2, 4, 6\}) = 0.9957$ with a missing probability of 0.00099. Thus, when acting autonomously in a distributed manner, SU 4 prefers to merge with $\{1, 2, 6\}$ rather than with $\{3, 5\}$ regardless of the optimal structure for the network as a whole. In brief, Fig. C.5 shows how the cognitive network

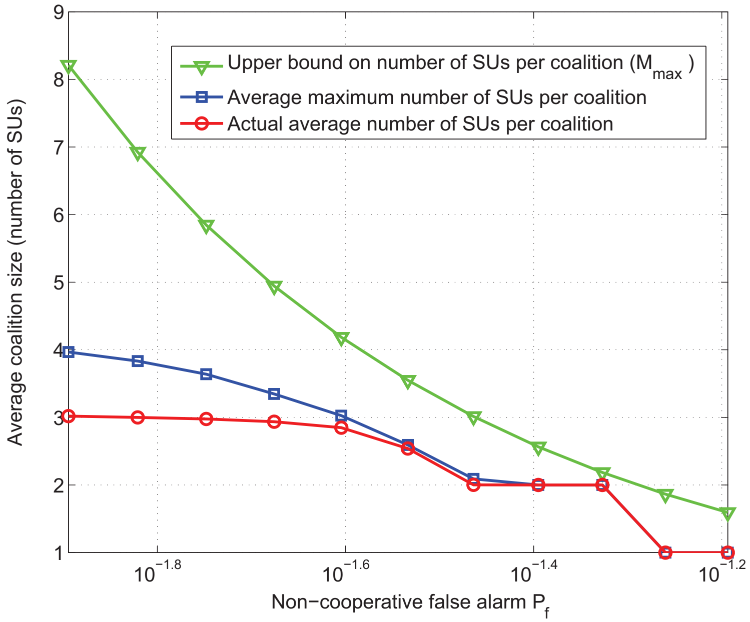

Fig. C.7: Maximum and average coalition size vs. non-cooperative false alarm P_f (or energy detection threshold λ) for the distributed solution for $N = 30$ SUs.

structures itself for both centralized and distributed approaches.

Furthermore, in Fig. C.6 we show how our distributed algorithm in Table C.I handles mobility during Phase 2 (adaptive coalition formation). For this purpose, after the network structure in Fig. C.5 has formed, we allow SU 1 to move horizontally along the positive x-axis while other SUs are immobile. In Fig. C.6, at the beginning, the utilities of SUs $\{1, 2, 4, 6\}$ are similar since they belong to the same coalition. These utilities decrease as SU 1 distances itself from $\{2, 4, 6\}$. After moving 0.8 km SUs $\{1, 6\}$ split from coalition $\{1, 2, 4, 6\}$ by Pareto order as $\phi_1(\{1, 6\}) = 0.9906 > \phi_1(\{1, 2, 4, 6\}) = 0.99$, $\phi_6(\{1, 6\}) = 0.9906 > \phi_6(\{1, 2, 4, 6\}) = 0.99$, $\phi_2(\{2, 4\}) = 0.991 > \phi_2(\{1, 2, 4, 6\}) = 0.99$ and $\phi_4(\{2, 4\}) = 0.991 > \phi_4(\{1, 2, 4, 6\} = 0.99)$ (this small advantage from splitting increases as SU 1 moves further). As SU 1 distances itself further from the PU, its utility and that of its partner

SU 6 decrease. Subsequently, as SU 1 moves 1.4 km it finds it beneficial to split from $\{1, 6\}$ and merge with SU 7. Through this merge, SU 1 and SU 7 improve their utilities. Meanwhile, SU 6 rejoins SUs $\{2, 4\}$ forming a 3-SU coalition $\{2, 4, 6\}$ while increasing the utilities of all three SUs. In a nutshell, this figure illustrates how adaptive coalition formation through merge and split operates in a mobile cognitive radio network. Similar results can be seen whenever all SUs are mobile or even the PU is mobile but they are omitted for space limitation.

In Fig. C.7, for a network of $N = 30$ SUs, we evaluate the sizes of the coalitions resulting from our distributed algorithm and compare them with the the upper bound $M_{\max}$ derived in Theorem 1. First and foremost, as the non-cooperative P_f increases, both the maximum and the average size of the formed coalitions decrease converging towards the non-cooperative case as P_f reaches the constraint $\alpha = 0.1$. Through this result, we can clearly see the limitations that the detection-false alarm probabilities trade off for collaborative sensing imposes on the coalition size and network topology. Also, in Fig. C.7, we show that, albeit the upper bound on coalition size $M_{\max}$ increases drastically as P_f becomes smaller, the average maximum coalition size achieved by the proposed algorithm does not exceed 4 SUs per coalition for the given network with $N = 30$. This result shows that, in general, the network topology is composed of a large number of small coalitions rather than a small number of large coalitions, even when P_f is small and the collaboration possibilities are high.

6 Conclusions

In this paper, we proposed a novel distributed algorithm for collaborative spectrum sensing in cognitive radio networks. We modeled the collaborative sensing problem as a coalitional game with non-transferable utility and we derived a distributed algorithm for coalition formation. The proposed coalition formation algorithm is based on two simple rules of merge-and-split that enable SUs in a cognitive network to cooperate for improving their detection probability while taking into account the cost in terms of false alarm probability. We characterized the network structure resulting from the proposed algorithm, studied its stability and showed that a maximum number of SUs per coalition exists for the proposed utility model. Simulation results showed that the proposed distributed algorithm reduces the average missing probability per SU up to 86.6% compared to the non-cooperative case. The results also showed how, through the proposed algorithm, the SUs can autonomously adapt the network structure to environmental changes such as mobility. Through simulations, we also compared the performance of the proposed algorithm with that of an optimal centralized solution.

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Paper D

Coalitional Games in Partition Form for Joint Spectrum Sensing and Access in Cognitive Radio Networks

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Abstract

Unlicensed secondary users (SUs) in cognitive radio networks are subject to an inherent tradeoff between *spectrum sensing* and *spectrum access*. Although each SU has an incentive to sense the primary user (PU) channels for locating spectrum holes, this exploration of the spectrum can come at the expense of a shorter transmission time, and, hence, a possibly smaller effective capacity for data transmission. In this paper, we investigate the impact of this tradeoff on the cooperative strategies of a network of SUs that seek to cooperate in order to improve their view of the spectrum (sensing), reduce the possibility of interference among each other, and improve their transmission capacity (access). We model the problem as a coalitional game in *partition form* and we propose an algorithm for coalition formation. Through the proposed algorithm, the SUs can take individual distributed decisions to join or leave a coalition while maximizing their utility that accounts for the average time spent for sensing as well as the capacity achieved while accessing the spectrum. We show that, by using the proposed algorithm, the SUs can self-organize into a Nash-stable network partition composed of disjoint coalitions, with the members of each coalition cooperating to jointly optimize their sensing and access performance. Simulation results show that the proposed algorithm yields a performance gain, in terms of the average payoff per SU per time slot reaching up to 77.25% relative to the non-cooperative case for a network of 20 SUs. The results also show how the algorithm allows the SUs to self-adapt to changes in the environment such as the change in the traffic of the PUs, or slow mobility.

1 Introduction

With the ongoing growth in wireless services, the demand for the radio spectrum has significantly increased. However, the spectrum resources are scarce and most of them have been already licensed to existing operators. Numerous studies done by agencies such as the Federal Communications Commission (FCC) in the United States have shown that the actual licensed spectrum remains unoccupied for large periods of time [1]. Thus, *cognitive radio* systems were proposed [2] in order to efficiently exploit these spectrum holes. Cognitive radios or secondary users (SUs) are wireless devices that can intelligently monitor and adapt to their environment, hence, they are able to share the spectrum with the licensed primary users (PUs), operating whenever the PUs are idle. Implementing cognitive radio systems faces various challenges [3], notably, for spectrum sensing and spectrum access. Spectrum sensing mainly deals with the stage during which the SUs attempt to learn their environment prior to the spectrum access stage where the SUs actually transmit their data.

Existing literature has tackled various aspects of spectrum sensing and spectrum access, individually. In [4], the performance of spectrum sensing, in terms of throughput, is investigated when the SUs share their instantaneous knowledge of the channel. The work in [5] studies the performance of different detectors for spectrum sensing while in [6], the optimal sensing time which maximizes the achievable throughput of the SUs, given the detection-false alarm tradeoff is derived. The authors in [7] study the use of sounding signals to detect primary systems with power control. Different cooperative techniques for improving spectrum sensing performance are discussed in [8–13]. Further, spectrum access has also received an increased attention [14–21]. In [14], a dynamic programming approach is proposed to allow the SUs to maximize their channel access time while taking into account a penalty factor from any collision with the PU. The work in [14] (and the references therein) establish that, in practice, the sensing time of CR networks is *large* and affects the access performance of the SUs. The authors in [15] propose a novel multiple access scheme that takes into account the physical layer transmission in cognitive networks. In [16], the authors model the spectrum access problem as a non-cooperative game, and propose learning algorithms to find the correlated equilibria of the game. Non-cooperative solutions for dynamic spectrum access are also proposed in [17] while taking into account changes in the SUs' environment such as the arrival of new PUs, among others. Additional challenges of spectrum access are tackled in [18–21].

Clearly, the spectrum sensing and spectrum access aspects of cognitive networks have been widely investigated in the literature, independently. However, a key challenge which remains relatively unexplored is to study the tradeoff between *spectrum sensing* and *spectrum access* when the SUs seek to improve both aspects, *jointly*. This tradeoff arises from the fact that the sensing time for the SUs is non-negligible [14], and can reduce their transmission performance. Thus, although each SU has an incentive to sense as many PU channels as possible for locating access opportunities, this spectrum exploration may come at the expense of a smaller transmission time, and, hence, a possibly smaller effective capacity for data transmission. Also, due to the limited capability of the cognitive devices, each SU, on its own, may not be able to explore more than a limited number of channels. As a result, the SUs can rely on cooperation for sharing the spectrum knowledge with nearby cognitive radio. Therefore, it is important to design cooperative strategies which allow the SUs to improve their performance while taking into account *both* sensing and access metrics.

The main contribution of this paper is to devise a cooperative scheme among the SUs in a multi-channel cognitive network, which enables them to improve their performance jointly at the sensing and access levels. From a sensing perspective, we propose a scheme through which the SUs cooperate in order to share their channel knowledge, and, hence, improve their view of the spectrum, consequently, reducing their sensing time. From an access perspective, the proposed cooperation protocol allows the SUs to improve their access capacities by: (i)- Learning from their cooperating partners the existence of alternative channels with better conditions, (ii)- Reducing the interference among each other, and (iii)- Exploiting multiple channels simultaneously, when possible. We model the problem as a coalitional game in *partition form*, and we propose an algorithm for coalition formation. Albeit coalitional games in partition form have been widely used in game theory, to the best of our knowledge, no existing work has utilized the partition form of coalitional game theory in the design of wireless protocols and systems. The proposed coalition formation algorithm allows the SUs to take distributed decisions to join or leave a coalition, while maximizing their utility which accounts for the average time needed to locate an unoccupied channel (spectrum sensing) and the average capacity achieved when transmitting the data (spectrum access). Thus, the SUs self-organize into disjoint coalitions that constitute a Nash-stable network partition. Within every formed coalition, the SUs act cooperatively by sharing their view of the spectrum, coordinating their sensing order, and distributing their powers over the seized channels whenever possible. Also, the proposed coalition formation algorithm allows the SUs to adapt the topology to environmental changes such as the changes in the availability of the PU channels or the slow mobility of the SUs. Simulation results show that the proposed algorithm increases the average payoff of the SUs up to 77.25% relatively to the non-cooperative case.

The rest of this paper is organized as follows: Section 2 presents the non-cooperative spectrum sensing and access model. In Section 3, we present the proposed cooperation model for joint spectrum access and sensing while in Section 4, we model the problem using coalitional games in partition form and we devise a distributed algorithm for coalition formation. Simulation results are presented and analyzed in Section 5. Finally, conclusions are drawn in Section 6.

2 Non-cooperative Spectrum Sensing and Access

In this section, we present the non-cooperative procedure for spectrum sensing and access in a cognitive network, prior to proposing, in the next sections, cooperation strategies for improving the performance of the SUs jointly for sensing and access.

Consider a cognitive radio network with N secondary users (SUs) engaged in the sensing of K primary users' (PUs) channels in order to access the spectrum and transmit their data to a common base station (BS). Let $\mathcal{N}$ and $\mathcal{K}$ denote the set of SUs and the set of PUs (channels), respectively. Due to the random nature of the traffic of the PUs and to the dynamics of the PUs, each channel $k \in \mathcal{K}$ is available for use by the SUs with a probability of θ_k (which depends on PU traffic only and not on the SUs). Although for very small K the SUs may be able to learn the statistics (probabilities θ_k) of all K channels, we consider the generalized case where each SU $i \in \mathcal{N}$ can only have accurate statistics regarding a subset $\mathcal{K}_i \subseteq \mathcal{K}$ of $K_i \leq K$ channels (e.g., via standard learning algorithms), during the period of time the channels remain stationary. We consider a frequency selective channel, whereby the channel gain $g_{i,k}$ of any SU $i \in \mathcal{N}$ perceived at the BS when SU i transmits over channel $k \in \mathcal{K}_i$ is $g_{i,k} = a_{i,k} \cdot d_i^{-\mu}$, with d_i the distance between SU i and the BS, μ the path loss exponent, and

$a_{i,k}$ a Rayleigh distributed fading amplitude for SU i on channel k with a variance of 1. We consider a channel with *slow* fading which varies independently over the frequencies (quasi-static over the frequency band). Note that other channel types can also be accommodated.

For transmitting its data, each SU $i \in \mathcal{N}$ is required to sense the channels in $\mathcal{K}_i$ persistently, one at a time, in order to locate a spectrum opportunity. We consider that each SU $i \in \mathcal{N}$ is opportunistic which implies that SU i senses the channels in $\mathcal{K}_i$ in a certain order, sequentially, and once it locates a spectrum hole it ends the sensing process and transmits over the first channel found unoccupied (by a PU). For the purpose of finding a preferred order for sensing, each SU i assigns a weight $w_{i,k}$ to every channel $k \in \mathcal{K}_i$ which will be used in sorting the channels. When assigning the weights and ordering the channels, the SUs face a tradeoff between improving their sensing time by giving a higher weight to channels that are often available, and improving their access performance by giving a higher weight to channels with a better condition. To capture this tradeoff, the weight $w_{i,k}$ assigned by an SU i to a channel $k \in \mathcal{K}_i$ will be taken as

$$w_{i,k} = \theta_k \cdot g_{i,k}, \tag{D.1}$$

where $g_{i,k}$ is the channel gain perceived by SU i over channel k and θ_k is the probability that channel k is available. Clearly, the weight given in (D.1) provides a balance between the need for quickly finding an available channel and the need for good channel conditions. Given the channel weights, each SU $i \in \mathcal{N}$ sorts its channels in a decreasing order of weights and begins sensing these channels in an ordered manner. Hence, each SU i senses the channels consecutively starting by the channel with the highest weight until finding an unoccupied channel on which to transmit, if any. The set of channels used by an SU $i \in \mathcal{N}$ *ordered* in decreasing weights is denoted by $\mathcal{K}_i^{\text{ord}} = \{k_1, \ldots, k_{K_i}\}$ where $w_{i,k_1} \geq w_{i,k_2} \geq \ldots \geq w_{i,k_{K_i}}$.

We consider a time-slotted spectrum sensing and access process whereby, within each slot, each SU $i \in \mathcal{N}$ spends a certain fraction of the slot for sensing the channels, and, once an available channel is found, the remainder time of the slot is used for spectrum access. In this regard, we consider that the channel available/busy time is comparable or larger to the duration of a slot, which is a common assumption in the literature [4, 12, 14, 22]. Given the ordered set of channels $\mathcal{K}_i^{\text{ord}}$, the average fraction of time τ_i spent by any SU $i \in \mathcal{N}$ for locating a free channel, i.e., the average sensing time, is given by (the duration of a slot is normalized to 1)

$$\tau_i(\mathcal{K}_i^{\text{ord}}) = \sum_{j=1}^{K_i} \left(j \cdot \alpha \cdot \theta_{k_j} \prod_{m=1}^{j-1} (1 - \theta_{k_m}) \right) + \prod_{l=1}^{K_i} (1 - \theta_{k_l}) \tag{D.2}$$

where $\alpha < 1$ is the fraction of time needed for sensing a single channel, and θ_{k_j} is the probability that channel $k_j \in \mathcal{K}_i^{\text{ord}}$ is unoccupied. The first term in (D.2) represents the average time spent for locating an unoccupied channel among the known channels in $\mathcal{K}_i^{\text{ord}}$, and the second term represents the probability that no available channel is found (in this case, the SU remains idle in the slot). Note that $\tau_i(\mathcal{K}_i^{\text{ord}})$ is function of $\mathcal{K}_i^{\text{ord}}$ and, hence, depends on the assigned weights and the ordering. For notational convenience, the argument of τ_i is dropped hereafter since the dependence on the channel ordering is clear from the context.

When the SUs are acting in a non-cooperative manner, given the ordered set of channels $\mathcal{K}_i^{\text{ord}}$, the average capacity achieved by an SU $i \in \mathcal{N}$ is given by

$$C_i = \sum_{j=1}^{K_i} \theta_{k_j} \prod_{m=1}^{j-1} (1 - \theta_{k_m}) \cdot \mathbb{E}_{I_{i,k_j}} \left[C_{i,k_j} \right] \tag{D.3}$$

where $\theta_{k_j} \prod_{m=1}^{j-1} (1 - \theta_{k_m})$ is the probability that SU i accesses channel $k_j \in \mathcal{K}_i^{\text{ord}}$ given the ordered set $\mathcal{K}_i^{\text{ord}}$, and $\mathbb{E}_{I_{i,k_j}} \left[C_{i,k_j} \right]$ is the expected value of the capacity achieved by SU i over channel k_j with the expectation taken over the distribution of the total interference I_{i,k_j} perceived on channel k_j by SU i from the SUs in $\mathcal{N} \setminus \{i\}$.

For evaluating the capacity in (D.3), every SU $i \in \mathcal{N}$ must have perfect knowledge of the channels that the other SUs are using, as well as the order in which these channels are being sensed and accessed (to compute the expectation) which is quite difficult in a practical network. To alleviate the information needed for finding the average capacity, some work such as [23, 24] consider, in (D.3), the capacities under the worst case interference, instead of the expectation over the interference. However, applying this assumption in our case requires considering the capacities under worst case interference on *every* channel for every SU i which is quite restrictive. Thus, in our setting, as an alternative to the expectation in (D.3), for any SU $i \in \mathcal{N}$ we consider the capacity $\bar{C}_{i,k_j}$ achieved over channel $k_j \in \mathcal{K}_i^{\text{ord}}$ under the average interference perceived from the SUs in $\mathcal{N} \setminus \{i\}$, given by

$$\bar{C}_{i,k_j} = \log_2 (1 + \Gamma_{i,k_j}). \tag{D.4}$$

Here, Γ_{i,k_j} is the SINR achieved by SU i when using channel k_j given an average total interference $\bar{I}_{i,k_j}$ perceived from the SUs in $\mathcal{N} \setminus \{i\}$ and is given by

$$\Gamma_{i,k_j} = \frac{g_{i,k_j} \cdot P_{i,k_j}}{\sigma^2 + \bar{I}_{i,k_j}}, \tag{D.5}$$

where P_{i,k_j} is the maximum transmit power of SU i used on channel k_j, and σ^2 is the variance of the Gaussian noise. In the non-cooperative setting, $P_{i,k_j} = \tilde{P}$ where $\tilde{P}$ is the maximum transmit power of any SU ($\tilde{P}$ is assumed the same for all SUs with no loss of generality). In a practical cognitive network, through measurements, any SU $i \in \mathcal{N}$ can obtain from its receiver an estimate of the average total interference $\bar{I}_{i,k_j}$ perceived on any channel $k_j \in \mathcal{K}_i^{\text{ord}}$ [25], and, thus, SU i is able to evaluate the capacity in (D.4). By using (D.4), we define the average capacity $\bar{C}_i$ in a manner analogous to (D.3) as follows

$$\bar{C}_i = \sum_{j=1}^{K_i} \theta_{k_j} \prod_{m=1}^{j-1} (1 - \theta_{k_m}) \cdot \bar{C}_{i,k_j}. \tag{D.6}$$

Clearly, given the measurement of the external interference, every SU i can easily evaluate its capacity in (D.6). Due to reasons such as Jensen's inequality, (D.6) represents a lower bound of (D.3) but it provides a good indicator of the access performance of the SUs. Hereafter, we solely deal with capacities given the measured average interference.

Consequently, the non-cooperative utility achieved by any SU $i \in \mathcal{N}$ per slot is given by

$$u(\{i\}, \mathcal{N}) = \bar{C}_i \cdot (1 - \tau_i), \tag{D.7}$$

where the dependence on $\mathcal{N}$ indicates the dependence of the utility on the external interference when the SUs are non-cooperative, τ_i is the fraction of time used for sensing given by (D.2), and $\bar{C}_i$ the average capacity given by (D.6). This utility captures the tradeoff between exploring the spectrum, i.e., sensing time, and exploiting the best spectrum opportunities, i.e., capacity achieved during spectrum access.

3 Joint Spectrum Sensing and Access Through Cooperation

To improve their joint sensing and access performance, the SUs in the cognitive network can cooperate. Hence, any group of SUs can cooperate by forming a *coalition* $S \subseteq \mathcal{N}$ in order to: (i)- Improve their sensing time and learn the presence of channels with better conditions by exchanging information on the statistics of their known channels, (ii)- Jointly coordinate the order in which the channels are accessed to reduce the interference on each other, and (iii)- Share their instantaneous sensing results to improve their capacity by distributing their total power over multiple channels, when possible.

First and foremost, whenever a coalition S of SUs forms, its members exchange their knowledge on the channels and their statistics. Hence, the set of channels that the coalition is aware of can be given by $\mathcal{K}_S = \cup_{i \in S} \mathcal{K}_i$ with cardinality $|\mathcal{K}_S| = K_S$. By sharing this information, each member of S can explore a larger number of channels, and, thus, can improve its sensing time by learning channels with better availability and by reducing the second term in (D.2). Moreover, as a result of sharing the known channels, some members of S may be able to access the spectrum with better channel conditions, hence, possibly improving their capacities as well.

Once the coalition members share their knowledge of the channels, the SUs will jointly coordinate their order of access over the channels in $\mathcal{K}_S$ in order to minimize the probability of interfering on each other. In this context, analogous to the non-cooperative case, the SUs in S proceed by assigning different weights on the channels in $\mathcal{K}_S$ using (D.1). Then, the SUs in coalition S *cooperatively* sort their channels, in a manner to reduce interference as much as possible. Thus, the SUs jointly *rank* their channels on a rank scale from 1 (the first channel to sense) to K_S (the last channel to sense). For every SU $i \in S$, let $\mathcal{Q}_{i,r}$ denote the set of channels that SU i selected *until* and including rank r. Further, we denote by $\mathcal{R}_r$ the set of SUs that selected a channel for rank r and by $\mathcal{K}_{r,S}$ the set of channels that have been selected for rank r by members of S. Given this notation, we propose the sorting procedure in Algorithm 1 for any coalition S. The gist of Algorithm 1 is that every SU in the coalition starts by using the non-cooperative weighing procedure over the set of channels $\mathcal{K}_S$. In the event where a set of SUs $G \subseteq S$ select the same channel for the same rank r, SU $j \in G$ with the highest weight is given this channel at rank r, and this

Algorithm 1 Proposed sorting algorithm for any coalition $S \subseteq \mathcal{N}$

$\mathcal{Q}_{i,0} \leftarrow \emptyset$
for $r = 1$ to K_S **do** {For rank $r = 1$ we find all the channels that SUs in S sense first, for $r = 2$ the channels that they sense second, and so on.}
  $\mathcal{Q}_{i,r} \leftarrow \mathcal{Q}_{i,r-1}$, $\mathcal{K}_{r,S} \leftarrow \emptyset$, $\mathcal{R}_r \leftarrow \emptyset$
  For rank r, each SU $i \in S$ proposes to select the channel k_i^r in $\mathcal{K}_S \setminus \mathcal{Q}_{i,r}$ which has the highest weight, i.e., $k_i^r = \underset{k \in \mathcal{K}_S \setminus \mathcal{Q}_{i,r}}{\arg\max}\ w_{i,k}$.
  for all $i \in S$ s. t. $k_i^r \neq k_j^r,\ \forall j \in S,\ i \neq j$ **do**
    SU i fixes its selection for this rank, and, hence:
    $\mathcal{Q}_{i,r} \leftarrow \mathcal{Q}_{i,r} \cup k_i^r$, $\mathcal{K}_{r,S} \leftarrow \mathcal{K}_{r,S} \cup k_i^r$, $\mathcal{R}_r \leftarrow \mathcal{R}_r \cup \{i\}$.
  end for
  for all $G \subseteq S \setminus \mathcal{R}_r,$ s. t. $k_i^r = k_j^r = k_G^r,\ \forall i, j \in G$ **do**
    a) The SU $j \in G$ which has the highest weight for k_G^r, i.e., $j = \underset{j \in G}{\arg\max}\ w_{j,k_G^r}$, selects channel k_G^r for rank r.
    b) $\mathcal{Q}_{j,r} \leftarrow \mathcal{Q}_{j,r} \cup k_G^r$, $\mathcal{K}_{r,S} \leftarrow \mathcal{K}_{r,S} \cup k_G^r$, $R_r \leftarrow R_r \cup \{j\}$.
    if $\mathcal{R}_r \neq S$ **then** {SUs with unselected channels for r exist}
      The SUs in $S \setminus \mathcal{R}_r$ repeat the previous procedure, but each SU $i \in S \setminus \mathcal{R}_r$, can only use the channels in $\mathcal{K}_\mathcal{S} \setminus \mathcal{K}_{r,S} \cup \mathcal{Q}_{i,r}$. However, if for any SU $i \in S \setminus \mathcal{R}_r$, we have $\mathcal{K}_\mathcal{S} \setminus \mathcal{K}_{r,S} \cup \mathcal{Q}_{i,r} = \emptyset$, then this SU will simply select the channel that will maximize its weight from the set $\mathcal{K}_\mathcal{S} \setminus \mathcal{Q}_{i,r}$, regardless of the other SUs selection.
    end if
  end for
end for

is repeated for all such sets G. After these channel selections are made at rank r, if a number of SUs have still not made any selection, i.e., $\mathcal{R}_r \neq S$, then these SUs repeat the procedure but can only use channels that their partners have not selected at rank r. However, if for any SU $i \in S \setminus \mathcal{R}_r$, this is not possible, it is inevitable that this SU i interferes with some of its partners at rank r, then SU i simply selects, at rank r, the channel in $\mathcal{K}_\mathcal{S} \setminus \mathcal{Q}_{i,r}$ with the highest weight. As a result of the sorting process, each SU $i \in S$ will have an *ordered set of channels* $\mathcal{K}_i^S$ of cardinality K_S which reflects the result of Algorithm 1.

Given this new ordering resulting from the sorting procedure of Algorithm 1, for every SU $i \in S$, the total average sensing time τ_i^S will still be expressed by (D.2). However, the sensing time τ_i^S is function of the channel ordering based on the set $\mathcal{K}_i^S$ which is ordered cooperatively, rather than $\mathcal{K}_i^{\text{ord}}$ which is the non-cooperative ordering.

Using Algorithm 1, the SUs that are members of the same coalition are able to reduce the interference on each other, by minimizing the possibility of selecting the same channel at the same rank (although they can still select the same channel but at different ranks). However, as a result of this joint sorting, some SUs might need to give a high rank to some channels with lower weights which can increase the sensing time of these SUs. Hence, this cooperative sorting of the channels highlights the fact that some SUs may trade off some gains in sensing performance (obtained by sharing channel statistics) for obtaining access gains (by avoiding interference through joint sorting). As we will see later in this section, in addition to the interference reduction, some SUs in a coalition S can also obtain access gains by using multiple channels simultaneously.

For every coalition S, we define $\mathcal{B}_S = \{b_1, \ldots, b_{|S|}\}$ as the multiset with every element b_i representing a channel in $\mathcal{K}_i^S$ selected by SU $i \in S$. Denote by $\mathfrak{B}_S$ as the family of all such multisets for coalition S which corresponds to the family of all permutations, with repetition, for the SUs in S over the channels in $\mathcal{K}_S$. Each multiset $\mathcal{B}_S \in \mathfrak{B}_S$ is chosen by SUs in S with a certain probability $p_{\mathcal{B}_S}$ given by

$$p_{\mathcal{B}_S} = \begin{cases} \prod_{k \in \cup_{i=1}^{|S|} b_i,\ b_i \in \mathcal{B}_S} \theta_k \prod_{j \in \cup_{i=1}^{|S|} \mathcal{K}_{i,b_i}^S} (1-\theta_j), & \text{if } \cup_{i=1}^{|S|} b_i \cap \cup_{i=1}^{|S|} \mathcal{K}_{i,b_i}^S = \emptyset \\ 0, \text{ otherwise.} \end{cases} \tag{D.8}$$

where, for any SU $i \in S$, the set $\mathcal{K}_{i,b_i}^S = \{j \in \mathcal{K}_i^S |\ \text{rank}(j) < \text{rank}(b_i)\}$ represents the set of channels that need to be busy before SU i selects channel $b_i \in \mathcal{B}_S$, i.e., the set of channels ranked higher than b_i (recall that the set $\mathcal{K}_i^S$ is ordered as a result of Algorithm 1). If $\cup_{i=1}^{|S|} b_i \cap \cup_{i=1}^{|S|} \mathcal{K}_{i,b_i}^S \neq \emptyset$, it implies that, for the selection $\mathcal{B}_S$, a channel needs to be available and busy at the same time which is impossible, and, hence, the probability for selecting any multiset $\mathcal{B}_S \in \mathfrak{B}_S$ having this property is 0. Due to this property, the SUs of any coalition $S \subseteq \mathcal{N}$, can only achieve a transmission capacity for the multisets $\mathcal{B}_S \in \bar{\mathfrak{B}}_S$ where $\bar{\mathfrak{B}}_S$ is the family of all *feasible* multisets for coalition S such that $\cup_{i=1}^{|S|} b_i \cap \cup_{i=1}^{|S|} \mathcal{K}_{i,b_i}^S = \emptyset$, which corresponds to the multisets which have a non-zero probability of occurrence as per (D.8). Note that, the multiset corresponding to the case where no SU $i \in S$ finds an unoccupied channel has also a non-zero probability, but is omitted as its corresponding capacity is 0 and, thus, it has no effect on the utility.

For every channel selection $\mathcal{B}_S \in \bar{\mathfrak{B}}_S$, one can partition coalition S into a number of *disjoint* sets $\{S_1, \ldots, S_L\}$ with $\cup_{l=1}^{L} S_l = S$ such that, for a given $l \in \{1, \ldots, L\}$, the channels in $\mathcal{B}_S$ selected by any $i \in S_l$ are of the same rank. Thus, the SUs belonging to any S_l access their selected channels *simultaneously* and, for this reason, they can coordinate their channel access. In the event where $|S_l| = 1$, then the SU in S_l simply transmits using its maximum power $\tilde{P}$ over its selected channel in $\mathcal{B}_S$. In contrast, for any $l \in \{1, \dots, L\}$ with $|S_l| > 1$, the SUs in S_l can share their sensing result (since they find their available channels simultaneously) and improve their access performance by distributing their powers cooperatively over the channels in the set $\mathcal{K}_{S_l}$ that corresponds to the channels selected by S_l given $\mathcal{B}_S$. For every SU $i \in S_l$, we associate a $1 \times |\mathcal{K}_{S_l}|$ vector $\boldsymbol{P}_i^{\mathcal{B}_S}$ where each element $P_{i,k}^{\mathcal{B}_S}$ represents the power that SU $i \in S_l$ will use on channel $k \in \mathcal{K}_{S_l}$ given the selection $\mathcal{B}_S$. Let $\boldsymbol{P}_{\mathcal{K}_{S_l}}^{\mathcal{B}_S} = [\boldsymbol{P}_1^{\mathcal{B}_S} \ \dots \ \boldsymbol{P}_{|S_l|}^{\mathcal{B}_S}]^T$. Hence, for every $S_l,\ l \in \{1, \dots, L\}$ such that $|S_l| > 1$, the SUs can distribute their power to maximize the total sum-rate that they achieve as a coalition, i.e., the social welfare, by solving [19]:

$$\max_{\boldsymbol{P}_{\mathcal{K}_{S_l}}^{\mathcal{B}_S}} \sum_{i \in S_l} \sum_{k \in \mathcal{K}_{S_l}} C_{i,k}, \tag{D.9}$$

$$\text{s.t. } P_{i,k}^{\mathcal{B}_S} \geq 0,\ \forall i \in S_l, k \in \mathcal{K}_{S_l},\ \sum_{k \in \mathcal{K}_{S_l}} P_{i,k}^{\mathcal{B}_S} = \tilde{P},\ \forall i \in S_l,$$

with $\tilde{P}$ the maximum transmit power and $C_{i,k}$ the capacity achieved by SU $i \in S_l$ over channel $k \in \mathcal{K}_{S_l}$ and is given by

$$C_{i,k} = \log\left(1 + \frac{P_{i,k}^{\mathcal{B}_S} \cdot g_{i,k}}{\sigma^2 + I_{i,k}^{S_l} + I_{i,k}^{S \setminus S_l} + \bar{I}_{S,k}}\right) \tag{D.10}$$

where $I_{i,k}^{S_l} = \sum_{j \in S_l, j \neq i} g_{j,k} P_{j,k}^{\mathcal{B}_S}$ is the interference between SUs in S_l on channel $k \in \mathcal{K}_{S_l}$, and $I_{i,k}^{S \setminus S_l} = \sum_{j \in S \setminus S_l} g_{j,k} P_{j,k}^{\mathcal{B}_S}$ is the interference from SUs in $S \setminus S_l$ on channel $k \in \mathcal{K}_{S_l}$ (if any). Further, $\bar{I}_{S,k}$ represents the average interference perceived by the members of coalition S, including SU i from the SUs *external* to S, which, given a partition Π of $\mathcal{N}$ with $S \in \Pi$, corresponds to the SUs in $\mathcal{N} \setminus S$ (which can also be organized into coalitions as per Π). Similar to the non-cooperative case, this average external interference can be estimated through measurements from the receiver (the receiver can inform every SU in S of the interference it perceived, and then the SUs in S can easily deduce the interference from the external sources).

[19]Other advanced optimization or game theoretical methods such as non-cooperative Nash equilibrium or Nash bargaining can also be used for distributing the powers, but are out of the scope of this paper and will be tackled separately in future work.

Subsequently, given that, for any $\mathcal{B}_S \in \bar{\mathfrak{B}}_S$, S is partitioned into $\{S_1, \ldots, S_L\}$ as previously described, the average capacity achieved, when acting cooperatively, by any SU $i \in S$, with $i \in S_l,\ l \in \{1, \ldots, L\}$ (for every $\mathcal{B}_S \in \bar{\mathfrak{B}}_S$), is

$$\bar{C}_i^S = \sum_{\mathcal{B}_S \in \bar{\mathfrak{B}}_S} p_{\mathcal{B}_S} \cdot C_i^{\mathcal{B}_S}, \tag{D.11}$$

where $p_{\mathcal{B}_S}$ is given by (D.8), and $C_i^{\mathcal{B}_S}$ is the total capacity achieved by SU $i \in S_l$ when the SUs in S select the channels in $\mathcal{B}_S$ and is given by

$$C_i^{\mathcal{B}_S} = \sum_{k \in \mathcal{K}_{S_l}} C_{i,k}^{\mathcal{B}_S}, \tag{D.12}$$

where $\mathcal{K}_{S_l} \subseteq \mathcal{K}_S$ is the set of channels available to $S_l \subseteq S$. Further, $C_{i,k}^{\mathcal{B}_S}$ is the capacity achieved by SU $i \in S_l$ on channel $k \in \mathcal{K}_{S_l}$ given the channel selection $\mathcal{B}_S$ and is a direct result (upon computing the powers) of (D.9) which is a standard constrained optimization problem that can be solved using well known methods [26].

Hence, the utility of any SU i in coalition S is given by

$$v_i(S, \Pi) = \bar{C}_i^S (1 - \tau_i^S) \tag{D.13}$$

where Π is the network partition currently in place which determines the external interference on coalition S, and τ_i^S is given by (D.2) using the set $\mathcal{K}_i^S$ which is ordered by SU i, cooperatively with the SUs in S, using Algorithm 1. Note that the utility in (D.13) reduces to (D.7) when the network is non-cooperative. Finally, we remark that, although cooperation can benefit the SUs both in the spectrum sensing and spectrum access levels, in many scenarios forming a coalition may also entail *costs*. From a spectrum sensing perspective, due to the need for re-ordering the channels to reduce the interference, the sensing time of some members of a coalition may be longer than their non-cooperative counterparts. From a spectrum access perspective, by sharing information, some SUs may become subject to new interference on some channels (although reduced by the sorting algorithm) which may degrade their capacities. Thus, there exists a number of tradeoffs for cooperation, in different aspects for both sensing and access. In this regard, clearly, the utility in (D.13) adequately captures these tradeoffs through the gains (or costs) in sensing time (spectrum sensing), and the gains (or costs) in capacity (spectrum access).

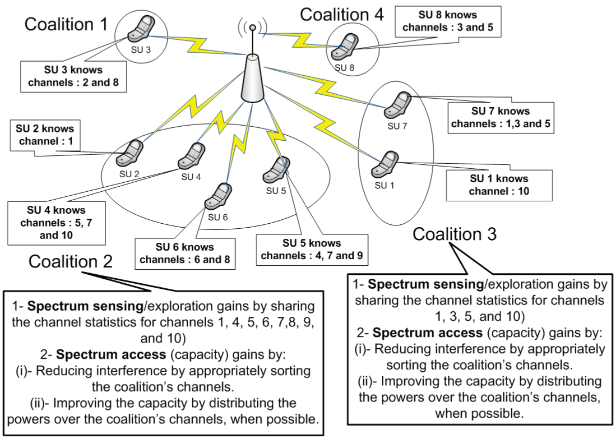

Fig. D.1: An illustrative example of coalition formation for joint spectrum sensing and access for $N = 8$ SUs and $K = 10$ channels.

In a nutshell, with these tradeoffs, for maximizing their utility in (D.13), the SUs can cooperate to form coalitions, as illustrated in Fig. D.1 for a network with $N = 8$ and $K = 10$. Subsequently, the next section provides an analytical framework to form SUs' coalitions such as in Fig. D.1.

4 Joint Spectrum Sensing and Access as a Coalitional Game in Partition Form

In this section, we cast the proposed joint spectrum sensing and access cooperative model as a coalitional game in partition form and we devise an algorithm for coalition formation.

4.1 Coalitional Games in Partition Form: Concepts

For the purpose of deriving an algorithm that allows the SUs to form coalitions such as in Fig. D.1, in a distributed manner, we use notions from cooperative game theory [27]. In this regard, denoting by $\mathfrak{P}$ the set of all partitions of $\mathcal{N}$, we formulate the joint spectrum sensing and access model of the previous section as a coalitional game in *partition form* with non-transferable utility which is defined as follows [27, 28]:

Definition 19 *A coalitional game in* partition form *with* non-transferable *utility (NTU) is defined by a pair* $(\mathcal{N}, V)$ *where* $\mathcal{N}$ *is the set of players and* V *is a mapping such that for every partition* $\Pi \in \mathfrak{P}$ *, and every coalition* $S \subseteq \mathcal{N}$*,* $S \in \Pi$*,* $V(S, \Pi)$ *is a closed convex subset of* $\mathbb{R}^S$ *that contains the payoff vectors that players in* S *can achieve.*

Hence, a coalitional game is in partition form if, for any coalition $S \subseteq \mathcal{N}$, the payoff of every player in the coalition depends on the partition Π, i.e., on the players in S as well as on the players in $\mathcal{N} \setminus S$. Further, the game has NTU if the utility received by S cannot be expressed by a single value which can be arbitrarily divided among the coalition members, but is rather expressed as a set of vectors representing the payoffs that each member of S can achieve when acting within S.

For the proposed joint spectrum sensing and access problem, given a partition Π of $\mathcal{N}$ and a coalition $S \in \Pi$, and denoting by $x_i(S, \Pi)$ the payoff of SU $i \in S$ received when acting in coalition S when Π is in place, we define the coalitional value set, i.e., the mapping V as follows

$$V(S, \Pi) = \{\boldsymbol{x}(S, \Pi) \in \mathbb{R}^S | \forall i \in S, x_i(S, \Pi) = v_i(S, \Pi)\}, \tag{D.14}$$

where $v_i(S, \Pi)$ is given by (D.13). Using (D.14), we note:

Remark 2 *The proposed joint spectrum sensing and access game can be modeled as a* $(\mathcal{N}, V)$ *coalitional game in partition form with non-transferable utility where the mapping* V *is a singleton set as given by* (D.14)*, and, hence, is a closed and convex subset of* $\mathbb{R}^S$*.*

Coalitional games in partition form have, recently, attracted interest in game theory [27–31]. Partition form games are characterized by the dependence of the payoffs on externalities, i.e., on the way the network is partitioned. Unlike coalitional games in characteristic form where the focus is on studying the stability of the grand coalition of all players [27], games in partition form are a richer and more complex framework since any coalitional structure can be optimal [28, 29]. In this regard, coalitional games in partition form are often classified as *coalition formation* games [28]. Hence, traditional solution concepts for coalitional games, such as the core or the Shapley value [27], are inapplicable to coalitional games in partition form [27–29]. For instance, for coalition formation games in partition form, there is a need for devising algorithms to form the coalitional structure that can potentially emerge in the network. In particular, for the proposed joint spectrum sensing and access coalitional game, due to the tradeoffs between the benefits and costs of cooperation as captured by (D.13) and explained in Section 3, we remark the following:

Remark 3 *In the proposed joint spectrum sensing and access* $(\mathcal{N}, V)$ *coalitional game in partition form, due to the dependence on externalities and the benefit-cost tradeoffs from cooperation as expressed in* (D.13) *and* (D.14)*, any coalitional structure may form in the network and the grand coalition is seldom beneficial due to increased costs. Hence, the proposed joint sensing and access game is classified as a coalition formation game in partition form.*

Most coalition formation algorithms in game theory literature [28, 29] are built for games in characteristic form. Although some approaches for the partition form are presented in [29], but most of these are targeted at solving problems in economics with utilities quite different from the one dealt with in this paper. In order to build a coalition formation algorithm suitable for joint spectrum sensing and access, we borrow concepts from [32], where the players build coalitions based on preferences (in a characteristic form game), and extend them to accommodate the partition form.

Definition 20 *For any SU* $i \in \mathcal{N}$*, a* preference relation *or* order $\succeq_i$ *is defined as a complete, reflexive, and transitive binary relation over the set of all coalition/partition pairs that SU* i *can be a member of, i.e., the set* $\{(S_k, \Pi) | S_k \subseteq \mathcal{N},\ i \in S_k,\ S_k \in \Pi,\ \Pi \in \mathfrak{P}\}$.

Consequently, for any SU $i \in \mathcal{N}$, given two coalitions and their respective partitions $S_1 \subseteq \mathcal{N},\ S_1 \in \Pi$ and, $S_2 \subseteq \mathcal{N},\ S_2 \in \Pi'$ such that $i \in S_1$ and $i \in S_2$, $(S_1, \Pi) \succeq_i (S_2, \Pi')$ indicates that player i prefers to be part of coalition S_1 when Π is in place, over being part of coalition S_2 when Π' is in place, or at least, i prefers both coalition/partition pairs equally. Further, using the asymmetric counterpart of $\succeq_i$, denoted by $\succ_i$, then $(S_1, \Pi) \succ_i (S_2, \Pi')$, indicates that player i *strictly* prefers being a member of S_1 within Π over being a member of S_2 with Π'. We also note that the preference relation can be used to compare two coalitions in the same partition, or the same coalition in two different partitions.

For every application, an adequate preference relation $\succeq_i$ can be defined to allow the players to quantify their preferences depending on their parameters of interest. In this paper, we propose the following preference relation for any SU $i \in \mathcal{N}$

$$(S_1, \Pi) \succeq_i (S_2, \Pi') \Leftrightarrow \phi_i(S_1, \Pi) \geq \phi_i(S_2, \Pi') \tag{D.15}$$

where $S_1 \in \Pi,\ S_2 \in \Pi'$, with $\Pi, \Pi' \in \mathfrak{P}$, are any two coalitions that contain SU i, i.e., $i \in S_1$ and $i \in S_2$ and ϕ_i is a preference function defined for any SU $i \in \mathcal{N}$ as follows (S is a coalition containing i)

$$\phi_i(S, \Pi) = \begin{cases} x_i(S, \Pi), & \text{if } (x_j(S, \Pi) \geq x_j(S \setminus \{i\}, \Pi), \forall j \in S \setminus \{i\} \ \&\ S \notin h(i)) \text{ or } (|S| = 1) \\ 0, & \text{otherwise,} \end{cases} \tag{D.16}$$

where $x_i(S, \Pi)$ is given by (D.13) through (D.14) and it represents the payoff received by SU i in coalition S when partition Π is in place and $h(i)$ is the history set of SU i which is a set that contains the coalitions of size larger than 1 that SU i was member of (visited) in the past, and had parted.

The main rationale behind the preference function ϕ_i is that any SU i assigns a preference equal to its achieved payoff for any coalition/partition pair (S, Π) such that either: (i)- S is the singleton coalition, i.e., SU i is acting non-cooperatively, or (ii)- The presence of SU i in coalition S is not detrimental to any of the SUs in $S \setminus \{i\}$, and coalition S has not been previously visited by SU i, i.e., is not in the history $h(i)$. Otherwise, the SU assigns a preference value of 0 to any coalition whose members' payoffs decrease due to the presence of i, since such a coalition would refuse to have i join the coalition. Also, any SU i assigns a preference of 0 to to any coalition which it already visited in the past and *left* since an SU i has no incentive to revisit a coalition previously left.

Having defined the main ingredients of the proposed game, in the next subsection, we devise an algorithm for coalition formation.

4.2 Coalition Formation Algorithm

In order to devise a coalition formation algorithm based on the SUs' preferences, we propose the following rule:

Definition 21 ***Switch Rule*** - *Given a partition $\Pi = \{S_1, \ldots, S_M\}$ of the set of SUs $\mathcal{N}$, an SU i decides to leave its current coalition S_m, for some $m \in \{1, \ldots, M\}$ and join another coalition $S_k \in \Pi \cup \{\emptyset\}$, $S_k \neq S_m$, hence forming $\Pi' = \{\Pi \setminus \{S_m, S_k\}\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$, if and only if $(S_k \cup \{i\}, \Pi') \succ_i (S_m, \Pi)$. Hence, $\{S_m, S_k\} \to \{S_m \setminus \{i\}, S_k \cup \{i\}\}$ and $\Pi \to \Pi'$.*

For any partition Π, the switch rule provides a mechanism whereby any SU can leave its current coalition S_m and join another coalition $S_k \in \Pi$, forming a new partition Π', given that the new pair $(S_k \cup \{i\}, \Pi')$ is strictly preferred over (S_m, Π) through the preference relation defined by (D.15) and (D.16). That is, an SU would *switch* to a new coalition if it can strictly improve its payoff, *without* decreasing the payoff of any member of the new coalition. Thus, the switch rule can be seen as an individual decision made by an SU, to move from its current coalition to a new coalition while improving its payoff, given the *consent* of the members of this new coalition as per (D.15). Further, whenever an SU decides to switch from its current coalition $S_m \in \Pi$ to join a different coalition, coalition S_m is stored in its history set $h(i)$ (if $|S_m| > 1$).

Consequently, we propose a coalition formation algorithm composed of three main phases: Neighbor discovery, coalition formation, and joint spectrum sensing and access. In the first phase, the SUs explore neighboring SUs (or coalitions) with whom they may cooperate. For discovering their neighbors, neighbor discovery algorithms suited for cognitive radio such as in [33, 34] may be used. Once neighbor discovery is complete, the next phase of the algorithm is the coalition formation phase. First, the SUs start by investigating the possibility of performing a switch operation by engaging in pairwise negotiations with discovered SUs/coalitions. Once an SU identifies a potential switch operation (satisfying (D.15) and (D.16)), it can make a *distributed* decision to switch and join a new coalition. In this phase, we consider that, the order in which the SUs make their switch operations is random but sequential (dictated by who requests first to cooperate). For any SU, a switch operation is easily performed as the SU can leave its current coalition and join the new coalition whose members already agree on the joining of this SU as per (D.15) and (D.16). The convergence of the proposed coalition formation algorithm during this phase is guaranteed as follows:

Theorem 1 *Starting from any initial network partition Π_{init}, the coalition formation phase of the proposed algorithm always converges to a final network partition Π_f composed of a number of disjoint coalitions of SUs.*

Proof: *Denote by $\Pi_{n_{l,i}}^{l,i}$ as the partition formed at iteration l during the time SU $i \in \mathcal{N}$ needs to act after the occurrence of $n_{l,i}$ switch operations by one or more SUs up to the turn of SU i in iteration l. Consider that the SUs act in ascending order, i.e., SU 1 acts first, then SU 2, and so on. Given any initial starting partition $\Pi_{\mathit{init}} = \Pi_0^{1,1}$, the coalition formation phase of the* *proposed algorithm consists of a sequence of switch operations as follows (as an example)*

$$\Pi_0^{1,1} \to \Pi_1^{1,2} \to \ldots \to \Pi_{n_{1,N}}^{1,N} \ldots \to \Pi_{n_{l,N}}^{l,N} \to \ldots, \tag{D.17}$$

where the operator $\to$ *indicates a switch operation. Based on* (D.15)*, for any two partitions* $\Pi_{n_{l,i}}^{l,i}$ *and* $\Pi_{n_{m,j}}^{m,j}$ *in* (D.17)*, such that* $n_{l,i} \neq n_{m,j}$*, i.e.,* $\Pi_{n_{m,j}}^{m,j}$ *is a result of the transformation of* $\Pi_{n_{l,i}}^{l,i}$ *(or vice versa) after a number of switch operations, we have two cases: (C1)-* $\Pi_{n_{l,i}}^{l,i} \neq \Pi_{n_{m,j}}^{m,j}$*, or (C2)- An SU revisited its non-cooperative state, and thus* $\Pi_{n_{l,i}}^{l,i} = \Pi_{n_{m,j}}^{m,j}$*.*

If (C1) is true for all $i, k \in \mathcal{N}$ *for any two iterations* l *and* m*, and, since the number of partitions of a set is* finite *(given by the Bell number [29]), then the number of transformations in* (D.17) *is finite. Hence, in this case, the sequence in* (D.17) *will always terminate after* L *iterations and converge to a final partition* $\Pi_f = \Pi_{n_{L,N}}^{L,N}$ *(without oscillation). If case (C2) also occurs in* (D.17)*, future switch operations (if any) for any SU that reverted to act non-cooperatively will always result in a new partition as per* (D.15)*. Thus, even when (C2) occurs, the finite number of partitions guarantees the algorithm's convergence to some* Π_f*. Hence, the coalition formation phase of the proposed algorithm always converges to a final partition* Π_f*.* □

The stability of the partition Π_f resulting from the convergence of the proposed algorithm can be studied using the following stability concept (modified from [32] to accommodate the partition form):

Definition 22 *A partition* $\Pi = \{S_1, \ldots, S_M\}$ *is* Nash-stable *if* $\forall i \in \mathcal{N}$ *s. t.* $i \in S_m, S_m \in \Pi$, $(S_m, \Pi) \succeq_i (S_k \cup \{i\}, \Pi')$ *for all* $S_k \in \Pi \cup \{\emptyset\}$ *with* $\Pi' = (\Pi \setminus \{S_m, S_k\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\})$*.*

Hence, a partition Π is Nash-stable, if no SU has an incentive to move from its current coalition to another coalition in Π or to deviate and act alone.

Proposition 1 *Any partition* Π_f *resulting from the coalition formation phase of the proposed algorithm is Nash-stable.*

Proof: *If the partition* Π_f *resulting from the proposed algorithm is* not *Nash-stable then, there* $\exists i \in \mathcal{N}$ *with* $i \in S_m$, $S_m \in \Pi_f$*, and a coalition* $S_k \in \Pi_f$ *such that* $(S_k \cup \{i\}, \Pi') \succ_i (S_m, \Pi)$*, hence, SU* i *can perform a* switch *operation which contradicts with the fact that* Π_f *is the result of the convergence of the proposed algorithm (Theorem* 1*). Thus, any partition* Π_f *resulting from the coalition formation phase of the proposed algorithm is Nash-stable.* □

Algorithm 2 One round of the proposed coalition formation algorithm

Initial State
The network is partitioned by $\Pi_{\text{initial}} = \{S_1, \ldots, S_M\}$. At the beginning of all time, the network is non-cooperative, hence, $\Pi_{\text{init}} = \mathcal{N}$.

Phase 1 - Neighbor Discovery:
Each SU in $\mathcal{N}$ surveys its neighborhood for existing coalitions, in order to learn the partition Π in place using existing neighbor discovery algorithms such as in [33, 34].

Phase 2 - Coalition Formation:

repeat
Each SU $i \in \mathcal{N}$ investigates potential switch operations using the preference in (D.15) by engaging in pairwise negotiations with existing coalitions in partition Π (initially $\Pi = \Pi_{\text{init}}$).
Once a switch operation is found:
a) SU i leaves its current coalition.
b) SU i updates its history $h(i)$, if needed.
c) SU i joins the new coalition with the consent of its members.

until convergence to a Nash-stable partition

Phase 3 - Joint Spectrum Sensing and Access:
The formed coalitions perform joint cooperative spectrum sensing and access as per Section 3.

By periodic runs of these phases, the algorithm allows the SUs to adapt the network structure to environmental changes (see Section 4.2).

Following the convergence of the coalition formation phase to a Nash-stable partition, the third and last phase of the algorithm entails the joint spectrum sensing and access where the SUs operate using the model described in Section 3 for locating unoccupied channels and transmitting their data cooperatively. A summary of one round of the proposed algorithm is given in Algorithm 2. The proposed algorithm can adapt the coalitional structure to environmental changes such as a change in the PU traffic or slow channel variations (e.g., due to slow mobility). For this purpose, the first two phases of the algorithm shown in Algorithm 2 are repeated periodically over time, allowing the SUs, in Phase 2, to take distributed decisions to adapt the network's topology through new switch operations (which would converge independent of the starting partition as per Theorem 1). Thus, for time varying environments, every period of time η the SUs assess whether it is possible to switch from their current coalition. Note that the history set $h(i)$ for any SU $i \in \mathcal{N}$ is also reset every η.

The proposed algorithm can be implemented in a distributed way, since, as already explained, the switch operation can be performed by the SUs independently of any centralized entity. First, for neighbor discovery, the

SUs can either utilize existing algorithms such as in [33, 34], or they can rely on information from control channels such as the recently proposed cognitive pilot channel (CPC) which provides frequency, location, and other information for assisting the SUs in their operation [35, 36]. Following neighbor discovery, the SUs engage in pairwise negotiations, over control channels, with their neighbors. In this phase, given a present partition Π, for every SU, the computational complexity of finding its next coalition, i.e., locating a switch operation, is easily seen to be $O(|\Pi|)$ in the worst case, and the largest value of $|\Pi|$ occurs when all the SUs are non-cooperative, in that case $|\Pi| = N$. Clearly, as coalitions start to form, the complexity of locating a potential switch operation becomes smaller. Also, for performing a switch, each SU and coalition have to evaluate their potential utility through (D.13), to determine whether a switch operation is possible. For doing so, the SUs require to know the external interference and to find all feasible permutations to compute their average capacity. Each SU in the network is made aware of the average external interference it perceives through measurements fed back from the receiver to the SU. As a result, for forming a coalition, the SUs compute the average external interference on the coalition by combining their individual measurements. Alternatively, for performing coalition formation, the SUs can also rely on information from the CPC which can provide a suitable means for gathering information on neighbors and their transmission schemes. Moreover, although, at first glance, finding all feasible permutations may appear complex, as per Section 3, the number of feasible permutations is generally small with respect to the total number of permutations due to the condition in (D.8). Further, as cooperation entails costs, the network eventually deals with small coalitions (as will be seen in Section 5) where finding these feasible permutations will be reasonable in complexity.

5 Simulation Results and Analysis

For simulations, the following network is set up: The BS is placed at the origin of a $3\text{km} \times 3\text{km}$ square area with the SUs randomly deployed in the area around it. We set the maximum SU transmit power to $\tilde{P} = 10$ mW, the noise variance to $\sigma^2 = -90$ dBm, and the path loss exponent to $\mu = 3$. Unless stated otherwise, we set the fraction of time for sensing a single

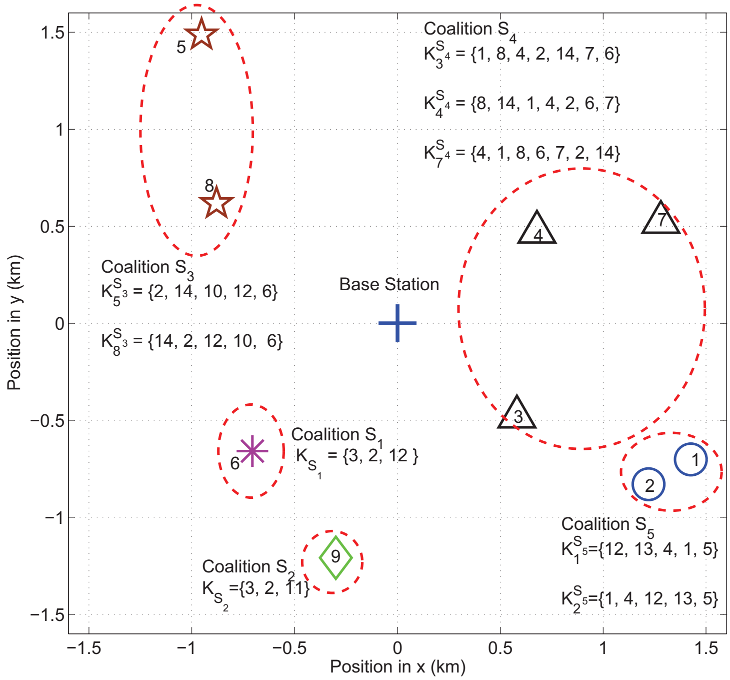

Fig. D.2: A snapshot of a network partition resulting from the proposed algorithm with $N = 9$ SUs and $K = 14$ channels.

channel to $\alpha = 0.05$ and we consider networks with $K = 14$ channels[20]. In addition, non-cooperatively, we consider that each SU can accurately learn the statistics of $K_i = 3$ channels, $\forall i \in \mathcal{N}$ (for every SU i these non-cooperative K_i channels are randomly picked among the available PUs[21]).

Fig. D.2 shows a snapshot of the network structure resulting from the proposed coalition formation algorithm for a randomly deployed network with $N = 9$ SUs and $K = 14$ channels. The probabilities that the channels

[20]As an example, this can map to the total channels in 802.11b, although the actual used number varies by region (11 for US, 13 for parts of Europe, etc.) [37].

[21]This method of selection is considered as a general case, other methods for non-cooperatively picking the PU channels can also be accommodated.

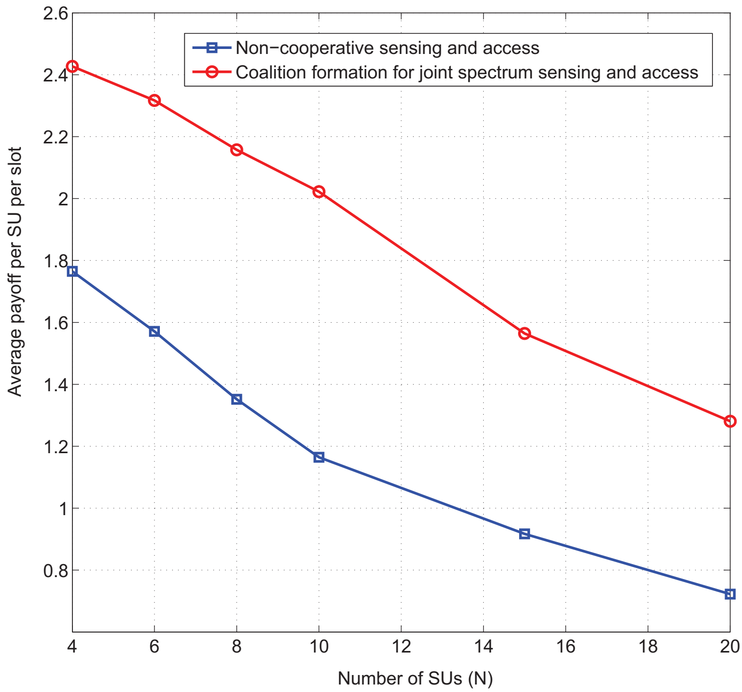

Fig. D.3: Average payoff achieved per SU per slot (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for a network with $K = 14$ channels as the network size N varies.

are unoccupied are: $\theta_1 = 0.98$, $\theta_2 = 0.22$, $\theta_3 = 0.64$, $\theta_4 = 0.81$, $\theta_5 = 0.058$, $\theta_6 = 0.048$, $\theta_7 = 0.067$, $\theta_8 = 0.94$, $\theta_9 = 0.18$, $\theta_{10} = 0.25$, $\theta_{11} = 0.17$, $\theta_{12} = 0.15$, $\theta_{13} = 0.23$, $\theta_{14} = 0.36$. In Fig. D.2, the SUs self-organize into 5 coalitions forming partition $\Pi_f = \{S_1, S_2, S_3, S_4, S_5\}$. For each coalition in Π_f, Fig. D.2 shows the *sorted* (by Algorithm 1) set of channels used by the SUs in the coalitions (note that channel 9 was not learned by any SU non-cooperatively). By inspecting the channel sets used by S_3, S_4, and S_5, we note that, by using Algorithm 1 the SUs sort their channels in a way to avoid selecting the same channel at the same rank, when possible. This is true for all ranks of these coalitions with two exceptions: The last rank for coalition S_3 where SUs 5 and 8 both rank channel 6 last since it is rarely available as $\theta_6 = 0.048$, and, similarly, the last rank for coalition S_5 where SUs 1 and 2 both select channel 5 (ranked lowest by both SUs) since it is also seldom available as $\theta_5 = 0.058$. The partition Π_f in Fig. D.2 is Nash-stable, as no SU has an incentive to change its coalition. For example, the non-cooperative utility of SU 9 is $x_9(\{9\}, \Pi_f) = 1.1$, by joining with SU 6, this utility drops to 0.38, also, the utility of SU 6 drops from $x_6(\{6\}, \Pi_f) = 1.79$ to 1.63. This result shows that cooperation can entail a cost, notably, due to the fact that that both SUs 6 and 9 know, non-cooperatively, almost the same channels (namely, 3 and 2), and hence, by cooperating they suffer a loss in sensing time which is not compensated by the access gains. Due to the cooperation tradeoffs, the utility of SU 9, drops to 0.797, 0.707, and 0.4624, if SU 9 joins coalitions S_3, S_4, or S_5, respectively. Thus, SU 9 has no incentive to switch its current coalition. This property can be verified for all SUs in Fig D.2 by inspecting the variation of their utilities if they switch their coalition, thus, partition Π_f is Nash-stable.

In Fig. D.3, we show the average payoff achieved per SU per slot for a network with $K = 14$ channels as the number of SUs, N, in the network increases. The results are averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$. Fig. D.3 shows that, as the number of SUs N increases, the performance of both cooperative and non-cooperative spectrum sensing and access decreases due to the increased interference. However, at all network sizes, the proposed coalition formation algorithm maintains a better performance compared to the non-cooperative case. In fact, the proposed joint spectrum sensing and access presents a significant performance advantage over the non-cooperative case, increasing with N as the SUs are more likely (and willing, due to increased interference) to find cooperating partners when N increases. This performance advantage reaches up to 77.25% relative to the non-cooperative case at $N = 20$ SUs.

In Fig. D.4, we show the average and average maximum coalition size (averaged over the random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) resulting from the proposed algorithm as the number of SUs, N, increases, for a network with $K = 14$ channels. Fig. D.4 shows that, as N increases, both the average and maximum coalition size increase with the average having a slower increase slope. Further, we note that the average and average maximum coalition size reach around 2.5 and 5 at $N = 20$, respectively. Hence, Fig. D.4 demonstrates that, although some large coalitions are emerging in the network,

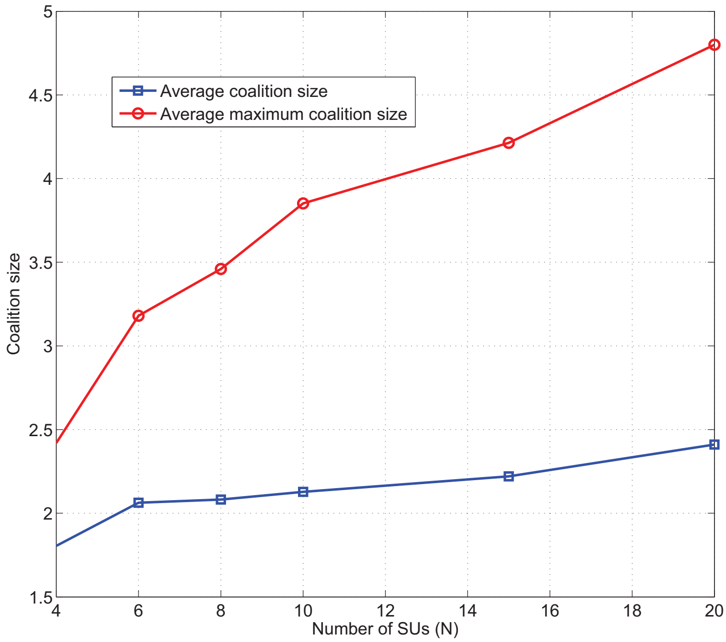

Fig. D.4: Average and average maximum coalition size (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for a network with $K = 14$ channels as the network size N varies.

in average, the size of the coalitions is relatively small. This result is due to the fact that, as mentioned in Section 3, although cooperation is beneficial, it is also accompanied by costs due to the needed re-ordering of the channels, the occurrence of new interference due to channel sharing, and so on. These costs limit the coalition size in average. Thus, Fig. D.4 shows that, when using coalition formation for joint spectrum sensing and access, the resulting network is, in general, composed of a large number of small coalitions rather than a small number of large coalitions. In brief, Fig. D.4 provides an insight on the network structure when the SUs cooperative for joint spectrum sensing and access.

In Fig. D.5, we show the average payoff achieved per SU per slot for

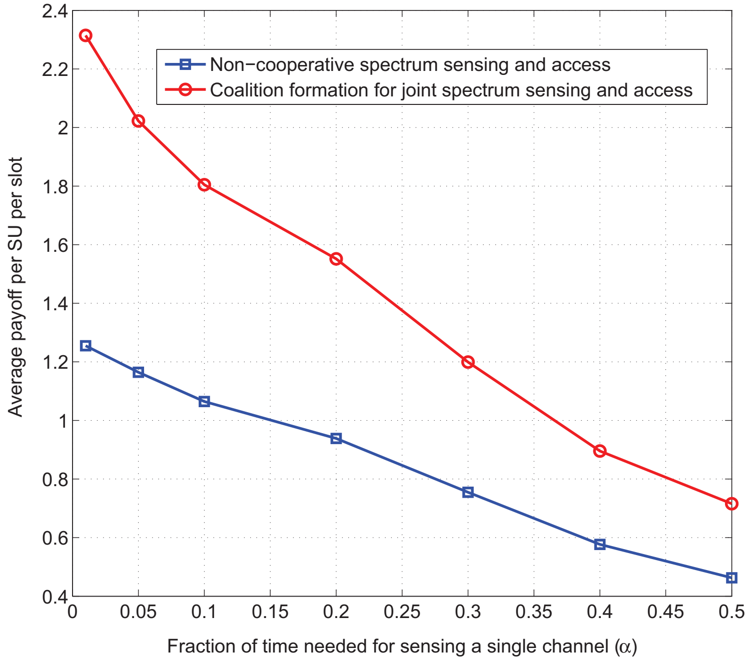

Fig. D.5: Average payoff achieved per SU per slot (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for a network with $N = 10$ SUs and $K = 14$ channels as the fraction of time needed for sensing a single channel α varies.

a network with $N = 10$ SUs and $K = 14$ channels as the fraction of time needed for sensing a single channel α increases. The results are averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$. Fig. D.5 demonstrates that, as the amount of time α dedicated for sensing a single channel increases, the time that can be allotted for spectrum access is reduced, and, thus, the average payoff per SU per slot for both cooperative and non-cooperative spectrum sensing and access decreases. In this figure, we can see that, at all α, the proposed joint spectrum sensing and access through coalition formation exhibits a performance gain over the non-cooperative case. This advantage decreases

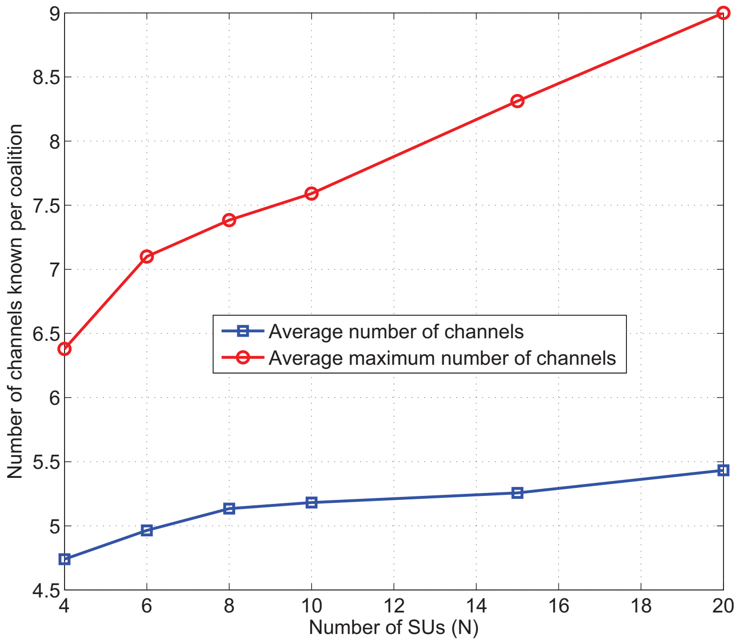

Fig. D.6: Average and average maximum number of channels (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) known per coalition for a network with $K = 14$ channels as the network size N varies.

with α, but it does not go below an improvement of 54.7% relative to the non-cooperative scheme at $\alpha = 0.5$, i.e., when half of the slot is used for sensing a single channel.

Fig. D.6 shows the average and average maximum number of channels known per coalition (averaged over the random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) as the number of SUs, N, increases, for a network with $K = 14$ channels. Fig. D.6 demonstrates that both the average and average maximum number of known channels per coalition increase with the network size N. This increase is due to the fact that, as more SUs are present in the network, the cooperation possibilities increase and the number of channels that can be shared

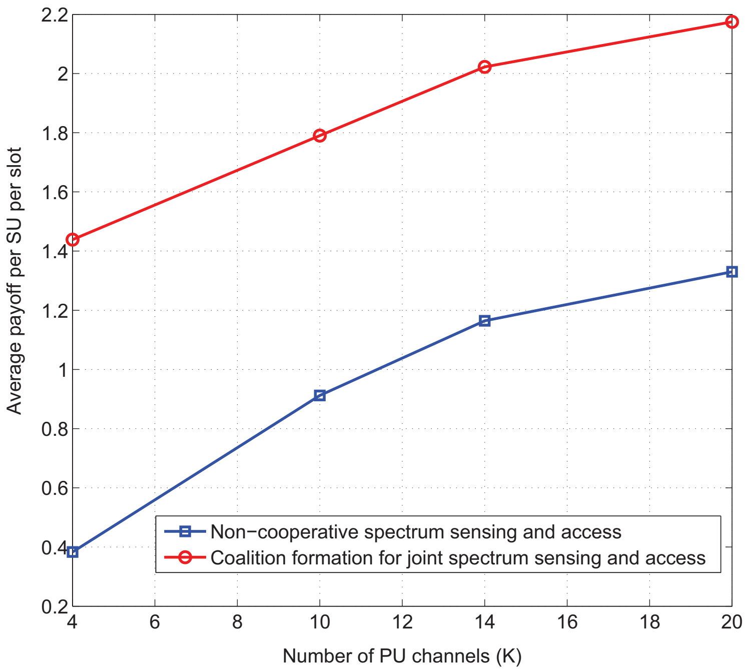

Fig. D.7: Average payoff achieved per SU per slot (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for a network with $N = 10$ SUs as the number of channels K varies.

per coalition also increases. In this regard, the average number of known channels ranges from around 4.75 for $N = 4$ to around 5.5 for $N = 20$, while the average maximum goes from 6.4 at $N = 4$ to 9 at $N = 20$. This result shows that the increase in the average number of known channels is small while that of the maximum is more significant. This implies that, due to the cooperation tradeoffs, in general, the SUs have an incentive to share a relatively moderate number of channels with the emergence of few coalitions sharing a large number of channels.

Fig. D.7 shows the average payoff achieved per SU per slot for a network with $N = 10$ SUs as the number of PU channels, K, increases. The

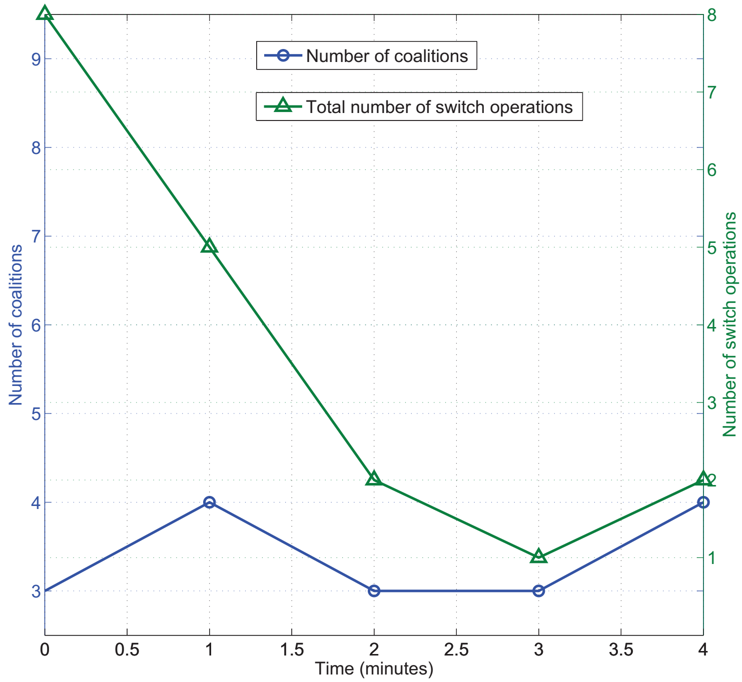

Fig. D.8: Network structure evolution with time for $N = 10$ SUs, as the traffic of the PUs, i.e., $\theta_k \forall k \in \mathcal{K}$ varies over a period of 4 minutes.

results are averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$. In this figure, we can see that as the number of channels K increases, the performance of both cooperative and non-cooperative spectrum sensing and access increases. For the non-cooperative case, this increase is mainly due to the fact that, as more channels become available, the possibility of interference due to the non-cooperative channel selection is reduced. For the proposed coalition formation algorithm, the increase in the performance is also due to the increased amount of channels that the SUs can share as K increases. Furthermore, Fig. D.7 demonstrates that the proposed joint spectrum sensing and access presents a significant performance advantage over the non-

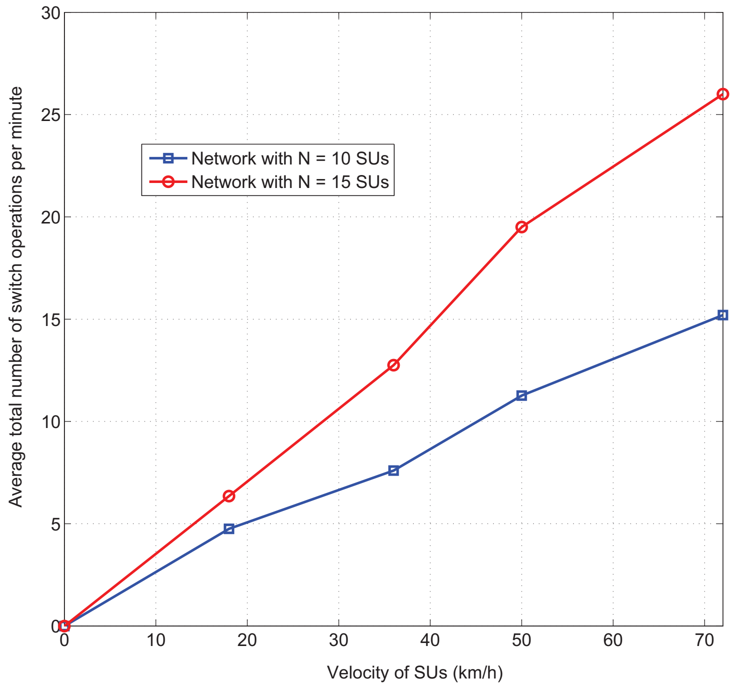

Fig. D.9: Average frequency of switch operations per minute (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for different speeds in a network with $K = 14$ channels for $N = 10$ SUs and $N = 15$ SUs.

cooperative case which is at least 63.5% for $K = 20$ and increases for networks with smaller channels. The increase in the performance advantage highlights the ability of the SUs to reduce effectively the interference among each others through the proposed coalition formation algorithm.

In Fig. D.8, we show, over a period of 4 minutes (after the initial network formation), the evolution of a network of $N = 10$ SUs and $K = 14$ channels over time when the PUs' traffic, i.e., the probabilities $\theta_k, \forall k \in \mathcal{K}$ vary, independently, every 1 minute. As the channel occupancy probability varies, the structure of the network changes, with new coalitions forming and others breaking due to switch operations occurring. The network starts with

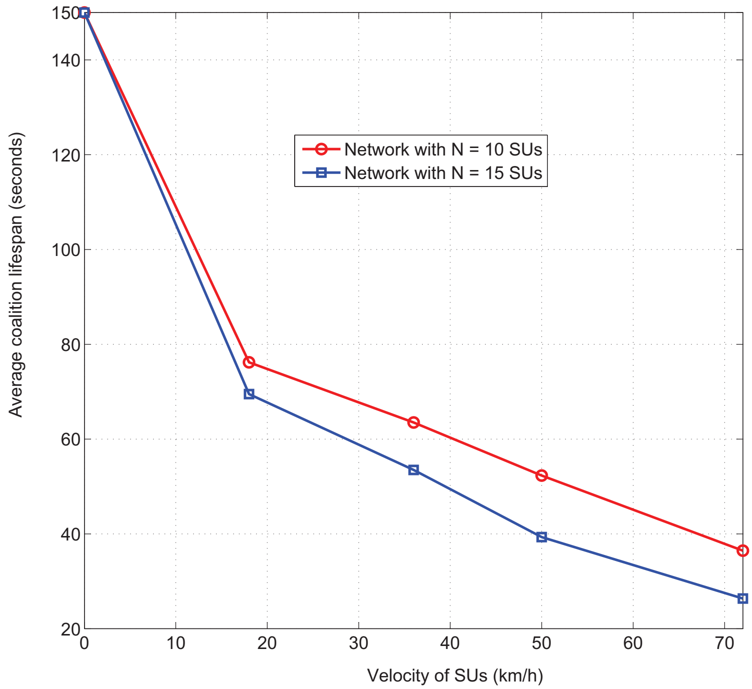

Fig. D.10: Average coalition lifespan in seconds (averaged over random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for different speeds in a network with $K = 14$ channels for $N = 10$ SUs and $N = 15$ SUs.

a non-cooperative structure made up of 10 non-cooperative SUs. First, the SUs self-organize in 3 coalitions upon the occurrence of 8 switch operations as per Fig. D.8 (at time 0). With time, the SUs can adapt the network's structure to the changes in the traffic of the PUs through adequate switch operations. For example, after 1 minute has elapsed, the number of coalitions increase from 3 to 4 as the SUs perform 5 switch operations. After a total of 18 switch operations over the 4 minutes, the final partition is made up of 4 coalitions that evolved from the initial 3 coalitions.

In Fig. D.9, we show the average total number of switch operations per minute (averaged over the random positions of the SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) for various speeds of the SUs for networks with $K = 14$ channels and for the cases of $N = 10$ SUs and $N = 15$ SUs. The SUs are moving using a random walk mobility model for a period of 2.5 minutes with the direction changing every $\eta = 30$ seconds. As the velocity increases, the average frequency of switch operations increases for all network sizes due to the dynamic changes in the network structure incurred by more mobility. These switch operations result from that fact that, periodically, every $\eta = 30$ seconds, the SUs are able to reengage in coalition formation through Algorithm 2, adapting the coalitional structure to the changes due to mobility. The average total number of switch operations per minute also increases with the number of SUs as the possibility of finding new cooperation partners becomes higher for larger N. For example, while for the case of $N = 10$ SUs the average frequency of switch operations varies from 4.8 operations per minute at a speed of 18 km/h to 15.2 operations per minute at a speed of 72 km/h, for the case of $N = 15$ SUs, the increase is much steeper and varies from 6.4 operations per minute at 18 km/h to 26 operations per minute at 72 km/h.

The network's adaptation to mobility is further assessed in Fig. D.10 where we show, over a period of 2.5 minutes, the average coalition lifespan (in seconds) achieved for various velocities of the SUs in a cognitive network with $K = 14$ channels and different number of SUs. The mobility model is similar to the one used in Fig. D.9 with $\eta = 30$ seconds. We define the coalition lifespan as the time (in seconds) during which a coalition operates in the network prior to accepting new SUs or breaking into smaller coalitions (due to switch operations). Fig. D.10 shows that, as the speed of the SUs increases, the average lifespan of a coalition decreases due to the fact that, as mobility becomes higher, the likelihood of forming new coalitions or splitting existing coalitions increases. For example, for $N = 15$ SUs, the coalition lifespan drops from around 69.5 seconds for a velocity of 18 km/h to around 53.5 seconds at 36 km/h, and down to about 26.4 seconds at 72 km/h. Furthermore, Fig. D.10 shows that as more SUs are present in the network, the coalition lifespan decreases. For instance, for any given velocity, the lifespan of a coalition for a network with $N = 10$ SUs is larger than that of a coalition in a network with $N = 15$ SUs. The main reason behind the decrease in coalition lifespan with N is that, for a given speed, as N increases, the SUs are more apt able to finding new partners to join with as they move. In a nutshell, Fig. D.10 provides an interesting assessment of the topology adaptation aspect of the proposed coalition formation algorithm through switch operations.

Finally, we note that, in order to highlight solely the changes due to mobility, the fading amplitude was considered constant in Fig. D.9 and Fig. D.10. Similar results can be seen when the fading amplitude also changes.

6 Conclusions

In this paper, we introduced a novel model for cooperation in cognitive radio networks, which accounts for both the spectrum sensing and spectrum access aspects. We modeled the problem as a coalitional game in partition form and we derived an algorithm which allows the SUs to take individual decisions for joining or leaving a coalition, depending on their achieved utility which accounts for the average time to find a unoccupied channel (spectrum sensing) and the average achieved capacity (spectrum access). We showed that, by using the proposed coalition formation algorithm, the SUs can self-organize into a Nash-stable network partition, and adapt this topology to environmental changes such as a change in the traffic of the PUs or slow mobility. Simulation results showed that the proposed algorithm yields gains, in terms of average payoff per SU per slot, reaching up to 77.25% relative to the non-cooperative case for a network with 20 SUs.

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Paper E

Hedonic Coalition Formation for Distributed Task Allocation among Wireless Agents

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Abstract

Autonomous wireless agents such as unmanned aerial vehicles, mobile base stations, cognitive devices, or self-operating wireless nodes present a great potential for deployment in next generation wireless networks. While current literature has been mainly focused on the use of agents within robotics or software engineering applications, this paper proposes a novel usage model for self-organizing agents suited to wireless communication networks. In the proposed model, a number of agents are required to collect data from several arbitrarily located tasks. Each task represents a queue of packets that require collection and subsequent wireless transmission by the agents to a central receiver. The problem is modeled as a *hedonic coalition formation* game between the agents and the tasks that interact in order to form disjoint coalitions. Each formed coalition is modeled as a polling system consisting of a number of agents, designated as collectors, which move between the different tasks present in the coalition, collect and transmit the packets. Within each coalition, some agents might also take the role of a relay for improving the packet success rate of the transmission. The proposed hedonic coalition formation algorithm allows the tasks and the agents to take distributed decisions to join or leave a coalition, based on the achieved benefit in terms of effective throughput, and the cost in terms of polling system delay. As a result of these decisions, the agents and tasks structure themselves into independent disjoint coalitions which constitute a Nash-stable network partition. Moreover, the proposed coalition formation algorithm allows the agents and tasks to adapt the topology to environmental changes such as the arrival of new tasks, the removal of existing tasks, or the mobility of the tasks. Simulation results show how the proposed algorithm allows the agents and tasks to self-organize into independent coalitions, while improving the performance, in terms of average player (agent or task) payoff, of at least 30.26% (for a network of 5 agents with up to 25 tasks) relatively to a scheme that allocates nearby tasks equally among agents.

1 Introduction

Next generation wireless networks will present a highly complex and dynamic environment characterized by a large number of heterogeneous information sources, and a variety of distributed network nodes. This is mainly due to the recent emergence of large-scale, distributed, and heterogeneous communication systems which are continuously increasing in size, traffic, applications, services, etc. For maintaining a satisfactory operation of such networks, there is a constant need for dynamically optimizing their performance, monitoring their operation and reconfiguring their topology. Due to the ubiquitous nature of such wireless networks, it is inherent to have self-organizing autonomous nodes (agents), that can service these networks at different levels such as data collection, monitoring, optimization, management, maintenance, among others [1–8]. These nodes belong to the authority maintaining the network, and must be able to survey large scale networks, and perform very specific tasks at different points in time, in a distributed and autonomous manner, with very little reliance on any centralized authority [1–3, 6–8].

While the use of such autonomous agents has been thoroughly investigated in robotics, computer systems or software engineering, research models that tackle the use of such agents in wireless communication networks are few. However, recently, the need for such agents in wireless networks has become of noticeable importance as many next generation networks encompass several wireless node types, such as cognitive devices or unmanned aerial vehicles, that are autonomous and self-adapting [1–8]. A key problem in this context is the problem of task allocation among a group of agents that need to execute a number of tasks. This problem has been already tackled in areas such as robotics control [9–11], or software systems [12, 13]. However, most of these existing models are unsuitable for task allocation problems in the context of wireless networks due to various reasons: (i)- The task allocation problems studied in the existing papers are mainly tailored for military operations, computer systems, or software engineering and, thus, cannot be readily applied in models pertaining to wireless networks, (ii)- the tasks are generally considered as static abstract entities with very simple characteristics and no intelligence (e.g. the tasks are just points in a plane) which is a major limitation, and (iii)- the existing models do not consider any aspects of wireless communication networks such as the characteristics of the wireless channel, the presence of data traffic, the need for wireless data transmission, or other wireless-specific specifications. In this context, numerous applications in next generation wireless networks require a number of agent-nodes to perform specific wireless-related tasks that emerge over time and are not pre-assigned. One example of these applications is the case where a number of wireless nodes are required to monitor the operation of the network or perform relaying at different times and locations [1, 2, 4–8]. In such applications, the objective is to provide algorithms that allow the agents to autonomously share the tasks among each other. The main existing contributions within wireless networking in this area [14–18], are focused on deploying unmanned aerial vehicles (UAVs) which can act as self-deploying autonomous agents that can efficiently perform *pre-assigned* tasks in numerous applications such as connectivity improvement in ad hoc network [15], routing [16, 17], and medium access control [18]. However, these contributions focus on *centralized* solutions for specific problems such as finding the optimal locations for the deployment of UAVs or devising efficient routing algorithms in ad hoc networks in the presence of UAVs (one or more). Moreover, in these papers, the tasks that the agents must accomplish are *pre-assigned and pre-determined*. In contrast, as previously mentioned, many applications in wireless networks require the agents to autonomously assign the tasks among themselves. Hence, it is inherent to devise algorithms, in the context of wireless networks, that allow an autonomous and distributed task allocation process among a number of *wireless agents*[22] with little reliance on centralized entities.

The main contribution of this paper is to propose a novel wireless communication-oriented model for the problem of task allocation among a number of autonomous agents. The proposed model considers a number of wireless agents that are required to collect data from arbitrarily located tasks. Each task represents a source of data, i.e., a queue with a Poisson arrival of packets, that the agents must collect and transmit via a wireless link to a central receiver. This formulation is deemed suitable to model several problems in next generation networks such as video surveillance in wireless networks, self-deployment of mobile relays in IEEE 802.16j networks [2], data collection in ad hoc and sensor networks [8], operation

[22]The term *wireless agent* refers to any node that can act autonomously and can perform wireless transmission. Examples of wireless agents are unmanned aerial vehicles [15], mobile base stations [6–8], cognitive wireless devices [3], or self-deploying mobile relay stations [2].

of mobile base stations in vehicular ad hoc networks [6] and mobile ad hoc networks [7] (the so called *message ferry* operation), wireless monitoring of randomly located sites, autonomous deployment of unmanned air vehicles in military ad hoc networks, and many other applications. For allocating the tasks, we introduce a novel framework from coalitional game theory, known as *hedonic coalition formation*. Albeit hedonic games have been widely used in game theory, to the best of our knowledge, no existing work utilized this framework in a communication or wireless environment. Thus, we model the task allocation problem as a hedonic coalition formation game between the agents and the tasks, and we introduce an algorithm for forming coalitions. Each formed coalition is modeled as a polling system consisting of a number of agents, designated as *collectors*, which act as a single server that moves continuously between the different tasks (queues) present in the coalition, gathering and transmitting the collected packets to a common receiver. Further, within each coalition, some agents can act as *relays* for improving the packet success rate during the wireless transmission. For forming coalitions, the agents and tasks can autonomously make a decision to join or leave a coalition based on well defined individual preference relations. These preferences are based on a coalitional value function that takes into account the benefits received from servicing a task, in terms of effective throughput (data collected), as well as the cost in terms of the polling system delay incurred from the time needed for servicing all the tasks in a coalition. We study the properties of the proposed algorithm, and show that it always converges to a Nash-stable network partition. Further, we investigate how the network topology can self-adapt to environmental changes such as the deployment of new tasks, the removal of existing tasks, and low mobility of the tasks. Simulation results show how the proposed algorithm allows the network to self-organize, while ensuring a significant performance improvement, in terms of average player (task or agent) payoff, compared to a scheme that assigns nearby tasks equally among the agents.

The remainder of this paper is organized as follows: Section 2 presents and motivates the proposed system model. In Section 3, we model the task allocation problem problem as a transferable utility coalitional game and propose a suited utility function. In Section 4, we classify the task allocation coalitional game as a hedonic coalition formation game, we discuss its key properties and we introduce the algorithm for coalition formation. Simulation results are presented, discussed, and analyzed in Section 5. Finally, conclusions are drawn in Section 6.

2 System Model

Consider a network consisting of M wireless agents that belong to a single network operator and that are controlled by a central command center (e.g., a central controller node or a satellite system). These agents are required to service T tasks that are arbitrarily located in a geographic area having an associated central wireless receiver connected to the command center. The tasks are entities that, in general, belong to one or more independent owners[23]. These tasks' owners can be, for example, service providers or third party operators. We denote the set of agents, and tasks by $\mathcal{M} = \{1, \ldots, M\}$, and $\mathcal{T} = \{1, \ldots, T\}$, respectively. In this paper, we consider only the case where the number of tasks is larger than the number of agents, hence, $T > M$. The main motivation behind this consideration is that, for most networks, the number of agents assigned to a specific area is generally small due to cost factors for example. Each task $i \in \mathcal{T}$ represents an M/D/1 queueing system[24], whereby packets of constant size B are generated using a Poisson arrival with an average arrival rate of λ_i. Hence, in the proposed model, we consider different classes of tasks each having its corresponding λ_i. The tasks we consider are sources of data that cannot send their information to the central receiver (and, subsequently, to the command center) without the help of an agent. These tasks can represent a group of mobile devices, such as sensors, video surveillance devices, or any other static or dynamic wireless nodes that have limited power and are unable to provide long-distance transmission. Hence, these devices (tasks) need to buffer their data locally and await to be serviced by an agent that can collect the data. For example, an agent such as a mobile base station, a mobile relay or a UAV can provide a line-of-sight link that can facilitate the transmission from the tasks to the central receiver. The tasks can also be mapped to any other source of packet data that require collection by an agent for transmission[25]. For servicing a task, each agent is required to move to the task location, collect the data, and transmit it using a wireless link to the central receiver. The command center periodically downloads this data from the receiver, e.g., through a backbone network. Each agent $i \in \mathcal{M}$ offers a link transmission capacity of μ_i, in packets/second, with which the agent can service the data from any task. Hence, the quantity $\frac{1}{\mu_i}$

[23]The scenario where all tasks and agents are owned by the same entity is a particular case of this generic model.

[24]Other types of queues, e.g., M/M/1, can also be considered without loss of generality in the coalition formation process proposed in this paper.

[25]The tasks can be moving with a periodic low mobility, as will be seen in later sections.

would represent the well known service time for a single packet that is being serviced by agent i. The agent which is collecting the data from a task is referred to as *collector*. In addition, each agent $i \in \mathcal{M}$ can transmit the data to the receiver with a maximum transmit power of $P_i = \tilde{P}$, assumed the same for all agents [26].

The proposed model allows each task to be serviced by multiple agents, and also, each agent (or group of agents) to service multiple tasks. Whenever a task is serviced by multiple agents, each agent can act as either a *collector* or a *relay*. Any group of agents that act together for data collection from the same task, can be seen as a single collector with improved link transmission capacity. In this paper, we consider that the link transmission capacity depends solely on the capabilities of the agents and not on the nature of the tasks. In this context, given a group of agents $G \subseteq \mathcal{M}$ that are acting as collectors for any task, the total link transmission capacity with which tasks can be serviced with by G can be given by

$$\mu^G = \sum_{j \in G} \mu_j. \tag{E.1}$$

For forming a single collector, multiple agents can easily coordinate the data extraction, and then transmission from every task, so as to allow a larger link transmission capacity for the serviced task as per (E.1). Moreover, the transmission of the packets by the agents from a task $i \in \mathcal{T}$ to the central receiver is subject to packet loss due to the fading on the wireless channel. In this regard, in addition to acting as collectors, some agents may act as *relays* for a task. These relay-agents would locate themselves at equal distances from the task (given that the task is already being served by *at least one* collector), and, hence, the collectors transmit the data to the receiver through multi-hop agents, improving the probability of successful transmission. In this context, in Rayleigh fading, the probability of successful transmission of a packet of size B bits from the collectors present at a task $i \in \mathcal{T}$ through a path of m agents, $Q_i = \{i_1, \ldots, i_m\}$, where $i_1 = i$ is the task being serviced, i_m is the central receiver (CR), and any other $i_h \in Q_i$ is a *relay*-agent, can be given by

$$\Pr_{i,\mathrm{CR}} = \prod_{h=1}^{m-1} \Pr^B_{i_h, i_{h+1}}, \tag{E.2}$$

[26]Note that, different maximum transmit power values can be easily accommodated in the coalition formation algorithm proposed in the next sections.

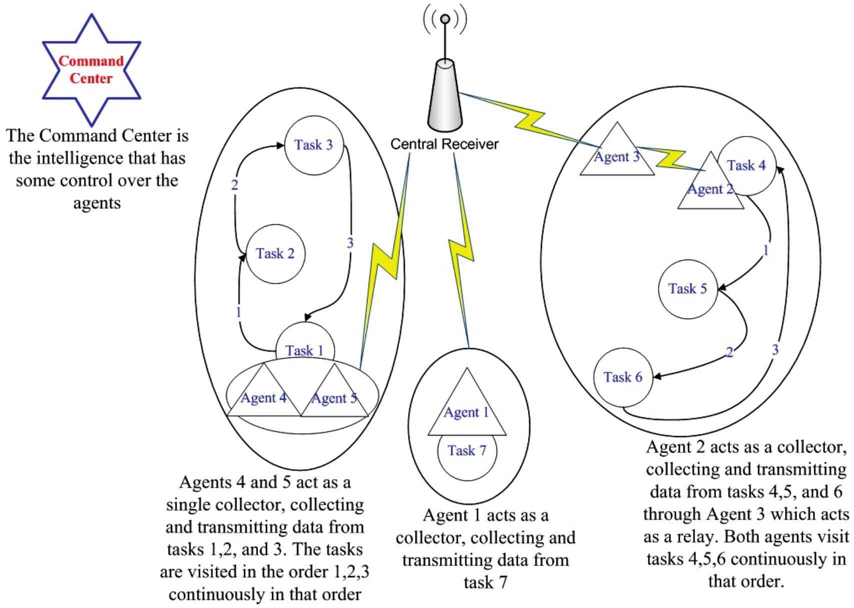

Fig. E.1: An illustrative example of the proposed model for task allocation in wireless networks (the agents are dynamic, i.e., they move from one task to the other continuously).

where $\mathrm{Pr}_{i_h,i_{h+1}}$ is the the probability of successful transmission of a single bit from agent i_h to agent (or the central receiver) i_{h+1}. This probability can be given by the probability of maintaining the SNR at the receiver above a target level ν_0 as follows [19]

$$\mathrm{Pr}_{i,i_{h+1}} = \exp\left(-\frac{\sigma^2 \nu_0 (D_{i_h,i_{h+1}})^\alpha}{\kappa \tilde{P}}\right), \tag{E.3}$$

where σ^2 is the variance of the Gaussian noise, κ is a path loss constant, α is the path loss exponent, $D_{i_h,i_{h+1}}$ is the distance between nodes i_h and i_{h+1}, and $\tilde{P}$ is the maximum transmit power of agent i_h.

For servicing a number of tasks $C \subseteq \mathcal{T}$, a group of agents $G \subseteq \mathcal{M}$ (collectors and relays) can sequentially move from one task to the other in C with a constant velocity η. The group G of agents, servicing the tasks in

C, stop at each task, with the collectors collecting and transmitting the packets using the relays (if any). The collectors would move from one task to the other, only if all the packets in the queue at the current task have been transmitted to the receiver (the process through which the agents move from one task to the other for data collection is cyclic). Simultaneously with the collectors, the relays also move, positioning themselves at equal distances on the line connecting the task being currently served by the collectors, and the central receiver. With this proposed model, the final network will consist of groups of tasks serviced by groups of agents, continuously. An illustration of this model is shown in Figure E.1.

Consequently, given this proposed model, the main objective is to provide an algorithm for distributing the tasks between the agents, given the operation of the agents previously described and shown in Figure E.1. In other words, the main goal is to allow the agents and task to autonomously form the coalitional structure in Figure E.1 and adapt it to environmental changes. For this purpose, the following sections formulate a game theoretic approach for achieving this objective.

3 Coalitional Game Formulation

In this section, we model the proposed problem as a coalitional game with transferable utility, and we propose a suitable utility function.

3.1 Game Formulation

By inspecting Figure E.1, one can clearly see that the task allocation problem among the agents can be mapped into the problem of the formation of coalitions. In this regard, coalitional game theory [20, Ch. 9] provides a suitable analytical tool for studying the formation of cooperative groups, i.e., coalitions, among a number of players. For the proposed model, the coalitional game is played between the agents and the tasks. Hence, the players set for the proposed task allocation coalitional game is denoted by $\mathcal{N}$, and contains both agents and tasks, i.e., $\mathcal{N} = \mathcal{M} \cup \mathcal{T}$. In the remainder of this paper, we use the term *player* to indicate either a task or an agent.

For any coalition $S \subseteq \mathcal{N}$ containing a number of agents and tasks, the agents belonging to this coalition can structure themselves into collectors and relays. Subsequently, as explained in the previous section, within each coalition, the collector-agents will continuously move from one task to the other, stopping at each task, and transmitting all the packets available in the queue to the central receiver, through the relay-agents (if any). This proposed task servicing scheme can be mapped to a well-known concept that is ubiquitous in computer systems, which is the concept of a *polling system* [21]. In a polling system, a single server moves between multiple queues in order to extract the packets from each queue, in a sequential and cyclic manner. Models pertaining to polling systems have been widely developed in various disciplines ranging from computer systems to communication networks, and different strategies for servicing the queues exist [21–24]. In the proposed task allocation model, the collectors of every coalition in our model are considered as a single server that is servicing the tasks (queues) sequentially, in a cyclic manner, i.e., after servicing the last task in a coalition $S \subseteq \mathcal{N}$, the collectors of S return to the first task in S that they previously visited hence repeating their route continuously. Further, whenever the collectors stop at any task $i \in S$, they collect and transmit the data present at this task until the queue is empty. This method of allowing the server to service a queue until emptying the queue is known as the *exhaustive* strategy for a polling system, which is applied at the level of every coalition $S \subseteq \mathcal{N}$ in our model. Moreover, the time for the server to move from one queue to the other is known as the *switchover* time. Consequently, we highlight the following property:

Property 4 *In the proposed task allocation model, every coalition $S \subseteq \mathcal{N}$ is a polling system with an* exhaustive *polling strategy and deterministic non-zero* switchover *times. In each such polling system S, the collector-agents are seen as the polling system server, and the tasks are the queues that the collector-agents must service.*

Furthermore, for any coalition S, once the queue at a task $i \in S$ is emptied, the collectors and relays in a coalition move from task i to the next task $j \in S$ with a constant velocity η, hence, incurring a switchover time $\theta_{i,j}$. The switchover time in our model corresponds to the time it takes for all the agents (collectors and relays) to move from one task to the next, which, assuming all agents start their mobility at the same time, maps to the time needed for the farthest agent to move from one task to the next. Since we consider only straight line trajectories for collectors and relays, and due to the fact that the relays always position themselves at equal distances on the line connecting the tasks in a coalition to the receiver, we have the following property (clearly seen through the geometry of Figure E.1).

Property 5 *Within any given coalition S, the switchover time between two tasks corresponds to the* constant *time it takes for one of the* collectors *to move from one of the tasks to the next.*

Having modeled every coalition $S \subseteq \mathcal{N}$ as a polling system, we investigate the average delay incurred per coalition. In fact, for polling systems, finding exact expressions for the delay at every queue is a highly complicated task, and hence, no general closed-form expressions for the delay at every queue in a polling system can be found [21, 22][27]. In this regard, a key criterion used for the analysis of the delay incurred by a polling system is the *pseudo-conservation* law which provides closed-form expressions for weighted sum of the means of the waiting times at the queues [21, 22]. For providing the pseudo-conservation law for a coalition $S \subseteq \mathcal{N}$ composed of a number of agents and a number of tasks, we make the following definitions. First, within coalition S, a group of agents $G_S \subseteq S \cap \mathcal{M}$ are designated as collectors. Second, for each task $i \in S \cap \mathcal{T}$ with an average arrival rate of λ_i, and served by a number of collectors $|G_S|$ with a link transmission capacity of μ^{G_S} (as given by (E.1)), we define the utilization factor of task i $\rho_i = \frac{\lambda_i}{\mu^{G_S}}$. Further, we define $\rho_S \triangleq \sum_{i \in S \cap \mathcal{T}} \rho_i$. Given these definitions, for a coalition S, the weighted sum of the means of the waiting times by the agents at all the tasks in the coalition are given by the pseudo-conservation law as follows [22, Section. VI-B] (taking into account that our switchover and service times are deterministic)

$$\sum_{i \in S \cap \mathcal{T}} \rho_i \bar{W}_i = \rho_S \frac{\sum_{i \in S \cap \mathcal{T}} \frac{\rho_i}{\mu^{G_S}}}{2(1-\rho_S)} + \rho_S \frac{\theta_S^2}{2} + \frac{\theta_S}{2(1-\rho_S)} \left[\rho_S^2 - \sum_{i \in S \cap \mathcal{T}} \rho_i^2 \right], \tag{E.4}$$

where $\bar{W}_i$ is the mean waiting time at task i and $\theta_S = \sum_{h=1}^{|S \cap \mathcal{T}|} \theta_{i_h, i_{h+1}}$ is the sum of the switchover times given a path of tasks $\{i_1, \ldots, i_{|S \cap \mathcal{T}|}\}$ followed by the agents, with $i_h \in S \cap \mathcal{T}$, $\forall\ h \in \{1, \ldots, |S \cap \mathcal{T}|\}$ and $i_{|S \cap \mathcal{T}|+1} = i_1$. The first term in the right hand side of (E.4) is the well known expression for the average queueing delay for M/D/1 queues, weighed by ρ_S. The second and third terms in the right hand side of (E.4) represent the average delay increase incurred by the travel time required for the collectors to move from one task to the other, i.e., the delay resulting from the switchover period. Further, for any coalition S that must form in the system, the following condition must hold:

$$\rho_S < 1. \tag{E.5}$$

[27] Note that some approximations [22] exist for polling systems under heavy traffic or large switchover times, but in our problem, they are not suitable as we require a more general delay expression.

This condition is a requirement for the stability of any polling system [21–24] and, thus, must be satisfied for any coalition that will form in the proposed model. In the event where this condition is violated, the system is considered unstable and the delay is considered as infinite. In this regard, the analysis presented in the remainder of this paper will take into account this condition and its impact on the coalition formation process (as demonstrated in the next sections, a coalition where $\rho_S \geq 1$ will never form).

3.2 Utility Function

In the proposed game, for every coalition $S \subseteq \mathcal{N}$, the agents must determine the order in which the tasks in S are visited, i.e., the path $\{i_1, \dots, i_{|S \cap \mathcal{T}|}\}$ which is an ordering over the set of tasks in S given by $S \cap \mathcal{T}$. Naturally, the agents must select the path that minimizes the total switchover time for one round of data collection. This can be mapped to the following well-known problem:

Property 6 *The problem of finding the path that minimizes the total switchover time for one round of data collection within a coalition $S \subseteq \mathcal{N}$ is mapped into the* traveling salesman *problem [25], where a salesman, i.e., the agents $S \cap \mathcal{M}$, is required to minimize the time of visiting a series of cities, i.e., the tasks $S \cap \mathcal{T}$.*

It is widely known that the solution for the traveling salesman problem is NP-complete [25], and, hence, there has been numerous heuristic algorithms for finding an acceptable near-optimal solution. One of the simplest of such algorithms is the nearest neighbor algorithm (also known as the greedy algorithm) [25]. In this algorithm, starting from a given city the salesman chooses the closet city as his next visit. Using the nearest neighbor algorithm, the ordering of the cities which minimizes the overall route is selected. The nearest neighbor algorithm is sub-optimal, however, it can quickly find a near-optimal solution (in most cases) and its computational complexity is small (linear in the number of cities) [25], hence making it suitable for complicated problems such as the task allocation problem we are considering. Therefore, in the proposed model, for every coalition S, the agents can easily work out the nearest neighbor route for the tasks, and operate according to it.

Having modeled each coalition as a polling system, the pseudo-conservation law in (E.4) allows to evaluate the cost, in terms of average waiting time (or delay), from forming a particular coalition. However, for every coalition, there is a benefit, in terms of the average effective throughput that the coalition is able to achieve. The average effective throughput for a coalition S is given by

$$L_S = \sum_{i \in S \cap \mathcal{T}} \lambda_i \cdot \Pr_{i,\mathrm{CR}}, \tag{E.6}$$

with $\Pr_{i,\mathrm{CR}}$ given by (E.2). By closely inspecting (E.1), one can see that adding more collectors improves the transmission link capacity, and, thus, reduces the service time that a certain task perceives. Based on this property and by using (E.4) one can easily see that, adding more collectors, i.e., improving the service time, reduces the overall delay in (E.4) [21–24]. Further, adding more relays would reduce the distance over which transmission is occurring, thus, improving the probability of successful transmission as per (E.2) [6, 19]. In consequence, using (E.6), one can see that this improvement in the probability of successful transmission is translated into an improvement in the effective throughput. Hence, each agent role (collector or relay) possesses its own benefit for a coalition.

A suitable criterion for characterizing the utility in networks that exhibit a tradeoff between the throughput and the delay is the concept of system *power* which is defined as the ratio of some power of the throughput and the delay (or a power of the delay) [26]. Hence, the concept of power is an attractive notion that allows to capture the fundamental tradeoff between throughput and delay in the proposed task allocation model. Power has been used thoroughly in the literature in applications that are sensitive to throughput as well as delay [27–29]. Mainly, for the proposed game, the utility of every coalition S is evaluated using a coalitional value function based on the power concept from [29] as follows

$$v(S) = \begin{cases} \delta \frac{L_S^{\beta}}{\left(\sum_{i \in S \cap \mathcal{T}} \rho_i \bar{W}_i\right)^{(1-\beta)}}, & \text{if } \rho_S < 1 \text{ and } |S| > 1, \\ 0, & \text{otherwise,} \end{cases} \tag{E.7}$$

where $\beta \in (0, 1)$ is a throughput-delay tradeoff parameter. In (E.7), the term δ represents the price per unit power that the network offers to coalition S. Hence, δ represents a generic control parameter that allows the network operator to somehow monitor the behavior of the players. The use of such control parameters is prevalent in game theory [30–34]. In certain scenarios, δ would represent physical monetary values paid by the operator to the different entities (agents and tasks). In such a case, on one hand, for the tasks, the operator simply would pay the tasks' owners for the amount of data (and its corresponding quality as per (E.7)) each one of their tasks had generated. On the other hand, for the agents, the payment would, for example, represent either a reward for the behavior of the agents or the proportion of maintenance or servicing that each agent would receiver from its operator. In this sense, the utility function in (E.7) would, thus, represents the total revenue achieved by a coalition S, given the network power that coalition S obtains. For coalitions that consist of a single agent or a single task, i.e., coalitions of size 1, the utility assigned is 0 due to the fact that such coalitions generate no benefit for their member (a single agent can collect no data unless it moves to task, while a single task cannot transmit any of its generated data without an agent collecting this data). Further, any coalition where condition (E.5) is not satisfied is also given a zero utility, since, in this case, the polling system that the coalition represents is unstable, and hence has an infinite delay.

Consequently, given the set of players $\mathcal{N}$, and the value function given in (E.7), we define a coalitional game $(\mathcal{N}, v)$ with transferable utility (TU). The utility in (E.7) represents the amount of money or revenue received by a coalition, and, hence, this amount can be arbitrarily apportioned between the coalition members, which justifies the TU nature of the game. For dividing this utility between the players, we adopt the equal fair allocation rule, whereby the payoff of any player $i \in S$, denoted by x_i^S is given by

$$x_i^S = \frac{v(S)}{|S|}. \tag{E.8}$$

The payoff x_i^S represents the amount of revenue that player $i \in S$ receives from the total revenue $v(S)$ that coalition S generates. The main motivation behind adopting the equal fair allocation rule is in order to highlight the fact that the agents and the tasks value each others equally. As seen in (E.7), the presence of an agent in a coalition is crucial in order for the tasks to obtain any payoff, and, vice versa, the presence of a task in a coalition is required for the agent to be able to obtain any kind of utility. Nonetheless, the proposed model and algorithm can accommodate any other type of payoff allocation rule. For instance, although in traditional coalitional games, the allocation rule may have a strong impact on the game's solution, for the proposed game, other allocation rules can be used with little impact on the analysis that is presented in the rest of the paper. This is mainly due to the nature and class of the proposed game which is quite different from traditional coalitional games. In fact, as clearly seen from (E.4) and (E.7), whenever the number of tasks in a coalition increases, the total delay increases, hence, reducing the utility from forming a coalition. Further, in a coalition where the number of tasks is large, the condition of stability for the polling system, as given by (E.5), can be easily violated due to heavy traffic incoming from a large number of tasks, thus, yielding a zero utility as per (E.7). Hence, forming coalitions between the tasks and the agents entails a cost that can limit the size of a coalition. In this regard, traditional solution concepts for TU games, such as the core [20], may not be applicable. In fact, in order for the core to exist, as a solution concept, a TU coalitional game must ensure that the grand coalition, i.e., the coalition of all players will form. However, as seen in Figure E.1 and corroborated by the utility in (E.7), in general, due to the cost for coalition formation, the grand coalition will not form. Instead, independent and disjoint coalitions appear in the network as a result of the task allocation process. In this regard, the proposed game is classified as a *coalition formation game* [30–34], and the objective is to find an algorithm that allows to form the coalition structure, instead of finding only a solution concept, such as the core, which aims mainly at stabilizing a grand coalition of all players.

4 Task Allocation as a Hedonic Coalition Formation Game

In this section, we map the proposed task allocation problem to a hedonic coalition formation game with an underlying transferable utility, and we propose a distributed algorithm for forming the coalitions using concepts from hedonic games.

4.1 Hedonic Coalition Formation: Concepts and Model

As already mentioned, the proposed task allocation model entails the formation of disjoint coalitions, and, hence, the proposed game is classified as a coalition formation game. In fact, coalition formation has been a topic of high interest in game theory [30–34]. Notably, in [32–34], a class of coalition formation games known as *hedonic coalition formation games* is investigated. This class of games entails several interesting properties that can be applied, not only in economics such as in [32–34], but also in wireless networks as we will demonstrate in this paper. The two key requirements for classifying a coalitional game as a *hedonic* game are as follows [32]:

Condition 1 - *(Hedonic Conditions) - A coalition formation game is classified as hedonic if*

1. *The payoff of any player depends* solely *on the members of the coalition to which the player belongs.*
2. *The coalitions form as a result of the* preferences *of the players over their possible coalitions' set.*

These two conditions characterize the framework of hedonic games. Mainly, the term *hedonic* pertains to the first condition above, whereby the payoff of any player i, in a hedonic game, must depend only on the identity of the players in the coalition to which player i belongs, with no dependence on the other players. For the second condition, in the remainder of this section, we will formally define how the preferences of the players over the coalitions can be used for the formation process.

Prior to investigating the application of hedonic games in the proposed model, we introduce some definitions, taken from [32].

Definition 23 *A* coalition structure *or a* coalition partition *is defined as the set $\Pi = \{S_1, \ldots, S_l\}$ which partitions the players set $\mathcal{N}$, i.e., $\forall\ k\ , S_k \subseteq \mathcal{N}$ are disjoint coalitions such that $\cup_{k=1}^{l} S_k = \mathcal{N}$ (an example of a partition Π is shown in Figure* E.1*).*

Definition 24 *Given a partition Π of $\mathcal{N}$, for every player $i \in \mathcal{N}$ we denote by $S_{\Pi}(i)$, the coalition to which player i belongs, i.e., coalition $S_{\Pi}(i) = S_k \in \Pi$, such that $i \in S_k$.*

In a hedonic game setting, each player must build preferences over its own set of possible coalitions. In other words, each player must be able to compare the coalitions, and order them based on which coalition the player prefers being a member of. For evaluating these preferences of the players over the coalitions, we define the concept of a preference relation or order as follows [32]:

Definition 25 *For any player $i \in \mathcal{N}$, a* preference relation *or* order $\succeq_i$ *is defined as a complete, reflexive, and transitive binary relation over the set of all coalitions that player i can possibly form, i.e., the set $\{S_k \subseteq \mathcal{N} : i \in S_k\}$.*

Consequently, for a player $i \in \mathcal{N}$, given two coalitions $S_1 \subseteq \mathcal{N}$ and, $S_2 \subseteq \mathcal{N}$ such that $i \in S_1$ and $i \in S_2$, $S_1 \succeq_i S_2$ indicates that player i prefers to be part of coalition S_1, over being part of coalition S_2, or at least, i

prefers both coalitions equally. Further, using the asymmetric counterpart of $\succeq_i$, denoted by $\succ_i$, then $S_1 \succ_i S_2$, indicates that player i *strictly* prefers being a member of S_1 over being a member of S_2. For every application, an adequate preference relation $\succeq_i$ can be defined to allow the players to quantify their preferences. The preference relation can be a function of many parameters, such as the payoffs that the players receive from each coalition, the weight each player gives to other players, and so on. Given the set of players $\mathcal{N}$, and a preference relation $\succeq_i$ for every player $i \in \mathcal{N}$, a hedonic coalition formation game is formally defined as follows [32]:

Definition 26 *A hedonic coalition formation game is a coalitional game that satisfies the two hedonic conditions previously prescribed, and is defined by the pair* $(\mathcal{N}, \succ)$ *where* $\mathcal{N}$ *is the set of players* ($|\mathcal{N}| = N$)*, and* $\succ$ *is a* profile of preferences*, i.e., preference relations,* $(\succeq_1, \ldots, \succeq_N)$ *defined for every player in* $\mathcal{N}$*.*

Having laid out and defined the main components of hedonic coalition formation games, we utilize this framework in order to provide a suitable solution for the task allocation problem proposed in Section 2. For instance, the proposed task allocation problem is easily modeled as a $(\mathcal{N}, \succ)$ hedonic game, where $\mathcal{N}$ is the set of agents and tasks previously defined, and $\succ$ is a profile of preferences that we will shortly define. First and foremost, for the proposed game model, given a network partition Π of $\mathcal{N}$, the payoff of any player i, depends only on the identity of the members of the coalition to which i belongs. In other words, the payoff of any player i depends solely on the players in the coalition in which player i belongs $S_{\Pi}(i)$ (easily seen through the formulation of Section 3). Hence, our game verifies the first hedonic condition.

Furthermore, for modeling the task allocation problem as a hedonic coalition formation game, the preference relations of the players must be clearly defined. In this regard, we define two types of preference relations, a first type suited for indicating the preferences of the agents, and a second type suited for the tasks. Subsequently, for evaluating the preferences of any agent $i \in \mathcal{M}$, we define the following operation (this preference relation is common for all agents, hence we denote it by $\succeq_i = \succeq_{\mathcal{M}},\ \forall i \in \mathcal{M}$)

$$S_2 \succeq_{\mathcal{M}} S_1 \Leftrightarrow u(S_2) \geq u(S_1), \tag{E.9}$$

where $S_1 \subseteq \mathcal{N}$ and $S_2 \subseteq \mathcal{N}$ are any two coalitions that contain agent i, i.e., $i \in S_1$ and $i \in S_2$ and $u : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is a preference function defined as follows

$$u(S) = \begin{cases} \infty, & \text{if } S = S_{\Pi}(i) \ \& \ S \setminus \{i\} \subseteq \mathcal{T}, \\ 0, & \text{if } S \in h(i), \\ x_i^S. & \text{otherwise,} \end{cases} \tag{E.10}$$

where Π is the *current* coalition partition which is in place in the game, x_i^S is the payoff received by player i from any division of the value function among the players in coalition S such as the equal fair division given in (E.8), and $h(i)$ is the history set of player i. At any point in time, the history set $h(i)$ is a set that contains the coalitions that player i was a part of in the past, prior to the formation of the current partition Π. Note that, by using the defined preference relation, the players can compare any two coalitions S_1 and S_2 independently of whether these two coalitions belong to Π or not.

The main rationale behind the preference function u in (E.10) is as follows. In this model, the agents, being entities owned by the operator, seek out to achieve two conflicting objectives: (i)- Service all tasks in the network for the benefit of the operator, and (ii)- Maximize the quality of service, in terms of power as per (E.7), for extracting the data from the tasks. The preference function u must be able to allow the agents to make coalition formation decisions that can capture this tradeoff between servicing all tasks (at the benefit of the operator) and achieving a good quality of service (at the benefit of both agents and operator). For this purpose, as per (E.10), any agent i that is the *sole* agent servicing tasks in its current coalition $S = S_{\Pi}(i)$ such that $S_{\Pi}(i) \cap \mathcal{M} = \{i\}$, assigns an infinite preference value to S. Hence, in order to service all tasks, the agent always assigns a maximum preference to its current coalition, if this current coalition is composed of only tasks and does not contain other agents. This case of the preference function u forbids the agent from leaving a group of tasks, already assigned to it, unattended by other agents. In this context, this condition pertains to the fist objective (objective (i) previously mentioned) of the agents and it implies that, whenever there is a risk of leaving tasks without service, the agent do not act selfishly, in contrast, they act in the benefit of the operator and remain with these tasks, independent of the payoff generated by these tasks. Such a decision allows an agent to avoid making a decision that can incur a risk of ultimately having tasks with no service in the network, in which case, the network operator would lose revenue from these unattended tasks and it may, for example, decide to replace the agent that led to the presence of such a group of tasks with no service. Otherwise, the agents' preference relation u would highlight the second objective of the agent, i.e., maximize its own payoff which maps into the revenue generated from the quality of service, i.e., the power as per (E.7). with which the tasks are being serviced. In this case, the preference relation is easily generated by the agents by comparing the value of the payoffs they receive from the two coalitions S_1 and S_2. Further, we note that no agent has any incentive to revisit a coalition that it had left previously, and hence, the agents assign a preference value of 0 for any coalition in their history (this can be seen as a basic learning process). In summary, taking into account the conflicting goals of the agents, between two coalitions S_1 and S_2, an agent i prefers the coalition that gives the better payoff, given that the agent is not alone in its current coalition, and the coalition with a better payoff is not in the history of agent i.

For the preferences of the tasks, an analogous approach can be taken. Formally, for evaluating the preferences of any task $j \in \mathcal{T}$, we define the following operation (this preference relation is common for all tasks, hence, we denote it by $\succeq_j = \succeq_{\mathcal{T}}, \forall j \in \mathcal{T}$)

$$S_2 \succeq_{\mathcal{T}} S_1 \Leftrightarrow w(S_2) \geq w(S_1), \tag{E.11}$$

with the tasks' preference function w defined as follows.

$$w(S) = \begin{cases} 0, & \text{if } S \in h(j), \\ x_j^S, & \text{otherwise.} \end{cases} \tag{E.12}$$

The preferences of the tasks are easily captured using the function w. The preference function of the tasks is different from that of the agents since the tasks are, in general, independent entities that act solely in their own interest. Thus, based on (E.12), each task prefers the coalition that provides the larger payoff x_j^S unless this coalition was already visited previously and left. In that case, the preference function of the tasks assigns a preference value of 0 for any coalition that the task had already visited in the past (and left to join another coalition). Using this preference relation, every task can evaluate its preferences over the possible coalitions that the task can form.

Consequently, the proposed task allocation model verifies both hedonic conditions, and, hence, we have the following:

Property 7 *The proposed task allocation problem among the agents is modeled as a $(\mathcal{N}, \succ)$ hedonic coalition formation game, with the preference rela-*

*tions given by (*E.9*) and (*E.11*).*

Note that the preference relations in (E.9) and (E.11) are also dependent on the underlying TU coalitional game described in Section 3. Having formulated the problem as a hedonic game, the final task is to provide a distributed algorithm, based on the defined preferences, for forming the coalitions.

Prior to deriving the algorithm for coalition formation, we highlight the following proposition of the proposed hedonic coalition formation model as a result of the polling system stability indicated in (E.5)

Proposition 1 *For the proposed hedonic coalition formation model for task allocation, assuming that all collector-agents have an equal link transmission capacity $\mu_i = \mu,\ \forall i \in \mathcal{M}$, any coalition $S \subseteq \mathcal{N}$ with $|S \cap \mathcal{M}|$ agents, must have at least $|G_S|_{min}$ collector-agents ($G_S \subseteq S \cap \mathcal{M}$) as follows*

$$|G_S| > |G_S|_{min} = \frac{\sum_{i \in S \cap \mathcal{T}} \lambda_i}{\mu}. \tag{E.13}$$

Further, when all the tasks in S belong to the same class, we have

$$|G_S|_{min} = \frac{|S \cap \mathcal{T}| \cdot \lambda}{\mu}, \tag{E.14}$$

which constitutes an upper bound on the number of collector-agents as a function of the number of tasks $|S \cap \mathcal{T}|$ for a given coalition S.

Proof: *As per the defined preference relations in (*E.10*) and (*E.12*), any coalition that will form in the proposed model must be stable since no agent or task has an incentive to join an unstable coalition, hence, we have, for every coalition $S \subseteq \mathcal{N}$ having $|G_S|$ collectors with $G_S \subseteq S \cap \mathcal{M}$, we have from (*E.5*) $\rho_S < 1$, and thus*

$$\sum_{i \in S \cap \mathcal{T}} \frac{\lambda_i}{\mu^{G_S}} < 1,$$

which, given the assumption that $\mu_i = \mu, \forall i \in \mathcal{M}$ yields

$$\frac{1}{|G_S| \cdot \mu} \cdot \sum_{i \in S \cap \mathcal{T}} \lambda_i < 1,$$

which yields

$$|G_S| > |G_S|_{min} = \frac{\sum_{i \in S \cap \mathcal{T}} \lambda_i}{\mu}. \tag{E.15}$$

*Further, if we assume that all the tasks belong to the same class, hence, $\lambda_i = \lambda,\ \forall i \in S \cap \mathcal{T}$, we immediately get, from (*E.15*)*

$$|G_S|_{min} = \frac{|S \cap \mathcal{T}| \cdot \lambda}{\mu}. \tag{E.16}$$

□

Consequently, for any proposed coalition formation algorithm, the bounds on the number of collector-agents in any coalition S as given by Proposition 1 will always be satisfied.

4.2 Hedonic Coalition Formation: Algorithm

In the previous subsection, we modeled the task allocation problem as a hedonic coalition formation game. Having laid out the main building blocks, the remaining objective is to devise an algorithm for forming the coalitions. While literature that studies the characteristics of existing partitions in hedonic games, such as in [32–34], is abundant, the problem of forming the coalitions both in the hedonic and non-hedonic setting is a challenging problem [30]. In this paper, we introduce an algorithm for coalition formation that allows the players to make *selfish* decisions as to which coalitions they decide to join at any point in time. The proposed algorithm will exploit the concepts of the hedonic game model formulated in the previous subsection.

In this regard, for forming coalitions between the tasks and the agents, we propose the following rule for coalition formation:

Definition 27 ***Switch Rule*** *- Given a partition $\Pi = \{S_1, \ldots, S_l\}$ of the set of players (agents and tasks) $\mathcal{N}$, a Player i decides to leave its current coalition $S_\Pi(i) = S_m$, for some $m \in \{1, \ldots, l\}$ and join another coalition $S_k \in \Pi \cup \{\emptyset\}$, $S_k \neq S_\Pi(i)$, if and only if $S_k \cup \{i\} \succ_i S_\Pi(i)$. Hence, $\{S_m, S_k\} \to \{S_m \setminus \{i\}, S_k \cup \{i\}\}$.*

Through a single switch rule made by any player i, any current partition Π of $\mathcal{N}$ is transformed into $\Pi' = (\Pi \setminus \{S_m, S_k\}) \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$. In simple terms, for every partition Π, the switch rule provides a mechanism through which any task or agent, can leave its current coalition $S_\Pi(i)$, and join another coalition $S_k \in \Pi$, given that the new coalition $S_k \cup \{i\}$ is strictly preferred over $S_\Pi(i)$ through any preference relation that i is using (in particular using the preference relations defined in (E.9) and (E.11)). Independent of the preference relations selected, the switch rule can be seen as a *selfish* decision made by a player, to move from its current coalition to a new coalition, regardless of the effect of this move on the other players. Furthermore, we consider that, whenever a player decides to switch from one coalition to another, the player updates its history set $h(i)$. Hence, given a partition Π, whenever a player i decides to leave coalition $S_m \in \Pi$ to join a different coalition, coalition S_m is automatically stored by player i in its history set $h(i)$.

Consequently, we propose a coalition formation algorithm composed of three main phases: Task discovery, hedonic coalition formation, and data collection. In the first phase, the central command receives information about the existence of tasks that require servicing and informs the agents of the locations and characteristics of the tasks (e.g., the arrival rates). Hence, the agents start by having full knowledge of the initial partition Π_{initial}. Once the agents are aware of the tasks, they broadcast their own presence to the tasks. Consequently, the players can interact with each other, for performing coalition formation. Hence, the second phase of the algorithm is the hedonic coalition formation phase. In this phase, all the players (tasks and agents) investigate the possibility of performing a switch operation. For identifying potential switch operations, given complete knowledge about the network (which can be gathered in different methods as will be discussed in Subsection 4.3)), each agent investigates its top preference, and decides to perform a switch operation, if possible through (E.9). As one can easily see through (E.7), in the proposed model, no coalition composed of tasks-only would ever form since such a coalition would always generate a 0 utility. As a direct result of this property, the tasks are only interested in switching to coalitions that contain at least a single agent. Hence, from a tasks' perspective, for determining its preferred switch operation, each task needs only to negotiate with existing agents in order to enquire on the amount of utility it can obtain by joining with this agent. By doing so, each task can determine the switch operation it is interested in making at a given time. We consider that, the players make sequential switch decisions[28]. For any agent, a switch operation is easily performed as the agent can leave its current coalition and join the new coalition, if (E.9) is satisfied. For the tasks, any task that finds out a possibility to switch, can autonomously request, over a control channel with the concerned agent, to be added to the coalition of interest (which would always contain at least one agent with whom the task previously negotiated). The convergence of the proposed hedonic coalition formation

[28]This order of switch operations is referred to as the *order of play* hereafter.

algorithm during this phase is guaranteed as follows:

Theorem 1 *Starting from any initial network partition $\Pi_{initial}$, the proposed hedonic coalition formation phase of the proposed algorithm always converges to a final network partition Π_f composed of a number of disjoint coalitions.*

Proof: *For the purpose of this proof, we denote $\Pi^k_{n_k}$ as the partition formed during the time k when player $i \in \mathcal{N}$ decides to act after n_k switch operations have previously occurred (the index n_k denotes the number of switch operations performed by one or more players up to time k). Given any initial starting partition $\Pi_{initial} = \Pi^1_0$, the hedonic coalition formation phase of the proposed algorithm consists of a sequence of switch operations. As per Definition 27, every switch operation transforms the current partition Π into another partition Π', hence, the hedonic coalition formation is composed of a sequence of switch operations, yielding the following transformations (as an example)*

$$\Pi^1_0 = \Pi^2_0 \rightarrow \Pi^3_1 \rightarrow \ldots \rightarrow \Pi^L_{n_L} \ldots \rightarrow \ldots \rightarrow \Pi^T_{n_T}, \tag{E.17}$$

where the operator $\rightarrow$ indicate the occurrence of a switch operation. In other words, $\Pi^k_{n_k} \rightarrow \Pi^{k+1}_{n_{k+1}}$, implies that during turn k, a certain player i made a single switch operation which yielded a new partition $\Pi^{k+1}_{n_{k+1}}$ at the turn $k+1$. By inspecting the preference relations defined in (E.9) and (E.11), it can be seen that every single switch operation leads to a partition that has not yet been visited (new partition). Hence, for any two partitions $\Pi^k_{n_k}$ and $\Pi^l_{n_l}$ in the transformations of (E.17), such that $n_k \neq n_l$, i.e., $\Pi^l_{n_l}$ is a result of the transformation of $\Pi^k_{n_k}$ (or vice versa) after a number of switch operations $|n_l - n_k|$, we have that $\Pi^k_{n_k} \neq \Pi^l_{n_l}$ for any two turns k and l.

Given this property and the well known fact that the number of partitions of a set is finite and given by the Bell number [30], the number of transformations in (E.17) is finite, and, hence, the sequence in (E.17) will always terminate and converge to a final partition $\Pi_f = \Pi^T_{n_T}$. Hence, the hedonic coalition formation phase of the proposed algorithm always converges to a final network partition Π_f composed of a number of disjoint coalitions consisting of agents and tasks, which completes the proof. □

The stability of the final partition Π_f resulting from the convergence of the proposed algorithm can be studied using the following stability concept from hedonic games [32]:

Definition 28 *A partition $\Pi = \{S_1, \ldots, S_l\}$ is* Nash-stable *if* $\forall i \in \mathcal{N},\ S_\Pi(i) \succeq_i$

$S_k \cup \{i\}$ *for all* $S_k \in \Pi \cup \{\emptyset\}$ *(for agents* $\succeq_i = \succeq_{\mathcal{M}}, \forall i \in \mathcal{N} \cap \mathcal{M}$ *and for tasks* $\succeq_i = \succeq_{\mathcal{T}}, \forall i \in \mathcal{N} \cap \mathcal{T}$*).*

In other words, a coalition partition Π is Nash-stable, if no player has an incentive to move from its current coalition to another coalition in Π or to deviate and act alone. Furthermore, a Nash-stable partition Π implies that there does not exist any coalition $S_k \in \mathcal{N}$ such that a player i strictly prefers to be part of S_k over being part of its current coalitions, while all players of S_k do not get hurt by forming $S_k \cup \{i\}$. This is the concept of individual stability, which is formally defined as follows [32]:

Definition 29 *A partition* $\Pi = \{S_1, \ldots, S_l\}$ *is* individually stable *if there do not exist* $i \in \mathcal{N}$*, and a coalition* $S_k \in \Pi \cup \{\emptyset\}$ *such that* $S_k \cup \{i\} \succ_i S_{\Pi}(i)$ *and* $S_k \cup \{i\} \succeq_j S_k$ *for all* $j \in S_k$ *(for agents* $\succeq_i = \succeq_{\mathcal{M}}, \forall i \in \mathcal{N} \cap \mathcal{M}$ *and for tasks* $\succeq_i = \succeq_{\mathcal{T}}, \forall i \in \mathcal{N} \cap \mathcal{T}$ *for tasks).*

As already noted, a Nash-stable partition is individually stable [32]. For the proposed hedonic coalition formation phase of the proposed algorithm, we have the following:

Proposition 2 *Any partition* Π_f *resulting from the hedonic coalition formation phase of the proposed algorithm is Nash-stable, and, hence, individually stable.*

Proof: *First and foremost, for any partition* Π*, no player (agent or task)* $i \in \mathcal{N}$ *has an incentive to leave its current coalition, and act alone as per the utility function in (*E.7*). Assume that the partition* Π_f *resulting from the proposed algorithm is not Nash-stable. Consequently, there exists a player* $i \in \mathcal{N}$*, and a coalition* $S_k \in \Pi_f$ *such that* $S_k \cup \{i\} \succ_i S_{\Pi_f}(i)$*, hence, player* i *can perform a* switch *operation which contradicts with the fact that* Π_f *is the result of the convergence of the proposed algorithm (Theorem* 1*). Consequently, any partition* Π_f *resulting from the hedonic coalition formation phase of the proposed algorithm is Nash-stable, and, hence, by [32], this resulting partition is also individually stable.* □

Following the formation of the coalitions and the convergence of the hedonic coalition formation phase to a Nash-stable partition, the last phase of the algorithm entails the actual data collection by the agents. In this phase, the agents move from one task to the other, in their respective coalitions, collecting the data and transmitting it to the central receiver, similar to a polling system, as explained in Sections 2 and 3. A summary of the proposed algorithm is shown in Table E.I.

Table E.I: The proposed hedonic coalition formation algorithm for task allocation in wireless networks.

Initial State

The network is partitioned by $\Pi_{\text{initial}} = \{S_1, \ldots, S_k\}$. At the beginning of all time Π_{initial} = $\mathcal{N}$ = $\mathcal{M} \cup \mathcal{T}$ with no tasks being serviced by any agent.

Three Phases for the Proposed Hedonic Coalition Formation Algorithm

Phase I - Task Discovery:

a) The command center is informed by one or multiple owners about the existence and characteristics of new tasks.

b) The central command center conveys the information on the initial network partition Π_{initial} by entering the information in appropriate databases such as those used for example, in cognitive radio networks for primary user information distribution [35], or in unmanned aerial vehicles operation [36, 37].

Phase II - Hedonic Coalition Formation:

In this phase, hedonic coalition formation occurs as follows:

repeat

For every player $i \in \mathcal{N}$, given a current partition Π_{current} (in the first round $\Pi_{\text{current}} = \Pi_{\text{initial}}$).

a) Player i investigates possible switch operations using the preferences given, respectively, by (E.9) and (E.11) for the agents and tasks.

b) Player i performs the switch operation that maximizes its payoff as follows:

b.1) Player i updates its history $h(i)$ by adding coalition $S_{\Pi_{\text{current}}}(i)$, before leaving it.

b.2) Player i leaves its current coalition $S_{\Pi_{\text{current}}}(i)$.

b.3) Player i joins the new coalition that maximizes its payoff.

until convergence to a final Nash-stable partition Π_f.

Phase III - Data Collection

a) The network is partitioned using Π_{final}.

b) The agents in each coalition $S_k \in \Pi_{\text{final}}$ *continuously* perform the following operations, i.e., act as a polling system with exhaustive strategy and switchover times:

b.1) Visit a first task in their respective coalitions.

b.2) The collector-agents collect the data from the task that is being visited.

b.3) The collector-agents transmit the data using wireless links to the central receiver either directly, or through other relay-agents.

b.4) Once the queue of the current is empty, visit the next task.

The order in which the tasks are visited is determined by the nearest neighbor solution to the traveling salesman problem (Section 3, Property 6). This third phase is continuously repeated and performed by all the agents in Π_f for a fixed period of time Ψ (for static environments $\Psi = \infty$).

Adaptation to environmental changes (periodic process)

a) In the presence of environmental changes, such as the deployment of new tasks, the removal of existing tasks, or periodic low mobility of the tasks, the third phase of the algorithm is performed continuously only for a *fixed* period of time Ψ.

b) After Ψ elapses, the first two phases are repeated to allow the players to self-organize and adapt the topology to these environmental changes.

c) This process is repeated periodically for networks where environmental changes may occur.

The proposed algorithm, as highlighted in Table E.I, can adapt the network topology to environmental changes such as the deployment of new tasks, the removal of a number of existing tasks, or a periodic low mobility of the tasks (in the case where the tasks represent mobile sensor devices for example). For this purpose, the first two phases of the algorithm shown in Table E.I are repeated periodically over time, to adapt to any changes that occurred in the environment. With regards to mobility, we only consider the cases where the tasks are mobile for a fixed period of time with a velocity that is smaller than that of the agents η. In the presence of such a mobile environment, the central command center, through Phase I of the algorithm in Table E.I informs the agents of the new tasks locations (periodically) and, thus, during Phase II of the proposed algorithm, both agents and tasks can react to the environment changes, and modify the existing topology. As per Theorem 1 and Proposition 2, regardless of the starting position, the players will always self-organize into a Nash-stable partition, even after mobility, the deployment of new tasks or the removal of existing tasks. In summary, in a changing environment, the first two phases of the algorithm in Table E.I are repeated periodically, after a certain fixed period of time Ψ has elapsed during which the players were involved in Phase III and the actual data collection and transmission occurred. Finally, whenever a changing environment is considered, the players are also allowed to periodically clear their history, so as to allow them to explore all the new possibilities that the changes in the environment may have yielded.

4.3 Distributed Implementation Possibilities

For implementation, first and foremost, in the proposed model, as shown in Figure E.1, we clearly distinguish between two inherently different entities: The command center, which is the intelligence that has some control over the agents and the central receiver which is a node in the network that is connected to the command center and which would receive the data transmitted by the agents (this distinction can be, for example, analogous to the distinction between a radio network controller and a base station in cellular networks). In practice, the central command can be, for example, a node that owns a number of agents and controls a large area which is divided into smaller areas with each area represented by the illustration of Figure E.1. Hence, each such small area is a region having its own central receiver and where a subset of agents needs to operate and perform coalition formation using our model. In other scenarios, the command center can also be a satellite system that controls groups of agents with each group deployed in a different area (notably when the agents are unmanned aerial vehicles for example). In contrast, the central receiver is simply a wireless node that receives the data from the agents and, subsequently, the command center can obtain this data from all receivers in its controlled area (e.g., through a backbone network)[29].

For performing coalition formation, the agents and tasks are required to know different information. From the agents perspective, in order to perform a switch operation, each agent is required to obtain data on the location of the tasks (hence, consequently deducing the hop distance D_{ij} between any two tasks i and j) as well as on the arrival rates $\lambda_i, i \in \mathcal{T}$ of these tasks. As a first step, whenever a tasks' owner (e.g., a service provider or a third party) requires that its tasks be serviced, it will give the details and characteristics of these tasks to the network operator (through service-level agreements for example) which would enter these details into the command center. Subsequently, the command center can insert this information into appropriate databases that the agents can access through, for example, an Internet connection. Such a transfer of information through active databases has been recently utilized in many communication architectures, for example, in cognitive radio network for primary user information distribution [35], or in unmanned aerial vehicles operation [36, 37]. In cases where the command center controls only a single set of agents and a single area, this information can be, instead, broadcast directly to the agents. Further, the agents are also required to know the capabilities of each others, notably, the link transmission capacity $\mu_i, \forall i \in \mathcal{M}$ and the velocity (which can be used to deduce the switchover times). As the agents are all owned by a single operator, this information can be easily fed to the agents at the beginning of all time prior to their deployment, and, thus, does not require any additional communication during coalition formation.

From the tasks perspective, the amount of information that needs to be known is much less than that of the agents, notably since the tasks are, in general, resource-limited entities. For instance, as mentioned in the previous section, for performing coalition formation, the tasks do not need to know about the existence or the characteristics of each others. The main information that needs to be known by the tasks is the actual presence of agents. The agents can initially announce/broadcast their presence to the tasks as soon as they enter into the network. Subsequently, the tasks need only to be able to enquire, over a control channel, about the potential utility they would receive from joining the coalition of a particular agent (which can contain other tasks or agents but this is transparent from the

[29]This model can easily accommodate the particular case where the command center and the central receiver coincide, e.g., in a small single-area network.

perspective of the tasks). The main reason for this is that the tasks have no benefit in forming coalitions that have no agents since such coalitions generate 0 utility for the tasks. Hence, from the point of view of the tasks, they would see every agent as a black box which can provide a certain payoff (communicated over a control channel during negotiation phase), and, based on this, they decide to join the coalition one or another agent (if multiple agents are in the same coalition then they would offer the same benefit from the tasks perspective).

Consequently, given the information that needs to be known by each player, the proposed algorithm can be implemented in a distributed way since the switch operation can be performed by the tasks or the agents independently of any centralized entity. In this regards, given a partition Π, in order to determine its preferred switch operation, an agent would assess the payoff it would obtain by joining with any coalition in Π, except for singleton coalitions composed of agents only. For the tasks, given Π, each task negotiates with only the agents (and the coalition to which they belong) in the network in order to evaluate its payoff and decide on a switch operation. By adopting a distributed implementation, one would reduce the overhead and computational load on the command center, notably when this command center is controlling numerous areas with different groups of agents (each such area is represented by the model of Figure E.1). Further, the distributed approach allows to decentralize the intelligence, and, thus, reduces the detrimental effects on the network and the tasks' owners that can be caused by failures or malicious behavior at the command center level. It is also important to note that the distributed approach complies better with the nature of both the agents and the tasks. On one hand, the agents are inherently autonomous nodes (partially controlled by the command center) that need to operate on their own and, thus, make distributed decisions [1–3, 6–8]. On the other hand, the tasks are independent entities that belong to different owners. Consequently, the tasks are apt to make their own decisions regarding coalition formation and are, generally, unwilling to accept a coalitional structure imposed by an external entity such as the command center. Nonetheless, we note that a centralized approach can also be adopted for the proposed algorithm notably in small networks where, for example, the command center coincides with the central receiver and owns all the tasks.

Regarding complexity, the main complexity lies in the switch operation, the solution to the traveling salesman problem, i.e., determining the order in which the tasks are visited within a coalition in order to evaluate the utility function, and the assignment of agents as either collectors or relays. For instance, given a present coalitional structure Π where each coalition in Π has *at least one task*, for every agent, the computational complexity of finding its next coalition, i.e., performing a switch operation, is easily seen to be $O(|\Pi|)$, and the worst case scenario is when all the tasks act alone, in that case $|\Pi| = T$. In contrast, for the tasks, the worst case complexity is $O(M)$ since, in order to make a switch operation, the tasks need only to negotiate with agents. With regards to the traveling salesman solution, the complexity of the used nearest neighbor solution is well known to be linear in the number of cities, i.e., tasks [25], hence, for a coalition $S_k \in \Pi$, the complexity of finding the traveling salesman solution is simply $O(|S_k|)$. It must be noted that, in static environments, finding the solution of the traveling salesman problem is done only once, which reduces the complexity. Additionally, for determining whether a agent acts as a collector or relay within any coalition, we consider that the players would compute this configuration by inspecting all combinations and selecting the one that maximizes the utility in (E.7). This computation is done during coalition formation for evaluating the potential utility, and, upon convergence, is maintained during network operation. As the number of agents in a single coalition is generally small, this computation is straightforward, and has reasonable complexity. Finally, in dynamic environments, as the algorithm is repeated periodically and since we consider only periodic low mobility, the complexity of the coalition formation algorithm is comparable to the one in the static environment, but with more runs of the algorithm.

5 Simulation Results and Analysis

For simulations, the following network is set up: A central receiver is placed at the origin of a 4 km $\times 4$ km square area with the tasks appearing in the area around it. The path loss parameters are set to $\alpha = 3$ and $\kappa = 1$, the target SNR is set to $\nu_0 = 10$ dB, the pricing factor is set to $\delta = 1$, and the noise variance $\sigma^2 = -120$ dBm. All packets are considered of size 256 bits which is a typical IP packet size. The agents are considered as having a constant velocity of $\eta = 60$ km/h, a transmit power of $\tilde{P} = 100$ mW, and a transmission link capacity of $\mu = 768$ kbps (assumed the same for all agents). Further, we consider two classes of tasks in the network. A first class that can be mapped to voice services having an arrival rate of 32 kbps, and a second class that can be mapped to video services, such as the widely known Quarter Common Intermediate Format (QCIF) [38],

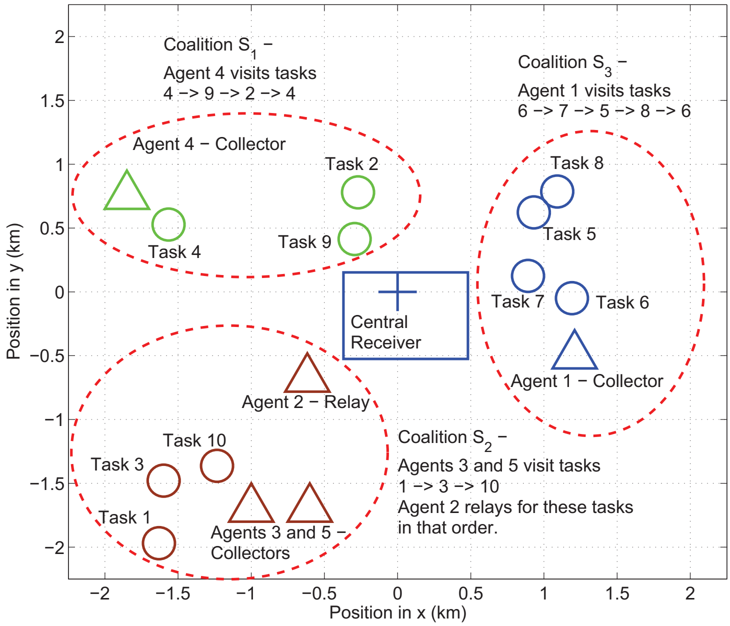

Fig. E.2: A snapshot of a final coalition structure resulting from the proposed hedonic coalition formation algorithm for a network of $M = 5$ agents and $T = 10$ tasks. In every coalition, the agents (collectors and relays) visit the tasks continuously in the shown order.

having an arrival rate 128 kbps. Tasks belonging to each class are generated with equal probability in the simulations. Unless stated otherwise, the throughput-delay tradeoff parameter β is set to 0.7, to indicate services that are reasonably delay tolerant.

In Figure E.2, we show a snapshot of the final network partition reached through the proposed hedonic coalition formation algorithm for a network consisting of $M = 5$ agents and $T = 10$ arbitrarily located tasks. In this figure, Tasks 1, 3, and 8 belong to the QCIF video class with an arrival rate of 128 kbps, while the remaining tasks belong to the voice class with an arrival rate of 32 kbps. In Figure E.2, we can easily see how the agents and tasks can agree on a partition whereby a number of agents service a group of nearby tasks for data collection and transmission. For the network of Figure E.2, the tasks are distributed into three coalitions, two of which (coalitions S_1 and S_3) are served by a single collector-agent. In contrast, coalition S_2 is served by two collectors and one relay. The agents in coalition 2 distributed their roles (relay or collector) depending on the achieved utility. For instance, for coalition S_2, having two collectors and one relay provides a utility of $v(S_2) = 52.25$ while having three collectors yields a utility of $v(S_2) = 10.59$, and having one collector and two relays yields a utility of $v(S_2) = 45.19$. As a result, the case of two collectors and one relay maximizes the utility and is agreed upon between the players. Further, the coalitions in Figure E.2 are dynamic, in the sense that, within each coalition, the agents move from one task to the other, collecting and transmitting data to the receiver continuously. The order in which the agents visit the tasks, as indicated in Figure E.2, is generated using a nearest neighbor solution for the traveling salesman problem as given by Property 6. For example, consider coalition S_2 in Figure E.2. In this coalition, agents 3 an 5 act as a single collector and move from task 1, to task 3, to task 10, and then back to task 1 repeating these visits in a cyclic manner. Concurrently with the collectors movement, agent 2 of coalition S_2, moves and positions itself at the middle of the line connecting the task being serviced by agents 3 and 5 to the central receiver. In other words, when the collectors are servicing task 1 agent 2 is at the middle of the line connecting task 1 to the central receiver, subsequently when the collectors are servicing task 3 agent 2 takes position at the middle of the line connecting task 3 to the central receiver and so on. Finally, note that, for all the coalitions in Figure E.2 one can verify that the minimum number of collectors, as per Proposition 1 is approximately 1, (e.g., for coalition S_2, we have $|G_{S_2}|_{\min} = \frac{9}{24}$ thus 1 collector is a minimum), and, hence, this condition is easily satisfied by the coalition formation process.

In Figure E.3, we assess the performance of the proposed hedonic coalition formation algorithm, in terms of the payoff (revenue) per player (agent or task) for a network having $M = 5$ agents, as the number of tasks increases. The figure shows the statistics (averaged over the random positions of the tasks), in terms of maximum, average, and minimum over the random order of play. In this figure, we compare the performance with an algorithm that assigns the tasks equally among the agents (i.e., an equal group of neighboring tasks are assigned for every agent). Fig-

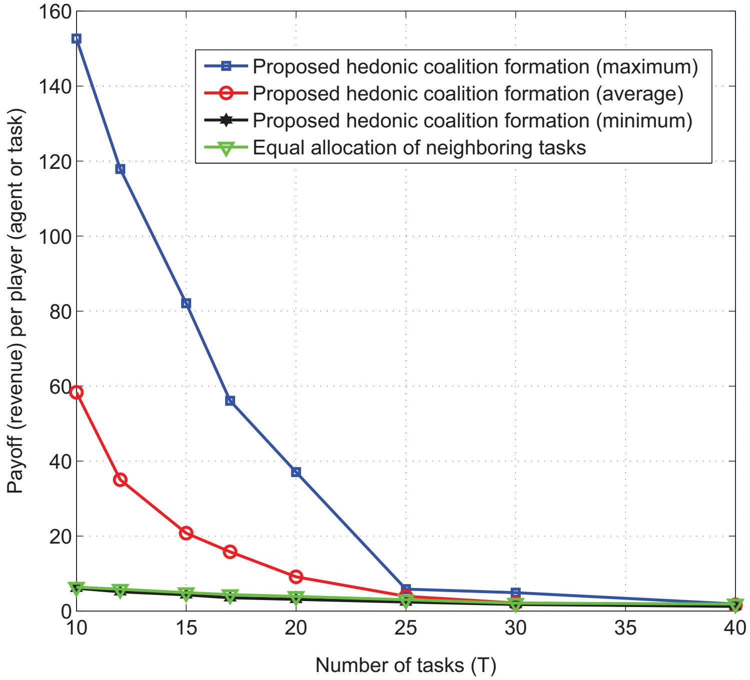

Fig. E.3: Performance statistics, in terms of maximum, average and minimum (over the order of play) player payoff (revenue), of the proposed hedonic coalition formation algorithm compared to an algorithm that allocates the neighboring tasks equally among the agents as the number of tasks increases for $M = 5$ agents. All the statistics are also averaged over the random positions of the tasks.

ure E.3 shows that the performance of both algorithms is bound to decrease as the number of tasks increases. This is mainly due to the fact that, for networks having a larger number of tasks, the delay incurred per coalition, and, thus, per user increases. This increase in the delay is not only due to the increase in the number of tasks, but also to the increase in the distance that the agents need to travel within their corresponding coalitions (increase in switchover times). In Figure E.3, we note that the minimum payoff achieved by the proposed algorithm is comparable to that of the equal allocation. Hence, the performance of the proposed algorithm is clearly lower bounded by the equal allocation algorithm. However, Figure E.3 shows that the average and maximum payoff resulting by the proposed algorithm is significantly better than the equal allocation at all network sizes up to $T = 25$ tasks. Albeit this performance improvement decreases with the increase in the number of tasks, the performance, in terms of average payoff per player, yielded by the proposed algorithm is no less than 30.26% better than the equal allocation for up to $T = 25$ tasks. Beyond $T = 25$ tasks, Figure E.3 shows that the average and maximum performance of the proposed algorithm is comparable to that of the equal allocation, notably at $T = 40$ tasks. The reduction in the performance gap between the two algorithms for large networks stems from the fact that, as more tasks exist in the network, for a fixed number of agents, the possibility of forming large coalitions, through the proposed hedonic coalition formation algorithm is reduced, and, hence, the structure becomes closer to equal allocation.

In Figure E.4, we show the statistics (averaged over the random positions of the tasks), in terms of maximum, average, and minimum (over the random order of play) payoff per player for a network with $T = 20$ tasks as the number of agents M increases. The performance is once again compared with an algorithm that assigns the tasks equally among the agents (i.e., an equal group of neighboring tasks are assigned for every agent). Figure E.4 shows that the performance of both algorithms increases as the number of agents increases. This is mainly due to the fact that when more agents are deployed, the tasks can be better serviced as the delay incurred per coalition decreases and the probability of successful transmission improves. For instance, as more agents enter the network, they can act as either collectors (for improving the delay) or relays (for improving the success probability). We note that, at $M = 3$, the performance statistics of the proposed algorithm converge to the equal allocation algorithm since, for a small number of agents, the flexibility of forming coalitions is quite restricted and equal allocation is the most straightforward coalitional structure. Nonetheless, Figure E.4 shows that, as M increases, the performance of the proposed algorithm, in terms of maximum and average payoff achieved, becomes significantly larger than that of the equal allocation algorithm, and this performance advantage increases as more agents are deployed. Finally, Figure E.4 also shows that the minimum

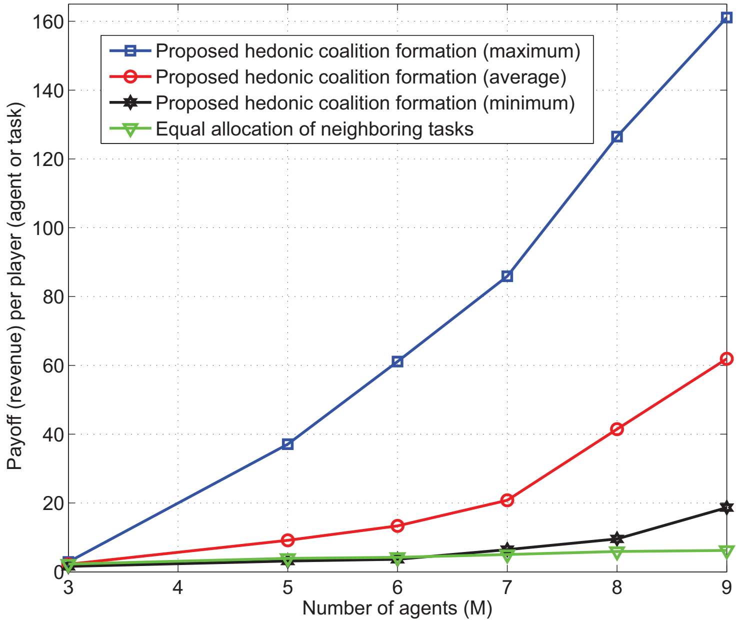

Fig. E.4: Performance statistics, in terms of maximum, average and minimum (over the order of play) player payoff (revenue), of the proposed hedonic coalition formation algorithm compared to an algorithm that allocates the neighboring tasks equally among the agents as the number of agents increases for $T = 20$ tasks. All the statistics are also averaged over the random positions of the tasks.

performance of the proposed algorithm is comparable to the equal allocation algorithm for network with a small number of agents, but as the number of agents increases, the minimum performance of hedonic coalition formation is 29% better than the equal allocation case at $M = 7$, and this advantage increases further with M.

In Figure E.5, we show the average and maximum (over the random order of play) coalition size resulting from the proposed algorithm as the number of tasks T increases, for a network of $M = 5$ agents and arbitrarily

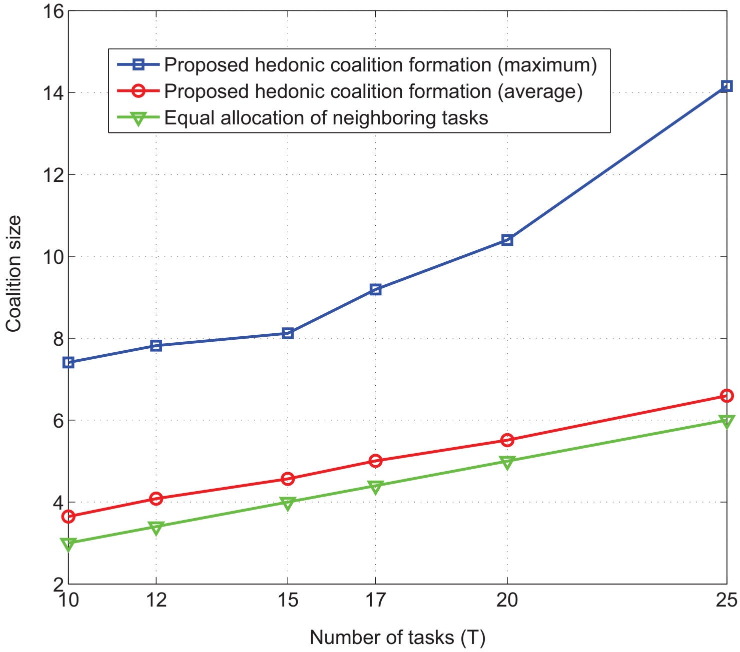

Fig. E.5: Average and maximum (over order of play) coalition size yielded by the proposed hedonic coalition formation algorithm and an algorithm that allocates the neighboring tasks equally among the agents, as a function of the number of tasks T for a network of $M = 5$ agents. These statistics are also averaged over random positions of the tasks.

deployed tasks. These results are averaged over the random positions of the tasks and are compared with the equal allocation algorithm. In this figure, we note that, as the number of tasks increases, the average coalition size for both algorithms increases. For the proposed algorithm, the maximum coalition size also increases with the number of tasks. This is an immediate result of the fact that, as the number of tasks increases, the probability of forming larger coalitions is higher and, hence, our proposed algorithm yields larger coalitions. Further, at all network sizes, the proposed algorithm yields coalitions that are relatively larger than the equal allocation algorithm. This result implies that, by allowing the players (agents and tasks) to selfishly select their coalitions, through the proposed algorithm, the players have an incentive to structure themselves in coalitions with average size lower bounded by the equal allocation. In a nutshell, through hedonic coalition formation, the resulting topology mainly consists of networks composed of a large number of small coalitions as demonstrated by the average coalition size. However, in a limited number of cases, the network topology can also be composed of a small number of large coalitions as highlighted by the maximum coalition size shown in Figure E.5.

In Figure E.6, we show, over a period of 5 minutes, the frequency in terms of average switch operations per minute per player (agent or task) achieved for various velocities of the tasks in a mobile wireless network with $M = 5$ agents and different number of tasks. As the velocity of the tasks increases, the frequency of the switch operations increases for both $T = 10$ and $T = 20$ due to the changes in the positions of the various tasks incurred by mobility. Figure E.6 shows that the case of $T = 20$ tasks yields a frequency of switch operations significantly higher than the case of $T = 10$ tasks. This result is interpreted by the fact that, as the number of tasks increases, the possibility of finding new partners as the tasks move increases significantly, hence yielding an increase in the topology variation as reflected by the number of switch operations. In summary, this figure shows that the proposed hedonic coalition formation algorithm allows the agents and the tasks to self-organize and adapt their topology to mobility, through adequate switch operations.

The network's adaptation to mobility is further assessed in Figure E.7, where we show, over a period of 5 minutes, the average coalition lifespan (in seconds) achieved for various velocities of the tasks in a mobile wireless network with $M = 5$ agents and different number of tasks. The coalition lifespan is defined as the time (in seconds) during which a coalition is present in the mobile network prior to accepting new members or breaking into smaller coalitions (due to switch operations). In this figure, we can see that, as the velocity of the tasks increases, the average lifespan of a coalition decreases. This is due to the fact that, as mobility increases, the possibility of forming new coalitions or splitting existing coalitions increases significantly. For example, for $T = 20$, the coalition lifespan drops from around 124 seconds for a tasks' velocity of 10 km/h to just under a

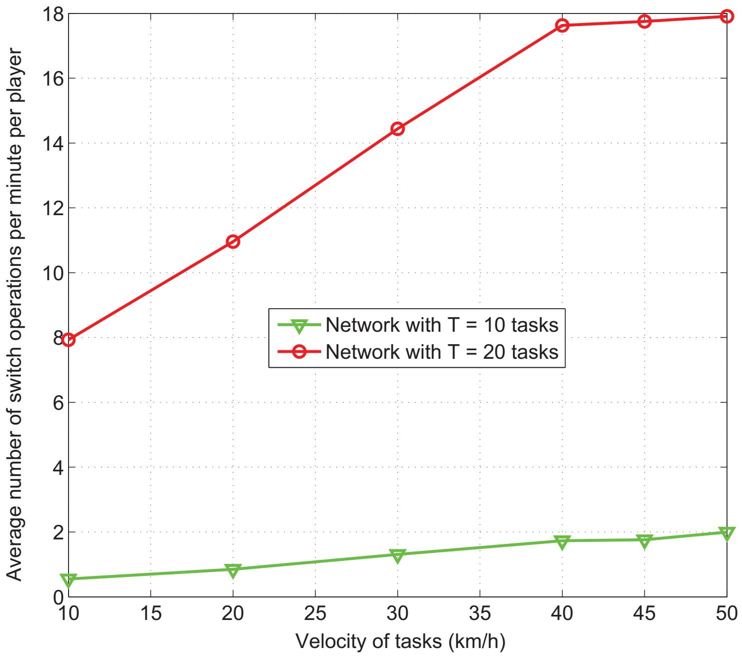

Fig. E.6: Frequency of switch operations per minute per player (agent or task) achieved over a period of 5 minutes for different tasks' velocities in wireless network having $M = 5$ agents and different number of mobile tasks.

minute as of 30 km/h, and down to around 42 seconds at 50 km/h. Furthermore, Figure E.7 shows that as more tasks are present in the network, the coalition lifespan decreases. For instance, for any given velocity, the lifespan of a coalition for a network with $T = 10$ tasks is significantly larger than that of a coalition in a network with $T = 20$ tasks. This is a direct result of the fact that, for a given tasks' velocity, as more tasks are present in the network, the players are able to find more partners to join with, and hence the lifespan of the coalitions becomes shorter. In brief, Figure E.7 provides an interesting assessment of the topology adaptation aspect of the proposed algorithm through the process of forming new coalitions or

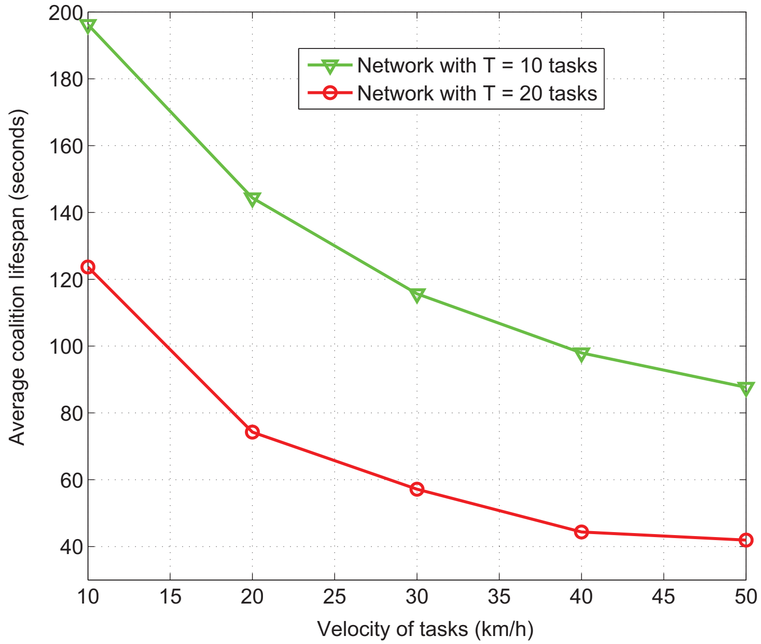

Fig. E.7: Average coalition lifespan (in seconds) achieved over a period of 5 minutes for different tasks' velocities in wireless network having $M = 5$ agents and different number of mobile tasks. The coalition lifespan indicates the time during which a coalition is present in the network before accepting new partners or breaking into smaller coalitions (due to switch operations).

breaking existing coalitions.

Moreover, for further analysis of the self-adapting aspect of the proposed hedonic coalition formation algorithm, we study the variations of the coalitional structure over time for a network where tasks are entering and leaving the network. For this purpose, in Figure E.8, we show the variations of the average (over the random positions of the tasks) number of players per coalition, i.e., the average coalition size, over a period of 10 minutes, as new tasks join the network and/or existing tasks leave the

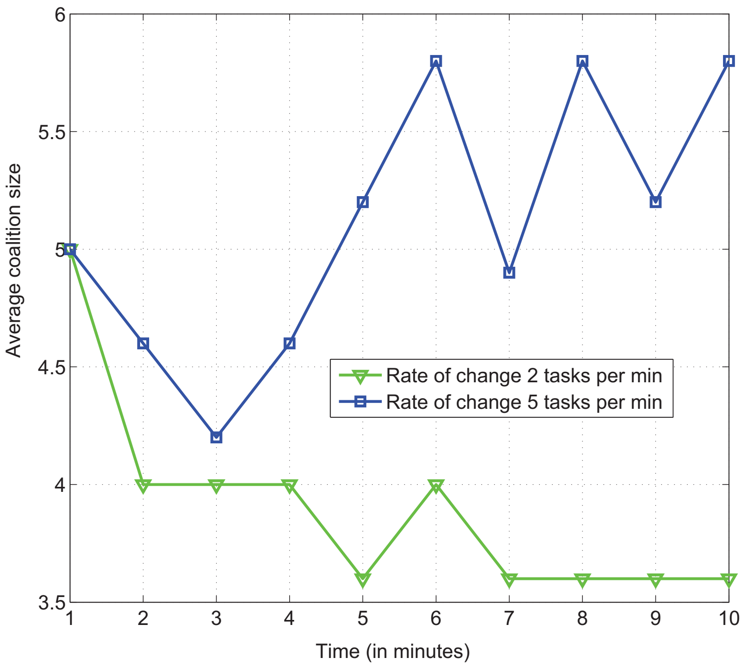

Fig. E.8: Topology variation over time as new tasks join the network and existing tasks leave the network with different rates of tasks arrival/departure for a network starting with $T = 15$ tasks and having $M = 5$ agents.

network. The considered network in Figure E.8 possesses $M = 5$ agents and starts with $T = 15$ tasks. The results are shown for different rates of change which is defined as the number of tasks that have either entered the network or left the network per minute. For example, a rate of change of 2 tasks per minute indicates that either 2 tasks enter the network every minute, 2 tasks leave the network every minute, or 1 tasks enters the network and another tasks leaves the network every minute (these cases may occur with equal probability). In this figure, we can see that, as time evolves, the structure of the network is changing, with new coalitions

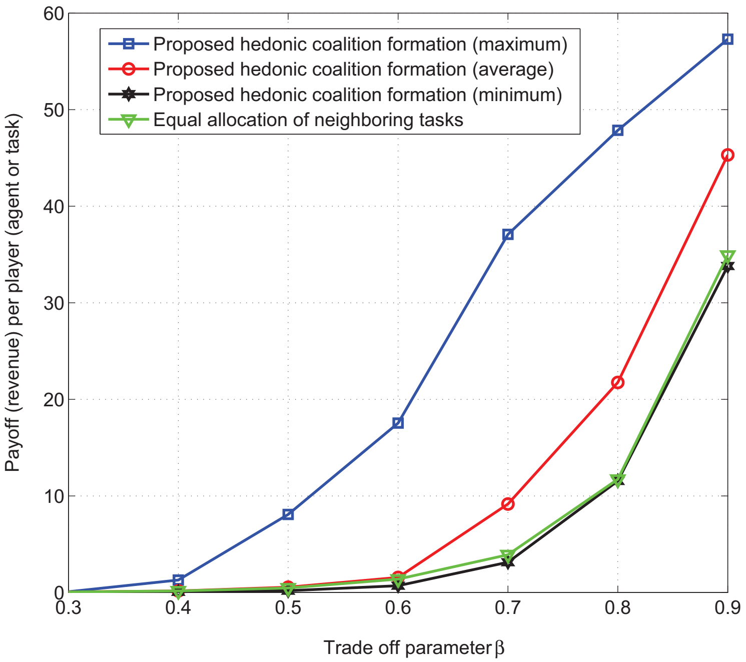

Fig. E.9: Performance statistics, in terms of maximum, average and minimum (over the order of play) player payoff (revenue), of the proposed hedonic coalition formation algorithm compared to an algorithm that allocates the neighboring tasks equally among the agents as the throughput-delay tradeoff parameter β increases for $M = 5$ agents and $T = 20$ tasks. All the statistics are also averaged over the random positions of the tasks.

forming and other breaking as reflected by the change in coalitions size. Furthermore, we note that, as the rate of change increases, the changes in the topology increase. For instance, it is seen in Figure E.8 that for a rate of change of 5 tasks per minute, the variations in the coalition size are much larger than for the case of 2 tasks per minute (which is almost constant for many periods of time). In summary, Figure E.8 shows the network topology variations as tasks enter or leave the network. Note that, after the 10 minutes have elapsed, the network re-enters in the Phase III of the algorithm where data collection and transmission occurs.

In Figure E.9, we assess the performance of the proposed hedonic coalition formation algorithm, in terms of the payoff (revenue) per player (agent or task) for a network having $M = 5$ agents and $T = 20$ tasks, as the throughput-delay tradeoff parameter β increases. The figure shows the statistics, in terms of maximum, average, and minimum over the random order of play between the players. In this figure, we can see that, for small β, the performance of the proposed algorithm is comparable to the equal allocation algorithm and the payoffs are generally small. This result is due to the fact that, for small β, the tasks are highly delay sensitive, and the delay component of the utility governs the performance. Hence, for such tasks, the proposed algorithm yields a performance similar to equal allocation. However, as the tradeoff parameter β increases, the maximum and average utility yielded by our proposed algorithm outperforms the equal allocation algorithm significantly. For instance, as of $\beta = 0.5$, hedonic coalition formation is highly desirable, and presents a performance improvement in terms of average payoff of around 19.56% relative to the equal allocation algorithm (at $\beta = 0.55$, the proposed algorithm has an average payoff of 0.55 while equal allocation has an average payoff of 0.46). This advantage increases with β. Note that, for all tradeoff parameters, the performance of the proposed algorithm, in terms of minimum (over order of play) payoff gained by a player is lower bounded by the equal allocation algorithm and, in average, outperforms the equal allocation algorithm.

6 Conclusions

In this paper, we introduced a novel model for task allocation among a number of autonomous agents in a wireless communication network. In the introduced model, a number of wireless agents are required to service several tasks, arbitrarily located in a given area. Each task represents a queue of packets that require collection and wireless transmission to a centralized receiver by the agents. The task allocation problem is modeled as a hedonic coalition formation game between the agents and the tasks that interact in order to form disjoint coalitions. Each formed coalition is mapped to a polling system which consists of a number of agents continuously collecting packets from a number of tasks. Within a coalition, the agents can act either as collectors that move between the different tasks present in the coalition for collecting the packet data, or relays for improving the wireless transmission of the data packets. For forming the coalitions, we introduce an algorithm that allows the players (tasks or agents) to join or leave the coalitions based on their preferences which capture the tradeoff between the effective throughput and the delay achieved by the coalition. We study the properties and characteristics of the proposed model, we show that the proposed hedonic coalition formation algorithm always converges to a Nash-stable partition, and we study how the proposed algorithm allows the agents and tasks to take distributed decisions for adapting the network topology to environmental changes such as the deployment of new tasks, the removal of existing tasks or the mobility of the tasks. Simulation results show how the proposed algorithm allows the agents and tasks to self-organize into independent coalitions, while improving the performance, in terms of average player (agent or task) payoff, of at least 30.26% (for a network of 5 agents with up to 25 tasks) relatively to a scheme that allocates nearby tasks equally among the agents. In a nutshell, by combining concepts from wireless networks, queueing theory and novel concepts from coalitional game theory, we proposed a new model for task allocation among autonomous agents in communication networks which is well suited for many practical applications such as data collection, data transmission, autonomous relaying, operation of message ferry (mobile base stations), surveillance, monitoring, or maintenance in next generation wireless networks.

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Paper F

Distributed Coalition Formation Games for Secure Wireless Transmission

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Abstract

Cooperation among wireless nodes has been recently proposed for improving the physical layer (PHY) security of wireless transmission in the presence of multiple eavesdroppers. While existing PHY security literature answered the question "what are the link-level *secrecy rate* gains from cooperation?", this paper attempts to answer the question of "how to achieve those gains in a practical decentralized wireless network and in the presence of a cost for information exchange?". For this purpose, we model the PHY security cooperation problem as a coalitional game with non-transferable utility and propose a distributed algorithm for coalition formation. Through the proposed algorithm, the wireless users can cooperate and self-organize into disjoint independent coalitions, while maximizing their secrecy rate taking into account the security costs during information exchange. We analyze the resulting coalitional structures for both decode-and-forward and amplify-and-forward cooperation and study how the users can adapt the network topology to environmental changes such as mobility. Through simulations, we assess the performance of the proposed algorithm and show that, by coalition formation using decode-and-forward, the average secrecy rate per user is increased of up to 25.3% and 24.4% (for a network with 45 users) relative to the non-cooperative and amplify-and-forward cases, respectively.

1 Introduction

With the recent emergence of ad hoc and decentralized networks, higher-layer security techniques such as encryption have become hard to implement. This led to an increased attention on studying the ability of the physical layer (PHY) to provide secure wireless communication. The main idea is to exploit the wireless channel PHY characteristics such as fading or noise for improving the reliability of wireless transmission. This reliability is quantified by the rate of secret information sent from a wireless node to its destination in the presence of eavesdroppers, i.e., the so called *secrecy rate*. The maximal achievable secrecy rate is referred to as the *secrecy capacity*. The study of this security aspect began with the pioneering work of Wyner over the wire-tap channel [1] and was followed up in [2, 3] for the scalar Gaussian wire-tap channel and the broadcast channel, respectively.

Recently, there has been a growing interest in carrying out these studies unto the wireless and the multi-user channels [4–10]. For instance, in [4] and [5], the authors study the secrecy capacity region for both the Gaussian and the fading broadcast channels and propose optimal power allocation strategies. In [6], the secrecy level in multiple access channels from a link-level perspective is studied. Further, multiple antenna systems have been proposed in [8] for ensuring a non-zero secrecy capacity. The work in [9, 10] presents a performance analysis for using cooperative beamforming (with no cost for cooperation), with decode-and-forward and amplify-and-forward relaying, to improve the secrecy rate of a single cluster consisting of one source node and a number of relays. Briefly, the majority of the existing literature is devoted to the information theoretic analysis of link-level performance gains of secure communications with no information exchange cost, notably when a source node cooperate with some relays as in [9, 10]. While this literature studied the performance of some cooperative schemes, no work seems to have investigated how a number of users, each with its own data, can interact and cooperate at network-wide level to improve their secrecy rate.

The main contribution of this work is to propose distributed cooperation strategies, through coalitional game theory [11], which allow to study the interactions between a network of users that seek to secure their communication in the presence of multiple eavesdroppers. Another major contribution is to study the impact on the network topology and dynamics of the inherent tradeoff that exists between the PHY security cooperation gains in terms of secrecy rate and the information exchange costs. In other words, while the earlier work answered the question "what are the secrecy rate gains from cooperation?", here, we seek to answer the question of "how to achieve those gains in a practical decentralized wireless network and in the presence of a cost for information exchange?". We model the problem as a non-transferable coalitional game and propose a distributed algorithm for autonomous coalition formation based on well suited concepts from cooperative games. Through the proposed algorithm, each user autonomously decides to form or break a coalition for maximizing its utility in terms of secrecy rate while accounting for the loss of secrecy rate during information exchange. We show that independent disjoint coalitions form in the network, due to the cooperation cost, and we study their properties for both the decode-and-forward and amplify-and-forward cooperation models.Simulation results show that, by coalition formation using decode-and-forward, the average secrecy rate per user is increased of up to 25.3% and 24.4% relative to the non-cooperative and amplify-and-forward cases, respectively. Further, the results show how the users can self-organize and adapt the topology to mobility.

The rest of this paper is organized as follows: Section 2 presents the system model. Section 3 presents the game formulation and properties. In Section 4 we devise the coalition formation algorithm. Simulation results are presented and analyzed in Section 5. Finally, conclusions are drawn in Section 6.

2 System Model

Consider a network having N transmitters (e.g. mobile users) sending data to M receivers (destinations) in the presence of K eavesdroppers that seek to tap into the transmission of the users. Users, receivers and eavesdroppers are unidirectional-single-antenna nodes. We define $\mathcal{N} = \{1, \ldots, N\}$, $\mathcal{M} = \{1, \ldots, M\}$ and $\mathcal{K} = \{1, \ldots, K\}$ as the sets of users, destinations, and eavesdroppers, respectively. In this work, we consider only the case of multiple eavesdroppers, hence, we have $K > 1$. Furthermore, let h_{i,m_i} denote the complex baseband channel gain between user $i \in \mathcal{N}$ and its destination $m_i \in \mathcal{M}$ and $g_{i,k}$ denote the channel gain between user $i \in \mathcal{N}$ and eavesdropper $k \in \mathcal{K}$. We consider a line of sight channel model with

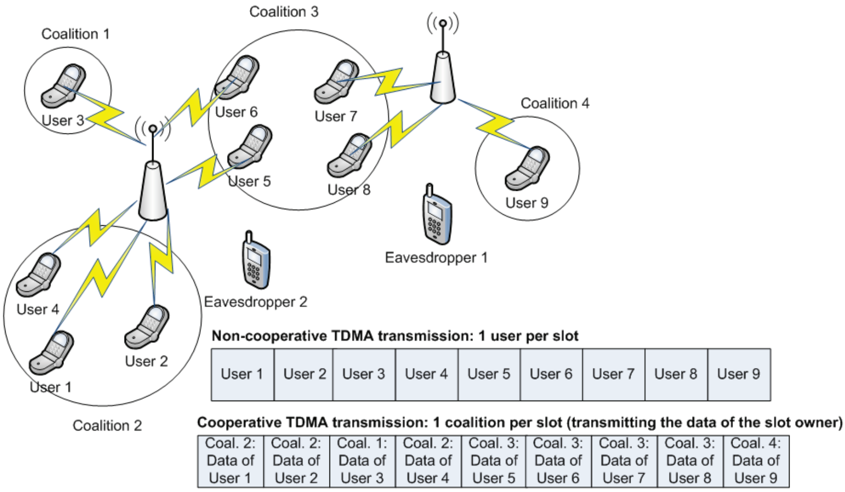

Fig. F.1: System model for physical layer security coalitional game.

$h_{i,m_i} = d_{i,m_i}^{-\frac{\mu}{2}} e^{j\phi_{i,m_i}}$ with d_{i,m_i} the distance between user i and its destination m_i, μ the pathloss exponent, and ϕ_{i,m_i} the phase offset. A similar model is used for the user-eavesdropper channel. Note that other channel models can also be accommodated.

Further, we consider a TDMA transmission, whereby, in a non-cooperative manner, each user occupies a single time slot. Within a single slot, the amount of reliable information transmitted from the user i occupying the slot to its destination m_i is quantified through the *secrecy rate* C_{i,m_i} defined as follows [4]:

$$C_{i,m_i} = \left(C_{i,m_i}^d - \max_{1 \le k \le K} C_{i,k}^e \right)^+, \tag{F.1}$$

where C_{i,m_i}^d is the capacity for the transmission between user i and its destination $m_i \in \mathcal{M}$, $C_{i,k}^e$ is the capacity of user i at the eavesdropper $k \in \mathcal{K}$, and $a^+ \triangleq \max(a, 0)$. Note that the secrecy rate in (F.1) is shown to be achievable in [12] using Gaussian inputs.

In a non-cooperative approach, due to the broadcast nature of the wireless channel, the transmission of the users can be overheard by the eavesdroppers which reduces their secrecy rate as clearly expressed in (F.1). For improving their performance and increasing their secrecy rate, the users can collaborate by forming coalitions. Within every coalition, the users can utilize collaborative beamforming techniques for improving their secrecy rates. In this context, every user i member of a coalition S can cooperate with its partners in S by dividing its slot into two durations:

1. In the first duration, user i broadcasts its data to the other members of coalition S.

2. In the second duration, coalition S performs collaborative beamforming. Thus, all the members of coalition S relay a weighted version of user i's signal to its destination.

Although finding an optimal cooperation scheme that maximizes the secrecy rate is quite complex [9], one approach for cooperation is to null the signal at the eavesdroppers, i.e., impose $C^{e}_{i,k} = 0, \forall k \in \mathcal{K}$, hence, improving their secrecy rate as compared to the non-cooperative rate in (F.1) [9]. Each coalition $S \subseteq \mathcal{N}$ that forms in the network is able to transmit within all the time slots previously held by its users. Thus, in the presence of cooperating coalitions, the TDMA system schedules one coalition per time slot. During a given slot, the coalition acts as a single entity for transmitting the data of the user that owns the slot. Fig. F.1 shows an illustration of this model for $N = 9$ users, $M = 2$ destinations, and $K = 2$ eavesdroppers.

Furthermore, we define a fixed transmit power *per time slot* $\tilde{P}$ which constrains *all the users* that are transmitting within a given slot. In a non-cooperative manner, this power constraint applies to the single user occupying the slot, while in a cooperative manner this *same* power constraint applies to the entire coalition occupying the slot. Such a power assumption is typical in TDMA systems comprising mobile users and is a direct result of ergodicity and the time varying user locations [13–15]. For every coalition S, during the time slot owned by user $i \in S$, user i utilizes a portion of the available power $\tilde{P}$ for information exchange (first stage) while the remaining portion P^{S}_{i} is used by the coalition S to transmit the actual data to the destination m_i of user i (second stage). For information exchange, user $i \in S$ can broadcast its information to the farthest user $\hat{i} \in S$, by doing so all the other members of S can also obtain the information due to the broadcast nature of the wireless channel. This information exchange incurs a power cost $\bar{P}_{i,\hat{i}}$ given by

$$\bar{P}_{i,\hat{i}} = \frac{\nu_0 \cdot \sigma^2}{|q_{i,\hat{i}}|^2}, \tag{F.2}$$

where ν_0 is a target average signal-to-noise ratio (SNR) for information exchange, σ^2 is the noise variance and $q_{i,\hat{i}}$ is the channel gain between users i and $\hat{i}$. The remaining power that coalition S utilizes for the transmission of the data of user i during the remaining time of this user's slot is

$$P_i^S = (\tilde{P} - \bar{P}_{i,\hat{i}})^+. \tag{F.3}$$

For every coalition S, during the transmission of the data of user i to its destination, the coalition members can cooperate, using either decode-and-forward (DF) or amplify-and-forward (AF), and, hence, weigh their signals in a way to *completely null* the signal at the eavesdroppers. In DF, the coalition members that are acting as relays decode the received signal in the information exchange phase, then re-encode it before performing beamforming. In contrast, for AF, coalition members that are acting as relays perform beamforming by weighing the noisy version of the received signal in the information exchange phase. For any coalition S the signal weights and the "user-destination" channels are represented by the $|S| \times 1$ vectors $\boldsymbol{w}_S = [w_{i_1}, \ldots, w_{i_{|S|}}]^H$ and $\boldsymbol{h}_S = [h_{i_1,m_1}, \ldots, h_{i_{|S|},m_{|S|}}]^H$, respectively. By nulling the signals at the eavesdropper through DF cooperation within coalition S, the secrecy rate achieved by user $i \in S$ at its destination m_i during user i's time slot becomes [9, Eq. (14)]

$$C_{i,m_i}^{S,\text{DF}} = \frac{1}{2}\log_2\left(1 + \frac{(\boldsymbol{w}_S^{*,\text{DF}})^H \boldsymbol{R}_S \boldsymbol{w}_S^{*,DF}}{\sigma^2}\right), \tag{F.4}$$

where $\boldsymbol{R}_S = \boldsymbol{h}_S \boldsymbol{h}_S^H$, σ^2 is the noise variance, and $\boldsymbol{w}_S^{*,\text{DF}}$ is the weight vector that maximizes the secrecy rate while nulling the signal at the eavesdropper with DF cooperation and can be found using [9, Eq. (20)]. In (F.4), the factor $\frac{1}{2}$ accounts for the fact that half of the slot of user i is reserved for information exchange.

For AF, we define, during the transmission slot of a user $i \in S$ member of a coalition S, the $|S| \times 1$ vector $\boldsymbol{a}_S^i$ with every element $a_{S,j}^i = \sqrt{\bar{P}_{i,\hat{i}}} q_{i,j} h_{j,m_j}$, $\forall j \neq i$ ($q_{i,j}$ is the channel between users i and j and $\bar{P}_{i,\hat{i}}$ is the power used by user i for information exchange as per (F.2)) and $a_{S,i}^i = \sqrt{\bar{P}_{i,\hat{i}}} h_{i,m_i}$ and the $|S| \times |S|$

diagonal matrix $\boldsymbol{U}_S^i$ with every diagonal element $u_{S,j,j}^i = |h_{j,m_j}|^2 \ \forall j \neq i$ and $u_{S,i,i}^i = 0$. Given these definitions and by nulling the signals at the eavesdropper through AF cooperation within coalition S, the secrecy rate achieved by user $i \in S$ at its destination m_i during user i's time slot becomes [10, Eq. (3)]

$$C_{i,m_i}^{S,\mathrm{AF}} = \frac{1}{2}\log_2\left(1 + \frac{(\boldsymbol{w}_S^{*,\mathrm{AF}})^H \boldsymbol{R}_a \boldsymbol{w}_S^{*,\mathrm{AF}}}{((\boldsymbol{w}_S^{*,\mathrm{AF}})^H \boldsymbol{U}_S^i \boldsymbol{w}_S^{*,\mathrm{AF}} + 1)\sigma^2}\right), \tag{F.5}$$

where $\boldsymbol{R}_a = \boldsymbol{a}_S^i(\boldsymbol{a}_S^i)^H$, and $\boldsymbol{w}_S^{*,\mathrm{AF}}$ is the weight vector that maximizes the secrecy rate while nulling the signal at the eavesdropper with AF cooperation and can be found using [10, Eqs.(14)-(15)]. Note that for AF, as seen in (F.5) there is a stronger dependence on the channels (through the matrix $\boldsymbol{R}_a$) between the cooperating users in both the first and second phase of cooperation, unlike in DF, where this dependence is solely through the power in (F.2) during the information exchange phase. Further, for AF, as the cooperating users amplify a noisy version of the signal, the noise is also amplified, which can reduce the cooperation gains, as seen through the term $(\boldsymbol{w}_S^{*,\mathrm{AF}})^H \boldsymbol{U}_S^i \boldsymbol{w}_S^{*,\mathrm{AF}}$.

Further, it must be stressed that, although the models for AF and DF cooperation in (F.4) and (F.5) are inspired from [9, 10], our work and contribution differ significantly from [9, 10]. While the work in [9, 10] is solely dedicated to finding the optimal weights in (F.4) and (F.5), and presenting a link-level performance analysis for a single cluster of neighboring nodes with no cost for cooperation, our work seeks to perform a network-level analysis by modeling the interactions among a network of users that seek to cooperate, in order to improve their performance, using either the DF or AF protocols in the presence of costs for information exchange. Hence, the main focus of this paper is modeling the user's behavior, studying the network dynamics and topology, and analyzing the network-level aspects of cooperation in PHY security problems. In this regard, the remainder of this paper is devoted to investigate how a network of users can cooperate, through the protocols described in this section, and improve the security of their wireless transmission, i.e., their secrecy rate.

Finally, note that, in this paper, we assume that the users have perfect knowledge of the channels to the eavesdroppers which is an assumption commonly used in most PHY security related literature, and as explained in [16] this channel information can be obtained by the users through a constant monitoring of the behavior of the eavesdroppers. Alternatively, the eavesdroppers in this work can also be seen as areas where the transmitters suspect the presence of malicious eavesdropping nodes and, hence, need to secure these locations. Hence, our current analysis can serve as an upper bound for future work where the analysis pertaining to the case where the eavesdroppers and their locations are not known will be tackled (in that case although the cooperation model needs to be modified, the PHY security coalitional game model presented in the following sections can be readily applied).

3 Physical Layer Security as A Coalitional Game

The proposed PHY security problem can be modeled as a $(\mathcal{N}, V)$ coalitional game with a non-transferable utility [11, 17] where V is a mapping such that for every coalition $S \subseteq \mathcal{N}$, $V(S)$ is a closed convex subset of $\mathbb{R}^{|S|}$ that contains the payoff vectors that players in S can achieve. Thus, given a coalition S and denoting by $\phi_i(S)$ the payoff of user $i \in S$ during its time slot, we define the coalitional value set, i.e., the mapping V as follows

$$\begin{aligned} V(S) = \{\boldsymbol{\phi}(S) \in \mathbb{R}^{|S|} |\ \ \forall i \in S\ \phi_i(S) = (v_i(S) - c_i(S))^+ \\ \text{if } P_i^S > 0, \text{ and } \phi_i(S) = -\infty \text{ otherwise.}\}, \end{aligned} \tag{F.6}$$

where $v_i(S) = C^S_{i,m_i}$ is the gain in terms of secrecy rate for user $i \in S$ given by (F.4) while taking into account the available power P_i^S in (F.3) and $c_i(S)$ is a secrecy cost function that captures the loss for user $i \in S$, in terms of secrecy rate, that occurs during information exchange. Note that, when *all* the power is spent for information exchange, the payoff $\phi_i(S)$ of user i is set to $-\infty$ since, in this case, the user has clearly no interest in cooperating.

With regard to the secrecy cost function $c_i(S)$, when a user $i \in S$ sends its information to the farthest user $\hat{i} \in S$ using a power level $\bar{P}_{i,\hat{i}}$, the eavesdroppers can overhear the transmission. This security loss is quantified by the capacity at the eavesdroppers resulting from the information exchange and which, for a particular eavedropper $k \in \mathcal{K}$, is given by $\hat{C}^e_{i,k} = \frac{1}{2}\log\left(1 + \frac{\bar{P}_{i,\hat{i}} \cdot |g_{i,k}|^2}{\sigma^2}\right)$ and the cost function $c(S)$ can be defined as

$$c_i(S) = \max\left(\hat{C}^e_{i,1}, \ldots, \hat{C}^e_{i,K}\right). \tag{F.7}$$

In general, coalitional game based problems seek to characterize the properties and stability of the grand coalition of all players since it is generally assumed that the grand coalition maximizes the utilities of the players [17]. In our case, although cooperation improves the secrecy rate as per (F.6) for the users in the TDMA network; the utility in (F.6) also accounts for two types of cooperation costs:(i)- The fraction of power spent for information exchange as per (F.3) and, (ii) the secrecy loss during information exchange as per (F.7) which can strongly limit the cooperation gains. Therefore, for the proposed $(\mathcal{N}, v)$ coalitional game we have:

Proposition 1 *For the proposed $(\mathcal{N}, V)$ coalitional game, the grand coalition of all the users* seldom *forms due to the various costs for information exchange. Instead, disjoint independent coalitions will form in the network.*

Proof: *The proof is found in [18, Property 2].* □

Due to this property, traditional solution concepts for coalitional games, such as the core [17], may not be applicable [11]. In fact, in order for the core to exist, as a solution concept, a coalitional game must ensure that the grand coalition, i.e., the coalition of all players will form. However, as seen in Figure F.1 and corroborated by Property 1, in general, due to the cost for coalition formation, the grand coalition will not form. Instead, independent and disjoint coalitions appear in the network as a result of the collaborative beamforming process. In this regard, the proposed game is classified as a *coalition formation game* [11], and the objective is to find the coalitional structure that will form in the network, instead of finding only a solution concept, such as the core, which aims mainly at stabilizing the grand coalition.

Furthermore, for the proposed $(\mathcal{N}, V)$ coalition formation game, a constraint on the coalition size, imposed by the nature of the cooperation protocol exists as follows:

Remark 4 *For the proposed $(\mathcal{N}, V)$ coalition formation game, the size of any coalition $S \subseteq \mathcal{N}$ that will form in the network must satisfy $|S| > K$ for both DF and AF cooperation.*

This is a direct result of the fact that, for nulling K eavesdroppers, at least $K+1$ users must cooperate, otherwise, no weight vector can be found to maximize the secrecy rate while nulling the signal at the eavesdroppers.

4 Distributed Coalition Formation Algorithm

4.1 Coalition Formation Algorithm

Coalition formation has recently attracted increased attention in game theory [11, 19, 20]. The goal of coalition formation games is to find algorithms for characterizing the coalitional structures that form in a network where the grand coalition is not optimal. For constructing a coalition formation process suitable to the proposed $(\mathcal{N}, V)$ PHY security cooperative game, we require the following definitions [11, 20]

Definition 30 *A* collection *of coalitions, denoted by $\mathcal{S}$, is defined as the set $\mathcal{S} = \{S_1, \ldots, S_l\}$ of mutually disjoint coalitions $S_i \subset \mathcal{N}$. In other words, a collection is any arbitrary group of disjoint coalitions S_i of $\mathcal{N}$ not necessarily spanning all players of $\mathcal{N}$. If the collection spans* all *the players of $\mathcal{N}$; that is $\bigcup_{j=1}^{l} S_j = \mathcal{N}$, the collection is a* partition *of $\mathcal{N}$.*

Definition 31 *A preference operator or* comparison relation $\triangleright$ *is an order defined for comparing two collections $\mathcal{R} = \{R_1, \ldots, R_l\}$ and $\mathcal{S} = \{S_1, \ldots, S_p\}$ that are partitions of the same subset $\mathcal{A} \subseteq \mathcal{N}$ (i.e. same players in $\mathcal{R}$ and $\mathcal{S}$). Therefore, $\mathcal{R} \triangleright \mathcal{S}$ implies that the way $\mathcal{R}$ partitions $\mathcal{A}$ is preferred to the way $\mathcal{S}$ partitions $\mathcal{A}$.*

For the proposed PHY security coalition formation game, an individual value order, i.e. an order which compares the individual payoffs of the users, is needed due to the non-transferable utility of the game. For this purpose, for the proposed game, we utilize the following order for defining the preferences of the users

Definition 32 *Consider two collections $\mathcal{R} = \{R_1, \ldots, R_l\}$ and $\mathcal{S} = \{S_1, \ldots, S_m\}$ that are partitions of the same subset $\mathcal{A} \subseteq \mathcal{N}$ (same players in $\mathcal{R}$ and $\mathcal{S}$). For a collection $\mathcal{R} = \{R_1, \ldots, R_l\}$, let the utility of a player j in a coalition $R_j \in \mathcal{R}$ be denoted by $\Phi_j(\mathcal{R}) = \phi_j(R_j) \in V(R_j)$. $\mathcal{R}$ is preferred over $\mathcal{S}$ by* Pareto order, *written as $\mathcal{R} \triangleright \mathcal{S}$, iff*

$$\mathcal{R} \triangleright \mathcal{S} \Longleftrightarrow \{\Phi_j(\mathcal{R}) \geq \Phi_j(\mathcal{S}) \; \forall \; j \in \mathcal{R}, \mathcal{S}\},$$

with at least one strict inequality *(>) for a player k.*

In other words, a collection is preferred by the players over another collection, if at least one player is able to improve its payoff without hurting the other players. Subsequently, for performing autonomous coalition formation between the users in the proposed PHY security game, we construct a distributed algorithm based on two simple rules denoted as "merge" and "split" [11, 20] defined as follows.

Table F.I: One round of the proposed PHY security coalition formation algorithm

Initial State

The network is partitioned by $\mathcal{T} = \{T_1, \ldots, T_k\}$ (At the beginning of all time $\mathcal{T}$ = $\mathcal{N}$ = $\{1, \ldots, N\}$ with non-cooperative users).

Three phases in each round of the coalition formation algorithm

Phase 1 - Neighbor Discovery:

a) Each coalition surveys its neighborhood for candidate partners.
b) For every coalition T_i, the candidate partners lie in the area represented by the intersection of $|T_i|$ circles with centers $j \in T_i$ and radii determined by the distance where the power for information exchange does not exceed $\tilde{P}$ for any user (easily computed through (F.2)).

Phase 2 - Adaptive Coalition Formation:

In this phase, coalition formation using merge-and-split occurs.

repeat

a) $\mathcal{F}$ = Merge($\mathcal{T}$); coalitions in $\mathcal{T}$ decide to merge based on the algorithm of Section 4.1.
b) $\mathcal{T}$ = Split($\mathcal{F}$); coalitions in $\mathcal{F}$ decide to split based on the Pareto order.

until merge-and-split terminates.

Phase 3 - Secure Transmission:

Each coalition's users exchange their information and transmit their data within their allotted slots.

The above three phases are repeated periodically during the network operation, allowing a topology that is adaptive to environmental changes such as mobility.

Definition 33 ***Merge Rule*** - *Merge any set of coalitions $\{S_1, \ldots, S_l\}$ whenever the merged form is preferred by the players, i.e., where $\{\bigcup_{j=1}^{l} S_j\} \triangleright \{S_1, \ldots, S_l\}$, therefore, $\{S_1, \ldots, S_l\} \to \{\bigcup_{j=1}^{l} S_j\}$.*

Definition 34 ***Split Rule*** - *Split any coalition $\bigcup_{j=1}^{l} S_j$ whenever a split form is preferred by the players, i.e., where $\{S_1, \ldots, S_l\} \triangleright \{\bigcup_{j=1}^{l} S_j\}$, thus, $\{\bigcup_{j=1}^{l} S_j\} \to \{S_1, \ldots, S_l\}$.*

Using the above rules, multiple coalitions can merge into a larger coalition if merging yields a preferred collection based on the Pareto order. This implies that a group of users can agree to form a larger coalition, if at least one of the users improves its payoff without decreasing the utilities of any of the other users. Similarly, an existing coalition can decide to split into smaller coalitions if splitting yields a preferred collection by Pareto order. The rationale behind these rules is that, once the users agree to sign a merge agreement, this agreement can only be broken if all the users approve. This is a family of coalition formation games known as coalition formation games with partially reversible agreements [19]. Using the rules of merge and split is highly suitable for the proposed PHY security game due to many reasons. For instance, each merge or split decision can be taken in a distributed manner by each individual user or by each already formed coalition. Further, it is shown in [20] that any arbitrary iteration of merge and split rules terminates, hence, these rules can be used as building blocks in a coalition formation process for the PHY security game.

Accordingly, for the proposed PHY security game, we construct a coalition formation algorithm based on merge-and-split and divided into three phases: Neighbor discovery, adaptive coalition formation, and transmission. In the neighbor discovery phase (Phase 1), each coalition (or user) surveys its environment in order to find possible cooperation candidates. For a coalition S_k the area that is surveyed for discovery is the intersection of $|S_k|$ circles, centered at the coalition members with each circle's radius given by the maximum distance $\bar{r}_i$ (for the circle centered at $i \in S_k$) within which the power cost for user i as given by (F.2) does not exceed the total available power $\tilde{P}$. This area is determined by the fact that, if a number of coalitions $\{S_1, \ldots, S_m\}$ attempt to merge into a new coalition $G = \cup_{i=1}^{m} S_i$ which contains a member $i \in G$ such that the power for information exchange needed by i exceeds $\tilde{P}$, then the payoff of i goes to $-\infty$ as per (F.6) and the Pareto order can never be verified. Clearly, as the number of users in a coalition increases, the number of circles increases, reducing the area where possible cooperation partners can be found. This implies that, as the size of a coalition grows, the possibility of adding new users decreases, and, hence, the complexity of performing merge also decreases.

Following Phase 1, the adaptive coalition formation phase (Phase 2) begins, whereby the users interact for assessing whether to form new coalitions with their neighbors or whether to break their current coalition. For this purpose, an iteration of sequential merge-and-split rules occurs in the network, whereby each coalition decides to merge (or split) depending on the utility improvement that merging (or splitting) yields. Starting from an initial network partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ of $\mathcal{N}$, any random coalition (individual user) can start with the merge process. The coalition $T_i \in \mathcal{T}$ which debuts the merge process starts by enumerating, sequentially, the possible coalitions, of size greater than K (Remark 1), that it can form with the neighbors that were discovered in Phase 1. On one hand, if a new coalition $\tilde{T}_i$ which is preferred by the users through Pareto order is identified, this coalition will form by a merge agreement of all its members. Hence, the merge ends by a final merged coalition T_i^{final} composed of T_i and one or several of coalitions in its vicinity. On the other hand, if T_i is unable to merge with any of the discovered partners, it ends its search and $T_i^{\text{final}} = T_i$.

The algorithm is repeated for the remaining $T_i \in \mathcal{T}$ until all the coalitions have made their merge decisions, resulting in a final partition $\mathcal{F}$. Following the merge process, the coalitions in the resulting partition $\mathcal{F}$ are next subject to split operations, if any is possible. In the proposed PHY security problem, the coalitions are only interested in splitting into structures that include either singleton users or coalitions of size larger than K or both (Remark 1). Similar to merge, the split is a local decision to each coalition. An iteration consisting of multiple successive merge-and-split operations is repeated until it terminates. The termination of an iteration of merge and split rules is guaranteed as shown in [20]. It must be stressed that the merge or split decisions can be taken in a distributed way by the users/coalitions without relying on any centralized entity.

In the final transmission phase (Phase 3), the coalitions exchange their information and begin their secure transmission towards their corresponding destinations, in a TDMA manner, one coalition per slot. Every slot is owned by a user who transmits its data with the help of its coalition partners, if that user belongs to a coalition. Hence, in this phase, the user perform the actual beamforming, while transmitting the data of every user within its corresponding slot. Each run of the proposed algorithm consists of these three phases, and is summarized in Table F.I. As time evolves and the users, eavesdroppers and destinations move (or new users or eavesdroppers enter/leave the network), the users can autonomously self-organize and adapt the network's topology through appropriate merge-and-split decisions during Phase 2. This adaptation to environmental changes is ensured by enabling the users to run the adaptive coalition formation phase periodically in the network.

The proposed algorithm in Table F.I can be implemented in a distributed manner. As the user can detect the strength of other users' uplink signals (through techniques similar to those used in the ad hoc routing discovery) [21], nearby coalitions can be discovered in Phase 1 for potential cooperation. In fact, during Phase 1, each coalition in the network can easily work out the area within which candidates for merge can be found, as previously explained in this section. Once the neighbors are discovered, the coalitions can perform merge operations based on the Pareto order in Phase 2. The complexity of the merge operation can grow exponentially with the number of candidates with whom a user i is able to merge (the number of coalitions in the neighboring area which is in general significantly smaller than N). As more coalitions form, the area within which candidates are found is smaller, and, hence, the merge complexity reduces. In addition, whenever a coalition finds a candidate to merge with, it automatically goes through with the merge operation, hence, avoiding the need for finding all possible merge forms and reducing further the complexity. Further, each formed coalition can also internally decides to split if its members find a split form by Pareto order. By using a control channel, the distributed users can coordinate and then cooperate using our model.

4.2 Partition Stability

The result of the proposed algorithm in Table F.I is a network partition composed of disjoint independent coalitions. The stability of this network partition can be investigated using the concept of a defection function [20].

Definition 35 *A* defection *function* $\mathbb{D}$ *is a function which associates with each partition* $\mathcal{T}$ *of* $\mathcal{N}$ *a group of collections in* $\mathcal{N}$. *A partition* $\mathcal{T} = \{T_1, \ldots, T_l\}$ *of* $\mathcal{N}$ *is* $\mathbb{D}$-stable *if no group of players is interested in leaving* $\mathcal{T}$ *when the players who leave can only form the collections allowed by* $\mathbb{D}$.

We are interested in two defection functions [11, 20]. First, the $\mathbb{D}_{hp}$ function which associates with each partition $\mathcal{T}$ of $\mathcal{N}$ the group of all partitions of $\mathcal{N}$ that can form through merge or split and the $\mathbb{D}_c$ function which associates with each partition $\mathcal{T}$ of $\mathcal{N}$ the group of all collections in $\mathcal{N}$. This function allows any group of players to leave the partition $\mathcal{T}$ of $\mathcal{N}$ through *any* operation and create an arbitrary *collection* in $\mathcal{N}$. Two forms of stability stem from these definitions: $\mathbb{D}_{hp}$ stability and a stronger $\mathbb{D}_c$ stability. A partition $\mathcal{T}$ is $\mathbb{D}_{hp}$-stable, if no player in $\mathcal{T}$ is interested in leaving $\mathcal{T}$ through merge-and-split to form other partitions in $\mathcal{N}$; while a partition $\mathcal{T}$ is $\mathbb{D}_c$-stable, if no player in $\mathcal{T}$ is interested in leaving $\mathcal{T}$ through *any* operation (not necessarily merge or split) to form other collections in $\mathcal{N}$.

Hence, a partition is $\mathbb{D}_{hp}$-stable if no coalition has an incentive to split or merge. For instance, a partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ is $\mathbb{D}_{hp}$-stable, if the following two necessary and sufficient conditions are met [11, 20] ($\not\triangleright$ is the non-preference operator, opposite of $\triangleright$): (i)- For each $i \in \{1, \ldots, m\}$ and for each partition $\{R_1, \ldots, R_m\}$ of $T_i \in \mathcal{T}$ we have $\{R_1, \ldots, R_m\} \not\triangleright T_i$, and (ii)- For each $S \subseteq \{1, \ldots, l\}$ we have $\bigcup_{i \in S} T_i \not\triangleright \{T_i | i \in S\}$. Using this definition of $\mathbb{D}_{hp}$ stability, we have

Theorem 1 *Every partition resulting from our proposed coalition formation algorithm is $\mathbb{D}_{hp}$-stable.*

Proof: *The proof is given in [18, Theorem 1].* □

Furthermore, a $\mathbb{D}_c$-stable partition $\mathcal{T}$ is characterized by being a strongly stable partition, which satisfies the following properties: (i)- A $\mathbb{D}_c$-stable partition is $\mathbb{D}_{hp}$-stable, (ii)- A $\mathbb{D}_c$-stable partition is a *unique* outcome of any iteration of merge-and-split and, (iii)- A $\mathbb{D}_c$-stable partition $\mathcal{T}$ is a unique $\triangleright$-maximal partition, that is for all partitions $\mathcal{T}' \neq \mathcal{T}$ of $\mathcal{N}$, $\mathcal{T} \triangleright \mathcal{T}'$. In the case where $\triangleright$ represents the Pareto order, this implies that the $\mathbb{D}_c$-stable partition $\mathcal{T}$ is the partition that presents a *Pareto optimal* utility distribution for all the players.

Clearly, it is desirable that the network self-organizes unto a $\mathbb{D}_c$-stable partition. However, the existence of a $\mathbb{D}_c$-stable partition is not always guaranteed [20]. The $\mathbb{D}_c$-stable partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ of the whole space $\mathcal{N}$ exists if a partition of $\mathcal{N}$ that verifies the following two necessary and sufficient conditions exists [20]:

1. For each $i \in \{1, \ldots, l\}$ and each pair of disjoint *coalitions* S_1 and S_2 such that $\{S_1 \cup S_2\} \subseteq T_i$ we have $\{S_1 \cup S_2\} \triangleright \{S_1, S_2\}$.
2. For the partition $\mathcal{T} = \{T_1, \ldots, T_l\}$ a coalition $G \subset \mathcal{N}$ formed of players belonging to different $T_i \in \mathcal{T}$ is $\mathcal{T}$-incompatible if for no $i \in \{1, \ldots, l\}$ we have $G \subset T_i$.

In summary, $\mathbb{D}_c$-stability requires that for all $\mathcal{T}$-incompatible coalitions $\{G\}[\mathcal{T}] \triangleright \{G\}$ where $\{G\}[\mathcal{T}] = \{G \cap T_i \ \forall \ i \in \{1, \ldots, l\}\}$ is the projection of coalition G on $\mathcal{T}$. If no partition of $\mathcal{N}$ can satisfy these conditions, then no $\mathbb{D}_c$-stable partition of $\mathcal{N}$ exists. Nevertheless, we have

Lemma 1 *For the proposed $(\mathcal{N}, v)$ PHY security coalitional game, the proposed algorithm of Table* F.I *converges to the optimal $\mathbb{D}_c$-stable partition, if such a partition exists. Otherwise, the final network partition is $\mathbb{D}_{hp}$-stable.*

Proof: *The proof is a consequence of Theorem 1 and the fact that the $\mathbb{D}_c$-stable partition is a unique outcome of any merge-and-split iteration [20] which is the case with any partition resulting from our algorithm.* □

Moreover, for the proposed game, the existence of the $\mathbb{D}_c$-stable partition cannot be always guaranteed. For instance, for verifying the first condition for existence of the $\mathbb{D}_c$-stable partition, the users that are members of each coalitions must verify the Pareto order through their utility given by (F.6). Similarly, for verifying the second condition of $\mathbb{D}_c$ stability, users belonging to all $\mathcal{T}$-incompatible coalitions in the network must verify the Pareto order. Consequently, the existence of such a $\mathbb{D}_c$-stable partition is strongly dependent on the location of the users and eavesdroppers through the individual utilities (secrecy capacities). Hence, the existence of the $\mathbb{D}_c$-stable partition is closely tied to the location of the users and the eavesdroppers, which, in a practical ad hoc wireless network are generally random. However, the proposed algorithm will always guarantee convergence to this optimal $\mathbb{D}_c$-stable partition when it exists as stated in Lemma 1. Whenever a $\mathbb{D}_c$-stable partition does not exist, the coalition structure resulting from the proposed algorithm will be $\mathbb{D}_{hp}$-stable (no coalition or individual user is able to merge or split any further).

5 Simulation Results and analysis

For simulations, a square network of 2.5 km $\times$ 2.5 km is set up with the users, eavesdroppers, and destinations randomly deployed within this area[30]. In this network, the users are always assigned to the closest destination, although other user-destination assignments can be used without any loss of generality. For all simulations, the number of destinations is taken as $M = 2$. Further, the power constraint per slot is set to $\tilde{P} = 10$ mW, the noise level is -90 dBm, and the SNR for information exchange is $\nu_0 = 10$ dB which implies a neighbor discovery circle radius of 1 km per user. For the channel model, the propagation loss is set to $\mu = 3$. All statistical results are averaged over the random positions of the users, eavesdroppers and destinations.

In Fig. F.2, we show a snapshot of the network structure resulting from the proposed coalition formation algorithm for a randomly deployed network with $N = 15$ users and $K = 2$ eavesdroppers for both DF (dashed

[30]This general network setting simulates a broad range of network types ranging from ad hoc networks, to sensor networks, WLAN networks as well as broadband or cellular networks.

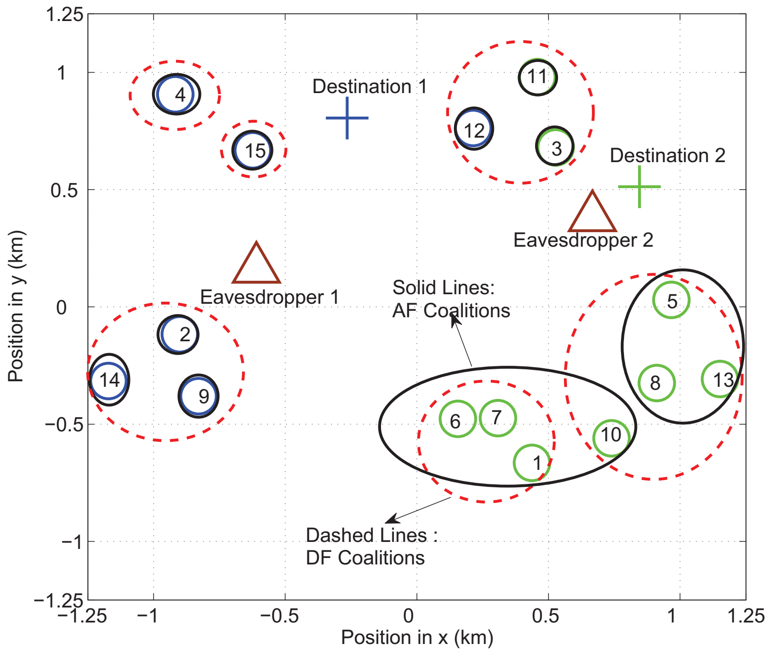

Fig. F.2: A snapshot of a coalitional structure resulting from our proposed coalition formation algorithm for a network with $N = 15$ users, $M = 2$ destinations and $K = 2$ eavesdroppers for DF (dashed lines) and AF (solid lines).

lines) and AF (solid lines) protocols. For DF, the users self-organized into 6 coalitions with the size of each coalition strictly larger than K or equal to 1. For example, Users 4 and 15, having no suitable partners for forming a coalition of size larger than 2, do not cooperate. The coalition formation process is a result of Pareto order agreements for merge (or split) between the users. For example, in DF, coalition $\{5, 8, 10, 13\}$ formed since all the users agree on its formation due to the fact that $V(\{5, 8, 10, 13\}) = \{\phi(\{5, 8, 10, 13\}) = [0.356\ 0.8952\ 1.7235\ 0.6213]\}$ which is a clear improvement on the non-cooperative utility which was 0 for all four users (due to proximity to eavesdropper 2). For AF, Fig. F.2 shows that

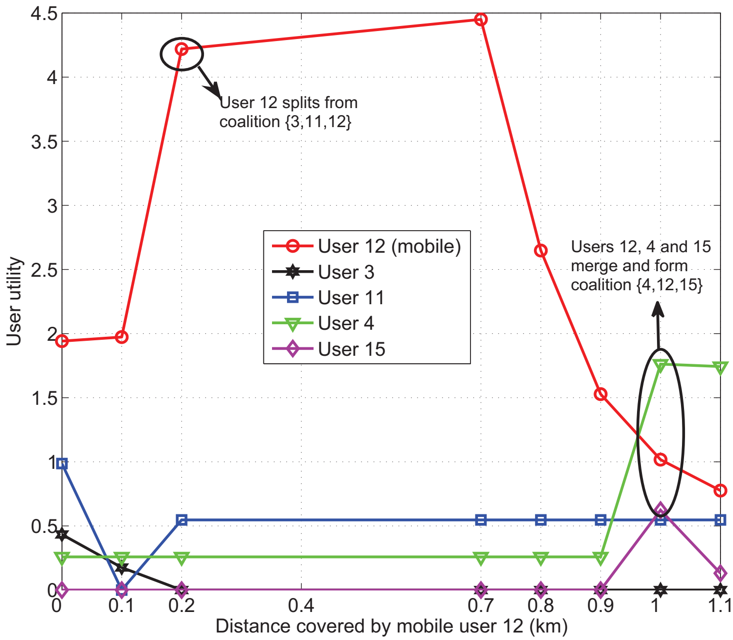

Fig. F.3: Self-adaptation of the network's topology to mobility as User 12 in Fig. F.2 moves horizontally on the negative x-axis (for DF).

only users $\{5, 8, 13\}$ and users $\{1, 6, 7, 10\}$ cooperate while all others remain non-cooperative. The main reason is that, in AF, the users need to amplify a noisy version of the signal using the beamforming weights. As a consequence, the noise can be highly amplified, and, for AF, cooperation is only beneficial in very favorable conditions. For example, coalitions $\{5, 8, 13\}$ and $\{1, 6, 7, 10\}$ have formed for AF due to being far from the eavesdroppers (relatively to the other users), hence, having a small cost for information exchange. In contrast, for coalitions such as $\{3, 11, 12\}$, the benefit from cooperation using AF is small compared to the cost, and, thus, these coalitions do not form.

In Fig. F.3 we show how the algorithm handles mobility through appropriate coalition formation decisions. For this purpose, the network setup of Fig. F.2 is considered for the DF case while User 12 is moving horizontally for 1.1 km in the direction of the *negative* x-axis. First of all, User 12 starts getting closer to its receiver (destination 2), and, hence, it improves its utility. In the meantime, the utilities of User 12's partners (Users 3 and 11) drop due to the increasing cost. As long as the distance covered by User 12 is less than 0.2 km, the coalition of Users 3, 11 and 12 can still bring mutual benefits to all three users. After that, splitting occurs by a mutual agreement and all three users transmit independently. When User 12 moves about 0.8 km, it begins to distance itself from its receiver and its utility begins to decrease. When the distance covered by User 12 reaches about 1 km, it will be beneficial to Users 12, 4, and 15 to form a 3-user coalition through the merge rule since they improve their utilities from $\phi_4(\{4\}) = 0.2577$, $\phi_{12}(\{12\}) = 0.7638$, and $\phi_{15}(\{15\}) = 0$ in a non-cooperative manner to $V(\{4, 12, 15\}) = \{\boldsymbol{\phi}(\{4, 12, 15\}) = [1.7618\ 1.0169\ 0.6227]\}$.

In Fig. F.4 we show the performance, in terms of average utility (secrecy rate) per user, as a function of the network size N for both the DF and AF cases for a network with $K = 2$ eavesdroppers. First, we note that the performance of coalition formation with DF is increasing with the size of the network, while the non-cooperative and the AF case present an almost constant performance. For instance, for the DF case, Fig. F.4 shows that, by forming coalitions, the average individual utility (secrecy rate) per user is increased at all network sizes with the performance advantage of DF increasing with the network size and reaching up to 25.3% and 24.4% improvement over the non-cooperative and the AF cases, respectively, at $N = 45$. This is interpreted by the fact that, as the number of users N increases, the probability of finding candidate partners to form coalitions with, using DF, increases for every user. Moreover, Fig. F.4 shows that the performance of AF cooperation is comparable to the non-cooperative case. Hence, although AF relaying can improve the secrecy rate of large clusters of nearby cooperating users when no cost is accounted for such as in [10], in a practical wireless network and in the presence of a cooperation cost, the possibility of cooperation using AF for secrecy rate improvement is rare as demonstrated in Fig. F.4. This is mainly due to the strong dependence of the secrecy rate for AF cooperation on the channel between the users as per (F.5), as well as the fact that, for AF, unless highly favorable conditions exist (e.g. for coalitions such as $\{1, 6, 7, 10\}$ in Fig. F.2) , the amplification of the noise resulting from beamforming using AF relaying hinders the gains from cooperation relative to the secrecy cost during the information exchange phase.

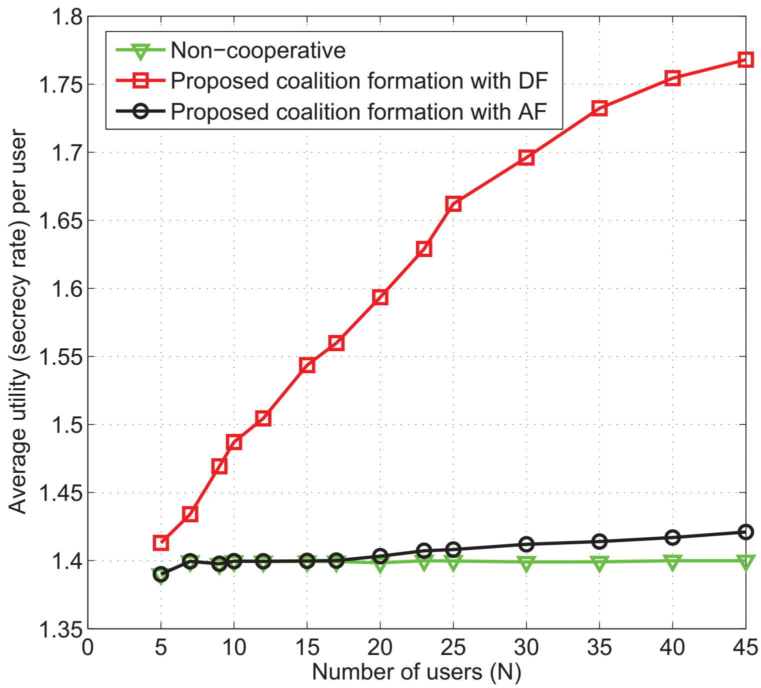

Fig. F.4: Performance in terms of the average individual user utility (secrecy rate) as a function of the network size N for $M = 2$ destinations and $K = 2$ eavesdroppers.

In Fig. F.5, we show the performance, in terms of average utility (secrecy rate) per user, as the number of eavesdroppers K increases for both the DF and AF cases for a network with $N = 45$ users. Fig. F.5 shows that, for DF, AF and the non-cooperative case, the average secrecy rate per user decreases as more eavesdroppers are present in the area. Moreover, for DF, the proposed coalition formation algorithm presents a performance advantage over both the non-cooperative case and the AF case at all K. Nonetheless, as shown by Fig.F.5, as the number of eavesdroppers increases, it becomes quite difficult for the users to improve their secrecy rate through coalition formation; consequently, at $K = 8$, all three

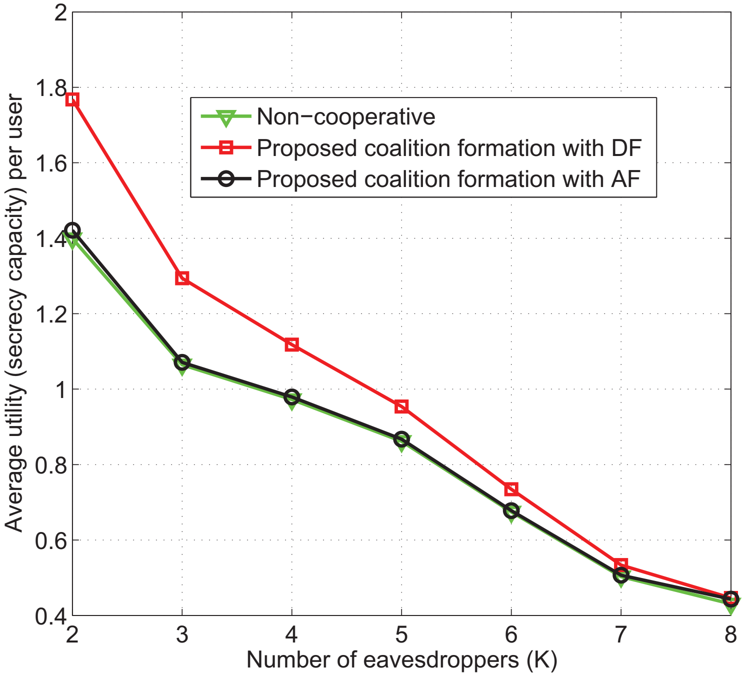

Fig. F.5: Performance in terms of the average individual user utility (secrecy rate) as a function of the number of eavesdroppers K for $N = 45$ users and $M = 2$ destinations.

schemes exhibit a similar performance. Finally, similar to the results of Fig. C.2, coalition formation using the AF cooperation protocol has a comparable performance with that of the non-cooperative case at all K as seen in Fig. F.5.

In Fig. F.6, for DF cooperation, we show the average and average maximum coalition size resulting from the proposed algorithm as the number of users, N, increases, for a network with $K = 2$ eavesdroppers. Fig. F.6 shows that both the average and average maximum coalition size increase with the number of users. This is mainly due to the fact that as N increases, the number of candidate cooperating partners increases. Further,

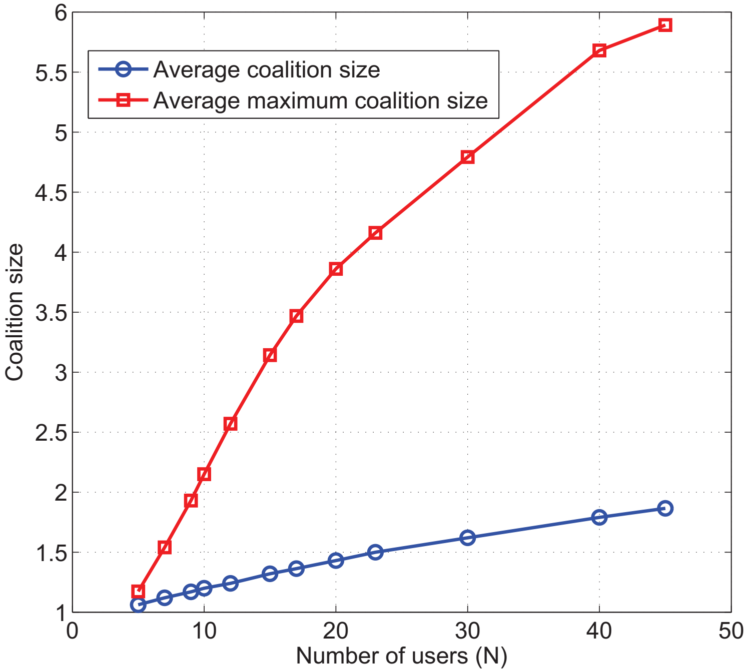

Fig. F.6: Average and average maximum coalition size as the network size N varies for $M = 2$ destinations and $K = 2$ eavesdroppers and DF cooperation.

through Fig. F.6 we note that the formed coalitions have a small average size and a relatively large maximum size reaching up to around 2 and 6, respectively, at $N = 45$. Since the average coalition size is below the minimum of 3 (as per Remark 1 due to having 2 eavesdroppers) and the average maximum coalition size is relatively large, the network structure is thus composed of a number of large coalitions with a few non-cooperative users.

In Fig. F.7, the performance, in terms of average utility (secrecy rate) per user, of the network for different cooperation costs, i.e., target average SNRs ν_0 is assessed. Fig. F.7 shows that cooperation through coalition formation with DF maintains gains, in terms of average secrecy rate per

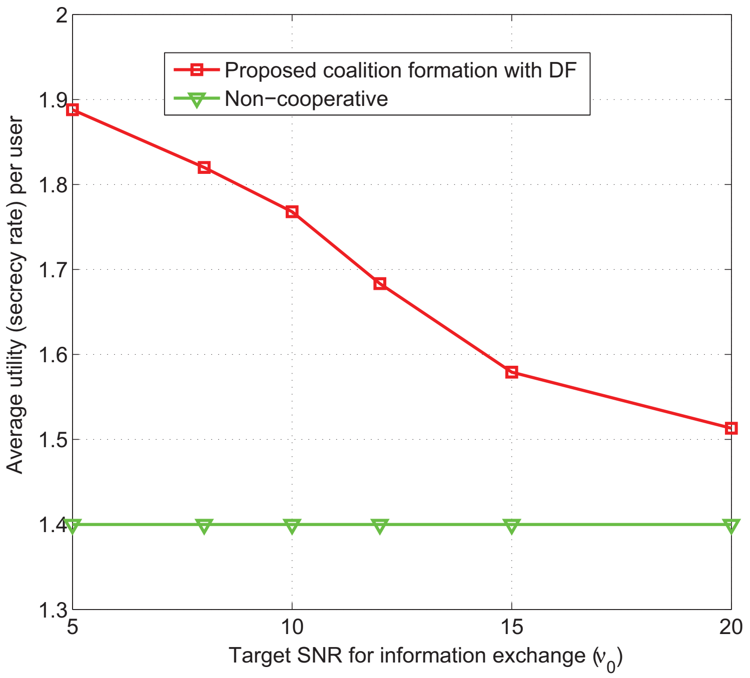

Fig. F.7: Average individual user utility as a function of the target SNR ν_0 for information exchange for a network with $N = 45$ users, $K = 2$ eavesdroppers and $M = 2$ destinations for DF.

user, at almost all costs (all SNR values). However, as the cost increases and the required target SNR becomes more stringent these gains decrease converging further towards the non-cooperative gains at high cost since cooperation becomes difficult due to the cost. As seen in Fig. F.7, the secrecy rate gains resulting from the proposed coalition formation algorithm range from 8.1% at $\nu_0 = 20$ dB to around 34.9% at $\nu_0 = 5$ dB improvement relative to the non-cooperative case.

The proposed algorithm's performance is further investigated in networks with $N = 20$ and $N = 45$ *mobile* users (random walk mobility) for a period of 5 minutes in the presence of $K = 2$ stationary eavesdroppers.

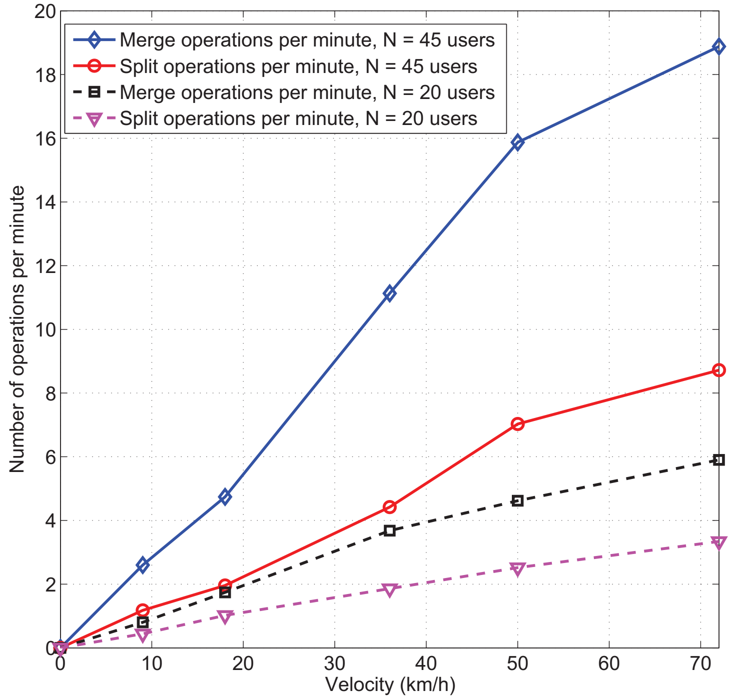

Fig. F.8: Frequency of merge and split operations per minute vs. speed of the users for different network sizes and $K = 2$ eavesdroppers with DF cooperation.

During this period, the proposed algorithm is run periodically every 30 seconds. The results in terms of the frequency of merge and split operations per minute are shown in Fig. F.8 for various speeds. As the speed increases, the frequency of both merge and split operations per minute increases due to the changes in the network structure incurred by the increased mobility. These frequencies reach up to around 19 merge operations per minute and 9 split operations per minute for $N = 45$ at a speed of 72 km/h. Finally, Fig. F.8 demonstrates that the frequency of merge and split operations increases with the network size N as the users become

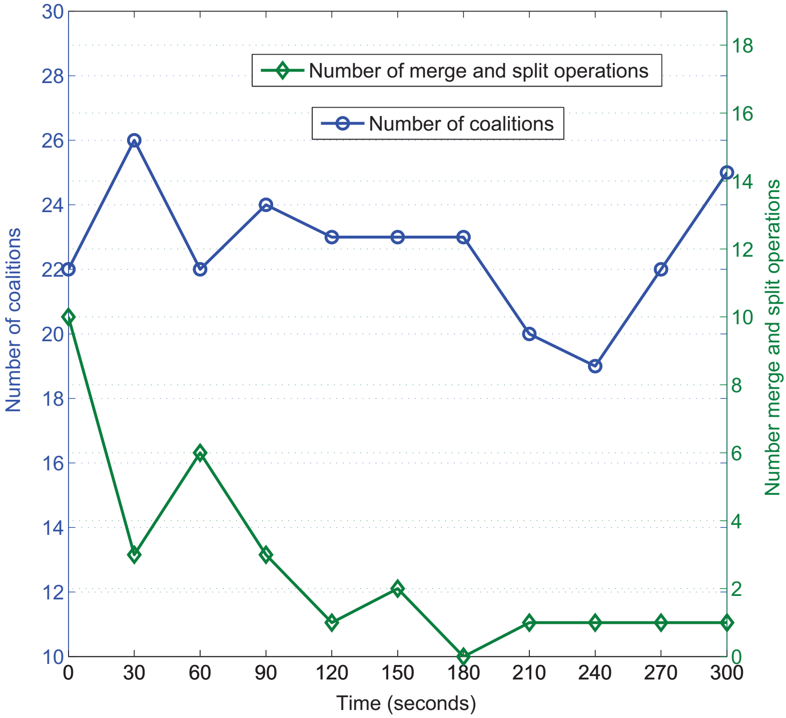

Fig. F.9: Evolution over time for a network with $N = 45$ users, $M = 2$ destinations, and $K = 2$ eavesdroppers with DF cooperation when the eavesdroppers are moving with a speed of 50 km/h.

more apt to finding new cooperation partners when moving which results in an increased coalition formation activity.

Fig. F.9 shows, for DF, how the structure of the wireless network with $N = 45$ users and $K = 2$ mobile eavesdroppers evolves and self-adapts over time (a period of 5 minutes), while both eavesdroppers are mobile with a constant velocity of 50 km/h. The proposed coalition formation algorithm is repeated periodically by the users every 30 seconds, in order to provide self-adaptation to mobility. First, the users self-organize into 22 coalitions after the occurrence of 10 merge and split operations at time $t = 0$. As time evolves, through adequate merge and split operations the network structure is adapted to the mobility of eavesdroppers. For example, at time $t = 1$ minute, through a total of 6 operations constituted of 5 merge and 1 split, the network structure changes from a partition of 26 coalitions back to a partition of 22 coalitions. Further, at $t = 3$ minutes, no merge or split operations occur, and, thus, the network structure remain unchanged. In summary, Fig. F.9 illustrates how the users can take adequate merge or split decisions to adapt the network structure to the mobility of the eavesdroppers.

6 Conclusions

In this paper, we have studied the user behavior, topology, and dynamics of a network of users that interact in order to improve their secrecy rate through both decode-and-forward and amplify-and-forward cooperation. We formulated the problem as a non-transferable coalitional game, and proposed a distributed and adaptive coalition formation algorithm. Through the proposed algorithm, the mobile users are able to take a distributed decision to form or break cooperative coalitions through well suited rules from cooperative games while maximizing their secrecy rate taking into account various costs for information exchange. We have characterized the network structure resulting from the proposed algorithm, studied its stability, and analyzed the self-adaptation of the topology to environmental changes such as mobility. Simulation results have shown that, for decode-and-forward, the proposed algorithm allowed the users to self-organize while improving the average secrecy rate per user up to 25.3% and 24.4% (for a network with 45 users) relative to the non-cooperative and amplify-and-forward cases, respectively.

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Paper G

Network Formation Games among the Relay Stations in Next Generation Wireless Networks

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Abstract

The introduction of relay station (RS) nodes is a key feature in next generation wireless networks such as 3GPP's long term evolution advanced (LTE-Advanced), or the forthcoming IEEE 802.16j WiMAX standard. This paper presents, using game theory, a novel approach for the formation of the tree architecture that connects the RSs and their serving base station in the *uplink* of the next generation wireless multi-hop systems. Unlike existing literature which mainly focused on performance analysis, we propose a distributed algorithm for studying the *structure* and *dynamics* of the network. We formulate a network formation game among the RSs whereby each RS aims to maximize a cross-layer utility function that takes into account the benefit from cooperative transmission, in terms of reduced bit error rate, and the costs in terms of the delay due to multi-hop transmission. For forming the tree structure, a distributed myopic algorithm is devised. Using the proposed algorithm, each RS can individually select the path that connects it to the BS through other RSs while optimizing its utility. We show the convergence of the algorithm into a Nash tree network, and we study how the RSs can adapt the network's topology to environmental changes such as mobility or the deployment of new mobile stations. Simulation results show that the proposed algorithm presents significant gains in terms of average utility per mobile station which is at least 21.5% better relatively to the case with no RSs and reaches up to 45.6% improvement compared to a nearest neighbor algorithm (for a network with 10 RSs). The results also show that the average number of hops does not exceed 3 even for a network with up to 25 RSs.

1 Introduction

Cooperation has recently emerged as a novel networking paradigm that can improve the performance of wireless communication networks at different levels. For instance, in order to mitigate the fading effects of the wireless channel, several nodes or relays can cooperate with a given source node in the transmission of its data to a far away destination, thereby, providing spatial diversity gains for the source node without the burden of having several antennas physically present on the node. This class of cooperation is commonly referred to as cooperative communications [1]. It has been demonstrated that by deploying one or multiple relays [1–3] a significant performance improvement can be witnessed in terms of throughput, bit error rate, capacity, or other metrics. In this regard, existing literature studied various aspects of cooperative transmission such as resource allocation [4], or link-level performance assessment [1–3]. Consequently, due to this performance gain that cooperative communications can yield in a wireless network, recently, the incorporation of relaying into next generation wireless networks has been proposed. In this context, the deployment of relay station (RS) nodes, dedicated for cooperative communications, is a key challenge in next generation networks such as 3GPP's long term evolution advanced (LTE-Advanced) [5] or the forthcoming IEEE 802.16j WiMAX standard [6].

For an efficient deployment of RSs in next generation networks, several key technical challenges need to be addressed at both the uplink and downlink levels. For the downlink of 802.16j networks, in [7], the authors study the optimal placement of one RS which maximizes the total rate of transmission. In [8], the authors study the capacity gains and the resource utilization in a multi-hop LTE network in the presence of RSs. Further, the performance of different relaying strategies in an LTE-Advanced network is studied in [9]. In [10], the use of dual relaying is studied in the context of 802.16j networks with multiple RSs. Resource allocation and network planning techniques for 802.16j networks in the presence of RSs are proposed in [11]. Furthermore, the authors in [12] study the possibility of coverage extension in an LTE-Advanced system, through the use of relaying. In [13], the communication possibilities between the RSs and the base station is studied and a need-basis algorithm for associating the RSs to their serving BS is proposed for LTE-Advanced networks. The possibilities for handover in an LTE network in the presence of RSs are analyzed in [14]. Other aspects of RS deployment in next generation networks are also considered in [15–19].

Although the performance assessment and operational aspects of RS deployment in next generation multi-hop networks such as LTE-Advanced or 802.16j has been thoroughly studied, one challenging area which remains relatively unexplored is the formation of the tree architecture connecting the BS to the RSs in its coverage area. One contribution toward tackling this problem in 802.16j networks has been made in [17] through a centralized approach. However, the work in [17] does not provide a clear algorithm for the tree formation nor does it consider cooperative transmission or multi-hop delay. In addition, a centralized approach can yield some significant overhead and complexity, namely in networks with a rapidly changing environment due to RS mobility or incoming traffic load. In our previous work [18, 19], we proposed game theoretical approaches to tackle the formation of a tree structure in an 802.16j network. However, the model in [18] does not account for the costs in terms of the delay incurred by multi-hop transmission while [19] is limited to delay tolerant VoIP networks and does not account for the effective throughput of the nodes. In order to take into account both the effective throughput and the delays in the network due to the traffic flow (queueing and transmission delay) for generic services, new models and algorithms, inherently different from [18, 19], are required.

The main contribution of this paper is to study the distributed formation of the network architecture connecting the RSs to their serving base station in next generation wireless systems such as LTE-Advanced or WiMAX 802.16j. Another key contribution is to propose a cross-layer utility function that captures the gains from cooperative transmission, in terms of a reduced bit error rate and improved effective throughput, as well as the costs incurred by multi-hop transmission in terms of delay. For this purpose, we formulate a network formation game among the RSs in next generation networks, and we build a myopic algorithm in which each RS selects the strategy that maximizes its utility. We show that, through the proposed algorithm, the RSs are able to self-organize into a Nash network tree structure rooted at the serving base station. Moreover, we demonstrate how, by periodic runs of the algorithm, the RSs can take autonomous decisions to adapt the network structure to environmental changes such as incoming traffic due to new mobile stations being deployed as well as mobility. Through simulations, we show that the proposed algorithm leads to a performance gain, in terms of average utility per mobile station, of at least 21.5% compared to the case with no RSs and up to 45.6% compared to a nearest neighbor algorithm.

The rest of this paper is organized as follows: Section 2 presents the system model and the game formulation. In Section 3, we introduce the cross-layer utility model and present the proposed network formation algorithm. Simulation results are presented and analyzed in Section 4. Finally, conclusions are drawn in Section 5.

2 System Model and Game Formulation

Consider a network of M RSs that can be either fixed, mobile, or nomadic. The RSs transmit their data in the uplink to a central base station (BS) through multi-hop links, and, therefore, a tree architecture needs to form, in the uplink, between the RSs and their serving BS. Once the uplink network structure forms, mobile stations (MSs) can hook to the network by selecting a serving RS or directly connecting to the BS. In this context, we consider that the MSs deposit their data packets to the serving RSs using direct transmission. Subsequently, the RSs in the network that received the data from the external MSs, can act as source nodes transmitting the received MS packets to the BS through one or more hops in the formed tree, using cooperative transmission. The considered direct transmission between an MS and its serving RS enables us to consider a tree formation algorithm that can be easily incorporated in a new or existing wireless networks without the need of coordination with external entities such as the MSs.

To perform cooperative transmission between the RSs and the BS, we consider a decoded relaying multi-hop diversity channel, such as the one in [3]. In this relaying scheme, each intermediate node on the path between a transmitting RS and the BS combines, encodes, and re-encodes the received signal from all preceding terminals before relaying (decode-and-forward). Formally, every MS k in the network constitutes a source of data traffic which follows a Poisson distribution with an average arrival rate λ_k. With such Poisson streams at the entry points of the network (the MSs), for every RS, the incoming packets are stored and transmitted in a first-in first-out (FIFO) fashion and we consider that we have the Kleinrock independence approximation [20, Chap. 3] with each RS being an M/D/1

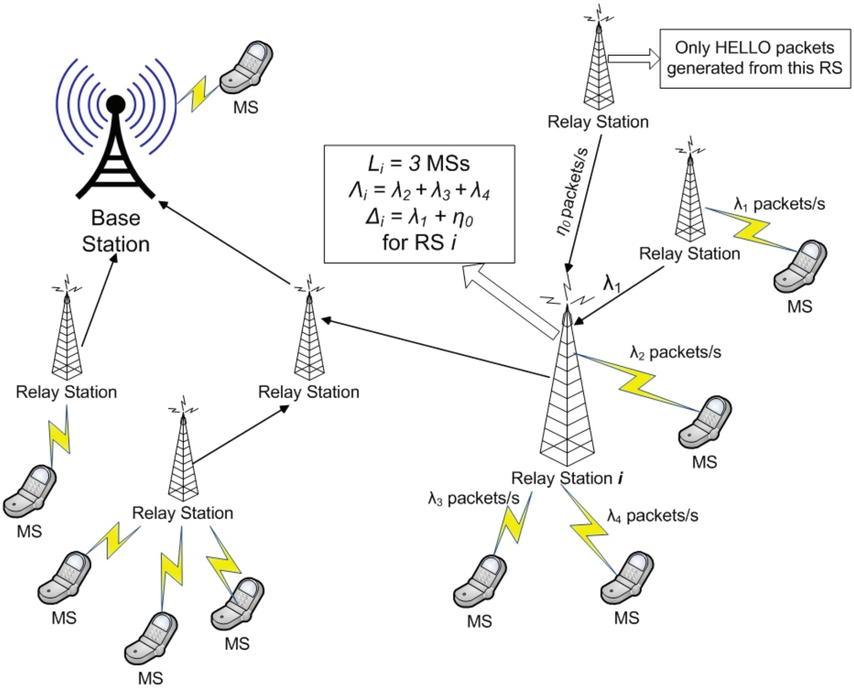

Fig. G.1: A prototype of the uplink tree model.

queueing system[31]. With this approximation, the total traffic that an RS i receives from the MSs that it is serving is a Poisson process with an average total arrival rate of $\Lambda_i = \sum_{l \in \mathcal{L}_i} \lambda_l$ where $\mathcal{L}_i$ is the set of MSs served by an RS i of cardinality $|\mathcal{L}_i| = L_i$. Moreover, RS i also receives packets from RSs that are connected to it with a total average rate Δ_i. For these Δ_i packets (received from other RSs), the sole role of RS i is to relay them to the next hop. In addition, any RS i that has no assigned MSs and no connected RSs ($\mathcal{L}_i = \emptyset$, $\Lambda_i = 0$, and $\Delta_i = 0$), transmits "HELLO" packets, generated with a Poisson arrival rate of η_0 in order to maintain its link to the BS active during periods of no actual traffic in the network. An illustrative example of this model is shown in Fig. G.1.

Given this network, the main objective is to provide a formulation that can adequately model the interactions between the RSs that seek to form

[31]Any other queueing model, e.g., M/M/1, can also be accommodated.

the uplink multi-hop tree architecture. For this purpose, we refer to the analytical framework of network formation games [21–24]. Network formation games constitute a subclass of problems which involve a number of independent decisions makers (players) that interact in order to form a suited graph that connects them. The final network graph G that results from a given network formation game is highly dependent on the goals, objectives, and incentives of every player in the game. Consequently, we model the proposed uplink tree formation problem as a network formation game among the RSs where the result of the interactions among the RSs is a *directed* graph $G(\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{1, \ldots, M+1\}$ denoting the set of all vertices (M RSs and the BS) that will be present in the graph and $\mathcal{E}$ denoting the set of all edges (links) that connect different pairs of RSs. Each directed link between two RSs i and j, denoted $(i, j) \in \mathcal{E}$, corresponds to an uplink traffic flow from RS i to RS j. We define the following notion of a path:

Definition 36 *Given any network graph $G(\mathcal{V}, \mathcal{E})$, a* path *between two nodes $i \in \mathcal{V}$ and $j \in \mathcal{V}$ is defined as a sequence of nodes $i_1, \ldots, i_K$ (in $\mathcal{V}$) such that $i_1 = i$, $i_K = j$ and each directed link $(i_k, i_{k+1}) \in G$ for each $k \in \{1, \ldots, K-1\}$.*

In this paper, we consider solely multi-hop tree (or forest, if some parts of the graph are disconnected) architectures, since such architectures are ubiquitous in next generation networks [6, 8, 9]. In this regard, throughout the paper we adopt the following convention:

Convention 2 *Each RS i is connected to the BS through at most* one *path, and, thus, we denote by q_i the path between any RS i and the BS whenever this path exists.*

Finally, we delineate the possible actions or strategies that each RS can take in the proposed network formation game. In this regard, for each RS i, the action space consists of the RSs (or the BS) that RS i wants to use as its next hop. Therefore, the strategy of an RS i is to select the link that it wants to form from its available action space. We note that, an RS i cannot connect to an RS j which is already connected to i, in the sense that if $(j, i) \in G$, then $(i, j) \notin G$. Hence, for a given graph G that governs the current network architecture, we let $\mathcal{A}_i = \{j \in \mathcal{V} \setminus \{i\} | (j, i) \in G\}$ denote the set of RSs from which RS i accepted a link (j, i), and $\mathcal{S}_i = \{(i, j) | j \in \mathcal{V} \setminus (\{i\} \bigcup \mathcal{A}_i)\}$ denote the set of links corresponding to the nodes (RSs or the BS) with whom i wants to connect (note that i cannot connect to RSs that are already connected to it, i.e., RSs in $\mathcal{A}_i$). Accordingly, the strategy of an RS i is to select the link $s_i \in \mathcal{S}_i$ that it wants to form, i.e., choose the RS that it will connect to. Based on Convention 1, an RS can be connected to at most *one* other node in our game so selecting to form a link s_i implicitly implies that RS i will *replace* its previously connected link (if any) with the new link s_i. Further, to each selection s_i by an RS i corresponds a path q_i to the BS (if $s_i = \emptyset$, then the RS chooses to be disconnected from the network).

3 Network Formation Game: Utility Function and Algorithm

3.1 Cross-layer Utility Function

Given the proposed network formation game model, the next step is to define a utility function that can capture the incentives of the RSs to connect to each others. In this context, we propose a cross-layer utility function that takes into account the performance measures in terms of the packet success rate (PSR) as well as the delay induced by multi-hop transmission. Hence, considering any tree network graph G, each RS in the network will be given a positive utility for every packet that is transmitted/relayed successfully to the BS out of all the packets that this RS has received from the external MSs. In this regard, every packet transmitted by any RS is subject to a bit error rate (BER) due to the communication over the wireless channel using one or more hops. For any data transmission between an RS $V_1 \in \mathcal{V}$ to the BS, denoted by V_{n+1}, going through $n-1$ intermediate RSs $\{V_2, \ldots, V_n\} \subset \mathcal{V}$, let N_r be the set of all receiving terminals, i.e., $N_r = \{V_2 \ldots V_{n+1}\}$ and $N_{r(i)}$ be the set of terminals that transmit a signal received by a node V_i. Hence, for an RS V_i on the path from the source V_1 to the destination V_{n+1}, we have $N_{r(i)} = \{V_1, \ldots, V_{i-1}\}$. Therefore, given this notation, the BER achieved at the BS V_{n+1} between a source RS $V_1 \in \mathcal{V}$ that is sending its data to the BS along a path $q_{V_1} = \{V_1, \ldots, V_{n+1}\}$ can be calculated through the tight upper bound given in [3, Eq. (10)] for the decoded relaying multi-hop diversity channel with Rayleigh fading and BPSK modulation[32] as follows

$$P^e_{q_{V_1}} \leq \sum_{N_i \in N_r} \frac{1}{2} \left(\sum_{N_k \in N_{r(i)}} \left[\prod_{\substack{N_j \in N_{r(i)} \\ N_j \neq N_k}} \frac{\gamma_{k,i}}{\gamma_{k,i} - \gamma_{j,i}} \left(1 - \sqrt{\frac{\gamma_{k,i}}{\gamma_{k,i}+1}} \right) \right] \right). \tag{G.1}$$

[32]The approach in this paper is not restricted to this channel and BPSK signal constellation since the algorithm proposed in the following section can be tailored to accommodate other types of relay channels as well as other modulation techniques.

Here, $\gamma_{i,j} = \frac{P_i \cdot h_{i,j}}{\sigma^2}$ is the average received SNR at node j from node i where P_i is the transmit power of node i, σ^2 the noise variance and $h_{i,j} = \frac{1}{d_{i,j}^{\mu}}$ is the path loss with $d_{i,j}$ the distance between i and j and μ the path loss exponent. Finally, for RS i which is connected to the BS through a *direct transmission* path q_i^d with no intermediate hops, the BER can be given by $P^e_{q_i^d} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_{i,BS}}{1+\gamma_{i,BS}}}\right)$ [2], [3]; where $\gamma_{i,BS}$ is the average received SNR at the BS from RS i. Using the BER expression in (G.1) and by having no channel coding, the PSR ρ_{i,q_i} perceived by an RS i over any path q_i is defined as follows

$$\rho_{i,q_i}(G) = (1 - P^e_{q_i})^B, \tag{G.2}$$

where B is the number of bits per packet. The PSR is a function of the network graph G as the path q_i varies depending on how RS i is connected to the BS in the formed network tree structure.

Communication over multi-hop wireless links yields a significant delay due to multi-hop transmission as well as buffering. Therefore, we let τ_{i,q_i} denote the average delay over the path $q_i = \{i_1, \ldots, i_k\}$ from an RS $i_1 = i$ to the BS. Finding the exact average delay over a path of consecutive queues is a challenging problem in queueing systems [20]. One possible approach for measuring the average delay along a path q_i in a network with Poisson arrivals at the entry points is to consider the Kleinrock approximation as mentioned in the previous section. In this context, the average delay over any path q_i can be given by [20, Chap. 3, Eqs. (3.42), (3.45), and (3.93)]

$$\tau_{i,q_i}(G) = \sum_{(i_k,i_{k+1}) \in q_i} \left(\frac{\Psi_{i_k,i_{k+1}}}{2\mu_{i_k,i_{k+1}}(\mu_{i_k,i_{k+1}} - \Psi_{i_k,i_{k+1}})} + \frac{1}{\mu_{i_k,i_{k+1}}} \right). \tag{G.3}$$

where $\Psi_{i_k,i_{k+1}} = \Lambda_{i_k} + \Delta_{i_k}$ is the total traffic (packets/s) traversing link $(i_k, i_{k+1}) \in q_i$ between RS i_k and RS i_{k+1} and originating from the L_{i_k} MSs in the set $\mathcal{L}_{i_k}$ of MSs connected to RS i_k ($\Lambda_{i_k} = \sum_{i \in \mathcal{L}_{i_k}} \lambda_i$) and from all RSs that are connected to i_k ($\Delta_{i_k} = \sum_{j \in \mathcal{A}_{i_k}} \Lambda_j$). The ratio $\frac{1}{\mu_{i_k,i_{k+1}}}$ represents the average transmission time (service time) on link $(i_k, i_{k+1}) \in q_i$ with $\mu_{i_k,i_{k+1}}$ being the service rate on link (i_k, i_{k+1}). This service rate is given by $\mu_{i_k,i_{k+1}} = \frac{C_{i_k,i_{k+1}}}{B}$ with $C_{i_k,i_{k+1}} = W \log(1 + \nu_{i_k,i_{k+1}})$ the capacity of the direct transmission between RS i_k and RS i_{k+1}, where $\nu_{i_k,i_{k+1}} = \frac{P_{i_k} h_{i_k,i_{k+1}}}{\sigma^2}$ is the received SNR from RS i_k at RS i_{k+1}, and W is the bandwidth available for RS i_k which is assumed the same for all RSs in the set of vertices $\mathcal{V}$, without loss of generality. Similar to the PSR, the delay depends on the paths from the RSs to the BS, and, hence, it is a function of the network graph G.

A suitable criterion for characterizing the utility in networks where the users' quality of service is sensitive to throughput as well as to delay is the concept of *system power*. In this context, power is defined as the ratio of some power of the throughput and the delay [25]. Hence, the concept of power is an attractive notion that allows one to capture the fundamental tradeoff between throughput and delay in the proposed network formation game. In fact, the concept of power has been used thoroughly in the literature to model applications where there exists a tradeoff between throughput and delay [26–29]. Consequently, given the delay and the PSR, we define the utility of an RS i with L_i connected MSs, as the power achieved by i which is given by

$$u_i(G) = \begin{cases} \dfrac{(\Lambda_i \cdot \rho_{i,q_i}(G))^{\beta}}{\tau_{i,q_i}(G)^{(1-\beta)}}, & \text{if } L_i > 0, \\ \dfrac{(\eta_0 \cdot \rho_{i,q_i}(G))^{\beta}}{\tau_{i,q_i}(G)^{(1-\beta)}}, & \text{if } L_i = 0, \end{cases} \tag{G.4}$$

where $\tau_{i,q_i}(G)$ is the delay given by (G.3), $\Lambda_i \cdot \rho_{i,q_i}(G)$ represents the effective throughput of RS i and $\beta \in (0,1)$ is a tradeoff parameter. The utility in (G.4) can model a general class of services, with each class of service having a different β. As β increases, the service becomes more delay tolerant and more throughput demanding. Note that, unless stated otherwise, throughout the rest of the paper the term "power" will refer to the ratio of throughput to delay and not to the transmit power of the RSs or MSs unless clearly stated as "transmit power".

Once the RSs form the tree topology, one needs to assess the performance of the MSs in terms of the power achieved by these MSs (considered as MS utility). In order to compute the utility of the MSs, the PSR as well as the delay over the whole transmission from MS to BS must be taken into account. Hence, given the proposed network model in Section 2, for each MS $i \in \mathcal{L}_j$ served by an RS j, the PSR is given by

$$\zeta_{i,j}(G) = \rho_{i,(i,j)} \cdot \rho_{j,q_j}(G), \tag{G.5}$$

where $\rho_{i,(i,j)}$ is the PSR on the direct transmission between MS i and RS j (which does not depend on the existing network graph G between the RSs) and $\rho_{j,q_j}(G)$ is the PSR from RS j to the BS along path q_j given by

(G.2) (the path q_j can be either a multi-hop path or a direct transmission depending on how RS j is connected in the graph G that governs the RSs' network). Furthermore, for any MS $i \in \mathcal{L}_j$ connected to an RS j, the delay for transmitting the data to the BS is given by (G.3) by taking into account, in addition to the delay on the RS's path q_j, the data traffic on the link (i, j) between the MS and the RS, i.e., the buffering and transmission delay at the MS level. Having the PSR given by (G.5) and the delay, the utility of a MS i connected to RS j is given by

$$v_i(G) = \frac{(\lambda_i \cdot \zeta_{i,j}(G))^{\beta}}{\tau_{i,q_j}(G)^{(1-\beta)}}. \tag{G.6}$$

Consequently, throughout the paper (unless stated otherwise) we consider that whenever an MS enters the network, it will connect to the RS which maximizes its utility in (G.6) given the current network topology G. This MS assignment is considered fixed as long as the RSs' network does not change, otherwise, the MSs can re-assess their utilities and change their assignment once to adapt to the changes in the RSs' network. Although more advanced techniques such as a non-cooperative Nash game can be used for assigning the MSs to the RS, these techniques are out of the scope of this paper and will be the subject of future work.

3.2 Network Formation Algorithm

Given the devised utility functions in the previous subsection, the next step in the proposed RSs' network formation game is to find an algorithm that can model the interactions among the RSs that seek to form the network tree structure. First, we show that, for any network formation algorithm, the resulting graph in the proposed game is a connected tree structure as follows:

Property 8 *The network graph resulting from any network formation algorithm for the proposed RSs game is a* connected *directed tree structure rooted at the BS.*

Proof: *Consider an RSs network graph G whereby an RS i is disconnected from the BS, i.e., no path of transmission (direct or multi-hop) exists between i and the BS. In this case, one can see that, the delay for all the packets at the disconnected RS i is infinite, i.e., $\tau_{i,q_i}(G) = \infty$, and, thus, the corresponding power is 0 as per the utility function in (G.4). As a result, there is no incentive for any RS in the network to disconnect from the BS* *since such a disconnection will drastically decrease its utility. Hence, any network graph G formed using the proposed RSs network formation game is a connected graph and due to Convention 1, this graph is a tree rooted at the BS.* □

A direct result of this property is that, if any RS is unable to connect to another suitable RSs for forming a link, this RS will connect to the BS using direct transmission. In this regards, we consider that the initial starting point for our network formation game is a star topology whereby all the RSs are connected directly to the BS, prior to interacting for further network formation decisions.

Whenever an RS i plays a strategy $s_i \in \mathcal{S}_i$ while all the remaining RSs maintain a vector of strategies $\boldsymbol{s}_{-i}$, we let $G_{s_i,\boldsymbol{s}_{-i}}$ denote the resulting network graph. By inspecting the RS utility in (G.4), one can clearly notice that, whenever an RS j accepts a link, due to the increased traffic that it receives, its utility may decrease as the delay increases. In this context, although each RS $i \in \mathcal{N}$ can play any strategy from its strategy space $\mathcal{S}_i$, there might exist some link $s_i = (i, j) \in \mathcal{S}_i$ where the receiving RS, i.e., RS j, does not accept the formation of s_i, if this leads to a significant decrease in its utility. In this regard, denoting by $G + s_i$ as the graph G modified by adding link $s_i = (i, j)$, we define the concept of a *feasible* strategy as follows:

Definition 37 *A strategy $s_i \in \mathcal{S}_i$ is a* feasible strategy *for an RS $i \in \mathcal{V}$ if and only if $u_{s_i}(G_{s_i,\boldsymbol{s}_{-i}} + s_i) \geq u_{s_i}(G_{s_i,\boldsymbol{s}_{-i}}) - \epsilon$ where ϵ is a small positive number. For any RS $i \in \mathcal{V}$, the set of all feasible strategies is denoted by $\hat{\mathcal{S}}_i \subseteq \mathcal{S}_i$.*

In a nutshell, given a network graph G, a feasible strategy for any RS $i \in \mathcal{V}$ is to select an RS among all the RSs that are willing to accept a connection from RS i, i.e., a feasible path, which maximizes its utility. On the other hand, any RS $j \in \mathcal{V}$ is willing to accept a connection from any other RS $i \in \mathcal{V}$ as long as the formation of the link (i, j) does not decrease the utility of j by more than ϵ. The main motivation for having $\epsilon > 0$ (sufficiently small) is that, in many cases, e.g., when the network has only HELLO packets circulating (no MS traffic), RS j might be willing to accept the formation of a link which can slightly decrease its utility at a given moment, but, as more traffic is generated in the network, this link can entail potential future benefits for RS j stemming from an increased effective throughput (recall that the utility in (G.4) captures the tradeoff between effective throughput and delay).

For any RS $i \in \mathcal{V}$, given the set of feasible strategies $\hat{\mathcal{S}}_i$, we define the *best response* for RS i as follows [23].

Definition 38 *A strategy $s_i^* \in \hat{\mathcal{S}}_i$ is a* best response *for an RS $i \in \mathcal{V}$ if $u_i(G_{s_i^*,\boldsymbol{s}_{-i}}) \geq u_i(G_{s_i,\boldsymbol{s}_{-i}})$, $\forall s_i \in \hat{S}_i$. Thus, the best response for RS i is to select the* feasible *link that maximizes its utility given that the other RSs maintain their vector of feasible strategies $\boldsymbol{s}_{-i}$.*

Subsequently, given the various properties of the RS network formation game, we devise a network formation algorithm based on the feasible best responses of the RSs. For this purpose, first, we consider that the RSs are myopic, such that each RS aims at improving its utility given only the current state of the network without taking into account the future evolution of the network. Developing an optimal myopic network formation algorithm is highly complex since there exists no formal rules for network formation in the literature [21]. For instance, depending on the model, utilities, and incentives of the players, different network formation rules can be applied. In this context, the game theoretical literature on network formation games presents various myopic algorithms for different game models with directed and undirected graphs [21–23]. For the network formation game among the RSs, we build a myopic algorithm for network formation inspired from those in [21] and [23]. In this regard, we define an algorithm where each round is mainly composed of three phases: a fair prioritization phase, a myopic network formation phase, and a multi-hop transmission phase. Hence, the proposed algorithm starts with a fair prioritization phase where each RS is given a priority depending on different criteria. For the purpose of exposition, in this paper, we consider a priority scheme that depends on the BER of the RSs as follows: RSs with a higher BER are assigned a higher priority. The main rationale behind this selection of priority is in order to fairly allow RSs that are perceiving a bad channel to possess an advantage in selecting their partners, for the purpose of improving their BER. By giving a priority advantage to RSs with high BER, these RSs can have a larger space of strategies out of which they can select a partner during the myopic network formation phase. Other priority functions can also be used, and in a general case, a random priority function can be defined.

The second phase of the proposed algorithm is the myopic network formation phase. During myopic network formation, the RSs engage in pairwise interactions, sequentially by order of priority, in order to make their network formation decisions. In this phase, each RS i can select a certain feasible strategy which allows it to improve its payoff. An *iteration* consists of a single sequence of plays during which all M RSs have made their strategy choice to myopically react to the choices of the other RSs. The myopic network formation phase can consist of one or more iteration. For every iteration t, we define the set $\mathcal{G}_t$ of all graphs that were reached at the end of all the iterations up to t, i.e., the set of all graphs that were formed at the end of iterations 1 till $t-1$ (at the beginning of all time, i.e., at $t = 0$, $\mathcal{G}_0 = \emptyset$). In every iteration t, during its turn, each RS i chooses to play its best response $s_i^* \in \hat{\mathcal{S}}_i \setminus \mathcal{S}_{\mathcal{G}_t}$ in order to maximize its utility at each round given the current network graph resulting from the strategies of the other RSs. The set $\mathcal{S}_{\mathcal{G}_t}$ represents the set of all strategies that RS i can take and which will yield a graph in $\mathcal{G}_t$. The main motivation behind excluding all the strategies that yield a graph in $\mathcal{G}_t$ is that, an RS i has no incentive to revisit a graph that was already left in the past. This can be seen as a basic learning scheme that the RSs can implement with low complexity (each RS can be made aware of the graph reached at the end of any iteration t by the BS or neighboring RSs). The best response of each RS can be seen as a *replace* operation, whereby the RS will replace its current link to the BS with another link that maximizes its utility (if such a link is available). Multiple iterations will be run until convergence to the final tree structure G_T where the RSs can no longer improve their utility through best responses. The convergence of the myopic network formation phase of the proposed algorithm is given by the following Theorem:

Theorem 1 *Given any initial network graph G_0, the myopic network formation phase of the proposed algorithm converges to a final network graph G_T after T iterations.*

Proof: *Every iteration t of the myopic network formation phase of the proposed algorithm can be seen as a sequence of best responses played by the RSs. In this regard, denoting by G_t the graph reached at the* end of *any iteration t, the myopic network formation phase consists of a sequence such as the following (as an example)*

$$G_0 \to G_1 \to G_2 \to \cdots \to G_t \to \cdots$$

At any iteration t, each RS i selects its best response out of the strategy space $\hat{\mathcal{S}}_i \setminus \mathcal{S}_{\mathcal{G}_t}$, hence, if any RS i plays a best response at iteration t, then surely $G_t \neq G_l$, $\forall l < t$. This process continues until finding an iteration where no RS can find any strategy to play. Given this property and the fact that the number of spanning trees for any graph is finite, *then the sequence* *in (3.2) will always converge to a final graph G_T after T iterations. Hence, the myopic network formation phase of our proposed algorithm always converges.* □

After the convergence of the network formation phase of the algorithm, the RSs are connected through a tree structure G_T and the third phase of the algorithm begins. This phase represents the actual data transmission phase, whereby the multi-hop network operation occurs as the RSs transmit the data over the existing tree architecture G_T. A summary of the proposed algorithm is given in Table G.I.

To study the stability of any graph G_T resulting from the proposed network formation algorithm, we utilize the concept of Nash equilibrium applied to network formation games as follows [23]:

Definition 39 *At an iteration T, a network graph $G_T(\mathcal{V}, \mathcal{E})$ in which no node i can improve its utility by a unilateral change in its strategy $s_i \in \mathcal{S}_i \setminus \mathcal{G}_T$ is a* Nash network *in the strategy space $\mathcal{S}_i \setminus \mathcal{G}_T,\ \forall i \in \mathcal{V}$.*

Therefore, in our proposed game, a Nash network is a network where no RS can improve its utility given the current strategies of all other RSs. For the proposed algorithm, we have the following property:

Lemma 1 *The final tree structure G_T resulting from the proposed algorithm is a Nash network in the strategy space $\mathcal{S}_i \setminus \mathcal{G}_T,\ \forall i \in \mathcal{V}$.*

Proof: *This lemma is a direct consequence of Theorem 1. Since the myopic network formation phase of the proposed algorithm is based on the best responses of the RSs at each iteration t in their strategy spaces $\mathcal{S}_i \setminus \mathcal{G}_t,\ \forall i \in \mathcal{V}$, then the convergence of the algorithm, as guaranteed by Theorem 1 reaches a Nash network (the convergence of a best response algorithm reaches a Nash equilibrium [30]) where no RS can unilaterally deviate from its strategy.* □

Furthermore, as the RSs can engage in the myopic network formation phase prior to any MS deployment, we consider the following convention throughout the rest of this paper:

Convention 3 *At the beginning of all time, once the operator deploys the network, the RSs engage in the network formation game by taking into account their utilities in terms of HELLO packets, prior to any mobility or presence of MSs.*

The main motivation behind Convention 3 is that the RSs can form an initial tree structure which shall be used by any MSs that will be deployed

Table G.I: Proposed network formation algorithm.

Initial State

The starting network is a graph where the RSs are directly connected to the BS (star network).

The proposed algorithm consists of three phases

Phase 1 - Fair Prioritization:

Each RS is given a priority depending on different criteria.
One example priority is to prioritize the RSs based on their BER (a lower BER implies a higher priority).

Phase 2 - Myopic Network Formation:

repeat

By order of priority, the RSs engage in a network formation game.

a) In every iteration t of Phase 2, each RS i plays its feasible best response $s_i^* \in \hat{\mathcal{S}}_i \setminus \mathcal{S}_{\mathcal{G}_t}$ (with $\mathcal{G}_t$ being the set of all graphs visited at the end of iterations 1 till $t-1$), maximizing its utility.

b) The best response s_i^* of each RS is a *replace* operation through which an RS i splits from its current parent RS and replaces it with a new RS that maximizes its utility, given that this new RS *accepts* the formation of the link.

until convergence to a final Nash tree G_T after T iterations.

Phase 3 - Multi-hop Transmission:

During this phase, data transmission from the MSs occurs using the formed network tree structure G_T.

For changing environments (e.g. due to mobility or the deployment of new MSs), multiple rounds of this algorithm are run *periodically* every time period θ, allowing the RSs to adapt the network topology.

in the network. If any adaptation to this structure is needed, periodic runs of the proposed algorithm can occur as discussed further in this section.

The proposed algorithm can be implemented in a distributed way within any next generation wireless multi-hop network, with a little reliance on the BS. For instance, the role of the BS in the proposed network formation algorithm is two-fold: (i)- to inform the RSs of their priorities during the prioritization phase, and (ii)- to inform the RSs of the graphs reached during past iterations. For both cases, the BS can simply send this information through a control channel. Due to the fact that the number of RSs within the area of a single BS is relatively small, the signalling and overhead for this information exchange between the BS and the RSs is minimal. With regards to the priorities, if the environment is invariant (e.g. the RSs are static), then the BS can inform the RSs of their priorities at the beginning of all time without any need to resend this information. Beyond this, the algorithm relies on distributed decisions taken by the RSs. Within every iteration t, during its turn, each RS can engage in pairwise negotiations with the surrounding RSs in order to find its best response, among the set of feasible strategies and given the graphs that were reached in previous iterations. In this regard, the worst case complexity of finding a suited partner for any RS i is $O(M)$ where M is the total number of RSs. In practice, the complexity is much smaller as the RSs do not negotiate with the RSs that are connected to them, nor with the RSs that can lead to a graph visited at previous iterations. In order to evaluate its utility while searching for the best response, each RS can easily acquire the BER and an estimate of the delay that each neighbor can provide. As a result, each RS i can take an individual decision to select the link s_i^* that can maximize its utility. The signaling required for gathering this information can be minimal as each RS can measure its current channel towards the BS as well as the flowing traffic and feed this information back to any RS that requests it during the pairwise negotiations. In dynamically changing environments, following the formation of the initial tree structure as per Convention 3, the network formation process is repeated periodically every θ allowing the RSs to take autonomous decisions to update the topology adapting it to any environmental changes that occurred during θ such as the deployment of MSs, mobility of the RSs and/or MSs, among others. In fact, engaging in the network formation game periodically rather than continuously reduces the signalling in the network, while allowing the topology to adapt itself to environmental changes. As the period θ is chosen to be smaller, the network formation game is played more often, allowing a better adaptation to networks with rapidly changing environments at the expense of extra signalling and overhead. Note that, when the RSs are mobile, and/or when new MSs are entering and leaving the network, the MSs can also, periodically, change their serving RS, to adapt to this change in the network.

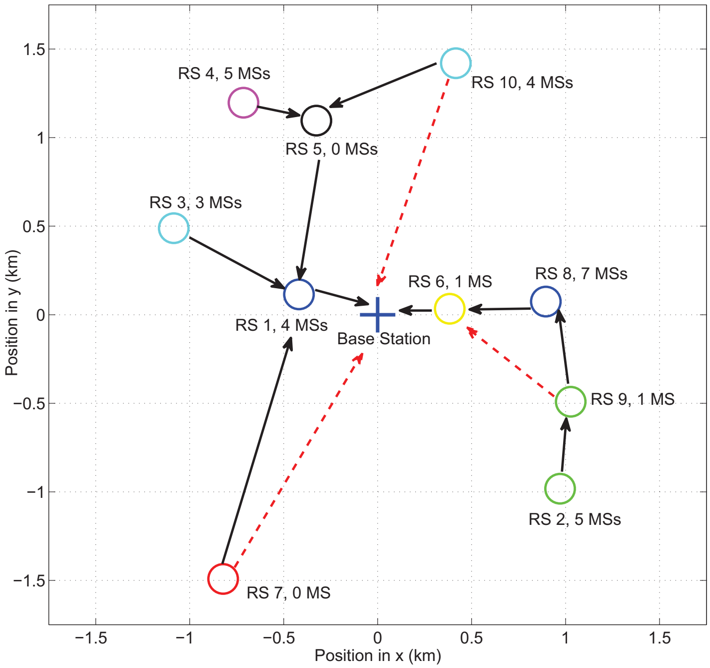

Fig. G.2: Snapshot of a tree topology formed using the proposed network algorithm with $M = 10$ RSs before (solid line) and after (dashed line) the random deployment of 30 MSs (the positions of the MSs are not shown for clarity of the figure).

4 Simulation Results and Analysis

For simulations, we consider a square area of 3 km $\times$ 3 km with the BS at the center. We deploy the RSs and the MSs within this area. The transmit power is set to 50 mW for all RSs and MSs, the noise level is -100 dBm, and the bandwidth per RS is set to $W = 100$ kHz. For path loss, we set the propagation loss to $\mu = 3$. We consider a traffic of 64 kbps, divided into packets of length $B = 256$ bits with an arrival rate of 250 packets/s. For the HELLO packets, we set $\eta_0 = 1$ packet/s with the same packet length of $B = 256$ bits. Unless stated otherwise, the tradeoff parameter is set to $\beta = 0.7$ to imply services that are slightly delay tolerant. Further, the parameter ϵ is selected to be equal to 1% of any RS's current utility, i.e., an RS accepts the formation of a link if its utility does not decrease by more than 1% of its current value.

In Fig. G.2, we randomly deploy $M = 10$ RSs within the area of the BS. The network starts with an initial star topology with all the RSs connected directly to the BS. Prior to the deployment of MSs (in the presence of HELLO packets only), the RSs engage in the proposed network formation algorithm and converge to the final Nash network structure shown by the solid lines in Fig. G.2. Clearly, the figure shows that through their distributed decisions the RSs select their preferred nearby partners, forming the multi-hop tree structure. Furthermore, we deploy 30 randomly located MSs in the area, and show how the RSs self-organize and adapt the network's topology to the incoming traffic through the dashed lines in Fig. G.2. For instance, RS 9 improves its utility from 266.74 to 268.5 by disconnecting from RS 8 and connecting to RS 6 instead. This improvement stems from the fact that, although connecting to RS 8 provides a better BER for RS 9, in the presence of the MSs, choosing a shorter path, i.e. through RS 6, the delay perceived by the traffic of RS 9 is reduced, hence, improving the overall utility. Moreover, due to the delay generated from the traffic received by RS 5 from the MSs connected to RS 4, RS 10 improves its utility from 189.57 to 259.36 by disconnecting from RS 5 and connecting directly to the BS. Similarly, in order to send its HELLO packet, RS 7 finds it beneficial to replace its current link with the congested RS 1 with a direct link to the BS. In brief, Fig. G.2 summarizes the operation of the proposed adaptive network formation algorithm with and without the presence of external traffic from MSs.

In Fig. G.3, we assess the effect of mobility on the network structure. For this purpose, we consider the network of Fig. G.2 *prior to the deployment of the MSs* and we consider that RS 9 is moving horizontally in the direction of the negative x-axis while the other RSs remain static. The variation in the utilities of the main concerned RSs during the mobility of RS 9 are shown in Fig. G.3. Once RS 9 starts its movement, its utility increases since its distance to its serving RS, RS 8, decreases. Similarly, the utility of RS 2, served by RS 9 also increases. As RS 9 moves around 0.2 km, it finds it beneficial to replace its current link with RS 8 and connect to RS 6 instead. In this context, RS 6 would accept the incoming connection from RS 9 since this acceptance does not affect its utility negatively as shown

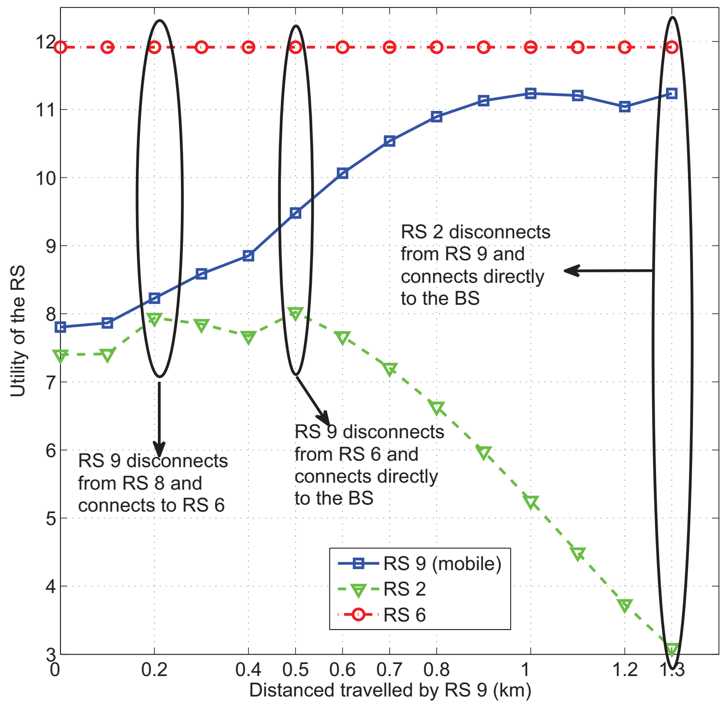

Fig. G.3: Adaptation of the network's tree structure to mobility of the RSs shown through the changes in the utility of RS 9 of Fig. G.2 as it moves on the x-axis in the negative direction prior to any MS presence.

in Fig. G.3 at 0.2 km. As RS 9 pursues its mobility, its utility improves as it gets closer to RS 6 while the utility of RS 2 decreases since RS 9 is distancing itself from it. After moving for a distance of 0.5 km, RS 9 becomes quite close to the BS, and, thus, it maximizes its utility by disconnecting from RS 6 and connecting directly to the BS. This action taken by RS 9 at 0.5 km also improves the utility of RS 2. Meanwhile, RS 9 continues its movement and its utility as well as that of RS 2 start to drop as RS 9 distances itself from the BS. As soon as RS 9 moves for a total of 1.3 km, RS 2 decides to disconnect from RS 9 and connect directly to the BS since the

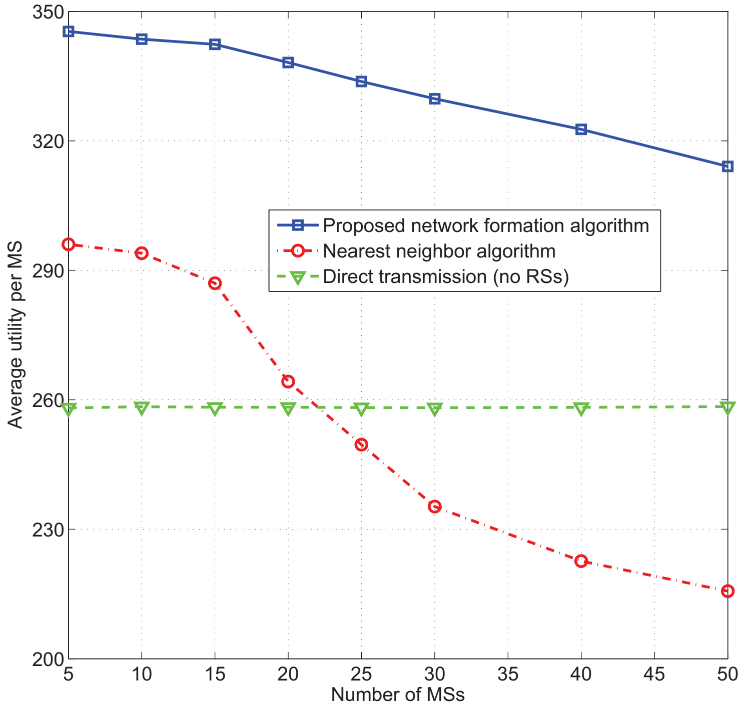

Fig. G.4: Performance assessment of the proposed network formation algorithm, in terms of average utility per MS, for a network having $M = 10$ RSs as the number of MSs varies (average over random positions of MSs and RSs).

direct transmission can provide a better utility at this point. In a nutshell, by inspecting the results of Fig. G.3, we clearly illustrate how the RSs can take distributed decisions that allow them to self-organize and adapt the topology to mobility.

Fig. G.4 shows the average achieved utility per MS for a network with $M = 10$ RSs as the number of MSs in the network increases. The results are averaged over random positions of the MSs and the RSs in the network. The performance of the proposed network formation algorithm is compared against the direct transmission performance, i.e., the case where no RSs exist in the network, as well as a nearest neighbor algorithm whereby each node selects the closest partner to connect to. In this figure, we can see that, as the number of MSs in the network increase, the performance of both the proposed algorithm as well as that of the nearest neighbor algorithm decrease. This result is due to the fact that, as more MSs are present in the network, the delay from multi-hop transmission due to the additional traffic increases, and, thus, the average payoff per MS decreases. In contrast, in the case of no RSs, the performance is unaffected by the increase in the number of MSs since no delay exists in the network. We also note that, due to the increased traffic, the performance of the nearest neighbor algorithm drops below that of the direct transmission at around 20 MSs. Further, Fig. G.4 shows that, at all network sizes, the proposed network formation algorithm presents a significant advantage over both the nearest neighbor algorithm and the direct transmission case. This performance advantage is of at least 21.5% compared to the direct transmission case (for 50 MSs) and it reaches up to 45.6% improvement relative to the nearest neighbor algorithm at 50 MSs.

The performance of the proposed network formation algorithm is further assessed in Fig. G.5, where we show the average utility per MS as the number of RSs M in the network varies, for a network with 40 MSs. Fig. G.5 shows that, as M increases, the performance of the proposed algorithm as well as that of the nearest neighbor algorithm increase. This is due to the fact that, as the number of RSs increase, the possibilities of benefiting from cooperative transmission gains increase, and, thus, the average utility per MS increase. In contrast, for the direct transmission scheme, the performance is constant as M varies, since this scheme does not depend on the number of RSs. Fig. G.5 demonstrates that, at all network sizes, the proposed network formation algorithm presents a significant performance gain reaching, respectively, up to 57.1% and 42.4% relative to the nearest neighbor algorithm and the direct transmission case.

In Fig. G.6, we show the average and the average maximum number of hops in the resulting network structure as the number of RSs M in the network increases for a network with 40 MSs (results are averaged over random positions of MSs and RSs). The number of hops shown in this figure represents the hops connecting RSs or the RSs to the BS, without accounting for the MS-RS hop. Fig. G.6 shows that, as the number of RSs M increases, both the average and the average maximum number of hops

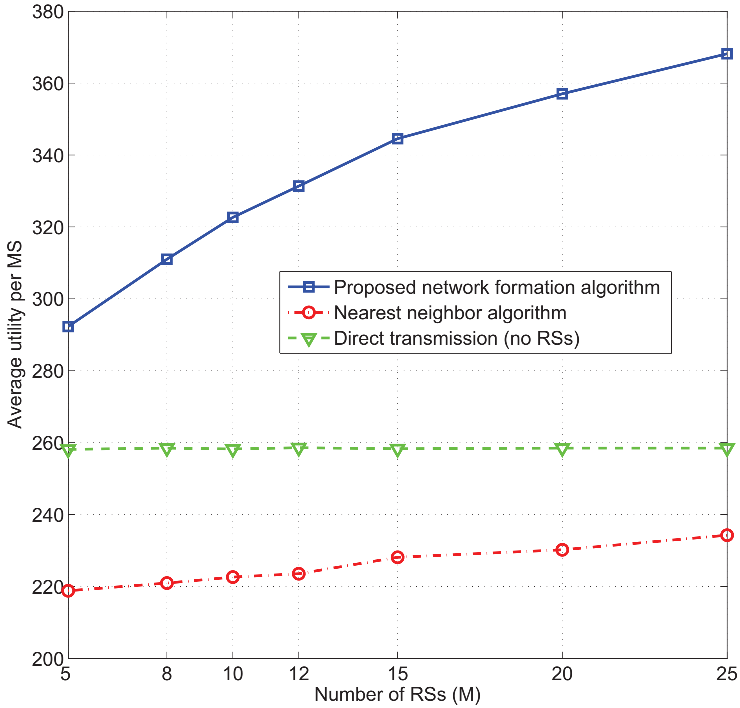

Fig. G.5: Performance assessment of the proposed network formation algorithm, in terms of average utility per MS, for a network having 40 MSs as the number of RSs M varies (average over random positions of MSs and RSs).

in the tree structure increase. The average and the average maximum number of hops vary, respectively, from 1.81 and 2.43 at $M = 5$ RSs, up to around 3 and 4.75 at $M = 25$. Consequently, as per Fig. G.6, due to the delay cost for multi-hop transmission, both the average and average maximum number of hops increase very slowly with the network size M. For instance, one can notice that, up to 20 additional RSs are needed in order to increase the average number of hops of around 1 hops and the

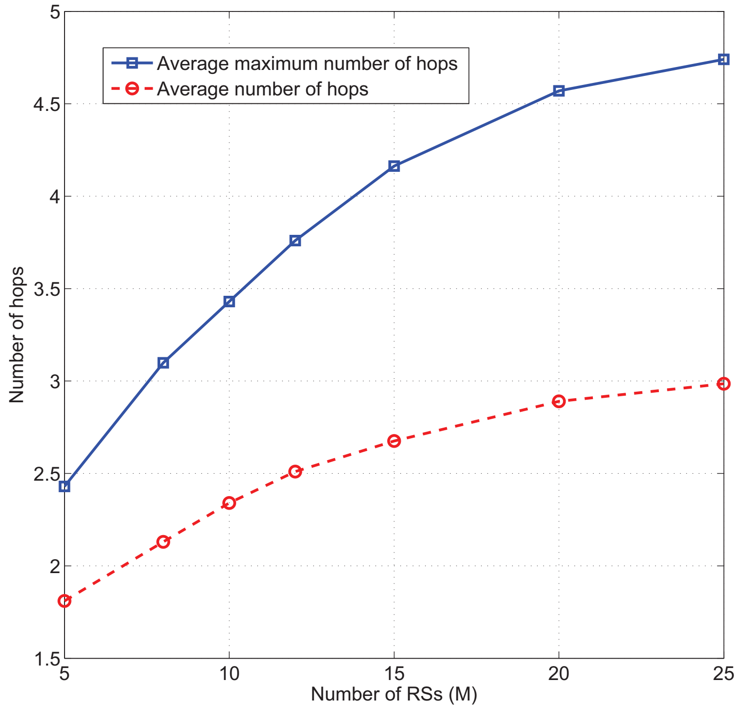

Fig. G.6: Average and average maximum number of hops in the final tree structure for a network with 40 MSs vs. number of RSs M in the network (average over random positions of MSs and RSs).

average maximum number of hops of only around 2 hops.

Fig. G.7 shows the average and the maximum number of iterations needed till convergence of the algorithm to the initial network structure prior to the deployment of any MSs, as the size of the network M increases. This figure shows that, as the number of RSs increase, the total number of iterations required for the convergence of the algorithm increases. This result is due to the fact that, as M increases, the cooperation options for every RS increase, and, thus, more actions are required prior to con-

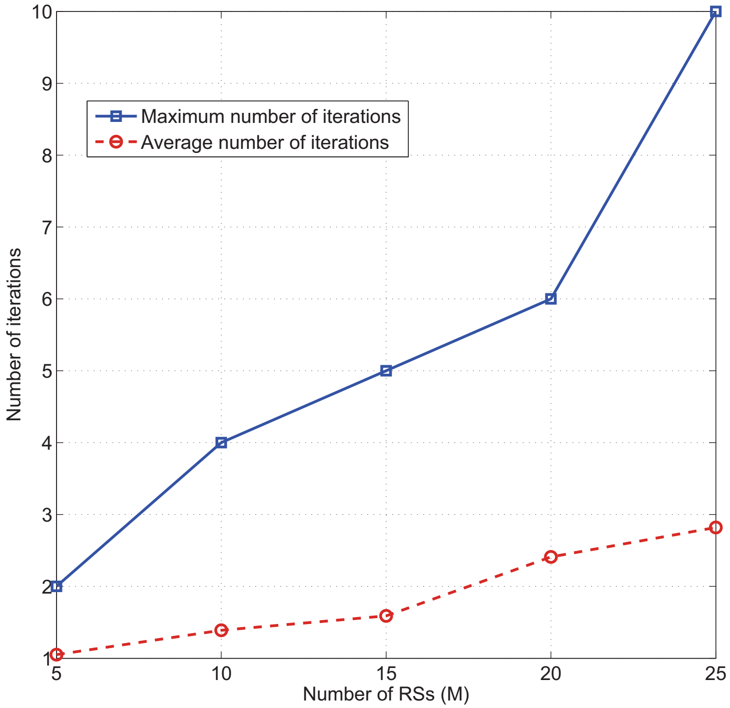

Fig. G.7: Average and maximum number of iterations till convergence vs. number of RSs M in the network (average over random positions of RSs).

vergence. Fig. G.7 shows that the average and the maximum number of iterations vary, respectively, from 1.05 and 2 at $M = 5$ RSs up to 2.82 and 10 at $M = 25$ RSs. Hence, this result demonstrates that, in average, the speed of convergence of the proposed algorithm is quite reasonable even for relatively large networks. Similar results can be seen for the convergence of the algorithm when MSs are deployed or when the RSs are moving.

In Fig. G.8, we show the average and the average maximum number of hops for a network with $M = 10$ RSs and 40 MSs as the tradeoff parameter β varies (results are averaged over random positions of MSs and RSs). Fig. G.8 shows that, as the tradeoff parameter increases, both the average

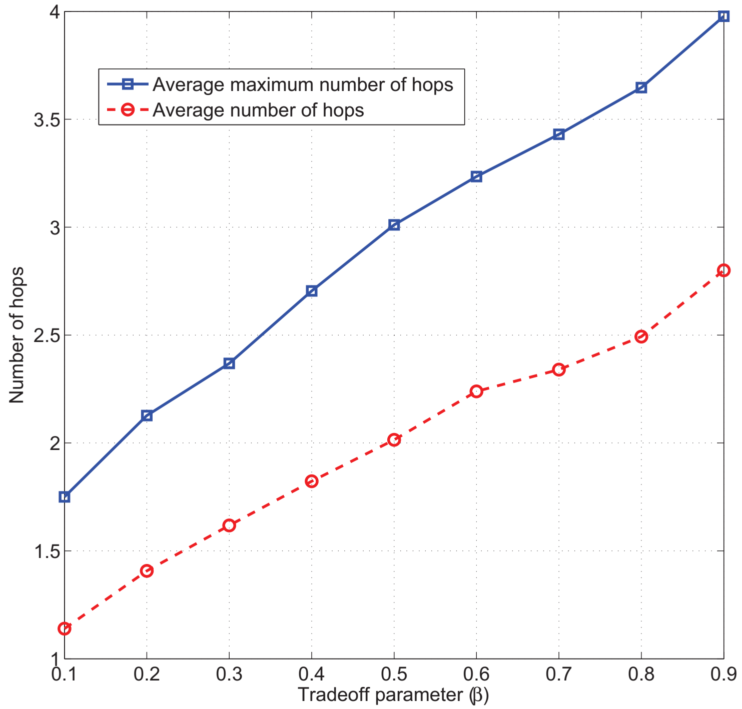

Fig. G.8: Average and average maximum number of hops in the final tree structure for a network with 10 RSs and 40 MSs as the tradeoff parameter β varies (average over random positions of MSs and RSs).

and the average maximum number of hops in the tree structure increase. For instance, the average and the average maximum number of hops vary, respectively, from 1.14 and 1.75 at $\beta = 0.1$, up to around 2.8 and around 4 at $\beta = 0.9$. The increase in the number of hops with β is due to the fact that, as the network becomes more delay tolerant (larger β) the possibilities for using multi-hop transmission among the RSs increases. In contrast, as the network becomes more delay sensitive, i.e., for small β, the RSs tend to self-organize into a tree structure with very small number of hops. For instance, at $\beta = 0.1$, the average number of hops is quite close to 1, which

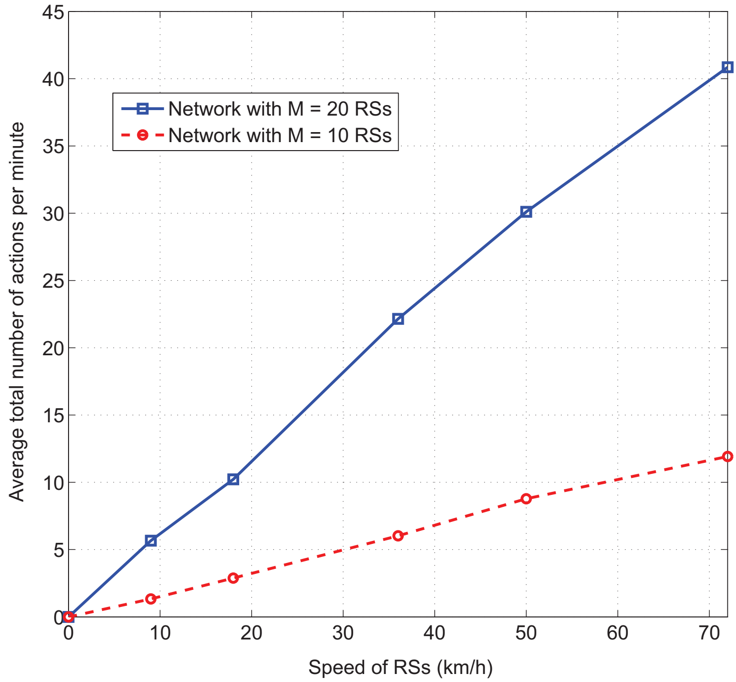

Fig. G.9: Average total number of actions (taken by all RSs) per minute for different RS speeds in networks with different sizes with 40 MSs.

implies that, for highly delay sensitive services, direct transmission from the RSs to the BS, i.e., the star topology, provides, on the average, the best architecture for communication.

In Fig. G.9, we show, over a period of 5 minutes, the average total number of actions taken by all RSs for various velocities of the RSs in a wireless network with 40 MSs and different number of RSs. The proposed network formation algorithm is repeated by the RSs, periodically, every $\theta = 30$ seconds, in order to provide self-adaptation to mobility. As the speed of the RSs increases, the average total number of actions per minute increases for both $M = 10$ RSs and $M = 20$ RSs. This result corroborates the fact that, as more mobility occurs in the network, the chances of changes in the network structure increase, and, thus, the RSs take more actions. Also,

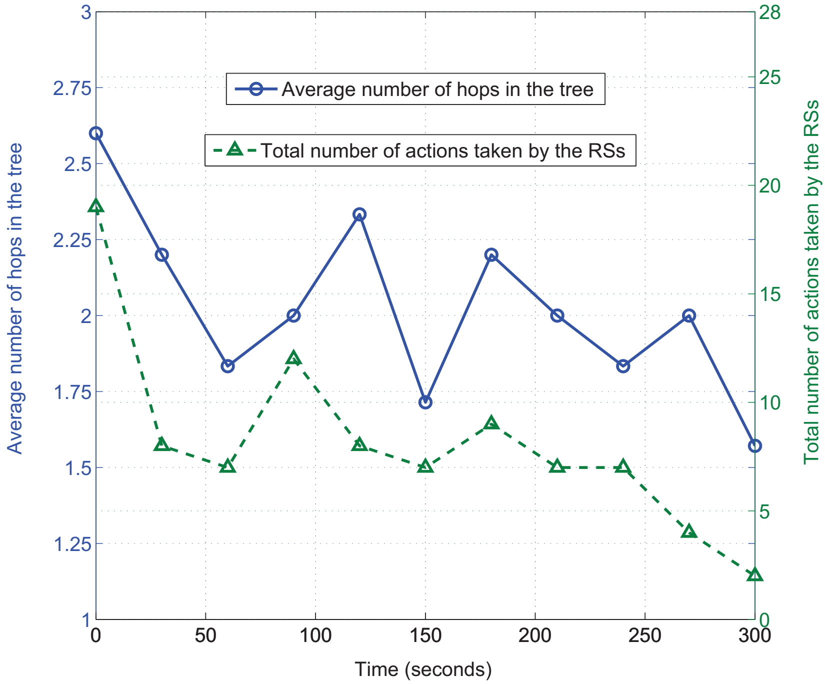

Fig. G.10: Evolution of the network tree structure over time as the RSs are moving with a speed of 72 km/h over a period of 5 minutes for a network with $M = 10$ RSs and 40 MSs.

Fig. G.9 shows that the case of $M = 20$ RSs yields an average total number of actions significantly higher than the case of $M = 10$ RSs. The reason of this difference is that, as the number of RSs M increases, the possibility of finding new partners when the RSs move increases significantly, hence yielding an increase in the topology variation as reflected by the average total number of actions. In this regard, for $M = 20$ RSs, the average total number of actions per minute varies from around 5.7 at 9 km/h to around 41 at 72 km/h while for $M = 10$ RSs, this variation is from 1.3 at 9 km/h to around 12 at 72 km/h. In summary, Fig. G.9 demonstrates how, through periodic runs of the proposed network formation algorithm, the RSs can adapt the topology through appropriate decisions.

Fig. G.10 shows how the tree structure in a network with $M = 10$ RSs, moving at a speed of 72 km/h, evolves and self-adapts over time for a period of 5 minutes. The proposed network formation algorithm is repeated by the RSs, periodically, every $\theta = 30$ seconds, in order to provide self-adaptation to mobility. Fig. G.10 shows that, after 19 actions taken by the RSs, the network starts with a tree structure with an average number of 2.6 hops in the tree at time $t = 0$. As time evolves, the RSs engage in the proposed network formation algorithm, and, through adequate actions, the tree structure is adapted to this environment change. For example, after 2.5 minutes have elapsed, the tree structure has an average number of 1.71 hops (after having 2.33 hops at 2 minutes), due to the occurrence of a total of 7 actions by the RSs. Once all the 5 minutes have passed, the network tree structure is finally made up of an average of 1.57 hops after a total of 90 actions played by the RSs during the whole 5 minutes duration.

5 Conclusions

In this paper, we have introduced a novel approach for forming the tree architecture that governs the uplink network structure of next generation wireless systems such as LTE-Advanced or WiMAX 802.16j. For this purpose, we formulated a network formation game among the RSs and we introduced a cross-layer utility function that takes into account the gains from cooperative transmission in terms of improved effective throughput as well as the delay costs incurred by multi-hop transmission. To form the tree structure, we devised a distributed myopic algorithm. Using the proposed network formation algorithm, each RS can take an individual decision to optimize its utility by selecting a suited next-hop partner, given the approval of this partner. We showed the convergence of the algorithm to a Nash network structure and we discussed how, through periodic runs of the algorithm, the RSs can adapt this structure to environmental changes such as mobility or incoming traffic. Simulation results demonstrated that the algorithm presents significant gains in terms of average achieved mobile station utility which is at least 21.5% better than the case with no RSs and reaches up to 45.6% improvement compared to a nearest neighbor algorithm. The results also show that the average number of hops in the tree does not exceed 3 even for a network with up to 25 RSs.

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