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## Open texture clarified

Joost Jacob Vecht

Department of Philosophy, Classics, History of Art and Ideas, University of Oslo, Oslo, Norway

### ABSTRACT

'Open texture' is the property of concepts or terms that they are not fully defined with regard to unexpected questions. Even though it is a ubiquitous phenomenon, there is no consensus on which concepts are subject to open texture. I present three equivalent definitions of open texture, connecting the concept to the notions of analyticity and algebraicity. I show that the claim that a concept is open-textured is equivalent to the claim that the class of analytic sentences using it can be changed, which in turn is equivalent to the claim that its definition is not algebraic. While this equivalence is interesting on its own, with it I can show the extent of open texture: all concepts that are not algebraically defined mathematical concepts are open textured. Finally, I show that this has unfortunate consequences for attempts to provide a foundation of mathematics.

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Concepts are 'open textured' if it is unclear whether they apply to some objects or not, not because of a vagueness in the concept but because the objects are new and unexpected in some way, and the concept simply is not capable of handling this odd case. Though the term has been around since it was coined in Waismann (1945), there is still no consensus about its extent: what concepts do and do not feature open texture. Whether 'hard' scientific concepts or even mathematical concepts are included is subject to discussion. I argue that in order to answer this question, we need to provide a clearer definition of what exactly open texture is. The best available definition of open texture can be sharpened, and alternative definitions can be given. I provide such a sharpening, and two alternative definitions: one by connecting the notion of open texture to the notion of analyticity and another by connecting it to the notion of algebraicity in the philosophy of

**CONTACT** Joost Jacob Vecht  joost@vecht.net

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mathematics. Surprisingly, these definitions turn out to be equivalent. Moreover, with these definitions in hand, I can answer the question of the extent of open texture and offer insights on the possibility of foundations for mathematics.

## 1. What is open texture?

*Open texture* (*Porosität*) is a term introduced by the Vienna Circle's Friedrich Waismann. Waismann was critical of the verificationist programme within the Circle, wherein the meaning of terms is to be found in sense data, and the truth value of sentences is to be determined by empirical verification of these sense data. His criticism focuses not on the process of verification or the reduction of terms to sense data *per se*, but on the open-ended nature of empirical terms.

According to Waismann, (empirical) concepts<sup>1</sup> are never fully delimited. Any definition, no matter how apparently sharp, always has some indeterminacy (Waismann 1945, 123). This is so because it cannot ever be fully foreseen in which ways a concept may require clarification in the future. New objects may arise for which it is unclear whether the old, 'clear' concept applies. The concept, previously crystal-clear, now seems like it had a flawed, incomplete definition all along.

Using a modern example, the concept of MONEY seemed clear, until suddenly challenged by the existence of digital, decentralised currency such as BitCoin. Whether it is money or not, whether it is currency or not, and why it should or should not be called such, is all part of a newer debate. One may observe that it is used as a currency with no other value than *as* a currency to buy or sell things with, and decide it is money. On the other hand, one could observe that there is nothing backing up the claim of value, such as a government, gold, or central bank, or that the value of BitCoin fluctuates too much for it to function as a reliable measure of price in the way that money typically does. On those grounds, one might feel that it is not money after all. Perhaps there is an appealing answer to the question, and we will collectively decide one way or the other. That is all fine. The point is that the concept of money itself turned out not to be fully delimited. There turned out to be a question on both the extension and intension of the concept MONEY,

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<sup>1</sup>Throughout this essay, I do not systematically distinguish between concepts and terms (following e.g. Prinzing 2018; Tanswell 2018). Nothing in this essay hinges on the distinction. In particular, the definitions given for open texture apply equally to both terms and concepts, and the conclusions drawn about, for example, the extent of open texture apply to both concepts and terms.

no matter how clear we thought it was in both ordinary and professional speech.

Waismann's claim is that this is a universal feature of empirical terms; they are not delimited by flawless, defined walls, but are porous in nature; they are *open textured*.

Open texture is a very fundamental characteristic of most, though not of all, empirical concepts, and it is this texture which prevents us from verifying conclusively most of our empirical statements. Take any material object statement. The terms which occur in it are non-exhaustive; that means that we cannot foresee completely all possible conditions in which they are to be used; there will always remain a possibility, however faint, that we have not taken into account something or other that may be relevant to their usage; and that means that we cannot foresee completely all the possible circumstances in which the statement is true or in which it is false. (Waismann 1945, 123)

One can imagine the 'porosity' of concepts quite vividly. A concept might have boundaries, and they might even be sharp, for the moment. But these boundaries are always porous: they are made of limestone, not steel. Things that are within the boundaries clearly fall under the concept, and things that fall outside it very clearly do not, but there is always the possibility that something is not clearly either in- or outside the boundaries. The boundaries cannot be made sharp in a future-proof way, because new questions may arise, the boundaries may turn out to have a hole in them that was not visible before.

Certain new kinds of objects may appear, and it may not be certain whether they fall under an established concept or not. Does my possession of some amount of BitCoin mean I possess some money? This calls into question the previous extensional boundaries of the concept. Likewise, are cryptocurrency part of the concept of money or not? This more conceptual question – 'does concept A fall under concept B?' – is left open as well. The intension of the concept MONEY shows the same open texture that its extension shows.

Summarising open texture we can say the following. Open texture is the feature of concepts, that they are never clearly delimited in any definite, permanent way. There is always the possibility for a re-evaluation caused by new circumstances. There can always be objects that are not clearly on the in- or outside of the boundaries of the concept. Likewise, there can always be *concepts* that are not clearly in- or outside the boundaries of the concept. Open texture is not a temporary fault; it does not mean that we have failed to properly delimit a concept. Rather, open texture sticks: concepts with open texture *cannot* be delimited in any

definite, permanent way. Defining any new boundaries will solve the current uncertainties, and may be useful, but does not eliminate the possibility for unexpected, new meaning change caused by new circumstances.

For example, we might 'fix' MONEY by deciding as a community that cryptocurrencies are money, BitCoin is money, and it is not important that money have an extremely stable value. We might make enough of these decisions, and codify them well enough, to eliminate any uncertainty on the subject. All is fine – until a few decades from now, someone invents SchmitCoins, based on a completely new concept, not quite a cryptocurrency, not quite traditional money, but different in some new, unforeseen way. The discussion starts again: are SchmitCoins money or not? Perhaps we decide yes, perhaps no. Perhaps we find quite useful and ingenious ways to delimit the concept of money again. All is fine, until a few decades later, there is a new sort of payment method introduced...

## 2. The question of the extent

A natural question that arises on open texture is simply: which concepts have open texture? Waismann's intention was clearly to speak of only empirical concepts, as his original formulation in the above citation betrays. But without a delineation for which concepts truly are 'empirical', this does not let us draw a clear line. What is worse, Waismann himself provided all the ammunition we need to shoot down any clear delineation between empirical and non-empirical concepts.<sup>2</sup> Moreover, it seems that Waismann's own stated limitation, that open texture does not apply to *all* empirical terms, seems hard to defend: I know of no empirical term that does not feature open texture.

While Waismann himself limited this property of open texture to empirical terms specifically, an investigation of other concepts reveals that it is applicable outside this sphere. Shapiro (2006a, 434) has an example of open texture for a decidedly non-empirical, mathematical concept:

The boundaries of the notion of 'natural number' are, perhaps, as rigorous and sharp as can be – hard as rails, as Wittgenstein might put it. Harder than rails. What of the more general notion of 'number'? Are complex numbers numbers? Surely. But this was once controversial. If it is a matter of proof or simple definition, why should there ever have been controversy? And what of quaternions? Are those numbers? Perhaps the jury is out on that one. Perhaps there is no real need to decide whether quaternions are numbers.

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<sup>2</sup>This can be done using much the same tactics Waismann uses to complicate terms like 'definition', 'expression', etc. in his *Analytic-Synthetic* series. We will see these arguments later.

We see two different kinds of terms here, one with an apparently closed texture and another, closely related one with open texture. Given the apparent correctness of this observation by Shapiro, where exactly does the distinction lie? What feature makes only the first concept ‘hard as rails’ without eliminating open texture in the second? It cannot be the fact that one notion is mathematical, while the other is not; NUMBER is as mathematical a notion as they get, as Shapiro points out (Shapiro 2006a, 434).

The difference cannot lie in the fact that one has a strict mathematical definition, while the other does not; for we can have meaningful discussions on what constitutes a set, even though there is a fine mathematical characterisation of it. It seems that there is discussion possible on the notion of SET in a way that is not possible with NATURAL NUMBER.<sup>3</sup> That suggests that if we were to draw a line between open textured and closed textured concepts, it should cross mathematical concepts in some way, leaving SET and NATURAL NUMBER on different sides. So far though, no such line has been drawn; rather, examples have been given and observations have been made on a case-by-case basis. To get a clear idea of where to draw it, we will need a slightly sharper notion of open texture.

### 3. Definitions of open texture

While the general idea of open texture has been gaining increasing attention recently, there is no consensus on a clear definition for it. One of the more difficult matters seems to be its relation to *vagueness*. On the one hand, open texture cases seem to point at a certain kind of semantic indeterminacy, at a possibility for a language user to use a term in various different ways. After all, the ‘odd cases’, like Bitcoin was for MONEY, allow for divergent use, and for the community to eventually decide one way or another. We seem to be currently free to use MONEY in such a way that it in- or excludes BitCoin.<sup>4</sup> Thus, it seems completely in line with a broad understanding of vagueness to call open texture a certain form of

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<sup>3</sup>It is interesting to note that perhaps, even the notion of natural number has not always been ‘hard as rails’. The question whether ‘0’ constitutes a natural number or not seems like it required a weighed answer at some point. Likewise, whether ‘1’ is a natural number or not was subject to discussion, since it is not a plurality that needs to be counted, unlike other numbers. Husserl (2003, 136–141) argued that one and zero are distinct concepts from number concepts, for example. I go into more depth about the replacement of a previously open-textured concept by a closed texture formalisation in Section 5.

<sup>4</sup>In fact, usage can become so vague that we use qualifiers to specifically exclude BitCoin: i.e. ‘traditional money’.

vagueness. On the other hand, it seems that there is a clear distinction between certain kinds of vagueness and open texture specifically.

Sorites vagueness, in particular, can and ought to be distinguished from open texture proper. In fact, Waismann (1945, 123) took care to distinguish the two even while introducing open texture:

*Vagueness* should be distinguished from *open texture*. A word which is actually used in a fluctuating way (such as 'heap' or 'pink') is said to be vague; a term like 'gold', though its actual use may not be vague, is non-exhaustive or of an open texture in that we can never fill up all the possible gaps through which a doubt may seep in. Open texture, then is something like the *possibility of vagueness*.

I can illustrate the difference by adapting a rather wonderful example due to Fenner Tanswell: the concept of BALDNESS (Tanswell 2018, 885). This is a vague concept in the sense that the Sorites paradox applies: losing a single hair does not change someone who is not bald to someone who is, and yet the process of losing hairs one by one would eventually turn a man bald. It's also open textured, but *not* because of this Sorites vagueness; it is open textured because we can imagine an 'odd case' outside the usual domain of application. Imagine a two-headed individual, one of which has a full head of hair, and one of which has no hair whatsoever. Clearly, it is indeterminate whether we would call this person bald or not, but it has nothing to do with Sorites: the heads are individually clearly either bald or not-bald, there is no vague in-between state involved.

The best currently available definition of is probably due to Tanswell (2018). In order to capture the notion of open texture as sharply as possible, he presents two definitions of it, one of which he ascribes to Waismann:<sup>5</sup>

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<sup>5</sup>The other definition, which he ascribes to Stewart Shapiro, was broader and included some notion of vagueness: 'A concept or term displays open texture iff there are cases for which a competent, rational agent may acceptably assert either that the concept applies or that it disapplies'. Tanswell (2018, 885) While I hold, with Shapiro, that open texture can be seen as a form of vagueness in that it certainly embodies a certain sort of indeterminacy, it is crucial to differentiate it from the narrower notion of Sorites vagueness at the very least, as we argue above. Hence, I choose to omit this second definition. Tanswell uses the example of BALDNESS to illustrate that these definitions agree on the extent of open texture. I do not think that the equivalence holds. The definition attributed to Shapiro is exceedingly broad, since it conflates certain kinds of vagueness *within* the standard domain of application with open texture. It seems that NATURAL NUMBER is closed textured according to Tanswell's definition. NATURAL NUMBER indeed does not have the possibility for undecided cases *outside* the normal domain of application. However one could easily imagine a world where a mathematician is completely free to include 0 in the naturals or not. In fact, the inclusion or exclusion of 0 in the naturals was subject to some debate. It then seems like it would be open-textured on the definition attributed to Shapiro.

**(OT Tanswell) Open texture, Tanswell's definition.** *A concept or term displays open texture iff there are possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept or not.*

Tanswell does not clarify what exactly is meant by a 'standard domain of application'. Perhaps it is not too difficult to sharpen the idea a bit. I can take the standard domain of application to be those objects of which we will commonly say that it holds (its extension), those objects of which we say that it does not hold (its anti-extension), and those objects which are accepted vague cases. For example, a green apple would fall inside the standard domain of application of GREEN because it is green; a red apple would fall inside it because it is *not* green; a greenish-yellowish apple would fall inside it because it is a borderline case, we could say that it is green; but *the thought of an apple* would not fall inside its standard domain of application. It is not the sort of thing of which we would say whether it is green or not. The open texture of a concept then becomes visible when there are objects outside this domain for which we decide, for whichever reason, to apply or disapply the concept anyway, and extend the domain accordingly.

Tanswell's definition seems to capture what we are aiming at, with the sole exception that it does not capture the temporally open-ended nature of open texture. By (OT Tanswell), a concept may have open texture if there is one single possible object falling outside the standard domain of application, and once we change our concept to deal with that object, it is 'fixed'. For example, WHOLE NUMBER might then be considered open textured simply because at a certain point, it had to be decided whether 0 was a number or not. Open texture tends to stick, however: any change in the concept to solve a difficult borderline case will only be a temporary patch. Recall the example of MONEY; even if we resolve the issue with BitCoin, there may still occur new difficult issues in the future. We can see this especially clearly when Waismann speaks Waismann (1945, 124) of the *essential incompleteness* of physical descriptions:

If I had to describe the right hand of mine which I am now holding up, I may say different things of it: I may state its size, its shape, its colour, its tissue, the chemical compound of its bones, its cells, and perhaps add some more particulars; but however far I go, I shall never reach a point where my description will be completed: logically speaking, it is always possible to extend the description by adding some detail or other. Every description stretches, as it were, into a horizon of open possibilities: however far I go, I shall always carry this horizon with me.



Open textured concepts are supposed to always carry this horizon with them; they cannot be closed just by giving a better definition, providing a better theory. No matter how far you walk, there is still going to be that horizon. Thus I may edit the proposed definition of open texture once more, getting as close to Waismann's definition of open texture as possible:

**(OT Modified) Open texture, Modified Tanswell's Definition.** *A concept or term displays open texture iff there are **always** possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept or not.*

Here, 'always' is to be taken in a robust temporal way: no matter what the future looks like, once we go there, there will be possible objects outside the standard domain of application. With this small change, it seems that we have an accurate and hopefully useful sharpening of the notion of open texture.<sup>6</sup>

#### 4. Defining open texture by analyticity

I will aim next to connect the notion of open texture to that of analyticity. By analyticity, I mean something akin to Waismann's idea of analyticity. Waismann took a critical attitude towards the concept, thinking that it could never be made sufficiently sharp. In contrast to Quine (1951), he did not push this to the point of outright scepticism about the use of the analytic/synthetic distinction: rather, he took a constructive attitude towards its apparently 'fraying off at the edges', stating that we could in fact learn from the act of *declaring* a sentence to be analytic (Waismann 1950, 128).

It is interesting to note that, even as he expresses the same sort of concerns in his papers on analyticity as he did in *Verifiability*, he does not mention open texture by name. And yet, it seems to take a central role, as Shapiro (2006b, 211–212) points out:

The phrase 'open-texture' does not appear in Waismann's treatment of the analytic–synthetic distinction in a lengthy article (or articles) published serially in *Analysis* from December 1949 until March 1953. Nevertheless, the notion clearly plays a central role in the essay(s). [...] Waismann proposes that the

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<sup>6</sup>On certain theories of tense and modality, if there are possible objects with certain properties, then there always are such possible objects:  $\diamond\exists x\phi(x)$  entails *Always*  $\diamond\exists x\phi(x)$ . On these theories, my addition can safely be left out. However, on any other theory, or when not yet considering a temporal aspect at all, it is important to make this extra condition explicit.

notion of 'definition' be left open, and that we should not seek for a hard and fast characterisation of the notion, giving necessary and sufficient conditions for something to be a definition. Flexibility can be a virtue.

In Waismann (1949), Waismann goes through a few definitions of analyticity, none of which are to this liking. He ends up with a definition that should suit our purposes:

**(Analyticity) Definition.** *A statement is analytic iff it can be transformed to a truth of logic by means of mere definitions and operators.*

Operators, here, may be either strictly logical or linguistic/idiomatic: Waismann considers 'predicativation' a linguistic operator, to move from phrases such as 'There is a red planet' to 'There is a thing that is red and that is a planet' (Waismann 1949, 35). This already puts us in relatively vague territory, since we cannot delimit exactly idiomatic operators like we can perhaps list the logical ones. Waismann point out that any vagueness or ambiguity in the terms like 'operator' and 'truth of logic' will mean a similar vagueness in the notion of analyticity (Waismann 1950, 25). But the real trouble starts when we consider what a 'definition' might be. We start out with a rather neat proposed clarification of what it could be:

We may sum up the discussion by saying that definitions are substitution licences of a particular sort (leaving the sense of this somewhat open), and that every substitution licence can be re-written as an equivalence. (Waismann 1949, 39–40).

But at this stage, open texture starts playing a role. In these substitution licenses, we find the 'descriptive horizon' he had employed in his open texture paper. No matter how many valid substitutions we may find for a term like 'time' in a specific context, we cannot exhaust the term 'time' with them. There is no single word or sentence that does the job that 'time' does (Waismann 1950, 27). With the idea of open texture in mind, we can see that there *could not* be. Indeed, Waismann holds that the concept of 'definition' itself cannot be captured by definition, and likewise 'meaning', 'explanation', 'precise', 'describing', and so forth Waismann (1951, 49). He pays particular attention to the notion of 'definition'. He sees the very phenomenon of a 'definition' as one characterised by variety. This makes it impossible to quite pin down the notion. Mathematical definitions, ostensive definitions, physical definitions, dictionary definitions, formal definitions *etc.* all behave slightly differently, and some definitions may straddle these divides or be read in multiple

different ways. Depending on the choices we make with regards to our definitions, the class of analytic statements may shrink or grow.

With these considerations in mind, we can try to flip our perspective, so to speak, and define open texture in terms of analyticity. An open textured concept always has the possibility for ‘undecided cases’ outside the standard domain of application. Consider the possibility that all sentences expressing these ‘undecided cases’ one way or another (i.e. uses of the term outside the standard domain of application) were synthetic. Then none of them involved the introduction of a new substitution licence of any sort. They would all simply be applications of the term, as it is currently conceived of, without further changes. But then they could not really be uses outside the standard term of application after all! In other terms, if there are only new *synthetic applications* of a term, that does not entail that it has changed.

By contrast, open texture requires that something changed about the concept: either the proposition outside the standard domain is analytic itself, i.e. it gives a new (partial) definition (e.g. by defining that BitCoin is money) or another analytic statement was added with cascading effects to our new use case (i.e. by defining that cryptocurrency is money, and hence, synthetically, Bitcoin is money, since it happens to be cryptocurrency.) But what is not an option is that no analytic sentence was introduced or changed at all (i.e. to say that Bitcoin is, synthetically, money, even though nothing has changed about the old concept MONEY and BitCoin falls outside its standard domain of application). Using these insights, I can define open texture in terms of analyticity as follows:

**(OT Analyticity) Open texture, Analyticity definition.** *A concept or term displays open texture iff the class of analytic sentences involving this concept or term can always be changed to address use cases outside the standard domain of application.*

One question is sure to arise at this point: why even try to give a new definition of open texture, given that the very concept of open texture means that we ought not to search for necessary and sufficient conditions? This is because a proposed sharpening of the notion can nevertheless serve to enlighten. In this case, it allows us to connect two imprecise but important notions – analyticity and open texture – to increase our understanding of both of them. We do not have to fear that by switching to this definition, we will have ignored the open texture of the term ‘open

texture' itself.<sup>7</sup> This new definition still leaves us with the wiggle room we want: the analytic–synthetic distinction is not a sharp one, as argued famously by Quine (1951), and at length by Waismann in his Analytic–Synthetic series.<sup>8</sup> There are bound to be propositions for which it is unclear whether they are analytic or synthetic, much as there are bound to be propositions for which it is unclear whether they are cases of open texture or not.

In effect, I am offering two possible sharpenings on the concept of open texture. While I believe they are plausible, this does not mean that there are not other possible ways to sharpen or clarify the concept of open texture. Neither does it mean that there is something wrong with accepting that open texture is, fundamentally, a concept with some inherent vagueness. But what these sharpenings offer is the possibility to connect the notions of analyticity and open texture, thereby giving us a better grip on both these slippery concepts. And as we shall find in the next section, this particular sharpening also falls in line with an existing distinction within the philosophy of mathematics (or at least, again, one plausible way to sharpen such a distinction).

## 5. Delimiting extent: defining by algebraicity

What we have, at this point, are two different definitions of open texture, one based on the modal notion of possible objects, the other based on the notion of analyticity. What we will find is that these definitions are equivalent: they will call the very same concepts open textured. Moreover, we will find that they overlap with an existing concept in the philosophy of mathematics, that of an *assertory* definition. It turns out that exactly those concepts which are open textured have such a definition.

Certain mathematical objects, especially in algebra, are defined *algebraically*, that is to say, their definitions do not assert the existence of any objects. For example, a group is any collection that has a unit and an addition operation on which certain axioms hold. By contrast, *assertory* definitions say that certain mathematical objects exist or not. For example, some of the axioms of Zermelo-Fraenkel set theory assert that, say, the empty set exists, an infinite set exists, and that whenever you

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<sup>7</sup>Since terms like 'term' and 'concept' are bound to keep changing in the future, it seems unavoidable that 'open texture', too, is subject to change.

<sup>8</sup>Particularly, in the first paragraphs of Waismann (1950), he announces that if there is any uncertainty in the notion of 'definition', this will mean an uncertainty in the notion of analyticity. He then goes on to problematise the notion of 'definition' at length to exactly that purpose.

have two sets, there exists another set containing exactly the elements in either of those two sets.

The distinction between assertory and algebraic definitions has its roots in the debate between Frege and Hilbert on the nature of mathematics, and the terms 'assertory' and 'algebraic' themselves appeared when this debate was revived relatively recently, when a new discussion was held about the suitability of category theory as a foundation of mathematics. Category-theoretic terms are typically considered algebraic, but this has not prevented their proposed use (Awodey 2004) as a foundation for mathematics.

A common view in this debate tends to be that having an assertory definition is a positive (or even a straight-up requirement) for a concept to function as a foundation for mathematics (see e.g. Hellman 2003; Linnebo and Pettigrew 2011; for the opposing view, see Awodey 2004; McLarty 2011, 2012). To illustrate the contrast between categories and sets: categories are defined in a typical algebraic manner. *Anything* with a morphism defined on it is a category. The concept of a category is defined absolutely, regardless of any background domain. By contrast, sets are defined in an assertory manner, because their axioms establish a domain of sets that exist; and relations are defined in an assertory manner, because they are defined on a background domain of sets. These concepts are not defined 'absolutely'; they either provide an exact domain of sets that exist, or they are defined over such a domain.

If we think back to the motivation for our original definition (OT Modified), the intuition behind using *this* distinction, of all possible distinctions within mathematics, should become clear. What characterises the open textured concept is the 'moving horizon of description'; the fact that no characterisation will ever fully capture the concept. Indeed, some mathematical objects behave like this: we can try to characterise what a set is, but we could always add more axioms that give us more sets. For example, we can extend our set-theoretic universe by adding axioms stating that there are large cardinals that are *this* big, or *this* big, or *this* big...All of this is possible in virtue of the fact that a set is defined in an assertory way. This leaves open the possibility for us to assert ever more: we have a descriptive 'horizon', even if it is not an empirical description. Algebraically defined concepts, by contrast, do not have this. Indeed, Waismann mentions (Waismann 1945, 125) some mathematical objects which have a *complete* description: triangles, geometrical figures (if its pattern is captured in some notation), and chess games (once their pattern is captured using chess notation). Certain

mathematical objects can be fully described, in virtue of having a definition of some  $X$  that picks out the  $X$ s on any background domain whatsoever (or by ignoring the problem of a background domain completely).

In the case of a geometrical pattern or a chess game, Waismann emphasises how we have to ‘capture’ a pattern in a mathematical notation before we can have a complete description. I see this as a proposal to change the topic from some object or phenomenon to a complete closed textured abstraction. An actual, physical geometric figure on a carpet has a moving descriptive horizon; but if we ignore its physicality and talk instead of this pattern as some geometric group, we can talk of a closed textured concept instead. Lakatos (1976)’s polyhedron case displays the same pattern, where the final move in the game of defining the polyhedron fundamentally alters the notion of ‘polyhedron’.

Concepts that today seem closed textured may have gone through many revisions, discussions, and unexpected twists and turns, even if they settled on a steady definition in the end. We can see this in Lakatos’ famous dialogue on the definition of the polyhedron (Lakatos 1976), or in the history of many a concept: Tanswell cites Raman-Sundström’s (2015) history of the concept of open-cover compactness. Some of these do seem to be cases in mathematics where apparently ‘open textured’ terms become closed textured. Take Lakatos’ discussion on the definition of a polyhedron. In this rational reconstruction of a historical debate on the notion of a polyhedron, some mathematics students propose various definitions and bring forth various counter-examples of unexpected polyhedrons that make these various definitions fail, and in turn inspire new ‘monster-barring’ definitions that exclude these unexpected cases. So far, so good: we see an ostensibly incompletable series of definitions for an apparently open textured term. This is the behaviour we would expect in dealing with open textured concepts. But in the end, a peculiar move is made: an abstract, set-theoretic definition of a polyhedron is given, and this settles the debate; afterwards, the discussion of polyhedrons concerns only objects which abide by this definition. But if the term ‘polyhedron’ was really open textured, it shouldn’t be possible to close it in this way!

A way to interpret what happens here is to see the final move in this game of definition-replacing as distinct from all previous moves. It is not an attempt to capture the use of the pre-existing term ‘polyhedron’ in the mathematical community. Rather, it is a suggestion to replace it. The proposal is to ignore whatever intuitions we have about what polyhedrons

really are, and, as a community, to shift our focus to this new definition instead. Maybe it does not fully line up with all the intuitions we had. Maybe we feel that some of the objects that are not polyhedrons according to our new definition *were*, before. But this is irrelevant: the definition gives us something to work with. Switching to it is then a change in topic, if a useful one: we decide to stop focusing on capturing the term ‘polyhedron’ as she is used, and focus instead on the new, closed textured concept POLYHEDRON.

Burgess (2015) provides further support for my claim that making a concept fully rigorous is tantamount to changing the topic. This is because of what he dubs the *paradox of rigour*: any treatment of a given subject matter that is genuinely rigorous will ipso facto cease to be a treatment of that subject matter alone. After all, once such a treatment is fully rigorous, it will apply to *any* subject matter that can be modelled using it. It will concern anything that satisfies the postulates involved (Burgess 2015, 65–66). In the case of POLYHEDRON, this is evident in the fact that its new definition is fully divorced from our spatial intuitions. It might be used for utterly different concerns, e.g. for algebraic rather than geometric problems.

Now let us show that the open textured concepts are exactly those which have an assertory definition. Recall the two definitions of open texture we have seen so far:

**(OT Modified) Open texture, Tanswell’s definition modified.** *A concept or term displays open texture iff there are always possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept or not.*

**(OT Analyticity) Open texture, Analyticity definition.** *A concept or term displays open texture iff the class of analytic sentences involving this concept or term can always be changed to address use cases outside the standard domain of application.*

And let us add a definition of what it means for a definition to be ‘assertory’:<sup>9</sup>

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<sup>9</sup>There is no consensus definition of ‘assertory’. Shapiro (2005) claims that to be assertory is to have certain, fixed truth values, but this cannot be right because certain assertory statements can have varying truth values in i.e. a pluralist framework. Hellman (2003) speaks of ‘having an intended interpretation’, which may be better, but leaves open an issue of intention in definitions seen traditionally as algebraic (e.g. groups). Definition (Assertory) is meant to follow the crease of the existing debate in the foundations of mathematics.

**(Assertory) Definition.** *A concept or term has an assertory definition if its definition asserts the existence of a certain domain of objects, or asserts properties of such objects; otherwise it is algebraic.*

For ease of use, I shall call a concept ‘assertory’ itself if it has an assertory definition, and likewise with ‘algebraic’. I can now show that all these definitions are equivalent. This claim would entail that a concept being assertory is another way of claiming that it is open textured:

**(OT Assertory) Open Texture, definition by being assertory.** *A concept or term is open textured if it is assertory, i.e. if its definition asserts the existence of a certain domain of objects, or asserts properties of such objects.*

I can then establish the equivalence of these three statements by proving three conditionals:

(*OT Modified*)  $\implies$  (*OT Assertory*): If a concept is open textured by (*OT Modified*), then there is always possible objects falling outside the standard domain of application. That means that its standard domain of application cannot be *absolutely* everything – it cannot be a concept that applies or disapplies regardless of any domain it is defined on. But then it cannot be an algebraically defined concept, which is exactly such a concept. Thus, the concept must have an assertory definition.

(*OT Assertory*)  $\implies$  (*OT Analyticity*): If a concept is assertory, then its definition asserts the existence of a certain domain of objects. For a sentence to express something of such an object, it must refer to one of the objects thus defined. That means that if there are use cases outside the standard domain of application, we will need to assert further existence claims in order to refer to them. This would mean that the set of analytic propositions expressed of the concept must be extended, since these existence claims will need to be analytic. Thus this would mean that the concept is open textured by definition (*OT Analyticity*).

This leaves us with the task of excluding the option that there are *no* use cases outside the standard domain of application. In such cases, an assertory definition would have to become broad enough to cover *every* possible use case and to apply absolutely on any object whatsoever. To assert the impossibility of such cases, we must accept Waismann’s premise of the ‘moving descriptive horizon’, that any further specification of a concept or term will still leave open the possibility for future change. A counterexample would be an assertory definition that is so broad that the domain to which it applies becomes absolutely large, *only by asserting*



*the possibilities one by one.* Without taking an algebraic approach and giving a definition that applies absolutely to anything, we would be left with this impossible task of positively expressing absolutely all possibilities that might occur in the future.

(*OT Analyticity*)  $\implies$  (*OT Modified*): If the set of analytic statements concerning a concept  $C$  can be extended to deal with use cases outside the standard domain of application, then there must *be* the possibility of such use cases. In other words, there must be possible objects falling outside of the standard domain of application for which there is no fact of the matter as to whether they fall under the concept or not. Thus, (*OT Modified*) holds.

In this way, I can clearly delineate the extent of open texture: all concepts are open textured, except for the small class of algebraic concepts within mathematics are open textured. A fortiori, any term in natural science features open texture, as do any other empirical terms.

## 6. Consequences

What we see, then, is that a concept is open textured by the traditional definition if and only if it is open textured by the analyticity definition, if and only if it is defined in an assertory manner. There are some interesting consequences to this.

First and foremost, I can establish the extent of open texture. Since only mathematical concepts – and even then, only a fragment of mathematical concepts – are defined algebraically, non-mathematical terms all feature open texture. This should be unsurprising, since non-mathematical concepts typically concern certain real-world objects, phenomena, or ideas. There should then always be the *possibility* at least for the world to surprise us with new objects and situations that challenge the boundaries of our concepts. Thus, all such concepts have open texture, including concepts defined as seemingly strictly as those employed in theoretical physics. Moreover, I find that we can justify the various contradictory intuitions in the literature about open texture in mathematics. On one hand, we can find with Tanswell and Shapiro that there is open texture in math. However, it does not extend to *all* of mathematics: certain concepts, like *GROUP* or *NATURAL NUMBER* turn out to be ‘hard as rails’ after all.

Interestingly, *NATURAL NUMBER*, being ‘hard as rails’ and closed textured, ends up on the algebraic side of this divide. This is in spite of the fact that the naturals are often seen as clear (platonic) objects of one sort or another, and their definitions as assertions of their existence. In other

words, arithmetic, which deals with the naturals, is often seen as an assertory theory, perhaps even archetypically so. Fortunately, one need not resort to all sort of uncomfortable rhetorical devices to consider it an algebraic theory: a mere structuralist approach to the objects of arithmetic will suffice. On this view, it is not individual ‘cases’ of the natural numbers, such as the ordinals in set theory, or one of the many theories that model first order arithmetic, that can be identified with the natural numbers. Rather, ‘the natural numbers’ as such are what all these different ‘instances’ of the natural numbers have in common. In this specific case, there is a categoricity proof to strengthen the case. Dedekind characterises natural numbers as ‘simply infinite systems’: infinite collections with a successor relation and a first item, 1, that is not in the range of this relation. He then shows that any two such systems are isomorphic. In other words, whenever a truth of arithmetic holds of one, it holds of any of the others; all these different systems are, for the purposes of arithmetic, identical. This strengthens our intuitions that we are, in effect, dealing with ‘one thing’: *the* natural numbers. Hence, we get a strong intuition that we are talking about something particular, that statements of arithmetic assert something about certain objects.<sup>10</sup> And yet, through this very proof, Dedekind shows that the natural numbers are characterised, in the end, in a thoroughly algebraic manner. The natural numbers are *anything* meeting Dedekind’s definition of a simply infinite system, in much the same way groups are *anything* meeting the axioms of group theory. Thus, like GROUP, NATURAL NUMBER comes out algebraic.

This particular sharpening of the notion of algebraicity then shows a (reductive) structuralist account of mathematics.<sup>11</sup> It characterises mathematical objects such as numbers as the structure that different objects or systems have in common. I see no great trouble in interpreting the notions of ‘algebraic’ and ‘assertory’ in this manner. The distinction is not always fully sharp in current literature, but it aims at something real: there seems to be a real difference between groups and categories on one hand and sets on the other. In order to capture that as best we can, the concept of algebraicity was always going to need to be sharpened. If this can be done in a way that sheds more light on the nature of algebraic definitions, all the better.

Second, this framework can provide a new perspective on the discussion about alternative foundations of mathematics. So far, the discussion about algebraic and assertory foundations has concerned issues

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<sup>10</sup>See Dedekind (1901); for an overview of his effect on the foundations of mathematics, see Reck (2017).

<sup>11</sup>Benacerraf (1965) sets the stage for a structuralist account of the naturals convincingly and perhaps most famously. A thorough overview of structuralism in mathematics may be found in Shapiro (1997).

surrounding the lack of a background domain Hellman (2003) and presupposed notions of collection and relation (Feferman 1977; Hellman 2003) versus the claim that these things are not needed (Awodey 2004). With the equivalence with open texture in hand, I can illuminate some other aspects of the issue. For example, we may prefer the use of assertory concepts if we aim for our foundations to reflect the ever-evolving nature of mathematics, and if we are comfortable admitting that foundations of mathematics can never be 'done' per se. After all, assertory concepts have open texture, so there will be new cases that challenge the boundaries of our old definitions. Our foundations will be, at least theoretically, ever shifting. On the other hand, algebraic foundations, such as category-theoretic ones, are the only ones that can ever meet strict requirements of 'hardness' – that is to say, if we want our mathematics to be 'hard as rails' like natural numbers, we will have to use an algebraic foundation. Of course, some surprising new developments in mathematics may then simply not be expressible in the chosen foundational system.

Third, this equivalence allows us to investigate the relationship between open texture and the analytic/synthetic distinction. Closed textured concepts appear in a number of clearly analytic statements: groups have a unit. (That does not, of course, exclude the possibility of synthetic statements: 'groups are useful' or of borderline cases: 'groups are sets'.) The algebraic definition of a group provides us with a core of sentences concerning groups that are analytic. That does not mean that there cannot be other statements analytic of *GROUP*, such as 'groups are algebra' and 'groups are mathematical'; but these statements seem like they can be, in theory, subject to open texture. The meanings of 'mathematics' and 'algebra' might change in surprising ways over time. Of course they will: they are clearly open textured, assertory concepts. So statements like 'groups are algebra' are not analytic in any future-proof way, but this is not because of open texture in the notion of *GROUP* per se. Rather, it is open textured only as it relates to the concepts of *ALGEBRA*. By contrast, the analytic statement that 'a group has a unit' does not feature any concept that does not feature in the very definition of a group. A unit, here, is only defined with respect to the concept of a group itself. In other words, we only use terms that are part of the very mathematical definition of a group. And the norm for this definition is that it is never to be changed! We do not leave the fully delineated space of our definition, of our schema. Nothing in the definition relies on anything outside it. It is herein that we can find a statement that is clearly analytic, and that is going to *stay* analytic, no matter what else happens in mathematics.

Finally, I may make a rather pessimistic note for the foundations of mathematics: it seems that it may not be possible to find a satisfactory foundation for all of mathematics if one's goal is to find a proper domain of mathematical discourse.<sup>12</sup> Concepts with assertory definitions are open textured and thus can never provide a once-and-for-all domain of discourse for mathematics. Any such domain is always subject to change to deal with new use cases. On the other hand, concepts with algebraic definitions beg the question for any sort of background domain of discourse and thus cannot provide what we would want either. From a more optimistic perspective, however, both options could be lived with without too much trouble. Temporary, ever-evolving assertory foundations do not sound too bad; and tools built on algebraic definitions may still *provide* for a domain of discourse, even if they themselves do not necessarily have one themselves (i.e. the Elementary Theory for the Category of Sets is assertory in the sense that its definitions define sets, but it relies on a theory of categories which is defined strictly algebraically). And fully algebraic foundations may do fine, if we decide we are not interested in defining a domain for mathematical discourse.

## 7. Conclusion

While open texture is a widespread phenomenon, no sharp definition of Waismann's concept has gained any traction. By sharpening the notion and providing two suggestions for a definition, we get the ability to connect the notion of open texture to that of analyticity, which interested Waismann himself, and to the assertory/algebraic distinction, which is relevant in current discussions in the foundations of mathematics.<sup>13</sup> Moreover, it allows us to get a clearer view on the extent of open texture: it applies even within mathematics, but only on objects and theories defined in an assertory manner. And finally, we can now see beforehand which ambitions a proposed foundation of mathematics ought to keep or to forego.

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<sup>12</sup>Compare Penelope Maddy's 'generous arena' in Maddy (2017).

<sup>13</sup>Interestingly, Waismann himself seems to have had tendencies in this direction: in Waismann (1982), he expresses viewpoints surprisingly close to a contemporary eliminative structuralist approach to mathematics, relying on apparent and potential ambiguities in common definitions.

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