Investigating the underlying item characteristics in NIFU's 1+1 tests for elementary mathematics

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Popular abstract

Large amounts of text might make mathematical word problems more complex for lower elementary students. This, and other factors, such as comparative words and mathematical content, that can relate to a math problems difficulty, were studied by analyzing the responses to two mathematics tests, one for grade two and one for grade three. The main finding is that number of words and the use of comparative words such as 'more', or 'less' in a math word problem can contribute to the overall difficulty of the problem. It is therefore recommended that number of words and comparative terms in math problems are taken into consideration in math education and research, and that future research includes and further investigates the relationship between language factors and mathematical problem difficulty.

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Ymkje Haverkamp

Investigating the underlying item characteristics in NIFU's 1+1 tests for elementary mathematics

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Abstract

There are multiple item characteristics, some unrelated to mathematics, that can have an impact on item difficulty. Research into the effect of item characteristics such as number of words and comparative language has already been performed in larger state assessments in an American context but has not yet been implemented in a Norwegian setting. In this paper, the relationship between mathematical and linguistic item characteristics to variation in item difficulty is investigated in two tests of elementary mathematics via an explanatory item response modelling approach. The results show that number of words are the biggest driver of item difficulty in the second-grade test, and that comparative terms and number of words combined are the biggest drivers of item difficulty in the third-grade test, explaining 38% and 45% of the variance respectively. A higher number of words was related to a higher expected difficulty in both tests, and the presence of a comparative term in an item was related to a higher expected difficulty in the third-grade test. This finding indicates that the number of words should be considered while creating new test items both in research and in practice, as this might have an unexpected impact on item difficulty. The next stage would be to further investigate the item characteristics in a mathematical and linguistic framework-based test and extend the mathematical framework to distinguish better between different mathematical content.

Keywords: Elementary school mathematics, item analysis, word problems (mathematics), language of mathematics, item difficulty

Investigating the underlying item characteristics in NIFU's 1+1 tests for elementary mathematics

Better knowledge on what a test measures gives a better view on the general construct validity of the test (Whitely, 1983), which ultimately leads to a better understanding of the outcome of an intervention, research study or class instruction. Ensuring a test is valid and contains items of appropriate difficulty however, is not always an easy task, as there can be multiple facets to the construct that is measured, and multiple item characteristics driving item difficulty such as mathematical content, text length and complexity (Ferrara et al., 2011; Schneider et al., 2013; Shaftel et al., 2006). Story problems in mathematics provide an interesting example. These can be used to showcase realistic examples of everyday mathematics, incorporate new mathematical content to the students by extending on known problems, and have the potential to engage students more than a simple calculation might (Haylock, 2010; McNeil et al., 2009; Verschaffel, 2000). However, the extra text in an item could also introduce extra demands on the students, increasing the item's difficulty (Walkington et al., 2018).

Previous content analyses and research into the relationship between different item characteristics and item difficulty has mainly been done in the USA, where the alignment demands between a states educational core standards and educational assessments are strict (Porter et al., 2008). In Norway, curriculum standards are less defined, and the responsibility for educational assessment in lower elementary education lies mainly with the teachers (Tveit, 2014). In the Norwegian context, the 1+1 tests, created by The Nordic Institute for Studies in Innovation, Research and Education (NIFU) in order to monitor an intervention in their 1+1 project, are a good example of teacher-made and curriculum-inspired tests(Nordisk institutt for studier av innovasjon, forskning og utdanning (NIFU), n.d.). The items were created in cooperation with teachers experienced in item writing, and modeled to resemble

Norway's national test in elementary mathematics (Utdanningsdirektoratet, 2019). In this study, two of NIFU's 1+1 tests will be investigated in order to assess the relationship between linguistic and mathematical characteristics on item difficulty.

Linguistics in elementary mathematics

Various research has shown that a relationship exists between reading skills and mathematical skills. Reading skills are positively related to math achievement, especially on the subject of word problems (Grimm, 2008; Vilenius-Tuohimaa et al., 2008). But this correlation is a complicated and interesting one, as there are many components in reading and mathematics that could relate to each other in different ways, such as fluent retrieval from memory, underlying reasoning skills necessary in both reading and problem solving, and a person's phonological processing abilities, needed in both reading and computational skills (Hecht et al., 2001; Koponen et al., 2007; Vilenius-Tuohimaa et al., 2008).

In practice we see that word problems are often included in mathematics teaching and testing, and when done well, word problems are considered easier to grasp and are more interesting for students to participate in (McNeil et al., 2009). Word problems also have an important role to help young children extend the concept of addition and multiplication (Verschaffel, 2000). The use of word problems should therefore not be immediately shelved, but we should stay mindful of the extra demands the linguistic component of word problems can put on children.

Word problems do require a student to make the assumption that there is in fact always a solvable mathematical problem hidden in the story, and that some story elements incongruous with daily life can be ignored (Verschaffel, 2000). These assumptions make a large variety of word problems possible in the context of the classroom, but this habituation can in some cases lead to children trying to solve impossible word problems, such as the famous (and impossible) 'age of the captain' problem (Verschaffel, 2000).

Sometimes students struggle with grasping the full information from the text that is needed to solve the problem (Cummins et al., 1988; Verschaffel, 2000). Children with mathematical difficulties that are good readers have an advantage over children with both mathematical and reading difficulties on mathematical areas related to language, and they also show a faster progression in mathematics achievement over time than children with both mathematical and reading difficulties (Hanich et al., 2001; Jordan et al., 2002). Children with only mathematical difficulties perform equally on word problems compared to students with only reading difficulties; Jordan and colleagues (2003) hypothesise that these children can draw from their respective strengths to compensate for the weaker skills in either mathematics or reading.

Mathematics content or linguistic features are prime examples of demands that can impact the difficulty of an item in a mathematics test (Schneider et al., 2013; Shaftel et al., 2006; Warren, 2006). Higher reading load, meaning longer and more complex text, in mathematics items was found to be related to item difficulty (Ferrara et al., 2011; Walkington et al., 2018). Children also struggle more with comparative items (e.g., ‘Anna has 4 pencils. She has 2 pencils more than Jane does. How many pencils does Jane have?’), due to a potential lack of understanding of the meaning of comparative words (Schumacher & Fuchs, 2012; Warren, 2006). Shaftel and colleagues (2006) found that higher linguistic demands in mathematics items, specifically containing difficult mathematics vocabulary, are related to higher item difficulty. As certain words and vocabulary might be related to specific mathematical content, and for example in the case of comparative terms possibly indicate a specific operation (Hanich et al., 2001; Haylock, 2010), it is important to consider the relationship between both linguistic demands and mathematical content demands and item difficulty (Haghverdi et al., 2012; Shaftel et al., 2006). In this research, the focus will mainly lie on mathematical demands in terms of mathematical content and complexity, and on linguistic demands in terms of word count, and comparative terms.

Mathematical demands

Haghverdi and colleagues (2012) reported that students made more errors in the algebra and geometry categories than in the arithmetic category in their research. Schneider and colleagues (2006) included a similar distinction as predictor of item difficulty in their research into item difficulty in the US National Assessment of Educational Progress. This mathematical content variable, Applying Math Knowledge (Schneider et al., 2013), was divided into two hierarchical categories: the first category referring to numeration items, and the second category referring to items with other content areas found in mathematics curricula (e.g., measurement, probability, algebra and data analysis).

Multi-digit skills were labeled the most important skill needed for mathematical achievement in third-grade students by Cowan and Powell (2014), where students with mathematical difficulties especially struggled with. Multi-digit numbers are also processed at a different speed and in a different way than single-digit numbers (Brysbaert, 1995; Nuerk et al., 2011). To account for increasing difficulty due to the use of single- versus multi-digit numbers in the item, numerical complexity should be considered (Daroczy et al., 2015; Nuerk et al., 2011).

Linguistic demands

Reading demands in mathematics and science items has been the topic of interest in multiple research projects (e.g. Ferrara et al., 2011; Haghverdi et al., 2012; Shaftel et al., 2006; Stiller et al., 2016, 2016; Walkington et al., 2018). Ferrara and colleagues (2011) found that a higher reading load, in their paper defined as a combination of amount of complicated text and the presence of visual displays, is positively related to item difficulty. Moreover, Martiniello (2009) and Walkington and colleagues (2018), pointed out that the relationship between linguistic item characteristics and item difficulty is stronger for second language learners, students speaking a different language at home, and low-achieving students**.** Shaftel and colleagues (2006) studied the effect of language characteristics such as number of words, number of sentences, comparative terms and vocabulary in grade four, seven and ten, and reported that language characteristics had the most impact in grade four. If it is indeed the case that language characteristics are of more impact in the lower grades, it makes research like this only the more relevant.

It is not only the amount of words that can impact item difficulty, the type of words matters as well. Comparative words such as ‘more’ ‘less’ and ‘equal’ are difficult for young children to understand (Schumacher & Fuchs, 2012; Warren, 2006). Young children often lack a full conception of the meaning of the words, and might forget the comparative term easily, which makes it harder for them to grasp what is asked of them (Schumacher & Fuchs, 2012; Warren, 2006).

Hanich and colleagues (2001) categorized four types of word problems in their research into mathematical cognition in children with learning difficulties − Change, Equalize, Combine, and Compare − where equalize and compare word problems were considered as having the most complicated semantics. Shaftel and colleagues (2006) found that comparative terms added to an item’s difficulty for their seventh-grade sample, but noted that this finding is difficult to attribute to the linguistics alone considering comparative terms can be an indicator for specific mathematical content. It should be interpreted as being both a mathematical and linguistic demand (Shaftel et al., 2006). A step further within comparative items there are comparison-to-ratio items (Haylock, 2010). In early grades word problems work well to expand from addition to multiplication, and comparative items transfer through ‘double or tripple the amount of’ items into multiplication items (Haylock, 2010; Verschaffel, 2000).

Visual images could have an effect on item difficulty as well. Ferrara and colleagues (2011) included visual images as additional complexity in the coding for reading load, and Stiller and colleagues (2016) found in their research into item features affecting science item difficulty that visual images increased an item's difficulty.

Aims and expectations

The first aim of the study is to present the content of NIFU's 1+1 tests in terms of the theoretical framework. In order to do this, items and predictors in two versions of the 1+1 test will be mapped, and a reverse-engineered blueprint of the test will be provided. Given the nature of the test we expect to mainly find numeration items.

The second aim of the study is to investigate the relationship between the different item characteristics and item difficulty. This will be done by analyzing the item responses of second and third-grade students to their corresponding grade-level 1+1 test, using an explanatory item response approach. Based on the aforementioned theory, word count is expected to be a strong predictor of item difficulty, as well as comparative terms. In mathematical content we expect the non-numeration items to be more difficult.

Method

The data for this study came from NIFU's 1 plus 1 project (Nordisk institutt for studier av innovasjon, forskning og utdanning (NIFU), n.d.). The 1 plus 1 project was set up to assess the effect of additional small group mathematics instruction on lower elementary students' mathematical skills. This randomized controlled trial intervention study includes 163 schools from 10 of Norway's larger municipalities and runs over multiple grades and year cohorts. Randomization took place at the school level. In order to monitor the students' progress, the 1+1 tests were created to measure mathematical ability at grade one, two, and three in elementary school.

Sample

The sample for this study consisted of lower elementary school students from the elementary school cohort born in 2009 included in NIFU's 1+1 project. Only control group students were included to avoid that the original project's intervention introduced a confounding effect. The students were tested once in grade 2 ($N = 3985$, $N_{schools} = 82$), and once in grade 3 ($N = 3617$, $N_{schools} = 78$). The use of the data for the current study was approved under the General Data Protection Regulation (GDPR) by the Norwegian Centre for Research Data, see Appendix I for the information letter and application (reference number 885257).

1+1 tests

The 1+1 tests are built to fall in line with Norway's national test in elementary mathematics, and is, like many educational measures in Norwegian context, designed in cooperation with teachers with experience in test item generation. The measures used in this study were the second-grade version and third-grade version of NIFU's 1+1 test. Both tests contained 19 items, after recoding two items into one in the third-grade test due to local item dependence issues (for examples of test items see Appendix III part A). Three items in the second-grade test and four items in the third-grade test that were originally scored with partial credit were recoded dichotomously (i.e., over 50% partial credit scored 1, otherwise 0).

Framework

To code the items within the 1+1 tests, two item demands frameworks were assembled from multiple research studies in the field of mathematics education and item writing (Daroczy et al., 2015; Ferrara et al., 2011; Haylock, 2010; Schneider et al., 2013; Shaftel et al., 2006; Warren, 2006). One framework with a focus on mathematical item characteristics, and one framework with a focus on linguistic item characteristics. The tests are not expected to follow a strict structure in terms of how often certain types of items are represented. Both frameworks are described below in more detail.

Mathematical demands

For the Mathematical demands framework (see Table 1) in this study, elements of frameworks from Daroczy and colleagues (2015) and Schneider and colleagues (2013) were used. The item descriptions were adapted to suit the sample and data, and not all categories used in the original research studies were included. Applying Mathematical Knowledge relates to the mathematics content within the item. It has a hierarchical nature, with Applying Mathematical Knowledge 1 (dummy coded as 0) considered easier, and Applying Mathematical Knowledge 2 (dummy coded as 1) considered more difficult (Haghverdi et al., 2012; Schneider et al., 2013).

Daroczy and colleagues (2015) pointed out the importance of including the property of numbers, such as the range, or single versus multi-digit numbers, as one of the factors that contributes to item difficulty through numerical complexity. In the current study numerical complexity is represented by coding for multi-digit numbers. Multi-digit is a dummy coded variable that relates to the distinction in numerical complexity between single- and multi-digit numbers (Daroczy et al., 2015; Haghverdi et al., 2012; Nuerk et al., 2011), with 1 indicating multi-digit numbers, 0 indicating otherwise.

Linguistic demands

The linguistic demands framework (see Table 2) contains adapted elements from a wide range of prior studies and frameworks (Ferrara et al., 2011; Haylock, 2010; Shaftel et al., 2006; Stiller et al., 2016). The coding for Number of words was a centered variable, showing the number of words in the problem statement, as done in research by Shaftel and colleagues (2006). Visual images is a dummy-coded variable, with 1 indicating a photograph or drawing is included in the item, as done in research by Stiller and colleagues (2016). The

Comparative variable is another dummy-coded variable, where 1 indicates the presence of a comparative term, such as ‘how many more’ in the item. Multiplicative comparatives were included as a separate variable with the theory in mind that during the first years of mathematics education multiplication is introduced, with comparative terms being one of the tools for transitioning from adding and subtracting to multiplication and division (Haylock, 2010). Ratio-comparatives is dummy-coded, with 1 indicating the occurrence of a multiplicative comparative term, such as ‘three times as many as’.

Table 1

Mathematical demands framework

Code	Description
Applying Mathematical Knowledge (AMK)	AMK is scored 0 if items require students to apply mathematics content and processes that are usually learned in their respective grade or before in number and number relations, computation and numerical estimation, and operation concepts (numeration). AMK is scored 1 if items require students to apply mathematics content and processes that are usually learned in their respective grade in measurement; geometry and spatial sense; data analysis, statistics and probability; and patterns, functions and algebra. It also includes some more advanced or complex applications of knowledge learned in their respective grade and before. This covers algorithms, procedures, translation between verbal expressions and equations, graphing, definitions, and terminology.
Multi-digit	Multi-digit response required.

Note. adapted from (Daroczy et al., 2015; Schneider et al., 2013)

Table 2

Linguistic demands framework

Code	Description
Number of words	Total number of words in the items problem statement
Visual images	Item includes picture or table
Comparative	Item contains comparative terms such as ‘greater than’ ‘smaller than’ ‘more’, ‘equal’ or ‘less’
Ratio-comparative	Item contains ratio-comparative terms such as ‘twice as many’, ‘double of’, ‘half of’, ‘three times as many’

Note. adapted from (Ferrara et al., 2011; Haylock, 2010; Shaftel et al., 2006; Stiller et al., 2016)

Statistical analysis

For both tests, the same procedure was followed after basic data management and preparation (e.g., excluding duplicate entries or fixing other obvious data entry errors). In a first step the items were coded by the author with the use of both frameworks. To provide an item demand blueprint of both tests, the distribution and intercorrelations of item demands are described. Secondly, the psychometric properties of both tests will be described using an item response theory approach, including model and item fit, and conditional reliability and targetting of the test (Lord, 2012). Missing item responses were recoded into 0 if in-between answered items. In case of not-reached items, missings remained and were treated as missing-at-random (Mislevy & Wu, 1996), except for the last seen item, which was recoded into 0. Item response patterns with a high amount of missing (proportion missing larger than .66), were considered problematic and were omitted. A total of 27 students were excluded in the second-grade test, and 28 students in the third-grade test, giving an effective sample size of n = 3958 and n = 3589 respectively. These descriptive IRT analyses are conducted via the mirt package (Chalmers, 2012), and illustrated with the WrightMap package (Torres Irribarra & Freund, 2014) in the statistical software environment R (R Core Team, 2020).

To examine how the different item characteristics relate to item difficulty, an explanatory item response approach was used (Wilson & De Boeck, 2004). Multiple item response models were fitted in the lme4 package in R (Bates et al., 2015; De Boeck et al., 2011). First, single predictor models were estimated for each item characteristic of the mathematical and linguistic demands frameworks, then a full model including all item characteristics was estimated, followed by three grouped models, combining specific linguistic demands and mathematical demands highlighted as important predictors in previous research.

The one-parameter logistic item response model (1 PL model) was chosen for its compatability with the practical use of the 1+1 tests, as it assumes equally discriminating items and the 1+1 tests are graded with sum scores. In the one parameter logistic item response model, the probability of a certain person (*p*) answering correctly to a certain item (*i*) is derived from that person's ability (θ_p) and that item's difficulty (β_i):

$$\Pr\,(Y_{pi} = 1|\theta_p, \beta_i = \frac{\exp(\theta_p - \beta_i)}{1 + \exp(\theta_p - \beta_i)},$$

The higher a person's ability is compared to the item's difficulty, the higher the probability of answering correctly. The approach used in this research builds on this and extends the model by including item characteristics as predictors for the item's difficulty, such that the item difficulty is now predicted based on *K* item predictors, $\beta_i = \sum_{k=1}^{K} X_{ik} b_k + \varepsilon_i$ (Janssen et al., 2004). In this explanatory model, part of the differences in difficulty between items is related to differences in the included item characteristics (X_1 to X_k) and to other remaining factors (as represented by the residual term ε_i).

Results

Test Blueprint

The two 1+1 tests include items both on basic numeration and on mathematical content such as measurement, data analysis, statistics and probability (i.e., AMK, see Table 3 and Table 4). Ratio-comparative items occurred least, with three items in the second-grade test, and four items in the third-grade test. Approximately half of the items in both tests included a visual representation and over half of the items required the student to work with multi-digit numbers. The number of items with multi-digit numbers was higher in the third-grade test, 16 out of 19 items, compared to the second-grade test, 12 out of 19 items.

The correlations between number of words and comparative terms and between mathematical knowledge and comparative terms were relatively strong (see Table 3 and Table 4). This is in line with the theory, as previous research found that comparative items both relate to the linguistic item features and to the mathematical content of the item (Shaftel et al., 2006).

Table 3

Blueprint Grade 2 Test

	Proportion in test	Correlation				
Characteristics		1.	2.	3.	4.	5.
1. AMK	0.37					
2. Number of words	15 (11)[a]	.03				
3. Comparatives	0.32	−.52	.5			
4. Ratio comparatives	0.16	−.33	.25	.64		
5. Visual	0.53	.29	.03	−.49	−.46	
6. Multi-digit	0.63	−.77	−.36	.28	.33	−.29

Note. [a] Mean (Standard Deviation) of number of words per item.

Table 4

Blueprint Grade 3 Test

Characteristics	Proportion in test	Correlation 1.	2.	3.	4.	5.
1. AMK	0.47					
2.Number of words	13 (9)[a]	.09				
3.Comparatives	0.32	.26	.46			
4.Ratio-comparatives	0.21	.03	.52	.76		
5.Visual	0.58	.17	.00	.12	−.08	
6.Multi-digit	0.84	−.46	−.11	−.33	−.13	−.08

Note. [a] Mean (Standard Deviation) of number of words per item.

Psychometric characteristics

Missingness

Item missingness was investigated, both per item and per person. There was a correlation between missingness per item and item order in both tests (0.59 in grade 2, 0.73 in grade 3), indicating a higher amount of missingness towards the end of the test. No systematic missingness was found when relating missingness at the person level to available background information on municipality and school of the student.

Model fit

Model fit was assessed, following recommended fit statistics (Maydeu-Olivares, 2013). The Rasch model fitted the data reasonably well in both grade 2 ($M2(170) = 1090$, $p < .001$, RMSEA 95% CI = [0.040 - 0.044], SRMSR = 0.06, CFI = 0.951), and in grade 3 ($M2(170) = 1045$, $p < .001$, RMSEA 95% CI = [0.041 - 0.046], SRMSR = 0.057, CFI = 0.945). Empirical item characteristic curves were used to inspect item misfit. Some items were discriminating slightly more than average, but no extreme discrepancies or anomalies were observed. Yen (1984)'s Q3 statistic was examined for both tests, but no local item dependency was indicated. More details on the item fit can be found in Appendix III part B.

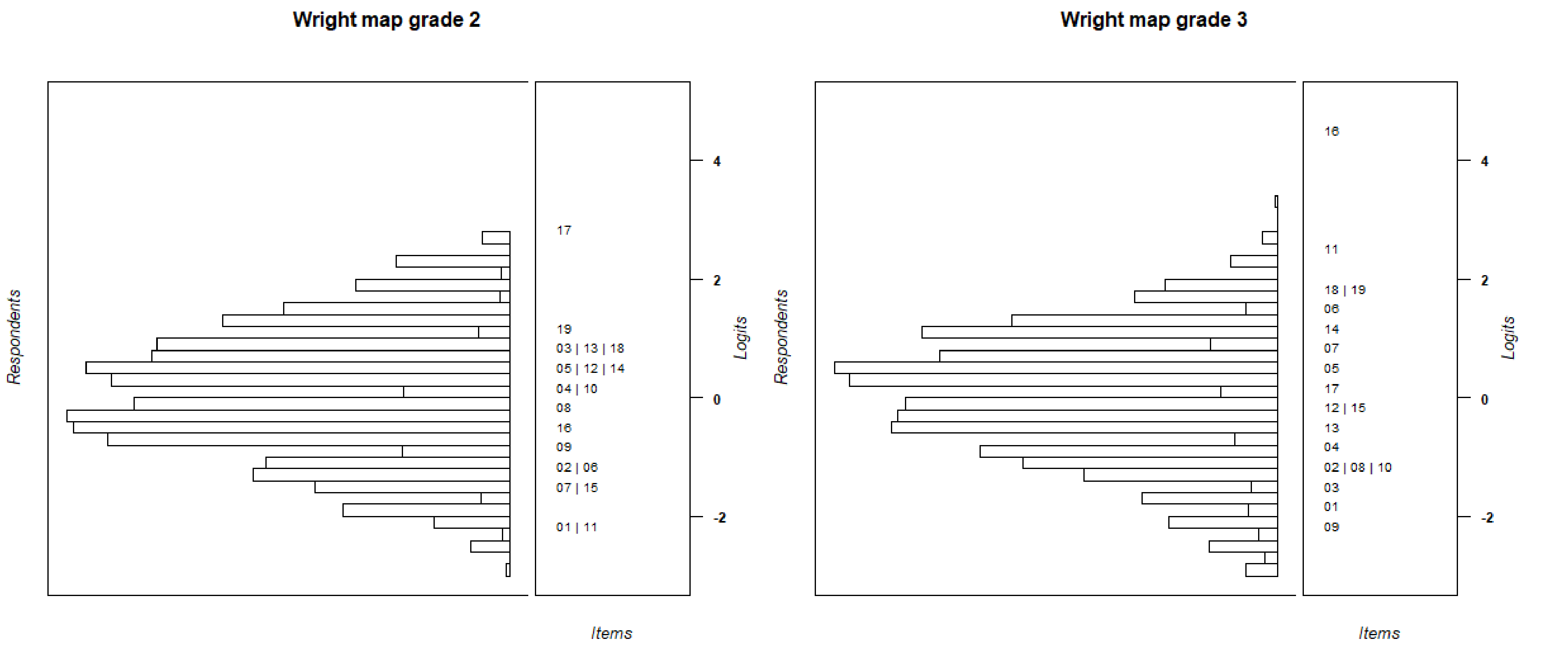
Test information and reliability

Both 1+1 tests adequately cover the lower-to-average ability level (see Wright maps Figure 1). The second-grade test is lacking some more difficult items, with the exception of item 17. Grade three has some more items at the upper end of the scale, with yet again one exceptionaly difficult item, item 16. Note that these two items did not show extreme problems with item fit, and hence, both items were kept throughout the analysis.

The test information plot in Figure 2 (left), shows that both tests give the most information around the average ability level, where most items were located. The tests are most informative on the lower-to-average ability level, indicating that it targets lower and average-achieving students slightly better than high-achieving students. The reliability of the test for students around the average on the latent ability scale lies between .70 and .80 (Figure 2, right). Measurement of lower and higher-achieving students is less precise with reliabilities dropping to .60 or lower for students close to two standard deviations away from the mean.

Figure 1

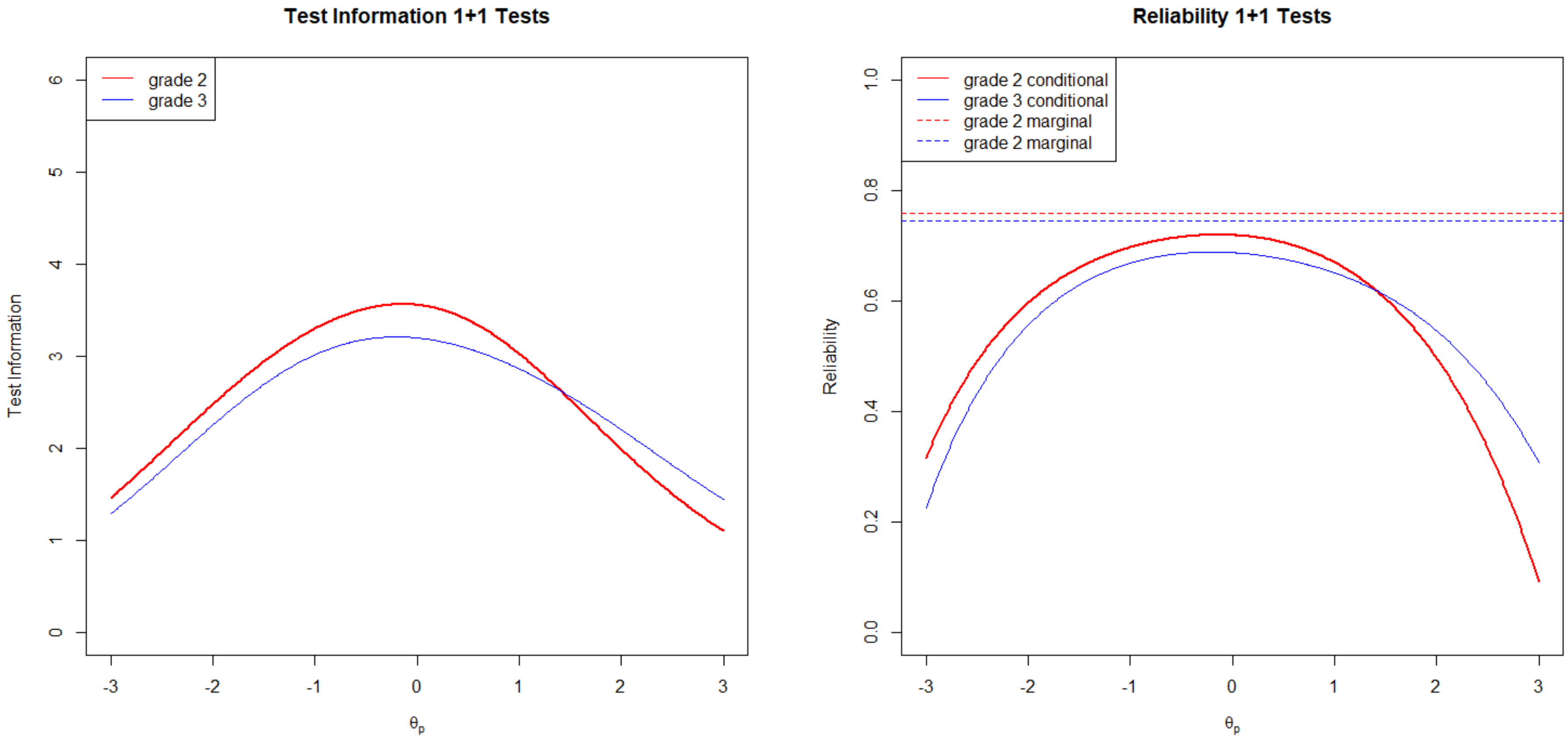
Wright Map of Grade 2 and Grade 3 Tests



Note. In the Wrigh map, each item's difficulty and the respondents' ability is mapped on the same (ability) scale (Callingham & Bond, 2006).

Figure 2

Test Information and Reliability Plots for the 1+1 Tests.



Note. θ_p = Ability level, Conditional reliability, the reliability conditional on the estimated ability level. This value differs for different response patterns. Marginal reliability, the general reliability of the test.

Relationship between item characteristics and item difficulty

Null model

In order to assess the relationship between item characteristics and item difficulty in the 1+1 tests, first a null model was created. It showed that the average second-grader has 54% chance of giving a correct response to an average item in the second-grade 1+1 test, for the average third-grader the chance of correctly answering an average third-grade-1+1 test item was 45%. Of the response variation in the second-grade test, 22% could be attributed to person differences, and 26% to item differences. Of the response variation in the third-grade test, 20% could be attributed to person differences, and 36% to item differences. The difference between the response variation attributed to item differences does indicate a certain disparity in the character of the two tests.

Single predictor models

The top right section of Table 5 shows the parameter estimates and model fit of the single predictor explanatory item response models for the second-grade 1+1 test. In each model, only one item characteristic was included as a predictor of item difficulty to examine to what extent this characteristic could explain item difficulty. Of all single predictors in the second-grade test, only Number of words showed a better fit than the null model when compared on AIC. The differences in number of words between different items explained 38% of the variance in item difficulty ($\chi^2(1) = 9.06$, $p = 0.003$) and had a negative relationship with item easiness ($b_{1(k)} = -.08$, $Z = -3.41$, $p < .001$). In this test, the expected difference in item difficulty between an item that has 10 words extra compared to another item, is .8 ability points (for the ability scale, see Figure 1). In general for the second-grade test this means that the more words an item contains, the more difficult the item can be expected to be.

In contrast, for the third-grade test, none of the single predictors showed a better model fit than the null model when compared on AIC (see Table 6, top right). Number of words explained the most variance in item difficulty, at 10% , closely followed by Comparatives, with nine percent; however, none of the predictors were significant.

Full model

For the full model an explanatory item response model including all predictors was fitted for each of the second and the third-grade tests. The full model for grade two explained 52% of the variance in item difficulty ($\chi^2(6) = 13.87$, $p = 0.032$), and AIC, indicated a better fit than the null model and almost all single predictor models (see Table 5, top left). The single predictor model with Number of words as a predictor remains the stronger model as the full model did not show a better model fit ($\chi^2(6) = 4.81$, $p = 0.440$).

The full model results for the third-grade test can be seen in the top left section of Table 6. The full model explained 48% of the variance in item difficulty, and had a better fit than both the null model and all single predictor models compared on the AIC, but a likelihood ratio test showed it was not a significantly better model than the null model ($\chi^2(6) = 12.34$, $p = .055$).

Table 5

Parameter Estimates and Model Fit for the Second-Grade 1 +1 Test - Explanatory Item Response Model

	Full Model		Single Predictor Model		
Predictors	b_k	SE	$b_{1(k)}$	R^2 (%)	AIC
(Intercept)	−0.18	0.84			79073.6 [a]
Applying Mathematical Knowledge (AMK)	0.04	0.74	−0.20	1	79075.5
Number of words	−0.12***	0.03	−0.08***	38	79066.5
Comparatives	1.82*	0.74	−0.26	1	79075.4
Ratio-comparatives	−0.49	0.73	−0.47	2	79075.3
Visual images	0.46	0.45	−0.09	0	79075.6
Multi-digit	−0.63	0.81	0.55	4	79074.8
Full model R^2				52	79071.7
Language+Compare				47	79067.6
	No. Words		−0.10***		
	Comparatives		1.20*		
	Ratio-comp.		−0.76		
Math type+Compare				4	79078.9
	AMK		−0.45		
	Comparatives		−0.28		
	Ratio-comp.		−0.44		
Complexity+Compare				9	79077.8
	Multidigit		0.77		
	Comparatives		−0.13		
	Ratio-comp.		−0.70		

Note. b regression coefficients, *SE* standard errors, R^2 % explained variance in item difficulty, *AIC* Akaike's information criterion

[a] Fit value for null model

*** $p <.001$, ** $p < .01$, *$p < .05$

Table 6

Parameter Estimates and Model Fit for the Third-Grade 1 +1 Test - Explanatory Item Response Model

	Full Model		Single Predictor Model		
Predictors	b_k	SE	$b_{1(k)}$	R^2 (%)	AIC
(Intercept)	−0.06	0.89			66797.0 [a]
Applying Mathematical Knowledge	−0.38	0.54	−0.72	5	66798.0
Number of words	−0.08*	0.04	−0.06	10	66797.1
Comparatives	−2.87**	1.08	−1.03	9	66797.2
Ratio-comparatives	3.84***	1.32	0.37	1	66798.8
Visual images	0.48	0.63	−0.16	0	66798.9
Multi-digit	−0.18	0.77	0.87	4	66798.2
Full model R^2				48	66796.6
Language+Compare				45	
	No. Words		−0.08*		66791.5
	Comparatives		−2.82***		
	Ratio-comp.		3.70***		
Math type+Compare				32	66795.5
	AMK		−0.05		
	Comparatives		−3.00**		
	Ratio-comp.		2.97*		
Complexity+Compare				32	66795.6
	Multidigit		0.05		
	Comparatives		−3.01**		
	Ratio-comp.		2.98**		

Note. b regression coefficients, *SE* standard errors, R^2 % explained variance in item difficulty, *AIC* Akaike's information criterion

[a] Fit value for null model

*** p <.001, ** p < .01, *p < .05

Grouped predictor models

Shaftel and colleagues (2006) noted in their discussion that comparative terms can be both seen in terms of (mathematical) language and as mathematical content. One model with Number of words and both comparative predictors was tested, as Ferrara and colleagues (2011) found that reading load and mathematics vocabulary consistently related to item difficulty. Two models, pairing both comparative predictors with one of the mathematical predictors, were used to test a situation where comparing is seen as a type of mathematical problem (Hanich et al., 2001).

In the grade two test, the Language+Compare model explained 47% of the variance in item difficulty, and outperformed the null model ($\chi^2(3) = 12.01$, $p = .007$). A likelihood ratio test with Number of words as a single predictor model indicated the use of the sparser model ($\chi^2(3) = 2.95$, $p = .229$). In the grade three test, Language+Compare explained 45% of the variance in item difficulty and outperformed the null model ($\chi^2(3) = 11.41$, $p = .010$) and all single predictor models. With a non-significant likelihood ratio test with the full model ($\chi^2(3) = 0.94$, $p = .817$), the sparser Language+Compare model is kept.

In the second-grade, Number of words as a single predictor proofed to be the strongest model. This means that according to this model, verbose items are expected to be more difficult than a brief item. In the third-grade, the Language+Compare model was the strongest model, indicating that in the third-grade test, an item with a comparative term is expected to be more difficult than a different item similar on word count and ratio-comparatives without a comparative term. The Number of words remains a predictor of item difficulty in the third-grade test as well, with an increase of item difficulty expected for a verbose item versus brief item, with Comparatives and Ratio-comparatives held constant.

The Math type+Compare model and the Complexity+Compare model were not significantly better than the null model in both tests. The results indicate that number of words is the driving force of item difficulty in the second-grade 1+1 test, and that Number of words, Comparative and Ratio-comparative terms are the driving force of item difficulty in the third-grade 1+1 tests.

Discussion

The current study was aimed to map the contents of the 1+1 tests and investigate the relationship between item characteristics and item difficulty in the 1+1 tests. Previous research has indicated that word count and mathematical content in mathematics items can increase item difficulty, and that children potentially have difficulties understanding comparative terms in mathematics (Ferrara et al., 2011; Schneider et al., 2013; Schumacher & Fuchs, 2012; Shaftel et al., 2006; Warren, 2006). This type of research was mainly done in an American context with larger-scale assessments based on prescribed competence demands. In the USA, proof of alignment between educational assessments and state mandated content standards is required by law, and a number of councils and research groups exist to create and assess procedures for content analysis (Porter et al., 2008). In Norway however, the dynamic between government and teachers differs, as teachers carry the main responsibility for student assessment in primary school, and there are few government-mandated assessments for this grade range (Tveit, 2014). The 1+1 tests are good examples of the type of assessments used in a Norwegian context. New assessments are created by teachers without the involvement of the test industry and, as Norway has a less explicitly stated elementary school curriculum, are more based on mutual consensus of what measures the curriculum and what the test should look like.

Item and predictor mapping of the 1+1 tests

The mapping of the 1+1 tests to the frameworks showed that most items in the third-grade test included multi-digit numbers; 16 of 19 items required the students to work with multi-digit numbers, compared to 12 of 19 items in the second-grade test. The magnitude effect describes the phenomenon that it takes more time to complete a problem as the magnitude of the numbers in the problem increases (Brysbaert, 1995). Cognition studies also show that multi-digit numbers require different processing than single-digit numbers (e.g. Nuerk et al., 2011). The difference in amount of multi-digit items is possibly one of the reasons the correlation between proportion missing per item and item order was stronger in the third-grade test. The test was timed, and it might have taken the students longer to complete each problem in the third-grade test than in the second-grade test. The low stakes nature of the test is another possible explanation for the higher missingness towards the end in both tests. Students may have become tired, or lost interest or motivation.

Word count and Comparative items were positively correlated in both tests. Shaftel and colleagues (2006) saw a similar effect in their study. This could possibly be due to language ability as an underlying skill. Fuchs and colleagues (2010) found language to be a unique predictor of word problems development, and in research specifically into comparative terms, Schumacher and Fuchs (2012) and Warren (2006) found that with instruction on the language demands of comparative problems, a student's understanding of, and performance on, comparative word problems increased.

Shaftel and colleagues (2006) also adressed a possible relationship between comparative items and mathematical content. This was not seen in the current study, with the strength and direction of the correlation between mathematical content and comparative items differing in both tests. This could be due to the presence of other possible mathematical factors, such as number properties as magnitude, role of the number and the required operation (Brysbaert, 1995; Daroczy et al., 2015; Nuerk et al., 2011) (all not coded for in the current study due to limitations in the design of the test).

In the mapping of the items to the students' ability level, two items proved remarkably difficult compared to the other items. Item 17 in the second-grade test, was an item very similar to item 9, an item earlier in the test, but with one difference. Instead of having to look at the increase, as was asked in the earlier item, this time the student was asked to look at the decrease. A possible explanation that may have added to the item's difficulty is that students overlooked the change in direction and assumed a similar task as seen earlier, as item 9 itself was of average difficulty in the test. Item 16 in the third-grade test had quite ambiguous wording with multiple possible interpretations, where only one of these led to the correct answer. This ambiguity might explain the notable difficulty of the item.

Overall, both tests mapped better, and provided most information, for students located at the lower to average ability level. Considering that the conditional reliability reduced substantially at two standard deviations from the mean, and that the targetting missed out on high achievers, the 1+1 tests are less suited for individual assessment. They do however serve their intended purpose of monitoring the general mathematical skill during the intervention at the macro level.

Relationship between item characteristics and item difficulty

Word count explained a substantial part of the variation in item difficulty as a single predictor in the second-grade test, and remained the strongest model throughout. This finding was not replicated in the third-grade test, where none of the single predictor models outperformed the null model. The findings in the second-grade test align with the findings by Ferrara and colleagues (2011) that reading load was a significant predictor of item difficulty.

It is possible that in the third-grade test the effect of word count was obscured by other variables, as word count did predict item difficulty in the Language+Compare model.

In order to better understand the comparative predictors, three separate grouped models were tested. The Language+Compare model was the strongest for the third-grade test. The combination of word count, comparative terms and ratio-comparative terms as predictors explained almost half of the variance in item difficulty for both tests, and outperformed both the null model and the full model. In the second grade test however, the single predictor model with number of words was still the best fitting model.

A comparative story problem is considered semantically more complicated (Hanich et al., 2001; Jordan & Hanich, 2000). This was seen in the third-grade test only to a certain extent, as comparative and ratio-comparative terms showed opposing directions. With all other predictors held constant, an item with a comparative term in the third-grade test is expected to be more difficult than an item without a comparative term, whereas an item including a ratio-comparative term is expected to be less difficult than a similar item without a ratio-comparative term. In the second-grade test, number of words was the strongest driver of item difficulty.

Different models fitted the different tests better, although number of words was related to item difficulty in both occasions. Shaftel and colleagues (2006) noted in their research that comparative language and number of words shared predictive variance, this can have been at play in the current study as well. It could also be due to the inherent differences between the two tests. Type of mathematical operation in the item could not be accounted for, and the type of ratio questions differed as well. In the second-grade test the ratio items related to the double of, and half of certain amounts, whereas the third-grade also included the multiplication table of three and four.

Limitations and implications

The main limitation of the current study was that the 1+1 tests were not created with the mathematical and linguistic demands frameworks in mind. For the purpose of the intervention it was not strictly necessary to create a set of items with balanced item characteristics. A well-balanced test created following the mathematical and linguistic demands frameworks would better allow for further investigating the impact of item characteristics on item difficulty in mathematics.

Some potentially relevant item characteristics did not occur in one of the tests and would not have allowed for the same frameworks to be applied in both versions of the 1+1 test. An example of this is the comparison between two numeration items (see Appendix III part A). The first item of both tests required the student to count with amounts of money. In the second-grade test the student had to select one picture out of two options, with the highest total amount of money. In the third-grade test the student had to fill in the exact total amount in an open response format. It is conceivable that through the difference in response format, multiple choice versus open response, the difficulty of the second-grade test item is lower than the difficulty of the third-grade test item. But due to a lack of variation in the third-grade test, only two items were multiple choice, it was not possible to code for response format.

It is possible that including more information on item format or other content related predictors would account for part of the still unexplained variance. Other elements that could be included in future research would be for example the four types of story problems − Change, Equalize, Combine and Compare − as defined by Hanich and colleagues (2001) , mathematical factors such as number properties as recommended by Daroczy and colleagues (2015), or through covering all strands of lower elementary mathematics as stated in the Norwegian curriculum − Numeration, Geometry, Measurement, and Statistics and Probability − (Utdanningsdirektoratet, 2020).

The inclusion of a teacher rated difficulty variable, as Schneider and colleagues (2013) included in their research, would also be a valuable addition. Having a group of experienced teachers rate a set of items on difficulty by placing them in order, or assigning an estimated proportion correct adds an element of how an item is expected to behave for a certain group of students. This could offer more insight in specific item types that are found to be more complicated in practice than expected by the teachers.

The finding that the number of words in an item was related to item difficulty in both tests, with an increase in word count leading to a decrease in item easiness, and that the use of comparatives was related to item difficulty in the third-grade test, is important for both research and practice. It shows that word count should not be overlooked as a predictor of item difficulty in mathematics in future frameworks and that the comparative and ratio-comparative predictor should be investigated further in a more structured design to untangle the direction of the relationship to item difficulty and the correlation with word count. It also indicates the importance for teachers, item writers and other experts in the field to consider the amount of words and comparative terms that are used while creating new mathematics items. Martiniello (2009) and Walkington (2018) pointed out in their research that this effect of added reading load might even be larger for second language learners. This element could not be adressed in the current study, but in the future, it is advised that this effect should be explored.

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Appendix I – A. GDPR Documents and Ethical Approval

NSD NORSK SENTER FOR FORSKNINGSDATA

Notification Form 885257

Last updated

24.03.2020

Which personal data will be processed?

- National ID number or other personal identification number

Type of data

Will you be processing special categories of personal data or personal data relating to criminal convictions and offences?

No

Project information

Project title

Masters thesis - Mathematics ability in primary school children in Norway.

If the collected personal data will be used for other purposes, please describe

Secondary data only, and will only be used in this project. The original data collection over a broader scope is done by NIFU and is part of a larger multiple year project (which has received approval from NSD).

Explain why the processing of personal data is necessary

The students' Norwegian personal ID is collected, which is used to link test ID occurrences and changes across time points in the original project. They exist in a securely stored register on an NSD-approved storage medium. After using this to link test data across time points, the working data files with test responses will be safely anonymized.

External funding

Type of project

Student project, Master's thesis

Contact information, student

Ymkje Elisabeth Haverkamp, ymkjeeh@student.uv.uio.no, tlf: 0047[redacted]

Data controller

Data controller (institution responsible for the project)

NIFU – Nordisk institutt for studier av innovasjon, forskning og utdanning

Project leader (academic employee/supervisor or PhD candidate)

Stephan Daus, stephan.daus@nifu.no, tlf: 90[redacted]

Will the responsibility of the data controller be shared with other institutions (joint data controllers)?

No

Sample 1

Describe the sample

Lower primary grade students (grade 3 data for this project specifically)

Recruitment or selection of the sample

Secondary data is the only data in this project. Sample originally selected by NIFU under the 1 plus 1 prosjektet.

Age

7 - 9

Will you include adults (18 years and over) who do not have the capacity to consent?

No

Personal data relating to sample 1

- National ID number or other personal identification number

How will you collect data relating to sample 1?

Data from another research project

Legal basis for processing general categories of personal data Consent (art. 6 nr. 1 a)

Who will give consent for children under 16 years?

Parents/guardians

Information for sample 1

Will you inform the sample about the processing of their personal data?

No

Explain why you will not inform the sample about the processing of their personal data.

Only secondary data is used in this project, Communications with sample run via the original datacontroller NIFU.

Third Persons

Will you be processing data relating to third persons?

No

Documentation

How will consent be documented?

- Electronically (email, e-form, digital
- signature) Manually (on paper)

How can consent be withdrawn?

contact as specified by original datacontroller NIFU

How can data subjects get access to their personal data or have their personal data corrected or deleted?

contact as specified by original datacontroller NIFU

Total number of data subjects in the project

5000-9999

Approvals

Will you obtain any of the following approvals or permits for the project?

Processing

Where will the personal data be processed?

- Computer belonging to the data controller
- Mobile device belonging to the data controller

Who will be processing/have access to the collected personal data?

- Project leader
- Student (student project)
- External co-workers/collaborators inside the EU/EEA
- Internal co-workers

Will the collected personal data be transferred/made available to a third country or international organisation outside the EU/EEA?

No

Information Security

Will directly identifiable data be stored separately from the rest of the collected data (e.g. in a scrambling key)?

Yes

Which technical and practical measures will be used to secure the personal data?

- Personal data will be anonymised as soon as no longer
- needed Restricted access

Duration of processing

Project period

01.09.2019 - 31.12.2020

Will personal data be stored after the end of the project?

No, the collected data will be stored in anonymous form

Which anonymization measures will be taken?

- Other

Secondary data, Storage and anonymization measures are done by data controller NIFU

Will the data subjects be identifiable (directly or indirectly) in the thesis/publications from the project?

No

Additional information

Due to using secondary, already existing data, not all points relate well to the project. While the project by NIFU uses a wider range of data for a longer time period, this project only contains results from mathematics tests and a code to track students. NIFU uses a securely stored register on an NSD-approved storage medium with the students' Norwegian personal ID, which is used to link test ID occurrences and changes across time points.

The dataset for this project contains the following variables:
AssessmentRoundID
Title - test title
UserID - generated ID
CandidateExternalID - a generated ID for the duration of the project (løpenr)
Questionnumber
Questiontitle
QuestionID
Score
Duration
Starttime
Endtime
Municipality
School
Class - a number indicating the cohort the students are in (09 meaning the 2009 cohort etc) StrataID
Tschool - dichotomous variable indicating wether the participant is in a test school or not.

NIFU considers this work falling under the 1+1 project, reference number 47196

NORSK SENTER FOR FORSKNINGSDATA

NSD's assessment

Project title
Masters thesis - Mathematics ability in primary school children in Norway.

Reference number
885257

Registered
16.01.2020 av Ymkje Elisabeth Haverkamp - ymkjeeh@uio.no

Data controller (institution responsible for the project)
NIFU – Nordisk institutt for studier av innovasjon, forskning og utdanning

Project leader (academic employee/supervisor or PhD candidate)
Stephan Daus, stephan.daus@nifu.no, tlf: 90[redacted]

Type of project
Student project, Master’s thesis

Contact information, student
Ymkje Elisabeth Haverkamp, ymkjeeh@student.uv.uio.no, tlf: 0047[redacted]

Project period
01.09.2019 - 31.12.2020

Status
26.03.2020 - Assessed

Assessment (1)

26.03.2020 - Assessed

Our assessment is that the processing of personal data in this project will comply with data protection legislation, so long as it is carried out in accordance with what is documented in the Notification Form and attachments, dated 26.03.2020, as well as in correspondence with NSD.

Everything is in place for the processing to begin.

NOTIFY CHANGES
If you intend to make changes to the processing of personal data in this project it may be necessary to notify NSD. This is done by updating the information registered in the Notification Form. On our website we explain which changes must be notified. Wait until you receive an answer from us before you carry out the

 changes.

TYPE OF DATA AND DURATION
The project will be processing general categories of personal data until 31.12.2020.

LEGAL BASIS

The project will gain consent from data subjects to process their personal data. We find that consent will meet the necessary requirements under art. 4 (11) and 7, in that it will be a freely given, specific, informed and unambiguous statement or action, which will be documented and can be withdrawn. The legal basis for processing personal data is therefore consent given by the data subject, cf. the General Data Protection Regulation art. 6.1 a).

PRINCIPLES RELATING TO PROCESSING PERSONAL DATA

NSD finds that the planned processing of personal data will be in accordance with the principles under the General Data Protection Regulation regarding:

- lawfulness, fairness and transparency (art. 5.1 a), in that data subjects will receive sufficient informationabout the processing and will give their consent
- purpose limitation (art. 5.1 b), in that personal data will be collected for specified, explicit and legitimatepurposes, and will not be processed for new, incompatible purposes
- data minimisation (art. 5.1 c), in that only personal data which are adequate, relevant and necessary for thepurpose of the project will be processed
- storage limitation (art. 5.1 e), in that personal data will not be stored for longer than is necessary to fulfilthe project's purpose

THE RIGHTS OF DATA SUBJECTS

Data subjects will have the following rights in this project: transparency (art. 12), information (art. 13), access (art. 15), rectification (art. 16), erasure (art. 17), restriction of processing (art. 18), notification (art.
19), data portability (art. 20).

These rights apply so long as the data subject can be identified in the collected data.

NSD finds that the information that will be given to data subjects about the processing of their personal data will meet the legal requirements for form and content, cf. art. 12.1 and art. 13.

We remind you that if a data subject contacts you about their rights, the data controller has a duty to reply within a month.

FOLLOW YOUR INSTITUTION'S GUIDELINES

NSD presupposes that the project will meet the requirements of accuracy (art. 5.1 d), integrity and confidentiality (art. 5.1 f) and security (art. 32) when processing personal data.

To ensure that these requirements are met you must follow your institution's internal guidelines and/or consult with your institution (i.e. the institution responsible for the project).

FOLLOW-UP OF THE PROJECT

NSD will follow up the progress of the project at the planned end date in order to determine whether the processing of personal data has been concluded.

Good luck with the project!

Contact person at NSD: Gry Henriksen
Data Protection Services for Research: +47 [redacted]

Appendix I – B. Information to parents and teachers.

1+1 prosjektet

NIFU/Samtykke s. 1

Informasjon til foresatte om skolens deltakelse i 1+1 prosjektet

Din kommune deltar i forskningsprosjektet 1+1 prosjektet: Smågruppeundervisning i matematikk på småskoletrinnet. Til sammen deltar 160 skoler fordelt over 10 kommuner i Norge. Hensikten med prosjektet er å undersøke om elevene oppnår bedre ferdigheter i matematikk når skolene tildeles mer lærerressurser slik at matematikkundervisningen kan gis i små grupper på 4-6 elever i deler av skoleåret. Prosjektet er finansiert av Norges forskningsråd og utføres i et samarbeid mellom tre forskningsmiljøer: NIFU Nordisk institutt for studier av innovasjon, forskning og utdanning, Senter for økonomisk forskning (SØF) og Institutt for samfunnsforskning (ISF).

Som del av prosjektet vil alle skolene gjennomføre korte prøver i matematikk i

begynnelsen og mot slutten av hvert skoleår. Dette gjøres for å undersøke hvordan elevenes ferdigheter i matematikk har utviklet seg i løpet av skoleåret. Elevene vil også få spørsmål om læringsmiljøet sitt. Prøvene vil utgjøre et pedagogisk verktøy for matematikklærerne i utformingen av tilpasset opplæring for hver elev. Prosjektet vil videre innhente resultater fra nasjonale prøver på 5. og 8. trinn, samt karakterer på 10. trinn. Enkelte av skolene vil få besøk av forskere i løpet av prosjektperioden, disse vil snakke med matematikklærere og observere undervisningen.

I prosjektet vil det være viktig å kunne koble resultater på tvers av prøvene, ved hjelp av et identifikasjonsnummer for eleven. Prosjektet vil koble på informasjon om foreldres utdanningsnivå, kjønn og innvandringsstatus (fødeland) for elever og foreldre. Dette hentes fra Statistisk Sentralbyrås befolkningsregister.

Sammenkoblingen av resultater fra ulike tester og kobling med data fra befolkningsregistret krever aktivt samtykke fra foreldre/foresatte. Prosjektet vil gi verdifull informasjon om hvordan kvaliteten i norsk skole kan forbedres. Vi håper derfor at du/dere er villige til å delta.

Hva skjer med informasjonen om elevene?

Alle personopplysninger vil bli behandlet konfidensielt. Kun forskere i prosjektgruppen vil ha tilgang til personopplysningene. Dataene lagres på et sikret område, hvor kun prosjektdeltakere har tilgang, via passord. Elever og skoler anonymiseres i analysene. Det betyr at ingen elever eller skoler vil kunne gjenkjennes i noen publikasjoner fra prosjektet.

Prosjektet skal avsluttes 31.07.2021. Data vil opprettholdes i sin opprinnelige form fram til 31.12.2025. Etter det vil dataene bli anonymisert. Datainnsamlingen er godkjent av Personvernombudet for forskning ved Norsk samfunnsvitenskapelig datatjeneste (NSD).

Frivillig deltakelse

Det er frivillig å delta i studien, og du kan når som helst trekke ditt samtykke uten å oppgi noen grunn. Dersom du trekker samtykket, vil alle personopplysninger om barnet ditt slettes. Beslutningen om å delta eller ikke vil ikke ha noen innvirkning på ditt barns undervisning.

For mer informasjon om 1+1 prosjektet, se vår hjemmeside: http://1pluss1prosjektet.no På forhånd takk for ditt bidrag til dette prosjektet!

Vennlig hilsen
Vibeke Opheim
Prosjektleder

Samtykke

Jeg har mottatt informasjon om prosjektet, og gir tillatelse til at mitt barn deltar.

Det er til stor hjelp for oss dersom du kan fylle ut barnets og skolens navn med BLOKKBOKSTAVER.

Barnets navn: ____

Klassenavn: ____

Skolens navn: ____

Foresattes signatur: ____

(Skjemaet leveres til skolen).

Appendix II – Data Management and Analysis Code

```
###################################
##SETUP & DATA IMPORT
###################################
#install.packages('WrightMap')
pkg = c("mirt","tidyverse", 'dplyr', 'tidyr', 'car', 'corrplot',
'WrightMap',  'lme4', 'psych')     #vector with packages you use
sapply(pkg,library,character.only=TRUE) #load in relevant packages at start


missingRecoder <- function(df) { # df must be a dataframe, not a matrix, in
this function
  for(row in 1:nrow(df)) { ## Recoding omitted as incorrect, leaving not-
reached as missing and the rest as is
    for(col in (ncol(df)-1):3) { # Going backwards
      if(is.na(df[row, col])) { # If this cell is empty
        if(!all(is.na(df[row, col:ncol(df)]))) { # and all cells from this
cells to the last column are not all empty
          df[row, col] <- 0L
        } else if(!is.na(df[row, col-1])) { # and the cell after this is
not empty
          df[row, col] <- 0L
        }
      }
    }
  }
  df
}

#1. A clean item response dataset: Persons - by - Items with first column
the person ID
    #ID variable for persons: "Person"
    #All binary coded items
    #clean implies unique codes, no duplicates or missing ID
#2. A clean item covariates dataset: Items - by - Features with first
column the item ID
    #ID variable for items: "Item"
    #AMK dummy: 0 = basic, 1 =rest, word count, recoded, item order, ...
#3. A clean person covariates dataset: Persons - by - Characteristics


dataIR17 = read.csv("dataIRwide17.csv")
dataIR17 = cbind(dataIR17$Person,missingRecoder(dataIR17[,-c(1,2)]))
    names(dataIR17)[1]="Person"

dataIR18 = read.csv("dataIRwide18.csv")
    dataIR18 = cbind(dataIR18$Person,missingRecoder(dataIR18[,-c(1)]))
    names(dataIR18)[1]="Person"


dataP17=read.csv('rowpersons (1).csv')
dataP18= read.csv('rowpersons18 (1).csv')
dataI17=read.csv('newframework17.csv')
dataI18=read.csv('newframework18.csv')

###################################
##ITEM RESPONSE DATA SUMMARY
###################################
```

```
#####BASIC#################################################################
####
    #Compute simple proportion correct per item and add as covariate to
dataI dataset
    dataI17$PCORRECT.I = apply(dataIR17[,-1],2,mean,na.rm=TRUE)
    dataI18$PCORRECT.I = apply(dataIR18[,-1],2,mean,na.rm=TRUE)


    #Compute simple proportion correct per person and add as covariate to
dataP dataset
    all.equal(dataP17$Person,dataIR17$Person)
    dataP17$PCORRECT.P = apply(dataIR17[,-1],1,mean,na.rm=TRUE)
    all.equal(dataP18$Person,dataIR18$Person)
    dataP18$PCORRECT.P = apply(dataIR18[,-1],1,mean,na.rm=TRUE)

    #Missing per item and add as covariate to dataI dataset
    dataI17$PMISS.I = apply(is.na(dataIR17[,-1]),2,mean)
    dataI18$PMISS.I = apply(is.na(dataIR18[,-1]),2,mean)

    #Missing per person and add as covariate to dataP dataset
    all.equal(dataP17$Person,dataIR17$Person)
    dataP17$PMISS.P = apply(is.na(dataIR17[,-1]),1,mean)
    all.equal(dataP18$Person,dataIR18$Person)
    dataP18$PMISS.P = apply(is.na(dataIR18[,-1]),1,mean)

    #Over 2/3 missing, considered as problematic

    EXCLUDE17 = dataP17$Person[dataP17$PMISS.P>(2/3)]; length(EXCLUDE17)
#27
    dataP17[EXCLUDE17,]

    EXCLUDE18 = dataP18$Person[dataP18$PMISS.P>(2/3)]; length(EXCLUDE18)
    dataP18[EXCLUDE18,]

    #EXCLUDE FROM DATASET
    dataP17 = dataP17[!dataP17$Person%in%EXCLUDE17,]
    dataIR17 = dataIR17[!dataIR17$Person%in%EXCLUDE17,]
    dataP18 = dataP18[!dataP18$Person%in%EXCLUDE18,]
    dataIR18 = dataIR18[!dataIR18$Person%in%EXCLUDE18,]

    #Item distribution? Person distribution?
    par(mfrow=c(2,2))

hist(dataP17$PCORRECT.P);abline(h=median(dataP17$PCORRECT.P),lwd=2);mean(da
taP17$PCORRECT.P>.50,na.rm=TRUE)

hist(dataP18$PCORRECT.P);abline(h=median(dataP18$PCORRECT.P),lwd=2);mean(da
taP18$PCORRECT.P>.50,na.rm=TRUE)
    barplot(dataI17$PCORRECT.I, names.arg =
c(1:19));barplot(dataI18$PCORRECT.I, names.arg = c(1:19))

        #item 17 extremely difficult?

    dev.off()    #reset par

#####Item response
patterns##################################################
PAT17 = dataIR17[,!names(dataIR17)%in%"Person"]
PAT18 = dataIR18[,!names(dataIR18)%in%"Person"]
PAT17[is.na(PAT17)]=9
PAT18[is.na(PAT18)]=9
```

```
PAT17 = apply(PAT17,1,paste0,collapse="")    #Item response pattern strings
with 9 as missing
PAT18 = apply(PAT18,1,paste0,collapse="")
    #Do the following:
    #count number of unique response patterns
    #compare to theoretically possible number and your number of persons
    #show those with high frequency
pat17=table(PAT17) #2^19=524288 possible response patterns + the not
reached possibilities but keep in mind n
length(pat17)    #unique responses
pat17[order(pat17,decreasing=TRUE)[1:15]]

pat18=table(PAT18) #2^19=524288 possible patterns + the not reached ones,
but keep in mind n
length(pat18)    #unique responses
pat18[order(pat18,decreasing=TRUE)[1:15]]

######Missing data
patterns#####################################################

    summary(dataP17$PMISS.P)
    summary(dataP18$PMISS.P)

    #make school variable unique
dataP17$school <- apply(dataP17[, c('munic', 'school')], 1, paste, collapse
= '')
dataP18$school <- apply(dataP18[, c('munic', 'school')], 1, paste, collapse
= '')
dataP17[,'school'] <- sapply(dataP17[,'school'], as.numeric)
dataP18$school<- as.numeric(dataP18$school)

describe(dataI17)
describe(dataI18)
    #Does person missingness go together with   person covariates?
    round(cor(dataP17,use="pairwise.complete.obs"),2)
    round(cor(dataP18,use="pairwise.complete.obs"),2)

    #Does item missingness go together with item covariates?
    round(cor(dataI17,use="pairwise.complete.obs"),2)
    round(cor(dataI18,use="pairwise.complete.obs"),2)

#inspect missingnes per school.
    options(max.print=999999)
    table(dataP17$PMISS.P,dataP17$school)
  table(dataP18$PMISS.P,dataP18$school)

  unique(dataP17$school) #82 schools
  unique(dataP18$school) #78 schools

  #inspect missing per municipality
  table(dataP17$PMISS.P,dataP17$munic)
  table(dataP18$PMISS.P,dataP18$munic)

######DESCRIPTIVE
IRT#######################################################################
    #using MIRT
    #Model comparison, Rasch, 2PL, 2dim 2PL
M1.17<-mirt(dataIR17[,-1],1,itemtype="Rasch") #Rasch chosen
M1.18<-mirt(dataIR18[,-1],1,itemtype="Rasch") # Rasch chosen.
#M2.17<-mirt(dataIR17[,-1],1,itemtype="2PL")
#M2.18<-mirt(dataIR18[,-1],1,itemtype="2PL")
```

```
#M3.17<-mirt(dataIR17[,-1],2,itemtype="2PL")
#M3.18<-mirt(dataIR18[,-1],2,itemtype="2PL")

sapply(lapply(c(M1.17,M1.18),M2,na.rm=TRUE),round,3)
M2(M1.17, na.rm = TRUE)

    #Summarize:

coef(M2.17,IRT=TRUE,simplify=TRUE)
coef(M2.18,IRT=TRUE,simplify=TRUE)

itemestimates17 <- as.data.frame(coef(M1.17,IRT=TRUE,simplify=TRUE))
itemestimates18 <- as.data.frame(coef(M1.18,IRT=TRUE,simplify=TRUE))

ITEMFIT = lapply(c(M1.17,M1.18),mirt:::itemfit,na.rm=TRUE)
ITEMFIT
fun<-function(x){
        alpha = .05 / nrow(x)
        return( which(x$p.S_X2<=alpha) )
}
MISFIT = lapply(ITEMFIT,fun)
MISFIT
#inspect empirical plots for possible misfit items
mirt:::itemfit(M1.17, empirical.plot = 1)
mirt:::itemfit(M1.17, empirical.plot = 2)
mirt:::itemfit(M1.17, empirical.plot = 3)
mirt:::itemfit(M1.17, empirical.plot = 5)
mirt:::itemfit(M1.17, empirical.plot = 7)
mirt:::itemfit(M1.17, empirical.plot = 9)
mirt:::itemfit(M1.17, empirical.plot = 14)
mirt:::itemfit(M1.17, empirical.plot = 15)
mirt:::itemfit(M1.17, empirical.plot = 18)

mirt:::itemfit(M1.18, empirical.plot = 5)
mirt:::itemfit(M1.18, empirical.plot = 7)
mirt:::itemfit(M1.18, empirical.plot = 12)
mirt:::itemfit(M1.18, empirical.plot = 13)
mirt:::itemfit(M1.18, empirical.plot = 14)
mirt:::itemfit(M1.18, empirical.plot = 16)
mirt:::itemfit(M1.18, empirical.plot = 18)

Q3.M1.17<-mirt:::residuals(M1.17, type = "Q3")
corrplot(Q3.M1.17, title = 'Q3 plot grade 2', mar=c(0,0,2,0))
#Clean :)
                                #check for 2dimensional structure
                                summary(M3.17, rotate= 'promax')

Q3.M1.18<-mirt:::residuals(M1.18, type = "Q3")
corrplot(Q3.M1.18, title = 'Q3 plot grade 3', mar=c(0,0,2,0))       #clean
now 1&2 is 1 item
                                #check for 2 dimensional structure
                                summary(M3.18, rotate= 'promax')


    #marginal reliability & test information / conditional reliability
function
    #test information

    plot(seq(-3, 3, by = 0.01), testinfo(M1.17, seq(-3, 3, by = 0.01)),
type = "l", xlab = expression(theta[p]), ylab = "Test Information", ylim =
c(0, 6), lwd = 2.5, col = "red", main = "Test Information 1+1 Tests")
```

```
    lines(seq(-3, 3, by = 0.01), testinfo(M1.18, seq(-3, 3, by = 0.01)),
col="blue")
    legend('topleft', legend= c('grade 2','grade 3'), col = c('red',
'blue'), lwd = 0.2)

    #Conditional reliability is a function of testinformation
    plot(seq(-3, 3, by = 0.01), 1-1/testinfo(M1.17, seq(-3, 3, by = 0.01)),
type = "l", xlab = expression(theta[p]), ylab = "Reliability", ylim = c(0,
1), lwd = 2.5, col = "red", main = "Reliability 1+1 Tests")
    lines(seq(-3, 3, by = 0.01), 1-1/testinfo(M1.18, seq(-3, 3, by =
0.01)), col="blue")
    legend('topleft', legend= c('grade 2 conditional','grade 3
conditional', 'grade 2 marginal', 'grade 2 marginal'),
           col = c('red', 'blue', 'red', 'blue'), lty = c(1,1,2,2), lwd =
0.2)
    #marginal reliability with assumed normal distribution for theta =
population-based
    abline(h = marginal_rxx(M1.17),col="red",lty=2)
    abline(h=marginal_rxx(M1.18),col="blue",lty=2)

        #targetting: person ability vs item difficulty
 #wrightmap
 Thetaset17<- mirt::fscores(M1.17, method = 'EAP' )
 Thetaset18<- mirt::fscores(M1.18, method = 'EAP')
 difficulties17  <- itemestimates17$items.b
 difficulties18 <- itemestimates18$items.b
 wrightMap(thetas = Thetaset17, thresholds = difficulties17, main.title =
'Wright map grade 2',
           item.prop = .25, item.side = 'itemClassic', person.side =
'personHist', min.l = -3, max.l = 5)
 wrightMap(thetas = Thetaset18, thresholds = difficulties18, main.title =
'Wright map grade 3',
           item.prop = .25, item.side = 'itemClassic', person.side =
'personHist', min.l = -3, max.l = 5)

##################################
##ITEM SUMMARY
##################################
#inspection
    describe(dataI17)
    describe(dataI18)
    round(cor(dataI17,use="pairwise.complete.obs"),2)
ic.c17<-    round(cor(dataI17,use="pairwise.complete.obs"),2)
corrplot(ic.c17)
    round(cor(dataI18,use="pairwise.complete.obs"),2)
    ic.c18 <-round(cor(dataI18,use="pairwise.complete.obs"),2)
    corrplot(ic.c18)
#relevant predictors
    describe(dataI17)
    describe(dataI18)
ic.17<- round(cor(dataI17[,c('AMK', 'WORDCOUNT', 'COMPARE', 'RATIOCOMPARE',
'VISUAL', 'MULTIDIGIT')],use="pairwise.complete.obs"),2)
    corrplot(ic.17, method = 'number', title = 'Correlation Table grade 2',
mar=c(0,0,1,0))

ic.18 <-round(cor(dataI18[,c('AMK', 'WORDCOUNT', 'COMPARE', 'RATIOCOMPARE',
'VISUAL', 'MULTIDIGIT')],use="pairwise.complete.obs"),2)
    corrplot(ic.18, method = 'number', title = 'Correlation Table grade 3',
mar=c(0,0,1,0), col = 'black')
```

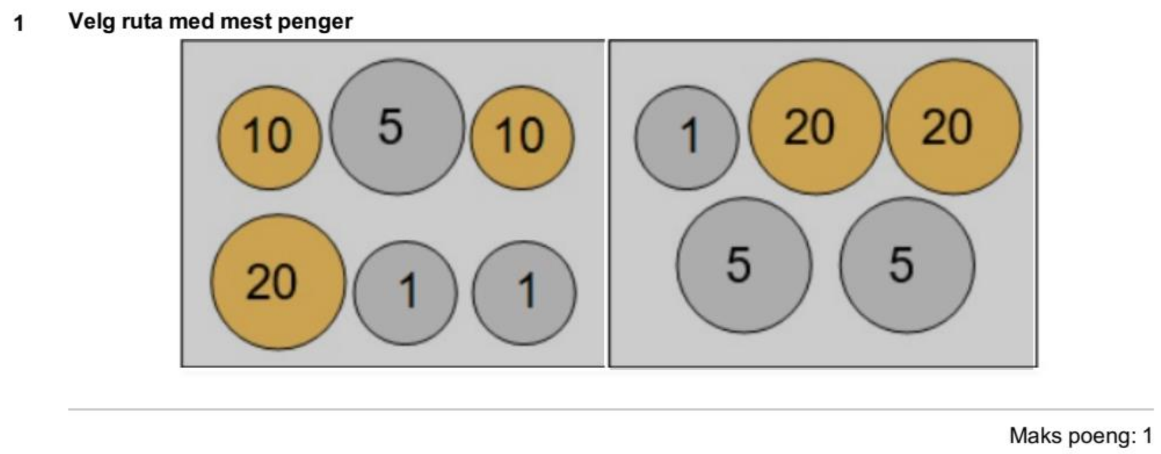
```
###################################
##Explanatory IRT
###################################
    #DATA PREP

        #center continuous
      dataI17$WORDCOUNT = dataI17$WORDCOUNT-mean(dataI17$WORDCOUNT)
      dataI18$WORDCOUNT = dataI18$WORDCOUNT-mean(dataI18$WORDCOUNT)
##############################
#explanatory IRT in LME4
##############################

#match item labels and create combined dataset
dataIR17 <- pivot_longer(data = dataIR17, cols = c('X1', 'X2', 'X3', 'X4',
'X5', 'X6', 'X7', 'X8',
                                                    'X9', 'X10','X11',
'X12', 'X13', 'X14', 'X15', 'X16',
                                                    'X17', 'X18', 'X19'),
names_to = 'Item', values_to = 'SCORE')
    dataIR18 <- pivot_longer(data = dataIR18, cols = c('X1', 'X2', 'X3',
'X4', 'X5', 'X6', 'X7', 'X8',
                                                        'X9', 'X10','X11',
'X12', 'X13', 'X14', 'X15', 'X16', 'X17', 'X18', 'X19'), names_to = 'Item',
values_to = 'SCORE')


dataI17$Item <- c('X1', 'X2', 'X3', 'X4', 'X5', 'X6', 'X7', 'X8',
                 'X9', 'X10','X11', 'X12', 'X13', 'X14', 'X15', 'X16',
                 'X17', 'X18', 'X19')
dataI18$Item <- c('X1', 'X2', 'X3', 'X4', 'X5', 'X6', 'X7', 'X8',
                  'X9', 'X10','X11', 'X12', 'X13', 'X14', 'X15', 'X16',
                  'X17', 'X18', 'X19')
dataIR.I18 <- merge(x = dataI18, y = dataIR18, by = 'Item', all = TRUE)
dataIR.I17 <- merge(x = dataI17, y = dataIR17, by = 'Item', all = TRUE)

#null models
M0.0.17<- glmer( SCORE ~ 1 + (1|Person) + (1|Item), data=dataIR.I17,
family=binomial(logit))
summary(M0.0.17)

M0.0.18<- glmer( SCORE ~ 1 + (1|Person) + (1|Item), data=dataIR.I18,
family=binomial(logit))
summary(M0.0.18)

#single predictor models
#2017
colnames(dataI17)
M0.1.17<- glmer( SCORE ~ 1 + (1|Person) + AMK+(1|Item), data=dataIR.I17,
family=binomial(logit))
summary(M0.1.17)
M0.2.17<- glmer( SCORE ~ 1 + (1|Person) + WORDCOUNT+(1|Item),
data=dataIR.I17, family=binomial(logit))
summary(M0.2.17)
M0.3.17<- glmer( SCORE ~ 1 + (1|Person) + RECODED+(1|Item),
data=dataIR.I17, family=binomial(logit))
summary(M0.3.17)
M0.4.17<- glmer( SCORE ~ 1 + (1|Person) + COMPARE+(1|Item),
data=dataIR.I17, family=binomial(logit))
summary(M0.4.17)
M0.5.17<- glmer( SCORE ~ 1 + (1|Person) + RATIOCOMPARE+(1|Item),
data=dataIR.I17, family=binomial(logit))
```

```
summary(M0.5.17)
M0.6.17<- glmer( SCORE ~ 1 + (1|Person) + ORDER+(1|Item), data=dataIR.I17,
family=binomial(logit))
summary(M0.6.17)
M0.7.17<- glmer( SCORE ~ 1 + (1|Person) + VISUAL+(1|Item), data=dataIR.I17,
family=binomial(logit))
summary(M0.7.17)
M0.8.17<- glmer( SCORE ~ 1 + (1|Person) + MULTIDIGIT+(1|Item),
data=dataIR.I17, family=binomial(logit))
summary(M0.8.17)

#single vs null
anova(M0.0.17, M0.2.17)

#2018
M0.1.18<- glmer( SCORE ~ 1 + (1|Person) + AMK+(1|Item), data=dataIR.I18,
family=binomial(logit))
summary(M0.1.18)
M0.2.18<- glmer( SCORE ~ 1 + (1|Person) + WORDCOUNT+(1|Item),
data=dataIR.I18, family=binomial(logit))
summary(M0.2.18)
M0.3.18<- glmer( SCORE ~ 1 + (1|Person) + RECODED+(1|Item),
data=dataIR.I18, family=binomial(logit))
summary(M0.3.18)
M0.4.18<- glmer( SCORE ~ 1 + (1|Person) + COMPARE+(1|Item),
data=dataIR.I18, family=binomial(logit))
summary(M0.4.18)
M0.5.18<- glmer( SCORE ~ 1 + (1|Person) + RATIOCOMPARE+(1|Item),
data=dataIR.I18, family=binomial(logit))
summary(M0.5.18)
M0.6.18<- glmer( SCORE ~ 1 + (1|Person) + ORDER+(1|Item), data=dataIR.I18,
family=binomial(logit))
summary(M0.6.18)
M0.7.18<- glmer( SCORE ~ 1 + (1|Person) + VISUAL+(1|Item), data=dataIR.I18,
family=binomial(logit))
summary(M0.7.18)
M0.8.18<- glmer( SCORE ~ 1 + (1|Person) + MULTIDIGIT+(1|Item),
data=dataIR.I18, family=binomial(logit))
summary(M0.8.18)

#full model
M1.2.17<- glmer( SCORE ~ 1 + (1|Person) + AMK + WORDCOUNT +  COMPARE +
RATIOCOMPARE + VISUAL + MULTIDIGIT+(1|Item), data=dataIR.I17,
family=binomial(logit))
summary(M1.2.17)

M1.2.18<- glmer( SCORE ~ 1 + (1|Person) + AMK + WORDCOUNT +  COMPARE +
RATIOCOMPARE + VISUAL + MULTIDIGIT+(1|Item), data=dataIR.I18,
family=binomial(logit))
summary(M1.2.18)

#full vs null
anova(M0.0.17, M1.2.17)
anova(M0.0.18, M1.2.18)
#full vs single
anova(M0.2.17, M1.2.17)


#grouped
#2017
```

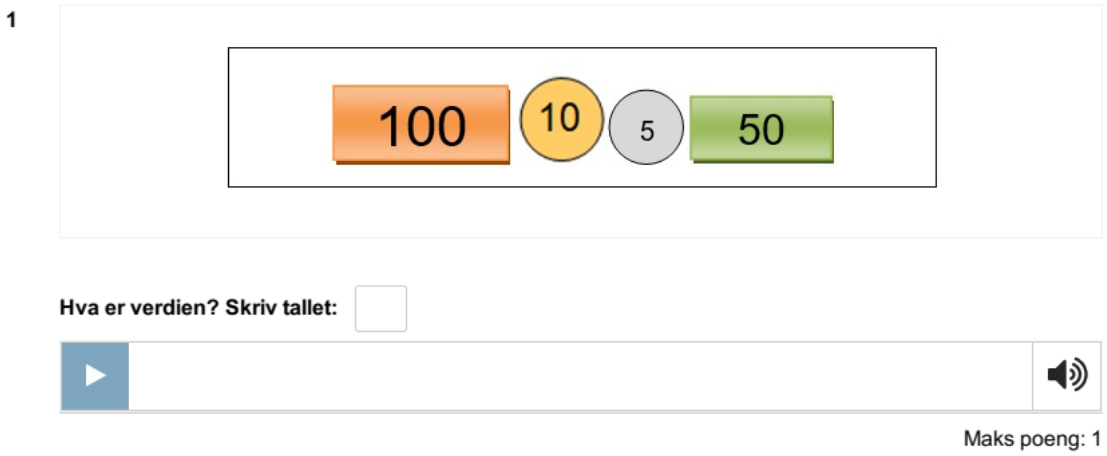
```
m3.4.17 <- glmer(SCORE ~ 1 + (1|Person) + WORDCOUNT + COMPARE +
RATIOCOMPARE +(1|Item), data = dataIR.I17, family=binomial(logit))
summary(m3.4.17)

m3.5.17 <- glmer(SCORE ~ 1 + (1|Person) + AMK + COMPARE + RATIOCOMPARE
+(1|Item), data = dataIR.I17, family=binomial(logit))
summary(m3.5.17)

m3.6.17 <- glmer(SCORE ~ 1 + (1|Person) + MULTIDIGIT + COMPARE +
RATIOCOMPARE +(1|Item), data = dataIR.I17, family=binomial(logit))
summary(m3.6.17)

#group vs null
anova(M0.0.17, m3.4.17)
anova(M0.0.17, m3.5.17)
anova(M0.0.17, m3.6.17)
#Language+compare group vs language single
anova(M0.2.17, m3.4.17)

#group vs full
#anova(M1.2.17, m3.4.17)

#2018
m3.4.18 <- glmer( SCORE ~ 1 + (1|Person) + WORDCOUNT + COMPARE +
RATIOCOMPARE +(1|Item), data=dataIR.I18, family=binomial(logit))
summary(m3.4.18)

m3.5.18 <- glmer( SCORE ~ 1 + (1|Person) + AMK + COMPARE + RATIOCOMPARE
+(1|Item), data=dataIR.I18, family=binomial(logit))
summary(m3.5.18)

m3.6.18 <- glmer( SCORE ~ 1 + (1|Person) + MULTIDIGIT+ COMPARE +
RATIOCOMPARE +(1|Item), data=dataIR.I18, family=binomial(logit))
summary(m3.6.18)

#group vs null
anova(M0.0.18, m3.4.18)
anova(M0.0.18, m3.5.18)
anova(M0.0.18, m3.6.18)
#Language+compare group vs full
anova(M1.2.18, m3.4.18)

invlogit<-function(x){
  Pr = 1/(1+exp(-x))
  return(Pr)
}
ICC.irt<-function(m,reqVC=FALSE){
  VC = c(unlist(lapply(VarCorr(m), diag)),(pi^2)/3)
  if(reqVC){return(VC);}
  ICC = VC/sum(VC)
  return(ICC)
}
R2item<-function(m1,m0){
  vc1=unlist(lapply(VarCorr(m1), diag))[2]
  vc0=unlist(lapply(VarCorr(m0), diag))[2]
  R2=1-vc1/vc0
  return(R2)
}

summary(M0.0.17)
invlogit(fixef(M0.0.17)[1])
```

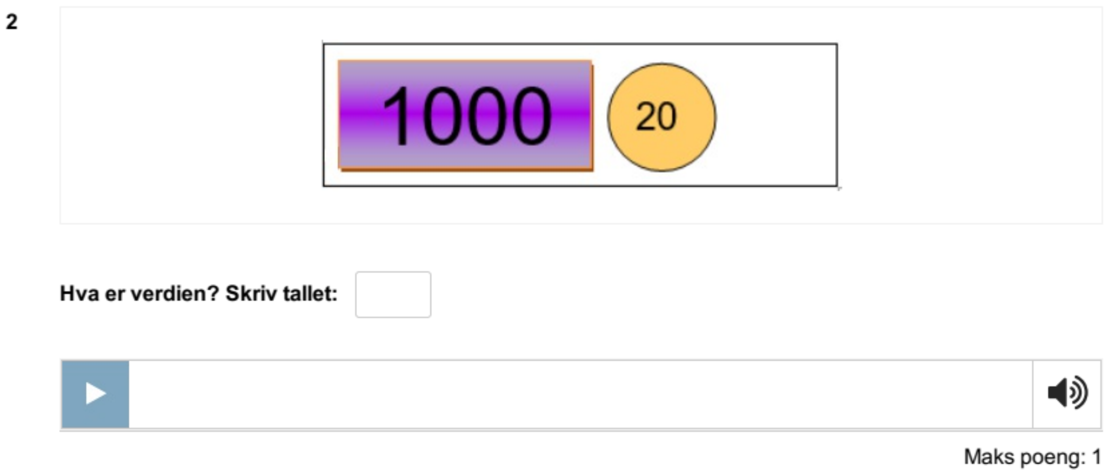
```
invlogit(fixef(M0.0.18)[1])#Average probability correct on an average item
by an average person
ICC.irt(M0.0.17)    #Response variation due to person differences, item
differences, and interaction between them (residual)
ICC.irt(M0.0.18)

#2017

m1 = c(M0.1.17,M0.2.17,M0.4.17,M0.5.17,M0.7.17,M0.8.17)
m3 = c(M1.2.17,M0.1.17)
m5 = c(m3.4.17, m3.5.17, m3.6.17)
lapply(m1,summary)
lapply(m3,summary)
lapply(m5,summary)
#Compute reduction in item variation for each model compared to baseline
model m0
round( sapply(m1,R2item,m0=M0.0.17) ,2)
round( sapply(m3,R2item,m0=M0.0.17) ,2)
round( sapply(m5,R2item,m0=M0.0.17) ,2)

#2018
m2 = c(M0.1.18,M0.2.18,M0.4.18,M0.5.18,M0.7.18,M0.8.18)
m4 = c(M1.2.18, M0.1.18)
m6 = c(m3.4.18, m3.5.18, m3.6.18)
lapply(m2,summary)
lapply(m4,summary)
lapply(m6,summary)
round( sapply(m2,R2item,m0=M0.0.18) ,2)
round( sapply(m4,R2item,m0=M0.0.18) ,2)
round( sapply(m6,R2item,m0=M0.0.18) ,2)


citation()
citation(package = "mirt")
citation(package = "WrightMap")
citation(package = 'lme4')
```

Appendix III – A. Supplemental Material – Example Items

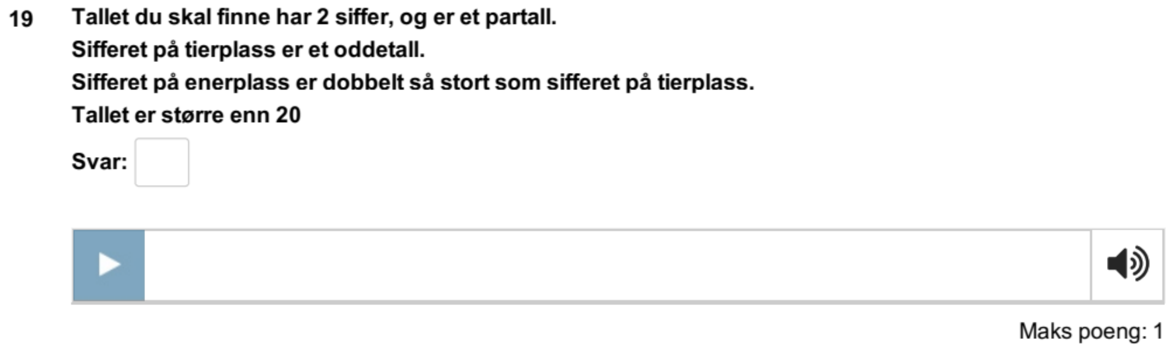


Item 1 in the second-grade 1+1 test





Originally item 1 and 2 in the third-grade 1+1 test. Recoded into item 1, partial score to avoid local item dependence issues


19 Tallet du skal finne har 2 siffer, og er et partall.
Sifferet på tierplass er et oddetall.
Sifferet på enerplass er dobbelt så stort som sifferet på tierplass.
Tallet er større enn 20

Svar:

Maks poeng: 1

An example of a more verbose item in the second-grade 1+1 test


15 En pose inneholder 52 klinkekuler i 4 ulike farger.
Det er like mange røde, gule, blå og grønne klinkekuler i posen.

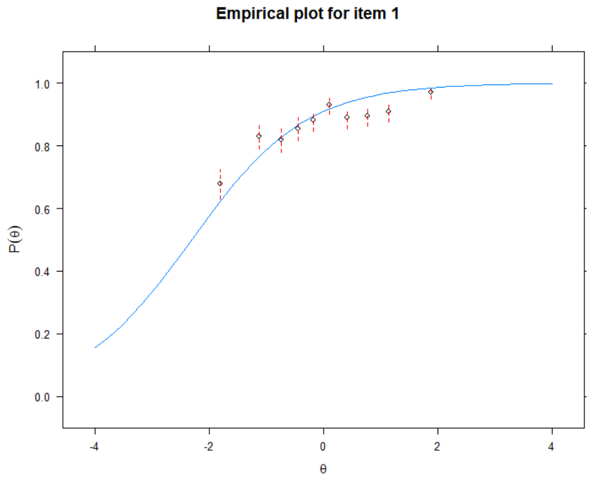
Hvor mange klinkekuler er blå?

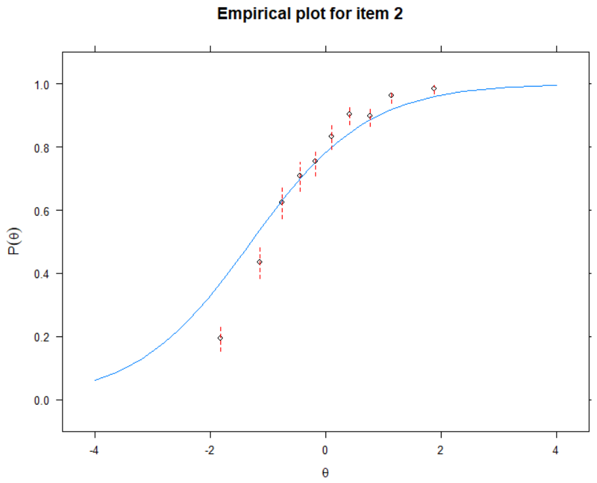
Maks poeng: 1

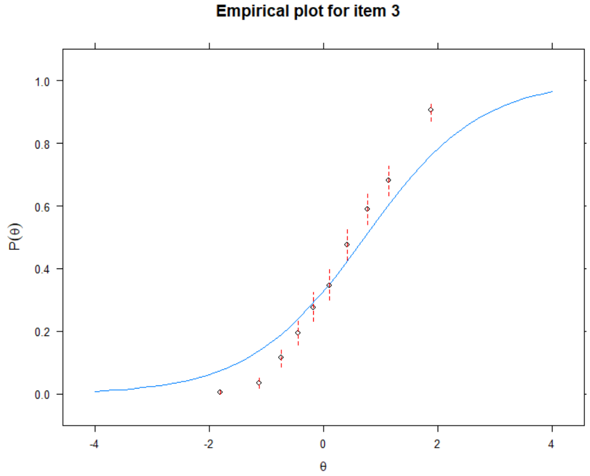
An example of a more verbose item in the third-grade 1+1 test

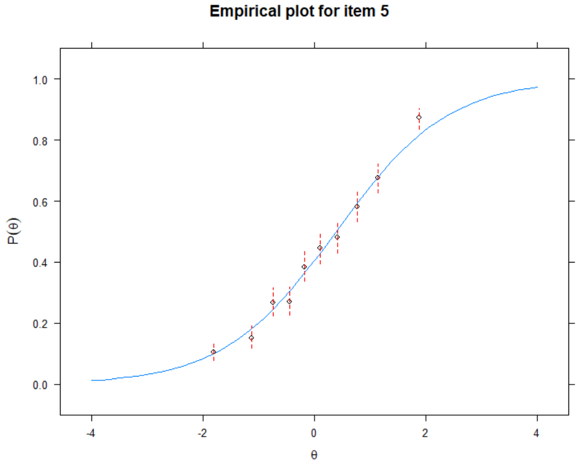
Appendix III – B. Supplemental Material – Additional Item Fit information.

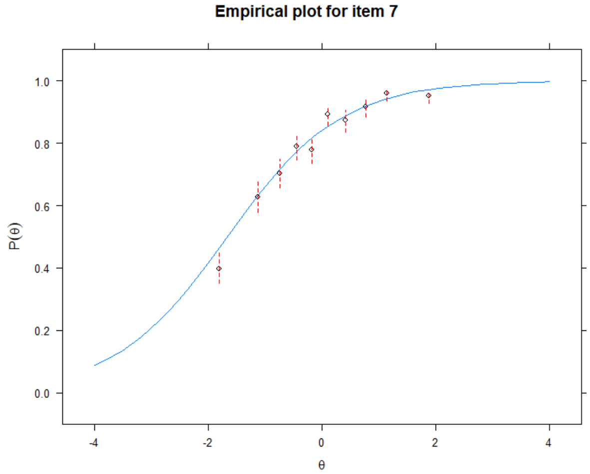
Second-grade 1+1 test fit results

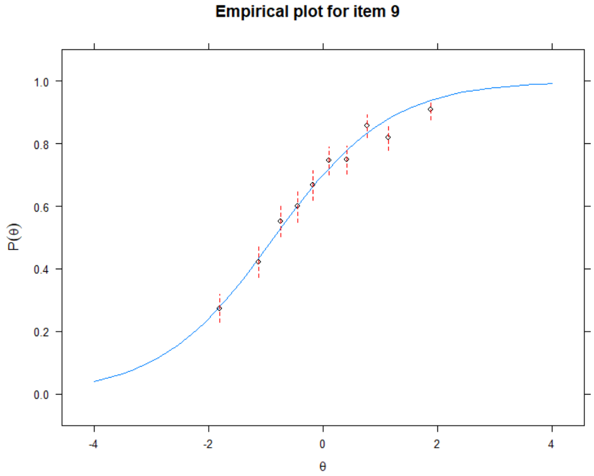




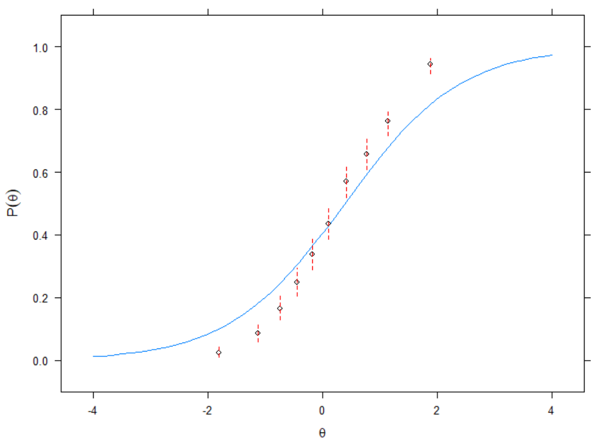




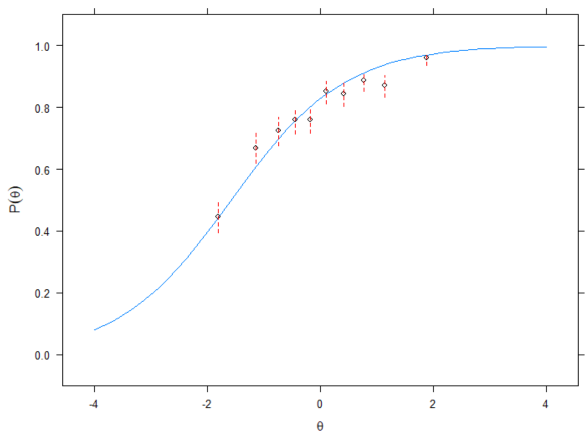




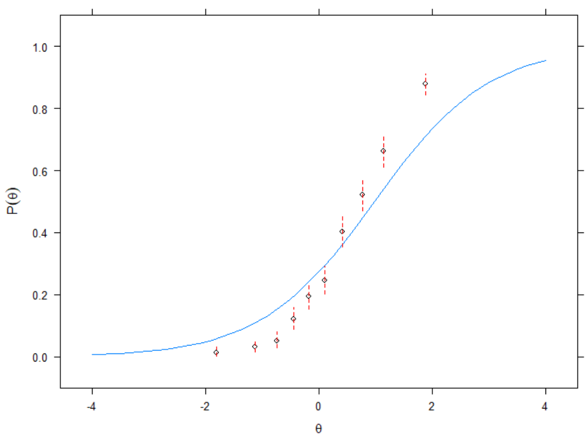
Empirical plot for item 14



Empirical plot for item 15

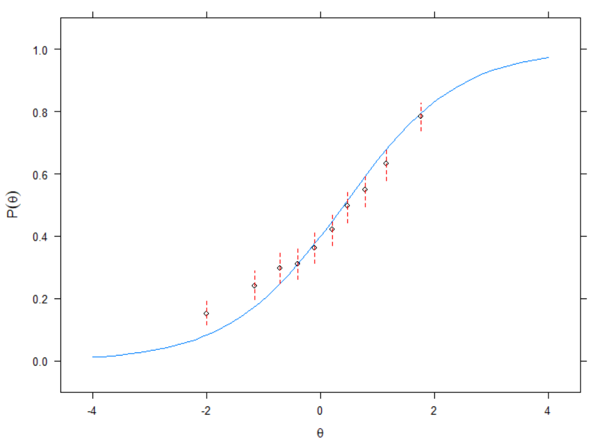


Empirical plot for item 18

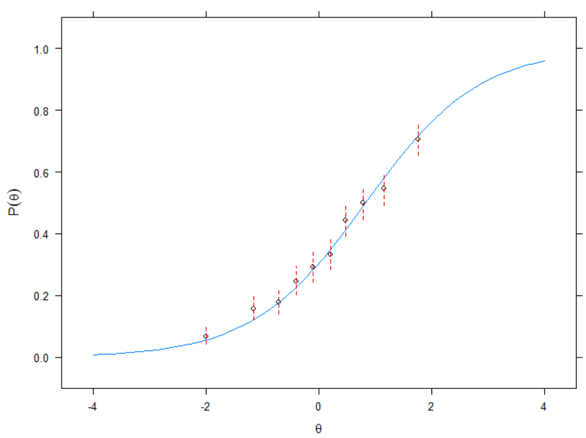


Third-grade 1+1 test fit results

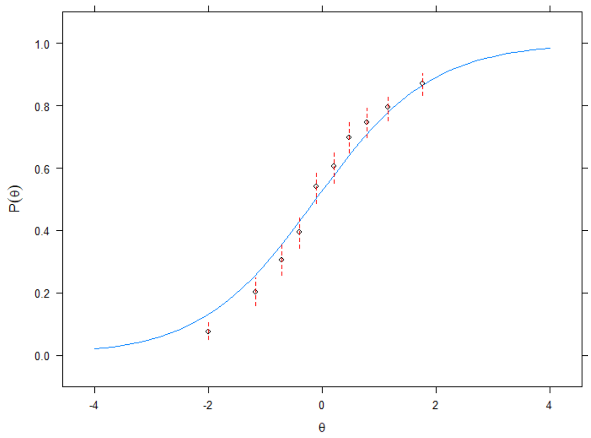
Empirical plot for item 5



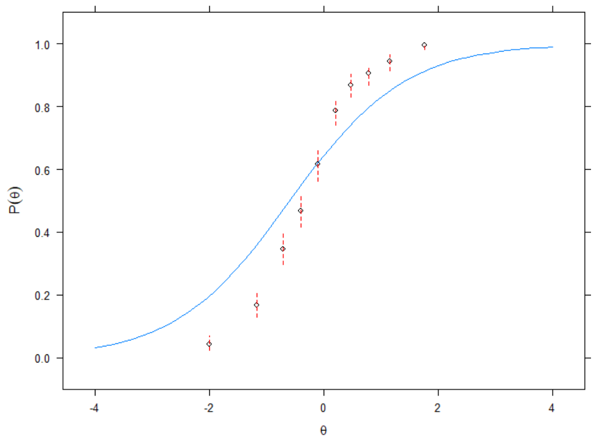
Empirical plot for item 7



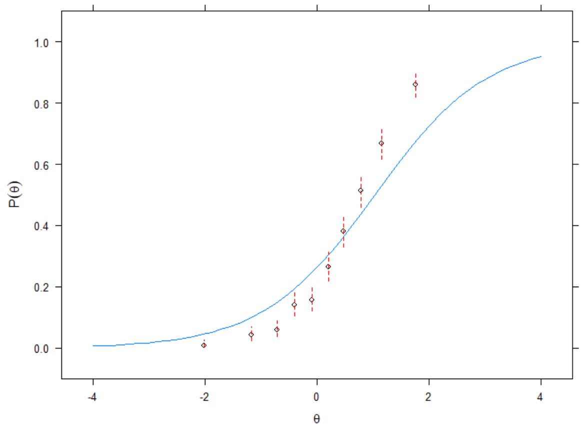
Empirical plot for item 12



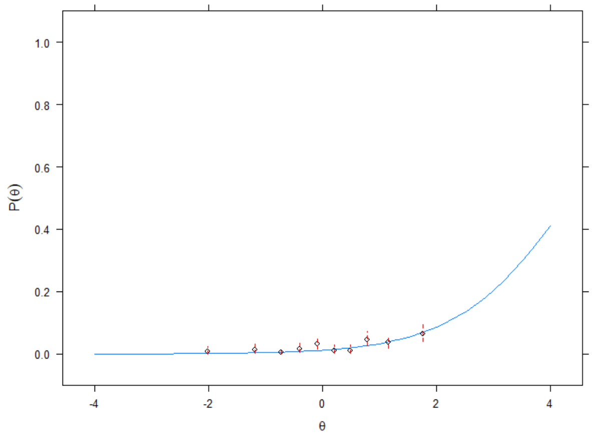
Empirical plot for item 13



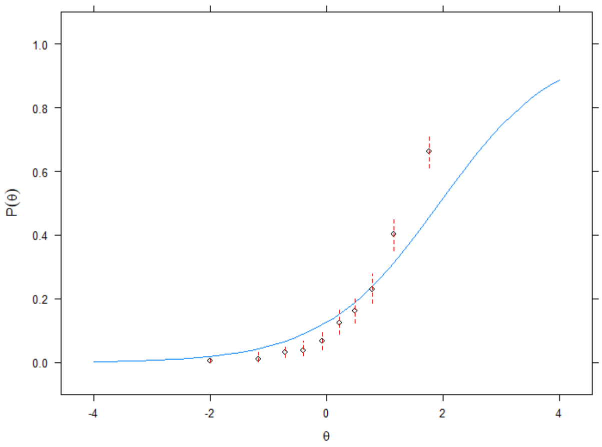
Empirical plot for item 14



Empirical plot for item 16



Empirical plot for item 18



Yen's Q3 statistics plotted for grade 2 and grade 3