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Free fall in curved spacetime—how to visualise gravity in general relativity

Magdalena Kersting[1,2]

[1] Department of Physics, University of Oslo, Oslo, Norway
[2] Department of Physics, OzGrav-UWA, The University of Western Australia, Perth, Australia

E-mail: magdalena.kersting@fys.uio.no

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Abstract

The first direct observation of gravitational waves in 2015 has led to an increased public interest in topics of general relativity (GR) and astronomy. Physics teachers and educators respond to this interest by introducing modern ideas of gravity and spacetime to high school students. Doing so, they face the challenge of finding suitable models that visualise gravity as the geometry of curved spacetime. Most models of GR, such as the popular rubber sheet model, only address spatial curvature. Yet, according to Albert Einstein, gravitational phenomena stem from deformations both in space and time. This paper presents a new model that builds on a relativistic generalisation of Newton's first law. We use Einstein's free fall thought experiment and a classical height-time diagram to explain how warped time gives rise to gravity. Our warped-time model acts as a convenient supplement to the rubber sheet model. To support teachers in integrating the model into their classroom practice, we have implemented the model as an interactive simulation that is freely accessible. The model is the result of a three-year period of developing and trialling digital learning resources in Norwegian high schools. Based on these trials, we suggest specific instructional strategies on how to use the warped-time model successfully in science classrooms.

1. Introduction

Einstein's general theory of relativity is our current best description of gravity. According to general relativity (GR), gravity is the result of the dynamic interplay between space, time, and the mass and energy content in the universe. Spacetime curves and ripples under the influence of massive objects. The first direct detection of gravitational waves in 2015 [1] has led to a new interest in topics of gravity and gravitational astronomy. This interest leads to new opportunities for teachers and educators to engage students and the general public [2–4]. Indeed, topics of GR and astronomy seem to motivate high school students to a great extent [5–7].

However, with great opportunities come great educational responsibilities. Physics teachers face the challenge of having to translate an abstract scientific theory into a qualitative description without oversimplifying the concepts too much. This paper responds to the challenge of educating and engaging high school students in topics of GR by presenting an interactive warped-time model. While the popular rubber sheet model uses curved

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space to explain planetary movement in an intuitive way, the model ignores deformations in time. Our warped-time model presents an alternative strategy to explain gravity. The model thus acts as a useful supplement to the rubber sheet to visualize how warped time makes objects fall.

The presentation of this paper follows a threefold structure: First, Einstein's key ideas on gravity and spacetime are summarised by presenting two models of GR: the traditional rubber sheet model and our warped-time model. The presentation lists advantages and limitations of each model as well. Second, the development of the warped-time model are contextualised as part of the greater design-based research project ReleQuant that develops digital learning resources in modern physics [8]. Finally, the last section reports on students' experiences with the warped-time model and discusses instructional implications to improve teaching and learning of GR.

2. Gravity and spacetime

This section summarises key ideas of GR and relates these ideas to two instructional models. The warped-space model has become synonymous with GR, whereas the warped-time model is our novel approach to visualising curved spacetime.

2.1. Warped-space model

At the heart of GR lies Einstein's field equation that describes the interplay between space, time, and massive objects [9]. The popular phrase 'spacetime tells matter how to move, matter tells spacetime how to curve' aptly encapsulates this equation [10]. The widely used rubber sheet model visualises this dynamic interplay through an intuitive hands-on activity [11].

The analogy compares the fabric of the universe to a stretched rubber sheet. Gravity is illustrated by placing a bowling ball and marbles on the rubber sheet. The bowling ball produces a warp of the rubber, which results in an inward tug that influences the movement of the marbles. It is the warp of the rubber sheet that creates the gravitational tug.

The rubber sheet model, sometimes also denoted spacetime simulator or pillow model [11, 12], offers an intuitive explanation of gravity.

The deformed sheet provides a mechanism of how gravity arises and the model has great explanatory power: it is suitable to show orbital motions, curved space, and photon trajectories [13]. Yet, no instructional model comes without limitations. Research suggests that the rubber sheet might be misleading despite its visual power and simplicity: The rubber sheet obscures that spacetime is 4D; in particular, the model obscures that spacetime has a temporal dimension [13].

2.2. Warped-time model

The warped-time model addresses limitations of the rubber sheet model by offering a strategy to visualise gravity as an effect of warped time. The warped-time model builds on another important equation of GR, the geodesic equation. The geodesic equation is an equation of motion that can be thought of as a generalisation of Newton's first law. In an attempt to introduce the geodesic equation to science classrooms, physics educators recently coined the term 'Einstein's first law' [14]: Objects that are not influenced by forces move along geodesic curves in spacetime.

A geodesic curve is the spacetime generalization of a straight line. The usefulness of geodesic curves in GR is that they are the paths followed by particles in free fall [15]. There is one important thing to note when formulating Einstein's first law: In contrast to classical mechanics, Einstein did not consider gravity to be a force. Thus, objects in free fall are indeed free—no force in the classical sense acts on them. Einstein's happiest thought, namely that a person in free fall will experience a state of weightlessness, is an everyday example of Einstein's first law: Objects in free fall follow geodesic curves in spacetime.

Building on Einstein's first law, a new teaching strategy makes the warping of time visible. The interactive warped-time model is part of a digital learning environment in GR that is freely accessible at www.viten.no/relativity. The warped-time model invites students to explore the physics of free fall both from a classical and from a relativistic perspective. As starting point, the model takes a digital height-time diagram and presents students with two different scenarios (figures 1 and 2): Einstein stands on top of a 45 m high tower and ponders the nature of gravity. In the first case, he remains standing on top of the

tower. In the second case, he steps off the tower in line with his famous thought experiment. To familiarise students with the digital height-time diagram, they are asked to draw trajectories into the height-time diagram. This task serves as a warm-up: Remaining on top of the tower corresponds to a straight line in the height-time diagram and stepping off the tower corresponds to a parabola.

The second part of the warped-time model shifts the two scenarios to a relativistic setting. This time, students have to take warped spacetime into account. Before they can draw trajectories students have to move a slider to warp the time-axis (figures 3 and 4). In this warped diagram, remaining on top of the tower corresponds to a curved line and stepping off the tower corresponds to a straight line.

The difference between the classical height-time diagram and its warped counterpart is that free-fall trajectories either look curved or straight. Students learn that a straight path through spacetime does not necessarily look like a 'straight line' in a given representation. Students learn to shift their perspective to understand that objects in free fall follow the straightest possible path through spacetime. Is it a force that pulls objects towards the ground? According to Einstein, there is no force pulling objects to the ground—it is the geometry of curved spacetime.

In a last step, the warped-time model invites students to move between the Newton and Einstein models of gravity (figure 5). By moving a slider up and down, students can compare how both physicists explain the physics of free fall in two different ways: Newton treats gravity as a force that accelerates objects in free fall towards the centre of the Earth. The corresponding trajectory in the space-time diagram is a parabola. Einstein treats gravity as a geometric phenomenon. Objects in free fall follow geodesic curves in spacetime. In a warped height-time diagram trajectories are straight indicating that there is no force acting on the object.

To help teachers use the warped-time model successfully, it is important to list its strengths and limitations. One important limitation of the warped-time model relates to the depiction of curvature. First, the warping of the time-axis is greatly exaggerated. Relativistic effects of warped time are very small on the surface of the

Earth [16]. Second, the curvature of the time-axis is chosen in such a way as to make a free-fall trajectory in the height-time diagram straight. Thus, the time-axis curves somewhat arbitrarily and the curvature does not accurately correspond to the way spacetime is warped around the Earth[3].

Another limitation of the warped-time model relates to the double nature of gravity. The model does not distinguish between the two aspects of gravity that affect an object—one aspect due to acceleration and one part representing tidal forces. The free fall thought experiment demonstrates the principle of equivalence: gravity and acceleration are locally indistinguishable: To describe a single idealised object in free fall one does not have to evoke curved spacetime explanations. In this case, one can describe gravity by shifting to an accelerated frame of reference. Yet, in reality, objects have an extension and will experience tidal forces. Tidal forces arise from non-uniformities in the gravitational field and cannot be removed in free fall. These forces relate to spacetime curvature. A more thorough discussion of tidal forces can be found in [17].

Despite its limitations, the warped-time model has several strengths that make it an ideal supplement to spatial visualisations of GR:

(1) The warped-time model makes use of one of Einstein's most famous thought experiments and thought experiments are powerful tools to communicate relativistic concepts to high school students [18].
(2) The model explicitly addresses and illustrates the time dimension in its depiction of curved spacetime. Moreover, the model links time dilation to the phenomenon of gravity: The warped height-time diagram illustrates that clocks run faster (or tick more often) higher up in a gravitational field (figure 6).
(3) The model compares Einstein and Newton's theories of gravity by using a representational tool that high school students are familiar with. Height-time diagrams allow students to link their previous knowledge of movement in a gravitational field to a relativistic model of gravity.

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[3] 'How Gravity Makes Things Fall' is an award-winning video by Edward Current that presents a different way of warping time to illustrate gravity: https://youtube.com/watch?v=jlTVIMOix3I.
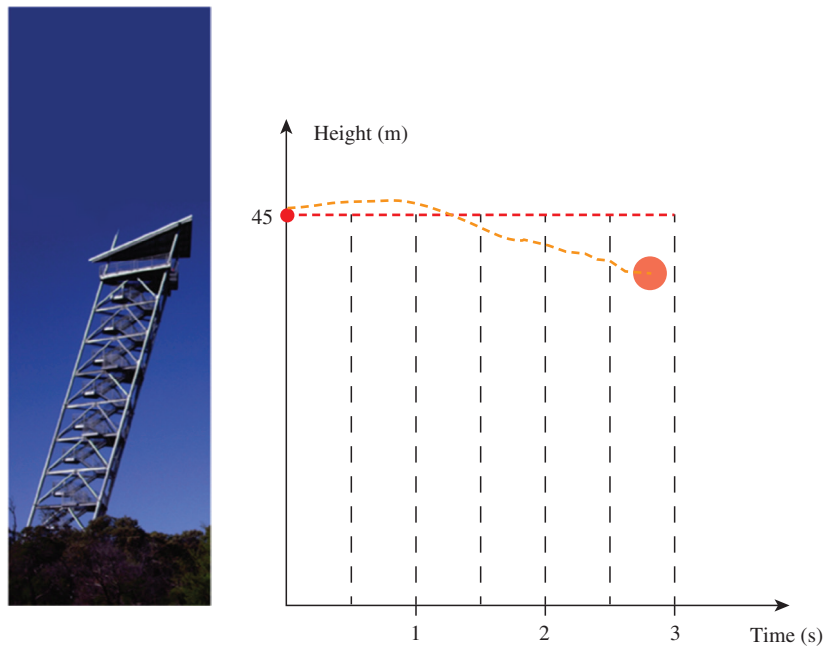
Figure 1. The warped-time model invites students to explore Einstein's free fall experiment from a classical perspective: remaining on top of the tower corresponds to a straight line.
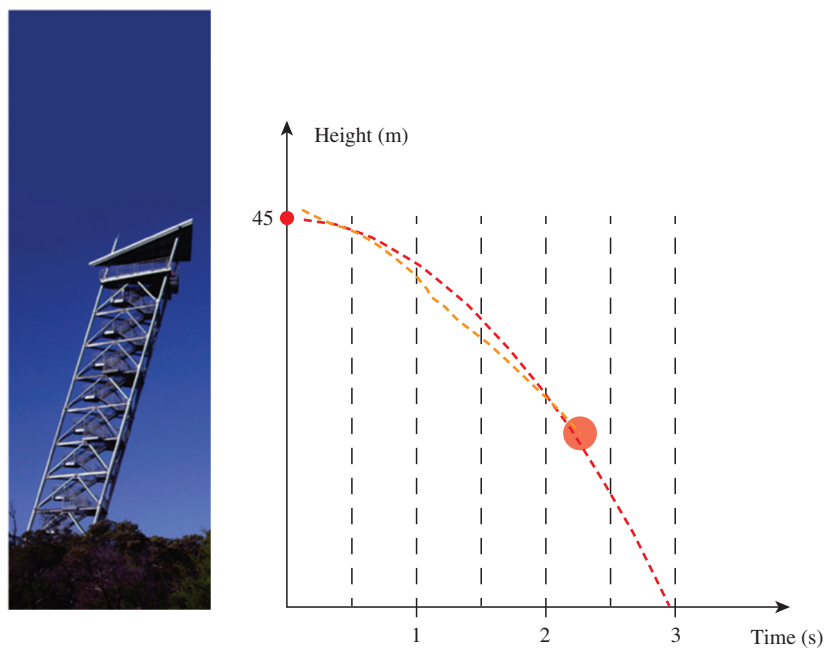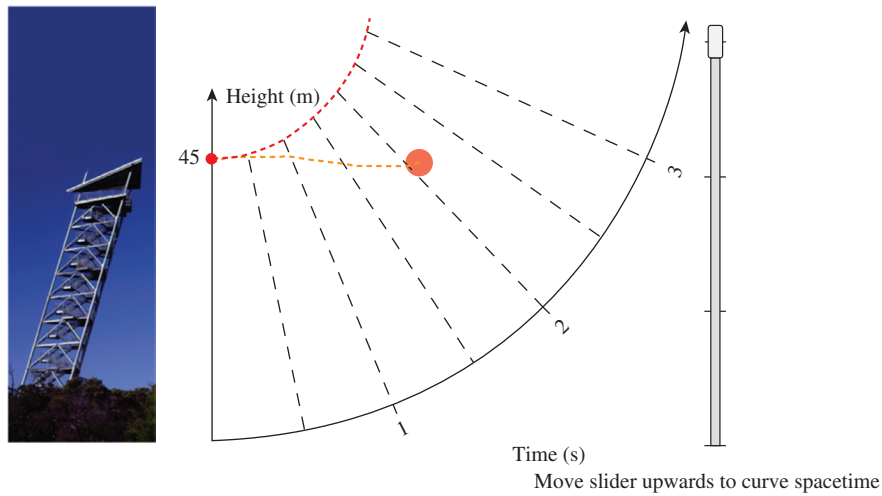

Figure 2. The warped-time model invites students to explore Einstein's free fall experiment from a classical perspective: stepping off the tower corresponds to a parabola in a height-time diagram. The interactive model can be found at www.viten.no/relativity.

Figure 3. The warped-time model invites students to explore Einstein's free fall experiment from a relativistic perspective: remaining on top of the tower corresponds to a curved line. The interactive model can be found at www.viten.no/relativity.
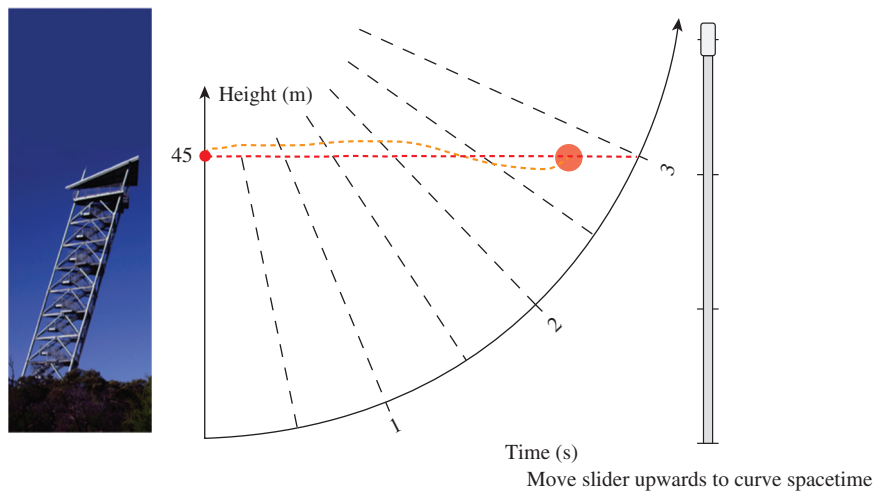

Figure 4. The warped-time model invites students to explore Einstein's free fall experiment from a relativistic perspective: stepping off the tower corresponds to a straight line.

(4) The warped-time model is flexible in its mode of presentation: Even though this paper presents a digital model of a warped height-time diagram, similar ideas can be implemented using hands-on activities only. It is for example possible to draw height-time diagrams on curved surfaces such as balloons or balls to illustrate the effect of a curved geometry on straight lines[4].

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[4] The Perimeter Institute Outreach Program presents a hands-on activity that illustrates the warping of spacetime using beach balls: https://resources.perimeterinstitute.ca.

3. Educational context

The warped-time model is the result of a design-based research approach to developing learning resources in modern physics [6]. Topics of modern physics place high demands on students' understanding of abstract and often counter-intuitive concepts. In response to these challenges project ReleQuant was established to study novel and innovative ways of teaching modern physics in Norwegian high schools [8]. In close collaboration with teachers and teacher students, the ReleQuant team developed a digital learning
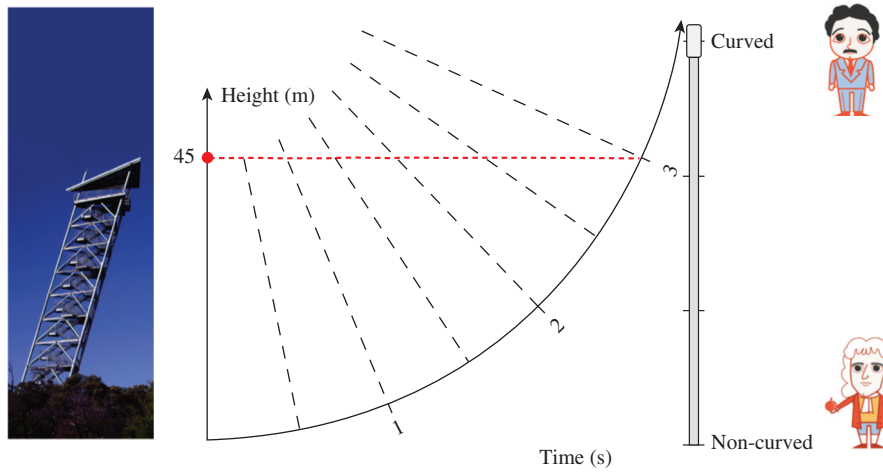
5

Figure 5. The warped-time model allows students to compare Newton and Einstein's theories of gravity by moving a slider between a classical and a warped height-time diagram.
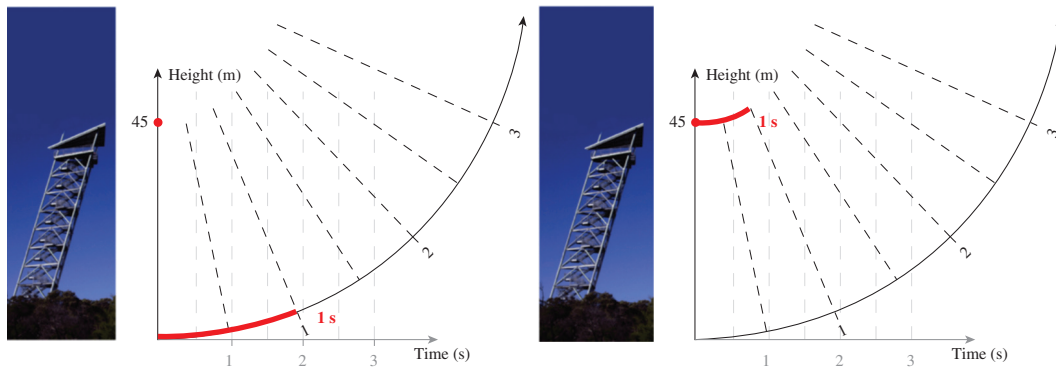

Figure 6. The warped-time model illustrates that clocks run faster higher up in a gravitational field: there are more ticks of a clock in the same stretch of time.

environment in GR. The learning resources were trialled in 12 upper secondary physics classes over a 3 year period. The Norwegian Centre for Science Education hosts the learning environment that is freely available in English and Norwegian on the open-source learning platform Viten: www.viten.no/relativity

In addition to having been developed within ReleQuant, the warped-time model pools experience from Einstein-First and the Gravity Discovery Centre. Einstein-First is an Australian educational project that aims to introduce young learners to topics of relativity and quantum physics by developing simple models and hands-on activities [19]. The Einstein-First team coined the notion of 'Einstein's first law' in reference to the geodesic

equation [14]. The Gravity Discovery Centre is an outreach facility and science museum co-located at the Australian International Gravitational Research Centre in Gin Gin, Western Australia. The centre features the so-called 'Leaning Tower of Gin Gin' which allows visitors to recreate free fall experiments [20]. The warped-time model presented in this paper takes a digital version of the Leaning Tower as a setting to explore Einstein's law and free-fall motion in curved spacetime.

3.1. Student experiences

The final design of the warped-time model is a result of three iterative rounds of developing and testing learning resources in 12 Norwegian

physics classrooms. In this section, key insights from the classroom trials are summarised to guide instruction based on the warped-time model.

Generally, the classroom trials showed that students felt motivated and engaged by curved spacetime even though many admitted that the concept was challenging [6]. The first trial of the learning resources suggested that students struggled to conceptualise movement along the time-dimension [21]. The warped-time model makes movement along the time dimension more visible for students by asking them to draw the trajectory of an object that remains spatially at rest. Understanding that objects always move in spacetime is an important insight that helps students integrate ideas of time and gravity into a relativistic framework.

The second trial of the learning resources targeted a prototype of the warped-time activity specifically. Analysis of small group discussions showed that even though many students seemed to be comfortable with the idea of movement in space and time, only few groups were able to connect geodesic curves to the physical state of being in free fall [22]. Thus, successful instruction should aim to link the geometric description of GR to the physics of free fall. Focus group interviews supported the findings from the classroom discussions during the second trial. Students perceived a gap between relativistic and classical descriptions of free fall. Moreover, the interviews revealed that students continued to find it difficult to visualise time even though the warped-time model helped them to get a better picture of this abstract concept.

Not all students approved of the warped-time model though. Some criticised the model for not being representative of relativistic phenomena. This criticism reveals an understanding of the limitations of this model as well as of the scope of Einstein's theory. Successful instruction of warped time should therefore complement the warped-time model with other examples from cosmology and astrophysics where relativistic phenomena have a more significant effect.

4. Discussion and conclusion

Every instructional model has limitations. In learning domains such as GR where concepts are very abstract or impossible to visualise, it is crucial to develop different models that can complement each other [23]. This paper presents a new instructional model to visualise gravity as a manifestation of warped time. The model acts as a supplement to spatial models of GR such as the rubber sheet model. In addition to addressing the time dimension, the model introduces students to Einstein's first law.

Based on our classroom trials, we suggest four specific instructional strategies to use the warped-time model successfully:

(1) It is important to emphasise that every object moves both in space and in time. The insight that objects always move in time (in other words, they age) helps students link geometric descriptions of gravity to their everyday experience of gravity.
(2) Einstein's thought experiment of freely falling objects is a popular introduction to GR. We suggest capitalising on this thought experiment and using the warped-time model as a second step to explain gravity as a manifestation of curved spacetime.
(3) In everyday life, relativistic phenomena cannot be observed directly. To help students make sense of warped time, we suggest using the warped-time model to discuss gravitational time dilation as well.
(4) The warped-time model allows for a direct comparison between the Newton and Einstein models of gravity by showing how two different models can describe the same physical phenomenon. Teachers should use this opportunity to help students build awareness of the nature of scientific models.

The detection of gravitational waves and applications of GR create a fantastic vision of physics for the future. It is up to teachers and physics educators to bring this vision into science classrooms. By offering novel instructional models that visualise gravity and curved spacetime, we hope to support teachers in engaging and inspiring the next generation of scientists.

Acknowledgments

ORCID iDs

Magdalena Kersting https://orcid.org/0000-0003-3568-8397

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Magdalena Kersting is a science educator and mathematical physicist who has developed a digital learning environment about Albert Einstein's general theory of relativity. She works as a PhD researcher in the physics education group at the University of Oslo and with the ARC Centre of Excellence for Gravitational Wave Discovery at the University of Western Australia. She studies learning processes in Einsteinian physics to understand how learners make meaning of abstract scientific concepts.