Errata to the paper: Stochastic Control of Memory Mean-Field Processes [1]

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- In Definition 1.1 the space $\mathcal{M}$ is not a Hilbert space, but a *pre-Hilbert* space. Throughout the paper $\mathcal{M}$ denotes the completion of this space.

- The proofs of Theorem 4.5 and Theorem 4.6 are not correct as stated.
  However, the proofs can easily be corrected - and even simplified - as follows:
  For a given Frechét differentiable operator $G : \mathcal{M} \mapsto \mathbb{L}^2(\mathbb{P})$ let $\nabla_m G$ denote its Fréchet derivative at the measure $m \in \mathcal{M}$, and for $X \in \mathbb{L}^2(\mathbb{P})$ let $M = \mathcal{L}(X)$ denote the law of X. Define the dual operator $\nabla_m^* G(\cdot) \in \mathbb{L}^2(\mathbb{P})$ by the property

  $$\mathbb{E}[< \nabla_m G, M >] = \mathbb{E}[\nabla_m^* G X]; \quad X \in \mathbb{L}^2(\mathbb{P}), \tag{0.1}$$

  and let the dual operator $\nabla_{\bar{m}} H^t$ be defined in the similar way as $\nabla_{\bar{x}} H^t$ in (4.6)-(4.7) in the paper. The existence and the uniqueness of $\nabla_m^* G$ and $\nabla_{\bar{m}} H^t$ follow by the Riesz representation theorem, as in the paper.

  With the introduction of these dual operators the proofs of Theorem 4.5 and Theorem 4.6 can be corrected and simplified. Specifically, there is no need for the adjoint variables p^1, q^1 and r^1, if we modify the backward stochastic differential equation (BSDE) for the adjoint variables p^0, q^0, r^0 accordingly. More precisely, everything concerning the processes p^1, q^1, r^1 can be deleted throughout the paper if we change the BSDE

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(4.13) to

$$
\begin{aligned}
dp^0(t) = &-\Big\{\frac{\partial H}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}} H^t|\mathcal{F}_t] + \nabla^*_m H(t) + \mathbb{E}[\nabla_{\bar{m}} H^t|\mathcal{F}_t]\Big\}dt + q^0(t)dB(t) \\
&+ \int_{\mathbb{R}_0} r^0(t,\zeta]\tilde{N}(dt,d\zeta); \quad t \in [0,T] \\
&p^0(t) = \frac{\partial h}{\partial x}(X(T),M(T)) + \nabla^*_m h(X(T),M(T)); \quad t \geq T \\
&q^0(t) = 0; \quad t > T \\
&r^0(t,\cdot) = 0; \quad t > T,
\end{aligned} \tag{0.2}
$$

- The BSDE (4.14) is not needed and should be deleted.
- The expression (4.23) should be corrected to

$$
\begin{aligned}
=&\mathbb{E}\Big[\int_0^T \hat{p}^0(t)\tilde{b}(t) - \int_0^T \Big\{\frac{\partial \hat{H}}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}}\hat{H}^t|\mathcal{F}_t] + \nabla^*_m \hat{H}(t) + \mathbb{E}[\nabla_{\bar{m}}\hat{H}^t|\mathcal{F}_t]\Big\}\tilde{X}(t)dt \\
&+ \int_0^T \hat{q}^0(t)\tilde{\sigma}(t)dt + \int_0^T\int_{\mathbb{R}_0} \bar{r}^0(t,\zeta)\tilde{\gamma}(t,\zeta)\nu(d\zeta)dt\Big]
\end{aligned} \tag{0.3}
$$

- Equation (4.24) can be deleted.
- The first part of the proof of Theorem 4.6 (up to and including (4.29)) can be deleted. All terms in the proof involving p^1 should be deleted. The equation for $\mathbb{E}[p^0(T)Z(T)]$ should be corrected to

$$
\begin{aligned}
\mathbb{E}[p^0(T)Z(T)] &= \mathbb{E}[\int_0^T p^0(t)dZ(t) + \int_0^T Z(t)dp^0(t) + [p^0, Z]_T] \\
&= \mathbb{E}[\int_0^T p^0(t)(\nabla b(t))^T\ (Z(t), Z_t, DM(t), DM_t, \pi(t), \pi_t)dt \\
&- \int_0^T \{\tfrac{\partial H}{\partial x}(t) + \mathbb{E}[\nabla_{\bar{x}} H^t|\mathcal{F}_t] + \nabla^*_m H(t) + \mathbb{E}[\nabla_{\bar{m}} H^t|\mathcal{F}_t]\}Z(t)dt \\
&+ \int_0^T q^0(t)(\nabla\sigma\,(t))^T\ (Z(t), Z_t, DM(t), DM_t, \pi\,(t)\,, \pi_t)dt \\
&+ \int_0^T\int_{\mathbb{R}_0} r^0(t,\zeta)(\nabla\gamma\,(t,\zeta))^T\ (Z(t), Z_t, DM(t), DM_t, \pi(t), \pi_t)\nu(d\zeta)dt].
\end{aligned}
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References

[1] Agram, N. and Øksendal, B.: Stochastic control of memory mean-field processes. Applied Mathematics and Optimization 2017, DOI 10.1007/s00245-017-9425-1.