

# Logic and Plurals

## 1. The logical analysis of plural expressions

Many natural languages contain plural vocabulary such as ‘we’, ‘those’, ‘the philosophers’, ‘cooperate’, and ‘gathered’. What is the correct logical analysis of sentences involving such vocabulary?

Before we can attempt to answer the question, we need to comment briefly on how we understand logical analysis. Logical analysis generally proceeds by paraphrasing sentences of natural language in a way that provides a more perspicuous representation of logically relevant features of those sentences. Often, the paraphrase is given in a formal language that is equipped with a deductive system and a model-theoretic semantics. However, as Quine observed, “to paraphrase a sentence of ordinary language into logical symbols is virtually to paraphrase it into a special part still of ordinary or semi-ordinary language [...]” (Quine 1960: 159) This is because, in many important cases, the sentences of the formal language are obvious counterparts of particular sentences of natural language (or natural language augmented with some mathematical locutions). The process of paraphrase into a more logically perspicuous fragment of natural language is known as *regimentation*.

Though the logical study of plurals is a relatively recent phenomenon, semantic questions concerning plurals were already entertained by the founders of modern logic.<sup>1</sup> Frege himself, for instance, addressed the question of the proper logical analysis of sentences with a plural subject, such as (1).

(1) Socrates and Plato are philosophers.

He wrote:

[W]e have two thoughts: Socrates is a philosopher and Plato is a philosopher, which are only strung together linguistically for the sake of convenience. Logically, Socrates and Plato is not to be conceived as the subject of which being a philosopher is predicated.

(Frege 1980: 40)

In effect, Frege proposes to analyze (1) as (2).

(2) Socrates is a philosopher and Plato is a philosopher.

However, he realizes that this strategy is not always available. Sentences such as (3) and (4) are not amenable to the conjunctive analysis proposed for (1).

(3) Bunsen and Kirchhoff laid the foundations of spectral analysis.

(4) The Romans conquered Gaul.

Frege remarked:

Here we must regard *Bunsen and Kirchhoff* as a whole. ‘The Romans conquered Gaul’ must be conceived in the same way. The Romans here are the Roman people, held together by custom, institutions, and laws. (Frege, *ibidem*)

While Frege provided no additional indications as to the nature of the objects that should serve as ‘wholes’ in the logical analysis of plurals, the subsequent literature has offered a number of alternatives. Sets, mereological sums, and groups are just some of the more popular candidates. By way of illustration, let us briefly consider the appeal to sets. The most famous advocate of this approach is Quine. One of the sentences he grapples with is known as the *Geach-Kaplan sentence*.<sup>2</sup>

(GK) Some critics admire only one another.

According to Quine, by “invoking classes and membership, we can do justice to [the Geach-Kaplan sentence]” (Quine 1982: 293). The regimentation Quine proposes may be informally glossed as (5).<sup>3</sup>

(5) There is a non-empty set such that any member of the set is a critic who admires some other member of the set.

(6)  $\exists s(\exists x(x \in s) \ \& \ \forall x(x \in s \rightarrow C(x)) \ \& \ \forall x\forall y[(x \in s \ \& \ x \text{ admires } y) \rightarrow (y \in s \ \& \ x \neq y)])$

To understand what is distinctive about Quine's position, consider the following sentence, which appears to be a set-theoretic truism.

(7) There are some sets that are self-identical, and every set that is not a member of itself is one of them.

It is reasonable to demand that no proper regimentation of this sentence render it obviously false.<sup>4</sup> However, a strict application of Quine's method of set-theoretic paraphrase would turn (7) into (8), which is inconsistent with standard set-theoretic principles.

(8) There is a non-empty set such that every member of it is a self-identical set, and every set that is not a member of itself is a member of it.

James Higginbotham aptly labels this problem *the paradox of plurality* (Higginbotham 1998: 265).

In linguistics, an influential approach to plurals is that of Godehard Link, who uses mereological sums to analyze plurals. In his framework, the formal language contains a special mereological relation ( $\leq$ ), corresponding to the notion of individual parthood: being an *atomic* part of. This notion is not to be confused with that of material parthood: being a *material* part of. For example, while Annie is an atomic part of the mereological sum (in the individual sense) of

Annie and Bonnie, she is not an atomic part of it in the material sense. So the plurality of Annie and Bonnie is the mereological fusion of Annie and Bonnie *taken as atomic individuals*. Let ' $\oplus$ ' stand for the binary operation of mereological fusion relative to individual parthood. Let  $\sigma x.\varphi(x)$  be the mereological fusion of the individuals satisfying the formula  $\varphi(x)$ .<sup>5</sup> Then we may formalize some basic plural sentences as displayed below (see Link 1983, Link 1998, as well as Moltmann 1997 and Champollion forthcoming for more details and applications of the framework).

(3) Bunsen and Kirchhoff laid the foundations of spectral analysis.

(3\*)  $F(b \oplus k)$

(9) The Romans conquered Gaul.

(9\*)  $C(\sigma x.R(x), g)$

(7) There are some sets that are self-identical, and every set that is not a member of itself is one of them.

(7\*)  $\exists x [\forall y (y \leq x \rightarrow (\text{Set}(y) \ \& \ y = y)) \ \& \ \forall y ((\text{Set}(y) \ \& \ y \notin y) \rightarrow y \leq x)]$

A final ‘singularizing’ strategy we should mention is based on Davidson’s analysis of predication (hence also plural predication) in terms of events, broadly understood to include states (see Higginbotham and Schein 1989, Schein 1993, and Moltmann’s chapter in this handbook). This approach enables us to eliminate a plural subject by reducing it to the single co-agents of the underlying event. To illustrate it, let us look at the treatment of one of Frege’s examples.

(3) Bunsen and Kirchhoff laid the foundations of spectral analysis.

(3\*\*) There is an event  $e$  of laying the foundations of spectral analysis such that Bunsen is a co-agent of  $e$ , Kirchhoff is a co-agent of  $e$ , and there is no other co-agent of  $e$ .

Are these ‘singularizing’ strategies successful? Many philosophical logicians believe that the answer is negative. Some of their main arguments will be outlined in section 3. First we will consider an altogether different analysis of plurals.

## **2. Taking plurals at face value**

George Boolos championed an approach to plurals that completely rejects Frege’s attempt to render plural discourse in terms of ‘wholes’.

Abandon, if one ever had it, the idea that use of plural forms must always be understood to commit one to the existence of sets [...] of those things to which the corresponding singular forms apply. There are, of course, quite a lot of Cheerios in that bowl, well over two hundred of them. But is there, in addition to the Cheerios, also a set of them all? [...]

It is haywire to think that when you have some Cheerios, you are eating a set [...]. [I]t doesn't follow just from the fact that there are some Cheerios in the bowl that, as some who theorize about the semantics of plurals would have it, there is also a set of them all.

(Boolos 1984: 448-449)

In fact, Boolos's rejection of the singularizing approach has a distinguished pedigree featuring, most prominently, (Russell 1938) (see Klement 2014). Russell distinguished between a *class as one* and a *class as many*. A class as one is a multiplicity of objects thought of as a single whole, as is done in traditional first-order set or class theory. In contrast, a class as many is a multiplicity of objects *as such*. There is no single entity that represents, collects, or goes proxy for the objects that make up the multiplicity. Russell emphasized the usefulness of the second way of thinking about multiplicities. In more recent history, (Black 1971) and (Simons 1982; Simons 1997) have advocated a treatment of plurals in the spirit of classes as many.<sup>6</sup>

What is the broader significance of Boolos's attack on singularizing analyses and of Russell's much earlier non-singularizing approach based on the notion of classes as many? At the heart of their remarks is the simple idea that plurals should be taken at face value. That is, we should allow certain forms of plural discourse in the regimentation of natural language. Frege, Quine, and others were simply wrong to think that plurals need to be paraphrased away. Rather,

plurals deserve to be regimented in their own terms by employing a type distinction between singular and plural expressions in the regimenting language. The standard implementation of this proposal is known as *plural logic*.

### 3. The language of plural logic

We first introduce a formal language that may be used to regiment a wide range of natural language uses of plurals. This language captures Boolos's and Russell's suggestion and will enable us to represent many valid patterns of reasoning that essentially involve plural expressions. The language is known in the philosophical literature as PFO+, which is short for *plural first-order logic plus plural predicates*. In one variant or another, it is the most common regimenting language for plurals in philosophical logic.<sup>7</sup>

Start with the standard language of first-order logic. We expand this language by making the following additions.

A. Plural variables ( $vv, xx, yy, \dots$ , and variously indexed variants thereof) and plural constants ( $aa, bb, \dots$ , and variants thereof), roughly corresponding to the natural language pronoun 'they' and to plural proper names, respectively.

B. Quantifiers that bind plural variables ( $\forall xx, \exists yy, \dots$ ).

C. A binary predicate  $<$  for plural membership, corresponding to the natural language 'is one of'



or ‘is among’. This predicate is treated as logical.

D. Symbols for collective plural predicates with numerical superscripts representing the predicate’s arity:  $P^1, P^2, \dots, Q^1, \dots$  (and variously indexed variants thereof). Examples of collective plural predicates are ‘...cooperate’, ‘...gather’, ‘...meet ...’, ‘...outnumber ...’. For economy, we leave the arity unmarked.

The fragment of PFO+ containing items A-C, i.e. PFO+ minus plural predicates, is known as PFO. The following chart summarizes which linguistic items are added to the standard language of first-order logic to obtain PFO+.

	natural language	symbolization
	equivalent	
<i>plural variables</i>	they <sub>1</sub> , they <sub>2</sub> , ...	$vv, vv_0, \dots, xx, \dots$
<i>plural constants</i>	the Hebrides, the Channel Islands	$aa, bb, \dots, aa_1, \dots$

*plural quantifier*          there are some (things)           $\exists vv, \exists xx, \dots$

*plural membership*          is one of, is among           $\prec$

*collective plural*          are two, cooperate,           $T(vv), C(vv),$

*predicates*          met, wrote together           $M(vv), WT(vv, x)$

The recursive clauses defining a well-formed formula of PFO+ are the obvious ones. However, some clarifications about the language are in order.

Firstly, one may require a rigid distinction between the argument places of predicates. An argument place that is open to a singular argument could be reserved *only* to such arguments. A similar restriction could be imposed on argument places open to plural arguments. Would this rigid distinction between singular and plural argument places reflect a feature of natural language? Different natural language predicates suggest different answers. Some predicates are flexible and are capable of combining felicitously with both singular and plural terms. Examples include ‘own a house’, ‘lifted a boat’, or, as in Frege’s example, ‘laid the foundations of spectral analysis’. (Of course, the conjugations of the verbs will have to be adjusted.) Other predicates

appear to lack this flexibility and combine felicitously *only* with plural terms, e.g. ‘cooperate with one another’ and ‘are two in number’. There is an interesting linguistic question as to the source of these felicity judgments: are they of syntactic, semantic, or pragmatic origin? We don’t wish to take a stand on these matters. For our purposes, we can leave things open, noting that the two kinds of argument place—apparently flexible and apparently inflexible—suggest different regimentation strategies, namely to admit flexible plural predicates, or not.

Secondly, collective plural predicates are contrasted with distributive ones, such as ‘are prime’, ‘are students’, ‘have visited Rome’. Roughly speaking, these are predicates that apply to a collection if and only if they apply to each of its members. How best to make this precise will depend on one’s stand on the issue of flexible plural predicates mentioned just above. If all plural predicates are allowed to be flexible, then a plural predicate  $P$  is distributive just in case the following is analytic (or near enough):

$$P(xx) \leftrightarrow \forall x (x \prec xx \rightarrow P(x))$$

In the presence of inflexible plural predicates, however, a slight modification is needed. Then a plural predicate  $P$  is distributive just in case its singular analogue  $P^s$  is such that the following is analytic (or near enough):

$$P(xx) \leftrightarrow \forall x (x \prec xx \rightarrow P^s(x))$$

If  $P$  has no singular analogue (as is arguably the case for ‘cooperate with one another’ and ‘are two in number’), then  $P$  is collective by default.<sup>8</sup> Distributive plural predicates in this sense may thus be obtained by paraphrase from their corresponding singular forms. For this reason, distributive plural predicates may be omitted from PFO+ without any loss of expressibility—

although admittedly with some violence to style.

It might be helpful to close this section by providing some basic examples of regimentation in PFO+.

(10) Some students cooperated.

(10')  $\exists xx (S(xx) \& C(xx))$

(11) Bunsen and Kirchhoff laid the foundations of spectral analysis.

(11')  $\exists xx (\forall y (y < xx \leftrightarrow (y = b \vee y = k)) \& L(xx))$

(GK) Some critics admire only one another.

(12)  $\exists xx (\forall x (x < xx \rightarrow C(x)) \& \forall x \forall y [(x < xx \& A(x, y)) \rightarrow (y < xx \& x \neq y)])$

#### **4. The theory of plural logic**

As a formal language, PFO+ comes equipped with logical rules of inference and axioms aimed at capturing correct reasoning in the fragment of natural language regimented by this formal language. The rules associated with the singular vocabulary—logical connectives and quantifiers—are the usual ones, i.e. introduction and elimination rules for each logical expression. Plural quantifiers are associated with introduction and elimination rules mirroring those for the singular quantifiers. Two principles may be added. One captures the fact that pluralities are not empty:

$$\text{(Non-empty)} \quad \forall xx \exists y y \prec xx$$

The other is an indiscernibility principle. It expresses the fact that coextensive pluralities satisfy the same formulas:

$$\text{(Indiscernibly)} \quad \forall xx \forall yy (\forall x(x \prec xx \leftrightarrow x \prec yy) \rightarrow (\varphi(xx) \leftrightarrow \varphi(yy)))$$

Finally, there is a principle sanctioning which pluralities there are. This is the axiom schema of *plural comprehension*. For any formula  $\varphi(x)$  of PFO+ containing  $x$  free, we have the axiom stating that, if  $\varphi(x)$  is satisfied by at least one thing, then the things each of which satisfies  $\varphi(x)$  exist. The formula  $\varphi(x)$  may contain parameters. So, in symbols, we have the universal closure of the following axiom schema:

$$\text{(P-Comp)} \quad \exists x \varphi(x) \rightarrow \exists xx \forall y (y \prec xx \leftrightarrow \varphi(y))$$

The notion of derivation is defined inductively in the usual way as an appropriate sequence of formulas.

## 5. The semantics of plural logic

The formal language of plural logic also comes equipped with a model-theoretic semantics that captures the notion of logical consequences in terms of models (also called interpretations or structures): a sentence  $\varphi$  is the logical consequence of some premises just in case there is no model of the language in which the premises are true and  $\varphi$  is false. So the central task is to characterize a suitable notion of model and a correlative notion of truth in a model (also called satisfaction).

Traditional model-theoretic semantics is based on set theory. A model is defined as an ordered pair  $(d, f)$ , where  $d$  is a non-empty set representing the domain of discourse of the model (i.e. what there is according to the model) and  $f$  is an interpretation function from the non-logical terminology of the language to set-theoretic constructions based on  $d$ .<sup>9</sup> Let  $\llbracket E \rrbracket_f$  be the denotation of the expression  $E$  according to the function  $f$ . A natural setup is one in which a proper name denotes an object in the domain and a monadic predicate denotes a subset of the domain. So for any proper name  $a$  and singular predicate  $S$ , the sentence  $S(a)$  is true in the model  $(d, f)$  if and only if  $\llbracket a \rrbracket_f \in \llbracket S \rrbracket_f$

There is an obvious way of extending this semantics to PFO and PFO+. The interpretation function  $f$  can be augmented so that plural constants are assigned non-empty

subsets of the domain and (monadic) plural predicates are assigned sets of subsets of the domain. The treatment of plural predication is then parallel to that of singular predication: for any plural term  $aa$  and plural predicate  $P$ , the sentence  $P(aa)$  is true in the model  $(d, f)$  if and only if  $\llbracket aa \rrbracket_f \in \llbracket P \rrbracket_f$ . Plural membership is treated as logical in that it is not subject to reinterpretation, always corresponding to set-theoretic membership. That is, a sentence of the form  $b < aa$  is true in the model  $(d, f)$  if and only if  $\llbracket b \rrbracket_f \in \llbracket aa \rrbracket_f$ .

On the semantics just developed, plural logic has metalogical properties that distinguish it from first-order logic. It is not compact, hence it is incomplete, and it fails to have the Löwenheim-Skolem property.<sup>10</sup>

However, the set-theoretic model theory for PFO and PFO+ has received two main criticisms. First, since plural terms are taken to denote sets, the semantics is said to introduce ontological commitments that are arguably absent in ordinary discourse. Second, by construing domains as sets, the set-theoretic model theory rules out models whose domain is too big to form a set. This means that there are no models that correspond to some intuitive interpretations of the language, such as those with a domain of quantification encompassing absolutely everything.<sup>11</sup>

As a reaction to these criticisms, an alternative approach to the semantics of PFO and PFO+ has gained increasing popularity. Instead of letting the values of plural terms be subsets of the first-order domain, the semantic values of plural terms are represented by plural terms in the metalanguage. A plural term now stands for *many* objects. To state this view, one needs of course to make use of plural resources in the metalanguage. This semantic proposal traces back to (Boolos 1985) who insisted that the value of a plural variable not be a set (or any kind of set-

like object). For him a plural variable has many values from the ordinary, first-order domain and thus ranges plurally over it. The semantics based on Boolos's new approach may be called *plurality-based* to highlight the contrast with the set-based semantics described above.

How are the criticisms to the set-based semantics avoided on the plurality-based approach? First, since plural terms no longer denote sets, the charge of introducing spurious ontological commitments to sets does not arise. A plural term denotes many objects as such, without the need of collecting those objects into a single entity. Second, by using plural resources in the metalanguage one may define a domain of quantification to be *some things* rather than a set. In turn, this enables one to capture a domain of quantification encompassing absolutely everything. That would be the domain consisting of the things such that everything is among them, which can be obtained by plural comprehension. Therefore, the plurality-based semantics sidesteps the two main criticisms leveled against the set-based approach.

In closing this section, we would like to make two further remarks. First, the plurality-based semantics sanctions the same metalogical properties as the set-based semantics. In particular, the resulting logic is not compact, is incomplete, and it fails to have the Löwenheim-Skolem property. Second, these metalogical properties, both in the case of the plurality-based semantics and in the case of the set-based semantics, depend on a *standard* treatment of quantification according to which plural quantifiers range over *all* subpluralities or *all* non-empty subsets of the first-order domain. But there is a *non-standard* (i.e. Henkin) treatment of plural logic. By allowing quantification over some but not all subpluralities of the first-order domain, or some but not all subsets of the first-order domain, we obtain respectively a Henkin plurality-



based semantics and a Henkin set-based semantics (see Florio and Linnebo forthcoming).

## 6. Arguments against singularizing strategies

Plural logic provides an appealing alternative to the singularizing strategies surveyed in section

1. But is the resort to plural logic inevitable? Can we successfully analyze plurals by paraphrasing them away according to one of those strategies? Singularizing approaches face some serious objections. *The argument from incorrect existential consequences* points out that, for a broad range of singularizing approaches, some translations will have first-order existential consequences that the initial plural sentence appears not to have. In those cases, we are able to transition *as a matter of logic* from *some objects* to some sort of ‘collection’ or single object that comprises or somehow represents those objects. But a number of examples suggest that this transition is not always licit. Consider these two sentences.

(13) Bill and Hillary are two. (Yi 1999)

(14) Russell and Whitehead wrote *Principia Mathematica*. (Oliver and Smiley 2001)

Set-theoretic and mereological paraphrases offer translations along the following lines.

(15) {Bill, Hillary} is two-membered.

(16) Russell $\oplus$ Whitehead wrote\* *Principia Mathematica*.

where ‘wrote\*’ stands for the appropriate mereological rendering of the predicate ‘wrote’. Thus, in both cases, the translation has a singular existential consequence, (17), that is intuitively neither a consequence of (13) or (14).

(17) There is a set or there is a mereological sum.

Boolos proposed a famous variant of this argument when he remarked that ‘I am eating some Cheerios’ does not logically entail ‘I am eating a set’ (Boolos 1984: 72).

Another argument is *the paradox of plurality* introduced in section 1. It purports to show that plausible singularizing strategies are bound to regiment intuitively true sentences of the object language as logical falsehoods (Boolos 1984; Lewis 1991; Schein 1993; Higginbotham 1998; Oliver and Smiley 2001; Rayo 2002; McKay 2006; Oliver and Smiley 2013).

While the paradox of plurality threatens a wide array of singularizing strategies, it does not threaten all of such strategies. Notably, the mereological approach escapes the paradox. The paradox assumes that the relation regimenting plural membership is not reflexive. Unlike the set-theoretic singularist, the mereological singularist need not grant this assumption. She can model plural membership by means of the reflexive relation of individual parthood ( $\leq$ ).

However, the very feature that immunizes the mereological approach from the paradox of plurality makes it vulnerable to another sort of objection. According to this objection, mereology doesn't have the resources to represent the more complex structure associated with plural expressions, thus validating intuitively invalid inferences. In particular, the mereological singularist faces difficulties when regimenting plural talk involving the very mereological notions that are at the core of her regimentation strategy. In such cases, there can be more structure than can be represented by the mereological strategy (see, e.g., Schein 1993, Oliver and Smiley 2001, and Rayo 2002).

The criticisms against singularizing strategies have varying degrees of force. Whether or not they are ultimately compelling, we hope that this brief exposition will suffice to appreciate the standard motivations for plural logic.

## **7. Logicality and plural logic**

One of the central disputes about plural logic concerns its status as *logic*. Does the logical system outlined above qualify as "pure logic"? Since the debate about what counts as pure logic is vast, we cannot do full justice to it here. Rather, we focus on three important marks of logicality: topic-neutrality, formality, and epistemic access.

The requirement of topic-neutrality is based on a simple, intuitive idea, namely that logical principles should be applicable to reasoning about any subject matter. In contrast, other principles are only applicable to particular domains of individuals. The laws of physics, for instance, concern the physical world and cannot be applied to reasoning about natural numbers

or other abstract entities. Plural logic seems to satisfy this intuitive notion of topic-neutrality. The validity of the principles of plural logic does not appear to be restricted to specific domains of individuals.

Another mark of logicality is formality. Logical principles are often thought to hold in virtue of their form, and not of their content. There are different ways of articulating the notion of formality, some of which are connected to the notion of topic-neutrality just discussed (see MacFarlane 2000). However, the following conditions are commonly associated with formality. The first is that formal principles are ontologically innocent: they do not commit us to the existence of particular objects. The second is that formal principles are unable to discriminate between objects: they cannot single out particular objects or classes thereof.

Is plural logic ontologically innocent? In particular, are plural quantifiers ontologically innocent? The traditional answer to these questions is affirmative. Plural logic indeed originated as an ontologically innocent alternative to second-order logic, a system that adds to first-order logic quantification into predicate position (Boolos 1984; Boolos 1985). This view is sustained by the plurality-based semantics developed in section 5. According to that semantics, plural quantifiers do not range over a special domain of plural entities. They range plurally over entities in the domain of the singular quantifiers and thus do not introduce commitments beyond those incurred by the first-order quantifiers.<sup>12</sup> However, both earlier critics of the ontological innocence of plural logic (e.g. Parsons 1990, Hazen 1993, and Shapiro 1993) and more recent ones (Linnebo 2003; Florio and Linnebo forthcoming) have emphasized that the conclusion follows only if ontological innocence is understood in terms of commitments to objects, i.e. to

entities in the range of the first-order quantifiers. However, there is a broader notion of ontological commitment tied to the presence of existential quantifiers of any logical category in a sentence's truth conditions. According to this notion, plural locutions incur additional ontological commitments even on a plurality-based semantics. The resulting view is an analogue of that espoused by Frege when he held that quantification into predicate position incurs its own distinctive kind of commitment, not to objects but rather to (what he called) concepts.

Another condition on formality is that formal principles should not discriminate between objects. A standard way of making the condition precise is to claim that logical principles are those that remain true no matter how the non-logical expressions of the language are reinterpreted. This presupposes a distinction between logical and non-logical expressions of the language, which is typically captured by defining logical notions in terms of isomorphism invariance (e.g. Tarski 1986, Sher 1991, McGee 1996) and then characterizing as logical the expressions that are suitably related to logical notions.<sup>13</sup> The notions corresponding to plural quantification and plural membership come out as logical on this account.

The last mark of logicality we would like to consider concerns *epistemic access*. It is often thought one must be able to grasp and accept logical principles (or logical notions) without relying on non-logical principles (or non-logical notions). Moreover, logical truths, if knowable, must be knowable independently of non-logical truths. In the context of plural logic, the prime suspect has been plural comprehension (see Linnebo 2003). The claim is that our knowledge and acceptance of plural comprehension is mediated by our knowledge and acceptance of set theory.

## 8. Plural logic and second-order logic

Both plural logic and second-order logic have found a number of philosophical applications. As first shown by Boolos, monadic second-order logic can be interpreted in PFO. The converse is true as well: PFO can be interpreted in monadic second-order logic. So, from a syntactic point of view, the two theories are equivalent. In light of this equivalence, one question (also relevant in the context of collective intentionality) is whether there is a genuine choice between them. When considering particular applications, are they interchangeable? Could plural logic be replaced by second-order logic in the formalization of talk about collections or collective entities? Or should one system be preferred over the other?

One reason to keep the two systems separate concerns natural language. Plural logic is typically motivated by the need to capture natural language plurals while avoiding the problems incurred by the singularizing strategies. Second-order logic can be said to enjoy a parallel motivation. There are examples strongly suggestive of variable binding of predicate positions (e.g. ‘John is everything we wanted him to be’, see Higginbotham 1998: 251, and Rayo and Yablo 2001), which includes predicate positions of plural predicates (‘John and Mary are everything we wanted them to be’). Thus plural logic and second-order logic might be needed as distinct formalisms for the regimentation of natural language.

Another reason to keep the two systems separate is that, as naturally interpreted, the semantic values of plural and second-order terms seem to have different modal profiles. While pluralities are modally rigid, properties are not. Compare the following sentences, where ‘them’ in (18) refers to Annie and Bonnie.

(18) Annie is one of them but might not have been.

(19) Annie is a philosopher but might not have been.

While (18) appears to be false, (19) appears to be true. This is symptomatic of the fact that we regard pluralities as modally rigid: if  $x$  is one of  $xx$ , then necessarily  $x$  is one of  $xx$ . In the jargon of possible worlds, we may say that pluralities retain their members across worlds (for more details, see Rumfitt 2005; Williamson 2010; Uzquiano 2011; an unorthodox view is defended by Hewitt 2012). Not so for all properties. Some properties do not retain their extension across worlds. For instance, the property of being a philosopher has a non-empty extension but might have had an empty one.<sup>14</sup>

### **Further readings**

The classic references for Boolos's pioneering work on plural logic are (Boolos 1984) and (Boolos 1985). More recent developments include (Yi 1999), (Oliver and Smiley 2001), (Rayo 2002), (Linnebo 2003), (Yi 2005) and (Yi 2006). There are three philosophical monographs on plurals, all embracing plural logic: (Yi 2002), (McKay 2006), and (Oliver and Smiley 2013). The latter is the most comprehensive philosophical treatment of the subject to date.

There is also a rich literature on plurals in linguistic semantics. Among the most

influential works, often adopting a singularizing approach, are (Link 1998), (Schein 1993), (Schwarzschild 1996), and (Landman 2000).

### **Related topics**

The interested reader is also invited to consult Ritchie's and Moltmann's chapters in this handbook.

### **Biographical note**

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## References

- Black, M. (1971) "The Elusiveness of Sets," *Review of Metaphysics*, 24, 614–636.
- Boolos, G. (1984) "To Be Is to Be a Value of a Variable (or to Be Some Values of Some Variables)," *Journal of Philosophy*, 81, 430–450.
- (1985) "Nominalist Platonism," *Philosophical Review*, 94, 327–344.
- Burgess, J. and Rosen, G. (1997) *A Subject with No Object. Strategies for Nominalistic Interpretations of Mathematics*, Oxford: Oxford University Press.
- Champollion, L. (forthcoming) *Parts of a Whole: Distributivity as a Bridge Between Aspect and Measurement*, Oxford: Oxford University Press.
- Enderton, H. B. (2002), *A Mathematical Introduction to Logic*, San Diego: Academic Press.
- Florio, S. and Linnebo, Ø. (forthcoming) "On the Innocence and Determinacy of Plural Quantification," *Noûs*.
- Frege, G. (1980) *Philosophical and Mathematical Correspondence*, Oxford: Basil Blackwell.
- Hazen, A. (1993) "Against Pluralism," *Australasian Journal of Philosophy*, 71, 132–144.
- Hewitt, S. (2012), "Modalising Plurals," *Journal of Philosophical Logic*, 41, 853–875.
- Higginbotham, J. (1998) "On Higher-Order Logic and Natural Language," in T. Smiley (ed.) *Philosophical Logic*, Oxford: Oxford University Press.
- Higginbotham, J. and Schein, B. (1989) "Plurals," in J. Carter and R.-M. Dechaine (eds.) *Proceedings of the North East Linguistics Society*, Amherst: Graduate Linguistics Students Association, University of Massachusetts.

Klement, K. (2014) “Early Russell on Types and Plurals,” *Journal for the History of Analytical Philosophy*, 2, 1–21.

Kreisel, G. (1967) “Informal Rigour and Completeness Proofs,” in I. Lakatos (ed.) *Problems in the Philosophy of Mathematics*, Amsterdam: North Holland.

Landman, F. (2000) *Events and Plurality*, Dordrecht: Kluwer Academic Publishers.

Lewis, D. (1991) *Parts of Classes*, Oxford: Basil Blackwell.

Link, G. (1983) “The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach,” in R. Bäuerle, C. Schwarze, and A. von Stechow (eds.) *Meaning, Use and the Interpretation of Language*, Berlin: De Gruyter.

----- (1998) *Algebraic Semantics in Language and Philosophy*, Stanford: CSLI Publications.

Linnebo, Ø. (2003) “Plural Quantification Exposed,” *Noûs*, 37, 71–92.

----- (2010) “Pluralities and Sets,” *Journal of Philosophy*, 107, 144–164.

MacFarlane, John. 2000. *What Does It Mean to Say That Logic Is Formal?*, PhD Thesis, University of Pittsburgh.

McCarthy, T. (1981) “The Idea of a Logical Constant,” *Journal of Philosophy*, 78, 499–523.

McGee, V. (1996) “Logical operations,” *Journal of Philosophical Logic*, 25, 567–580.

McKay, T. J. (2006) *Plural Predication*, Oxford: Oxford University Press.

Moltmann, F. (1997) *Parts and Wholes in Semantics*, Oxford: Oxford University Press.

- Oliver, A. and Smiley, T. (2001) "Strategies for a Logic of Plurals," *Philosophical Quarterly*, 51, 289–306.
- (2013) *Plural Logic*, Oxford: Oxford University Press.
- Parsons, C. (1990) "The Structuralist View of Mathematical Objects," *Synthese*, 84, 303–346.
- Quine, W. V. O. (1982) *Methods of Logic*, Cambridge: Harvard University Press.
- Rayo, A. (2002) "Word and Objects," *Noûs*, 36, 436–464.
- Rayo, A. and Williamson, T. (2003) "A Completeness Theorem for Unrestricted First-Order Languages," in JC Beall (ed.) *Liars and Heaps*, Oxford: Oxford University Press.
- Rayo, A. and Yablo, S. (2001) "Nominalism Through De-Nominalization," *Noûs*, 35, 74–92.
- Resnik, M. (1988) "Second-Order Logic Still Wild," *Journal of Philosophy*, 85, 75–87.
- Rumfitt, I. (2005) "Plural Terms: Another Variety of Reference?" in J. L. Bermúdez (ed.) *Thought, Reference, and Experience: Themes from the Philosophy of Gareth Evans*, Oxford: Oxford University Press.
- Russell, B. (1938) *The Principles of Mathematics*, New York: Norton.
- Sagi, G. (2015) "The Modal and Epistemic Arguments Against the Invariance Criterion for Logical Terms," *Journal of Philosophy*, 112, 159-167.
- Schein, B. (1993) *Plurals and Events*, Cambridge: MIT Press.
- Schwarzschild, R. (1996) *Pluralities*, Dordrecht: Kluwer Academic Publishers.
- Shapiro, S. (1993) "Modality and Ontology," *Mind*, 102, 455–481.

- Sher, G. (1991) *The Bounds of Logic: A Generalized Viewpoint*, Cambridge: MIT Press.
- Simons, P. (1982) "Plural Reference and Set Theory," in B. Smith (ed.) *Parts and Moments: Studies in Logic and Formal Ontology*, München: Philosophia Verlag.
- (1997) "Higher-Order Quantification and Ontological Commitment," *dialectica*, 51, 255–271.
- Tarski, A. (1986) "What Are Logical Notions?" *History and Philosophy of Logic*, 7, 143–154.
- Uzquiano, G. (2011) "Plural Quantification and Modality," *Proceedings of the Aristotelian Society*, 111, 219–250.
- Williamson, T. (2010) "Necessitism, Contingentism, and Plural Quantification," *Mind*, 119, 657–748.
- Winter, Y. and Scha, R. (2015) "Plurals," in S. Lappin and C. Fox (eds.), *The Handbook of Contemporary Semantic Theory*, Oxford: Wiley-Blackwell.
- Yablo, S. (2006) "Circularity and Paradox," in T. Bolander, V. F. Hendricks, and S. A. Pedersen (eds.), *Self-Reference*, Stanford: CSLI Publications.
- Yi, B.-U. (1999) "Is Two a Property?" *Journal of Philosophy*, 96, 163–90.
- (2002) *Understanding the Many*, London: Routledge.
- (2005) "The Logic and Meaning of Plurals. Part I." *Journal of Philosophical Logic*, 34, 459–506.
- (2006). "The Logic and Meaning of Plurals. Part II." *Journal of Philosophical Logic*, 35,

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- <sup>1</sup> For historical details, see (Oliver and Smiley 2013: chapter 2).
  - <sup>2</sup> As shown by Boolos, who credits David Kaplan, there is no paraphrase of this sentences comprising only singular vocabulary and the predicates occurring in it (Boolos 1984: 432-433).
  - <sup>3</sup> See (Resnik 1988) for a similar view.
  - <sup>4</sup> According to the view defended in (Linnebo 2010), (7) is false—but only for non-obvious reasons having to do with the ‘definiteness’ of any plurality, contrasted with the ‘indefiniteness’ of the notion of a self-identical set.
  - <sup>5</sup> If desired, both fusion constructions can be defined in terms of the parthood relation by exploiting the fact that the fusion is the smallest object whose parts include the things to be fused.
  - <sup>6</sup> Again, for historical details, see (Oliver and Smiley 2013: chapter 2).
  - <sup>7</sup> For systems that employ the notation for variables adopted here, see (Rayo 2002) and (Linnebo 2003). An ancestor of this notation is found in (Burgess and Rosen 1997). Variants include variables in boldface (Oliver and Smiley), capitalized (McKay), or pluralized with the letter ‘s’ (Yi).
  - <sup>8</sup> Notice that our definition of distributivity takes the form of (analytic) equivalences. Some authors (e.g. McKay 2006: 6) tie distributivity solely to the left-to-right implication. For discussion and references, see (Oliver and Smiley 2013: 112-113). For an overview of linguistic treatments of distributivity, see (Winter and Scha 2015).
  - <sup>9</sup> The logical terminology is not subject to reinterpretation. Its meanings are characterized inductively through the characterization of the notion of truth in a model.
  - <sup>10</sup> For an explanation of these properties, see any advanced introduction to logic, e.g. (Enderton 2002).
  - <sup>11</sup> Since the completeness theorem fails for plural logic with standard semantics, the famous Kreisel ‘squeezing argument’ (Kreisel 1967) is not available. See (Rayo and Williamson 2003) for discussion.
  - <sup>12</sup> Note that the picture is different if one adopts the set-based semantics or attempts to regiment plurals along one of the singularizing strategies presented in section 1.
  - <sup>13</sup> Denoting a logical notion has been claimed to be necessary but not sufficient for the logicity of an expression. An additional semantic connection would be required (as argued, for instance, by McCarthy 1981 and McGee 1996; see also Sagi 2015 for a critical evaluation of these arguments).
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