No. 19 June 1976

THE VERTICAL ATTENUATION COEFFICIENT OF SUBMARINE IRRADIANCE

by

Eyvind Aas

INSTITUTT FOR GEOFYSIKK

UNIVERSITETET I OSLO

INSTITUTE REPORT SERIES

No.19 June 1976

THE VERTICAL ATTENUATION COEFFICIENT OF SUBMARINE IRRADIANCE

by

Eyvind Aas

ABSTRACT

An approximated formula for the vertical attenuation coefficient as a function of the absorption coefficient, the scattering coefficient, the sun height, the cloudiness and the depth is deduced. A relation between only the vertical attenuation coefficient, sun height, cloudiness and depth is also proposed.

THE VERTICAL ATTENUATION COEFFICIENT OF SUBMARINE IRRADIANCE.

1. INTRODUCTION.

While analyzing irradiance observations at 465 nm from the Norwegian Sea and the Barents Sea (AAS and BERGE, 1976), a relation between the vertical attenuation coefficient and the sun height was needed. A usual assumption is that close to the surface the vertical attenuation coefficient K of downward irradiance will be related to the zenith angle j of the refracted sun rays by the relation

$$K = K_o/\cos j = K_o \sec j \qquad (1)$$

K_o is then the coefficient for a zenith sun.

This relation has been tested by KOSLYANINOV and PELEVIN (1966), JERLOV and NYGÅRD (1969), and HØJERSLEV (1973, 1974 a). KOSLYANINOV and PELEVIN found the relation valid down to an optical depth $cz = 2.5$ (c is the beam attenuation coefficient, z is the depth). JERLOV and NYGÅRD found it valid down to 10 m for green light in the Baltic Sea and to 75 m for blue light in the Sargasso Sea. Both these depths correspond roughly to an optical depth 4. According to HØJERSLEV (1974a) it should be expected from theory that K was essentially independent of the sun height for sun heights less than 25°. He found for green light in the Baltic little variation for heights less than 40°. In the Mediterranean he found for blue light little dependence upon the sun height for heights less than 50° (1974 b).

It is, however, clear that it should be a gradual transition from the state when the "secans relation" (eq.1) is valid to the state when it is not.

In connection with the before-mentioned irradiance measurements in the Norwegian Sea and the Barents Sea there were no measurements of the optical depth cz, so the formula which is developed here is based finally only on observed values of vertical attenuation coefficient, cloudiness, sun height and depth.

2. THE FUNCTION $K_d(a, R, D_D, j, r)$.

For an ocean which is optically homogeneous in horrisontal directions, the GERSHUN equation (1939) yields

$$\frac{d}{dz}(E_d - E_u) = -aE_o \tag{2.1}$$

E_d is downward irradiance, E_u upward irradiance, a is the absorption coefficient and E_o is the scalar irradiance. By introducing the vertical attenuation coefficients K_d and K_u, defined by

$$K_d = -\frac{1}{E_d}\frac{dE_d}{dz}, \quad K_u = -\frac{1}{E_u}\frac{dE_u}{dz} \tag{2.2}$$

and the downward and upward part of the scalar irradiance,

$$E_o = E_{od} + E_{ou} \tag{2.3}$$

the equation becomes

$$K_d E_d - K_u E_u = a(E_{od} + E_{ou}) \tag{2.4}$$

By means of the irradiance ratio

$$R = \frac{E_u}{E_d} \tag{2.5}$$

and the scalar irradiance ratio

$$R_o = \frac{E_{ou}}{E_{od}} \tag{2.6}$$

eq. 2.4 may be written

$$K_d = a \frac{E_{od}(1+R_o)}{E_d(1-R^{K_u/K_d})} = a \frac{D_d(1+R_o)}{1 - R^{K_u/K_d}} \tag{2.7}$$

The function

$$D_d = {}^{E_{od}}/_{E_d} \tag{2.8}$$

is termed the downward distribution function by TYLER and PREISENDORFER (1962). In Table 1, which is calculated and interpolated from TYLER's radiance observations at 480 nm in Lake Pend Oreille (TYLER and PREISENDORFER, 1962), it is seen that the relative standard deviation, S_{rel}, of the ratio K_u/K_d is 4%, and that the mean value of the ratio is 1.02. It is also seen that the standard deviation of the ratio R_o/R is 2%, and that the mean value is 2.13. In lack of other knowledge I then make the assumptions

$$\frac{K_u}{K_d} \approx 1 \tag{2.9}$$

$$\frac{R_o}{R} \approx 2 \tag{2.10}$$

and eq. 2.7 becomes, since from Table 1 $R << 1$,

$$K_d \approx \frac{a\ D_d(1+2R)}{1-R} \approx a(1+3R)D_d \tag{2.11}$$

The equation (2.11) may also be obtained in the following way: Table 1 gives that the upward distribution function

$$D_u = \frac{E_{ou}}{E_u} \tag{2.12}$$

has the mean value 2.8 with a standard deviation of 2%. If we assume that this value has a general validity, we may return to eq. 2.4 and write it

$$K_d E_d - K_u E_u \approx a(E_{od} + 2.8\, E_u) \qquad (2.13)$$

This may be rewritten as

$$K_d E_d(1 - \frac{K_u}{K_d} R - \frac{2.8\, a}{K_d} R = a\, E_{od} \qquad (2.14)$$

From Table 1 it is seen that the mean value of a/K_d in this case is 0.7, so that

$$K_d E_d(1 - R - 2R) = K_d E_d(1 - 3R) = a\, E_{od}, \qquad (2.15)$$

or again

$$K_d \approx a(1 + 3R) D_d. \qquad (2.16)$$

The distribution function D_d is a function of the downward radiance L, given by

$$D_d = \frac{E_{od}}{E_d} = \frac{\int_{2\pi} L\, d\,\omega}{\int_{2\pi} L \cos \Theta d\omega} = \frac{1}{\cos \Theta_M} = \sec \Theta_M \qquad (2.17)$$

$d\omega$ is an infinitesimal solid angle in the direction of L, and the integral is performed in the upper hemisphere. Θ is the angle between L and the vertical. Θ_M is the mean value of Θ in the integral. When the two integrals in eq. 2.17 are dominated by the radiance from the sun, Θ_M will be close to j, and eq. 2.16 will be of the same form as eq. 1. In the case of a clear sky in Table 1, the sun height was 56.6°. This makes $\sec_j = 1.10$, which is not equal to the values of D_d.

The distribution function may be separated into two main factors, namely the contribution from the direct sun light and from the diffuse light. If the radiance from the diffuse light is termed L_D and the radiance from the sun L_S, we get from eq. 2.17

$$\frac{E_{od}}{E_d} = \frac{L_S d\omega_S + \int_{2\pi - d\omega_S} L_D d\omega}{L_S \cos j \, d\omega_S + \int_{2\pi - d\omega_S} L_D \cos \theta d\omega} \tag{2.18}$$

The direct contribution to the irradiance from the sun is

$$E_{dS} = L_S \cos j \, d\omega_S \tag{2.19}$$

which makes

$$L_S d\omega_S = E_{dS} \sec j. \tag{2.20}$$

The contribution to the irradiance from the diffuse light is

$$E_{dD} = \int_{2\pi - d\omega_S} L_D \cos \theta d\omega = \cos \theta_D \int_{2\pi - d\omega_S} L_D d\omega \tag{2.21}$$

where θ_D is the mean value of θ in the integration. We may write eq. 2.21 as

$$\int_{2\pi - d\omega_S} L_D d\omega_S = E_{dD} \sec \theta_D \tag{2.22}$$

and eq. 2.18 then becomes

$$\frac{E_{od}}{E_d} = \frac{E_{dS} \sec j + E_{dD} \sec \theta_D}{E_{dS} + E_{dD}} \tag{2.23}$$

Since

$$E_{dD} = E_d - E_{dS}, \tag{2.24}$$

eq. 2.23 may be written

$$D_d = \frac{E_{od}}{E_d} = \frac{E_{dS} \sec j + (E_d - E_{dS}) \sec \theta_D}{E_d}$$

$$= \sec \theta_D + (\sec j - \sec \theta_D) \frac{E_{dS}}{E_d} \tag{2.25}$$

The unknown distribution function D_d has then been separated into a known function of the solar altitude, sec j, a still unknown distribution function of the diffuse light,

$$\sec \Theta_D = D_D \approx \frac{\int_{2\pi} L_D d\omega}{\int_{2\pi} L_D \cos \Theta d\omega} \tag{2.26}$$

and an unknown ratio between the solar contribution to the irradiance and the total irradiance. The relation between the sun height h and the sun ray's angle of refraction j is given by SNELL's law in the form

$$\sec j = 1/\sqrt{1-(\tfrac{3}{4})^2 \cos^2 h} \tag{2.27}$$

Eq. 2.16 and 2.25 give

$$K_d = a(1+3R)(\sec \Theta_D + (\sec j - \sec \Theta_D) \frac{E_{dS}}{E_d}) \tag{2.28}$$

For convenience, we may now omit the index d and introduce

$$a_R = a(1 + 3R) \tag{2.29}$$

and

$$r = E_S/E \tag{2.30}$$

We then have for the vertical attenuation coefficient

$$K = a_R(\sec \Theta_D + (\sec j - \sec \Theta_D)r) \tag{2.31}$$

I assume that the distribution function $D_D = \sec \Theta_D$ of the diffuse light may be regarded as independent of the sun height, at least compared with the function sec j of the direct sun rays. The ratio r, however, should be a function of the sun height, the cloudiness, the depth and the turbidity of the water.

When the sun is dominating the irradiande, r will be close to 1 and K will be given by

$$K \approx a_R \sec j \tag{2.32}$$

When the diffuse light is dominating, r will be close to 0, and K will be given by

$$K \approx a_R \sec \theta_D \tag{2.33}$$

3. THE FUNCTION $r_o(h,C)$.

In eq. 2.30 the downward irradiance E may be expressed as

$$E(z) = E(o)\ e^{-Kz} \tag{3.1}$$

where z is the depth.

Similarly we may express E_S as

$$E_S(z) = E_S(o)\ e^{-K_S z} \tag{3.2}$$

where K_S is the still unknown vertical attenuation coefficient of the sun rays. Eq. 2.30 may then be written

$$r = \frac{E_S(z)}{E(z)} = \frac{E_S(o)}{E(o)}\ e^{-(K_S-K)z} = r_o e^{-(K_S-K)z} \tag{3.3}$$

where

$$r_o = \frac{E_S(o)}{E(o)} \tag{3.4}$$

The function r_o is the value of r just beneath the sea surface, and will depend on E_S and E in air, and on their transmission through the sea surface. We shall first consider their ratio in air

$$r_a = \frac{E_{sa}}{E_a} \tag{3.5}$$

In clear weather this ratio is mainly a function of wavelength and sun height, Fig. 1, but it depends also on the transmittance of the air, that is on the dust and vapour contents of the air (KIMBALL, 1924, HINZPETER, 1955,1956,1957, GATES, 1966), and on the albedo of the ground (DEIRMENDJIAN and SEKERA, 1954). Due to the low albedo of the sea, the last effect will only reduce r_a with maximum 2% at small sun heights at sea, and this effect shall be disregarded here.

r_o is obtained from r_a by

$$r_o = \frac{\tau_S r_a}{\tau_S r_a + \tau_D (1 - r_a)} \tag{3.6}$$

Here τ_S is the Fresnel transmittance of the sun rays through the sea surface and τ_D is the corresponding transmittance of the diffuse irradiance. I have chosen $\tau_D = 0.93$ (isotropic radiance distribution).

The three cases in Fig. 1 may then be expressed as

a) $r_o = 1/(1 + 0.13 \text{ cosec } h \; e^{0.20 \text{ cosec } h}$ (3.7)

b) $r_o = 1/(1 + 0.20 \text{ cosec } h \; e^{0.22 \text{ cosec } h})$ (3.8)

c) $r_o = 1/(1 + 0.27 \text{ cosec } h \; e^{0.38 \text{ cosec } h})$ (3.9)

In lack again of other knowledge I have chosen

$$r_o = 1/(1 + 0.2 \text{ cosec } h \; e^{0.3 \text{ cosec } h}) \tag{3.10}$$

When there are clouds, the solar irradiance may be reduced to a mean value of

$$E_S' = E_S(1 - C) \tag{3.11}$$

C is the cloudiness in parts of one. Correspondingly the contribution of diffuse light from the blue part of the sky will be $E_D(1 - C)$. The contribution of diffuse light from the cloudy part of the sky may be written $E_C C$. E_C is the irradiance when the cloudiness is 1. By comparison with data by KALITIN (KONDRATYEV, 1969, p. 459) and ROBINSON (1966, p. 214-216), I have chosen as a mean value $E_C \approx 0.5 E$, where E is the irradiance of a clear sky. r_o then becomes

$$r_o' = \frac{E_S'}{E_S' + E_D'} = \frac{E_S(1-C)}{ES(1-C)+E_D(1-C)+0.5EC}$$

$$= \frac{ES(1-C)}{E(1-C+0.5C)} = r_o \frac{1-C}{1-0.5C} \qquad (3.12)$$

Thus the effect of clouds is the correction term $(1 - C)/(1- 0.5C)$. Eq. 3.10 and 3.12 give

$$r_o' = \frac{1 - C}{1-0.5C} / (1 + 0.2 \operatorname{cosec} h \, e^{0.3 \operatorname{cosec} h}) \qquad (3.13)$$

4. THE FUNCTION $r(r_o,a,b,K,j,z)$.

In eq. 3.2 the sun rays are attenuated by the coefficient

$$K_S = (a + b) \sec j \qquad (4.1)$$

However, as about 50% of the scattered light will be contained within a 10^o cone around the rays (JERLOV, 1968, p. 39) it is questionable if all of this light should be considered as lost from the solar irradiance. If not, it may be more correct to write

$$K_S = (a + \alpha b) \sec j \qquad (4.2)$$

where α is the fraction of b which contributes to the attenuation.

In Chapter 5 it is found that this factor α is 0.86.

With this value eq. 3.3 becomes

$$r = r_o \, e^{-((a+0.86b)\sec j \, - \, K)z} \qquad (4.3)$$

5. THE FUNCTION $D_D(a,b,K,j,z)$.

Measurements of the downward distribution function of diffuse light, $D_D = \sec \Theta_D$, are rather few. JERLOV and LILJEQVIST (1938) found that the ratio E_o/E_d at 530 nm in cloudy weather had values between 1.33 and 1.37. TYLER's observations at 480 nm, quoted in Table 1, give values of D_d in cloudy weather between 1.29 and 1.34. In the last case the values may seem to increase asymptotically from the surface and down towards a fixed value.

From HØJERSLEV's observations at 477 nm (1973, 1974a), the least squares method gives (Table 2) that the sum

$$\Sigma(K-(a_R(\sec \Theta_D + (\sec j - \sec \Theta_D)r_o e^{-(K_S-K)z})))^2 \qquad (5.1)$$

where $K_S = (a + \alpha b)\sec j$, obtains its lowest value for $\alpha = 0.72$ and $\sec \Theta_D = 1.38$. However, if we solve for $\sec \Theta_D$ from eq. 2.31, we get

$$\sec \Theta_D = \frac{K/a_R - r \sec j}{1 - r} \qquad (5.2)$$

and the ratio on the right side of this equation does not seem to be constant. I will, as already mentioned, assume that sec Θ_D has less variation with the sun height than sec j, but that it may have the same kind of variation with depth as TYLER's measurements indicate. From eq. 5.2 it seem natural that sec Θ_D should have the same dependence on depth as r, and I have then chosen the simple form

$$\sec \Theta_D = \beta - \gamma e^{-(K_S - K)z} \qquad (5.3)$$

The sum 5.1 will now get a still lower value for $\alpha = 0.86$, $\beta = 1.41$, and $\gamma = 0.21$ (Table 2). The last form of sec Θ_D,

$$\sec \Theta_D = 1.41 - 0.21\, e^{-((a+0.86\, b)\sec j - K)z} \qquad (5.4)$$

has the interesting result that sec $\Theta_D = 1.20$ just beneath the surface. If we assume that the diffuse radiance is constant inside the critical solid angle of total reflection and zero outside, then sec Θ_D becomes

$$\sec \Theta_D = \frac{2\pi \int_0^{\Theta_c} L_D \sin \Theta \, d\Theta}{2\pi \int_0^{\Theta_c} L_D \sin \Theta \cos \Theta \, d\Theta} = \frac{1-\cos \Theta_c}{\sin^2 \Theta_c / 2} = \frac{2}{1+\cos \Theta_c} \qquad (5.5)$$

$$= 1.20$$

since $\Theta_c = 48.3^o$.

Thus from this result the diffuse radiance seems to be contained within the critical solid angle at zero depth. With increasing depth the value of sec Θ_D increases asymptotically from 1.20 towards 1.41.

While this result was obtained for light at 477 nm, I shall assume that it is also valid at 465 nm.

6. PRACTICAL FORMS OF THE FUNCTION K.

We have now obtained that

$$K = a_R(\sec \Theta_D + (\sec j - \sec \Theta_D)r) \tag{6.1}$$

where

$$a_R = a(1 + 3R), \tag{6.2}$$

$$\sec \Theta_D = 1.41 - 0.21\, e^{-(K_S-K)z} \tag{6.3}$$

$$r = r_o\, e^{-(K_S - K)z} \tag{6.4}$$

$$r_o = \frac{1 - C}{1-0.5C}/(1 + 0.2 \operatorname{cosec} h\, e^{0.3 \operatorname{cosec} h}) \tag{6.5}$$

$$\sec j = (1 - (\tfrac{3}{4})^2 \cos^2 h)^{-\frac{1}{2}} \tag{6.6}$$

$$K_S = (a + 0.86\, b)\sec j \tag{6.7}$$

This gives a functional relation between K, R, a, b, h, C and z. However, if we should want to calculate K, eq. 6.1 does not give an explicit solution for K. It would be more practical if R and K on the right side of the equation were substituted with approximated functions of a, b, h and z. In Chapter 7 it is deduced that

$$R \approx 0.009\, b/a \tag{6.8}$$

It is also decuced that

$$K \approx (a + 0.022\, b)D_d \tag{6.9}$$

so that

$$K_S - K \approx a(\sec j - D_d) + b(0.86 \sec j - 0.022\, D_d) \tag{6.10}$$

In Table 2 it is shown that an approximated form of this is

$$K_S - K \approx -0.01\, a + 0.73\, b \sec j \approx 0.73\, b \sec j \tag{6.11}$$

Thus the equations 6.1-6.6, 6.8 and 6.11 give K(a,b,h,c,z). Fig. 2 shows calculated and observed values of K, based on the same data by HØJERSLEV as used in Table 2.
If we have observations of a and b then K_S is determined from eq. 6.7. However, in my case, I have measurements only for K, so that K_S should rather be expressed as a function of K. The contents of yellow substance in the Norwegian Sea and the Barents Sea is very low, except in the coastal areas. The attenuation should then be dominated by particles, and if we assume that there is a fairly constant ratio between the scattering and absorption coefficients, there may also be an almost constant ratio between (a + 0.86b)sec j and K. Table 2 gives for Mediteranean waters with low yellow substance contents that this ratio is 2.12. Eq. 6.7 then becomes

$$K_S \approx 2.12\ K \qquad (6.12)$$

The difference K_S - K may then just as well be approximated to the simple expression

$$K_S - K \approx K \qquad (6.13)$$

The equations 6.1-6.6 and 6.13 then defines a relation between K, a_R, h and z. In Table 3 are given calculated values of K for 3 different values of a_R in 0, 10 and 20 m depth. It is seen that according to this model the maximum value of K will not occure for $h = 0^o$, but for $h \approx 20^o$. It is also seen that the mean value of K in the range 0 - 20 m coincides very well with the value of K calculated for 10 m. In the before-mentioned work by AAS and BERGE the mean value of K between the surface and 20 m depth was derived from irradiance observations.

To obtain the value of K for $h = 35^\circ$ and $C = 0$, the relation

$$\frac{\overline{K(35,0,)}}{\overline{K(h,C)}} = \frac{(\sec \Theta_D + (\sec j - \sec \Theta_D)r)_{35,0,10}}{(\sec \Theta_D + (\sec j - \sec \Theta_D)r)_{h,C,10}} \qquad (6\ 14)$$

where

$$(\sec j)_{35} = 1.27 \qquad (6.15)$$

$$r_{o_{35,0}} = 0.63 \qquad (6.16)$$

was used. This relation may then be written

$$\overline{K(35,0)} = \overline{K(h,C)} \; \frac{1.41 - 0.298\ e^{-10K} + 0.132\ e^{-20K}}{1.41 - (0.21 + (1.41 - \sec j)r_o)e^{-10K} + 0.21 r_o e^{-20K}} \qquad (6.17)$$

whre r_o is given by eq. 6.5.

7. APPENDIX 1.

THE MODEL OF KOZLYANINOV AND PELEVIN.

The following is a very simplified presentation of the model of KOZLYANINOV and PELEVIN (1966).

$$\frac{dE_d}{dz} = -(a' + b')E_d + b'E_u \tag{7.1}$$

$$-\frac{dE_u}{dz} = -(a' + b')E_u + b'E_d \tag{7.2}$$

By differensiating the first of these two simplified equations of transfer with regard to z, and then eliminating dE_u/dz by means of the second, one obtains

$$\frac{d^2E_d}{dz^2} = (a'^2 + 2a'b')E_d \tag{7.3}$$

KOZLYANINOV and PELEVIN term a' the "efficient" coefficient of absorption and b' the "efficient" coefficient of scattering, and to obtain a practical formula, they have made the approximation that these coefficients are the same for downward and upward irradiance. By definition a' is

$$a' = a \frac{\int_{2\pi} L \, d\omega}{\int_{2\pi} L \cos \theta \, d\omega} = a D \tag{7.4}$$

With the assumption

$$\frac{b'}{b_b} = \frac{a'}{a} = D \tag{7.5}$$

where b_b is the backward scattering coefficient, and by assuming that the distribution function D varies only little with depth in the upper layer of the sea, eq. 7.3 may be integrated to

$$E_d(z) = E_d(o)\ e^{-\sqrt{a^2+2ab_b}\ Dz} \qquad (7.6)$$

By inserting this expression in eq. 7.1 and solving for E_u one gets

$$E_u = \frac{a+b_b-\sqrt{a^2+2ab_b}}{b_b}\ E_d(o)\ e^{-\sqrt{a^2+2ab_b}\ Dz} \qquad (7.7)$$

Thus

$$K_d = K_u = \sqrt{a^2+2ab_b}\ D \qquad (7.8)$$

and

$$R = \frac{E_u}{E_d} = \frac{a+b_b-\sqrt{a^2+2ab_b}}{b_b} \qquad (7.9)$$

By solving for b_b from eq. 7.9, one gets

$$b_b = \frac{2a\ R}{(1-R)^2} \qquad (7.10)$$

and substitution of this value in eq. 7.8 gives

$$K = a\ \frac{1+R}{1-R}\ D \qquad (7.11)$$

In order to see better the resemblance between this expression and eq. 2.16, we should make some simplifications. By dividing numerator and denumerator in eq. 7.9 with $a+b_b$, and noting that $a^2+2ab_b = (a+b_b)^2-b_b^2$, one gets

$$R = \frac{1-\sqrt{1-b_b^2/(a+b_b)^2}}{b_b/(a+b_b)} \qquad (7.12)$$

Now the ratio $b_b/(a+b_b) = (b_b/a)/(1+_b/a)$ is very small. For particles $b_b/b \approx 0.02$ (JERLOV 1968, p. 39).

So when particles are dominating the attenuance with a and b of the same order, the ratio b_b/a too will be of order 0.02. When yellow substance is dominating the attenuance, the ratio will be even smaller. For pure sea water at 465 nm b_b/a will have its highest value, about 0.1. The square root in eq. 7.12 may then be expanded to

$$R = \frac{1 - (1-\frac{1}{2}\ {b_b}^2/(a + b_b)^2)}{b_b/(a + b_b)} = \frac{1}{2}\ \frac{b_b/a}{1+b_b/a} \approx \frac{1}{2}\ \frac{b_b}{a}\ (1- \frac{b_b}{a}) \qquad (7.13)$$

$$\approx \frac{1}{2}\ b_b/a$$

Since R is of the same order as b_b/a, eq. 7.11 may be approximated to

$$K = a(1 + 2R)D \qquad (7.14)$$

Thus by eq. 7.8, the model of KOZLYANINOV and PELEVIN justifies the earlier assumption in eq. 2.9: $K_d \approx K_u$, and eq. 7.14 confirms the result in eq. 2.16.

The only difference is the factor 1 + 2R instead of 1 + 3R, but since R is a small number, this difference is small. The difference originates from the observa ion in eq. 2.10 which yields

$$\frac{R_o}{R} = \frac{E_{ou}E_d}{E_{od}E_u} = \frac{D_u}{D_d} \approx 2 \qquad (7.15)$$

or

$$D_u \approx 2D_d \qquad (7.16)$$

However, according to eq. 7.1, 7.2, and 7.4 in the present model

$$a'_u = aD_u = a'_d = aD_d \qquad (7.17)$$

or

$$D_u = D_d = D \tag{7.18}$$

which is certainly less correct.

Expansion of the square root in eq. 7.8 gives the interesting result

$$\begin{aligned} K &= a\sqrt{1 + 2b_{b/a}}\, D \approx a(1 + b_{b/a})D \\ &= (a + b_b)D = (a + \varepsilon\, b)D \end{aligned} \tag{7.19}$$

where

$$\varepsilon = b_b/b \tag{7.20}$$

As already mentioned, ε may be about

$$\varepsilon = b_{b/b} \approx 0.02 \tag{7.21}$$

except when the water is very pure.

From similtaneous observations of K,a,b, and D, we should then expect that

$$K = (a + 0.02\ b)D \tag{7.22}$$

HØJERSLEV (1973, 1974a) has presented such observations, and by applying the least square method as shown in Table 2, his data give

$$K = (a + 0.022\ b)D_d \tag{7.23}$$

With the assumption in eq. 7.21, one should expect that

$$R \approx \frac{1}{2}\,\frac{0.02\ b}{a + 0.02\ b} \tag{7.24}$$

or

$$R \approx \frac{1}{2}\,\frac{0.02\ b}{a} \tag{7.25}$$

In fact, the same set of data gives, as shown in Table 2, that

$$R \approx \frac{1}{2}\,\frac{0.015\ b}{a + 0.015\ b} \tag{7.26}$$

or

$$R \approx \frac{1}{2}\,\frac{0.017\ b}{a} \tag{7.27}$$

8. APPENDIX 2.

THE MODEL OF LUNDGREN AND HØJERSLEV.

LUNDGREN and HØJERSLEV (1971) deduced the approximated equation

$$K_d \simeq a(4R + \sec j) \tag{8.1}$$

By following the assumptions in their model, but not their procedure, we may obtain a result which is more similar to eq. 6.1.

Their simplified model of radiance distribution consists of three radiance types:

1) The direct radiance L_S from the sun.
2) The diffuse constant radiance L_D inside the solid angle of total reflection.
3) The diffuse constant radiance L_u outside the solid angle of total reflection.

The downward solar irradiance E_S then becomes

$$E_S = L_S \cos j \, d\omega_S \tag{8.2}$$

The downward diffuse irradiance E_D becomes

$$E_D = \int_0^{\Theta_{cr}} 2\pi L_D \sin\Theta \cos\Theta \, d\Theta + \int_{\Theta_{cr}}^{\pi/2} 2\pi L_u \sin\Theta \cos\Theta \, d\Theta$$

$$= \pi L_D \sin^2\Theta_{cr} + \pi L_u (1 - \sin^2\Theta_{cr}) \tag{8.3}$$

The upward irradiance E_u becomes

$$E_u = \int_0^{\pi/2} 2\pi L_u \sin\Theta \cos\Theta \, d\Theta = \pi L_u \tag{8.4}$$

and the scalar irradiance E_o becomes

$$E_o = L_S d\omega_S + \int_0^{\Theta_{cr}} 2\pi\ L_D \sin\Theta\ d\Theta + \int_{\Theta_{cr}}^{\pi} 2\pi\ L_u \sin\Theta\ d\Theta$$

$$= L_S d\omega_S + 2\pi\ L_D(1 - \cos\Theta_{cr}) + 2\pi\ L_u(\cos\Theta_{cr} + 1) \qquad (8.5)$$

By inserting for L_S, L_D and L_u from eq. 8.2, 8.3 and 8.4, eq. 8.5 may be written

$$E_o = \frac{E_S}{\cos j} + \frac{2}{\sin^2\Theta_{cr}}(E_D - E_u(1 - \sin^2\Theta_{cr}))(1 - \cos\Theta_{cr})$$

$$+ 2\ E_u(\cos\Theta_{cr} + 1) \qquad (8.6)$$

With $\Theta_{cr} = 48.6^o$, E_o becomes

$$E_o = E_S \sec j + 1.20\ E_D + 2.80\ E_u \qquad (8.7)$$

We may now return to eq. 2.4 and insert for E_o

$$K_d E_d - K_u E_u = a(E_S \sec j + 1.20(E_d - E_S) + 2.80\ E_u \qquad (8.8)$$

This may be rewritten as

$$K_d E_d (1 - (\frac{K_u}{K_d} + 2.8\ \frac{a}{K_d})\ \frac{E_u}{E_d}) = a(1.2\ E_d + (\sec j - 1.2)E_S) \qquad (8.9)$$

If we insert the values $\overline{K_u/K_d} \approx 1$, $\overline{a/K_d} \approx 0.7$ from Table 1, eq 8.9 becomes

$$K_d E_d(1 - (1 + 2.0)R) = a(1.2\ E_d + (\sec j - 1.2)E_S) \qquad (8.10)$$

or

$$K_d \approx a(1 + 3R)(1.2 + (\sec j - 1.2)r) \qquad (8.11)$$

Thus the radiance distribution model of LUNDGREN and HØJERSLEV may lead to the same result as in eq. 6.1-3, with one exception: - eq. 6.3 gives $\sec\Theta_D = 1.2$ only just beneath the surface.

REFERENCES

AAS, E. and BERGE, G., 1976. Irradiance observations in the Norwegian Sea and the Barents Sea. (In prep.)

DEIRMENDJIAN, D. and SEKERA, Z., 1954. Global radiation resulting from multiple scattering in a Rayleigh atmosphere. Tellus, 4: 382-398.

GATES, D.M., 1966. Spectral distribution of solar radiation at the earth's surface. Science. 151. No. 3710:523-529.

GERSHUN, A., 1939. The light field. J. Math. Phys., 18: 51-151.

HINZPETER, H., 1955,1956,1957. Einfache Rechnungen und Messungen zur Global- und Himmelsstrahlung. I,II,III. Zeitschr. f. Meteor, 9, Heft.10: 308-315, 10,4:100-110, 11, 1: 1-10.

HØJERSLEV, N.K., 1973. Inherent and apparent optical properties of the Western Mediterranean and the Hardangerfjord. Rep.Inst. Fysisk Oceanografi, Un. Copenhagen, 21. 23 p.p.

HØJERSLEV, N.K. 1974a. Inherent and apparent optical properties of the Baltic. Rep. Inst. Fysisk Oceanografi, Un. Copenhagen, 23. 41 p.p.

HØJERSLEV, N.K. 1974b. Daylight measurements for photosynthetic studies in the western Mediterranean. Rep.Inst. Fysisk Oceanogr., Un. Copenhagen, 26. 38 p.p.

JERLOV, N.G., 1968. Optical oceanography. Elsevier Publ. Comp. Amsterdam.194 p.p.

JERLOV (JOHNSON), N.G. and LILJEQUIST, G., 1938. On the angular distribution of submarine daylight and on the total submarine illumination. Svenska Hydrograf.Biol. Komm. Skrifter, Ny ser. Hydrog., 14.15 p.p.

JERLOV, N.G. and NYGÅRD, K., 1969. Influence of solar elevation on attenuation of underwater irradiance. Rep. Inst. Fysisk Oceanografi, Un. Copenhagen, 4: 6 p.p.

KIMBALL, H.H., 1924: Records of total solar radiation intensity and their relation to daylight intensity. Monthly Weather Review, vol. 52, No. 10: 473-479.

KONDRATYEV, K.YA., 1969: Radiation in the atmosphere. Academic Press, New York. 912 p.p.

KOSLYANINOV, M.V. and PELEVIN, V.N., 1966. On the application of a one-dimensional approximation in the investigation of the propagation of optical radiation in the sea. U.S. Dept. Comm., Joint Publ. Res.Ser., Rept., 36: 54-63.

LUNDGREN, B. and HØJERSLEV, N., 1971. Daylight measurements in the Sargasso Sea. Results from the "Dana" Expedition, January-April 1966. Rep. Inst. Fysisk Oceanografi, Univ. Copenhagen, 14. 33 p.p.

MOREL, A., 1974. Optical properties of pure water and pure sea water. In: Optical aspects of oceanography. Editors: N.G. JERLOV and E. STEEMANN-NIELSEN. Academic Press Inc., London. p.1-24.

ROBINSON, N., 1966. Solar radiation. Elsevier Publishing Company, Amsterdam. 347 p.p.

TYLER, J.E. and PREISENDORFER, R.W., 1962. Light. In: The Sea, Vol.1, ed. M.N. HILL, Interscience Publishers, London: 397-451.

TABLE 1

Depth m	D_d	D_u	R %	R_o %	R_o/R	K_u/K_d	a/K_d
				Clear sunny sky, $h = 57^\circ$			
5	1.25	2.70	2.10	4.55	2.16		
10	1.28	2.73	1.87	4.00	2.12	0.95	0.76
20	1.30	2.78	2.10	4.52	2.15	1.06	0.68
30	1.31	2.78	2.27	4.82	2.12	1.03	0.69
40	1.32	2.76	2.34	4.90	2.10	1.01	0.71
50	1.31	2.76	2.34	4.93	2.10	1.02	0.71
60	1.31	2.86	2.12	4.59	2.18		
Mean value	1.30	2.77	2.16	4.62	2.13	1.01	0.71
				Overcast sky, $h = 40^\circ$			
5	1.29	2.77	2.19	4.70	2.15		
10	1.30	2.79	2.30	4.93	2.14	1.05	0.74
20	1.33	2.81	2.52	5.34	2.11	1.04	0.69
30	1.33	2.77	2.65	5.52	2.08	1.03	0.63
40	1.34	2.79	2.76	5.74	2.08	1.00	0.67
50	1.34	2.87	2.67	5.72	2.14	0.96	0.72
60	1.34	2.97	2.50	5.60	2.23		
Mean value	1.32	2.82	2.51	5.36	2.13	1.02	0.69
Total mean	1.31	2.80	2.34	4.99	2.13	1.02	0.70
S_{rel} %	2	2	11	10	2	4	5

The table is inter- and extrapolated from TYLER's observations at 480 nm of D_d, D_u, R, K_d and a in Lake Pend Oreille.

R_o, R_o/R and K_u/K_d were calculated from

$$R_o/R = D_u/D_d$$

$$\frac{K_u}{K_d} = 1 + \frac{\ln(R(z_2)/R(z_1))}{(z_2-z_1)K_d}$$

TABLE 2

Y	ΣY^2 = min for	B=Baltic Sea M=Mediterranean N= Number of observations
$K-(a_R(\sec\Theta_D + (\sec j - \sec\Theta_D)r_o e^{-(K_S-K)z})$ where $K_S = (a + \alpha b)\sec j$	$\alpha = 0.72$ $\sec\Theta_D = 1.38$	B + M N = 39
as above, but $\sec\Theta_D = \beta - \gamma e^{-(K_S-K)z}$	$\alpha =$ 0.86 β 1.41 $\gamma =$ 0.21	B + M N = 39
$K - (a + \varepsilon b)D_d$	$\varepsilon = 0.0219$	B + M N = 24
$R(a + \varepsilon b) - 0.5\,\varepsilon b$	$\varepsilon = 0.0145$	B + M N = 24
$R - 0.5\,\varepsilon b/a$	$\varepsilon = 0.0166$	B + M N = 24
$((a + 0.86\,b)\sec j - K) - (\lambda a + \mu b \sec j)$	$\lambda = -0.010$ $\mu =$ 0.734	B + M N = 24
$(a + 0.86\,b)\sec j - \nu K$	$\nu = 2.12$	M N = 14

K, E_o, E_d, E_u and a are observed at 477 nm, b is observed at 655 nm (HØJERSLEV 1973, 1974a). By the assumption that scattering by particles is independent of the wavelength, b_{477} can be calculated from

$$b_{477} = b_{655} + (b_{w477} - b_{w655}) = b_{655} + 0.0026\ m^{-1}$$

b_w is the scattering coefficient of pure sea water (MOREL, 1974).
FROM eq. 2.4, 2.9 and 2.11 it may be deduced that

$$D_d = \frac{1}{1+3R}\,\frac{E_o}{E_d - E_u}$$

TABLE 3

	$a_R = 0.015$				$a_R = 0.15$			
h	K_o	K_{10}	K_{20}	$\bar{K}$	K_o	K_{10}	K_{20}	$\bar{K}$
0	.0180	.0186	.0190	.0185	.180	.207	.211	.204
5	.0180	.0186	.0190	.0185	.180	.207	.211	.204
10	.0186	.0189	.0193	.0190	.186	.208	.211	.205
15	.0191	.0193	.0195	.0193	.191	.208	.211	.207
20	.0193	.0195	.0196	.0195	.193	.208	.211	.207
25	.0192	.0194	.0195	.0194	.192	.207	.211	.207
30	.0190	.0191	.0193	.0192	.190	.207	.211	.207
35	.0186	.0188	.0190	.0188	.186	.206	.211	.205
40	.0182	.0184	.0187	.0185	.182	.205	.211	.205
45	.0178	.0180	.0183	.0181	.178	.204	.211	.204
50	.0174	.0177	.0180	.0177	.174	.204	.211	.203
55	.0170	.0174	.0177	.0174	.170	.204	.211	.202
60	.0166	.0170	.0174	.0170	.166	.203	.211	.201
65	.0163	.0168	.0172	.0168	.163	.202	.211	.200
70	.0161	.0165	.0171	.0166	.161	.202	.211	.200
75	.0159	.0164	.0169	.0164	.159	.201	.211	.200
80	.0158	.0162	.0168	.0163	.158	.201	.211	.199
85	.0156	.0162	.0167	.0162	.156	.201	.211	.199
90	.0156	.0162	.0167	.0162	.156	.201	.211	.199

$a_R = 1.5$				
h	K_o	K_{10}	K_{20}	$\bar{K}$
0	1.80	2.11	2.11	2.10
5	1.80	2.11	2.11	2.10
10	1.86	2.11	2.11	2.10
15	1.91	2.11	2.11	2.11
20	1.93	2.11	2.11	2.11
25	1.92	2.11	2.11	2.11
30	1.90	2.11	2.11	2.11
35	1.86	2.11	2.11	2.10
40	1.82	2.11	2.11	2.10
45	1.78	2.11	2.11	2.10
50	1.74	2.11	2.11	2.10
55	1.70	2.11	2.11	2.10
60	1.66	2.11	2.11	2.10
65	1.63	2.11	2.11	2.10
70	1.61	2.11	2.11	2.10
75	1.59	2.11	2.11	2.10
80	1.57	2.11	2.11	2.10
85	1.56	2.11	2.11	2.10
90	1.56	2.11	2.11	2.10

The vertical attenuation coefficients just beneath the surface and at 10 and 20 meters depth, K_0, K_{10} and K_{20}, were determined from eq.6.1-6.6 and 6.13 by means of a computer. Although K changes with depth, the same K was used on both sides of eq.6.1. The error introduced in this way was assumed to be of the same order as the other approximations or less.

To obtain an expression for the mean value of K we consider the more simple form

$$K = a_R \left(1.38 + (\sec j - 1.38) r_0 e^{-Kz} \right)$$

which may be written

$$K = A + B e^{-Kz} \quad . \qquad ((I)$$

The mean value in the range 0 - 20 m becomes

$$\overline{K} = \frac{1}{20}\int_0^{20} K\, dz \approx A + B\frac{(1 - e^{-20K_{20}})}{20\ K_{20}} \quad . \qquad (II)$$

Eq.I gives that

$$K_0 = A + B \qquad (III)$$

$$K_{20} \approx A + B e^{-20K_{20}} \quad . \qquad (IV)$$

Insertion from eq.III and IV in II gives

$$\overline{K} \approx \frac{K_{20} - K_0\ e^{-20K_{20}}}{1 - e^{-20K_{20}}} + \frac{K_0 - K_{20}}{20\ K_{20}} \quad .$$

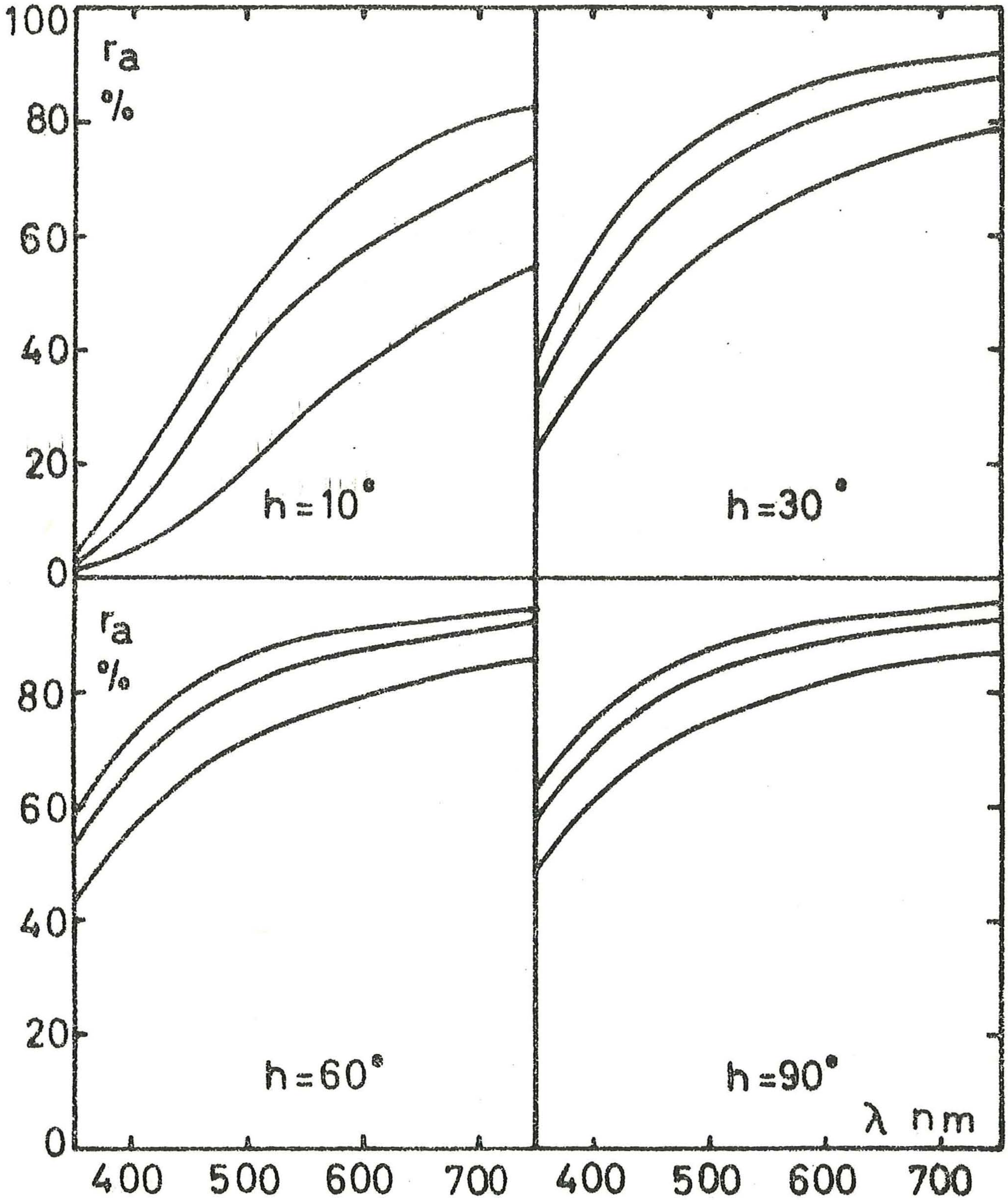

Fig.1. The function r_a for three different water vapour contents. (Data from HINZPETER 1957).

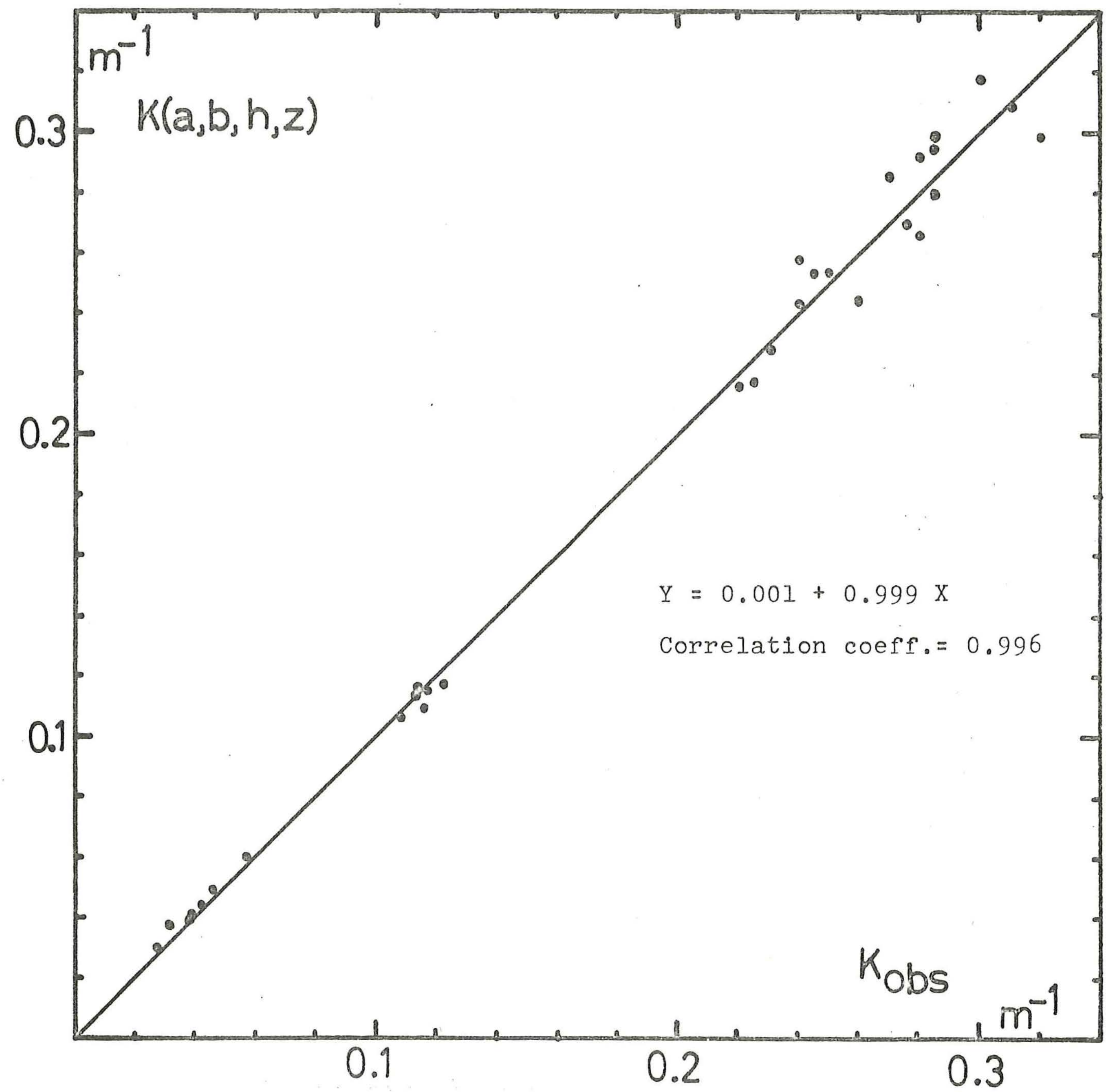

Fig.2. Calculated and observed vertical attenuation coefficients.