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OF CLEAR OCEAN WATER

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**ABSTRACT**

The absorption coefficient of JERLOVs Ocean water type I is calculated for different wavelengths by means of the vertical attenuation coefficient of downward irradiance and the irradiance ratio. The values are partly larger and partly smaller than those given by other authors for pure water. It is concluded that the absorption coefficient of pure water is not yet known with sufficient accuracy.

It is demonstrated that the average cosine of upward travelling photons can be estimated with good accuracy from observations of maximum and minimum horizontal radiance and radiance from nadir.

## 1. INTRODUCTION

Although optically pure freshwater or pure sea water is not found anywhere in nature, the optical properties of pure water are still very important and should be known, since they constitute the limits for the variation of clear natural water. References to earlier investigations of optical properties of pure water are given by e.g. MOREL (1974), TAM and PATEL (1979), and SMITH and BAKER (1981), and shall not be repeated here. While the volume scattering function  $\beta_w$  and the total scattering coefficient  $b_w$  for pure water given by MOREL (1974) seem to be generally accepted, it is my impression that established values for the attenuation coefficient  $c_w$  and the absorption coefficient  $a_w$  of pure water are still lacking.

The purpose of the present paper is just to show how the absorption coefficient  $a$  for very clear ocean water can be estimated from observations of the vertical attenuation coefficient of downward irradiance,  $K_d$ , and the irradiance ratio,  $R$ , and how these results partly confirm and partly disagree with other measurements.

## 2. SOME RECENT VALUES OF THE ABSORPTION COEFFICIENT

The classical method to determine the absorption coefficient of pure water,  $a_w$ , has been to measure the attenuation coefficient of pure water,  $c_w$ , by standard spectrophotometry and then subtract the scattering coefficient of pure water,  $b_w$ , since

$$a_w = c_w - b_w \quad (1)$$

The accuracy of  $a_w$  depends then on the accuracy of  $c_w$  and  $b_w$ .

The spectrophotometric measurements of  $c_w$  by CLARKE and JAMES from 1939 have been among the most quoted, and are included here for comparison. Table 1 presents values of  $a_w$  that are based on their values of  $c_w$  (as selected by JERLOV, 1976), and corresponding values of  $b_w$  for freshwater, as given by MOREL (1974).  $\lambda$  is the wavelength of light in air. MOREL and PRIEUR (1977) have given the most recent measurements of  $c_w$  by this method, and their calculated values of  $a_w$  are given in Table 1.

QUERRY et al. (1978) applied a split-pulse laser method to

avoid some of the problems of standard spectrophotometry. Their values of  $a_w$ , calculated from measured  $c_w$ , are given in Table 1.

A perhaps more "direct" way to calculate the absorption coefficient, is to measure the quantities of the Gershun equation, which for a horizontally stratified medium yields

$$a = - \frac{1}{E_0} \frac{d}{dz} (E_d - E_u) \quad (2)$$

Here  $a$  is the absorption coefficient,  $z$  is the vertical coordinate, positive downwards, and  $E_d$ ,  $E_u$  and  $E_0$  are the downward, upward and scalar irradiances, respectively. The irradiances can either be measured directly or obtained from integration of the radiance distribution. Absorption coefficients calculated by this method have been presented for Sargasso Sea water by LUNDGREN and HØJERSLEV (1971), and for Western Mediterranean water by HØJERSLEV (1973). Some of their results for Sargasso Sea water are given in Table 1.

A quite different technique has been employed by HASS and DAVISSON (1977). Absorbed light will result in a temperature rise of the water, and by calorimetry the absorption coefficient can be found. Their results are presented in Table 1.

A pulsed laser beam in water will produce a sound signal, and this optoacoustic effect makes it possible to calculate  $a_w$ . The results obtained in this way by TAM and PATEL (1979) are presented in Table 1. The authors estimated the uncertainty of their  $a_w$  values to be about 10%.

Most recently SMITH and BAKER (1981) have combined their own measurements of the vertical attenuation coefficient for downward irradiance,  $K_d$ , with the results of other authors, to form a new set of data for  $a_w$ . They apply the inequality

$$a_w \leq K_d - \frac{1}{2} b_w \quad (3)$$

to obtain an upper limit for  $a_w$  (Table 1).

However, a more precise relation than eq. 3 can be obtained from the Gershun equation, and this will be demonstrated in the next section.

### 3. THE ESTIMATION OF THE ABSORPTION COEFFICIENT FROM THE VERTICAL ATTENUATION COEFFICIENT AND THE IRRADIANCE RATIO

The vertical attenuation coefficient of downward irradiance,  $K_d$ , is defined as

$$K_d = - \frac{1}{E_d} \frac{dE_d}{dz} \quad (4)$$

Similarly the attenuation coefficient of upward irradiance,  $K_u$ , is defined as

$$K_u = - \frac{1}{E_u} \frac{dE_u}{dz} \quad (5)$$

The irradiance ratio  $R$  is given by

$$R = \frac{E_u}{E_d} \quad (6)$$

The scalar irradiance  $E_o$  may be separated into the downward scalar irradiance  $E_{od}$  and the upward scalar irradiance  $E_{ou}$ , so that

$$E_o = E_{od} + E_{ou} \quad (7)$$

The average cosine of downward travelling photons,  $\bar{\mu}_d$ , is defined as

$$\bar{\mu}_d = \frac{E_d}{E_{od}} \quad (8)$$

while the average cosine of upward travelling photons,  $\bar{\mu}_u$ , is given by

$$\bar{\mu}_u = \frac{E_u}{E_{ou}} \quad (9)$$

Now we may introduce  $K_d$  and  $K_u$  into eq. 2 by means of eqs. 4-5, and eliminate  $E_d$ ,  $E_u$  and  $E_o$  by means of eqs. 6-9. The result may be written

$$a = K_d \bar{\mu}_d \frac{1 - R K_u / K_d}{1 + R \bar{\mu}_d / \bar{\mu}_u} \quad (10)$$

Since JERLOV (1976) has given values of  $K_d$  and  $R$  for very clear ocean water, it may be interesting to see the values of  $a$  that can be obtained with eq. 10. In order to use the equation, however, we



must first estimate the values of  $K_u/K_d$  and  $\bar{\mu}_d/\bar{\mu}_u$ .

Table 2 shows values of the ratio  $K_u/K_d$  from the surface layer of the Sargasso Sea and the Western Mediterranean (LUNDGREN and HØJERSLEV, 1971, HØJERSLEV, 1973). The ratio seems to obtain its highest values in the blue part of the spectrum and to decrease towards longer wavelengths. However, the standard deviation of the ratio is 0.18, and the mean value is 0.98, and as a first approximation we shall use  $K_u/K_d \approx 1$ . With reference to eq.10 we see that the accuracy of the ratio is not important, since it is multiplied with  $R$  which does not exceed 6%. Even an error in  $K_u/K_d$  of 30% will not result in an error in  $a$  greater than 2%, and this is about the same uncertainty that  $K_d$  contributes.

The average cosine of the downward travelling photons,  $\bar{\mu}_d$ , probably is close to  $\cos \theta_s$ , where  $\theta_s$  is the zenith distance of the sun in water. According to JERLOV (1976, p.102)  $\bar{\mu}_d$  varies between 0.95 and 0.60, and we have chosen the value 0.95 for  $\bar{\mu}_d$  at all wavelengths here. This choice is most important, since an error in  $\bar{\mu}_d$  will produce the same relative error in  $a$ .

The average cosine of the upward travelling photons,  $\bar{\mu}_u$ , have been estimated from radiance observations in the surface layer of the Sargasso Sea (LUNDGREN and HØJERSLEV, 1971). The results that are presented in Table 3 indicates that  $\bar{\mu}_u$  may decrease somewhat from the UV part of the spectrum towards longer wavelengths. (It should be noted, however, that the values at 625 nm are higher than at the preceding wavelengths). The calculations that lead to the estimates are given in the Appendix. The standard deviation of the values of  $\bar{\mu}_u$  in Table 3 is 0.04, and the mean value is 0.43. For the ratio  $\bar{\mu}_d/\bar{\mu}_u$  we have then, as a first approximation, applied the value 2. It should be pointed out that the choice of this value is not crucial for the value of  $a$ , since the ratio  $\bar{\mu}_d/\bar{\mu}_u$  is multiplied with  $R$  in eq. 10.

The values of  $K_d$  for Ocean water type I are taken from table 27 in JERLOV's "Marine Optics" (1976). Values of  $R$  are taken from his table 33, and from his fig. 40 in the original presentation of the data (JERLOV, 1951). The data are given in Table 4. Eq. 10 has then been used with  $\bar{\mu}_d \approx 0.95$ ,  $K_u/K_d \approx 1$  and  $\bar{\mu}_d/\bar{\mu}_u \approx 2$  in order to obtain the values of  $a$  presented in Table 1 and 4. The equation now has the simple form

$$a = 0.95 \frac{1 - R}{1 + 2R} K_d \quad (11)$$

$K_d$  and  $R$  are not independent optical quantities, but are linked together mainly by the absorption coefficient  $a$ . For high solar altitudes and clear ocean water it has been shown (AAS, 1987) that

$$R \approx \frac{1}{1 + (\bar{\mu}_d/\bar{\mu}_u)} \frac{b_b}{a}, \quad (12)$$

which for our value of  $\bar{\mu}_d/\bar{\mu}_u$  becomes

$$R \approx \frac{1}{3} \frac{b_b}{a} \quad (13)$$

Elimination of  $a$  between eqs. 11 and 13 gives

$$b_b \approx 2.85 \frac{R(1-R)}{1+2R} K_d \quad (14)$$

The calculated values of  $b_b$  are presented in Table 4. From these values are subtracted the backscattering coefficient of pure sea water,  $b_{wb}^s$  (MOREL, 1974), and the difference is the backscattering coefficient  $b_{pb}$  for particles. As it is seen in Table 4, the irregular variation of  $b_{pb}$  can be explained as due to the uncertainty of  $K_d$  and  $R$ , which are given with only two figures. The values of  $K_d$  and  $R$  given by JERLOV thus form a consistent data set. The mean value of  $b_{pb}$  in Table 4 is  $0.2 \cdot 10^{-3} \text{ m}^{-1}$ .

#### 4. CONCLUSION

Table 1 illustrates that the variation between the different series of the absorption coefficient at all wavelengths is larger than 10%. Although it may be tempting to think that a measurement with a smaller value has purer water than a measurement with a higher value, so that the smaller value is closer to the true value than the higher one, the explanation may of course also be that the smaller value is due to an error of method. Until different methods of equal accuracy are able to produce similar results, I think the conclusion must be that we do not know the absorption coefficient of pure water with the necessary accuracy. An acceptable uncertainty may be less than 10%, preferably less than 5%.

TABLE 1. THE ABSORPTION COEFFICIENT OF PURE WATER  
AND CLEAR OCEAN WATER

$\lambda$ (nm)	$b_w^f$ ( $10^{-3} \text{ m}^{-1}$ )	$a_w$ ( $10^{-3} \text{ m}^{-1}$ )					$a$ ( $10^{-3} \text{ m}^{-1}$ )		
		M	C&J	H&D	M&P	Q&A	T&P	L&H	S&B
350	10.35	-	-	-	-	-	-	46	51
375	7.68	37	-	-	-	-	23	26	31
400	5.81	37	-	18	-	-	-	17	23
425	4.47	29	-	16	44	-	24	15	18
450	3.49	16	-	15	39	23	-	15	16
475	2.76	15	-	17	36	18	16	17	15
488	-	-	17	20	37	19	-	19	-
500	2.22	34	-	26	45	23	-	26	24
525	1.79	39	-	50	63	39	32	49	39
542	-	-	29	58	70	49	-	57	-
550	1.49	68	-	64	76	57	-	64	59
575	1.25	90	-	94	98	82	-	94	85
600	1.09	185	-	245	240	205	-	244	223
625	-	228	-	315	308	296	250	314	290
633	-	-	180	323	-	304	-	322	-
650	-	288	-	350	-	324	-	349	340
675	-	367	-	440	-	407	-	440	400
700	-	500	-	650	-	590	-	650	530

Some of the numbers above are rounded off and interpolated to simplify comparison between the series.

#### References

- M: MOREL, 1974. T&P: TAM and PATEL, 1979.  
 C&J: CLARKE and JAMES, 1939. L&H: LUNDGREN and HØJERSLEV, 1971.  
 H&D: HASS and DAVISSON, 1977. S&B: SMITH and BAKER, 1981.  
 M&P: MOREL and PRIEUR, 1977. J: Based on data from JERLOV, 1976.  
 Q&A: QUERRY et al., 1978.



TABLE 2. THE RATIO  $K_U / K_D$ 

Sargasso Sea					Mediterranean. St.A				
$h_s$ (deg)	Depth (m)	$\lambda$ (nm)	$K_d$ ( $10^{-3} m^{-1}$ )	$K_u/K_d$	$h_s$ (deg)	Depth (m)	$\lambda$ (nm)	$K_d$ ( $10^{-3} m^{-1}$ )	$K_u/K_d$
56	0-25	375	44.7	1.13	68	5-15	372	173	0.71
"	"	430	22.7	1.34	74	5-25	427	82.4	1.12
"	"	484	27.4	1.07	68	"	477	114	0.91
"	"	514	52.7	0.85	70	"	532	73.3	0.86
"	0-10	574	152	0.68	71	"	572	122	0.8
Mediterranean. St.J1A					Mediterranean. St.J2				
$h_s$ (deg)	Depth (m)	$\lambda$ (nm)	$K_d$ ( $10^{-3} m^{-1}$ )	$K_u/K_d$	$h_s$ (deg)	Depth (m)	$\lambda$ (nm)	$K_d$ ( $10^{-3} m^{-1}$ )	$K_u/K_d$
62	0.5-25	372	84.9	1.02	72	0.5-15	372	62.6	1.12
74	"	427	44.3	1.01	64	"	427	34.2	1.23
61	"	477	35.9	1.06	60	"	477	26.3	1.17
53	10-25	532	57.7	1.06	73	"	532	58.3	0.84
74	"	572	94.0	0.78	68	"	572	90.5	0.73

Sargasso Sea data are from LUNDGREN and HØJERSLEV, 1971.

Mediterranean data are from HØJERSLEV, 1973.

TABLE 3. THE AVERAGE COSINE OF UPWARD IRRADIANCE IN THE SARGASSO SEA

$\lambda$ (nm)	Depth (m)	$h_s$ (deg)	$\theta_s$ (deg)	$\rho$	$\bar{\mu}_u$	$1/\bar{\mu}_u$
375	2	58	23	0.91	0.51	1.97
"	10	"	"	1.12	49	2.04
"	25	"	"	1.42	47	12
425	2	56	25	1.50	0.47	2.15
"	10	"	"	1.68	46	19
"	25	"	"	1.96	44	25
475	2	58	23	3.43	0.40	2.50
"	10	"	"	3.50	40	51
"	25	"	"	4.70	38	66
525	2	48	30	3.30	0.40	2.48
"	10	"	"	3.08	41	45
"	25	"	"	3.05	41	44
575	2	58	23	3.85	0.39	2.56
"	10	"	"	4.05	39	58
"	25	"	"	4.38	38	62
625	2	56	25	1.88	0.45	2.23
"	10	"	"	2.14	44	29
"	20	"	"	2.52	43	34

The Table is based on radiance observations by LUNDGREN and HØJERSLEV (1971).  
 $\rho$  is the radiance ratio  $\bar{L}(90^\circ)/L(180^\circ)$ .

TABLE 4. THE ABSORPTION COEFFICIENT OF OCEAN WATER TYPE I

$\lambda$ (nm)	$K_d$ ( $10^{-3} \text{ m}^{-1}$ )	R $10^{-3}$	a ( $10^{-3} \text{ m}^{-1}$ )	$b_b$ ( $10^{-3} \text{ m}^{-1}$ )	$b_{wb}^s$ ( $10^{-3} \text{ m}^{-1}$ )	$b_{pb}$ ( $10^{-3} \text{ m}^{-1}$ )
350	62	47	51	7.2	6.7	0.5
375	38	55	31	5.1	5.0	0.1
400	28	58	23	3.9	3.8	0.1
425	22	58	18	3.1	2.9	0.2
450	19	52	16	2.4	2.3	0.1
475	18	41	15	1.9	1.8	0.1
500	27	26	24	1.9	1.5	0.4
525	43	13	39	1.5	1.2	0.3
550	63	7	59	1.2	1.0	0.2
575	89	-	85	-	-	-
600	235	-	223	-	-	-
625	305	-	290	-	-	-
650	360	-	340	-	-	-
675	420	-	400	-	-	-
700	560	-	530	-	-	-

The values for  $K_d$  and R are from JERLOV (1976, 1951), and the values for  $b_{wb}^s$  are from MOREL (1974).



TABLE 5. COMPARISON OF THE OBSERVED AND ESTIMATED AVERAGE COSINE OF UPWARD IRRADIANCE IN LAKE PEND OREILLE, IDAHO.

Clear sunny sky, $h_s=57^\circ$				Overcast sky, $h_s=40^\circ$			
Depth (m)	$\rho$	$\bar{\mu}_u$		Depth (m)	$\rho$	$\bar{\mu}_u$	
		Obs.	Est.			Obs.	Est.
4	4.65	0.370	0.377	6	5.59	0.361	0.363
10	4.99	367	372	-	-	-	-
17	5.38	360	366	18	6.11	355	357
29	5.43	360	365	31	5.42	361	366
41	5.32	363	367	43	5.62	358	363
54	5.41	362	366	55	7.74	340	340
66	7.72	339	340	-	-	-	-

The data are from TYLER and PREISENDORFER, 1962.  $\lambda = 480$  nm.  $\rho$  is the radiance ratio  $\bar{L}(90^\circ)/L(180^\circ)$ .

## APPENDIX

## 5. THE AVERAGE AZIMUTHAL RADIANCE

LUNDGREN and HØJERSLEV (1971) found that a fair approximation of the radiance  $L$  versus azimuth angle  $\varphi$  would be

$$L(\varphi) = A \frac{\varepsilon + 1}{\varepsilon - \cos\varphi} \quad (15)$$

where  $A$  and  $\varepsilon$  are constants. These constants will be determined by the maximum radiance  $L(0)$  and the minimum radiance  $L(\pi)$ :

$$A = L(0) \quad (16)$$

$$\varepsilon = (L(0) + L(\pi)) / (L(0) - L(\pi)) \quad (17)$$

LUNDGREN and HØJERSLEV applied this relation for the deepest measurements of each colour in the Sargosso Sea, and not in the surface layer where the influence of the direct sun rays would cause to large deviations from the approximation.

The average azimuthal radiance  $\bar{L}$  is defined by

$$\bar{L} = \frac{1}{\pi} \int_0^{\pi} L(\varphi) d\varphi = \frac{1}{\pi} \int_0^{\pi} A \frac{\varepsilon + 1}{\varepsilon - \cos\varphi} d\varphi \quad (18)$$

By the substitution

$$t = \left( \frac{\varepsilon + 1}{\varepsilon - 1} \right)^{1/2} \tan \frac{\varphi}{2} \quad (19)$$

which gives

$$\cos \varphi = \frac{1 - \frac{\varepsilon - 1}{\varepsilon + 1} t^2}{1 + \frac{\varepsilon - 1}{\varepsilon + 1} t^2} \quad (20)$$

$$d\varphi = \frac{2 \left( \frac{\varepsilon - 1}{\varepsilon + 1} \right)^{1/2} dt}{1 + \left( \frac{\varepsilon - 1}{\varepsilon + 1} \right) t^2} \quad (21)$$

eq. 18 becomes

$$\bar{L} = A \frac{2}{\pi} \left( \frac{\varepsilon + 1}{\varepsilon - 1} \right)^{1/2} \int_0^{\infty} \frac{dt}{1 + t^2} = A \frac{2}{\pi} \left( \frac{\varepsilon + 1}{\varepsilon - 1} \right)^{1/2} \left[ \arctan t \right]_0^{\infty}$$

$$= A \left( \frac{\epsilon+1}{\epsilon-1} \right)^{1/2} = (L(0) L(\pi))^{1/2} \quad (22)$$

Thus, if the radiance distribution follows eq. 15, the average azimuthal radiance  $\bar{L}$  becomes the geometric mean of  $L(0)$  and  $L(\pi)$ .

LUNDGREN and HØJERSLEV applied this result for all zenith distances at their deepest measurements. In the next section we shall extend the form of approximation given by eq. 15 to the variation of the radiance with zenith distance within the surface layer, but only for the upward radiance, that is for zenith distances between  $\pi/2$  and  $\pi$ . It will be demonstrated that the three values of maximum and minimum horizontal radiance and radiance from nadir, seem to be sufficient to estimate the average cosine  $\bar{\mu}_u$ .

## 6. THE AVERAGE COSINE OF UPWARD TRAVELLING PHOTONS

We shall now assume that the average azimuthal radiance  $\bar{L}(\theta)$  from the zenith distance  $\theta$  varies approximately as a function of the form

$$\bar{L}(\theta) = B \frac{1+\alpha}{1-\alpha \cos \theta} \quad , \quad (23)$$

when  $\theta$  varies between  $\pi/2$  and  $\pi$ . The constants  $B$  and  $\alpha$  are determined by the radiance from nadir  $\bar{L}(\pi)$  and the average horizontal radiance  $\bar{L}(\pi/2)$ :

$$B = \bar{L}(\pi) \quad , \quad (24)$$

$$\alpha = \frac{\bar{L}(\pi/2)}{\bar{L}(\pi)} - 1 \quad , \quad (25)$$

The upward scalar irradiance  $E_{ou}$  becomes, with the radiance distribution (23),

$$E_{ou} = 2\pi \int_{\pi/2}^{\pi} \bar{L}(\theta) \sin \theta \, d\theta = 2\pi B(1+\alpha) \int_{\pi/2}^{\pi} \frac{\sin \theta \, d\theta}{1-\alpha \cos \theta} \quad (26)$$

By the substitution

$$t = \left( \tan \frac{\theta}{2} \right)^2 \quad , \quad (27)$$

we have that



$$\sin \theta \, d\theta = 2 \, dt / (1+t)^2 \quad , \quad (28)$$

$$\cos \theta = (1-t)/(1+t) \quad . \quad (29)$$

We will also write

$$\gamma = (1-\alpha)/(1+\alpha). \quad (30)$$

Eq.26 can now be written

$$\begin{aligned} E_{ou} &= 4\pi B \int_1^{\infty} \frac{dt}{(1+t)(\gamma+t)} = \frac{4\pi B}{\gamma-1} \int_1^{\infty} \left( \frac{dt}{1+t} - \frac{dt}{\gamma+t} \right) \\ &= \frac{4\pi B}{\gamma-1} \left[ \ln \frac{1+t}{\gamma+t} \right] = \frac{4\pi B}{\gamma-1} \ln \frac{\gamma+1}{2} \\ &= 2\pi B \frac{1+\alpha}{\alpha} \ln (1+\alpha) \end{aligned} \quad (31)$$

The upward irradiance  $E_u$  is defined as

$$E_u = 2\pi \int_{\pi/2}^{\pi} \bar{L}(\theta) (-\cos \theta) \sin \theta \, d\theta \quad (32)$$

With the radiance distribution (23) and the substitutions (27)-(30) the upward irradiance becomes

$$\begin{aligned} E_u &= -4\pi B \int_1^{\infty} \frac{dt - t \, dt}{(\gamma+t)(1+t)^2} \\ &= \frac{4\pi B}{(\gamma-1)^2} \int_1^{\infty} \left[ \frac{-2(\gamma-1)}{(1+t)^2} + \frac{\gamma+1}{1+t} - \frac{\gamma+1}{\gamma+t} \right] dt \\ &= \frac{4\pi B}{(\gamma-1)^2} \left[ \frac{2(\gamma-1)}{1+t} + (\gamma+1) \ln \frac{1+t}{\gamma+t} \right] \\ &= \frac{4\pi B}{(\gamma-1)^2} \left[ (1-\gamma) + (\gamma+1) \ln \frac{\gamma+1}{2} \right] \\ &= 2\pi B \frac{1+\alpha}{\alpha} \left[ 1 - \frac{1}{\alpha} \ln (1+\alpha) \right] \end{aligned} \quad (33)$$

The average cosine of upward travelling photons,  $\bar{\mu}_u$ , is the ratio between the expressions (33) and (31):

$$\bar{\mu}_u = \frac{E_u}{E_{ou}} = \frac{1 - \frac{1}{\alpha} \ln(1+\alpha)}{\ln(1+\alpha)} \quad (34)$$

Rather than to use  $\alpha$ , it may be more convenient to operate with

$$\rho = \alpha + 1 = \bar{L}(\pi/2) / \bar{L}(\pi) \quad (35)$$

Eq. 34 then obtains the form

$$\bar{\mu}_u = \left[ 1 - \frac{\ln \rho}{\rho - 1} \right] / \ln \rho \quad (36)$$

It can be shown that if the radiance becomes isotropic, that is  $\rho \rightarrow 1$ , then  $\bar{\mu}_u \rightarrow 1/2$  as it should.

The very simple relation above can be tested against the radiance observations in Lake Pend Oreille, presented by TYLER (1960). From the radiance distribution the average cosine  $\bar{\mu}_u$  has been obtained by integration (TYLER and PREISENDORFER, 1962,  $D_+ = 1/\bar{\mu}_u$  is given in their table), and can be compared with the estimates obtained from eq. 36. The values are shown in Table 5.

The surprising result is that the deviation between the observed and the estimated values is nowhere larger than 2%. I have then concluded that the relation (36) can also be applied with fairly good accuracy on the corresponding radiance observations from the Sargasso Sea, given by LUNDGREN and HØJERSLEV. The results have already been presented in Table 3.

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