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PROPOSITIONAL CONTINGENTISM

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Abstract. According to propositional contingentism, it is contingent what propositions there are. This paper presents two ways of modeling contingency in what propositions there are using two classes of possible worlds models. The two classes of models are shown to be equivalent as models of contingency in what propositions there are, although they differ as to which other aspects of reality they represent. These constructions are based on recent work by Robert Stalnaker; the aim of this paper is to explain, expand, and, in one aspect, correct Stalnaker's discussion.

§1. Introduction. Propositional contingentism is the view that it is contingent what propositions there are. Many of those who have held this view have been motivated by an argument roughly along the following lines:

I could have failed to be.
Had I not been, there would not have been the proposition that I am me.
Therefore, the proposition that I am me could have failed to be.

Early instances of such arguments can be found in Prior's writings, e.g., in (Prior, 1967, pp. 150–151), where he gives such an argument for contingency in what facts there are. Later examples of at least tentative endorsements of such arguments can be found in Fine (1977b), Adams (1981), Fitch (1996), Bennett (2005), David (2009), Speaks (2012), Stalnaker (2012), and Nelson (2014). Relatedly, Williamson (2002, 2013, chapter 6) endorses the second premise of the argument on the assumption of the truth of the first, which he rejects. An exception in the literature is Lindström (2009), who argues for propositional contingentism on the basis of a puzzle about possible world semantics due to Kaplan (1995). Some, like Williamson, deny the first premise, but few have explicitly denied the second premise; examples are Plantinga (1983) and Bealer (1993, 1998).

This indicates that propositional contingentism is widely regarded as an interesting and plausible view. Yet, while some aspects of propositional contingentism, such as its implications for semantics, have been discussed at length, there have been surprisingly few investigations into the seemingly more basic issue of developing a systematic theory of what propositions there are and what propositions there could have been. One exception is Fine (1980), who develops such a theory on the assumption that propositions are individuated relatively finely. As far as I am aware, there are only two such investigations which assume a more coarse-grained theory of propositions according to which propositions are identical if they are strictly equivalent (i.e., according to which p is q if necessarily, p if and only if q). These are Fine (1977b) and Stalnaker (2012, Appendix A).

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Both Fine and Stalnaker proceed model-theoretically, constructing classes of possible worlds models in which propositions are identified with sets of possible worlds. (Or rather, these are models in which sets of representatives of worlds represent propositions. As usual in possible worlds model theory, the entities representing worlds and propositions will be spoken of as if they were in fact worlds and propositions, although this is of course not required.) In principle, such a model theory is straightforward to define, following the variable domain possible worlds model theory of Kripke (1963), by associating each world with a domain of propositions. The model theory might therefore simply be the class of tuples $\langle W, D \rangle$ such that W is a set and $D : W \to \mathcal{P}(\mathcal{P}(W))$. However, this model theory does not limit contingency in what propositions there are in any interesting way. For instance, it does not enforce the natural constraint that necessarily, the propositions there are are closed under negation.

Both the model theories of Fine and Stalnaker are more informative, and it turns out that they are closely related in both philosophical and formal respects. However, neither of them is easily accessible, although for very different reasons. Fine's model theory not only represents propositions but also individuals and relations in a complex hierarchy of intensional and extensional relations, and the development is dense and technical. Stalnaker's model theory is only sketched in a very short appendix, and the formal definitions are not related in any detail to the preceding philosophical discussion. Furthermore, Stalnaker gives two variants of his model theory which he claims to be equivalent; however, as will be shown below, one of his definitions must be corrected to establish the equivalence.

The aim of this paper is to provide an accessible but rigorous development of model theories for propositional contingentism along the lines of Stalnaker and Fine. In the interest of clarity, they are introduced on their own terms, with minimal references to the literature. An appendix states Stalnaker's original definitions, shows how they differ from the ones proposed here, and argues for the latter. The models developed here are related to Fine's models in Fritz (unpublished a), based on the work in Fritz & Goodman (forthcoming).

The remainder of this paper is structured as follows: In Section 2, a possible worlds model theory is developed whose models, called *equivalence systems*, associate with every world an equivalence relation of indistinguishability between worlds; they are interpreted as models of contingency in what propositions there are by taking the propositions at a world w to be the sets of worlds which contain either both or neither of two worlds indistinguishable at w. In Section 3, a second possible worlds model theory is developed whose models, called *permutation systems*, associate with every world a set of permutations representing the symmetries of modal space from the perspective of this world; they are interpreted as models of contingency in what propositions there are by mapping them to equivalence systems, associating each world w with the equivalence relation which holds between two worlds if one is mapped to the other by some symmetry of w.

In Sections 2 and 3, a restriction of coherence is imposed on each class of models. In Section 4, it is proven that an equivalence system is coherent if and only if it is determined by a coherent permutation system. This shows that the two classes of coherent models represent the same patterns of contingency in what propositions there are. Section 5 shows that the two kinds of models nevertheless differ in what they represent, as different coherent permutation systems may determine the same coherent equivalence system. That this is in line with our philosophical interpretation of the systems is shown using a simple example. Section 6 delves deeper into the structural relations between the two kinds of systems, investigating both the structures formed by the two classes of coherent systems under natural orders and some relations between these two structures.

This paper is part of a larger body of work by Jeremy Goodman and myself; connections to related papers are discussed in the concluding Section 7. Appendix A discusses Stalnaker's models, and shows that the present definition of coherent permutation systems matches Stalnaker's corresponding definition, whereas the present definition of coherent equivalence systems is more restrictive than Stalnaker's corresponding definition. Using a simple example, it is shown that Stalnaker's philosophical considerations support the present definition rather than his own. Since much of the following is formulated in terms of possible worlds, Appendix B considers how such talk may be understood. It is argued that the version of propositional contingentism discussed here is incompatible with taking talk of worlds at face value. A well-known strategy for understanding such talk in terms of propositions is adapted to fit propositional contingentism, but it is noted that the strategy is limited in generality. Whether this lack of generality is a serious problem for the theory is left open.

§2. Equivalence Systems. Consider again the second premise of the above argument for propositional contingentism: Why should there not have been the proposition that I am me, had I not been? An answer which motivates both classes of models to be explored is that without me, there would not have been the resources required to draw the distinction drawn by the proposition that I am me. This idea is best elaborated using a simpler, albeit more artificial, example: Consider the possibility of there being two fundamental particles a and b which actually are nothing. Assume that for both a and b, there is a world in which this particle makes up the only matter in an otherwise completely homogenous space-time continuum. Let w_a and w_b be such a pair of worlds. Had there been a and b, then w_a and w_b could be distinguished in terms of a and b, but since actually there are neither a nor b and w_a and w_b differ only in which individual they contain, w_a and w_b can actually not be distinguished. Thus in particular, they cannot be distinguished by any proposition, so all propositions are either true in both or neither of w_a and w_b. Of course, if there had been a and b, w_a and w_b could be distinguished, and so there would be propositions true in only one of them.

This line of thought motivates the idea that what propositions there are at a given world depends on which distinctions among worlds can be drawn at it. Both classes of models to be explored take up this idea and model what distinctions among worlds can be drawn at a given world, from which what propositions there at that world is derived. (Note that the metaphors of distinguishing worlds in terms of certain individuals and drawing distinctions at a certain world should not be read epistemically, or as straightforwardly relating to the abilities of agents. Rather, they should be understood metaphysically, and so as directly describing features of reality.) Both classes of models identify propositions with sets of worlds, taking such a set to be true at a world if it contains the world, and to be necessary if it is the set of all worlds.

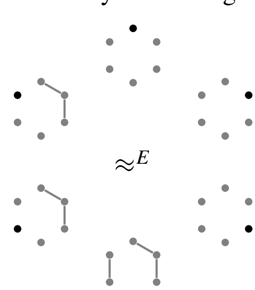
The first class of models represents what distinctions among worlds can be drawn at a given world in the most straightforward manner: such a model associates with each world w a relation $\approx_w$, which relates two worlds if and only if they cannot be distinguished at w. Since the relevant notion of indistinguishability is plausibly reflexive, symmetric and transitive, $\approx_w$ will be assumed to be an equivalence relation. This determines what propositions there are as follows: At w, there are those propositions P such that for all worlds v and u related by $\approx_w$, P is true in v if and only if P is true in u. Equivalently, the propositions at w are the unions of sets of equivalence classes of $\approx_w$; this is the unique complete atomic field of sets whose atoms are the equivalence classes of $\approx_w$. Formally, define:

DEFINITION 2.1. *For every set W, an* equivalence system on W *is a function $\approx$ mapping every $w \in W$ to an equivalence relation $\approx_w$ on W.*

As an example of an equivalence system, consider the function $\approx^E$ on $\{1, \ldots, 6\}$ which maps 1, 2 and 3 to the identity relation, which will be called id (letting the context determine its domain); which maps 4 to the equivalence relation on $\{1, \ldots, 6\}$ which relates two elements just in case they are both strictly less than 4, both identical to 4 or both strictly greater than 4; and which maps 5 and 6 to the equivalence relation on $\{1, \ldots, 6\}$ which relates two elements just in case they are identical or both strictly less than 4.

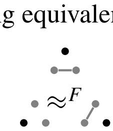
It will be helpful to represent such systems pictorially. Here is a natural way of drawing any equivalence system $\approx$ based on a set of worlds $\{1, \ldots, n\}$ for some natural number n: Draw representations of the worlds in a circle, starting with 1 at the top and turning clockwise. In this circle, each world i is represented by a smaller circle of dots, each of which represents a world; again, start with 1 at the top and turn clockwise. In this smaller circle representing i, indicate which worlds are related by $\approx_i$ by drawing a line connecting dots which represent worlds related by $\approx_i$. There is no need to indicate a direction since $\approx_i$ is symmetric; dots don't have to be connected to themselves as $\approx_i$ is reflexive; and two dots need not be connected if they are already connected by a path since $\approx_i$ is transitive.

As an example, the following is one way of drawing $\approx^E$:



Here, for any $i \leq 6$, the dot representing i in the circle representing i has been distinguished by drawing it black instead of gray; this is to make it easier to see which dots correspond to which circles. The center of the big circle is used to label the system.

Not all equivalence systems are plausible models of which worlds can be distinguished at a given world. Consider the following equivalence system:



According to this system, 2 and 3 are indistinguishable at 1. Yet, 2 and 3 differ structurally in what can be distinguished at them: At 2, the other two worlds *cannot* be distinguished, although at 3 the other two worlds *can* be distinguished. Thus 2 and 3 can be distinguished purely in terms of what can be distinguished at them, and therefore 2 and 3 can't be indistinguishable at 1.

Here is another version of the same argument: According to $\approx^F$, there are four propositions in 1 and 2, and eight propositions in 3. So the proposition that there are exactly four propositions is the set $\{1, 2\}$. Delineating the worlds in which there are exactly four propositions seems not to depend on any special resources, so in every world, there should be the proposition that there are exactly four propositions. In particular, there should be this proposition in 1, so there should be the proposition $\{1, 2\}$ in 1. This conflicts with the fact that $2 \approx_1^F 3$, and therefore shows that $\approx^F$ is not a plausible model. This line of thought could also be spelled out more formally using so-called *comprehension principles*; see Fritz & Goodman (forthcoming) and Fritz (unpublished c) for further discussion.

The upshot is that in a coherent equivalence system, worlds indistinguishable at a given world must in particular be indistinguishable in terms of indistinguishability. In general, worlds indistinguishable at a given world w must be indistinguishable in terms of all resources available in w, including the notion of indistinguishability. To turn this into a formal criterion, two questions must be answered. First, what are the resources available at a world, as represented by an equivalence system? And second, how can indistinguishability given those resources be understood?

Concerning the first question, three resources can be identified as being represented by equivalence systems: First, each world contains a set of propositions, given by its relation of indistinguishability. Second, as noted above, the notion of indistinguishability is a resource available at any world. Finally, it is natural to count each world as being one of the resources available at itself.

The natural answer to the second question is that v and u are indistinguishable given certain resources if v and u are symmetric with respect to them; that is, if there is a way of reconfiguring worlds which maps v to u but respects the given resources, in the sense of these resources being invariant under this reconfiguration. Formally, such a reconfiguration is a *permutation* – a bijection from worlds to worlds, i.e., a function from worlds to worlds which is both surjective (onto) and injective (one-to-one). It only remains to specify what it takes for a permutation of worlds to respect the three resources identified above. This is obvious in the case of the world itself: w is invariant under f just in case f maps w to itself. The other two resources require a bit more thought.

For propositions, note that it is straightforward to extend a permutation of worlds to a permutation of propositions, by letting the image of a proposition P under a permutation f be the set of the images of members of P under f: $f.P = \{f(w) : w \in P\}$. (The notation $f.P$ indicates that from a group-theoretic perspective, the extension of f from worlds to sets of worlds can be understood as an action.) Thus, a permutation f respects the propositions at w just in case it maps every union of a set of equivalence classes of $\approx_w$ to itself. It is easy to see that this is equivalent to requiring f to map each world v to one $\approx_w$-related to v. Taking f to be the set of pairs $\langle v, u\rangle$ such that $f(v) = u$ and $\approx_w$ as the set of pairs $\langle v, u\rangle$ such that $v \approx_w u$, this is most concisely written as $f \subseteq \approx_w$.

For a permutation f to respect the notion of indistinguishability, facts about which worlds can distinguished at a given world must be invariant under permuting the worlds using f. That is, v and u must be indistinguishable at w just in case $f(v)$ and $f(u)$ are indistinguishable at $f(w)$; i.e., $v \approx_w u$ if and only if $f(v) \approx_{f(w)} f(u)$. In this case, f is called an *automorphism of* $\approx$. If $\approx$ is represented as a ternary relation, then this is the familiar notion of an automorphism, i.e., an isomorphism from a set associated with a relation to itself.

The coherence constraint can now be stated formally; it requires that if v and u are indistinguishable at w, then there is a permutation f mapping v to u which (i) is a subset of $\approx_w$, (ii) is an automorphism of $\approx$, and (iii) maps w to itself. To state this more concisely, let aut($\approx$) be the set of automorphisms of $\approx$; this is a group, a fact which will be useful later. Further, let $\text{aut}(\approx)_w$ be the set of elements of $\text{aut}(\approx)_w$ which map w to itself; this is called the *stabilizer of* w. With this, the condition can be formulated as follows:

DEFINITION 2.2. *An equivalence system* $\approx$ *on a set* W coheres *if for all* $w, v, u \in W$ *such that* $v \approx_w u$*, there is an* $f \in \text{aut}(\approx)_w$ *such that* $f(v) = u$ *and* $f \subseteq \approx_w$*.*

A system cohering will in the following sometimes also be phrased as it *being coherent*. Note that since every element of $\text{aut}(\approx)_w$ maps w to itself, w can only be $\approx_w$-related to itself in a coherent equivalence system. Consequently, w's equivalence class under $\approx_w$ is its singleton: Every world contains its singleton proposition.

To illustrate how coherence is applied, it is helpful to introduce *cycle-notation* of permutations, which is best explained by examples: Considering permutations on $W = \{1, \ldots, 6\}$, the permutation which maps 1 to 2, 2 to 3, 3 to 1 and all other elements of W to themselves can be written (123); the permutation which maps each $i < 5$ to itself and 5 and 6 to each other can be written (56).

Consider again the systems $\approx^F$ and $\approx^E$ depicted above. $\approx^F$ is easily seen to be incoherent: Since $2 \approx^F_1 3$, coherence requires there to be an $f \in \text{aut}(\approx^F)_1$ such that $f(2) = 3$ and $f \subseteq \approx^F_1$. The only permutation of $\{1, 2, 3\}$ mapping 1 to itself and 2 to 3 is $f = (23)$; however, $f \notin \text{aut}(\approx^F)$: $3 \approx^F_2 1$ holds, but $f(3) \approx^F_{f(2)} f(1)$, i.e., $2 \approx^F_3 1$ does not. Thus $\approx^F$ is incoherent.

In contrast, $\approx^E$ is coherent. This follows from the fact that for all $w, v, u \in \{1, \ldots, 6\}$, if $v \approx^E_w u$ then $(vu) \in \text{aut}(\approx)_w$. (While this is evidently a sufficient condition for coherence, $\approx^B$ in the proof of Proposition 6.8 shows that it is not a necessary condition.) Although somewhat laborious, checking that this claim holds is a straightforward matter using the above definition of automorphisms and stabilizers.

The definition of a coherent equivalence system and the notation used here to state it are closely modeled on Stalnaker (2012, Appendix A), although the coherence constraint differs in a crucial respect from Stalnaker's definition. Appendix A below motivates this deviation. The concluding Section 7 notes that coherent equivalence systems correspond exactly to the propositional fragment of Fine's more general model theory of higher-order contingency, as well as some of its variants.

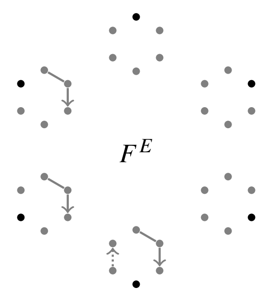
§3. Permutation Systems. Models of the second class represent, for each world, which permutations of worlds respect *all* distinctions among worlds which can be drawn at that world. Call a permutation which does so a *symmetry* of the world. A model of the second class is therefore a function mapping each world to the set of its symmetries. Clearly, the identity permutation is a symmetry of every world. Further, if a permutation f respects certain distinctions, then so does its inverse f^{-1}, and if two permutations f and g respect these distinctions, then so does their composition fg. Imposing these three constraints on the symmetries of each world is equivalent to requiring that the symmetries of each world form a *permutation group* on the set of worlds. Thus, define formally:

DEFINITION 3.1. *For every set* W*, a* permutation system on W *is a function* F *mapping every* $w \in W$ *to a permutation group* F_w *on* W*.*

To give an example of a permutation system, write, for a set of permutations G, $\langle G \rangle$ for the permutation group generated by G, i.e., the set of permutations which can be obtained by finite combinations of elements of G by inverses and composition. E.g, $\langle \{(123)\} \rangle = \{(123), (321), \text{id}\}$. Here is an example of a permutation system: Let F^E be the function on $\{1,\ldots,6\}$ which maps 1, 2 and 3 to $\{\text{id}\}$, 4 to $\langle \{(123), (56)\} \rangle$ and 5 and 6 to $\langle \{(123)\} \rangle$.

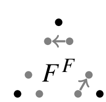
It will also be helpful to be able to draw any permutation system F on $\{1, \ldots, n\}$, for some natural number n. Again, draw a circle of n circles of n dots. For each $i \leq n$, choose a set of permutations G generating F_i, and assign to each member of G a different style of arrow, such as solid versus dotted lines. Draw each permutation in G in the circle of dots representing i by arrows indicating which worlds are mapped to which worlds. Such arrows will be allowed to go through several worlds, leaving it implicit that the dot at the tip of an arrow represents the world mapped to the world represented by the dot at the start of the arrow; arrows from a world to itself will be omitted. E.g., to draw (123), draw a single arrow starting at 1, going through 2 and pointing to 3.

As an example, the following represents F^E, choosing $\{(123), (56)\}$ to generate F_4 and $\{(123)\}$ to generate F_5 and F_6:



Note that for a given permutation system F, there might be several choices of associating each world i with a set of permutations G which generates F_i. Consequently, it is not always straightforward to tell whether two drawings represent the same permutation system.

Again, not every such system is plausible:



According to F^F, switching 2 and 3 is a symmetry of 1. But 2 and 3 can be distinguished in terms of their symmetries, since 2 does but 3 does not have a symmetry other than the trivial identity permutation.

A coherence constraint must be imposed which ensures that the permutations associated with a given world respects its resources, as represented by the permutation system. These resources are only the notion of a symmetry and the world itself. Note in particular that the symmetries of a world need not be resources available at that world: the symmetries associated with a world only *describe* what distinctions can drawn using the resources available at this world; they need not themselves be resources available at the world. In the model theory of Fritz & Goodman (forthcoming), which extends the present treatment of contingency in what propositions there are to a type hierarchy of relations, this can be demonstrated more concretely by treating permutations of worlds as a special kind of binary relations among nullary relations. A related issue is discussed in Appendix B.

What does it take for a permutation of worlds f to respect the notion of a symmetry? To spell this out, f must first be extended to a permutation of permutations g of worlds. Let $f.g$ be the result of applying f to g. One can reason as follows to argue that $f.g = fgf^{-1}$: $f.g$ should behave on the elements permuted by f as g behaves on the original elements. So if g maps w to v, then the result of applying f to g should map $f(w)$ to $f(v)$. Thus it should be the case that $(f.g)f(w) = f(v)$, and since $v = g(w)$, $(f.g)f(w) = fg(w)$.

Let $u = f(w)$; since f is a bijection, $w = f^{-1}(u)$. Then $f.g(u) = fgf^{-1}(u)$, and so in general $f.g = fgf^{-1}$; this is called the *conjugation of g by f*.

This definition is naturally extended to sets of permutations, letting $f.G = \{f.g : g \in G\}$. The required constraint on f is now formulated straightforwardly by requiring f to map F_w, for any world w, to $F_{f(w)}$, i.e., $f.F_w = F_{f(w)}$. As above, call such permutations *automorphisms of* F, and the set of such permutations aut(F). Again, the set of members of aut(F) mapping w to itself is the *stabilizer of* w, written aut$(F)_w$. Thus the members of aut$(F)_w$ are exactly the permutations which satisfy the constraint of respecting the resources available at w, as represented by the permutation system. Consequently, coherence can be defined as follows:

DEFINITION 3.2. *A permutation system F on a set W* coheres *if for all $w \in W$, $F_w \subseteq$ aut$(F)_w$.*

Consider again the systems F^F and F^E depicted above: F^F is incoherent, since $f = (23)$ is a member of F^F_1 but not an automorphism of F^F: $(31) \in F^F_2$ holds, while $f.(31) \in F_{f(2)}$, i.e., $(21) \in F_3$, does not hold. And although somewhat laborious, it is routine to show that F^E is coherent using the above definitions.

How do permutation systems model contingency in what propositions there are? They do so by determining an equivalence system, which itself can be seen as a model of contingency in what propositions there are, as described above. To see how a permutation systems F determines an equivalence system, note first that if there is an $f \in F_w$ which maps v to u, then there is a symmetry of w which maps v to u; consequently, v and u must be indistinguishable at w. Conversely, it was noted above that if v and u are indistinguishable at w, then they must be indistinguishable in terms of all resources available at w, so there must be a symmetry of w mapping v to u, i.e. an $f \in F_w$ such that $f(v) = u$. Thus the equivalence system determined by F counts v and u as indistinguishable at w if and only if there is an $f \in F_w$ such that $f(v) = u$. Taking relations and functions to be sets of pairs as noted above, this can be summed up as follows:

DEFINITION 3.3. *For every permutation system F on a set W, the* equivalence system determined by *F, written $\varepsilon(F)$, is such that for all $w \in W$:*

$$\varepsilon(F)_w = \bigcup F_w.$$

It is straightforward to check that this is well-defined, i.e., that $\varepsilon(F)_w$ is an equivalence relation for every $w \in W$. To illustrate the definition, note that $\varepsilon(F^F) = \approx^F$ and $\varepsilon(F^E) = \approx^E$.

It might be illustrative to consider the following alternative description of the equivalence system determined by a permutation system F. To state it, some further basic group-theoretic notions are needed (which won't be used in subsequent sections). For any permutation group G on a set X and $x \in X$, define $G.x$, the *orbit of* x, to be the set of elements to which x is moved by members of G, i.e., $\{g(x) : g \in G\}$. Let X/G be the set of orbits, i.e., $\{G.x : x \in X\}$. This must be a partition of X, i.e., a set of pairwise disjoint subsets of X whose union is X. Such a partition corresponds uniquely to the equivalence relation whose equivalence classes are all and only the members of the partition. $\varepsilon(F)$ can now be described as the equivalence system which maps every $w \in W$ to the equivalence relation corresponding to W/F_w, the set of orbits of F_w.

As with equivalence systems, the definition of a coherent permutation system and the notation used to state it are closely modeled on Stalnaker (2012, Appendix A), and, as noted in Appendix A, this coherence constraint essentially corresponds exactly to Stalnaker's definition.

§4. Equivalence. Do coherent equivalence systems and coherent permutation systems encode the same theory of propositional contingency, in the sense of admitting the same patterns of contingency in what propositions there are? This section shows that this is so, by showing that an equivalence system is coherent if and only if it is determined by a coherent permutation system. That every coherent permutation system determines a coherent equivalence system is easy to show using the following lemma:

LEMMA 4.1. *For any permutation system F,* $\mathrm{aut}(F) \subseteq \mathrm{aut}(\varepsilon(F))$.

Proof. Let $f \in \mathrm{aut}(F)$, and consider any $w, v, u \in W$ such that $v\varepsilon(F)_w u$. Then there is a $g \in F_w$ such that $g(v) = u$. Since $f \in \mathrm{aut}(F)$, $f.g \in F_{f(w)}$, so $f(v)\varepsilon(F)_{f(w)} f.g(f(v))$. As $f.g(f(v)) = fg(v) = f(u)$, it follows that $f(v)\varepsilon(F)_{f(w)} f(u)$, as required. The converse direction follows by a symmetric argument for f^{-1}. □

THEOREM 4.2. *Every coherent permutation system determines a coherent equivalence system.*

Proof. Let F be a coherent permutation system on a set W, and consider any $w, v, u \in W$ such that $v\varepsilon(F)_w u$. Then there is an $f \in F_w$ such that $f(v) = u$. Since F is coherent, $f \in \mathrm{aut}(F)_w$, and so by Lemma 4.1, $f \in \mathrm{aut}(\varepsilon(F))_w$. By construction of $\varepsilon(F)$, $f \subseteq \varepsilon(F)_w$. □

To show that every coherent equivalence system is determined by a coherent permutation system, a mapping from equivalence systems to permutation systems will be used. The idea behind this mapping is to associate each world w with the set of automorphisms which respect the propositions at w, using the above extension of permutations of worlds to propositions. It is easy to see that these are exactly the automorphisms which respect the equivalence classes at w. Formally, write $W/{\approx_w}$ for the set of equivalence classes under W, which is called the *quotient set of W by $\approx_w$*, and write $\mathrm{aut}(\approx)_{(W/\approx_w)}$ for the set of automorphisms of $\approx$ which map each member of $W/{\approx_w}$ to itself, which is called the *pointwise stabilizer of* $W/{\approx_w}$. It is easy to see that $\mathrm{aut}(\approx)_{(W/\approx_w)} = \{f \in \mathrm{aut}(\approx) : f \subseteq {\approx_w}\}$. Thus, define formally:

DEFINITION 4.3. *For every equivalence system $\approx$ on a set W, the* permutation system determined by $\approx$, *written $\pi(\approx)$, is such that for all $w \in W$:*

$$\pi(\approx)_w = \mathrm{aut}(\approx)_{(W/\approx_w)}.$$

The desired result can now be obtained from two lemmas. The first shows that every coherent equivalence system determines a coherent permutation system. The second shows that every coherent equivalence system is determined by the permutation system it determines. To prove the first, permutations of worlds, already extended to propositions, are analogously extended once more to sets of propositions.

LEMMA 4.4. *Every coherent equivalence system determines a coherent permutation system.*

Proof. Let $\approx$ be a coherent equivalence system on a set W, and consider any $w \in W$ and $f \in \pi(\approx)_w$. To prove that $f \in \mathrm{aut}(\pi(\approx))$, consider any $v \in W$; we prove that $f.\pi(\approx)_v = \pi(\approx)_{f(v)}$. First, $f.\pi(\approx)_v = f.\mathrm{aut}(\approx)_{(W/\approx_v)}$; by a general principle for stabilizers, this is $\mathrm{aut}(\approx)_{(f.W/\approx_v)}$, which, as $f \in \mathrm{aut}(\approx)$, is $\mathrm{aut}(\approx)_{(W/\approx_{f(v)})}$, i.e., $\pi(\approx)_{f(v)}$. As noted above, $\{w\} \in W/{\approx_w}$, so $f(w) = w$, and thus $f \in \mathrm{aut}(\pi(\approx))_w$, as required. □

LEMMA 4.5. *For every coherent equivalence system* $\approx$, $\approx = \varepsilon(\pi(\approx))$.

Proof. If $v \approx_w u$, then there is an $f \in \text{aut}(\approx)_w$ such that $f(v) = u$ and $f \subseteq \approx_w$. So $f \in \pi(\approx)_w$, and hence there is an $f \in \pi(\approx)_w$ such that $f(v) = u$. Therefore $v\varepsilon(\pi(\approx))_w u$. If $v\varepsilon(\pi(\approx))_w u$, then there is an $f \in \pi(\approx)_w$ such that $f(v) = u$. Since $f \in \pi(\approx)_w$, $f \subseteq \approx_w$; in particular $v \approx_w f(v)$, so $v \approx_w u$. □

THEOREM 4.6. *Every coherent equivalence system is determined by a coherent permutation system.*

Proof. If $\approx$ is a coherent equivalence system, then by Lemma 4.4, $\pi(\approx)$ is a coherent permutation system, and by Lemma 4.5, $\approx$ is the equivalence system determined by it. □

Together, Theorems 4.2 and 4.6 show that as models of contingency in what propositions there are, coherent equivalence systems and coherent permutation systems are equivalent.

§5. Relating Coherent Systems. Given Theorems 4.2 and 4.6, one might conjecture that the two kinds of coherent systems are equivalent in a stronger sense, namely that the two determination relations are bijections and mutual inverses. This, however, is not the case; the relations between the two kinds of coherent systems are more interesting. To explore them in more detail in the following, fix an arbitrary set W as the set of worlds, and consider ε as a function from coherent permutation systems to coherent equivalence systems, and π as a function from coherent equivalence systems to coherent permutation systems, all of them on W – this will be left tacit in this section and the next.

Theorem 4.6 shows that every coherent equivalence system is determined by a coherent permutation system, so ε is surjective. But as the following result shows, ε is not injective, at least not for every choice of set of worlds W:

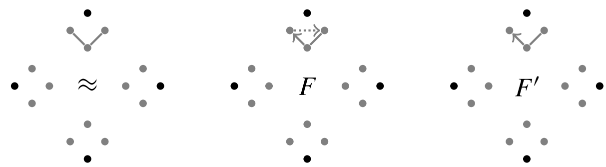
THEOREM 5.1. *For some set* W*, there are distinct coherent permutation systems on* W *which determine the same equivalence system.*

Proof. Let F and F^ω be the permutation systems on an infinite set W such that for every $w \in W$, F_w is the set of permutations of W which map w to itself, and F^ω_w is the set of permutations of W which map w to itself and only finitely many worlds not to themselves. It is routine to check that F and F^ω are coherent. Both determine the equivalence system on W which maps each $w \in W$ to the equivalence relation whose equivalence classes are $\{w\}$ and $W\backslash\{w\}$. □

This also shows that π is not surjective: If it were, there would be coherent equivalence systems $\approx$ and $\approx'$ determining permutation systems F and F' which determine the same equivalence system, contradicting Lemma 4.5. However, Lemma 4.5 shows that π is injective, and that the inverse of π is the restriction of ε to coherent permutation system which are determined by coherent equivalence systems.

Theorem 5.1 should not come as a surprise: given the interpretation of equivalence and permutation system, there was no reason to expect distinct coherent permutation systems to determine distinct equivalence systems. For recall that equivalence systems encode *which* worlds can be distinguished at a given world, whereas permutation systems encode *how* worlds can be permuted in ways which constitute a symmetry of the given world. While it is natural to think that the former information is contained in the latter information, there is no reason to expect the converse.

To illustrate this more concretely, it is helpful to consider witnesses of the existential claim of Theorem 5.1 which are perhaps less elegant than the ones used in the above proof. Let F and F' be the permutation systems on $W = \{1, 2, 3, 4\}$ such that F_1 is the set of permutations of W which map 1 to itself, which can be represented as $\langle\{(234), (42)\}\rangle$, $F'_1 = \langle\{(234)\}\rangle$, and for all $w \in \{2, 3, 4\}$, $F_w = F'_w = \{\mathrm{id}\}$. It is routine to check that these are coherent and determine the same equivalence system $\approx$:



$F'_1 = \{(234), (432), \mathrm{id}\}$ does not contain (23), (24) or (34). So F' might seem like a curious permutation system: How could it be that at 1, any two of 2, 3 and 4 are indistinguishable, yet not every way of permuting these is a symmetry of 1? How such a permutation system might arise can be motivated by considering individuals and their relations. To facilitate the comparison between F and F', the following story does so for both of these permutation systems. Let a, b and c be three possible electrons, and R some qualitative relation in which electrons can stand. Consider four worlds, labeled 1, 2, 3 and 4, which are so simple that in some sense, all there is to be said about any one of them is which of a, b and c there are in it and which individuals are related by R in it. In 1, there is nothing; in 2, there are a and b; in 3, there are b and c; and in 4, there are c and a. Consider two ways of adding a pattern of instantiation for R; according to the left, in each world, both individuals there are at this world stand in R to each other; according to the right, Rab in 2, Rbc in 3 and Rca in 4:

1:					1:				
2:	$a \longleftrightarrow b$				2:	$a \longrightarrow b$			
3:		$b \longleftrightarrow c$			3:		$b \longrightarrow c$		
4:	$a \longleftarrow\!\!\longrightarrow c$				4:	$a \longleftarrow c$			

The symmetries of a given world w were introduced above as the permutations of worlds which respect *all* distinctions among worlds which can be drawn using the resources at w. Clearly these resources include the individuals there are at w, and plausibly, they include the qualitative relation R as well as the notion of being, i.e., which individuals there are at a world. As above, a symmetry is understood to respect these resources if it maps them to themselves. Permutations of worlds are not obviously extended to permutations of, e.g., individuals, in the way they were extended to propositions above. Rather, once individuals are considered, a reconfiguration of modal reality should be understood as consisting of a permutation of worlds f as well as a permutation of individuals g. Such a reconfiguration can be understood to respect an individual if g maps the individual to itself; it can be understood to respect R if at any world w, individuals x and y stand in R if and only if $g(x)$ and $g(y)$ stand in R in $f(w)$, and it can be understood to respect the notion of being if at any world w, there is an individual x if and only if there is $g(x)$ at $f(w)$. A permutation of worlds being a symmetry of a world can now be understood as it being part of a more comprehensive reconfiguration of modal space.

This way of deriving the symmetries of a world from what individuals there are in it and some distinguished qualitative relations among individuals goes back to Fine (1977b), and further developments can be found in Fritz & Goodman (forthcoming). What has been said so far suffices to indicate how F is derived from the left configuration and how F' is derived from the right configuration. E.g., the permutation of worlds (23) is a symmetry of 1 in F, since it can be extended by a permutation of individuals, namely (ac), to form a reconfiguration of modal space which respects all resources at 1. That it respects all individuals at 1 is trivial, since there are none. That it respects R is easily verified: since Rab at 2, Rcb must hold at 3, which is the case; similarly for the other instances of the condition. Finally, the notion of being is respected: a and b (the individuals at 2) are mapped to c and b (the individuals at 3), and similarly for the other instances of the condition. In contrast, (23) is not a symmetry of 1 in F': For this to be the case, there would have to be a permutation g of individuals with which (23) forms a reconfiguration of modal space which respects all resources at 1. In particular, this would have to respect the notion of being, and so, since 2 is mapped to 3, map each of a and b to one of b and c. Since R must be respected as well and Rab holds in 2, $Rg(a)g(b)$ must hold in 3, so g must map a to b and b to c. Since g is a permutation, it must map c to a. But this means that another instance of the reconfiguration respecting R is not satisfied: 3 is mapped to 2 and Rbc holds in 3, but $Rg(b)g(c)$, i.e., Rca, does not hold in 2. So (23) is not a symmetry of 1 in F'.

To summarize, *how* worlds can be permuted in ways which constitute a symmetry of a world goes beyond *which* worlds can be distinguished at it. But then, what does π, the function which maps every coherent equivalence system to a coherent permutation system, do? Lemma 4.5 says that the permutation system determined by a coherent equivalence system $\approx$ is a coherent permutation system which determines $\approx$. But as the above examples witness, there might be more than one such permutation system, so the question is: which one does π take us to? The definition of the permutation system determined by a given equivalence system works by associating with each world the set of *all* automorphisms of the equivalence system which respect the propositions at that world according to the equivalence system. The natural guess is therefore that $\pi(\approx)$ is the most inclusive among the coherent permutation systems which determine $\approx$, in the sense that for any world w, $\pi(\approx)_w$ contains all permutations in F_w for any such permutation system F. In the next section, it is shown that this conjecture is correct. To do so, the idea of ordering permutation systems according to how inclusive they are is first made precise. It is clear that equivalence systems can be ordered analogously, which provides a new perspective on equivalence systems and permutation systems, namely as two ordered sets connected by two functions.

§6. Ordering Coherent Systems. To start, the order among coherent permutation systems is defined formally:

DEFINITION 6.1. *$\sqsubseteq$ is the binary relation on the set of coherent permutation systems such that $F \sqsubseteq F'$ just in case for all $w \in W$, $F_w \subseteq F'_w$.*

It is easy to see that $\sqsubseteq$ is a partial order, i.e., that it is reflexive, transitive and antisymmetric. In such an order, an element x which is greater than or equal to all elements of a subset C of the ordered set is called an *upper bound* of C. An upper bound of C which is an element of C is called the *greatest* element of C. There need not always be such an element, but if there is one, it is unique. The conjecture ventured above is that for every coherent equivalence system $\approx$, $\pi(\approx)$ is the greatest element of the set of coherent permutation systems which determine $\approx$. To state this more concisely, define $\varepsilon^{-1}(\approx)$ to be the preimage of $\approx$ under ε, i.e., the set of permutation systems F such that $\varepsilon(F) = \approx$. As the next lemma shows, $\pi(\approx)$ is an upper bound of $\varepsilon^{-1}(\approx)$:

LEMMA 6.2. *For every coherent equivalence system $\approx$ and $F \in \varepsilon^{-1}(\approx)$, $F \sqsubseteq \pi(\approx)$.*

Proof. Consider any $w \in W$ and $f \in F_w$. As $\pi(\approx)_w = \mathrm{aut}(\approx)_{(W/\approx_w)}$, it suffices to show that $f \in \mathrm{aut}(\approx)_{(W/\approx_w)}$. Since F is coherent, $f \in \mathrm{aut}(F)$, and so by Lemma 4.1, $f \in \mathrm{aut}(\varepsilon(F))$. $\varepsilon(F) = \approx$ by assumption, so $f \in \mathrm{aut}(\approx)$. By definition of ε, $f \subseteq \varepsilon(F)_w$, so $f \subseteq \approx_w$, and therefore $f \in \mathrm{aut}(\approx)_{(W/\approx_w)}$. □

For every coherent equivalence system $\approx$, $\pi(\approx) \in \varepsilon^{-1}(\approx)$ by Lemma 4.5, so the conjecture follows immediately from this and the previous lemma:

THEOREM 6.3. *For every coherent equivalence system $\approx$, $\pi(\approx)$ is the greatest element of $\varepsilon^{-1}(\approx)$ under $\sqsubseteq$.*

While this shows that every preimage of a coherent equivalence system has a greatest element, this is clearly not the case for sets of coherent permutation systems in general: a set of two permutation systems F and F' such that for some $w \in W$, neither $F_w \subseteq F'_w$ nor $F'_w \subseteq F_w$, has no greatest element. But something closely related holds: Every set C of coherent permutation systems has a *least upper bound*, written $\bigvee C$, i.e., an upper bound of C which is less than or equal to all upper bounds of C. (Least upper bounds are also unique, and in general, there need not be one. If a set has a greatest element, this is its least upper bound.) C also has a greatest lower bound, written $\bigwedge C$, i.e., a lower bound of C which is greater than or equal to all lower bounds of C, where a lower bound of C is of course an element which is less than or equal to all elements of C. A partial order in which every set C has both a least upper bound and a greatest lower bound is called a *complete lattice*. In such an order, there are in particular the least upper bound and the greatest lower bound of the set of all elements; these can be thought of as the greatest and least elements overall and are written $\top$ and $\bot$. The following proposition shows that coherent permutation systems form a complete lattice, specifying greatest lower bounds, $\top$ and $\bot$. To state it, write $(S_W)_w$ for the set of permutations of W which map w to itself – again, this is the stabilizer of w, now with respect to S_W, the set of permutations of W, which is called the *symmetric group on W*.

PROPOSITION 6.4. *The set of coherent permutation systems ordered by $\sqsubseteq$ is a complete lattice, where for any subset C, $\bigwedge C$, $\top$ and $\bot$ are the permutation systems such that for all $w \in W$:*

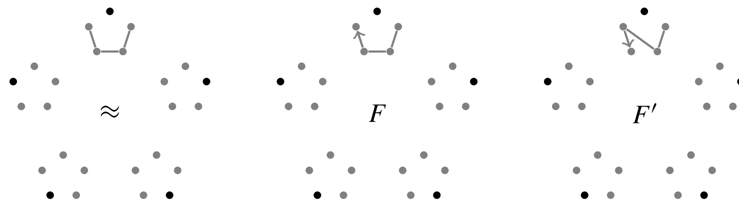
$$\bigwedge C_w = (S_W)_w \cap \bigcap_{F \in C} F_w$$
$$\top_w = (S_W)_w$$
$$\bot_w = \{\mathrm{id}\}$$

Proof. It is routine to show that the function which maps every $w \in W$ to $(S_W)_w \cap \bigcap_{F \in C} F_w$ is a coherent permutation system and the greatest lower bound of C. Any partial order in which every set, including the empty set, has a greatest lower bound (i.e., any complete meet-semilattice) is also a complete lattice. It is again routine to show that the functions which map every $w \in W$ to $(S_W)_w$ and $\{\mathrm{id}\}$ are $\top$ and $\bot$, respectively. □

Since sets of coherent permutation systems have both least upper bounds and greatest lower bounds and any preimage of a coherent equivalence system contains its least upper bound, one might wonder whether any such set also contains its greatest lower bound. It turns out that this is not the case, at least in the sense that for some sets W, the claim does not hold:

PROPOSITION 6.5. *For some set W and coherent equivalence system $\approx$ on W, $\bigwedge \varepsilon^{-1}(\approx) \notin \varepsilon^{-1}(\approx)$.*

Proof. F and F' are coherent and $\varepsilon(F) = \varepsilon(F') = \approx$, but $F \wedge F' = \bot \notin \varepsilon^{-1}(\approx)$:



□

This shows that there is in general no function analogous to π which maps every coherent equivalence system to the least element of its preimage. To be more precise, x is the *least* element of a set C partially ordered by $\leq$ just in case $x \in C$ and $x \leq y$ for all $y \in C$. Such a least element is the greatest lower bound, so by Proposition 6.5, the preimages of some coherent equivalence systems do not have least elements.

Consider now the corresponding order on coherent equivalence systems:

DEFINITION 6.6. *$\preccurlyeq$ is the binary relation on the set of coherent equivalence systems such that $\approx \preccurlyeq \approx'$ just in case for all $w \in W$, $\approx_w \subseteq \approx'_w$.*

Again, it is easy to see that this is a partial order. What else can be said about it? A natural conjecture is that it is isomorphic to the image of π (the set of permutation systems determined by coherent equivalence systems) ordered by $\sqsubseteq$. More specifically, one might conjecture that for any coherent equivalence systems $\approx$ and $\approx'$, $\approx \preccurlyeq \approx'$ if and only if $\pi(\approx) \sqsubseteq \pi(\approx')$. This turns out not to be the case; while the 'if' direction holds, the 'only if' direction does not. To establish the former claim, it will be shown that ε is order-preserving, in the sense that for all coherent permutation systems F and F', if $F \sqsubseteq F'$ then $\varepsilon(F) \preccurlyeq \varepsilon(F')$:

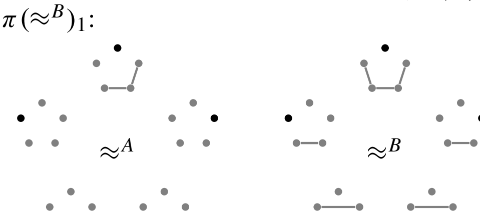
PROPOSITION 6.7. *ε is order-preserving.*

Proof. Assume that F and F' are coherent permutation systems such that $F \sqsubseteq F'$. If $v\varepsilon(F)_w u$, then there is an $f \in F_w$ such that $f(v) = u$, so given $F \sqsubseteq F'$, $f \in F'_w$ and therefore $v\varepsilon(F')_w u$. □

In particular, for all coherent equivalence systems $\approx$ and $\approx'$, if $\pi(\approx) \sqsubseteq \pi(\approx')$ then $\varepsilon(\pi(\approx)) \preccurlyeq \varepsilon(\pi(\approx'))$, and so by Lemma 4.5, $\approx \preccurlyeq \approx'$. To show that the other direction of the conjecture does not hold, it will be shown that π is not guaranteed to be order-preserving, in the sense that for some coherent equivalence systems $\approx$ and $\approx'$, $\approx \preccurlyeq \approx'$ while $\pi(\approx) \not\sqsubseteq \pi(\approx')$:

PROPOSITION 6.8. *For some set W, π is not order-preserving.*

Proof. $\approx^A$ and $\approx^B$ are coherent and $\approx^A \preccurlyeq \approx^B$, but $\pi(\approx^A) \not\sqsubseteq \pi(\approx^B)$ since $(234) \in \pi(\approx^A)_1$ and $(234) \notin \pi(\approx^B)_1$:



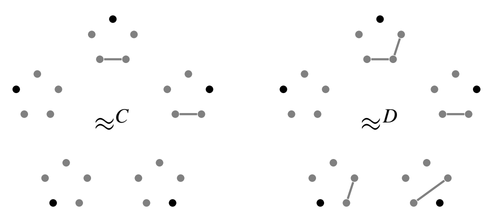
□

So while π is a bijection from coherent equivalence systems to its image, it is not an isomorphism between the two sets ordered by $\preccurlyeq$ and $\sqsubseteq$.

What structure do coherent equivalence systems ordered by $\preccurlyeq$ form? It turns out that in contrast to coherent permutation systems, they do not form a complete lattice. It can even be shown that they neither form a join- nor a meet-semilattice, i.e., that two coherent equivalence systems are neither guaranteed to have a least upper bound nor guaranteed to have a greatest lower bound:[1]

PROPOSITION 6.9. *For some set W, coherent equivalence systems on W ordered by $\preccurlyeq$ form neither a join- nor a meet-semilattice.*

Proof. Let $\approx^A$ and $\approx^B$ be as in the proof of Proposition 6.8, and consider in addition the following coherent equivalence systems:



We argue (i) that $\approx^A$ and $\approx^C$ have no least upper bound, and (ii) that $\approx^B$ and $\approx^D$ have no greatest lower bound.

For (i), note that $\approx^B$ and $\approx^D$ are both upper bounds of $\approx^A$ and $\approx^C$. $\approx^D$ is the only upper bound $\approx$ of $\approx^A$ and $\approx^C$ such that $\approx \preccurlyeq \approx^D$, since it is the only coherent one of the four equivalence systems $\approx$ such that $\approx^A \preccurlyeq \approx$, $\approx^C \preccurlyeq \approx$ and $\approx \preccurlyeq \approx^D$. Since $\approx^D \not\preccurlyeq \approx^B$, it follows that there is no upper bound $\approx$ of $\approx^A$ and $\approx^C$ such that $\approx \preccurlyeq \approx^B$ and $\approx \preccurlyeq \approx^D$, and therefore no least upper bound of $\approx^A$ and $\approx^C$.

For (ii), note that $\approx^A$ is the only lower bound $\approx$ of $\approx^B$ and $\approx^D$ such that $\approx^A \preccurlyeq \approx$: Any such $\approx$ maps 3, 4 and 5 to the identity relation; since also $2 \approx_1 3$, $\approx_2$ is the identity relation; finally, since $\approx^A_1 = \approx^D_1$, $\approx_1 = \approx^A_1$. Since $\approx^C \not\preccurlyeq \approx^A$, it follows that there is no lower bound $\approx$ of $\approx^B$ and $\approx^D$ such that $\approx^A \preccurlyeq \approx$ and $\approx^C \preccurlyeq \approx$, and therefore no greatest lower bound of $\approx^B$ and $\approx^D$. □

§7. Conclusion. Two kinds of models for propositional contingentism were developed above, which were shown to be equivalent as models of contingency in what propositions there are, but not equivalent overall. Permutation systems were shown to draw finer distinctions than equivalence systems, and a philosophically motivated example was given for this difference using individuals and their relations. The details of the example suggest that the present treatment of contingency in what propositions there are can be expanded into a more comprehensive theory of higher-order contingency, i.e., contingency in what propositions, properties and relations there are. An investigation of this kind was already carried out in great detail in Fine (1977b). Fritz & Goodman (forthcoming) explore variants of Fine's proposal which take up some further ideas from Stalnaker (2012), and argue that Fine's proposal must be revised to take contingency in what relations there are seriously. The discussion in Fritz & Goodman (forthcoming) shows that there are a number of choice points in how to develop a theory of higher-order contingency. Fritz (unpublished a) shows that many but not all of them agree on the patterns of contingency in what propositions

[1] The results established in this section also immediately show that π and ε do not form a Galois connection, as one might have conjectured.

there are, which exactly correspond to the kinds of models developed here, i.e., coherent equivalence systems. Some similar results for patterns of symmetries are established there as well, showing that it depends on the particular details of the theory whether they exactly correspond to the class of coherent permutation systems.

Two aspects of the present model theory for propositional contingentism are explored elsewhere. First, Fritz (unpublished c) interprets two extensions of propositional modal logic on coherent equivalence systems. The first is an extension by propositional quantifiers, which are naturally interpreted at a given world as ranging over the propositions which there are at the world according to the system, i.e., the unions of the sets of equivalence classes of the equivalence relation associated with the world. This logic is shown not to be recursively axiomatizable, since it is recursively isomorphic to second-order logic. The second extension adds a modality which expresses that there is the proposition expressed by the formula it operates on, which can be seen as a fragment of the first extension.

Second, the ramifications of propositional contingentism on the semantics of counterfactuals are explored in Fritz & Goodman (unpublished a). It is argued that the present models of propositional contingentism are straightforwardly combined with the theory of counterfactuals of Lewis (1973), but that they are in tension with the theory of counterfactuals of Stalnaker (1968). The main point of tension arises from the principle of conditional excluded middle, which turns out to hold only for propositions there are at a world, not for propositions there could be.

Appendices

Appendix A: Stalnaker's Models. (Stalnaker, 2012, Appendix A) presents two classes of models on which the above development of coherent equivalence and permutation systems is based. The present appendix describes the differences between the above definitions and Stalnaker's definitions, and argues for the former. In inessential respects, Stalnaker's notation is modified to simplify the comparison.

Before considering the formal definitions, one merely terminological difference between Stalnaker (2012) and the present article must be mentioned: What is called a "world" here is called a "point (of logical space)" by Stalnaker. Stalnaker uses "world" for maximally strong non-trivial propositions at a world, which in equivalence systems are represented by equivalence classes.

Stalnaker defines the class of models corresponding to coherent permutation systems as follows: For each member w of a set W, let F_w be a set of permutations on W such that:

(1') If $f \in F_w$, then $f(w) = w$.

(2') F_w is closed under inverse and composition.

(3') If $f \in F_w$ and $g \in F_v$, then $f.g \in F_{f(v)}$.

It is routine to show that all of these constraints are satisfied if F is a coherent permutation system; in particular, (2') follows from permutation systems mapping worlds to permutation groups, and (3') follows from the fact that the coherence constraint requires F_w to be a set of automorphisms of F. As Stalnaker formulates his condition, the converse cannot be established, as his condition does not rule out F_w being empty. This is ruled out for permutation systems, since permutation groups must contain the identity function. But Stalnaker seems committed to the stronger requirement as well, since he implicitly relies on it on p. 138, where he argues that the relation of a world being mapped to another by a member of F_w is an equivalence relation. If the condition of F_w not being empty is added to (2'), it is routine to show that any F satisfying Stalnaker's constraints is a coherent permutation system: That F is a permutation system follows from the fact that it is a function from W to sets of permutations on W satisfying the strengthened version of (2'). That each member of F_w maps w to itself is required by (1'), so it only remains to show that each $f \in F_w$ is an automorphism of F. So let $v \in W$, and consider any $g \in F_v$. Then by (3'), $f.g \in F_{f(v)}$, so $f.F_v \subseteq F_{f(v)}$. As by (2'), $f^{-1} \in F_w$, an analogous argument establishes that $f^{-1}.F_{f(v)} \subseteq F_{f^{-1}f(v)}$ and thus $F_{f(v)} \subseteq f.F_v$. Hence $F_{f(v)} = f.F_v$, as required.

Stalnaker defines the class of models corresponding to coherent equivalence systems as follows: For each member w of a set W, let $\approx_w$ be an equivalence relation on W such that:

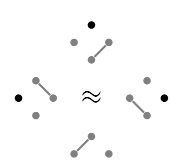
(1) If $w \approx_w v$ then $w = v$.

(2) If $v \approx_w u$, then there exists a permutation function f from W onto W meeting these two constraints:

 (2a) $f(v) = u$

 (2b) for any x, y, and z, $y \approx_x z$ if and only if $f(y) \approx_{f(x)} f(z)$

It is routine to show that all of these constraints are satisfied if $\approx$ is a coherent equivalence system; in particular, (1) follows from the fact that if $w \approx_w v$ then there must be an automorphism mapping w to itself as well as w to v, and (2) follows from the fact that (2b) is equivalent to the condition of f being an automorphism. Considering the converse direction, note that the permutation f whose existence is required in (2) is not required to be a subset of $\approx_w$, as in the definition of coherent equivalence systems. This suggests the possibility of an equivalence system satisfying Stalnaker's constraints without being coherent. The following example shows that there are such equivalence systems:



This evidently satisfies condition (1) of Stalnaker's constraints; for condition (2), note that either (1234) or (4321) witnesses the existential claim in any non-trivial case. To see that $\approx$ is not coherent, consider the fact that $2 \approx_1 3$. The only non-trivial permutation f which is a subset of $\approx_1$ is $f = (23)$, but this is not an automorphism: $3 \approx_2 4$ holds, but $f(3) \approx_{f(2)} f(4)$, i.e., $2 \approx_3 4$ does not.

Thus, Stalnaker's constraints on equivalence systems are strictly weaker than being coherent. With Theorem 4.2, it follows that not every equivalence system satisfying Stalnaker's constraints is determined by a coherent permutation system. This can also be shown directly by noting that the only permutation system which determines $\approx$ is incoherent. Thus Stalnaker's claim that his two models of propositional contingency are equivalent is incorrect; to reinstate it, the stronger condition of coherence for equivalence systems must be imposed.

Stalnaker does provide a formal argument to show that his constraints on F and $\approx$ are equivalent. However, Stalnaker only shows that if a permutation system F satisfies his constraint, then so does $\varepsilon(F)$, and if an equivalence system $\approx$ satisfies his constraint then so does $\pi(\approx)$. While the first of these results is to the point (cf. Theorem 4.2), the second results is strictly speaking irrelevant – what is required is that every equivalence system $\approx$ which satisfies his constraint is determined by some permutation system F which satisfies his constraint (cf. Theorem 4.6), and this turns out not to be the case.

The philosophical discussion in Section 2 already motivates coherence as defined here. It can be supported further by considering the above example of $\approx$ in more detail. According to $\approx$, 2 and 3 are indistinguishable at 1. But at 1, there is a proposition, namely $\{1\}$, which there is at 2 but not at 3. Thus 2 and 3 *can* be distinguished in terms of resources available at 1, and so cannot be indistinguishable at 1. As in the case of $\approx^F$, this line of thought can also be put in terms of what propositions there are: at 1, there should be the proposition that there is the proposition $\{1\}$, which is $\{1, 2\}$, contradicting the fact that $2 \approx_1 3$.

Appendix B: Worlds. Each equivalence or permutation system is based on a set W, whose members were called "worlds" in incautious formulations, and said to "represent worlds" in more cautious formulations. Both of these formulations seem to presuppose that there are these entities – worlds. And this seems to be in conflict with the talk of indistinguishability between worlds engaged in above: Two worlds were said to be indistinguishable if they cannot be distinguished in terms of the resources available, i.e., in terms of what there is. Thus any two worlds should be distinguished in terms of themselves, and so there should not be any indistinguishable worlds. This conclusion might seem like a *reductio* of the whole project. But this is too quick, since even if there *are* no indistinguishable worlds, this does not rule out that there *could be* worlds which actually are indistinguishable. This suggests that talk of there being a certain world should be understood as it being possible that there is such a world.

To illustrate this understanding of world-talk in more detail, the following takes worlds to be maximally strong non-trivial propositions, as suggested, e.g., in Stalnaker (1976). For brevity, call such propositions "maximal". Furthermore, quantification over propositions will be understood as quantification into sentence position, so the proposal will be spelled out in a formal language with propositional variables $p, q, \ldots$, the usual Boolean operators, $\Box$ for necessity and quantifiers $\exists$ and $\forall$ binding propositional variables. As the above models did not include an accessibility relation, the correctness of the modal logic **S5** will be assumed.

In such a setting, a proposition p can be understood to be maximal if it is possible and strictly entails each proposition or its negation: $\Diamond p \wedge \forall q(\Box(p \rightarrow q) \vee \Box(p \rightarrow \neg q))$ (cf. Fine (1970) and Kaplan (1970)). If there is contingency in what propositions there are, being maximal might not suffice to count as a world, since a proposition might be maximal without being necessarily maximal. Given the models of propositional contingency developed here, being necessarily maximal suffices for being counted as a world, so define the following syntactic abbreviation:

$$world(p) := \Diamond p \wedge \Box\forall q(\Box(p \rightarrow q) \vee \Box(p \rightarrow \neg q))$$

Correspondingly, a claim being true at a world can be understood as being strictly entailed by it:

$$@w\varphi := \Box(w \rightarrow \varphi)$$

Talk of there being a world satisfying a certain condition is to be understood as talk of it being possible that there is a proposition which is necessarily maximal and actually satisfies the condition. Adapting a strategy found in Fine (1977a), the following spells this out formally in a way which makes sure that "actually" is interpreted in such a way that the construction can be embedded in modal operators:

$$\exists v(world(v) \wedge v \wedge \Diamond\exists w(world(w) \wedge @v \ldots))$$

This provides a way of understanding talk of there being a world satisfying a certain condition in terms of propositional quantification without begging the question of propositional contingentism. Some other forms of quantification such as universal quantification can of course be treated similarly. But the strategy is limited; it is neither obvious how to treat a generalized quantificational claim like "there are uncountably many worlds such that ..." nor higher-order quantificational claim like "there is a binary relation among worlds such that ...".

The latter is especially worrying, since on the present proposal of understanding world-talk, quantification over relations and permutations among worlds, which the above model-theoretic discussion freely engaged in, is naturally understood in terms of higher-order quantification. This issue is most naturally investigated in the richer setting of higher-order modal logic, in which there are quantifiers over individuals, as well quantifiers over relations in a type hierarchy of relations, treating propositions as nullary relations. As mentioned in Section 5, a fuller treatment of higher-order contingentism in such a setting is developed in Fritz & Goodman (forthcoming). There, it is shown that the present worries about being able to make sense of quantification over relations among worlds are well-founded, since reformulating a theory of higher-order contingency in such a way that the relations used to formulate these theories are required to satisfy the constraints of the relevant theory themselves restricts which structures the theory admits. Whether this also restricts the patterns of indistinguishability and symmetries determined by these structures, and so puts additional restrictions on the coherence constraints developed here, is a difficult issue; see Fritz (unpublished a).

The formal results are more conclusive in the case of the generalized quantifier "there are uncountably many worlds such that ...". The results obtained in Fritz (unpublished b) show that claims of this form cannot be expressed even in a higher-order modal language in which quantifiers are available for all types of relations, all generalized quantifiers are available as primitive expressions for all types, and which is infinitary in the sense of allowing conjunctions of arbitrary sets of formulas and universal and existential quantifiers binding sets of variables of arbitrary cardinality, and containing variables of all types.

Whether these limitative results tell against the present theory of propositional contingency is not obvious, and may depend on the use to which the models developed here are put. Fritz & Goodman (unpublished b) use analogous limitations of talk of possible individuals to argue against higher-order contingentism in general, and so in particular against propositional contingentism.

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