Topic Study Group No. 46: Knowledge in/for Teaching Mathematics at the Secondary Level

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The program of TSG 46 focused on three themes:

1. Conceptualization and theorization of knowledge in/for teaching mathematics at the secondary level.
2. Methods for measuring, assessing, evaluating and comparing knowledge in/for teaching mathematics at the secondary level.
3. Connections between knowledge and practice of teaching mathematics at the secondary level.

The first three sessions centred on the three themes, while the fourth was devoted to summary, discussion and reflections. Below we describe the activities that took place during the four sessions.

Session 1: Conceptualization and Theorization of Knowledge in/for Teaching Mathematics at the Secondary Level (Chairs: Nils Buchholtz and Tim Rowland)

The problem: A number of international studies investigate the professional knowledge for teaching mathematics at the secondary level and for this purpose draw back on various different theoretical conceptualizations. Within the session the similarities and differences of different conceptualizations were analyzed and discussed, but also the current challenges of these conceptualizations were faced, especially with regard to the interaction between theoretically-assumed knowledge

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facets and their visible manifestation in the practice of teaching. The current challenge seems to be to differentiate rather better current conceptualizations for teaching mathematics at the secondary level according to the theoretically sound and empirically-based integration of action-based knowledge facets. Orienting theoretical conceptualizations more to practical school-based contexts offers a basis for empirical research that is oriented more to the realities of mathematics teaching in school. The invited contributors of the session presented promising perspectives in this field, with a valuable overview from the first presenter:

Presentation 1: *Conceptualization and Theorization of knowledge in/for teaching mathematics at secondary level*, by Michael Neubrand from the University of Oldenburg in Germany.
Presentation 2: *Academic mathematics or school mathematics? What kind of content knowledge do mathematics teachers need?*, by Aiso Heinze and Anika Dreher from IPN—Leibniz Institute for Science and Mathematics Education in Germany (together with Anke Lindmeier, IPN, Germany).
Presentation 3: *Analysing secondary mathematics teaching with the knowledge quartet*, by Tim Rowland from the Universities of Cambridge and East Anglia in the UK (together with Anne Thwaites and Libby Jared from the University of Cambridge, UK).

Session 2: Methods for Measuring, Assessing, Evaluating and Comparing Knowledge in/for Teaching Mathematics at the Secondary Level (Chairs: Charalambos Y. Charalambous and Xinrong Yang)

The problem: The last two decades have seen considerable work not only in theorizing the knowledge needed for the work of teaching mathematics, but also in operationalizing and measuring this knowledge. These last two facets pose significant challenges to scholars working on exploring teacher knowledge and its effects on instructional quality and student learning, since at least two critical questions need to be addressed when it comes to considering these issues: (a) what (aspects of teacher knowledge) to measure—especially given the multifaceted nature of (recent) teacher knowledge conceptualizations—and (b) how best to measure them to ensure that valid and reliable data are collected, and legitimate inferences are drawn. Although these questions have attracted significant scholarly interest for elementary school grades, the field of measuring teacher knowledge at the secondary school is still developing. These issues were taken up by both Session-2 presentations. The first of these pointed to the importance of focusing on the knowledge entailments of key mathematical teaching tasks as opposed to simply attending to different types of knowledge; further capitalizing on videos to measure teacher knowledge as embedded in practice; and measuring teacher knowledge in cost-efficient ways. The second presentation made a case about the importance of measuring both generic teaching tasks as well as content-specific tasks, and the knowledge entailments associated with them.

Presentation 1: *Measuring Secondary Teachers' Knowledge of Teaching Mathematics: Developing a Field,* by Heather C. Hill of the Harvard Graduate School of Education, the USA.
Presentation 2: *Measuring Instructional Quality in Mathematics Education*, by Lena Schlesinger and Armin Jentsch of the University of Hamburg, Germany.

Session 3: Connections Between Knowledge and Practice of Teaching Mathematics at the Secondary Level (Chair: Ruhama Even)

The problem: That expertise in mathematics teaching requires adequate mathematical knowledge is a trivial statement, but what "adequate" means is not clear. In many countries, the education of secondary school mathematics teachers traditionally includes a strong emphasis on advanced mathematics courses at the college or university level, taught by mathematicians, assuming that it would contribute to the quality of classroom instruction. This tradition, however, has been reconsidered in recent years, and the relevance of advanced mathematics courses to the quality of secondary school mathematics teaching is being debated. Is there a need for advanced mathematics studies in the professional education and development of secondary school mathematics teachers? What might be the relevance of advanced mathematics courses taught by research mathematicians to teaching secondary school mathematics? This issue was the focus of the three presentations in session 3, all of which reported on studies that addressed the overarching question: *What are the relevance and the contribution of advanced mathematics studies to secondary school mathematics teaching?*

Presentation 1: *Accommodation of teachers' knowledge of inverse functions with the group of invertible functions*, by Nicholas H. Wasserman from Teachers College in the USA.
Presentation 2: *Senior secondary school teachers' advanced mathematics knowledge and their teaching in china*, by Haode Zuo and Frederick K.S. Leung from the University of Hong Kong in China.
Presentation 3: *Teachers' views on the relevance of advanced mathematics studies to secondary school teaching*, by Ruhama Even from the Weizmann Institute of Science in Israel.

Session 4: Summary, Discussion and Reflections (Chairs: Xinrong Yang and Ruhama Even)

In this session, Xinrong Yang from Southwest University in China, Nils Buchholtz from the University of Hamburg in Germany, and Charalambos Charalambous from the University of Cyprus in Cyprus reflected on the first three sessions. Below is a summary of their reflections.

Current theoretical conceptualizations of knowledge in/for teaching mathematics at the secondary level primarily focus on knowledge as a personal disposition that can be tapped for empirical surveys. At the theoretical level, drawing on the seminal work of Shulman (1986), various dimensions of knowledge are often distinguished and segregated depending on assumed content-related aspects, or on aspects of practical teaching. When such knowledge is operationalized in empirical studies, it becomes

more possible to separate these different facets empirically. Michael Neubrand and Tim Rowland pointed out that the more context-oriented knowledge gets analyzed in such studies, the harder it gets to empirically differentiate the knowledge in actu from other factors such as the teacher's personality or the affective level, which leads us to look more at the performance of mathematics teachers and at classifications of situations in which mathematical knowledge surfaces in teaching.

In retrospect, thirty years after Shulman's (1986) pioneering work, we can now claim that much has been accomplished on different fronts. Reflecting on this rapidly accumulating work, Charalambos Charalambous argued that the polyphony in the different theoretical frameworks and conceptualizations advanced thus far seems to be productive; he nevertheless voiced concern as to whether this polyphony will eventually be turned into cacophony, in the sense that we might run the risk of creating a Tower of Babel when it comes to talking about, studying, and measuring teacher knowledge. He thus suggested that scholars invest more in *exploring synergies* between different conceptualizations. Given that a shift seems to be observed from studying components of teacher knowledge to investigating tasks of teaching and the knowledge requirements these tasks impose on teachers (cf. Gitomer & Zisk, 2015), the need to develop a comprehensive framework encompassing such tasks and detailing their knowledge requirements for teachers was also underlined. Finally, the merit of employing the different approaches pursued so far to study teacher knowledge was highlighted. At the same time, a series of open issues was also outlined. For example, at what level of granularity should teacher knowledge be measured to ensure both its predictive validity and generalizability? To what extent might certain measures be culturally specific, and what might the implications of this specificity be? To what extent might certain items used in teacher-knowledge measures function differently when used in different contexts? To what extent does the knowledge measured actually impact teachers' teaching practice and students' mathematics achievement? This indicative list of questions indicates that there remains significant uncharted terrain to explore when working on studying teacher knowledge, and its effects on instruction and student learning.

References

Gitomer, D. H., & Zisk, R. C. (2015). Knowing what teachers know. *Review of Research in Education, 39*(1), 1–53.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.