Cosmological perturbations from statistical thermal fluctuations

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Cosmological perturbations due to statistical thermal fluctuations in a single fluid characterized by an arbitrary equation of state are computed. Formulas to predict the scalar and tensor perturbation spectra and non-Gaussianity parameters at a given temperature are derived. These results are relevant to cosmological scenarios, such as cyclic or emergent universes, where cosmic structures may have been seeded thermally instead of originating purely from quantum vacuum fluctuations.

I. INTRODUCTION

In the standard inflationary paradigm, the seeds for cosmic structure are generated as quantum fluctuations. During inflation, the quantum fluctuations of the fields present are stretched by the cosmic expansion to macroscopic sizes and become classical [1]. These small inhomogeneities are then amplified in the later evolution of the Universe by gravitational collapse and eventually form the galaxies and other structures we observe around us today. The predictions of the simplest inflationary models can be matched with observations that require a nearly scale-invariant but slightly red-tilted spectrum with *only* upper limits having been set on gravitational waves and any deviations from the simplest statistical properties in terms of non-Gaussianity (NG) or statistical anisotropy [2].

Thermal fluctuations introduce another possible origin for small inhomogeneities and anisotropies. Thermal fluctuations are different from fluid hydrodynamical fluctuations [1,3]. In general, fluid fluctuations can arise from two different sources. There can be fluctuations in energy density and the associated temperature driven, for instance, by quantum fluctuations; this is what is traditionally discussed in the literature. However, even if one can define a unique temperature in a given volume, there are fluctuations in energy within the volume due to the statistical nature of thermal physics. These are random fluctuations in all finite-temperature systems that arise already at the classical level, and this is what is commonly referred to as thermal fluctuations.[1] In the early Universe the temperatures could be very high, and therefore these fluctuations could be significant. The reason why in typical inflationary scenarios we do not worry about these fluctuations is that once inflation begins any "preinflationary" thermal matter is expected to dilute away rapidly leaving us with an almost pure vacuum state. However, there are cosmological models where thermal fluctuations could be solely or to a significant amount responsible for the initial seeds of inhomogeneities. For instance, in cyclic inflationary scenarios [4,5], where particle/entropy production keeps up with the inflationary dilution, thermal fluctuations become relevant[2]; for some of the interesting results we have found the reader is referred to our companion paper Ref. [8]. In bouncing cosmologies, where the big bang singularity is replaced by a smooth evolution from a contracting to an expanding phase, different matter sources become important near the bounce (for a recent review see Ref. [9]), again making the thermal fluctuations relevant.

In the present study we shall not refer to particular models but rather strive for generality. The purpose is to develop the general formalism to tackle cosmological perturbations due to thermal fluctuations. We are going to make the following assumptions:

(i) The universe contains a thermal fluid in "significant" abundance, i.e. the interactions within the fluid are able

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[1]In Appendix B, we provide the condition when the statistical thermal fluctuations dominate over the fluid fluctuations, clarifying some of the physics issues in the process.

[2]For other inflationary scenarios where thermal matter is relevant see Refs. [6,7], but for such complex systems where there are multiple fluids interacting with one another we do not expect our analysis to be directly applicable.

to maintain thermal equilibrium; this requires both *kinetic* and *chemical equilibrium* (see Refs. [10–12], and for a more recent discussion Refs. [13,14]). Typically this means that the relevant scattering rates have to be larger than the Hubble expansion rate. This requirement is often referred to as the ability of the fluid to maintain "local" thermal equilibrium.

(ii) For the sub-Hubble modes, (i.e. physical wavelength smaller than the Hubble radius, or the appropriate cosmological time scale), the statistical thermal fluctuations dominate over the quantum vacuum fluctuations inherent in the fluid. We will provide a quantitative criteria for this to occur in Appendix B.

(iii) There is no significant isocurvature perturbations due to the possible presence of other fluids.

(iv) There are no anisotropic stresses in any of the fluids.

(v) There is some cosmological mechanism in place for the modes to exit from the sub-Hubble to the super-Hubble phase, after which the fluctuations evolve according to the usual hydrodynamical equations coupled to gravity.[3] For a general discussion on different ways of realizing such a mechanism see Ref. [17]. As in previous literature dealing with thermal fluctuations [15,18–20], we also assume that the transition from the sub- to super-Hubble phase is instantaneous. To study the precise nature of the transition would involve nonequilibrium thermodynamics in curved space-time which is clearly out of the scope of the present paper, but we do not expect the results to be affected beyond $\mathcal{O}(1)$ factors.

(vi) At least, near the sub to super transition we can trust general relativity (GR) and the usual laws of thermodynamics.

In previous literature thermal statistical fluctuations have been considered in a variety of contexts. The earliest

hint that these could be relevant for CMB was perhaps provided by Peebles [21], and further developed in Refs. [18,20,22] (see also Ref. [23]). Since then thermal fluctuations have found applications in several models: string cosmology [16,19], inflation (in particular warm inflation [24,25]—for reviews see e.g. Refs. [6,7]—but see also Refs. [26–29]), bouncing cosmologies [15] and the Milne/holographic universe [30–32]. In the present paper we first generalize the calculation of the curvature perturbations to the case when the thermal matter could have an arbitrary equation of state,[4] in the process clarifying several conceptual issues related to gauge choices, and the transfer of perturbations from sub- to super-Hubble phase. These generalizations can be particularly important and interesting for early-Universe cosmology where stringy thermodynamics [33–40] and/or phase transitions may be relevant, neither of which is described by a constant equation-of-state parameter which is what previous studies have been mostly confined to. Next, we provide general formulas to compute not only the scalar power spectrum, but also the spectrum of gravity waves and higher-point correlation functions. These results when applied to phase transitions in cyclic inflationary models turn out to produce interesting signatures for Planck, and will be discussed in Ref. [8].

The paper is organized as follows. In Sec. II we will derive the curvature perturbation in a universe filled with a thermal fluid with an arbitrary equation of state. We also compute the spectrum of gravity waves expected in this setup, or equivalently the tensor-to-scalar ratio. These derivations are somewhat technical, but only familiarity with standard cosmological perturbation theory is assumed.[5] In Sec. III, we will derive the non-Gaussianity parameters due to the thermal fluctuations, and in Sec. IV we illustrate the application of our formulas for a radiation-dominated contracting universe. Section V briefly concludes the paper. Appendix A concerns a technical issue of going over from real to Fourier space that is needed to make contact with the usual cosmological perturbation analysis, in Appendix B we compare the relative strengths between quantum/hydrodynamical and statistical thermal fluctuations, and in Appendix C we calculate thermal pressure fluctuations for completeness.

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[3]Most conservatively, if one only wants to consider a single-ideal-fluid scenario within GR, then the consistency of thermodynamic analysis dictates that we restrict ourselves to a contracting universe. This is because if the equation-of-state parameter ω is negative, so is the specific heat, typically indicating some sort of instability in the system, but in the expanding phase to have the modes exit one requires $\omega < -1/3$. If one admits multiple components, however, various other possibilities may open up. For instance, the presence of a cosmological constant in addition to a thermal fluid is perfectly consistent with all of our assumptions. This is why our analysis can be applied directly to the cyclic inflation model which contains a negative cosmological constant. If a positive cosmological constant is present, one could even have modes exit in the expansion phase. More generally, if one has fluids which are not interacting with each other, then one should be able to apply our analysis as long as the isocurvature perturbations can be ignored, and whether the latter is true or not needs to be checked on a case-by-case basis. A similar setup was previously considered in Refs. [15,16]. If, on the other hand, the fluids start to interact, as for example in the warm inflation scenario [6,7], the analysis becomes more involved, and we do not expect our calculations to remain valid.

II. THE CURVATURE PERTURBATION FROM THERMAL FLUCTUATIONS

A. Curvature perturbation and the appropriate gauge choice

We are going to consider a cosmological setup where the dominant fluid component of the universe is thermal, i.e.

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[4]This is going to be particularly relevant for applications to cyclic inflation models.

[5]For a very pedagogical and transparent introduction, see http://www.theory.physics.helsinki.fi/~genrel/CosPerShort.pdf.

there exists local thermal equilibrium, so that as long as the wavelengths of fluctuations are smaller than the cosmological time scale their power spectrum is determined by the thermal fluctuations in the thermal fluid. Once the modes become super-Hubble, thermal correlations over the relevant physical wavelengths can no longer be maintained; instead the fluctuations evolve according to the usual hydrodynamical differential equations coupling the metric and the matter fluctuations. Essentially, in this setup the thermal fluctuations act as initial conditions to seed the super-Hubble fluctuations.

Now, the super-Hubble modes are easy to track because they behave as zero modes and it is well known [1] that the curvature perturbation, ζ_k, remains a constant even if the equation-of-state parameter does not.[6] In fact, the above statement is true even if general relativity is not valid [44], but as long as we are only looking at adiabatic super-Hubble perturbations. This makes our analysis applicable to several bouncing/cyclic models which resort to modifying gravity to obtain a nonsingular bounce (modulo the caveats about mode mixing mentioned in the previous footnote). However, if the universe does contain more than one type of fluid/field, then isocurvature perturbations could be important, but we are not going to consider them in this paper.

Our goal in this section will be to compute ζ_k arising from thermal fluctuations in the sub-Hubble phase where thermal correlations can exist. In particular we will evaluate this at the Hubble crossing which, according to our previous discussion, will provide us with the primordial spectrum for the cosmic microwave background radiation (CMBR). In the next section, we are going to calculate the two- and the three-point correlation functions as well as the gravity-wave spectrum. We do not make any assumptions about whether we have an expanding or a contracting universe (again modulo the previous comments about mode matching) or whether there is a single thermal fluid or several energy components. However, we shall make the crucial assumption that the fluctuations are dominated by the thermal fluctuations of a single fluid, $\delta\rho$, where ρ is the energy density of the thermal fluid. We parametrize the contribution of the thermal fluid to the energy budget by

$$\Omega = \frac{a^2\rho}{3M_p^2\mathcal{H}^2}, \tag{1}$$

where a is the scale factor of the universe, $\mathcal{H} = \dot{a}/a$ is the conformal Hubble rate, M_p is the reduced Planck mass,

and here and in the following the overdot denotes the derivative with respect to conformal time. Thus, if the thermal fluid is the only component in the universe, $\Omega = 1$.

We would now like to relate the perturbations in the fluid to a gauge-invariant degree of freedom describing the metric perturbations. The scalar perturbations in the metric can be parametrized as [1]

$$ds^2 = a^2(\eta)[-(1 + 2\phi)d\eta^2 + B_{,i}d\eta dx^i$$
$$+ ((1 - 2\psi)\delta_{ij} + E_{,ij})dx^i dx^j]. \tag{2}$$

We also need to parametrize the perturbations in the matter content, which will be treated as a perfect fluid, so no anisotropic stresses are present. Then the energy-momentum tensor can be written as

$$T^0{}_0 = -\rho(1 + \delta), \quad T^0{}_i = \rho(1 + w)v_{,i}, \quad T^i{}_j = \rho(w + c_s^2\delta). \tag{3}$$

Since we assume a thermal fluid, all the background quantities are given solely by the temperature,

$$\rho = \rho(T), \qquad p = p(T), \tag{4}$$

and so is $w(T)$ and $c_s^2(T)$.[7] In the following the prime will always denote the derivative with respect to the temperature, and we often drop the explicit argument T. An overdot will refer to a derivative with respect to the conformal time η.

Now we want to calculate the gauge-invariant curvature perturbation, which in a general gauge reads as

$$\zeta = -\Psi - \mathcal{H}(v - B). \tag{5}$$

Since it is a gauge-invariant quantity we can evaluate it in any gauge of our choice. Let us choose to work in the longitudinal gauge where $E = B = 0$. In this gauge the metric can be written in terms of the gauge-invariant Bardeen potentials

$$\Phi \equiv \phi - \frac{1}{a}\left[\left(-B + \frac{\dot{E}}{2}\right)a\right]^\bullet \tag{6}$$

and

$$\Psi \equiv \psi + \frac{1}{6}\nabla^2 E - \frac{\dot{a}}{a}\left(B - \frac{\dot{E}}{2}\right) \tag{7}$$

as

$$ds^2 = a^2(\eta)[-(1 + 2\Phi)d\eta^2 + (1 - 2\Psi)dx^2], \tag{8}$$

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[6]In a contracting universe one of the two modes of ζ_k is growing (the one which is decaying in an expanding universe), while the second is constant [41,42]. We will here assume that the growing mode in the contracting phase couples only to the decaying mode in the expanding phase. Whether this is the case or not depends on the specific model of the transition between contraction and expansion. For a discussion of this issue see e.g. Ref. [43].

[7]Unlike in the case of hydrodynamical fluctuations, the pressure fluctuation is not given by the adiabatic value, $\delta p = (p'(T)/\rho'(T))\delta\rho$, but is rather determined via stress-energy conservation; see Appendix C for the precise formula. We will see shortly that to compute the spectra we do not need the explicit form of the sound speed squared. However, as the modes exit the horizon, the perturbations are expected to relax to their adiabatic value which guarantees the constancy of the curvature perturbation at large scales.

and the $0i$ component of the Einstein's equations determines v in terms of the Bardeen potentials,

$$\dot{\Psi} + \mathcal{H}\Phi = \frac{a^2}{2M_p^2}(1 + w)\rho v. \tag{9}$$

Using this in Eq. (5) we have that

$$\zeta = -\Psi - \frac{2M_p^2\mathcal{H}}{(1 + w)a^2\rho}(\dot{\Psi} + \mathcal{H}\Phi). \tag{10}$$

Please note that the above equation is written in a completely gauge-invariant form and is therefore valid in any gauge. This is important for us because it will become necessary for us to switch to the comoving gauge on physical grounds. This is actually a subtle issue which, to our knowledge, has not been explained before. The point is that all thermodynamic calculations, such as those relevant when we will derive the energy fluctuations in a given volume, are typically carried out in Minkowski space-time. In order to generalize the analysis to the Friedmann-Lemaître-Robertson-Walker metric (or any other metric for that matter) one has go to a frame where the background fluid is at "rest" [45], which is none other than the comoving gauge. This gives a gauge-invariant definition of $\delta\rho$ which is consistent with the Minkowski calculations; it is related to the Bardeen potential via the relativistic Poisson equation,

$$\Psi = -\frac{1}{2}\left(\frac{a}{kM_p}\right)^2\delta\rho^C. \tag{11}$$

The superscript "C" refers to the comoving gauge which we are going to subsequently drop as all the thermodynamic calculations implicitly assume this same gauge choice for the perturbation in the matter density field.

At this point it is useful to set $\Phi = \Psi$, since we assumed that the anisotropic stresses can be neglected. We can then compute $\dot{\Phi}_k$ in terms of the density fluctuations from Eq. (11) and substitute it in Eq. (10) to obtain

$$\zeta = \frac{1}{2}\left(\frac{a}{kM_p}\right)^2\left[1 + \frac{2M_p^2H^2}{(1 + w)\rho}(3 + r)\right]\delta\rho, \tag{12}$$

where the time evolution of the density fluctuation is parametrized as

$$r = \frac{d\log\delta\rho}{d\log a} = \frac{(\delta\rho)'}{\delta\rho}\frac{\dot{T}}{\mathcal{H}}. \tag{13}$$

We remind the readers that the prime corresponds to the derivative with respect to the temperature.

More succinctly,

$$\zeta_k = \frac{A(T_k)}{H_k^2M_p^2}\delta\rho_k, \tag{14}$$

where we have defined a time-/temperature-dependent proportionality coefficient

$$A(T) \equiv \frac{1}{2}\left[1 + \frac{2(3 + r)}{3(1 + w)\Omega}\right], \tag{15}$$

for later convenience. These quantities will depend on the temperature at the time of the "exit" of a given comoving mode.

B. Thermal density fluctuations

We will now use the thermodynamics to quantify fluctuations in the fluid and then use the results of the previous section to relate them to the metric perturbation spectra. This is more of a review of what has been discussed in the previous literature [15,19,20], and our results agree to within $\mathcal{O}(1)$ factors, until a crucial step highlighted at the end of this subsection.

One defines the average fluctuation in energy, ΔE, via

$$\langle\Delta E\rangle_L^2 \equiv \langle E^2\rangle - \langle E\rangle\rangle^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial\beta^2} - \left(\frac{1}{Z}\frac{\partial Z}{\partial\beta}\right)^2 = \frac{\partial^2\ln Z}{\partial\beta^2}$$

$$= -\frac{\partial\langle E\rangle}{\partial\beta} = T^2 C_L, \tag{16}$$

$$\langle\delta\rho^2\rangle_L = \frac{T^2 C_V}{L^6} = \frac{T^2}{L^3}\frac{\partial\rho}{\partial T}, \tag{17}$$

where C_L is the heat capacity of the thermal system for a given volume L^3. Note we have also introduced a subscript L in $(\Delta E)_L^2$ to denote that we are considering fluctuations in a given volume.

The next step is to go from real space to momentum space. This is a tricky procedure as it depends to some extent on the window function one chooses. In Appendix A, we consider in detail this procedure using a Gaussian window function[8] and obtain

$$\delta\rho_k^2 = \frac{\gamma^2}{k^3}\langle\delta\rho^2\rangle_{L=a/k} \quad \text{with} \quad \gamma = 2\sqrt{2}\pi^{3/4} \approx 6.7. \tag{18}$$

Thus we have

$$\delta\rho_k^2 = \frac{\gamma^2}{a^3}T^2\rho', \tag{19}$$

leading to

$$\zeta_k^2 = A^2(T_k)\frac{\gamma^2}{a^3}\frac{T^2\rho'}{H_k^4 M_p^4}, \tag{20}$$

and eventually

$$\mathcal{P}_\zeta = k^3\langle\zeta_k^2\rangle = A^2(T_k)\gamma^2\frac{T_k^2\rho_k'}{H_k M_p^4}$$

$$= \sqrt{3\Omega}\gamma^2 A^2(T_k)\frac{T_k^2\rho_k'}{M_p^3\sqrt{\rho_k}}, \tag{21}$$

using the standard definitions of the power spectrum. The subscript k refers to the fact that all these quantities have to

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[8]Different schemes typically yield slightly different values for γ.

be evaluated at the Hubble crossing condition, $H_k = k/a$, which we have also used along with Eq. (1) to eliminate the Hubble factors. A few comments are now in order. Firstly, as in all previous literature on the subject, in deriving Eq. (21) we have implicitly assumed an instantaneous transition from the thermally correlated sub-Hubble phase to the hydrodynamical super-Hubble phase. This is obviously not realistic but a careful investigation of such a transition is very challenging because it will involve non-equilibrium thermodynamics on curved space-time, which is clearly out of the scope of the present paper.

Secondly, one can see that the factor $A(T)$ basically tells us what is the difference between the spectra of the gravitational potential (or, via the Poisson equation, the density perturbation) and the spectra of the gauge-invariant curvature perturbation. It is crucial to take this into account: one can imagine physical situations where A can even vanish or diverge. This is where our results differ significantly from previous studies which typically only computed Φ_k and are directly applicable only for constant equation-of-state parameters when making comparisons with observations.

C. The prefactor $A(T)$

The last missing piece required to obtain the power spectrum is an expression for $A(T)$. Explicitly, our definition is

$$A(T) = \frac{3(1 + w)\Omega + 2(3 + r)}{6(1 + w)\Omega}. \qquad (22)$$

w can be obtained straightforwardly as a function of temperature from the partition function, or $p(T)$. A useful thermodynamic relation in this context is

$$\rho(T) = T\frac{dp(T)}{dT} - p(T), \qquad (23)$$

so that

$$w = \frac{p}{Tp' - p}. \qquad (24)$$

The computation of Ω at the exit temperature depends on the specific model under consideration, and one cannot make any further simplifications at this point. Obviously, if the thermal fluid is the only energy component in the universe, then $\Omega = 1$.

We are finally left with the evaluation of r. From the expression we obtained for thermal energy density fluctuations (19) we first find

$$r = -\frac{3}{2} + \left(\frac{2\rho' + T\rho''}{2\rho'}\right)\frac{d\ln T}{d\ln a}.$$

Now, recalling the continuity equation

$$\dot{\rho} + 3\mathcal{H}(1 + w)\rho = 0, \qquad (25)$$

we see

$$\frac{d\ln T}{d\ln a} = -3(1 + w)\frac{\rho}{T\rho'}. \qquad (26)$$

Thus we finally have

$$r = -\frac{3}{2}\left[1 + \frac{(1 + w)\rho(2\rho' + T\rho'')}{T\rho'^2}\right]. \qquad (27)$$

Note that the sign of this also remains the same in the contracting phase; then the temperature is getting lower with time, but the Hubble rate is also negative. We will illustrate the computation of the power spectrum for the special case of radiation towards the end of the next section.

D. Gravity waves

Another interesting probe of our early Universe are the primordial gravitational waves, which are stretched like the scalar perturbations. However, for a linearized Einstein's gravity there is no source term for the gravitational waves. In principle the initial conditions for the gravitational waves could be set purely classically [46] or from the quantum vacuum condition [1].[9] Assuming that the initial conditions for the primordial gravitational waves are set by quantum vacuum, i.e. Bunch-Davis, the gravitational-wave spectrum is given by

$$\mathcal{P}_h = \frac{1}{4\pi^2}\left(\frac{H}{M_p}\right)^2 = \frac{\rho}{12\pi^2 M_p^4\Omega}. \qquad (28)$$

The tensor-to-scalar ratio is then given by

$$r_{t/s} \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = \frac{1}{\gamma^2}\frac{1}{12\sqrt{3}\pi^3\Omega^{\frac{3}{2}}A^2(T)}\frac{\rho^{\frac{3}{2}}}{M_pT^2\rho'}. \qquad (29)$$

In general, the temperature dependence of this and the scalar spectra depend very nonlinearly on the properties of the thermal fluid, but these are straightforward to compute once we know $\rho(T)$.

III. NON-GAUSSIANITY: BI- AND TRISPECTRUM FOR THE CURVATURE PERTURBATIONS

In the previous section we evaluated the CMB power spectrum as a two-step process. In Sec. II B we calculated the thermal density fluctuations from the partition function (or pressure) governing the thermodynamics of the fluid in the comoving gauge in which the fluid is "at rest" and therefore the Minkowski space-time calculations can be applied [45]. In Sec. II A we found how these thermal fluctuations are related to the curvature fluctuations, which then allowed us to obtain the two-point correlation function in the CMB. We can apply the same prescription to obtain higher-point correlation functions—we just have to compute the appropriate higher thermodynamic cumulants

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[9]In fact, any thermal matter can only act as sources of gravitational waves if its partition function is nonextensive [19,34], a scenario not considered here.

(see also Ref. [18] for an earlier study of non-Gaussianities from thermal fluctuations).

The third- and the fourth-order centered cumulants are given by

$$-\frac{\partial^3 \ln Z}{\partial \beta^3} = \langle E^3 \rangle - 3\langle E^2 \rangle \langle E \rangle + 2\langle E \rangle^3 \equiv \langle \Delta E^3 \rangle, \quad (30)$$

$$-\frac{\partial^4 \ln Z}{\partial \beta^4} = \langle E^4 \rangle - 4\langle E^3 \rangle \langle E \rangle + 6\langle E^2 \rangle \langle E \rangle^2 - 4\langle E \rangle \langle E \rangle^3$$
$$\equiv \langle \Delta E^4 \rangle. \quad (31)$$

From the above thermodynamics we infer that

$$\langle \delta\rho^3 \rangle_L = \frac{T^3(2\rho' + T\rho'')}{L^6}, \quad (32)$$

$$\langle \delta\rho^4 \rangle_L = \frac{2T^4(3\rho' + 3T\rho'' + \rho''')}{L^9}, \quad (33)$$

where one considers fluctuations in a box of size L. These formulas hold in the real space, but can be converted to momentum space using window functions as before[10]; we have

$$\langle \delta\rho^3 \rangle = \frac{\gamma^3}{k^{\frac{9}{2}}}\langle \delta\rho^3 \rangle_L, \qquad \langle \delta\rho^4 \rangle = \frac{\gamma^4}{k^6}\langle \delta\rho^4 \rangle_L. \quad (34)$$

Using the standard definitions of the spectrum and non-Gaussianity parameters (see Refs. [2,47]), we are ready to write down the results using Eq. (20) in Eq. (34),

$$f_{NL} = \frac{5}{8}k^{-\frac{3}{2}}\frac{\langle \xi_k^3 \rangle}{\langle \xi_k^2 \rangle^2} = \frac{1}{\Omega\gamma A(T)}\left[\frac{5\rho(2\rho' + T\rho'')}{24T(\rho')^2}\right]$$
$$\equiv \frac{F(T)}{\Omega\gamma A(T)}, \quad (35)$$

$$g_{NL} = \frac{25}{54}k^{-3}\frac{\langle \xi_k^4 \rangle}{\langle \xi_k^2 \rangle^3}$$
$$= \frac{1}{\Omega^2\gamma^2 A^2(T)}\left[\frac{25\rho^2[3(\rho' + T\rho'') + T^2\rho''']}{243T^2(\rho')^3}\right]$$
$$\equiv \frac{G(T)}{\Omega^2\gamma^2 A^2(T)}. \quad (36)$$

In deriving the expressions for the non-Gaussianity parameters we have used Eq. (14) to relate fluctuations in the energy density to the curvature fluctuations, and also Eq. (1) to eliminate the Hubble factors.

Physically, the most important aspect about the thermal NG parameters is that they depend on how the pressure/density varies as a function of the temperature. Moreover,

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[10]The relations to the Fourier-space spectra are tricky, and the γ factors for the different correlation functions could be different depending upon the window functions used, but the difference is only expected to provide $\mathcal{O}(1)$ modulations.

the higher the order of the correlation function, the higher the derivatives that becomes relevant. It is not hard to realize then that if the exit temperature is close to a thermal phase transition, we might be able to see its effects in the enhancement of NG parameters. This is precisely what we observed in the context of a cyclic inflation scenario [8]. For the present, we will just illustrate the various computations of the cosmological observables by considering the ordinary relativistic fluid. In particular, we will see that for pure radiation, the non-Gaussianities and the tensor-to-scalar ratio are too small to be observed by the Planck experiment.

IV. EXAMPLE: RADIATION

For the purpose of illustration let us consider a radiation-dominated contracting universe. Since in this case the Hubble radius contracts faster (as $1/t$) than the physical wavelengths (as $1/\sqrt{t}$), the latter is pushed out of the Hubble radius and the various perturbative spectra become imprinted at the time of the mode exit. We can use the formulas of the previous section to compute the different cosmological observables.

For relativistic radiation fluid we have that

$$\rho(T) = gT^4 \quad \text{and} \quad p(T) = \frac{g}{3}T^4, \quad (37)$$

where g depends on the number of relativistic degrees of freedom. It follows immediately that

$$w = \frac{1}{3} \quad \text{and} \quad r = -4, \Rightarrow A = \frac{1}{4}. \quad (38)$$

In general, these parameters need not be constant, but they happen to be in this simple case, or whenever pressure is a power law in temperature.

The amplitude of the primordial spectrum, according to our convention, is then given by

$$\mathcal{P}_\zeta = \frac{\sqrt{3g}\gamma^2}{4}\left(\frac{T}{M_p}\right)^3, \quad (39)$$

where T corresponds to the temperature when the given mode exit becomes super-Hubble. Evidently, the spectrum is not scale invariant because the amplitude depends strongly on the temperature, and $T \propto 1/a$ giving rise to a very large blue tilt. We should point out that our claim that Eq. (39) is the primordial spectrum relevant for CMBR relies crucially on the fact that there is no mixing between the mode of ζ_k which is growing in the contracting phase with the dominant mode in the expanding phase—the constant mode—and in addition on the assumption that there are no entropy modes which become important and which could change the spectrum of the curvature fluctuations on super-Hubble scales. If there is unsuppressed mixing between the growing mode in the contracting phase and the constant mode in the expanding phase (see Ref. [48] for examples where this is the case), then the

amplitude of the resulting curvature fluctuations changes, but the spectrum remains as given by Eq. (39). The reason that there is no change in the shape of the spectrum (in contrast to the case of a matter-dominated phase of contraction where the index of the power spectrum changes by -2; see Refs. [41,42]) comes from the fact that for a radiative equation of state the canonical fluctuation variable v [49,50] which is related to ζ via $\zeta \sim a^{-1} v$ has a vanishing squeezing factor and hence remains conserved. Thus, there is no preferential growth of long-wavelength modes compared to short-wavelength modes which would come from the fact that long-wavelength modes spend more time on super-Hubble scales in the contracting phase. For a discussion of this point see Ref. [48].

For radiation we obtain the following numbers for the non-Gaussianity parameters:

$$f_{NL} = \frac{25}{24\gamma} \approx 0.16, \tag{40}$$

$$g_{NL} = \frac{25}{216\gamma^2} \approx 0.003. \tag{41}$$

Not surprisingly for radiation, which is free from any internal scales, both the f_{NL} and the g_{NL} parameters turn out to be scale invariant. The above approximate values correspond to the γ value for the Gaussian window function (see Appendix A) which is unfortunately beyond Planck's sensitivity.

Let us compute the tensor-to-scalar ratio for radiation. We readily obtain

$$r_{t/s} = \frac{4\sqrt{g}}{75\sqrt{3}\pi^3 \gamma^2} \frac{T}{M_p}. \tag{42}$$

For a given temperature, we can fix the unknown g by matching the amplitude of perturbations (39) with the observed one. This then fixes the tensor-to-scalar ratio (42). In other words, we can deduce the primordial temperature from observations,

$$\frac{T}{M_p} = \frac{8}{75\gamma^2 \pi^{\frac{3}{2}}} \sqrt{\frac{2A_0}{r_{t/s}}} = \frac{1}{75\pi^3} \sqrt{\frac{2A_0}{r_{t/s}}} > 6.1 \times 10^{-8}. \tag{43}$$

In the second equality we have used the Gaussian window value for γ, and the lower bound is obtained from the present best-fit WMAP value for the amplitude $A_0 = 2.4 \times 10^{-9}$ and the bound $r_{t/s} < 0.24$, which both apply at the scale $k = 0.002/\text{MpC}$. The minimal allowed temperature corresponds to a huge number of effective degrees of freedom, $g \sim 10^{22}$. For $g \sim 1$ of order unity, the observed amplitude requires $T/M_p \sim 10^{-4}$, in which case the tensor-to-scalar ratio will be too low to be measured.

To conclude, thermal fluctuations in usual radiation cannot account for the CMBR spectrum as the spectrum is heavily tilted, not surprisingly. We also found that such fluctuations cannot produce large non-Gaussinities or

gravity-wave signals. Similar conclusions hold for simple polytropic thermal fluids, but as we will see in Ref. [8] richer thermodynamics may indeed yield detectable non-Gaussianities.

V. CONCLUSIONS

We considered statistical thermal fluctuations as a possible source for cosmological large-scale structures. We presented a robust derivation of scalar and tensor spectra in this context. We also explicitly provided the formulas for the bi- and trispectrum, and outlined the procedure which is straightforward to implement in order to obtain non-Gaussianity at an arbitrary order. The results were applied for the case of radiation for illustration, and they are easily applied to any other fluid, given nothing but its thermodynamic properties specified by the equation of state, or equivalently, the temperature dependence of its energy density. Fundamentally, these follow from the partition function.

Another question is whether there are realistic cosmological models in which statistical thermal fluctuations are dominant instead of the usual quantum fluctuations to seed the large-scale structures. We believe the cyclic inflationary scenario presents a plausible framework where this indeed turns out to be the case, and in a companion paper we shall apply the results obtained here to study this scenario in detail and show that there are parameter regions compatible with the present observations and falsifiable predictions for both the tensor-to-scalar ratio and non-Gaussianity.

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APPENDIX A: WINDOW FUNCTION

Here we are going to take a closer look at how the fluctuations in energy in a given physical volume are converted to the power spectrum. We should clarify that this is essentially a review of what has already been discussed in the literature [15,18–20]; although the algorithm for converting real-space fluctuations to momentum space is a well-known subject and included in several books [51], since it is an important part of the discussion on thermal fluctuations, we felt it would be convenient for the readers to include this sub-derivation and make the overall presentation self-contained. For a slightly different approach to the problem, see Ref. [23]. To begin with we note that

the thermodynamic computation of energy fluctuations is done in an "adiabatic" approximation scheme where we ignore the cosmological evolution. Thus δ_L tells how in a given "Euclidian" time slice the "relative energy" in a given physical volume L^3 fluctuates. The question we are interested in asking is that for a given time slice, if we know how δ_L depends on L, how we can compute the Fourier components of relative density fluctuations in that same time slice. To understand such a "kinematical/statistical" relationship we can therefore work with physical coordinates and momenta. Once we derive the relationship, it is relatively easy to convert the results into "comoving" language, which is more useful for cosmology.

To keep things finite we first choose a "fiducial" volume in our universe, $\bar{V}$, which is big enough that it contains all the relevant scales we are interested in i.e., $L \ll \bar{L}$. We are not going to provide all the rigorous details/justifications for doing this, which are presented in Ref. [51]. We are also going to assume periodic boundary conditions, so that physical momenta are confined to integral values,

$$\mathbf{k} = \frac{2\pi}{\bar{L}}\mathbf{n}. \tag{A1}$$

We can now define the Fourier components via

$$\tilde{\delta}_n = \frac{1}{\bar{V}} \int_{\bar{V}} \delta(x) \exp\left(i2\pi\mathbf{n} \cdot \mathbf{x}/\bar{L}\right) d^3x, \tag{A2}$$

$$\delta(x) = \sum \tilde{\delta}_n \exp\left(-i2\pi\mathbf{n} \cdot \mathbf{x}/\bar{L}\right), \tag{A3}$$

and the relative density fluctuations are defined in the usual way,

$$\delta(x) \equiv \frac{\rho(x) - \langle\rho\rangle}{\langle\rho\rangle}, \quad \text{where } \langle\rho\rangle \equiv \frac{\int_{\bar{V}} \rho(x) d^3x}{\bar{V}}. \tag{A4}$$

Now, what we are interested is to obtain a statistical measure of δ. However, statistically one expects

$$\langle\delta\rangle = 0. \tag{A5}$$

Thus, one has to obtain an estimate of $\delta_\mathbf{k}$ using a root-mean-square approach. The standard approach is to use the correlation function,

$$\zeta(\mathbf{r}) \equiv \langle\delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r})\rangle_{\bar{V}} = \sum_{\mathbf{n}\mathbf{n}'}\left\langle\tilde{\delta}_\mathbf{n}\tilde{\delta}_{\mathbf{n}'}\exp\left[-\frac{i2\pi\mathbf{n} \cdot (\mathbf{x} + \mathbf{r})}{\bar{L}}\right]\exp\left[-\frac{i2\pi\mathbf{n}' \cdot \mathbf{x}}{\bar{L}}\right]\right\rangle$$

$$= \sum_{\mathbf{n}\mathbf{n}'}\left\langle\tilde{\delta}_\mathbf{n}\tilde{\delta}_{\mathbf{n}'}^*\exp\left[-\frac{i2\pi(\mathbf{n} - \mathbf{n}') \cdot \mathbf{x}}{\bar{L}}\right]\exp\left[-\frac{i2\pi\mathbf{n} \cdot \mathbf{r}}{\bar{L}}\right]\right\rangle = \sum_{\mathbf{n}}\langle|\tilde{\delta}_n|^2\rangle\exp\left[-\frac{i2\pi\mathbf{n} \cdot \mathbf{r}}{\bar{L}}\right]. \tag{A6}$$

Our final step is to go from the discretized momentum space to a continuum limit ($\bar{V} \to \infty$ limit). To achieve this, as is always done in statistical mechanics, we have to convert sums into integrals and go from the n-space to $\mathbf{k}$-space,

$$\sum_{\mathbf{n}}\langle|\tilde{\delta}_\mathbf{n}|^2\rangle \to \int d^3n\langle|\tilde{\delta}_\mathbf{n}|^2\rangle \equiv \int d^3k\,\delta_\mathbf{k}^2 \Rightarrow \delta_\mathbf{k}^2 = \frac{\bar{V}}{(2\pi)^3}\langle|\tilde{\delta}_\mathbf{n}|^2\rangle, \tag{A7}$$

since the n- and k-spaces are related via the density of states,

$$d^3n = \frac{\bar{V}}{(2\pi)^3}d^3\mathbf{k}. \tag{A8}$$

Thus the correlation function can now be expressed as

$$\zeta(r) = \sum_{\mathbf{n}}\langle|\tilde{\delta}_\mathbf{n}|^2\rangle\exp\left[-\frac{i2\pi\mathbf{n} \cdot \mathbf{r}}{\bar{L}}\right]$$

$$= \frac{1}{(2\pi)^3}\int d^3k\,\delta_\mathbf{k}^2\exp\left(-i\mathbf{k} \cdot \mathbf{r}\right). \tag{A9}$$

In other words, $\delta_\mathbf{k}^2$ is just the Fourier transform of the correlation function. Since the correlation function is a physical quantity i.e., it does not depend on the fiducial

volume over which the averaging is performed, it is also clear that $\delta_\mathbf{k}^2$ is also a physical quantity.

Using a similar analysis one can also statistically compute the mass/energy variance in a given physical volume. We find (the derivation is straightforward and given in Sec. 13.3 of Ref. [51])

$$\delta_L^2 = \sum_n \tilde{\delta}_\mathbf{n}^2 W^2(2\pi nL/\bar{L}), \tag{A10}$$

where the window function is defined as

$$W[\mathbf{k}L] \equiv \left[\frac{1}{V}\int_V \exp\left[i\mathbf{k} \cdot \mathbf{y}\right]d^3y\right]. \tag{A11}$$

Again we can pass from the discrete n-space to the continuum momentum k-space to find

$$\delta_L^2 = \frac{1}{(2\pi)^3}\int d^3k\,\delta_\mathbf{k}^2 W^2(\mathbf{k}L)$$

$$= \frac{1}{2\pi^2}\int d\mathbf{k}\,k^2\delta_\mathbf{k}^2 W^2(\mathbf{k}L) \equiv \frac{1}{2\pi^2}\int d\mathbf{k}\,k^2 P(\mathbf{k})W^2(\mathbf{k}L). \tag{A12}$$

One can write down an analytical expression for the window function (A11) in terms of Bessel functions. However, to avoid some technical complications in

previous studies other window functions are often used, such as

$$W_G(\mathbf{k}L) \equiv \exp\left[-\frac{1}{2}(\mathbf{k}L)^2\right]. \tag{A13}$$

We emphasize that in the above formulas $\mathbf{k}$ and L represent physical momenta and lengths respectively. The inversion thus gives us

$$\delta_{\mathbf{k}}^2 = P(\mathbf{k}) \sim \frac{T^2 \partial \rho}{\rho^2 \partial T}. \tag{A14}$$

The final step involves going from the physical Fourier components to the comoving Fourier modes. The two are defined via

$$\Phi_{\vec{k}}^2 = \frac{1}{(2\pi)^3} \int d^3\vec{x} \, \Phi^2(\vec{x}) e^{i\vec{k}\cdot\vec{x}}, \tag{A15}$$

$$\Phi_{\mathbf{k}}^2 = \frac{1}{(2\pi)^3} \int d^3\mathbf{x} \, \Phi^2(x) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{A16}$$

where we now denote the physical coordinates as vectors $\vec{x} = a(t)\mathbf{x}$. It is now clear that

$$\Phi_{\mathbf{k}}^2 = a^{-3}(t)\Phi_{\vec{k}}^2. \tag{A17}$$

APPENDIX B: QUANTUM VS THERMAL FLUCTUATIONS

To estimate the relative contributions to the density perturbations originating from thermal statistical vs the quantum vacuum fluctuations, let us consider an ideal fluid with a constant equation-of-state parameter $0 < w < 1$. This corresponds to having a polynomial pressure as a function of temperature,

$$p(T) = m^4\left(\frac{T}{m}\right)^{\frac{1+w}{w}} \quad \text{and} \quad p = w\rho, \tag{B1}$$

where m is a mass scale associated with the fluid. Thus according to our formula for the spectrum, its parametric dependence on the temperature is given by

$$\mathcal{P}_{\text{th}} \propto \left(\frac{m}{M_p}\right)^3 \left(\frac{T}{m}\right)^{\frac{3w+1}{2w}}. \tag{B2}$$

In contrast, the quantum vacuum fluctuations yield a power spectrum that is proportional to the density of the background fluid [1],

$$\mathcal{P}_{\text{vac}} \propto \left(\frac{\rho}{M_p^4}\right) \sim \left(\frac{m}{M_p}\right)^4 \left(\frac{T}{m}\right)^{\frac{1+w}{w}}. \tag{B3}$$

We immediately notice that the vacuum fluctuations have an extra Planck suppression,

$$\mathcal{R} \equiv \frac{\mathcal{P}_{\text{vac}}}{\mathcal{P}_{\text{th}}} \sim \left(\frac{T}{M_p}\right)^{\frac{1-w}{2w}} \left(\frac{m}{M_p}\right)^{\frac{3w-1}{2w}}. \tag{B4}$$

For instance, for radiation, since $w = 1/3$, we have

$$\mathcal{R} \sim \frac{T}{M_p}. \tag{B5}$$

More generally, for the range $1 \geq w \geq 1/3$, since both the exponents in Eq. (B4) are positive, as long as T, $m < M_p$, the vacuum fluctuations are suppressed as compared to the random thermal fluctuations. If $1/3 \geq w > 0$, then depending upon the value of w, if T is sufficiently larger than m, the vacuum fluctuations may be able to dominate over the thermal fluctuations. However, in most physical scenarios one expects $T \lesssim m$ for the validity of the physics involving the thermal fluid. Thus in most physical scenarios we actually expect the thermal fluctuations to dominate the show, but the formal condition for this to happen is given by

$$\left(\frac{m}{M_p}\right)\left(\frac{T}{m}\right)^{\frac{1-w}{2w}} < 1. \tag{B6}$$

APPENDIX C: PRESSURE PERTURBATION

For a fluid in thermal equilibrium, the thermodynamic properties of the fluid determine the density fluctuation. This, through Einstein's field equations it is related to the gravitational potential, which in turn determines the pressure fluctuation. We can thus compute the explicit form of the latter. This is a straightforward but tedious task and we omit the details of the algebra here.[11] The result is

$$
\begin{aligned}
c_s^2 &= \frac{\delta p}{\delta \rho} \\
&= \frac{a^2 \rho}{4k^2 M_p^2 T^2 (\rho')^4}[9\rho^2 T^2 (w+1)^2 (\rho'')^2 + T^2(5w-2)(\rho')^4 \\
&\quad + \rho(w+1)(\rho')^2(T^2(1-15w)\rho'' + 24\rho(w+1)) \\
&\quad - 2\rho T(w+1)(15w+2)(\rho')^3 \\
&\quad - 6\rho^2 T(w+1)^2 \rho'(\rho^{(3)}T - 2\rho'')].
\end{aligned}
\tag{C1}
$$

For radiation this yields $c_s^2 = (2/3)(aT^2/kM_p)^2$. This is certainly different from c_{adi}^2, which again highlights the difference between thermal and hydrodynamic perturbations. In particular, the properties of the fluid and the background expansion need to be taken into account (both of which in principle are determined by the temperature), but in addition the relation is scale dependent. In fact, approaching the small-volume limit $k \to \infty$ the pressure fluctuation becomes negligible, and at the largest-scale limit $k \to 0$ the result formally diverges, but of course thermal correlations are only expected to exist as long as $k/a < H$.

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[11]A convenient starting point is Eq. (15.21) in http://www.theory.physics.helsinki.fi/~genrel/CosPerShort.pdf.

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