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VISCOSITY OF SUSPENSION: SOME ASPECTS OF THE PROBLEM

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Viscosity of suspension: some aspects of the problem

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Abstract

The influence of the formation and destruction of the agregates on viscosity of magnetic fluid is considered. The kinetic equations are used to describe the process of formation and destruction of the agregates. The coefficient of kinetic equations are calculated for shear and uniacial extensional or compression flows.

1 Introduction

The coagulation of particles is one of the intereseting phenomena in suspension. This phenomenon is studied by a lot of authors in different articles, e.g. [1–7]. Influence of the different forces of interaction (the forces of attraction, the forces of repulsion), the hydrodynamical interaction, Brownian motion and flow of liquid on process of coagulation is studied usually. The problem of coagulation is very important not only for studying the effect of instability of suspension. The mechanical properties of suspension, viscosity in particular, are changed by coagulation also. It is well-known that viscosity of suspension depends on form and size of particles. But the particles can form the agregates in process of coagulation. The destruction of the agregates by mechanical forces (in particular, by viscous forces) can take place in suspension also. That is why destraction of agregates by viscous forces is a very interesting theoretical problem and solution of it may be used in practical applications.

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2 General equations and assumptions

Let us have a suspension in which the formation and destruction of agregates take place. We shall consider the agregates as the spheroids with basic dimensions a and b $(a \geq b)$. Let Γ be the volume fraction of particles and agregates of particles in suspension and $\Gamma \ll 1$. The particles and agregates of particles are immersed in incompressible fluid of viscosity η_0. Viscosity of suspension η may be presented by following expression

$$\eta = \eta_0(1 + \alpha_1 n_1 + \alpha_2 n2 + \cdots + \alpha_N n_N) \qquad (1)$$

Here, n_i is the number of particles or agregates of the same form and dimensions in unit of volume of fluid, α_i is a coefficient which depends on form and size of particles or agregates. The number n_i of particles and agregates of the same form and sizes is changed and we must write the kinetic equations for it. The Smoluchowski kinetic equations are usually used when the coagulation in suspension is considered. The equations are written as [1,6]

$$\frac{\partial n_i}{\partial t} = -\sum_j A_j n_j n_i + \sum_k B_k n_k n_{i-k} \qquad (2)$$

These equations describe the formation of the agregates only. We must write the new addends in equation (2) if we want to describe the process of destruction of the agregates. The kinetic equation for common case is written as

$$\frac{\partial n_i}{\partial t} = -\sum_j A_j n_j n_i + \sum_k B_k n_k n_{i-k} + \sum_l C_{il} n_l - D_i n_i\,; \quad (k<i,\ l>i) \qquad (3)$$

Having used the equations (3) and (1) we get the expression for viscosity of suspension. But the calculation of the coefficients A_j, B_k, C_{il}, D_i is a very difficult problem. The expressions for coefficients A_j, B_k are considered in the works [1,6,7]. The coefficient D_{il} was calculated in the work [9]. We shall calculate coefficients C_{il}, D_i using another assumption than the one used in work [9].

Let the ambient flow field arround a spheroid have velocity $\vec{u}$ which can be characterized instantaneously by a uniform rate-of-strain tensor

$$\gamma_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and a rigid-body rotation with angular velocity $\vec{\omega}$

$$\omega_i = -e_{ijk}\omega_{jk}$$

$$\omega_{jk} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_k} - \frac{\partial u_k}{\partial x_j}\right)$$

We may write the following expression for the velocity $\vec{u}$

$$u_i = \gamma_{ij}x_j + \omega_{ij}x_j$$

The orientation of spheroid in the flow of fluid is defined by vector $\vec{e}$ along the large axis a. The fluid acts on the surface of the spheroid by a viscous force. The expression of the viscous force acting on a unit area of surface may be written as [8]

$$P_i = -p_0 s\frac{x_i}{a_i^2} - 4\eta_0 s\left(\sum_{j=1}^{3}\alpha_{0j}A_{jj}\right)\frac{x_i}{a_i^2} + \frac{8\pi\eta_0 s}{ab^2}\sum_{e=1}^{3}A_{ie}\frac{x_e}{a_e^2} \qquad (4)$$

Here $s = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{b^4}\right)^{-1/2}$,

$$A_{ii} = \frac{3\alpha''_{i0}a_{ji}a_{li}\gamma_{jl} - \sum_{k=1}^{3}\alpha''_{k0}a_{jk}a_{lk}\gamma_{jl}}{6\beta''_0(\beta''_0 + 2\alpha''_0)}$$

$$A_{ik} = \frac{\alpha_{i0}(a_k^2 - a_i^2)a_{ji}a_{lk}\gamma_{jl}}{2(\alpha_{0i} - \alpha_{k0})(a_i^2\alpha_{i0} + a_k^2\alpha_{k0})} + \frac{a_k^2 a_{ji}a_{lk}(\omega_{jl} - \Omega_{jl})}{2(a_i^2\alpha_{i0} + a_k^2\alpha_{k0})}, \quad (i \neq k)$$

$$\alpha_0 = \alpha_{10} = \int_0^\infty \frac{d\xi}{(b^2+\xi)(a^2+\xi)^{3/2}},$$

$$\alpha_{20} = \alpha_{30} = \beta_0 = \int_0^\infty \frac{d\xi}{(b^2+\xi)^2(a^2+\xi)^{1/2}},$$

$$\alpha''_0 = \alpha''_{10} = \int_0^\infty \frac{\xi d\xi}{(b^2+\xi)^3(a^2+\xi)^{1/2}},$$

$$\alpha''_{20} = \alpha''_{30} = \beta''_0 = \int_0^\infty \frac{\xi d\xi}{(b^2+\xi)^2(a^2+\xi)^{3/2}},$$

$$a_1 = a, \quad a_2 = a_3 = b,$$

p_0 is the pressure in fluid without spheroid, Ω_{ij} is the angular velocity of spheroid, a_{ij} are the special coefficients which can be found from following relations

$$a_{lj}a_{ki} = \delta_{ij}\,, \quad a_{jk}a_{ik} = \delta_{ij}$$

where δ_{ij} is the Kronecker delta. Angular velocity of spheroid Ω_{ij} is equal [8]

$$\Omega_{lm} = \frac{3}{16\pi\eta_0}\left[\left(\frac{a^2\alpha_0 + b^2\beta_0}{a^2+b^2} - \beta_0\right)(e_m e_k \delta_{lj} + e_l e_j \delta_{mk})L_{jk} + \beta_0 L_{lm}\right]$$
$$+ \lambda(e_m \gamma_{ls} - e_l \gamma_{ms})e_s + \omega_{lm} \qquad (5)$$

where L_{ij} is a moment of external force, $\lambda = (a^2 - b^2)/(a^2 + b^2)$, $e_i = a_{i1}$.

The viscous force acting on all surface of spheroid is equal to zero, of course. But the viscous force acting on part of surface is not equal to zero. It means that particles which are contained in agregate can be teared off by viscous force.

Let the particles be teared off from tip of spheroid. The viscous force acting on these particles is equal [9]

$$F_{\|} = \frac{8\pi\eta_0}{a} A_{11} \sin^2\psi^*$$
$$F_{\perp} = \frac{8\pi\eta_0}{a} \sin^2\psi^* \sqrt{A_{21}^2 + A_{31}^2} \qquad (6)$$

Here $F_{\|}, F_{\perp}$ are forces which act along and perpendicular the large axis of spheroid respectively, the ψ^* is a spatial angle which corresponds to surface of particle on the tip of spheroid. The particles will be teared off from agregate in the following case

$$F_{\|} - F_a + m_p a_n \geq 0 \qquad (7)$$

where F_a is the attraction force between the particle on the tip of spheroid and other particles which form the agregate, m_p, a_n are mass and centripetal acceleration of particle, respectively. For small particles we may write condition (7) as [9]

$$F_{\|} \geq F_a \qquad (8)$$

The value of force $F_{\|}$ depends upon orientation of spheroid in fluid flow. The multitude of the orientations for which condition (8) is correct, is denoted by Ω_d. Consider the function f representing the probability of direction of the vector $\vec{e}$. We shall write the following expression for f

$$f = \begin{cases} 0, & \text{for all the orientations from multitude } \Omega_d \\ \text{solution of the equation (9)} & \end{cases}$$

The equation for function f is [8]

$$\frac{\partial f}{\partial t} + \lambda(e_s\gamma_{sl} - e_l e_m e_s \gamma_{ms} + \omega_{ls}e_s)\frac{\partial f}{\partial e_l} -$$
$$- 3\lambda e_l e_s \gamma_{ls} f - D\kappa[(e_j e_m h_m - h_j)\frac{\partial f}{\partial e_j} + 2e_j h_j f] =$$
$$= D\left(\vec{e}^2\frac{\partial^2 f}{\partial e_j^2} - 2e_s\frac{\partial f}{\partial e_s} - e_i e_s \frac{\partial^2 f}{\partial e_j \partial e_s}\right)$$
$$\kappa = mH/KT, \quad \vec{h} = \vec{H}/H \tag{9}$$

Here we consider the case when the agregates have vector of magnetization $\vec{m}$ along the vector $\vec{e}$, $\vec{m}=m\vec{e}$, $\vec{H}$ is vector of external magnetical field, KT is temperature, D is diffusion coefficient of Brownian motion. The moment of external force is equal for this case: $L_{ij}=mH(e_j h_j - e_i h_j)$. The solution of the equation (9) contains an arbitrary multipying constant. We choose this constant so that

$$\int_{4\pi-\Omega_d} f d\omega = 1 \tag{10}$$

where $d\omega$ is element of solid angle.

The multitude Ω_d defines some line $\mathcal{L}$ on the surface of sphere with unit radius. The number of agregates M which have been destroyed in the unit volume of suspension is equal the flux across the line $\mathcal{L}$. The flux $\vec{j}$ has following expression

$$\vec{j} = -D\nabla_s f + f\vec{\Gamma}$$

Here $\nabla_s f$ is gradient of the function f along the surface of unit sphere, vector $\vec{\Gamma}$ is equal

$$\Gamma_l = \Omega_{ls} e_s$$

We obtain

$$M = n\int_{\mathcal{Y}} \vec{j}\vec{\tau} dl \tag{11}$$

where $\vec{\tau}$ is unit vector along the surface of unit sphere and perpendicular to line $\mathcal{L}$, n is number of agregates in unit volume of suspension.

Let n_1 denote the concentration of particles which tear off from agregates and n_{i-2} $(i \geq 2)$ the concentration of agregates which form by destruction the agregates of concentration n_i. Then we may write the expressions for coefficients C_{il} and D_i:

$$C_{il}\ n_l = \begin{cases} 2M_l & \text{for } i=1, \quad l\geq 2 \\ M_l & \text{for } i=l-2, \quad l>2 \end{cases}$$
$$D_i\ n_i = M_i$$

Here,

$$M_t = n_t \int_{\mathcal{L}_i} \vec{j}_t \vec{\tau}_t dl\,.$$

The index t is mean the kind of agregates.

3 Application

Let us consider two cases of fluid flows:

1. Couette flow.

$$[\gamma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma \\ 0 & \gamma & 0 \end{bmatrix}\,, \quad \vec{\omega} = (-\gamma, 0, 0)$$

We obtain from equation (6) and condition (8)

$$e_2 e_3 > \frac{3a\beta_0'' F_a}{8\pi\eta_0\gamma \sin^2\psi^*}$$

The multitude Ω_d is defined by the last relation. The common solution of equation (5) for Couette flow is unknown.

We may find the function f for small values of $\kappa, \epsilon\,(\epsilon = \gamma/D)$. The function f of the first order in κ, ϵ is equal

$$f = C(1 + \kappa\vec{e}\vec{h} + \frac{\lambda\gamma}{D} e_2 e_3)$$

This relation may be introduced into equation (10), giving

$$C = \left(\int_{4\pi-\Omega_d} (1 + \kappa\vec{e}\vec{h} + \frac{\lambda\gamma}{D} e_2 e_3) d\omega\right)^{-1}$$

The C may be approximated for the small values B,
$(B = 3a\beta_0'' F_a / 8\pi\eta_0\gamma \sin^2\psi^*)$ by

$$C \approx \left[2\pi\left(1 - \frac{\kappa h_2}{2} + \frac{\kappa h_3}{2} - \frac{2}{3}\frac{\lambda\gamma}{\pi D}\right)\right]^{-1}$$

and, hence, we obtain

$$M = \frac{D n_0}{2\pi}\left[\kappa h_2(\pi - 2) + \kappa h_3(\pi - 2) + \frac{\lambda\gamma}{D}(4 - \pi)\right]$$

2. Uniaxial extensional flow.

$$[\gamma] = \begin{bmatrix} -\gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & 2\gamma \end{bmatrix}, \quad \vec{\omega} = (0,0,0)$$

The condition (8) is represented by

$$e_3^2 \geq \frac{1}{3} + \frac{\beta_0'' a F_a}{4\pi\eta_0\gamma}$$

The function of probability f of the first order in κ, ϵ is equal

$$f = C\Big[1 + \kappa\vec{e}\vec{h} + \frac{\lambda\gamma}{2D}(3e_3^2 - 1)\Big]$$

and for constant C we may write in spherical system of coordinates

$$C = \Big[4\pi\cos\theta^*(1 - \frac{\lambda\gamma}{2D}\sin^2\theta^*)\Big]^{-1}$$

where $\theta^* = \arccos\sqrt{\frac{1+B}{3}}$, $(0 \leq \theta^* \leq \pi)$. The expression for M may be written as

$$M = \frac{\lambda\gamma}{2}\Big(\frac{1+B}{3}\Big)\sqrt{\frac{2-B}{3}}$$

The analogous calculations may be made for a uniaxial compression flow. The expression for M in this case is equal

$$M = \frac{3\lambda\gamma}{2}\left[\Big(\frac{2+B}{3}\Big)\Big(\frac{1-B}{3}\Big)^3\right]^{1/2} \cdot \left[1 - \Big(\frac{1-B}{3}\Big)^{1/2}\right]^{-1}.$$

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