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WAVE DRIFT DAMPING AND LOW-FREQUENCY OSCILLATIONS OF AN ELLIPTIC CYLINDER IN IRREGULAR WAVES

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ABSTRACT

Wave drift damping and low-frequency oscillations of a moored elliptic cylinder is examined. The cylinder is restricted to move along a horizontal frictionless constraint. The moorings are simulated by slack linear springs. The fluid layer is infinitely deep and the motion is two-dimensional. The fluid flow around the body and the forces acting upon it are computed in the frame of reference following the low-frequency position of the body. The slowly varying force is calculated by the approximation due to Marthinsen (1983). One part of the resulting slowly varying force is a damping force being proportional to the low-frequency velocity of the body. This force is closely related to wave drift force damping. For long incoming waves, we obtain positive damping. For short incoming waves, however, the damping force becomes negative. Also slowly varying frequency of encounter of the incoming waves is introduced due to the slowly varying velocity of the body. The equation of motion for the low-frequency oscillations is solved by numerical integration. Results for several sea states are examined. The impact of the damping force obtained here is compared with the impact of wave drift force damping and viscous damping.

1. INTRODUCTION

An irregular sea will due to non-linearities give rise to wave forces oscillating with the sum and difference frequencies of every two waves. If the wave spectrum is narrow-banded, the force due to the difference frequencies will be slowly varying in time. The slowly varying forces are of small magnitude compared to the total wave forces. However, if there is a moored body in the waves with resonance at low frequencies, the slowly varying forces may excite large low-frequency horizontal displacements of the body, introduce severe strain in the moorings and cause difficulties in positioning the body.

Slowly varying forces are extensively studied in the literature, and a general review can be found in Ogilvie (1983). However, mechanisms which are damping low-frequency oscillations are not yet fully understood. The present account is therefore devoted to studying damping of low-frequency oscillations. The body is a slender elliptic cylinder being moored with weak linear springs. The major axis is horizontal and the cylinder is restricted to move along a frictionless horizontal constraint. The flow around the cylinder is two-dimensional.

The damping force studied here is closely related to wave drift force damping, which is studied by Wichers and Huismans (1984), Nakamura et al. (1986), Faltinsen et al. (1986), Faltinsen and Sortland (1986) and Huismans (1986). They find that this is an important damping force for lengthwise motions of slender bodies where viscous damping turns out to be very small. Wave drift force damping is due to change in the mean second order force upon a body when it moves with a constant speed against or in waves. Mean second order force upon a cylinder is found by the scattered waves in the far field, as described in Grue and Palm (1985), and in Grue and Palm (1986) where impact of a uniform current upon slowly varying forces is discussed. Incompressible, inviscid fluid and irrotational motion is assumed. The water depth is infinite. The scattered waves are calculated by applying source distribution along the body surface. The source distribution is approximated by piecewise qubic B-splines, as described by Mo and Palm (1987). The first order forces, necessary for the equation for the first order motions of the body, are found as described in Grue (1986).

In section 2 we obtain values of the mean second order force due to the submerged ellipse and formulate the slowly varying force. It is found that the mean second order force increases considerably for increasing current speed, when the body moves against long incoming waves. On the other hand, the mean second order force decreases when the body moves against waves with wave length shorter than about three times the horizontal extent of the cylinder section. This may be a characteristic feature for submerged bodies and leads to negative damping.

Equation of motion for low-frequency oscillations is formulated and results for two incoming waves are discussed in section 3. Results for an irregular sea are given in section 4. Different sea states are studied. The results show that the wave drift force damping (1.1) is the most important contribution from taking into account the slowly varying position and velocity of the body in calculating the slowly varying force. Finally sections 5 and 6 are respectively devoted to discussion and conclusion.

The slowly varying position $x(t)$ introduces encounter frequency $\sigma(t)$ given by

$$\sigma(t) = \omega + k\dot{x}(t) \qquad (2.4)$$

$\sigma(t)$ and $x(t)$ are slowly varying functions of time provided that

$$|\dot{x}/c| << 1 \qquad (2.5)$$

where $c=\omega/k$ denotes the phase velocity of the incoming waves in the fixed frame of reference.

In the relative frame of reference the cylinder undergoes small first order oscillations with periods $2\pi/\sigma(t)$ on a slowly varying current with speed

$$U(t) = \dot{x}(t) \qquad (2.6)$$

directed along the negative x_1-axis when $\dot{x}(t)>0$. When (2.5) is fulfilled, we may as a first approximation calculate the first order forces upon the cylinder, the first order motions of the cylinder, and the scattered waves by assuming U being uniform in time. The interesting part of the second order force is then found by the first order flow in the far-field applying the impulse equation.

2b. Calculation of mean second order force in regular waves

The mean second order force $\bar{F}$ upon a cylinder moving with constant speed U against the incoming wave (2.1), may for small U be written order terms in (2.7) are very small for $U/\sqrt{gR} < 0.2$.

Figure 2 shows that the reflected wave amplitude a^- gets a pronounced increase for kR less than unity, due to a forward velocity of the body. For larger value of kR the reflected wave amplitude decreases. Figure 3 reveals that α is negative for $kR<0.9$. Hence, the value of the mean second order force increases when the cylinder moves with a constant speed against the waves. For $kR>0.9$ the converse is true. Compared to $U=0$, the force then decreases when the cylinder moves against the waves. This occurs when $r_1(k)-2\sqrt{k/g}\,r_0(k)$ becomes negative. The fact that $\bar{F}$, as a function of U, decreases for moderate and short incoming waves, is also true for submerged cylinders with other contours, as discussed in Grue and Palm (1986). We note that only a small change in the speed $U/\sqrt{gR}$ leads to a relatively large change in $\bar{F}(k,U)$ for $kR<0.9$.

2c. Slowly varying force due to irregular waves

If the incoming waves consist of N wave components we may write the surface elevation at x_1 by

$$\eta(x_1,t) = \mathrm{Re}\sum_{m=1}^{N} A_m \exp(i\omega_m t+k_m x(t)+k_m x_1) \tag{2.11}$$

where A_m are complex amplitudes, $k_m=\omega_m^2/g$ is wave number of component m, and Re denotes real part. We now assume that the amplitudes A_m are given from a narrow-banded power spectrum $S(\omega)$ by

$$|A_m|^2 = 2S(\omega_m)\Delta\omega \tag{2.12}$$

$\psi_{x(t)}(t,x(t))$. The term $Ua^2(t,x(t))\alpha(k_L)$ is a damping term when $\alpha(k_L)<0$. This term leads, however, to negative damping when $\alpha(k_L)>0$, which in the present case occurs for $k_L R>0.9$.

3. EQUATION OF MOTION

Let us apply a linear mooring force $-Cx$, where C denotes the spring constant. The added mass force $-m_{11}\ddot{x}$ must be added to the force (2.17) due to the slowly varying accelerations of the body. m_{11} denotes added mass of the submerged elliptic cylinder as the frequency of oscillation tends to zero. Calculation gives $m_{11}=0.1434\rho R^2$ for $b/R=0.2$, $d/R=1$. We denote the mass of the cylinder by m (which equals the displaced water mass). The equation of motion for the low-frequency oscillations then becomes

$$m\ddot{x} = -m_{11}\ddot{x} - Cx + F(t) \qquad (3.1)$$

$F(t)$ is given by (2.17) with $U=\dot{x}$.

In (3.1) we have neglected the impact of viscous drag forces, which is very small in the present example. We shall, for comparison, in some cases study solutions of (3.1) with the damping term in (2.17) replaced by viscous drag. Value of viscous drag upon an elliptic cylinder may be found from modern Development in Fluid Dynamics (1950, p. 415). For large Reynolds number the drag force is

$$\rho b C_D \dot{x}|\dot{x}| \qquad (3.2)$$

where $C_D \simeq 0.1$ for $b/R=0.2$.

The solution of (3.7) is

$$x^{(0)}(t) = -a^{(0)}\sin(\Delta\omega t+\delta^{(0)}) \tag{3.9}$$

where $a^{(0)}$ and $\delta^{(0)}$ are unknown. Inserting (3.9) into (3.8) and avoiding secular solutions of $x^{(1)}(t)$, $a^{(0)}$ and $\delta^{(0)}$ are determined by

$$a^{(0)} = \frac{T(k_L)}{\Delta\omega\alpha(k_L)} \frac{1}{1 + \frac{T(k_L)}{4\alpha(k_L)\frac{\Delta\omega}{\Delta k}}} \tag{3.10}$$

$$\delta^{(0)} = 0 \tag{3.11}$$

The solution for $x^{(1)}(t)$ becomes

$$x^{(1)} \simeq \bar{x} + a^{(1)}\cos 2\Delta\omega t \tag{3.12}$$

where

$$\bar{x} = \frac{a^2}{C}((2+\Delta k a^{(0)})T(k_L)-\Delta\omega a^{(0)}\alpha(k_L)) \tag{3.13}$$

$$a^{(1)} = \frac{a^2}{3C}(\Delta k a^{(0)}T(k_L) + \Delta\omega a^{(0)}\alpha(k_L)) \tag{3.14}$$

Hence, the solution of (3.3) becomes approximately

$$x(t) \simeq \bar{x} - a^{(0)}\sin\Delta\omega t + a^{(1)}\cos 2\Delta\omega t \tag{3.15}$$

The amplitudes of higher order harmonic solutions of (3.3) are very small.

The oscillation $a^{(0)}\sin\Delta\omega t$ is mainly damped by the wave drift force damping (1.1), which in (3.3) is the term $\dot{x}2a^2\alpha(k_L)$. We note that $a^{(0)}$ is independent of the amplitude a of the incoming waves. The validity of the solution (3.15) is based upon the inequalities (2.5) and (3.4), which corresponds to the solution of (3.18) becomes

$$x(t) \approx -\frac{a}{\Delta\omega}\left(\frac{3\pi|T(k_L)|}{4\rho b C_D}\right)^{\frac{1}{2}}\sin \Delta\omega t + \frac{2a^2T(k_L)}{C} \qquad (3.20)$$

In this case the amplitude of the oscillation increases linearly with the incoming wave amplitude. This solution is also displayed in figure 4. We note that, for smaller value of a/R, the solutions of (3.3) and (3.18) become very close to each other. This means that the damping force in (3.3) and the viscous drag force are of equal magnitude.

4. RESULTS FOR IRREGULAR WAVES.

The equation of motion is now

$$(m+m_{11})\ddot{x} - \dot{x}a^2(t,x)\alpha(k_L) + Cx = a^2(t,x)T(k_L) \qquad (4.1)$$

where $a(t,x)$ is given by (2.14), (2.15) and k_L by (2.16). We shall first examine stability of solutions of this equation. This may be achieved by studying the homogeneous version of (4.1), i.e.

$$(m+m_{11})\ddot{x} - \dot{x}a^2(t,x)\alpha(k_L) + Cx = 0 \qquad (4.2)$$

We multiply this equation by $\dot{x}$ and integrate from t_0 to t. We then obtain

$$\frac{1}{2}(m+m_{11})\dot{x}^2 + \frac{C}{2}x^2 = E_0 + \int_{t_0}^{t} \dot{x}^2a^2(t,x)\alpha(k_L)dt \qquad (4.3)$$

where $E_0 = \frac{1}{2}(m+m_{11})\dot{x}^2 + \frac{C}{2}x^2$ at $t=t_0$. Hence, every induced

We shall give the sea state by either a narrow-banded Gauss curve spectrum, which is very close to the JONSWAP spectrum, or by the more broad-banded Pierson-Moskowitz spectrum. Let us denote significant wave height of the incoming waves by H_s. Gauss curve spectrum with mean frequency ω_p, spread 2δ and area $m_0=H_s^2/16$ is then given by

$$S_G(\omega) = \frac{H_s^2}{16\sqrt{2\pi}\delta} \exp(\frac{(\omega-\omega_p)^2}{2\delta^2}) \tag{4.6}$$

From Houmb and Overvik (1976) the spread 2δ is related to the spectral moments $m_n=\int_0^\infty S(\omega)\omega^n d\omega$ by

$$\delta^2 = \frac{m_2 m_0 - m_1^2}{m_1^2} \tag{4.7}$$

In the present account we choose $\delta=0.15\omega_p$, and $\omega_p=0.40144\sqrt{g/H_s}$ equals the peak frequency of the Pierson-Moskowitz spectrum. The Pierson-Moskowitz spectrum, given in e.g. Newman (1977), may be expressed by the significant wave height by

$$S_{pm}(\omega) = \frac{0.0081g^2}{\omega^5} \exp(-\frac{0.0324g^2}{H_s^2\omega^4}) \tag{4.8}$$

The Gauss curve spectrum and the Pierson-Moskowitz spectrum are displayed in figure 5 for $H_s/R=0.4, 0.6$.

The amplitudes of the wave components are given by (2.12) with $\Delta\omega=(\omega_{max}-\omega_{min})/N$. ω_{max} and ω_{min} denote respectively the upper and lower truncation of the spectrum. N is the number of wave components. The frequencies ω_m of the different wave components are chosen randomly in the N intervals

$$\omega_{min} + (m-1)\Delta\omega < \omega_m < \omega_{min} + m\Delta\omega \tag{4.9}$$

To examine the difference between the solutions of (4.1), (4.5) and (4.11) more closely, we have run long time series for x(t) and computed the time average of x(t) and standard deviation $\sigma_x=(\overline{(x-\bar{x})^2})^{\frac{1}{2}}$. A bar denotes time avarage. The resonance frequency of the system is given by $\Omega\sqrt{R/g}=(CR/(m+m_{11})g)^{\frac{1}{2}}=0.05$, which corresponds to a period $P=125.7\sqrt{R/g}$. The time series are run for 119 periods. The transients vanish after 6-7 periods. Calculation of $\bar{x}$ and σ_x are based upon the 110 final periods of the different time histories. Gauss curve spectrum with $H_s/R=0.4,0.6$ is applied. Results for various choice of wave components are obtained for σ_x in tables 1 and 2, and for $\bar{x}$ in tables 3 and 4.

Number of wave components	Eq.(4.1)	Eq.(4.5)	Eq.(4.11)
100	1.04	1.16	1.14
200	1.08	1.23	1.17
400	0.98	1.15	1.07

Table 1. Standard deviation σ_x/R for solutions of eqs. (4.1), (4.5),(4.11). Gauss curve spectrum with $H_s/R=0.4$. 100, 200 or 400 wave components.

Number of wave components	Eq.(4.1)	Eq.(4.5)	Eq.(4.11)
100	0.96	0.98	0.98
200	1.12	1.24	1.19
400	1.10	1.16	1.14

Table 2. Same as table 1, but Gauss curve spectrum with $H_s/R=0.6$.

5. DISCUSSION

Slowly varying forces are most commonly calculated by the approximation due to Newman (1974). Applying corresponding derivation as in section 2, this approximation leads to the following expression for the slowly varying force

$$F(t) = -F_0 + \mathrm{Re}\sum_{m=1}^{N}\sum_{\substack{n=1\\ n\neq m}}^{N} A_m A_n^{*} T_{mm} \exp(i((\omega_m-\omega_n)t+(k_m-k_n)x(t)))$$

$$-B\dot{x} + \dot{x}\mathrm{Re}\sum_{m=1}^{N}\sum_{\substack{n=1\\ n\neq m}}^{N} A_m A_n^{*} \alpha_{mm} \exp(i((\omega_m-\omega_n)t+(k_m-k_n)x(t))) \qquad (5.1)$$

where

$$F_0 = -2\int_0^{\infty} S(\omega)T(k(\omega))d\omega \qquad (5.2a)$$

$$B = -2\int_0^{\infty} S(\omega)\alpha(k(\omega))d\omega \qquad (5.2b)$$

$$T_{mm} = T(k_m) \qquad (5.2c)$$

$$\alpha_{mm} = \alpha(k_m) \qquad (5.2d)$$

New term in the present account is

$$\dot{x}\mathrm{Re}\sum_{m=1}^{N}\sum_{\substack{n=1\\ n\neq m}}^{N} A_m A_n^{*} \alpha_{mm} \exp(i((\omega_m-\omega_n)t+(k_m-k_n)x(t))) \qquad (5.3)$$

and $x(t)$ occurs in the phase function

$$(\omega_m-\omega_n)t + (k_m-k_n)x(t) \qquad (5.4)$$

The term (5.3) corresponds to (4.12). In the previous section we demonstrated that the part of the slowly varying force (2.17) corresponding to the sum of (5.3) and the slow drift force damping $-B\dot{x}$, is a damping force. The values of the slowly varying forces

$$S_F(\omega) = 8\int_\omega^\infty S(\omega_1)S(\omega_1-\omega)\,|T(k(\omega_1))|^2 d\omega_1 \qquad (5.9)$$

where $S(\omega)$ denotes the power spectrum due to the incoming waves. The value of $\bar{x}$ is simply obtained by

$$\bar{x} = -F_0/C \qquad (5.10)$$

Let us then calculate σ_x by (5.8) and (5.9) and $\bar{x}$ by (5.10) and compare with results from time series calculations presented in secton 4. Applying $\Omega\sqrt{R/g}=0.05$ and Gauss curve spectrum with $H_s/R=0.4,0.6$ we obtain the following table

H_s/R	0.4	0.6
σ_x/R	1.02	1.20
$\bar{x}/R$	-0.48	-0.52

Table 5. Standard deviation σ_x and mean value $\bar{x}$ calculated by (5.8),(5.9) and (5.10). Gauss curve spectrum with $H_s/R=0.4,0.6$.

These values shall be compared with the results obtained in tables 1-4 for solutions of eq. (4.11), based upon the approximation due to Marthinsen.

Application of slowly varying forces calculated by the approximations due to Newman or Marthinsen leads to an error in the displacement $x(t)$ being of order $O(\Omega\sqrt{R/g})$. Hence, for $\Omega\sqrt{R/g}=0.05$, we may expect that the theoretical predictions of $x(t)$ are

APPENDIX. Mean second order force for small value of current speed.

Mean second order force $\bar{F}$ upon a cylinder submerged in harmonic waves in a uniform current is discussed in Grue and Palm (1985). We let the current with speed U be directed along the negative x_1-axis. The incoming waves with amplitude a and wave number k is propagating in the direction of the current. For small value of U it may be shown from the results in Grue and Palm (1985) that there is a reflected wave upstream of the body with wave number k^- and amplitude a^-, and only one transmitted wave downstream with wave number k and amplitude a^+. The mean second order force is then given by

$$\bar{F} = -E^- \frac{c_g^- - U}{c^-} + (E^+ - E)\frac{c_g - U}{c} \qquad (A.1)$$

where

$$c^- = (g/k^-)^{\frac{1}{2}}, \quad c = -(g/k)^{\frac{1}{2}} \qquad (A.2a)$$

$$c_g^- = \frac{1}{2}c^-, \quad c_g = \frac{1}{2}c \qquad (A.2b)$$

$$E^- = \frac{1}{2}\rho g a^{-2}, \quad E^+ = \frac{1}{2}\rho g a^{+2}, \quad E = \frac{1}{2}\rho g a^2 \qquad (A.2c)$$

Balance of energy is also derived in Grue and Palm (1985) and reads

$$E^- \frac{c^- - U}{c^-}(c_g^- - U) = (E^+ - E)\frac{c - U}{c}(c_g - U) \qquad (A.3)$$

Inserting (A.3) into (A.1) we obtain

$$\bar{F} = E^- \frac{c_g^- - U}{c^-} \frac{c^- - c}{c - U} \qquad (A.4)$$

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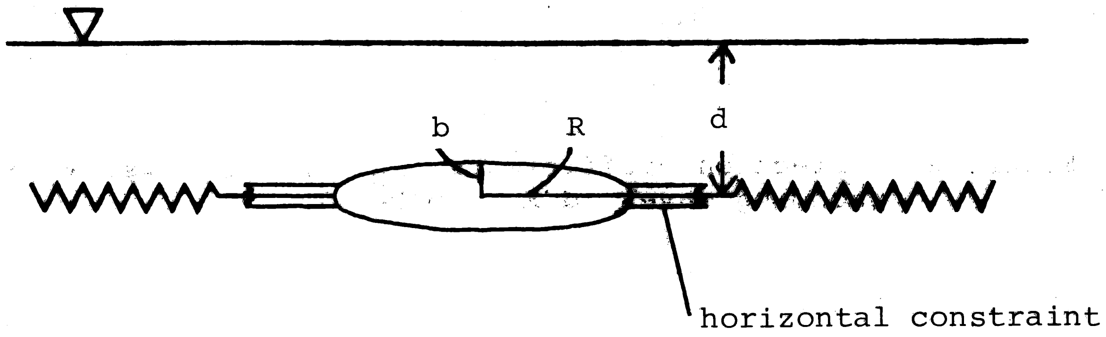

Figure 1. The moored elliptic cylinder which can move along a horizontal constraint.

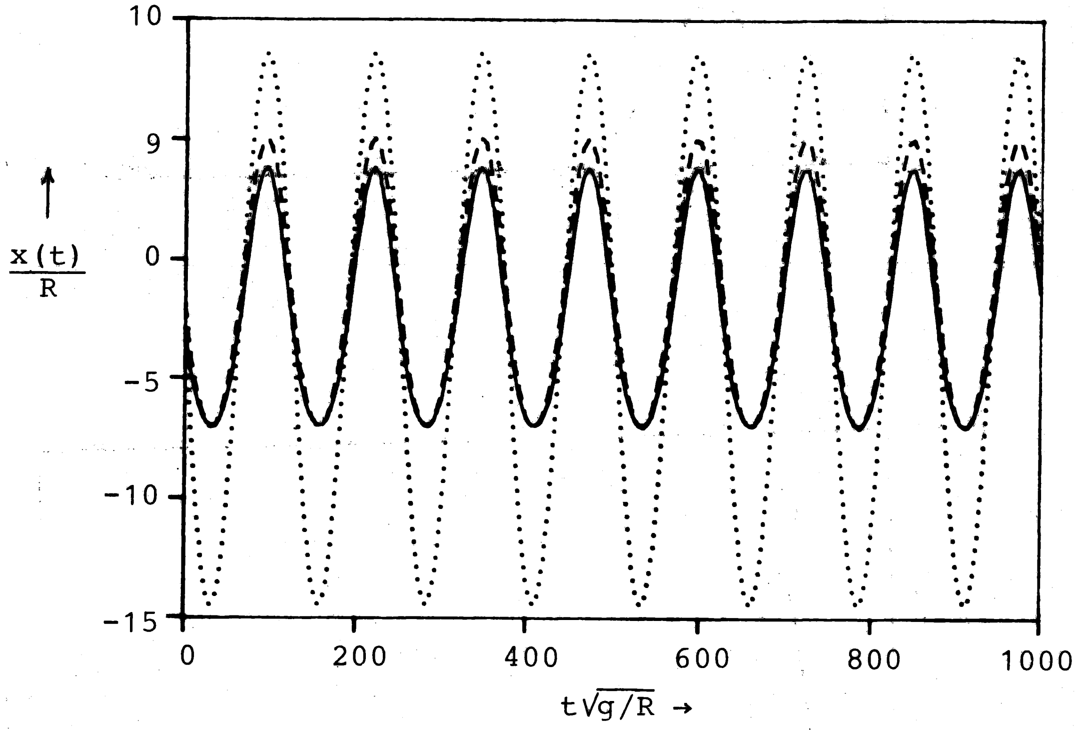

Figure 4. Time histories for solutions of x(t). Solution (3.15) (solid line), solution (3.15) with $\Delta k=0$ (dashed line), solution (3.20) (dotted line).

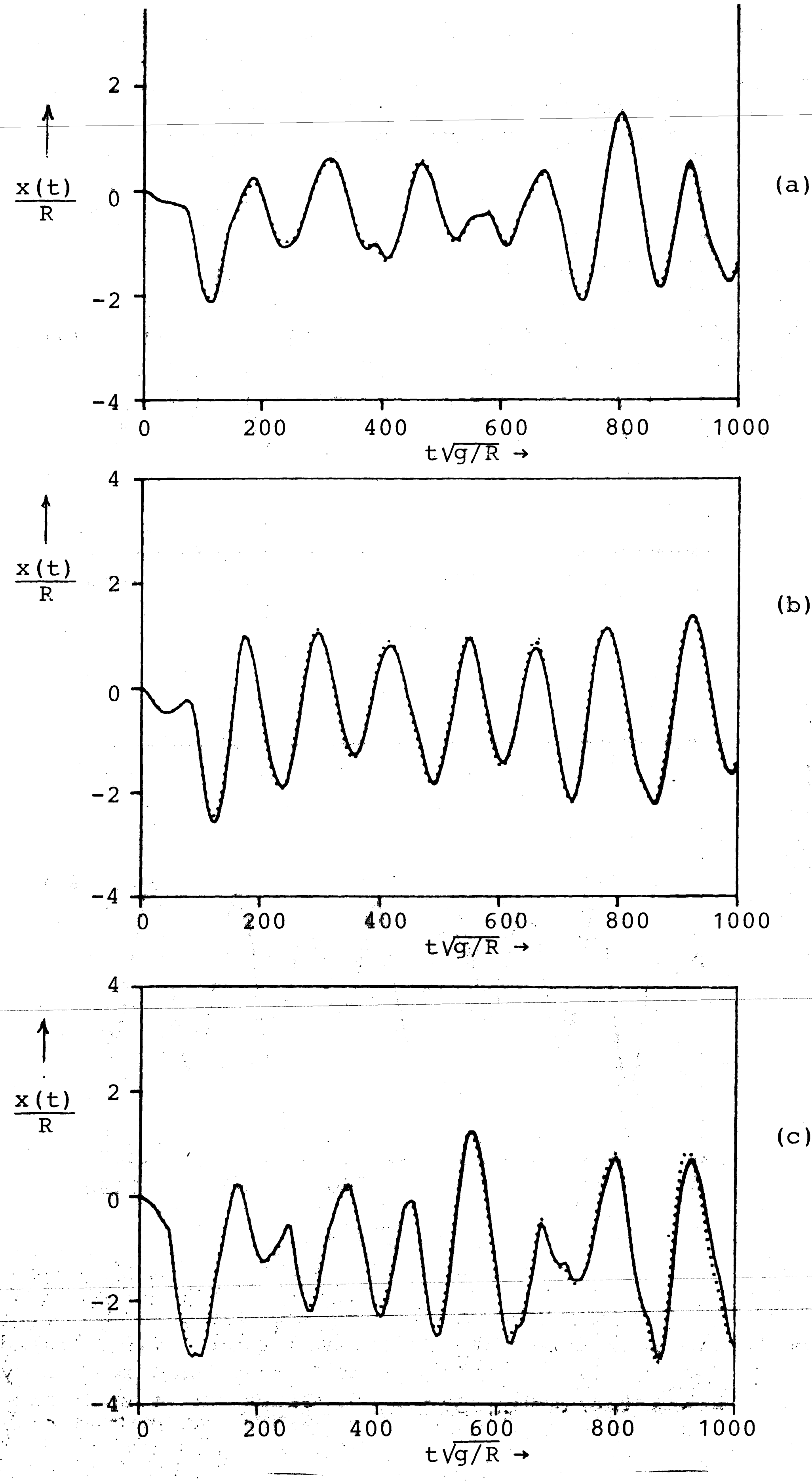

Figure 6. Comparison between solutions of eq. (4.1) (solid line) and eq. (4.10) (dotted line). Resonance frequency $\Omega\sqrt{R/g}=(CR/(m+m_{11})g)^{\frac{1}{2}}=0.05$. 100 wave components.

a) Gauss curve spectrum, $H_s/R=0.4$,

b) Gauss curve spectrum, $H_s/R=0.6$,

c) Pierson-Moskowitz spectrum, $H_s/R=0.6$.

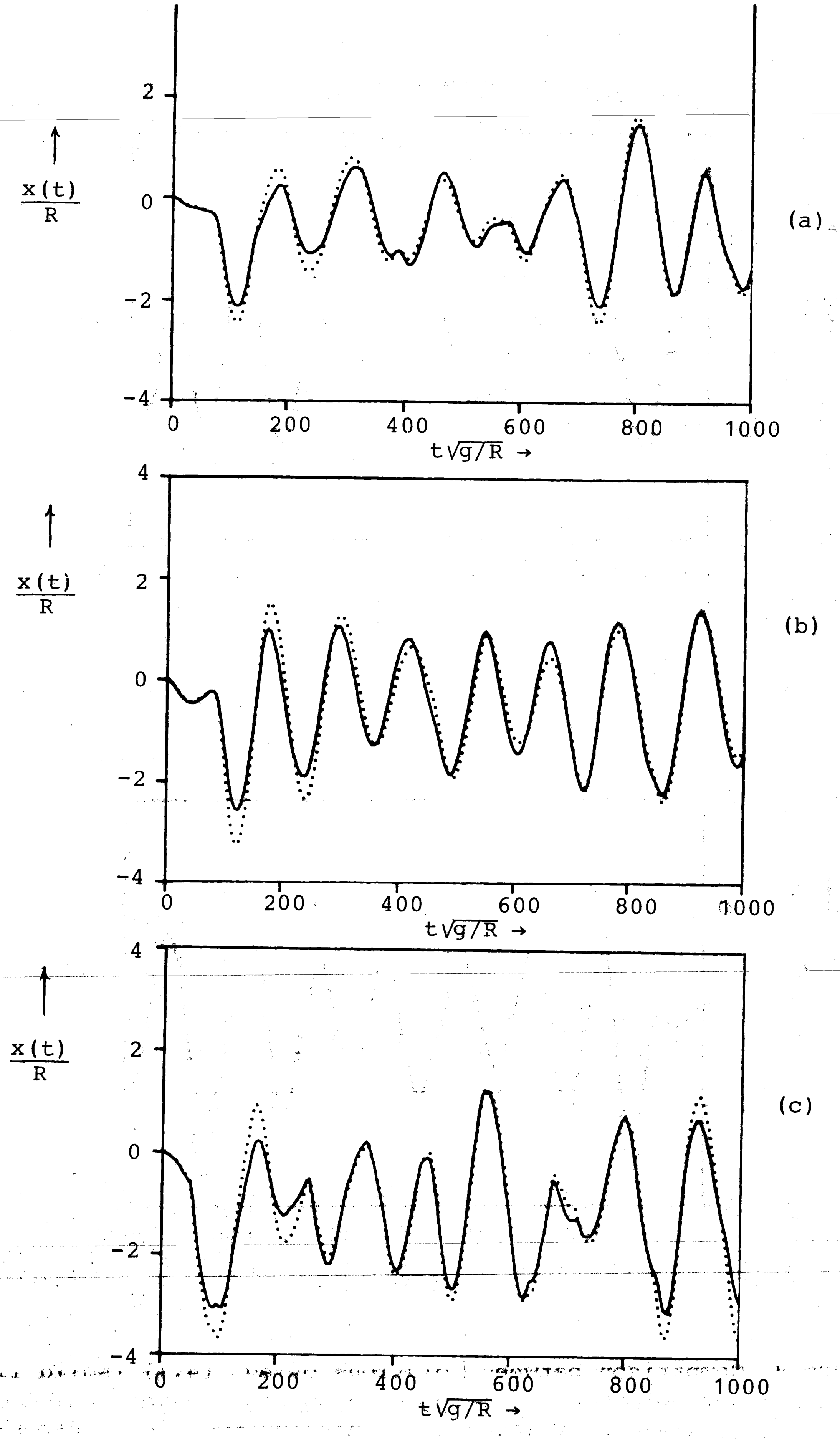

Figure 8. Comparison between solutions of eq. (4.1) (solid line) and eq. (4.11) (dotted line). Resonance frequency $\Omega\sqrt{R/g}=(CR/(m+m_{11})g)^{\frac{1}{2}}=0.05$. 100 wave components.
a) Gauss curve spectrum, $H_s/R=0.4$,
b) Gauss curve spectrum, $H_s/R=0.6$,
c) Pierson-Moskowitz spectrum, $H_s/R=0.6$.

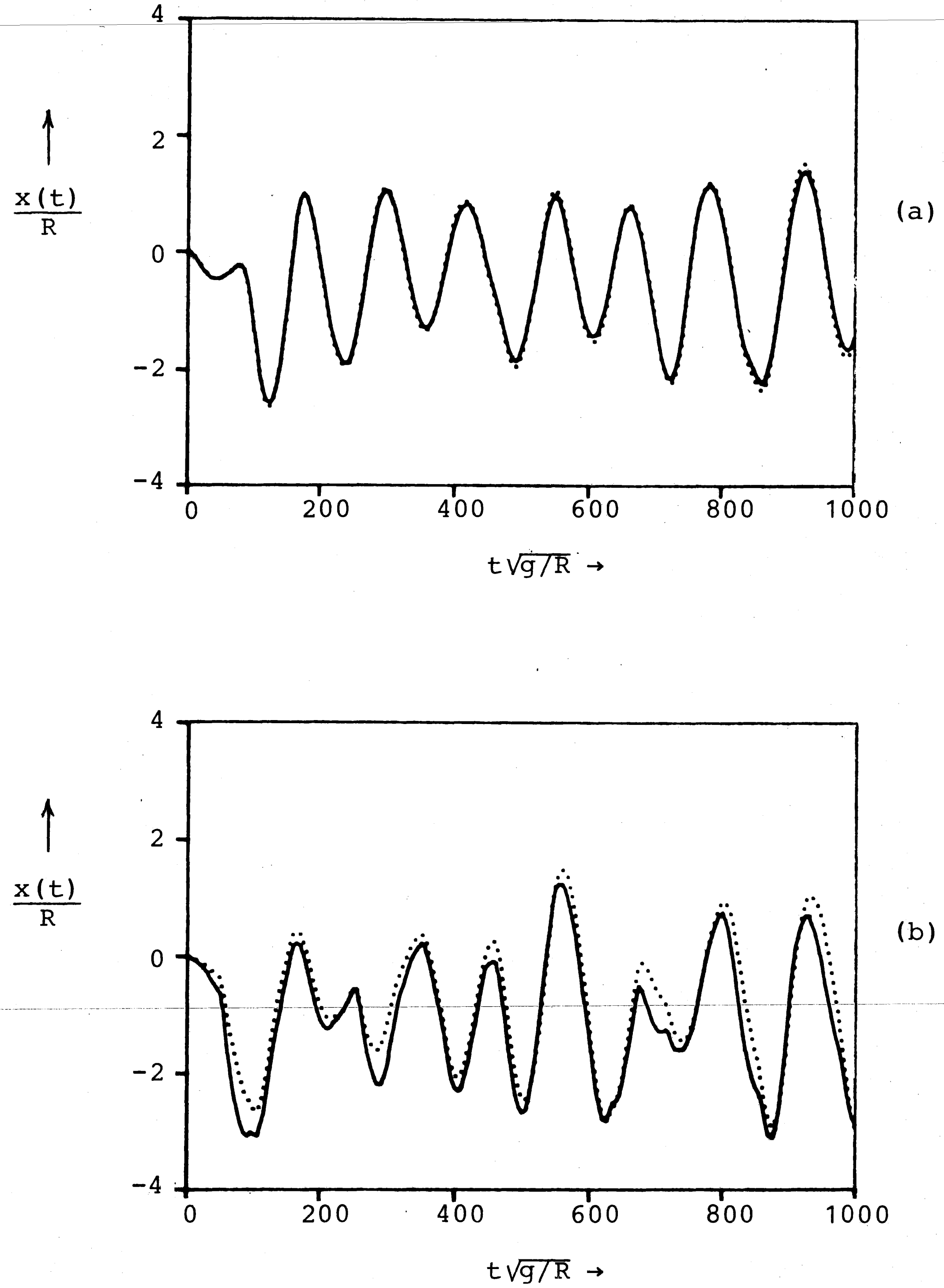

Figure 10. Comparison between solutions of eq. (4.1) (solid line) and eq. (5.6) with $F(t)$ given by eq. (5.1) (dotted line). Resonance frequency $\Omega\sqrt{R/g}=(CR/(m+m_{11})g)^{\frac{1}{2}}=0.05$. 100 wave components, $H_s/R=0.6$. a) Gauss curve spectrum, b) Pierson-Moskowitz spectrum.