ISBN 82-553-0367-7
Applied Mathematics
No 5 - December 8 1978

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PREPRINT SERIES - Matematisk institutt, Universitetet i Oslo

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Abstract.

This is the first of two papers on the problems and possibilities connected with the use of magnetic monopoles in the interpretation of elementary particle properties.

In this paper it is shown that the main difficulty which has hitherto prevented the calculation of the energy levels in a system of two magnetic monopole charges (in C.G.S. units equal to or greater than the Dirac monopole charge $g = \frac{137}{2}e$, where e designates the elementary electric charge) is related to the implicit assumption generally applied in the calculation of energy levels, that magnetic monopoles behave as coulombian point charges. If this assumption is removed, for example by considering each magnetic monopole as an object whose magnetic charge is distributed in a spherically symmetric fashion within a certain volume with a standard radius r_o, comparable to the classical radius one would have to ascribe to it if one wants to interprete its mass as magnetostatic energy, the problem of calculating the energy levels can be solved.

A few examples of coulombian field approximations consistent with this interpretation of magnetic charge distribution are presented, and some of the lowest energy levels of particles formed by the association of two magnetic monopoles are calculated for circular orbits by the Bohr and Sommerfeld procedure. Such associations of magnetic monopoles obtain particularely appealing properties for the interpretation of elementary particles if we use the simple assumptions:

(1) That all magnetic monopoles have the same standard radius r_o.

(2) That their masses can be interpreted as magnetostatic energy.

elementary magnetic charges values roughly 5.000 times greater than forces between elementary electric charges, in the case formula 1 applies. A consequence of these conspicuous interaction forces is that relativistic effects may have to play a prominent role in all calculations of energy levels of interacting monopoles.

2. If we were to interprete (for the sake of argument) the mass M_o of a magnetic monopole as magnetostatic energy of a spherical uniformly distributed magnetic charge g_o , its classical radius r_o would be:

(2) $$r_o = \frac{3g_o^2}{5M_oc^2}$$

It is however well known that in electron scattering experiments electrostatic forces within distances shorter than the classical radius of the electron appear to follow Coulombs law as if all the electronic charge were concentrated in a very small region (singularity) in the center of the electron. If might be tempting to assume that the same could be the case for magnetic monopoles, namely that the magnetostatic charge could be concentrated in a very small region (singularity) in the center of the monopole(see also Appendix).

3. This hypothesis meets however with additional difficulties in the case of magnetic monopoles, rising major problems which do not arise in the case of the much smaller electric charges. The problems arise if one attempts to calculate the bindings energies in a system of two magnetic monopoles in orbit around each other, for example in order to investigate the possibilities of applying magnetic monopoles in the interpretation of the properties of elementary particles (Schwinger, 1969). The difficulty arises both in the

$$a_n = \frac{53.33n^2}{(137)^2} r_o$$

This semi-major axis is far shorter than the classical radius r_o in the case $n < \frac{137}{8\sqrt{2}}$, in which the bindings energy E_s is greater than $(M_1+M_2)c^2$. Both potential and bindings energy could never be that high if we had not made the assumption (see above) that the coulombian inverse square low would apply that far inside the classical radius r_o of each monopole.

A similar kind of problem arises in the relativistic case if we try to calculate the energy levels E_d derived from Dirac's equations instead of E_s. We don't have to look for sofisticated general formulae. Since the problem seem to be related to the size of the magnetic charges rather than the masses of the particles, we can use the ordinary formula for energy levels in a two particle system consisting of an atomic nucleus and a single electron after replacing the product Ze^2 of the nuclear and electron charges by the product g_1g_2 of two monopole charges. We obtain:

(5)
$$E_d = m_o c^2 \sqrt{1 + \frac{g_1^2 g_2^2}{\left(\sqrt{k^2 - \frac{g_1^2 g_2^2}{\hbar^2 c^2}} + n - k\right)^2}}$$

$k+\frac{1}{2}$ and $n+\frac{1}{2}$ being positive integer quantum numbers ($k \leq n$), and m_o being the rest mass of the smaller particle (electron mass in the atomic case) which is assumed to be much smaller than that of the other particle. Even in the case in which k obtain its maximum value ($k = n$), which in the original Bohr-Sommerfeld model was (before the spin of the electron had been detected) ascribed to circular orbits, the square root in the denominator of formula (5) becomes imaginary when $n < \frac{137}{4}$, for $g_1^2 = g_2^2 = \frac{137\,\hbar c}{4}$ (formula 1). As a result all orbits have complex binding energies E_d for magnetic monopoles this size, when the energy quantum n is smaller than $\frac{137}{4}$. Even orbits with $n > \frac{137}{4}$ may have complex bindinngs energies if $k < \frac{137}{4}$ according to formula (5).

It will be sufficient to establish the existence of the lower angular momentum limit for the circular orbits, since every elliptical orbit has a circular orbit tangent to it at its periapsis, whose orbital velocity is locally lower. If that velocity exceeds the speed od light (the main argument used in order to prove the orbits ipossibility) the same will apply to the periapsis velocity of the elliptical orbit. In the process we will also establish a few useful formulae.

We call r the (constant) distance between two particles of masses M_1 and M_2, and magnetic charges g_1 and g_2 moving in circular orbits around a common baricenter. r_1 and r_2 will be their distances from the baricenter, v_1 and v_2 their velocities relative to the baricenter.

The formulae for a circular movement in a central field may conveniently be written in the following manner:

(6) $$\frac{M_1 v_1^2}{r_1} = \frac{M_2 v_2^2}{r_2} = F$$

where

(7) $$r = r_1 + r_2$$

r being the distance between the two particles.

(8) $$r_1 M_1 = r_2 M_2$$

(9) $$v_1 M_1 = v_2 M_2$$

the last equation stating that the impulses of the two particles relative to the baricenter are equal. If in these equations we replace M_1 and M_2 by their relativistic expressions

(10) $$M_1 = \frac{M_{1o}}{\sqrt{1 - \frac{v_1^2}{c^2}}}, \qquad M_2 = \frac{M_{2o}}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

M_{1o} and M_{2o} being the rest-masses of the two particles, we obtain a system of equations which can be used to calculate the 4 unknowns r_1, r_2, v_1, v_2 when M_{1o}, M_{2o}, F and r are given.

In case F is a coulombian force defined by

$$F = \frac{g_1 g_2}{r^2}$$

formula (14) becomes:

(15) $$A = \frac{g_1 g_2}{v_1 + v_2}$$

Obviously this angular momentum will not approach zero when $r \longrightarrow 0$ and $v_1, v_2 \longrightarrow c$. It will instead approach the value $\left|\frac{g_1 g_2}{2c}\right|$ which is its lower limit

(16) $$A > \frac{g_1 g_2}{2c} \quad \text{for } r > 0$$

This formula explains why we were unable to find real number solutions of the Dirac equation with $k < \frac{137}{4}$ in the case $|g_1 g_2| = \frac{137\,\hbar c}{4}$. As a matter of fact the bindings energy formula (5) was derived from Dirac's equations by ignoring the movement of the nucleus (mass M_1) relative to that of a smaller particle (mass $m_0 = M_2$). This procedure, translated in semi classical terms is equivalent to disregarding the velocity v_1 relative to the velocity $v = v_2$ of the lighter particle, in which case the formula (15) is replaced by

$$A \cong \left|\frac{g_1 g_2}{v}\right|$$

and formula (16) by

$$A \geq \frac{g_1 g_2}{c}$$

Since the angular momentum is $k\hbar$ in formula (5), this means that there are no real solutions with $k\hbar < \frac{g_1 g_2}{c}$ or $k < 137/4$ in case $g_1 g_2 = \frac{137\,\hbar c}{4}$.

3. The poly-exponential coulombian field approximations.

Our next problem is to find an approximation for the coulombian force field which approaches a coulombian field when $r \to \infty$, but behaves in a manner comparable to that between two charges each distributed with a constant or gradually variable but finite density within a region of a certain standard radius r_o. This standard radius is used as a substitute for the classical radius of the particles or a comparable parameter. There are many approximations of coulombian fields fulfilling this conditions, and many of them would also have comparable properties within a standard radius r_o if they fulfill all the requirements (see below) we impose to them. We shall present two of them, which have the additional advantage of being easy to handle mathematically. They will be designated as the "exponential and the double-exponential coulombian field approximation" respectively.

The simplest case will be to assume that all magnetic monopole charges have the same standard radius r_o. This assumption gives to the exponential fields properties particularely appealing in view of possible applications in the interpretation of elementary particles (see last section).

A basic property of two equal and opposite magnetic or electric charges with the same finite density distribution assumed to be spherically symmetric* is that they will cancel out if they are placed in the same spot, so that the distance r between their centers is equal to zero. This has a series of implications, such as:

1. The attraction force between two charges of opposite sign will not go to infinity, but on the contrary it will approach zero when $r \to o$. The same applies to the repultion force between charges of equal sign.

2. The magnetostatic energy of two equal charges of opposite sign will approach zero when $r \to 0$. As a result the potential energy of the two charges will approach but never exceed the negative sum $- 2 M_o c^2$ of their magnetostatic

* Spherical symmetry is not likely in particles with spin different from 0. We are only presenting examples to illustrate the method.

$\frac{g_1 g_2}{r_o}$ used in both of the above formulae is selected in order to fulfil the usual requirement that $U \to 0$ when $r \to \infty$. Another convenient selection of this arbitrary constant is obtained by adding to U the magnetostatic energies W_{1o} and W_{2o} of the respective particles when they are at an infinite distance from each other and from every magnetic monopole. This way we obtain their total magnetostatic energy W(r) when they are at a distance r from each other:

(20) $$W(r) = U + W_{1o} + W_{2o}$$

How W_{1o} and W_{2o} can be calculated will be shown below. First we shall however give the force F between the two particles, which can be obtained from the respective formulae (17) and (18) by derivation.

Exponential

(21) $$F = -\frac{g_1 g_2}{r^2} \mathrm{Exp}(-r_o/r)$$

Double-exponential

(22) $$F = -\frac{g_1 g_2}{r^2} \mathrm{Exp}(1 + r_o/r - \mathrm{Exp}(r_o/r))$$

Both U and F are plotted in fi. 1 for exponential approximation, in fig. 2 for the double exponential approximation. In each case it is assumed that g_1 and g_2 are charges of opposite sign, both equal to the elementary magnetic monopole charge g.

We notice in the figures that the attraction force between two charges reaches a maximum value for $r = \frac{1}{2} r_o$ in fig. 1, for $r \approx 0.9\, r_o$ in fig. 2 and decreases below that distance to reach 0 when r = 0.

According to formula (17) or (18) when r = 0 two elementary charges of opposite sign will have a potential

4. Some low energy levels in binary systems.

We shall proceed with the calculation of energy levels for circular orbits in binary systems.

If in formula (14) the force F is calculated by formula (21) or (22) which apply respectively in the exponential and the double-exponential approximation, we obtain an expression for the angular momentum A which does not have a finite lower limit. In each case the lower limit is 0 and is reached when r = 0. We have therefore no problem of the kind rised by formula (16) for the coulombian case, which prohibited the selection of energy levels with angular momentum $n\hbar$ or $k\hbar$ lower than the lower limit $\left|\frac{e_1 e_2}{2c}\right|$ imposed by relativity theory for that case.

Since we are dealing with circular orbits we can use Bohr's quantisation formula* A = $n\hbar$ or

$$M_1 v_1 r_1 + M_2 v_2 r_2 = n\hbar \tag{28}$$

n being a non negative quantum number.

By using (7) and (9) this formula yeilds $M_1 v_1 (r_1 + r_2) = M_1 v_1 r = n\hbar$ or according to (10):

$$\frac{M_{1o}}{\sqrt{1 - \frac{v_1^2}{c^2}}} v_1 r = n\hbar$$

By solving this equation with respect to v_1 we obtain:

$$v_1 = \frac{c}{\sqrt{1 + \frac{M_{1o}^2 c^2 r^2}{n^2 \hbar^2}}} \tag{29}$$

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* This formula does not require the use of point charges and can be used for relativistic as well as non-relativistic applications.

Tables 5 and 6 give for various n, g_1 and g_2 the values of $W[v, g_1, g_2]$ defined as

(34) $$W[n, g_1, g_2] = E + M_{1o}c^2 + M_{2o}c^2$$

which is the total energy of the two-particle system and is, linked to the rest mass $M_o[n]$ of the system by the relation

(35) $$M_o[n] = \frac{1}{c^2} W[n, g_1, g_2]$$

As an example of application let us consider the case of two coulombian electric charges in which the first particle is an electron ($M_{1o} = m$, $g_1 = e$) and the second particle an atomic nucleus ($M_{2o} >> m$, $g_2 = Ze$). The velocity v_2 (formula 30) will then be much smaller than v_1 (formula 29) and can be disregarded, so that formula (31) will become

$$1 + \frac{c}{\sqrt{1 + \frac{m^2c^2r^2}{n^2h^2}}} = \frac{Ze^2}{nh}$$

T_2 can also be disregarded relative to T_1, so that formula (32) becomes

$$E = U + T_1$$

where $U = -rF$ in the coulombian case or according to (11), (13) and (33):

$$E = M_oc^2\left(\sqrt{1 - \frac{v_1^2}{c^2}} - 1\right)$$

But according to (15) disregarding v_2 and putting $g_1g_2 = Ze^2$ and $A = nh$ we have $v_1 = \frac{Ze^2}{nh}$ which gives:

$$E = M_oc^2\left(\sqrt{1 - \frac{Z^2e^4}{n^2h^2c^2}} - 1\right)$$

Table 1. R/RU, EXPONENTIAL CASE

A (N=1)

M20	g2	M10 g1	1*M0 1*g	4*M0 2*g	9*M0 3*g	16*M0 4*g	25*M0 5*g	36*M0 6*g
1*M0	1*g		0.23465920	0.19133101	0.17612475	0.16757628	0.16176847	0.15742044
4*M0	2*g		0.19133101	0.15209169	0.13864158	0.13167646	0.12724720	0.12407844
9*M0	3*g		0.17612475	0.13864158	0.12503569	0.11800547	0.11365357	0.11063694
16*M0	4*g		0.16757628	0.13167646	0.11800547	0.11078343	0.10630535	0.10323278
25*M0	5*g		0.16176847	0.12724720	0.11365357	0.10630535	0.10169508	0.09852489
36*M0	6*g		0.15742044	0.12407844	0.11063694	0.10323278	0.09852489	0.09526369

B (N=2)

M20	g2	M10 g1	1*M0 1*g	4*M0 2*g	9*M0 3*g	16*M0 4*g	25*M0 5*g	36*M0 6*g
1*M0	1*g		0.31272989	0.24242314	0.21865851	0.20563491	0.19693828	0.19050891
4*M0	2*g		0.24242314	0.18538140	0.16624063	0.15650148	0.15038045	0.14603809
9*M0	3*g		0.21865851	0.16624063	0.14750948	0.13799638	0.13217413	0.12817014
16*M0	4*g		0.20563491	0.15650148	0.13799638	0.12838819	0.12249986	0.11849229
25*M0	5*g		0.19693828	0.15038045	0.13217413	0.12249986	0.11650262	0.11241316
36*M0	6*g		0.19050891	0.14603809	0.12817014	0.11849229	0.11241316	0.10823854

Table 2. R/R0, DOUBLE EXPONENTIAL CASE

A (N=1)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g		0.47979773	0.44891030	0.43746364	0.43081771	0.42620298	0.42269161
4*M0	2*g		0.44891030	0.41740741	0.40591015	0.39974378	0.39573484	0.39282141
9*M0	3*g		0.43746364	0.40591015	0.39365165	0.38705849	0.38287392	0.37992282
16*M0	4*g		0.43081771	0.39974378	0.38705849	0.38005871	0.37559666	0.37247609
25*M0	5*g		0.42620298	0.39573484	0.38287392	0.37559666	0.37089323	0.36759128
36*M0	6*g		0.42269161	0.39282141	0.37992282	0.37247609	0.36759128	0.36413239

B (N=2)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g		0.53128390	0.48634068	0.47040688	0.46135268	0.45514860	0.45047183
4*M0	2*g		0.48634068	0.44398407	0.42906715	0.42120158	0.41613817	0.41248289
9*M0	3*g		0.47040688	0.42906715	0.41354251	0.40533409	0.40017504	0.39655997
16*M0	4*g		0.46135268	0.42120158	0.40533409	0.39672857	0.39129781	0.38752464
25*M0	5*g		0.45514860	0.41613817	0.40017504	0.39129781	0.38561903	0.38165928
36*M0	6*g		0.45047183	0.41248289	0.39655997	0.38752464	0.38165928	0.37753484

TABLE 3

V1/C AND V2/C, EXPONENTIAL CASE

A (N=1)

			M1U	1*MU	4*MU	9*MU	16*MU	25*MU	36*MU
M2U	g2	g1		1*g	2*g	3*g	4*g	5*g	6*g
1*MU	1*g	V1/C		0.24148212	0.07607878	0.03681392	0.02177378	0.01443744	0.01030345
		V2/C		0.24148212	0.29190719	0.31470390	0.32905724	0.33953012	0.34778721
4*MU	2*g	V1/C		0.29190719	0.09554600	0.04674749	0.02770604	0.01835304	0.01307174
		V2/C		0.07607878	0.09554600	0.10471805	0.11019160	0.11397818	0.11684975
9*MU	3*g	V1/C		0.31470390	0.10471805	0.05182138	0.03091290	0.02054729	0.01465953
		V2/C		0.03681392	0.04674749	0.05182138	0.05489962	0.05699507	0.05854384
16*MU	4*g	V1/C		0.32905724	0.11019160	0.05489962	0.03292601	0.02196694	0.01571071
		V2/C		0.02177378	0.02770604	0.03091290	0.03292601	0.03431141	0.03533158
25*MU	5*g	V1/C		0.33953012	0.11397818	0.05699507	0.03431141	0.02296228	0.01646122
		V2/C		0.01443744	0.01835304	0.02054729	0.02196694	0.02296228	0.02370071
36*MU	6*g	V1/C		0.34778721	0.11684975	0.05854384	0.03533158	0.02370071	0.01702455
		V2/C		0.01030345	0.01307174	0.01465953	0.01571071	0.01646122	0.01702455

B (N=2)

			M1U	1*MU	4*MU	9*MU	16*MU	25*MU	36*MU
M2U	g2	g1		1*g	2*g	3*g	4*g	5*g	6*g
1*MU	1*g	V1/C		0.34984836	0.11957439	0.05924164	0.03547392	0.02371413	0.01702624
		V2/C		0.34984836	0.43401493	0.47112347	0.49385053	0.51007438	0.52264271
4*MU	2*g	V1/C		0.43401493	0.15557957	0.07782169	0.04658962	0.03104978	0.02220873
		V2/C		0.11957439	0.15557957	0.17298373	0.18339678	0.19059564	0.19604811
9*MU	3*g	V1/C		0.47112347	0.17298373	0.08763205	0.05282081	0.03532172	0.02530294
		V2/C		0.05924164	0.07782169	0.08763205	0.09362191	0.09770739	0.10072923
16*MU	4*g	V1/C		0.49385053	0.18339678	0.09362191	0.05676147	0.03810731	0.02736807
		V2/C		0.03547392	0.04658962	0.05282081	0.05676147	0.05948045	0.06148468
25*MU	5*g	V1/C		0.51007438	0.19059564	0.09770739	0.05948045	0.04006589	0.02884689
		V2/C		0.02371413	0.03104978	0.03532172	0.03810731	0.04006589	0.04152098
36*MU	6*g	V1/C		0.52264271	0.19604811	0.10072923	0.06148468	0.04152098	0.02995850
		V2/C		0.01702624	0.02220873	0.02530294	0.02736807	0.02884689	0.02995850

TABLE 4

V1/C AND V2/C, DOUBLE EXPONENTIAL CASE

A (N=1)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g	V1/C	0.12081431	0.03250277	0.01482986	0.00847111	0.00548033	0.00383743
		V2/C	0.12081431	0.12899305	0.13230993	0.13431443	0.13574208	0.13684867
4*M0	2*g	V1/C	0.12899305	0.03495295	0.01598238	0.00912956	0.00590225	0.00412922
		V2/C	0.03250277	0.03495295	0.03594172	0.03649541	0.03686463	0.03713766
9*M0	3*g	V1/C	0.13230993	0.03594172	0.01647995	0.00942874	0.00610050	0.00426941
		V2/C	0.01482986	0.01598238	0.01647995	0.01676059	0.01694372	0.01707529
16*M0	4*g	V1/C	0.13431443	0.03649541	0.01676059	0.00960238	0.00621870	0.00435476
		V2/C	0.00847111	0.00912956	0.00942874	0.00960238	0.00971644	0.00979784
25*M0	5*g	V1/C	0.13574208	0.03686463	0.01694372	0.00971644	0.00629755	0.00441263
		V2/C	0.00548033	0.00590225	0.00610050	0.00621870	0.00629755	0.00635412
36*M0	6*g	V1/C	0.13684867	0.03713766	0.01707529	0.00979784	0.00635412	0.00445454
		V2/C	0.00383743	0.00412922	0.00426941	0.00435476	0.00441263	0.00445454

B (N=2)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g	V1/C	0.21469671	0.05992632	0.02757516	0.01581947	0.01026321	0.00720142
		V2/C	0.21469671	0.23349877	0.24095583	0.24540246	0.24854219	0.25096080
4*M0	2*g	V1/C	0.23349877	0.06561981	0.03022965	0.01732703	0.01122521	0.00786462
		V2/C	0.05992632	0.06561981	0.06789081	0.06915260	0.06998993	0.07060708
9*M0	3*g	V1/C	0.24095583	0.06789081	0.03136339	0.01800511	0.01167293	0.00818038
		V2/C	0.02757516	0.03022965	0.03136339	0.03199789	0.03240997	0.03270511
16*M0	4*g	V1/C	0.24540246	0.06915260	0.03199789	0.01839554	0.01193771	0.00837110
		V2/C	0.01581947	0.01732703	0.01800511	0.01839554	0.01865076	0.01883229
25*M0	5*g	V1/C	0.24854219	0.06998993	0.03240997	0.01865076	0.01211348	0.00849974
		V2/C	0.01026321	0.01122521	0.01167293	0.01193771	0.01211348	0.01223914
36*M0	6*g	V1/C	0.25096080	0.07060708	0.03270511	0.01883229	0.01223914	0.00859259
		V2/C	0.00720142	0.00786462	0.00818038	0.00837110	0.00849974	0.00859259

Table 5. MASS OF BINARY SYSTEM OVER MASS M0 OF ELEMANTARY CHARGE, EXPONENTIAL CASE

A (N=1)

M20	g2	M10 g1	1*M0 1*g	4*M0 2*g	9*M0 3*g	16*M0 4*g	25*M0 5*g	36*M0 6*g
1*M0	1*g		0.08919689	1.07865131	4.08016151	9.08325493	16.08643222	25.08940458
4*M0	2*g		1.07865131	0.04792672	1.04080856	4.03870475	9.03817677	16.03825331
9*M0	3*g		4.08016151	1.04080856	0.03026986	1.02625489	4.02446079	9.02360678
16*M0	4*g		9.08325493	4.03870475	1.02625489	0.02120566	1.01874685	4.01741982
25*M0	5*g		16.08643222	9.03817677	4.02446079	1.01874685	0.01586866	1.01424789
36*M0	6*g		25.08940458	16.03825331	9.02360678	4.01741982	1.01424789	0.01242447

B (N=2)

M20	g2	M10 g1	1*M0 1*g	4*M0 2*g	9*M0 3*g	16*M0 4*g	25*M0 5*g	36*M0 6*g
1*M0	1*g		0.21662937	1.20355335	4.21147549	9.22192001	16.23198748	25.24119568
4*M0	2*g		1.20355335	0.13495362	1.11789274	4.11326754	9.11264026	16.11353040
9*M0	3*g		4.21147549	1.11789274	0.08998370	1.07917368	4.07441664	9.07225752
16*M0	4*g		9.22192001	4.11326754	1.07917368	0.06493506	1.05794764	4.05419564
25*M0	5*g		16.23198748	9.11264026	4.07441664	1.05794764	0.04953861	1.04478407
36*M0	6*g		25.24119568	16.11353040	9.07225752	4.05419564	1.04478407	0.03932858

Table 6. MASS OF BINARY SYSTEM OVER MASS M0 OF ELEMANTARY CHARGE, DOUBLE EXPONENTIAL CASE

A (N=1)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g		0.01651317	1.01155588	4.01073307	9.01053667	16.01050687	25.01053381
4*M0	2*g		1.01155588	0.00526327	1.00399387	4.00355172	9.00335586	16.00325727
9*M0	3*g		4.01073307	1.00399387	0.00259638	1.00209093	4.00185609	9.00172997
16*M0	4*g		9.01053667	4.00355172	1.00209093	0.00155616	1.00130343	4.00116587
25*M0	5*g		16.01050687	9.00335586	4.00185609	1.00130343	0.00104094	1.00089645
36*M0	6*g		25.01053381	16.00325727	9.00172997	4.00116587	1.00089645	0.00074768

B (N=2)

M20	g2	M10 / g1	1*M0 / 1*g	4*M0 / 2*g	9*M0 / 3*g	16*M0 / 4*g	25*M0 / 5*g	36*M0 / 6*g
1*M0	1*g		0.05538704	1.04001492	4.03752452	9.03703761	16.03707123	25.03726935
4*M0	2*g		1.04001492	0.01889133	1.01447958	4.01294112	9.01226640	16.01193285
9*M0	3*g		4.03752452	1.01447958	0.00951219	1.00770009	4.00685692	9.00640512
16*M0	4*g		9.03703761	4.01294112	1.00770009	0.00576091	1.00484252	4.00434065
25*M0	5*g		16.03707123	9.01226640	4.00685692	1.00484252	0.00388050	1.00334930
36*M0	6*g		25.03726935	16.01193285	9.00640512	4.00434065	1.00334930	0.00279999

$M_o = g^2/2r_o^2$ (formula 24) of the Dirac monopole is used as a unit. We notice at once that magnetically neutral systems (diagonal figures in the tables) have relatively low masses compared with the mass 1 of a Dirac monopole. On the other hand the masses of magnetically charged systems are slightly higher but close to the masses (1, 4, 9 .... etc.) of magnetic monopoles with the same magnetic charges (g, 2g, 3g, ...etc. respectively). The difference between the mass of a two/or more particle-system and the mass of a magnetic monopole with the same magnetic charge is, what we call, its mass-residual.

The very fact that it has ben possible to calculate the masses of binary systems for the two lowest energy levels is evidence that there is no theoretical barier against the application of quantum theory even within distances smaller than the classical radius of magnetic monopol charges, if due consideration is given to a finite density distribution of magnetic charge within this distance.

The main assumptions used in this paper are:

(1) That the masses of magnetic monopoles are magnetostatic energies of charges distributed with spherical symmetry around the centers of the respective particles.

(2) That the linear dimentions of the various monopoles are the same and can be described by a finite common standard radius r_o.

(3) That some approximations of coulombian fields with properties similar to those we have presented can be used as approximate means to describe the interactions between magnetic monopoles.

With these assumptions we have shown that it is possible to identify some important properties, particularely masses and energies, of particles formed by associations of several magnetic monopoles. This opens the possibility to obtain theoretic predictions of various theories for the interpretation of elementary particles by the association of several magnetic monopoles. In a subsequent paper (Barricelli 1978) a few suggestions for such an interpretation will be discussed and theoretical calculations of the masses of various elementary particles will be compared with experimental data.

monopole), irrespective of the question whether or not this radius can be subject to a direct measurement by the method indicated above.

The proportion P_r between attraction force F and its coulombian value $\frac{g^2}{r^2}$ for two magnetic monopoles of opposite equal charges g and -g is defined by the formula

(A) $$P_r = \frac{r^2 F}{g^2}$$

We want to find the r value r^* where this proportion reaches a certain value $P < 1$ used in order to define the particle radius r^*.

F must be equal to the centrifugal force:

(B) $$F = \frac{mv^2}{r}$$

m being the mass and v the velocity of each particle.

By eliminating F between (A) and (B) we obtain:

(C) $$P_r = \frac{mv^2 r}{g^2}$$

or

(D) $$v = g\sqrt{\frac{P_r}{mr}}$$

We define the angular momentum A by the formula

(E) $$A = mvr$$

or according to (D)

(F) $$A = g\sqrt{mrP_r}$$

If m is the mass of a magnetic monopole given by formula (24) in section 4:

$$m = \frac{g^2}{2r_o c^2}$$

then formula (M) becomes

(N) $$\sqrt{r} = \sqrt{r^*} \pm \frac{\hbar c}{a^2}\sqrt{\frac{2r_o}{P}}$$

If g is a Dirac monopole given by formula 1 section 1, $g^2 = \frac{137}{4}\hbar c$ and the above formula becomes:

$$\sqrt{r} = \sqrt{r^*} \pm \frac{4}{137}\sqrt{\frac{2r_o}{P}}$$

Since r_o is supposed to be comparable to r^* and P is comparable to 1, the error would be slightly lower than $\frac{10}{137}\sqrt{r^*}$ or less than 10 %.

This is a rather conspicuous error but not so large as to make the radius of a magnetic monopole a meaningless concept.

A diametrally opposite result would be obtained if in stead of calculating the error for a Dirac monopole-radius we had calculated the error for an electron-radius. In that case g would have to be replaced by e in formula (N) and since $e^2 = \frac{\hbar c}{137}$, formula (N) would become

$$\sqrt{r} = \sqrt{r^*} \pm 137\sqrt{\frac{r_e}{P}}$$

The error would be more than two orders of magnitude greater than the quantity $\sqrt{r^*}$ we want to measure (since $P < 1$).

The radius of the electron, as defined above, is not a measurable quantity according to Heisemberg's indetermination principle. This would seem to put a question mark behind every statement to the effect that the electron has a specific radius finite or zero. One may argue that the results of high energy scattering experiments fit the notion that the radius of the electron is zero or at least insignificant compared with the classical electron radius. But then one can argue with the same right that the spectrum of the hydrogen atom fit the notion that the errors in Bohr's theoretic values for the orbital velocity

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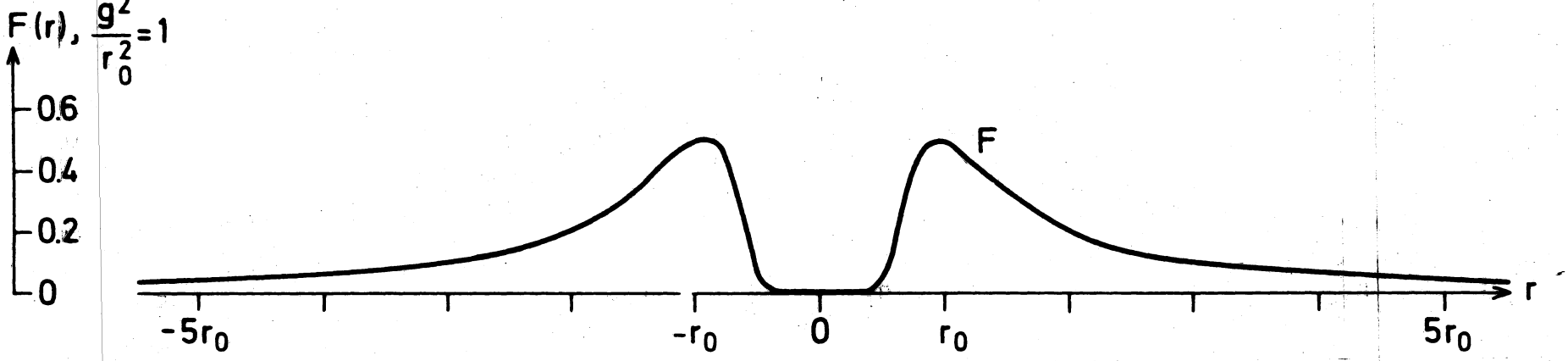
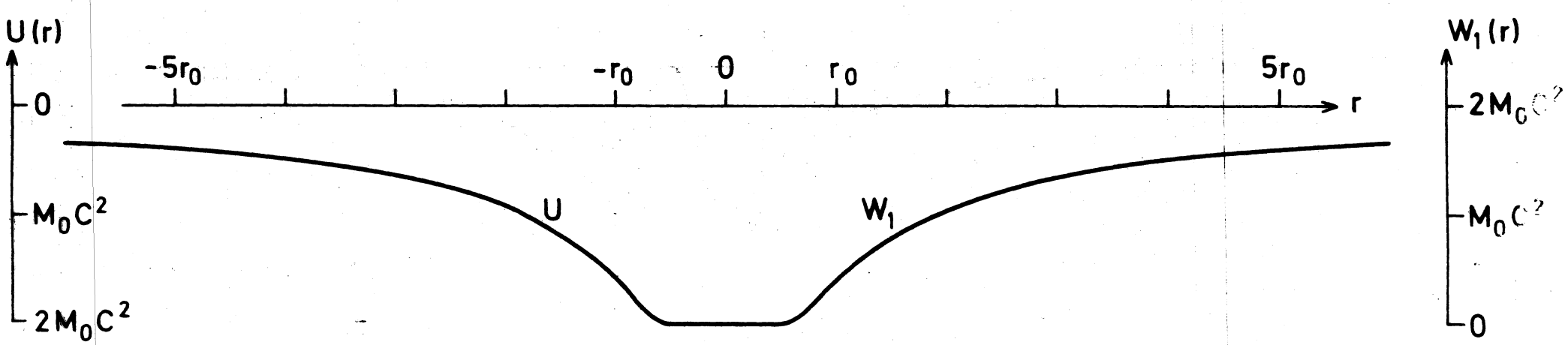
LEGENDS

Fig. 1
Expenential approximation force F (upper diagram) and potential energy U or W_1 (lower diagram), as a function of distance r, between two magnetic monopoles of charge g (g = Dirac monopole). F is measured in units of g^2/r_o^2, U and W_1 in units of $M_o c^2 = g^2/r_o$. W_1 is defined by $W_1 = U + 2M_o c^2$, and is equal to 0 when r = 0, and equal to $2M_o c^2$ when r = ∞. W_1 is therefore a measure of the total magnetostatic energy of the two monopoles as a function of their distance.

Fig. 2
Same as fig. 1 for double-exponential case.


F(r), $\frac{g^2}{r_0^2}=1$
0.6
0.4
0.2
0
F
r
$-5r_0$
$-r_0$
0
r_0
$5r_0$


U(r)
0
M_0C^2
$2M_0C^2$
U
W_1
r
$-5r_0$
$-r_0$
0
r_0
$5r_0$
$W_1(r)$
$2M_0C^2$
M_0C^2
0