"The interaction between water waves and marine structures."

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1 Introduction

The study of waves and wave-body interaction has become very important in recent years due to the increase in offshore activities related to oil extraction under the sea floor. Earlier the main interest in this field was focused on ship and ship motion. Now also the study of the forces acting on oil platforms, risers, pipe-lines etc., and their motions, has become important. At the same time there has been a renewed interest in ships, especially in high-speed ships.
Ships are charecterized by their slender form, i.e. the length is much larger than the maximum lateral dimension of the body. An essential part of the ship resistance, the wave resistance, is due to the steady wave system observed behind the ship. The wave pattern is confined to a section symmetrical about the ship's length axis, with included semiangles of $19\frac{1}{2}$. In addition there will also be a viscous resistance. When ocean waves are present there will also be a so-called added resistance.

Oil platforms may be of rather different type. Some are penetrating the sea floor, namely the jacket type and the gravity platform. Platforms fixed to the bottom have been built for water depths up to about 300 m. The semi-submersible, and also the floating production ship, are moored, but are floating relatively freely. The tension leg platform (TLP), which is a modern concept among platforms, is connected to the bottom by vertical tethers, but is still able to perform horizontal oscillations and small vertical oscillations.

The incident waves will be diffracted by the body and the change in momentum transport will create a force on the body. The waves will also set up viscous forces. These may be form drag forces due to vortex shedding or friction forces. Let us assume that the body is restrained from moving and that the fluid depth is infinte. The dominating body force is determined by the relative magnitude on the incident wave length λ and wave amplitude A, and the characteristic body dimension L. Let L_{part} denote the maximal particle displacement in the incident wave, T the period and

ω the frequency. A measure of the maximal displacement is then

$$L_{part} = \omega AT = 2\pi A \tag{1}$$

Let us introduce the Keuligan-Carpenter number KC, defined by

$$KC = \frac{L_{part}}{L} = 2\pi \frac{A}{L} \tag{2}$$

Laboratory experiments tell us that vortex shedding caused by waves takes place when KC is of order unity or larger. It is noted that KC is proportional to $\frac{A}{L}$. If $KC >> 1$, viscous effects will dominate and the wave force will be of the form

$$F = \frac{1}{2}\rho L^2 C_D U|U| \tag{3}$$

where ρ is fluid density and U is a characteristic particle velocity.
On the other side, when $KC << 1$, inertial forces will dominate. For $\frac{A}{\lambda} << 1$ the problem may be linearized. It may then be shown that

$$F = \rho g L^3 (C_M \dot{U} + C_D U) \tag{4}$$

where C_M and C_D are the apparant mass and apparant damping coefficient. The latter is due to generation of waves.

For $\frac{A}{L} = O(1)$ no satisfactory formulae exist for the wave force. This is, practically speaking, an important parameter region (pipe lines, columns on a TLP-platform etc). The most used formula is the Morrison formula, which is a combination of the two formulae above

$$F = \rho g L^3 C_M \dot{U} + \frac{1}{2}\rho L^2 C_D U|U| \tag{5}$$

where C_M and C_D are determined experimentally.

We start by discussing the first order problem, i.e. linear waves ($\frac{A}{\lambda} << 1$). The solution of this problem solves many practical problems met in ocean engineering.

2 Linear waves

We consider a large floating, or submerged body so that $\frac{A}{L} << 1$. Viscous effects may then be neglected. The fluid is assumed incompressibel and the motion irrotational. Hence a velocity potential $\Phi(x, y, z, t)$ exits satisfying the Laplace equation

$$\nabla^2 \Phi = 0 \tag{6}$$

Here (x, y, z) denote a reference system with the undisturbed free surface in the xy-plane and the z-axis vertically upwards. t is time. The free surface boundary condition expressing that the individual derivation of pressure is zero is

$$\Phi_{tt} + g\Phi_z + 2\nabla\Phi\nabla\Phi_t + \frac{1}{2}\nabla\Phi \cdot \nabla(\nabla\Phi\nabla\Phi) = 0 \qquad (z = \hat{\zeta}) \tag{7}$$

and from the Bernoulli equation

$$g\hat{\zeta}(x, y, t) = -\Phi_t(x, y, \zeta, t) - \frac{1}{2}(\nabla\Phi(x, y, \zeta, t))^2 \tag{8}$$

where $\hat{\zeta}$ denotes the elevation of the free surface and g the acceleration of gravity. Linearizing (7) we obtain

$$\Phi_{tt} + g\Phi_z = 0 \qquad (z = 0) \tag{9}$$

Furthermore, at the body the kinematic boundary condition requires that the normal velocity of the body, V_n, equals the normal velocity of the fluid $\frac{\partial \Phi}{\partial n}$. Hence

$$\frac{\partial \Phi}{\partial n} = V_n \qquad \text{on } S_B \tag{10}$$

where S_B is the wetted boundary of the body (see figure 1).
Assuming that the sea is of constant depth h we also have

$$\frac{\partial \Phi}{\partial z} = 0 \qquad (z = -h) \tag{11}$$

At last, the radiation conditions must be fulfilled at $(x, y) = \infty$.

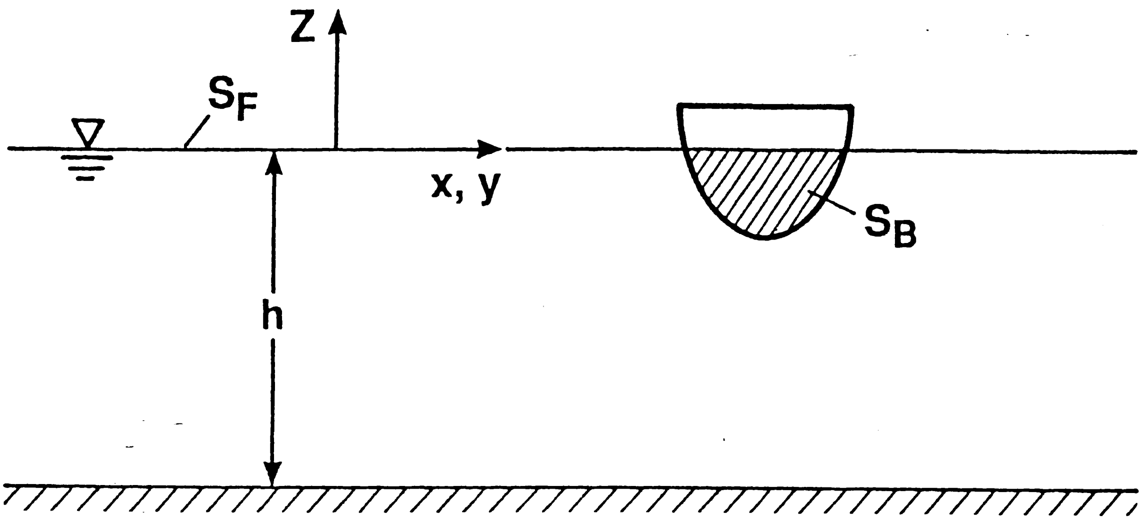

Figure 1

To evaluate the motion of a floating body we also apply the equations of linear and angular momentum for the body :

$$m\ddot{r}_G = \int_{S_B} pnds + mg + F_0 \tag{12}$$

$$\int_{V_B} r \times \ddot{r}dm = \int_{S_B} r \times pnds + r_G \times mg + M_0 \tag{13}$$

Here m is the mass of the body, dot denotes time-differentiation, G center of gravity, n unit vector positive out of the fluid, p pressure, obtained from the linearized Bernoulli equation

$$p = -\rho(\Phi_t + gz) \tag{14}$$

F_0 and M_0 are constraining force and moment from external support.
The sea is irregular and the mean amplitude distribution is given by the wave spectrum. Since the problem is linear we may consider one regular incident wave with velocity profile

$$\Phi_0 = Re(\phi_0 e^{i\omega t}) \tag{15}$$

where

$$\phi_0 = \frac{igA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}e^{-ik\cdot r} \tag{16}$$

with

$$k \cdot r = k_x x + k_y y = k(x\cos\beta + y\sin\beta) \tag{17}$$

Here ω is incident frequency, k wave number. ω and k are connected by the dispersion relation

$$k\tanh kh = \frac{\omega^2}{g} \tag{18}$$

β denote the angle of incident.
We assume that the transient motion has died out so that both fluid and floating body has a time-dependence of form $\exp(i\omega t)$
Let us consider a stable body which moves in response to the incident wave. The body has six degrees of freedom, as illustrated in figure 2. In the figure is introduced a frame of references (x_1, x_2, x_3) fixed to the body. Under the assumption that (16) is a first-order quantity, the distinction between this coordinate system and the (x, y, z)-system introduced earlier may be neglected.

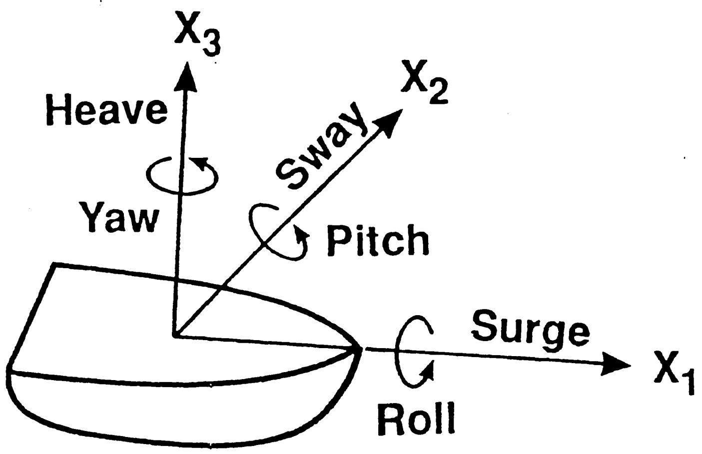

Figure 2

In figure 2 surge, sway and heave denote the translational motions along $(x, y, z) \equiv (x_1, x_2, x_3)$ with velocity (U_1, U_2, U_3), and roll, pitch and yaw denote the rotational motions along the same axis with angular velocities (U_4, U_5, U_6), respectively. Following now Newman (1977) we write

$$U_j(t) = Re(i\omega\xi_j e^{i\omega t}), \qquad j = 1, 2, ..., 6 \tag{19}$$

where ξ_j is the complex amplitude of the displacement. The velocity potential Φ may be written

$$\Phi(x, y, z, t) = Re\{(\sum_{j=1}^{6}(\xi_j\phi_j(x, y, z) + \tag{20}$$

$$A(\phi_0(x, y, z) + \phi_D(x, y, z))e^{i\omega t}\} \tag{21}$$

where ϕ_D denotes the velocity potential for the diffracted motion.

2.1 The radiation problem

The velocity potential $\phi_j, j = 1, 2, ..., 6$ appearing in (19) denote the velocity potential due to a unit body displacement . The boundary conditions for ϕ_j are given by (9) at $z = 0$ and (11) at $z = -h$. At the body boundary equation (10) gives

$$\frac{\partial\phi_j}{\partial n} = i\omega n_j \qquad (j = 1, 2, 3) \tag{22}$$

$$\frac{\partial\phi_j}{\partial n} = i\omega(r \times n)_{j-3} \qquad (j = 4, 5, 6) \tag{23}$$

where r is the vector (x, y, z). ϕ_j also satisfies the Laplace equation and at infinity the radiation conditions which express that the wave energy flux is outwards. This problem to obtain $\phi_j, (j = 1, 2, ..., 6)$, satisfying the conditions above is called the radiation problem.

The dynamic term $-\rho\Phi_t$ in the Bernoulli equation (14) gives a force and moment on the body given by

$$\begin{pmatrix} F \\ M \end{pmatrix} = -\rho Re \sum_{j=1}^{6} i\omega\xi_j e^{i\omega t} \iint_{S_B} \begin{pmatrix} n \\ r \times n \end{pmatrix} \phi_j ds \tag{24}$$

Introducing the boundary conditions (22)-(23) and replacing M by (F_4, F_5, F_6), equation (24) takes form

$$F_i = Re \sum_{j=1}^{6} \xi_j e^{i\omega t} f_{ij} \qquad (i = 1, 2, ..., 6) \tag{25}$$

where

$$f_{ij} = -\rho \iint_{S_B} \frac{\partial \phi_i}{\partial n} \phi_j ds \tag{26}$$

Here f_{ij} is complex and may be written in the form

$$f_{ij} = \omega^2 a_{ij} - i\omega b_{ij} \tag{27}$$

where a_{ij} and b_{ij} are real quantities. a_{ij} is force component proportional to the acceleration of the body and is the added mass coefficient. b_{ij} is propotrional to the velocity of the body and is called the damping coefficient. The physical reason for the existence of this damping is that the motion of the body generates waves which radiate energy. This imply that the diagonal element in b_{ij} are always positive. For an infinite fluid field, the diagonal elements of a_{ij} is also always positive, since the kinetic energy has a positive definite form. In the case of a free surface, the diagonal terms are usually positive, but not always. The possibility for negative diagonal terms can be inferred from the formula

$$T - V = \frac{1}{4} \sum a_{ij} u_i u_j^* \tag{28}$$

derived by Falnes (1983). Here T and V are the total mean kinetic and potential wave energy, respectively. u_i denotes the complex velocity of the body and a star denotes complex conjugate. (28) is a generalization of the well known formula for an infinte homogenous fluid where V is zero. We see from (28) that if V is larger than T, some of diagonal elements of a_{ij} must be negative. This may, for example occur for a catamaran where resonance leads to large values of V.

Added mass and the damping coefficient are important concepts in fluid mechanics. The former are determined by the near-field of ϕ_j and the latter by the far field values. Still there exist several integral relations between the added mass and the damping coefficients, the so-called Kramers-Kroniger relations (see Greenhow 1989).

To summarize we note that the radiation problem gives rise to two different kind of forces: (i) one which is in phase with the acceleration of the body and formally lead to a change in the body mass, and (ii) one which is in phase with the velocity of the body and is a damping force.

2.2 The diffraction problem

The two last terms in (21) give through the dynamic pressure term $-\rho\Phi_t$ a new set of forces and moments :

$$\begin{pmatrix} F \\ M \end{pmatrix} = -\rho Re[i\omega e^{i\omega At} \int_{S_B} (\phi_0 + \phi_D) \begin{pmatrix} n \\ r \times n \end{pmatrix} ds] \tag{29}$$

Using the boundary conditions (22)-(23) the equation may be written

$$F_i = Re(Ae^{i\omega t} K_i) \qquad (i = 1, 2, ..., 6) \tag{30}$$

where

$$K_i = -\rho \int_{S_B} (\phi_0 + \phi_D) \frac{\partial \phi_i}{\partial n} ds \tag{31}$$

Since the problem is linearized, we may here assume that the body is at rest.
The forces and moments defined by (29) are often called the wave-exciting forces and moments. The first part of the integral is due to the (known) incident wave and is the Froude-Krylov force and moment. This force (moment) has sometimes in ship theory been taken as approximation for (29), but normally this is a very rough approximation. Exceptions are thin or slender ships (see Newman, 1977).
The last term contains ϕ_D which satisfies the Laplace equation, the boundary conditions (9) at $z = 0$ and (11) at $z = -h$, and the radiation conditions at infinity. At the body boundary (10) requires

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_0}{\partial n} \tag{32}$$

The problem to find ϕ_D is called the diffraction problem.
It is often troublesome to solve the diffraction problem. It is therefore important that it is possible to obtain the forces (29) without really solving the diffraction

problem. To show this we use Green's theorem for the functions ϕ_D and ϕ_i on the closed surface indicated in figure 3.

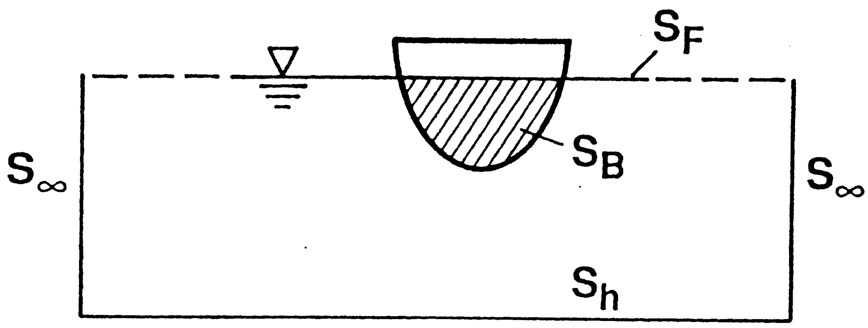

Figure 3

We have

$$\int_\Gamma (\phi_i \frac{\partial \phi_D}{\partial n} - \phi_D \frac{\partial \phi_i}{\partial n}) = 0 \qquad (i = 1, 2, ..., 6) \tag{33}$$

where Γ denotes integration over $S_\infty + S_F + S_B + S_h$.
Since ϕ_D and ϕ_i satisfy the same boundary conditions except at the body, (33) reduces to

$$\iint_{S_B} (\phi_i \frac{\partial \phi_D}{\partial n} - \phi_D \frac{\partial \phi_i}{\partial n}) ds = 0 \qquad (i = 1, 2, ..., 6) \tag{34}$$

Using (34) and the boundary conditions (32), equation (31) takes the form

$$K_i = -\rho \iint_{S_B} (\phi_0 \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial \phi_0}{\partial n}) ds \tag{35}$$

which is indepedent of ϕ_D. ϕ_i is known from the radiation problem.
By Green's theorem equation (35) may also be written

$$K_i = \rho \int_{S_\infty} (\phi_0 \frac{\partial \phi_i}{\partial n} - \phi_i \frac{\partial \phi_0}{\partial n}) ds \tag{36}$$

where S_∞ is the control surcface at infinity. This formula has the advantage that the value of ϕ_i is only needed at infinity. The radiation potential ϕ_i is proportional to the square root of wave energy flux. Hence (36) suggests a close connection between the excitation force K_i and the wave damping b_{ii}. In two dimensions it is found that

$$b_{ii} = \frac{|K_i|^2}{2\rho g C_g} \tag{37}$$

where C_g is the group velocity. It is interesting to note that a body's ability to radiate wave energy is intimately connected to the magnitude of the exitation force.

The formula (36) is the Haskind-Newman relation. Consequences of this formula, as for example (37), has been discussed by Wehausen (1971) and Newman (1976).

Some interesting general relations between two diffraction problems may be obtained by using Green's theorem. Let ϕ_1 and ϕ_2 be the velocity potentials for waves with the same frequencies but different angles of incidence, satisfying the boundary conditions (9), (11) and the radiation conditions. Since at the body $\frac{\partial\phi_1}{\partial n} = \frac{\partial\phi_2}{\partial n} = 0$, it follows that

$$\int_{S_\infty} (\phi_1 \frac{\partial\phi_2}{\partial n} - \phi_2 \frac{\partial\phi_1}{\partial n}) ds = 0 \tag{38}$$

where S_∞ denotes the surface of the vertical, circular cylinder at infinity.

We consider the two-dimensional case. ϕ_1 and ϕ_2 is then the velocity potential for two regular waves with the same frequency, being incident from left and right, respectively.

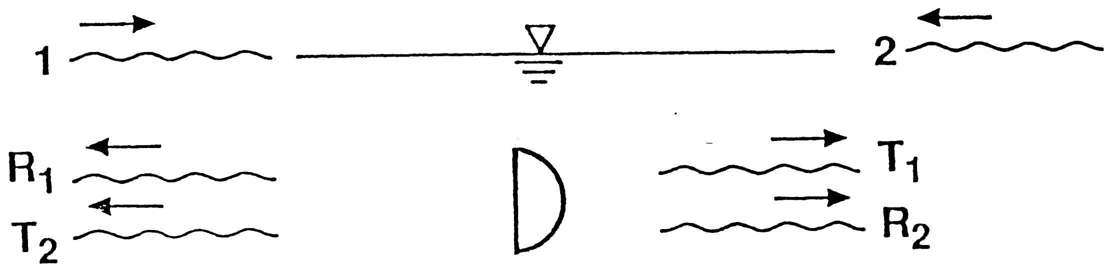

Figure 4

It may be shown from (38) that the transmission coefficients for the two waves are identical :

$$T_1 = T_2 \tag{39}$$

independent of the form of the body. Replacing ϕ_2 in (38) by ϕ_2^* we obtain

$$R_1 T_2^* + R_2^* T_1 = 0 \tag{40}$$

which combined with (39) gives

$$|R_1| = |R_2| \tag{41}$$

Hence also the numerical value of the reflection coefficient is dependent on the direction of the incident wave (for other results, see Mei, 1989).

2.3 Comments on the numerical methods in the diffraction and radiation problems.

To solve the diffraction and radiation problems it is necassary to apply numerical methods. The most used method is the boundary element method, which has the merit that the dimension of the problem is reduced by one such that three-dimensional problems result in integrating over surfaces and two-dimensional problems along curves. The starting point is to introduce a Green function G, defined by

$$\nabla^2 G(r, r') = \frac{\delta(R)}{4\pi R^2}, \quad R = r - r' \tag{42}$$

where r and r', the field and source points, respectively, as elements in the fluid or at the body boundary. Furthermore G fulfils the boundary conditions (9) and (11), and the radiation conditions. Physically, the Green function is a local, oscillatory source of unit strength. The mathematical form is given in Wehausen & Laitone (1960). Applying Green's theorem to the velocity potential ϕ and G it is found that

$$\int_{S_B} (\phi \frac{\partial G}{\partial n'} - G \frac{\partial \phi}{\partial n'}) ds' = \left\{ \begin{array}{c} -\frac{1}{2}\phi(x, y, z) \\ -\phi(x, y, z) \end{array} \right\} \tag{43}$$

for (x, y, z) on the body boundary S_B and in the fluid, respectively. Let $\frac{\partial \phi}{\partial n} = f$ at the body boundary where f is known. It then follows from (43) that

$$\frac{1}{2}\phi(x, y, z) + \int_{S_B} \phi \frac{\partial G}{\partial n'} ds' = \int_{S_B} G f ds' \tag{44}$$

for (x, y, z) on S_B. This is a Fredholm integral equation of the second kind. Solving (44) we find ϕ at the surface. The value of ϕ in the fluid is then obtained from (43). We notice that by this procedure the velocity potential in the fluid is expressed in the form of sources and normal dipoles distributed over the body boundary. The strength of these are $\frac{\partial \phi}{\partial n}$ and ϕ, respectively.

A related method is based on expressing ϕ in the fluid only by sources and sinks distributed over the body surface :

$$\phi(r) = \int_{S_B} \sigma G ds' \tag{45}$$

where r is a point in the fluid or on the body boundary. Here σ is the unknown source strength. ϕ given by (45) satisfies the Laplace equation and all boundary conditions, except at the body boundary S_B. This condition is also fulfilled if

$$-\frac{1}{2}\sigma + \int_{S_B} \sigma \frac{\partial G}{\partial n} ds' = f \tag{46}$$

(46) is a Fredholm integral equation of the second kind. When σ is obtained from (46), the velocity potential ϕ in the fluid is found from (45).
It is also a possible way of proceeding to express ϕ in the fluid only by normal dipoles on the body boundary

$$\phi(r) = \int_{S_B} \kappa \frac{\partial G}{\partial n'} ds' \tag{47}$$

where κ is the strength of the dipole. Hence

$$f = \frac{\partial \phi}{\partial n} = \frac{\partial}{\partial n} \int_{S_B} \kappa \frac{\partial G}{\partial n'} ds' \tag{48}$$

which is a Fredholm integral equation of the first kind.
Instead of using a Green function we may use a simple fundamental solution. In three dimensions a natural choice would be $\frac{1}{4\pi}\frac{1}{r}$. Applying the Green theorem for a fundamental function and ϕ, we end up with a Fredholm integral equation of the second kind with a simple kernel. However, in addition we must now also integrate over the free surface, which introduces more unknown quantities. This way of proceeding may be preferable in non-linear problems.

The method of integral equations applied on floating bodies introduces by discretizising of the problem a set of irregular frequencies. For these values of the frequency the determinant of the system becomes zero and the obtained values of added mass and damping coefficients have substansial errors. The irregular frequencies may cause considerable trouble. It is not known on beforehand for which frequencies they occur. For bodies where the forces are varying strongly with the frequency, it may be difficult to decide whether the variance is due to, for example, resonance or the non-physical irregular frequncies.

2.4 The hydrostatic forces and the equations of motion

The Bernoulli equation contains also a hydrostatic tem $-\rho g z$ which gives rise to hydrostatic forces. Since the problem is linearized, the free surface may be taken as plane. The only effect of the incident wave, is to give the body a small translation and rotation with amplitude ξ_i. The hydrostatic forces are obtained by elementary considerations and may be written (see Newman 1977)

$$F_i = -\sum_{j=1}^{6} C_{i,j}\xi_j \qquad (i = 1, 2, ..., 6) \tag{49}$$

where $C_{33} = \rho g S$, $C_{44} = \rho g V[\frac{S_{22}}{V} + z_B - z_G]$, $C_{55} = \rho g V[\frac{S_{11}}{V} + z_B - z_G]$. For other values of (i, j), $C_{ij} = 0$. It is here assumed that the origin of the frame of references is situated at the midpoint of the waterplane (center of flotation) and the x- and y-axis directed along the principle axis of the waterplane. Furtermore S is the waterplane area, V the displaced volum, z_B and z_G the vertical coordinates for the center of buoyancy and the center of gravity, respectively. $S_{ii}, (i = 1, 2)$ denotes the moment of inertia of the waterplane with respect to the x- and y-axis.

An equation for determining the oscillation of the body is now obtained by introducing the expressions for the forces into the equations for conservation of linear and angular momentum. The inertial forces due to the body mass may appropriately be written

$$F_i = \sum_{j=1}^{6} -m_{i,j}\omega^2\xi_j \tag{50}$$

where

$$m_{ij} = \left\{ \begin{array}{cccccc} m & 0 & 0 & 0 & mz_G & 0 \\ 0 & m & 0 & -mz_G & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_G & 0 & I_{11} & I_{12} & I_{13} \\ mz_G & 0 & 0 & I_{21} & I_{22} & I_{23} \\ 0 & 0 & 0 & I_{31} & I_{32} & I_{33} \end{array} \right\} \tag{51}$$

Here m is the body mass, and I_{ij} denotes the moment of inertia for the body. It is obtained that

$$\sum_{j=1}^{6} \xi_j[-\omega^2(m_{ij} + a_{ij}) + i\omega b_{ij} + c_{ij}] = AK_i \tag{52}$$

which is the form given by Newman (1977). Equations (52) is a system of 6 equations with 6 unknown. The analogy with the harmonic oscillator is noted.

2.5 Waves and current

If in addition to incident waves a current occurs, the problem to find the inertial forces become more complicated. A current has also a great impact on whether vortex shedding takes place or not. This has been demonstrated in laboratory experiments by Zhao, Faltinsen, Krokstad and Aanesland (1988), for a floating half-sphere moving with constant velocity U. In the case no current ($U = 0$), vortex shedding occured when the Keulegan-Carpenter number KC was larger than about $2 - 3$. For $U \neq 0$, the possibility for observing vortex shedding was very dependent on the ratio $\frac{U}{U_m}$ where U_m is the maximal particle velocity in the incident wave in the current direction. Roughly speaking, it was found that if $U > U_m$, seperation took place for all values of KC. If, on the other side $U < U_m$ seperation did not take place if KC was sufficiently small.
We shall for simplicity assume that no vortex shedding occur and that the current is uniform. It is two reasons why it is important to take into account a current firstly in connection with offshore operations (for example in the Northern Sea) a current about $1 - 2ms^{-1}$ is often observed.
Secondly, a moving ship or platform lead in a body-fixed frame of references to a problem with a (uniform) current.

Wave forces on floating bodies inbedded in a uniform current (or equivalently, with a uniform forward velocity) have recently been studied by Zhao, Faltinsen, Krokstad and Aanesland (1988), Zhao and Faltinsen (1989), Wu and Eatock-Taylor (1990) and Nossen, Grue and Palm (1991). In all these papers it is assumed that $\frac{U\omega}{g}$ is small. In the two first mentioed papers a hybrid method are used. Close to the body a boundary element method with Rankine sources is applied and this region is matced to an outer regime where a multipole expasion is used.

Wu and Eatock-Taylor (1990) develops the velocity potential in a Taylor series after $\frac{U\omega}{g}$ and retains the two first terms. In Nossen et al. (1991) also the Green function is developed in a series after $\frac{U\omega}{g}$ and the two first terms are retained.
The actual Green function may then as shown by Huijsmans and Hermans (1985), be expressed by the Green function for $U = 0$ and it is derivatives, for which effective subroutines exist. Using Green's theorem it is after some manipulation shown that the velocity potential ϕ_1 for the diffraction potential satisfies the integral equation

$$2\pi\phi^1 + \int_{S_B} \phi^1 \frac{\partial G^0}{\partial n} ds = 2i \int_{S_F} \phi^0 (\nabla_1 G^0 \cdot \nabla_1 \chi + \frac{1}{2} G^0 \nabla_1^2 \chi) ds - \int_{S_B} \phi^0 \frac{\partial G^1}{\partial n} ds \quad (53)$$

where the velocity potential ϕ and the Green function G is written as

$$\phi = \phi^0 + \frac{U\omega}{g}\phi^1 \quad (54)$$

$$G = G^0 + \frac{U\omega}{g} G^1 \quad (55)$$

Here ϕ^0 and G^0 is the velocity potential and Green function for $U = 0$, respectively. ϕ^0 is obtained by standard procedure and is considered known. χ is the steady velocity potetial set up by the body due to the current, and is obtained by a simple integral equation. Subscripts 1 indicates that the derivatives are horizontal.
Equation (53) is a Fredholm integral equation of the second kind. We note that the right hand side is in principle known. Hence (53) is an integral equation where the unknown is only located over the wetted part of the body. A similar

equation is obtained for the radiation problem.

It turns out that a small current has a relatively great impact on the magnitude of the hydrodynamical forces acting on the body. This is particular true for the horizontal drift forces. These are defined as time-averaged forces and are responsible for the horizontal drift of the body. The time-averaged value of the first order forces vanish, and the lowest order of the drift force is of second order. The force is obtained by integrating the pressure over the body, using the Bernoulli equation

$$p = -\rho \frac{\partial \phi}{\partial t} - \frac{1}{2}\rho \nabla \phi \cdot \nabla \phi - gz \tag{56}$$

Instead of using the near field, we may use the principle of conservation of momentum and compute the momentum transport at infinity. The last method is numerically the most robust one.

Let the mean second order drift force in the x-direction be denoted by F. Since the current U is small, we write

$$F = F_0 + \frac{\partial F}{\partial U} U \tag{57}$$

where F_0 is the drift force for $U = 0$.

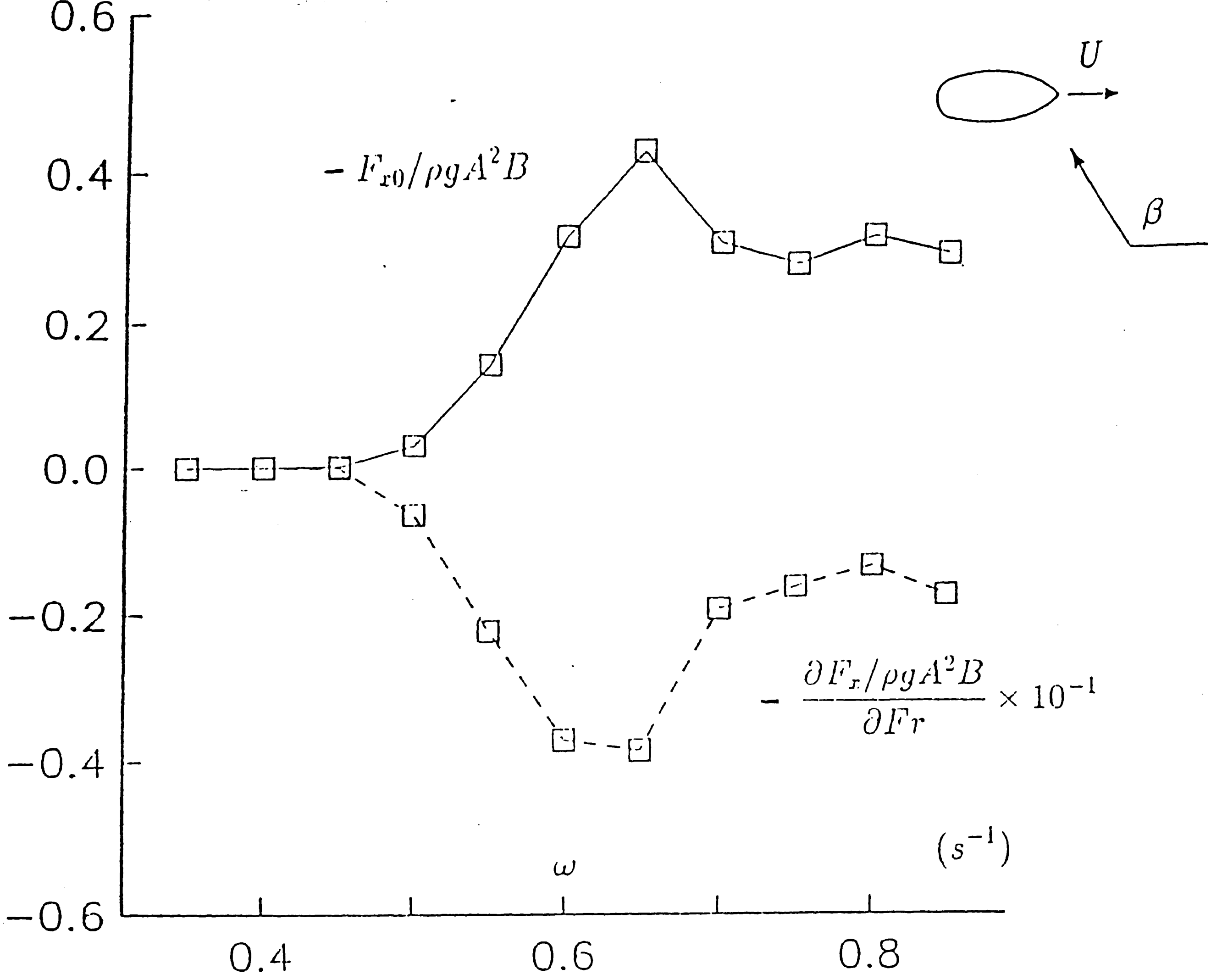

Figure 5

The effect of U is demonstrated in the two following examples. In figure 5 (from Grue and Palm, 1991) we consider a ship with length $L = 230$m and a beam $B = 41$m, moving along the positive x-axis. The wave incidence angle $\beta = 140^o$, and Fr denotes the Froude number $\frac{U}{\sqrt{gB}}$. We note from the figure that for a Froude number $Fr = 10^{-1}$, the effect of the current leads to a change in the drift force of about $50 - 100\%$. $Fr = 10^{-1}$ corresponds to $U = 2ms^{-1}$. As a second example (from Nossen et al.) we consider a tension leg platform composed of four vertical cylinders, each of radius a, draft $3a$ and mounted to a ring-like pontoon, as indicated in figure 6.

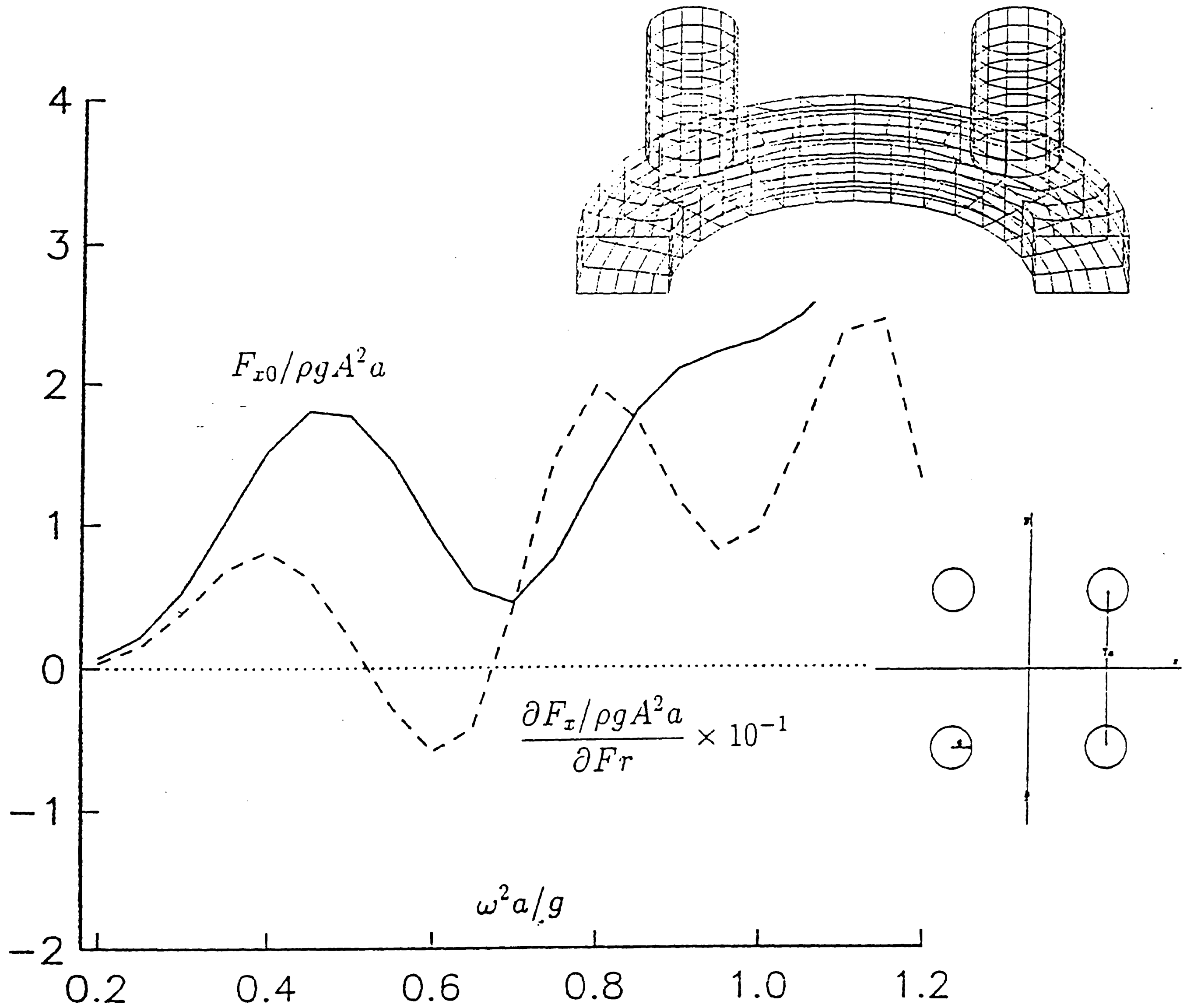

Figure 6

The circular pontoon has a rectangular cross-section with breath $2a$ and height $1.4a$. The columns are placed on the pontoon such that their centers form a square with sides $7a$. The Froude number is now defined by $Fr = \frac{U}{\sqrt{ga}}$. We notice that $Fr = 0.1$ gives a change of about $50 - 100\%$ in the drift force. For $a = 15$m, $Fr = 0.1$ corresponds to $U = 1.2ms^{-1}$. It is noted that the value of the drift force as a function of ω, oscillates strongly obviously due to interaction between the cylinders.

3 Second-order waves

Linear theory gives a very useful approxmation to many of the problems met with in marin fluid dynamics. But some problems are strongly non-linear, as for example breaking waves. This is also true for problems of the entry - exit type, i.e. the body is entering or leaving the water. One example, this type, from naval architecture, is the impact loads on ship bows which are performing large motion relative to the waves. Another example, from ocean engineering, is crane operation where the body is lowered through the free surface. Some progress has been obtained in the solution of the problem, see Faltinsen (1990). Other practical problems in marin fluid dynamics may be explained by using second order theory to the wave motion. We have already discussed the steady (mean) second order force, the drift force, which was obtained from products of first-order quantities only and do not require knowledge of the second-order solution. However, for $U \neq 0$ this seems to be true only if the force is computed in the far field, and not when the force is found by pressure integration over the body.

But also the second-order oscillatory forces are important: Observations of moored platforms and ships show that the bodies may perform horizontal oscillations with typical periods of order 1-2 minutes, often called slowdrift oscillations. The incident wave spectrum has a peak at about 10 seconds, and practically no energy at the characteristic periods of the platform motion. Furthermore, a tension leg platform may be vertical oscillations with periods about 2-4 seconds, called ringing or springing. The incident wave spectrum has no energy on these periods either. To explain these observations, terms with difference- and sum-frequencies due to product of two first order waves must be taken into account. Let Φ_1 and Φ_2 denote the first order velocity potential due to two incident waves

$$\Phi_1 = Re[\phi_1(x, y, z)e^{i\omega_1 t}] = \frac{1}{2}\phi_1 e^{i\omega_1 t} + cc \tag{58}$$

$$\Phi_2 = Re[\phi_2(x, y, z)e^{i\omega_2 t}] = \frac{1}{2}\phi_2 e^{i\omega_2 t} + cc \tag{59}$$

where cc denotes complex conjugate. Hence the product of Φ_1 and Φ_2 gives

$$\Phi_1\Phi_2 = \frac{1}{4}\phi_1\phi_2 e^{i(\omega_1+\omega_2)t} + \frac{1}{4}\phi_1\phi_2^* e^{i(\omega_1-\omega_2)t} + cc \tag{60}$$

where a star denotes complex conjugate.
Hence by considering two incident waves of different frequencies, i.e. biharmonic incident waves, non-linear effects produce loads of frequencies outside the (linear) wave spectrum, which may be in resonance with the oscilating system.
To study these phenomenons we usually have to solve the second order oscillatory problem with biharmonic incidents wave. This is complicated and time consuming. There exists, however, an important and very much used approximation for the slow drift problem (Newman 1974). Let us consider an irregular sea which we approximate by a finite sum

$$\xi(t) = Re \sum_{m=1}^{N} A_m \exp(i\omega_m t) \tag{61}$$

The horizontal slowly varying second-order force, which is only due to difference frequencies, may be written

$$F(t) = Re \sum_{m=1}^{N}\sum_{n=1}^{N} A_m A_n^* T_{mn} \exp i(\omega_m - \omega_n)t \tag{62}$$

where T_{mn} is the (unknown) transfer function. If the wave spectrum is narrow, it seems plausible that a good approximation is obtained by putting $T_{mn} = T_{nm} = T_{mm}$. It is seen from (62) by letting $m = n$, that T_{mm} is the transfer function for the steady second order drift force for incident monochromatic waves of frequency ω_m, and hence is completely given by first order quantities. Newman's approximation therefore is a very effective tool when the wave spectrum is narrow.
Another effective method is proposed by Marthinsen (1983). For a narrow wave spectrum, the spectrum may be replaced by a single sinusoidal wave with slowly varying amplitude and a local wave number and frequency varying with time and place. In this approximation the local values of the amplitude and frequency is introduced into the expression for $T_{mm}(\omega_m)A_m A_m^*$.
The equation for the slowdrift oscillations may be written

$$(m + a_{11})\ddot{x} + k|\dot{x}|\dot{x} + cx = F(t) \tag{63}$$

where m is the body mass and a_{11} the added mass. The second term is a damping term, which often is small. The term cx is the restoring force due to the mooring system. The load $F(t)$ is the slowly varying second order force, given by (62). Wichers and Sluijs (1979) observed that the damping for slowly varying horizontal oscillations differs significantly from the viscous damping measured in calm water. They suggested that the difference is due to $F(t)$ being a function of U. For small U we have

$$F(t) = F_0(t) + (\frac{\partial F}{\partial U})U \tag{64}$$

Introducing (64) in (63) and replacing U with $-\dot{x}$, we see that the last term in (64) acts as a damping term, provided $\frac{\partial F}{\partial U}$ is positive. This artificial damping term is important and is called the wave-drift damping. It is usually positive, but not always.
Using Newman's approximation, the knowledge of the mean second order force is sufficient to find the load as well as the wave drift damping. In special cases, for example for the tension leg platform, as we have seen, $\frac{\partial F}{\partial U}$ becomes negative and acts destabilizing. Negative wave drift damping is also found for submerged bodies of elliptical shape.

For second order oscillations with sum-frequenices and also for difference-frequencies in the case of a broad wave spectrum, no effective approximation is available. To solve these problems it is necassary to solve the whole second order problem for two incident waves. A numerical solution for axissymmetric bodies has recently been given by Kim and Yue (1990). Before this paper appeared, works on the second order problem for monochromatic incident waves were published by Molin (1979), Eatock-Taylor & Hung (1987), Kim & Yue (1989). In all these works the forward velocity is zero ($U = 0$).
Let us for simplicity consider monochromatic incidents waves, assuming that the body is restrained and the fluid depth is infinty. The second order oscillatory velocity potential $\Phi^{(2)}$ is written as

$$\Phi^{(2)} = Re(\phi^{(2)} e^{2i\omega t}) \tag{65}$$

Furthermore

$$\Phi^{(2)} = \Phi_I^{(2)} + \Phi_D^{(2)} \tag{66}$$

where $\Phi_I^{(2)}$ is the second order incident wave potential and $\Phi_D^{(2)}$ the second order diffracted potential. Since for infinte depth $\Phi_I^{(2)}$ is zero, we may drop the subscript D. We have

$$\nabla^2 \phi^{(2)} = 0 \tag{67}$$

$$\phi_z^{(2)} - 4K\phi^{(2)} = q(x,y) \qquad (z=0) \tag{68}$$

where $K = \frac{\omega^2}{g}$ and q is the quadratic forcing function, defined in terms of the first-order solution by

$$q = \left(-\frac{i\omega}{g}\nabla\phi^{(1)} \cdot \nabla\phi^{(1)} + \frac{i\omega}{2g}\phi^{(1)}(\phi_{zz}^{(1)} - K\phi_z^{(1)})\right)_{z=0} \tag{69}$$

Furthermore

$$\phi_n^{(2)} = 0 \qquad \text{at the body} \tag{70}$$

$$\phi^{(2)} = 0 \qquad (z = -\infty) \tag{71}$$

It is appropriate to split $\phi^{(2)}$ in two terms

$$\phi^{(2)} = \phi_1^{(2)} + \phi_2^{(2)} \tag{72}$$

Here $\phi_1^{(2)}$ is a particular solution of the problem, fulfilling all boundary conditions, except the body boundary condition (70). $\phi_2^{(2)}$ then fulfils the homogeneous boundary equation

$$(\phi_2^{(2)})_z - 4K\phi_2^{(2)} = 0 \qquad (z=0) \tag{73}$$

and

$$\frac{\partial\phi_2^{(2)}}{\partial n} = -\frac{\partial\phi_1^{(2)}}{\partial n} \qquad \text{at the body} \tag{74}$$

together with (71). $\phi_2^{(2)}$ at infinty is composed of free waves which must travel outwards. $\phi_1^{(2)}$ is a solution forced by the free surface conditions, but contains also free waves which must travel outwards. The solution is uniquely specified by the requirement that the free waves must travel outwards, which is the radiation

conditions to be used.
In principle, and formally, there is no difficults in obtaining $\phi_1^{(2)}$ and $\phi_2^{(2)}$. In fact, the problem to find $\phi_1^{(2)}$ is identical to find the velocity potential for a given pressure load on the free surface. Using Green's theorm for $\phi_1^{(2)}$ and the Green function with frequency 2ω, we obtain

$$\phi_1^{(2)} = \frac{1}{2\pi} \int_{S_F} q ds \int_0^\infty \frac{k}{k - 4K} e^{kz} J_0[k(r^2 + \rho^2 - 2r\rho\cos(\theta - \alpha))^{1/2}] dk \quad (75)$$

where S_F denotes integration over the free surface. Here j_0 is the Bessel function of the first kind, (r, θ) and (ρ, α) (dummy variables), are polar coordinates. The evaluation of (75) is however complicated numerically, due to oscillations of the integrand for large values of ρ. This problem is avoided by using numerical methods for moderate values of ρ and analytical methods for large values of ρ. Still the problem is highly time-consuming. When $\phi_1^{(2)}$ is found, $\phi_2^{(2)}$ is easily obtained.
It is important to find some simpler, approximate methods for evaluating the integral (75) and the corresponding one for incident biharmonic waves. Interesting results, valid for large depths, has been obtained by Newman (1990). Let us consider the two-dimensional case. At large positve x-values

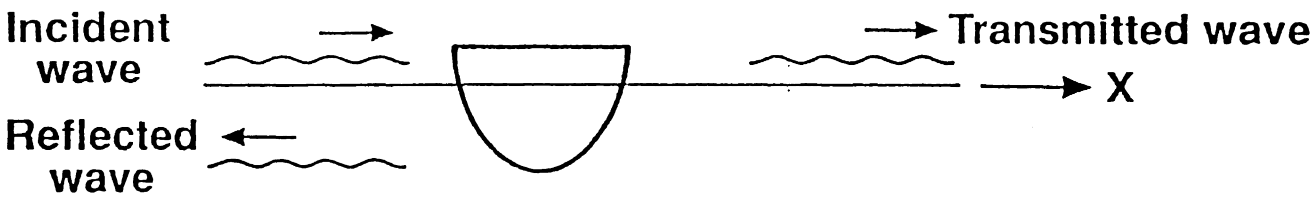

Figure 7

the first order solution consits of one single wave, the transmitted wave. Hence for $x = \infty$, $q = 0$. For large negatives values of x, the first order solutions consits of the incident wave and the reflected wave :

$$\phi^{(1)} \sim \frac{qA}{\omega}(e^{Kz-iKx} + Re^{Kz+iKx}) \tag{76}$$

where R is the reflection coefficient. Introducing (76) in (74), we obtain

$$q \sim -4i\omega KA^2R \qquad (x=-\infty) \tag{77}$$

To find a formula for $\phi_1^{(2)}$ we use Green's theorem for $\phi_1^{(2)}$ and the Green function of frequency 2ω on the $ABCD$

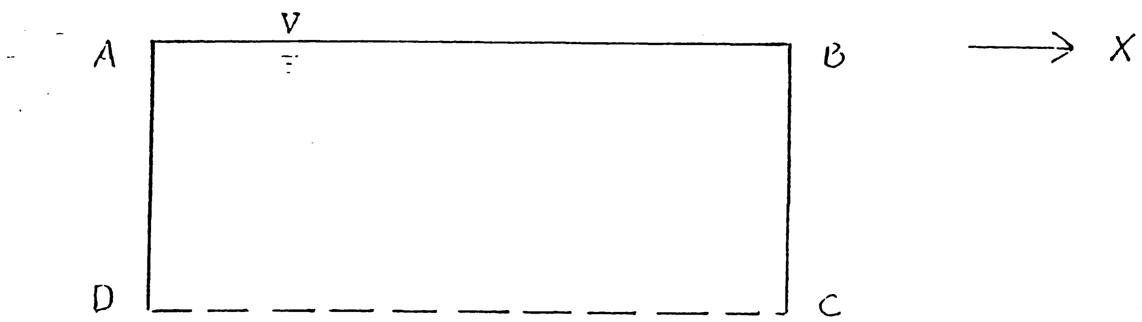

Figure 8

which gives

$$\phi_1^{(2)} = \frac{1}{\pi}\int_{-x_1}^{+x_2} q(\xi)d\xi \int_0^\infty \frac{\cos k(x-\xi)e^{kz}}{k-4K}dk + L_1 + L_2 \tag{78}$$

where L_1 and L_2 are the contribution from the integrals along DA and CB, respectively. The latter is zero due to the radiation conditions, and the former becomes

$$L_1 = -i\omega RA^2e^{-i4Kx_1}e^{i4Kx+4Kz} \tag{79}$$

For large values of z, (79) is exponentially small and will be neglected. Letting $(x_1, x_2) \rightarrow \infty$ we then obtain

$$\phi_1^{(2)} = \frac{1}{\pi}\int_{-\infty}^{+\infty} q(\xi)d\xi \int_0^\infty \frac{\cos k(x-\xi)e^{kz}}{k-4K}dk \tag{80}$$

which is the form used by Newman.
For large values of z the exponential function in (80) becomes very small, and only small values of k give contributions. Hence in the inner integral we may develop $(k-4K)^{-1}$ in powers of k and integrate term-by-term. Neglecting contributions which are exponential small, it follows that

$$\phi_1^{(2)} \sim \frac{1}{4\pi K}\sum_{m=0}(4K)^{-m}\frac{\partial^m}{\partial z^m}\int_{-\infty}^{+\infty}\frac{z}{z^2+(x-\xi)^2}q(\xi)d\xi \tag{81}$$

Retaining only the first term ($m=0$), we have

$$\phi_1^{(2)} \sim \frac{1}{4\pi K}\int_{-\infty}^{+\infty}\frac{z}{z^2+(x-\xi)^2}q(\xi)d\xi \tag{82}$$

Introducing $\frac{\xi}{z}=v$, (82) may be written

$$\phi_1^{(2)} \sim \frac{1}{4\pi K}\int_{-\infty}^{0}\frac{q(vz)}{1+(\frac{x}{z}-v)^2}dv+\int_{0}^{\infty}\frac{q(vz)}{1+(\frac{x}{z}-v)^2}dv \tag{83}$$

Sinze z is large

$$\phi_1^{(2)} \sim \frac{q(-\infty)}{4\pi K}\int_{-\infty}^{0}\frac{1}{1+(\frac{x}{z}-v)^2}dv \tag{84}$$

since $q(\infty)$ is zero. (84) reduces to

$$\phi_1^{(2)} = \frac{C}{2}[1-\frac{2}{\pi}\tan^{-1}(\frac{x}{|z|})] \tag{85}$$

where

$$C = i\omega A^2 R \tag{86}$$

This formula was derived by Newman for two-dimensional problems. It is observed from (86) that for large values of $\frac{x}{|z|}$, $\phi_1^{(2)}=0$. For large negative values of $\frac{x}{|z|}$, $\phi_1^{(2)}=C$ whereas for small numerical values of $\frac{x}{|z|}$, $\phi_1^{(2)}=\frac{C}{2}$, exactly the average of the values for large positive and large negative $\frac{x}{|z|}$-values.
It may be a surprising result that in the two-dimensional case the second order velocity potential approaches a constant values different from zero as z increases. Hence at large depth the second order pressure may give rise to relatively large oscillating vertical forces on floating body extending deep in the

water.
Comparison between the force computed from (85) and laboratory experiments have only been possible in one case, viz. experiments by Johansson (1989) for a fixed rectangular cylinder. The incident wave length is 1.2m, the other data as shown in figure 9.

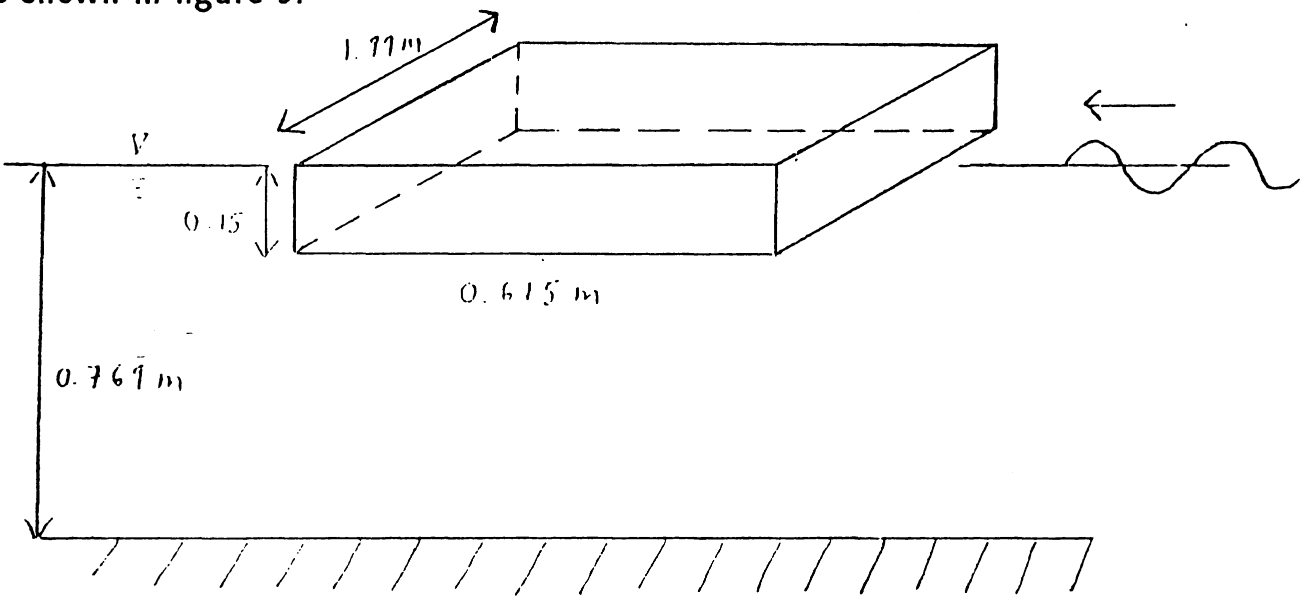

Figure 9

The vertical forces show a excellent agreement.
It is interesting to observe that the value of the second order velocity potential at large depth is determined uniquely by the value of q at $x = -\infty$. This value is different from zero due to reflection of the incident wave. If the incident wave is not reflected, which is true for a submerged, horizontal sircular cylinder, $q = 0$ at $x = -\infty$. In this case the second order velocity potential approaches zero as z increases.
It has been known for many years that two plane sinusoidal waves travelling in opposite directions with same frequencies have a second order velocity potential, oscillating with time, which does not decay with depth. This second order motion was analyzed by Longuet-Higgins (1950) who suggested that the phenomenon could be used to forecast tropical cyclones by observing the microseismic waves generated at the sea floor.
In three dimensions a similar effect occurs. The solution is then given by (75). In polar coordinates the asymptotic value of q for large r is found to be

$$q(r,\theta) \sim F(\theta)(Kr)^{-\frac{1}{2}}e^{-iKr(1+\cos\theta)} \tag{87}$$

Exactly as in the two-dimensional case, the asymptotic value of the second order velocity potential for large z, is only depending on the value of q at infinity, given by (87). We notice that for $\theta = \pi$, i.e. on the weather side, $q(r,\theta)$ is not oscillating in space, decaying uniformly as $r^{-\frac{1}{2}}$. In this direction we have reflection and standing waves, and it may be shown that it is only for this direction at infinity $q(r,\theta)$ gives contribution to the asymptotic value of the second order velocity potential. Hence only $F(\pi)$ in (87) is needed for finding the asymptotic value of $\phi_1^{(2)}$. It is found that

$$\phi_1^{(2)} \sim \frac{F(\pi)e^{i\frac{\pi}{4}}}{4(2\pi)^{\frac{1}{2}}K^2} \frac{z}{R(R+x)} \tag{88}$$

where

$$R = (x^2 + y^2 + z^2)^{\frac{1}{2}} \tag{89}$$

Thus for $x^2 + y^2 << z^2$

$$\phi_1^{(2)} \sim \frac{F(\pi)e^{i\frac{\pi}{4}}}{4(2\pi)^{\frac{1}{2}}K^2} \frac{1}{z} \tag{90}$$

Hence in the three-dimensional case the second order velocity potential decays as $\frac{1}{z}$.

The formula (88) have been used to compute for example the second-order, second-harmonic pressure distribution on a vertical circular cylinder as function of depth, on the weather side ($\theta = \pi$, upper curve) and the lee side ($\theta = 0$, lower curve), see figure 10. The results are given for a cylinder of draught $4a$ and $Ka = 1.52$, with a being the radius of the cylinder. The pressure is normalized by $\frac{\rho g A^2}{a}$. The crosses denote the corresponding results from the numerical solution by Kim & Yue (1988).

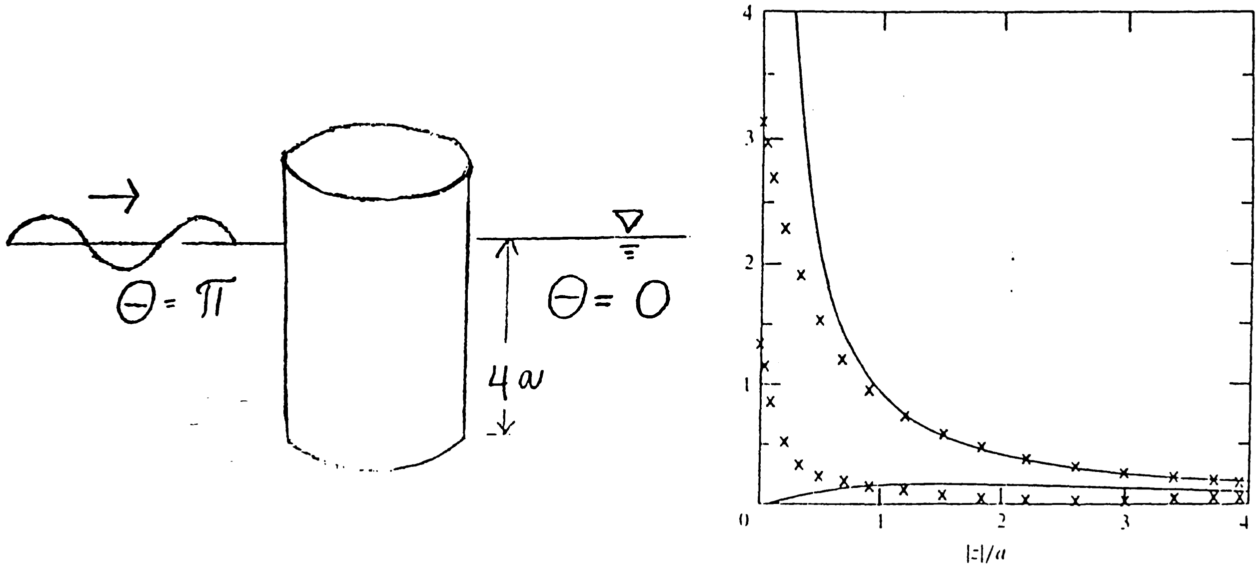

Figure 10

We note that the asymptotic approxmation gives very good reults on the weather side for $\frac{|z|}{a} > 1$. On the lee side however, the results are not satisfying.

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