THE INFLUENCE OF A UNIFORM CURRENT ON SLOWLY VARYING FORCES

by

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ABSTRACT

The impact of a uniform current on slowly varying forces is examined. The model in consideration is two-dimensional, and the body is a restrained submerged circular or elliptic cylinder. Newman's approximation (1974) is applied to approximate the slowly varying force. Time histories of the slowly varying force are obtained for different speeds of the current. It is found that the slowly varying force, for incoming waves travelling <u>against</u> the current, becomes much larger for moderate speeds of the current than for small speeds of the current. For incoming waves travelling <u>with</u> the current the slowly varying force is of the same order of magnitude for moderate and low speeds of the current.

1. INTRODUCTION

Waves of different frequencies will due to second order effects give rise to wave forces with frequencies equal to the sum and difference of the frequencies of every two waves. If the wave spectrum is narrow-banded, the wave force corresponding to the difference frequencies will be slowly varying in time. Usually these slowly varying forces will be relatively small. They may, however, be of great importance if the system they are acting on has low eigen frequencies. This will, for example, often be true for moored bodies. For a thorough discussion of slowly varying forces we refer to Ogilvie (1983).

In many actual cases where slowly varying forces are important, a current may be present. To our knowledge the impact of a current on the magnitude of these forces has not yet been studied in the litterature. This is most likely due to the fact that the occurrence of a current complicates the problem seriously. In this note we shall therefore restrict ourselves to a rather simple model: The problem is two-dimensional, the current is uniform in space and time, and the slowly varying forces act on a restrained, submerged body. The assumption of two-dimensionality means that the incoming waves have crests parallel to the cylinder axis. We shall only consider submerged body with circular or elliptic contour. The assumption that the cylinder is restrained is relevant in practical cases where the cylinder is part of a construction performing sufficiently small first order motions. Example of such a construction is an oil platform with potoons or bracings being long cylinders.

Viscous effects may also give rise to slowly varying forces. We shall here, however, restrict ourselves to studying wave forces. One of the main effects of a uniform current on the wave field is that one incoming wave gives rise to several waves. Thus for $\tau = U\sigma/g < 1/4$ (σ frequency, g acceleration due to gravity and U speed of the current), three new waves normally are generated by the body, and for $\tau > 1/4$ one wave. However, in the special case where the submerged body is a circular cylinder, only one new wave is generated for all values of τ. The diffraction properties of the circular cylinder imbedded in a uniform current are treated in Grue and Palm (1984). The corresponding problem for the elliptic cylinder is studied in Mo and Palm (1985). Based on the results from these papers and by applying Newman's hypothesis (1974) we shall discuss the horizontal slowly varying force acting on the cylinder.

2. THEORY

Let coordinates be taken with x-axis in the mean free surface and y-axis positive upwards. The uniform velocity of the water, U, is horizontal and along the negative x-axis. For a harmonic wave at a fixed point the surface elevation η may be written

$$\eta(t) = \text{Re}\, A \exp(i\sigma t) \tag{2.1}$$

where t denotes time, Re real part, A complex amplitude, i imaginary unit and σ frequency (frequency of encounter). There are four possible kind of incoming waves (see Grue and Palm 1984). Here we shall consider two of them: i. waves travelling upstream with positive group velocity and ii. waves travelling downstream. For simplicity we assume that the fluid layer is of infinite depth. Let ω be the intrinsic frequency. Hence $\omega^2 = gk$ where k is the wave number. σ and ω are for the two cases in consideration connected by

$$\text{i.} \quad \sigma = \omega - Uk \qquad \left(\tau = \frac{U\sigma}{g} < \frac{1}{4}\right) \tag{2.2}$$

$$\text{ii.} \quad \sigma = \omega + Uk \tag{2.3}$$

The two other classes of incoming waves are given by iii. $\sigma = \omega - Uk$ (formally the same as i, but with negative group velocity) and iv. $\sigma = Uk - \omega$. For moderate values of U these cases correspond to short waves which in practical applications usually carry modest wave energy.

Let us then consider an irregular sea, which we approximate by an infinite sum

$$\eta(t) = \text{Re}\ \Sigma\ A_m \exp(i\sigma_m t) \tag{2.4}$$

Following Newman (1974) the horizontal slowly varying force acting on a body may be written

$$F(t) = \text{Re} \sum_{m,n} A_m \bar{A}_n T_{mn} \exp(i(\sigma_m - \sigma_n)t) \tag{2.5}$$

where T_{mn} is the (complex) transfer function and a bar denotes the complex conjugate. It is a very time consuming task to compute T_{mn} in actual cases. Some simpler procedure is therefore needed. Such a simplification is obtained by Newman's approximation (1974), approximately valid for narrow-banded spectra and expressing that

$$T_{mn} = T_{mm} \tag{2.6}$$

T_{mm} is closely related to F, the mean steady, second order force acting on the body due to an incoming harmonic wave with frequency σ_m, by

$$F = |A_m|^2 T_{mm} \tag{2.7}$$

Applying (2.6), (2.5) reduces to

$$F(t) = \text{Re} \sum_{m,n} A_m \bar{A}_n T_{mm} \exp(i(\sigma_m - \sigma_n)t) \tag{2.8}$$

3. NUMERICAL EXAMPLES

It is appropriate to make a slight change in notation, writing $T_{mm} = T(k_m)$, with k_m denoting the wave number of the incoming wave. $F(t)$ is determined by the transfer function T and the magnitude of the various amplitudes. Let D denote the distance between the uppermost point of the cylinder and the free surface, $2R$ the diameter of the circular cylinder as well as the length of the major axis of the ellipse (which is parallel to the free surface). The minor axis B is chosen to be R. In all cases considered here $D/R = 0.8$. As actual example $R = 5$ m, $D = 4$ m, which are realistic dimensions of pontoons or bracings of an oil platform.

In figures 1a, 1b and 2 $T(k)$ due to a submerged circular or elliptic cylinder is displayed for incoming waves travelling against or with the current for various values of the Froude number $U/\sqrt{gR}$. In the figures ρ denotes the density of the fluid. It is noted from fig. 1a that T is practically zero for $U/\sqrt{gR} < 0.3$. This is in agreement with the classical result that the coefficient of reflection is zero for a submerged circular cylinder when $U = 0$. For $U/\sqrt{gR}$ larger than about 0.4 T is no longer small and increases rapidly with increasing Froude number. We therefore expect a rather abrupt change in the magnitude of the slowly varying force for Froude numbers larger than about 0.4. It is also worth noting that T (and thereby F defined by (2.7)) is negative in spite of the fact that the incoming wave is travelling in positive direction. We notice from fig. 1b that for a submerged ellipse, and $U = 0$, T (and F) is positive for incoming waves travelling in the positive direction, as expected. For increasing values of the Froude number, however, T becomes soon negative and approaches a form similar valid for a circular cylinder.

In fig. 2 T is displayed for an incoming wave travelling downstream, i.e. along negative x-axis. For this incoming wave T (and F) is negative for all Froude numbers. We also note that T has a strong tendency to be narrow-banded for increasing Froude numbers. The same tendency, but not so pronounced is also found in fig. 1. However, for moderate values of the Froude number the magnitude of T is much smaller in fig. 2 than in the former case. For the circle, $T \approx 0$ for incoming waves travelling downstream.

To evaluate $F(t)$ from (2.8) we also need to know the amplitudes. We write

$$A_m = a(k_m)\exp(i\delta_m') \tag{2.9}$$

To obtain realistic estimates for the real amplitude $a(k_m)$ we let $a(k_m)$ be given by a typical power spectrum for the wind sea. The phase δ_m is chosen as a random angle. Denoting the spectral energy density for $S(\omega)$, we have

$$a(k_m) = a(\omega_m) = (2S(\omega_m)\Delta\omega_m)^{\frac{1}{2}} \tag{2.10}$$

As characteristic wind sea spectrum we have chosen the truncated tail spectrum (for a discussion of the truncated tail spectrum see Gran (1982))

$$S(\omega) = \begin{matrix} \alpha g^2/\omega^5 & \omega > \Omega \\ 0 & \omega < \Omega \end{matrix} \tag{2.11}$$

where $\alpha = 0.0081$ (Phillips constant). The peak frequency Ω is chosen so that $\Omega^2 R/g = 0.45$ which corresponds to a wave length $\lambda = 14R$, significant wave height $H = 0.4R$ and average zero crossing wave period $T = \pi\sqrt{2}/\Omega$. With $R = 5$ m; $\Omega = 0.94\ s^{-1}$, $\lambda = 70$ m, $H = 2$ m, $T = 4.7$ s. The truncated tail spectrum is shown in figure 3.

The time history of the slowly varying force $F(t)$ is displayed for various Froude numbers in figures 4 for incoming waves travelling against the current. $F(t)$ is evaluated by using 40 wave components, equally spaced in the wave number space. In figures 4(a,e) we consider a submerged elliptical cylinder and in 4(f,g) a submerged circular cylinder. A typical feature in case of the elliptical cylinder (and most likely also for a body of general contour) is that the magnitude of $F(t)$ first decreases for increasing values of the Froude number. For $U/\sqrt{gR} = 0.35$, (for $R = 5$ m, $U = 2.45\ ms^{-1}$) $F(t)$ is very close to zero. For larger values of the Froude number, the magnitude of $F(t)$ increases rapidly. Already for $U/\sqrt{gR} = 0.4$ ($U = 2.8\ ms^{-1}$) $F(t)$ has obtained appreciable amplitude. For the case of a circular cylinder $F(t)$ is very small for $U/\sqrt{gR} < 0.35$ ($U < 2.45\ ms^{-1}$). For larger values of the Froude number the force on the elliptic and circular contour behaves rather similar.

In fig. 5 is shown the slowly varying force acting on an elliptic cylinder due to an incoming wave travelling with the current when $U/\sqrt{gR} = 0.2$ ($U = 1.4\ ms^{-1}$). Comparing with figure 4a we note that the magnitude of $F(t)$ is of the same order in the two cases. For larger values of the Froude number, $F(t) \approx 0$. In figures 4 and 5 $F(t)/\rho g R^2 = 0.002$ corresponds to $F(t) = 490$ N/m and $t\sqrt{g/R} = 1120$ corresponds to $t = 800$ s for $R = 5$ m.

CONCLUSION

In the present paper we have studied the horizontal slowly varying forces on a submerged restrained body when a uniform current is present. One main result is that the force due to incoming waves travelling against the current is an order larger than the force for incoming waves travelling with the current.

Another typical feature is that the transfer function T has a marked tendency to become narrow-banded for increasing values of the Froude number. This tendency is more pronounced for waves travelling with the current than for waves travelling against the current. Hence for not too small values of the Froude number only a small part of the wave energy spectrum is responsible for the magnitude of the slowly varying force. The third, and perhaps the most important result, is that for the submerged bodies considered, the slowly varying force increases very much when the Froude number becomes greater than a certain value, in the actual example found to be about 0.4.

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FIGURE LEGENDS

Figure 1. Transfer functions T versus wavenumber of the incoming waves, which are travelling <u>against</u> the current.

(a) The circular cylinder, $D/R = 0.8$, $U/\sqrt{gR} = 0.3, 0.4, 0.45, 0.5$

(b) The elliptical cylinder, $D/R = 0.8$, $B/2R = 0.5$, $U/\sqrt{gR} = 0, 0.2, 0.4, 0.45, 0.5$. The small arrows denote from right to left the value of kR corresponding to $\tau = 0.25$, for $U/\sqrt{gR} = 0.45, 0.5$, respectively.

Figure 2. Transfer functions T for the elliptic cylinder versus wavenumber of the incoming waves, which are travelling <u>with</u> the current. $D/R = 0.8$, $B/2R = 0.5$, $U/\sqrt{gR} = 0, 0.2, 0.3, 0.4$. The small arrows denote from right to left the value of kR corresponding to $\tau = 0.25$, for $U/\sqrt{gR} = 0.2, 0.3, 0.4$, respectively.

Figure 3. The truncated tail function with $\Omega^2 R/g = 0.45$.

Figure 4. Time history of the slowly varying force pr. unit length of the cylinder for incoming waves travelling <u>against</u> the current with $D/R = 0.8$, and $B/2R = 0.5$ for the ellipse.

(a) ellipse, $U = 0$, (b) ellipse, $U/\sqrt{gR} = 0.2$ ($U = 1.4\ ms^{-1}$),

(c) ellipse, $U/\sqrt{gR} = 0.3$ ($U = 2.1\ ms^{-1}$),

(d) ellipse, $U/\sqrt{gR} = 0.4$ ($U = 2.8\ ms^{-1}$),

(e) ellipse, $U/\sqrt{gR} = 0.45$ ($U = 3.15\ ms^{-1}$),

(f) circle, $U/\sqrt{gR} = 0.4$ ($U = 2.8\ ms^{-1}$),

(g) circle, $U/\sqrt{gR} = 0.45$ ($U = 3.15\ ms^{-1}$).

Figure 5. Time history of the slowly varying force $F(t)$ pr. unit length of the elliptical cylinder for incoming waves travelling with the current. $D/R = 0.8$, $B/2R = 0.5$, $U/\sqrt{gR} = 0.2$ $(U = 1.4\ \mathrm{ms}^{-1})$.

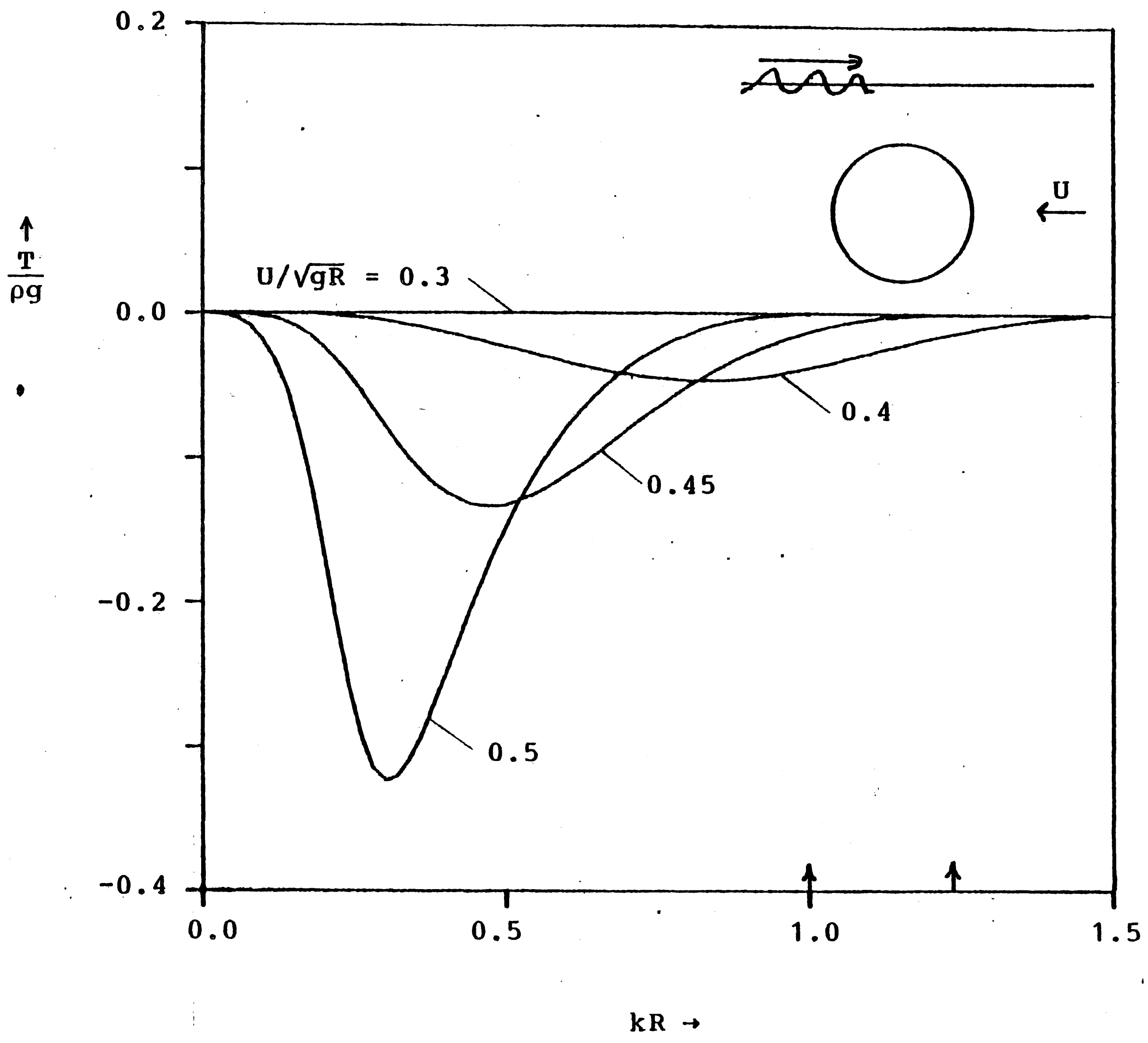

Figure 1a.

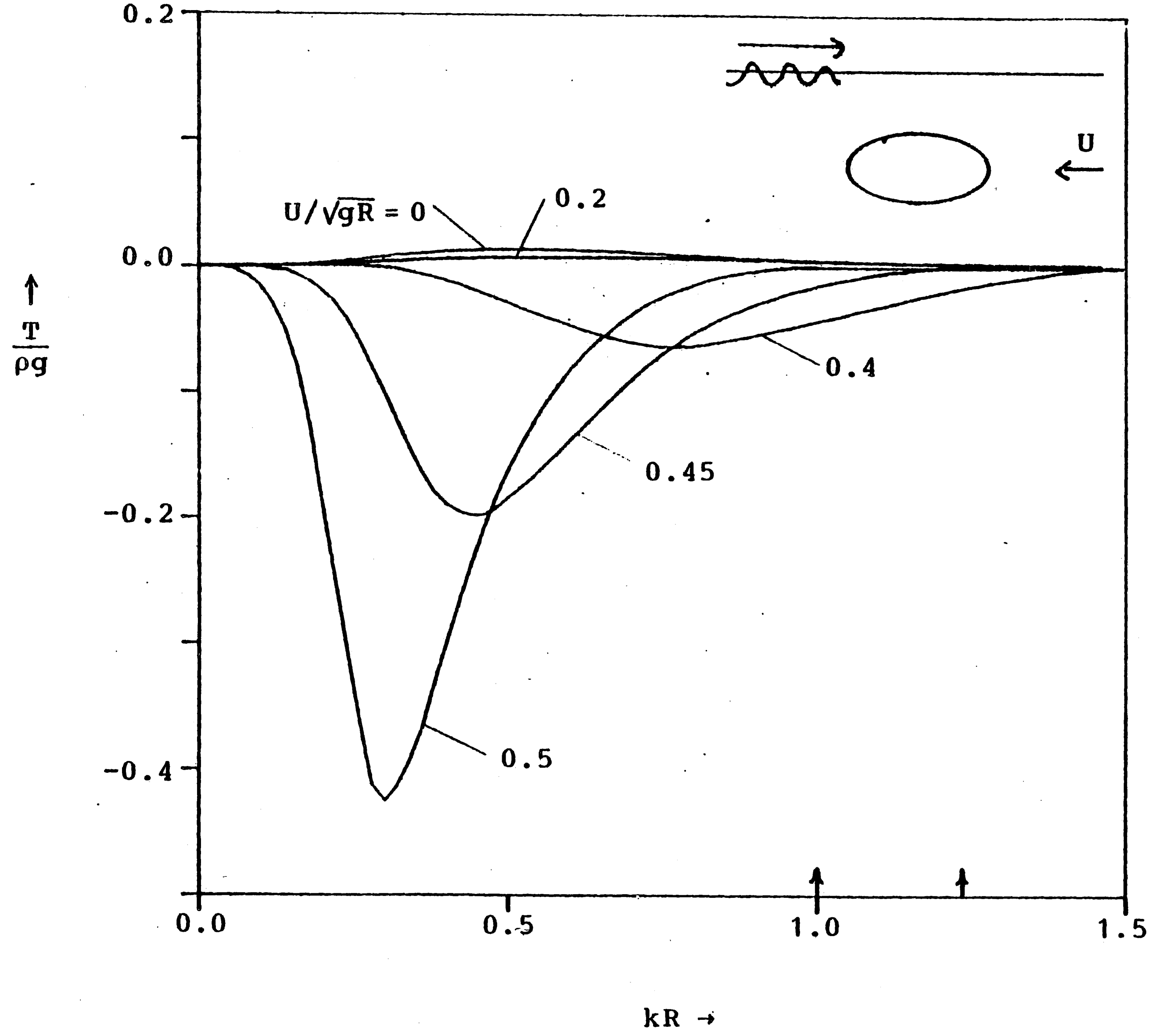

Figure 1b.

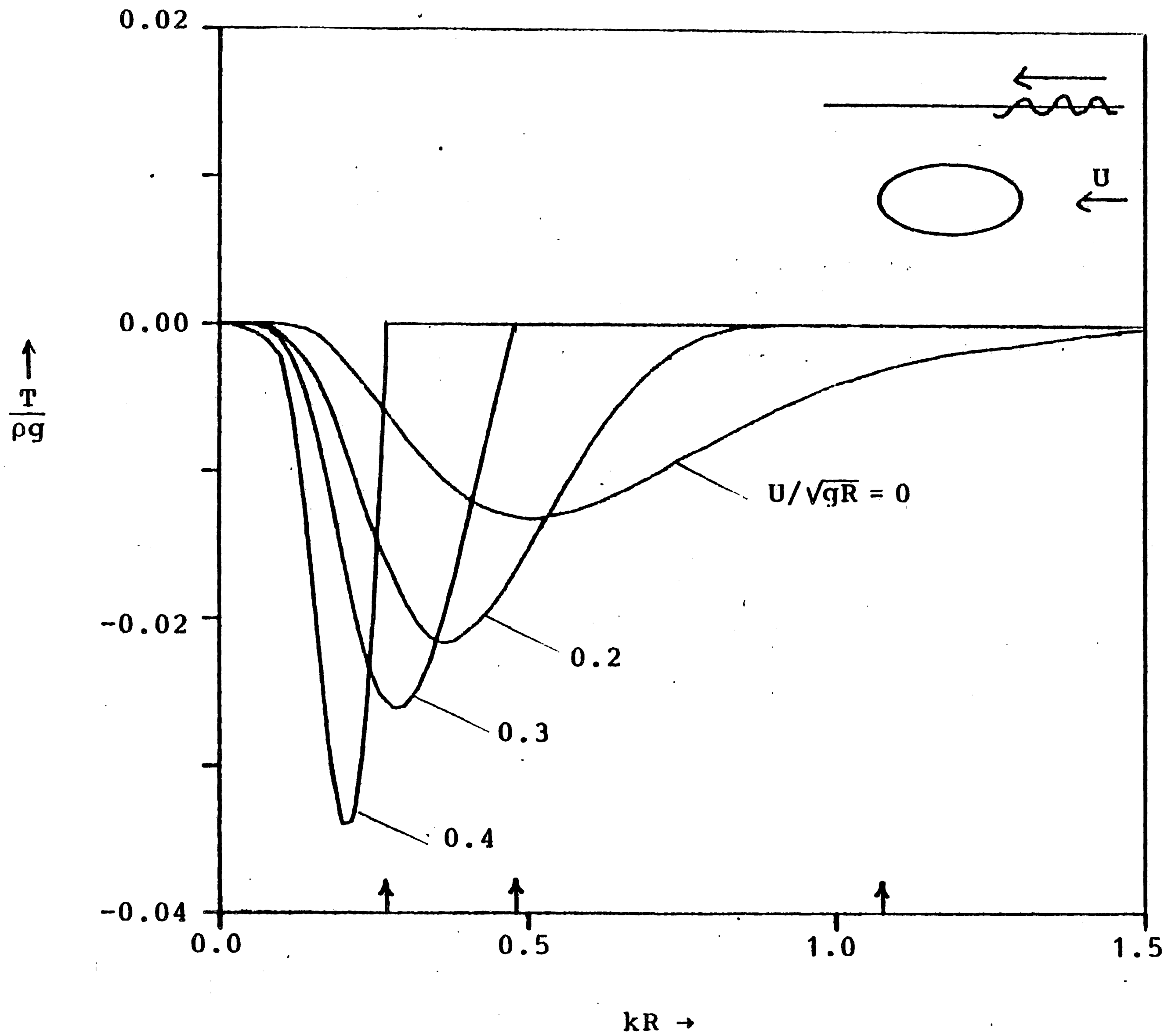

Figure 2.

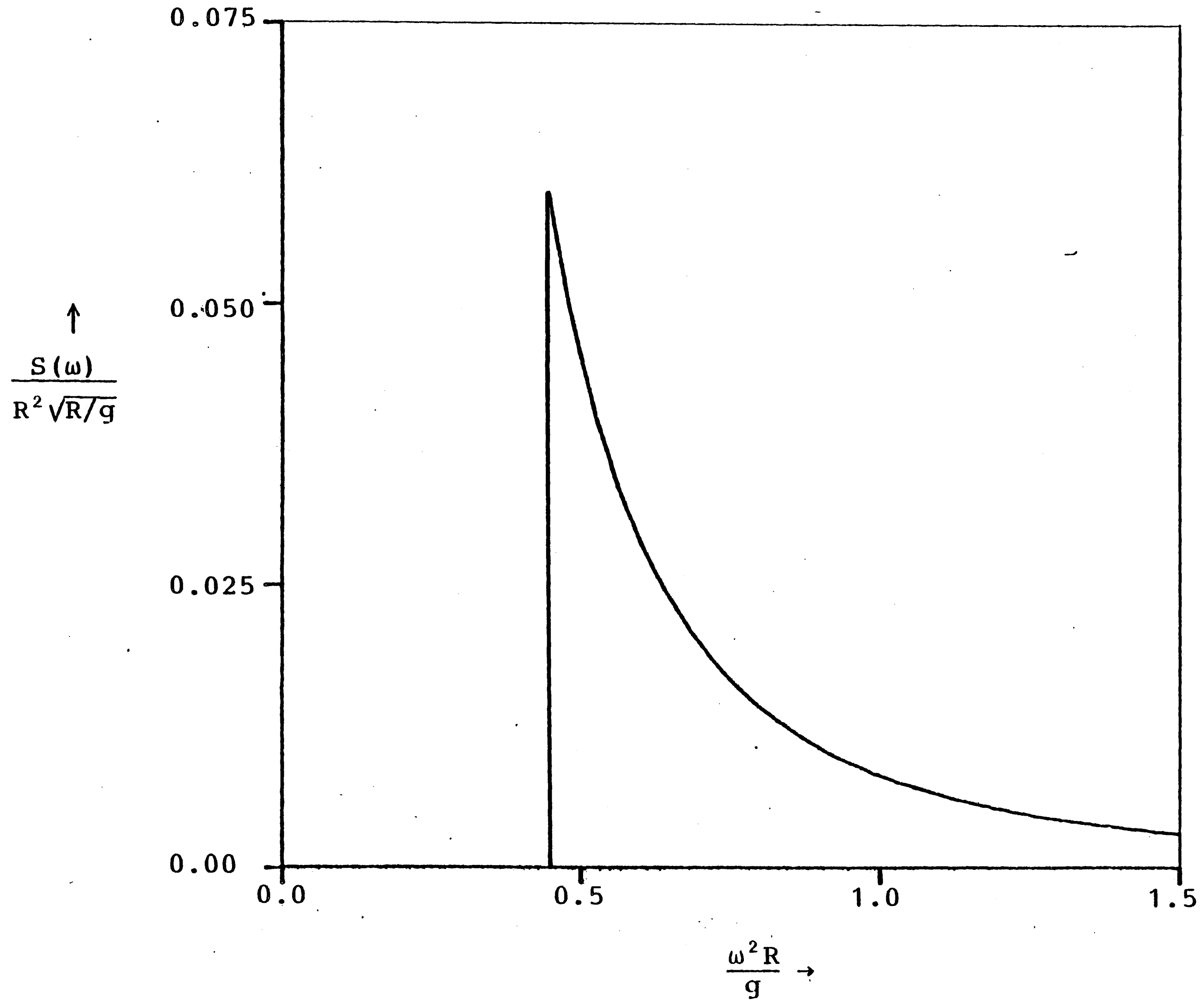
0.075
0.050
0.025
0.00
0.0
0.5
1.0
1.5
$\frac{S(\omega)}{R^2\sqrt{R/g}}$ ↑
$\frac{\omega^2 R}{g}$ →

Figure 3.

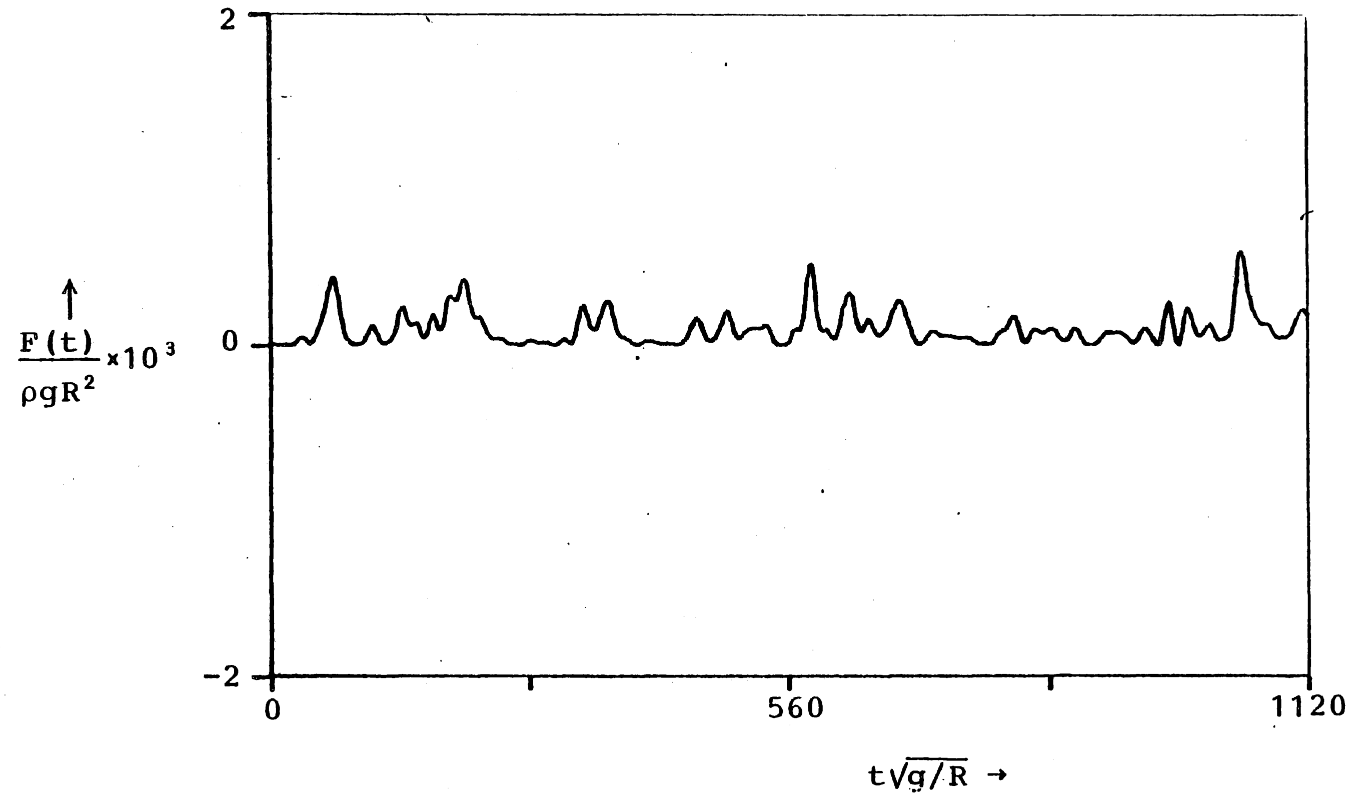

Figure 4a.

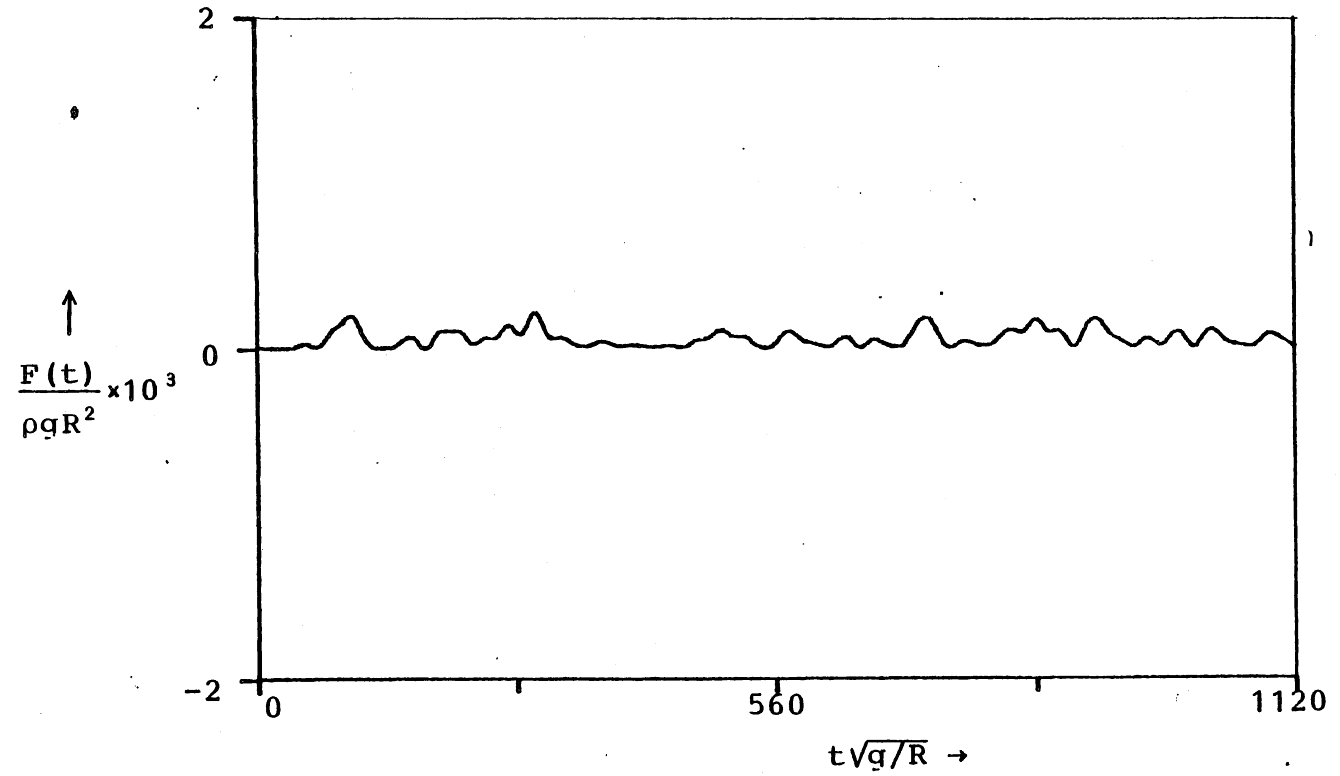

Figure 4b.

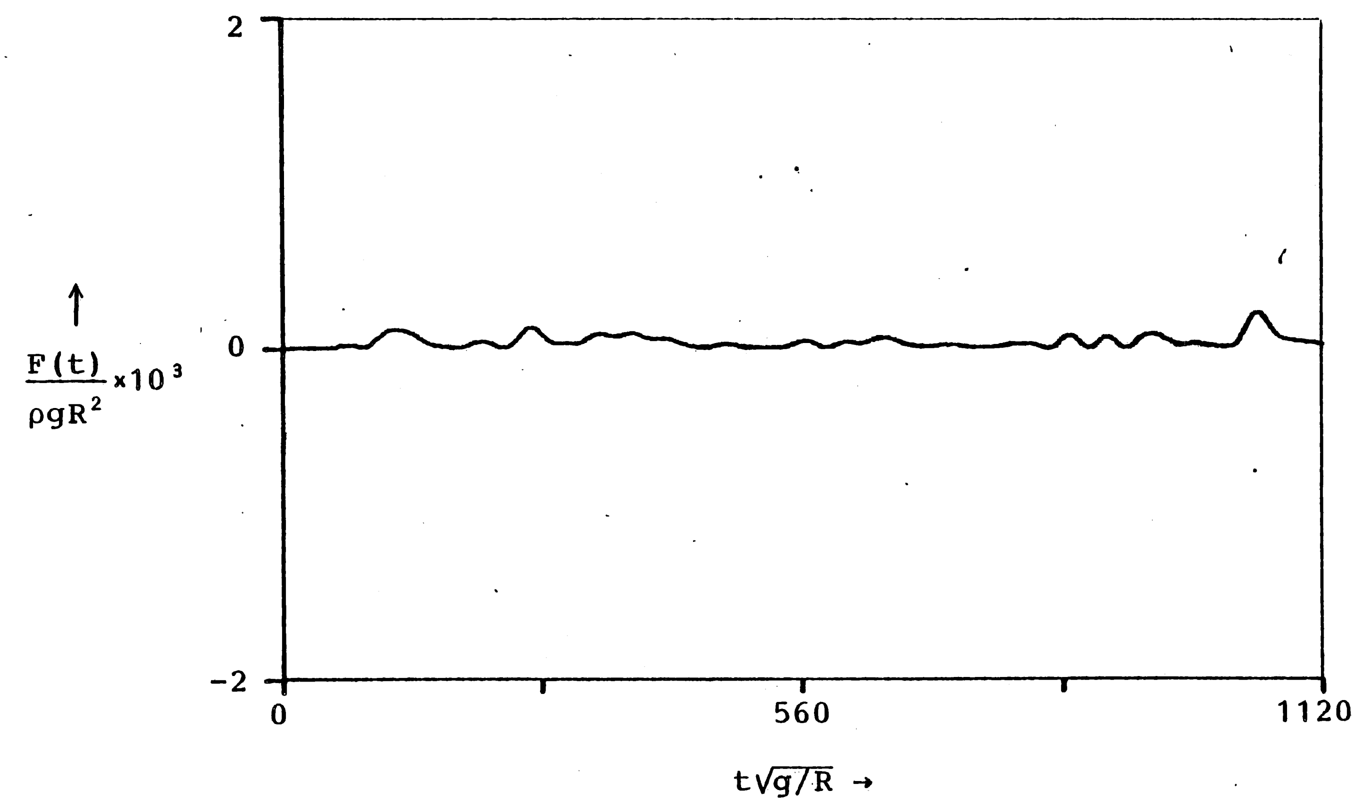

Figure 4c.

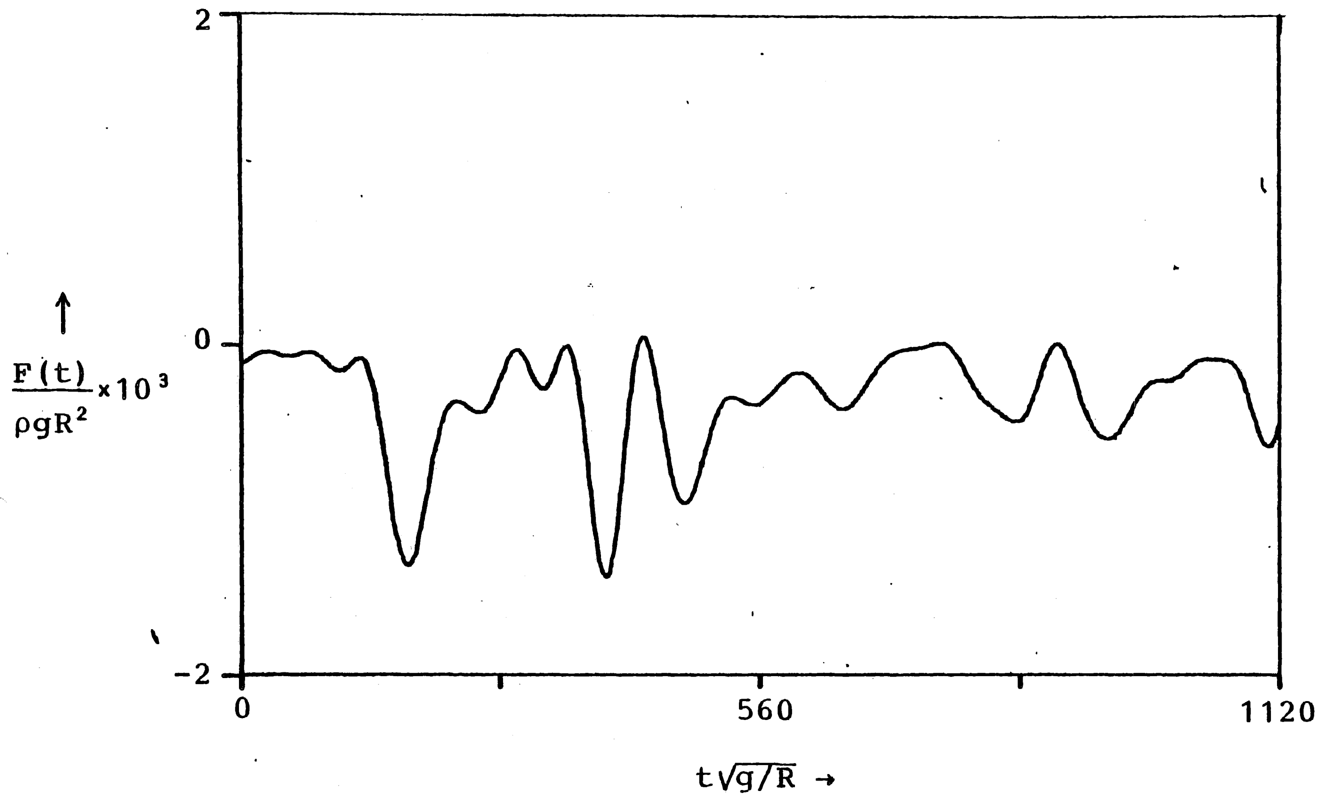

Figure 4d.

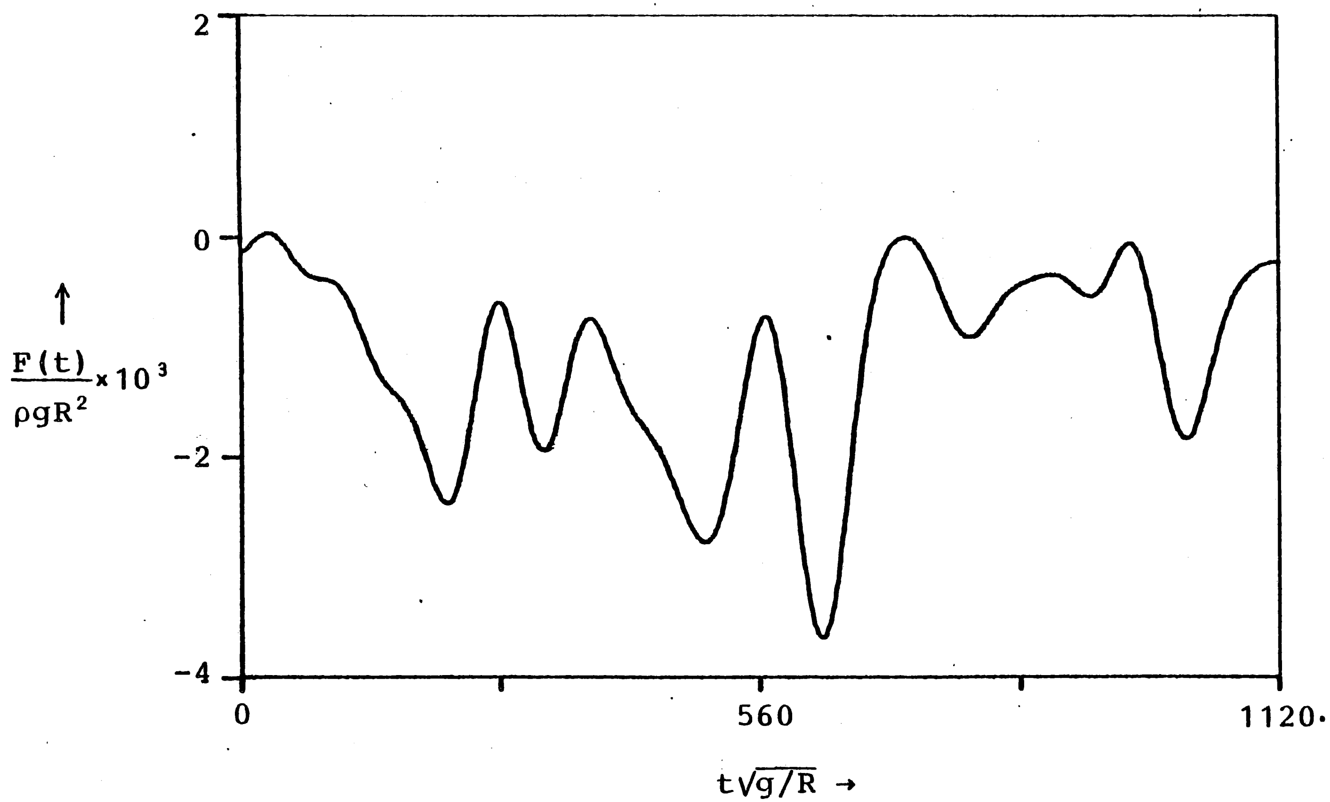

Figure 4e.

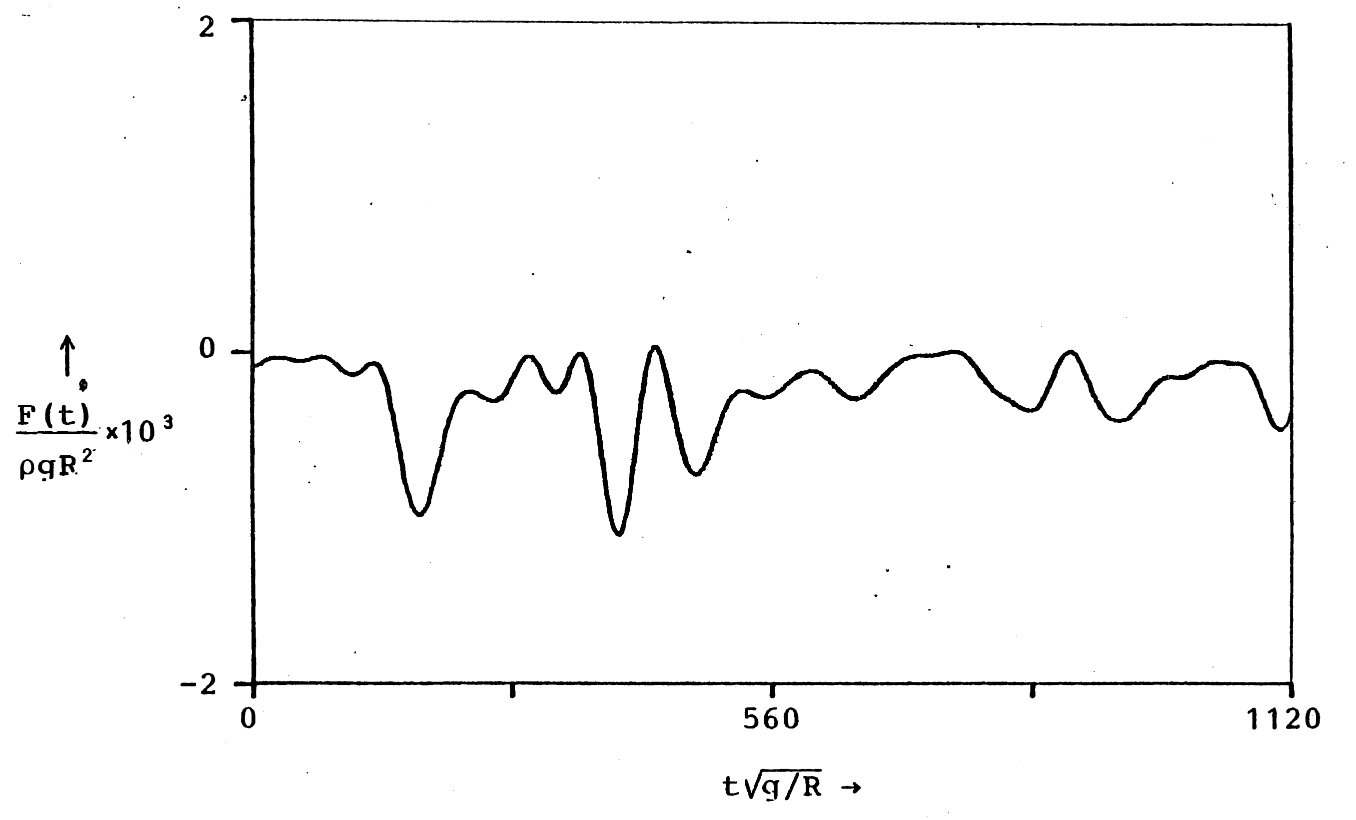

Figure 4f.

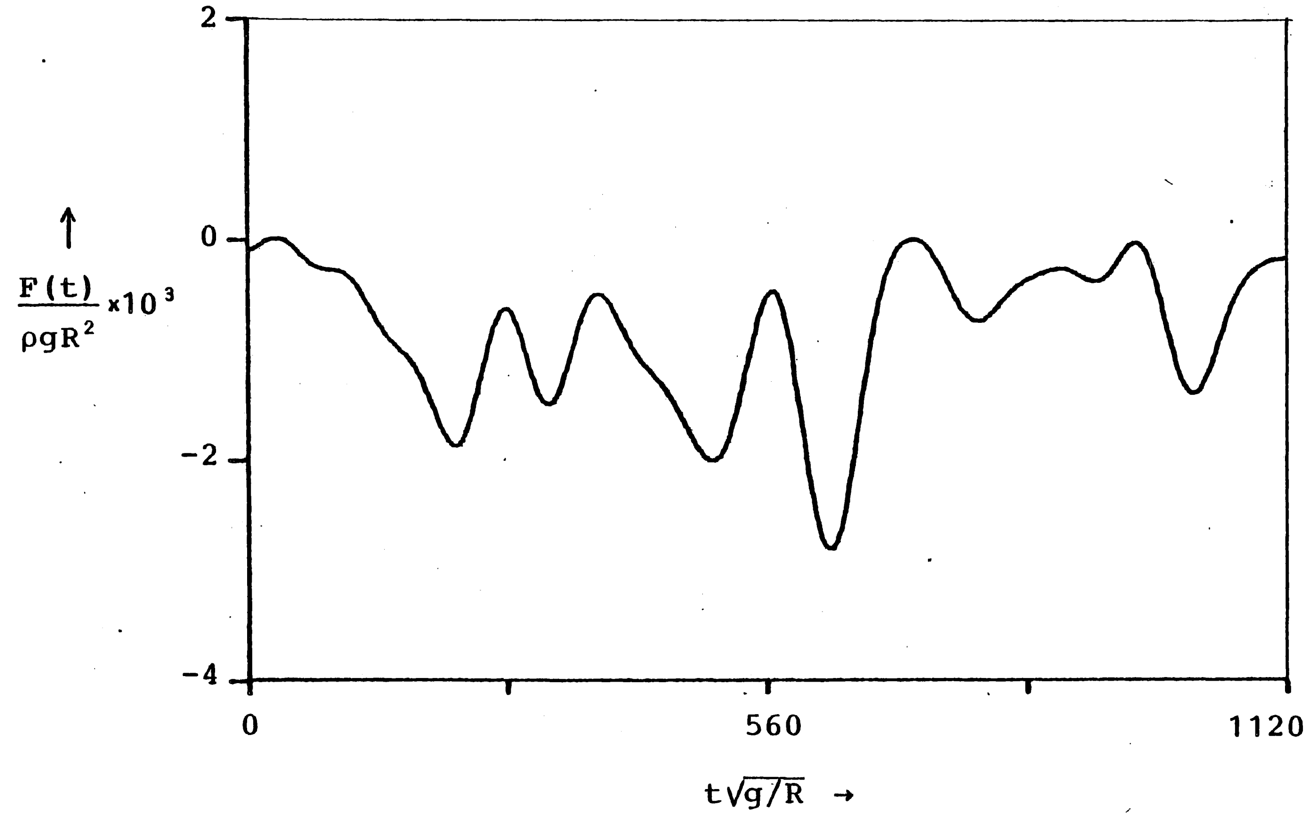

Figure 4g.

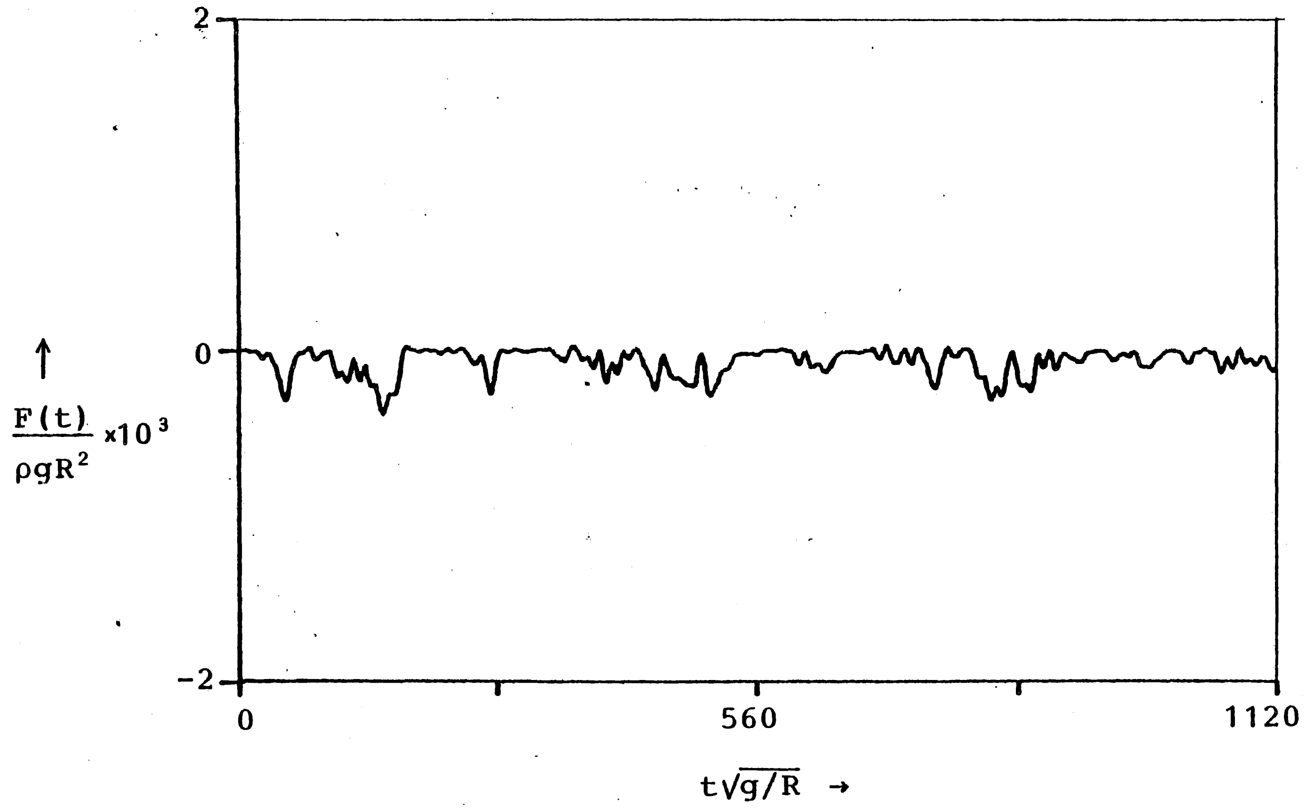

Figure 5.