Admissible investment strategies in continuous trading

by

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Abstract

We consider a situation where relative prices of assets may change continuously and also have discrete jumps at random time points. The problem is the one of portfolio optimization. If the utility function used is the logarithm, we first argue that an optimal investment plan exists. Secondly, we show that any such plan has a certain optimality property known to hold also in discrete time models. Moreover, we show that this optimality criterion can be simplified significantly. In particular we show how admissibility can be related directly to observable characteristics of the investment strategy.

1. Introduction

We assume an agent is faced with d different investment alternatives. The price of alternative ℓ at time t is denoted by $p_\ell(t)$, $\ell = 1,2,\ldots,d$, and the dynamic equations for these prices are

$$(1)\qquad \frac{dp_\ell(t)}{p_\ell(t)} = \mu_\ell(t,\omega)dt + \sum_{j=1}^{d} \sigma_{\ell j}(t,\omega)db_j(t) + \sum_{k=-m}^{m} \beta_{\ell k} dN_{\ell k}(t) \qquad \ell = 1,2,\ldots,d$$

Here $p(t) = (p_1(t), p_2(t), \ldots, p_d(t))'$, $\mu = (\mu_1, \mu_2, \ldots, \mu_d)'$ is the drift vector, and $\sigma = (\sigma_{\ell j})$ is the diffusion matrix for the continuous part of the relative price vector, where $b(t) = (b_1(t), \ldots, b_d(t))'$ is a vector of independent standard Brownian motions. (The transposed of a matrix A is denoted by A'.)

$N_{\ell k}(t))$ are orthogonal (i.e., they do not have simultaneous jumps) point processes (e.g., Poisson) counting the number of price changes of relative size $\beta_{\ell k}$ that occurred during [0,t] for asset ℓ, $k = -m, -m+1, \ldots, m-1, m$ and $-1 < \beta_{\ell,-m} < \ldots < \beta_{\ell,m}$, $\ell = 1,2,\ldots,d$.

Equation (1) is a stochastic differential equation, and the model can be interpreted as a time series model in continuous time. By this we mean that the model is not a result of economic equilibrium theory, or similar theoretical analysis, rather it may be used in practice to fit real price data. Since the present model allows for jumps, it is likely that good fits may be easier to obtain for a stretch of data $\{p(t), t \in [0,T]\}$, than in the case where only diffusion and drift components determine the model. A further advantage for practical purposes is that estimation problems for this model have been considered (Aase and Guttorp (1984)). Portfolio optimization is treated in Aase (1984), (1985b) and economic equilibrium theory for semimartingales can be found in Harrison and Pliska (1981) and Huang (1985). Stochastic control can be found in Aase (1986a), applications to insurance in Aase (1985c), and applications to R&D in Aase (1985a). Option pricing formulas for such combined processes can be found in Aase (1986b).

The organization of the paper is as follows: In Section 2 we define an optimal investment strategy, which is shown to exist in Section 2.1. In Section 2.2 we use Breiman's definition of an admissible investment strategy, where we show in Theorem 1 how it is related to optimal portfolio rules. Finally, in Theorem 2 we demonstrate how admissibility relates directly to observable characteristics of the investment strategy.

2. Portfolio optimization

Let W_t be the wealth of the agent at time t. The associated dynamic equation for W_t is

$$dW_t = W_{t-} \sum_{\ell=1}^{d} \rho_\ell(t,\omega) \frac{dp_\ell(t)}{p_\ell(t)} \tag{2}$$

where $\rho_\ell(t,\omega)$ is the fraction of the agents wealth which is invested in alternative ℓ at time t.

We consider utility functions U(x) from R into R satisfying the usual $U'>0$, $U''<0$. For models in discrete time, some optimality results are known when $U(x) = \ln x$. In this connection we think of optimality in a normative sense (Thorp (1975)).

As usual a probability space (Ω,F,P) is given as well as a filtration $\{F_t, t \geq 0\}$, and we assume that all strategies $\rho(t,\omega) = (\rho_\ell(t,\omega))$, $\ell=1,\ldots,d$, are F_t-predictable stochastic processes satisfying $\Sigma\rho_\ell = 1$.

Our goal is to find a strategy (an investment policy) $\rho(t,\omega)$ which maximizes

$$\limsup_{t\to\infty} \frac{1}{t} E\{U(W_t)\}. \tag{3}$$

(We use limsup here to avoid complications if the limit does not exist.)

By using $V(t) = \ln W_t (= U(W_t))$ we may note the following:.

Formally we define (Markowitz (1976))

$$Rate\ of\ growth\ (of\ wealth) = exp\left[\frac{1}{t}\int_0^t dV(s)\right] - 1$$

$$Average\ return = \frac{1}{t}\int_0^t W_{s-}^{-1}\, dW_s$$

By maximizing

$$\frac{1}{t} E\left[\int_0^t dV(s)\right]$$

we are certain to make the expected rate of growth large, since by Jensen's inequality

$$E\left\{exp\left[\frac{1}{t}\int_0^t dV(s)\right] - 1\right\} \geq exp\left\{E\left[\frac{1}{t}\int_0^t dV(s)\right]\right\} - 1. \tag{4}$$

On the other hand, by maximizing the expected average return it does not follow that the expected rate of growth becomes large. Further, by maximizing the expected rate of growth, the expected average return becomes large.

Other reasons for using the logarithmic utility function are: (a) the existence of an optimal investment strategy ρ^*, (b) computational convenience. The latter fact includes that we avoid using the Bellman optimality principle in order to find ρ^* (see Aase (1985b)). This is very fortunate, since the Bellman equation is generally very hard to solve (in this case it is possible, see Aase (1984)). Using the logarithm as the utility function is sometimes called the Kelly criterion (Thorp (1971)).

We shall call a strategy ρ^* **optimal** if it maximizes

$$E\left[ln\, W_t\right] \; for\ all\ t \geq 0.$$

If ρ^* is optimal, the wealth W corresponding to ρ^* is denoted by W^{ρ^*} or simply W^*.

2.1 Existence of an optimal policy

In this section we shall discuss conditions sufficient for an optimal portfolio choice to exist. First we notice that by use of the Ito-Meyer's lemma (or by the Doleans-Dade's exponential formula) we have for $t \geq s$

$$(5) \quad ln\, W_t = ln\, W_s + \int_s^t f_\rho(r)dr + \int_s^t \sum_{ij} \rho_i \sigma_{ij} db_j(r) + \int_s^t \sum_{ik} ln\left(1 + \beta_{ik}\rho_i\right)dM_{ik}(r)$$

where

$$(6) \quad f_\rho = \sum_{i=1}^{d} (\rho_i\mu_i - \frac{1}{2}\rho_i^2 \sum_{\ell=1}^{d} \sigma_{i\ell}^2) - \sum_{i>j}^{d} \rho_i\rho_j \sum_{\ell=1}^{d} \sigma_{i\ell}\sigma_{j\ell} + \sum_{i=1}^{d} \sum_{k=-m}^{m} ln(1+\rho_i\beta_{ik})\lambda_{ik}$$

and

$$(7) \quad M_{ik}(t) = N_{ik}(t) - \int_0^t \lambda_{ik}(r)dr$$

Here $M_{ik}(t)$ are F_t-margingales. (See Aase (1984) and Aase (1985b) eqn. (17).) Also, $\lambda_{ik}(t)$ are the F_t-intensity processes of $N_{ik}(t)$.

In particular it follows that

$$E(\ln W_t) = \ln W_0 + E\left[\int_0^t f_\rho(r)dr\right] \qquad (8)$$

The coefficients μ_i, $\sigma_{i\ell}$ and λ_{ik} are all assumed to be F_t-predictable (for example, it sufficies that they are left continuous and F_t-adapted). From these assumptions it follows that the conditional expectation of the integrands in (8) given F_r is exactly $f_\rho(r)$. Now, given that the process $\{p_t(\omega), t\in[0,T]\}$ is well-behaved (sufficient conditions are given below), we may maximize $E\{\ln W_t\}$ by choosing for each $\omega\in\Omega$, $s\leq t$, $\rho(s,\omega)$ such that f_ρ is maximized subject to the constraints $\Sigma\rho_i=1$, $\rho_i \geq 0$, $i = 1,2, ..., d$. Since the set

$$\left\{\rho : \sum_{i=1}^{d} \rho_i = 1, \rho_i \geq 0, i = 1,2, ..., d\right\}$$

is a compact simplex of R^d, and f_ρ is continuous on this set, it is clear that (at least) one such optimal strategy $\rho^* = \rho^*(s,\omega)$ exists.

For sufficient conditions on the process $p_t(\omega)$ for this procedure to provide an optimal solution, we rely on a theorem of Beneš (1970) and on results in Bremaud (1981): Let $D^d[0,T]$ be the space of sample functions on $[0,T]$ which are continuous on the right having limits on the left (corlol = cadlag) with values in R^d. We then assume

(a) $$\int_0^T E\|p_t\|^2\, dt < \infty, \text{ where } \|\ \| \text{ is the Euclidean norm on } R^d$$

(b) $$0 \leq f_\rho(t) \leq C\left(1 + \int_0^t \|p_s\|^2 ds\right), \forall t\in[0,T], p\in D_+^d[0,T] \text{ where } C \text{ is some positive constant.}$$

Notice that a linearly boundedness assumption (see Gihman and Skorohod (1979), p. 120) is a sufficient condition for (a) to hold and for the upper bound in (b) to hold. In order that $0 \leq f_\rho(t)$, $\forall t\in[0,T]$, the expected returns on the stocks cannot be too small as compared to the volatility of the stocks.

Notice that ρ^* does not depend on t, i.e., $\rho^*(s,\omega)$ maximizes $E\{\ln W_t\}$ for all t. hence

$$\limsup_{t\to\infty} \frac{1}{t} E\left\{\ln W_t^{\rho^*}\right\} \geq \limsup_{t\to\infty} \frac{1}{t} E\left\{\ln W_t^{\rho}\right\}, \forall \rho$$

and therefore ρ^* also solves the problem of maximizing (3).

In the present case the solution may be found explicitly subject to certain conditions being met (see Aase (1984), (1985b)).

2.2 Optimal strategies and admissible strategies

In this section we follow Breiman (1960) in order to establish an optimality property for the Kelly criterion itself.

Under certain conditions there is, in the discrete time version, a fixed fraction strategy, independent of t, $t = 0, 1, 2, \ldots$, which maximizes $E[\ln W_t]$ (see Breiman (1961)). In the present model the process p(t) is not assumed to be time homogeneous (stationary), and the resulting optimal strategy will in general depend on time.

Below we let W_t' be the fortune using the investment strategy ρ'.

Historically the concept of admissibility has received much attention in the finance literature: Following Breiman (1960), we formalize as follows:

Definition 1

The strategy ρ is **inadmissible** if there is a fixed number $\alpha > 0$ such that for every $\varepsilon > 0$ there exists a competing strategy ρ' such that on a set of probability greater than α-ε $\limsup_{t\to\infty}(W_t/W_t') = 0$ and except on a set of probability at most ε

$$\limsup_{t\to\infty}(W_t/W_t') \leq 1.$$

We say that ρ is **admissible** if ρ is not inadmissible.

Theorem 1. Let ρ be a given strategy with corresponding wealth W^ρ. Consider the following statements:

$$(I)\ P\left[\limsup \frac{W_t}{W_t^*} = 0\right] > 0 \qquad \textit{for some } \textbf{optimal} \textit{ strategy } \rho^*.$$

$$(II)\ P\left[\limsup \frac{W_t}{W_t''} = 0\right] > 0 \qquad \textit{for some strategy } \rho''.$$

(III) ρ *is inadmissible*

$$(IV) \quad P\left[\liminf \frac{W_t}{W_t^*} = 0\right] > 0 \qquad \textit{for } \textbf{all} \textit{ optimal strategies } \rho^*.$$

We have the following relations between these statements:

$$(I) \Rightarrow (II) \Leftrightarrow (III) \Rightarrow (IV).$$

However, the implications (III) $\Rightarrow$ (I) and (IV) $\Rightarrow$ (III) are **not** true in general.

Before we prove Theorem 1 we note the following immediate consequence:

Corollary 1. All optimal strategies are admissible.

Proof of Corollary 1. If ρ is optimal, then by choosing $\rho^* = \rho$ we see that (IV) cannot hold. Hence (III) does not hold, i.e., ρ is admissible.

In the proof of Theorem 1 we will need the following auxiliary results:

Lemma 1. The two processes $\ln \frac{W_t^*}{W_t}$ and $\frac{W_t^*}{W_t}$ are both submartingales wrt. $\{F_t\}_{t \geq 0}$.

Proof. By (5), (6) and (7) we have, for $t \geq s$,

$$E\left[\ln \frac{W_t^*}{W_t} \middle| F_s\right] = \ln \frac{W_s^*}{W_s} + E\left[\int_s^t (f_{\rho^*}(r) - f_\rho(r))dr \middle| F_s\right] \geq \ln \frac{W_s^*}{W_s},$$

since ρ^* is chosen to maximize f.

Thus

$$X_t = \ln \frac{W_t^*}{W_t} \textit{ is a submartingale.}$$

Since $\phi(x) = \exp x$ is convex and increasing it follows from Jensen's inequality that

$$\phi(X_s) \leq \phi(E[X_t|F_s]) \leq E[\phi(X_t)|F_s], \textit{ i.e., that } \phi(X_t) = \frac{W_t^*}{W_t} \textit{ is a submartingale.}$$

Lemma 2. $\limsup_{t \to \infty} \frac{W_t^*}{W_t} > 0$ a.s.

Proof. Put $X_t = \ln \dfrac{W_t^*}{W_t}$ and for all integers n define

$$\tau_n = \inf\{t > 0; X_t \geq n\}$$

Then τ_n is an F_t - stopping time and therefore $X_{t\wedge\tau_n}$ is a submartingale by Lemma 1. Since $X_{t\wedge\tau_n}$ is bounded above, it follows from Doob's submartingale convergence theorem (Doob (1953)) that

$$\lim_{t\to\infty} X_{t\wedge\tau_n}$$

exists (and is finite) a.s.

Since this holds for all n, we conclude that $P[\lim_{t\to\infty} X_t = -\infty] = 0$, so that

$$\limsup_{t\to\infty} X_t > -\infty \text{ a.s. and Lemma 2 follows.}$$

Proof of Theorem 1

(I) $\Rightarrow$ (II) and (III) $\Rightarrow$ (II) are trivial.

(II) $\Rightarrow$ (III): Assume (II) holds. Put

$$B = \left\{\omega; \mathit{limsup} \frac{W_t}{W_t''} = 0\right\}, P(B) = \alpha > 0.$$

Let $\varepsilon > 0$.

Then by choosing s large enough we can find $B_s \in F_s$ such that $P(B_s \Delta B) < \varepsilon$, where Δ denotes the symmetric set difference. Now define ρ' as follows:

$$\rho'(t,\omega) = \begin{cases} \rho(t,\omega) & \text{if } t < s \\ \rho''(t,\omega) & \text{if } t \geq s \text{ and } \omega \in B_s \\ \rho(t,\omega) & \text{if } t \geq s \text{ and } \omega \notin B_s \end{cases}$$

Since the occurrence of jumps of W_t does not depend on ρ, we see from (5) that on B_s we have

$$ln \frac{W_t''}{W_t'} = ln \frac{W_s''}{W_s'} \qquad for\, all\, t \geq s\,,$$

since $\rho' = \rho''$ for $t \geq s$.

Therefore

$$\lim_{t\to\infty} ln \frac{W_t''}{W_t'} = ln \frac{W_t''}{W_s'} < \infty \; on \; B_s.$$

Thus

$$\limsup_{t\to\infty} \frac{W_t}{W_t'} = \limsup_{t\to\infty} \left(\frac{W_t}{W_t''} \cdot \frac{W_t''}{W_t'} \right) = 0 \; on \; B_s \cap B.$$

Outside B_s we clearly have $W_t = W_t'$.

Hence

$$\limsup_{t\to\infty} \frac{W_t}{W_t'} \leq 1 \quad outside\, B\Delta B_s.$$

Since $P(B\Delta B_s) < \varepsilon$ we have shown that ρ is inadmissible.

<u>(III) ⇒ (IV):</u> Assume (III) holds. Then there exists a strategy ρ' with

$$P\left[\limsup_{t\to\infty} \frac{W_t}{W_t'} = 0\right] > 0$$

Hence

$$\liminf_{t\to\infty} \frac{W_t}{W_t^*} = \liminf_{t\to\infty} \left(\frac{W_t}{W_t'} \cdot \frac{W_t'}{W_t^*} \right) = 0$$

with positive probability, since $\liminf \dfrac{W_t'}{W_{t^*}} < \infty$ a.s. by Lemma 2. Hence (IV) holds.

To prove the last two assertions it suffices to point out a situation where we have (III) but not (I) and a situation where we have (IV) but not (III):

Choose $\beta_{ij} = 0$ for all i,j and choose

$$\sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \mu = ((1+\varepsilon)u, u, 0) \text{ where}$$

$u = u(t) \geq 0$, $\varepsilon = \varepsilon(t) \geq 0$ are to be determined.

Then

$$f_\rho(t) = \sum_i \rho_i\mu_i - \frac{1}{2}\sum_{i,j} \rho_i\rho_j(\sigma\sigma')_{ij} = \rho_1(1+\varepsilon)u + \rho_2 u - \frac{1}{2}\rho_1^2$$

so it is easily seen that

$$\rho^* = (\varepsilon u, 1-\varepsilon u, 0), f_{\rho^*} = \frac{1}{2}\varepsilon^2 u^2 + u.$$

Let $0 \leq \delta(t) \leq 1$ be a function to be determined and put

$$\rho = (\delta, 1-2\delta, \delta) \textit{ so that } f_\rho = (1+\delta\varepsilon-\delta)u$$

and

$$\rho' = (\delta, 1-\delta, 0) \textit{ so that } f_{\rho'} = (1+\delta\varepsilon)u.$$

Then

$$(9) \qquad ln\frac{W_t}{W_t^*} = \int_0^t (f_\rho - f_{\rho^*})dr + \int_0^t (\delta-\varepsilon u)db_1 = \int_0^t ((\delta\varepsilon-\delta)u - \frac{1}{2}\varepsilon^2u^2)dr + \int_0^t (\delta-\varepsilon u)db_1$$

and

$$(10) \qquad ln\frac{W_t}{W_t'} = -\int_0^t \delta u\ dr \leq 0 \ \textit{i.e.,}\ W_t \leq W_t' \textit{ for all } t \geq 0.$$

Now choose $u(r) = \varepsilon(r) = r^{-3/4}$

and $\qquad \delta(r) = r^{-1/4}$ for $r \geq 1$.

Then

$$ln\frac{W_t}{W_t'} \rightarrow -\infty \ a.s. \ as\ t \rightarrow \infty,$$

so

(11) $$\frac{W_t}{W'_t} \to 0 \quad a.s. \; as \; t \to \infty,$$

Moreover, note that

(12) $$\int_0^t ((\delta\varepsilon - \delta)u - \frac{1}{2}\varepsilon^2 u^2) dr \sim -\log t \text{ for large } t.$$

In general an Ito integral of the form $Y_t = \int_0^t \sum_i a_i db_i$ is a time change of one-dimensional Brownian motion; more precisely we have

(13) $$Y_t = \int_0^t \sum_i a_i db_i = \tilde{b}_{\beta_t},$$

for some one-dimensional Brownian motion b, where

(14) $$\beta_t = \int_0^t (\sum_i a_i^2(r)) dr \qquad (\text{Øksendal}(1985))$$

The law of iterated logarithm for Brownian motion (Karlin and Taylor (1975)) states that

(15) $$\limsup_{T \to \infty} \frac{\tilde{b}_T}{\sqrt{2T \ln \ln T}} = 1 \; a.s.$$

This gives that

(16) $$\limsup_{t \to \infty} \frac{\int_0^t (\delta - \varepsilon u) db_1}{\left(\int_0^t (\delta - \varepsilon u)^2 dr \right)^{1/2}} = \limsup_{t \to \infty} \frac{\int_0^t (\delta - \varepsilon u) db_1}{t^{1/4}} = \infty \; a.s.$$

Combining (12) and (16) we get from (9) that

(17) $$\limsup_{t \to \infty} \left(\ln \frac{W_t}{W_t^*} \right) = \infty \; a.s.$$

From (10) and (11) we have that ρ is inadmissible, i.e., ρ satisfies (III), and from (17) we conclude that (I) does not hold. Thus (III) ⇒ (I) does not hold.

On the other hand, if we change to $\rho = (1,0,0)$, then for any strategy $\rho'' = (\delta, 1\text{-}\delta\text{-}\eta, \eta)$ we have

$$ln \frac{W_t}{W_t''} = \int_0^t (\varepsilon(1-\delta)+\eta)u\, dr + \int_0^t (1-\delta)db_1$$

Since $\varepsilon(1\text{-}\delta)+\eta \geq 0$ we conclude that

$$\underset{t\to\infty}{limsup} \frac{W_t}{W_t''} > 0 \ a.s.$$

Thus ρ is admissible. On the other hand

$$ln \frac{W_t}{W_t^*} = \int_0^t \varepsilon u(1 - \frac{1}{2}\varepsilon u)dr + \int_0^t (1-\varepsilon u)db_1,$$

so, again by using (15), we conclude that

$$\underset{t\to\infty}{liminf}\left(ln \frac{W_t}{W_t^*} \right) = -\infty \ a.s.$$

So ρ satisfies (IV) but not (III).

That completes the proof of Theorem 1.

The last example illustrates that a strategy ρ is admissible if it is close enough to ρ^* in a sense to be made precise in the next theorem. We use the notation

$$h_\rho(r) = \sum_{i=1}^{d} \sum_{k=-m}^{m} ln\,(1 + \rho_i(r)\beta_{ik})\ \lambda_{ik}(r)$$

and similarly for ρ^*. Let $h_\rho = h_\rho \vee 0$ and $h_\rho = (-h_\rho)\vee 0$ so that $h_\rho = h_\rho - h_\rho$. Furthermore, let $g_\rho(r) = (g_\rho^{(j)}(r)) = (\Sigma\rho_i(r)\sigma_{ij}(r))$ and similarly for ρ^*.

For the next result we need conditions guaranteeing that the processes $\ln W_t$ and $\ln W_t^*$ do not explode, i.e., that they do not hit $\pm\infty$ at a finite time point. These processes can explode for several reasons:

(i) from too much drift μ_i;

(ii) from too much local variance σ_{ij};

(iii) from too many jumps λ_{ij};

(iv) from too large jumps β_{ik}.

Conditions guaranteeing that the three first cases do not occur are given in Aase (1985b) and we assume these to hold here. The last case will not happen here from our assumptions on the relative jump sizes β_{ik} ($\rho_i \geq 0$). We also need the following:

$$\text{(18)} \qquad \int_0^t h_\rho^+(r)\,dr \to \infty, \int_0^t h_\rho^-(r)dr \to \infty \ a.s. \ \ as\, t \to \infty$$

and similarly for ρ^*.

This condition only says that the "jumps matter". Mathematically it means that the point processes will have an infinite number of jumps on $[0,\infty)$, which is the usual assumption for point processes. (For this to hold, the processes should not explode).

Theorem 2. Assume that (18) holds.

(a) If there exists an optimal ρ^* such that with probability one

$$\text{(19)} \qquad \int_0^\infty (f_{\rho^*}-f_\rho)dr < \infty, \int_0^\infty (g_{\rho^*}-g_\rho)^2 dr < \infty, and \int_0^\infty (h_{\rho^*}-h_\rho)dr < \infty,$$

then ρ is admissible.

(b) Conversely, if ρ is an admissible strategy, then for every optimal ρ^* we have

$$\text{(20)} \quad \limsup_{t\to\infty} \left\{ \sqrt{\left(\int_0^t \left(g_\rho - g_{\rho^*}\right)^2 dr\right) ln \ ln\left(\int_0^t \left(g_\rho - g_{\rho^*}\right)^2 dr\right)} - \int_0^t (f_{\rho^*}-f_\rho)dr - \int_0^t \left(h_{\rho^*}-h_\rho)dr\right) \right\} > -\infty \ a.s.$$

Proof. We use the law of the iterated logarithm as above to estimate the second term on the right hand side in

$$ln\,\frac{W_t}{W_t^*} = \int_0^t (f_\rho - f_{\rho^*})dr + \int_0^t (g_\rho - g_{\rho^*})db + \int_0^t \sum_{i=1}^{d} \sum_{k=-m}^{m} ln(1+\rho_i\beta_{ik})\ dN_{ik}(r)$$

The last term can be estimated as follows: By a random time change we can transform this term to a standard Poisson process. From the strong law of large numbers for renewal processes it follows that

$$\frac{\int_0^t \sum_{i=1}^{d} \sum_{k=-m}^{m} ln(1+\rho_i\beta_{ik})dN_{ik}(r)}{\int_0^t h_\rho(r)\,dr} \xrightarrow[t\to\infty]{a.s.} 1$$

provided (18) holds. (Aase and Guttorp (1984).) If (19) holds, then (IV) in Theorem 1 does not hold and hence ρ is admissible.

Conversely, if ρ is admissible, then (I) in Theorem 1 does not hold and (20) follows.

Remark.

Whereas earlier results on admissibility have only given conditions on the wealths W_t and W_t^*, the importance of Theorem 2 rests on the fact that this concept is now related directly to the characteristics of the investment strategy ρ.

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