"New physics in charm sector"

by

Arturo Bernardo González Alatorre

UiO : Universitetet i Oslo

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Department of Physics
Faculty of Mathematics and Natural Sciences
University of Oslo

Advisor: Jan Olav Eeg

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"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are, if it doesn't agree with experiment, it's wrong."

Richard P. Feynman

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Chapter 1

Introduction

The twentieth century marked a period of enormous growth in the field of particle physics. Throughout the century, newer and more exotic particles were discovered, which naturally led to the question of how they interact. A complete and well-established quantum field theory describing particles and their interactions emerged: the *Standard Model* of particle physics.

However, particle physics is at a turning point today. On the one hand, the Standard Model (SM) is presently the best description of particle phenomenology, but on the other hand, the Standard Model is not considered a complete theory and that has lead to the necessity of Physics Beyond the Standard Model (BSM), often also referred to as "New Physics", which the model can not explain. A proof of its incompleteness comes from the inability to provide an explanation for the dark matter, the fermion masses hierarchy and the quantitative asymmetry between matter and antimatter of the universe, among others.

The start of the LHC opened an exciting time for particle physics culminating with the discovery of a new particle consistent with the predicted Higgs boson. The results so far achieved are just the beginning of a new exciting time during which we expect to improve our understanding of fundamental matter and find answers to the open questions of particle physics.

Today, indirect search for signs of new physics is done at the LHCb experiment, which is one of the four large experiments at the LHC, by making high precision measurements. The LHCb collaboration investigates observables such as the decay-time-dependent Charge-Parity (CP) asymmetry $A_{CP}(f;t)$ defined as [**1**]

$$A_{CP}(f;t) = \frac{\Gamma(P(t) \to f) - \Gamma(\bar{P}(t) \to \bar{f})}{\Gamma(P(t) \to f) + \Gamma(\bar{P}(t) \to \bar{f})} \tag{1.1}$$

where A_{CP} quantifies the difference between matter and antimatter for the decay of a particle P to a final state f.

The main topic of this thesis are the studies of CP violation in the charm sector, and in particular the non-leptonic decayment of D^0 mesons to K^+K^- and $\pi^+\pi^-$. Generally, the magnitudes of CP asymmetries in decays to these final states are expected to be small in SM, with predictions of up to $\mathcal{O}(10^{-3})$ [**2**]. However, recent studies have shown that larger asymmetries may be expected in the SM [**3, 4**]. Namely, it was surprising when in 2012 the LHCb collaboration announced the following result

$$\begin{aligned} \Delta A_{CP} &\equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) \\ &= [-0.82 \pm 0.21(stat.) \pm 0.11(sys.)]\% \end{aligned} \tag{1.2}$$

giving a 3.5 σ signal of CP violation; a sample of 600 pb^{-1} of data taken during 2011 at $\sqrt{s} = 7$ TeV was used [**5**]. Precise measurements to date of the time-integrated CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ were made by the CDF, BaBar and Belle collaborations and the results are summarized as

Experiment	ΔA_{CP}	Reference
CDF (2012)	$[-0.62 \pm 0.21(stat.) \pm 0.10(sys.)]\%$	[**6**]
BaBar (2008)	$[+0.24 \pm 0.62(stat.) \pm 0.26(sys.)]\%$	[**7**]
Belle (2012)	$[-0.87 \pm 0.41(stat.) \pm 0.06(sys.)]\%$	[**8**]

Table 1.1. Previous experimental results on A_{CP}.

New measurements, based on a pp collision data collected during 2011 at $\sqrt{s} = 7$ TeV, corresponding to an integrated luminosity of 1 fb^{-1} , was recently presented in 2013 by the LHCb collaboration [**9, 10**]

$$\Delta A_{CP} = [-0.34 \pm 0.15(stat.) \pm 0.10(sys.)]\% \tag{1.3}$$

$$\Delta A_{CP} = [+0.49 \pm 0.30(stat.) \pm 0.14(sys.)]\%^1 \tag{1.4}$$

A comparison of the different measurements of ΔA_{CP} are presented in Figure 1.1.

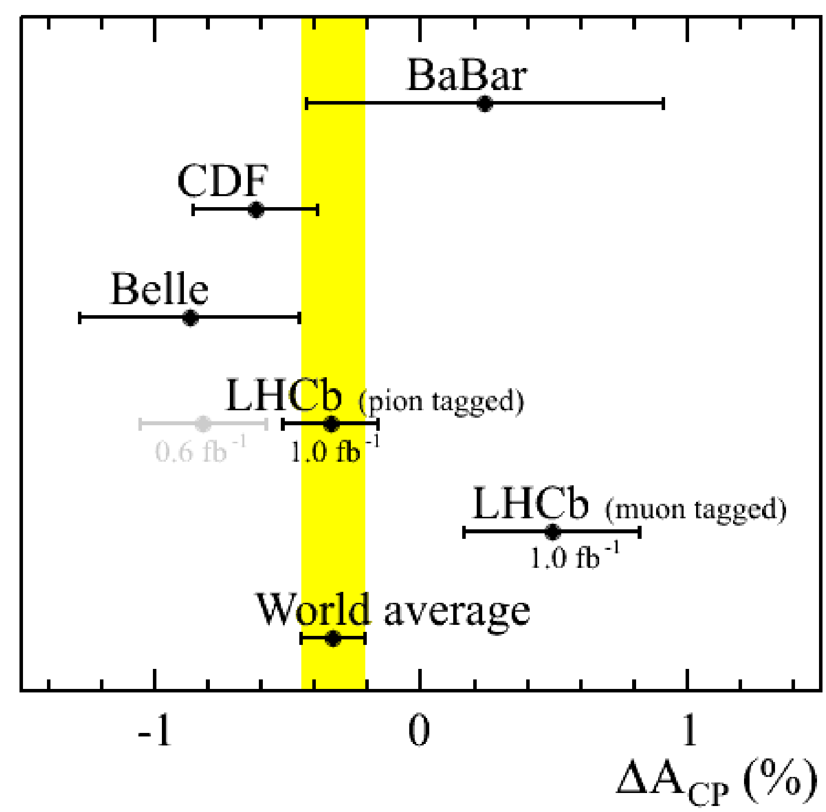


Figure 1.1. Comparison of different measurements of ΔA_{CP}. The previous LHCb result is shown as the shaded grey point. A naive world average is shown as the yellow band [2].

[1] D^0 mesons are produced in semileptonic $\bar{B} \to D^0 \mu^- \bar{\nu}_\mu X$ and the charge of the muon is used to tag the flavor of the D^0 meson [**10**].

[2] Taken from http://lhcb-public.web.cern.ch/lhcb-public/Images2013/DeltaAcp.png

As shown in the Figure 1.1, the two new LHCb results are consistent with each other and with the previous ones showed in in Table 1.1 at the 2σ level, but do not confirm the previous evidence of CP violation in the charm sector that had previously been reported.

In this thesis we considered for CP violation in the Cabbibo suppressed decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$. In Chapter 2, a brief overview of the Standard Model, C, P, and T symmetries and the Cabbibo-Kobayashi-Maskawa mixing-matrix is presented. The theory of CP violation is presented in Chapter 3. The Weak decays of D mesons and charm quark physics are described in Chapter 4. In chapter 5, the analysis for CP violation in the SM for singly Cabbibo suppressed processes is presented, including QCD and Penguin contributions. New physics in the neutral meson D^0 system is treated in Chapter 6, that is based in the work by Altmannshofer, Primulando, Yu and Yu [**11**]. Finally in Chapter 7 the conclusions are presented.

Chapter 2

Theoretical overview

1. The Standard Model

The main goal in particle physics is to understand the structure of the universe, its fundamental constituents and the laws governing its behavior. There is a theory that fits this prescription, and describes with high precision the laws governing the fundamental particles, the *Standard Model* of particle physics.

The Standard Model of particle physics is the most successful explanation of the fundamental structure of matter that exists today. Formulated as a Lorentz-covariant quantum field theory, combining *Quantum Chromodynamics* (QCD) with the *Electroweak theory* (EW) developed by Glashow, Weinberg and Salam [**12, 13, 14**]. It is invariant under transformations of the group

$$\mathbf{SU(3)_{color}} \otimes \mathbf{SU(2)_L} \otimes \mathbf{U(1)_Y} \tag{2.1}$$

where the three symmetries are known as *Color*, *Weak Isospin* and *Weak Hypercharge* respectively [**15**]. It describes three of the four fundamental interactions, the electromagnetic, weak and strong ones among different fundamental particles, providing for the most part an elegant and coherent theoretical framework and a set of precise and well-tested predictions.

1.1. Fundamental constituents. In the Standard Model the fermions, spin-$\frac{1}{2}$ particles, are divided into two classes, leptons and quarks. Leptons and quarks are grouped into three generations of doublets, where each generation is identical in all quantum numbers differing only by the masses of these ones (see Figure 2.1). Each generation consists of a pair of leptons, whose interactions are governed by the electroweak forces, and a quark doublet, which are subject to both electroweak and strong forces. In analogy with fermions, there exists a class of particles with integer spin called bosons. The gauge bosons in the Standard Model are particles that mediate the interactions between fundamental particles.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

Figure 2.1. Electroweak doublets. In units of electric charge, e^-, μ^- and τ^- has charge 1, while neutrinos has neutral electrical charge. For the case of quarks, u-type quarks (u, s, t) and d-type quarks (d, c, b) has charge $+2/3$ and $-1/3$ respectively. The primes on the d, s, b quarks refers to the fact that the mass and weak interactions eigenstates are not necessarily the same, as will be explain in section 3.

1.2. Fundamental interactions. When two matter particles interact through a fundamental force, in the Standard Model, the proccess are described by the exchange or emission of "force particles" called gauge bosons and comprises three interactions: the electromagnetic, weak and strong.

Electroweak interaction. All the particles with electric charge, all quarks and the three charged leptons (e^-, μ^-, τ^-), interact through the electromagnetic force being this force the responsible to binds atoms and molecules together.

The weak force acts on particles with weak charge, all leptons and quarks, and accounts for some of the spontaneous transformation of particles into others with lower mass (i.e., the β-decay of a radioactive nucleus). It is due of the weak force that all the massive particles created at the origin of the universe, have decayed to the less massive particles that compose the universe we can see today.

The weak and electromagnetic interactions can be unified and described by a $SU(2)_L \otimes SU(1)_Y$ gauge symmetry, in what is commonly known as Electroweak interaction. It involves four gauge bosons: $\mathbf{W}^+$ and $\mathbf{W}^-$ which are responsible for flavor changing charged current interaction, $\mathbf{Z^0}$ which leads to flavor conserving weak neutral currents and the $\boldsymbol{\gamma}$ (photon) that mediates the electromagnetic interaction between charged particles.

The Lagrangian for the Electroweak interactions is made up of a *charged current* and a *neutral current.*

$$\mathscr{L}_{int}^{EW} = \mathscr{L}_{CC} + \mathscr{L}_{NC} = -\frac{g}{\sqrt{2}}[J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}] - eJ_\mu^{em} A^\mu - \frac{g}{\cos\theta_W}[J_\mu^0 Z^\mu] \tag{2.2}$$

The neutral current part of the Lagrangian is made up of the neutral electromagnetic J_μ^{em} and weak currents J_μ^0, which are given in terms of the electric charge and isospin of the fermions, with $J_\mu^{em} = Q_f \bar{f}\gamma_\mu f$ and $J_\mu^0 = \bar{f}\gamma_\mu[(I_z^f - 2Q_f \sin^2\theta_W) - I_z^f \gamma_5]f$ summing over all flavors. The charged current in the quark sector is given by

$$J_\mu^+ = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

where the L subscript represents the left-handed projector $\frac{1}{2}(1-\gamma_5)$ which reflects the vector - axial-vector (V - A) structure of the weak interaction. V_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix (see section 3).

Strong interaction. While leptons are detected as discrete particles in nature, quarks can not be found free in nature. Quarks, which are massive and have fractional electric charge, are further characterized by a quantum number known as *color charge*, there are 3 color charges commonly *red, green* and *blue*. The strong interaction is described by quantum chromodynamics (QCD) which is based on the non-abelian gauge group $SU(3)_{color}$ gauge theory. There are eight gauge bosons, corresponding to eight generators of the $SU(3)_{color}$ group, called gluons **g** which mediate the strong force between particles with color charge and is responsible for the confinement of the quarks to form $SU(3)_{color}$-singlet bound-states called *hadrons*, and on a larger scale, for the binding of the hadrons in a nucleus. Hadrons must be color neutrals and are found in two classes; *baryons*, which contain three quarks (or three anti-quarks) and *mesons*, which contain a quark and an anti-quark. Mesons have integer spin (bosons), while baryons have half integer spin (fermions). The Lagrangian of QCD can be written as

$$\mathscr{L}_{QCD} = \bar{Q}_\alpha(i(\gamma^\mu D_\mu)_{\alpha\beta} - m\delta_{\alpha\beta})Q_\beta - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} \tag{2.3}$$

Here $G^a_{\mu\nu}$ is the field strength tensor and defined as

$$G^a_{\mu\nu} = \partial_\mu \partial^a_\nu - \partial_\nu \partial^a_\mu + g_s f^{abc} A^a_\mu A^a_\nu \tag{2.4}$$

and $iD_\mu = i\partial_\mu - g_s T^a A^a_\mu$, where A^a_μ is the vector field representing the gluon and g_s is the strong coupling constant. Q_α is a column vector of six quark fields which correspond to the six flavors, the quark fields are color triplets, so that the indices α and β run over the colors. The T^a are the generators of the group $SU(3)_C$. The sum over repeated indices is understood. An extensive treatment of QED and QCD can be found in the books by Peskin and Schroeder, and Mandl and Shaw [**16, 17**].

The γ (photon) and gluons are massless, while the $W^\pm$ and Z^0 boson are massive. Each of these fermions and gauge bosons has a corresponding antiparticle which has the same mass but the opposite electric charge. The neutral gauge bosons, photons, gluons and Z^0 are their own antiparticles. The electromagnetic and the strong interaction conserve quark and lepton flavor while in the weak interaction is not. The Higgs boson is the last particle, predicted by the Standard Model, confirmed by experiments. The Higgs boson is a special bosonic particle, because it explains why the gauge bosons and fermions are massive, through their interaction with the Higgs boson. Figure 2.2 shows the fundamental particles of the Standard Model and summarize some of their physical properties measured so far as masses, charges and spin.

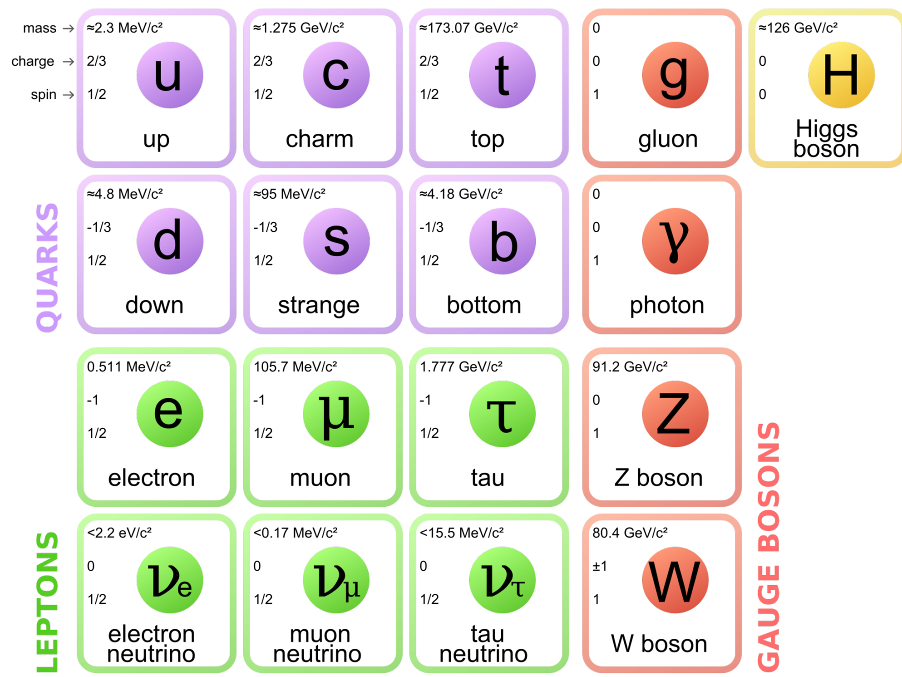


Figure 2.2. Fundamental particles of the Standard Model: quarks, leptons and force carriers[3].

In the standard model, during a weak decay a fermion (lepton or quark) transforms into its doublet partner by emission of a charged weak boson $W^{\pm}$. The $W^{\pm}$ can then either materialize a fermion-anti-fermion pair belonging to the same doublet, or couple to another fermion and transform it in its doublet partner (see Figure 2.3). A weak decay can therefore be represented as the interaction of two fermion currents (either leptonic or hadronic), mediated by a charged $W^{\pm}$

[3]Figure taken from Wikipedia: http://en.wikipedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg

bosonic current. Since only transitions between doublet partners are posible, the weak current mediating the decay process is always charged. Obviously, a weak decay can take place only if it is energetically possible, i.e. if the parent fermion has a larger mass than the daughter fermion. For this reason the quark u and the lepton e^-, being the lowest mass quark and lepton, do not decay.

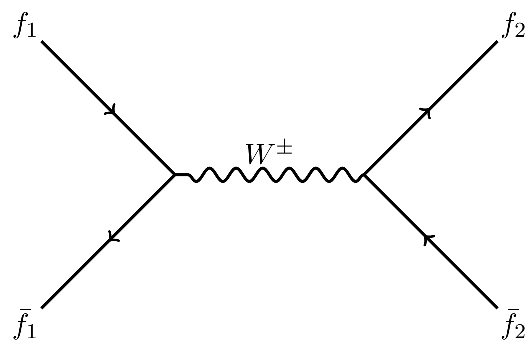


Figure 2.3. Schematic representation of a weak decay. Either $(f_1, \bar{f}_1)$ and $(f_2, \bar{f}_2)$ are fermions that belong to the same electroweak doublet.

Even though the Standard Model is not a complete theory. It does not take into account some experimental evidences, such as the presence of dark matter, the fermion masses hierarchy and the quantitative asymmetry between matter and antimatter of the universe. Current experiments are designed to measure even more accurately the parameters of the model in an effort to search for Physics Beyond the Standard Model (BSM), often also referred to as "New Physics", which the model can not explain.

2. C, P and T Symmetry

Symmetries are very important in physics, since they play an important role with respect to the laws of nature. A symmetry in a physical system is any type of transformation applied to the Lagrangian $\mathscr{L}$ which does not change under the transformation, in other words it leaves it invariant. Noether's Theorem **[18]** states that for a system descried by a Lagrangian $\mathscr{L}$, any symmetry which leaves the action invariant implies the existence of a conservation law. There are many types of symmetries that can be seen in nature. For example gauge symmetries, and the discrete symmetries of parity, charged conjugation, and time-reversal.

The Standard Model is based on $SU(3)_{color} \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetries. The $U(1)$ group is a one-dimensional phase rotation. $SU(2)_L \otimes U(1)_Y$ are the symmetry groups governing the electroweak interactions, known as the Glashow-Weinberg-Salam model [**12, 13, 14**]. $SU(3)_{color}$ is the symmetry group of the strong interaction, or quantum chromodynamics (QCD). The Lagrangian for this theory is then the sum of the strong interactions term $\mathscr{L}_{QCD}$ and the term for electroweak interactions $\mathscr{L}_{EW}$.

The discrete symmetries of charged conjugation (C), parity (P), and time-reversal (T) are very important symmetries in the Standard Model, and play an important role in particle physics. On a state, described by a four 4-vector $(x^0, \mathbf{x})$, operations are define as:

- **Charge conjugation C:** the particle is transformed into its antiparticle;
- **Parity P:** $\mathcal{P}(x^0, \mathbf{x}) = (x^0, -\mathbf{x})$, the space coordinates are reversed (reverses all momenta, but leaves spin unchanged);
- **Time-Reversal T:** $\mathcal{T}(x^0, \mathbf{x}) = (-x^0, \mathbf{x})$, the time coordinates are reversed (reverses both momenta and spin).

Those symmetries can be combined, for example the transformation $\mathcal{CP}$ changes a particle in its antiparticle and then inverts its momentum and helicity. While it is possible to predict violations of individual symmetries, the combined transformation $\mathcal{CPT}$ is always symmetric in the Standard Model [**19**] and up to now, it is confirmed to be conserved by all experimental searches.

There is no experimental evidence that interactions governed by the strong and electromagnetic forces violate C, P or T separately, while weak interactions violate both, C and P [**20**]. This is due to the vector - axial-vector (V - A) structure of the weak coupling. However, CP violation of the weak interactions was observed for first time, in the neutral kaon system $K_L \to \pi^+\pi^-$, by

Cronin and Fitch in 1964 [**21**]. Since then, CP violation has been one of the most interesting and active fields in particle physics. Recently CP violation was also observed in the and D^0 meson decay systems (CDF [**6**], BaBar [**7**] and Belle [**8**]).

3. The CKM matrix

The transformation between the quark mass eigenstates d, s, b and the weak interaction eigenstates d', s', b' is given by a 3×3 unitary matrix, usually referred to as the V_{CKM} (Cabibbo-Kobayashi-Maskawa) quark mixing matrix [**22, 23**], and can be written as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where $V_{q_1 q_2}$ is the coupling related to the transition $q_1 \to q_2$. Many parametrizations exist in the literature, but the most used are the *standard parametrization* [**24**], and the *Wolfenstein parametrization* [**25**].

In the standard parametrization, also used by the Particle Data Group [**26**], the CKM matrix is written as

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = cos\theta_{ij}$ and $s_{ij} = sin\theta_{ij}$ with θ_{ij} the mixing angles between the different families and δ as the CP violating phase. The standard choice for the four independent parameters, due to s_{13} and s_{23} are small and of the order of $\mathcal{O}(10^{-3})$ and $\mathcal{O}(10^{-2})$, is $s_{12} = |V_{us}|$, $s_{13} \sim |V_{ub}|$, $s_{23} \sim |V_{cb}|$ and δ.

The single complex phase enters because of the three-generations nature of the CKM matrix. A mass-mixing matrix involving only two quark generations would be parametrizable solely in terms of real rotations while more quark generations would lead to a more complex parametrization involving multiple phases. The fact that δ is non-zero allows for the Standard Model mechanism of CP violation in weak decays. Since $|V_{ub}| \sim \sin\theta_{13}$ multiplies every term in the CKM matrix carrying that phase, the Standard Model mechanism for CP violations requires $|V_{ub}|$ to be non-zero.

The square of the magnitude of $|V_{ij}|$ is the relative probability for a weak transition between quarks of flavor i and j. The fact that the off-diagonal elements of CKM matrix are not zero has a number of phenomenological implications, including that the possibility for flavor-changing transitions between quarks of different generations are allowed in weak charged-current interactions.

Another frequently used parameterization of the CKM matrix is the so-called Wolfenstein parametrization. Starting from the consideration that the mixing angles are small, the Wolfenstein parametrization [**25**] emphasizes in the magnitudes of the V_{CKM} elements. The parameters on the diagonal elements are ≈ 1, while off-diagonal elements are small. Transitions within the same quark generation are the most likely, while cross-generation transitions are suppressed. In the Wolfenstein parametrization, the matrix elements are the result of a power series expansion in terms of a small parameter $\lambda = |V_{us}| = sin\theta_C \approx 0.22$, where θ_C is called the Cabibbo angle [**22**]. To the order $\mathcal{O}(\lambda^3)$, it is expressed as

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where the four independent parameters are in this case λ, A, ρ and η, with η as the CP violating phase. Those parameters are defined as

$$s_{12} = \lambda,\ s_{23} = A\lambda^2,\ s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

Leaving the strong *CP* violation aside, the only source remaining for *CP* violation in the SM is the CKM matrix. Therefore, investigation of the magnitude of *CP* violation in the weak interaction is a well suited area in search for Physics BSM.

Chapter 3

Theory of *CP* violation

After a chapter which mostly contained a basic theoretical overview of particle physics, we now explore *CP* violation. One distinguishes direct and indirect CP violation. At last, we discuss the observable ΔA_{CP} and how the different kinds of CP violation contribute to it. This chapter is mostly based on the text books by Branco, Lavoura and Silva, and Bigi and Sanda [**27, 28**] .

CP violation in D decay processes has been observed and is highly suppressed in the Standard Model. Therefore, in general, observation of an appreciable *CP* violating effect in the charm system would be a signature of new physics contributions. The measurement of asymmetries between a decay and its *CP* conjugate is a way to investigate the magnitude of this *CP* violation and study if it exceeds the Standard Model prediction, giving hints towards New Physics. The advantage of measuring asymmetries is that due to their definition many systematical effects cancel. That way high precision can be achieved.

CP violation in decay appears on the amplitude level. The general form of the time integrated *CP* asymmetry is given by Equation 1.1

$$A_{CP}(f;t) = \frac{\Gamma(P \to f) - \Gamma(\bar{P} \to \bar{f})}{\Gamma(P \to f) + \Gamma(\bar{P} \to \bar{f})} \tag{3.1}$$

where $\Gamma(P \to f)$ is the decay rate of the particle P to the final state f. The decay rates take following form

$$\Gamma(P \to f) = \tilde{\Gamma}_f \, |\mathcal{M}(P \to f)|^2, \qquad \Gamma(\bar{P} \to \bar{f}) = \tilde{\Gamma}_f \, |\mathcal{M}(\bar{P} \to \bar{f})|^2 \tag{3.2}$$

where $\mathcal{M}(P \to f)$ is the decay amplitude for the decay of a particle P to a final state f and $\tilde{\Gamma}_f$ is a phase space factor.

The time integrated CP asymmetry receives contributions from

- A^m CP violation in mixing.
- A^i CP violation in interference between decays with and without mixing.
- A^d_f CP violation in decay.

The "indirect" CP asymmetries A^m and A^i are approximately independent of the final state and depends only on $D^0 - \bar{D}^0$ mixing parameters. The "direct" CP asymmetry A^d_f is instead sensitive to the final state.

1. Direct *CP* violation (CP Violation in Decay).

We obtain the general definition of a direct CP asymmetry as

$$A_{CP}(f) = a^{dir}_{CP} \equiv \frac{1 - \left|\frac{\mathcal{M}(\bar{P}\to\bar{f})}{\mathcal{M}(P\to f)}\right|^2}{1 + \left|\frac{\mathcal{M}(\bar{P}\to\bar{f})}{\mathcal{M}(P\to f)}\right|^2} \tag{3.3}$$

what gives the defining condition for direct CP violation as

$$\left|\frac{\mathcal{M}(\bar{P} \to \bar{f})}{\mathcal{M}(P \to f)}\right| \neq 1 \tag{3.4}$$

Direct CP violation occurs if two different amplitudes contribute to a single decay, i.e. a tree and a higher order process, such that the interference of these two contributing amplitudes leads to a different decay rate for the CP-conjugated process. To show this we assume that the final state is a CP eigenstate such that $\mathcal{CP}A(P \to f) = \eta^f_{CP} A(P \to f)$, where $\eta^f_{CP} = \pm$ for f being an even/odd CP eigenstate. Furthermore, the amplitudes, which represent the decay of P and its CP-conjugate $\bar{P}$ to the final state f, can be parametrized in the following way [**1**]

$$A_f = A_f^T \left(1 + r_f e^{i(\delta_f + \phi_f)}\right) \tag{3.5}$$

$$\bar{A}_f = \eta^f_{CP} A_f^T \left(1 + r_f e^{i(\delta_f - \phi_f)}\right) \tag{3.6}$$

where A_f^T is the dominant tree level amplitude, and r_f the subleading penguin amplitudes. The penguin amplitudes have a weak phase ϕ_f and a strong phase δ_f. Under the assumption that r_f is small, the direct CP asymmetry becomes

$$A_f^d = 2 r_f \sin\delta_f \sin\phi_f \tag{3.7}$$

2. Indirect *CP* violation

In contrast to direct CP violation, which is possible for neutral and charged meson decays, indirect CP violation is only possible in the decays of neutral mesons, because they can transform into their antiparticle.

2.1. *CP* violation in mixing. CP violation in mixing occurs if a neutral particle P^0 cannot decay into a final state $\bar{f}$ but its CP-conjugate $\bar{P}^0$ can. Consequently, P^0 needs first to oscillate to the antiparticle state before decaying into the given final state $\bar{f}$. As example we have the semileptonic decays

$$P^0 \to \bar{P}^0 \to l^+ + X^- \not\leftarrow P^0 \tag{3.8}$$

$$\bar{P}^0 \to P^0 \to l^- + X^+ \not\leftarrow \bar{P}^0 \tag{3.9}$$

where $f = l^- + X^+$ and $\bar{f} = l^+ + X^-$.

2.2. *CP* violation in interference between decays with and without mixing. If mixing followed by decay and direct decay interfere this creates an additional form of *CP* violation. The final state must be common to P^0 and $\bar{P}^0$. An example for this kind of *CP* violation are the decays $D^0 \to K^+K^-$, $\bar{D}^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$ and $\bar{D}^0 \to \pi^+\pi^-$.

3. *CP* violation in D^0 decays

In this section we are going to explore the structure of ΔA_{CP} and which of the three types of *CP* violation contribute to ΔA_{CP}.

The direct *CP* asymmetry is dependent on the final state, since it is dependent on the decay amplitude which is different for every final state. The indirect *CP* violation is universal for all final states to a good approximation, since a_{CP}^{ind} is a function of the mixing parameters only. The effective decay time is also dependent on the final state this is because different decay processes have smaller/larger phase spaces and therefore are less/more likely to occur after a certain time.

3.1. $\boldsymbol{\Delta A_{CP}}$. For $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, the difference ΔA_{CP} can then be written as

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \tag{3.10}$$

$$= \Delta a_{CP}^{dir} + a_{CP}^{ind} \frac{\Delta\langle t\rangle}{\tau_D} \tag{3.11}$$

where Δa_{CP}^{dir} is defined by $\Delta a_{CP}^{dir} = a_{CP}^{dir}(K^+K^-) - a_{CP}^{dir}(\pi^+\pi^-)$, in the same way $\Delta\langle t\rangle$ is defined as $\Delta\langle t\rangle = \langle t_{KK}\rangle - \langle t_{\pi\pi}\rangle$. The value for a_{CP}^{dir} and a_{CP}^{ind} can be taken from HFAG **[29]**

$$a_{CP}^{dir} = (-0.333 \pm 0.120)\% \tag{3.12}$$

$$a_{CP}^{ind} = (0.015 \pm 0.052)\% \tag{3.13}$$

and the values for $\frac{\Delta\langle t\rangle}{\tau}$ can be read of from the following table

Experiment	$\frac{\Delta\langle t\rangle}{\tau}$	$\frac{\overline{\langle t\rangle}}{\tau}$	Reference
CDF	0.25	2.58	[**6**]
LHCb	0.11	2.10	[**9**]

Table 3.1. Experimental values for $\frac{\Delta\langle t\rangle}{\tau}$.

Due to the small value for a_{CP}^{ind}, it is a good approximation to take

$$\Delta A_{CP} \approx \Delta a_{CP}^{dir} \tag{3.14}$$

We can then say that ΔA_{CP} mainly consists of the difference of direct CP asymmetries. The collected world data with uncertainties are summarized in Figure 3.1.

[4]Taken from http://www.slac.stanford.edu/xorg/hfag/charm/CHARM13/DCPV/direct_indirect_cpv.html

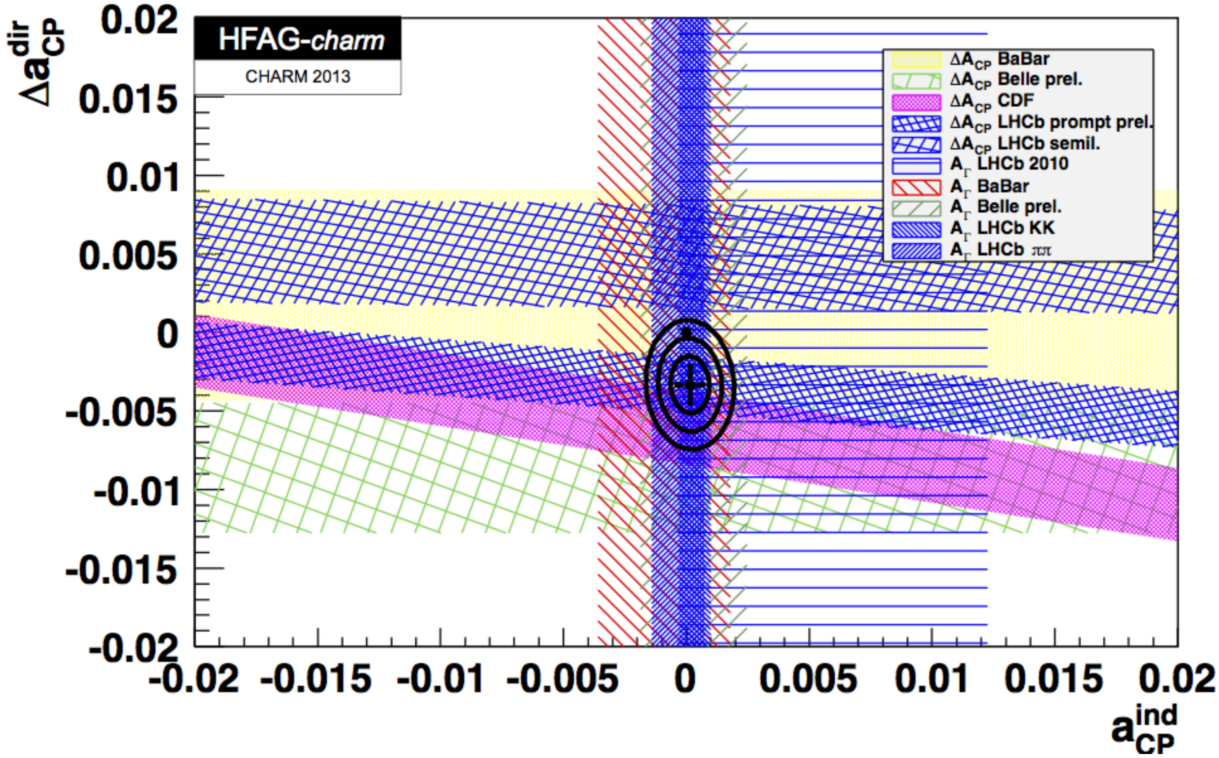


Figure 3.1. The combination plot shows the measurements listed in the Table above for ΔA_{CP}, where the bands represent $\pm 1\sigma$ intervals. The point of no CP violation (0,0) is shown as a filled circle, and two-dimensional 68% CL, 95% CL, and 99.7% CL regions are plotted as ellipses with the best fit value as a cross. The measurements of ΔA_{CP} can be used to write Δa^{dir}_{CP} as function of a^{ind}_{CP} as in Equation 3.11. The plot was taken from HFAG [**29**]. From the fit, the change in χ^2 from the minimum value is consistent with no CP violation at 2.0% CL [4].

Chapter 4

Weak decays of D meson

In the previous chapter, we established that ΔA_{CP} is the difference of the two direct asymmetries, we also found that direct asymmetries are generated by the interference of at least two different processes that contribute to one decay. Yet, we did not discuss the decay processes themselves, we will take up this task in this chapter. In principle, the decay amplitudes can be derived from the SM Lagrangian, nevertheless, this involves some complications. Corrections of the strong interaction "QCD corrections" make the analysis difficult because not all QCD corrections can be computed from first principles. This also hinders us from making precise predictions for ΔA_{CP} as we will see in the third section of this chapter.

1. Charm quark physics

Since the discovery of the J/ψ, forty years ago (1974) [**30, 31**], charm physics has been an interesting and active area of investigation. The charm quark is the lightest of the heavy quarks ($m_c \approx 1.275 GeV/c^2$) and provides an intermediate state extending the knowledge from lighter flavors to heavy quark physics. Also since many more hadrons are produced containing a charm quark than heavier flavors, it is easier and more accessible to study experimentally than other heavy quarks.

1.1. Charmed mesons decays. There are three types of decays of the D meson, categorized according to the final state particles produced. Firstly there are the leptonic decays, such as $D^+ \to l^+\nu_l$ and semi-leptonic decays such as $D^0 \to K^-e^+\nu_e$. Finally, and most important in the context of this work, there are the fully hadronic, a.k.a. non-leptonic decays, i.e. $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$.

The study of charmed meson decays are important for improving our knowledge of the SM. In particular, the study of leptonic and semileptonic decays allow us to measure the CKM matrix elements V_{cs} and V_{cd} to a high level of precision. Leptonic and semileptonic decays are also used to test theoretical predictions describing the strong interaction (QCD) in heavy quark systems.

In the SM, the charm quark, and hence mesons containing valence c quark, decays through the weak charge current into a light quark with charge $-\frac{1}{3}$; i.e. the strange (s) or down (d) quark. The coupling constant is proportional to the element V_{cq} of the CKM mixing matrix and the decay rate is proportional to $|V_{cq}|^2$. The W^+ boson emmited by the charm quark may decay leptonically, to a lepton - neutrino (l, ν_l) pair, or hadronically, to a quark - anti-quark $(q_1, \bar{q}_2)$ pair, which then hadronize into a daughter meson (K or π). Therefore, when the weak interaction is responsible for the decay of charmed particles, strong interactions play an important role in the hadronization process which determines the final state. The lowest order diagram (i.e. neglecting gluon emission) through which the decay can proceed are shown in Figure 4.1 (for the case of a charm meson $D^0 = (c\bar{u})$).

1.2. Non-leptonic weak decays for particles with charm. Non-leptonic decays can be described as two hadronic charged currents coupled by the exchange of W^+ gauge bosons. The final state are only hadrons and the lowest order interaction is written as

$$L_{eff} = -4\frac{G_F}{\sqrt{2}}J^\mu J^\dagger_\mu + h.c. \tag{4.1}$$

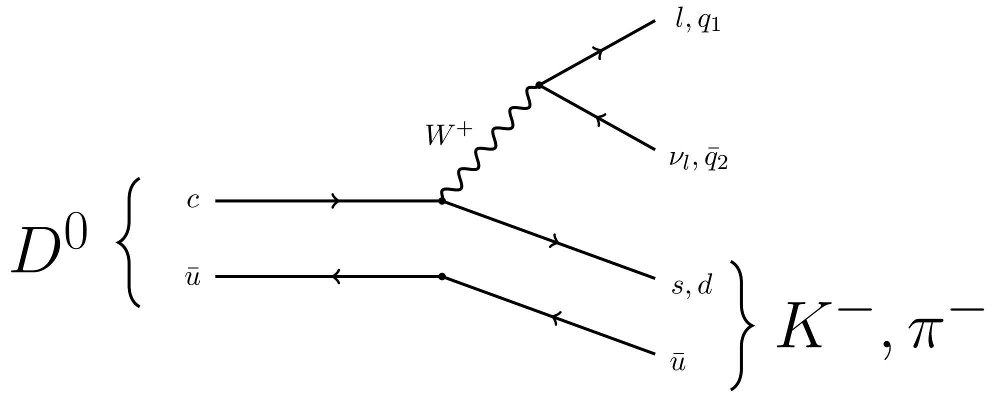


Figure 4.1. Charmed quarks decay under the weak interaction into s or d quarks. The W^+ boson produced may decay leptonically [to a (l, ν_l) pair] or hadronically [to a $(q_1, \bar{q}_2)$ pair].

where G_F is the Fermi coupling constant.

In charm decays, considering only the first two generations of quarks, the CKM matrix can be approximated by a 2×2 unitary matrix with one real angle

$$V_C = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

where $\theta_c \approx 13°$ is the Cabbibo angle and determines the level of mixing between the two generations and hence the decay rate. The mass eigenstates of the d and s quarks can be expressed as

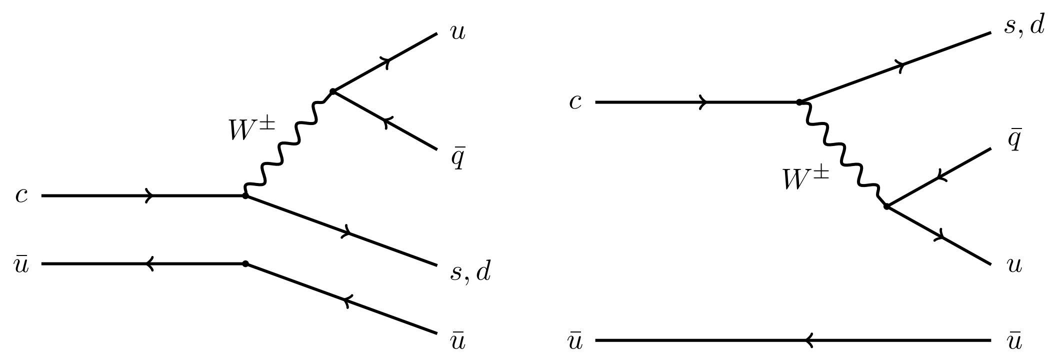
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = V_C \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\begin{aligned} d' &\equiv d\cos\theta_c + s\sin\theta_c \\ s' &\equiv -d\sin\theta_c + s\cos\theta_c \end{aligned}$$

In this approximation $c \to s$ and $u \to d$ transitions, proportional to $\cos\theta_c$ (probability $\sim \cos^2\theta_c \approx 0.95$), are favored with respect to $c \to d$ and $s \to u$ transitions, proportional to

$\sin\theta_c$ (probability $\sim \sin^2\theta_c \approx 0.05$). These two types of transitions are called Cabbibo-favored (CF) and singly-Cabbibo-suppressed (SCS), respectively.

In the spectator decay Figure 4.2, the light anti-quark $\bar{u}$ does not take part in the weak interaction. It is a spectator to the charm decay process and afterwards combines with the daughter quark, either from the c-quark decay or W-boson decay, to form another daughter meson. In the spectator mechanism, the dacay rate into any $q-\bar{q}$ pair is favor by a factor of three over the decay rate into $l-\nu_l$ pair, because there are three color degrees of freedom. In the same way, in the **external spectator**, a.k.a. "*color allowed*" or "*factorizable*", color is automatically conserved, while in the **internal spectator**, a.k.a. "*color suppressed*" or "*non-factorizable*", amplitude color is suppressed since the color of the quarks from the virtual W most match the color of the quarks from the parent meson. $\zeta = \frac{1}{N_c} = \frac{1}{3}$.



a) External spectator. b) Internal spectator.

Figure 4.2. Spectator decay for charm quark. A) External spectator. B) Internal spectator.

2. Low-Energy Effective Lagrangian

In order to describe non-leptonic decays we have to take into account corrections of the strong interaction to the weak interaction. The strong interaction acts on the quark constituents of the hadrons and is the most difficult part in the evaluation of decay amplitudes.

In this section we will give the final Lagrangian which describes weak decays, where the weak and strong interaction are combined in effective operators Q_i, by making use of the *Operator product expansion* (OPE). The idea of *Low-Energy Effective Lagrangian* (LEEL) is to divide in short and long-distance interactions. The short-distance interactions describe the contributions of the heavy degrees of freedom such as the top and bottom quark and the W boson. They can be evaluated and will be encoded into the Wilson coefficients. The long-distance effects, will be discussed a little bit further.

2.1. Operator Product Expansion (OPE). Since we want to describe D meson decays in which CP violation appears we consider the charged current Lagrangian $\mathscr{L}_{CC}$. The basic tree-level for SCS decays has the generic flavor structure $c \to qu\bar{q}$ for $q = s, d$, and gives the amplitude $\mathcal{M}$

$$\mathcal{M} = \frac{ig_w^2}{2(q^2 - M_W^2)} V_{cq} V_{uq}^* \langle M_1 M_2 | \, (\bar{q}\gamma^\mu Lc) \, (\bar{u}\gamma^\nu Lq) \, |D^0\rangle \tag{4.2}$$

where M_W is the mass of the W boson, g_w is the weak coupling constant, V_{cq} and V_{uq} are CKM factors and $L = \frac{1-\gamma_5}{2}$ the left-handed spinor in the Dirac space.

In the limit where $|q^2| = M_D^2 \ll M_W^2$ ($M_D \sim 2$ GeV, $M_W = 80.4$ GeV), the propagator of the boson line become simply $\frac{g_{\mu\nu}}{M_W^2}$, and the matrix element simplifies to Equation 4.3 where the identity $\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$ was used

$$\mathcal{M} = -i4\frac{G_F}{\sqrt{2}} V_{cq} V_{uq}^* \langle M_1 M_2 | (\bar{q}\gamma^\mu Lc)(\bar{u}\gamma_\mu Lq) | D^0\rangle \tag{4.3}$$

This amplitude could also have been obtained from the following effective Lagrangian

$$\mathscr{L}_{eff} = 4\frac{G_F}{\sqrt{2}} V_{cq} V_{uq}^* (\bar{q}\gamma^\mu Lc)(\bar{u}\gamma_\mu Lq) \tag{4.4}$$

which is valid in the low energy limit where the quark interactions with the W boson can be approximated to be point-like. The lowest order diagram for $c \to qu\bar{q}$ transition is shown in Figure 4.3. The same figure also shows the approximated weak vertex as a pointlike interaction. Higher order corrections, which correspond to operators of higher dimension can usually be neglected, we will neglect all higher order corrections in this thesis.

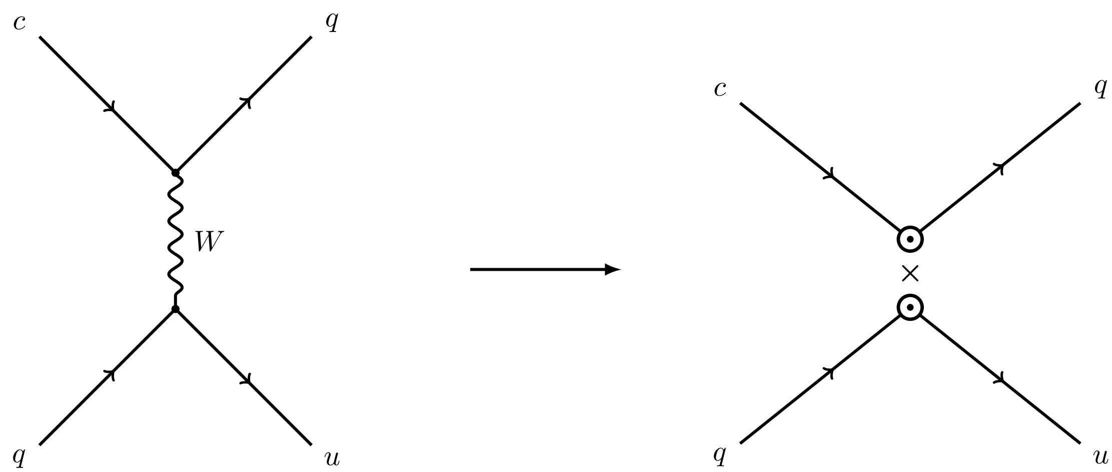


Figure 4.3. Tree-level diagram for $c \to qu\bar{q}$ transitions. The figure on right shows the approximated weak vertex as a pointlike interaction.

2.2. Low-Energy Effective Lagrangian (LEEL) for SCS decays. In SCS decays the quark level transition has the form $c \to pu\bar{p}$ with $p = d, s, b$ so that penguin diagrams as in Figure 4.4 do contribute. In Figure 4.4, the gluon creates an additional quark pair, this type of diagram is called a QCD penguin.

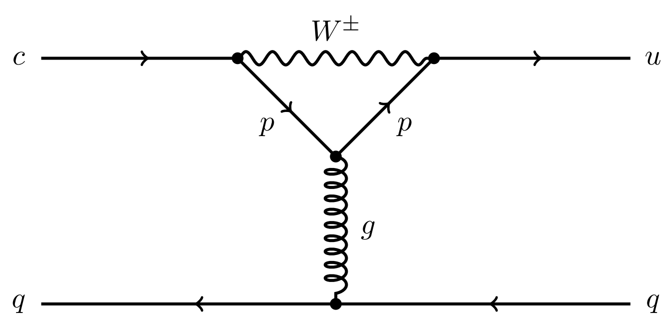


Figure 4.4. Penguin diagram.

Because of their flavor structure penguin diagrams contribute only to SCS decays. If QCD corrections and penguin interactions additional to the basic tree interaction are taken into account of the order α_s, the total effective non-leptonic quark level Lagrangian at a scale μ for SCS charm decay is given by [**4, 32**]

$$\mathscr{L}_{eff} = -4\frac{G_F}{\sqrt{2}}\left[\sum_{q=s,d} V_{uq}V^*_{cq}(C_1 Q^q_1 + C_2 Q^q_2) - V_{ub}V^*_{cb}\sum_{n=3}^{6} C_n Q_n + C_{8g}Q_{8g}\right] + h.c. \tag{4.5}$$

where C_i are the Wilson coefficients containing loop effects from scales above μ. The current-current operators $Q_{1,2}$ are defined as

$$Q^q_1 = (\bar{q}_\alpha q_\beta)_{V-A}(\bar{u}_\beta c_\alpha)_{V-A} \qquad Q^q_2 = (\bar{q}c)_{V-A}(\bar{u}q)_{V-A} \tag{4.6}$$

and the so-called "penguin" operators Q_{3-6} are given as

$$\begin{aligned} Q_3 &= (\bar{u}c)_{V-A}\sum_q(\bar{q}q)_{V-A} \qquad & Q_4 &= (\bar{u}_\alpha c_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A} \\ Q_5 &= (\bar{u}c)_{V-A}\sum_q(\bar{q}q)_{V+A} \qquad & Q_6 &= \bar{u}_\alpha c_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A} \end{aligned} \tag{4.7}$$

where summation over color indices α, β understood and $p = d, s$.

Since the quark pairs in the penguins couple to a gluon $V - A$ and $V + A$ currents contribute, yet it is common to write these contributions separately. At last there is the magnetic penguin operator which arises through the mass of the charm quark

$$Q_{8g} = -\frac{g_s}{8\pi^2} m_c \bar{u}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}c \tag{4.8}$$

Evaluated at the charm quark mass $\mu \approx m_c$ the Wilson coefficients take the following numerical values at next-to-leading order (NLO) [**4**]

$$\begin{gathered} C_1 = -0.41, \;\; C_2 = 1.21, \;\; C_3 = 0.02, \;\; C_4 = -0.04 \\ C_5 = 0.01, \;\; C_6 = -0.05, \;\; C_{8g} = -0.06 \end{gathered} \tag{4.9}$$

The Q_i denote the relevant local operators which govern the particular decay in question and can be considered as effective point-like vertices while the Wilson coefficients are then seen as "coupling constants" of these effective vertices, summarizing the contributions from physics at scales higher than μ.

With the Lagrangian in Equation 4.5 we can describe all decays of a D meson into two light pseudoscalars. The problem of the LEEL is that the matrix elements $\langle Q_i \rangle$ cannot be computed *a posteriori*. Yet, in some cases they can be extracted from experiment, how this can be done is presented in the next section.

3. Factorization

In this section, we will introduce the "*naive factorization*" to give an approximate value to the hadronic matrix elements. In order to do this we need to define some non-perturbative quantities, which can be extracted from leptonic and semileptonic decays.

3.1. Naive Factorization. To evaluate the hadronic matrix elements of the operators in the effective Lagrangian that is relevant for SCS decays, we introduce the *"naive factorization"* [**33**]. For two-body non-leptonic decays of $D^0 \to M_1 M_2$, the major difficulty involves the evaluation of the hadronic matrix elements

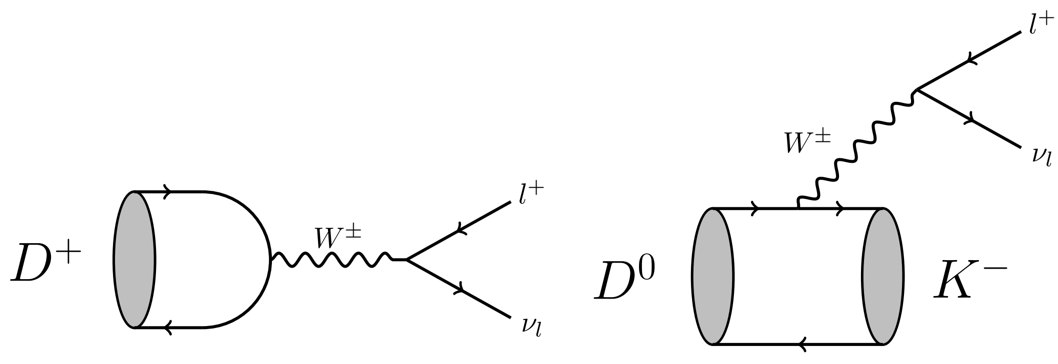
$$\langle M_1 M_2 | \mathscr{L}_{eff} | D^0 \rangle \tag{4.10}$$

It results very convenient to separate the full matrix element into a product of matrix elements of two quark currents. Then for the basic-tree transition of $c \to q\bar{u}\bar{q}$ we have

$$\langle M_1 M_2 | (\bar{q}\gamma^\mu L c)(\bar{u}\gamma_\mu L q) | D^0 \rangle \longrightarrow \langle M_1 | (\bar{q}\gamma^\mu L c) | D^0 \rangle \langle M_2 | (\bar{u}\gamma_\mu L q) | 0 \rangle \tag{4.11}$$

The first product represents the transition matrix element between the D^0 meson and one of the final state mesons, and the second is the matrix element of the other final state meson being "created" from the vacuum.

3.2. Non-perturbative quantities. As said before, in order to give an approximate value to the hadronic matrix elements we need to define some non-perturbative quanties, which can be extracted from leptonic and semileptonic decays. This subsection is based and adapted from [**34**].



a) Leptonic decay. b) Semileptonic decay.

Figure 4.5. Feynman diagrams. a) Leptonic decay. b) Semileptonic decay.

Leptonic decays. The simplest example of a leptonic decay for a D meson is given $D^+ \to l^+\nu_l$, as depicted in Figure 6.1 a). From the corresponding Feynman diagram, the decay amplitude for this process can be written as

$$\mathcal{M} = i\,\frac{g_w^2}{2} V_{cd}^* \left[\bar{u}_{\nu_l}\gamma^\mu L v_l\right] \frac{g_{\mu\nu}}{q^2 - M_W^2} \langle 0|\bar{s}\gamma^\nu Lc|D^+\rangle \tag{4.12}$$

where M_W is the mass of the W boson, g_w is the weak coupling constant, V_{cd}^* the corresponding element of the CKM matrix and $L = \frac{1-\gamma_5}{2}$ the left-handed spinor in the Dirac space. Here again, in the limit where the four-momentum q that is carried by the W boson satisfies $|q^2| \sim M_D^2 \ll M_W^2$ ($M_D \sim 2$ GeV, $M_W = 80.4$ GeV), the propagator of the boson line become simply $\frac{g_{\mu\nu}}{M_W^2}$, and the matrix element simplifies to

$$\mathcal{M} = -i4\,\frac{G_F}{\sqrt{2}} V_{cd}^* \left[\bar{u}_{\nu_l}\gamma^\mu L v_l\right] \langle 0|\bar{s}\gamma_\mu Lc|D^+\rangle \tag{4.13}$$

where the identity $\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$ was used. Ignoring the higher order terms, all the hadronic physics are enclosed in the *hadronic matrix element* $\langle 0|\bar{s}\gamma_\mu Lc|D^+\rangle$ which describes the decay of a D^+ meson. Since D^+ is a pseudoscalar particle, the part $\langle 0|\bar{s}\gamma_\mu c|D^+\rangle$ of the hadronic matrix element must be zero $\langle 0|\bar{s}\gamma_\mu c|D^+\rangle = 0$, so we can write

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D^+(P)\rangle = if_{D^+}P_\mu^{D^+} \tag{4.14}$$

where f_{D^+} is the D^+ meson *decay constant*.

Semileptonic decays. For semileptonic decays we can take the example $D^0 \to K^- l^+ \nu_l$, which is illustrated in 6.1 b). The amplitude of the decay $D^0 \to K^- l^+ \nu_l$ can be written as

$$\mathcal{M} = -i4\,\frac{G_F}{\sqrt{2}} V_{cs}^* \left[\bar{u}_{\nu_l}\gamma^\mu L v_l\right] \langle K^-|\bar{s}\gamma_\mu Lc|D^+\rangle \tag{4.15}$$

where as above $q^2 \sim M_D^2 \ll M_W^2$ was used. Here again, all the hadronic physics are enclosed in the hadronic matrix element $\langle K^-|\bar{s}\gamma_\mu Lc|D^+\rangle$. Since K^- and D^+ are pseudoscalars, we have $\langle K^-|\bar{s}\gamma_\mu\gamma_5 c|D^+\rangle = 0$, so we can write

$$\begin{aligned}
\langle K^-|\bar{s}\gamma_\mu c|D^+\rangle &= f_+(q^2)(P_D + P_K)_\mu + f_-(q^2)\,q_\mu && (4.16)\\
&= f_+(q^2)\left[(P_D + P_K)_\mu - \frac{M_D^2 - M_K^2}{q^2} q_\mu\right] + f_0(q^2)\frac{M_D^2 - M_K^2}{q^2} q_\mu && (4.17)
\end{aligned}$$

where $f_0(q^2) \equiv f_+(q^2) + f_-(q^2)\frac{q^2}{M_D^2 - M_K^2}$ and $q \equiv P_D - P_K$, with $P_D^2 = M_D^2$, $P_K^2 = M_K^2$ and q^2 is the invariant mass of the dilepton pair (l^+, ν_l). In Equation 4.17 $f_+(q^2)$ and $f_0(q^2)$ are the *form factors* of the $D^0 \to K^-$transitions.

Chapter 5

The Standard Approach to SCS decays

In this chapter, the singly Cabbibo suppressed processes are going to be presented but up to now we have just considered $c \to qu\bar{q}$ processes in general. So now, we take a look in the two meson decays governed by $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, not including the contributions for the final states with η and π^0 mesons.

Using the OPE, we examine the decays at tree level, followed by one-loop order QCD corrections to the products of quark currents, where gluons are exchanged between the weak interaction vertices in all possible ways. Following the same general strategy as used by Grinstein et al. [**35**].

1. The decays at tree level.

The amplitude at tree-level for Cabibbo suppressed processes $c \to su\bar{s}$ and $c \to du\bar{d}$ can be written as Equation 4.2

$$\mathcal{M}_{K^+K^-} = -i4\frac{G_F}{\sqrt{2}}V_{cs}V_{us}^*\langle K^+K^-|(\bar{s}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ls)|D^0\rangle \tag{5.1}$$

$$\mathcal{M}_{\pi^+\pi^-} = -i4\frac{G_F}{\sqrt{2}}V_{cd}V_{ud}^*\langle \pi^+\pi^-|(\bar{d}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ld)|D^0\rangle \tag{5.2}$$

and the effective Lagrangian as Equation 4.4

$$\mathscr{L}_{eff}^{weak} = -\tilde{G}_\theta[(\bar{s}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ls) - (\bar{d}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ld)] \tag{5.3}$$

$$= -\tilde{G}_\theta\, Q_2 \tag{5.4}$$

with $\tilde{G}_\theta \equiv 4\frac{G_F}{\sqrt{2}}\cos\theta_c \sin\theta_c$, and

$$Q_2 \equiv (\bar{s}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ls) - (\bar{d}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ld) \tag{5.5}$$

$$= Q_2^s - Q_2^d \tag{5.6}$$

These give contributions to $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ where CP violation has been seen. Utilizing the naive factorization previously presented, the matrix elements can be separated into a product of matrix elements.

$$\langle K^+K^-|\, Q_2^s\,|D^0\rangle = \langle K^-|\,\bar{s}\gamma^\mu Lc\,|D^0\rangle\langle K^+|\,\bar{u}\gamma_\mu Ls\,|0\rangle \tag{5.7}$$

$$\langle \pi^+\pi^-|\, Q_2^d\,|D^0\rangle = \langle \pi^-|\,\bar{d}\gamma^\mu Lc\,|D^0\rangle\langle \pi^+|\,\bar{u}\gamma_\mu Ld\,|0\rangle \tag{5.8}$$

The first of the products is the transition matrix element between the D^0 meson and K^- (π^-) meson, and the second is the matrix element of the K^+ (π^+) meson being "created" from the vacuum via the axial current, proportional to the kaon (pion) decay constant f_K (f_π). For the K^+ we have

$$\langle K^+|\,\bar{u}\gamma_\mu Ls\,|0\rangle = -\frac{1}{2}\langle K^+|\,\bar{u}\gamma_\mu\gamma_5 s\,|0\rangle \tag{5.9}$$

$$= -\frac{1}{2}(if_K P_\mu^K) \tag{5.10}$$

and, for the case of π^+

$$\langle \pi^+|\,\bar{u}\gamma_\mu Ld\,|0\rangle = -\frac{1}{2}\langle \pi^+|\,\bar{u}\gamma_\mu\gamma_5 d\,|0\rangle \tag{5.11}$$

$$= -\frac{1}{2}(if_\pi P_\mu^\pi) \tag{5.12}$$

The hadronic current between the D^0 meson and K^- meson is related to

$$\langle K^-|\,\bar{s}\gamma^\mu Lc\,|D^0\rangle = \frac{1}{2}\langle K^-|\,\bar{s}\gamma^\mu c\,|D^0\rangle \tag{5.13}$$
$$= \frac{1}{2}[f_+(P_D+P_K)^\mu + f_-(P_D-P_K)^\mu] \tag{5.14}$$

and for π^-

$$\langle \pi^-|\,\bar{d}\gamma^\mu Lc\,|D^0\rangle = \frac{1}{2}\langle \pi^-|\,\bar{d}\gamma^\mu c\,|D^0\rangle \tag{5.15}$$
$$= \frac{1}{2}[f_+(P_D+P_\pi)^\mu + f_-(P_D-P_\pi)^\mu] \tag{5.16}$$

with $f_\pm$ relatively well known. Equations 5.14 and 5.16 can also be written in terms of the decay factors f_+ and f_0 as Equation 4.17.

In the next section QCD corrections to one-loop order are going to be considered. So defining a new the local operator Q_2^c, which represents the QCD correction to Q_2, as

$$Q_2^c \equiv (\bar{q}_i T^a_{ij}\gamma^\mu Lc_j)(\bar{u}_k T^a_{kl}\gamma_\mu Lq_l)$$

with T^a as color matrices with color indices i, j, k, l. The product of color matrices is given by $T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$ (see Appendix C, Equation C.7), so

$$Q_2^c = (\bar{q}_i\gamma^\mu Lc_k)(\bar{u}_k\gamma_\mu Lq_i) - \frac{1}{N_c}(\bar{q}_i\gamma^\mu Lc_i)(\bar{u}_k\gamma_\mu Lq_k) \tag{5.17}$$

the second term, is equivalent to Q_2 suppressed by a factor $\frac{1}{N_c}$, where N_c is the number of colors. The first term, after Fierz transformation is apply can be define as a new local operator Q_1 as

$$Q_1 \equiv (\bar{s}\gamma^\mu Ls)(\bar{u}\gamma_\mu Lc) - (\bar{d}\gamma^\mu Ld)(\bar{u}\gamma_\mu Lc) \tag{5.18}$$

Q_1 gives contributions to η and π^0 in final state, and it is related to the Figure 4.2 b), which corresponds to a color suppressed transition, even though Q_1 is generated by "loop" contributions via Q_2^c (see section 2).

2. One loop QCD corrections

To obtain QCD corrections at one-loop order for $c \to q u \bar{q}$ process. We must calculate the contribution to the Lagrangians for each of the diagrams in Figure 5.1, utilizing HQET (Heavy Quark Effective Theory) in the energy range $m_c > \mu$. The charm quark mass is taken to be infinity and its four-velocity fixed, so we can replace the quark field c to a reduced heavy quark field $c \to h_v^{(c)}$. The feynman diagrams in Figure 5.1 show the lowest order QCD loop contributions for Q_2. Once we calculate the 4 diagrams we sum these to find the total contribution [**36, 37**].

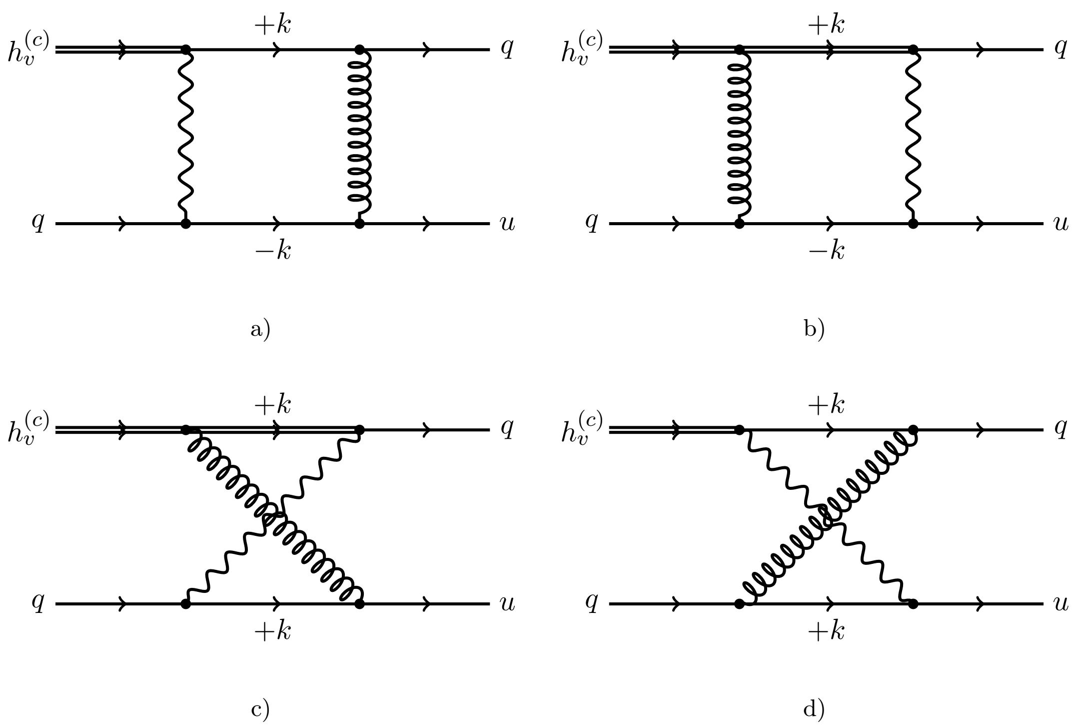


Figure 5.1. QCD corrections at one-loop order.

2.1. Diagram a) contribution. In the Table A.1 and A.2 are listed the Feynman rules necessaries to write the mathematical terms for these diagrams. The diagram 5.1 a) can be written in a mathematical way as

$$\Gamma_a = i\int \frac{d^4p}{(2\pi)^4}\left[\bar{q}(-ig_s\gamma^\nu T^a)\frac{i(\gamma^\sigma p_\sigma + m_q)}{p^2 - m_q^2 + i\epsilon}\frac{-ig_w}{\sqrt{2}}\gamma^\mu L V_{cq} c\right] \times \frac{ig_{\nu\lambda}}{p^2 + i\epsilon}$$
$$\frac{ig_{\mu\kappa}}{p^2 - M_W^2 + i\epsilon} \times \left[\bar{u}(-ig_s\gamma^\lambda T^a)\frac{i(-\gamma^\rho p_\rho + m_u)}{p^2 - m_u^2 + i\epsilon}\frac{-ig_w}{\sqrt{2}}\gamma^\kappa L V_{uq} q\right] \quad (5.19)$$

where all external momenta corrections are negligible compared to M_W^2, as $|p_{ext}^2| \sim M_D^2 \ll M_W^2$. Rearrenging all the terms we get

$$\Gamma_a = i4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^* g_s^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 + i\epsilon}\frac{M_W^2}{p^2 - M_W^2 + i\epsilon}$$
$$\left[\bar{q}T^a\gamma^\nu\frac{\gamma^\sigma p_\sigma + m_q}{p^2 - m_q^2 + i\epsilon}\gamma^\mu L h_v^{(c)}\right]\left[\bar{u}T^a\gamma_\nu\frac{-\gamma_\rho p^\rho + m_u}{p^2 - m_u^2 + i\epsilon}\gamma_\mu L q\right] \quad (5.20)$$

neglecting the mass of the u-quark, q-quark in the numerator, and with the following relation **[16, 17]**

$$\int \frac{d^4p}{(2\pi)^4}p^\sigma p^\rho\, f(p^2) = \frac{\delta^{\sigma\rho}}{4}\int \frac{d^4p}{(2\pi)^4}p^2\, f(p^2) \quad (5.21)$$

the integral simplifies to

$$\Gamma_a = -i\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\, g_s^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 + i\epsilon}\frac{M_W^2}{p^2 - M_W^2 + i\epsilon}\frac{1}{p^2 - m_q^2 + i\epsilon}$$
$$\left[\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu L h_v^{(c)}\right][\bar{u}T^a\gamma_\nu\gamma_\rho\gamma_\mu L q] \quad (5.22)$$

Introducing μ as the low energy cutoff, and replacing m_q with μ. The integral is explicitly calculated in Apendix B, using the Feynman parametrization, and Wick rotation, resulting on the following

$$\Gamma_a = \frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\frac{g_s^2}{(4\pi)^2}Log\left[\frac{M_W^2}{\mu^2}\right](\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu L h_v^{(c)})(\bar{u}T^a\gamma_\nu\gamma_\rho\gamma_\mu L q) \quad (5.23)$$

the fine coupling constant is given by $g_s^2 = 4\pi\alpha_s$, and with $\frac{g_s^2}{(4\pi)^2}Log\left[\frac{M_W^2}{\mu^2}\right]$] being contained in the Wilson coefficients showed in Equation 4.9.

The products of Dirac bilinear $(\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu L h_v^{(c)})(\bar{u}T^a\gamma_\nu\gamma_\rho\gamma_\mu Lq)$ can be simplified using γ-matrices identites, such as C.3. The results are shown in Apendix C, introducing them the whole term simplifies to

$$\Gamma_a = 2\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]\left[(\bar{q}\gamma^\mu Lq)(\bar{u}\gamma_\mu Lh_v^{(c)}) - \frac{1}{N_c}(\bar{q}\gamma^\mu Lh_v^{(c)})(\bar{u}\gamma_\mu Lq)\right] \quad (5.24)$$

$$= 2\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]\left(Q_1 - \frac{1}{N_c}Q_2\right) \quad (5.25)$$

$$= 4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]Q_2^c \quad (5.26)$$

with Q_2^c, corrections to the local operator Q_2, is (see Appendix C)

$$Q_2^c = (\bar{q}T^a\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\mu Lq) = \frac{1}{2}\left(Q_1 - \frac{1}{N_c}Q_2\right). \quad (5.27)$$

2.2. Diagrams b) and c) contribution. The diagram 5.1 b) can be written in a mathematical way as

$$\Gamma_b = i4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*g_s^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+i\epsilon}\frac{M_W^2}{p^2-M_W^2+i\epsilon}\frac{1}{v\cdot p+i\epsilon}$$
$$\left[\bar{q}T^a\gamma^\nu Lv^\mu h_v^{(c)}\right]\left[\bar{u}\gamma_\nu L\frac{-\gamma_\rho p^\rho + m_u}{p^2-m_u^2+i\epsilon}\gamma_\mu T^a q\right] \quad (5.28)$$

and for the diagram 5.1 c)

$$\Gamma_c = i4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*g_s^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+i\epsilon}\frac{M_W^2}{p^2-M_W^2+i\epsilon}\frac{1}{v\cdot p+i\epsilon}$$
$$\left[\bar{q}T^a\gamma^\nu Lv^\mu h_v^{(c)}\right]\left[\bar{u}\gamma_\mu L\frac{\gamma_\rho p^\rho + m_u}{p^2-m_u^2+i\epsilon}\gamma_\nu T^a q\right] \quad (5.29)$$

The integral of the kind

$$I^\rho = \int\frac{d^4p}{(2\pi)^4}\frac{M_W^2\,p^\rho}{(p^2+i\epsilon)(p^2-M_W^2+i\epsilon)(v\cdot p+i\epsilon)(p^2-m_u^2+i\epsilon)} \quad (5.30)$$

$$= I_0 v^\rho \quad (5.31)$$

with I_0 given by B.34.

$\lrcorner$

Introducing 5.31 in the Equations 5.28 and 5.29 is evidently that they differ only by a sign, for that reason both terms cancel each other.

$$\Gamma_b = -\Gamma_c \tag{5.32}$$

2.3. Diagram d) contribution. The diagram 5.1 d) can be written in a mathematical way as

$$\begin{aligned}\Gamma_d = i\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^* g_s^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+i\epsilon}\frac{M_W^2}{p^2-M_W^2+i\epsilon}\frac{1}{p^2-m_q^2+i\epsilon}\\ \left[\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu L h_v^{(c)}\right]\left[\bar{u}\gamma_\mu L\gamma_\sigma\gamma_\nu T^a q\right]\end{aligned} \tag{5.33}$$

here again external momenta corrections are negligible compared to M_W^2. Replacing m_q with μ and using B.34 and C.12, Γ_d simplifies to

$$\Gamma_d = \frac{G_F}{2\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]\left[(\bar{q}\gamma^\mu Lq)(\bar{u}\gamma_\mu L h_v^{(c)}) - \frac{1}{N_c}(\bar{q}\gamma^\mu L h_v^{(c)})(\bar{u}\gamma_\mu L q)\right] \tag{5.34}$$

$$= \frac{G_F}{2\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]\left(Q_1 - \frac{1}{N_c}Q_2\right) \tag{5.35}$$

$$= \frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^*\frac{\alpha_s}{\pi}Log\left[\frac{M_W^2}{\mu^2}\right]Q_2^c \tag{5.36}$$

so, it is easy to see that

$$\Gamma_d = -\frac{1}{4}\Gamma_a \tag{5.37}$$

2.4. Total contribution. Now, we have evaluated each individual diagram, so to know the total QCD contribution at one-loop level, we just need to sum all those individual contributions given by Equations 5.26, 5.32 and 5.37 we get

$$\Gamma = -4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^{*}\left(\frac{3\alpha_s}{4\pi}Log\left[\frac{M_W^2}{\mu^2}\right]\right)Q_2^c \tag{5.38}$$

3. $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays

The effective Lagrangian at tree level for processes with these final states can be expresses as Equation 4.5

$$\mathscr{L}_{eff} = -4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^{*}\left[C_1Q_1 + C_2Q_2\right]$$

where the local operators Q_1 and Q_2 were defined as

$$Q_1 = (\bar{s}\gamma^{\mu}Ls)(\bar{u}\gamma_{\mu}Lc) - (\bar{d}\gamma^{\mu}Ld)(\bar{u}\gamma_{\mu}Lc)$$

$$Q_2 = (\bar{s}\gamma^{\mu}Lc)(\bar{u}\gamma_{\mu}Ls) - (\bar{d}\gamma^{\mu}Lc)(\bar{u}\gamma_{\mu}Ld)$$

Using the color matrix identity C.7 and the Fierz transformation, the Lagrangian can be expressed as

$$\mathscr{L}_{eff} = -4\frac{G_F}{\sqrt{2}}V_{cq}V_{uq}^{*}\left[\left(C_2 + \frac{1}{N_c}C_1\right)Q_2 + 2C_1(\bar{q}\gamma^{\mu}T^aLc)(\bar{u}\gamma_{\mu}T^aLq)\right] \tag{5.39}$$

Inserting the hadronic states between the currents we get

$$\begin{aligned}\mathcal{M}_{K^+K^-} = -i4\frac{G_F}{\sqrt{2}}V_{cs}V_{us}^{*}\Bigg[&\left(C_2 + \frac{1}{N_c}C_1\right)\langle K^-|\,\bar{s}\gamma^{\mu}Lc\,|D^0\rangle\langle K^+|\,\bar{u}\gamma_{\mu}Ls\,|0\rangle \\ &+2\,C_1\langle K^+K^-|(\bar{s}\gamma^{\mu}T^aLc)(\bar{u}\gamma_{\mu}T^aLs)|D^0\rangle\Bigg]\end{aligned} \tag{5.40}$$

and

$$\begin{aligned}\mathcal{M}_{\pi^+\pi^-} = -i4\frac{G_F}{\sqrt{2}}V_{cd}V_{ud}^{*}\Bigg[&\left(C_2 + \frac{1}{N_c}C_1\right)\langle \pi^-|\,\bar{d}\gamma^{\mu}Lc\,|D^0\rangle\langle \pi^+|\,\bar{u}\gamma_{\mu}Ld\,|0\rangle \\ &+2\,C_1\langle \pi^+\pi^-|(\bar{d}\gamma^{\mu}T^aLc)(\bar{u}\gamma_{\mu}T^aLd)|D^0\rangle\Bigg]\end{aligned} \tag{5.41}$$

where the term proportional to $2\,C_1$ with color matrices inside the matrix elements are the genuinely non-factorizable contribution to the process. In the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ processes the decay products of the W^+ boson hadronize without mixing, corresponding to the external spectator or "color allow" diagram, while color suppressed processes corresponding to final states with π^0 and η mesons were not considered.

The contributions to the purely non-factorizable term can be calculated with either the Lattice gauge or quark models such as the heavy-light chiral quark model (HLχQM) and the large energy light chiral quark model (LELχQM) which has been widely used for the study of B meson decays [**38, 39, 40**]. For the purpose of this work such contributions are not going to be calculated.

Then 5.40 and 5.41 simplifies to

$$\mathcal{M}_{K^+K^-} = -i4\frac{G_F}{\sqrt{2}}V_{cs}V_{us}^*\left(C_2+\frac{1}{N_c}C_1\right)\langle K^-|\,\bar{s}\gamma^\mu Lc\,|D^0\rangle\langle K^+|\,\bar{u}\gamma_\mu Ls\,|0\rangle \tag{5.42}$$

$$= -\frac{G_F}{\sqrt{2}}\cos\theta_c\sin\theta_c\,C_A f_K f_0^{D\to K}(M_K^2)(M_D^2-M_K^2) \tag{5.43}$$

and

$$\mathcal{M}_{\pi^+\pi^-} = -i4\frac{G_F}{\sqrt{2}}V_{cd}V_{ud}^*\left(C_2+\frac{1}{N_c}C_1\right)\langle \pi^-|\,\bar{d}\gamma^\mu Lc\,|D^0\rangle\langle \pi^+|\,\bar{u}\gamma_\mu Ld\,|0\rangle \tag{5.44}$$

$$= +\frac{G_F}{\sqrt{2}}\cos\theta_c\sin\theta_c\,C_A f_\pi f_0^{D\to \pi}(M_\pi^2)(M_D^2-M_\pi^2) \tag{5.45}$$

where $C_A = \left(C_2 + \frac{1}{N_c}C_1\right) \approx 1$.

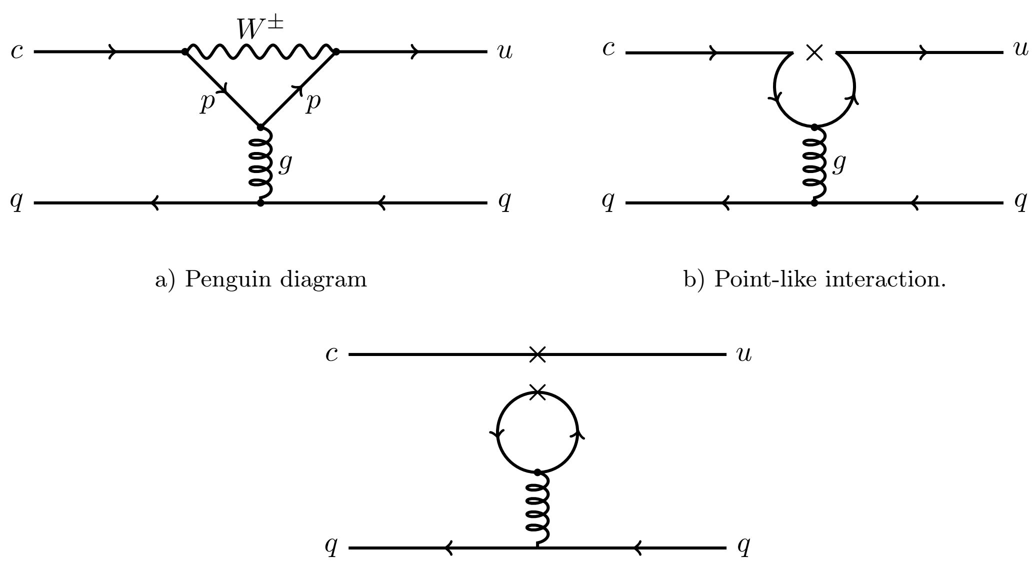
3.1. CP violation. The tree level decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ only involve the first two quark generations, it means, that it does not have the CP violating Kobayashi - Maskawa (KM) phase. The CP violation does appears in the SM in the CKM matrix when we consider 3 quark generations. It does enter by the so-called "penguin" diagrams for SCS decays of neutral D mesons that thus can provide both the required weak and strong phase difference relative to the leading SM tree amplitude.

The difference between the time-integrated CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ measured by LHCb is given by [**5**]

$$\Delta A_{CP} = A^d_{K^+K^-} - A^d_{\pi^+\pi^-} + \frac{\Delta\langle t\rangle}{\tau}\left(A^m + A^i\right) \tag{5.46}$$

In the limit where $\Delta\langle t\rangle$ vanishes, ΔA_{CP} is equal to the difference in the direct CP asymmetry between the two decays. However, if the time acceptance is different for K^+K^- and $\pi^+\pi^-$ final states, then a contribution from indirect CP violation remains. Given the dependence of ΔA_{CP} on the direct and indirect CP asymmetries, and the measured value for $\frac{\Delta\langle t\rangle}{\tau}$ showed in Table 3.1, the contribution from indirect CP violation is suppressed and ΔA_{CP} is primary sensitive to direct CP violation [**5**]. Even though, the contribution of penguin diagram corrections to CP violation is small for $c \to su\bar{s}$ and $c \to du\bar{d}$ decays, but not entirely negligible.

3.2. Penguin contributions. To calculate the contributions from penguins to CP violation, first we must evalute the penguin diagram showed in Figure 5.2 a).



a) Penguin diagram

b) Point-like interaction.

c) After Fierz transformation.

Figure 5.2. Penguin diagram evolution. a) Original penguin diagram, when W is integrated out, it turns to a point-like interaction b), and finally after Fierz transformation c).

So, evaluating the diagram and rearreanging terms

$$\mathcal{M}^{pen} = i4\frac{G_F}{\sqrt{2}}g_s^2 V_{cp}V_{up}^* \int \frac{d^4\tilde{p}}{(2\pi)^4}\frac{1}{k^2+i\epsilon}\cdot\frac{M_W^2}{p_W^2-M_W^2+i\epsilon}\left[\bar{u}\gamma^\mu L\frac{\gamma\cdot p_{p_2}+m_p}{p_{p_2}^2-m_p^2+i\epsilon}\right. \tag{5.47}$$

$$\left.\gamma^\kappa T^a\,\frac{\gamma\cdot p_{p_1}+m_p}{p_{p_1}^2-m_p^2+i\epsilon}\,\gamma_\mu Lc\right](\bar{q}\gamma_\kappa T^a q) \tag{5.48}$$

with $p_W = \tilde{p}-k-p_u$ and $p_{p_{1,2}} = \tilde{p}\pm k$. Using C.7, and the fact that the last term is just vector so $\gamma_\kappa = \gamma_\kappa(R+L)$

$$\mathcal{M}^{pen} = i4\frac{G_F}{\sqrt{2}}g_s^2 V_{cp}V_{up}^* \int \frac{d^4\tilde{p}}{(2\pi)^4}\frac{1}{k^2+i\epsilon}\cdot\frac{M_W^2}{p_W^2-M_W^2+i\epsilon}\left[\bar{u}\gamma^\mu L\frac{\gamma\cdot p_{p_2}+m_p}{p_{p_2}^2-m_p^2+i\epsilon}\right. \tag{5.49}$$

$$\left.\gamma^\kappa\,\frac{\gamma\cdot p_{p_1}+m_p}{p_{p_1}^2-m_p^2+i\epsilon}\,\gamma_\mu Lc\right](\bar{q}\gamma_\kappa q)\left(\delta_{il}\delta_{kj}-\frac{1}{N_c}\delta_{ij}\delta kl\right). \tag{5.50}$$

the product of hadronic currents is proportional to

$$[\bar{u}_i\gamma^\mu Lc_j][\bar{q}_k\gamma_\mu(R+L)q_l]\left(\delta_{il}\delta_{kj}-\frac{1}{N_c}\delta_{ij}\delta kl\right) \tag{5.51}$$

$$[\bar{u}_i\gamma^\mu Lc_k][\bar{q}_k\gamma_\mu(R+L)q_i]-\frac{1}{N_c}[\bar{u}_i\gamma^\mu Lc_i][\bar{q}_k\gamma_\mu(R+L)q_k] \tag{5.52}$$

applying the Fierz transformation in the first term, we get

$$[\bar{u}_i\gamma^\mu Lq_i][\bar{q}_k\gamma_\mu(R+L)c_k]-\frac{1}{N_c}[\bar{u}_i\gamma^\mu Lc_i][\bar{q}_k\gamma_\mu(R+L)q_k] \tag{5.53}$$

from here we can get the four local operators, which were previously defined

$$Q_3 = [\bar{u}\gamma^\mu Lc][\bar{q}\gamma_\mu Lq] \qquad Q_4 = [\bar{u}\gamma^\mu Lq][\bar{q}\gamma_\mu Lc] \tag{5.54}$$

$$Q_5 = [\bar{u}\gamma^\mu Lc][\bar{q}\gamma_\mu Rq] \qquad Q_6 = [\bar{u}\gamma^\mu Lq][\bar{q}\gamma_\mu Rc] \tag{5.55}$$

4. Renormalization Group Equation (RGE)

The perturbative correction to the decay amplitude goes $\alpha_s Log\left[\frac{M_W^2}{\mu}\right]$, which is of order 1. Thus, we should include higher-order corrections, corresponding to more gluons exchanged. The renormalization group method can be adapted for this, letting us sum over leading logarithmic terms to all orders of α_s.

For the one-loop QCD corrections we have

$$Q_2 \Longrightarrow Q_2 + \underbrace{\frac{\alpha_s}{4\pi} Log\left[\frac{M_W^2}{\mu^2}\right]\left(Q_1 - \frac{1}{N_c}Q_2\right)}_{QCD\ \ corrections}, \tag{5.56}$$

We then improve our results with an adoption of RGE and scaling following the same procedure as [**41**]. The new coefficients for RGE are defined as

$$c_{\pm}(\mu) = \left[\frac{\alpha_s(M)}{\alpha_s(\mu)}\right]^{\frac{2d_{\pm}}{b_0}} c_{\pm}(M) \tag{5.57}$$

with $d_{\pm}$ is a constant obtained from $\gamma_{\pm} = d_{\pm}\frac{\alpha_s(\mu)}{\pi}$, and $b_0 = 11N_C - \frac{2}{3}N_f$ comes from the first term of the QCD beta function, where N_C is the number of quark colors, and N_f the number of quark flavors at energy μ.

Changing to the $Q_{\pm}$ basis, with $Q_{\pm} = \frac{1}{2}\left(Q_2 \pm Q_1\right)$

$$\begin{pmatrix} Q_+ \\ Q_- \end{pmatrix} \Longrightarrow \left[1 + \frac{\alpha_s}{2\pi} Log\frac{M_W^2}{\mu^2}\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}\right]\begin{pmatrix} Q_+ \\ Q_- \end{pmatrix}, \tag{5.58}$$

Inserting the new $c_{\pm}$ values into the equation and calculating the first order in α_s/π, we get

$$\gamma_+ = \mu\frac{d}{d\mu} Log\left[1 + \left(\frac{\alpha_s}{2\pi} Log\frac{M_W^2}{\mu}\right)\right] \approx -\frac{\alpha_s}{2\pi}, \tag{5.59}$$

$$\gamma_- = \mu\frac{d}{d\mu} Log\left[1 - 2\left(\frac{\alpha_s}{2\pi} Log\frac{M_W^2}{\mu}\right)\right] \approx +\frac{\alpha_s}{\pi} \tag{5.60}$$

giving $d_+ = -\frac{1}{2}$, $d_- = 1$. With the charm quark integrated out $N_f = 3$, then $b_0 = \frac{27}{3}$. For the M_W^2 integrated out $d_\pm \to 2d_\pm$, $N_f = 4$, and $b_0 = \frac{25}{3}$. With this, the coefficients $c_\pm$ are given by

$$c_+ = \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{-\frac{3}{27}} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_c)}\right)^{\frac{-6}{25}} \tag{5.61}$$

$$c_- = \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{\frac{6}{27}} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_c)}\right)^{\frac{12}{25}} \tag{5.62}$$

and then we can write the coefficients for the operator Q_2 as,

$$c_2(\mu) = \frac{1}{2}\left[c_+(\mu) + c_-(\mu)\right] \tag{5.63}$$

$$= \frac{1}{2}\left[\left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{-\frac{3}{27}} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_c)}\right)^{-\frac{6}{25}} + \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{\frac{6}{27}} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_c)}\right)^{\frac{12}{25}}\right] \tag{5.64}$$

for the scaling in the region $m_c > \mu$.

5. Partial conclusions

As it was said before, tree level diagrams for charm quark decays do not contain CP violation as, in a good approximation, it can be explain with 2 quark generations. So if CP wants to be explained, we need to introduce high order contributions, such as penguin diagrams, as CP appears in form of the Kobayashi - Maskawa phase when we consider 3 quark generations.

However, it seems that even considering penguin contribution it is not enough to explain the results obtained by the LHCb experiment [**5, 9, 10**]. The contribution due to penguin diagrams is small, and loop suppressed, so this leads to the necessity of new models, including those ones beyond the Standard Model.

"New physics"

This chapter is based in the article *"New physics model of direct* CP *violation in charm decays"* by Almannshofer et al. [**11**]. The wok of Altmannshofer et al. presents a comparative study of the impact that NP degrees of freedom would have on the direct CP asymmetries in singly Cabbibo suppressed D meson decays. All this motivated by the recent LHCb measurements of ΔA_{CP}, the difference between the time-integrated CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$.

Altmannshofer et al., consider models with new massive neutral gauge bosons that have flavor changing tree level couplings to quarks, models with extended scalar sectors, and models where the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays are modified at the loop level by gluon penguins.

1. New Physics contributions at tree level.

In this section, we analyze a model proposed by Almannshofer et al. where one new field, a color flavor changing scalar, is added to the SM. We will discuss extensions of the SM in which a scalar octet ϕ_8^a leads to tree level contributions to the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decay amplitudes.

The Lagrangian for this process can be written as (for $q = s, d$)

$$\mathscr{L} = G(c \to u)\,(\bar{u}_L T^a \phi^a c_R + X_q \bar{q}_L T^a q_R \phi^a)\,. \tag{6.1}$$

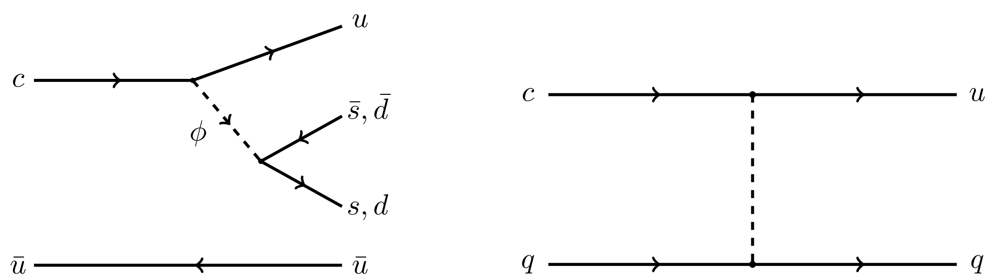


Figure 6.1. Color suppressed transition for ϕ_8^a octet model.

If we evaluate the correspoding Feynman diagram, the effective Lagrangian can be written as

$$i\mathscr{L}_{eff}^{\phi} = iG(c \to u)(\bar{u}_L T^a c_R)\left[\frac{i}{(-M_\phi^2)}\right](iX_q \bar{q}_L T^a q_R) \tag{6.2}$$

$$\mathscr{L}_{eff}^{\phi} = \frac{G(c \to u)X_q}{M_\phi^2}(\bar{u}_L T^a c_R)(\bar{q}_L T^a q_R) \tag{6.3}$$

$$= \frac{G(c \to u)X_q}{M_\phi^2}(\bar{u} T^a Rc)(\bar{q} T^a Rq) \tag{6.4}$$

using the identity C.7 and defining $G_{\phi q} \equiv \frac{G(c\to u)X_q}{M_\phi^2}$, the effective Lagrangian can be rewritten as

$$\mathscr{L}_{eff}^{\phi} = \frac{G_{\phi q}}{2}\left[(\bar{q}_i R q_j)(\bar{u}_k R c_l)\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)\right] \tag{6.5}$$

$$= \frac{G_{\phi q}}{2}\left[(\bar{q}_i R q_k)(\bar{u}_k R c_i) - \frac{1}{N_c}(\bar{q}_i R q_i)(\bar{u}_k R c_k)\right] \tag{6.6}$$

then Fierz tranforming both terms in the previous effective Lagrangian an additional factor of $\frac{1}{2}$ is gotten [**42**].

$$\mathscr{L}_{eff}^{\phi} = \frac{G_{\phi q}}{4}\left[(\bar{q}_i R c_i)(\bar{u}_k R q_k) - \frac{1}{N_c}(\bar{q}_i R c_k)(\bar{u}_k R q_i)\right] \tag{6.7}$$

Inserting the hadronic states in the effective Lagrangian, the second term in the right-hand side of the Lagrangian gives a factor $\propto \frac{\delta_{ik}\delta_{ki}}{N_c^2}$ where $\delta_{ik}\delta_{ki} = N_c$ because the color of the quarks in the currents must match, and using the naive factorization, we can write the hadronic matrix elements as

$$\langle P^+P^-| \mathscr{L}^{\phi}_{eff} |D^0\rangle = \frac{G_{\phi q}}{4}\left[\langle P^+| \bar{u}Rq |0\rangle\langle P^-| \bar{q}Rc |D^0\rangle \left(1-\frac{1}{N_c^2}\right)\right]. \tag{6.8}$$

1.1. Case 1: $D^0 \to K^+K^-$. For the D^0 meson decay with K^+K^- in the final state, we can rewrite Equation 6.8 with $P = K$ and $q = s$.

$$\langle K^+K^-| \mathscr{L}^{\phi}_{eff} |D^0\rangle = \frac{G_{\phi q}}{4}\left[\langle K^+| \bar{u}Rs |0\rangle\langle K^-| \bar{s}Rc |D^0\rangle \left(1-\frac{1}{N_c^2}\right)\right] \tag{6.9}$$

The hadronic matrix elements can be obtain using primitive techniques as presented in the following

For the first hadronic matrix element, K^+ is a pseudoscalar particle, then the hadronic matrix element simplifies to

$$\langle K^+| \bar{u}Rs |0\rangle = \frac{1}{2}\langle K^+| \bar{u}\gamma_5 s |0\rangle \tag{6.10}$$

using the Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi$, $\{\gamma^5, \gamma^\mu\} = 0$ and $\{\partial_\mu, \gamma^\mu\} = 0$

$$i\partial_\mu (\bar{u}\gamma^\mu\gamma_5 s) = (i\partial_\mu \bar{u})\gamma^\mu\gamma_5 s + \bar{u}\gamma^\mu\gamma_5(i\partial_\mu s) \tag{6.11}$$

$$= (m_s + m_u)\, \bar{u}\gamma_5 s \tag{6.12}$$

then

$$\bar{u}\gamma_5 s \sim \frac{i\partial_\mu (\bar{u}\gamma^\mu\gamma_5 s)}{(m_s + m_u)} \tag{6.13}$$

so inserting the hadronic states into the currents, and using the decay constant found in Equation 4.14

$$\langle K^{+}|\,\bar{u}Rs\,|0\rangle = \frac{1}{2}\langle K^{+}|\,\bar{u}\gamma_5 s\,|0\rangle \tag{6.14}$$

$$= \frac{1}{2}\left(\frac{m_K^2 f_K}{m_s+m_u}\right) = \frac{1}{2}\,X_K. \tag{6.15}$$

For the second hadronic matrix element the evaluation is a little bit more elaborated. As K^- and D^0 are pseudoscalar particles, the hadronic matrix element simplifies to

$$\langle K^{-}|\,\bar{s}Rc\,|D^0\rangle = \frac{1}{2}\langle K^{-}|\,\bar{s}c\,|D^0\rangle \tag{6.16}$$

and using the Dirac equation and $\{\partial_\mu, \gamma^\mu\} = 0$

$$i\partial_\mu\,(\bar{s}\gamma^\mu c) = (i\partial_\mu\,\bar{s})\gamma^\mu c + \bar{s}\gamma^\mu(i\partial_\mu\,c) \tag{6.17}$$

$$= (m_c - m_s)\,\bar{s}c \tag{6.18}$$

then

$$\bar{s}c \sim \frac{1}{m_c - m_s} i\partial(\bar{s}\gamma^\mu c) \tag{6.19}$$

inserting the hadronic states into the left-hand side term of Equation 6.17

$$i\partial_\mu\langle K^{-}|\,\bar{s}\gamma^\mu c\,|D^0\rangle = (P_c - P_s)_\mu\langle K^{-}|\,\bar{s}\gamma^\mu c\,|D^0\rangle \tag{6.20}$$

and now evaluating $\langle K^{-}|\,\bar{s}\gamma^\mu c\,|D^0\rangle$

$$\langle K^{-}|\,\bar{s}\gamma^\mu c\,|D^0\rangle = \left[f_+(q^2)(P_c+P_s)^\mu + f_-(q^2)(P_c-P_s)^\mu\right] \tag{6.21}$$

$$= 2E(\zeta n^\mu + \zeta_1 v^\mu) \tag{6.22}$$

where in the last equation the form factors in the *Large Energy Effective Theroy* (LEET) limit were parametrized as [**43, 44**]. $P_c^\mu = M_D v^\mu$ and $P_s^\mu = En^\mu$ where $m_s \ll E$, with E and m_s as the energy and mass of the s quark. The four vectors v, n are given by $v = (1; \vec{0})$ and $n = (1; 0, 0, 1)$ in the rest frame of the decaying heavy meson. A peculiar feature of exclusive heavy-to-light transitions is the large energy E given to the daughter by the parent hadron in almost the whole physical phase space except the vicinity of the zero- recoil point [**43**], so $2E \simeq M_D$ and $\zeta_D \sim 2/3$ [**44**], so

$$i\partial_\mu \langle K^- | \, \bar{s}\gamma^\mu c \, | D^0 \rangle = (P_c - P_s)_\mu \langle K^- | \, \bar{s}\gamma^\mu c \, | D^0 \rangle \tag{6.23}$$

$$= 2E(M_D v - En)_\mu (\zeta n^\mu + \zeta_1 v^\mu) \tag{6.24}$$

$$= 2E(\zeta M_D - E\zeta_1) \simeq 2EM_D\zeta \tag{6.25}$$

where $v^2 = 1$, $v \cdot n = 1$ and $n^2 \simeq 0$ were used, then

$$i\partial_\mu \langle K^- | \, \bar{s}\gamma^\mu c \, | D^0 \rangle = M_D^2 \zeta_D \tag{6.26}$$

with $\zeta = \zeta_D$.

Inserting the hadronic states in the Equation 6.19 and using Equation 6.26 we get

$$\langle K^- | \, \bar{s}c \, | D^0 \rangle \simeq \frac{M_D^2}{m_c} \zeta_D \tag{6.27}$$

for $m_c \gg m_s$. Then the right-hand side of Equation 6.16 simplifies to

$$\langle K^- | \, \bar{s}Rc \, | D^0 \rangle = \frac{1}{2} \langle K^- | \, \bar{s}c \, | D^0 \rangle \simeq \frac{1}{2} \frac{M_D^2}{m_c} \zeta_D \tag{6.28}$$

⌟

Finally, the Equation 6.9 simplifies to

$$\langle K^+ K^- | \mathscr{L}^{\phi}_{eff} | D^0 \rangle \simeq \frac{G_{\phi q}}{4} \frac{1}{2} X_K \frac{1}{2} \frac{M_D^2}{m_c} \zeta_D \frac{8}{9} \tag{6.29}$$

$$= \frac{G_{\phi q}}{18} X_K \frac{M_D^2}{m_c} \zeta_D \tag{6.30}$$

where the factor $\frac{8}{9}$ came from the term $(1 - \frac{1}{N_c^2})$ with $N_c = 3$. $G_{q\phi} = \frac{G(c\to u)X_s}{M_\phi^2}$, $X_s = \zeta_d \frac{m_s}{v}$ and $G(c \to u) = \zeta_u \frac{m_c}{v} \lambda e^{i\phi_K}$ with $e^{i\phi_K}$ as the weak phase.

$$\langle K^+K^-|\mathscr{L}^{\phi}_{eff}|D^0\rangle = \frac{1}{18}\frac{1}{M_\phi^2}\zeta_u \frac{m_c}{v}\lambda e^{i\phi_K}\zeta_d \frac{m_s}{v}\frac{m_K^2}{m_s} f_K \frac{M_D^2}{m_c}\zeta_D \tag{6.31}$$

$$= \frac{1}{18}\frac{\zeta_u \zeta_d}{M_\phi^2}\lambda \frac{m_K^2 f_K M_D^2 \zeta_D}{v^2} e^{i\phi_K} \tag{6.32}$$

with m_u and CP violation in penguin diagrams neglected. According to Equation 3.5 in order to account for CP violation, a strong phase $e^{i\delta_K}$ must be explicitly contained in the amplitude, which could be obtain via other contributions such as "*Meson loops*".

Meson loops. Considering the work presented by Eeg, Fajfer and Prapotnik [**38**] were "*Chiral perturbative theories*" (XPT) were used to study the non-leptonic decays of $B \to D\bar{D}$, we will try to make an analogy to get information for the non-leptonic decays of $D^0 \to K^+K^-$.

In principle, we can make a rough approximation between the two decay modes,

$$\begin{aligned} B(M_B) &\to D(M_D) \\ M_D &\to M_K \end{aligned}$$

and we could even consider Feynman diagrams beyond factorization such as,

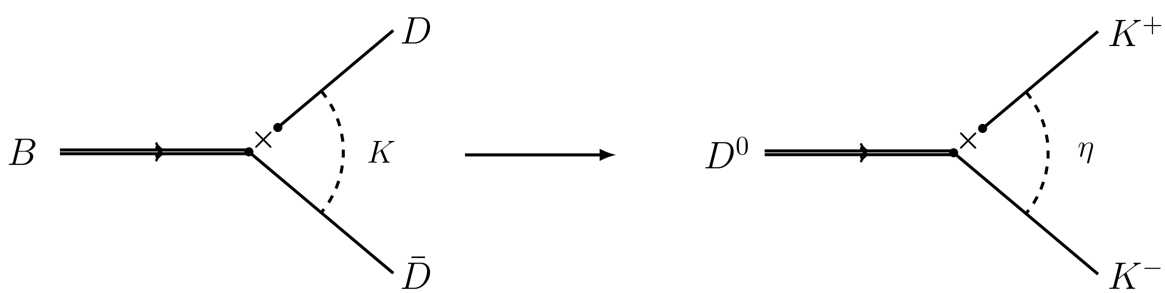


Figure 6.2. Meson loop diagrams. The figure on left shows the diagram for $B \to D\bar{D}$ decay modes, in the right for $D^0 \to K^+K^-$.

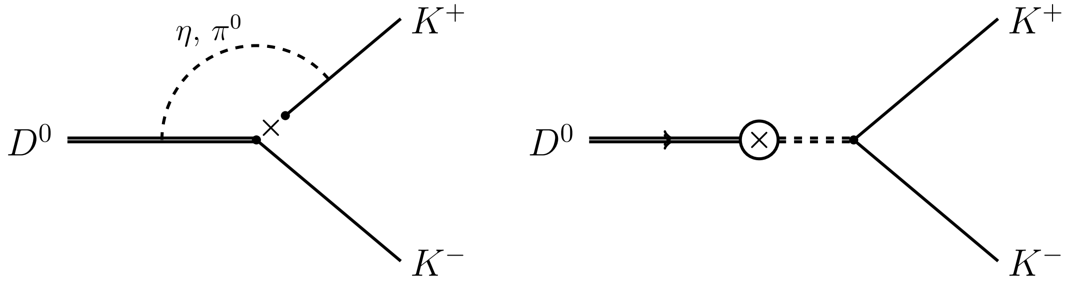


Figure 6.3. Meson loop diagrams. The figure on left shows the diagram for $D^0 \to K^+K^-$ decay modes, in the right for resonance of $D^0 \to K^+K^-$. The Imaginary part of the propagator of unstable resonance $\propto iM_R\Gamma_R$, from where the strong phase could be obtained.

Those diagrams give complex amplitudes, such that the Imaginary parts give phases that can be related to the strong phase we need. Equation 3.7 gives the direct asymmetry when the subleading penguin amplitude is small $A_f^d = 2r_f \sin(\delta_f) \sin\phi_f$, so either the weak and strong phases are needed, and must be different from zero in order to get CP violation.

Considering the amplitude given in Equation 6.32 + the complex amplitudes from meson loops and SM, we can include both the weak and strong phase, so we can write the total amplitude, in general, as

$$A^{Tot} = A^{SM} + A^{NP} + A^{Meson} \tag{6.33}$$

which can be related to Equation 3.5.

$$A_f^{Tot} \quad = \quad A_f^T \left(1 + r_f e^{i(\delta_f + \phi_f)}\right) \tag{6.34}$$

Standard Model matrix elements. Remembering the last chapter, the SM counterpart to the hadronic matrix element is written as

$$\langle K^+K^-|\mathscr{L}_{eff}^{SM}|D^0\rangle = -i4\frac{G_F}{\sqrt{2}}\underbrace{V_{cs}V_{us}^*}_{\sim\lambda}\langle K^+K^-|(\bar{s}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ls)|D^0\rangle \tag{6.35}$$

$$\langle K^+K^-|(\bar{s}\gamma^\mu Lc)(\bar{u}\gamma_\mu Ls)|D^0\rangle = -\frac{1}{4}\langle K^-|(\bar{s}\gamma^\mu c)|D^0\rangle\langle K^+|(\bar{u}\gamma^\mu\gamma_5 s)|0\rangle \tag{6.36}$$

with $\langle K^-|\,\bar{s}\gamma^\mu c\,|D^0\rangle = 2E(\zeta_D n^\mu + \zeta_1 v^\mu)$ and $\langle K^+|\,\bar{u}\gamma^\mu\gamma_5 s\,|0\rangle = if_K P_K^\mu$ where $P_K^\mu = E\tilde{n}^\mu$, then

$$-\frac{1}{4}\langle K^-|\,\bar{s}\gamma^\mu c\,|D^0\rangle\langle K^+|\,\bar{u}\gamma_\mu\gamma_5 s\,|0\rangle = -i\frac{1}{4}[2E\zeta_D \underbrace{n\cdot\tilde{n}}_{\sim 2} Ef_K] = -i\frac{1}{4}f_K M_D^2\zeta_D \tag{6.37}$$

where $\tilde{n}$ was defined as $\tilde{n} = (1;0,0,-1)$ **[44]**, then

$$\langle K^+K^-|\mathscr{L}_{eff}^{SM}|D^0\rangle = -\frac{G_F}{\sqrt{2}}\lambda f_K M_D^2\zeta_D \tag{6.38}$$

Subleading penguin amplitude r_{KK}. To calculate $r_{KK}e^{i\Delta}$ we just take the ratio of amplitudes of the octed ϕ_8^a model and Standard Model.

$$r_{KK}e^{i\Delta} = \frac{\langle K^+K^-|\mathscr{L}_{eff}^{\phi}|D^0\rangle}{\langle K^+K^-|\mathscr{L}_{eff}^{SM}|D^0\rangle} \simeq \frac{1}{18}\frac{\zeta_u\zeta_d}{M_\phi^2}\frac{m_K^2\sqrt{2}}{v^2 G_F}\cdot factor \tag{6.39}$$

where CP violation in penguin diagrams was neglected. $\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$, $M_W = \frac{1}{2}gv$ so $M_W^2 = \frac{1}{4}g^2v^2$, $g_w^2 = \frac{1}{2}g^2$ then $M_W^2 = \frac{1}{2}g_w^2v^2$, so we can rewrite $\frac{G_F}{\sqrt{2}} = \frac{1}{4v^2}$.

$$r_{KK}e^{i\Delta} = \frac{2}{9}\frac{\zeta_u\zeta_d}{M_\phi^2}m_K^2. \tag{6.40}$$

Conclusions

Recently, the LHCb collaboration presented the first evidence for CP violation in charm quark decays [**5**]. In specific, the difference between the time-integrated CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$

$$\begin{aligned} \Delta A_{CP} &= A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \\ &= [-0.34 \pm 0.15(stat.) \pm 0.10(sys.)]\% \end{aligned} \tag{7.1}$$

was reported. This measurement is consistent at about the 1σ level with previos measurements from CDF [**6**], Babar [**7**] and Belle [**8**]. The interpretation of this measurements as a sign of NP require a well-understood SM calculation of this observable.

As only the first two quark generations are necessary for the main contributions of the process in the SM, it is therefore CP conserving. In other words, the Standard Model CP violation in these decays is CKM suppressed.

The CP violation contribution to the $c \to qu\bar{q}$ decays are both CKM and loop-suppressed and, therefore, entirely negligible. Sizable direct CP asymmetries in the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays are only possible in the SM if the relevant hadronic matrix elements are strongly enhanced.

Precise SM calculations, however, are difficult to accomplish. Although tree level and loop level SM contributions to the quark level processes $c \to qu\bar{q}$ are readily calculated, the evaluation of the hadronic matrix elements is not easily performed.

In the simplest approach, naive factorization, the hadronic matrix elements are "factorized" into the product of matrix elements, which is formally the leading term in the heavy charm quark limit. As the charm mass is close to Λ_χ, chiral symmetry scale, where the perturbative QCD breaks down, however, this approach suffers from large $1/m_c$ power corrections. In particular, so-called anihilation diagrams are ignored, where quarks are pair-produced from the vacuum to complete the K or π mesons in the final state, as are long-range QCD effects such as final state rescattering, where constituent s-quarks of a $D^0 \to K^+K^-$ decay rescatter into d-quarks if a $\pi^+\pi^-$ final state. Several recent papers have discussed improved estimates for ΔA_{CP} in the SM.

The charm sector has been considered an excellent way to prove new physics beyond the Standard Model, even since before the result of LHCb and interference between the two processes is a key ingredient in CP violation that can be enhanced by the participation of new heavy particles in the penguin loop. Even when there is a large uncertainty in the SM value for ΔA_{CP}, it is nevertheless important and exciting to consider the possibility that in effect it is evindence of NP.

Appendices

Appendix A

Feynman rules for one-loop corrections

Element	Descripción
$i \int \frac{d^4p}{(2\pi)^4}$	Integral over internal momentum p
$\bar{q}$	Outgoing q-quark
$-ig_s\gamma^\nu T^a$	qqg-vertex
$\frac{i(\gamma^\sigma p_\sigma + m_q)}{p^2 - m_q^2 + i\epsilon}$	q-quark propagator
$\frac{-ig_w}{\sqrt{2}}\gamma^\mu L V_{cq}$	qcW-vertex
c	Incoming c-quark
$\frac{ig_{\nu\lambda}}{p^2 + i\epsilon}$	Gluon propagator in Feynman gauge
$\frac{ig_{\mu\kappa}}{p^2 - M_W^2 + i\epsilon}$	W propagator in Feynman gauge
q	Incoming q-antiquark

Table A.1. Feynman rules for one-loop corrections.

Element	Descripción
$h_v^{(c)}$	Incoming reduced heavy quark with velocity v
$\bar{h}_v^{(c)}$	Ougoing reduced heavy quark with velocity v
$\frac{i}{v\cdot p}$	Reduced heavy quark with velocity v propagator
$-ig_sT^av^\mu$	Gluon - reduced heavy quark vertex

Table A.2. Feynman rules for one-loop corrections with heavy quark.

Loop integrals and Dimensional Regularization

B.1. Feynman Parametrization

The Feynman parametrization is a way to write fractions with a product in the denominator and was invented by Richard Feynman to calculate loop integrals. Starting from the generalized Feynman parametrization for an arbitrary number of factors

$$\frac{1}{A_1 A_2 ... A_n} = \int_0^1 dx_1 ... dx_n \, \delta\left(\sum x_i - 1\right) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + ... + x_n A_n]^n} \tag{B.1}$$

and it is easy to prove B.1 directly by induction.

For our purpose we are gonna prove B.1 just for 3 factors

$$\int_0^1 dx_1 \, dx_2 \, dx_3 \, \delta(x_1 + x_2 + x_3 - 1) \frac{(3-1)!}{[x_1 A_1 + x_2 A_2 + x_3 A_3]^3} \tag{B.2}$$

$$2 \int_0^1 dx_2 \int_0^{1-x_2} \frac{dx_3}{[(1 - x_2 - x_3) A_1 + x_2 A_2 + x_3 A_3]^3} \tag{B.3}$$

rearranging in terms of x_2 and x_3

$$2\int_0^1 dx_2 \int_0^{1-x_2} \frac{dx_3}{[A_1 + x_2(A_2 - A_1) + x_3(A_3 - A_1)]^3} \tag{B.4}$$

To easily calculate the integral we can do a change of variable. Defining the new variables as

$$u = A_1 + x_2(A_2 - A_1) + x_3(A_3 - A_1) \tag{B.5}$$

$$du = (A_3 - A_1)\, dx_3 \tag{B.6}$$

$$dx_3 = \frac{du}{(A_3 - A_1)} \tag{B.7}$$

so now the integral has the form

$$\frac{2}{(A_3 - A_1)} \int_0^1 dx_2 \int_a^b \frac{du}{u^3} \tag{B.8}$$

with the new limits of integration given by $a = A_1 + x_2(A_2 - A_1)$ and $b = x_2(A_2 - A_3) + A_3$.

$$\frac{1}{(A_3 - A_1)} \int_0^1 dx_2 \left[\frac{1}{u^2}\right]_b^a = \frac{1}{(A_3 - A_1)} \int_0^1 dx_2 \left[\frac{1}{[A_1 + x_2(A_2 - A_1)]^2} - \frac{1}{[x_2(A_2 - A_3) + A_3]^2}\right]$$

now, we can split the integral into the sum of integrals (or in our case the difference)

$$\frac{1}{(A_3 - A_1)} \int_0^1 \frac{dx_2}{[A_1 + x_2(A_2 - A_1)]^2} - \frac{1}{(A_3 - A_1)} \int_0^1 \frac{dx_2}{[x_2(A_2 - A_3) + A_3]^2} \tag{B.9}$$

The first integral can be calculated, as before, doing a change of variable

$$u = A_1 + x_2(A_2 - A_1) \tag{B.10}$$

$$du = (A_2 - A_1)\, dx_2 \tag{B.11}$$

$$dx_2 = \frac{du}{(A_2 - A_1)} \tag{B.12}$$

so now the integral has the form

$$\frac{1}{(A_3 - A_1)(A_2 - A_1)} \int_{A_1}^{A_2} \frac{du}{u^2} \tag{B.13}$$

$$\frac{1}{(A_3 - A_1)(A_2 - A_1)} \left[\frac{1}{u}\right]_{A_2}^{A_1} \tag{B.14}$$

$$\frac{1}{(A_3 - A_1)(A_2 - A_1)} \left[\frac{1}{A_1} - \frac{1}{A_2}\right] \tag{B.15}$$

$$\frac{1}{(A_3 - A_1)} \left[\frac{1}{A_1 A_2}\right] \tag{B.16}$$

It is evident that the second integral is calculated as before, giving the result

$$\frac{1}{(A_3 - A_1)} \left[\frac{1}{A_3 A_2}\right] \tag{B.17}$$

the last thing to do is subtract B.17 to B.16, resulting

$$\frac{1}{(A_3 - A_1)} \left[\frac{1}{A_1 A_2} - \frac{1}{A_3 A_2}\right] = \frac{1}{(A_3 - A_1)} \left[\frac{(A_3 - A_1)A_2}{(A_1 A_2 A_3)A_2}\right] \tag{B.18}$$

$$= \frac{1}{A_1 A_2 A_3} \tag{B.19}$$

So, the generalized Feynman parametrization for a product of 3 terms in the denominator was proven.

$$\frac{1}{A_1 A_2 A_3} = \int_0^1 dx_1\, dx_2\, dx_3\, \delta(x_1 + x_2 + x_3 - 1) \frac{(3-1)!}{[x_1 A_1 + x_2 A_2 + x_3 A_3]^3} \tag{B.20}$$

B.2. Wick rotation

After Feynamn parametrization the loop integrals are typically on the form

$$\int d^4 p \frac{(p^2)^\alpha}{(p^2 - A + i\epsilon)^\beta} \tag{B.21}$$

The $+i\epsilon$ term in the Equation B.21 places the poles slightly above the real line for $Re(p^0) < 0$, and slightly below the real axis for $Re(p^0) > 0$. This allow us to rotate the contour of the p^0 integration a quarter counterclockwise. With p^0 restricted to the imaginary axis the Minkowski metric becomes Euclidean: $ip^0 = p^0_E, \mathbf{p} = \mathbf{p_E}, p^2 = -p^2_E$. Then Equation B.21 can be rewritten and evaluated in the Euclidean space,

$$\int d^4p \frac{(p^2)^\alpha}{(p^2 - A + i\epsilon)^\beta} = i\int d^4p_E \frac{(-p_E^2)^\alpha}{(-p_E^2 - A)^\beta} \tag{B.22}$$

$$= i\int_0^\infty dp_E \int d\Omega^{(4)} p_E^3 \frac{(-p_E^2)^\alpha}{(-p_E^2 - A)^\beta} \tag{B.23}$$

$$= i\frac{(-1)^{\alpha-\beta}}{A^{\beta-\alpha-2}} \int d\Omega^{(4)} \int_0^\infty d\bar{p}_E \frac{\bar{p}_E^{2\alpha+3}}{(1+\bar{p}_E^2)^\beta} \tag{B.24}$$

with $p_E^2 = \bar{p}_E^2 A$. Using the Gamma function integral

$$\int_0^\infty dx \frac{x^\alpha}{(1+x^2)^\beta} = \frac{\Gamma\left[\frac{1}{2}(1+\alpha)\right]\Gamma\left[\frac{1}{2}(2\beta-\alpha-1)\right]}{2\Gamma(\beta)} \tag{B.25}$$

and the fact that $\int d\Omega^{(4)} = 2\pi^2$, the Equation B.21 simplifies to

$$\int d^4p \frac{(p^2)^\alpha}{(p^2 - A + i\epsilon)^\beta} = \frac{i\pi^2(-1)^{\alpha-\beta}}{A^{\beta-\alpha-2}} \frac{\Gamma(\alpha+2)\Gamma(\beta-\alpha-2)}{\Gamma(\beta)} \tag{B.26}$$

B.3. Perturbative QCD integrals

The loop integral with two light quarks exchanging a gluon and a W boson,

$$I_0 = \int \frac{d^4p}{(2\pi)^4} \frac{M_W^2}{(p^2 + i\epsilon)(p^2 - M_W^2 + i\epsilon)(p^2 - \mu^2 + i\epsilon)} \tag{B.27}$$

using Feynman parametrization, I_0 takes the form

$$
\begin{aligned}
I_0 &= \int \frac{d^4p}{(2\pi)^4} \int_0^1 dx_2 \int_0^{1-x_2} \frac{2M_W^2\, dx_3}{(1-x_2-x_3)(p^2+i\epsilon)+x_2(p^2-M_W^2+i\epsilon)+x_3(p^2-\mu^2+i\epsilon)} \\
&= \int \frac{d^4p}{(2\pi)^4} \int_0^1 dx_2 \int_0^{1-x_2} \frac{2M_W^2\, dx_3}{[p^2-[x_2M_W^2+x_3\mu^2]+i\epsilon]^3} \\
&= -\frac{i}{(4\pi)^2} \int_0^1 dx_2 \int_0^{1-x_2} \frac{M_W^2\, dx_3}{x_2M_W^2+x_3\mu^2}
\end{aligned}
$$

the last line was simplified with using B.26.

To calculate I_0 we change variables as

$$
\begin{aligned}
u &= x_2M_W^2+x_3\mu^2 \\
du &= \mu^2\, dx_3 \\
dx_3 &= \frac{du}{\mu^2}
\end{aligned}
$$

then

$$
I_0 = -\frac{i}{(4\pi)^2} \int_0^1 dx_2 \int_0^{1-x_2} \frac{M_W^2\, dx_3}{x_1M_W^2+x_2\mu^2} = -\frac{i}{(4\pi)^2}\frac{M_W^2}{\mu^2} \int_0^1 dx_2 \int_a^b \frac{du}{u} \tag{B.28}
$$

with $a = x_2M_W^2$ and $b = x_2(M_W^2-\mu^2)+\mu^2$.

$$
= -\frac{i}{(4\pi)^2}\frac{M_W^2}{\mu^2} \int_0^1 dx_2 \left\{Log[x_2(M_W^2-\mu^2)+\mu^2] - Log[x_2M_W^2]\right\} \tag{B.29}
$$

$$
= -\frac{i}{(4\pi)^2}\frac{M_W^2}{\mu^2} \int_0^1 dx_2 Log\left[1-\frac{\mu^2}{M_W^2}\left(1-\frac{1}{x_2}\right)\right] \tag{B.30}
$$

The integral of the type :

$$
\int Log\left[1-\frac{a}{b}\left(1-\frac{1}{x}\right)\right] dx = xLog\left[1-\frac{a}{b}\left(1-\frac{1}{x}\right)\right] + \frac{aLog\,(a-ax+bx)}{b-a} \tag{B.31}
$$

so

$$
\begin{aligned}
(B.32) \quad &= -\frac{i}{(4\pi)^2}\frac{M_W^2}{\mu^2}\int_0^1 dx_2\, Log\left[1-\frac{\mu^2}{M_W^2}\left(1-\frac{1}{x_2}\right)\right] \\
(B.33) \quad &= -\frac{i}{(4\pi)^2}\frac{M_W^2}{\mu^2}\left[x_2\, Log\left[1-\frac{\mu^2}{M_W^2}\left(1-\frac{1}{x_2}\right)\right]+\frac{\mu^2}{M_W^2}Log\left[(M_W^2-\mu^2)\,x_2+\mu^2\right]\right]_0^1
\end{aligned}
$$

the first term vanishes due to $xln(x) \to 0$ for $x \to 0$, so finally

$$
I_0 = -\frac{i}{(4\pi)^2}Log\left(\frac{M_W^2}{\mu^2}\right) \tag{B.34}
$$

Appendix C

Detailed calculations

C.1. Product of Dirac bilinears

The contribution of loop diagrams with two light quarks exchanging a gluon and a W boson, have terms related to products of Dirac bilinears on the form

$$(\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\nu\gamma_\sigma\gamma_\mu Lq) \tag{C.1}$$

$$(\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\nu L\gamma_\sigma\gamma_\mu q) \tag{C.2}$$

C.1.1. Product of γ-matrices. C.1 and C.2 can be simplify using the γ-matrices identities for the product of this ones.

$$\gamma^\nu\gamma^\sigma\gamma^\mu = g^{\nu\sigma}\gamma^\mu + g^{\sigma\mu}\gamma^\nu - g^{\nu\mu}\gamma^\sigma - i\epsilon^{\nu\sigma\mu\beta}\gamma_\beta\gamma_5 \tag{C.3}$$

so the bilinear products simplify to

$$(\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\nu\gamma_\sigma\gamma_\mu Lq) = 16\,(\bar{q}T^a\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\mu Lq) \tag{C.4}$$

$$(\bar{q}T^a\gamma^\nu\gamma^\sigma\gamma^\mu Lh_v^{(c)})(\bar{u}\gamma_\mu L\gamma_\sigma\gamma_\nu T^a q) = 4\,(\bar{q}T^a\gamma^\mu Lh_v^{(c)})(\bar{u}T^a\gamma_\mu Lq) \tag{C.5}$$

with

$$Q_2^c = (\bar{q}T^a\gamma^\mu L h_v^{(c)})(\bar{u}T^a\gamma_\mu L q). \tag{C.6}$$

where q and $c \to h_v^{(c)}$ are the quark and reduced heavy quark field, T^a a color matrix and L the left-handed projector in Dirac space.

C.1.2. Color matrices T^a. It is very helpful to use the relation for the product of two color matrices T^a of the group SU(3) and can be written as

$$T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \tag{C.7}$$

where i, j, k, l are color indices = 1,2,3.

Introducing the term given by C.7 in the right side term of C.6,

$$\frac{1}{2}\,(\bar{q}_i\gamma^\mu L h_v^{(c_j)})(\bar{u}_k\gamma_\mu L q_l) \times \left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \tag{C.8}$$

and rearranging the indices

$$\frac{1}{2}\left[(\bar{q}_i\gamma^\mu L h_v^{(c_k)})(\bar{u}_k\gamma_\mu L q_i) - \frac{1}{N_c}(\bar{q}_i\gamma^\mu L h_v^{(c_i)})(\bar{u}_k\gamma_\mu L q_k)\right] \tag{C.9}$$

the Fierz transformation must be used in the first term,

Fierz transformation:

$$(\bar{q}_i\gamma^\mu L h_v^{(c_k)})(\bar{u}_k\gamma_\mu L q_i) = (\bar{q}_i\gamma^\mu L q_i)(\bar{u}_k\gamma_\mu L h_v^{(c_k)}) \tag{C.10}$$

so finally C.1 and C.2 can be written as

$$8\left[(\bar{q}\gamma^\mu L q)(\bar{u}\gamma_\mu L h_v^{(c)}) - \frac{1}{N_c}(\bar{q}\gamma^\mu L h_v^{(c)})(\bar{u}\gamma_\mu L q)\right] \tag{C.11}$$

and

$$2\left[(\bar{q}\gamma^{\mu}Lq)(\bar{u}\gamma_{\mu}Lh_v^{(c)}) - \frac{1}{N_c}(\bar{q}\gamma^{\mu}Lh_v^{(c)})(\bar{u}\gamma_{\mu}Lq)\right] \tag{C.12}$$

Q_2^c defined in C.6, now can be expressed in terms of the quark operators Q_1 and Q_2

$$Q_2^c = \frac{1}{2}\left(Q_1 - \frac{1}{N_c}Q_2\right) \tag{C.13}$$

with

$$Q_1 = (\bar{q}\gamma^{\mu}Lq)(\bar{u}\gamma_{\mu}Lh_v^{(c)}) \tag{C.14}$$

$$Q_2 = (\bar{q}\gamma^{\mu}Lh_v^{(c)})(\bar{u}\gamma_{\mu}Lq) \tag{C.15}$$

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