INVESTIGATION OF SRS EQUATIONS BY SMALL SPHERICAL PARTICLES

by

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ABSTRACT

The equations for determination of the Stokes field for stimulated Raman scattering (SRS) by small spherical particles are considered. Solutions are constructed in the form of power series, and it is shown that they are convergent. A theorem about properties of the general solutions is proved.

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1. DESCRIPTION OF THE PROBLEM AND DERIVATION OF THE BASIC EQUATION

Assume that we have spherical particles whose dimensions are comparable to the wave-length of incident laser radiation. The distribution of the field inside the particles is described by a well-known theory [1]. It turns out that in certain domains inside the particle, the intensity of the field is considerably greater than the intensity of incoming waves. As a consequence of this a Stokes field $\vec{E}_s$ with the frequency of $\omega_s = \omega_L - \omega_m$ is generated, where ω_L is the frequncy of the optical phonons. The Stokes field inside a particle is described by the following system of Maxwell equations

$$\begin{aligned} \nabla\times\vec{H} &= \frac{1}{c}\frac{\partial\vec{D}}{\partial t} \\ \nabla\times\vec{E} &= -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \\ \vec{B} = \vec{H}, \quad \vec{D} &= \vec{E} + 4\pi\vec{P} = \varepsilon\vec{E} + 4\pi\vec{P}^{NL} \end{aligned} \tag{1.1}$$

This system reduces to the well-known wave equation for Fourier components:

$$\nabla\times\nabla\times\vec{E}_s - \frac{\omega_s^2}{c^2}\varepsilon\vec{E}_s = 4\pi\frac{\omega_s^2}{c^2}\vec{P}^{NL}. \tag{1.2}$$

As is known for stimulated Raman scattering (SRS) the dependence of $\vec{P}^{NL}$ on $\vec{E}_s$ is parametrical, that is $\vec{P}^{NL} = \chi|\vec{E}_L|^2\vec{E}_s$, where $\chi = \frac{N}{2m}\left(\frac{\partial\alpha}{\partial Q_m}\right)\cdot\frac{1}{\omega^2_{0m} - \omega^2_m - i\omega_m\Gamma_m}$ and $|\vec{E}_L|^2 = \vec{E}_L\cdot\vec{E}_L^*$. The influation field $\vec{E}_L$ has in this case the following components

$$E_r = \frac{E_0 \cos\phi}{k_i^2 r^2} \sum_{\ell=1}^{\infty} C_\ell \ell(\ell+1)\psi_\ell(k_i r)\vec{Q}_\ell(\theta)\sin\theta,$$

$$E_\theta = \frac{E_0 \cos\phi}{k_i r} \sum_{\ell=1}^{\infty} C_\ell \psi'(k_i r)\vec{S}_\ell(\theta) + B_\ell \psi_\ell(k_i r)\vec{Q}_\ell(\theta)) \qquad (1.3)$$

$$E_\phi = \frac{E_0 \operatorname{sos}\phi}{k_i r} \sum_{\ell=1}^{\infty} C_\ell \psi_\ell'(k_i r) Q_\ell + iB_\ell \psi_\ell(k_i r) S_\ell(\theta)),$$

where

$$C_\ell = i^\ell \cdot \frac{2\ell+1}{\ell(\ell+1)} \cdot \frac{m}{\zeta_\ell(\rho)\psi_\ell'(m\rho) - m\zeta_\ell(\rho)\psi_\ell(m\rho)} \qquad (1.4)$$

$$B_\ell = -i^\ell \cdot \frac{2\ell+1}{\ell(\ell+1)} \cdot \frac{m}{\zeta_\ell'(\rho)\psi_\ell(m\rho) - m\zeta_\ell(\rho)\psi_\ell(m\rho)}$$

Here ' is always the derivative with respect to the argument inside the function, $k_i = 2\pi m_i/\lambda_L$ is the wave number of the material of the particle with complex index of refraction $m_i = n - i æ$, $\rho = 2\pi m_1/\lambda_L$ is the parameter of diffraction, $m = m_i/m_1$, m_1 is the index of refraction of the surroundings over the particle, R is the radius of the particle, r, θ, ϕ are coordinates of a point inside the particle, $\psi_\ell(z) = \sqrt{\pi z/2} \cdot J_{\ell+\frac{1}{2}}(z)$, $J_{\ell+\frac{1}{2}}(z)$ is a Bessel function of the first kind of order $\ell+\frac{1}{2}$, $\zeta_\ell(z) = \sqrt{\pi z/2} \cdot H^{(2)}_{\ell+\frac{1}{2}}(z)$, $H^{(2)}_{\ell+\frac{1}{2}}(z)$ is a Hankel function of the second kind of order $\ell+\frac{1}{2}$. The angles functions $Q_\ell(\theta)$ and $S_\ell(\theta)$ are expressed in terms of Legendre polynomials $Q_\ell(\theta) = P_\ell^{(1)}(\cos\theta)/\sin\theta$, $S_\ell(\theta) = -(P_\ell^{(1)}(\cos\theta))'\sin\theta$.

Finally, the Stokes field inside the particle is described by the following equation

$$\nabla \times \nabla \times \vec{E}_s - K^2 \vec{E}_s = 4\pi \frac{\omega_s^2}{c^2} \cdot \frac{N}{2m} \left(\frac{\partial\alpha}{\partial Q_m}\right)^2 \frac{|\vec{E}_L|^2 \vec{E}_s}{\omega_{0m}^2 - \omega_m^2 - i\omega_m \Gamma_m} \qquad (1.5)$$

where $K^2 = \frac{\omega_s^2}{c^2} \varepsilon$.

The following approach is proposed. Let us apply our equation the operator of angular momentum $\vec{L}=-i\vec{r}\times\nabla$, which operating an gradients gives zero. Using the indentity $\nabla\times\nabla\times\vec{E}=\nabla(\nabla\cdot\vec{E}-\nabla^2\vec{E})$ we recieive the scalar equation

$$\nabla^2\eta + k^2\eta = -4\pi A|\vec{E}_L|^2\eta \quad , \tag{1.6}$$

where

$$A = \frac{\omega^2{}_s}{c^2} \cdot \frac{N}{2m} \frac{1}{\omega^2_{0m}-\omega^2{}_m-i\omega_m\Gamma_m} \quad \text{and} \quad \eta=\vec{L}\cdot\vec{E}_s .$$

We can write the integral equation for Green's function of the corresponding homogeneous problem as

$$\eta(\vec{r}) = -Ak^2\int\frac{|\vec{E}_L(\vec{r}\,')|^2\eta(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} e^{ik|\vec{r}-\vec{r}\,'|}d\vec{r}\,' + \eta_0(\vec{r}) \tag{1.7}$$

where η_0 is the solution of the homogeneous problem.

The function η will be found as a spherical harmonic expansion, that is

$$\eta = \sum_{\ell m} C_{\ell m}(r)Y_{\ell m}(\theta,\phi) \quad .$$

Substituting this expansion into (1.7), using the well-known expression for Green's function

$$\frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} = 4\pi ik\sum_{\ell m} j_\ell(kr^<)h_\ell^{(1)}(kr^>)Y_{\ell m}(\theta,\phi)Y^*_{\ell m}(\theta,\phi) \tag{1.8}$$

where $r^>$, $r^<$ are greater than or smaller than r and r', and taking into account the orthogonality of the spherical harmonics, we receive

$$C_{\ell m}(r) = C^{0}_{\ell m}(r) - 4\pi i k^3 \int_0^R \int_{\Omega'} |\vec{E}_L|^2 \gamma_\ell(kr^<) h^{(1)}(kr^>) \cdot$$
$$\cdot \sum_{\ell m} C_{\ell m}(r') Y^*_{\ell m}(\theta', \phi') Y_{\ell m}(\theta', \phi') r'^2 dr' d^2\Omega'$$

Knowing the angular dependence of $|\vec{E}|^2$, we can now integrate over the angles. As a result we have

$$C_{\ell m}(r) = C^{(0)}_{\ell m}(r) - 4\pi A i k^3 \int_0^R f_{\ell m}(r') C_{\ell m}(r') J_\ell(kr^<) h^{(1)}_\ell(kr^>) r'^2 dr', \quad (1.10)$$

$$\begin{aligned} f_{\ell m}(r) = {} & \frac{E_0^2}{r^4 k_i^2 k_i^{2*}} \beta_1(\ell,m) C_\ell C^*_\ell \ell^2(\ell+1)^2 \phi_\ell(k_i r) \phi_\ell(k_i r) + \\ & + \frac{E_0^2}{r^2 k_i k_i^*} (\beta_2(\ell,m) C_\ell C^*_\ell \phi'_\ell(k_i r) \phi'^*_\ell(k_i r) + \qquad (1.11) \\ & + \beta_3(\ell,m) i B_\ell C^*_\ell \phi(k_i r) \phi'^*_\ell(k_i r) - \beta_3(\ell,m) i C_\ell B^*_\ell \phi'(k_i r) \phi^*_\ell(k_i r) + \\ & + \beta_4 B_\ell B^*_\ell \phi_\ell(k_i r) \phi^*_\ell(k_i r)) \end{aligned}$$

where the coefficients $\beta_1, \ldots, \beta_4$ are obtained as the result of a direct integration.

Using the well-known differential relation

$$(\nabla^2_r + k^2 - \frac{\ell(\ell+1)}{r^2})(J_\ell(kr^<) h^{(1)}_\ell(kr^>)) = \frac{i\delta(r-r')}{kr'^2} \quad (1.12)$$

we have the equation

$$\nabla^2_r C_{\ell m}(r) + (k^2 - \frac{\ell(\ell+1)}{r^2} - 4\pi A k^2 A_{\ell m}(r)) C_{\ell m}(r) = 0 \quad (1.13)$$

for determination of the coefficients $C_{\ell m}(r)$. Knowing the functions $C_{\ell m}(r)$ we can then determine the function η.

The next step is to determine the Stokes field using the function η. But we shall only deal with the determination of the functions $C_{\ell m}(r)$.

2. THE CONSTRUCTION OF SOLUTIONS OF THE EQUATION (1.13) IN THE FORM OF POWER SERIES

The equation (1.13) is preferable written on the form

$$x^2 \frac{d^2 z}{dx^2} + 2x\left(\frac{dz}{dx}\right)^2 + (-\beta + x^2 f(x))z = 0 \tag{2.1}$$

(all symbols are clear from previous chapter), where $f(x)$ is supposed to be analytic for all sufficiently small $|x|$. In this case we have the expansion

$$f(x) = \sum_{i=0}^{\infty} f_i x^i \tag{2.2}$$

where all f_i are constants and the power series is convergent.

Let us find solutions of (2.1) on the form of the converging power series

$$z = \sum_{i=0}^{\infty} C_i x^i \tag{2.3}$$

with constant C_i. The existence of such solutions are not guarantieed by the general theorems of the existence of solutions of differential equations.

Substituting (2.3) into (2.1) we receive

$$\begin{aligned} & x^2(2C_2 + 2\cdot 3\cdot C_3 x + 3\cdot 4\cdot C_4 x^2 + \ldots + n(n-1)C_n x^{n-2}) + \\ & + (-\beta + x^2(f_0 + f_1 x + f_2 x^2 + \ldots + f_n x^n + \ldots))(C_0 + C_1 x + C_2 x^2 + \ldots) + \\ & + 2x(C_1 + 2C_2 x + 3C_3 x^2 + \ldots + nC_n x^{n-1})^2 \equiv 0 \end{aligned} \tag{2.4}$$

Comparing the coefficients of x and x^0 in this relation we have that $C_0 = 0$ and

$$C_1(2C_1 - \beta) = 0 \tag{2.5}$$

Hence $C_1 = 0$ or $C_1 = -\beta/2$.

2.1. The construction of solutions in the case $C_1=0$

Comparing the coefficients of $x^2, x^3, \ldots$ we receive $(2-\beta)C_2=0$; taking $C_2=0$ we have the following equation $(6-\beta)C_3=0$. Then, by $C_3=0$ we have $(12-\beta)C_4=0$ and so on. In other words, it is necessary to have $C_2=C_3=\ldots=C_\ell=0$ for finding $C_{\ell m}$ from the equation (1.13). We shall point out the method for finding such solutions for $\beta=2$ and discuss the convergence of the power series. In the case of another β the calculation formulas are the same.

So for $\beta=2$ the parameter C_2 may be chosen to be arbitrary (see (2.5)). The system for determination of the coefficients of the power series (2.3) is

$$
\begin{aligned}
&(2\cdot 3-2)C_3 + 8C_2^2 = 0 , \\
&(3\cdot 4-2)C_4 + 4\cdot 2\cdot 3C_2C_3 + f_0C_2 = 0 , \\
&(4\cdot 5-2)C_5 + 2\cdot 3^2C_3^2 + 4\cdot 2\cdot 4C_2C_4 + f_0C_3 + f_1C_2 = 0 , \\
&(6\cdot 5-2)C_6 + 4(4\cdot 4C_3C_4+2\cdot 5C_2C_5) + f_0C_4 + f_1C_3 + f_2C_2 = 0 , \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (2.6)\\
&(2n(2n-1)-2)C_{2n} + 4(2\cdot(2n-1)C_2C_{2n-1} + 3(2n-2)C_3C_{2n-2} + \ldots \\
&+ n(n+1)C_nC_{n+1}) + f_0C_{2n-2} + f_1C_{2n-3} + \ldots + f_{2n-4}C_2 = 0 , \\
&((2n+1)\cdot 2n-2)C_{2n+1}+2(n+1)^2C_{n+1}^2+4(2\cdot 2nC_2C_{2n}+3(2n-1)C_3C_{2n-1}+ \\
&+\ldots+n(n+2)C_nC_{n+2}) + f_0C_{2n-1} + f_1C_{2n-2} + \ldots + f_{2n-3}C_2 = 0 . \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

We receive from this system

$$
C_3 = -2C_2^2 , \quad C_4 = C_2 \cdot \frac{48C_2-f_0}{3\cdot 4-2} ,
$$

$$
C_5 = C_2 \cdot \frac{-72C_2^2+2f_0C_2-f_1}{4\cdot 5-2} \qquad (2.7)
$$

$$
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
$$

and hence, the coefficients C_i are obtainable.

Let us take an example which has general character and all principle questions of convergence will be clear.

Example. Let all coefficients of the power series of (2.2) be bounded, that is there exists a positive number M such that for all integer i $|f_i| \leqslant M$. Let us choose $|C_2| \leqslant \min\{0{,}01; 1/M\}$, then by $|C_i| \leqslant 1$ $(i=3,4,\ldots)$ we have $\sum_{i_1+i_2=K} |f_{i_1} C_{i2}| \leqslant K$. In this case we can calculate

$$
\begin{aligned}
|C_3| &= 2 \cdot 10^{-4} , \\
|C_4| &\leqslant \frac{4 \cdot 2 \cdot 3 |C_2| \, |C_3| + 1}{10} = 0.100 , \\
|C_5| &\leqslant \frac{2 \cdot 3^2 \cdot C_3^2 + 4 \cdot 8 \cdot |C_2| \cdot |C_4| + 2}{18} = 0.113 , \\
|C_6| &\leqslant \frac{4(3 \cdot 4 \cdot |C_3| \, |C_4| + 2 \cdot 5 |C_2| \cdot |C_5| + 3}{28} = 0.109 , \\
|C_7| &\leqslant 0.120, \quad |C_8| \leqslant 0.117, \quad |C_9| \leqslant 0.101 , \\
|C_{10}| &\leqslant 0.101, \quad |C_{11}| \leqslant 0.103 , \quad |C_{12}| \leqslant 0.090 , \\
|C_{13}| &\leqslant 0.074 , \quad |C_{14}| \leqslant 0.081 .
\end{aligned}
\tag{2.8}
$$

It is clear from the formulas (2.6)-(2.8), that the laws for constructing the coefficients with even and odd indices are different. The sequence of the coefficients from the example have maximal values at C_{10} and C_{11}. After that the sequence of coefficients is decreasing and provides the convergence of the power series of (2.3). The power series in this example is majorized by

$$x^2 + x^3 + \ldots \tag{2.9}$$

which is the sum of the geometric progression, and this provides the convergence for all $|x|<1$.

An appropriate power series for other values of β are constructed by analogy.

2.2. The construction of solutions in the case of $C_1=-\beta/2$

Comparing the coefficients by powers of x in the relation (2.4) we see the system

$$
\begin{aligned}
&C_2(2-5\beta) = 0 ,\\
&(2\cdot 3-\beta)C_3 + 2\cdot 2^2\cdot C_2 + 2\cdot 3\cdot C_2\cdot C_3 + f_0C_1 = 0 ,\\
&(3\cdot 4-\beta)C_4 - \tfrac{\beta}{2}\cdot f_1 + 2\cdot 2\cdot 3\cdot C_3 = 0 ,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (2.10)\\
&(2n(2n-1)-\beta)C_{2n}+4(2(2n-1)C_2C_{2n-1}+3(2n-2)C_3C_{2n-2}+\dots\\
&+n(n+1)C_nC_{n+1}) + f_0C_{2n-2} + f_1C_{2n-3} + \dots + f_{2n-2}C_2 = 0 ,\\
&((2n+1)\cdot 2n-\beta)C_{2n+1}+2(n+1)^2C_{n+1}^2+4(2\cdot 2nC_2C_{2n}+3(2n-1)C_3C_{2n-1}+\\
&+\dots+n(n+1)C_nC_{n+2}) + f_0C_{2n-1} + f_1C_{2n-2} + \dots + f_{2n-1}C_2 = 0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots
\end{aligned}
$$

which is similar to (2.6). We have from this system $C_2=0$ (because $\beta\neq 2/5$, β is an integer),

$$C_3 = \frac{f_0\beta}{2\cdot 3-\beta} \quad \text{by} \quad \beta\neq 6$$

and C_3 is arbitrary by $\beta=6$. But it is necessary to point out the additional restriction for equation (2.1); $f_0=0$ in the case of $\beta=6$. The rest of the coefficients are calculated by the recurrence formulas of (2.6), (2.7) where the denominators

$(2n(2n-2)-2)$ and $(2n(2n+1)-2)$ must be changed to $(2n(2n-2)-\beta)$ and $(2n(2n+1)-\beta)$ in both cases $\beta=6$ or $\beta\neq6$.

Let us take $\beta=3\cdot4$. It is necessary to require $f_1=4\cdot2\cdot3C_3$ and repeat the previous considerations in this situation. And so on.

The character of convergence of the received power series is similar to the character of convergence of the power series of the previous paragraph.

Finally we must answer the very important question whether the power series (2.3) is convergent. The answer is positive. We see from (2.6), (2.7) and (2.10) that the coefficients of the power series (2.3) are polynomials of C_2. The degree of this polynomial for each coefficient is not smaller than the degree of the polynomial of the previous coefficient. Hence, we have

$$\frac{C_n}{C_{n+1}} = \frac{A_n C_2^p + \dots}{B_{n+1} C^q + \dots} \qquad (A_n B_{n+1} \neq 0,\ q \geq p)$$

where the dots represent smaller powers. It is clear, that

$$\lim_{n\to\infty} \frac{C_n}{C_{n+1}} = \lim_{n\to\infty} \frac{A_n}{B_{n+1}} C_2^{p-q} > 0$$

if $|C_2|<1$ and (2.8) is convergent [2].

3. THE CONSTRUCTION OF THE GENERAL SOLUTION OF THE EQUATION (2.1) BY USING TWO KNOWN PARTICULAR SOLUTIONS

If you have two linearly independent solutions of a homogeneous linear differential equation of the second order $y_1(x)$ and $y_2(x)$ you can write the general solution of this equation on the form

$$y = C_1 y_1(x) + C_2 y_2(x) ,$$

where C_1 and C_2 are arbitrary constants. As for nonlinear equations, the problem of the construction of the general solution by using known particular solutions is very difficult and results in this domain are available only for some equations of the first order [3-5].

In this paper the problem will be solved for the equation (2.1) written in the more general form

$$\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right)^2 + f(x)y = 0 \qquad (3.2)$$

Let us find the general solution of this equation on the form

$$y = x_1(x)y_1(x) + x_2(x)y_2(x) \qquad (3.3)$$

where $y_1(x)$, $y_2(x)$ are different independent solutions of the equation (3.2), $x_1(x)$ and $x_2(x)$ are unknown functions, which will be determined. Let $W=y_1(x)y_2'(x)-y_2(x)y_1'(x)$ and

$$\frac{dx_1(x)}{dx} = \frac{2y_2(x)}{xW}(y'^2-(x_1(x)y_1^2(x)+x_2(x)y_2^2(x)) ,$$
$$\frac{dx_2(x)}{dx} = -\frac{2y_1(x)}{xW}(y^2-(x_1(x)y_1^2(x)+x_2(x)y_2^2(x)) , \qquad (3.4)$$

where y is on the form of (3.3).

We have to show that the general solution of the equation (3.2) is (3.3) in this case. Calculating, we see that

$$y' = x_1'(x)y_1(x) + x_2'(x)y_2(x) + x_1(x)y_1'(x) + x_2(x)y_2'(x) =$$

$$= \frac{2y_1(x)y_2(x)}{xW}(y'^2-(x_1(x)y_1^2(x)+x_2(x)y_2^2(x)) - \frac{2y_1(x)y_2(x)}{xW}(y'^2-$$

$$-(x_1(x)y_1^2(x)+x_2(x)y_2^2(x))) +$$

$$+ (x_1(x)y_1'(x)+x_2(x)y_2'(x) = x_1(x)y_1'(x) + x_2(x)y_2'(x) \ .$$

Taking into account the previous relation we transform the left hand side of equation (3.2)

$$y'' + \frac{2y'^2}{x} + f(x)y = x_1'(x)y_1'(x) + x_2'(x)y_2'(x) + x_1(x)y_1''(x) +$$

$$+ x_2(x)y_2''(x) + \frac{2y'^2}{x} + f(x)y =$$

$$= \frac{2y_2(x)y_1'(x)}{xW}(y'^2-(x_2(x)y_1(x)+x_2(x)y_2(x)) - \frac{2y_1(x)y_2'(x)}{xW}(y'^2-$$

$$+ (x_1(x)y_1^2(x)+x_2(x)y_2^2(x)) + \frac{2y'^2}{x} + f(x)y =$$

$$= -\frac{2}{x}(y'^2-(x_1(x)y_1^2(x)+x_2(x)y_2^2(x))) - x_1(\frac{2y_1^2(x)}{x} + f(x)y_1(x)) -$$

$$- x_1(\frac{2y_2'^2(x)}{x} + f(x)y_2(x)) + \frac{2(x_1(x)y_1(x)+x_2(x)y_2(x))^2}{x} +$$

$$+ f(x)(x_1(x)y_1(x)+x_2(x)y_2(x)) \equiv 0 \ .$$

Simplifying the right hand side of (3.4) we receive the following

<u>Theorem</u>. The general solution of the equation (3.2) is determined by the formulas (3.3), where $(x_1(x),x_2(x))$ is the general solution of the system

$$\begin{aligned} \frac{dx_1(x)}{dx} &= \frac{4x_1(x)x_2(x)(y_1(x)y_2^2(x)}{xW} \\ \frac{dx_2(x)}{dx} &= \frac{-4x_1(x)x_2(x)y_1^2(x)y_2(x)}{xW} \end{aligned} \tag{3.5}$$

Substituting $x_1(x)=t_1-t_2$, $x_2=t_1+t_2$ into the system (3.5) we have

$$\begin{aligned} \frac{dt_1}{dx} &= \frac{2y_1(x)y_2(x)(y_1(x)-y_2(x))(t_1^2-t_2^2)}{xW} \\ \frac{dt_2}{dx} &= \frac{-2y_1(x)y_2(x)(y_1(x)+y_2(x))(t_1^2-t_2^2)}{xW} \end{aligned} \tag{3.6}$$

This is the system of the Riccati equations. The main property of this system is that the solutions have moving singular points, which are moving poles [6]. The solutions have the form

$$x_i = \frac{A_i}{x-C} + \Phi_i(x-C) \ , \tag{3.7}$$

where A_i are constants, $\Phi_i(x-C)$ are analitic functions, C is an arbitrary constant $(i=1,2)$. This means, that every solution has its <u>own</u> singularity which does not depend on the singularities of the coefficients of the equation. This fact makes the above method for constructing the solutions of equations (1.13) more efficient than the numerical methods.

It should be noted in conclusion that according to the theorem above every initial value problem for equations of the form (2.1) with initial values $z(R) = const_1$, $z'(R) = const_2$, has a solution which is a power series.

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