ISBN 82–553–0869–5
Pure Mathematics

No. 34
December 1993

Four-dimensional pseudo-Riemannian homogeneous spaces. Classification of complex pairs.

by

B. Komrakov, Jnr.*

PREPRINT SERIES – Matematisk institutt, Universitetet i Oslo

*Submitted under the 1993 cooperation project between the Department of Mathematics, University of Oslo, and the International Sophus Lie Center.

Homogeneous spaces
B. Komrakov seminar

FOUR-DIMENSIONAL PSEUDO–RIEMANNIAN HOMOGENEOUS SPACES. CLASSIFICATION OF COMPLEX PAIRS

B. Komrakov, Jnr.

International Sophus Lie Centre

ISLC International Sophus Lie Centre
Address:
Belarus, 220123, Minsk, P.O. Box 70
Current address:
Centre for Advanced Study
at The Norwegian Academy of Science and Letters
P.O. Box 7606 Skillebekk
0205 Oslo
Norway
e-mail: islc@shs.no

This publication was typeset using $\mathcal{AMS}$-TEX
the American Mathematical Society's TEXmacro system.

Introduction

We consider classification of lower-dimensional homogeneous spaces an immediate continuation and global version of classification results obtained by Sophus Lie. Two-dimensional homogeneous spaces were classified locally by Sophus Lie [L1] and globally by G.D. Mostow [M]. (See also preprint [KTD], where the complete classification of two-dimensional homogeneous spaces, both locally and globally, is presented.) S. Lie also obtained some results in classification of three-dimensional homogeneous spaces and described all subalgebras in the Lie algebra $\mathfrak{so}(4,\mathbb{C})$ (in terms of vector fields). A detailed account of these classifications can be found in [L2]. The classification of all tree-dimensional isotropically-faithful homogeneous spaces was obtained in [KT].

The problem of classification of four-dimensional pseudo-Riemannian homogeneous spaces is interesting from the point of view of both geometry and physics, and not only in the case of signature $(1,3)$ (spaces of relativity theory) but also in the case of signature $(2,2)$ (twistors).

This preprint presents the results of the first part of our work devoted to classification of four-dimensional homogeneous spaces with an invariant pseudo-Riemannian metric of arbitrary signature. A similar classification for the case of Riemannian metric can be found in [I].

Let $(\overline{G},M)$ be a homogeneous space, $G=\overline{G}_x$ the stabilizer of an arbitrary point $x\in M$, and $(\bar{\mathfrak{g}},\mathfrak{g})$ the pair of Lie algebras corresponding to the pair $(\overline{G},G)$ of Lie groups.

Lemma. *Suppose that the homogeneous space $(\overline{G},M)$ admits an invariant pseudo-Riemannian metric. Then the isotropic representation of the pair $(\bar{\mathfrak{g}},\mathfrak{g})$*

$$\rho\colon\mathfrak{g}\to\mathfrak{gl}(\bar{\mathfrak{g}}/\mathfrak{g}),\qquad \rho(x)(\bar{x}+\mathfrak{g})=[x,\bar{x}]+\mathfrak{g}\quad(x\in\mathfrak{g},\ \bar{x}\in\bar{\mathfrak{g}})$$

is faithful. Moreover, there exists a basis of $\bar{\mathfrak{g}}/\mathfrak{g}$ such that $\rho(\mathfrak{g})$ lies in one of the following Lie algebras: $\mathfrak{so}(4)$, $\mathfrak{so}(3,1)$, or $\mathfrak{so}(2,2)$.

It is convenient, at first, to consider complexifications of pairs $(\bar{\mathfrak{g}},\mathfrak{g})$. In accordance with this, we divide solution of our problem into the following four parts:

(1) We find (up to conjugation) all possible forms the subalgebra $(\rho(\mathfrak{g}))^{\mathbb{C}}=\rho^{\mathbb{C}}(\mathfrak{g}^{\mathbb{C}})$ can assume. This is equivalent to classifying (up to conjugation) subalgebras $\mathfrak{p}$ in the Lie algebra $\mathfrak{so}(4,\mathbb{C})$.
(2) For each subalgebra $\mathfrak{p}$ obtained in (1), we find (up to equivalence of pairs) all complex pairs $(\bar{\mathfrak{g}}^{\mathbb{C}},\mathfrak{g}^{\mathbb{C}})$ such that the subalgebra $\rho^{\mathbb{C}}(\mathfrak{g}^{\mathbb{C}})$ is conjugate to $\mathfrak{p}$.
(3) For each complex pair $(\bar{\mathfrak{g}}^{\mathbb{C}},\mathfrak{g}^{\mathbb{C}})$, we find (up to equivalence of pairs) all its real forms $(\bar{\mathfrak{g}},\mathfrak{g})$.
(4) For each real pair obtained in (3), we construct all (up to isomorphism) corresponding homogeneous spaces.

A detailed description of techniques we use for constructing pairs $(\bar{\mathfrak{g}},\mathfrak{g})$ with a given faithful isotropic representation can be found in [KT]. Methods of finding all homogeneous spaces corresponding to a given pair $(\bar{\mathfrak{g}},\mathfrak{g})$ were described by G.D. Mostow in [M].

Example. Suppose that $\rho^{\mathbb{C}}(\mathfrak{g}^{\mathbb{C}})$ in a suitable basis has the form:

$$\mathfrak{so}(3,\mathbb{C}) = \left\{ \begin{pmatrix} 0 & x & y & 0 \\ -x & 0 & z & 0 \\ -y & -z & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \middle| \, x,y,z \in \mathbb{C} \right\}.$$

We identify $\mathfrak{g}^{\mathbb{C}}$ with $\rho^{\mathbb{C}}(\mathfrak{g}^{\mathbb{C}})$. Then the corresponding complex pairs (viewed up to equivalence) have the form:

(1) $\bar{\mathfrak{g}}^{\mathbb{C}} = \mathfrak{g}^{\mathbb{C}} \leftthreetimes \mathbb{C}^4 \cong (\mathfrak{so}(3,\mathbb{C}) \leftthreetimes \mathbb{C}^3) \times \mathbb{C}$;
(2) $\bar{\mathfrak{g}}^{\mathbb{C}} = \mathfrak{sl}(2,\mathbb{C}) \times \mathfrak{sl}(2,\mathbb{C}) \times \mathbb{C}$, $\mathfrak{g}^{\mathbb{C}} = \{ (x,x,0) \mid x \in \mathfrak{sl}(2,\mathbb{C}) \}$.

Every real form of these pairs is equivalent to one of the following pairs:

(1) $\bar{\mathfrak{g}} = (\mathfrak{so}(3) \leftthreetimes \mathbb{R}^3) \times \mathbb{R}$, $\mathfrak{g} = (\mathfrak{so}(3) \times \{0\}) \times \{0\}$;
(2) $\bar{\mathfrak{g}} = (\mathfrak{so}(2,1) \leftthreetimes \mathbb{R}^3) \times \mathbb{R}$, $\mathfrak{g} = (\mathfrak{so}(2,1) \times \{0\}) \times \{0\}$;
(3) $\bar{\mathfrak{g}} = \mathfrak{sl}(2,\mathbb{R}) \times \mathfrak{sl}(2,\mathbb{R}) \times \mathbb{R}$, $\mathfrak{g} = \{ (x,x,0) \mid x \in \mathfrak{sl}(2,\mathbb{R}) \}$;
(4) $\bar{\mathfrak{g}} = \mathfrak{su}(2) \times \mathfrak{su}(2) \times \mathbb{R}$, $\mathfrak{g} = \{ (x,x,0) \mid x \in \mathfrak{su}(2) \}$;
(5) $\bar{\mathfrak{g}} = \mathfrak{sl}(2,\mathbb{C})_{\mathbb{R}} \times \mathbb{R}$, $\mathfrak{g} = \mathfrak{sl}(2,\mathbb{R}) \times \{0\}$;
(6) $\bar{\mathfrak{g}} = \mathfrak{sl}(2,\mathbb{C})_{\mathbb{R}} \times \mathbb{R}$, $\mathfrak{g} = \mathfrak{su}(2) \times \{0\}$.

§1 Classification of subalgebras of the Lie algebra $\mathfrak{so}(4,\mathbb{C})$

Theorem 1. *Any nonzero subalgebra of the Lie algebra $\mathfrak{so}(4,\mathbb{C})$ is conjugate (with respect to $GL(4,\mathbb{C})$) to one and only one of the following subalgebras:*

$\dim \mathfrak{g} = 1$

1. $\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & \lambda x & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & 0 & -\lambda x \end{pmatrix}$ *2.* $\begin{pmatrix} x & x & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -x & -x \end{pmatrix}$

$|\lambda| \leqslant 1$; $\mathcal{R}e\lambda > 0$ or $\mathcal{R}e\lambda = 0$, $\mathcal{I}m\lambda \geqslant 0$.

3. $\begin{pmatrix} 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ *4.* $\begin{pmatrix} 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\dim \mathfrak{g} = 2$

1. $\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & 0 & -y \end{pmatrix}$ *2.* $\begin{pmatrix} x & y & 0 & 0 \\ 0 & \lambda x & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -y & -\lambda x \end{pmatrix}$, $|\lambda| \leqslant 1$

3. $\begin{pmatrix} x & y & 0 & x \\ 0 & -x & -x & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -y & x \end{pmatrix}$ *4.* $\begin{pmatrix} x & y & 0 & 0 \\ 0 & 0 & y & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

5. $\begin{pmatrix} 0 & x & 0 & y \\ 0 & 0 & -y & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -x & 0 \end{pmatrix}$

$\underline{\dim \mathfrak{g} = 3}$

1. $\begin{pmatrix} x & z & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -z & -y \end{pmatrix}$

2. $\begin{pmatrix} x & y & 0 & z \\ 0 & \lambda x & -z & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -y & -\lambda x \end{pmatrix}$

$\mathcal{R}e\lambda > 0$ *or* $\mathcal{R}e\lambda = 0,\ \mathcal{I}m\lambda \geqslant 0$

3. $\begin{pmatrix} 0 & y & 0 & z \\ 0 & x & -z & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -y & -x \end{pmatrix}$

4. $\begin{pmatrix} x & y & 0 & 0 \\ z & -x & 0 & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & x \end{pmatrix}$

5. $\begin{pmatrix} 2x & y & 0 & y \\ z & 0 & -y & 0 \\ 0 & -z & -2x & -z \\ z & 0 & -y & 0 \end{pmatrix}$

$\underline{\dim \mathfrak{g} = 4}$

1. $\begin{pmatrix} x & z & 0 & t \\ 0 & y & -t & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -z & -y \end{pmatrix}$

2. $\begin{pmatrix} x & y & 0 & 0 \\ z & t & 0 & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & -t \end{pmatrix}$

3. $\begin{pmatrix} x & y & 0 & t \\ z & -x & -t & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & x \end{pmatrix}$

$\underline{\dim \mathfrak{g} = 5}$

1. $\begin{pmatrix} x & y & 0 & u \\ z & t & -u & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & -t \end{pmatrix}$

$\underline{\dim \mathfrak{g} = 6}$

1. $\begin{pmatrix} x & y & 0 & u \\ z & t & -u & 0 \\ 0 & v & -x & -z \\ -v & 0 & -y & -t \end{pmatrix}$

§2 Classification of complex pairs

Theorem 2. *Any pair* $(\bar{\mathfrak{g}}^{\mathbb{C}}, \mathfrak{g}^{\mathbb{C}})$ *corresponding to an effective pseudo-Riemannian pair* $(\bar{\mathfrak{g}}, \mathfrak{g})$ *of codimension 4 is equivalent to one and only one of the following:*

0.1 $\mathfrak{g} = \{0\}$

$\bar{\mathfrak{g}}_1$	u_1	u_2	u_3	u_4
u_1	0	u_3	u_2	0
u_2	$-u_3$	0	u_1	0
u_3	$-u_2$	$-u_1$	0	0
u_4	0	0	0	0

$\bar{\mathfrak{g}}_2$	u_1	u_2	u_3	u_4
u_1	0	u_3	0	u_1
u_2	$-u_3$	0	0	αu_2
u_3	0	0	0	$(\alpha+1)u_3$
u_4	$-u_1$	$-\alpha u_2$	$-(\alpha+1)u_3$	0

$\bar{\mathfrak{g}}_3$	u_1	u_2	u_3	u_4
u_1	0	0	0	$2u_1$
u_2	0	0	u_1	u_2
u_3	0	$-u_1$	0	u_2+u_3
u_4	$-2u_1$	$-u_2$	$-u_2-u_3$	0

$\bar{\mathfrak{g}}_4$	u_1	u_2	u_3	u_4
u_1	0	0	0	u_1
u_2	0	0	0	u_1+u_2
u_3	0	0	0	u_2+u_3
u_4	$-u_1$	$-u_1-u_2$	$-u_2-u_3$	0

$\bar{\mathfrak{g}}_5$	u_1	u_2	u_3	u_4
u_1	0	0	0	u_1
u_2	0	0	0	u_1+u_2
u_3	0	0	0	αu_3
u_4	$-u_1$	$-u_1-u_2$	$-\alpha u_3$	0

$\bar{\mathfrak{g}}_6$	u_1	u_2	u_3	u_4
u_1	0	0	0	u_1
u_2	0	0	0	αu_2
u_3	0	0	0	βu_3
u_4	$-u_1$	$-\alpha u_2$	$-\beta u_3$	0

$\bar{\mathfrak{g}}_7$	u_1	u_2	u_3	u_4
u_1	0	0	u_1	0
u_2	0	0	0	u_2
u_3	$-u_1$	0	0	0
u_4	0	$-u_2$	0	0

$\bar{\mathfrak{g}}_8$	u_1	u_2	u_3	u_4
u_1	0	0	0	u_2
u_2	0	0	0	0
u_3	0	0	0	u_1
u_4	$-u_2$	0	$-u_1$	0

$\bar{\mathfrak{g}}_9$	u_1	u_2	u_3	u_4
u_1	0	0	0	u_1
u_2	0	0	0	0
u_3	0	0	0	u_2
u_4	$-u_1$	0	$-u_2$	0

$\bar{\mathfrak{g}}_{10}$	u_1	u_2	u_3	u_4
u_1	0	0	0	0
u_2	0	0	u_1	0
u_3	0	$-u_1$	0	0
u_4	0	0	0	0

$\bar{\mathfrak{g}}_{11}$	u_1	u_2	u_3	u_4
u_1	0	0	0	0
u_2	0	0	0	0
u_3	0	0	0	0
u_4	0	0	0	0

1.1

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & 0 & 0 & 0\\ 0 & \lambda x & 0 & 0\\ 0 & 0 & -x & 0\\ 0 & 0 & 0 & -\lambda x\end{pmatrix}\right| x\in\mathbb{C}\right\}$$

$$\lambda\in\mathbb{C},\ |\lambda|\leqslant 1,\ \mathcal{R}e\lambda>0 \text{ or } \mathcal{R}e\lambda=0,\ \mathcal{I}m\lambda\geqslant 0$$

$\lambda=0$

$\bar{\mathfrak{g}}_1$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	0	0
u_2	0	0	0	0	αu_2
u_3	u_3	0	0	0	u_3
u_4	0	0	$-\alpha u_2$	$-u_3$	0

$\bar{\mathfrak{g}}_2$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	u_2	0
u_2	0	0	0	0	u_2
u_3	u_3	$-u_2$	0	0	u_3
u_4	0	0	$-u_2$	$-u_3$	0

$\bar{\mathfrak{g}}_3$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	e_1+u_2	0
u_2	0	0	0	0	0
u_3	u_3	$-e_1-u_2$	0	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_4$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	u_2	0
u_2	0	0	0	0	0
u_3	u_3	$-u_2$	0	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_5$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	e_1	0
u_2	0	0	0	0	u_2
u_3	u_3	$-e_1$	0	0	0
u_4	0	0	$-u_2$	0	0

$\bar{\mathfrak{g}}_6$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	0	0
u_2	0	0	0	0	u_2
u_3	u_3	0	0	0	0
u_4	0	0	$-u_2$	0	0

$\bar{\mathfrak{g}}_7$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	0	$-u_3$	0
u_1	$-u_1$	0	0	e_1	0
u_2	0	0	0	0	0
u_3	u_3	$-e_1$	0	0	0
u_4	0	0	0	0	0

$\lambda = \frac{1}{2}$

$\bar{\mathfrak{g}}_8$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	$\frac{1}{2}u_2$	$-u_3$	$-\frac{1}{2}u_4$
u_1	$-u_1$	0	0	$-2e_1$	u_2
u_2	$-\frac{1}{2}u_2$	0	0	u_4	0
u_3	u_3	$2e_1$	$-u_4$	0	0
u_4	$\frac{1}{2}u_4$	$-u_2$	0	0	0

$\bar{\mathfrak{g}}_9$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	$\frac{1}{2}u_2$	$-u_3$	$-\frac{1}{2}u_4$
u_1	$-u_1$	0	0	0	u_2
u_2	$-\frac{1}{2}u_2$	0	0	0	0
u_3	u_3	0	0	0	0
u_4	$\frac{1}{2}u_4$	$-u_2$	0	0	0

$\lambda \in \mathbb{C},\ |\lambda| \leqslant 1,\ \mathcal{R}e\lambda \geqslant 0$ *or* $\mathcal{R}e\lambda = 0,\ \mathcal{I}m\lambda \geqslant 0$

$\bar{\mathfrak{g}}_{10}$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	λu_2	$-u_3$	$-\lambda u_4$
u_1	$-u_1$	0	0	0	0
u_2	$-\lambda u_2$	0	0	0	0
u_3	u_3	0	0	0	0
u_4	λu_4	0	0	0	0

1.2

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & x & 0 & 0\\ 0 & x & 0 & 0\\ 0 & 0 & -x & 0\\ 0 & 0 & -x & -x\end{pmatrix}\right| x\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	u_1	u_2	u_3	u_4
e_1	0	u_1	u_1+u_2	$-u_3-u_4$	$-u_4$
u_1	$-u_1$	0	0	0	0
u_2	$-u_1-u_2$	0	0	0	0
u_3	u_3+u_4	0	0	0	0
u_4	u_4	0	0	0	0

1.3

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} 0 & 0 & x & 0\\ 0 & 0 & 0 & x\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\right| x\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	u_1	u_2	u_3	u_4
e_1	0	e_1	0	u_1	u_2
u_1	$-e_1$	0	$-\frac{1}{2}u_2$	u_3	$\frac{1}{2}u_4$
u_2	0	$\frac{1}{2}u_2$	0	$\frac{1}{2}u_4$	0
u_3	$-u_1$	$-u_3$	$-\frac{1}{2}u_4$	0	0
u_4	$-u_2$	$-\frac{1}{2}u_4$	0	0	0

$\bar{\mathfrak{g}}_2$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$-\lambda e_1+(\lambda+1)u_1+\lambda u_2$	0
u_2	0	0	0	0	u_2
u_3	$-u_1$	$\lambda e_1-(\lambda+1)u_1-\lambda u_2$	0	0	0
u_4	$-u_2$	0	$-u_2$	0	0

$$|\lambda|<1 \text{ or } |\lambda|=1,\ 0\leqslant \arg\lambda\leqslant\pi$$

$\bar{\mathfrak{g}}_3$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	u_1	0
u_2	0	0	0	0	u_2
u_3	$-u_1$	$-u_1$	0	0	e_1
u_4	$-u_2$	0	$-u_2$	$-e_1$	0

$\bar{\mathfrak{g}}_4$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	u_1	u_2
u_3	$-u_1$	0	$-u_1$	0	$-u_3$
u_4	$-u_2$	0	$-u_2$	u_3	0

$\bar{\mathfrak{g}}_5$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	x	y
u_2	0	0	0	y	z
u_3	$-u_1$	$-x$	$-y$	0	0
u_4	$-u_2$	$-y$	$-z$	0	0 , *where*

$$x = \frac{1}{1+\lambda}e_1 + \frac{\lambda}{1+\lambda}u_1 - \frac{1}{1+\lambda}u_2,$$
$$y = -\frac{1}{1+\lambda}e_1 + \frac{1}{1+\lambda}u_1 + \frac{1}{1+\lambda}u_2,$$
$$z = -\frac{\lambda}{1+\lambda}e_1 + \frac{\lambda}{1+\lambda}u_1 + \frac{1+2\lambda}{1+\lambda}u_2.$$

$\bar{\mathfrak{g}}_6$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	λu_1	$-\lambda e_1+(\lambda+1)u_2$
u_3	$-u_1$	0	$-\lambda u_1$	0	$-\lambda u_3$
u_4	$-u_2$	0	$\lambda e_1-(\lambda+1)u_2$	λu_3	0

$\bar{\mathfrak{g}}_7$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	0	u_2
u_3	$-u_1$	0	0	0	e_1
u_4	$-u_2$	0	$-u_2$	$-e_1$	0

$\bar{\mathfrak{g}}_8$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	$-u_1$	e_1
u_3	$-u_1$	0	u_1	0	e_1+u_3
u_4	$-u_2$	0	$-e_1$	$-e_1-u_3$	0

$\bar{\mathfrak{g}}_9$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$-\lambda\mu e_1+(\lambda+\mu)u_1$	μu_2
u_2	0	0	0	u_2	0
u_3	$-u_1$	$\lambda\mu e_1-(\lambda+\mu)u_1$	$-u_2$	0	$(\mu-1)u_4$
u_4	$-u_2$	$-\mu u_2$	0	$(1-\mu)u_4$	0

$\bar{\mathfrak{g}}_{10}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$-\frac{\lambda}{2}e_1+(\lambda+\frac{1}{2})u_1$	$\frac{1}{2}u_2$
u_2	0	0	0	u_2	0
u_3	$-u_1$	$\frac{\lambda}{2}e_1-(\lambda+\frac{1}{2})u_1$	$-u_2$	0	$u_1-\frac{1}{2}u_4$
u_4	$-u_2$	$-\frac{1}{2}u_2$	0	$\frac{1}{2}u_4-u_1$	0

$\bar{\mathfrak{g}}_{11}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$\lambda(1-\lambda)e_1+u_1$	$(1-\lambda)u_2$
u_2	0	0	0	u_2	0
u_3	$-u_1$	$\lambda(\lambda-1)e_1-u_1$	$-u_2$	0	$e_1-\lambda u_4$
u_4	$-u_2$	$(\lambda-1)u_2$	0	$-e_1+\lambda u_4$	0

$\bar{\mathfrak{g}}_{12}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$-e_1+2u_1$	u_2
u_2	0	0	0	u_2	$-e_1+u_1$
u_3	$-u_1$	e_1-2u_1	$-u_2$	0	0
u_4	$-u_2$	$-u_2$	e_1-u_1	0	0

$\bar{\mathfrak{g}}_{13}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	0	u_1
u_3	$-u_1$	0	0	0	e_1
u_4	$-u_2$	0	$-u_1$	$-e_1$	0

$\bar{\mathfrak{g}}_{14}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	0	u_1
u_3	$-u_1$	0	0	0	0
u_4	$-u_2$	0	$-u_1$	0	0

$\bar{\mathfrak{g}}_{15}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	u_1
u_2	0	0	0	u_1	$-e_1+u_1+2u_2$
u_3	$-u_1$	0	$-u_1$	0	0
u_4	$-u_2$	$-u_1$	$e_1-u_1-2u_2$	0	0

$\bar{\mathfrak{g}}_{16}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	u_1	u_2-u_1
u_3	$-u_1$	0	$-u_1$	0	$-u_3$
u_4	$-u_2$	0	u_1-u_2	u_3	0

$\bar{\mathfrak{g}}_{17}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	u_1
u_2	0	0	0	λu_1	$-\lambda e_1+(1-\lambda)u_1+(1+\lambda)u_2$
u_3	$-u_1$	0	$-\lambda u_1$	0	$(1-\lambda)u_3$
u_4	$-u_2$	$-u_1$	$\lambda e_1+(\lambda-1)u_1-(1+\lambda)u_2$	$(\lambda-1)u_3$	0

$\bar{\mathfrak{g}}_{18}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	u_1
u_2	0	0	0	$\frac{1}{2}u_1$	$-\frac{1}{2}e_1+\frac{1}{2}u_1+\frac{3}{2}u_2$
u_3	$-u_1$	0	$-\frac{1}{2}u_1$	0	$u_2+\frac{1}{2}u_3$
u_4	$-u_2$	$-u_1$	$\frac{1}{2}e_1-\frac{1}{2}u_1-\frac{3}{2}u_2$	$-u_2-\frac{1}{2}u_3$	0

$\bar{\mathfrak{g}}_{19}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	u_1
u_2	0	0	0	0	u_1+u_2
u_3	$-u_1$	0	0	0	u_2+u_3
u_4	$-u_2$	$-u_1$	$-u_1-u_2$	$-u_2-u_3$	0

$\bar{\mathfrak{g}}_{20}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$(1-2\lambda)e_1+2\lambda u_1$	$(2\lambda-1)u_2$
u_2	0	0	0	λu_2	$\frac{2\lambda-1}{2(\lambda-1)}e_1-\frac{1}{2(\lambda-1)}u_1$
u_3	$-u_1$	$(2\lambda-1)e_1-2\lambda u_1$	$-\lambda u_2$	0	$(\lambda-1)u_4$
u_4	$-u_2$	$(1-2\lambda)u_2$	$\frac{1-2\lambda}{2(\lambda-1)}e_1+\frac{1}{2(\lambda-1)}u_1$	$(1-\lambda)u_4$	0

$\lambda \neq 1$

$\bar{\mathfrak{g}}_{21}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$-\frac{1}{3}e_1+\frac{4}{3}u_1$	$\frac{1}{3}u_2$
u_2	0	0	0	$\frac{2}{3}u_2$	$-\frac{1}{2}e_1+\frac{3}{2}u_1$
u_3	$-u_1$	$\frac{1}{3}e_1-\frac{4}{3}u_1$	$-\frac{2}{3}u_2$	0	$u_1-\frac{1}{3}u_4$
u_4	$-u_2$	$-\frac{1}{3}u_2$	$\frac{1}{2}e_1-\frac{3}{2}u_1$	$\frac{1}{3}u_4-u_1$	0

$\bar{\mathfrak{g}}_{22}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	$2u_1$	$2u_2$
u_2	0	0	0	u_2	$e_1-\frac{1}{2}u_1$
u_3	$-u_1$	$-2u_1$	$-u_2$	0	u_4
u_4	$-u_2$	$-2u_2$	$\frac{1}{2}u_1-e_1$	$-u_4$	0

$\bar{\mathfrak{g}}_{23}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	0	0
u_3	$-u_1$	0	0	0	e_1
u_4	$-u_2$	0	0	$-e_1$	0

$\bar{\mathfrak{g}}_{24}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	0	0
u_2	0	0	0	0	0
u_3	$-u_1$	0	0	0	0
u_4	$-u_2$	0	0	0	0

$\bar{\mathfrak{g}}_{25}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	u_2
u_1	0	0	0	x	y
u_2	0	0	0	y	z
u_3	$-u_1$	$-x$	$-y$	0	0
u_4	$-u_2$	$-y$	$-z$	0	0

where

$$
\begin{aligned}
x &= \frac{\lambda\mu(\lambda-1)}{\lambda+\mu-\lambda\mu}e_1 + \frac{\lambda^2+\mu-\lambda^2\mu}{\lambda+\mu-\lambda\mu}u_1 + \frac{\lambda(1-\lambda)}{\lambda+\mu-\lambda\mu}u_2,\\
y &= -\frac{\lambda\mu}{\lambda+\mu-\lambda\mu}e_1 + \frac{\mu}{\lambda+\mu-\lambda\mu}u_1 + \frac{\lambda}{\lambda+\mu-\lambda\mu}u_2,\\
z &= \frac{\lambda\mu(\mu-1)}{\lambda+\mu-\lambda\mu}e_1 + \frac{\mu(1-\mu)}{\lambda+\mu-\lambda\mu}u_1 + \frac{\lambda+\mu^2-\mu^2\lambda}{\lambda+\mu-\lambda\mu}u_2,\\
&\lambda+\mu-\lambda\mu \neq 0.
\end{aligned}
$$

Two pairs corresponding to parameters (λ_1, μ_1) and (λ_2, μ_2) are equivalent if and only if the points (λ_1, μ_1), $(\lambda_2, \mu_2) \in \mathbb{C}^* \times \mathbb{C}^*$ lie in the same orbit of the action of the symmetric group $\mathfrak{S}_3$ on $\mathbb{C}^* \times \mathbb{C}^*$ generated by the transformations

$$(\lambda, \mu) \mapsto (\mu, \lambda); \qquad (\lambda, \mu) \mapsto \left(\frac{1}{\lambda}, -\frac{\mu}{\lambda}\right).$$

1.4

$$\mathfrak{g} = \left\{ \left. \begin{pmatrix} 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right| x \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	e_1
u_1	0	0	u_1	u_2	u_1
u_2	$-u_1$	$-u_1$	0	0	u_3
u_3	$-u_2$	$-u_2$	0	0	$-u_3$
u_4	$-e_1$	$-u_1$	$-u_3$	u_3	0

$\bar{\mathfrak{g}}_2$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	e_1
u_1	0	0	0	0	αu_1
u_2	$-u_1$	0	0	0	$(\alpha-1)u_2$
u_3	$-u_2$	0	0	0	$(\alpha-2)u_3$
u_4	$-e_1$	$-\alpha u_1$	$(1-\alpha)u_2$	$(2-\alpha)u_3$	0

$\bar{\mathfrak{g}}_3$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	e_1
u_1	0	0	0	0	$2u_1$
u_2	$-u_1$	0	0	e_1	u_2
u_3	$-u_2$	0	$-e_1$	0	0
u_4	$-e_1$	$-2u_1$	$-u_2$	0	0

$\bar{\mathfrak{g}}_4$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	u_1	u_2	0
u_2	$-u_1$	$-u_1$	0	u_3	0
u_3	$-u_2$	$-u_2$	$-u_3$	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_5$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	u_1
u_2	$-u_1$	0	0	0	u_2
u_3	$-u_2$	0	0	0	u_1+u_3
u_4	0	$-u_1$	$-u_2$	$-u_1-u_3$	0

$\bar{\mathfrak{g}}_6$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	u_1
u_2	$-u_1$	0	0	0	u_2
u_3	$-u_2$	0	0	0	u_3
u_4	0	$-u_1$	$-u_2$	$-u_3$	0

$\bar{\mathfrak{g}}_7$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	u_1	0
u_2	$-u_1$	0	0	$\alpha e_1+u_2+u_4$	0
u_3	$-u_2$	$-u_1$	$-\alpha e_1-u_2-u_4$	0	βu_4
u_4	0	0	0	$-\beta u_4$	0

$\bar{\mathfrak{g}}_8$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	u_1	0
u_2	$-u_1$	0	0	αe_1+u_2	0
u_3	$-u_2$	$-u_1$	$-\alpha e_1-u_2$	0	βu_4
u_4	0	0	0	$-\beta u_4$	0

$\bar{\mathfrak{g}}_9$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	u_1	0
u_2	$-u_1$	0	0	$\alpha e_1+u_2+u_4$	0
u_3	$-u_2$	$-u_1$	$-\alpha e_1-u_2-u_4$	0	u_1-u_4
u_4	0	0	0	u_4-u_1	0

$\bar{\mathfrak{g}}_{10}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	u_1	0
u_2	$-u_1$	0	0	αe_1+u_2	0
u_3	$-u_2$	$-u_1$	$-\alpha e_1-u_2$	0	u_1-u_4
u_4	0	0	0	u_4-u_1	0

$\bar{\mathfrak{g}}_{11}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	αe_1+u_4	0
u_3	$-u_2$	0	$-\alpha e_1-u_4$	0	u_4
u_4	0	0	0	$-u_4$	0

$\bar{\mathfrak{g}}_{12}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	αe_1	0
u_3	$-u_2$	0	$-\alpha e_1$	0	u_4
u_4	0	0	0	$-u_4$	0

$\bar{\mathfrak{g}}_{13}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	e_1+u_4	0
u_3	$-u_2$	0	$-e_1-u_4$	0	u_1
u_4	0	0	0	$-u_1$	0

$\bar{\mathfrak{g}}_{14}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	u_4	0
u_3	$-u_2$	0	$-u_4$	0	u_1
u_4	0	0	0	$-u_1$	0

$\bar{\mathfrak{g}}_{15}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	e_1+u_4	0
u_3	$-u_2$	0	$-e_1-u_4$	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_{16}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	u_4	0
u_3	$-u_2$	0	$-u_4$	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_{17}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	e_1	0
u_3	$-u_2$	0	$-e_1$	0	u_1
u_4	0	0	0	$-u_1$	0

$\bar{\mathfrak{g}}_{18}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	0	0
u_3	$-u_2$	0	0	0	u_1
u_4	0	0	0	$-u_1$	0

$\bar{\mathfrak{g}}_{19}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	e_1	0
u_3	$-u_2$	0	$-e_1$	0	0
u_4	0	0	0	0	0

$\bar{\mathfrak{g}}_{20}$	e_1	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	0
u_1	0	0	0	0	0
u_2	$-u_1$	0	0	0	0
u_3	$-u_2$	0	0	0	0
u_4	0	0	0	0	0

2.1

$$\mathfrak{g} = \left\{ \left. \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & 0 & -y \end{pmatrix} \right| x, y \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	u_1	0	$-u_3$	0
e_2	0	0	0	u_2	0	$-u_4$
u_1	$-u_1$	0	0	0	e_1	0
u_2	0	$-u_2$	0	0	0	e_2
u_3	u_3	0	$-e_1$	0	0	0
u_4	0	u_4	0	$-e_2$	0	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	u_1	0	$-u_3$	0
e_2	0	0	0	u_2	0	$-u_4$
u_1	$-u_1$	0	0	0	e_1	0
u_2	0	$-u_2$	0	0	0	0
u_3	u_3	0	$-e_1$	0	0	0
u_4	0	u_4	0	0	0	0

$\bar{\mathfrak{g}}_3$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	u_1	0	$-u_3$	0
e_2	0	0	0	u_2	0	$-u_4$
u_1	$-u_1$	0	0	0	0	0
u_2	0	$-u_2$	0	0	0	0
u_3	u_3	0	0	0	0	0
u_4	0	u_4	0	0	0	0

2.2

$$\mathfrak{g} = \left\{ \left. \begin{pmatrix} x & y & 0 & 0 \\ 0 & \lambda x & 0 & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -y & -\lambda x \end{pmatrix} \right| x, y \in \mathbb{C} \right\} \qquad \lambda \in \mathbb{C},\ |\lambda| \leqslant 1$$

$\lambda = 0$

$\bar{\mathfrak{g}}_1$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	$-u_4$	$-2e_2$
u_1	$-u_1$	0	0	0	u_2	$-u_1$
u_2	0	$-u_1$	0	0	0	u_2
u_3	u_3	u_4	$-u_2$	0	0	$2u_3$
u_4	0	$2e_2$	u_1	$-u_2$	$-2u_3$	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	e_2	u_4	0
u_2	0	$-u_1$	$-e_2$	0	$(\alpha-1)u_3$	αu_4
u_3	u_3	u_4	$-u_4$	$(1-\alpha)u_3$	0	0
u_4	0	0	0	$-\alpha u_4$	0	0

$\bar{\mathfrak{g}}_3$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	0	0
u_2	0	$-u_1$	0	0	u_3	u_4
u_3	u_3	u_4	0	$-u_3$	0	0
u_4	0	0	0	$-u_4$	0	0

$\lambda = 1$

$\bar{\mathfrak{g}}_4$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	$-u_3$	$-u_4$
e_2	0	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	e_2	0
u_2	$-u_2$	$-u_1$	0	0	e_1	e_2
u_3	u_3	u_4	$-e_2$	$-e_1$	0	0
u_4	u_4	0	0	$-e_2$	0	0

$\bar{\mathfrak{g}}_5$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	u_1	u_2	$-u_3$	$-u_4$
e_2	0	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	0	0
u_2	$-u_2$	$-u_1$	0	0	e_2	0
u_3	u_3	u_4	0	$-e_2$	0	0
u_4	u_4	0	0	0	0	0

$\lambda = \frac{1}{2}$

$\bar{\mathfrak{g}}_6$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	$\frac{3}{2}e_2$	u_1	$-\frac{1}{2}u_2$	$-u_3$	$\frac{1}{2}u_4$
e_2	$-\frac{3}{2}e_2$	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	u_4	0	0
u_2	$\frac{1}{2}u_2$	$-u_1$	$-u_4$	0	0	0
u_3	u_3	u_4	0	0	0	0
u_4	$-\frac{1}{2}u_4$	0	0	0	0	0

$\lambda \in \mathbb{C},\ |\lambda| \leqslant 1$

$\bar{\mathfrak{g}}_7$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	$(1-\lambda)e_2$	u_1	λu_2	$-u_3$	$-\lambda u_4$
e_2	$(\lambda-1)e_2$	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	0	0
u_2	$-\lambda u_2$	$-u_1$	0	0	0	0
u_3	u_3	u_4	0	0	0	0
u_4	λu_4	0	0	0	0	0

2.3

$$\mathfrak{g} = \left\{ \begin{pmatrix} x & y & 0 & x \\ 0 & -x & -x & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -y & x \end{pmatrix} \middle| \; x, y \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	u_1	$-u_2$	$-u_2-u_3$	u_1+u_4
e_2	$-2e_2$	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	0	0
u_2	u_2	$-u_1$	0	0	0	0
u_3	u_2+u_3	u_4	0	0	0	0
u_4	$-u_1-u_4$	0	0	0	0	0

2.4

$$\mathfrak{g} = \left\{ \begin{pmatrix} x & y & 0 & 0 \\ 0 & 0 & y & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \middle| \; x, y \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	u_2	0
u_1	$-u_1$	0	0	u_1	u_2	0
u_2	0	$-u_1$	$-u_1$	0	u_3	0
u_3	u_3	$-u_2$	$-u_2$	$-u_3$	0	0
u_4	0	0	0	0	0	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	u_2	0
u_1	$-u_1$	0	0	0	0	u_1
u_2	0	$-u_1$	0	0	0	u_2
u_3	u_3	$-u_2$	0	0	0	u_3
u_4	0	0	$-u_1$	$-u_2$	$-u_3$	0

$\bar{\mathfrak{g}}_3$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	e_2	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	u_1	u_2	0
u_1	$-u_1$	0	0	0	0	0
u_2	0	$-u_1$	0	0	0	0
u_3	u_3	$-u_2$	0	0	0	0
u_4	0	0	0	0	0	0

2.5

$$\mathfrak{g}=\left\{\left.\begin{pmatrix}0&x&0&y\\0&0&-y&0\\0&0&0&0\\0&0&-x&0\end{pmatrix}\right| x,y\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	$-2e_1$
e_2	0	0	0	$-2e_2$	$-u_2$	u_1
u_1	0	0	0	$2e_2-u_1$	u_2+u_4	$2e_1-u_1$
u_2	$-u_1$	$2e_2$	u_1-2e_2	0	$-2u_3$	u_2-u_4
u_3	u_4	u_2	$-u_2-u_4$	$2u_3$	0	$2u_3$
u_4	$2e_1$	$-u_1$	u_1-2e_1	u_4-u_2	$-2u_3$	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	$-2e_2$	$-u_2$	u_1
u_1	0	0	0	$-u_1$	u_4	0
u_2	$-u_1$	$2e_2$	u_1	0	$-2u_3$	$-u_4$
u_3	u_4	u_2	$-u_4$	$2u_3$	0	0
u_4	0	$-u_1$	0	u_4	0	0

$\bar{\mathfrak{g}}_3$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	u_1	0
u_2	$-u_1$	0	0	0	$e_1+\alpha e_2+(1-\beta)u_2$	βu_1
u_3	u_4	u_2	$-u_1$	$-e_1-\alpha e_2+(\beta-1)u_2$	0	$-(\alpha+\beta)e_1+\gamma e_2-(1+\beta)u_4$
u_4	0	$-u_1$	0	$-\beta u_1$	$(\alpha+\beta)e_1-\gamma e_2+(1+\beta)u_4$	0

$\bar{\mathfrak{g}}_4$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	u_1	0
u_2	$-u_1$	0	0	0	$\alpha e_2+(1-\beta)u_2$	βu_1
u_3	u_4	u_2	$-u_1$	$-\alpha e_2+(\beta-1)u_2$	0	$-(\alpha+\beta)e_1-(1+\beta)u_4$
u_4	0	$-u_1$	0	$-\beta u_1$	$(\alpha+\beta)e_1+(1+\beta)u_4$	0

$\bar{\mathfrak{g}}_5$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	$e_1+\alpha e_2-u_2$	u_1
u_3	u_4	u_2	0	$-e_1-\alpha e_2+u_2$	0	$-\alpha e_1+\beta-u_4$
u_4	0	$-u_1$	0	$-u_1$	$\alpha e_1-\beta e_2+u_4$	0

$\bar{\mathfrak{g}}_6$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	αe_2-u_2	u_1
u_3	u_4	u_2	0	$-\alpha e_2+u_2$	0	$-\alpha e_1-u_4$
u_4	0	$-u_1$	0	$-u_1$	αe_1+u_4	0

$\bar{\mathfrak{g}}_7$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	e_1+e_2	0
u_3	u_4	u_2	0	$-e_1-e_2$	0	$-e_1+\alpha e_2$
u_4	0	$-u_1$	0	0	$e_1-\alpha e_2$	0

$\bar{\mathfrak{g}}_8$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	e_1	0
u_3	u_4	u_2	0	$-e_1$	0	e_2
u_4	0	$-u_1$	0	0	$-e_2$	0

$\bar{\mathfrak{g}}_9$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	e_2	0
u_3	u_4	u_2	0	$-e_2$	0	$-e_1$
u_4	0	$-u_1$	0	0	e_1	0

$\bar{\mathfrak{g}}_{10}$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	e_1	0
u_3	u_4	u_2	0	$-e_1$	0	0
u_4	0	$-u_1$	0	0	0	0

$\bar{\mathfrak{g}}_{11}$	e_1	e_2	u_1	u_2	u_3	u_4
e_1	0	0	0	u_1	$-u_4$	0
e_2	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0
u_2	$-u_1$	0	0	0	0	0
u_3	u_4	u_2	0	0	0	0
u_4	0	$-u_1$	0	0	0	0

3.1

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & z & 0 & 0\\ 0 & y & 0 & 0\\ 0 & 0 & -x & 0\\ 0 & 0 & -z & -y\end{pmatrix}\right| x,y,z\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	0	e_3	u_1	0	$-u_3$	0
e_2	0	0	$-e_3$	0	u_2	0	$-u_4$
e_3	$-e_3$	e_3	0	0	u_1	$-u_4$	0
u_1	$-u_1$	0	0	0	0	0	0
u_2	0	$-u_2$	$-u_1$	0	0	0	0
u_3	u_3	0	u_4	0	0	0	0
u_4	0	u_4	0	0	0	0	0

3.2

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & y & 0 & z\\ 0 & \lambda x & -z & 0\\ 0 & 0 & -x & 0\\ 0 & 0 & -y & -\lambda x\end{pmatrix}\right| x,y,z\in\mathbb{C}\right\},$$

$\lambda\in\mathbb{C},\ \mathcal{R}e\lambda>0$ **or** $\mathcal{R}e\lambda=0,\ \mathcal{I}m\lambda\geqslant 0$

$\lambda=0$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	e_2	e_3	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	0	u_1	$-u_4$	$-2e_2$
e_3	$-e_3$	0	0	0	$-2e_3$	$-u_2$	u_1
u_1	$-u_1$	0	0	0	$2e_3-u_1$	u_2+u_4	$2e_2-u_1$
u_2	0	$-u_1$	$2e_3$	u_1-2e_3	0	$-2u_3$	u_2-u_4
u_3	u_3	u_4	u_2	$-u_2-u_4$	$2u_3$	0	$2u_3$
u_4	0	$2e_2$	$-u_1$	u_1-2e_2	u_4-u_2	$-2u_3$	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	e_2	e_3	u_1	0	$-u_3$	0
e_2	$-e_2$	0	0	0	u_1	$-u_4$	$-2e_2$
e_3	$-e_3$	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	0	0	0	u_2	$-u_1$
u_2	0	$-u_1$	0	0	0	0	u_2
u_3	u_3	u_4	u_2	$-u_2$	0	0	$2u_3$
u_4	0	$2e_2$	$-u_1$	u_1	$-u_2$	$-2u_3$	0

$\lambda=1$

$\bar{\mathfrak{g}}_3$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	u_1	u_2	$-u_3$	$-u_4$
e_2	0	0	0	0	u_1	$-u_4$	0
e_3	$-2e_3$	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	0	0	0	0	0
u_2	$-u_2$	$-u_1$	0	0	0	e_2	0
u_3	u_3	u_4	u_2	0	$-e_2$	0	0
u_4	u_4	0	$-u_1$	0	0	0	0

$\lambda \in \mathbb{C}$, $\mathcal{R}e\lambda > 0$ **or** $\mathcal{R}e\lambda = 0, \mathcal{I}m\lambda \geqslant 0$

$\bar{\mathfrak{g}}_4$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$(1-\lambda)e_2$	$(1+\lambda)e_3$	u_1	λu_2	$-u_3$	$-\lambda u_4$
e_2	$(\lambda-1)e_2$	0	0	0	u_1	$-u_4$	0
e_3	$-(1+\lambda)e_3$	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	0	0	0	0	0
u_2	$-\lambda u_2$	$-u_1$	0	0	0	0	0
u_3	u_3	u_4	u_2	0	0	0	0
u_4	λu_4	0	$-u_1$	0	0	0	0

3.3

$$\mathfrak{g} = \left\{ \begin{pmatrix} 0 & y & 0 & z \\ 0 & x & -z & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -y & -x \end{pmatrix} \middle| \, x, y, z \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$-e_2$	e_3	0	u_2	0	$-u_4$
e_2	e_2	0	0	0	u_1	$-u_4$	0
e_3	$-e_3$	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	u_1	0
u_2	$-u_2$	$-u_1$	0	0	0	αe_3+u_2	0
u_3	0	u_4	u_2	$-u_1$	$-\alpha e_3-u_2$	0	$-\alpha e_2-u_4$
u_4	u_4	0	$-u_1$	0	0	αe_2+u_4	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$-e_2$	e_3	0	u_2	0	$-u_4$
e_2	e_2	0	0	0	u_1	$-u_4$	0
e_3	$-e_3$	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0	0
u_2	$-u_2$	$-u_1$	0	0	0	e_3	0
u_3	0	u_4	u_2	0	$-e_3$	0	$-e_2$
u_4	u_4	0	$-u_1$	0	0	e_2	0

$\bar{\mathfrak{g}}_3$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$-e_2$	e_3	0	u_2	0	$-u_4$
e_2	e_2	0	0	0	u_1	$-u_4$	0
e_3	$-e_3$	0	0	0	0	$-u_2$	u_1
u_1	0	0	0	0	0	0	0
u_2	$-u_2$	$-u_1$	0	0	0	0	0
u_3	0	u_4	u_2	0	0	0	0
u_4	u_4	0	$-u_1$	0	0	0	0

3.4

$$\mathfrak{g} = \left\{ \left(\begin{array}{cccc} x & y & 0 & 0 \\ z & -x & 0 & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & x \end{array} \right) \middle| \; x, y, z \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	$-2e_3$	u_1	$-u_2$	$-u_3$	u_4
e_2	$-2e_2$	0	e_1	0	u_1	$-u_4$	0
e_3	$2e_3$	$-e_1$	0	u_2	0	0	$-u_3$
u_1	$-u_1$	0	$-u_2$	0	0	0	0
u_2	u_2	$-u_1$	0	0	0	0	0
u_3	u_3	u_4	0	0	0	0	0
u_4	$-u_4$	0	u_3	0	0	0	0

3.5

$$\mathfrak{g} = \left\{ \left(\begin{array}{cccc} 2x & y & 0 & y \\ z & 0 & -y & 0 \\ 0 & -z & -2x & -z \\ z & 0 & -y & 0 \end{array} \right) \middle| \; x, y, z \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	$-2e_3$	$2u_1$	0	$-2u_3$	0
e_2	$-2e_2$	0	e_1	0	u_1	$-u_2-u_4$	u_1
e_3	$2e_3$	$-e_1$	0	u_2+u_4	$-u_3$	0	$-u_3$
u_1	$-2u_1$	0	$-u_2-u_4$	0	e_2	e_1	e_2
u_2	0	$-u_1$	u_3	$-e_2$	0	e_3	0
u_3	$2u_3$	u_2+u_4	0	$-e_1$	$-e_3$	0	$-e_3$
u_4	0	$-u_1$	u_3	$-e_2$	0	e_3	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	$-2e_3$	$2u_1$	0	$-2u_3$	0
e_2	$-2e_2$	0	e_1	0	u_1	$-u_2-u_4$	u_1
e_3	$2e_3$	$-e_1$	0	u_2+u_4	$-u_3$	0	$-u_3$
u_1	$-2u_1$	0	$-u_2-u_4$	0	0	0	0
u_2	0	$-u_1$	u_3	0	0	0	0
u_3	$2u_3$	u_2+u_4	0	0	0	0	0
u_4	0	$-u_1$	u_3	0	0	0	0

4.1

$$\mathfrak{g} = \left\{ \left. \begin{pmatrix} x & z & 0 & t \\ 0 & y & -t & 0 \\ 0 & 0 & -x & 0 \\ 0 & 0 & -z & -y \end{pmatrix} \right| x, y, z, t \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	e_4	u_1	u_2	u_3	u_4
e_1	0	0	e_3	e_4	u_1	0	$-u_3$	0
e_2	0	0	$-e_3$	e_4	0	u_2	0	$-u_4$
e_3	$-e_3$	e_3	0	0	0	u_1	$-u_4$	0
e_4	$-e_4$	$-e_4$	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	0	0	0	0	0	0
u_2	0	$-u_2$	$-u_1$	0	0	0	0	0
u_3	u_3	0	u_4	u_2	0	0	0	0
u_4	0	u_4	0	$-u_1$	0	0	0	0

4.2

$$\mathfrak{g} = \left\{ \left. \begin{pmatrix} x & y & 0 & 0 \\ z & t & 0 & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & -t \end{pmatrix} \right| x, y, z, t \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	e_4	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	$-2e_4$	u_1	$-u_2$	$-u_3$	u_4
e_2	0	0	0	0	u_1	u_2	$-u_3$	$-u_4$
e_3	$-2e_3$	0	0	e_1	0	u_1	$-u_4$	0
e_4	$2e_4$	0	$-e_1$	0	u_2	0	0	$-u_3$
u_1	$-u_1$	$-u_1$	0	$-u_2$	0	0	e_1+3e_2	$2e_3$
u_2	u_2	$-u_2$	$-u_1$	0	0	0	$2e_4$	$-e_1+3e_2$
u_3	u_3	u_3	u_4	0	$-e_1-3e_2$	$-2e_4$	0	0
u_4	$-u_4$	u_4	0	u_3	$-2e_3$	e_1-3e_2	0	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	e_4	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	$-2e_4$	u_1	$-u_2$	$-u_3$	u_4
e_2	0	0	0	0	u_1	u_2	$-u_3$	$-u_4$
e_3	$-2e_3$	0	0	e_1	0	u_1	$-u_4$	0
e_4	$2e_4$	0	$-e_1$	0	u_2	0	0	$-u_3$
u_1	$-u_1$	$-u_1$	0	$-u_2$	0	0	0	0
u_2	u_2	$-u_2$	$-u_1$	0	0	0	0	0
u_3	u_3	u_3	u_4	0	0	0	0	0
u_4	$-u_4$	u_4	0	u_3	0	0	0	0

4.3

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & y & 0 & t \\ z & -x & -t & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & x \end{pmatrix}\right| x,y,z,t\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	e_4	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	$-2e_3$	0	u_1	$-u_2$	$-u_3$	u_4
e_2	$-2e_2$	0	e_1	0	0	u_1	$-u_4$	0
e_3	$2e_3$	$-e_1$	0	0	u_2	0	0	$-u_3$
e_4	0	0	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	$-u_2$	0	0	0	0	0
u_2	u_2	$-u_1$	0	0	0	0	0	0
u_3	u_3	u_4	0	u_2	0	0	0	e_4
u_4	$-u_4$	0	u_3	$-u_1$	0	0	$-e_4$	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	e_4	u_1	u_2	u_3	u_4
e_1	0	$2e_2$	$-2e_3$	0	u_1	$-u_2$	$-u_3$	u_4
e_2	$-2e_2$	0	e_1	0	0	u_1	$-u_4$	0
e_3	$2e_3$	$-e_1$	0	0	u_2	0	0	$-u_3$
e_4	0	0	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	0	$-u_2$	0	0	0	0	0
u_2	u_2	$-u_1$	0	0	0	0	0	0
u_3	u_3	u_4	0	u_2	0	0	0	0
u_4	$-u_4$	0	u_3	$-u_1$	0	0	0	0

5.1

$$\mathfrak{g}=\left\{\left.\begin{pmatrix} x & y & 0 & u \\ z & t & -u & 0 \\ 0 & 0 & -x & -z \\ 0 & 0 & -y & -t \end{pmatrix}\right| x,y,z,t,u\in\mathbb{C}\right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	e_4	e_5	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	$-2e_4$	0	u_1	$-u_2$	$-u_3$	u_4
e_2	0	0	0	0	$2e_5$	u_1	u_2	$-u_3$	$-u_4$
e_3	$-2e_3$	0	0	e_1	0	0	u_1	$-u_4$	0
e_4	$2e_4$	0	$-e_1$	0	0	u_2	0	0	$-u_3$
e_5	0	$-2e_5$	0	0	0	0	0	$-u_2$	u_1
u_1	$-u_1$	$-u_1$	0	$-u_2$	0	0	0	0	0
u_2	u_2	$-u_2$	$-u_1$	0	0	0	0	0	0
u_3	u_3	u_3	u_4	0	u_2	0	0	0	0
u_4	$-u_4$	u_4	0	u_3	$-u_1$	0	0	0	0

6.1

$$\mathfrak{g} = \left\{ \left(\begin{array}{cccc} x & y & 0 & u \\ z & t & -u & 0 \\ 0 & v & -x & -z \\ -v & 0 & -y & -t \end{array} \right) \middle| \; x, y, z, t, u, v \in \mathbb{C} \right\}$$

$\bar{\mathfrak{g}}_1$	e_1	e_2	e_3	e_4	e_5	e_6	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	$-2e_4$	0	0	u_1	$-u_2$	$-u_3$	u_4
e_2	0	0	0	0	$2e_5$	$-2e_6$	u_1	u_2	$-u_3$	$-u_4$
e_3	$-2e_3$	0	0	e_1	0	0	0	u_1	$-u_4$	0
e_4	$2e_4$	0	$-e_1$	0	0	0	u_2	0	0	$-u_3$
e_5	0	$-2e_5$	0	0	0	$-e_2$	0	0	$-u_2$	u_1
e_6	0	$2e_6$	0	0	e_2	0	$-u_4$	u_3	0	0
u_1	$-u_1$	$-u_1$	0	$-u_2$	0	u_4	0	$2e_5$	e_1+e_2	$2e_3$
u_2	u_2	$-u_2$	$-u_1$	0	0	$-u_3$	$-2e_5$	0	$2e_4$	$-e_1+e_2$
u_3	u_3	u_3	u_4	0	u_2	0	$-e_1-e_2$	$-2e_4$	0	$2e_6$
u_4	$-u_4$	u_4	0	u_3	$-u_1$	0	$-2e_3$	e_1-e_2	$-2e_6$	0

$\bar{\mathfrak{g}}_2$	e_1	e_2	e_3	e_4	e_5	e_6	u_1	u_2	u_3	u_4
e_1	0	0	$2e_3$	$-2e_4$	0	0	u_1	$-u_2$	$-u_3$	u_4
e_2	0	0	0	0	$2e_5$	$-2e_6$	u_1	u_2	$-u_3$	$-u_4$
e_3	$-2e_3$	0	0	e_1	0	0	0	u_1	$-u_4$	0
e_4	$2e_4$	0	$-e_1$	0	0	0	u_2	0	0	$-u_3$
e_5	0	$-2e_5$	0	0	0	$-e_2$	0	0	$-u_2$	u_1
e_6	0	$2e_6$	0	0	e_2	0	$-u_4$	u_3	0	0
u_1	$-u_1$	$-u_1$	0	$-u_2$	0	u_4	0	0	0	0
u_2	u_2	$-u_2$	$-u_1$	0	0	$-u_3$	0	0	0	0
u_3	u_3	u_3	u_4	0	u_2	0	0	0	0	0
u_4	$-u_4$	u_4	0	u_3	$-u_1$	0	0	0	0	0

References

[I] S. Ishihara, *Homogeneous Riemannian spaces of four dimensions*, Jour. of the Math. Soc. of Japan **7** (1955), 345–370.

[KT] B. Komrakov, A. Tchourioumov and others, *Three-dimensional isotropically-faithful homogeneous spaces*, Preprint University of Oslo (Nov. 1993).

[KTD] B. Komrakov, A. Tchourioumov, B. Doubrov, *Two-dimensional homogeneous spaces*, Preprint University of Oslo **No. 17** (June 1993).

[L1] S. Lie, *Theorie der Transformationsgruppen.* III. *Bestimmung aller Gruppen einer zweifach ausgedehnten Punktmannigfaltigkeit*, Arch. for Math., Bd. III (1878), Kristiania, 93–165.

[L2] S. Lie, *Theorie der Transformationsgruppen. Ab.* III, Teubner, Leipzig, 1893.

[M] G. Mostow, *The extensibility of local Lie groups of transformations and groups on surfaces*, Ann. of Math. **32** (1950).