ON A GENERALIZATION OF THE MACKEY INDUCTION PROCEDURE

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ABSTRACT. This paper may be regarded as a continuation of earlier studies concerning generalizations of the Mackey-Blattner induction procedure. Replacing the subgroup H of the group G of the classical procedure by a locally compact group H which acts on the same locally compact space as G and the unitary representation of H by a bounded representation on a Banach space, we construct an isometric representation of G, depending only of the equivalence class of the initial representation. We extend the Theorem on Induction in Stages and the Tensor Product Theorem in this case.

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0. Preliminaries

The first study on infinite dimensional induced representations of locally compact (second countable) groups appears in the memoir of G.W. Mackey ([12]) published in 1952. Here, G.W. Mackey, first introduced a definition of induced representations for separable locally compact groups and Hilbert spaces. Using this definition he proved four main theorems which he called: the Induction in Stages Theorem, The Tensor Product Theorem, the Subgroup Theorem and the Intertwining number Theorem. Later, R. Blattner ([2]) gave an equivalent construction for arbitrary locally compact groups.
It is well known that the process of analytic continuation of Lie group representations leads one to consider representations on Banach spaces (or even, more generally, linear system representations), ([6]). Moreover, it can occur that the induced representation is unitary, whereas the initial one is not. Thus it seems natural trying to extend the induction procedure in various directions.
The present paper is related to two previous generalizations of the classical induction procedure. The first one is due to R.A. Fontenot and I. Schochetman ([7]). They started from the ideas of F. Bruhat ([4]) thereby extending the theory of induced isometric representations in Banach spaces of H. Kraljevic ([11]) to "p-q inductible representations". The second generalization made by H. Moscovici ([14]), even if treating only unitary representations, has the advantage of replacing the subgroup H of the group G by a group H (which need not be a subgroup of G) acting on the same locally compact space X as G. In order to indicate the usefulness of his generalized induction procedure, we mention that in [15], using this method H. Moscovici was able to recover the principal series of a semisimple Lie group (take $G = KAN$, $X = G/N$, $H = MA$, M=the centralizer of A in K, in the definition below.)
In the sequel we extend the theory of "generalized induced representations" ([14]) to Banach bounded representations in the case when the "small" group H of the original representation is non-compact (the compact case was considered in [16], but only for isometric representations). We note that it is straight forward to transfer all the results of this paper to Frechet (bounded or "p-q inductible" ([7])) representations (except the Tensor Product Theorem).
After the Preliminaries, we give, in the first part, an induction procedure in the following situation: H and G are locally compact groups acting on the same locally compact space X (called an (H-G) intermediate space) and V is a bounded representation of H in a Banach space, cf. Theorem 1.2. In the second section we state the Theorem on Induction in Stages and the Tensor Product Theorem for this induction procedure. We closely follow the classical proofs, emphasizing the technical modifications involved in this new situation.

Next, we shall recall some terminology and results required by our discussion and we shall establish some new notations. Throughout this paper, G and H will be locally compact groups. Let m_G (resp. m_H) be a (left) Haar measure for G (resp. H) and Δ_G (resp. Δ_H) the coresponding modular function. As in ([14]) we give the definition of an $(H-G)$ intermediate space.

Definition. *A locally compact Hausdorff space X is called $(H-G)$ intermediate if the following conditions are fulfilled:*

(1) *H acts continuously and properly by right translations on X and X/H is paracompact (the H-action is denoted by θ)*
(2) *G acts continuously by left translations on X (the G-action is denoted by φ)*
(3)
$$\varphi(g)\theta(h) = \theta(h)\varphi(g), \quad (g \in G,\ h \in H)$$
(4) *There exists a Radon measure m on X such that*
$$\varphi(g)m = m \ , \ (g \in G)$$
$$\theta(h)m = \Delta(h)m \ , \ (h \in H)$$
(i.e. m is G-invariant and H-relatively invariant with continuous multiplier Δ)

In the sequel, if needed, for showing that H and G (or only H) act on X, we shall use the notation ${}_G X_H$ (resp. X_H) = X.
Let $(E, \| \ \|)$ be a Banach space. As usual $\mathcal{C}(X, E)$ (resp. $\mathcal{C}_0(X, E)$) is the space of all continuous functions from X to E (resp. which are with compact support). If $E = \mathbb{R}$ we shall put only $\mathcal{C}(X)$ (resp. $\mathcal{C}_0(X)$).
We include here a result concerning the functions of $\mathcal{C}(X, E)$.

Lemma 1. *Let $f \in \mathcal{C}(X, E)$ and K be a compact subset of X. Then, for every $\varepsilon > 0$, there exists an open simmetric relatively compact neighbourhood W of the identity of G such that*
$$\| f(ak) - f(k) \| < \varepsilon, \quad \forall k \in K,\ a \in W$$

Proof. Let k be an arbitrary element of K. By the continuity of f, it follows that there exists an open neighbourhood U_k of k such that
$$\| f(x) - f(k) \| < \frac{\varepsilon}{2}, \quad \forall x \in U_k$$
The application $(g, x) \longmapsto gx$ from $G \times X$ to X being continuous in (e, k), we can find a symmetric relatively compact neighbourhood W_k of the identity in G and an open neighbourhood of k (denoted here also by U_k) such that
$$W_k U_k \subset U_k$$
The family $(U_k)_{k \in K}$ is an open covering of K, thus there exists a finite subcovering $(U_{k_i})_{\{1,2,\ldots,n\}}$ for K. If $W = \cap_{i=1}^{n} W_{k_i}$, it is clear that W is an (open) symmetric, relatively compact neighbourhood of the identity of G. Let a be in W, so a is in $W_{k_i}, \forall i = 1, 2 \ldots n$. On the other hand, for every k in K, $\exists j_0 \in \{1, 2, \ldots, n\}$ such that $k \in U_{k_{j_0}}$.Then
$$ak \in W_{k_{j_0}} U_{k_{j_0}} \subset U_{k_{j_0}}$$
and
$$k \in U_{k_{j_0}} \subset W_{k_{j_0}} U_{k_{j_0}} \subset U_{k_{j_0}}$$

Consequently, for $a \in W$ and $k \in K$ we have

$$\| f(ak) - f(k) \| \leq \| f(ak) - f(ak_{j_0}) \| + \| f(ak_{j_0}) - f(k) \| < \varepsilon$$

Now, let $X,\ H$ be as above. If $\pi : X \longrightarrow X/H$ is the canonical surjection, we shall put for $x \in X$, $\dot{x} = \pi(x)$. We shall also use the notation $KH = \pi^{-1}(\pi(K))$ for any (compact) subset K of X. It is well known that for any compact subset $\breve{K}$ of X/H, there exists a compact subset K in X such that $\pi(K) = \breve{K}$.
We also recall that, by virtue of the statements of [3] (Ch.VII, §2), there exist two functions on X and a measure on X/H (for convenience denoted by ρ, β and $\breve{m}$, resp.) such that:

(1) The function β (called a Bruhat function on X_H) is a continuous positive function on X whose support has the property that $supp\beta \cap KH$ is compact for any compact subset K of X and $\beta^\flat \equiv 1$ (that is $\int_H \beta(xh)dm_H(h) = 1, \forall x \in X$).

(2) The function ρ (called a ρ-function on X_H) is a continuous positive function on X with the properties

$$\rho(xh) = \frac{\Delta_H}{\Delta}(h)\rho(x), \quad \forall x \in X, h \in H$$

and

$$\int_{X/H} (\int_H f(xh)dm_H(h))d\breve{m}(\dot{x}) = \int_X f(x)\rho(x), \quad \forall f \in \mathcal{C}_0(X)$$

(3)

$$\rho(x) = \int_{X/H} \frac{\Delta}{\Delta_H}(h)\beta(xh)dm_H(h), \quad \forall x \in X$$

Lemma 2. *Let ${}_GX_H$ be an $(H-G)$ intermediate space and $a \in G$. Then*

$$\int_{X/H} (\int_H f(xh)dm_H(h))d\breve{m}(a\dot{x}) =$$

$$\int_{X/H} (\int_H f(xh)dm_H(h)\frac{\rho(ax)}{\rho(x)}d\breve{m}(\dot{x}), \quad \forall f \in \mathcal{C}_0(X).$$

Proof. Let f be arbitrary in $\mathcal{C}_0(X)$. Then:

$$\int_{X/H} (\int_H f(xh)dm_H(h))d\breve{m}(a\dot{x}) = \int_{X/H} (\int_H f(a^{-1}yh)dm_H(h))d\breve{m}(\cdot y) =$$

$$= \int_X f(a^{-1}y)\rho(y)dm(y) = \int_X f(z)\rho(az)dm(z) = \int_X f(z)\frac{\rho(az)}{\rho(z)}\rho(z)dm(z) =$$

$$= \int_{X/H} (\int_H f(zh)\frac{\rho(azh)}{\rho(zh)}dm_H(h))d\breve{m}(\dot{z}) = \int_{X/H} (\int_H f(xh)dm_H(h))\frac{\rho(ax)}{\rho(x)}d\breve{m}(\dot{x})$$

Finally, if E is a Banach space, let $\mathcal{L}(E)$ be the bounded invertible operators on E.

Definition. *A bounded representation V of H on the Banach space E is a homomorphism of H into $\mathcal{L}(E)$ which is strongly continuous and such that there exists a $c > 0$ with*

$$\| V(h) \| \leq c, \quad \forall h \in H$$

Remarks. 1. It is well known that the strong continuity of V is equivalent to weak continuity.
2. It is obvious that any isometric representation is a bounded representation.

Two representations V_i $(i = 1,2)$ of H on the Banach spaces E_i $(i = 1,2)$ are said to be equivalent if there exists an invertible bounded operator from E_1 to E_2 such that $V_1(h)T = TV_2(h)$, $\forall h \in H$ (called an intertwining operator of V_1 and V_2). If, in addition, T is an isometry, the representations V_1 and V_2 will be called isometrically equivalent ($V_1 \cong V_2$).

1. P-Induced representations

Assume that $p \in [1, \infty]$. We shall define $(p - V)$ homogeneous functions on X, where X is an $(H - G)$ intermediate space and V a bounded representation of H on the Banach space E. By starting from $(p - V)$ homogeneous functions, we shall construct (for any $p \in [1, \infty]$) an induced Banach representation by V up to G.

Definition. *A function $f : X \longrightarrow E$ is called $(p - V)$ homogeneous ([7]) if*

$$f(xh) = (\frac{\Delta_H}{\Delta}(h))^{\frac{1}{p}} V(h)f(x), \quad x \in X, h \in H$$

Let $\mathcal{C}_p^V(X, E)$ be the space of all functions $f : X \longrightarrow E$ with the following properties:

(1) f is $(p - V)$ homogeneous
(2) f is continuous
(3) f has compact support modulo H (i.e. $\pi(suppf)$ is compact).

Theorem 1.1. *With the previous notations, the next sets equality holds*

$$\mathcal{C}_p^V(X,E) = \{f_V : X \longrightarrow E | f_V(x) = \int_H (\frac{\Delta}{\Delta_H})^{\frac{1}{p}} V(h^{-1}) f(xh) dm_H(h), f \in \mathcal{C}_0(X,E)\}$$

Proof. First, we shall verify that f_V is a continuous, $(p - V)$ homogeneous function with compact support modulo H.
(1) For $x \in X$ and $h_1 \in H$, we have

$$f_V(xh_1) = \int_H (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}} V(h^{-1}) f(xh_1h) dm_H(h)$$

$$= \int_H (\frac{\Delta}{\Delta_H}(h_1^{-1}))^{\frac{1}{p}} (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}} V(h^{-1}h_1) f(xh) dm_H(h)$$

$$= (\frac{\Delta_H}{\Delta}(h_1))^{\frac{1}{p}} V(h_1)(\int_H (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}} V(h^{-1})f(xh)dm_H(h)) = (\frac{\Delta_H}{\Delta}(h_1))^{\frac{1}{p}} V(h_1)f(x)$$

So, as we easily have seen, the function f_V is $(p-V)$ homogeneous.
(2) Fix x_0 in X and $\varepsilon > 0$. Let f_{x_0} be the function on H to E defined by $f_{x_0} = f \circ L_{x_0}$, (where $L_{x_0} : H \longrightarrow X$ is the right translation in x_0, $L_{x_0}(h) = x_0 h$). From the fact that H acts properly on X and from the equality

$$suppf_{x_0} = L_{x_0}^{-1}(suppf)$$

it follows that $suppf_{x_0}$ is compact.
Then, if Z is a fixed compact neighbourhood of the identity e_H in H, there exists W a neighbourhood of e_H contained in Z such that

$$\| f(x_0h) - f(x_0l) \| < \frac{\varepsilon}{c \cdot m_H(A_{x_0}) \cdot \alpha_{x_0}}, \quad h, l \in H, \quad hl^{-1} \in W$$

(where $A_{x_0} = suppf_{x_0} \cdot Z$, so compact and $\alpha_{x_0} = \sup\limits_{h \in A_{x_0}} (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}}$).
Let U be the neighbourhood of x_0 in X defined by $U = x_0 W$. It is obvious to see that for any $y \in U$ and $h \notin A_{x_0}$, $f(yh) = 0$. Then, for arbitrary y in U, we have:

$$\| f_V(y) - f_V(x_0) \| = \| \int_H (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}} V(h^{-1})(f(yh) - f(x_0h))dm_H(h) \|$$

$$\leq \int_{A_{x_0}} (\frac{\Delta}{\Delta_H}(h))^{\frac{1}{p}} \cdot c \cdot \| f(yh) - f(x_0h) \| dm_H(h)$$

$$\leq \alpha_{x_0} \cdot c \cdot \frac{\varepsilon}{c \cdot m_H(A_{x_0}) \cdot \alpha_{x_0}} \cdot m_H(A_{x_0}) = \varepsilon$$

Hence, the continuity of f_V is proved.
(3) In order to see that the support of f_V is compact modulo H, we observe that the inclusion

$$\pi(suppf_V) \subseteq \pi(suppf)$$

easily holds.
Conversely, let f be in $\mathcal{C}_p^V(X, E)$. Then, there exists a compact subset B of X such that $\pi(B) = \pi(supp\ f)$; moreover, it is clear that BH is a (nonvoid) closed set. If ϕ is a positive continuous function on X such that $\phi|_B = 1$, we denote by D the open subset of X, $\{ x \in X \mid \int_H \phi(xh)dm_H(h) > 0 \}$.
First, we prove that BH is contained in D. Indeed, let x be an element in BH. Thus, there exists $b \in B$ and $h_1 \in H$ such that $xh_1 = b$. By the properties of ϕ, it follows there exists a neighbourhood of $b = xh_1$ such that $\phi|_V > 0$. Now, using the continuity of the application $(y, h) \longmapsto yh$ (of $X \times H$ to X) in xh_1, we find that there exists a compact neighbourhood of h_1 in H, denoted by W such that $xW \subset V$. Then, we have

$$0 < \int_W \phi(xh)dm_H(h) < \int_H \phi(xh)dm_H(h)$$

and that means $x \in D$.
Now, we define the function θ from X to E by

$$\theta(x) = \begin{cases} \dfrac{f(x)}{\int_H \phi(xh)dm_H(h)} & \text{for} \quad x \in D \\ 0 & \text{for} \displaystyle\int_H \phi(xh)dm_H(h) = 0 \end{cases}$$

Using the facts "$X = D \cup \complement(BH)$", "$D$ and $\complement(BH)$ are open sets", "$\theta|_D$ and $\theta|_{\complement(BH)}$ are continuous" it follows that θ is continuous on X. By taking

$$g = \phi \cdot \theta$$

it is clear that g is continuous and with compact support (because *supp* $g \subset$ *supp* ϕ).
It is straightforward now to see that $f = g_V$.
The theorem is proved.

Next we introduce a p-norm on $\mathcal{C}_p^V(X,E)$. First, we observe that if we fix $f \in \mathcal{C}_p^V(X,E)$, for every $x \in X$ the function $h \longmapsto \| f(xh) \| / \rho(xh)^{\frac{1}{p}}$ is bounded on H. Then, we define a positive application on X/H, denoted by $F_p(f)$, by

$$F_p(f)(\dot{x}) = \sup_{h \in H} \frac{\| f(xh) \|}{\rho(xh)^{\frac{1}{p}}}$$

It is clear that $F_p(f)$ is well defined.

Proposition 1.1. *For every $f \in \mathcal{C}_p^V(X,E)$, the following statements hold:*

1. *$suppF_p(f) \subset \pi(suppf)$*
2. *$F_p(f_a)(\dot{x}) = (\frac{\rho(ax)}{\rho(x)})^{\frac{1}{p}} F_p(f)(\dot{ax})$, $\forall a \in G$, $\dot{x} \in X/H$, (where for $a \in G$, the function f_a on X is defined by $f_a(x) = f(ax)$).*
3. *$F_p(f)$ is bounded on X/H.*

Proof. (1) Straightforward.
(2) For $a \in G$ and $\dot{x} \in G/H$, we have

$$F_p(f_a)(\dot{x}) = \sup_{h \in H} \frac{\| f(a(xh)) \|}{\rho(xh)^{\frac{1}{p}}} = \sup_{h \in H} \frac{\| f(a(xh)) \|}{(\frac{\rho(x)}{\rho(ax)})^{\frac{1}{p}} \rho^{\frac{1}{p}}(a(xh))}$$

$$= (\frac{\rho(ax)}{\rho(x)})^{\frac{1}{p}} \sup_{h \in H} \frac{\| f((ax)h) \|}{\rho^{\frac{1}{p}}((ax)h)} = (\frac{\rho(ax)}{\rho(x)})^{\frac{1}{p}} F_p(f)(\dot{ax})$$

(3) It is clear if one use (1).

Remark. From the fact that the application $F_p(f)(\cdot)$ is the supremum of the family of continuous functions, $(\frac{f(\cdot h)}{\rho(\cdot h)^{\frac{1}{p}}})_{h\in H}$, it follows that $F_p(f)(\cdot)$ is lower semicontinuous. Thus, for $1 \leq p < \infty$ we may define

$$\| f \|_p = (\int_{X/H} F_p(f)(\dot{x})^p d\breve{\mu}(\dot{x}))^{\frac{1}{p}}$$

For $p = \infty$, using once again the previous proposition, we may put

$$\| f \|_\infty = \sup_{x\in X} F_\infty(f)(\dot{x})$$

Proposition 1.2. *For every $p \in [1,\infty]$, the application $f \longmapsto \| f \|_p$ on $\mathcal{C}_p^V(X,E)$ is a norm with the property*

$$\| f_a \|_p = \| f \|_p, \quad \forall a \in G.$$

Proof. It is easy to verify that $F_p : \mathcal{C}_p^V(X,E) \longrightarrow [0,\infty)$ is a norm. Consequently, we shall check only the mentioned property. Let a be in G. If $p \in [1,\infty)$, with Lemma 2 (Preliminaries), we have:

$$\| f_a \|_p^p = \int_{X/H} (F_p(f_a)(\dot{x}))^p d\breve{m}(\dot{x}) = \int_{X/H} F_p(f)(a\dot{x})^p \frac{\rho(ax)}{\rho(x)} d\breve{m}(\dot{x})$$

$$= \int_{X/H} F_p(f)(a\dot{x})^p d\breve{m}(a\dot{x}) = \| f \|_p^p$$

If $p = \infty$ we have:

$$\| f_a \|_\infty = \sup_{x\in X} \sup_{h\in H} \| f(axh) \|$$

$$= \sup_{ax\in X} \sup_{h\in H} \| f(axh) \| = \| f \|_\infty$$

Notation. *For any $p \in [1,\infty]$, let $B_V^p(X,E)$ be the completion of $\mathcal{C}_p^V(X,E)$ with respect to the norm $\| \cdot \|_p$. For $a \in G$ we define on $\mathcal{C}_p^V(X,E)$ the application $U_p(G)(a)$ by*

$$U_p(G)(a)(f) = f_{a^{-1}}$$

which is clearly continuous. Consequently, we may consider the extension of $U_p(G)(a)$ to $B_V^p(X,E)$, which will be also denoted by $U_p(G)(a)$.

Theorem 1.2. *The mapping $a \longmapsto U_p(G)(a)$ on G is an isometric representation of G on the Banach space $B^p_V(X,E)$.*
For fixed $p \in [1,\infty]$, this representation is named the p-induced representation of the bounded representation V of the group H in the Banach space E and is denoted by $\underset{H\nearrow G^p}{ind}{}^X V$.

Proof. It is obvious that the application $a \longmapsto U_p(G)(a)$ is a group homomorphism. With the previous proposition it is also clear that $\forall a \in G$, $U_p(G)(a)$ is an isometry. We need only prove the (strong) continuity of the application $a \longmapsto U_p(G)(a)$. To do this, it is enough to show that if $f \in \mathcal{C}^V_p(X,E)$, for any $\varepsilon > 0$, there exists W, a neighbourhood of the identity e_G of G, such that:

$$\| U_p(G)(a)(f) - f \|_p < \varepsilon, \quad \forall a \in W$$

Therefore, let f be in $\mathcal{C}^V_p(X,E)$. First, we remark that if Z is a fixed symmetric compact neighbourhood of e_G, the set $\pi(Z(suppf)H)$ is compact in X/H. (Indeed, if we consider the application on $G \times X/H$ to X/H, $(g,x) \longmapsto g\dot{x}$, the set $\pi(Z(suppf)H)$ is the immage of the compact set $Z \times \pi(suppf)$ under this continuous application.) Consequently, there exists a compact K in X such that $KH = Z(suppf)H$. It results that for every $a \in Z$, $supp(f_{a^{-1}} - f) \subset KH$.
Now, let $\varepsilon > 0$ be. By virtue of Lemma 1 (Preliminaries), we obtain an open symmetric (relatively compact) neighbourhood W of e_G, $W \subset Z$ such that:

$$\| f(a^{-1}x) - f(x) \| < \varepsilon, \quad \forall x \in K, \ a \in W$$

Let p be in $[1,\infty)$. For $a \in W$, we have

$$\begin{aligned}\| U_p(G)(a)(f) - f \|_p^p &= \int_{X/H} (\sup_{h\in H} \frac{\| f(a^{-1}xh) - f(xh) \|}{\rho(xh)^{\frac{1}{p}}})^p d\breve{m}(\dot{x}) \\ &\leq c^p \int_{\pi(K)} \rho(x)^{-1} \| f(a^{-1}x) - f(x) \|^p \, d\breve{m}(\dot{x}) \\ &\leq c^p \cdot \inf_{x\in K} \rho(x) \cdot \breve{m}(\pi(K)) \cdot \varepsilon\end{aligned}$$

If $p = \infty$,

$$\begin{aligned}\| U_p(G)(a)(f) - f \|_\infty &= \sup_{x\in X} \sup_{h\in H} \| f(a^{-1}xh) - f(xh) \| \\ &\leq c \cdot \sup_{x\in K} \| f(a^{-1}x) - f(x) \| < \varepsilon \cdot c, \quad \forall a \in W\end{aligned}$$

With the previous calculus, the theorem is proved.

Remarks. 1. It is obvious to show that (in the spite of the fact that in appearance the definition of $\underset{H\nearrow G^p}{ind}{}^X V$ depends of the choice of the function ρ in X) up to isometric equivalence, $\underset{H\nearrow G^p}{ind}{}^X V$ is independent of the choice of ρ.
2. As it was expected, in the case where H is a closed subgroup of G, we refind the usual definitions of induced representations in Banach spaces ([11],[7]) by taking $X = G$.

The next result shows that the representation $\underset{H\nearrow G^p}{ind}{}^X V$ depends only on the equivalence class of the original representation.

Proposition 1.3. *Let V_i be two (isometrically) equivalent representations of the group H on the Banach spaces E_i, $(i = 1, 2)$. Then, the representations $\underset{H\nearrow G^p}{ind}{}^X V_i$, $(i = 1, 2)$ are also (isometrically) equivalent.*

Proof. It is appearent.

In order to simplify some parts of the proofs of the main theorems of the next section, we shall use a certain density property of the set functions $\{\epsilon_V(\phi, \xi) \mid \phi \in \mathcal{C}_0(X), \xi \in E\}$, where, for $\phi \in \mathcal{C}_0(X)$ and $\xi \in E$, the function $\epsilon_V(\phi, \xi)$ is defined by

$$\epsilon_V(\phi, \xi) = \int_H (\frac{\Delta_H}{\Delta}(h))^{\frac{1}{p}} \phi(xh) V(h) \xi dm_H(h)$$

(It is easy to see that $\epsilon_V(\phi, \xi) \in \mathcal{C}_p^V(X, E), \forall \phi \in \mathcal{C}_0(X), \xi \in E.$)
The proof of the next lemma is analogous to [11] (Theorem 1(c)).

Lemma 1.1. *If D is a total subset of the space E, then $\epsilon_V(\mathcal{C}_0(X), D)$ is total in $B_V^p(X, E)$.*

Finally, we include here another result which will be used also in the next section. This is similar to the classical one (Theorem 5.10,[7]).

Theorem 1.3. *If X is second countable and V an isometric representation of H on the separable Banach space E, then , for each p in $[1, \infty)$, the space $B_V^p(X, E)$ is isometrically isomorphic to $L^p(X/H, E)$.*

2.The Induction in Stages and Tensor Product Theorem

Let G_1, G_2, G be locally compact groups, X_{12} a $(G_1 - G_2)$ intermediate space and X_2 a $(G_2 - G)$ intermediate space. Assume V is a bounded representation of G_1 on the Banach space E. With the previous induction procedure we can construct the isometric representation of $\underset{G_1\nearrow G_2{}^p}{ind}{}^{X_{12}} V$ of G_2 and, also the induced "in stages" isometric representation $\underset{G_2\nearrow G^p}{ind}{}^{X_2} \underset{G_1\nearrow G_2{}^p}{ind}{}^{X_{12}} V$ of G. It is natural to pose the question if there exists a $G - G_1$ space X_1 such that the representation $\underset{G_1\nearrow G^p}{ind}{}^{X_1} V$ is equivalent with $\underset{G_2\nearrow G^p}{ind}{}^{X_2} \underset{G_1\nearrow G_2{}^p}{ind}{}^{X_{12}} V$. In the classical induction procedure, when $G_1 \subset G_2 \subset G$, $X_{12} = G_2$, $X_2 = G$, by taking X_1 equal to G, the well-known Theorem on Induction in Stages states that the representations $\underset{G_1\nearrow G_2}{ind} \underset{G_2\nearrow G}{ind} V$ and $\underset{G_1\nearrow G}{ind} V$ are equivalent, even in the case of representations on Banach spaces ([7],[11]). The theorem on induction in stages was also proved (for the above generalized induction procedure) in the particular case of unitary representations in [14] and of isometric representations only when G_1, G_2 are compact groups in [17]. Here, we state that this theorem holds also for bounded representations on Banach spaces without the compactness hypothesis on the groups G_1, G_2.
Let G_1, G_2, G be locally compact groups with left Haar measures m_{G_1}, m_{G_2}, m_G. Suppose X_{12}, (resp. X_2) is a $(G_2 - G_1)$, (resp. $(G - G_2)$) intermediate space equipped with the measure m_{12}, (resp. m_2), relatively invariant under the G_1 action, (resp. G_2 action) and with the continuous multiplier Δ_1, (resp. Δ_2). We shall construct a $(G - G_1)$ intermediate

space, denoted by X_1 (like in [14]). This space will be equipped with a measure m_1 which, assuming that X_2/G_2 is paracompact, will be invariant under the G action and relatively invariant under the G_1 action with the same multiplier as the measure m_{12} of X_{12}, that is Δ_1 ([14],Prop. 2.1). Consequently, defining the (left) continuous action of the group G_2 on the space $X_2 \times X_{12}$ by $\tau(g)(x,y) = (xg^{-1}, gy)$, $(g \in G_2, x \in X_2, y \in X_{12})$, let X_1 be the orbit space of $X_2 \times X_{12}$ with respect to this action ($X_1 = X_2 \times X_{12}/G_2$). The elements of X_1 are denoted by $x \circ y = q(x,y)$, where $q : X_2 \times X_{12} \longrightarrow X_1$ is the canonical surjection. We define the left (continuous) action of G on X_1 by $\varphi(s, x \circ y) = (sx, y)$, $(s \in G, x \circ y \in X_1)$ and the right (continuous) action of G_1 on X_1 by $\theta(s, x \circ y) = x \circ yh$, (actions for which we shall further use the notations $s(x \circ y) = \varphi(s, x \circ y)$, resp. $(x \circ y)h = \theta(h, x \circ y)$). We notice that X_1 is a $G - G_1$ intermediate space, the Radon measure on X_1, invariant under the G action and relatively invariant under the G_1 action with continuous multiplier Δ_1, being $m_2 \times m_{12}/m_{G_2}$ ([3], §2, Prop.4), which here will be denoted by m_1. By virtue of the results of [14] (§ 2), there exists on $X_2 \times X_{12}$ a continuous strictly positive function r such that

(1)
$$r(s^{-1}x, yh) = r(x,y), \quad \forall (x,y) \in X_2 \times X_{12}, s \in G, h \in G_1$$

(2)
$$r(xg, g^{-1}y) = \frac{\Delta_{G_2}}{\Delta_2}(g) r(x,y), \quad \forall (x,y) \in X_2 \times X_{12}, g \in G_2$$

(3)
$$\int_{X_2 \times X_{12}} frd(m_2 \times m_{12}) = \int_{X_1} dm_1(x \circ y) \int_{G_2} f(xg, g^{-1}y) dm_{G_2}(g), \quad \forall f \in \mathcal{C}_0(X_2 \times X_{12})$$

We shall observe that if β_2 is a Bruhat function on $X_{2_{G_2}}$, and β_{12} is a Bruhat function on $X_{12_{G_1}}$, then the positive function γ definite by

$$\gamma(x \circ y) = \int_{G_2} \beta_2(xg) \beta_{12}(g^{-1}y) dm_{G_2}(g)$$

is a Bruhat function on $X_{1_{G_1}}$.
Indeed, by its definition, it is clear that γ is continuous such that the intersection of its support with any set of the form KG_1 (K compact in X_1) is a compact set. We also have:

$$\int_{G_1} \gamma((x \circ y)h) dm_{G_1}(h) = \int_{G_1} (\int_{G_2} \beta_2(xg) \beta_{12}(g^{-1}yh) dm_{G_2}(g)) dm_{G_1}(h)$$

$$= \int_{G_2} \beta_2(xg) dm_{G_2}(g) \cdot \int_{G_1} \beta_{12}(g^{-1}yh) dm_{G_1}(h) = 1$$

In the sequel we preserve the above notations and, in addition ρ_{12}, (resp. ρ) will be a ρ-function on $X_{12_{G_1}}$ (resp. $X_{1_{G_1}}$).

We are now in a position to state and prove the theorem of "induction on stages".

Theorem 2.1. *If V is a bounded representation of the group G_1 on the Banach space E, X_{12} is a $(G_1 - G_2)$ intermediate space, X_2 is a $(G_2 - G)$ intermediate space, then, there exists a $(G_1 - G)$ intermediate space, X_1, such that, for any $p \in [1, \infty]$ the representation $\underset{G_1 \nearrow G^p}{ind}{}^{X_1} V$ is isometrically equivalent to $\underset{G_2 \nearrow G^p}{ind}{}^{X_2} \underset{G_1 \nearrow G_2{}^p}{ind}{}^{X_{12}} V$.*

Proof. Let X_1 be the space constructed as above, starting from the spaces X_{12}, X_2. To make the notations easier, we shall use ν for the representation $\underset{G_1 \nearrow G^p}{ind}{}^{X_1} V$, λ for $\underset{G_1 \nearrow G_2{}^p}{ind}{}^{X_{12}} V$ and χ for $\underset{G_2 \nearrow G^p}{ind}{}^{X_2} \lambda$. In order to prove that $\nu \cong \chi$ it is enough to define an intertwining isometry $A \in \mathcal{L}(B_V^p(X_1, E), \mathcal{B}_p^\lambda(X_2, B_V^p(X_{12}, E)))$ only for the elements of $\mathcal{C}_p^V(X_1, E)$. Therefore, for $f \in \mathcal{C}_p^V(X_1, E)$ and $x \in X_2$ we define on X_{12} the function $\hat{f}(x)$ by

$$\hat{f}(x)(y) = r(x, y)^{\frac{1}{p}} f(x \circ y)$$

Basically, by using the facts "X_1 equipped with the measure m_1 has the same multiplier Δ_1 as the measure m_{12} (on X_{12})" and "any compact in an homogeneous space is the image by the canonical surjection of a compact", it can be shown that $\hat{f}(x) \in \mathcal{C}_p^V(X_{12}, E)$. Hence, for any $f \in \mathcal{C}_p^V(X_1, E)$ we can define an application u_f on X_2 to $\mathcal{C}_p^V(X_{12}, E)$ by

$$u_f(x) = \hat{f}(x)$$

We state that $u_f \in \mathcal{C}_p^\lambda(X_2, B_V^p(X_{12}, E))$. It is straitghforward that u_f is $(p - \lambda)$ homogeneous and *supp* u_f is compact modulo G_2. In order to prove the continuity of u_f on X_2, let x_0 be fixed in X_2 and for arbitrary x in X_2, let us compute $\| u_f(x) - u_f(x_0) \|_p^p$. First, we consider $p \in [1, \infty)$.Then:

$$\| u_f(x) - u_f(x_0) \|_p^p = \int_{X_{12}/G_1} (\sup_{h \in G_1} \frac{\| (u_f(x) - u_f(x_0))(yh) \|}{\rho_{12}^{\frac{1}{p}}(yh)})^p \, d\check{m}_{12}(\dot{y})$$

$$= \int_{X_{12}/G_1} \rho_{12}^{-1}(y) (\sup_{h \in G_1} (\frac{\Delta_{G_1}}{\Delta_G}(h))^{-\frac{1}{p}} \| (u_f(x) - u_f(x_0))(yh) \|)^p d\check{m}_{12}(\dot{y})$$

$$\leq c^p \cdot \int_{X_{12}/G_1} \rho_{12}^{-1}(y) r(x, y) \| f(x \circ y) - f(x_0 \circ y) \|^p \, dm_{12}(y)$$

$$= c^p \cdot \int_{X_{12}/G_1} (\int_{G_1} \beta_{12}(yh) \rho_{12}^{-1}(y) r(x, y) \| f(x \circ y) - f(x_0 \circ y) \|^p \, dm_{G_1}(h)) d\check{m}_{12}(\dot{y})$$

$$= c^p \cdot \int_{X_{12}} \beta_{12}(y) r(x, y) \| f(x \circ y) - f(x_0 \circ y) \|^p \, dm_{12}(y)$$

For $x_0 \in X_{12}$, there exists a compact set $K_{12} \subset X_{12}$, such that *supp* $\hat{f}_{x_0} = K_{12} G_1$. Let $C =$ *supp* $\beta_{12} \cap K_{12} G_1$ be, therefore C is a compact set. For $\varepsilon > 0$ and $z \in C$, from the continuity in (x_0, z) of the application on $X_2 \times X_{12}$, $(x, y) \longmapsto r(x, y) f(x \circ y)$ it follows there exists a neighbourhood U_z of (x_0, z) such that, if $(x, y) \in U_z$

$$r(x, y)^{\frac{1}{p}} \| f(x \circ y) - f(x_0 \circ z) \| < \varepsilon^{\frac{1}{p}}$$

By taking into account that the projection on X_{12} of U_z denoted by U_{12}^z is a neighbourhood of z in the space X_{12}, by the compacity of $C \subset \bigcup_{z\in C} U_{12}^z$, it results that there exist $z_1, \ldots z_n \in C$ with $C \in \bigcup_{i=1}^n U_{12}^{z_i}$. We put $\breve{U} = \bigcap_{i=1}^n U_{z_i}$, where U_{z_i} is a neighbourhood of (x_0, z_i) such that the projection on X_{12} of U_{z_i} be $U_{12}^{z_i}$, $(i = 1, \ldots, n)$. Now, if U is the projection on X_{12} of $\breve{U}$ which is clearly a neighbourhood of x_0 in X_{12}, we have for any $y \in U$

$$r(x,y) \parallel f(x \circ y) - f(x_0 \circ y) \parallel^p < \varepsilon$$

For any $y \in U$, we obtain:

$$\parallel u_f(x) - u_f(x_0) \parallel_p^p \leq c^p \int_C \beta_{12}(y) r(x,y) \parallel f(x \circ y) - f(x_0 \circ y) \parallel^p dm_{12}(y)$$

$$\leq c^p \cdot \sup_{y\in C} \beta_{12}(y)\ \varepsilon.$$

It follows that for $p \in [1, \infty)$, $u_f \in \mathcal{C}_p^\lambda(X_2, B_V^p(X_{12}, E))$.
If $p = \infty$, by the relations

$$\parallel u_f(x) - u_f(x_0) \parallel_\infty = \sup_{y\in X_{12}} (\sup_{h\in G_1} \parallel (u_f(x) - u_f(x_0))(yh) \parallel)$$

$$\leq c \sup_{y\in X_{12}} \parallel f(x \circ y) - f(x_0 \circ y) \parallel$$

it results also (with arguments similar to the above) the continuity of u_f.
We define the linear operator A from $\mathcal{C}_p^V(X, E)$ to $\mathcal{C}_p^\lambda(X_2, B_V^p(X_{12}, E))$,

$$A(f) = u_f$$

We shall prove that A is an isometry. First, suppose $p \in [1, \infty)$. Then:

$$\parallel A(f) \parallel_p^p = \int_{X_2/G_2} \rho_2^{-1}(x) (\sup_{g\in G_2} (\frac{\Delta_{G_2}}{\Delta_2}(g))^{-\frac{1}{p}} \parallel u_f(xg) \parallel)^p d\breve{m}_{12}(\dot{y})$$

$$= \int_{X_2} \beta_2(x) (\sup_{g\in G_2} (\frac{\Delta_{G_2}}{\Delta_2}(g))^{-\frac{1}{p}} (\frac{\Delta_{G_2}}{\Delta_2}(g))^{\frac{1}{p}} \parallel \lambda(g) u_f(x) \parallel)^p dm_2(x) =$$

$$= \int_{X_2} \beta_2(x) \parallel u_f(x) \parallel^p dm_2(x)$$

$$= \int_{X_2} \beta_2(x) (\int_{X_{12}} \beta_{12}(y) (\sup_{h\in G_1} \frac{r^{\frac{1}{p}}(x, yh) f(x \circ yh) \parallel}{(\frac{\Delta_{G_1}}{\Delta}(h))^{\frac{1}{p}}})^p dm_{12}(y)) dm_2(x)$$

$$= \int_{X_2} \beta_2(x) (\int_{X_{12}} \beta_{12}(y) (\sup_{h\in G_1} r^{\frac{1}{p}}(x,y) \parallel V(h) f(x \circ y) \parallel)^p dm_{12}(y)) dm_2(x)$$

$$= \int_{X_2} \beta_2(x) (\int_{X_{12}} \beta_{12}(y) r(x,y) (\sup_{h\in G_1} \parallel V(h) f(x \circ y) \parallel)^p dm_{12}(y)) dm_2(x)$$

$$= \int_{X_2 \times X_{12}} \beta_2(x)\beta_{12}(y)r(x,y)(\sup_{h\in G_1} \| V(h)f(x \circ y) \|)^p d(m_2 \times m_{12})(x,y)$$

$$= \int_{X_1} dm_1(x \circ y)(\int_{G_2} \beta_2(xg)\beta_{12}(g^{-1}y)(\sup_{h\in G_1} \| V(h)f(x \circ y) \|)^p dm_{G_2}(g))$$

$$= \int_{X_1} \gamma(x \circ y)(\sup_{h\in G_1} \| V(h)f(x \circ y) \|)^p dm_1(x \circ y) = \| f \|_p^p$$

If $p = \infty$, we have:

$$\| A(f) \|_p^p = \sup_{x\in X_2} \sup_{g\in G_2} \| u_f(xg) \|_\infty = \sup_{x\in X_2} \sup_{g\in G} \| \lambda(g)u_f(x) \|_\infty$$

$$= \sup_{x\in X_2} \| u_f(x) \|_\infty = \sup_{x\in X_2} \sup_{y\in X_{12}} \sup_{h\in G_1} \| f(x \circ yh) \|$$

$$= \sup_{x\circ y\in X_1} \sup_{h\in G_1} \| f((x \circ y)h) \| = \| f \|_\infty$$

With arguments similar to the compact case ([17], §2, Theorem 2), by using Lemma 1.1, it is easy to see that the image of the operator A is dense in $\mathcal{B}_p^\lambda(X_2, B_V^p(X_{12}, E))$, hence it follows that A is onto.
A short computation proves that A is an intertwining operator for the representations ν and χ. Indeed, let $a \in G$ be and an arbitrary function f in $\mathcal{C}_p^V(X_1, E)$. Then:

$$(\chi(a)A)(f)(x)(y) = (\chi(a)u_f)(x)(y) = \chi(a)(f(x \circ y))$$

$$= f(a^{-1}(x \circ y)) = (A(_a f))(x)(y) = (A\nu(a))(f)(x)(y)$$

The theorem on induction in stages is proved.

We obtain next a Tensor Product Theorem ([12],[5]) for the present construction of induced representations. The circumstances of our definition lead us to consider the projective tensor product of Banach spaces (and implicitly of operators). In order to prove the analogue of the classical Tensor Product Theorem, we need an isometric isomorphism between $\mathcal{B}_p^{V_1}(X_1, E_1)\hat{\otimes}_\pi \mathcal{B}_p^{V_2}(X_2, E_2)$ and $\mathcal{B}_p^{V_1\otimes V_2}(X_1 \times X_2, E_1\hat{\otimes}_\pi E_2)$. Assuming that the intermediate space X is second countable and the Banach space E is separable, we have (Theorem 1.3) that $B_V^p(X, E)$ is isometrically isomorphic to $L^p(X/H, E)$. It is known, in general, the projective tensor product of "L^p" spaces "is not" an "L^p" space. With the above hypothesis, A. Kleppner ([10]) proved for $p = 1$ that there exists an isometric isomorphism between $L^1(X_1/H_1, E_1)\hat{\otimes}_\pi L^1(X_2/H_2)$ and $L^1(X_1/H_1 \times X_2/H_2, E_1\hat{\otimes}_\pi E_2)$. By virtue of this result we can generalise the Tensor Product Theorem to our context.
Therefore, for $i = 1, 2$, let H_i be a locally compact group with a (left) Haar measure m_{H_i}, $(E_i, \| \cdot \|_i)$ be a Banach space and V_i be a bounded representation of H_i on E_i. Let $E_1\hat{\otimes}_\pi E_2$ be the projective tensor product of the spaces E_1 and E_2 (that is the completion

of the algebraic tensor product $E_1 \otimes E_2$ under tht norm topology) and $V_1 \otimes V_2$ (defined on $H_1 \times H_2$) be the tensor product of the operators V_i, $i = 1, 2$:

$$(V_1 \otimes V_2)(h_1, h_2) = V_1(h_1) \otimes V_2(h_2), \quad h_1 \in H_1, h_2 \in H_2.$$

Since, for any $(h_1, h_2) \in H_1 \times H_2$ and $\xi_1 \in E_1$, $\xi_2 \in E_2$ we have

$$\| (V_1 \otimes V_2)(h_1, h_2)(\xi_1 \otimes \xi_2) \| = \| V_1(h_1)\xi_1 \otimes V_2(h_2)\xi_2 \|$$

$$= \| V_1(h_1)\xi_1 \|_1 \cdot \| V_2(h_2)\xi_2 \|_2 \leq c_1 \cdot c_2 \cdot \| \xi_1 \|_1 \cdot \| \xi_2 \|_2 = c_1 \cdot c_2 \cdot \| \xi_1 \otimes \xi_2 \|$$

it results that $V_1 \otimes V_2$ is a bounded representation of the group $H_1 \times H_2$ on the Banach space $E_1 \hat{\underset{\pi}{\otimes}} E_2$.

Now, let G_i be a locally compact group and X_i be an $(H_i - G_i)$ intermediate space (which is second countable), $i = 1, 2$. The Radon measure on X_i is denoted by m_i and its multiplier by Δ_i, $i = 1, 2$. It is appearent that $X_1 \times X_2$ become a $(H_1 \times H_2 - G_1 \times G_2)$ intermediate space if we define in a natural way the left and right actions of $G_1 \times G_2$, resp. $H_1 \times H_2$:

$$(g_1, g_2)(x_1, x_2) = (g_1 x_1, g_2 x_2), \quad \forall x_1 \in X_1, x_2 \in X_2, h_1 \in H_1, h_2 \in H_2$$

$$(x_1, x_2)(h_1, h_2) = (x_1 h_1, x_2 h_2) \quad \forall x_1 \in X_1, x_2 \in X_2, g_1 \in G_1, g_2 \in G_2$$

We can consider on $X_1 \times X_2$ the measure $m_1 \times m_2$ which is clearly $H_1 \times H_2$ invariant and $G_1 \times G_2$ quasiinvariant with the multiplier $\Delta_1 \Delta_2$.

Theorem 2.2. *(The Tensor Product Theorem)*
The representations $\underset{H_1 \nearrow G_1}{ind} {}_1^{X_1} V_1 \otimes \underset{H_2 \nearrow G_2}{ind} {}_1^{X_2} V_2$ *and* $\underset{H_1 \times H_2 \nearrow G_1 \times G_2}{ind} {}_1^{X_1 \times X_2} V_1 \otimes V_2$ *are isometrically equivalent.*

The proof of this theorem is in essence the same as in [17] because the compactness of the groups H_1, H_2 did not intervene in any of the arguments. For this reason we shall give here only a sketch. We preserve all our previous notations.

Proof. (Sketch) We need to find an isometry Φ from $\mathcal{B}_1^{V_1}(X_1, E_1) \hat{\underset{\pi}{\otimes}} \mathcal{B}_1^{V_2}(X_2, E_2)$ and $\mathcal{B}_1^{V_1 \otimes V_2}(X_1 \times X_2, E_1 \hat{\underset{\pi}{\otimes}} E_2)$ which intertwines $\underset{H_1 \nearrow G_1}{ind} {}_1^{X_1} V1 \otimes \underset{H_2 \nearrow G_2}{ind} {}_1^{X_2} V_2$ and $\underset{H_1 \times H_2 \nearrow G_1 \times G_2}{ind} {}_1^{X_1 \times X_2} V_1 \otimes V_2$.

Let us consider

$$\Omega = \{ f_1 \otimes f_2 \mid f_1 \in \mathcal{C}_1^{V_1}(X_1, E_1), f_2 \in \mathcal{C}_1^{V_2}(X_2, E_2) \} \subset \mathcal{B}_1^{V_1}(X_1, E_1) \hat{\underset{\pi}{\otimes}} \mathcal{B}_1^{V_2}(X_2, E_2)$$

and Φ defined on Ω by

$$\Phi(f_1 \otimes f_2) = f_{12}$$

where $f_{12} : X_1 \times X_2 \longrightarrow E_1 \hat{\underset{\pi}{\otimes}} E_2$, $f_{12}(x_1, x_2) = f_1(x_1) \otimes f_2(x_2)$.

It suffices to prove that the mapping Φ on Ω has the following properties:

(1) $\Phi(\Omega)$ is total in $\mathcal{B}_1^{V_1\otimes V_2}(X_1\times X_2, E_1\hat{\underset{\pi}{\otimes}}E_2)$.

(That is clear if we use: "$\Phi(\Omega)\subset \mathcal{C}_1^{V_1\otimes V_2}(X_1\otimes X_2, E_1\hat{\underset{\pi}{\otimes}}E_2)$", "$\epsilon_{V_1\otimes V_2}(\theta_{12},\xi_1\otimes\xi_2)(x_1,x_2) = \epsilon_{V_1}(\theta_1,\xi_1)(x_1)\otimes\epsilon_{V_2}(\theta_2,\xi_2)(x_2), \theta_i\in\mathcal{C}_0(X_i), \xi_i\in E_i, (i=1,2), \theta_{12}(x_1,x_2)=\theta_1(x_1)\cdot\theta_2(x_2)$" and Lemma 1.1).

(2) Φ is an intertwining operator on Ω.

(It is a short computation).

(3) The extension by linearity of Φ to $\mathcal{B}_1^{V_1}(X_1,E_1)\hat{\underset{\pi}{\otimes}}\mathcal{B}_1^{V_2}(X_2,E_2)$ is an isometry between $\mathcal{B}_1^{V_1}(X_1,E_1)\hat{\underset{\pi}{\otimes}}\mathcal{B}_1^{V_2}(X_2,E_2)$ and $\mathcal{B}_1^{V_1\otimes V_2}(X_1\times X_2, E_1\hat{\underset{\pi}{\otimes}}E_2)$

(That is appearent by the definition of Φ on Ω, if we use Theorem 3 and the result of [10] (see page 170)).

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