Cosmology of Symmetron Mass-Varying Neutrinos

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A man ought to read
just as his inclination leads him;
for what he reads as a task
will do him little good.

SAMUEL JOHNSON

Abstract

In this thesis we investigate a dynamical cosmological constant called the quintessence. Quintessence is a scalar field varying in both space and time following a chosen potential. To explain the coincidence problem, e.g. why the expansion of the universe is accelerating now, we couple the scalar field to neutrinos. The coupling changes the constant neutrino mass to a mass varying neutrino that depends on the scalar field. However, this coupling causes a fifth force on the neutrinos creating to much clumping on large scales. We suggest to use the screening potential of the symmetron model to prevent this clumping. The symmetron forces the scalar field to zero at high neutrino densities and spontaneously breaks the potential at low neutrino densities making the scalar field active. We modified a standard code for the cosmological evolution (CAMB) to include both the mass varying neutrino and the symmetron. Adding an cosmological constant is still needed to achieve the observed late time accelerated expansion. Simulation where done for three different scenarios, depending on the occurrence of the symmetry breaking. There is a notable effect from the symmetron-neutrino coupling and larger effects as one increases the neutrino mass. However, for all scenarios one observe small changes compared with the standard $LCDM$.

Acknowledgments

> Supervision is an opportunity to bring someone back to their own mind, to show them how good they can be.
>
> *Nancy Kline, 'Time to Think'*

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CHAPTER 1

Introduction

> “Begin at the beginning," the King said gravely, “and go on till you come to the end: then stop."
>
> *Lewis Carroll, Alice in Wonderland*

The High-Z Supernova Search Team and the Supernova Cosmology Project revolutionized the field of cosmology with their discovery that the expansion rate of the universe started to accelerate quite recently [1],[2]. These observations of *standard candles* are considered the beginning of the new Era of High-Precision Cosmology [3]. However, the cause of the acceleration is still unknown and is simply dubbed as the *dark energy*.

From observations of the cosmic microwave background (CMB), first by the Wilkinson Microwave Anisotropy Probe (WMAP) [4],[5],[6],[7],[8] and later by the Planck space observatory [9], infer that wethe lack an extra energy component to make the universe flat. This missing energy is considered to be dark energy, see figure 1.1. Evidence from large-sclace structure, e.g. from WiggleZ Dark Energy Survey [10] gives further support for the existence of dark energy.

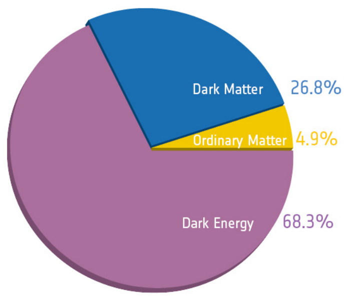


Figure 1.1: The relative amounts of the different constituents of the universe given by the Planck satellite. Figure from ESA.

In the title of the original paper of the High-Z SN group [1], a simpler term for the acceleration of the universe was used, namely *the cosmological constant* Λ (Lambda). The cosmological constant is a homogeneous energy density that fills the whole universe and works like "anti-gravity". The theoretical idea was first developed by Albert Einstein [11]. The cosmological constant was meant to counteract gravity making the

universe static, i.e. neither expanding nor contracting).
In this thesis I investigate one of the alternative theories of dark energy called quintessence. Quintessence is a dynamical scalar field with a potential energy suggested to cause the observed acceleration of the universe. Quintessence differs from the cosmological constant in that it can vary in space and time. One can imagine the field as a sea of energy filling the whole of space evolving according to the shape of its potential. The theory was first suggested in 1987 as a time-dependent cosmological constant [12]. Later the scalar field was introduced as the so-called "cosmon" [13],[14] or as a rolling homogeneous scalar field [15].

After the discovery of an accelerating universe and a non-zero cosmological constant the field re-appeared as "This fifth contribution to the cosmic energy density", the quintessence [16]. Later the work on cosmon, quintessence and dark energy was combined [17],[18],[19].

One suggestion to explain why the cosmological acceleration has set in so close to our own time is to couple the scalar field to matter. One can express the coupling such that one obtains *growing matter* [20]. The particle masses becomes time dependent, directly determined by the scalar field [21]. The focus of this thesis is the theory where only the masses of the neutrinos are varying [22], MaVans (Mass Varying neutrinos). The neutrino mass changes as the scalar field evolves and stops when the neutrinos become non-relativistic due to the expansion of the universe. "This leads to a transition from a cosmological scaling solution with dynamical dark energy at early time to a cosmological constant dominated universe at late time." [22].

But there is a problem with this scenario. The coupled particles experience the scalar field creating an extra contribution along its path in the universe compared to particles which are not coupled. This contribution is defined as the "fifth force" [13], [23]. Consequently, this extra force creates instabilities in the neutrino perturbations leading to a exponential growth in the perturbations. Consequently this instabilities create a substantial neutrino clustering on large-structures of the universe which contradicts with observations [24], [25]. Furthermore in N-body simulations with growing neutrino masses did the neutrinos exceed the speed of light. [26]. Improvements have been made, but the results of new N-body simulations still confirms the formation of large-scale neutrino structures and that "the velocities of neutrinos are accelerated to relativistic values during the process of structure formation" , [27].

Our suggestion to solve this problem is the inclusion of a screening mechanism which allows the force to mediate in regions with low density and to be decoupled and screened in regions of high density. To implement the screening mechanism, we use a spontaneous symmetry breaking potential: the symmetron [28],[29]. We calculate the linear background and perturbation and compare this to the Planck parameter space.

1.1 Structure of the thesis

In chapter 2, I give a short introduction to the notation and theoretical background used in this thesis. This includes quantum field theory, general relativity (GR) and the standard model of cosmology. In chapter 3 I present some of the different modifications to general relativity. In chapter 4 I included more details of the quintessence model, mass-varying neutrinos and symmetron, which is the main focus of this thesis. Chapter 5 consist of the background and changes on the code CAMB, which is used to run the simulations for this thesis. In chapter 6 I present the results of these simulations for different configurations of the model. Last in chapter 7 I present the final result and discuss future possible work beyond this thesis.

CHAPTER 2

Preliminaries

> If you can't explain something simply, then you haven't really understood it.
>
> *Albert Einstein*

The chapter gives a short introduction to the mathematical theory and usage of quantum field theory, special and general relativity and modern cosmology as well as the notation used in the thesis. First, I present the Lagrangian formalism and effective theory from quantum field theory which include the quantum corrections. Second I discuss briefly the theory of special relativity before I look at the mathematical background for the construction of GR. I then give an introduction to GR and use the variation principle to find the Einstein equations. Last I discuss modern cosmology with the Friedmann equations, $LCDM$-model before introducing the latest observations and challenges in cosmology.

2.1 Notation, Conventions and Acronyms

Vectors, Conformal time, redshift and scale factor

Space and time is classically divided into two separate quantities, t for the time and $\vec{x} = (x, y, z)$ for the three dimensions of space. But in modern era they have been combined into a bigger 4-dimensional space called Space-time. Space-time is then defined as the set of all events, where an event is a particular point at a particular time expressed by the 4-vectors, time-space $x_\mu = (t, x, y, z)$, four-momentum $p_\mu = (E, \vec{p})$ and four-gradient $\nabla_\mu = (\partial/\partial t, -\vec{\nabla})$ [30, page 6][31, page 13]. In cosmology one uses the conformal time $d\eta$ rather than the regular cosmic time t which is defined by [32, page 34]:

$$d\eta \equiv \frac{dt}{a(t)}.$$

Here a is the *scale factor*. Imaging the universe as a expanding grid will the grid point be the *comoving distance* and the physical distance be the scale factor times the scale factor, see figure 2.1 [32, page 2].

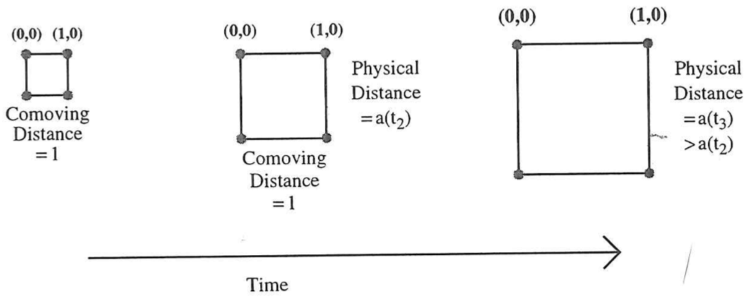


Figure 2.1: Expansion of the universe. The comoving distance between points in a hypothetical grid remains constant as the universe expands. The physical distance is proportional to the comoving distance times the scale factor, so it gets larger as time evolves.

The conformal time can be taught of as the comoving horizon. This is the total distance that light could have traveled since $t = 0$ given by $\eta = \int_0^t \frac{dt'}{a(t')}$. The conformal horizon, η therefore represents the size of the universe today, and objects that are separated by a distance greater than η are not per definition causally connected, [32, page 34].

Redshift is given by the Doppler effect as a change of the wavelength of light for an observer moving relative to the source. The received wavelength is shorter when the source is approaching and longer when the source is recessing compared to the emitted wavelength. The cosmological redshift z is a consequence of the expansion of the universe. Since the photon wavelength is inversely proportional to the frequency the wavelength of the photon will grow as it travels towards us through the expanding universe,

$$z = \frac{\lambda_O}{\lambda_e} - 1 = \frac{a_0}{a} - 1, \tag{2.1}$$

where z is the redshift, λ_O is the observed wavelength, λ_e the emitted wavelength, a_0 the scale factor of the universe today and a the scale factor of the universe when the photon was emitted. Redshift is used in cosmology as a time standard, having $z = 0$ today. Consequently is the scale factor also used as an time standard with $a_0 = 1$. Conventionally represent variables underscored with zero the variable value today.

The metric is Lorentzian, meaning that is has a signature of $(-, +, +, +)$. Note that x is used as a space-time coordinate where it is appropriate.

Einstein summation convention

In this thesis I frequently use the Einstein summation convention: If the same index name appears twice in any term, as both an upper and a lower index, that term is assumed to be summed over all possible values of that index [33, page 13]. Mathematically is this expressed as

$$g^{\mu\nu} R_{\mu\alpha} = \sum_{x=0}^{3} g^{x\nu} R_{x\alpha}.$$

Greek indices implies a summation over all four components of space-time, and Latin indices implies a summation over only the three space indices.

Derivatives

The following notation has been used for different types of derivatives:

$$\begin{aligned}
&\text{The total derivative:} && \frac{df}{dx},\ f^{'} \\
&\text{The partial derivatives:} && \frac{\partial f}{\partial x},\ f_{,x} \\
&\text{The covariant derivatives:} && \nabla_x f,\ f_{;x} \\
&\text{The d'Alembert operator:} && \Box \equiv \nabla_\mu \nabla^\mu \\
&\text{The Laplacian operator:} && \nabla^2 \equiv \sum_i \frac{d^2}{dx_i^2}, \\
&\text{The functional derivate:} && \frac{\delta}{\delta J(x)},
\end{aligned} \tag{2.2}$$

where x_i is the i'th component of the 4-vector. When we evaluate the derivative at a particular point, we sometimes use is $\frac{\partial f}{\partial x}\Big|_{x=x_i} \equiv f_{,x_i}$. For a scalar A and a tensor F this notation imply

$$\begin{aligned}
&\frac{dA}{dt} \equiv \dot{A}\ ,\ \frac{dA}{dx} \equiv A_{,x}\ ,\ \frac{d^2A}{dx^2} \equiv A_{,xx}\ ,\frac{dA}{dx^i} \equiv \partial_i\ , \\
&\partial_{(i}F_{j)} = \partial_i F_j + \partial_j F_i\ ,\ \partial_{[i}F_{j]} = \partial_i F_j - \partial_j F_i.
\end{aligned} \tag{2.3}$$

Other Conventions

Units

In this thesis I use natural units, $c = \hbar = 1$. All basic quantities like energy, mass, time and length are expressed in terms of the energy unit *electron volt* (eV), where 1 eV $= 1.602 \times 10^{-19}$J. [34]

$$\text{energy} = \text{mass} = \text{time}^{-1} = \text{length}^{-1} \tag{2.4}$$

Transforming from SI-units to natural units gives us:

$$\begin{aligned}
1 \text{ kilogram} &= 5.61 \times 10^{26} \text{ GeV} \\
1 \text{ second} &= 1.52 \times 10^{24} \text{ GeV}^{-1} \\
1 \text{ meter} &= 5.07 \times 10^{15} \text{ GeV}^{-1}
\end{aligned} \tag{2.5}$$

Acronyms and List of frequently used symbols

The following acronyms and symbols is used throughout this thesis:

- CMB - Cosmic microwave background
- GR - General Relativity
- $LCDM$ - Lambda Cold Dark Matter
- MaVaNs - Mass Varying Neutrinos
- 1PI - One Particle Irreducible

Symbol	Name	Definition or value
$\hbar$	Reduced Planck constant	$\hbar = 1$
c	Speed of light	$c = 1$
G	Newton's Gravitational constant	$G = 6.71 \times 10^{-39}$ GeV^{-2}
κ	Commonly used constant	$\kappa = 8\pi G = 1/M_{Pl}^2$
M_P	Planck mass	$M_P = \sqrt{\hbar c/G} = 1/\sqrt{G}$
		$= 1.2209 \times 10^{19}$ GeV
M_{Pl}	Reduced Planck mass	$M_{Pl} = 1/\sqrt{8\pi G}$
		$= 2.435 \times 10^{18}$ GeV
ϕ	Scalar field	
$g_{\mu\nu}$	Metric tensor	
$\eta_{\mu\nu}$	Minkowski metric (Flat spacetime)	$\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, 1, 1)$
$\Gamma^{\sigma}_{\mu\nu}$	Christoffel Symbol	
$R_{\mu\nu}$	Ricci tensor	
R	Ricci Scalar	

Symbol	Name	Definition or value
$G_{\mu\nu}$	Einstein tensor	$G_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/2$
$T_{\mu\nu}$	Stress-energy tensor	
a	Cosmic scale factor	$a_0 = 1$ (today)
H	Hubble parameter	$H = \dot{a}/a$
ρ	Energy (Mass) Density	
ρ_c	Critical density of the universe	$\rho_c = 3H^2/(8\pi G) = 3H^2 M_{Pl}^2$
P	Pressure	
ω	Equation of state	$\omega = P/\rho$

2.2 Field Theory

2.2.1 Lagrangian formalism and Noether's theorem

The Lagrangian formalism is a widely used technique, first developed in classical mechanics, but used in everything from quantum field theory to cosmology. I have used for this section well-known classical references for the formalism, [35] and [36]. For the quantum fields have I used [35] and [37].

The Lagrangian $L\left(\vec{q}(t), \dot{\vec{q}}(t), t\right)$ is a single function, which contains all physical information concerning a system and the forces acting on it. The Lagrangian is a function of the degrees of freedom which is expressed trough $\vec{q} = (q_1(t), \ldots, q_i(t), \ldots, q_n(t))$ the generalized coordinates in configuration space, and $\dot{\vec{q}}(t) = (\dot{q_1}(t), \ldots, \dot{q_i}(t), \ldots, \dot{q_n}(t))$ the generalized velocity. The evolution of $\vec{q}$ with time, the dynamics of the physical system, can be found by solving the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{q}}}\right) - \frac{\partial L}{\partial \vec{q}} = 0,$$

where the Lagrangian is expressed as $L = T - U$, with T as the kinetic energy, U the potential energy, $\frac{\partial L}{\partial \dot{\vec{q}}}$ the generalized momentum and $\frac{\partial L}{\partial \vec{q}}$ the generalized forces.

The dynamics of the physical system, the equations of motion, coincide with the extremal of the action functional

$$S = \int_{t_1}^{t_2} L \; dt.$$

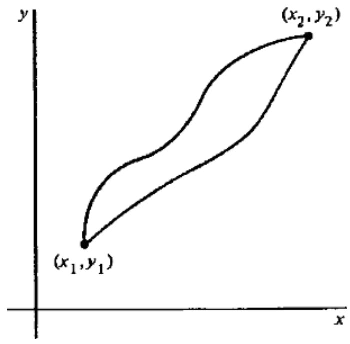


Figure 2.2: Illustration of a small perturbation on the original path in configuration space. Taken from [35, page 36].

A *functional* is mathematically a set of functions of the real numbers, e.g. the possible paths in the configuration space. The extremal of the action is formally found by varying the action by a small perturbation $q_i(t) \to q_i(t) + \delta q_i(t)$ with the constraints on the endpoints of the path in configuration space $\delta q_i(t_1) = \delta q_i(t_2) = 0$, see figure 2.2. This process of variation is called the *Hamiltionan's principle of least action* or the *Principle of Stationary Action*, see Appendix A.

To take the step from the classical theory to field theory require two alternations. Firstly one expands from classical spatial vectors to the continuous four-dimensional space-time by introducing the Lagrangian density $\mathcal{L}$,

$$L = \int \mathcal{L}\ d^3x$$

where the total Lagrangian L is integrate over the respective spatial dimensions. The action then become

$$S = \int_\Omega \mathcal{L}\ d^4x,$$

where Ω is a region (volume) of space-time and d^4x stands for the four-dimensional element $dtd^3x = dx^0d^3x$.

Secondly one introduce the space-time dependent fields $\Phi_i(x_\mu)$, $i = 1, \ldots, N$ that describe our system. These can be multiple fields, or components of a vector field. When introducing these fields the Lagrangian density and the action functional S is no longer a function of the generalized coordinates and momentum but is replaced by the set of fields Φ_i and its space-time derivatives $\partial_\mu \Phi_i$. One can now vary the fields itself $\Phi^i \to \Phi^i + \delta\Phi_i$ with the constraint, on the surface $\Gamma(\Omega)$ of our space-time region Ω, such that $\delta\Phi_i(x_\mu) = 0$ for all x_μ on the boundary $\Gamma(\Omega)$. Using Hamilton's principle again (see Appendix A), one ends with the Euler-Lagrange equations for fields,

$$\frac{\partial \mathcal{L}}{\partial \Phi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \Phi_{i,\mu}} \right) = 0.$$

Note that the Lagrangian density later will be referred mainly as the Lagrangian.

Another very important concept is the conversation of certain quantities, currents, due to true symmetries of the Lagrangian. This is summarized in the following theorem taken from [38, Chapter 1.3].

Noether's Theorem:
Continuous symmetry of the Lagrangian gives rise to a conserved current $j^\mu(x)$.

See Appendix B for a more thorough review and proof.

2.2.2 Effective Action and Potential

In this section I have used [39, Chapter 9 and 11], [40] and [41] as my main references.

Path integrals is a formulation, or method, which describes quantum field theory by generalizing the principle of stationary action from classical mechanics (see subsection 2.2.1). It replaces the classical trajectory with a *functional integral* over the amplitudes of all possible motions to compute a final *quantum amplitude* of the process at hand. The modulus squared of the quantum amplitude represents the probability or probability density of the problem.

The Hamiltonian of the problem at hand is given by the free fields and the interaction, $H = H_{free} + H_{int} = H_0 + H_{int}$, where $|0\rangle$ is the vacuum state (ground state) of the free fields H_0 and $|\Omega\rangle$ is the vacuum state with the interactions H. Note that the two vacuum states are in general different.

The first building block for a theory which is governed by the Lagrangian $\mathcal{L}$, is the *correlation function* defined by the vacuum expectation value of the time-ordered (for time-dependent fields) products of fields,

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \langle \Omega | \, T\phi(x_1) \cdots \phi(x_n) \, |\Omega\rangle \, .$$

One can make this general defining the *generating functional of correlation functions*

$$Z[J] = \int \mathcal{D}\phi \exp\left[i \int d^4x (\mathcal{L} + J\phi) \right], \tag{2.6}$$

where the time integration in the exponent runs from $-T$ to T with $T \to \infty(1-i\epsilon)$. This is a functional integral over the field, ϕ, and the spatially varying *source term* or *current* $J(x)$ so that $J\phi \equiv J(x)\phi(x)$. From the generating functional of correlation functions, $Z[J]$, one can calculate the *two-point correlation functions*, or *two-point Green's function*, by taking the functional derivative of $Z[J]$,

$$\begin{aligned} \langle \Omega | \, T\phi(x_1)\phi(x_2) \, |\Omega\rangle &= Z[J]^{-1} \left(-i \frac{\delta}{\delta J(x_1)} \right) \left(-i \frac{\delta}{\delta J(x_2)} \right) Z[J] \\ &= Z[J]^{-1} \int \mathcal{D}\phi e^{i \int d^4x (\mathcal{L} + J\phi)} \phi(x_1)\phi(x_2), \end{aligned}$$

and for a free field

$$\langle 0| T\phi(x_1)\phi(x_2) |0\rangle = Z[J]^{-1} \left(-i\frac{\delta}{\delta J(x_1)}\right) \left(-i\frac{\delta}{\delta J(x_2)}\right) Z[J]\Big|_{J=0}.$$

Here $\frac{\delta}{\delta J(x)}$ is functional derivative which obeys the basic axiom (in four dimensions)

$$\frac{\delta}{\delta J(x)} J(y) = \delta^{(4)}(x-y) \quad \text{or} \quad \frac{\delta}{\delta J(x)} \int d^4y J(y)\phi(y) = \phi(x).$$

To compute higher order correlation functions one simply take further functional derivatives.

Physically the two-point correlation function is interpreted as the amplitude for the propagation of a particle between x_1 and x_2. In free theory (without interactions), the two-point correlation is exactly the *Feynman propagator*,

$$\langle 0| T\phi(x)\phi(y) |0\rangle = D_F(x-y).$$

The propagator is the inverse of the wave operator appropriate to the particle and in Feynman diagrams it is the propagators represented by virtual particles on the internal lines as illustrated in figure 2.3.

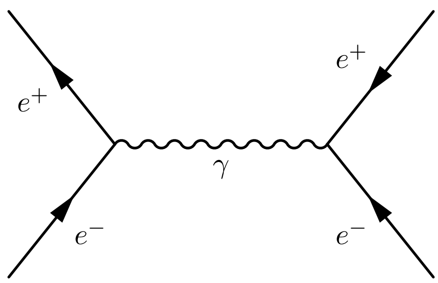


Figure 2.3: Feynman diagram for electron-positron scattering, with a photon propagator.

With this interpretation of the generating functional (2.6) one define the energy functional $E[J]$ by

$$Z[J] = e^{-iE[J]}. \tag{2.7}$$

Since the right hand side is the functional integral representing the amplitude of vacuum to vacuum, $E[J]$ becomes the *vacuum energy* as a function of the external source. Taking the functional derivative of $E[J]$ gives the expectation value of ϕ in the presence of the source J,

$$\frac{\delta}{\delta J(x)} E[J] = -\langle \Omega| \phi(x) |\Omega\rangle_J .$$

One can then define the *classical field* as

$$\phi_{cl}(x) = \langle\Omega|\,\phi(x)\,|\Omega\rangle_J\,. \tag{2.8}$$

Note that the classical field depends on the external source $J(x)$. However, when one set the source to zero one obtain the one-point correlation function, or the expectation value of ϕ,

$$\frac{\delta}{\delta J(x)}E[J]\bigg|_{J=0} = \langle\phi(x)\rangle\,. \tag{2.9}$$

Continuing by taking the second functional derivative and setting the source to zero again one end up with

$$\frac{\delta^2 E[J]}{\delta J(x)\delta J(y)}\bigg|_{J=0} = -i\left[\langle\phi(x)\phi(y)\rangle - \langle\phi(x)\rangle\langle\phi(y)\rangle\right]. \tag{2.10}$$

The first term of equation (2.10), $\langle\phi(x)\phi(y)\rangle$, consist of the contribution of two Feynman diagrams shown in figure 2.4,

Figure 2.4: Feynman diagram for $\langle\phi(x)\phi(y)\rangle$, connected plus disconnected diagrams

where each shaded circle corresponds to the *sum of connected diagrams*. This means that the second diagram is the sum of all disconnected diagrams. This last term is in turn canceled by the second term in the equation (2.10), $\langle\phi(x)\rangle\langle\phi(y)\rangle$. This means that the second functional derivative of $E[J]$, equation (2.10), corresponds to the *two-point connected correlation function*,

$$\frac{\delta^2 E[J]}{\delta J(x)\delta J(y)} = -i\,\langle\phi(x)\phi(y)\rangle_{connected} = -iD(x,y),$$

which is identified as the exact propagator for the field ϕ. This relation continues for each higher order functional derivative of $E[J]$ giving us the general formula for the n'ht functional derivative of $E[J]$,

$$\frac{\delta^n E[J]}{\delta J(x_1)\cdots\delta J(x_n)} = (i)^{n+1}\,\langle\phi(x_1)\cdots\phi(x_n)\rangle_{connected}\,. \tag{2.11}$$

From the general equation (2.11) one conclude that $E[J]$ is the *generating functional of connected correlati*

Further, taking the Legendre transform of $E[J]$,

$$\Gamma\left[\phi_{cl}\right] = -E[J] - \int d^4y J(y)\phi_{cl},$$

gives us the expression for the *the effective action*. This effective action is then a modification of the action which takes into account the quantum-mechanical corrections, or *loop corrections*. The functional derivative of the effective action gives the current,

$$\frac{\delta}{\delta\phi_{cl}(x)}\Gamma\left[\phi_{cl}\right] = -J(x). \tag{2.12}$$

Setting the external source to zero in (2.12) one obtain

$$\frac{\delta}{\delta\phi_{cl}(x)}\Gamma\left[\phi_{cl}\right]\bigg|_{\phi_{cl}=\langle\phi\rangle} = 0. \tag{2.13}$$

The solutions to (2.13) are the values of ϕ_{cl} which are considered to be stable. The solutions of ϕ_{cl} which are independent of x are the one where the vacuum state is translation-invariant. Solutions corresponding to localized lumps of field held together by their self-interaction are solutions which depends on x and are called *solitons*.

Taking the functional derivative of (2.12) gives us

$$\begin{aligned}
\frac{\delta}{\delta J(y)}\frac{\delta\Gamma}{\delta\phi_{cl}(x)} = \frac{\delta J(x)}{\delta J(y)} &= -\delta^{(4)}\left(x-y\right) \\
&= -\int d^4z \frac{\delta\phi_{cl}(z)}{\delta J(y)}\frac{\delta^2\Gamma}{\delta\phi_{cl}(z)\delta\phi_{cl}(x)} \\
&= \int d^4z \frac{\delta^2 E}{\delta J(y)\delta J(z)}\frac{\delta^2\Gamma}{\delta\phi_{cl}(z)\delta\phi_{cl}(x)}.
\end{aligned}$$

Setting $J=0$ gives us that $\phi_{cl}=\langle\phi\rangle$ and $\delta^2E/\delta J(y)\delta J(z)$ becomes the exact propagator of the field, resulting in the relation,

$$\int d^4z D(y,z)\frac{\delta^2\Gamma}{\delta\phi_{cl}(z)\delta\phi_{cl}(x)} = i\delta^{(4)}\left(x-y\right).$$

This means that the second derivative of the effective action, setting the source to zero, becomes the inverse of the propagator

$$\frac{\delta^2\Gamma}{\delta\phi_{cl}(x)\delta\phi_{cl}(y)}\bigg|_{\phi_{cl}=\langle\phi\rangle} = iD^{-1}(x,y).$$

This is the exact kinetic operator and can be identified with the 1PI (*One Particle Irreducible*) two-point function. This relation continues for each higher order derivative of the effective action. One therefore conclude that the effective action is *the generating functional for 1PI correlation functions*. One-particle irreducible diagrams are any diagram that cannot be split in two by removing a single line shown in figure 2.5.

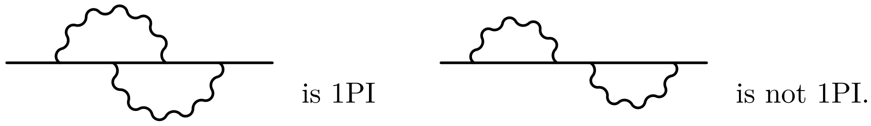


Figure 2.5: Feynman diagram for a 1PI (One-particle irreducible) and a Feynman diagram which is not a 1PI.

Normally one denote the sum of all 1PI diagrams with two external line (scalar, fermion or photon) by a blob of 1PI shown in figure 2.6.

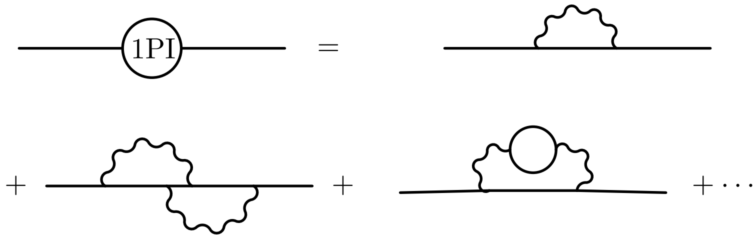


Figure 2.6: Feynman diagram of the sum of One-particle irreducible. Represented by the gray blob.

In turn the exact propagator can be written as the geometric series of 1PI functions and summed as shown in figure 2.7 .

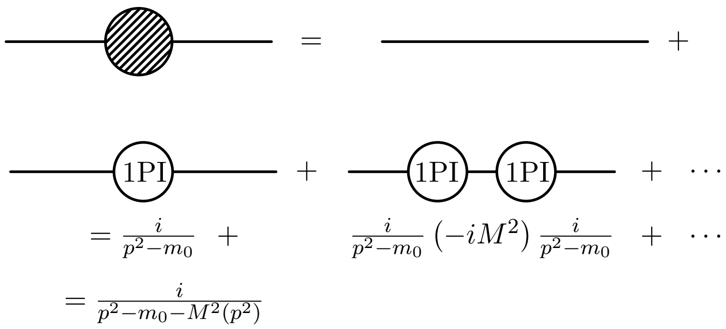


Figure 2.7: Feynman diagram for the exact propagator with self-energy mass M, or full two point function, as a geometric series of 1PI (One-particle irreducible). Taken from [39, Equation: (7.43)].

The effective action is proportional to the volume of space-time over which the functional integral is taken. If T is the time part of this region and $V = \int d^4x$ is the space part, one write the effective action as

$$\Gamma\left[\phi_{cl}\right] = -\left(VT\right) V_{eff}\left(\phi_{cl}\right) = -\int d^4x\ V_{eff}, \tag{2.14}$$

where the coefficient V_{eff} is called the *effective potential*. The condition of (2.13) is reduces to

$$\frac{\partial}{\partial \phi_{cl}} V_{eff}\left(\phi_{cl}\right) = 0, \tag{2.15}$$

where each solution of (2.15) corresponds to a translation-invariant state with $J = 0$. This means that every extrema of V_{eff} is independent of x.

2.3 Special and General Relativity

In this section have I used [42], [43], [44] and [45] as references. The books [42] and [43] is recommended as an introduction for new beginners, while [44] and [45] is a more comprehensive classical text.

2.3.1 Newtonian Gravity and The Theory of Special Relativity

Newtonian gravity and special relativity are both theories which is needed to describe flat universe. First Newtonian gravity for the gravitational part, and special relativity for the incorporation of Maxwell's equations.

Newtonian gravity, and mechanics, gives in an reference frame, the so-called inertial frames. The inertial reference frames is described by a Cartesian position vector, $\vec{x} = (x_1, x_2, x_3)$ and a time t and transforms invariant between the inertial frames by a transformation from the Galileo group [45, Chapter 1.3].

The Principle of Galilean Relativity: The laws of motion must be invariant under Galilean transformation.

One distinct between the mass exerted by an object trough gravity, gravitational mass, and the inertial mass of an object. However, in Newtonian gravity does all matter attract each other. Consider an object of inertial mass M acting by gravity on another object with inertial mass m. The object M will exert a force on the object m by the inverse-square law

$$F = \frac{GMm}{r^2}, \tag{2.16}$$

where F is the force, r is the distance between the objects and G is the Newton's gravitational constant. The gravitational force acting on M by m is the same. One observe that the gravitational mass and the inertial mass is the same. This is summarized in the principle called the *Weak Equivalence Principle*, [42, Page 48].

The Weak Equivalence Principle: The inertial mass and of gravitational mass of any object are equal.

By Newton's second law $F = ma$ one conclude that the acceleration the object feels under gravity is independent of its own mass. The force excreted by the two objects creates a gravitational potential between them on the form

$$\Phi = -\frac{GMm}{r},$$

with the force (2.16) being exerted in the direction which decreases the potential energy the fastest. The gravitational acceleration is given by the gradient of the potential and is related to the mass density in the universe ρ by the Poisson equation

$$\nabla^2\Phi = 4\pi G\rho.$$

The *Universality of Free Fall* [46], is an alternative equivalent formulation of the weak equivalence principle saying that any two object fall with the same acceleration in a external gravitational field.

The Universality of Free Fall:
The acceleration imparted to a body by a gravitational field is independent of the nature of the body.

The equations of electromagnetism, Maxwell's equations, and the speed of light are however not invariant under the Galilean transformation. In 1905 Albert Einstein proposed that the Galilean transformation should be replaced by the *Lorentz transformation*.

The Principle of Invariant Light Speed:
The speed of light in empty space is the same in all inertial frames and independent of the motion of the light source. From [47, Page 226].

Under the Lorentz transformation where Maxwell's equations and the speed of light is invariant, but not the Newtonian laws of motion. Einstein modifies the laws of motion such that the new equations and Maxwell's equation where invariant [45, Chapter 1.3].

The Principle of Special Relativity:
The physical laws must be invariant under Lorentz transformation, or then that they are the same in all inertial frames.

For a short historical overview for both theories see [45, Chapter 1], and for the mathematical content of Lorentz transformation and other relativistic consequences of special relativity see [45, Chapter 2].

2.3.2 Differential Geometry

An alternative formulation of gravity is geometrically, trough curvature. The description of special relativity is given by Minkowski space and is flat. General relativity (GR) includes both the theory of special relativity and general curvature. For a background on the geometry of Minkowski space see [48]. For this subsection have I used [42], [48], [49] and [33] as my references.

Differential geometry is the mathematical description of differentiation in geometry, on a manifold like our four dimensional space-time. One define a *contravariant vector* with an upper index, V^ν, which under the coordinate transformation $x \to x^{'}$ transforms as

$$V^{'\mu} = \frac{\partial x^{'\mu}}{\partial x^\nu} V^\nu.$$

The *covariant vector* is defined with an lower index, U_ν. It transforms under the same coordinate transformation as

$$U'_\mu = \frac{\partial x^\nu}{\partial x'^\mu} U_\nu.$$

Note that a scalar field is a covariant vector. The generalization of a contra-variant and covariant vector is the *tensors*, with both upper and lower indexes.

The geometry of the universe described by GR is given by the metric. The metric is defined by the scalar product, also called inner product, which is a non-degenerate, symmetric, bilinear form g. Having $V = V^\mu \mathbf{e}_\mu$ and $W = W^\mu \mathbf{e}_\mu$ as two vectors, in an arbitrary basis $\mathbf{e}_\mu$ of the manifold, the inner product is defined as

$$g(v, w) = v \cdot w = g_{\mu\nu} v^\mu w^\nu,$$

where $g_{\mu\nu} = \mathbf{e}_\mu \cdot \mathbf{e}_\nu$ is our metric. Another definitions of the metric is the infinitesimal *line element*,

$$ds^2 = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu$$

where ds is the infinitesimal interval. The infinitesimal interval describing the distance in our space-time between any event x^α and any nearby event $x^\alpha + dx^\alpha$. For the scalar product to be invariant under the Lorentz transformation, will the *norm*, the inner product of the vector itself, not be positively defined [42, Page 23]:

$$\eta_{\mu\nu} V^\mu V^\nu \begin{cases} < 0, & V^\mu \text{ is timelike} \\ = 0, & V^\mu \text{ is lightlike or null} \\ > 0, & V^\mu \text{ is spacelike} \end{cases}$$

An inverse metric is defined as the counter metric giving the *Kronecker delta*,

$$\eta_{\mu\nu} \eta^{\mu\rho} = \delta^\mu_\rho = \begin{cases} 0, & \text{if } \mu \neq \rho \\ 1, & \text{if } \mu = \rho \end{cases}$$

The metric is a $(0, 2)$ tensor, while the inverse metric and the Kronecker delta is a $(2, 0)$ and $(1, 1)$ tensor, respectively. The *Levi-Civita symbol*, a $(0, 4)$ tensor,

$$\epsilon_{\mu\nu\rho\sigma} = \begin{cases} +1, & \text{if } \mu\nu\rho\sigma \text{ is a even permuation of } 0123 \\ -1, & \text{if } \mu\nu\rho\sigma \text{ is a odd permuation of } 0123 \\ 0, & \text{otherwise.} \end{cases}$$

To describe curvature one need the *connection*, a tool to relate vectors in the tangent space of nearby points. The *covariant derivative*, ∇, performs the operations of the partial derivative in curved space and is independent of the coordinates. The covariant derivative is for each direction μ given by the partial derivative in the respective direction plus a correction term. This correction term is given by the connection coefficients, the *Christoffel Symbols*, $\Gamma^{\sigma}_{\mu\nu}$. The covariant derivative of a contra-variant vector V^{ν} and a covariant vector W_{ν} are defined as [50, Page 239],

$$\nabla_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \Gamma^{\nu}_{\mu\sigma}V^{\sigma}, \quad \nabla_{\mu}W_{\nu} = \partial_{\mu}W_{\nu} - \Gamma^{\sigma}_{\mu\nu}W_{\sigma}. \tag{2.17}$$

Note that for a scalar field is reduced to the partial derivative, $\nabla_{\mu}\phi = \partial_{\mu}\phi$.

The connection may or may not depend on the metric. But the metric itself imply a own unique symmetric connection. The connection ρ is compatible with the metric if the covariant derivative of the metric with respect to the connection is zero, e.g. $\nabla_{\rho}g_{\mu\nu} = 0$. The metric $g_{\mu\nu}$ defines a natural derivative, which is the one used in GR. The Christoffel symbols is expressed by the metric $g_{\mu\nu}$ as,

$$\Gamma^{\sigma}_{\mu\nu} = \frac{g^{\sigma\alpha}}{2}\left(g_{\nu\alpha,\mu} + g_{\alpha\mu,\nu} - g_{\mu\nu,\alpha}\right). \tag{2.18}$$

In GR the connection is also torsion-free, meaning that the Christoffel symbols are symmetric in the lower indexes, $\Gamma^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\nu\mu} = 0$.

Moving a vector along a path, a curve $x^{\mu}(\tau)$ with parametrization τ, keeping the vector constant, is known as *parallel transport*. A path whose tangent vectors are connected by parallel transport is called a *geodesic* if with respect to the covariant derivative the acceleration is zero. The geodesics are therefore the "straight" lines of curved space, and follow the *geodesic equation*,

$$\frac{d^2x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\sigma}\frac{dx^{\alpha}}{d\tau}\frac{dx^{\sigma}}{d\tau}.$$

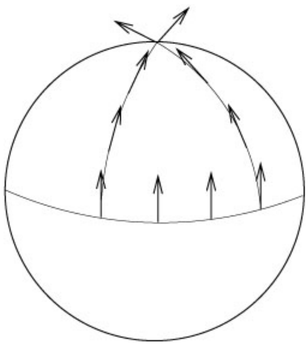


Figure 2.8: Parallel transport of a vector on the sphere. Figure from [51].

Parallel transporting an vector on curved space is however different than parallel transporting on flat space. The vector will be affected by the intrinsic curvature and will be depended on the path of parallel transport, see figure 2.8.

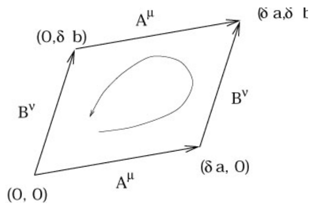


Figure 2.9: Infinitesimal loop defined by the vectors A^μ and B^μ. Figure from [51].

One can parallel transports the vector V^μ around an infinitesimal loop, first in the direction A^μ then in the direction B^μ and back, see figure 2.9. The change of the vector V^μ is

$$\delta V^\alpha = R^\alpha{}_{\sigma\mu\nu} V\sigma A^\mu B^\nu,$$

where $R^\alpha{}_{\sigma\mu\nu}$ is a $(1,3)$ tensor, the *Riemann Curvature Tensor*. The Riemann curvature tensor is identified as

$$R^\alpha{}_{\sigma\mu\nu} = \Gamma^\alpha_{\nu\sigma,\mu} - \Gamma^\alpha_{\mu\sigma,\nu} + \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}.$$

The *Ricci tensor*, is defined trough the contraction of the Riemann curvature tensor,

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu},$$

which is symmetric in the two indexes $R_{\mu\nu} = R_{\nu\mu}$. Further is the trace of the Ricci tensor the *Ricci Scalar* or the curvature scalar,

$$R = R^\mu{}_\mu = g^{\mu\nu} R_{\mu\nu}. \tag{2.19}$$

From this one finally end up with the *Einstein Tensor* as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{2.20}$$

where is it symmetric since the Ricci tensor and metric is also symmetric. Note that because of the Bianchi identities the Einstein tensor is divergence free [45, Chapter 6.8],

$$\nabla^\mu G_{\mu\nu} = 0. \tag{2.21}$$

2.3.3 The General Relativity

Einstein arrived in 1907 at the *Einstein Equivalence Principle*.

Einstein Equivalence Principle:
The outcome of any local, non-gravitational test experiment is independent of the experimental apparatus velocity relative to the gravitational field and is independent of where and when in the gravitational field the experiment is performed. From [52].

An alternative formulation is the *Principle of General Covariance*.

<u>The Principle of General Covariance:</u>

1. The equations, laws of physics, holds in the absence of gravitation. That is it agrees with the laws of special relativity.
2. The equations (law of physics) is generally covariant, that is preserves its form under a general coordinate transformation.

From [45, Chapter 4] and [44, Chapter 4.1-4.2].

The principle says that there is no way of distinguishing between a free fall in a uniform gravitational field (on the surface of the Earth) and in a uniform acceleration (in an accelerating rocket). The laws of physics should also be formulated by tensors, which are invariant objects.

Einstein equivalence principle arise naturally from the *universallity* of gravity. It affects all particles *and* energy the same and is manifested trough the curvature of space-time, described by the Einstein Tensor (2.20). The energy (also mass) and momentum content of the universe is given by the $(2,0)$ symmetric [49, Section 5.7] *energy-momemtum tensor*, sometimes also called the *stress-energy tensor*, $T_{\mu\nu}$. The *Einstein field equations* or the Einstein equations for GR is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2.22}$$

where $\kappa = 8\pi G$.

The left hand side describes the geometry of the universe, while the right hand side described the energy-momentum content of the universe. The path that particles moves is affected by the curvature. But the curvature is dictated in turn by the energy content and distribution in the universe.

The energy-momentum is conserved (locally) $\nabla^\mu T_{\mu\nu}$, since the Einstein tensor is divergence free (2.21). The *Vacuum Einstein Equations* describes the Einstein equations in vacuum $T_{\mu\nu} = 0$ and is given by

$$R_{\mu\nu} = 0.$$

Note that the Einstein equations can also be obtained from Lagrangian formalism, decried in subsection 2.2.1., using the Einstein-Hilbert action combined with the matter action. See Appendix C. for details.

2.3.4 Friedmann Equations

One distinguish between three categories of curvature, namely positive, negative and flat curvature, see figure 2.10. The Friedmann-Lemaître-Robertson-Walker metric, which describes a homogeneous and isotropic universe, gives the line-element

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right], \tag{2.23}$$

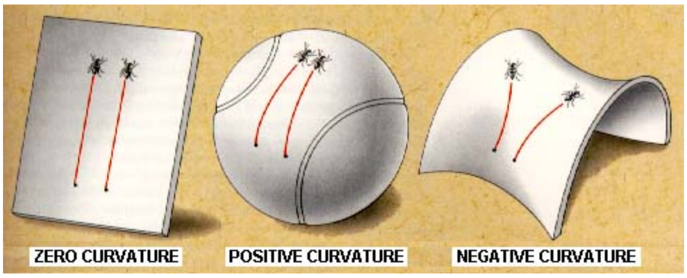


Figure 2.10: Flat, positive and negative curvature. Taken from [53, page 69].

where K is the curvature, a the expansion factor or scale factor and $d\Omega = d\theta^2 + \sin^2\theta d\phi$ is the differential solid angle. The negatively curved (open, hyperbolic), flat and positively curved (closed, sphere) universe corresponds to a curvature of $K < 0$ $(K = -1)$, $K = 0$ and $K > 0$ $(K = 1)$, respectively. In a flat Friedmann-Lemaître-Robertson-Walker universe the metric reduces to

$$ds^2 = -dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right). \tag{2.24}$$

The content of the universe is approximated as a *perfect fluid*. A perfect fluid is defined as an continuous distribution of matter with the stress-energy tensor [44, Page 62],[42, Chapter 8.3],

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + P g_{\mu\nu}, \tag{2.25}$$

where in comoving coordinates $u_\mu = (1, 0, 0, 0)$ is the four-velocity, $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}\,(-1, 1, 1, 1)$ the metric, ρ the energy density and P the isotropic pressure. Raising one index up one finds $T^\mu{}_\nu = \text{diag}\,(-\rho, P, P, P)$ and the trace as $T = T^\mu{}_\mu = -\rho + 3p$.

The *Equation of State* gives us the relationship between the density and pressure and is given by,

$$P = \omega\rho. \tag{2.26}$$

The zero component of the conservation of energy is given by

$$\nabla_\mu T^\mu{}_0 = -\dot{\rho} - 3\frac{\dot{a}}{a}(\rho + P) = 0,$$

and can be expressed as

$$\dot{\rho} + 3H(\rho + P) = 0, \tag{2.27}$$

which is in literature is called both the *fluid equation* and the *continuity equation*. Here is $H = \dot{a}/a$ the *Hubble parameter*.

Inserting the stress-energy tensor (2.25) and the Friedmann-Lemaître-Robertson-Walker metric (2.23) into the Einstein equations (2.22), gives from the time-component, (00)-component, the first Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{2.28}$$

From the space-component, (ii)-component, and the continuity equation (2.27) on obtain the second Friedmann equation, the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right). \tag{2.29}$$

From the first Friedmann equation (2.28) one can obtain the *critical density*, the density for which the universe will be flat,

$$\rho_c(t) = \frac{3H^2}{8\pi G}.$$

The critical density defines an natural scale for the densities the *density parameter*, Ω, which is defined by

$$\Omega = \frac{\rho}{\rho_c},$$

for a component with density ρ. Note that both the densities of the individual components of the universe ρ and the critical density ρ_c is be dependent on time and therefore also the density parameter.

2.4 Modern Cosmology

For an easy introduction for new-beginners in modern cosmology I recommend Andrew Liddle's [54], and Sean M. Carroll's [42] books. Robert W Mald's [44] and Steven Weinberg's [45] books are a more throughout background for this section. All of these book have I used as an reference for this section.

2.4.1 Short History of the Cosmos

> I'm trying to understand cosmology, why the Big Bang had the properties it did. And it's interesting to think that connects directly to our kitchens and how we can make eggs, how we can remember one direction of time, why causes precede effects, why we are born young and grow older. It's all because of entropy increasing.
>
> *Sean M. Carroll*

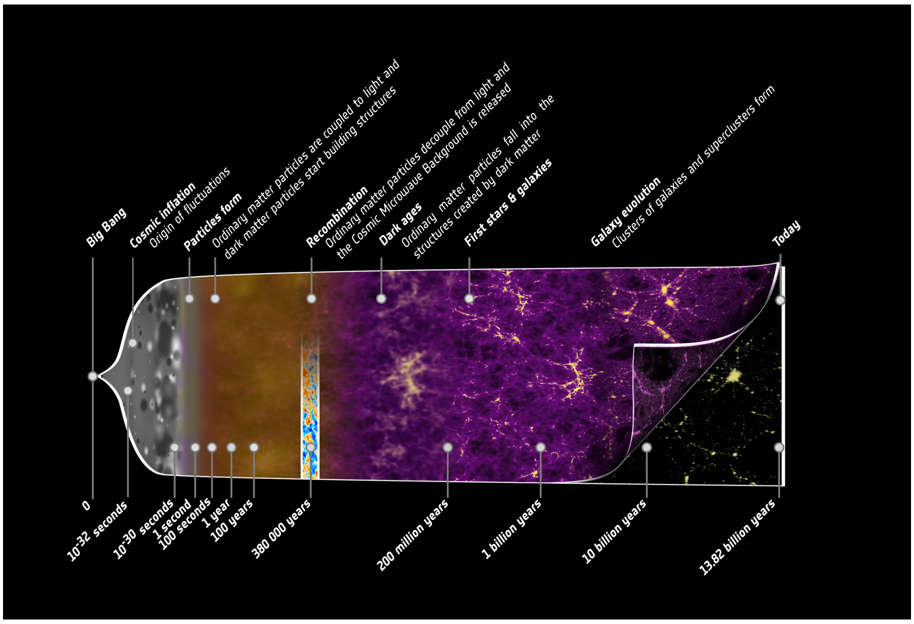


Illustration of the history of the Universe, from the Big Bang to today. Figure from [55].

In the standard model of cosmology the universe is flat, homogeneous and isotropic. It is filled with photons, neutrinos, cold dark matter, baryons and a cosmological constant *Lambda*. This model we call Lambda-Cold-Dark-Matter ($LCDM$) for the dominating elements of this universe.The standard model starts with the Big Bang. Its followed by an period of rapid expansion of space called Inflation. The rapid expansion explains for cosmologist why the universe have some important and fundamental properties of our known universe from a random configuration given at Big Bang.

1. The Flatness Problem.
   How could one explain why the spatial curvature of the universe is so flat from a wide possibilities of initial condition set by the Big Bang?
2. The Horizon Problem.
   Why is the universe homogeneous and isotropic on very larges scales?
3. The Monopole Problem.
   This problem arrives from the fact that the universe is so small and so dense that particle physics "take over" and predicts a magnetic monopole that we don't observe.

How these problems is solved and more details about inflation see [56],[57] and[58]. Inflation sets the initial conditions for the evolution of our visible universe as a flat, homogeneous and isotropic universe, washing out the conditions at the Big Bang. After inflation is the universe hot, dense and chaotic, but is dominated by radiation. Nuclei can't to form because of the high temperature and the universe is opaque. However, the universe has not stopped expanding and cool down. Using the equation of state (2.26) one can express the continuity equation (2.27) as

$$\frac{\dot{\rho}}{\rho} = -3\,(1+\omega)\,\frac{\dot{a}}{a}.$$

If the equation of state ω is constant, can on integrating from an arbitrary scale factor a to today $a_0 = 1$ and obtain

$$\rho = \rho_0 \left(\frac{1}{a}\right)^{3(1+\omega)},$$

where ρ_0 is the energy density of today. The densities of the different components of the universe, photon, baryons, dark matter, cosmological constant will evolves differently as the universe expands depending on their equation of state, see figure 2.11.

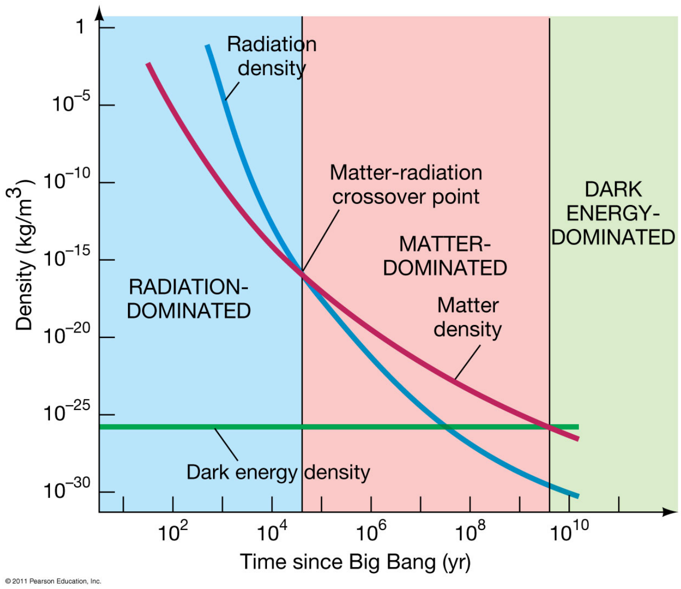


Figure 2.11: Illustration of the different epochs of the Universe, Radiation- , Matter- and Dark Energy-Dominated period. Figure from [59].

Non-relativistic matter has an equation of state $\omega \approx 0$, while both relativistic matter and radiation have an equation of state of $\omega = 1/3$. The vacuum energy, the cosmological constant, has an equation of state $\omega = -1$. Neutrinos are first relativistic and becomes non-relativistic when there temperature have become smaller then their rest mass. The universe will therefore be first dominated by radiation (photons) then by matter (dark matter) before being dominated by dark energy.

Under the radiation dominated period did the temperature decrease enough such that charged electrons and protons became bound to form electrically neutral hydrogen atoms. This epoch is called recombination. At recombination the universe became transparent and the remaining photons started to travel freely. These photons are what we observe as the Cosmic Microwave Background (CMB) radiation and is the photon relic from recombination.

2.4.2 The $LCDM$ model

Observational evidence of dark energy is substantial and growing, see [60, Chapter 4-7], [61, Chapter 5] for a more comprehensive documentation. However we do not know the nature of dark energy. In the standard model of cosmology is dark energy described as an cosmological constant, Lambda. Following the Lagrangian formulation, in section 2.2.1. on obtain the $LCDM$ model by adding an extra term -2Λ to the Einstein-Hilbert action, see C,

$$S_G = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R-2\Lambda)\,.$$

The action principle gives the Einstein equations [61, Chapter 6.1-6.2]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu},$$

and the Friedmann equations,

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right) + \frac{\Lambda}{3}.$$

The density parameter for the cosmological constant is given by

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}.$$

Using the first Friedmann equation (2.28), rewriting and rearranging gives

$$\Omega + \Omega_\Lambda - 1 = \frac{K}{a^2H^2},$$

where Ω is the density parameter for the other components of the universe, baryons, cold dark matter, photons and neutrinos. An open, flat and closed universe will then be given by $0 \le \Omega + \Omega_\Lambda < 1$, $\Omega + \Omega_\Lambda = 1$ and $\Omega + \Omega_\Lambda > 1$, respectively. If the Universe is dominated by a pressure less matter, $P = 0$, one can make the Universe static, e.g. $\dot a = \ddot a = 0$, if

$$\rho = \frac{\Lambda}{\kappa}, \quad \frac{K}{a^2} = \Lambda.$$

To obtain the late time cosmological acceleration we require that the cosmological constant is in the order of the square of the present Hubble parameter, H_0, [61, Chapter 6.3]

$$\Lambda \approx H_0^2, \quad \rho_\Lambda \approx 10^{-47}\ \mathrm{GeV}^4.$$

However, this does not fit the vacuum energy density given by particle physics

$$\rho_{vac} \simeq 10^{74}\ \mathrm{GeV}^4.$$

This problem of fitting the value of the cosmological constant to a known physical quantity is known as the *fine tuning problem.* The cosmological constant is also almost identical to the present matter energy density even if there no direct relation between the two. This is called the *coincidence problem* or *why now* problem e.g., Why is the cosmological constant dominating now, so close to our time?

2.4.3 Perturbations

The universe as we know it is homogeneous and isotropic on large scales, following the Friedmann-Lemaître-Robertson-Walker metric. However, a homogeneous and isotropic universe do not form any structures like galaxies and solar systems. To explain the structures we see today one assumed that there were some small perturbations after inflation given by

$$\delta(\vec{x}, t) = \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)},$$

where $\bar{\rho}(t)$ is the average density of the component [62], [32]. The perturbations are effected by the expansion of the universe and can either grow to form structure or dilute into the nothing. The equations for the perturbations are obtained from solving, as before, the Einstein equations but included an perturbation on the metric. In this thesis we study the perturbations in Fourier space. The transformation between from the spacial position $\vec{x}$ to the wavelength $\vec{k}$ in Fourier space is given as

$$\delta(\vec{x}, t) = \sum_{\vec{k}} \delta(\vec{k}, t) e^{i\vec{k}\cdot\vec{x}}.$$

CHAPTER 3

Alternative theories

> No amount of experimentation can ever prove me right; a single experiment can prove me wrong. *Albert Einstein*

In this chapter I present some of the different extensions of general relativity. Quintessence is a dynamical scalar field, which provides the extension used in this thesis. The generalization of scalar-tensor theory, the Brans-DIcke model and the $f(r)$-gravity are sketched . Last I shortly explain the mechanism of chameleon screening.

For the extensions of GR or any other theory to be viable it must fulfill the following criteria [49, Chapter 39]:

1. Self-consistency: The result of the calculation for an experiment should be consistent and give a unique result in different approaches within the theory.
2. Completeness: From first principle of the theory, i.e. by the theory itself, one should be able to calculate the outcome of any experiment.
3. Agreement with past experiments: For example the theory must be relativistic, following special relativity in the absence of gravity, and have the correct Newtonian limit to be consistent with earlier experiments.

A restricted class of gravitational theories is called *metric theories* [49, Chapter 39] with the following properties:

a. Space-time possesses a metric.
b. The metric satisfies the Einstein equivalence principle, meaning that in each local Lorentz frame the theory of special relativity is valid.

Some of the alternative theories are presented in following sections. In this chapter I have used [61], [63] and [64] as main references for the different theories.

3.1 Quintessence

3.1.1 Quintessence in simple terms

In section 2.4. one observe that the dark energy is given by a cosmological constant, fixed static energy density and equation of state $\omega = -1$. Quintessence, however, is a scalar field ϕ, with pressure P_ϕ and density ρ_ϕ given by the potential energy $V(\phi)$ and the kinetic term $\dot{\phi}$ of the field. Hence quintessence is a dynamical theory with a time-dependent, and spatially inhomogeneous density, pressure and equation of state. One can consider quintessence as an *dynamical cosmological constant* and it is the alternative model investigated in this thesis. The basis of the theory, the field equations, are presented in section 3.1.2. while the dynamics of quintessence field are presented in section 4.1.

3.1.2 Equations for Quintessence

The quintessence field [61, Chapter 7.1],[60, Chapter 3.1] is governed by its field equations which are obtained by taking the variation of the action S given as,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}R + \mathcal{L}_\phi\right] + S_M(\tilde{g_{\mu\nu}}, \psi_i) \tag{3.1}$$

where

$$\mathcal{L}_\phi = -\frac{1}{2}(\nabla\phi)^2 - V(\phi) = -\frac{1}{2}g^{\mu\nu}\partial_\nu\phi\partial_\mu\phi - V(\phi).$$

Here $\mathcal{L}_\phi$ is the Lagrangian of the quintessence field, g the determinant of the Einstein metric $g_{\mu\nu}$, S_M the matter action, R the Ricci scalar and $\kappa = 8\pi G$. The different matter fields ψ_i, representing particles, are coupled to the Jordan frame metric $\tilde{g_{\mu\nu}}$. The full derivation with coupling is done in 4.2.1. However, for now we do not assume any coupling i.e. $\tilde{g_{\mu\nu}} = g_{\mu\nu}$, between the quintessence field and the matter fields. The energy-momentum tensor of quintessence [65, Chapter 8.3] is given by

$$T_{\mu\nu}^{(\phi)} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_\phi\right)}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right].$$

In the flat Friedmann-Lemaître-Robertson-Walker background, this gives the energy density ρ_ϕ and the pressure P_ϕ of the field as

$$\rho_\phi = -T_0{}^{0(\phi)} = \frac{1}{2}\dot{\phi} + V(\phi), \quad P_\phi = \frac{1}{3}T_i{}^{i(\phi)} = \frac{1}{2}\dot{\phi} - V(\phi). \tag{3.2}$$

The equation of state is

$$\omega_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}, \tag{3.3}$$

and is therefore restricted to

$$-1 \leq \omega_\phi \leq 1.$$

In a flat Friedmann-Lemaître-Robertson-Walker universe, the Einstein (Friedmann) equations (2.28) and (2.29) for a universe with quintessence given by

$$
\begin{aligned}
H^2 &= \frac{\kappa}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_M\right], \\
\dot{H} &= -\frac{\kappa}{2}\left[\dot{\phi}^2 + \rho_M + P_M\right],
\end{aligned}
$$

where $\kappa = 8\pi G$. Taking the variation of the action (3.1) with respect to the scalar field ϕ, $\phi \to \phi + \delta\phi$, vanishing at infinity leads us to the field equation, *Klein-Gorden equation*

$$
\Box\phi - V_{,\phi} = 0,
$$

(see Appendix D). In the flat Friedmann-Lemaître-Robertson-Walker background the metric is given by (2.24),

$$
ds^2 = -dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right),
$$

and the d'Alembert operator $\Box$ on our scalar field is then expressed as

$$
\begin{aligned}
\Box\phi &= \nabla^\mu\nabla_\mu\phi = \nabla^\mu\left(\partial_\mu\phi\right) = g^{\mu\nu}\nabla_\nu\left(\partial_\mu\phi\right) \\
&= g^{\mu\nu}\left[\partial_\nu\partial_\mu\phi + g^{\mu\nu}\Gamma^\sigma_{\nu\sigma}\partial_\mu\phi\right] = \partial^\mu\partial_\mu\phi + g^{\mu\nu}\Gamma^\sigma_{\nu\sigma}\partial_\mu\phi \\
&= -\frac{d^2\phi}{dt^2} + \frac{1}{a^2}\vec{\nabla}^2\phi + g^{\mu\nu}\Gamma^\sigma_{\nu\sigma}\phi_{,\mu} \\
&= -\ddot{\phi} + g^{00}\Gamma^\sigma_{0\sigma}\phi_{,0} = -\ddot{\phi} - 3H\dot{\phi},
\end{aligned}
$$

where I have used the definition of the covariant derivative (2.17), the Christoffel symbols d(2.18) and that $g^{00} = -1$. The FLRW metric describes a homogeneous universe, which gives a homogeneous (no spatial variation) scalar field and therefore the spatial derivative $\vec{\nabla}$ of the scalar field is zero. This result leads us to the field equations on the form

$$
\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0.
$$

Note that one can also obtain this equation from the continuity equation of the quintessence field,

$$
\dot{\rho_\phi} + 3H\left(\rho_\phi + P_\phi\right) = 0.
$$

3.2 Scalar-Tensor Theories

The generalization of the scalar field theory (quintessence) including a tensor field (the metric) is called a *scalar-tensor theory.* The general form of the scalar-tensor Lagrangian is given by

$$
\mathcal{L} = \frac{1}{2\kappa}\sqrt{-g}\left[f(\phi, R) + g(\phi)\nabla_\mu\nabla^\mu\phi - 2\Lambda(\phi)\right] + \mathcal{L}_m\left(\Psi_i,\ A^2(\phi)g_{\mu\nu}\right),
$$

where g, A and Λ are arbitrary functions of the scalar field ϕ, f a function of both the scalar field and the Ricci tensor and $\mathcal{L}_m$ is the Lagrangian of the matter field(s) Ψ_i. Regular quintessence is obtained by choosing the values $f(\phi, R) = A(\phi) = 1$, $g(\phi) = \frac{1}{2}$ and $\Lambda(\phi) = \frac{V(\phi)}{2}$, where $V(\phi)$ is the potential of the scalar field. Note that the function $h(\phi)$ can be absorbed into the metric by an conformal transformation, $h(\phi)g_{\mu\nu} \to \tilde{g_{\mu\nu}}$. It can also be considered as the coupling interaction between the scalar field and the matter field(s) in the Jordan frame, see subsection 4.2.1. A few of the specific theories are described in subsection 3.2.1. and 3.2.2.

3.2.1 Brans-Dicke Model

The *Brans-Dicke Model* is a known scalar-tensor theory where the gravitational interaction is mediated by the metric, i.e. the tensor field, as well as a scalar field. In this model the Newtonian gravitational constant G is not constant but depends on the scalar field ϕ as $\tilde{G} = \frac{G}{\phi}$ [66]. The Brans-Dicke model Lagrangian can be obtained by choosing $f(\phi, R) = \phi R$, $h(\phi) = 1$, $\Lambda(\phi) = 0$ and $g(\phi) = \frac{\omega}{\phi}$ where ω is an constant known as the *Brans-Dicke parameter* or Brans-Dicke coupling constant. This gives

$$\mathcal{L} = \frac{1}{2\kappa}\sqrt{-g}\left[\phi R + \frac{\omega}{\phi}\nabla_\mu \nabla^\mu \phi\right] + \mathcal{L}_m\left(\Psi_i,\ g_{\mu\nu}\right),$$

where the variation of the action gives

$$\phi G_{\mu\nu} + \left[\Box\phi + \frac{1}{2}\frac{\omega}{\phi}\nabla_\mu \phi \nabla^\mu \phi\right] g_{\mu\nu} = \kappa T_{\mu\nu} + \nabla_\mu \nabla \nu \phi + \frac{\omega}{\phi}\nabla_\mu \phi \nabla \nu \phi.$$

Note that one can generalize the Brans-Dicke by adding a potential $2\Lambda(\phi) = V(\phi)$.

3.2.2 $f(r)$-Gravity

Another theory is the $f(r)$-gravity where $f(\phi, R) = f(R)$, i.e. a general function of the Ricci scalar, $h(\phi) = 1$, $\Lambda(\phi) = g(\phi) = 0$. The variation of the $f(R)$ action gives

$$\frac{df(R)}{dR}R_{\mu\nu} - \left[\Box\frac{df(R)}{dR} + \frac{1}{2}f(R)\right] g_{\mu\nu} = \nabla_\mu \nabla_\nu \frac{df(R)}{dR} + \kappa T_{\mu\nu}$$

3.3 Screening - Chameleon Model

Some theories include a minimal coupling between the scalar field and matter, i.e. $A(\phi) \neq 0$. This coupling results in an extra fifth force acting on the coupled matter field

$$F_\phi = -\frac{d\ln A(\phi)}{d\phi}\vec{\nabla}\phi.$$

See section 4.2.3 for more details on the fifth force. However, tests of the equivalence principle on the solar system scales sets tight constrains on the coupling to matter, making it very small [67]. However, to avoid these constrains one have proposed several screening mechanism. One of these screening mechanism is the *chameleon model* presented in [68]. The chameleon model uses an specific coupling to matter, given by $A(\phi) = e^{\beta\phi/M_{Pl}}$, which changes the dynamics of the field ϕ such that it is not governed by the potential $V(\phi)$ alone, but by the effective potential given as

$$V_{eff}(\phi) = V(\phi) + \rho e^{\beta\phi/M_{Pl}}. \tag{3.4}$$

Note that the effective potential in Eq. 3.4 is not the same as the quantum field effective potential in Eq. 2.14. See figure 3.1 for an illustration of the effective potential in chameleon screening.

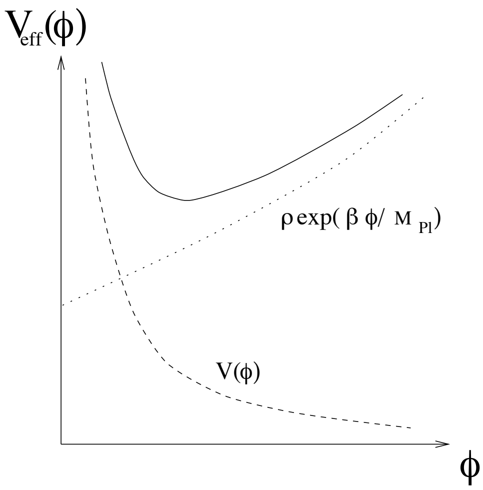


Figure 3.1: Illustration of the effective potential for the chameleon screening. The dashed line represent the original potential $V(\phi)$, the dotted line the contribution of the coupling $\rho e^{\beta\phi/M_{Pl}}$ and the solid line the effective potential V_{eff}. Figure from [68].

The effective potential in Eq. 3.4 affect the mass of the scalar field depending on the local matter density. In regions with high density the field will have a large mass and consequently obtain a low mass in regions of low density.One have therefore large effects from the scalar field in low densities regions while at the same time furfull the constrains in the solar system. See figure 3.2 for illustration of the effect in different density regions.

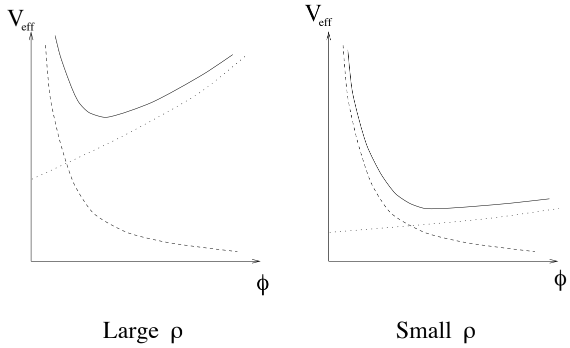


Figure 3.2: Illustration of the effective potential for the chameleon screening in different regions of densities. Left side represent regions of high density and is dominated by the coupling term. Right side represent regions of low densities and is dominated by the original potential. Figure from [68].

CHAPTER 4

Review of Quintessence, Mass varying neutrino and Symmetron

> Quintessence: (in classical and medieval philosophy) quinta essentia, the fifth essence, a fifth substance in addition to the four elements (fire, air, water and earth), thought to compose the heavenly bodies and to be latent in all things.
>
> *Oxford Dictionary of English*

In this chapter I go through the details of the quintessence model, mass varying neutrinos and symmetron. The main references for further details are [61, Chapter 7], [63] for the quintessence model, [22] for mass varying neutrinos, and [28],[29] for the symmetron potential and model.

4.1 The dynamics of Quintessence

Since quintessence is a dynamical theory, see section 3.1., the energy density doesn't have to be very small under the radiation and matter dominated periods like it is required for the cosmological constant in $LCDM$. However, to ensure that quintessence is dominating at late times the quintessence field should possess one of two behavior: A *scaling behaviour* or a *tracker behaviour*. If the field possesses a scaling behavior, or scaling solution, the ratio between the field and background density (matter or radiation) $\Omega_\phi/\Omega_{Back}$ will be a non-zero constant [69]. If on the other hand, the field possesses a tracking behavior, the field density will track, but be less than, the radiation density. The field density will only at very late times grow to dominate and drive the universe into a period of accelerated expansion [70].

To achieve a late-time acceleration the equation of state must be $\omega < -\frac{1}{3}$. This implies that the field has to be shallow enough such that $\dot{\phi}^2 < V(\phi)$. To quantify the shallowness one can introduce the slow-roll parameters, usually from inflationary cosmology

[71, II D]

$$\epsilon_s \equiv \frac{1}{2\kappa}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta_s \equiv \frac{V_{,\phi\phi}}{\kappa V}.$$

For the field to be sufficiently slow such that $\dot{\phi}^2 \ll V(\phi)$ both of the conditions $\epsilon_s \ll 1$ and $|\eta_s| \ll 1$ must be satisfied. However, the field should also be steep enough for it to possess the tracker behavior. This is necessity because "the tracker solution is an attractor in the sense that a very wide range of initial conditions for ϕ and $\dot{\phi}$ rapidly approach a common evolutionary track, so that the cosmology is insensitive to the initial conditions. Tracking has an advantage similar to inflation in that a wide range of initial conditions is funneled into the same final condition" [70]. If $\dot{\phi}^2/2 \gg 1$ tracking will always be satisfied and the field equation of state will be $\omega_\phi \simeq 1$.

The slope of the field is characterized by the quantity λ defined by

$$\lambda = -\frac{V_{,\phi}}{\kappa V},$$

and obeys the following equation

$$\frac{d\lambda}{dN} = -\frac{\lambda^2 (\Gamma - 1)\kappa\dot{\phi}}{H},$$

where $\Gamma = \frac{V V_{,\phi\phi}}{V_{,\phi}^2}$ and $N = \ln a$ is the number of e-folding. If λ is constant ($\Gamma = 1$), this will yield an exponential potential

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}.$$

This potential belongs to a class of quintessence models called the *freezing models*. These freezing models are fields that are rolling in the past but they freeze when reaching the epoch of cosmic acceleration. The other class is called *thawing models*. Here the field, characterized by the mass m_ϕ, has been frozen by the Hubble friction (Hubble damping). Once H drops below m_ϕ the field startsd to evolve and dominate the universe. Both classes are discussed in detail in [72]. Figure 4.1. illustrates the behavior of the two classes. For the potential to possess the tracking behavior with $\omega_\phi < \omega_{Background}$ it must fulfills the so called *tracking condition* [73]:

$$\Gamma > 1,$$

irrespective of the sign of $V_{,\phi}$. The slope λ will decrease toward 0 and become flat, thereby giving rise to an accelerated expansion at late times.

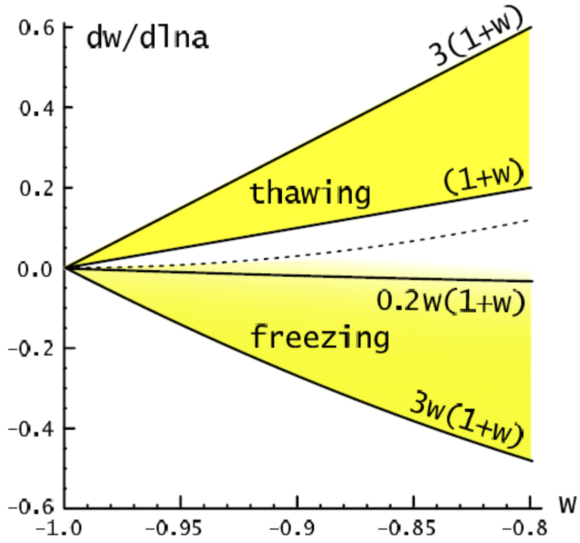


Figure 4.1: Illustration of the ω-$d\omega/dN$ phase space occupied by thawing and freezing fields indicated by the shaded regions. The fading at the top of the freezing region indicates the approximate nature of this boundary. Freezing models start above this line, but pass below it by a redshift $z \sim 1$. The short-dashed line shows the boundary between accelerating (thawing) and decelerating (freezing) down the potential. Figure from [72].

4.2 Coupling and Mass Varying Neutrinos

In this thesis am I investigating the interaction between neutrinos and dark energy described by the quintessence field. The motivation for the connection is that the energy scale of dark energy, $(\mathcal{O}\left(10^{-3}eV\right))$ is of the same order as the neutrino mass [74]. A coupling between quintessence and neutrinos can also stop the time evolution of the quintessence field by increasing the mass of the neutrinos. This can leads to a trigger for the cosmological constant dominated period in the universe at late time and explain the why now problem [22].

4.2.1 Jordan Frame

The general action for quintessence, Eq. (3.1), gives the possibility to introduce a coupling between the quintessence field and matter fields. One can couple the matter fields trough conformal rescaling

$$\tilde{g_{\mu\nu}}^{(i)} = A_i^2(\phi) g_{\mu\nu}^{(i)}, \tag{4.1}$$

where the subscript i specify which matter species is coupled, $A(\phi)$ a positive function describing coupling between the field and chosen matter i, $g_{\mu\nu}$ and $\tilde{g_{\mu\nu}}$ are the metric in the Einstein frame and Jordan frame, respectively.

Test particles will follow the metric of the conformal Jordan frame and this is the frame where the energy-momentum tensor is covariantly conserved. The scalar field is an ordinary scalar field in the conformal Einstein frame, which is the frame where the field equations take the form of the Einstein equations [64].

The conformal transformation between the Einstein and Jordan frame in Eq. (4.1) corresponds to the transformation of our cosmic ruler, or more specifically the line element given by $d\tilde{s}^2 = A^2\ ds^2$. In four dimensions the determinate of the metric transforms as $\sqrt{-\tilde{g}} = A^4\sqrt{-g}$.

The change in the field equation caused by the coupling, $A(\phi)$, between quintessence and the neutrino matter field is given by (see Appendix D)

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + A^{-3\omega_\nu} A_{,\phi}\left(1 - 3\omega_\nu\right)\rho_\nu = 0, \tag{4.2}$$

where ρ_ν is the rescaled energy density in the Einstein frame. This can be rewritten as an *effective potential* giving the potential that the quintessence field actually experience. In order not to confuse the quintessence effective potential with the effective quantum field theory potential, we rename the former as the *active potential*. Rewriting the Klein-Gordon equation with the active potential gives

$$\ddot{\phi} + 3H\dot{\phi} + V_{act,\phi} = 0, \tag{4.3}$$

where the definition of the active potential is

$$V_{act} = V(\phi) + A^{1-3\omega_\nu}(\phi)\rho_\nu.$$

4.2.2 Mass Varying Neutrinos

The coupling terms between the neutrinos and quintessence can be rewritten in terms the neutrino mass term such that the neutrino mass becomes a function of the quintessence field, i.e. $m_\nu = m_\nu(\phi)$.

The mass of the neutrinos will change as the field evolves and this theory and these neutrinos are dubbed the Mass Varying Neutrinos (MaVaNs) [75]. MaVaNs have also been called growing neutrino [22], or a more specific form of growing matter [20] (older version [76]). When the neutrinos become non-relativistic at late times (around $z \approx 2 \cdot 10^3 \frac{m_\nu}{eV}$ [77]) they experience a increase of the mass. Because of the coupling to the quintessence field does this event stop the evolution of the scalar field and start the late time acceleration of the expansion of the universe. This could explain the why now problem for dark energy. Note that it is possible to choose the coupling such that the neutrino mass decreases with time instead of growing.

Following [78][74], I assume that neutrinos follow the Fermi-Dirac phase space distribution f_0 and neglect any chemical potential. This leads to the following energy density, pressure and number density equations for the neutrinos:

$$\rho_v = \frac{1}{a^4}\int q^2 dq\ d\Omega\ \epsilon\ f_0, \tag{4.4}$$

$$P_\nu = \frac{1}{3a4}\int q^2 dq\ d\Omega\ f_0\ \frac{q^2}{\epsilon} \tag{4.5}$$

$$n_v = \frac{1}{a^3}\int q^2 dq\ f_0 = \frac{3\ \zeta(3) T_\nu^3}{2\pi^2},$$

where q is the comoving momentum, $\epsilon^2 = q^2 + m_\nu^2(\phi)a^2$ is the energy and T_ν the temperature of the neutrinos. For relativistic neutrinos, i.e. $q^2 \gg m_\nu^2(\phi)$, will the density take on the form of as a black body

$$\rho_\nu = \frac{7\pi^2}{120}T_\nu.$$

For non-relativistic neutrinos, i.e. $q^2 \ll m_\nu^2(\phi)$, the energy density and pressure reduce to

$$\rho_\nu \simeq m_\nu(\phi)n_\nu, \quad \text{and} \quad P_\nu \simeq 0.$$

Taking the time-derivative of the neutrino density Eq. (4.4) and using the definition of the pressure Eq. (4.5), gives the continuity equation of the quintessence-neutrinos (quintessence-neutrino energy conservation),

$$\dot{\rho_\nu} + 3H\left(\rho_\nu + P_\nu\right) = -\beta\dot{\phi}\left(\rho_\nu - 3P_\nu\right), \tag{4.6}$$

where

$$\beta = \frac{d\ln m_\nu(\phi)}{d\phi} \tag{4.7}$$

is the dimensionless quintessence-neutrino coupling, see Appendix E. The extra term on the right hand side of the continuity equation Eq. (4.6), is caused by the coupling between quintessence and the neutrinos[79],[74]. Note that from the definition of β, Eq. (4.7) the coupling can be a function of the field ϕ depending on the form of the neutrinos mass function [22].

Taking into account the energy conservation of the coupled neutrino-quintessence system, $\dot{\rho}_{coupled} + 3H\left(\rho_{coupled} + P_{coupled}\right) = 0$, where $\rho_{coupled} = \rho_\phi + \rho_\nu$, $P_{coupled} = P_\phi + P_\nu$, and the definitions of ρ_ϕ and P_ϕ from Eq. (3.2) one obtain the Klein-Gordon equation of the couple quintessence field as

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\beta\left(\rho_\nu + P_\nu\right), \tag{4.8}$$

where there is an extra source term due to the neutrino coupling. This can be rewritten to

$$\ddot{\phi} + 3H\dot{\phi} + V_{act,\phi} = 0,$$

where the active potential is given by

$$V_{act} = V(\phi) + e^{\beta\phi}\left(\tilde{\rho_\nu} - 3\tilde{P_\nu}\right)$$

where $\tilde{\rho_\nu} = \rho_\nu e^{-\beta\phi}$, $\tilde{P_\nu} = P_\nu e^{-\beta\phi}$ are density and pressure in the Jordan frame, and both are independent of ϕ. Comparing with Eq. (4.2) one observe that

$$\beta = A^{-3\omega_\nu}A_{,\phi}, \tag{4.9}$$

which is the same coupling as in Eq. (4.7) giving the form of the neutrino mass as

$$\begin{aligned} m_\nu &= M_0 A(\phi) \quad \text{for relativistic neutrinos, i.e. } \omega_\nu = \frac{1}{3}, \\ m_\nu &= M_0 e^{A(\phi)} \quad \text{for non-relativistic neutrinos, i.e. } \omega_\nu = 0, \end{aligned}$$

where M_0 is a renormalization constant.

4.2.3 Fifth force

Since test particles follow the metric in the Jordan frame, free falling test particles will follow the geodesic equation in the Jordan frame given by

$$\frac{d^2x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\alpha\sigma}\frac{dx^\alpha}{d\tau}\frac{dx^\sigma}{d\tau},$$

where $\tilde{\Gamma}^\mu_{\alpha\sigma}$ is the Christoffel symbol given by the Jordan metric. The transformation of the Christoffel symbol from the Jordan frame to the Einstein frame is [44][80],

$$\tilde{\Gamma}^\mu_{\alpha\sigma} = \Gamma^\mu_{\alpha\sigma} + \left(\delta^\mu_\alpha \nabla_\sigma \ln A(\phi) + \delta^\mu_\sigma \nabla_\alpha \ln A(\phi) - g_{\mu\nu}\nabla^\mu \ln A(\phi)\right).$$

The geodesic equation in the Einstein frame is therefore

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\sigma}\frac{dx^\alpha}{d\tau}\frac{dx^\sigma}{d\tau} + \frac{d\ln A(\phi)}{d\phi}\left(\frac{dx^\alpha}{d\tau}\frac{dx^\mu}{d\tau}\phi_{,\alpha} + \frac{dx^\sigma}{d\tau}\frac{dx^\mu}{d\tau}\phi_{,\sigma} - \frac{dx^\alpha}{d\tau}\frac{dx_\alpha}{d\tau}\phi^{,\mu}\right) \tag{4.10}$$

where the last term in Eq. (4.10) is an extra force term acting on the particles called the fifth force. In the Newtonian limit, i.e. $\frac{dx^i}{d\tau} \ll 1$, the fifth force is given by

$$F_\phi = -\frac{d\ln A(\phi)}{d\phi}\vec{\nabla}\phi. \tag{4.11}$$

In [74] they investigated a model with mass-varying neutrinos and coupling given by

$$m_\nu(\phi) = M_0 e^{\beta\phi}, \quad \tilde{g_{\mu\nu}} = e^{2\beta\phi}g_{\mu\nu},$$

where M_0 is a normalizing term for the neutrino mass, β is the dimensionless coupling constant between neutrinos and quintessence, and $\tilde{g_{\mu\nu}}$ and $g_{\mu\nu}$ are the Jordan and Einstein frame metric, respectively.

In this model the quintessence field is initially frozen and the neutrino are light and relativistic. At redshifts $z_{NR} \sim 5 - 10$ the neutrinos become non-relativistic, i.e. the temperature of neutrinos becomes lower then their rest mass [79]. This event exchange energy with the quintessence field via the quintessence-neutrino conservation equation given by Eq. (4.6) .This energy exchange constitute a cosmological trigger event which stops the time evolution of the quintessence field and gives the late time acceleration of the universe.

The coupling will also affect the CMB anisotropy spectrum for large angular scales, $l < 100$. On the scales of $10 < l < 100$ one observe an increase in power, while for $l < 10$ one finds an increase or decrease in power depending on the choice of parameters,[74]. See section 5.1. for details on the CMB and the moment l.

These results can be understood by the creation of too much clumping or growth of neutrino structures on very large scales [24][25],[79], [79], created by the extra fifth force acting on the neutrinos.

We therefore suggest a screening mechanism, more specifically the symmetron model described in section 4.3. This mechanism forces the quintessence field to zero at high densities and to a specific non-zero point at low densities. This will screen the fifth force in regions of high densities so only only standard gravity acts on the neutrinos and therefore one obtain no additional clumping of neutrinos at large scales.
Areas of low densities will consequently the field be non-zero and the neutrinos will be affected by the fifth force and the increase in mass caused by the quintessence-neutrino coupling. The field can be though of as being turned field on and off to give the desired dark energy in regions of low densities, while curing the problem of neutrino clumping on large scales.

4.3 Symmetron Potential

The *symmetron potential* [28],[29], is given by

$$V_{sym}(\phi) = V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \mathcal{O}\left(\frac{\phi^4}{M^4}\right),$$

where the last term includes all the higher order terms. In this thesis we only consider the simplest version of the symmetron potential

$$V_{sym}(\phi) = V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4. \tag{4.12}$$

The potential depends on the two mass scale μ, M and the dimensionless coupling constant λ. Using the symmetron potential the Lagrangian of the quintessence field Eq. (3.1) is invariant under the discrete symmetry transformation of the field $\phi \leftrightarrow -\phi$. The minima of the symmetron potential is determined by the conditions,

$$V_{,\phi} = 0, \quad V_{,\phi\phi} > 0. \tag{4.13}$$

If $\mu^2 < 0$ both terms of the symmetron potential (4.12) are positive and one obtains an absolute unique minimum at $\phi = 0$. However, if $\mu^2 > 0$ two different points exists for which the requirement (4.13) is satisfied. The minimum values, denoted by $\phi_\pm$, the value of the potential and second derivative at the minimum are given by

$$\phi_\pm = \pm\sqrt{\frac{\mu^2}{\lambda}}, \quad V(\phi_\pm) = -\frac{\mu^4}{4\lambda}, \quad V_{,\phi\phi}(\phi_\pm) = 2\mu^2.$$

Since the value of the potential (4.12) is the same for both points $V(\phi_+) = V(\phi_-)$, they are equivalent for a minima of the potential. There is an unstable extremum of the potential at $\phi = 0$, since $V_{,\phi}(0) = 0$ and $V_{,\phi\phi} < 0$. For the classical potential one would like to have an stable extremum and must choose the ground state as either

$\langle\phi\rangle = (\mu^2/\lambda)^{1/2}$ or $\langle\phi\rangle = -(\mu^2/\lambda)^{1/2}$. Therefore, the reflection symmetry $\phi \leftrightarrow -\phi$ present in the Lagrangian broken by the our choice of vacuum state[65, Chapter 7.1] as illustrated in figure ??.

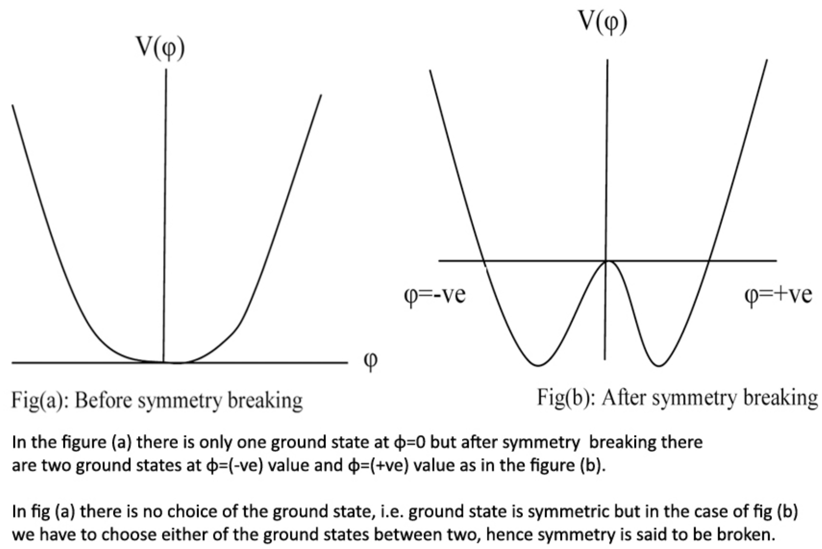


Figure 4.2: Illustration of Spontaneous symmetry breaking. Taken from [81].

"A symmetry of the Lagrangian not respected by the vacuum is said to be *spontaneously broken.*[65, Chapter 7.1]".

4.3.1 Higgs Mechanism

A well known spontaneous symmetry breaking is the *Higgs mechanism* (or more correctly the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism), the spontaneous symmetry breaking of gauge symmetries. However, a gauge symmetry is different than a true symmetry since the symmetry breaking of a gauge symmetry does not give rise to characteristic *massless Nambu-Goldstone bosons* [82] of a continuous symmetry, but to massive bosons [83],[84],[85].

Following [37, Chapter 8.1-8.2],[39, Chapter 20] have one a complex scalar field coupled to itself and the electromagnetic field trough the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 - V(\phi) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\phi)^*(D_\mu\phi) - \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2\,, \tag{4.14}$$

where * indicates the complex conjugate, $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ is the the electromagnetic field and $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative. This Lagrangian is invariant under the $U(1)$ gauge transformation

$$\begin{aligned} \phi(x) &\to \phi^{'}(x) = \phi(x)e^{-ief(x)} \\ \phi^*(x) &\to \phi^{*'}(x) = \phi^*(x)e^{ief(x)} \\ A_u(x) &\to A^{'}_{\mu}(x) = A_{\mu}(x) + \partial f(x). \end{aligned} \tag{4.15}$$

With $\mu^2 > 0$ the field acquire a vacuum expectation value that is not unique and the $U(1)$ global transformation will be spontaneously broken. To ensure an Lorentz invariance Lagrangian the vector field A_μ have to vanish for the vacuum and one obtain a circle of minimum field value at

$$\langle\phi\rangle = \phi_0 = \left(\frac{\mu^2}{2\lambda}\right)^{1/2} e^{i\theta} = \frac{v}{\sqrt{2}}e^{i\theta}, \quad 0 \leq \theta < 2\pi,$$

where the phase angle θ defines a direction in the complex ϕ-plane.

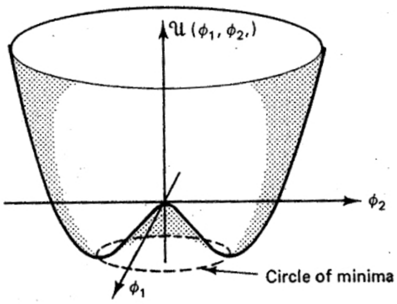


Figure 4.3: Illustration of the circle of minima field value for a complex scalar field $\phi = \frac{1}{\sqrt{2}}[\phi_1 + i\phi_2]$, ϕ_1 and ϕ_2 real. Taken from [86].

Decomposing the complex field $\phi(x) = \frac{1}{\sqrt{2}}[v + \phi_1 + i\phi_2]$ gives us the potential of the form

$$V(\phi) = -\frac{1}{2}\lambda v^2 + \lambda v^2\phi_1^2 + \mathcal{O}(\phi_i^3),$$

where the real field ϕ_1 corresponds to an neutral (uncharged) Klein-Gorden field (spin-0) with mass $\sqrt{2\lambda v^2}$ and ϕ_2 is the massless Goldstone boson of the theory.

However, for any complex field ϕ, a gauge transformation on the form of (4.15) can transform the field ϕ into a real field of the form

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \phi_1(x)\right], \tag{4.16}$$

where the field ϕ_2 has been removed from the theory. The gauge where the field has this form is called *unitary gauge*. Choosing this gauge takes makes it possible to divide the Lagrangian Eq. (4.14 into two parts, namely

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I. \tag{4.17}$$

Here

$$\mathcal{L}_0(x) = \frac{1}{2}\left[\partial^\mu \phi_1(x)\right]\left[\partial_\mu \phi_1(x)\right] - \frac{1}{2}\left(2\lambda v^2\right)\phi_1^2(x) \\ - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(ev)^2 A_\mu(x)A^\mu(x),$$

contains the quadratic terms and has no terms which couple $\phi_1(x)$ and $A_\mu(x)$ thus making it the free-field Lagrangian of a real Klein-Gordon field $\phi_1(x)$. While our second term

$$\mathcal{L}_I(x) = -\lambda v\phi_1^3(x) - \frac{1}{4}\lambda\phi_1^4(x) \\ + \frac{1}{2}e^2 A_\mu(x)A^\mu(x)\left[2v\phi_1(x) + \phi_1^2(x)\right],$$

is the Lagrangian containing all the interactions. When one quantizes $\mathcal{L}_0$, the real field ϕ_1 will remain the same, but the gauge vector boson A_μ will acquire the mass $|ev|$. Thus we have transformed a complex scalar field and a massless real vector field into a real scalar field and a massive real vector field. This mechanism by spontaneous symmetry breaking generates the mass for the gauge boson is what we referred in this section as the Higgs mechanism.

4.4 Symmetron Model

To be able to produce the spontaneous symmetry breaking at low and high density one require a specific coupling between the quintessence field and the neutrinos. This coupling and general equations can be found in [28],[29],[87] and are given by

$$\tilde{g_{\mu\nu}} = A^2(\phi)g_{\mu\nu} \quad A(\phi) = 1 + \frac{1}{2M^2}\phi^2,$$

where $A(\phi)$ is the coupling between neutrinos and quintessence, M is the mass scale of the theory, $\tilde{g_{\mu\nu}}$ and $g_{\mu\nu}$ are the Jordan and Einstein frame respectively. Because of the coupling between neutrinos and quintessence will the potential, Klein-Gordon equation and mass of the neutrinos change to the following quantities

$$V(\phi) \to V_{act} = V(\phi) + A(\phi)(\rho - 3P)$$
$$\Box\phi - V_\phi = 0 \to \Box\phi - V_{act,\phi} = 0$$
$$m_\nu = \text{constant} \ \to m_\nu(\phi) = M_0 A(\phi)$$

or more specifically as

$$V_{act} = \frac{1}{2}\left(\frac{\rho_\nu - 3P_\nu}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 + \rho_\nu - 3P_\nu \tag{4.18}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{act,\phi} = 0 \tag{4.19}$$

$$m_\nu = M_0\left(1 + \frac{\phi^2}{2M^2}\right), \tag{4.20}$$

where M_0 is an normalization constant for the neutrino mass. To break the active potential in (4.18) both μ^2 and λ must be positive. When the trace of the neutrinos $\rho_\nu - 3P_\nu$ is larger than $\mu^2 M^2$ the field will be dominated by the positive term $\frac{\rho_\nu - 3P_\nu}{M^2}\phi^2$, pushing the field to zero. However, when the trace is lower than $\mu^2 M^2$ the second negative term $-\mu^2\phi^2$ will dominate and therefore break the symmetry. The minimum of the field in the broken configuration is $\phi_0 = \pm\frac{\mu}{\sqrt{\lambda}} = \pm\phi_{VEV}$ as shown in section 4.3. This coupling gives us the desired screening mechanism in high densities of neutrinos and an active component in low densities. Note that in the original symmetron model in [28],[29],[87] the symmetron scalar field only is coupled to non-relativistic dark matter, neglecting the pressure, i.e. $\omega \approx 0 \to P \approx 0$.

Note that, as described in [87], one has to include an constant term to the symmetron potential, a cosmological constant C_0, to reproduce the late time acceleration. The screening mechanism makes the scalar field lose its dynamical probabilities since the symmetry is either broken or restored. The field value is therefore restricted by the solutions of the symmetry and symmetry breaking configuration, i.e. $\phi = 0$ and $\phi = \pm\phi_{VEV}$.

CHAPTER 5

CAMB

Controlling complexity is the essence of computer programming.

Brian Kernigan

In this chapter I will introduce the computational software CAMB which I used as a basis for my implementation of the model with MaVans and symmetron potential. More details can be found in [54] (Cosmology),[88] (CAMB-webpage), [89] (Notes on the CAMB code), [90] (Antony Lewis paper on CAMB), [91] (COSMOMC-webpage) and [92] (Antony Lewis paper on COSMOMC).

5.1 Short introduction to CAMB

CAMB is short for Code for Anisotropies in the Microwave Background. This is a code written in FORTRAN 90 which calculates the *theoretical* power spectrum from a set of initial cosmological parameters. The power spectrum, P, is the Fourier transform of the two-point correlation function of the Cosmic Microwave Background (CMB) temperature anisotropies i.e. the statistical correlation between the small variations in the temperature of the background radiation from point to point on the sky. More precisely, the CMB is is given as a function on a sphere and can therefore be expanded in terms of the spherical harmonic Y_{lm}. The coefficients a_{lm} are given by the CMB temperature fluctuations Θ as

$$a_{lm}(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}} \int d\Omega \; Y^*_{lm}(\hat{p}) \; \Theta(\vec{k},\hat{p},\eta).$$

Fixing the moment of the spherical harmonics l and calculating the variance of the distribution of a_{lm} gives us the desired CMB power for an given l as

$$C_l \; \delta_{ll'}\delta_{mm'} = \left\langle |a_{lm}|^2 \right\rangle.$$

The CMB power spectrum is therefore an *angular power spectrum* where l representing an angle on the sky or a scale of the universe. Low values of l are correlations between larges angles on the sky, i.e. large structural scales, and high l are correlations between small angles, i.e. small scales. In figure 5.1 the CMB map from the satellite Planck is shown [93].

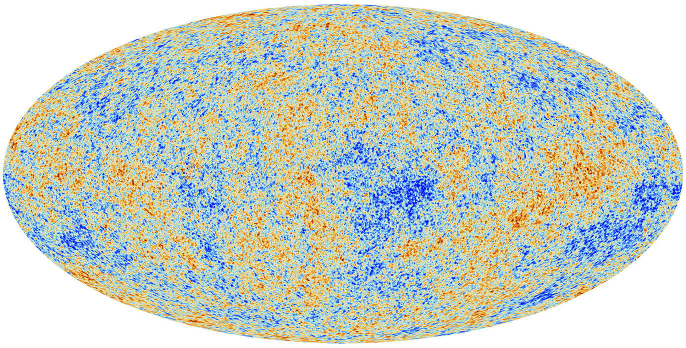


Figure 5.1: The cosmic microwave background as seen by Planck. Figure from [94].

CAMB also calculated the matter power spectrum which is the perturbations as a function of the wavenumber in Fourier space k, i.e. the density contrast $\delta = \delta_{matter}/\rho_{matter}$ (the difference between the mean density and the local density). More precisely it is the Fourier transform of the *matter* two-point correlation function. The wavenumber k represents also an scale and has dimensions $[h/Mpc]$. A value of $k = 0.1$ corresponds to structures of the order 7 Mpc and $k = 1$ to 0.7 Mpc, assuming that $h = 0.7$.

CAMB is a parallelized version of the earlier CMBFAST which uses the standard Boltzmann and Einstein equations [32] to calculate the cosmological evolution. In standard $LCDM$, CAMB uses the following cosmological parameters:

- Ω_b (baryon density)
- Ω_c (cold dark matter density)
- Ω_ν (neutrino density)
- n_s (spectral index - the tilt of the primordial power spectrum)
- A - ScalarPowerAmp (the amplitude of the primordial power spectrum)
- H_0 (Hubble constant)
- CMB parameters τ (opacity at reionization)
- T_{cmb} (CMB temperature, usually fixed)
- Y_P (primordial helium abundance, usually fixed)

The best fit from the Planck collaboration paper [9], shown in table 2, have been used to initialize the cosmological parameters for our simulations.

In this thesis we work only in the linear theory which describes the largest scales, that are dominated by the cosmic expansion. However, on small scales gravitation is non-linear and need to be computed more accurately using N-body simulations. This is left for future work.

5.2 Steps in CAMB

Modification to the newest version of CAMB, December 2013 version, was made to implement the MaVans with the symmetron coupling to quintessence. The formulas were taken from the modified equation in [74],[29],[87]. The changes were first made for the quintessence field with exponential potential and a exponential coupling to the neutrino mass, i.e. $V = V_0 e^{-\sigma\phi}$ and $m_\nu(\phi) = M_0 e^{\beta\phi}$. The slope of the exponential, $\sigma = 1.2257$, where chosen to be the same as the original setting in the module *equation_quint.f90.*

The changes were made in steps consisting of background, perturbation and MaVans. When the changes were fully implemented for the exponential potentiality I changed the coupling and potential to the symmetron model, i.e. $V = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + C_0$, $m_\nu(\phi) = M_0\left(1 + \frac{\phi^2}{2M^2}\right)$.

5.3 The Equations

Background

The background evolution of quintessence with the exponential potential, $V = V_0 e^{-\sigma\phi}$, coupled through MaVans in flat FLRW metric leads to a modification of the Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \beta\left(\rho_\nu - 3P_\nu\right), \tag{5.1}$$

where $\beta = \frac{d\ln m_\nu}{d\phi}$ is the dimensionless coupling between neutrinos and quintessence field. To determine the evolution of the cosmological background, the Friedman equations are changed to

$$H^2 = \frac{\kappa}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_M\right],$$
$$\dot{H} = -\frac{\kappa}{2}\left[\dot{\phi}^2 + \rho_M + P_M\right].$$

Note that naturally there should be a coupling term with the neutrinos in the first Friedman equation. However, this coupling is absorbed into the MaVans. The coupling strength between , β, the symmetron field and neutrinos is given as

$$\beta = M_{Pl}A_{,\phi}/A = \frac{M_{Pl}\ \phi}{M^2 + \phi^2/2}. \tag{5.2}$$

Note that we added a cosmological constant C_0 to the symmetron potential, as done in [87], to obtain the late time acceleration. We then expect a model close to $LCDM$ with some minor corrections caused by the interaction between the scalar field ϕ and the neutrinos.

Perturbation

The modified perturbation equations are the perturbed energy density of neutrinos The modifications where made for the neutrino density perturbation

$$\delta\rho_\nu = \frac{1}{a^4}\int q^2 dq d\Omega\ f_0 \epsilon \Psi + \delta\phi \frac{d\ln m_\nu}{d\phi}\left(\rho_\nu - 3P\nu\right)$$

and the neutrino pressure perturbation

$$\delta P_\nu = \frac{1}{3a^4}\int q^2 dq d\Omega f_0 \left(\frac{q^2}{\epsilon}\Psi - \delta\phi \frac{d\ln m_\nu}{d\phi}\frac{q^2 m_\nu^2 a^2}{\epsilon^3}\right),$$

where q is the comoving momentum, Ω the solid angle, $f_0(q)$ the Fermi-Dirac distribution, Ψ the linear order perturbation to the distribution function, $\epsilon = \sqrt{q^2 + a^2 m_\nu}$ the comoving energy of the neutrinos and $m_\nu(\phi)$ the MaVans. Finally the perturbed Klein-Gordon equation is given by

$$\begin{aligned}&\ddot{\delta\phi} + 2H\dot{\delta\phi} + \left(k^2 + a^2\frac{d^2V}{d\phi^2}\right)\delta\phi + \frac{1}{2}\dot{h}\dot{\phi} = \\ &- a^2\left[\frac{d\ln m_\nu}{d\phi}\left(\delta\rho_\nu - 3\delta P_\nu\right) + \frac{d^2\ln m_\nu}{d\phi^2}\delta\phi\left(\rho_\nu - 3P_\nu\right)\right],\end{aligned}$$

where k is the wavenumber and h is the scalar part of the metric perturbation. Note that these perturbation equations are taken directly from the paper, which is written in symmetron gauge.

MaVans

The MaVans with exponential coupling are given by

$$m_\nu(\phi) = M_0 e^{\beta\phi},$$

where M_0 is a normalization term for the neutrino mass. For symmetron, the coupling changes and the MaVans are therefore given by

$$m_\nu(\phi) = M_0 A = M_0\left(1 + \frac{\phi^2}{2M^2}\right),$$

where M_0 is again a normalization term for the neutrino mass.

CHAPTER 6

Parameters and Results

> Insanity: doing the same thing over and over again and expecting different results.
>
> *Albert Einstein*

In this chapter I present a selection of tested models and their cosmological implications.

6.1 Symmetron parameters

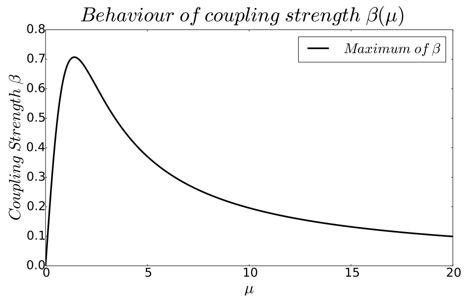
The symmetron parameters consist of two mass parameters μ and M and the dimensionless parameter λ. The symmetron potential has two different possible configuration, the spontaneously broken potential and the unbroken potential, see figures 6.3 and 6.4. The symmetron field is therefore restricted by the potentials minimums at $\phi = 0$ and $\phi_0 = \pm\mu/\sqrt{\lambda}$ respectively. The maximum mass difference is thereby given by as

$$\frac{\Delta m_\nu}{m_\nu} = \frac{|\phi_0|}{2M^2} = \frac{\mu^2}{2\lambda M^2}.$$

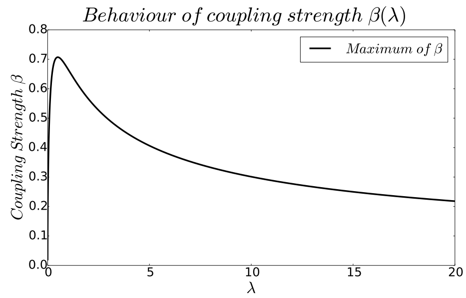
One observe that higher values of μ increases the mass difference, while for higher values of either λ or M the mass difference is lowered. The maximum coupling strength β between the symmetron field and the neutrinos is given as

$$\beta = \frac{M_{Pl}\,\phi_0}{M^2 + \phi_0^2/2}.$$

If one increase either the parameter μ or λ the strength will at first increase until it reaches a maximum. After this maximum the strength starts to decrease for higher values of μ and λ, see figure 6.1. Higher values of M will lower the coupling strength, see figure 6.2.



(a) The qualitative behavior of the coupling strength with increasing μ.



(b) The qualitative behavior of the coupling strength with increasing λ.

Figure 6.1: The qualitative behavior of the coupling strength β as a function of the mass parameter μ (a) and of λ (b). For both cases the strength grows to a maximum before decreasing for higher values.

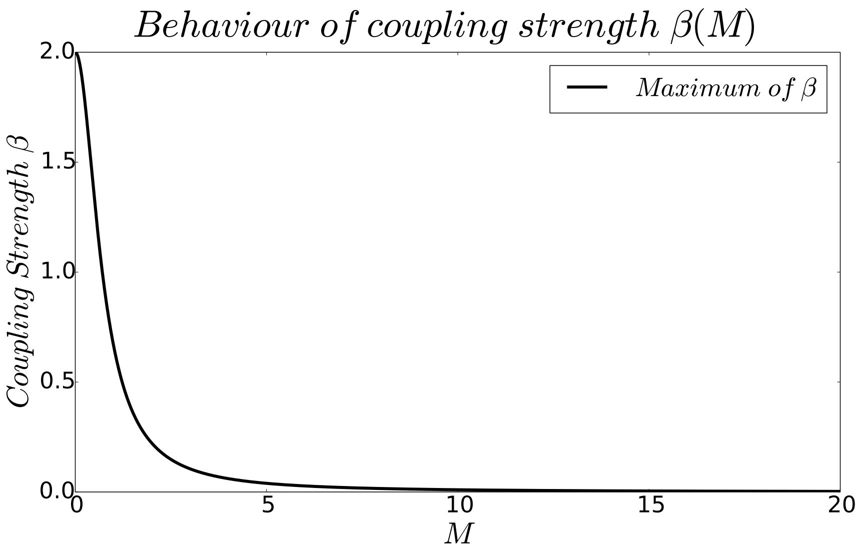


Figure 6.2: The qualitative behavior of the coupling strength β as a function of the mass parameter M. The strength always decreases for higher values of M.

The two parameters μ and λ determine the shape of the symmetron potential. Changing the parameter λ to a higher values for the unbroken potential ,squeezes the potential. Reducing λ makes the potential consequently flatter. Note that if the broken potential becomes to flat the perturbations in the symmetron field can move the field away from its minimum at $\phi = 0$, see figure 6.4.

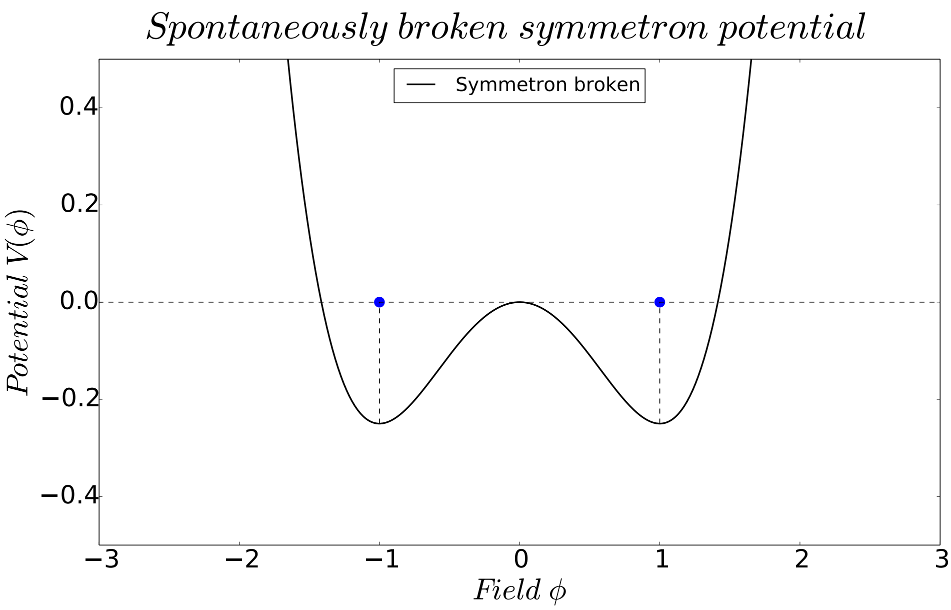


Figure 6.3: Illustration of the spontaneously broken symmetron potential, with its two possible minimums at $\phi_0 = \pm\mu/\sqrt{\lambda}$. Note that the potential value is below zero.

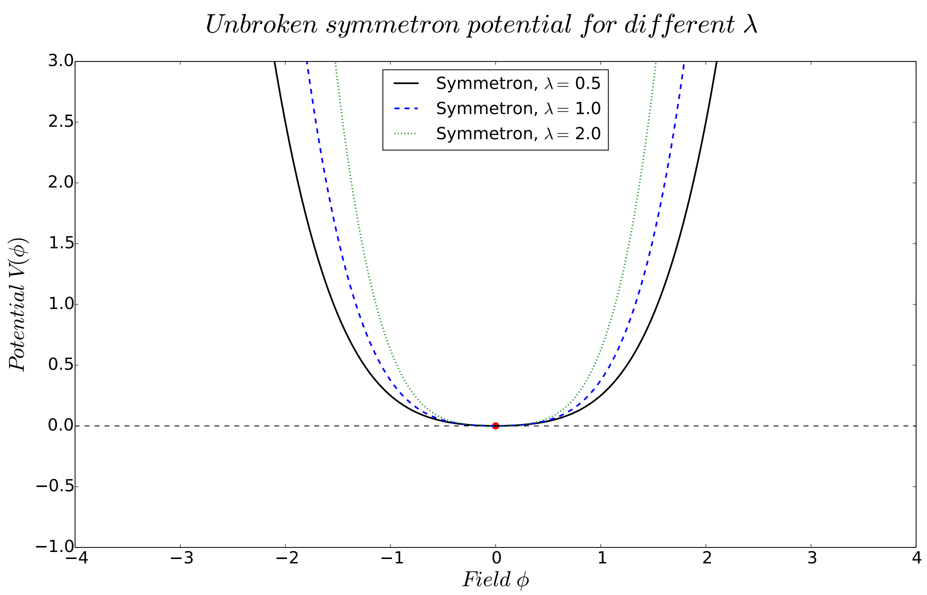


Figure 6.4: Illustration of the unbroken symmetron potential with different λ and its common minimum at $\phi = 0$. The potential becomes flatter as one decreases the value of λ.

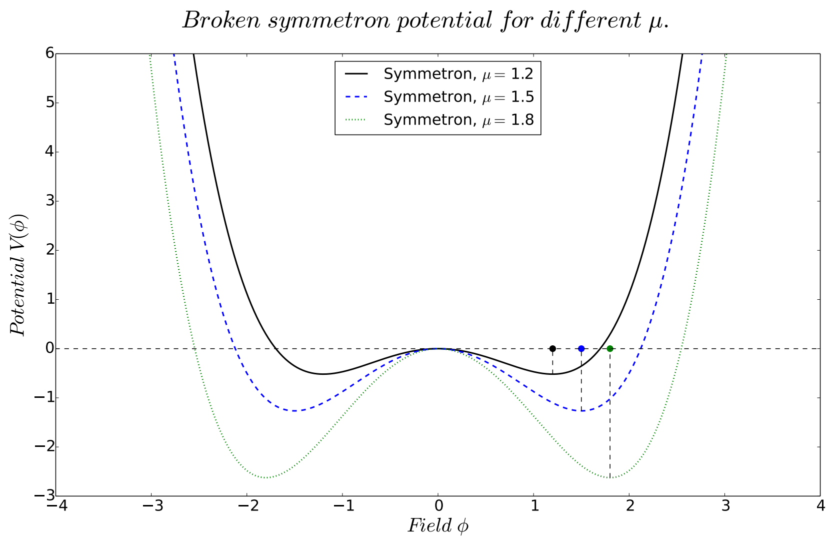


Figure 6.5: Illustration of the spontaneously broken symmetron potential for different choices of μ. The black, blue and green dots shows the respective minimum point for its respective case. The symmetron potential and field value at the potential minimum increases with higher values of the parameter μ.

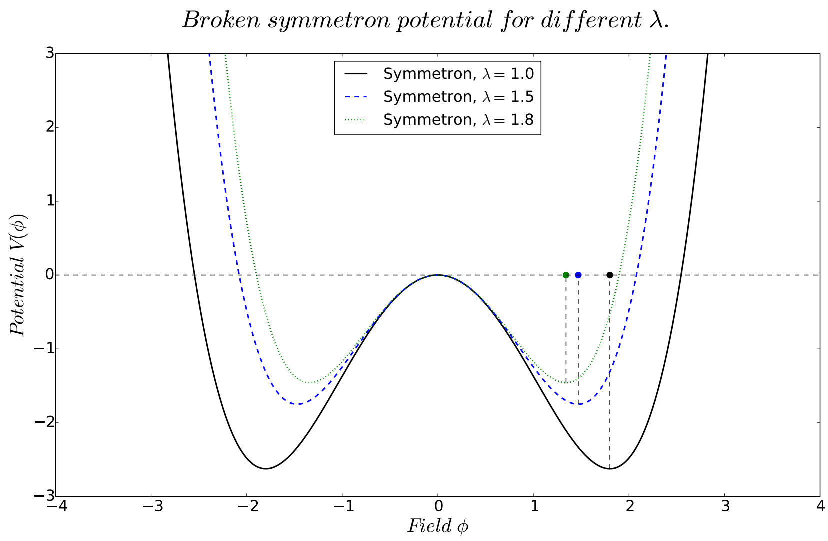


Figure 6.6: Illustration of the spontaneously broken symmetron potential for different choices of λ. The dots show the position of the minimum point in the respective case. The symmetron potential and field value at the potential minimum decreases with higher values of the parameter λ.

For the spontaneously broken potential one observe that a higher value of λ lowers the minimum of the potential and shift its torwards a smaller field values ϕ making it flatter, see figure 6.6. The other parameter, μ, have the opposite effect on the spontaneously broken potential than λ and increases both the potential and field at the potential minimum for higher values, see figure 6.5. Note that in the spontaneously broken configuration the potential value is below zero, see figure 6.3.

To obtain the dark energy *today* the cosmological constant can be set at a higher value than in standard $LCDM$, depending on which configuration the potential is in *today*. This leads to a different curvature of the universe at earlier time, when the potential configuration is different than today. However, the neutrino mass changes also with the configuration of the potential. But the neutrino mass depends on position of the potential minimum while the cosmological constant depends its value. Therefore the changes of the neutrino mass can compensate for changes in the symmetron potential, partially or entirely, depending on the settings of the symmetron parameters.

6.2 Physical parameters

Instead of the parameters μ, λ and M we would like to use the physical parameters, z_{break}, L, β, i.e. the redshift of the symmetry breaking, the range of the fifth force and the strength of the fifth force relative to gravity defined in Eq. (5.2). Following [87] we define the critical neutrino matter density for the spontaneous symmetry breaking and its redshift as

$$\rho_{break} = \mu^2 M^2 = 3H_0^2 M_{pl}^2 \Omega_{\nu,\text{today}} (1 + \omega(a_{break}))\,(1 + z_{break})^3\,,$$

where H_0 and $\Omega_{\nu,\text{today}}$ are the Hubble parameter and neutrino density today. In regions where the trace of the energy-momentum tensor of neutrinos is higher than the breaking density, i.e. $\rho_\nu - 3P_\nu > \rho_{break}$, the symmetry of the potential upheld. The scalar field ϕ therefore moves towards the minimum of the unbroken potential at $\phi_{min} = 0$. In regions where the trace is lower, i.e. $\rho_\nu - 3P_\nu < \rho_{break}$, the field symmetry is spontaneously broken and the new minimum will be at

$$\phi_{min} = \pm\phi_0 \sqrt{1 - \frac{\rho_\nu}{\rho_{break}}},$$

where we have chosen ϕ_0 to be the positive solution of the field $\phi_0 = \frac{\mu^2}{\lambda}$, see Section 4.3. for details. Note that since the neutrinos are initially relativistic, i.e. $\rho_\nu - 3P_\nu = 0$, the symmetry is already broken in the beginning. The symmetry is restored when the neutrinos become non-relativistic and breaks yet again when the neutrino density becomes lower than the breaking density. The mass, m_ϕ, of the small fluctuations around the minimum of the active potential is given as

$$m_\phi^2 \equiv V_{act,\phi\phi} = \left(\frac{\rho_\nu - 3P_\nu}{\rho_{break}} - 1\right)\mu^2 + 3\lambda\phi_{min}^2$$
$$= \begin{cases} \mu^2\left(\frac{\rho_\nu - 3P_\nu}{\rho_{break}} - 1\right), & \rho_\nu > \rho_{break} \\ 2\mu^2\left(1 - \frac{\rho_\nu - 3P_\nu}{\rho_{break}}\right), & \rho_\nu < \rho_{break} \end{cases}$$

The Compton wavelength, $\lambda_\phi = \frac{1}{m_\phi}$, for the longest range of the fifth force is therefore given by

$$\lambda_0 = \frac{1}{\sqrt{2}\mu},$$

and the maximum range of the fifth force mediated by the field in units of Mpc/h is

$$L \equiv \frac{\lambda_0}{\text{Mpc}/h}.$$

The parameters μ, λ and M are given in units of M_{Pl} whereas L is given in Mpc/h, while z_{break} and β are dimensionless. The dark energy of our model consists of two contributions. One is the symmetron field itself and the second is the added cosmological constant C_0. We obtained $C_0 = f\rho_\Lambda$ by adjusting the fraction of the cosmological density ρ_Λ given by $LCDM$ such that we retrieved the correct total dark energy density today. Here f is the fraction of cosmological density. For all symmetron models and $LCDM$ we had 3 equally massive neutrinos.

6.3 Models and Results

I have divided the results into three different symmetron parameter choices called Symmetron A, Symmetron B and Symmetron C. Symmetron A represent a tuning of the parameters to a coupling strength of about 1 and a breaking redshift in the future. In the case of Symmetron B the field is tuned to zero on the background, but affect the perturbations. Symmetron C is where both the strength and redshift are higher than in Symmetron A and B.

In Symmetron A and B the neutrino mass was normalized such the sum of the neutrino mass today where $\sum_\nu m_\nu = 0.04\ eV, 0.9\ eV$ and $3.0\ eV$. For Symmetron C and $LCDM$ we included also three scenarios where the sum of the neutrino mass where $\sum_\nu m_\nu = 0.1\ eV,\ 0.3\ eV$ and $6.0\ eV$.

6.3.1 Symmetron A

Symmetron A represent a tuning of the parameters to a coupling strength of 1 and a breaking redshift in the future. The parameters for Symmetron A are given in table 6.1.

Table 6.1: Symmetron A

$\sum_\nu m_\nu$	μ	λ	M	β	L	z_{break}	f
0.04	$5 \cdot 10^{-60}$	$2.5 \cdot 10^{-111}$	10^{-2}	0.999	371.34	$-9.99 \cdot 10^{-5}$	0.999
0.9	$5 \cdot 10^{-60}$	$2.5 \cdot 10^{-111}$	10^{-2}	0.999	371.34	$-9.99 \cdot 10^{-5}$	0.999
3.0	$5 \cdot 10^{-60}$	$2.5 \cdot 10^{-111}$	10^{-2}	0.999	371.34	$-9.99 \cdot 10^{-5}$	0.999

One observes that the fraction f is close to one. We conclude that the cosmological constant contributes to the entire dark energy component and confirms that the symmetron potential is indeed unbroken today. Since the neutrinos are initially relativistic the trace of the neutrino energy tensor is zero. Consequently the symmetron potential is initially broken and the symmetron field is at its maximum possible value of $\phi_0 = \mu^2/\lambda$, until the neutrinos becomes non-relativistic and the trace becomes non-zero. The symmetry is thereby restored and the symmetron field decrease to zero. The symmetron field and neutrino mass stay at its new minimum value until the redshift of symmetry breaking in the future. Figures 6.7 and 6.8 show the changes in the field and neutrino mass respectively for the scenario with $\sum_\nu m_\nu = 0.9\ eV$.

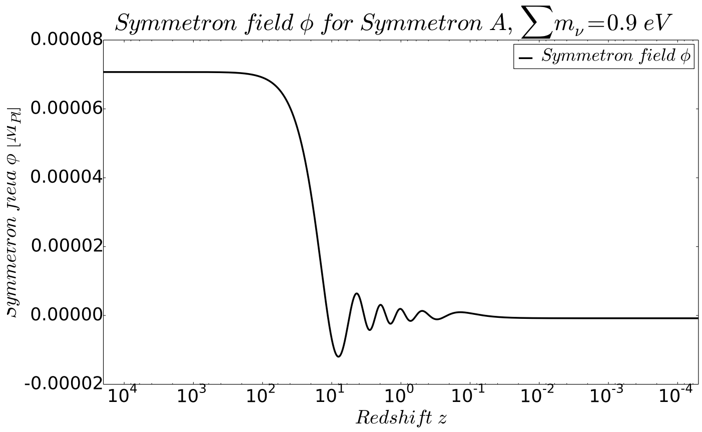


Figure 6.7: The change in the symmetron field ϕ for Symmetron A in the $\sum_\nu m_\nu = 0.9\ eV$ scenario. The symmetron potential is initially spontaneously broken and the symmetron field is at its maximum possible value. The neutrino becomes then non-relativistic, changing the symmetron potential to be unbroken, sending the field value to zero.

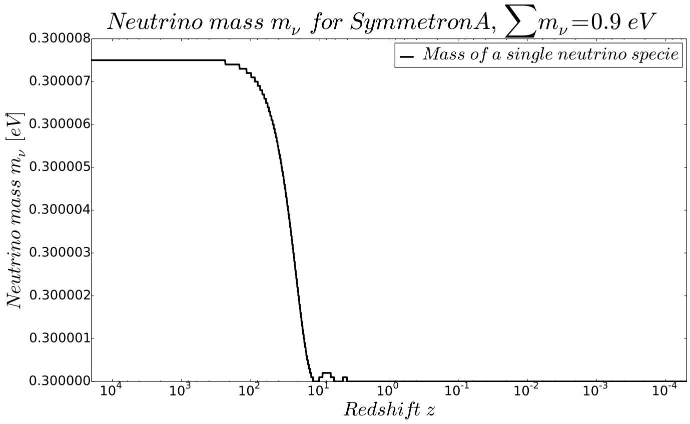


Figure 6.8: The change of the neutrino mass for one neutrino specie in Symmetron A for the $\sum_\nu m_\nu = 0.9\ eV$ scenario. The symmetron potential is initially spontaneously broken and the neutrino mass is at its maximum possible value. The neutrino becomes then non-relativistic, changing the symmetron potential to be unbroken, decreasing the neutrino mass to the normalized mass of 0.3 eV.

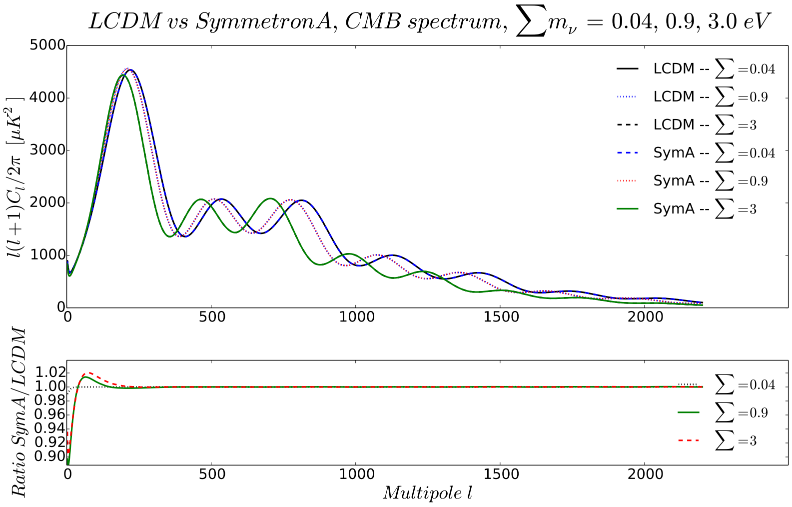


Figure 6.9: Top: The CMB power spectrum for $LCDM$ and Symmetron A for the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. The CMB power spectrum for the two models overlap each other. Bottom: The ratio between the CMB power spectrum of Symmetron A and $LCDM$, for the of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$ neutrino mass scenarios. One can observe a two percent changes in the two highest mass scenarios compared to $LCDM$. Larger effects are in areas with large cosmic variance and difficult to assess.

The effect of Symmetron A on the CMB power spectrum are presented in figure 6.9.

One can observe a two percent change in the two highest mass scenarios compared to $LCDM$. Effects on the largest scales, very low moments l, are in areas of large cosmic variance and difficult to assess since on obtain large error bars for the CMB.

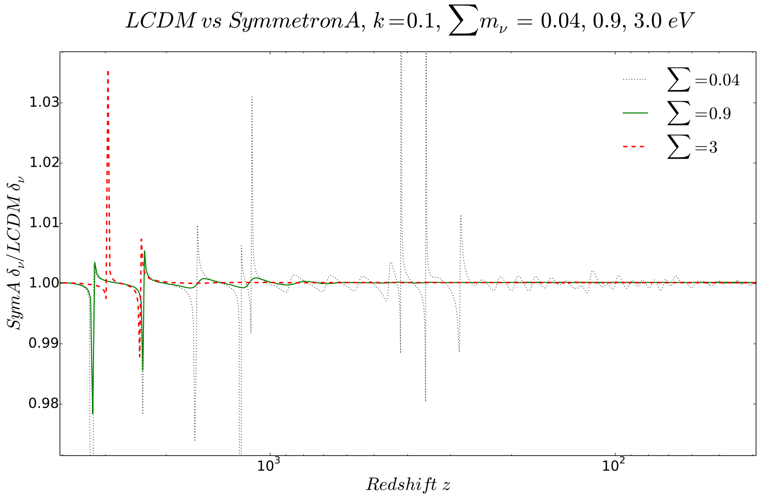


Figure 6.10: The ratio between Symmetron A δ_ν and $LCDM$ δ_ν for wavenumber $k = 0.1$ in the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. One observe fluctuations around the time when the neutrinos become non-relativistic. However, these fluctuations stabilizes and return to values close to the neutrino perturbations of $LCDM$.

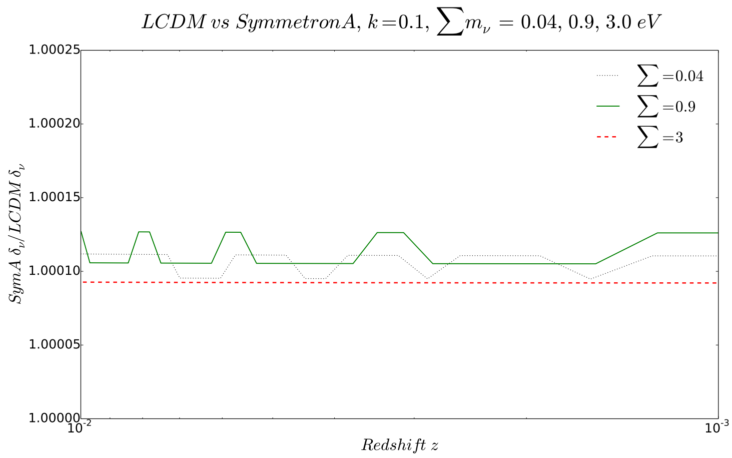


Figure 6.11: The ratio between Symmetron A δ_ν and $LCDM$ δ_ν around today for the wavenumber $k = 0.1$ in the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. On observes a minimum decrease in the neutrino perturbation compared with $LCDM$ today.

Even if the different in mass is extremely small, see figure 6.8, the symmetron-neutrino coupling does affect the neutrino perturbations. In figures 6.10 and 6.11 one can observe fluctuation around the time when the neutrino become non-relativistic. However, these fluctuations are quickly stabilized resulting in a minimum change to the neutrino perturbations today compared to $LCDM$.

The perturbations have an effect on the matter power spectrum, as seen in figure 6.12. The effects on the matter power spectrum are larger for higher neutrino mass. However, the changes are to small to be detected in the matter power spectrum.

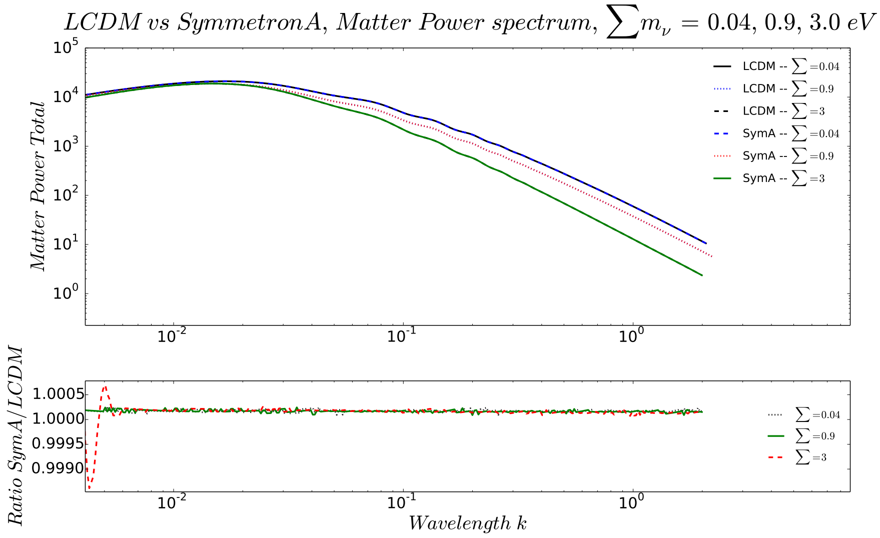


Figure 6.12: Top: Matter power spectrum for $LCDM$ and Symmetron A, for the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. Bottom: The ratio between the matter power spectrum for Symmetron A and $LCDM$, for the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. One observe larger effects on the matter power spectrum for higher neutrino mass.

6.3.2 Symmetron B

Symmetron B is almost the same as Symmetron A, but the symmetry of the potential is never broken because the breaking parameter μ is not included in the symmetron potential, i.e. $\mu = 0$. Consequently , the symmetron-neutrino coupling strength β and symmetron field ϕ are zero. The field is turned off in the background evolution. The parameters for symmetron B are given in table 6.2.

Table 6.2: Symmetron B

$\sum_\nu m_\nu$	μ	λ	M	β	L	z_{break}	f
0.04	0	$2.5 \cdot 10^{-111}$	10^{-2}	0	∞	∞	0.999
0.9	0	$2.5 \cdot 10^{-111}$	10^{-2}	0	∞	∞	0.999
3.0	0	$2.5 \cdot 10^{-111}$	10^{-2}	0	∞	∞	0.999

As shown in figure 6.13 is the symmetron field is indeed always zero both before and after the neutrino becomes non-relativistic. The neutrino mass stays at its normalized mass of 0.3 eV, see figure 6.8. Consequently there is no redshift at which the symmetry is spontaneously broken.

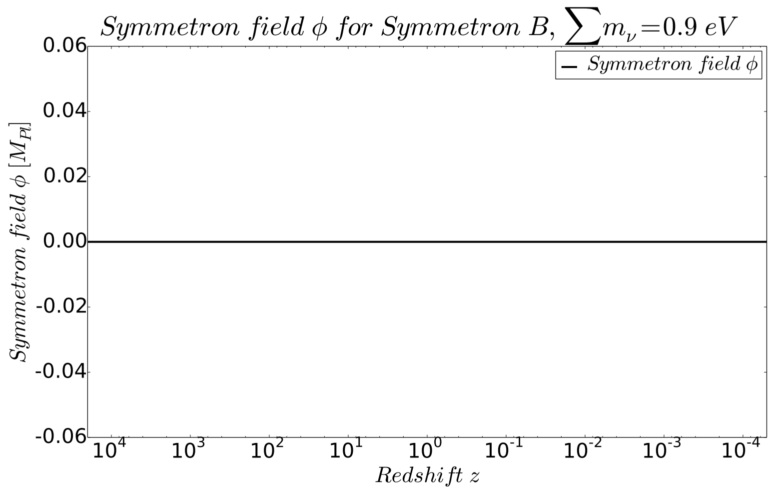


Figure 6.13: The change of the symmetron field in Symmetron B in the $\sum_\nu m_\nu = 0.9\ eV$ scenario. The field is always zero since the symmetron potential is always unbroken.

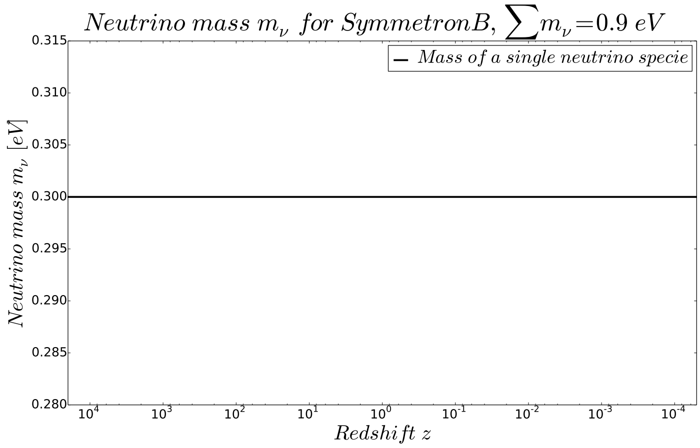


Figure 6.14: The change of the neutrino mass for one neutrino specie in Symmetron B in the $\sum_\nu m_\nu = 0.9\ eV$ scenario. The neutrino mass stay always at its normalized mass of 0.3 eV since the symmetron potential is always unbroken.

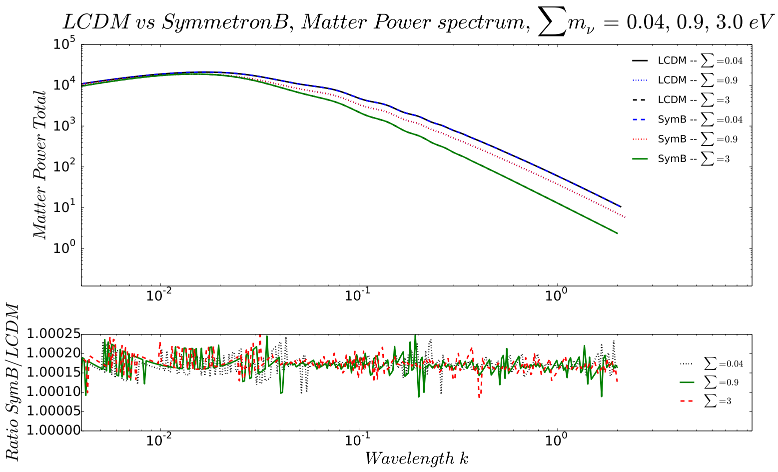


Figure 6.15: Top: The matter power spectrum for $LCDM$ and Symmetron B $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. neutrino mass scenarios. One observe that the two models are overlapping or extremely close. Bottom: The ratio between matter power spectrum of Symmetron B and the matter power spectrum of $LCDM$, for the $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$. neutrino mass scenarios.

Since the changes in neutrino mass in Symmetron A are extremely small is Symmetron A very close to Symmetron B, differing only in the perturbations. The difference in the matter power spectrum between Symmetron B and $LCDM$, see figure 6.15 are smaller compared to Symmetron A, see figure 6.12. This can be explained by difference in perturbation caused by the symmetron-neutrino coupling.

6.3.3 Symmetron C

Symmetron C has a larger coupling strength than Symmetron A. Consequently the breaking redshift z_{break} and mass difference are also higher than in Symmetron A. The settings for Symmetron C are given in table 6.3.

Table 6.3: Symmetron C

$\sum_\nu m_\nu$	μ	λ	M	β	L	z_{break}	f
0.04	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	15.879	1.353
0.1	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	11.389	1.353
0.3	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	8.309	1.353
0.9	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	6.068	1.353
3.0	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	4.231	1.350
6.0	10^{-56}	10^{-104}	$5 \cdot 10^{-4}$	392.15	0.185	3.398	1.347

As shown in table 6.3 the symmetron contribute to the dark energy (negatively) today. Consequently, one needs to adjust the cosmological constant to higher values than in Symmetron A, B and $LCDM$ to obtain the desired dark energy of today. The changes in the cosmological constant is lower in the two scenarios with high neutrino mass, $\sum_\nu m_\nu = 3.0, 6.0\ eV$. This is because the neutrino density in these two cases is more influential and therefore one needs a lower dark energy today.

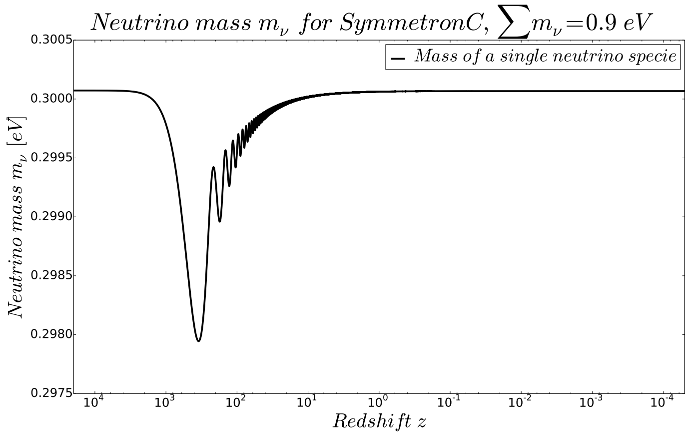


Figure 6.16: Results of the change of the neutrino mass for one neutrino specie in Symmetron C for the $\sum_\nu m_\nu = 0.9\ eV$ scenario.

In table 6.3 one observes also that the breaking redshift decreases as the neutrinos become heavier. The breaking density ρ_{break} stays the same for all the different neutrino mass scenarios, but the neutrino density rises as one increases the neutrino mass. One needs therefore a longer time of expansion for the neutrino density to be lower than the breaking density.

Figure 6.16 shows that the neutrino mass decreases when the neutrinos become non-relativistic, as in Symmetron A. The symmetry breaking density is higher than in the case of the Symmetron A model and therefore the symmetry is spontaneously broken at an earlier time. Once the symmetry is broken the field value and the neutrino mass increase to their former values of relativistic neutrinos. However, the transition from unbroken to spontaneously broken potential is not smooth. One observes in figure 6.16 oscillations between the broken and unbroken symmetry state, while the neutrino mass rises. Eventually the density of the neutrino become diluted and the mass is at its prior unbroken mass value such that the neutrino mass stays constant.

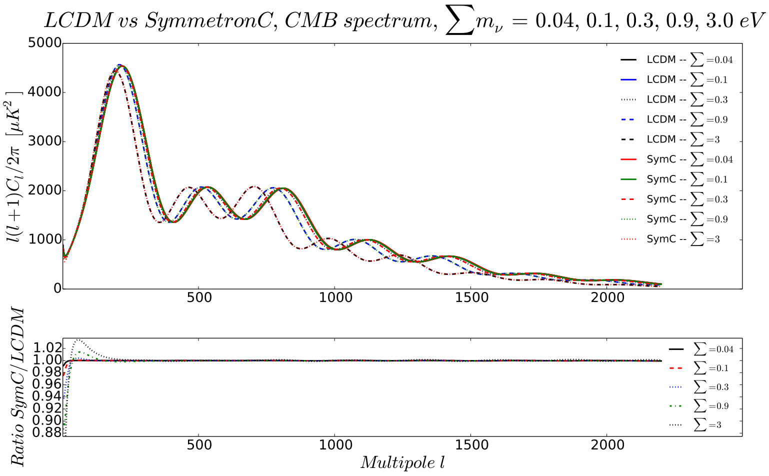


Figure 6.17: Top: The CMB power spectrum for Symmetron C and $LCDM$ for the $\sum_\nu m_\nu = 0.04, 0.1, 0.3, 0.9, 3.0\ eV$ neutrino mass scenarios. Bottom: The ratio between the CMB power spectrum of Symmetron C and $LCDM$ or the $\sum_\nu m_\nu = 0.04, 0.1, 0.3, 0.9, 3.0\ eV$ neutrino mass scenarios.

The CMB power spectrum of Symmetron C model is given in figure 6.17. For the neutrino mass scenario $\sum_\nu m_\nu = 6.0\ eV$ in Symmetron C one observes an substantially increase on larges scales in the matter power spectrum and the CMB power spectrum, see figure 6.18. The neutrino mass become so high that the neutrinos become non-relativistic before recombination. Therefore we can excluded this mass scenario with a Symmetron C configuration by observations.

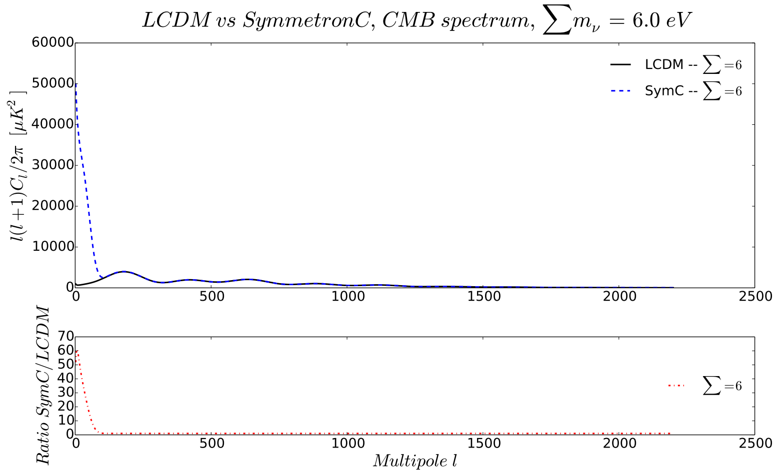


Figure 6.18: CMB power spectrum in Symmetron C for the $\sum_\nu m_\nu = 6.0\ eV$ neutrino mass scenario. One observe a large increase in the power spectrum at larges scales.

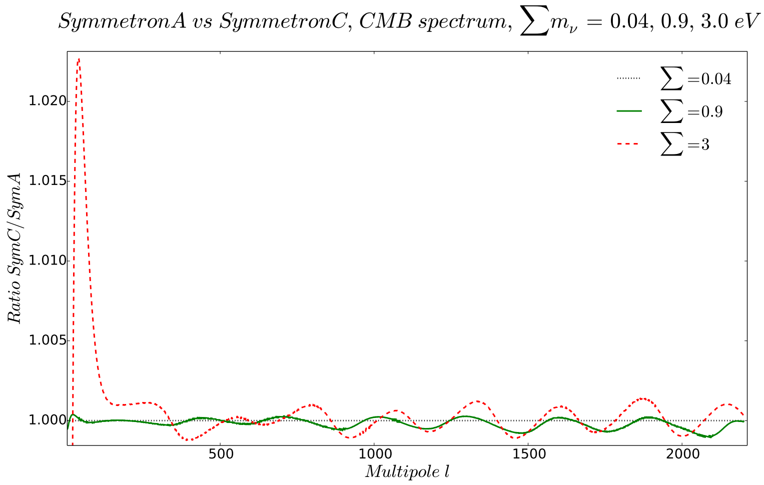


Figure 6.19: The difference between the CMB power spectrum for the Symmetron A and Symmetron C. One observe a clear change in the CMB power spectrum caused by the symmetron field acting on the neutrinos increase with the neutrino mass.

The difference between the CMB power spectrum of Symmetron C and Symmetron A is given in figure 6.19 and are in the same order as Symmetron A. One observes a clear change in the CMB power spectrum which is caused by the symmetron field acting on the neutrinos and increases for larger neutrino masses. The relative mass difference is much higher than in the case of Symmetron A.

Fluctuations in the neutrino perturbations are also observed in Symmetron C, see figure 6.20. As in Symmetron A and B the fluctuation stabilize, but the fluctuation themselves are larger than in the two previous cases.
One observes small but clear differences in the neutrino perturbation today for the various neutrino masses, see figure 6.21. For low neutrino masses the neutrino perturbations are smaller than $LCDM$, while being larger for high masses.

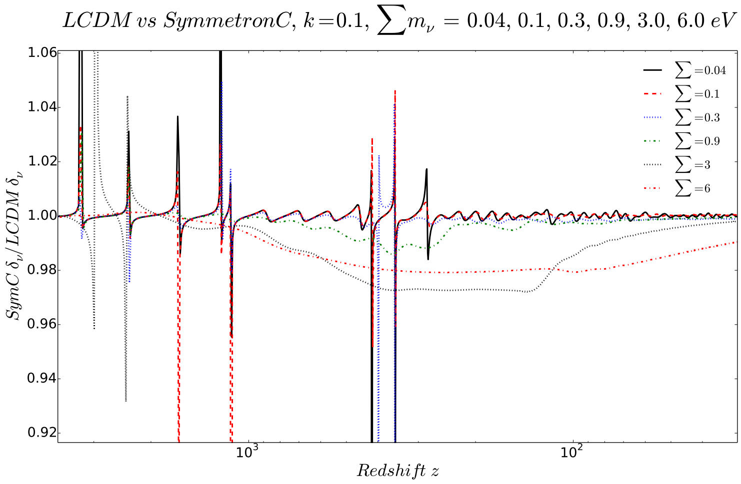


Figure 6.20: A closeup of the ratio Symmetron C δ_ν and $LCDM$ δ_ν at $k = 0.1$.

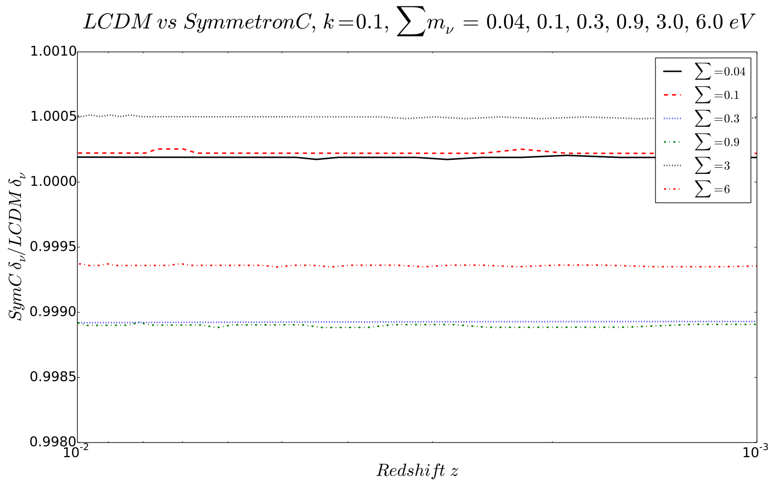


Figure 6.21: Symmetron C around today for the ratio between δ_ν and $LCDM$ δ_ν at $k = 0.1$.

For higher scales at $k = 0.01$ one observes the same trend for Symmetron C, see figure 6.22. However, for the $\sum_\nu m_\nu = 3.0\ eV$ scenario we observe an increase in the perturbation compared to $LCDM$ when it is affected by the neutrino mass changes. The neutrino perturbation is however not able to restore the perturbation to the $LCDM$ value, but remain higher, see figure 6.23.

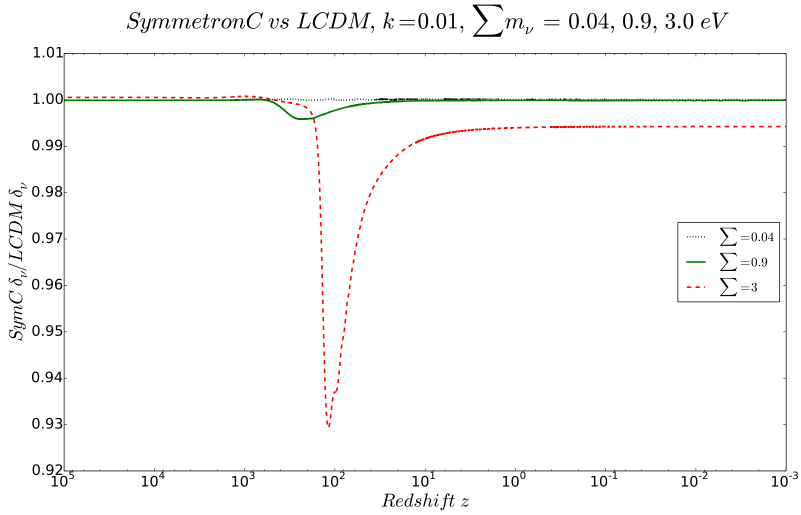


Figure 6.22: The ratio of Symmetron C δ_ν and $LCDM$ δ_ν at $k = 0.01$ in the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$.

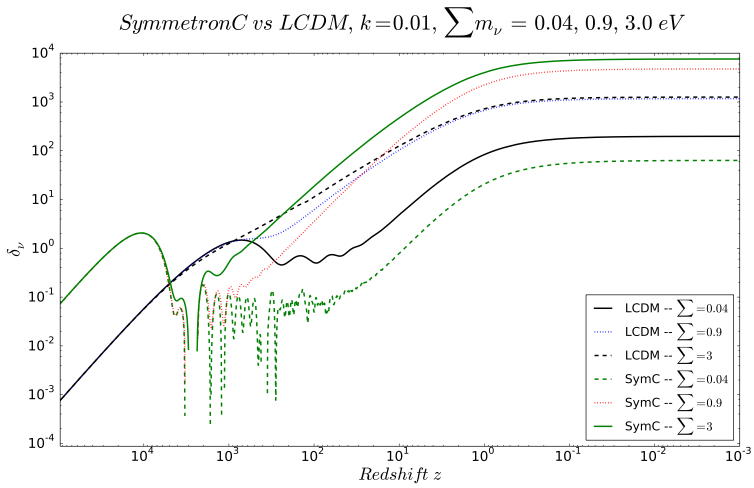


Figure 6.23: $LCDM$ δ_ν and Symmetron C δ_ν at $k = 0.01$ in the cases of $\sum_\nu m_\nu = 0.04, 0.9, 3.0\ eV$.

The effect on the matter spectrum on the neutrino mass scenarios with $\sum_\nu < 6.0\ eV$ in figure 6.24. One clearly observe an effect at the highest mass scenario of $\sum_\nu = 3.0\ eV$. However, for the other scenarios there are a only minimal changes in the matter power spectrum, see figure 6.25. One can observe a trend of more structures for higher mass scenarios caused by the neutrino perturbations. But the differences are so small that one can't make any conclusions from the matter power spectrum.

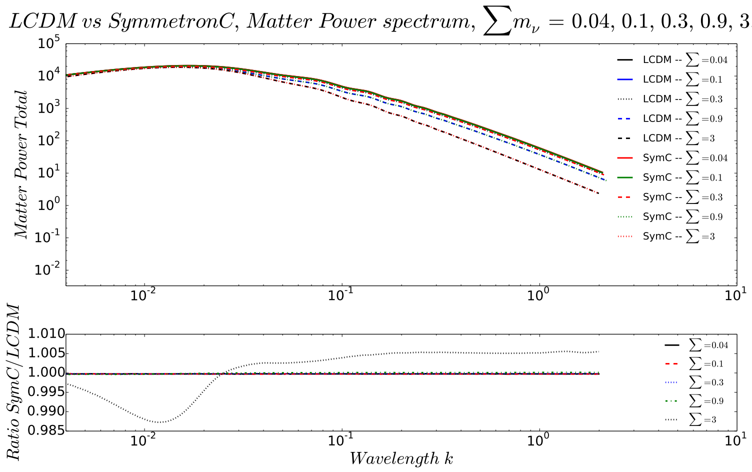


Figure 6.24: Matter power spectrum for $LCDM$, Symmetron C, and the ratio between Symmetron C and $LCDM$, in the $\sum_\nu m_\nu = 0.04, 0.1, 0.3, 0.9, 3.0\ eV$ neutrino mass scenarios.

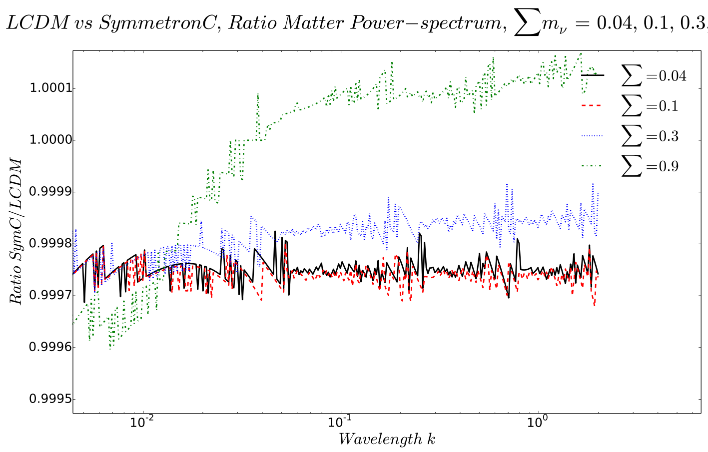


Figure 6.25: Closeup of the matter power spectrum for $LCDM$, Symmetron C, and the ratio between Symmetron C and $LCDM$, in the $\sum_\nu m_\nu = 0.04, 0.1, 0.3, 0.9\ eV$ neutrino mass scenarios.

CHAPTER 7

Outline and future work

> It is always wise to look ahead, but difficult to look further than you can see.
>
> *Winston Churchill*

The goal of this thesis was to investigate if the symmetron scheme could work with MaVans and cure the instabilities of the neutrino perturbations. The symmetron mechanism should screen out the fifth force created by coupling between the scalar field ϕ and the neutrino in regions of high density. We hoped that the effect would stabilize the perturbation and result in a structure formation closer to the observations. To investigate the problem I modified an already existing code which simulates the cosmological evolution, CAMB, by including both MaVans and the symmetron.

The results of the simulations done in CAMB for mass varying neutrino including the symmetron model show that there is a stabilizing effect on the perturbations. However, the scalar field dynamics is reduce because of the form of the symmetron potential and coupling. We are restricted to be either in a unbroken configuration or a spontaneously broken configuration. The scalar field is therefore trapped by the potential and looses its dark energy properties. To achieve the desired dark energy today we added an cosmological constant to the symmetron potential. The MaVans are also restricted by the configurations of the symmetron. Its therefore difficult to obtain a large maximum neutrino mass difference. However, we observe from the results that the effects from the symmetron increase with higher neutrino masses, as expected. One can also observe the restoring and breaking of the symmetron potential in the three different configurations. The largest measurable effects on the CMB power spectrum are about two percent and can therefore set constrains on the neutrino mass. Effects on the matter power spectrum are to small measure. The only exception is the scenario with $\sum_\nu m_\nu = 6.0$ for Symmetron C which is ruled out. Note that another choice of parameters might still work for this neutrino mass scenario.

Future work

Calculations of quantum field corrections are not included in our model. However, following the paper by Doran, Michael and Jäckel, Jörg [95] the quantum field effective potential was calculated with a mass dependent fermion. They found that the coupling between the scalar field and the fermions is quite restricted by quantum field theory. If one insert the symmetron potential in the calculations one obtain the effective potential $V_{eff} = V\left(1 + \frac{\Lambda}{32\pi^2}\frac{12\lambda - 4\mu^2/\phi^2}{\lambda\phi^2 - 2\mu^2}\right)$, which would lead to large quantum corrections. One needs therefore to assume that all the quantum corrections are included in the potential already and no further corrections are necessary.

In the current work we would investigate the parameter space using CosmoMC (Cosmological Markov-Chain Monte-Carlo). COSMOMC is an algorithm for sampling probability distributions from Markov Chains. These Markov chains are constructed by CosmoMC using CAMB. Predictions from CAMB is compared with data from the Planck Satellite allowing us to investigate the phase space of the different parameters.

Since many effects from this model are in the non-linear regime one would need to run MaVans with the symmetron model in N-body codes. This can be implemented in already existing codes which include modified gravity with the symmetron. One could also go further and test MaVans with other screening mechanisms, like the chameleon for comparison.

Appendices

APPENDIX A

Hamilton's principle of least action and the Euler-Lagrange equations

The action functional is given by

$$S = \int_{t_1}^{t_2} L dt, \tag{A.1}$$

where $L\left(\vec{q}(t), \dot{\vec{q}}(t), t\right)$ is the Lagrangian, see subsection (2.2.1) for definition.

Consider a small variation $q_i(t) \to q_i(t) + \delta q_i(t)$ with the constrains

$$\delta q_i(t_1) = \delta q_i(t_2) = 0 \tag{A.2}$$

on the action in Eq. ($A.1$). The action should be stationary under the variation [35], [36], and therefore requires a variation in the Lagrangian

$$\delta S = \int_{t_1}^{t_2} \delta L \; dt = 0. \tag{A.3}$$

The variation in the Lagrangian is given by (for all i's),

$$\begin{aligned} \delta L &= \frac{\partial L}{\partial q_i}\delta q_i + \frac{\partial L}{\partial \dot{q}_i}\delta \dot{q}_i \\ &= \frac{\partial L}{\partial q_i}\delta q_i - \frac{d}{dt}\frac{\partial L}{\partial q_i}\delta q_i + \frac{d}{dt}\left[\frac{\partial L}{\partial q_i}\delta q_i\right] \\ &= \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial q_i}\right]\delta q_i + \frac{d}{dt}\left[\frac{\partial L}{\partial q_i}\delta q_i\right]. \end{aligned} \tag{A.4}$$

Setting the variation of the Lagrangian from Eq. ($A.4$) into the variation of the action in Eq. ($A.3$) gives

$$\delta S = \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial q_i}\right]\delta q_i dt + \left[\frac{\partial L}{\partial q_i}\delta q_i\right]_{t_1}^{t_2} \tag{A.5}$$

$$= \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial q_i}\right]\delta q_i dt, \tag{A.6}$$

where one have used the constrains of Eq. ($A.2$) on the last term in Eq. ($A.5$). Since the variation of the action δS is to vanish for an arbitrary variation of the path in configuration space must the term inside the brackets in Eq. ($A.6$) be zero. This leads to the *Euler-Lagrange equations*,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \; \forall \; i.$$

Going from classical mechanics to field theory gives the action functional for fields

$$S = \int_\Omega \mathcal{L}\left(\Phi^i(x_\mu), \partial_\mu \Phi^i(x_\mu)\right) d^4x,$$

where Ω is a region of space-time, $\mathcal{L}$ the Lagrangian density, $\Phi_i(x_\mu)$ the i'th component of the field (or i'th field), $\partial_\mu \Phi_i(x_\mu)$ it the space-time derivative of the field(s) and d^4x the infinitesimal integration volume in four-dimension space-time. The variation of the field(s) $\Phi^i \to \Phi_i + \delta\Phi_i$ with the constrain

$$\delta\Phi^i(x_\mu) = 0 \; \forall \; x_\mu \text{ on the boundary } \Gamma(\Omega), \tag{A.7}$$

on the surface $\Gamma(\Omega)$ of Ω gives the variation of the Lagrangian density as

$$\delta S = \int_\Omega \delta\mathcal{L} \; d^4x = 0. \tag{A.8}$$

The variation of the Lagrangian density is given by (for all i's),

$$\begin{aligned} \delta\mathcal{L} &= \frac{\partial \mathcal{L}}{\partial \Phi_i}\delta\Phi_i + \frac{\partial L}{\partial \Phi_{i,\mu}}\delta\Phi_{i,\mu} \\ &= \left[\frac{\partial \mathcal{L}}{\partial \Phi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \Phi_i}\right]\delta\Phi_i + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial \Phi_{i,\mu}}\delta\Phi_i\right). \end{aligned} \tag{A.9}$$

Setting the variation of the Lagrangian density from ($A.9$) into the variation of the action ($A.8$) gives

$$\begin{aligned} \delta S(\Omega) &= \int_\Omega \left[\frac{\partial \mathcal{L}}{\partial \Phi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \Phi_i}\right]\delta\Phi_i d^4x + \int_\Omega \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial \Phi_{i,\mu}}\delta\Phi_i\right) \\ &= \int_\Omega \left[\frac{\partial \mathcal{L}}{\partial \Phi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \Phi_i}\right]\delta\Phi_i d^4x, . \end{aligned} \tag{A.10}$$

For the last integral term in Eq. ($A.10$) we have converted it to a surface integral over the surface $\Gamma(\Omega)$ by using Gauss's divergence theorem in four dimensions. Using the constraint in Eq. ($A.7$) will the surface integral over $\Gamma(\Omega)$ vanish. If the variation of the action functional $\delta S(\Omega)$ is to vanish for an arbitrary regions of space-time Ω and arbitrary variation of the field(s) gives this the *equation of motion* or the *Euler-Lagrange equation* for fields,

$$\frac{\partial \mathcal{L}}{\partial \Phi_i} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial \Phi_{i,\mu}}\right) = 0. \tag{A.11}$$

APPENDIX B

Noether's Theorem

Most of this section of the appendix has been taken from [38, Chapter 1.3] without much changes, but I have also used [35, Chapter 13.7] as second source for this text.

> Noether's Theorem:
> Continuous symmetry of the Lagrangian gives rise to a conserved current $j^{\mu}(x)$, such that the equations of motion imply
> $$\partial_{\mu} j^{\mu} = 0,$$
> in other words, $\partial j^0/\partial t + \vec{\nabla} \cdot \vec{j} = 0$.

A conserved current implies a conserved charge Q, defined as

$$Q = \int d^3 x j^0.$$

However, note that the existence of a current is a much stronger statement than the existence of a conserved charge, because it implies that charge is conserved locally.

Proof of Noether's Theorem:

The transformation (variation),

$$\delta\phi_a(x) = X_a(\phi), \tag{B.1}$$

is a symmetry if the Lagrangian changes by a total derivative,

$$\delta\mathcal{L} = \partial_{\mu} F^{\mu}, \tag{B.2}$$

for some set of functions $F^{\mu}(\phi)$. Taking an arbitrary transformation of the fields $\delta\phi_a$ one obtain the variation equation

$$\delta\mathcal{L} = \left[\frac{\partial\mathcal{L}}{\partial\phi_a} - \partial_{\mu}\frac{\partial\mathcal{L}}{\partial\phi_a}\right]\delta\phi_a + \partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial\phi_{a,\mu}}\delta\phi_a\right). \tag{B.3}$$

When the equations of motion in Eq. ($A.11$) are satisfied the square bracket term in Eq. ($B.3$) vanish. We are therefore left with the expression

$$\delta\mathcal{L} = \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial\phi_{a,\mu}} \delta\phi_a \right). \tag{B.4}$$

Equating this expression of Eq. ($B.4$) with the symmetry of Eq. ($B.1$) and Eq. ($B.2$) gives the result

$$\partial_\mu j^\mu = 0 \quad \text{with} \quad j^\mu = \frac{\partial\mathcal{L}}{\partial\phi_{a,\mu}} X_a(\phi) - F^\mu(\phi).$$

APPENDIX C

Einstein-Hilbert Action

This section of the appendix is based on the references, [44, AppendixE] and [45, Chapter 12].

The Lagrangian density of the *vacuum Einstein equation*, i.e. *the gravitational field equation*, is given by

$$\mathcal{L}_G = \frac{\sqrt{-g}}{2\kappa} R, \tag{C.1}$$

where $g = \det(g_{\mu\nu})$, $\kappa = 8\pi G$ and R the Ricci scalar defined in (2.19). To obtain the coupled Einstein-matter field equations, one adds to the gravitational field equation in Eq. (C.1) the Lagrangian density for the matter field, $\mathcal{L}_m$. Since the gravitational Lagrangian does not depend the matter field, can one vary the two components in the total action separately giving

$$S = S_G + S_m.$$

Here is S_G the *The Hilbert Action*, or the *Einstein-Hilbert action.* Varying the total action with respect to the metric $g_{\mu\nu}$ gives us,

$$\delta S = \delta S_G + \delta S_m = \frac{1}{2\kappa} \int d^4x \left[\delta\left(\sqrt{-g}\right) R + \left(\delta g^{\mu\nu} R_{\mu\nu} + g^{\mu\nu} \delta R_{\mu\nu}\right) \sqrt{-g}\right] + \delta S_m = 0, \tag{C.2}$$

where the variation of the matter fields is defined as

$$\delta S_m = -\frac{1}{2} \int d^4x \sqrt{-g} T_{\alpha\beta} \delta g^{\alpha\beta}.$$

Here the coefficient $T_{\alpha\beta}$ is defined to be the energy-momentum tensor $T_{\mu\nu}$. Since g is the determinant of $g_{\mu\nu}$, is its variation given by

$$\delta\left(\sqrt{-g}\right) = -\frac{1}{2\sqrt{g}} \delta g = \frac{1}{2}\sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}.$$

The variation of the Ricci tensor $R_{\mu\nu}$ is

$$\delta R_{\mu\nu} = \delta\Gamma^{\alpha}_{\nu\alpha,\mu} - \delta\Gamma^{\alpha}_{\mu\alpha,\nu} + \delta\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\alpha} + \Gamma^{\alpha}_{\mu\lambda}\delta\Gamma^{\lambda}_{\nu\alpha} - \delta\Gamma^{\alpha}_{\nu\lambda}\Gamma^{\lambda}_{\mu\alpha} - \Gamma^{\alpha}_{\nu\lambda}\delta\Gamma^{\lambda}_{\mu\alpha}. \tag{C.3}$$

One can simplify the expression in Eq. ($C.3$) using that the variation of the Christoffel symbol $\delta\Gamma^{\alpha}_{\mu\nu}$ is an tensor. Taking the covariant derivative of the Christoffel symbol gives

$$\delta\Gamma^{\sigma}_{\mu\nu;\alpha} = \delta\Gamma^{\sigma}_{\mu\nu,\alpha} + \Gamma^{\sigma}_{\alpha\beta}\delta\Gamma^{\beta}_{\mu\nu} - \Gamma^{\beta}_{\alpha\mu}\delta\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\beta}_{\alpha\nu}\delta\Gamma^{\sigma}_{\mu\beta},$$

and

$$\delta R_{\mu\nu} = \delta\Gamma^{\alpha}_{\nu\alpha;\mu} - \delta\Gamma^{\alpha}_{\mu\alpha;\nu}.$$

The last term in Eq. ($C.2$) is

$$\sqrt{-g}g^{\mu\nu}\delta R_{\mu\nu} = \sqrt{-g}\left[\left(g^{\mu\nu}\delta\Gamma^{\alpha}_{\nu\alpha;\mu}\right) - \left(g^{\mu\nu}\delta\Gamma^{\alpha}_{\mu\alpha;\nu}\right)\right],$$

which is zero when integrating over all space by the Gauss' theorem. Taking all the different terms together in Eq. ($C.2$) gives *the Einstein equation*

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}.$$

APPENDIX D

Quintessence Action

Without coupling

The quintessence action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}R + \mathcal{L}_\phi\right] + S_M(\tilde{g_{\mu\nu}}, \psi_i); \quad \mathcal{L}_\phi = -\frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi) = -\frac{1}{2}g^{\mu\nu}\partial_\nu\phi\partial_\mu\phi - V(\phi).$$

Here I assume no coupling between the quintessence field and the matter fields i.e. $\tilde{g_{\mu\nu}} = g_{\mu\nu}$. Taking the variation of the action with respect to the scalar field ϕ, $\phi \to \phi + \delta\phi$, vanishing at infinity leads to

$$\begin{aligned}
\delta_\phi S &= \int d^4x\sqrt{-g}\left[\delta_\phi \mathcal{L}_\phi\right] \\
&= -\int d^4x\sqrt{-g}\ \delta_\phi\left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + V(\phi)\right] \\
&= -\int d^4x\sqrt{-g}\ \delta_\phi\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi + V\right] \\
&= -\int d^4x\sqrt{-g}\left[\delta_\phi\left(\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi\right) + V_{,\phi}\delta\phi\right] \\
&= -\int d^4x\sqrt{-g}\left[\nabla^\mu\phi\nabla_\mu\delta\phi + V_{,\phi}\delta\phi\right] \qquad \text{(D.1)} \\
&= -\int d^4x\sqrt{-g}\left[\nabla^\mu\left(\nabla_\mu\phi\delta\phi\right) - \nabla^\mu\nabla_\mu\phi\delta\phi + V_{,\phi}\delta\phi\right] \qquad \text{(D.2)} \\
&= \int d^4x\sqrt{-g}\left[\Box\phi - V_{,\phi}\right]\delta\phi.
\end{aligned}$$

I have used in Eq. ($D.1$) the symmetry of the covariant derivative on the scalar field ϕ, such that $\nabla^\mu\phi\nabla_\mu\delta\phi = \frac{1}{2}\left(\nabla^\mu\delta\phi\nabla_\mu\phi + \nabla^\mu\phi\nabla_\mu\delta\phi\right)$. The first term in Eq. ($D.2$) vanishes by Gauss divergence theorem, since it is a total divergence and the variation of the field became zero at infinity. Since the variation of the action should vanish for an arbitrary variation $\delta\phi$ one obtain the field equation, or *Klein-Gorden equation*

$$\Box\phi - V_{,\phi} = 0. \qquad \text{(D.3)}$$

With coupling

Taking the variation of the action with respect to the scalar field ϕ, **with** a coupling $A(\phi)$ to the a matter field (neutrinos), will give a correction to the Klein-Gordon equation in Eq. ($D.3$) as

$$\begin{aligned}
\delta_\phi S &= \int d^4x\sqrt{-g}\left[\Box\phi - V_{,\phi} - \frac{1}{\sqrt{-g}}\frac{\mathcal{L}_\nu}{\partial\phi}\right]\delta\phi \\
&= \int d^4x\sqrt{-g}\left[\Box\phi - V_{,\phi} - \frac{1}{\sqrt{-g}}\frac{\mathcal{L}_\nu}{\partial\tilde{g_{\mu\nu}}}\frac{\partial\tilde{g_{\mu\nu}}}{\partial\phi}\right]\delta\phi \\
&= \int d^4x\sqrt{-g}\left[\Box\phi - V_{,\phi} - \frac{2}{\sqrt{-g}}\frac{\mathcal{L}_\nu}{\partial\tilde{g_{\mu\nu}}}\frac{A_{,\phi}}{A}\tilde{g_{\mu\nu}}\right]\delta\phi \\
&= \int d^4x\sqrt{-g}\left[\Box\phi - V_{,\phi} - A^3A_{,\phi}\frac{2}{\sqrt{-\tilde{g}}}\frac{\mathcal{L}_\nu}{\partial\tilde{g_{\mu\nu}}}\tilde{g_{\mu\nu}}\right]\delta
\end{aligned}$$

$$\Box\phi = V_{,\phi} + A^3A_{,\phi}\frac{2}{\sqrt{-\tilde{g}}}\frac{\mathcal{L}_\nu}{\partial\tilde{g_{\mu\nu}}}\tilde{g_{\mu\nu}}. \tag{D.4}$$

One recognize the last term in Eq. (D.4) as the trace of the neutrino energy-momentum tensor in the Jordan frame

$$\tilde{T^{\mu\nu}}_{\text{neutrino}} = -\frac{2}{\sqrt{-\tilde{g}}}\frac{\mathcal{L}_{\text{neutrino}}}{\partial\tilde{g_{\mu\nu}}}, \quad \tilde{T}_{\text{neutrino}} = \tilde{T^{\mu\nu}}_{\text{neutrino}}, \quad \tilde{g_{\mu\nu}} = \left(-\tilde{\rho}_\nu + 3\tilde{P}_\nu\right) = -\left(1-3\omega_\nu\right)\tilde{\rho}_\nu.$$

The changes in Klein-Gordon equation is therefore given more precisely by

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + A^3A_{,\phi}\left(1-3\omega_\nu\right)\tilde{\rho}_\nu = 0.$$

One can re-express the energy density trough the *rescaled energy density* of the Einstein frame

$$\rho_\nu = A^{3(1+\omega_\nu)}\tilde{\rho}_\nu$$

The rescaled energy density will still satisfies the usual conservation law in the Einstein frame, i.e. $\rho \sim a^{-3(1+\omega)}$ [29]. The field equations becomes

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + A^{-3\omega_\nu}A_{,\phi}\left(1-3\omega_\nu\right)\rho_\nu = 0.$$

The field equations can also be expressed as

$$\ddot{\phi} + 3H\dot{\phi} + V_{act,\phi} = 0,$$

where the active potential is

$$V_{act} = V + A^{1-3\omega_\nu}\rho_\nu.$$

APPENDIX E

Conservation of Quintessence-Neutrino

Taking the time-derivative of the neutrino density

$$\rho_v = \frac{1}{a^4}\int q^2 dq \; d\Omega \; \epsilon \; f_0,$$

and using the definition of the neutrino pressure

$$P_\nu = \frac{1}{3a4}\int q^2 dq \; d\Omega \; f_0 \; \frac{q^2}{\epsilon}$$

gives the continuity equation of quintessence-neutrino (quintessence-neutrino energy conservation) as

$$\begin{aligned}
\dot{\rho_\nu} &= \frac{d}{dt}\left(\frac{1}{a^4}\right)\int q^2 dq d\Omega \epsilon f_0 \\
&= \frac{d}{dt}\left(\frac{1}{a^4}\right)\int q^2 dq d\Omega \epsilon f_0 + \frac{1}{a^4}\int q^2 dq d\Omega \frac{d}{dt}(\epsilon) f_0 \\
&= -\frac{4H}{a^4}\int q^2 dq d\Omega \epsilon f_0 + \frac{1}{a^4}\int q^2 dq d\Omega \frac{1}{2\epsilon}\frac{d}{dt}\left(m_\nu a^2\right) f_0 \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{1}{2\epsilon}\frac{d}{dt}\left(m_\nu a^2\right) f_0 \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{2\epsilon}\left[2m_\nu a^2 \dot{m_\nu} + 2m_\nu^2 a\dot{a}\right] \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 \frac{\dot{m_\nu}}{m_\nu} + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 H \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 \frac{d}{dt}(\ln m_\nu) + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 H \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 \frac{d}{dt}(\ln m_\nu)\left[+\frac{q^2}{\epsilon} - \frac{q^2}{\epsilon}\right] + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 H \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{1}{a^4}\int q^2 dq d\Omega f_0 \epsilon \frac{d\ln m_\nu}{dt} - \frac{1}{a^4}\int q^2 dq d\Omega f_0 \frac{q^2}{\epsilon}\frac{d\ln m_\nu}{dt} + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 H \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{d\ln m_\nu}{dt}\rho_\nu - \frac{d\ln m_\nu}{dt} 3P_\nu + \frac{1}{a^4}\int q^2 dq d\Omega \frac{f_0}{\epsilon} m_\nu^2 a^2 H \left[+q^2H^2 - q^2H^2\right]
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4H}{a^4}\rho_\nu + \frac{d\ln m_\nu}{dt}\rho_\nu - \frac{d\ln m_\nu}{dt}3P_\nu + \frac{1}{a^4}\int q^2 dq d\Omega f_0 \epsilon H - \frac{1}{a^4}\int q^2 dq d\Omega f_0 \frac{q^2}{\epsilon} H \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{d\ln m_\nu}{dt}\rho_\nu - \frac{d\ln m_\nu}{dt}3P_\nu + H\rho_\nu - 3HP_\nu \\
&= -\frac{4H}{a^4}\rho_\nu + \frac{d\ln m_\nu}{d\phi}\dot{\phi}\rho_\nu - \frac{d\ln m_\nu}{d\phi}\dot{\phi}3P_\nu + H\rho_\nu - 3HP_\nu \Rightarrow
\end{aligned}
$$

$$
\dot{\rho_\nu} + 3H\left(\rho_\nu + P_\nu\right) = \beta\dot{\phi}\left(\rho_\nu - 3P_\nu\right),
$$

where

$$
\beta = \frac{d\ln m_\nu(\phi)}{d\phi}
$$

is the dimensionless quintessence-neutrino coupling.

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