EXPLORING THE INTER-WAR DEBATES ON BUSINESS CYCLE MODELING

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PREFACE

I would first and foremost like to thank my supervisor Olav Bjerkholt for extending me the patience regarding the time it has taken to complete this thesis.

It has also been a privilege to survey visionaries, like Ragnar Frisch, that helped further the study of the intermittence phenomenon in business cycles.

The inter-war period is fascinating, and I hope to continue finding historical anecdotes applicable to my practical endeavors in economics, for years to come.

Finally, I would like to thank my wife Emy for her invaluable support.

SUMMARY

In this paper, the debates surrounding Ragnar Frisch's business cycle models during the inter-war period, between the end of World War I and the beginning of World War II, will be discussed. The inter-war period provided a unique set of circumstances, during which a duality between the neoclassical and institutional approaches of business cycles emerged. While early analysis of business cycles focused on the cause of economic upturns and downturns, Frisch used the empirical findings from the structural estimation work on the intermittence of overlapping cycles, which he discovered through 'long waves'. He applied these findings to prevailing economic theory, and standardized the dynamic framework for macroeconomic modeling. The general preoccupation of economists shifted away from the debate of endogenous or exogenous causes, to developing methods for calibrating economic policy.

Emphasis will be placed on the concurrent debates between Frisch and his contemporaries, such as Knut Wicksell, Eugen Slutsky, Joseph Schumpeter and their contributions to the development of Frisch's business cycle approach. The paper will conclude with Richard Goodwin and Ping Chen's contribution by adapting Frisch's model to chaotic distributions. In the end, Frisch will come close to articulating a non-linear version of the 'impulse and propagation mechanism'.

The aim of the analysis is to answer these three questions:

1. What empirical and theoretical works concerning business cycles, surrounding the inter-war years, influenced Frisch?
2. What were the important debates that helped mold Frisch's dynamic model framework?
3. Why is shape and time path a central part of the debate on the problems surrounding non-linearity in econometric models?

Section 1 will address the first question by providing the historical background of business cycle modeling and the circumstances particular to the inter-war period. Earlier influences on Frisch's 'impulse and propagation' synthesis will be discussed, focusing on how empirical studies during the inter-war period (Kondratiev, 1984:1928a), on 'aperiodicity' and intermittence of cycles of different orders, found support in prevailing theory concerning capital formation (Tugan-Baranovsky, 1898). The influences on Frisch from the theory that 'moderate irregularity' in the oscillation in the propagation mechanism, represented through Knut Wicksell's adaptation of the rocking horse metaphor (Wicksell, 1918), will be introduced.

Section 2 will discuss how Frisch distinguishes 'dynamics' from 'statics', and how this contrasted to the definitions used by his predecessors. Frisch emphasized the distinction between 'dynamics' and 'statics' by defining 'dynamics' as a mode of analysis representing different points in time, and 'statics' as mode of analysis representing other things at the same point in time (Frisch, 2010/1930). How Frisch expresses the historical element within his deterministic framework, setting him apart from the endogenous representations of his predecessors, will be analyzed. Question 2 will be answered by the analysis of the debates of his contemporaries. Section 2.1 will discuss how Frisch adapted Walras' 'general equilibrium' to economic fluctuations. Section 2.2 will critique Johan Åkerman's use of conventional 'statistical decomposition techniques' in identifying business cycle turning points, signifying the end to the era of the 'peak to peak' cycle decomposition of the Institutionalists. Section 2.3 will explain how Frisch adapted Eugen Slutsky's random summation (Slutsky, 1937/1927) to achieve 'free but damped' oscillations in the 'impulse and propagation' mechanism. Section 2.4 will discuss the dialogue between Frisch and Schumpeter on the role innovations (Schumpeter, 1911/2008) played in the impulse mechanism in Wickell's rocking horse, and how this culminated in the non-linear representation of Frisch (1933). Section 2.5 will summarize the critique of Schumpeter's 'Business Cycles' in Kuznets (1940), and the difficulty in modeling cycles of different orders of capital formation movements within an equilibrium framework.

Section 3 will discuss efforts to represent non-linearity and chaotic distributions, building upon Frisch's dynamic framework. The solution provided by Henri Poincaré in the 'three-bodies problem' on structural instability eventually implied that models handling complex characteristics, like economic fluctuations, could not be represented by linear models where differential equations represented reality as a 'continuum'. Modeling the 'Schumpeter clock' satisfied the conditions posed by the 'initial conditions sensitivity', or Lyapunov exponent, improving model accuracy. Albeit, only for economic variates, which have a high degree of short-term sensitivity and power law tendencies. Question 3 will be addressed by an overview of Poincaré's findings on chaos, and how models using 'Schumpeter clock' synthesis improves the non-linear characteristics of Frisch (1933). Section 3.1 will illustrate the difficulties of defining the terms 'random' and 'chaos'. Section 3.2 will explain Poincaré's basic findings regarding the 'sensitivity dependence on initial conditions', and how Frisch comes close to satisfying these conditions, stopping just short of developing a full-fledged model in Frisch (1934a). In Section 3.3 Richard Goodwin's non-linear representation of Frisch (1933), using the 'Schumpeterian clock', will be discussed. Poincaré's observation on structural instability was addressed via mutual conditioning (Goodwin 1951). Still, the exogenous shock embedded in the model was not self-sustaining, just like in the findings of Zambelli (2004). In Section 3.4, Ping Chen will address the issue of historical input in modeling a 'Schumpeterian clock' in continuous time, using time-frequency analysis (Chen, 2005). Section 3.5 will discuss use of Lyapunov exponent measurement in economic time series, and its inherent limitations. Section 3.6 will discuss briefly, better estimation practices, using Lévy stable distributions, in gaging deterministic accuracy within short-intervals of local bifurcation points.

Section 4 concludes.

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1. Introduction and background to the emergence of business cycle modeling

Modeling of macroeconomic fluctuations has existed in embryonic form long before the inter-war (1919-1939) period, such as in the sunspot theory of Jevons, or other accounts drawing upon mechanical analogies such as the pendulum properties rooted in Leonardo and Galileo. At the turn of the 20th century, many economists involved in the study of trade cycles, understood intuitively that the 'social interaction' [1] that generated trade cycles that steered the economy away from equilibrium, was too complex to analyze using simple equilibrium models. Constant economic and geopolitical turmoil during the inter-war years, meant that the study of disequilibrium where "*such divergences can, of course, only be established by a study of the actual facts*" (Cassel, 1918/1932), eventually gave in, to stabilizing policy considerations. Taming business cycles became a priority of a higher order, particular at policy-making institutions, such as the League of Nations. Nikolai Kondratiev's 'long wave' research program on the empirical findings on the intermittence of cycles of different orders and magnitude, would also find support in theories on 'capital formation'. The studies of Slutsky-Yule showed the existence of "*spurious cycles*" (Pollock, 2014). [2] Ragnar Frisch took an immediate interest in Eugene Slutsky's (1927) findings, and eventually worked out they could be exploited to underpin his own approach to explaining the emergence of business cycles.

"It appears that what dynamic econometric theory gives us is not the time shape of standard curves with which the empirically observed time series are to be compared, but it gives us the weight system by which to perform the accumulation. The fundamental problem therefore rests on what is the harmonic nature of the time series produced by accumulation according to such a pre-assigned weight system". (Letter from Frisch to Slutsky 14 Dec 1934, National Library of Norway)

1 Cassel's use of the phrase 'social order', in the following quote, illustrates the wide awareness on empirical studies that suggested 'social interaction' played a large role in trade. Cassel (1918/1932), p.646. "Anyone who complains of trade cycles and condemns a 'social order' that facilitates or tolerates their existence, is really complaining of the advance of our material civilization." Priorities will change to policy considerations, as illustrated in Cassel (1927-1928).

2 Until Slutsky and Yule, no one had demonstrated how random events could mimic repetitive, regular oscillations in economic activity. Spurious cycles refer to the generation of these fictitious cycles. A typical macro model that is autoregressive (AR), describes a linear system where random shocks are temporary and decay over time. This contrasts with harmonic functions with sinusoid movements, which were stereotypical of business cycle depictions prior to the studies of Slutsky-Yule. It is considered one of the foundations of neoclassical business cycle theory. In Pollock (2014), the effect refers to the idea that fluctuations can be imparted to a data sequence by passing through a linear filter. AR series may generate cyclical patterns when there are no cyclical elements in the observation.

Nikolai Kondratiev's 'The Major Cycles of the Conjuncture' (1994:1928a) prominently featured an empirical approach to business cycles. His work helped popularize the notion that there was a certain steady causal underlying structure, which accounted for a certain orderliness of the phenomena. This roughly 50 year 'long wave' [3] phenomena, observed in statistical series such as production, trade, money, wages reflected large technology and social trends. The values underlying these data series would not have identical periodicity as other drivers of an economy, but they would rise and fall in phases with other series, based on supply versus demand, as underlying forces in a capitalist economy. The underlying mechanical representation was articulated prior to this, in Kondratiev (1925/1924) on statics and dynamics. His interpretation was that statics was considered the very essence of the phenomena, revolving around some equilibrium organizing notion. In contrast, reality was dynamic, as it adjusted for time. Dynamics in his realm consisted of two movements – an irreversible development in one direction like demographics, and a reversible development like interest rates and prices. Thus, the model required an endogenous [4] description of the variation. Supporting these reversible developments was 'capital formation', revolving around the replacement of capital goods. The more durable the investment, the longer the lead time was between the "demand impulse" and "production" of the capital goods (Van Duijn, 1983).

Clements Juglar [5] was one of the predecessors interested in the role price movements played in business cycles. At the focus was excessive speculative behavior, where a crisis allows for a self-correction mechanism, which flows into prosperity, crisis and depression phases of cycles. Speculation

[3] According to Day (1976), the 'long wave', was a result of Kondratiev's analysis of the 1920-21 crisis. It was meant to define the means for restoration of equilibrium, rather than an imminent failure of the capitalist structure. The term was meant to hypothesize the trend of capitalist development. When short-term oscillations of 7-11 years (reflecting the fixed investment cycle) were statistically smoothed, this revealed the pattern of 48-60 year long waves. Phases in the 'long wave' would be interpreted as reflecting technological innovation, wars and agricultural depression.

[4] Exogenous versus Endogenous: Exogenous, as in determined outside the model. Endogenous, as in determined within the model. The neoclassical economics view that business cycles were caused by exogenous factors, has one of its origins in Say's law. Say's law refers to the notion that production is the source of demand.

[5] Dal-Pont (2007) describes Juglar's focus on the link between credit and business cycles. The role of 'causes', as in 'shocks', was of secondary importance compared to that of properties describing fluctuations like periodicity. This would lead to an endogenous business cycle framework, with credit cycles causing the periodicity in business cycles. Juglar would become one of the first economists to use time-series in his empirical work to detect these credit cycles. It is important to note he did not subscribe to the monetary overinvestment theory views held by the Austrian school of economics, which held banks accountable. Rather, he attributed speculative and contagious behavior that contributed to credit misalignment.

and the contagion effects were depicted as the origins for credit cycles. However, his analysis focused on prices and rates of interest almost exclusively, and treated business cycles as a monetary phenomenon.

Conceptually, this was similar to Tugan-Baranovsky's synthesis, [6] which depicted large economic fluctuations, generated by periods of over and under-investment, where accumulated savings played an incremental role in the replacement cycle of capital goods. Tugan-Baranovsky would suggest that the peak of the Juglar cycle would start with the early stage of a recession as credit runs and investments shrink. Industries falter, before loanable funds at banks rise again, driving the cycle back up. By incorporating Karl Marx's "*reproduction process of capital*" via lock-step between the consumer and production sectors, Tugan-Baranovsky formulated an Institutionalist [7] 'capitalist mode of production' version of an endogenous industrial cycle (Reijnders, 2007).

Joseph Schumpeter, one of the first economists to explain the existence of Kondratiev's 'long waves', would later call the 7 to 11 year fixed investment cycle, the Juglar cycle (Schumpeter, 1939). He would tribute this to Tugan-Baranovsky's description of fluctuations in production and plant and equipment (Schumpeter, 1981/1954). The 'innovative' entrepreneur would become the primary agent in fixed investment cycle, in his 'treatise for circular flow' (Schumpeter, 1934/1911). The entrepreneur would disturb the equilibrium, seeking profitable investment opportunities arising from technical progress. Meanwhile, the entrepreneur would use credit to fund this investment activity (Dal-Pont Legrand & Hagemann, 2005).

[6] Tugan-Baranovsky was influenced by Karl Marx's periodic renewal of fixed capital. Bazhal (2013) cites Tugan-Baranovsky as one of the first to argue an endogenous cause in business cycles, in his empirical study of 'periodical crisis' of Ukraine in relation to the Russia Empire. Tugan-Baranovsky (1898) p.579 "*Capital accumulation is not merely a simple replacement of capital consumption with employee consumption…accumulated capital transforms not only into a salary, but also into means of production, which are not elements of consumption of whatever class of the public*".

[7] The American Institutionalist movement (1880-1920) applied political economy to the studies of capital formation. Thorstein Veblen used inductive empirical generalizations to derive economic theory, based on cultural and socio-economic traits and how they affect Institutions. Neoclassical economics, which embodies the universal analysis of socio-economic behavior, associated strictly with individualistic and competitive market behavior. Differences in neoclassical and Institutional synthesis on capital formation can be illustrated by the Cobb-Douglas aggregate production function, which derives technical progress from raising productivity of labor. The two factors of production in the classical form: capital and labor, does not allow for scientific advances as an input.

Similarly, Tugan-Baranovsky's, observations regarding the instability of the credit economy and its role in fluctuations in fixed capital investments would also have a strong influence in cementing Knut Wicksell's belief (Legrand, 2007) in technical progress and variations in real capital investments as principal cause of cycles. Later, Wicksell sought to combine the Eugen Böhm-Bawerk circulating-capital theory [8] and Walras's general-equilibrium theory [9]. By separating capital as a separate factor of production, Wicksell synthesized a measure of capital productivity, and showed that monetary factors amplified actual fluctuations. In this 'cumulative process' (Wicksell, 1962/1898), price level adjustments would be a direct result of the variation between equilibrium and market (loan) rates of interest (Humphrey, 2002). The equilibrium, or 'natural' rate of interest, in this case, would be the endogenous variable in a general-equilibrium solution with an economy without money.

Wicksell believed that an equilibrium rate of interest was decided by the balance of supply and demand in consumption goods, offset by technical progress (such as new products and irregular productions improvements). Rising productivity would raise the return on investment projects, and thus the equilibrium rate of interest. The exclusion [10] of the representation 'cumulative process' itself from Wicksell's trade cycles would have some strong implications. His reasons for opposing the monetary impulse theory of price dynamics would remain confusing. [11] Wicksell (1962/1898 p.96) contains analysis that delve into the effect of distortion of allocation and prices caused by differences in natural and real rates. (Leijonhufvud, 1997) This would also stall efforts by some of his students [12] to develop the 'cumulative process' into an endogenous business cycle theory.

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8 Circulating capital is defined as raw material stock, goods in process and finished goods (Böhm-Bawerk, 1930/1888).

9 In Walras' general-equilibrium theory, decisions are made based on expected prices. These price expectations affect decisions on labor, credit and input.

10 Boianovsky(1995) explains how the 'cumulative process' would be helpful in explaining why "the upper turning points can have the nature of a crisis, which results from wrong expectations induced by cumulative price movements".

11 Wicksell did not use 'money' in his 'cumulative process'. An interesting argument by Gustav Cassel, Wicksell's main rival in Swedish, on the merits of using Wicksell's cumulative process inclusive of 'money' to influence business cycles by regulating real interest-rate, can be found in Cassel (1927-1928). "*This observation throws a clear light on the futility of the mathematical wave-theory of life. It has become a fashion among economists, or rather statisticians without thorough economic training, to look upon everything that happens in economic life as subordinate to statistical curves and subject to being predicted by mathematical analyst of these curves…It cannot be doubted that with a bank rate regulated on these lines the conditions for the development of the trade cycles would be radically altered, in that indeed our familiar trade cycles would be a thing of the past.*." Comparing this to footnote 2 on Cassel (1918/1932) illustrates the shift in sentiment from identifying causality to combating cycles.

12 One such student was Johan Åkerman, which will be further elaborated in 2.2 Åkerman's statistical decomposition.

The Wicksell metaphor of the 'rocking horse' as a compact expression of his that would represent his trade cycle view originated in book review, Wicksell (1918). [13] He compared the trade cycle mechanism to that of a rocking horse that was pushed by a stick. The push of the stick was the external impulse that got the horse going. The rhythm of the rocking horse's movement was independent of the pattern of the pushes. Thus the strength of the impulse and the mechanism of the rocking horse were the two defining influences maintaining cycles. Ragnar Frisch brought Wicksell's rocking horse simile to the attention of English speaking economists.

If you hit a wooden rocking-horse with a club, the movement of the horse will be very different from that of the club. (Frisch, 1933, p.198)

The most frequently cited source for Wicksell's view on business cycles is his article "The Enigma of the Business Cycles" (Wicksell, 1953/1907). Wicksell (1917)'s view was that "*intelligent credit policy*" (Ohlin, 1962 reprint), under most circumstances would avoid the "*rocking tendency from growing violent*". The repetition of the cycle is explained by an expansion that would overshoot equilibrium and create a small crisis where proportionality is restored to setup for a new upswing. This idea of moderate irregularity would have a lasting influence on Frisch's mechanical business cycle representation of the rocking-horsing. [14]

One technical issue in determining empirically the Kondratiev wave, was that investment time series were shorter than industrial production time series. Since capital investment playing a major role in economic fluctuations was primarily a post Napoleonic, Industrial revolution phenomenon, there were limited empirical data-points for measuring a 50 year 'long wave' cycle. This may partly explain why the inner-mechanics of this cycle became a subject for further interpretation by the various actors involved in inter-war econometrics movement.

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[13] This book was Karl Petander's "Goda och Dårliga Tider" [good and bad times]. The rocking horse simile appears in foot note 1 in Wicksell(1918, p.71).

[14] Frisch (1952c) emphasizes Wicksell's influence on Frisch, particularly in monetary theory, especially with regards to monetary equilibrium and the difference between the actual and natural rate of interest.These concepts would be engrained in Frisch's assumptions in the rocking-horse metaphor in Frisch (1933b).

Kondratiev's historical, yet basic causal framework received avid endorsements from the emerging econometrics movement. Earlier historical analysis of business cycles had predominantly focused on the causality of economic upturns or downturns, rather than focusing in on the shape and duration of the cycle. Time series for measuring cycles of different length had already been established by then, with Warren Person's assertion that business cycles consisted of four components: 'cyclical, seasonal, long-term and accidental'. This would imply certain presumptions about economic behavior (Morgan, 1990). According to Bjerkholt (1995a), Ragnar Frisch criticized the implied inhomogeneity of Person's method for lacking analytical rigor, as well as its implied assumption of cycles with constant periodicity. Frisch subscribed, at least to begin with,[15] to Kondratiev's long wave theory, but found the seasonal representation lacking rigorous mathematical representation.

One thing suggesting there is existence of a super-cycle is the fact that the normal of the major cycle in some cases is found to ascend and in other cases to descend. The ordinary business cycle is itself a long time movement with respect to the seasonal fluctuation which we now consider to be accidental, are really due to small cycles, and only appear as accidental because out data are not available at intervals short enough to investigate the manner in which these "accidental fluctuations" are generated as an interference phenomenon composed of small and still smaller cycles (Frisch, 1927a).

Kondratiev stimulated fellow economists into pursuing the underlying presumptions on cycle causality, the natural next step that helped provide structure for the economic policy making pursuits that followed.

Prominently featured amongst the historical approach to economic fluctuations, was the role of 'innovation' in Schumpeter (1934/1911). This approach integrated the analysis of economic development over the long-run of business cycles, that dealt with the Kondratiev's 'impulses' following the disequilibrium path. Meanwhile, neoclassical economic theory, during the early part of the century, had come under attack by the American Institutionalists in particular, for lacking quantitative rigor.

[15] Frisch, et al. (1932e) p.135 endorses Kondratiev's conclusions on 'long waves'. His radio lectures on 13, 24 April and 11 May 1932, published as 'Konjukturene' depicts the relation of wars with wheat prices. Results were inconclusive, except that perhaps wars had contributed to high wheat prices.

Veblen would call neoclassical methods ‘unscientific’, and theory ‘teleological’ and ‘pre-Darwin’ in their inability to keep up with physics and biology (Landreth & Colander, 1994). It would take Frisch’s quantitative formulation of these theories, to pioneer the formal structure in the modeling of economic fluctuations. Current debate over the shape and time-path of economic fluctuations can be traced to debates, resulting in controversies and positions taken, during the inter-war period.

Slutsky (1927) formalized the shape of sinusoid oscillations found in random time-series distributions and made the mechanical structure elegant, upon which Frisch (1933) applied prevalent economic theory. This was in contrast to the non-linear characterization of price movements suggested by Frisch in the Poincaré lectures (Frisch, 2009/1933). From a practical point of view, not only was non-linear [16] mathematics difficult to solve, but use of differential equations required a continuum without any breaks. Frisch’s thoughts on the need to simplify, in order to solve the issue of time shape in business cycles, is reflected in the following quote:

It is always possible to give even a macro-dynamic analysis in detail if we confine ourselves to a purely formal theory... Such a theory, however, would only have a rather limited interest (Frisch, 1933, p.2).

One of the main ideological divides in business cycle analysis at the time, was whether the business cycle was driven by endogenous or exogenous forces, i.e by forces intrinsic in the economic system or by outside disturbances. Central in Kondratiev’s (1994:1928a) analysis was a certain struggle with the concept of equilibrium with empirical observations that stirred up a multi-generational debate of whether economic fluctuations were caused by exogenous or endogenous factors.

The protagonists of the endogenous cycle theory don’t have difficulty in accepting that exogenous factors do interfere with the basic oscillation pattern in a random fashion, whereas the impulse and

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[16] Wikipedia “*Non-linear systems of equations are simultaneous equations in which the unknowns appear as variables of a polynomial of degree higher than one*”.

propagation theorists readily accept that there is an internal mechanism that absorbs such external shocks by fluctuating around its equilibrium level, just like the tree that bends over and springs back to surmount the forces of wind. The consensus is that there is an oscillating mechanism that is characteristic of a modern economy and that can be known (Reijnders, 2007, p.13).

Louça (1999) argues in his survey of Ernest Mandel [17] that the effect of trend-cycle decomposition techniques [18] on exogenous and endogenous explanations for causal factors, was the result of Slutsky's 'random summation' on Frisch's 'Impulse and propagation' model. The formalization of a business cycle model provided a rigorous framework that led the focus away from the exogeneity or endogenity of causality. This was particularly relevant in the context of the 'long wave', in retrospect, as the two World Wars provided extremely large deviations for statistical data series at the time. Statistical decomposition methods allowed for lop-sided causal theories, both exogenous and endogenous based, were urgently in need of quantified verification. Simply said, exogenous theorists equated the two World Wars as excess deviations to random and external events, whereas endogenous arguments would ignore these events all together.

The structural estimation work marked a dramatic shift away from blind-sighted focus on identifying the causes of periodic cyclical instability in economic activity. Among mathematically-oriented economists, there was a strong underlying belief that the analysis of the interference phenomenon between different components – specifically short (Juglar), long (Kondratiev) cycles and trends of different orders, may hold answers to the causal question of business cycles. Statistical decomposition was one possibility in unlocking the causal explanation. One of the key issues highlighted, (Frisch, 1931c) was how to incorporate the historical element, which made the cyclical character of certain observed components non-constant over time.

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17 Mandel (Louça, 1999) argued that systemic shocks or exogenous factors were necessary to drive a new expansion cycle, whereas the internal capitalist mechanism relating to production could cause the actual inflection points. Louça argues that Mandel's hypothesis brings in social and political processes, and at the same time does not disregard existing 'economic laws'.

18 Trend cycle decomposition refers to an economic time series that is represented by a deterministic trend and random cycle or noise. The typical aim is to smooth the data to isolate the main driver of the data series from the randomness. The process of decomposition involves separating the trend cycle from seasonal factors.

A key divide was how structural growth fit in the cycle. The Institutional framework for endogenous business cycles centered around Thorstein Veblen's [19] framework of capitalist production, exchange and distribution. Veblen felt strongly that neoclassical economics failed to address growth as a structural phenomenon, and were simply satisfied with addressing it with purely quantitative means where long-run growth was exogenously driven. His efforts, however, were more focused on observations regarding culturally defined economic choices made by individuals or organizations, such as in his 'conspicuous consumption' (Veblen, 1992/1899).

This inductive method of American Institutionalists could be traced back to the German Historical school, where empirical work placed rigorous logic and general propositions to derive a hypothesis of reality. Rather than basing economics on Newtonian mechanics [20], inspiration lay in law and biology.

Wesley Mitchell and the National Bureau of Economic Research (NBER) [21] used in depth business cycle measurement to highlight the widening gap between this applied and pure theory. In particular, Mitchell opposed the general equilibrium paradigm in its application to economic fluctuations, especially in terms of validating the path to equilibrium. To him, the distinction between exogenous and endogenous fact bears little meaning. Frisch argued in the Yale lectures (Frisch, 2010/1930) that the Institutionalist approach had severe shortcomings. Their practitioners could not induce conclusions from the same object of observations. The accumulation of empirical facts could in no way compensate for the lack of a prior theoretical conception. Frisch argued for the preeminence of theory in the interpretation of empirical reality.

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[19] Thorstein Veblen was the spiritual leader of the American Institutional school, but it would be Mitchell who would put his philosophy into practice through the "NBER approach" (explained in footnote 7). Veblen is known for his synthesis of Darwinian evolution into economic theory. The Institutional approach also focused on the endogenous role of innovation.

[20] Newtonian mechanics, in the context of neoclassical economics used classical deterministic laws of energy physics from the eighteenth century. Mirowski (1989) gives the example of the "perpetual motion machine", which violates the 'conservation principle' that states "some quantity, e.g mass or energy, be kept constant in a given system."

[21] National Bureau of Economic Research: A non-profit research organization started by Wesley Mitchell, Frederick Mills, Simon Kuznets among others. NBER empirically documented business-cycle research. Morgan (1990) mentions that Mitchell's efforts to publish his 'Business Cycle' book series became a full ongoing project for NBER in what would be described later as the "NBER method" of specific and referencing of cycles.

Economic statistics tells us nothing if not viewed on the background of a broad theoretical-economic analysis. (Frisch, 1931a, p.281, transl. from Norwegian)

This ‘war of words’ was directed at the inductionist approach used by the Institutionalists. NBER’s statistical time-series on business cycles of Wesley Mitchell’s ‘peak to peak’ composition was seen as statistical verification without theoretical reasoning lacking proper structure. This ultimately would stalwart Johan Åkerman’s pursuit of a quantitative endogenous business cycle synthesis of the rock-horse analogy. Åkerman had spent time both under Warren Persons as well as Wesley Mitchell. Separating wave length into seasonal components would be considered an improvement from their methods, but the lack of continuum in the cycle premise was considered to be an inferior approach by Frisch (Bjerkholt, 2007).

A strong over-riding consideration, among the founding members of the Econometric society [22], was addressing the 'social ills' that arose from these large fluctuations observed during the inter-war Great Depression. By 1932-1933, with prolonged periods of high unemployment, stagflation and the emergence of Hitler as a disruptive force in Europe, Kondratiev’s empirical conclusions on ‘long waves’ were being actively promoted. (Frisch, et al., 1932e) One of the policymaking intuitions was that had oscillating mechanism been known, it would have been possible to assess how to react to a certain disturbance. If the impulse could be identified, the effects of the impulses could be counterbalanced via policy-measures. It did not help that a great deal of Mitchell’s work consisted of manipulative statistical methods to measure economic data. One example was the Mitchell’s empirical work on a 3.5 year cycle, which failed to predict such a severe depression at the time. Concurrently, validation of Slutsky’s summation techniques, which opened up the possibility of creating regular cycles not present in data, did not help the validity of the inductionist movement.

[22] Econometrics Society, was founded in 1930 for the advancement of economic theory in relation to statics and mathematics. It was sponsored by the academic journal Econometrica, which started as medium of communication between its members.

With the ‘error term’ structure in time series moving away from simple “estimation by-products of a priori formulated theoretical model” (Qin & Gilbert, 2001) to ‘aberrations’ and ‘stimuli’, prime focus shifted to the oscillation mechanism itself. Focus was placed on formalizing “the tools for determining the interrelations between stochastic variables”. A stochastic approach, where random interferences could be observed, could now include the ‘error term’ (Frisch, 1939d). One of the key constraints remaining for the ‘error term,’ [23] was how to incorporate the ‘historical aspect’ in cyclical movements, with Knut Wicksell’s impulse-propagation dichotomy. He would seek to integrate Schumpeter's theory of Innovation into the ‘error term’, with mixed results.

It was not only eloquent and forceful persuasion of the ‘impulse-propagation’ dichotomy, that gained the upper hand over pure endogenous explanations. ‘Impulse-propagation’ clearly offered theoreticians a compromise with observable business cycle data. Use of linear functional equation allowed comparative statics methods of analysis to remain relevant in mainstream economics, particularly as a development tool for policy making. Using non-linear equation was too difficult to solve, and was inconsistent with the assumption of continuity in neoclassical economic theory (Mirowski, 1990). This view is elaborated in Morgan (1990), where the framework allowed for free and damped oscillations, but at the same time the suggestion was that the fluctuations were kept going by external shocks. This was compatible with ‘irregular’ and ‘undamped’ data observed by empiricists at the time.

Zambelli (2004) argues that the model of Frisch (1933) does not actually oscillate. The model looks deceptively simple but the combination of differential and difference equations makes it very hard to penetrate. [24] By the time mathematically stable linear models, reliant on outside shocks to propagate, were being used as a macro modeler’s building-blocks, empirical data out of the Great Depression influenced Frisch’s senior colleague, Irving Fisher, into considering non-linear systems that could create “*aperiodic cyclical behavior*” (Keen, 1996). Fischer’s work on monetary policy was particularly

[23] “Errors” in Frisch’s terms include shocks as “errors in the rational behavior of individuals” or “any event which contradicts the assumptions of some pure economic theory and thus prevents the variables from following the exact course implied by that theory” (Frisch, 1939, p.639)

[24] Zambelli (2004, p.3) tested the model by simulation and found that “when the system is removed away from equilibrium (impulse) the return towards equilibrium (propagation) is not an oscillating one”.

important in relation to the swings of the rocking horse in Frisch (1933). Delays in capital investment were needed to satisfy increased consumer demand, causing recurring oscillations in economic output. Frisch does mention Fisher's argumentation (Fisher, 1933a) regarding the accelerant effect of debt. [25]

One idea would be to distinguish between savings and investment…in an actual situation, difference between the two will tend to produce a depression or expansion. This is Keynes' point of view…Another way is to introduce the fact that the existence of debt exerts a profound influence on the behavior of both consumers and producer. This is the lead idea of Irving Fisher's approach to the business cycle problem. (Frisch, 1933, p.11)

Tinbergen would mention in response to criticism, directed at deterministic models and complexity in economics, "*It is a relatively new area, although Ragnar Frisch came very close to it some decades ago…It is a subject for the* future" (Tinbergen, 1992, p.256).

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[25] Fisher describes disequilibrium, referred to as 'two paradoxes' in Fisher (1933a, p.344), "*the more debtors pay, the more they owe*", overriding the existing cycle theory. Fisher distinguishes between "*free*" cycles that are self-generating in a wave or pendulum like motion tending to equilibrium, and "*forced*" cycles that are seasonal and continue unabated, "*catastrophes*", where increased indebtedness triggers deflation.

2 Statics and Dynamics

2.1 Frisch's notion of Statics and Dynamics

For more than twenty years I have been telling my students that one of the widespread uses of "Statics" and "Dynamics" was to distinguish a writer's own work from that of his opponents against whom he tried to argue. Typically, "Statics was what those benighted opponents have been writing, "Dynamics" was one's own, vastly superior theory. *(Machlup, 1959, p.100)*

To provide a logical and constructive framework for the tools for modeling economic fluctuations, Frisch had an engineering mathematical construct, rather than an application of physics laws to methods of structural estimation. One of these important constructs was to make precise the concept of statics versus dynamics. Frisch felt that models, building upon prevailing economic theory, would be rendered obsolete in policy-making if they failed to distinguish the 'cause' from the 'effect' of causality. This not only accounted for changes taking place at the same point in time, but also other changes at other points in time (Frisch, 2010/1930). The definition and distinction of the often used terms, 'statics' and 'dynamics', should represent different 'modes of analysis', rather than 'different forms of description' of the phenomenon at large. Economic variables that referred to the same point in time vis-a-vis dynamic theorems that included component values belonging to different points in time had to be properly differentiated.

At heart of the different forms of description of the phenomenon at large, was the need to tackle the fixed nature, represented in Walras's system of simultaneous equations. This meant moving away from the 'dynamics' representation by way of changes in technology, endowment and preferences. Walras' mathematical treatment of dynamics, by adding a subscript "t" for time to statics, was equivalent to representing utility as being "*a variable functionally related to time*" (Walras, 1954/1874). Frisch felt this was clearly insufficient. He would mention that "*Dynamics is much more than statics plus time*" (Frisch, 2010/1930). Even Walras himself admitted that the general equilibrium hypothesis was an ideal, and not a 'real state' (Tieben, 2012).

Equilibrium in production, like equilibrium in exchange, is an ideal and not a real state. (Walras 1954/1874: 224-225). Such is the continuous market, which is perpetually tending towards equilibrium without ever actually attaining it, because the market has no other way of approaching equilibrium except for groping, and, before the goal is reached, it has to renew its efforts and start over again, all the basic data of the problem…For just as a lake is, at times, stirred to its depth by storm, so also the market is sometimes thrown into violent confusion by crisis, which are sudden and general disturbances of equilibrium (Walras 1954/1874: 380-381).

Frisch took a particularly hard stance against 'abstract-minded' dichotomy of the Institutionalist school and Wesley Mitchell. The rather general adoption of Veblen's 'dynamics' comes to mind.

Each new situation is a variation of what has gone before it and embodies causal factors that has been effected by what went before. (Veblen, 1919, p.241-242)

In typical abstract Veblen form, each identified cycle was analyzed as having its own unique history. It was Mitchell who linked the Institutional point of view with quantitative research. An early opponent to the general equilibrium hypothesis, Mitchell took the stochastic approach, built upon empirical evidence of non-Gaussian and irregular disturbances in studies of Business Cycles. Mitchell's NBER compiled empirical analysis that had concluded that the existence of normal equilibrium states were hard to prove. In his view, economic theory was irrelevant, making the distinction between endogenous and exogenous factors meaningless.

Frisch regarded Mitchell as being 'theory-less' and made cursory remarks regarding Mitchell's cycles, saying, 'facts don't speak for themselves'. Mitchell would fail to identify the interference phenomena amongst different orders of cycles in his empirical studies. In Frisch's view, the use of averaging by NBER, to attain certain typical cycle patterns, was neither analysis nor dynamics. Conceptually, it mimicked Antoine Cournot's four stage depiction of economic fluctuations. Analytically, it risked concealing the various combinations of interrelationships in different cycles, which in turn explained turning points. Mitchell's statistical decomposition of each curve into a unique phenomenon was predicated on giving scientific precision to his 'rough impression' of the causality. Slutsky would make a similar argument about Mitchell's "*crude description*" and "*denial*" of business cycle periodicity. (Slutsky, 1937) Regarding any concrete conclusions to his empirical work on business cycle causality, Arthur Burns [26] reveals that in Mitchell's uncompleted 'Studies in Business Cycles', he had intended

to develop a model of business cycles from carefully screened observations, to use it in describing how the cycles of experience are typically propagated, and then press on to account for the outstanding differences among them". (Burns, 1951, p.5)

The opus was never completed. Mitchell's argument that business cycles were recurrent, but not periodic, would lack a proper framework.

In contrast to the widespread acclaim Kondratiev received for his causal framework, he later took a conflicting view on equilibrium as the unifying concept behind business fluctuations (Kondratiev, 1925/1924). Change indicated the existence of the phenomena itself. Static equilibrium under Aristotle's 'essence' represented the "*core of the identity and invariance of phenomena*", while dynamic equilibrium represented "*change and difference*". The two movements within his dynamic process, "*irreversible processes*" and "*reversible processes*" implied that dynamics included statics. (Louça, 1999) Later in Kondratiev (1998/1926b), he carried trend decomposition procedures on long-term fluctuations that showed incompatible results with general equilibrium. This seemed to imply an endogenous approach. [27]

[26] Arthur Burns was a protégé of Wesley Mitchell, who focused on business cycle studies. He went on to publish 'Measuring Business Cycles' with Mitchell in 1946.

[27] Louça (1999) p.176 points out Kondratiev's inherent contradiction between the notion of equilibrium and irreversible changes, particularly in the use of Marshall' s 'third-order equilibrium' where conditions of production of capital goods varies along the way.

2.2 Åkerman's Institutional cycle decomposition

Johan Åkerman, [28] who was a student of Knut Wicksell, would fail to receive traction for his endogenous quantitative business cycle theory based on Wicksell's 'cumulative process' explaining cyclical fluctuations. This was despite claims of strong similarities by Åkerman.[29] Having been one of the first to suggest that erratic, random events were the impulse behind boom-bust traits of marked economies, Wicksell maintained that 'cumulative processes' could not explain cyclical fluctuations. Frisch would focus on methodology, rather than lingering on the 'theory-free' (Frisch, 1970c) mantra of the Institutionalists. Frisch's critique of Åkerman's dissertation focused on his application of harmonic analysis in his 'cumulative cycle' rocking horse analogy, explaining the periodicity of business cycles on seasonal changes.

The nature of changes. Instead of the classic doctrine of equilibrium and the theoretically unimportant factors, disturbing the equilibrium it is proposed to use the wave or the sinus curve as standard. The frame of the investigation is thus formed by periods of different length and amplitude. Just as Fourier's theorem states that every curve can be resolved in a number of sine curves, all economic changes may be reconstructed as a sum of periods of different length. (Åkerman, 1928, as quoted in Besomi, 2006, p.22)

Åkerman's point of view centered upon the interference between seasonal variations of geometrical proportions and progressivity. The idea was that small fluctuations could induce larger ones through an integral relationship. Åkerman's flaw ultimately came down to the use of Fourier's theory of harmonic analysis. Decomposition, into any number of sine curves of varying lengths, lacked 'historical accountability'. Using the illustration of Irving Fisher's 'hydrodynamics' to illustrate 'his stream of

[28] Åkerman defends his doctoral thesis 'Om det ekonomiska livets rytmik', with Frisch as his opponent. Goldstein, Joshua (1988): p.32 Åkerman would later become one of the strong advocates for war being the causality for Long Waves, compared to the majority, which advocated for capital investments or innovation (Goldstein, 1988).

[29] Jonung, Lars (1994): p.174 notes. Wicksell stuck with his assumption that only changes in real rates could explain secular fluctuations, despite examples used in his renowned 'cumulative process' suggesting otherwise. Wicksell remained reluctant to endorse cumulative process based theories on trade cycle from several students. Despite this, Frisch would formulate his model on Wicksell's monetary assumptions.

water' depiction of the business cycle, these cyclic components of different length and amplitude had to be multiples of seasonal components that were bound together.

Meanwhile, Frisch's methodological approach to cycle studies would draw heavy inspiration from the heated exchanges regarding Wicksell's rocking horse metaphor. Åkerman's mechanisms of 'progressivity' factors and repetitive seasonal variations would provide the analogy for Frisch's variant for 'impulse' and 'propagation'. Wicksell's rocking horse metaphor represented 'forced oscillations', contrary to the "*free oscillations*", which could be explained by Wicksell and Schumpeter's account for external shock being clarified by irregular technological advancement.

Further discussions of Åkerman's dissertation would focus on Frisch's conviction that cyclical movements were a continuous process. Economic fluctuations moved as free oscillations and were determined by the inner structure of the oscillating system. Irregular shocks that disrupted equilibrium had to be clearly distinguished from the 'rhythmic' phenomena. This stance was already quite cemented by the time he wrote Frisch (1927a). (Bjerkholt, 2012) Cycle properties were constantly changing and "*total methods*" (Morgan, 1990) that relied on regular sinusoidal parameters like Fourier analysis, were likely to fail.[30]

One of the outcomes of Frisch's theory was that it narrowed things down and provided clarity, away from the tendency of economists at the time to focus on the "*cause*". Whether they were exogenous or endogenous cycles, judgment was focused on their turning points. Åkerman's turning points were difficult to identify, because they were continuous (Boianovsky, 2007). Besides, applying Marshall's distinction of short and long runs of business cycles to analyze interaction between seasonal, cyclical and secular trends went against Wicksell's observation of business cycle fluctuations in contrast to his

[30] Frisch's critique of Fourier analysis is elaborated further in Bourmans (1999, p.91) on the use of mixed differential-difference equations, which led to a "*general solution of weighted sum of harmonic functions*". Similar complications arose from Fourier analysis as "*this can be any arbitrary movement when the weights are not further specified*".

‘*cumulative process of price change’ that “different laws apply to periods of different duration”* (Boianovsky & Trautwein, 2007).

2.3 Distortion of Slutsky’s random summation

Backing the notion of freely oscillating cycles was Frisch's distortion of Slutsky's random summation, in what became regarded as the equilibrium path friendly ‘Frisch-Slutsky’ approach to investigating causality on cycles. The summation of random, mutually-exclusive events could exhibit the appearance of strictly periodic fluctuations in time-series. Both real and artificial datasets could now exhibit serial correlation and exhibit sinusoid periodicity, making empirical work on business cycles into sheer statistics. Substituting the random ‘causes’ in Slutsky’s smoothing of statistical data with moving averages, with the word 'shocks' would allow Frisch to settle on a linear rather than a non-linear dynamic model for simulation purposes (Bjerkholt, 2007). Slutsky never really meant to refer to the neoclassical equilibrium notion of random causes of cycles (Barnett, 2006). What Frisch and Slutsky wanted to account for was that the impulses were fundamentally different. Frisch wanted the differential equation with impulses enhanced, whereas, Slutsky wanted the sum of impulses weighted.

As far as damped oscillating cycles were concerned, one of the earlier adopters of Slutsky's 'cumulation of random causes' in an endogenous synthesis, was Simon Kuznets. For him, statics related to establishing “*factors and conditions of economic equilibrium*”. Dynamics related to analyzing “*both the transitory and secular changes as deviations from that equilibrium*”, with the underlying caveat that the “*state of arts and of nature do not remain stable*” (Irawan, 2013). Kuznets would also make an analytical assessment over Åkerman’s ‘seasonal variation’ synthesis versus Slutsky’s random summation, in his attempt to replace “*hypothetical, qualitative and timeless economics of equilibrium*” (Meade, 1932). Kuznets would ultimately reject his claims, but would apply the Institutional statistical moving average approach to Slutsky’s random summation (Kuznets, 1933).

An extremely large deviation from the mean, since it would be included in a number of members of the moving average, would tend to raise or depress the level of all the members that included it, and, hence, would tend to form cyclical swings. Kuznets concluded that the shape of the distribution of the random causes and the period of the moving average would influence the amplitude and timing of the cycles generated (Barnett, 2006, p.418).

Kuznets would take Slutsky's synthesis further by formulating methods of differentiating the summation of random events periodicity into different economic processes.[31] Historical anecdotes do suggest that with statics and dynamics, Schumpeter was not able to define much beyond statics, other than it being equilibrium analysis and the core phenomena covered by that method. Having used a very broad qualification for the word 'dynamics', various categorical distinctions pre-TED[32] were offered as "*logical correlatives*", such as the 'theory of capital accumulation' or 'theory of endogenous change'. He gradually adopted Frisch's definition of dynamics, from his 1912 edition TED "*changing the data of the static system... through innovations by dynamic entrepreneurs*", to "*theorems which include in our functions values of variables which belong to different points in time*" (Machlup, 1959). As a symbolic gesture of his endorsement of Frisch's terminology, Schumpeter amended the terms 'statics economy' and 'dynamic phenomena' that were used in reference to the 'phenomena' in the 1934 edition of TED to 'circular flow of economic life' and 'economic development'.

The Stockholm lectures (1931) would reveal some of Frisch's Yale lectures insights regarding the "*systematic framework*" for dealing with 'Impulse problems and propagation problems with a free oscillation'. He would also clarify that Schumpeter's idea of dynamics was basically the 'Impulse', and that "*his own terminology was totally different...*" (Bjerkholt, 2007). Schumpeter's idea of dynamics included the evolutionary characteristic of "structural change", whereas Frisch "assumed reversible time, since all events were reduced to irrelevant shocks upon a repetitive mechanism".

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[31] Kuznets was a strong advocate against notion of equilibrium in business cycles, one of the first to describe Slutsky's work in Kuznets (1929). At one point he had considered the validity of Åkerman's "interesting connection between seasonal swings and timing of turning points in cyclical the notion of different types of random shocks impacting the business cycle" as well. His conclusion was that Åkerman underestimated the complexity of seasonal swings with business cycle. Kuznets claimed a demographic based cycle, now recognized as the 18 year land cycle.

[32] TED refers to Schumpeter's book Theory of Development (1934/1912). Connotations on statics and dynamics were amended from the first edition published in 1912 to the 1934 edition to align with Frisch's synthesis in Frisch (2010/1930).

(Louça, 2007) The distinction between static and dynamic analysis went beyond whether "*rates of growth*" or "*response rate with respect to time*" were used or not. (Frisch, 2010/1930) The account of the phenomena in itself, would not be categorized as dynamics.

Any phenomena as such, could be stationary or evolutionary. Any phenomena can be made subject to static as well as dynamic analysis. Furthermore, many phenomena which are evolutionary in microcosmos are stationary in macrocosmos.… Both the analytic and historic dynamics deal with the changes which take place in time. The economic changes in time which is not yet brought into or does not at all let themselves into sharply formulated theoretical laws may be said to belong to the historic dynamics. (Frisch, 1929d, transl. from Norwegian)

Frisch's synthesis understated the complexity of 'dynamic' versus 'static' procedures. By separating phenomena into those which can be statically versus dynamically analyzed, and conditioning that some phenomena are better analyzed statistically, there was a nature exclusion of new assumptions that could not be qualified by established theory. It seems that Schumpeter did not take enough heed into Frisch's definition of 'historical dynamics' when he reaffirmed that he was involved in analytical dynamics of an evolutionary phenomena, given "*no historical evolutionary factors will be indicated*" (Andersen, 1991, p.6). The definition of statics and dynamics would be detrimental in this regard.

2.4 Frisch's contemplation of Schumpeter's innovation

One of the central themes on time and shape of business cycles and their deterministic accuracy has revolved around the conditioning by empiricists regarding strict periodicity. A deterministic model by definition is:

"*One in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables (Wikipedia, u.d.).*

Techniques used to measure ‘regular periodicity’, like Fourier and spectral analysis, used the sine curve as the underlying shape of cyclicity.

Kondratiev’s view was that there was nothing like strict periodicity in social and economic phenomena, but that there was a certain regularity in the phenomena’s recurrence, over time. Goldstein (1988) took that synthesis a bit further, by identifying regular periodicity of cycles using inferential statistics and distinguishing ‘cycle time’ by way of descriptive statistics. Measuring ‘time cycles’ involved accounting for regularity by tracking lagged correlation between variables by historical phase periods; a rather subjective exercise at best.

The basic business model framework, represented by the swinging pendulum with its basic sinusoid oscillation, had already been prevalent with the extensive use of the pendulum metaphor, prior the adaptation of Wicksell's rocking-horse metaphor. The metaphor of the pendulum swing implied a ‘strictly periodic’ metaphor. What had not been part of the visual construct was the intermittence of these different cycles observed by post-Napoleonic empiricists. Frisch had chosen to simplify the construct, for modeling purposes, by assimilating a ‘systems of equation’ construct. Superimposition of erratic shocks occurred on a dampened, standard frictionless pendulum.

I am thinking of a procedure which would make it possible to trace a given component in its historical course so that we can compare a given historical swing in the component in question with the next swing of the same component. In many sorts of data, particularly in economic data, it is quite obvious that the cyclical character of a given component is not constant. (*Frisch, 1931c, p.75)*

The “*equilibrium property of the propagation part*”, which was not explicitly addressed in the Stockholm lectures, (Bjerkholt, 2007) was discussed one month prior, at the University of Minnesota lectures (1931). This required a procedure for curve fitting to reflect breaks in the trend, reflecting “*local properties, seeking information about oscillation around equilibria*” (Louça, 2007, p.133). Implicitly, the Slutsky-Frisch random summation allowed for ‘impulses’ that contained these ‘shocks’, which moved the economic system away from equilibrium, to avoid being subjected to ‘regular time’, rather than ‘cycle time’. This would add support to the improved ‘chain of pendula’ (Frisch, 1928a) and the ‘small pushes’ to the rocking horse metaphor in proposing a dynamic theory that reflected the historical course.

Frisch showed more interest in the deterministic components (the rocking horse) of the propagation mechanism (Duarte & Hoover, 2012). Few distinctions in the character of shocks were necessary within the impulse mechanism of Slutsky-Frisch. Regardless, dialogue on the conceptual and mathematical representation of 'Schumpeterian' innovations gave valuable insights, which one could consider as one of the first debates on the nature of randomness in the econometric movement. The historical element would be important with this regard.

Frisch argued that the key to identifying business cycles causality lay in analyzing the numerical relations between sets of different magnitudes of certain economic parameters, separating the 'impulse' from the 'propagation' problem. This fit into Frisch's pre-established interpretation of the empirical anecdotes, as a result of his general acceptance of ‘long waves’. As early as 'The Analysis of the Statistical Time Series' (1927), Frisch focuses in on the “Interference phenomenon” in the business cycle. This would involve studies such as the differential properties of the composite curve, decomposing cycles, trends of variety of lengths, and where the cyclical characteristics were changing (Frisch, 1931c). He went to great lengths to match the different cycles of length and elasticities of the Kitchin, Juglar and Kondratiev cycles.

Frisch's characterization of sinusoid movement of waves in business cycles, refers to the gradients in (Frisch, 1929d). Rather than a description of the phenomena, he considered it a method of analysis, where a “*sequence of equilibrium states*” could be analyzed. To help capture the dynamic cyclical movement using the random-shock variable, Frisch extended Wicksell’s (1907) synthesis of the Walrasian deterministic model by adding new elements. This was done first by capturing the ‘propagation’ problem, which derived a macro-dynamic system of mixed difference and differential equations. Then ‘erratic shocks’ shocks were introduced to capture the cyclical movement of economic variables. These shocks were considered ‘errors’ in the rational behavior of individuals, and were treated as residual error terms from regression equations in structural equation models.

Both Frisch and Schumpeter shared a strong interest in distinguishing between unimportant factors in the development process and important factors that characterized the strength of innovative capitalism. One of the impediments was Walras’ general equilibrium, which went only so far as to represent the static view. It remained insufficient in explaining the idea of dynamics within the context of an economic phenomenon. For Schumpeter, Walras’ account of tâtonnement [33] gave a false impression of the speed of market return to its equilibrium (Andersen, 2009). Concurrent beliefs in formalizing a model of the cyclical process, such as interference phenomena of different parameters of oscillation, prevailed early on.

The three sine curve approximations, [34] consisting of seemingly fitting Kitchin (40 months), Jugular (8-11 years) and Kondratiev (50 years+) cycles in Schumpeter (1939), were compatible with Frisch's 'recursive cycles' (Thalberg, 1998). Frisch accounted for the decomposition of the different order of cycles during the 1930 Yale lectures, using a mechanical illustration of interference, which depicted smaller pendulums attached to a larger pendulum. This also sounded quite similar to Schumpeter’s descriptions where “each Kondratiev should contain an integral number of Juglars and each Juglar an integral number of Kitchins” (Kuznets 1940).

In the end, agreement over the terminology of 'statics' and 'dynamics', remained inconclusive. Schumpeter's continued to use the term 'statics and dynamics' within the context of business-cycles as an evolutionary phenomena. This compares to the revision of TED (1934) [35] definition, which had adopted Frisch’s definition that abided by laws ruled by Newtonian mechanics (Andersen, 1991).

[33] Evolution of Walras’ ‘tâtonnement’ process, as an idea of equilibrium, disequilibrium, equilibration is discussed in Donzelli (2005).
[34] The three sine curve approximation is called the ‘the schema of three cycles’ in Schumpeter (1939).
[35] Schumpeter, had extensive revisions to The Theory of Economic Development in the 1934 edition.

Schumpeter's illustration, which continued to use words like 'endogenous, spontaneous and discontinuous' were inconsistent with a deterministic mechanical system where Innovation would be administered through 'erratic shocks', especially small and random ones.

At the center of their disagreement, was the second nature historical dynamics played in the deterministic structure of the model. Evolutionary phenomena, which fit into historical dynamics, were not eligible for formal models, under Frisch’s definition. This was despite the inspiration they both had received from Kondratieff's 'Long Cycle of Conjecture' earlier on the actual dynamic properties in the historical series.

Historical dynamics is an attempt to analyze by dynamic principles those phenomena which have not yet been brought into rigorously formulated theoretical laws [formal models], and which must therefore be treated by a more or less vague or subjective colored reasoning. From a theoretical point of view it is the analytical dynamics, which is dynamics in the restricted sense (Frisch, 2010/1930, p.62).

Using systems of equations allowed for two unsatisfactory outcomes. One interpretation allowed for a system that already incorporated innovation as an endogenously determined process as a derivation of Slutsky’s random summation process. This effectively reduced summation of exogenous shocks as unknowns. The other interpretation explained innovation as exogenous to the system, serving to explain the process, but this was inconsistent with their idea of a recursive cycle. Allowing for stochastical elements to be introduced into the model would later leave room open to further empirical scrutiny.

Frisch contemplated ‘Schumpeterian innovations’ in his late-stage deterministic representation of ‘impulses’ as ‘Schumpeterian innovations’ and ‘random shocks’, as a substitute for the “error term” in Frisch, (1933). But it never quite goes as far as representing Schumpeterian 'innovations' in his Cassel presentations. He had made a reference to the mechanical representations of the Schumpeter pendulum in lecture four at the Institute of Poincaré, [36] using innovation as the source of energy. The "*water*" in the model represented by "*pendulum suspended from a pivot… Above the pivot there is a vessel.. the vessel with water is supplied by a stream of water. e.g as a constant inflow*" (Frisch, 2009a, p.68) represented the Innovations. This would also mean allowing for forced oscillations. A forced pendulum could be achieved by a damped pendulum, but that would have taken the concept away from free oscillations, combined by exogenous shocks.[37]

In the end, Frisch required that a compromise on the variables representing innovation in the model did not alter the structure of the procedure, only producing a recursive cycle. So, instead, he "refers" to the need to research it further as

ideas that accumulate in a more or less continuous fashion, but are put into practical application on a larger scale only during certain phases of the cycle (Frisch, 1933, p.33)

The end result of their debate would be that Schumpeter valued the interior characteristics of change, whereas Frisch valued the interior characteristics of adjustments after change. For Schumpeter, adopting a exogenous explanation for this source of energy, that represented his innovations was unacceptable. His view gradually metamorphosed to a non-linear view on 'innovations' within the 'impulse' mechanism. That eventually required a deformed, non-regular and shifty nature of innovation that emerged from within the system, which was extremely challenging to represent mathematically

[36] Frisch delivered a series of lectures at Institut Henri Poincaré on the econometric approach titles "Problems and Methods of Econometrics" in March 1933, a few months ahead of Frisch (1933). These series sums up his work on dynamic movements, particularly on business cycles.
Lecture four was called "*Examples of dynamic econometric theories. Oscillation in closed systems. The theory of crises*" introduces the mechanism of the Schumpeter pendulum, following extensive dialogue between Frisch and Schumpeter with regards to its characteristics.

[37] Louca (2007) p.146 mentions that Schumpter had contemplated 'forced' instead of 'free' oscillations, before Frisch(1933) had completed his manuscript. The idea of innovations cumulating and creating the expansion could be expressed."in relation to the nature of energies maintaining the economic oscillations." i..e a forced pendulum Frisch would suggest "this could eventually be represented by van der Pol type of relaxation oscillations". Insight was based on correspondence between his colleague Arnold Rostad and Van Der Pol (25 Oct 1928) with regards to using his relaxation oscillation to "the periodic return of economic crisis". This methodology would be used later in Goodwin(1951).

Partly due to that reason, the two ‘different causes’ of impulses, 'Schumpeterian Innovations' and 'random shocks', would face empirical scrutiny, from endogenous cycle proponents in Simon Kuznets.

2.5 Kuznets critique in Schumpeter’s Innovation

Kuznet's critique (Kuznets 1940) of Schumpeter’s 'Business Cycles’ focuses in on the empirical impossibility of trying to identify the concurrence of the ‘Kondratieff-Juglar-Kitchin three-cycle’ scheme. Business Cycles (1939) argued that 'Innovations' was the decisive impulse, and that their periods of execution dictated the different cycle lengths. Identifying their points of inflection in the neighborhood of equilibrium levels using time-series analysis sounded eloquent, but was empirically difficult to show. Having them concur, within the schematic of different categories of investment goods, i.e fluctuations of inventories (Kitchin), in fixed capex (Juglar), in basic capital goods, all as the medium for basic innovations (Kondratiev) was conceptually impossible.

These three cycles tended to group in integral numbers. The implication would be that each upswing or downswing of a longer cycle commenced in the short term by the peaking or troughing of the shorter cycle.

Kuznets had pursued empirical studies on the inequality of time coefficients in different industries, and their associated appearance as cyclical fluctuations. His own ‘general theory of economic change’ reflected Paul Rosenstein-Rodan’s emphasis of inequality of time coefficients[38], but also Löwe’s positioning on equilibrium in that the disproportionality of the time coefficients, “*could aggravate the effects of time differences*” (Dal-Pont, 2005). In contrast to Frisch’s substitution of Slutsky’s random summation of ‘causes’ with ‘shocks’, Kuznets extends the summation where complications arise from the skew in trending of random events, affecting business behavior, and thus causing disproportional

[38] Rosenstein-Rodan (1934) addresses methodological issues on the time sequence of economic adjustments from an Austrian economics synthesis. It recognized three problems with time (i)economic period of an activity (ii)scarcity of time from consumer point of view (iii)pace of adjustment in demand and supply.

extended oscillations. Similarly, he rejected Schumpeter's 'bunching effect' in innovation, but rather envisioned a "*flowing in a continuous stream*" (Kuznets, 1940, p.263) of innovations, where the evolution itself showed no cyclical formation.

In what can be referred to as 'Löwe's dilemma' [39] (Besomi, 2006), Schumpeter justified the systematically unstable effect of innovation as the exception of a minority, with the majority behaving passively in behavior described by static equilibrium theory. This was somewhat a middle-way stance, as he did not accept the methodically unstable or inherently stable view. Instead, he argued that innovations by changing the state of affairs, shifted to a new equilibrium, only temporarily upsetting stability properties. Schumpeter's (1941) best described Institutionalism as a revolt against Alfred Marshall's (Foresti, 2010) methodological path, directed at showing the conditions for equilibrium was only a temporary state. Veblen, the founding philosophical founder of Institutionalism, argued that such a temporary state rejected the notion of cycles with periodicity.

Schumpeter, in his letters to Frisch, comments with recurring insistence on the lack of structural change in the system that resulted as a condition of its own movement. The mechanical representation could not account for that change. At heart was the issue of different treatments for change, whether they were exogenous or endogenous. Frisch's use of systems of equations in contrast to Schumpeter's treatment of 'external' and 'internal' forces illustrates this. (Festre, 2002) He elaborates the Schumpeterian distinction where innovation could be defined as "setting up of a new production function" with its own endogenous propagation mechanics.

[39] Löwe suggested substituting equilibrium with the cyclical movement of the economy. The methodological critique focuses on the strictly static concept of equilibrium in illustrating a dynamic process.

3 Non-linearity and chaotic distributions

3.1 Distinguishing the randomness and chaos in lieu of the Error Term

By no means did Frisch intend to avoid an endogenous explanation. The intention may have been to avoid a certain predominance of "*exogenous shifts imposed on the system or suggestions of a non-linear process*" (Louça F., 2007) that would ruin the normative simplicity. This is suggested by an earlier version in Frisch (1933), "*consisting of 37 to 38 mixed non-linear difference-differential-integral equations*" that was effectively unsolvable (Andvig, 1981). A 1936 letter exchange between Frisch and Åkerman on his presentation of 'Premises of Trade-Cycle Theory', contains a strong suggestion for Åkerman to pursue a non-linear model using 'undampened oscillation'' (Boianovsky & Trautwein, 2007). Frisch's clarifications that in linear systems, both undamped and damped oscillations, are independent of initial conditions (impulses) supports that claim.

This again, had something to do with the dynamics of linear modeling representation (Vijayamohanan, 2008), "*differential equations represent reality as a continuum, in which changes in time and place occur smoothly and uninterruptedly, even though sudden breaks and qualitative changes do take place in nature*". In retrospect, Keen(1996) and Zambelli(2004) both suggest that had Frisch modeled the metaphor of forced pendulum driven by 'continuously' replenishing to appease with Schumpeter's innovation driven cycle, Frisch would have introduced non-linear analysis to economics.

The non-linear deterministic representation of 'forced oscillations' becomes a lot more complex, as a substitution for 'free oscillations', reduces the clear distinction between endogenous variables explained by the inbuilt oscillating properties and exogenous maintenance of the movement. Schumpeter takes an indeterminate stance, defined as "*element of chance*" and "*personal elements*" (Yagi, 2008) - a stance on innovation. Representing this 'inner vibratory system', the 'deformations' was a definition more prone to a biological mutation. Discussions from Frisch's philosophy of chaos in his lecture 8 of the Poincare lectures, as well as the Nobel prize speech, point to awareness of the complexities that might arise from a non-linear solution to his problem. At the same time, hesitancy regarding any probabilistic notion of the "error term" reinforced his association of 'pure chaotic distributions' with 'indeterminate' or 'randomness'.

In biological evolution, however, errors arise from random mutations, and are selected through the interplay of social and natural forces. Exogenous and endogenous stochasticity are considered and, once a mutuation is selected, the 'error' may generate a path-dependent trajectory – the name of which is evolution. (Louça , 2007, p.255)

This would point to shortcomings in the Newtonian definition of the 'error' as simple errors of measurement. Preference for closed deterministic systems was an 'error term', conditioned on the probabilistic conditioning closer to problem solving in controlled measurement in astronomy rather than biology. Testing for degrees of freedom [40] on singular observations was more accurate, than in an evolutionary environment, where observations were time-dependent and required further accuracy in sample versus population. Distinction between the terms 'random' and 'chaos' remained quite blurred among economist circles. Slutsky refers to "*chaotically-random*" [41] elements, while considering the laws of physics and biology, when touching upon the source of regularity of cycles (Slutsky, 1937/1927).

Distinguishing 'deterministic chaos' from 'random temporal dynamics' in physics, is a start.

Random is defined as

"*irreproducible and unpredictable*" versus chaotic is "*irregular in time (aperiodic), deterministic (same initial condition leads to same final state, but final state is very different for small changes to initial conditions), difficult or impossible to make long-term prediction, complex and ordered in phase*

[40] Degrees of freedom refers to variables that are free to choose from. Three degrees of freedom can be simply illustrated by a+b+c+d=n. Choosing the first 3 leaves 1 left to equal n.

[41] Slutsky refers to the possibilities of structure between random fluctuations that takes into consideration laws of physics and biology like thermodynamics and Mendel's law relating to chance. Acceptance for a deterministic probability set had not developed yet and were mixed with the unknown.

space (fractal structure)” (Nikolic, u.d.).

Alfred Marshall’s [42] comes to mind when associating the representation of economics closer to biology than mechanics (Hodgson, 2013).

“The Mecca of the economist lies in the economic biology rather than in economic dynamics” (Marshall, 1920/1890).

3.2 Poincaré and initial conditions

Physics in the late 19th century was waking up to the reality that even simple dynamic systems were dictated by very complex motions. The traces for the study of geometrical differential equations, as well as ‘bifurcation theory’ that allowed for qualitative analysis of structural change of complex systems, points to Poincaré’s 1889 research on the ‘three-bodies problem’. [43] King Oscar II had extended a prize in the ‘three-bodies problem’, to anyone who could demonstrate whether the solar system was stable. Poincaré would introduce analytical techniques from geometry and topology to discuss the total property, rather than local properties of solutions. By considering the sun, earth and the moon orbiting in a plane under their mutual gravitational attractions, in most cases, the presence of a third-body acted to change the orbits course. In certain situations, the planet would move in a very erratic way. Just like the pendulum, the system contained some ‘unstable solutions’. Not all dynamic equations were integrable, limiting the predictability of the trajectory within the system. This initial-conditions reliance related to one of the core characteristics of chaotic systems, the ‘butterfly effect’. [44]

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42 Marshal had more of a biological, rather than a mechanical metaphor in mind for complexity of economic systems. He used Herbert Spencer’s description of evolution in Marshall (1920/1890). Coined the term “survival of the fittest” in Principles of Biology(1864).

43 Murzi, Mauro, Jules Henri Poincare (1854-1912): Internet Encyclopedia of Philosophy. “In his research on the three-bodies system’, Poincare discovers a chaotic deterministic system. Given the law of gravity and the initial positions and velocities of the only three bodies in all of space, the subsequent positions and velocities are fixed, so the ‘three-body system’ is deterministic. However, Poincare found that the evolution of such a system is often chaotic in the sense that a small perturbation in initial state such as a slight change in one body’s initial position may lead to a radically different later state than would be produced by the unperturbed system. If the slight change is not detectable by measuring instruments, then we will not be able to predict which final state will occur. So, Poincare’s research proved that the problem of determinism and the problem of predictability are distinct problems.”

44 Butterfly effect is the commonly used analogy to describe ‘sensitivity dependence on initial conditions’, coined by Edward Lorenz as the theoretical representation of a hurricane’s formation being contingent on some butterfly flapping

Illustration of this sensitivity dependence on ‘initial conditions’ from minor alterations in deterministic nonlinear systems, originates also from Henri Poincairé.

Small differences in the initial conditions produce very great ones in the final phenomenon. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon (Poincare, 1914/1908, p.68).

Poincaré would go on to suggest how to handle the issue of unidentified initial conditions as random in his ‘Method of Arbitrary Functions’, by identifying the element of ‘chance’ as the direct result of this uncertainty. His recurrence theorem, which “*states that certain systems will, after a sufficiently long but finite time, return to a state very close to the initial state*” (Wikipedia, 2014), provides a solution to the probabilistic independent versus ‘recurrent measures’ question posed by Ergodic theory. [45]

Frisch would fail to mention Poincaré’s notions regarding non-linear oscillations during his Poincaré lecture on Chaos in 1933. After Frisch (1933), two subsequential publications could explain why Tinbergen’s memoirs reflect the opinion that “Frisch had come close to chaos”. In Frisch’s representation of a complex system of market interaction between two agents (Frisch, 1934a), it was already quite clear that he was aware of the chaotic nature in the assumptions of the paper. The depicted system’s behavior depended essentially on the preconditions and the values of the parameter. What followed via collaborations with Koopman and Tinbergen, was a more complex version, consisting of six dimensional systems of equation model [46] with the flexibility of prices, defined by price multiplied by quantity, to reflect the evolutionary nature of a liberalistic economy.

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its wings earlier or not.

[45] Dajani (2008, p.5) on Ergodic Theory “it is the study of long term average behavior of systems evolved in time”. Berliner (1992, p.79) as an answer to question posed by Ergodic theory, he explained that “when can we expect the average of the data, over time, in the first experiment, to be the same (in expectation) as the average of the data, over the replicates at a fixed time”.

[46] Louça(2007, p.277) Refers to three page document dated 1 October 1935 “The Non-Curative Power of the Capitalistic Economy – A Non-Linear Equation System Describing how Buying Activity Depends on Previous Deliveries”, based on the discussion at Namur, 1935 meeting for the Econometrics Society. The paper was based on discussions between Tinbergen, Koopmans and Frisch, regarding a further adaptation of Frisch’s 1934 model. The aim was to represent a more realistic version of a simple economy, consisting of production and exchange.

3.3 Goodwin's synthesis of Schumpeter clock

One of the early manifestations of early non-linear deterministic models revolved around the 'Schumpeter clock', [47] which relies entirely on innovation, giving forced oscillations vis-à-vis mutual conditioning, "leading to an erratic clockwork" (Goodwin, 1951). This 'mutual conditioning' is a term that goes back to Christian Huygens' discovery of the two synchronizing pendulum clocks, which eventually would swing in synchrony, regardless of their initial conditions.

Regardless of initial conditions those two pendula converged after some transient to an oscillatory regime characterized by identical frequency of the oscillation, while the two pendula angles moved in anti-phase" (Kuznetsov, et al., 2007).

Still, Goodwin's non-linear adaptation of the Investment function in Hick's theory of trade cycles required an external shock to explain both the growth cycle and to distinguish each fluctuation.

Goodwin's aim with his non-linear accelerator was to account for the identified imperfections in early post-war research in non-linear dynamics, based on the empirical observation that business cycles were not exactly repetitive. Before anything else, the concept of structural stability had to be differentiated from structural instability (Chen, 1990/1989), just as Poincaré's correction to the 'three-bodies problem' would imply. Poincaré defined stability as the distance before and after a trajectory, resulting from a variation in its initial conditions. His stability criteria showed remarkable similarity with the prominent Lyapunov stability measure, which measured the distance between similar points between two orbits, corresponding to specific moments in time. By introducing an exogenous driver, Goodwin's nonlinear

[47] Schumpeter clock is Richard Goodwin's formal representation of the 'forced pendulum', where Schumpeterian innovations are administered in continuous fashion as the source of energy of the impulse mechanism reflected in Frisch (1933). The assumption of dampening is no longer sufficient for stability, leading to the problem of 'coupling' between two frequencies. Economic time is irregularly struck by swarms of innovations, i.e clusters of technological and economic shocks erratically but systematically administered to markets. Irregular oscillations is thus the unifying concept for dynamics.

accelerator-multiplier was able to endogenously create cyclical behavior, resulting in locally unstable steady-states and stable limit cycles. Limit cycles like the Goodwin accelerator required geometric observations, allowing for "*asymptotically stable cycles regardless of the initial shocks and lags*" (Bernard, Gevorkyan, Palley, & Semmler, 2013). The term 'limit cycles' were also called 'Van der Pol oscillators', using second order differential equations, based on his observation of deterministic chaos in Nature (1927). Their finding was based on the modeling of electrical oscillations, imitating an analog computer. Experimenting with different parameters, the finding was that initial conditions played a large role in generating different cycles.

3.4 Ping Chen's synthesis of Schumpeter clock

Ping Chen's non-linear model depicting a 'Schumpeter clock' can be used to illustrate some of the basic philosophical underpinnings of shape and time-paths in complex systems during the same time line. Chen bridges the gap between equilibrium and 'evolution-biological' model by defining business cycles from this Marshall distinction, as living rhythms with structural stability and dynamic resilience. He introduces the concept of 'color chaos', [48] adding a characteristic frequency similar to a biological clock to a non-linear oscillator in continuous time (Chen, 2005).

This contrasts with classical mechanical concepts like the Hamiltonian, where non integrable systems and continuous time cannot be addressed. The optimization problem implies that 'time symmetry' ignores historical input (Chen, 2013). A characteristic feature of the time-frequency analysis [49] used in this model was the clear pattern of deterministic spirals. The optimization issue can be illustrated by pointing out the 'time symmetry' problem found in traditional statistical techniques, such as Fourier and spectral analysis. Fourier relied on regular periodicity, or sine waves, in its underlying assumption

[48] Chen (2001, p.12) writing about an alternative explanation for 'color chaos' : "*Color Chaos is a nonlinear oscillator in continuous time. Color means a narrow frequency band in a power spectrum*."

[49] Wen(2005) illustrates the use of wavelet-based time-frequency analysis, typically used in signal process engineering. Time frequency-analysis attempts to reflect historical information that leads to the time-variation characteristics of business cycles.

on cyclicity. Frisch directly criticized trend and cycle decomposition techniques like Fourier and periodograms (Bjerkholt, 2007) in his Time Series treatise, as well as in Åkerman (1928).

Although this was not empirically illustrated, "total methods" techniques could not be applied to a dynamic economy where conditions were evolving. Instead, local properties, such as slope and convexity, needed to be measured at each point in time. Chen uses time-frequency analysis, an improvement on Fourier analysis, where signal frequencies can vary with time. More specifically, Fourier analysis assumes that signals are infinite, while in reality, many signals are short and can mutate substantially. Chen (2005) finds a key characteristic of 'color chaos' -- deterministic spirals that become apparent by a Gabor transformation -- to "determine the sinusoidal frequency and phase content of local sections of a signal as changes over time" (Wikipedia).

Many of the questions concerning the accuracy of deterministic non-linear systems revolve around 'strange attractors'. [50] Strange attracters refer to 'fractals'[51], or more subjectively, some kind of long term pattern found in bounded chaotic systems, which may repeat itself, despite having started with different initial conditions. Skeptics question whether chaos theory can "*reconcile randomness and determinism in economics*" (Mirowski, 1990, p.303). Such arguments are several-fold. Neoclassical theory does not allow for dissipative economic systems, as the consensus of 'ordinary attractors' (moving closer to the orbit of equilibrium) versus 'strange attractors' (moving away) is unclear. One can assume this refers to Mirowski's assumption that neo-classical methods do not meet the minimal procedural qualification for dissipative systems that contain a 'strange attractor' of 3 or more degrees of freedom.

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[50] Following is a simplified definition of 'strange attractors' from Petree (u.d.) 'Strange attractors' is a "pattern of the pathway, in visual form, produced by graphing the behavior of a system. An attractor is defined as "*a limited set that collects trajectories*". Because many, if not most, non-linear systems are unpredictable and yet patterned, it is called 'strange'. Since it tends to produce a fractal geometric shape, it is said to be attracted to that shape.

[51] Fractals are irregular geometric shapes that have properties of 'self-similarity', or the same degree of irregularity on all scales. Mandelbrot(1982/1975) "*Fractal Geometry plays two roles. It is the geometry of determinstic chaos and it can also describe the geometry of mountains, clouds and galaxies*".

Another argument seems to be directed more at having a deterministic approach, period. This is even if the model assumes a non-equilibrium approach, and is bound to qualify under ‘dissipative’, with the simulation of real life phenomena, like the biological ‘Schumpeter clock’. Chen argues that deterministic chaos has both regularity and irregularity. So a conclusion cannot be made that degrees of freedom are infinite, and one has to be open for nonlinear chaotic systems to be predictable (Chen, 1990).

The counter-argument is provided that deterministic models are once again misrepresenting reality by accounting for random phenomena, only in a more mathematically dynamic form.

Here it seems that the formalism of strange attractors may promise law-governed behavior independent of historical location (Mirowski, 1990, p.291).

3.5 Lyapunov exponent and the predictability horizon

One of the central arguments around the deterministic representation of chaos revolves around the positive Lyapunov exponent, a condition for verification of the ‘strange attractor’. The Lyapunov exponent quantifies the degree of ‘sensitivity of initial conditions’. The positive Lyapunov illustrates both the reliance on the ‘initial conditions’, as well as the magnification of those ‘initial differences’. This provides a statistical quantification of stability, in terms of the degree of chaos in the system (Peitgen, 1992). Skeptics can make plenty of arguments as to why using mathematical representations of fractal structure behavior does not necessarily improve the accuracy of deterministic modeling. Part of the answer lies inherently in the limitations of computing and evolutionary biology in fully making empirical usage of time series. One of these measures comes from the standpoint of ‘low’ versus ‘high dimension’ chaos, living up to Poincaré’s recurrence theorem over the longer-run. Low dimensional chaos can be characterized by singularity, whereas high dimensional chaos can be characterized by multiple positive Lyapunov exponents. Economic time series, generally speaking, do not meet the low dimensional chaos threshold, which is more commonly available in the physical sciences.

From the procedure used to calculate correlation dimension it seems apparent that a highly complex chaotic system greater than five dimensions is very difficult to detect. It is impossible to verify that the correlation dimension is infinite using a finite time series. Hence there is no practical difference between pure randomness and high dimensional chaos. One tentative conclusion seems to be that while analysis of chaotic dynamics being valuable insights into how economic systems behave, the empirical task of extracting evidence of chaotic dynamics from economic time series is more difficult than in the physical sciences because in the physical sciences once can use over 100,000 observations to detect low-dimensional chaos, whereas the largest time series usually available to economics consist of about 2,000 observations. (Dore, 2007, p.19)

Going back to Chen (2003), modifying a Fourier - Gabor Wavelet representation for the 'Schumpeter clock' provides a better representation for economic dynamics by making use of time-frequency analysis. This has to do with accounting for short-term time intervals of signals, improving the inherent accuracy posed to account for this inherent exponentiality.

According to Lyapunov exponent, the predictable horizon increases merely logarithmically with the precision of assessment. For example, an increase in the precision by a factor of ten, the prediction rises two more time units (Özün, 2006, p.182).

3.6 Stable Lévy Distribution

Part of the answer to the extending the predictability time-horizon may have to do with the evolving understanding of the mathematics revolving around the analysis of various discontinuous phenomena, particularly the bifurcation theory. [52] In the most generalist form, bifurcation theory can be characterized as changes in structure of a dynamic system, when the parameter changes. Local bifurcations are changes in the dynamic system that happen near a fixed point. The world moves in fits and starts. Bifurcation depiction tries to explain the butterfly effect in a graph, which is extremely difficult to draw.

Put more eloquently,

chaos is produced spontaneously, and in a chaotic system there are instances in which the mix of order assumes new directions. These concentrations are called bifurcation points..where the system branches off" (Nolan, u.d.).

Bifurcation was coined by Poincaré, ahead of what is now regarded as chaos theory, or fractal geometry. The expression that was used, 'forme de bifurcation', was initially used in reference to the term 'trifurcation', when studying the geodesics on the surface of railway rails and their singularity (Bruter, 1986). It was the 'initial conditions' sensitivity question Poincaré posed on structural stability that opened the way for qualitative analysis of structural change of systems, including the possibilities for 'strange attractors', which now can help identify bifurcation. The actual derivation of the necessary conditions did not come along until the Andronov-Pontryagin [53] criterion in 1937 (Veneziani, 2006).

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[52] Rosser (1991) p.7 Different methods of analyzing discontinuous phenomena include "catastrophe theory, chaos theory, synergetics theory, self-organizing criticality, spin glass theory, and emergent complexity".

[53] Andronov-Pontryagin criterion are the conditions for structural stability of dynamic systems in the plane. Structural stability refers to the property of the dynamic system in its entirety.

Elements of qualitative analysis on bifurcation in economic literature can be found in Irving Fisher's 'Debt deflation theory of Great Depressions', where "*over-indebtedness to start with and deflation following soon after*" (Fisher, 1933a), destabilizes the economic system. The over-indebtedness is exponential as interest exceeds principal repayments, and the initial disturbance is amplified by higher real cost of debt, as equity value of businesses diminish. Deflation would accelerate into a depression through massive liquidation of debt, as individuals also try to reduce their debt in the 'two paradoxes'. This same process would be replicated in Hyman Minsky's 'Financial stability hypothesis'. Prices driven by expectation would affect pro-cyclical movements in capital goods, whereas aggregate debt burdens would drive price expectations down and accelerate a depression. Qualitative examples of structural change of systems have been few and far between.

As to how realistic it is to reflect the bifurcation process in deterministic models beyond very short intervals, Mirowski in his synthesis on Mandelbrot offers some anecdotes. His suggestion is to go back to the empirical drawing boards. Mandelbrot's primary empirical insight being "*distributions of prices are approximately scale-invariant*" (Mirowski, 1990), this brings into question any element of Marshallian distinction with respect to the short versus the long-term. Fractals do not give you the time component for predictions. It just tells you the shape and form in how change takes place.

Mathematically, chaotic systems are represented by differential equations that cannot be solved, so that we are unable to calculate the state of the system at a specific future time 't' (Levy, 1994, p.168).

One of the best estimation practices for identifying the vicinity of chaos, in the form of discontinuity of prices up as well as down, is in Lévy stable distributions. The early scaling characteristics of the Lévy distribution can be found in Cauchy in 1853, with the distribution of random variables proportional, "*under non-random weighting, in form of addition*" (Mandelbrot, 2001). The Levy distribution shows tail properties similar to the 80/20 Pareto distribution [54], with much more weight on the long tail than normal distributions. It offers the three properties, which makes it useful within the confines of

[54] Chatterjee & Chakrabarti, (2007, p.216) gives historical empirical data on Pareto's law, as well as a brief modern

deterministic chaos: stability [55], power law [56] and asymptotics and fits within the confine of the central limit theorem[57]. This implies that they follow the shape of a power law of scale invariance, a lot like an exponential function. Levy was also Mandelbrot's teacher and a student of Jacques Hadamard, who showed that chaotic motion with particles moved exponentially from one another, with a positive Lyapunov exponent.

Lévy stable distributions are suitable for modeling processes with a high degree of short-term sensitivity which are strictly non-negative, and exhibit power law tendencies. Mirowski (1990) highlights Mandelbrot's findings pertaining to the tails in Lévy distributions resembling "*an aymptotic form of 'Pareto's Law*'… and he began to suspect that such hyperbolic distributions were endemic to economic variates." Still, the continued reliance on financial data that exhibit tail frequencies in Mandebrot (2010) is telling, in his use of 1900-1905 daily cotton prices (Mandelbrot, 1963).

description: significantly asymmetric frequency distribution of income levels in a monotonically declining curve.

55 The sum of two Lévy distributions equal to a stable Lévy distribution. Stability in the basic properties do not change, such as their fractal nature, where you can see the same behavior across different scales in length, time and price.

56 Wikipedia defines a Power law as a "functional relationship between two quantities, where one quantity varies as a power of another. Attributes are scale invariant, with no average and universal", as in phase transitions in thermo-dynamics like water approaching a boiling point.

57 de Moivre (1738/1718) introduced the general notion of the Central Limit Theorem (CLT), where if you sum up a sufficient series of independent, random variables, you end up with a normal distribution, de Moivre approximated the binomial distributional properties by using the integral of the density of a probability distribution, exp * (-x2). The term was coined in 1920 by George Pólya as the CLT approached its final form in probability theory. Mirowski (1990, p.294) argues that the CLT is only one of many stable distructions as it "restricts the outcome to normality by imposing the condition that each of the constituent random variables has finite variance".

4. Conclusion

Until Frisch (1933b), there was little formal framework for macro-modeling the general economy in some rigorous fashion, which accounted for the aperiodic movement observed in the time series decompositions that built upon prevalent economic theory. By adopting empirical findings from the intermittence of overlapping cycles that comprised of ‘long wave’ structural estimation work, the general preoccupation shifted away from the debate of endogenous or exogenous causes of business cycles. The deterministic framework would provide the formalism required for policy making considerations, a justified response to the huge volatility experienced during the Great Depression.

Frisch’s distinction between historical and dynamic analysis, in the debates surrounding the Schumpeter biological clock, would bring forth the difficulty in representing structural growth in a linear manner. Assumptions of continuity and reversible change was also gradually negated by the findings of instability in Poincaré’s three point problem, which was inconsistent with the underlying Newtonian assumption in prevailing economic theory.

Too much emphasis has been placed on validating whether the depiction of free but damped oscillation in Frisch form could create spurious cycles as promised. Chaotic processes can take place both as an endogenous or exogenous process. Tinbergen said “*Frisch had come close to chaos*” (Tinbergen, 1992). The sketch on the complex system of market interaction between two agents highly suggests that he realized early on that the predictions were heavily initial condition dependent. Frisch was skeptical towards using any probabilistic notions. However, it is questionable whether further distinction between random and chaos in the ‘error term’ would have allowed for non-linear deterministic systems to develop faster.

Improvements made on Schumpeter's biological clock in Goodwin's non-linear accelerator multiplies still required introducing an exogenous driver. In Hick's theory of trade cycles, this required an external shock to explain the growth cycle and distinguish each fluctuation. Ping Chen's mathematical representation of an "evolution-biological" approach goes further in addressing Goldstein's 'cycle time' in 'long waves', based on observing inferential statistics on lagged correlation between variables by historical phase periods. Applying time-frequency analysis in a Gabor Wavelet representation seems to improve accuracy in a non-linear deterministic framework. If the 'evolution-biological' approach has any backward looking reflection on Frisch's approach, I believe this suggests he realized that nonlinear models would require differential and difference equations that could not be solved, at an early stage.

This dilemma can be illustrated by how little that is understood about the sensitivity parameters of the so called initial conditions, or butterfly effect, to this day. The massive data set required to meet the minimum threshold of one positive Lynapov exponent, to measure deterministic chaos at the local bifurcation level, limits the fields that can be empirically tested. Mandelbrot's breakthrough in detecting fractal behavior in stock price series is telling. Applying chaos principles, to one or a few variables, improved model accuracy, in comparison to more complex macro-economic models.

Critiques of deterministic framework for non-linear modeling, such as Philip Mirowski, seeks answers by using technology to calibrate and process information via infomatics. A prime example today of this approach, where short-term predictions are made via algorithmic prowess in chaotic systems, is high-frequency trading (HFTs). HFTs actively arbitrate how individual buyers of stocks respond to certain news based on processing historical behavioral responses, by buying lower and selling higher. Arguably, 'quantification' is now superseded by the computerization of economics, perhaps just like mathematization superseded economic theory in econometrics during the inter-war years.

One of the many good reasons on focusing on the inter-war period is its unique set of circumstances. The inter-war period was time when endogenous and exogenous debates overlapped, and dramatic discontinuous events occurred -- the end and beginning of two World Wars, as well as the Great

Depression. This has provided a deterministic framework for a local bifurcation point that has been used by policy makers, particularly the central bankers, to calibrate macro dynamic models, to this very day. Perceptions of shape, time path, and methodology for analyzing economic fluctuations were all in flux. Now that there is a bigger awareness of chaos, I am reminded of a more simpler, less ambitious quote. “History doesn’t repeat itself, but it does rhyme” (Mark Twain). The correct underlying assumptions of a model may be more important than its accuracy, or a complex set of equations.

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