Resolution potential of surface wave phase velocity measurements at small arrays

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SUMMARY

The deployment of temporary arrays of broadband seismological stations over dedicated targets is common practice. Measurement of surface wave phase velocity across a small array and its depth-inversion gives us information about the structure below the array which is complementary to the information obtained from body-wave analysis. The question is however: what do we actually measure when the array is much smaller than the wave length, and how does the measured phase velocity relates to the real structure below the array? We quantify this relationship by performing a series of numerical simulations of surface wave propagation in 3-D structures and by measuring the apparent phase velocity across the array on the synthetics. A principal conclusion is that heterogeneities located outside the array can map in a complex way onto the phase velocities measured by the array. In order to minimize this effect, it is necessary to have a large number of events and to average measurements from events well-distributed in backazimuth. A second observation is that the period of the wave has a remarkably small influence on the lateral resolution of the measurement, which is dominantly controlled by the size of the array. We analyse if the artefacts created by heterogeneities can be mistaken for azimuthal variations caused by anisotropy. We also show that if the amplitude of the surface waves can be measured precisely enough, phase velocities can be corrected and the artefacts which occur due to reflections and diffractions in 3-D structures greatly reduced.

Key words: Surface waves and free oscillations; Seismic anisotropy; Wave Scattering and diffraction.

1 INTRODUCTION

Classical surface wave velocity analysis studies can be mainly separated into two groups. Tomographic methods where group or phase velocites are measured between source and stations (e.g. Nishimura & Forsyth 1988; Ekstrom *et al.* 1997), and network-methods, where the phase difference between two stations or more are analysed to remove the effect of the source (e.g. Nolet & Panza 1976; Friederich 1998; Prindle & Tanimoto 2006). Two-stations methods were originally designed to use data from two distant long-period stations located along the great-circle between the stations and the epicentre. Minimum distances between long-period stations were typically several hundreds of km, and the phase difference between two stations was typically larger than several cycles. With the advent of broadband network deployment, the distances between the stations used for such measurements have gradually been reduced down to a fraction of a wavelength. Phase-measurement errors are therefore becoming a major issue. Pedersen *et al.* (2003) presented a method for measuring the local dispersion of surface waves over small-aperture arrays based on the assumption of plane incoming waves and on averaging over azimuths to suppress the effect of noise and diffraction outside the array. They showed with synthetic examples that their procedure is stable over a large range of wavelengths, including eight times the size of the array with 5 per cent white noise. Another issue is the significance of the phase velocities measured over small arrays: do they represent the mean velocity of the structure just below the array or are they influenced by the surrounding. Pedersen *et al.* (2003) tested the resolution of the method in terms of structure using synthetic seismograms calculated in a 2-D structure, but tests have yet to be performed in 3-D structures. 3-D structures produce scattering which is more complicated than that produced by 2-D structures, and it is important to analyse this aspect of the problem as well.

The purpose of the present study is to analyse the performance of surface wave phase measurements over small arrays in the presence of 3-D heterogeneities. We will first analyse the characteristics of the wavefield at periods of 25, 50 and 100 s when they propagate over heterogeneities ranging from 20 to 140 km in size. Array measurements consist of measuring phase velocities by sampling the phase of the wavefield at a number of points. We will analyse how the location of the array with respect to the heterogeneity, as well as its size, influences the measured phase velocities. Taking the

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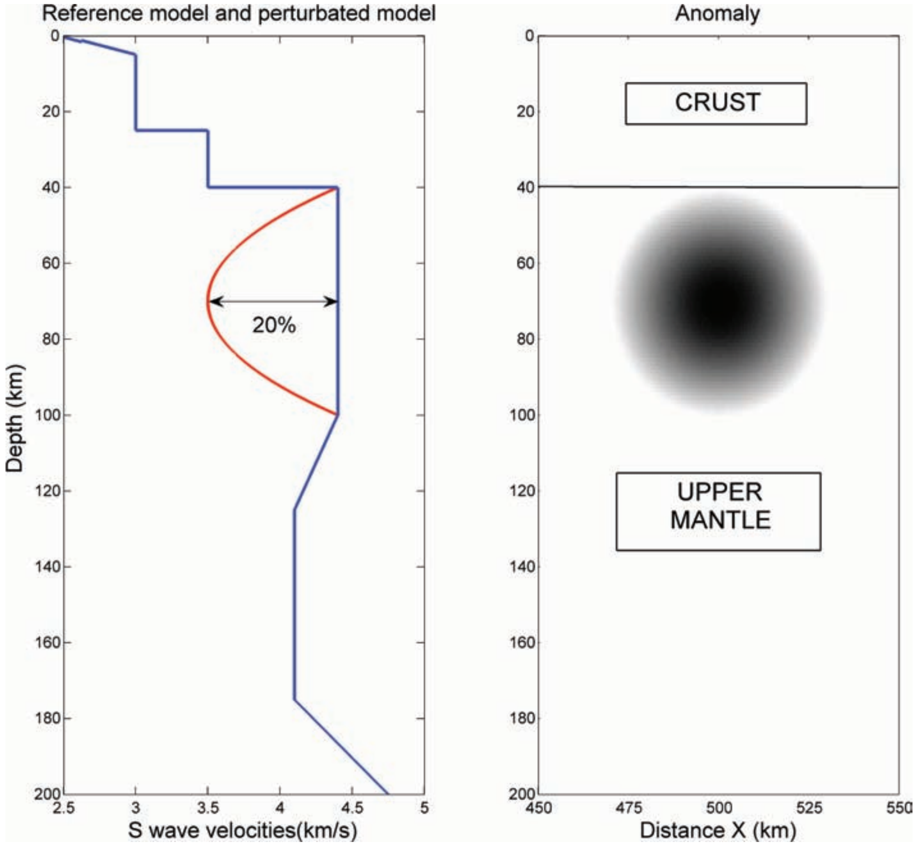

Figure 1. Left panel: *S*-wave velocity as a function of depth in the reference model used and amplitude of the perturbation with depth at the centre of the heterogeneity. Right panel: Cross-section showing the lateral structure of the heterogeneity.

average of the phase velocity measured from events arriving from different backazimuths is an important element in Pedersen *et al.* (2003)'s method. We will analyse the consequences of dropping this step in order to get the azimuthal variation of the phase velocity, as was done in Pedersen *et al.* (2006). Wielandt (1993) pointed out that it is only for plane waves that true structural phase velocities can be measured using the phase only. In case of non-plane waves, phase velocities must be corrected using terms related to the gradient and Laplacian of the amplitude. We analyse how these terms improve the determination of the measured phase velocities.

2 MODELS AND METHOD

We present results obtained in simple 3-D models with a single low-velocity spherical or ellipsoidal heterogeneity located at the top of the mantle. The heterogeneity has a maximum amplitude of 20 per cent at the centre, decreasing to zero at the outskirts as shown in Fig. 1. The depth extent of the heterogeneity is 60 km in all cases, but the lateral extend varies from 20 to 140 km. At the centre of the heterogeneity, the local Rayleigh wave phase velocity anomaly at 50 s period is decreased by 0.3 km s^{-1}. The heterogeneity, unless mentioned otherwise, is always located at the position 500–500 km, at the centre of the model, which has horizontal dimensions of 1000 km by 1000 km, sampled every 10 km. Another model configuration, with a crustal root, was tested in Bodin (2006) and yields similar results to the present case.

Phase velocities are calculated by measuring phase differences between different surface locations for Rayleigh waves propagating across the 3-D structure. The wavefield is computed using the multiple-scattering mode-coupling method of Maupin (2001). A pure fundamental mode of the Rayleigh wave is incident on the structure and interaction with the heterogeneity leads to coupling to overtones and Love waves, as well as self-coupling and diffraction. Since we observe that the effect of coupling to Love waves and to overtones is negligible on the final phase velocity that we measure, we will present results where the only coupling taken into account is the self-coupling of the fundamental mode Rayleigh wave to itself. Three iterations, corresponding to three orders of scattering, are accounted for in the examples shown below, but the phase varies very little after one order of scattering (corresponding to single scattering or Born approximation). The computations are done at single frequencies of 0.04, 0.02 and 0.01 Hz, corresponding to periods of 25, 50 and 100 s and wavelengths of approximatively 80, 200 and 400 km. Computations at neighbouring frequencies show that the phase of the wavefield is varying smoothly with period over the extent of our 3-D models, and that computations with a finite-frequency band would yield similar results to those at a single frequency.

Since the goal of this study is to analyse how surface waves registered on small arrays are able to resolve heterogeneities, the dimension of the array is an important element in our analysis. However, prior to analysing the phase velocity measured with arrays, we analyse the wavefield itself at the surface. This shows how the wavefield interacts with the heterogeneity and will help us understanding in the next sections how arrays are sensitive to the heterogeneity.

Phase velocites are usually evaluated by measuring phases, either by taking the phase difference between different stations, by measuring the phase difference between source and receiver or using

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the phase information in multimode synthetics. This is equivalent to defining the phase velocity as a rescaling of the inverse of the phase gradient by the classical equation:

$$c = \frac{2\pi}{T\,\|\nabla\phi\|} \tag{1}$$

where ϕ is the phase and T is the period.

Wielandt (1993) has shown that phase differences give a correct value for the phase velocity only in case of plane waves. When the amplitude is locally varying, a correction related to amplitude variations should be added to the value obtained with phase differences in order to get the correct value for the structural phase velocity. The phase velocity becomes:

$$c = \frac{2\pi}{T\sqrt{(\|\nabla\phi\|^2 - \tau)}} \tag{2}$$

where the correction, which we call the Wielandt correction, depends on the logarithm of the amplitude $a = \ln(A)$ through:

$$\tau = (\nabla a)^2 + \Delta a \tag{3}$$

With real data, it is very difficult to obtain measurements of the amplitudes which are precise enough for the correction to be viable. Friederich (1998) used some correction in his surface wave tomography of Germany but it is usually not possible to do this correction with small arrays at the present time. However, for the sake of completeness, we compute the phase velocity obtained by phase difference alone and the one obtained with the Wielandt correction. This shows the improvement we would have if we could measure the amplitudes on real data.

At any location in the heterogeneous structure, we can define the *local* phase velocity as the phase velocity of the fundamental Rayleigh mode in a laterally homogeneous structure having the depth-dependence of the model at that location. This gives us a reference to which to compare the measured phase velocities. The goal of phase velocity measurement methods is to bring the measured velocity as close as possible to the local phase velocity, such that inversion with depth of the phase velocity measured at one point yields the correct structure with depth at that location. We will therefore compare local phase velocity, possibly averaged over the surface of the array, to the measured values. The local phase velocites and the eigenfunctions of the modes in the reference model were calculated using the computer program of Saito (1988).

3 APPARENT PHASE VELOCITIES

The upper panels in Fig. 2 show the phase velocity calculated by taking the gradient of the phase of the vertical component of the total wavefield propagating through negative heterogeneities of 60 (left) and 140 km(right) in diameter. The gradient is measured by taking simple numerical differences between phases measured at neighbouring nodes (10 km apart) of the model. The wavefield is a plane Rayleigh wave fundamental mode of period 50 s, incident from the left. The maximum amplitude in measured phase velocity perturbation is the negative anomaly in the middle of the model, at the location of the heterogeneity. There are, however, quite a number of other elements in the phase of the wavefield which complicate the pattern. Reflected waves interfering with the direct wave create oscillating positive and negative anomalies to the left of the heterogeneity. This interference has a wavelength half that of the incoming wavefield and an amplitude which is almost as large as the negative anomaly located at the heterogeneity. To the right of the heterogeneity, we observe a slightly positive anomaly, most pronounced in the case of the 140 km wide heterogeneity. The difference between local and measured phase velocities can be seen more quantitatively in the bottom panels where profiles of the phase velocities at the position 500 km in the *y*-direction (perpendicular to incoming wave direction) are shown as solid lines together with the anomaly in local phase velocity. These results are in agreement with those of Friederich *et al.* (2000) who performed similar simulations with larger scale heterogeneities. They observe artefacts related to the diffraction behind the heterogeneity, as in our case, but much weaker effects related to the reflection which are smaller in their case due to the smoother variation of the structure laterally.

The middle panels show the phase velocity after the Wielandt correction, using eq. (2). The correction, which is done here using the simplest possible scheme to numerically evaluate the gradients and Laplacian of the amplitude of the vertical component sampled every 10 km, gives very good results. The artefacts related to the interference of direct and reflected and forward-scattered waves almost disappear. The negative anomalies are located very precisely at the position of the heterogeneity, as can be seen on the lower panels, where the measured phase velocities with correction are shown with dashed lines. The main difference with the local phase velocity is that the amplitude of the velocity anomaly is underestimated, especially for the narrower heterogeneity.

In the present examples, the lateral variations of the amplitudes are not large. In front of the heterogeneity, where we observe the artefacts related to the reflected waves, the amplitude of the vertical component varies by only 2 per cent (peak-to-peak) for the 60 km-wide heterogeneity, and 1 per cent for the 140 km-wide heterogeneity. The maximum amplification is located on the heterogeneity and is almost 3 per cent for the small heterogeneity and 7 per cent for the large one. It is interesting to notice than these moderate amplitude variations still create significant artefacts in the phase velocity measured with phase measurement alone, and that the Wielandt correction is able to correct this very well and to give a very good measurement of the phase velocity with one event only. These synthetics are done at one frequency only. The artefacts would probably be less severe far away from the heterogeneity in synthetics made with a finite frequency range. Tests with neighbouring frequencies show however that the phase pattern does not change very quickly with frequency and that the artefacts we get here close to the heterogeneity with phase measurements alone should be expected also with wider-band data.

4 ARRAY MEASUREMENTS

4.1 Procedure

We use an array configuration with seven stations, as shown on Fig. 3. This gives, with a small number of stations, the possibility of measuring not only the phase but also the amplitude gradient and Laplacian necessary for Wielandt correction, for events coming from all directions.

In order to simulate measurements of phase velocities using an array, we have used a two-step procedure similar to the one used for real data. In the first step, the phases measured at the different stations for a single event are used in a simple least-squares procedure to derive simultaneously the mean phase velocity for this data set and the direction of arrival of the wave. This is done independently for all the events that we want to analyse. In a second step, the direction of arrival for each event is fixed to the direction found

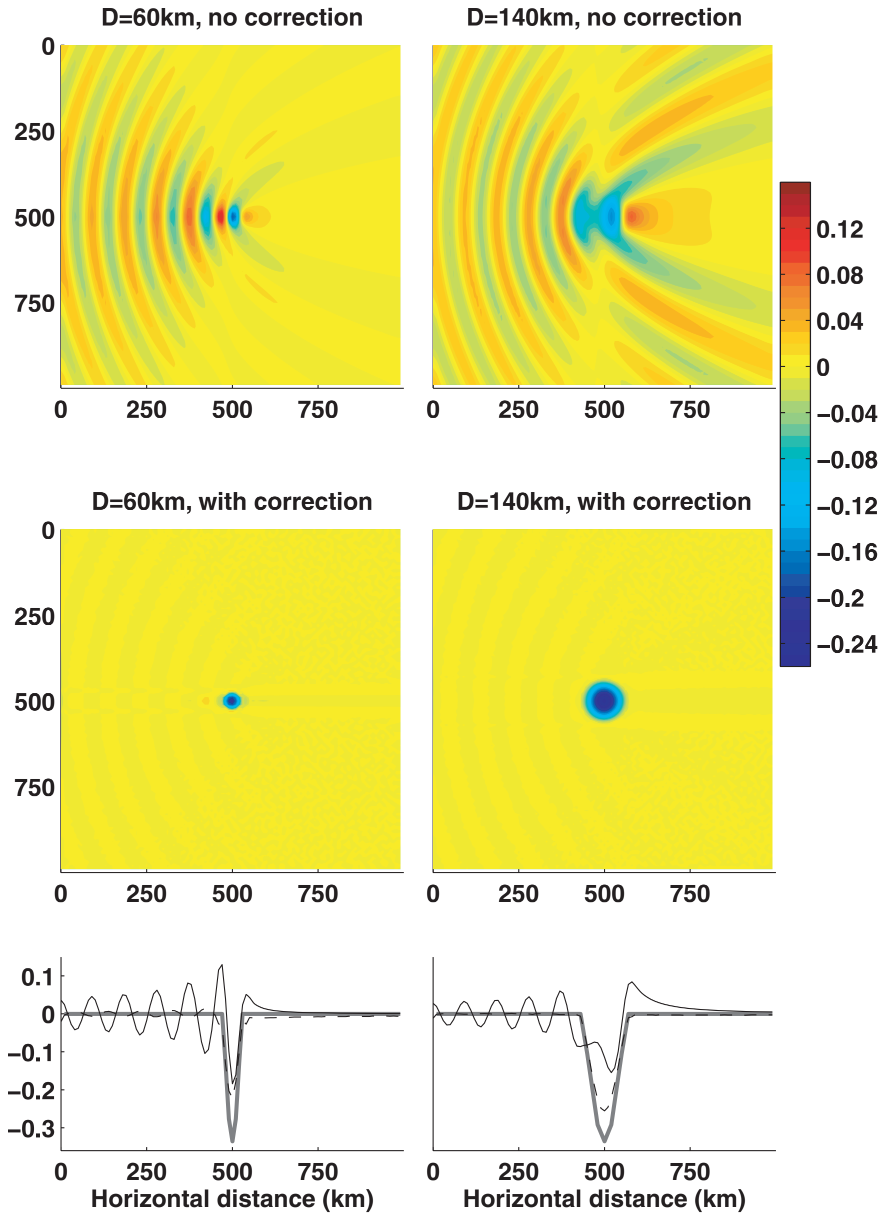

Figure 2. Local phase velocity perturbation in km s^{-1} for a Rayleigh wave with 50 s period incident on two heterogeneities with a diameter of 60 km (left plots) and 140 km (right plots). The upper plots show the phase velocity measured using phase differences only. The middle plots show the phase velocities after correction for amplitude variations, and the lower plots show the velocity along a line crossing the model in the direction of the incident wave at the position 500 km. The solid line shows the velocity measured with phases only. The dashed line shows the phase velocity after amplitude correction, and the thick grey line shows the local phase velocity, corresponding exactly to the structure at the vertical of the point considered.

in the previous step, and the phase data from all events are used simultaneously to evaluate the best phase velocity for the whole data set using a least-squares inversion. This phase velocity is then plotted at the centre of the array. By moving the array with respect to the heterogeneity, we can map how arrays located on or away from heterogeneities sense them.

4.2 Variation with array dimension

Fig. 4 shows such a mapping for two different array dimensions and an heterogeneity of 60 km in diameter. In this case, only one event has been used, and the resulting phase velocity of course closely resembles the phase velocity calculated in the previous section (left column of Fig. 2). The only difference is some smoothing due to the size of the array. This smoothing reduces the amplitude of the measured anomalies by a factor of about 2 with an array of 43 km in diameter, but does not modify the geometry of the phase velocity map or the relative amplitude of the maximum negative anomaly with respect to the other anomalies (which are undesirable artefacts). The correction related to amplitude variations improves the phase velocity significantly, reducing the effect of reflected waves, eliminating the forward-diffraction effect at 500 km and mapping well the negative anomaly both in width and in amplitude.

With an array of 100 km in diameter, the smoothing is strong and amplitude is further reduced, leading to a more complex pattern. The first interference maximum due to the reflection is reduced.

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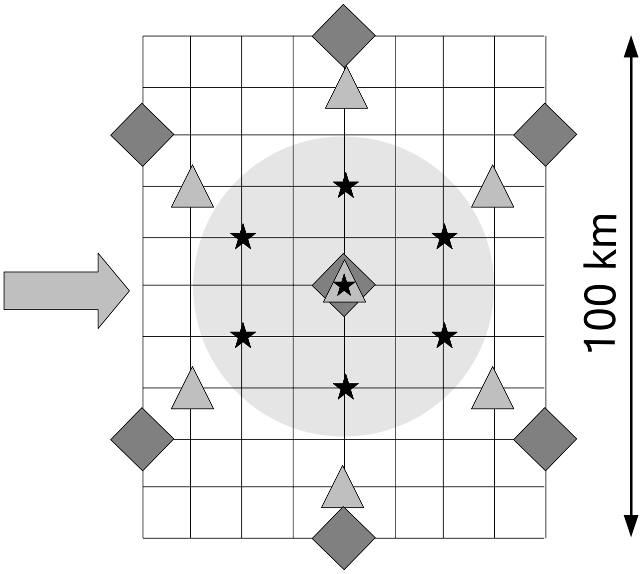

Figure 3. Array configurations used in this study.

4.3 Variation with period

We have tested how the phase velocity pattern varies with period in the range 10–100 s. Fig. 5 shows the phase velocity measured for an incident Rayleigh wave at 25 and 100 s period. Due to the depth location of the anomaly, the Rayleigh wave phase velocities are less perturbed by the heterogeneity at these periods than at 50 s period, giving phase velocity variations of 0.15 and 0.07 km s^{-1} respectively. At short periods, the reflections and the diffraction pattern behind the heterogeneity are less pronounced than at longer periods and

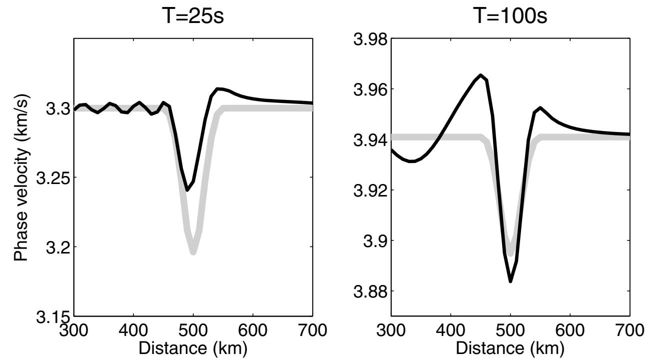

Figure 5. The same as bottom panels of Fig. 4, but for waves with periods 25 and 100 s. The measured phase velocity without Wieland correction are shown in solid line, and the local phase velocity averaged laterally with a 2-D box function of the size of the array are shown in thick grey line.

the measured phase velocity follows quite closely the local phase velocity. At 100 s on the other hand, the reflections are strong and bring significant positive values close to the heterogeneity. We note that in both cases the major negative anomaly is accurately located and has a lateral extent similar to that in the synthetic model. This is observed in the whole range of periods that we have analysed, from 10 to 100 s.

4.4 Variation with array and heterogenity dimensions for multi-backazimuths data

The previous sections show that phase velocities measured with phase data in one azimuth only are strongly perturbed by artefacts related to reflections and diffractions. The procedure proposed by

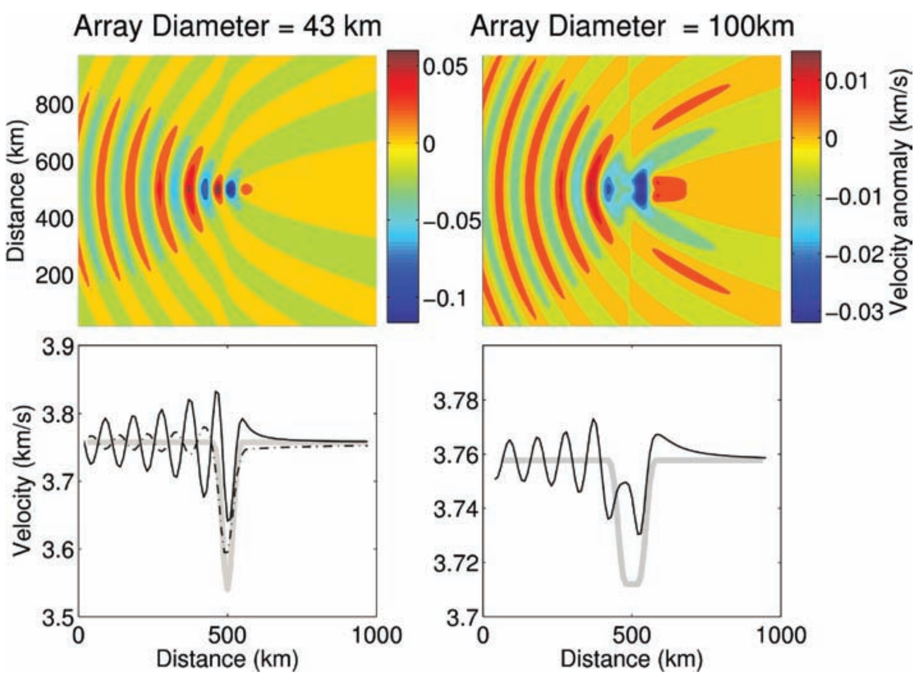

Figure 4. Phase velocity measured with arrays of diameters 43 km (left) and 100 km (right) for a 50 s Rayleigh wave incident from the left on an heterogeneity of 60 km in diameter, located at the centre of the map. The upper panels show a mapping of the phase velocity. The array is moved from point to point over the entire plane and phase velocities measured at different positions are plotted at the location of the centre of the array. The lower panels show the measured phase velocity without Wieland correction (solid line), together with the local phase velocity averaged laterally with a 2-D box function of the size of the array (thick grey line). The bottom left panel also shows the measured phase velocity with Wieland correction (dashed line).

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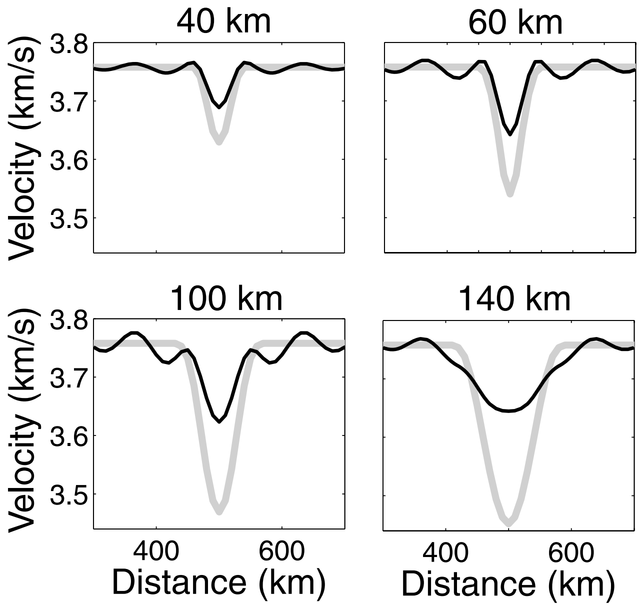

Figure 6. The same as Fig. 5, but for different lateral sizes of the heterogeneities and using data in 40 well-distributed backazimuths.

Pedersen *et al.* (2003) consists of averaging the data from events with different backazimuths and we will now test how this averaging procedure improves the measurements. We have therefore calculated the phase velocity measured at arrays located at different positions with respect to the heterogeneity and for wavefields coming from 40 events evenly distributed in backazimuths. The results for four different sizes of the heterogeneity and for a period of 50 s are shown in Fig. 6, together with the local phase velocity averaged over the size of the array, which has a dimension of 43 km in this case. The azimuthal smoothing considerably reduces the amplitude of the artefacts away from the heterogeneity. Although the detailed shape of the curve varies depending on interference patterns, the minimum phase velocity is well-located at the position of the heterogneity and the width of the region where the velocity is reduced generally corresponds well to the width of the heterogeneity. The largest discrepancy with the local phase velocity is in the amplitude which is a factor of 2 too small for heterogeneities from 60 to 140 km width. At 40 km width, the array has the same dimension as the heterogeneity, and the smoothing of the model by the size of the array further reduces the amplitude of the maximum phase velocity anomaly.

We tested the number of events necessary to obtain a sufficient averaging with azimuth. The error made on the phase velocity measured with a uniform distribution of events decrease asymptotically with the number of events. We found that 15 events are sufficient to get an error close within a 10 per cent interval to the error obtained with a very large number of events. Similar results are obtained with other periods, ranging from 10 to 100 s. The azimuthal averaging is able to reduce the artefacts away from the heterogeneity. The variation of the measured phase velocities with respect to the reference one follows well the local phase velocity variation even for the longest periods. This indicates that the lateral resolution of azimuthally averaged phase velocities can be considerably smaller than the wavelength of the waves involved.

Fig. 7 shows the amplitude of the phase velocity anomaly, as a perturbation with respect to the reference phase velocity value outside the heterogeneity, as a function of the heterogeneity and array dimensions. The plot to the right shows the values predicted by simple geometrical considerations, that is using simply the local phase velocity and smoothing it by a 2-D box-car function of the dimension of the array. The maximum phase velocity reduction at the centre of the heterogeneity is 0.3 km s^{-1}. This value is predicted to be seen by arrays which are much smaller in size than the heterogeneity, for example a 30 km array and an heterogeneity of 140 km. The smoothing effect of the array dimension gradually reduces the maximum amplitude to values very close to zero for large arrays (for ex. 100 km) and small heterogeneities (for ex. 20 km).

The measured phase velocity variations (left plot of Fig. 7) also account for the effect of the wavelength. In the present case of a 50 s period Rayleigh wave, the wavelength is larger than both the heterogeneity and the array. The measured amplitude is smaller than the predictions in the whole range of heterogeneity and array dimensions used here. The reduction is by an almost constant factor

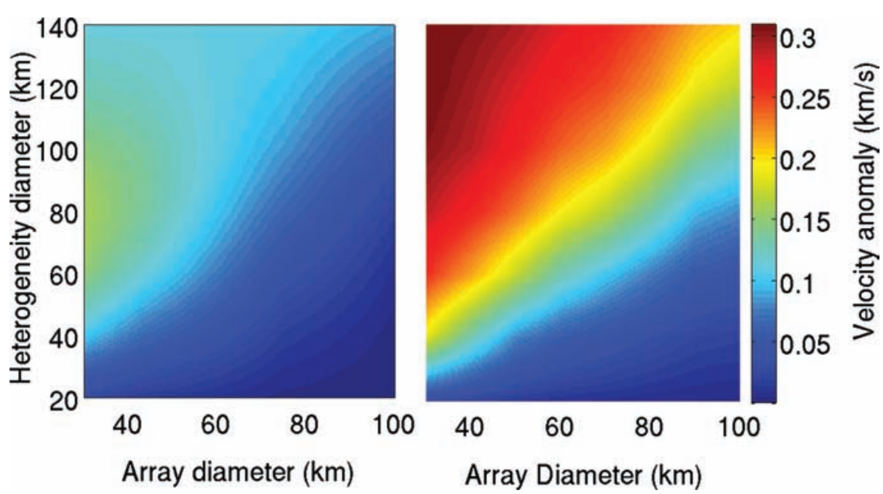

Figure 7. Summary figure showing how the maximum amplitude of the phase velocity anomaly is varying with the dimension of the heterogeneity and the dimension of the array. The left panel shows the measured maximum anomaly as a function of the dimension of the heterogeneity and the array using 50 s Rayleigh waves arriving at the array from different backazimuths. The right panel shows the expected maximum anomaly as a function of the dimension of the heterogeneity and the array. The variation on this panel is a pure geometrical effect due to the smoothing of the true phase velocity variation by the size of the array. The maximum anomaly is of 0.30 km s^{-1}, and is seen by small arrays over large heterogeneities.

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of two. The effect seen here is therefore dominantly an effect of the wavelength compared to the size of the heterogeneity. It can also been seen for a 43 km array on Fig. 6. The variation with array dimension and heterogeneity diameter is less regular than the purely geometric one shown in the right plot. We see in particular a maximum value for 80 km heterogeneities seen by small arrays followed by a slight decrease at larger heterogeneity sizes. For even larger scale heterogeneities with respect to both the array dimension and the wavelength, not shown here, we have verified that this reduction disappears and that we obtain the result predicted by ray-theory that surface waves propagate with their local phase velocity (Woodhouse 1974).

Similar results are obtained in Bodin (2006) for a crustal thickening of 15 km where the topography of the Moho has a 2-D-gaussian shape over a surface of 120 km in diameter. For 25 s period, with a wavelength of about 80 km, the observed maximum velocity anomaly variation is almost as large as the local phase velocity anomaly, while it is 50 per cent smaller at 50 s period. Longer periods show a more complex behaviour related to variations in the interference of reflected and diffracted waves. At 100 s period for example, the measured velocity anomaly is larger than the predicted one by a factor of almost 2. The width of the region with reduced velocities on the other hand corresponds closely to the width of the region where the heterogeneity is located, as in the examples presented here.

Our modelling shows that heterogeneities smaller than half the wavelength affect phase velocity measurements. The area over which a phase velocity anomaly can be measured matches the width of the heterogeneity even for waves with large wavelengths. On the other hand, the amplitude of the velocity variation is not completely reliable. It is usually smaller than the local phase velocity reduction, which implies that measurements are likely to underestimate the amplitude of the heterogeneities present below the array.

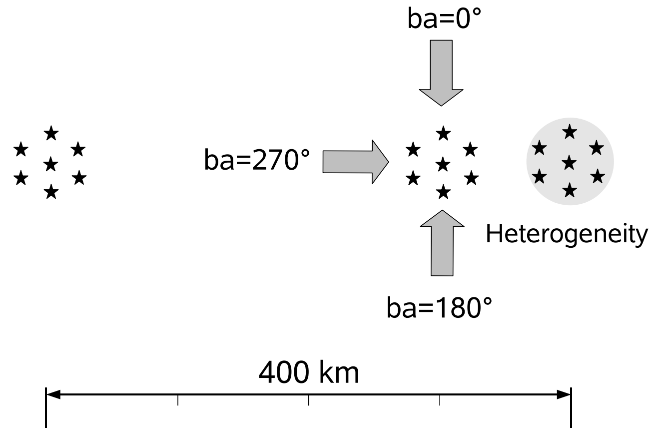

Figure 8. Scheme showing the locations of the arrays with respect to the location of the heterogeneity in the analysis of the variation of the phase velocity with backazimuth.

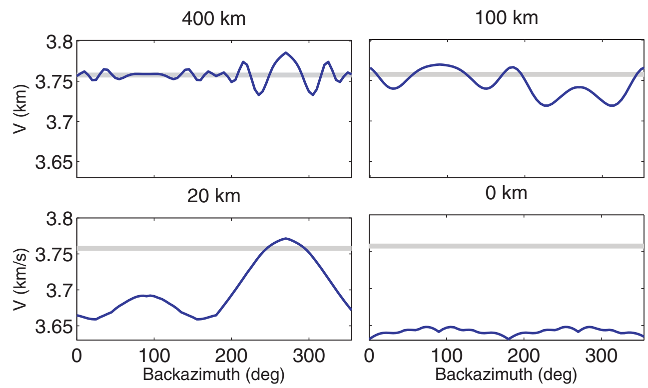

Figure 9. Variation of the phase velocity with backazimuth measured on an array located at different distances from the heterogeneity.

4.5 Azimuthal variation

The azimuthal variation of surface wave phase velocities is an important element in analysis of upper mantle anisotropy. Although most results concerning azimuthal variations originate from larger tomographic studies which should not suffer from the bias discussed here, a number of studies have been conducted using networks of broadband stations of limited aperture. Anisotropy-related azimuthal variations have π- and $\frac{\pi}{2}$-periodicity (Smith & Dahlen 1973). The component in $\frac{\pi}{2}$-periodicity is seldom measured and is predicted to be small for Rayleigh waves. Considering the importance of averaging over azimuth in order to avoid biases related to reflected or diffracted waves, we analyse here to what extent array measurements can be used to determine azimuthal variation of surface wave velocities related to anisotropy.

We analyse the azimuthal variation of the phase velocity measured at arrays located at different distances from the centre of the heterogeneity, as shown in Fig. 8. The phase velocity is measured as a function of azimuth by measuring phase velocites with the same procedure as above but also averaging over events located in moving backazimuth-windows of 20°, as in Pedersen *et al.* (2006). Removing the 20°-window averaging did not modify the results. The phase velocities as a function of backazimuths at arrays located at four different distances from the heterogeneity are shown in Fig. 9 for 50 s period Rayleigh waves. For the array located at 400 km from the heterogeneity, we observe a peak-to-peak azimuthal variation of 1.5 per cent. This amplitude is similar to what is usually attributed to anisotropy, but the variation with azimuth does not have the periodicity of anisotropic-related variations. The mean value of the velocity corresponds well to the mean value away from the heterogeneity. For a distance of 100 km, the azimuthal variation gets a 2π-periodic variation, which cannot be mistaken for anisotropy and a $\frac{\pi}{2}$-periodicity which could be mistaken. At 20 km from the heterogeneity, the phase velocity has strong 2π and π-periodic variations of about 2 and 1 per cent, respectively. The low velocites observed in the 0 to 180° backazimuth interval correspond to waves which have just propagated through the heterogeneity, while waves coming from 270° backazimuth have not traversed the heterogeneity and show a velocity similar to the reference velocity outside the heterogeneity. When the array is located on top of the heterogeneity (lower right panel), we do not observe azimuthal variation. This measurement has been repeated with many more events, giving similar results.

Heterogeneities located close to an array can introduce biases which are significant and which can be mistaken for azimuthal variations related to anisotropy. An important element here, however, is that the azimuthal dependence of the biases changes very quickly with the location of the array with respect to the heterogeneity. Averaging velocities using a larger array of for example 30 stations, as is quite common in broadband experiments, should give the possibility of reducing the biases and retrieving an average value which can be related to mean anisotropy under the area. One should, however, expect significant noise on more localized estimations of the azimuthal variation and be very cautious in interpretation of variations of the azimuthal variation over the network.

5 DISCUSSION AND CONCLUSION

Using synthetics in models with 3-D heterogeneities, we have analysed whether phase velocities of relatively long period surface waves registered on small arrays of dimension 30–100 km are able to detect heterogeneities of the size of the array or similar.

Using phases only and data from only one event or one azimuth, we observe that the measured phase velocities suffer from numerous artefacts related to the interference of the direct wave with diffracted and reflected waves. In the cases we have studied, with a single strong heterogeneity of up to 20 per cent in the mantle, these artefacts can be almost as large as the major phase velocity anomaly related directly to the heterogeneity. The major phase velocity anomaly is, however, well-located on the heterogeneity. We observe that the period of the wave has a large influence on the amplitude and shape of the artefacts, but little on the major anomaly. There are two kinds of main artefacts in our simulations. The ones behind the heterogeneity, related to diffracted waves, have opposite sign compared to the main anomaly. The same kind of artefact was observed by Friederich *et al.* (2000) with larger and smoother heterogeneities than in our case. This kind of artefact is likely to be common, consistent over a wide range of frequencies, and may lead to data misinterpretation. The second kind of artefact is related to the interference of the reflected and direct waves. The amplitude of the reflected waves is directly proportional to the lateral gradient of the structure. Although it is common to assume that reflection of surface waves is weak in the Earth, observations show that some structures are sharp enough to reflect a significant amount of surface wave energy (Stich & Morelli 2007, for a recent example). The pattern of oscillating positive and negative artefacts related to reflected energy will produce at a given location oscillating variations in the dispersion curves which are more likely to be interpreted as noise than the more stable pattern observed behind the heterogeneity. As such, reflections will introduce uncertainties in the models but may not produce any bias. The artefacts observed here are strongly correlated to the fact that we analyse the resolving power of arrays that are small compared with the length of the waves involved. Note that the errors and biases introduced by non-plane incoming waves are strongly reduced for station spacing of the same order of magnitude as the wavelength (Pedersen 2006).

If instead of a single wavefield, we use waves with well-distributed backazimuths around the array and calculate an average phase velocity over the array, we find that most of the artefacts related to reflected and diffracted waves are averaged out and that the measured phase velocities have the resolution to detect heterogeneities of the size of the array. The period of the waves have little influence on the resolution and we observe that anomalies which are smaller than a quarter-of-a-wavelength can in principle be detected. In the case of a 50 s period Rayleigh wave, the main limitation is that the amplitude of the recovered phase velocity anomaly is smaller than the true local phase velocity anomaly by a factor of almost two in a large range of dimensions for the array and the heterogeneity. This factor depends strongly on the wavelength compared to the size of the heterogeneity and can be smaller than one in other cases. This suggests that phase velocities measured over small arrays have the resolution of the dimension of the array itself, even with long-period waves, but that they fail to estimate the level of heterogeneity in a given region.

The necessity to average over azimuth in order to eliminate artefacts limits the possibility of using small arrays to measure azimuthal anisotropy. We analysed if the artefacts caused by diffracted and reflected waves can be mistaken for anisotropy by studying how the phase velocity varies with azimuth for different distances between the heterogeneity and the array. We observe that π-periodic variations of the order of magnitude of what can be expected from azimuthal anisotropy occur for arrays located in the vicinity but not exactly above an anomaly. We observe also that the phase velocity artefacts vary very quickly with array location. This suggests that averaging the azimuthal variation measured over several small arrays located close to each other, or using a network of many more than seven stations would be needed to extract information on azimuthal phase velocity variation.

Using not only the phase but also the amplitude of the waves to derive a correction to the velocity (Wielandt 1993) improves the measurements considerably. This has already been shown for larger heterogeneities by Friederich *et al.* (2000). We showed that the corrected phase velocities accurately reproduce the local phase velocity, up to a small reduction factor, even with data in one backazimuth only. The artefacts related to interfering waves disappear almost completely with the correction and the true structural phase velocity can be measured even in the presence of interfering wavetrains. This opens new possibilities for measuring local phase velocities at periods of 20 s and below from surface waves generated by teleseismic events, for which interference usually inhibit proper dispersion measurements. The major difficulty is of course to get precise measurements of the amplitudes. We show here that small-scale amplitude variation down to 1 per cent amplitude have a notable influence on the measured phase velocities. Friederich (1998) found, through a surface wave tomography taking into account phase and amplitude information, much larger variations of phase velocities up to 50 per cent at 26 and 51s period at the scale of southern Germany.

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