Research Article

Measures for Human Reproduction Should Be Linked to Both Men and Women

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We introduce the two-sex net reproduction rate (2SNRR) and the two-sex total fertility rate (2STFR)—two demographic indicators that reflect the number of children born, given age specific fertility and mortality of the adults. The main quality of these indicators is that they measure the childbearing behaviour of both women and men. The indicators have intuitive value, since they tell us to what extent adults are replaced by children. While the traditional net reproduction rate (NRR) describes general replacement trends among women only, the 2SNRR is an indicator of a population's growth potential, irrespective of sex. We demonstrate the use of the indicators with data from Bejsce parish in Poland for the period 1800–1967 and with data from UN projections for China for future years. We discuss the consequences for our understanding of fertility trends when sex ratios deviate from normal levels.

1. Motivation

The net reproduction rate (NRR) represents the mean number of surviving daughters of a woman, based on prevailing rates for age-specific fertility and mortality. In the 1880s, Richard Boeckh of the Berlin Statistisches Amt and William Farr of the General Register Office in London developed and used this measure—most probably independently of each other; see Kuczynski [1], Lewes [2], and De Gans [3]. The NRR gained popularity in the 1920s and 1930s, but after the Second World War, when period fertility rose rapidly in many industrialized countries and with it the NRRs, the measure was accused of being misleading. In fact, it was only that the NRR was interpreted in a naïve way, indicating the likely achievement of birth cohorts [4, page 109]. However, like the period total fertility rate (TFR), the period NRR says nothing about cohorts.

Although the TFR is a much more widely used measure, the NRR is regularly used in publications by national and international statistical offices, such as those from the United Nations Population Division. It allows one to understand generational replacement trends in the female part of the population: assuming constant fertility and mortality regimes and assuming that one can ignore migration, NRR-values above unity imply long-term population growth and values below one imply long-term decline. Indeed, it plays an important role in stable population theory, since it determines, together with the mean generation length, the intrinsic growth rate of the stable population that emerges under prevailing fertility and mortality conditions (see, e.g., [5, page 152]).

The NRR is a one-sex demographic measure as it only considers the female population. Thus the long-term growth rate of the population implied by the NRR is, in effect, the growth rate for the female part of the population only, not for the male part. Boeckh's student Robert Kuczynski [1] was probably the first one who defined and computed net reproduction rates for both men and women. The traditional "female" NRR characterizes fertility in terms of the average number of daughters born to women. Therefore, a straightforward extension is to define a male NRR likewise as the average number of sons per man. However, empirical values of these two reproduction rates are bound to be different. Hence, when taken to stability, male and female populations will grow or diminish at different rates, and one sex will eventually dominate the other, which is unrealistic. This so-called two-sex problem was extensively debated in the following years;

see, for instance, Myers [8], Karmel [9, 10], and Pollard [11]. The general outcome of this debate was that there is no intrinsic reason to prefer the female NRR to the male NRR, or the other way round. Rather, one should develop reproduction measures that include the behaviour of men and women simultaneously.

Soon demographers realized that the formation of couples lies at the heart of the two-sex problem, and the attention shifted towards marriage [9, 10, 12]. From this perspective, one can formulate the two-sex problem as follows: define a marriage (or union formation) rate that expresses marriages (new unions) by men aged x and women aged y relative to exposure time in the unmarried (without a partner) state of men and women of these ages. The problem is to find an appropriate expression for the combined exposure times of men and women with age combination x, y. Many have attempted to solve this issue, but at present there is no generally accepted solution; see Pollard [13] and Alho and Spencer [14, page 183].

Pending the discovery of truly two-sex measures of reproduction, analytical demographers traditionally focus on age-specific fertility of women (e.g., [5]). Much less attention is given to the childbearing behaviour of men. Zhang [15] discusses the most important reasons. Women, unlike men, have a more sharply and narrowly defined reproductive age range. Fertility data are usually gathered by interviewing women not men. Finally, the father's characteristics are frequently missing or are lacking the quality or detail of the mother's, particularly for extramarital births.

As long as sex ratios are more or less balanced, measures of female-based fertility correspond closely with those of men [16]. However, when there are strong imbalances between numbers of men and women in reproductive ages, the childbearing behaviour of both women and men needs to be studied, because the fertility levels of the two sexes will differ. Such imbalances may be caused by selective migration (as in the Gulf States), by past unbalanced sex ratios at birth (as in China), or by excess mortality of either men or women (as in France after World War 1). For example, Kuczynski [1] reports a net reproduction rate of women in France in the period 1920–23 equal to 0.977, while the NRR of men was 1.194. He explains this difference by the unbalanced sex ratio of the population in prime reproductive ages, as a consequence of World War 1. Data supplied by INSEE show a sex ratio of the population aged 20–39 on 1 January 1921 equal to 0.82.

When used as prediction tools, fertility models based on fertility rates of women only (and not of men) will predict a certain number of births, even in the absence of men. Thus, these fertility models lead to unrealistic results in such theoretical situations. Somewhat less theoretical is the situation where there is a shortage or an abundance of men in childbearing ages. When using a one-sex fertility model, a possible imbalance between the sexes has no effect on the predicted number of births. Hence, one-sex fertility models do not always reflect reality accurately enough.

The purpose of this paper is to revive some of Kuczynski's original ideas and to draw attention to the advantages of modelling male and female reproduction, instead of modelling female reproduction alone. It is *not* our intention to contribute to a solution of the two-sex problem. Our ambition is much more pragmatic, namely, to define fertility measures that combine childbearing behaviour of men and of women. As we will show, a population's growth potential may be severely under- or overestimated when fertility measures of one sex are included in the analysis, in particular when there is a strong imbalance between the numbers of men and women in prime reproductive ages. By considering the reproduction of men and women simultaneously, one is able to obtain a more complete picture of population growth than by restricting oneself to one sex. We present Kuczynski's two-sex measure and our own variant thereof, which is slightly different. We name the latter the two-sex net reproduction rate (2SNRR). The 2SNRR, to be defined below, is a person's average number of children (both daughters and sons) surviving to reproduce him or her, based on a given regime of age-specific fertility and mortality for men and women. Hence, it is a measure of population growth inferred from fertility and mortality of both sexes.

The paper proceeds as follows. First, we define the two-sex net reproduction rate (2SNRR) and the two-sex total fertility rate (2STFR) and discuss their advantages and disadvantages from a theoretical point of view. Next, we introduce approximate expressions for the 2SNRR and the 2STFR, which are useful in case information on male fertility is lacking. We illustrate the use of these indicators with data from the parish of Bejsce in Poland. For this region, detailed age-specific data are available on fertility (and mortality) not only for women, but also for men. The data apply to the period 1800–1967. In addition, we report some findings for China, where we expect for the future many more men in childbearing ages than women.

2. The Two-Sex Net Reproduction Rate (2SNRR)

We start with the net reproduction rate for the female part of the population defined in the usual way (e.g., [5, page 113])

$$\text{NRR} = \sum \text{fw}_x^g \cdot \text{Lw}_x. \tag{1}$$

The upper index g tells us that age-specific fertility rates for women fw_x^g are restricted to newly born girls. Age-specific mortality of these women is expressed by Lw_x, which represents the number of person-years lived by a woman in the age interval denoted by x. Lw_x is derived from a life table with radix equal to one. Summation is over all fertile ages.

When mortality is absent, the NRR gives the average number of live born daughters per woman when age-specific fertility would be constant. Mortality of women who are in their childbearing ages reduces that average number. When taken literally, the latter mortality accounts for daughters who were not born, because some women died before they could give birth to (another) daughter. However, the usual interpretation of the NRR is that it gives the mean number of surviving daughters per woman. This is correct, because the interpretation of the NRR is that for a *hypothetical* cohort. In other words, *both* age-specific fertility *and* age-specific mortality are assumed constant. Hence, the mortality regime

as expressed by Lw_x applies to the daughters as well, and the reduction factors Lw_x can be interpreted as the survival probability to an average childbearing age.

Kuczynski [1, page 36] defined a fertility measure that we call 2SNRRw; the net reproduction rate for women where children of both sexes are included

$$2SNRRw = \sum fw_x \cdot Lw_x. \tag{2}$$

As opposed to the NRR, age-specific fertility for women (fw_x) in this expression is not restricted to newly born girls. Hence, 2SNRRw gives the mean number of surviving offspring per woman.

For men, Kuczynski [1, page 37] proposed a similar measure, which we will call here the net reproduction rate of men, abbreviated as 2SNRRm. Age-specific fertility rates of men fm_x cover births of both genders. Mortality Lm_x is restricted to men. The expression is

$$2SNRRm = \sum fm_x \cdot Lm_x. \tag{3}$$

Its interpretation is similar to that of the 2SNRRw. Again, summation is over all fertile ages of men. The fertile age span of men is longer than that of women; see below.

We wish to combine male and female fertility into one indicator and define the two-sex net reproduction rate 2SNRR as the average of 2SNRRw and 2SNRRm. We take the geometric mean of the latter variables to find

$$2SNRR = \sqrt{(2SNRRw \cdot 2SNRRm)}. \tag{4}$$

We interpret the 2SNRR as the mean number of surviving offspring per person and define replacement fertility as a situation in which the 2SNRR equals two.

Our 2SNRR differs slightly from the two-sex net reproduction rate that Kuczynski [1, page 37] proposed. First, he multiplied the 2SNRRw with the proportion newly born girls to obtain the familiar female NRR (in terms of average number of newborn daughters per woman). In addition, he multiplied 2SNRRm with the proportion newly born boys to obtain the male NRR (average number of sons per man). Second, he took the simple arithmetic average of these two reproduction measures. Thus in his case replacement implies a two-sex rate equal to one.

As opposed to Kuczynski, we take the geometric average of the two one-sex measures. The reason is that the two-sex measure defined this way leads to a particularly simple approximate expression for gross reproduction in case information about male fertility is missing; see expression (7) below. As yet another alternative to the geometric mean in expression (4), we could have taken the harmonic mean of 2SNRRw and 2SNRRm. Keyfitz [17] and Schoen [18] show how geometric means and harmonic means of relevant variables for men and for women have been used in two-sex models of pair formation. The simple arithmetic mean, as used by Kuczynski, would have been less satisfactory, because it assigns a nonzero value to the 2SNRR thus defined; even in a hypothetical situation where age-specific fertility rates of one of the two sexes would be zero. In practice, when the imbalance between men and women in childbearing ages

is small, there is little difference between averages based on the simple arithmetic mean, the harmonic mean, or the geometric mean.

As indicated above, the 2SNRR has several advantages compared to the NRR.

(i) The NRR omits males, which implies that one cannot use it to infer statements for the entire population, such as growth. Given fertility rates fw_x^g, the NRR is insensitive for variations in the sex ratio at birth. For instance, a larger surplus of male births leads to an increase in the 2SNRR (other things being equal), while it does not affect the NRR.

(ii) Compared to a one-sex fertility measure, the use of a two-sex measure is more intuitive because it reflects family size.

At the same time, we are aware of the advantages of the traditional NRR. These include the following.

(i) Age-specific fertility rates (ASFRs) of a given year or for a given cohort of women are more often available than similar data for men.

(ii) The NRR measures precisely whether each generation of mothers is having enough daughters to replace themselves in the population based on fertility and mortality up to age 49, while one may need to observe fertility and mortality of older ages to replicate the study for men, as men have a longer reproductive life span.

(iii) There is often poorer information on parent-child linkages for men than for women, which makes estimates on male fertility, such as men's ages at childbearing more difficult.

3. The Two-Sex Total Fertility Rate

The 2SNRR accounts for mortality of men and women. Assume mortality at childbearing ages is so low that we can ignore it. This allows one to define the two-sex gross reproduction rate or, equivalently, the two-sex total fertility rate 2STFR. It is defined as the geometric mean of the total fertility rates for women and men, or

$$2STFR = \sqrt{(TFRw \cdot TFRm)}. \tag{5}$$

Here, TFRw is the traditional total fertility rate, being the sum of age-specific fertility rates for women ($\sum fw_x$). TFRm is defined likewise. The two-sex total fertility rate is a useful indicator, for two reasons.

(i) It simplifies analyses of the reproductive potential of a population when mortality can be ignored.

(ii) When we compare time trends in 2SNRR and 2STFR, we are able to show the role of mortality decline both for men and for women during the period of analysis.

4. Approximate Expressions

4.1. A First Approximation. In many cases, a good approximation to the NRR as given in expression (1) is $k_g \cdot TFRw \cdot lw_\mu$,

where TFRw is as defined above, k_g is the share of girls among all live births, and lw_μ is the survival probability of women from birth ($x = 0$) to the mean age at childbearing ($x = \mu$) [5]. The product $k_g \cdot$ TFRw is included in this approximate expression because the NRR is restricted to female births. Since 2SNRRw in expression (2) includes fertility rates for both male and female births, a similar approximation to 2SNRRw is TFRw $\cdot lw_\mu$. Note that the survival probability lw_μ is computed based on age-specific mortality rates for women. At the same time, the mean age at childbearing μ is computed based on age-specific fertility rates for women (fw) where children can be of any sex. In obvious notation, we find an approximate expression for 2SNRRm as TFRm. lm_μ, where lm_μ is the survival probability of men from birth to their mean age at childbearing.

With these approximations, we can write for the 2SNRR

$$2\text{SNRR} \approx \sqrt{(\text{TFRw} \cdot \text{TFRm})\left(lw_\mu \cdot lm_\mu\right)}. \qquad (6)$$

In the empirical part of the paper we will analyse how close the approximation in expression (6) is, compared to the exact value of 2SNRR defined in (4).

4.2. A Second Approximation Based on Regression Analysis (We Gratefully Acknowledge Research Assistance by Xin Le for This Analysis). The second approximation is useful when age-specific fertility rates and total fertility rates for men are not available. It is based on regression.

We have analysed the link between the total fertility rates of men (TFRm) and women (TFRw), and the role the sex ratio for the population aged 20–39 (SR20–39) plays.

The effect on fertility of a shortage of women in prime childbearing ages, and thus an excess of men is unclear. Some women, who otherwise would not have married (or, more generally, found a male partner) and not have had a child, will have one simply because there are many men on the marriage/partnership market. On the other hand, with an excess supply of men, many men will remain childless, and thus there are fewer births. Whether the sex ratio of adults boosts or depresses fertility is an empirical matter.

Table 10 of the 2008 issue of the United Nations' Demographic Yearbook gives values for the total fertility rate of men for 65 countries/regions from different parts of the world [6]. The data are from 2006, 2007, or 2008, with a few exceptions (2000: Armenia; 2003: United Kingdom; 2004: Uruguay, Azerbaijan, Albania, San Marino, Fiji). We added TFRw data from Table 11 of the same yearbook, or from earlier yearbooks if necessary.

Figure 1 presents a scatter plot of TFRm against TFRw. The graph reveals that the two indicators show little difference for most of the countries/regions, with a few notable exceptions: TFRw values for Qatar, Bahrain, and Åland Islands are much higher (2.43, 2.30, and 1.87, resp.) than the corresponding values of the TFRm (0.62, 1.13, and 0.67). At the same time, Hong Kong SAR has a TFRw of 1.06 and a TFRm of 1.62. We used other UN sources to check the data of these four countries/regions against errors. No TFR-information was found for Åland Islands. This is an autonomous group of islands at the South-West coast of Finland, with 28,000
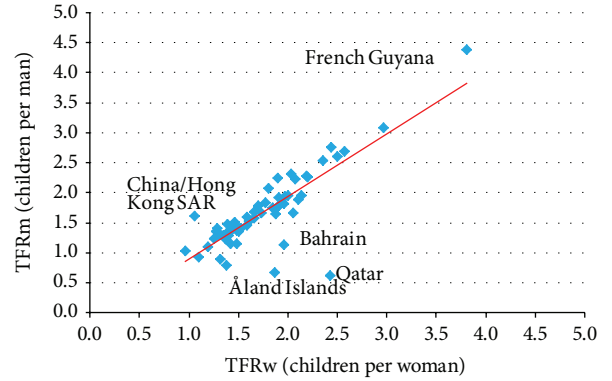

Figure 1: Scatter plot of TFR of women (TFRw) versus TFR of men (TFRm). Countries/regions around 2006–2008; see text for details. Source: UN [6].

Table 1: Estimation results for expression (7).

Parameter	Estimate	Heteroscedasticity robust standard error
Constant	−0.078	0.045
ln(TFRw)	1.094	0.062
ln(SR20-39)	−1.032	0.097

inhabitants in 2010 [19]. Because of its small population size, we omitted Åland Islands from our data set, and, for the same reasons, San Marino as well. The remaining three outlier countries/regions led us to hypothesize that the sex ratio of the adult population might have an impact on the link between the male and female TFR: Qatar and Bahrain have large surpluses of men in their populations, while Hong Kong SAR has relatively many women. By simulating a hypothetical population with a fixed number of births and a fixed age structure of the female part and changing numbers of men in each age group, we found that the link between the sex ratio of the adult population and TFRm was likely nonlinear. We collected data for the sex ratio of the adult population from UN sources. This left us with 63 countries/regions.

We formulated the following regression model:

$$\ln\left(\text{TFRm}_i\right) = a + b \cdot \ln\left(\text{TFRw}_i\right) + c \cdot \ln\left(\text{SR20–39}_i\right) + e_i. \qquad (7)$$

Here $\ln(\text{TFRm}_i)$ is the natural logarithm of the TFRm of country i (and similarly for TFRw_i); $\ln(\text{SR20–39}_i)$ is the natural logarithm of the sex ratio of the population aged 20–39, while e_i is a random error term for which we assume expectation zero and constant variance. It is assumed to be independent across countries. The parameters a, b, and c are to be estimated.

Least squares estimation gave the results in Table 1.

The coefficient estimates have the expected signs. This simple model fits well with the data: it explains 83% of the variation in $\ln(\text{TFRm})$. The estimated coefficients of $\ln(\text{TFRw})$ and $\ln(\text{SR20–39})$ are close to one and minus one, respectively. Given the corresponding standard errors, the confidence intervals for both estimates include the values one and minus one, respectively. We performed an F-test to check

whether the hypothesis $b = 1$, $c = -1$ could be rejected against the alternative $b \neq 1$ and/or $c \neq -1$. It turned out that we could not reject this hypothesis based on the available data (significance level of 5%). Therefore we reestimated the model with these restrictions; that is, we estimated the constant of the model

$$\ln(\text{TFRm}_i) = a + \ln(\text{TFRw}_i) - \ln(\text{SR20--39}_i) + e_i, \quad (8)$$

and we found an estimate equal to -0.030. By taking antilogs, we can use the model to predict the TFRm for a certain country as

$$\text{TFRm} = 0.971 \cdot \frac{\text{TFRw}}{\text{SR20--39}}. \quad (9)$$

Model (9) is our preferred model for the link between total fertility rates of men and women. It has great intuitive value. In a balanced population (SR20--39 $= 1$), the TFR of men is 3% lower than that of women. The reason is that men have a somewhat larger reproductive age span than women. Thus, when we compute the mean numbers of births per man or woman, we divide a certain number of births by slightly more men than women. When SR20--39 > 1, there are more men than women aged 20--39, and the number of men in the denominator becomes even larger, reducing the TFRm even further.

Assuming model (7) is correct, we can find a simple expression for the two-sex total fertility rate (cf. expression (5)) as follows. Take the natural logarithm of 2STFR in expression (5) and substitute ln(TFRm) by expression (7) to find

$$\ln(2\text{STFR}_i) = \frac{1}{2}a + \frac{1}{2}(b+1) \cdot \ln(\text{TFRw}_i)$$
$$+ \frac{1}{2}c \cdot \ln(\text{SR20--39}_i) + d_i, \quad (10)$$

$$\text{or} \quad \ln(2\text{STFR}_i) = p + q \cdot \ln(\text{TFRw}_i)$$
$$+ r \cdot \ln(\text{SR20--39}_i) + d_i,$$

where $p = (1/2)a$, $q = (1/2)(b+1)$, $r = (1/2)c$, and d_i is an error term with the usual properties. As we could expect, OLS-estimates of p, q, and r turned out to be equal to -0.039, 1.0471, and -0.516, respectively. A hypothesis that q equals 1 and r equals $-1/2$ cannot be rejected. We reestimated the model

$$\ln(2\text{STFR}_i) = p + \ln(\text{TFRw}_i) - \frac{1}{2}\ln(\text{SR20--39}_i) + d_i, \quad (11)$$

and we found an estimate of p equal to -0.015. Since $\exp(-0.015) = 0.985$, an approximate expression for model (5) becomes

$$2\text{STFR} = 0.985 \cdot \frac{\text{TFRw}}{\sqrt{\text{SR20--39}}}. \quad (12)$$

The simple form of this expression is one justification for taking geometric averages, rather than harmonic or other averages, when computing two-sex measures. For populations with a balanced sex structure we find that the 2STFR is about 1.5 percent lower than the TFRw. In case there are 10 percent more men than women aged 20--39, the model predicts a 2STFR which is six percent lower than the TFRw.

A numerical check of expression (12) will be given below.

How much information is lost when we ignore any difference between the female net reproduction rate 2SNRRw and the corresponding rate 2SNRRm for males? The empirical analysis in this section shows that very little is lost when the age structures of men and women in childbearing ages are balanced. In case we would assume that 2SNRRw = 2SNRRm, our two-sex reproduction measure 2SNRR would become equal to either of these (see expression (5)). Or, to put it differently, when we are willing to assume that male and female mortality and fertility patterns are similar, the approximate relationship (6) leads to

$$2\text{SNRR} \approx \frac{\text{NRR}}{k_g}. \quad (13)$$

To be more precise, once we assume that TFRw·$\text{lw}_\mu \approx$ TFRm·lm_μ, expression (13) follows from expression (6), together with the traditional approximation NRR $\approx k_g \cdot$ TFRw $\cdot$ lw_μ.

The error resulting from the approximation in expression (13) is the extent to which male age-specific mortality and fertility patterns differ from those of women. Until mid-reproductive ages, men tend to have higher mortality than women do. However, the differences are not large and globally from 2005--2010, UN [20] estimates that 89.4% of women survive to age 30, while 88.7% of men survive to age 30. In effect, only 0.7% points more women than men survive to this age.

Female fecundity is largely restricted to ages 15--49, but males can reproduce until higher ages [6]. For Canada, 99.8% of the children have fathers below age 50 in 2007. In Norway 98.5 percent of newly born babies in 2008 have fathers below 50. For the majority of the countries where male childbearing can be observed between 2004 and 2008, the average age at childbearing ranges from age 30 to 35. Male fertility also falls with age due to age similarity between women and men in formal unions and also due to age-related declines in adult male copulation frequency, semen volume, and sperm mobility [21].

All this assumes that the sex ratio SR20--39 is close to one. If that is not the case, expression (13) cannot be used to approximate the 2SNRR. The effect on two-sex fertility of a change in the sex ratio of adults in childbearing ages can be evaluated by computing elasticities. Doing so we note that a one percent increase in the SR20--39 causes the 2STFR to drop by one-half of a percent. This shows that the negative effect of more men on the male TFR (because more men will remain childless) is stronger than its positive effect on the female TFR (more women will find a partner and fewer will remain childless).

5. Empirical Illustrations

5.1. Bejsce Parish 1800–1967

5.1.1. General Description of the Database. In order to illustrate the concepts introduced in the previous section we

use a reconstitution of parish registers from Bejsce, a region located in south-central Poland. We selected this data source because the records of births contain information about both the mother and the father for a period of more than one and a half century. The data were collected and verified by a research team led by Edmund Piasecki from the Institute of Anthropology at the Polish Academy of Science around 1975. Piasecki's book of 1990 [22] contains detailed information on data collection procedures and descriptive results.

The reconstitution of parish data from Bejsce offers a unique possibility to observe a demographic development of the population from the mid-18th century up to the end of the first demographic transition. The parish registers from Bejsce contain information about births and deaths from the beginning of the 17th century. However, the coverage of the original data file is limited to the period between 1765 and 1967. This is mainly related to incompleteness of the registers before 1765 and thus the lack of a proper representation of the population in the parish books. Although, as analysed by Piasecki, the registration of births and deaths has been steadily improving over the whole 18th century, the data before the year 1765 should not be used because there were relatively few observations in that period, possibly caused by incomplete registration [22].

For the purposes of the present study, the data have been restricted to the period between 1800 and 1967. The main argument for this choice is the fact that the time series before the year 1800 might contain errors due to incomplete registration. On the other hand, by truncating the period under observation we do not lose any important information since the selected period seems to be the most important one with respect to the demographic transition in the Bejsce parish. Detailed discussions of the problems and limitations related to the use of register data (mostly due to its selectivity) may be found in Kasakoff and Adams [23], Saito [24], Voland [25], and Desjardins [26]. Fortunately, the issue of selectivity of registration is not of major concern in the case of Bejsce registers. The parish has not been exposed to any major depressions due to plagues, wars, or extensive migration. According to estimates by Piasecki [22], out- and inmigration in the parish through the whole period covered by reconstitution has not exceeded around 5% of the total number of cases in the database. Few scholars have used the data from Bejsce since the publication of the monograph by Piasecki [22]. The attempt to make the database known to a wider scientific community resulted in a series of papers, which focused on the various aspects of the interplay between mortality and fertility [27–30].

5.1.2. Findings. The long time period covered and the accuracy of reconstitution with respect to births and deaths allow us to calculate the two-sex measures introduced in the previous sections. Since the records of births in the parish contained information about both the mother and the father, it was possible to calculate demographic indicators for both sexes separately. In order to calculate fertility rates correctly we have retained in the database only those cases for which it was possible to identify both the mother and the father. We
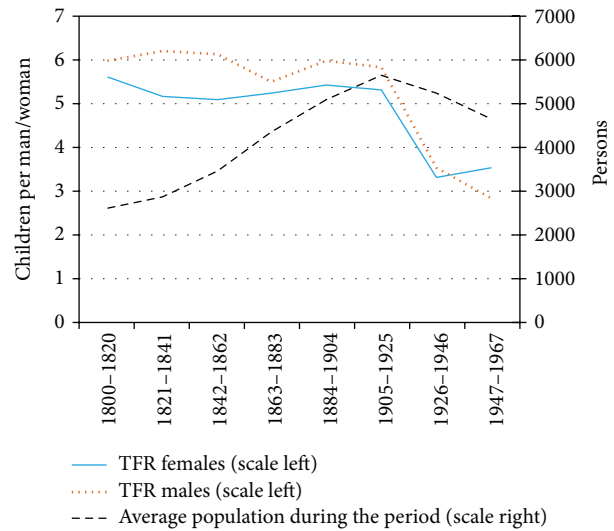

Figure 2: Total fertility rate for males and females and population size in Bejsce parish, 1800–1967.

have omitted approximately 12 percent of the total number of births due to lack of information about the mother or the father.

Within the period under analysis the population of Bejsce underwent a demographic transition resulting in a drop of the total fertility rate for females from 5.5 children at the beginning of the observation to a level of around 3 children in 1967 (Figure 2). A similar change in the total fertility rate can be observed for males although males had slightly higher fertility (except for the period between 1947 and 1967).

In the period under analysis the population of Bejsce parish has nearly doubled its size starting from 2400 inhabitants in 1800 to reach around 4600 individuals in 1967. However, the population size was the largest in the year 1912 when it reached 5600 inhabitants. This peak population size coincides with significant changes in the total fertility rate over time. Total fertility rate has been more or less constant up to 1905 (around a level of five children per woman on average). Declining fertility rates over the period of demographic transition lowered the TFR to a level around three.

These changes in fertility rates were accompanied by improved mortality conditions as measured by the life expectancy at birth (Figure 3). For both males and females, the life expectancy (e_0) fluctuated around 30 years up to the year 1905. It grew steadily to a level of 48 years for males and 51 years for females around 1967. The life expectancy in Bejsce was considerably lower than the national figure, because infant mortality was relatively high in this period.

When we compare the trends in 2SNRR and 2STFR (Figure 4), we see that mortality has played a major role over the period of the demographic transition in Bejsce parish. The main message of Figure 4 is twofold. First, net fertility has fluctuated around 2–2.5 children per adult—slightly above replacement level. Second, until the early 20th century, couples got about five children. This gross fertility
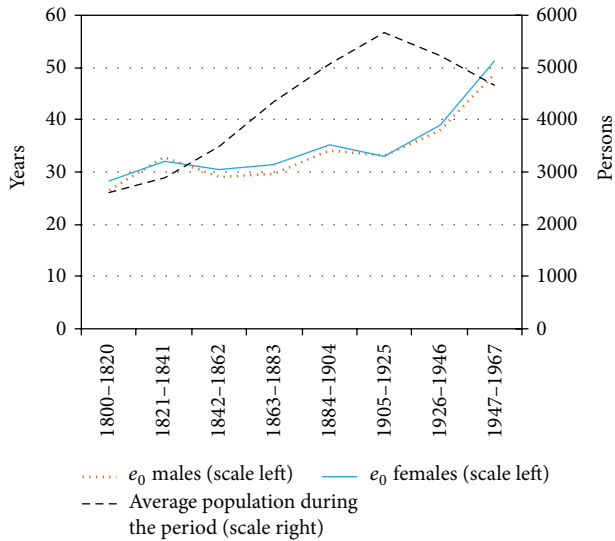
FIGURE 3: Life expectancy at birth (e_0) for males and females and population size in Bejsce parish, 1800–1967.
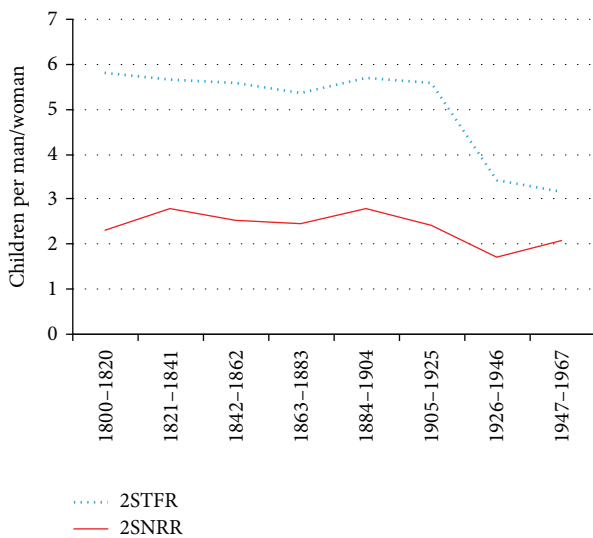

FIGURE 4: Trends in 2SNRR and 2STFR in Bejsce parish, 1800–1967.

level is much higher than net fertility as mortality was high, particularly in the early part of the period in question. As mortality declined in the 20th century, the gap between 2SNRR and 2STFR became much smaller. Although the gross number of children per adult declined from 1926–1946 to 1947–1967, net fertility increased slightly.

Finally, Figure 5 shows that expressions (6) and (12) approximate the 2SNRR and the 2STFR very well.

5.2. China 1990–2050.

In Section 4, we found that an unbalanced sex ratio of adults in prime reproductive ages goes together with a TFR of men, which is different from that of women. In the case of China, it leads to a lower TFR value for men, compared to that for women. Therefore, the fertility level for the whole population, that is, both men and women,
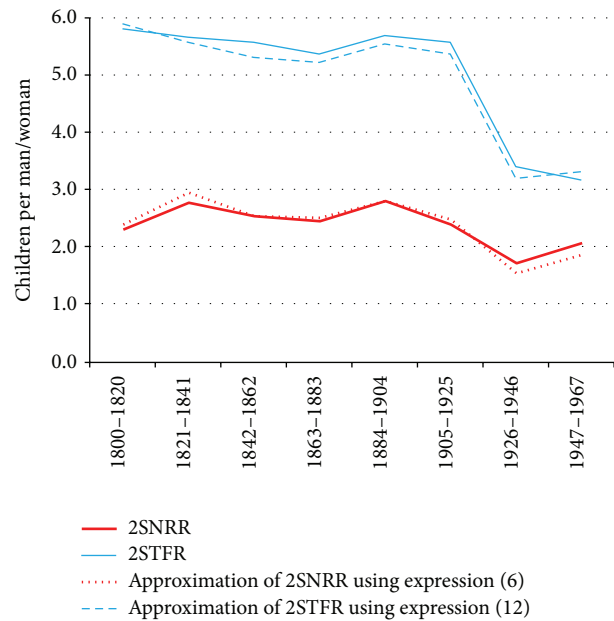

FIGURE 5: Approximations of 2SNRR and 2STFR using expressions (6) and (12).

is lower than that for the female part alone. This depresses the long-term growth rate, compared to the value obtained from female fertility levels alone.

In its population projections for China, the UN assumes that the TFR trend for women will reverse and increase slowly from its current level of 1.6 children per woman to 1.8 children per woman by 2050 [7]. At the same time, the SR20–39 will grow from 1.06 in the period 2005–2010 to 1.20 by 2030, as an immediate result of an increasing value of the sex ratio at birth in the past four decades. The sex ratio at birth, that is, the number of boys born as a ratio of the number of newborn girls, rose from a natural level of 104–106 boys per 100 girls in the mid-1970s to 120 boys per 100 girls in 2005 [31]. The 2010 Population Census of People's Republic of China gives 119 boys aged zero per 100 girls of that age (see http://www.stats.gov.cn/tjsj/pcsj/rkpc/6rp/indexch.htm).

We have assumed that the relationship between the TFR of men and women described above will also hold for China during the first half of this century and computed 2STFR estimates. Figure 6 plots our results.

Fertility is lower by 0.1-0.2 children when we measure it by the 2STFR, compared with the traditional TFRw. The gap between TFRw and TFRm rises from 0.2 in 1990–1995 to 0.4 by 2050.

Note that our two-sex fertility estimates in Figure 6 may be too high. Our estimates are directly linked to TFRw estimates published by the UN, which have been criticized. For instance, Retherford et al. [32] estimate that the true level of the TFRw in 2000 was between 1.5 and 1.6 children per woman, considerably lower than the UN estimate of 1.75. More recently, Zhao and Chen [33] analysed results of the Population Census of 2010. Compared with the UN estimates, they lowered the TFRw from 1.80 to 1.60 for 1995–2000, from

1.70 to 1.45 for 2000–2005, and from 1.64 to 1.45 for 2005–2010. Taking a TFRw of 1.45 children per woman for 2005–2010, our model predicts a TFRm equal to 1.33 children per man and a 2STFR of 1.39 children per adult or more than one-tenth of a child lower than the results in Figure 6. We also analysed the effects of a permanently high value of SR20–39 at 1.2 between 2035 and 2050 (instead of the decrease to 1.17 in Figure 6, caused by the rapid fall in the SRB assumed by the UN) on the TFRm and the 2STFR. These effects were only marginal.

As can be seen in Figure 6, the UN [7] predicts a TFRw in 2030–35 of 1.63 children per woman. If the average woman has her children around age 30, say, and this fertility regime would continue for a long period, the female part of the population would ultimately decline by $100 * [1 - (1.63/2)] = 29\%$ every 30 years. Here we ignore migration and mortality of persons in childbearing ages. Since 1990, the emigration rate has been around one to three emigrants per year per 10000 inhabitants (UN 2011). A 2009 life table of China published by the WHO gives a survival probability from birth to age 30 of almost 97 percent (http://apps.who.int/ghodata/?vid=60340). This implies an annual rate of decline of 0.7 percent. Our results suggest a two-sex fertility level of 1.47 children (the 2STFR value in 2030–35), accounting for an unbalanced SRB20–39 and a population decline that is much stronger: 1.0 percent per year. The UN predicts a rate of decline of the population of China equal to 0.2 percent in 2030–35 and 0.6 percent by 2050 ([7], Medium Variant). It turns out that long run annual population decline in China implied by the two-sex fertility measure for the period 2010–2050 is *up to a factor two* stronger than that based on female fertility only.

6. Conclusions and Discussion

The net reproduction ratio in regions of the world where the sex ratio of adults is unbalanced will create an incorrect measure of population dynamics and growth. We propose a two-sex net reproduction rate (2SNRR) and a two-sex total fertility rate (2STFR)—two demographic indicators that reflect childbearing behaviour of both women and men and consider both female and male offspring. In addition to considering the fertility of both sexes rather than only focusing on women, these measures have great intuitive value, since they tell us to what extent adults are replaced by children. While the traditional net reproduction rate (NRR) describes general replacement trends in the female part of the population only, the 2SNRR is an indicator of a population's growth potential, irrespective of sex.

Sex ratios vary over time and region (for instance in India, China, South Africa, and several Arab Gulf States). Causes include sex-selective migration [34, 35] which affect sex ratios in both sending and receiving countries [20], wars [36], or the consequences of natural catastrophes [37]. Birth control and abortion have also affected sex ratios for many [38] and the impact is particularly strong in Asia in recent years [20, 39]. The 2SNRR and the 2STFR could give a new understanding of cross-country variation in fertility and a better understanding of time trends in fertility when sex
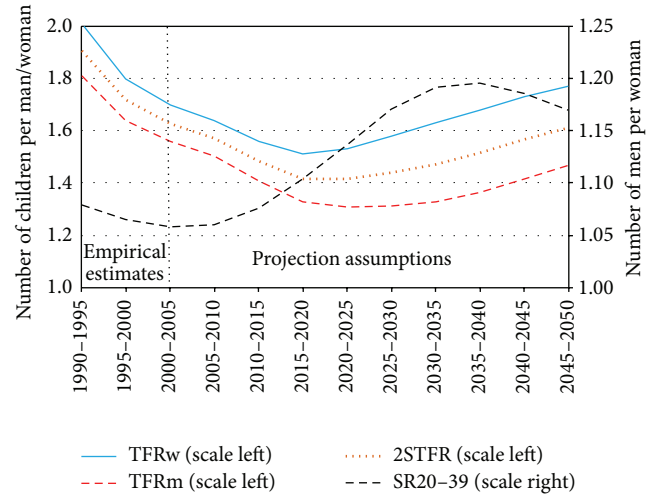

FIGURE 6: China 1990–2050: the total fertility rate of women (TFRw) and of men (TFRm), the two-sex total fertility rate (2STFR), and the sex ratio of the population aged 20–39 (SR20–39). Source: UN [7] for TFRw and SR20–39; own computations for TFRm and 2STFR.

ratios are skewed. When numbers of men and women in childbearing ages are about equal, the traditional female TFR will give a correct impression of overall period fertility.

Note that in many countries today, mortality for men and women less than fifty years of age is low. This means that the reproduction rate (in whatever form, male, female, or joint) offers little additional information to the analogous total fertility rate. However, for a study of the demographic transition, as in Bejsce parish, the reproduction rate has considerable interest.

The two-sex fertility measures proposed here account for unbalanced sex ratios of the population of childbearing ages. As such, they are an improvement compared to the female-dominant (the traditional TFR, here labelled as TFRw) or female-only (the traditional NRR) measures of fertility level and of reproduction. However, these two-sex measures also have a number of weaknesses, as pointed out in Section 3. One further problem to be mentioned is related to our claim that the two-sex measures can give us an indication of long run population growth: this is only correct when the population is stable. The latter condition is rarely fulfilled-one would need a population projection to obtain a more precise impression of future population growth. Such a projection may account for irregular age distributions, varying levels of fertility and mortality, and it would allow one to include migration as a component of population change. However, the two-sex fertility measures share this condition of stability with the traditional female-dominant and female-only measures of fertility.

The standard way of computing a population projection is by means of the so-called cohort-component method (e.g., [5, page 119]). This method is female-dominant in its usual form: births are projected based on assumed fertility rates combined with the size of only the female part of the population of childbearing years. When the male part of that population is much larger or much smaller than the female

part, this is an unrealistic approach. To solve that problem one has to take account of the unbalanced sex ratio of the population of childbearing ages, and one is confronted with the two-sex problem. To compute two-sex fertility measures is a useful first step, pending the discovery of a generally accepted solution of the two-sex problem.
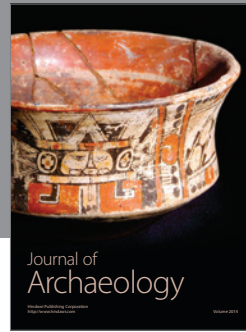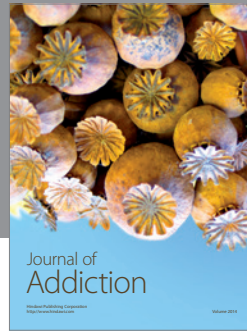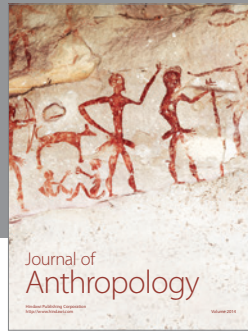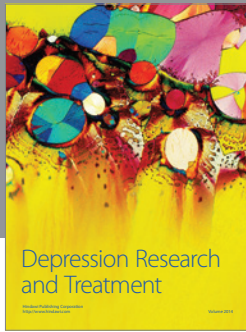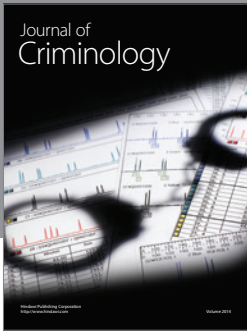
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] R. Kuczynski, *Fertility and Reproduction: Methods of Measuring the Balance of Births and Deaths*, Falcon Press, New York, NY, USA, 1932.

[2] F. M. M. Lewes, "A note on the origin of the net reproduction ratio," *Population Studies*, vol. 38, no. 2, pp. 321–324, 1984.

[3] H. De Gans, *Population Forecasting 1895–1945: The Transition To Modernity*, Kluwer Academic, Dodrecht, The Netherlands, 1999, European Studies of Population 5.

[4] C. Newell, *Methods and Models in Demography*, Belhaven Press, London, UK, 1988.

[5] S. H. Preston, P. Heuveline, and M. Guillot, *Demography: Measuring and Modeling Population Processes*, Blackwell Publishers, Oxford, UK, 2001.

[6] UN, *Demographic Yearbook 2008*, United Nations, New York, NY, USA, 60th edition, 2010, http://unstats.un.org/unsd/demographic/products/dyb/dybsets/2008%20DYB.pdf.

[7] UN, *World Population Prospects. the 2010 Revision. United Nations Population Division*, United Nations, New York, NY, USA, 2011.

[8] R. J. Myers, "The validity and significance of male net reproduction rates," *Journal of the American Statistical Association*, vol. 36, no. 214, pp. 275–282, 1941.

[9] P. H. Karmel, "The relations between male and female reproduction rates," *Population Studies*, vol. 1, no. 3, pp. 249–274, 1947.

[10] P. H. Karmel, "A rejoinder to Mr Hajnal's comments," *Population Studies*, vol. 2, no. 3, pp. 361–372, 1948.

[11] A. Pollard, "The measurement of reproductivity," *Journal of the Institute of Actuaries*, vol. 74, pp. 289–337, 1948.

[12] P. Vincent, "De la mesure du taux intrinsèque d'accroissement naturel dans les populations monogames," *Population*, vol. 4, pp. 699–712, 1946.

[13] J. H. Pollard, "Modelling the interaction between the sexes," *Mathematical and Computer Modelling*, vol. 26, no. 6, pp. 11–24, 1997.

[14] J. Alho and B. Spencer, *Statistical Demography and Forecasting*, Springer, New York, NY, USA, 2005.

[15] L. Zhang, *Male Fertility Patterns and Determinants*, vol. 27 of *The Springer Series on Demographic Methods and Population Analysis*, Springer, Dordrecht, The Netherlands, 2011.

[16] A. Chahnazarian, "Determinants of the sex ratio at birth: review of recent literature," *Social biology*, vol. 35, no. 3-4, pp. 214–235, 1988.

[17] N. Keyfitz, "The mathematics of sex and marriage," in *Proceedings of the 6th Berkeley symposium on mathematical statistics and probability*, vol. 4, pp. 89–108, 1971.

[18] R. Schoen, "The harmonic mean as the basis of a realistic two-sex marriage model," *Demography*, vol. 18, no. 2, pp. 201–216, 1981.

[19] Statistics and Research Åland 2012, Invånarantalet i byar och stadsdelar 2000, 2010 och 2011, (Population size in cities and regions 2000, 2010 and 2011). Statistics and Research Åland—Population, http://www.asub.ax.

[20] UN, *World Population Prospects. the 2008 Revision. United Nations Population Division*, United Nations, New York, NY, USA, 2009.

[21] B. Kühnert and E. Nieschlag, "Reproductive functions of the ageing male," *Human Reproduction Update*, vol. 10, no. 4, pp. 327–339, 2004.

[22] E. Piasecki, Populations of the Bejsce parish (Kielce voivodeship, Poland) in the Light of parish register of the 18th–20th centuries. A demographic study. Warsaw: PWN, 1990.

[23] A. B. Kasakoff and J. W. Adams, "The effect of migration on ages at vital events: a critique of family reconstitution in historical demography," *European Journal of Population*, vol. 11, no. 3, pp. 199–242, 1995.

[24] O. Saito, "Historical demography: achievements and prospects," *Population Studies*, vol. 50, no. 3, pp. 537–553, 1996.

[25] E. Voland, "Contributions of family reconstitution studies to evolutionary reproductive ecology," *Evolutionary Anthropology*, vol. 9, no. 3, pp. 134–146, 2000.

[26] B. Desjardins, "Bias in age at marriage in family reconstitutions: evidence from French-Canadian data," *Population Studies*, vol. 49, no. 1, pp. 165–169, 1995.

[27] K. Tymicki, "Kin influence on female reproductive behavior: the evidence from reconstitution of the Bejsce parish registers, 18th–20th centuries, Poland," *The American Journal of Human Biology*, vol. 16, no. 5, pp. 508–522, 2004.

[28] K. Tymicki, "The interplay between infant mortality and subsequent reproductive behavior. Evidence for the replacement effect from historical population of Bejsce parish, 18th–20th centuries, Poland," *Historical Social Research*, vol. 30, no. 5, pp. 240–264, 2005.

[29] K. Tymicki, "When do kinsmen really help? Examination of the cohort and parity specific kin effects on fertility behavior. Case of the Bejsce parish register reconstitution study, 18–20 centuries, Poland," in *Kinship and Demographic Behaviour in the Past*, T. Bengtsson, Ed., pp. 135–154, Springer, Berlin, Germany, 2008.

[30] K. Tymicki, "Correlates of infant and childhood mortality: a theoretical overview and new evidence from the analysis of longitudinal data of the Bejsce (Poland) parish register reconstitution study of the 18th-20th centuries," *Demographic Research*, vol. 20, no. 23, pp. 559–594, 2009.

[31] D. Bhattacharjya, A. Sudarshan, S. Tuljapurkar, R. Shachter, and M. Feldman, "How can economic schemes curtail the increasing sex ratio at birth in China?" *Demographic Research*, vol. 19, article 54, pp. 1831–1850, 2008.

[32] R. D. Retherford, M. K. Choe, J. Chen, L. Xiru, and C. Hongyan, "How far has fertility in China really declined?" *Population and Development Review*, vol. 31, no. 1, pp. 57–84, 2005.

[33] Z. Zhao and W. Chen, "China's far below-replacement fertility and its long-term impact: comments on the preliminary results of the 2010 census," *Demographic Research*, vol. 25, pp. 819–836, 2011.

[34] K. M. Donato, J. T. Alexander, D. R. Gabaccia, and J. Leinonen, "Variations in the gender composition of immigrant populations: How they matter," *International Migration Review*, vol. 45, no. 3, pp. 495–526, 2011.

[35] G. Stecklov, C. Carletto, C. Azzarri, and B. Davis, "Gender and migration from Albania," *Demography*, vol. 47, no. 4, pp. 935–961, 2010.

[36] T. Hesketh and Z. W. Xing, "Abnormal sex ratios in human populations: causes and consequences," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 103, no. 36, pp. 13271–13275, 2006.

[37] M. Fukuda, K. Fukuda, T. Shimizu, and H. Møller, "Decline in sex ratio at birth after Kobe earthquake," *Human Reproduction*, vol. 13, no. 8, pp. 2321–2322, 1998.

[38] S. Winston, "Birth control and the sex-ratio at birth," *The American Journal of Sociology*, vol. 38, no. 2, pp. 225–231, 1932.

[39] C. Park and N. Cho, "Consequences of son preference in a low-fertility society: imbalance of the sex ratio at birth in Korea," *Population & Development Review*, vol. 21, no. 1, pp. 59–84, 1995.