Stochastic modelling and pricing of energy related markets

With analysis of the weather and shipping markets

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To my family
Intan Mas Ayu, Adam Irfan and Humaira

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Imran Taib
Oslo, May 21, 2013

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Chapter 1

Introduction

The deregulation of energy industry since the beginning of 1990s has resulted in an enormous impact to the financial markets worldwide. Norway, New Zealand and United Kingdom are among the earliest countries having the liberalised electricity sector. A 24 out of 50 states in United States have deregulated electricity and 19 states with deregulated gas*. The deregulated market opens for competitiveness, resulting of revenue uncertainty to the energy producers, and on the other hand consumers are affected with volatile energy prices. As a consequence, the market for derivatives products emerges which provides possible hedging tools for both parties. The aim of this thesis is twofold. Firstly, we investigate some relevant stochastic models and secondly, we focus on pricing derivatives for commodities traded in energy related markets.

We will concentrate on weather and shipping, both are energy related markets. They are different in nature but share some similarities from a modelling point of view. Weather is obviously nonstorable and similarly, it is impossible to store shipping commodity because its underlying asset is a service. Furthermore, there is a close relation between these two commodities and electricity. Electricity and some other industries like agriculture and tourism are weather sensitive. For instance, electricity is needed in summer time to operate air-conditioning if the temperature is too hot and normally there is an extensive use of electricity in winter for heating if the temperature is too cold. The agricultural sector is totally dependent on weather. Bad weather will probably damage the crops. Ski resorts will lose money if weather conditions cannot attract the skiers. We refer to Benth and Šaltytė Benth [17] for the discussion on the impact of weather to industry. Shipping on the other hand can be linked to electricity in one way. The coal used to operate power plants for example are essentially transported from a country or continent using freight service, and the cost of hiring vessels becomes one of the important factors in electricity generation. All of these linkages pointing towards the importance of weather and shipping markets which received great attention nowadays.

Weather contracts are settled against an objectively measurable index such as cooling degree-day (CDD), heating degree-day (HDD) or cumulative average temperature (CAT) for the products traded at Chicago Mercantile Exchange (CME). The idea of binding the price to a certain index can be alternatively applied to insurance, where the way claims are made is not justified by real losses anymore but based on the index. Weather index insurance has gradually become an interesting hedging tool and designed for households in developing countries (see Barnett et

*Refer to www.quantumgas.com for a complete list of energy deregulated states in US.

al. [10, 11] and Skees [77]). In this thesis, we will concentrate on weather or more precisely temperature index insurance. For example, a farmer wants to insure the crops against unusual hot temperature in harvest season, say, can buy such a temperature index insurance contract. This contract pays the farmer some amount of money according to, for example the CDD-type index if temperature moves beyond a certain predefined threshold which will probably harm the crops. The advantages of using index based insurance are to avoid the difficulty of assessing the actual damages and preventing from moral hazard problem.

Since temperature is not tradedable on spot, the issue on modelling the futures arises. This thesis study the continuous time autoregressive moving average (CARMA) model for temperature futures and we base our study on the general class of Lévy-driven CARMA processes with stochastic volatility (see Brockwell [26]). Such CARMA models have been successfully applied in modelling electricity (see García et al. [40] and Benth et al. [15]), temperature (see Benth, Šaltytė Benth and Koekebakker [22]) and interest rate (see Andresen et al. [6]). As CARMA processes are stationary, the mean reversion plays a major role to measure how fast the processes are reverting. We investigate the speed of mean reversion through the *half life* concept which was introduced by Clewlow and Strickland [34]. The half life is defined as the time (on average) it takes for a process to revert back to half of its distance away from the average level. Our findings can be regarded as an extended version of Clewlow and Strickland where the half life is not a deterministic number anymore, but is dependent on the state of the process and also the stochastic volatility (see Benth and Taib [24]). We study the implication of half life to temperature futures, particularly for contracts written on the CAT index.

According to Ådland [2], the mean reversion is also one of the stylized facts for freight rate. This could be impetus to model the freight rate using the continuous time autoregressive (CAR) process, a subclass of CARMA. However, the best procedure to determine a well-structured stochastic model is to study the weaknesses of the traditional model used for financial asset dynamics which is based on stochastic process driving by Brownian motion; the so-called geometric Brownian motion (GBM). From empirical investigations (see Benth, Koekebakker and Taib [16]) of spot freight rates, it turns out that GBM is not appropriate. The GBM does not only fail to capture the peaky behaviour of the logreturns distribution, it even fails to model the heavy tails. A more reliable model can be obtained by relaxing the Brownian motion assumption, and allow for more general Lévy processes. Thus, we use an exponential Lévy process to model the price dynamics. We also consider a model with stochastic volatility since the shipping freight rates are observed to exhibit stochastic volatility.

Stochastic volatility is a common property in modelling the energy markets. We utilize the stochastic volatility model of Barndorff-Nielsen and Shephard [8] (BNS for short) in this thesis. The BNS stochastic volatility process is defined as the sum of Ornstein-Uhlenbeck process driven by a subordinator, the Lévy process with only positive increments. Even though the structure is simple, the BNS model provide a very flexible framework to deal with leptokurtic distributional and dependency structure properties (see Benth [13]). In addition, it gives room for analytic pricing of forwards.

The nonstorability property of weather and shipping markets eventually makes the cost of carry relationship between spot and forward/futures prices not applicable. From the arbitrage pricing theory in mathematical finance, one can have the price of the forward using spot-forward relationship framework as long as the forward prices are martingales under equivalent martingale measure. This thesis follows the aforementioned framework in pricing forward/futures for temperature and freight markets. Our starting point is the CARMA model introduced for temperature dynamics and various stochastic models for spot freight rates where we will infer the forward/futures thereof. Another popular way to model the forward is using Heath-Jarrow-Morton [46] (HJM) approach which is not to be considered in this thesis. We will mention some papers contributed to the literature in modelling the temperature futures and freight forwards in the relevant chapter.

1.1 The financial market for energy related commodity

Great attention in modelling the energy markets goes to electricity. Many theoretical and empirical studies have been done to clearly understand the evolution of electricity prices. For example, Lucia and Schwartz [63] have investigated regular patterns in the price dynamics. The analysis and modelling of electricity prices were studied by Cartea and Figueroa [31]. Further, Benth, Kallsen and Meyer-Brandis [14] and also a paper by Kiesel, Schindlmayr and Börger [60] have focused on the pricing of electricity futures/forward. An article by Hambly, Howison and Kluge [44] has contributed to the study related to options pricing of electricity market. We refer to Benth, Šaltytė Benth and Koekebakker [22] for a discussion on specific issues and challenging modelling problems in electricity markets.

The articles contributed to the markets related to electricity like natural gas and temperature are almost equally increasing, but the freight markets still get little attention. The nonstorability property makes these markets similar to electricity. Natural gas is more special since it can be stored but quite costly and limited. In this section, we will describe the financial market for the last two *exotic* markets: temperature and freight which shall be studied throughout the thesis. Our aim is to give a better understanding on the features of these energy related markets where we may include some parts on discussing the problems with the commodities from a modelling point of view.

1.1.1 The temperature market

The close relation between temperature and electricity is undoubtedly. For electricity producers in Norway, say, very low electricity price in summer period will affect their earnings and on the contrary, unexpected high electricity price in the winter is costly to the consumers. Both are concerned with uncertain electricity prices in the future and may be seeking for any available financial contract to hedge against temperature risk. The temperature derivatives market is a platform that provides for some possible solutions. It started informally in 1996 and began to trade over-the-counter (OTC) in 1997. Nowadays, the only market offering the temperature derivatives contracts is Chicago Mercantile Exchange (see Benth and Šaltytė Benth [17]) which will be the main topic of this Subsection.

As noted, the temperature futures contracts traded at CME are basically based on three different indices: the cooling degree-day, heating degree-day and cumulative average temperature. The two former indices are calculated against some threshold $\widetilde{T} = 65\,^{\circ}\mathrm{F} \approx 18\,^{\circ}\mathrm{C}$ (we use different notation in Chapter 2). The CDD is defined as the difference between the average temperature on particular day t and the threshold as long as this is positive. Otherwise, the index gives value zero. Mathematically, it can be expressed as

$$\text{CDD}(t) = \max(T(t) - \widetilde{T}, 0). \tag{1.1.1}$$

The threshold temperature is the starting point to compute the index. When the temperature goes above $\widetilde{T}$, then the CDD starts to calculate the amount of degrees exceeding that value. This is simply a measurement of electricity required for cooling.

The HDD index is almost similar to CDD, but it computes the amount of degrees below $\widetilde{T}$, being a measure of electricity demand due to for heating. If there is a day where temperature drop below $18\,^{\circ}\mathrm{C}$, meaning that the heating system should be switched on (which require the use of electricity), then HDD gives a positive value. The mathematical expression for the HDD index is

$$\text{HDD}(t) = \max(\widetilde{T} - T(t), 0). \tag{1.1.2}$$

The CAT index accumulates the average temperature for a certain predefined period $[\tau_1, \tau_2]$, given as

$$\text{CAT}(t) = \sum_{t=\tau_1}^{\tau_2} T(t)\,, \tag{1.1.3}$$

where the futures contracts based on this index are considered in monthly or seasonal basis. The more important thing here is how the index-based futures contract is converted into money. For US cities, the CDD futures are settled for \$20 per unit; or simply $\sum_{t=\tau_1}^{\tau_2} \text{CDD}(t) \times \$20$, while the HDD and CAT indices for European countries are settled at £20 for one unit (refer to Benth, Šaltytė Benth and Koekebakker [22]).

Temperature index insurance

There is a close resemblance between weather derivatives and weather index insurance since both are tied to a certain index measured at a specific weather station over a defined period of time. For weather insurance, the claims are made according to an objective measurable index of specific weather variable like rainfall or temperature. The latter variable will be discussed in this thesis, and one can refer to Barnett [11], Turvey [82] or Skees [77] for the use of other weather variables in designing the weather index insurance.

Weather index insurance is still a young field, but is slowly growing. It has been discussed in academic papers since early 1999 as the potential solution for agricultural economies in developing countries. The pilot study was conducted by World Bank's Commodity Risk Management Group (CRMG) between 2003 to 2006 and the first transaction of weather index insurance was in India in June 2003. There have been several other pilot projects afterwards, for instance the completed pilot projects in Ukraine, Ethiopia and Malawi (see Shynkarenko [73]). Until October 2012, there are 3 million farms in India† and nearly 100,000 farmers in nine countries in Sub-Saharan Africa as well as in Sri Lanka and South Asia (see Global Index Insurance Facility [43]) covered by weather index insurance. These indicate the increasing demand for weather-based insurance worldwide.

We will now illustrate how weather (precisely temperature) index insurance works. Con-

†The report was retrieved from http://www.cgap.org/blog/lessons-india-weather-insurance-small-farmers.

sider a farmer that lives in the area where drought is one of the serious threats in growing crops. This may be a result of extremely high temperatures. To hedge against the temperature risk, the farmer engages in a temperature-based insurance by signing a contract and paying an amount of premium, P. The payoff of the temperature index insurance is calculated using, for instance the CDD-type index, mathematically given by

$$X(\tau_1, \tau_2) = k \times \sum_{s=\tau_1}^{\tau_2} \max(T(s) - \widetilde{T}, 0)\,, \tag{1.1.4}$$

for a certain coverage period $[\tau_1, \tau_2]$. The daily average temperature at a specific location are recorded along the period, and the threshold is assumed to be equal $\widetilde{T} = 45°\mathrm{C}$ (just for illustration). If for any particular day where the observed temperature is more than $45°\mathrm{C}$, then the deviation is computed. At τ_2, all the deviations are summed up and then converted into money by a factor k. Setting $k = \$20$ and assume $\sum_{s=\tau_1}^{\tau_2} \mathrm{CDD}(s) = 15$, then the indemnity being paid to the farmer is $X(\tau_1, \tau_2) = \$300$. Equation (1.1.4) represents the loss for the temperature index insurance. In Chapter 2, we will focus on the issue of how much premium should be paid by the farmer. This is important since they are expected to be poor.

1.1.2 The shipping freight markets

Shipping industry contributes to approximately more than 75% of the volume of the world trade in commodities and manufactured products (see Alizadeh and Nomikos [5]). This indicates that the demand for the shipping transportation is considerably high with a huge number of shipowners, operators and charterers involved in the industry. The price of shipping services are very volatile. Remarkably, the annualised volatility of shipping freight rates varies between 59% to 79% in the years 2008 to 2011[‡]. This is relatively high compared to the other commodities such as crude oil and agricultural with the average of 50%. The implication of high volatility of freight rates to the shipowner is the risk of loosing in terms of revenues and it contributes to some additional costs to the charterer. Thus, understanding the features of the market is important to the market's participants.

There are five major categories of seaborne trade: namely, oil tanker, gas tanker, container, dry bulk and other. About 38% of seaborne trading are contributed from dry-bulk commodities which makes the dry bulk market a major segment of the entire shipping category (see Prokopczuk [67]). Depending on the size of vessel, dry bulk is classified into four major indices: the Handysize, Handymax, Capesize and Panamax. The major dry bulk commodities such as iron ore, coal and grains are transported using larger vessel and are in Capesize and Panamax categories. Meanwhile, the minor dry bulk commodities which consist of steel products, fertilizer, sugar, cement and other are transferred by smaller vessel and are in Handysize and Handymax classes.

There are three optional ways to charter a vessel which include bareboat, time-charter and voyage-charter. The flexibility is given to the charterer either to fully (bareboat) or semi (time-charter) rental or otherwise (voyage-charter). The bareboat charter method allows charterer to operate a vessel using their own capacity including crew. The charterer bears all costs and pays

[‡]See http://www.bbk.ac.uk/cfc/papers/nomikos.pdf for detailed report of commodities annualised volatility.

a monthly fee to the shipowner for leasing the vessel. For time-charter or semi rental method, the charterer can instruct the shipowner in the vessel operation but commercial management is still under the charterer's responsibility. The per-day fee plus costs such as fuel, port fees, food and others are covered by the charterer. Lastly, the voyage-charter provides freight service where the charterer pays the shipowner a per ton fee based on point-to-point basis. By using this method, all costs for vessel operation are at the expense of the shipowner.

The information about freight rates were initially provided by the Baltic Exchange in 1985 using Baltic Freight Index (BFI). The index covers 13 voyage routes ranging from 14,000 metric tons (mt) of fertiliser up to 120,000 mt of coal, with no time-charter routes. Until January 2013, there are 50 daily single routes being monitored by the Baltic Exchange encapsulated in six indices: the Baltic Capesize Index, Baltic Panamax Index, Baltic Dry Index, Baltic Supramax Index, Baltic Handysize Index and Baltic International Tanker Routes [§]. We will concentrate in this thesis on the two former indices. The empirical data of the Baltic Capesize Index and Baltic Panamax Index are analysed in Chapter 4 and a study on pricing shipping freight forward will be presented in Chapter 5. There are detail information of shipping freight markets in Alizadeh and Nomikos [5] and Kavussanov and Visvikis [59]. We refer to the textbook by Alizadeh and Nomikos [5] in the following explanation of the freight markets.

Baltic Capesize Index

The Baltic Capesize Index is the index based on 10 daily Capesize vessel assessments including voyage and time-charter rates. Table 1.1 describes the different BCI routes with 6 voyage-charter routes (C2 to C7 and C12) and 4 time-charter routes (C8_03 to C11_03). They are classified according to their cargo type and size. The voyage-charter routes are quoted in terms of US$/mt of cargo transported, while the time-charter routes are calculated in terms of US$/day (see Alizadeh and Nomikos [5]).

The voyage routes mainly cover transportation of iron ore and coal. The routes C3 and C5 are the most important where each represents 15% of the entire BCI routes. The C3 route transport cargo which contains iron ore from Tubarão in Brazil to Beilun and Baoshun in China, while C5 route operates from Western Australia to the same destinations as C3. The time-charter routes reflect the freight rates of four major trading routes: the Atlantic Trade (C8_03), Pacific Trade (C10_03), Continent to the Far East trip (C9_03) and trip back from the Far East to the Continent (C11_03). The most important time-charter route is C10_03 which represents 20% from the whole BCI routes. This route is operating between China and Japan for a Pacific round voyage.

Baltic Panamax Index

The Baltic Panamax Index is the index based on four Panamax vessel assessments with equal vessel size of 74,000 mt deadweight (dwt). The specification of the BPI are given in Table 1.2 covering the four time-charter routes namely the: P1A_03, P2A_03, P3A_03 and P4_03. The routes are of equal importance. The P1A_03 is designed for trans-Atlantic route, where the delivery and redelivery of the vessel are in the continent between Skaw in Denmark to Gibraltar.

[§]Detailed description of the index is provided at www.balticexchange.com.

Table 1.1: Description of the Baltic Capesize Index route (see Alizadeh and Nomikos [5])

Route	Cargo type and size	Route description	Weighting
C2	160,000 mt iron ore	Tubarão to Rotterdam	10%
C3	150,000 mt iron ore	Tubarão to Beilun-Baoshun	15%
C4	150,000 mt coal	Richards Bay to Rotterdam	5%
C5	150,000 mt iron ore	Western Australia to Beilun-Baoshun	15%
C7	150,000 mt coal	Bolivar to Rotterdam	5%
C8_03	172,000 mt deadweight time charter	Delivery Gibraltar-Hamburg for a trans-Atlantic round voyage, redelivery Gibraltar-Hamburg range. Duration: 30-45 days	10%
C9_03	172,000 mt deadweight time charter	Delivery ARA-Mediterranean for a trip to the Far East, redelivery China-Japan range. Duration: 65 days	5%
C10_03	172,000 mt deadweight time charter	Delivery China-Japan for a Pacific round voyage, redelivery China-Japan range. Duration: 30-40 days	20%
C11_03	172,000 mt deadweight time charter	Delivery China-Japan for a trip to ARA or the Mediterranean. Duration: 65 days	5%
C12	150,000 mt coal	Gladstone to Rotterdam	10%

The route P2A_03 is a trip to the Far East and cover the redelivery between Taiwan and Japan. Further, the other two routes, P3A_03 and P4A_03 encompass the vessel delivery between Japan and South Korea, but with a trans-Pacific round voyage and a trip to continental Europe respectively.

Table 1.2: Description of the Baltic Panamax Index route (see Alizadeh and Nomikos [5])

Route	Route description	Weighting
P1A_03	Delivery Skaw-Gibraltar range for a trans-Atlantic round voyage (including ECSA), redelivery Skaw-Gibraltar range. Duration: 45–60 days	25%
P2A_03	Delivery Skaw-Gibraltar range for a trip to the Far East, redelivery Taiwan-Japan range. Duration: 60–65 days	25%
P3A_03	Delivery Japan-South Korea for a trans-Pacific round voyage, either via Australia or NOPAC, redelivery Japan-South Korea range. Duration: 35–50 days	25%
P4_03	Delivery Japan-South Korea for a trip to continental Europe (via US West Coast-British Columbia range), redelivery Skaw-Gibraltar range. Duration: 50–60 days	25%

1.2 Stochastic modelling of energy related markets

From the modelling point of view, temperature and freight markets share two identical stylized facts: stochastic volatility and mean reversion. The heavy-tailed logreturns distribution is one of the main features for freight rates, while seasonality is the property for temperature which may be not so significant for the freight dynamics. To the best of our knowledge, the findings of seasonality behaviour in freight markets are mixed and varies according to the market segments. We will address this issue later, on our way in discussing the stylized facts of the temperature and freight markets.

1.2.1 Stylized facts

First, we discuss the *stochastic volatility* property. The price of a commodity is observed to have volatility which is changing stochastically over time. The study done by Hikspoors and Jaimungal [48] for NYMEX crude oil prices and Benth [13] for UK gas spot prices have found a stochastic volatility structure in the price dynamics. Our observation using the time series of Malaysian temperature data shows the sign of stochastic volatility (see Benth and Taib [24]). This finding is included in Chapter 3 and in line with the study by Benth and Šaltytė Benth [20] using Stockholm temperature data. In the freight markets, the stochastic volatility property can be explained by resorting to the supply and demand curve. The limitation of supply and demand inelasticity which determines the freight rates may drive the volatility to different levels over time. For example, in the situation when the supply of tonnage is extremely high, any shock in the market resulting from changes in demand for a very short time will not give significant impact to the freight rates since the effect of such shocks can be absorbed by the market. The volatility in this period stays at a relatively low level. On the contrary, when the supply is tight for the reason of tonnage shortage or excessive demand, any shock in the market due to the changes in demand may drive the price sharply and ultimately the volatility is high in this period. By simply checking the volatility clustering in the time series of the logreturns of freight rates, we conclude that stochastic volatility is significant for freight markets.

The second stylized fact is *mean reversion*, meaning that the temperature and freight rates tend to revert to the average level in the long run. This is common for temperatures since conservation of energy plays the role and for the freight rates, this reflects the marginal cost of providing the freight service. We concentrate the discussion here to the freight rates since this feature is rather natural for temperature. Mean reversion can be directly linked to the supply and demand of the freight service where any adjustment on the supply side will increase (decrease) the extremely low (high) freight rates. For example, when the freight rate is high, the supply will naturally be high. The number of vessels being demolished will decrease and may cease at certain time and there are new vessels brought into the market as new order takes place. To some extent, the oversupply of vessels will gradually bring the freight rates down to the average level. On the other hand, the supply will decrease when the freight rate is low. More vessels are carried out from the market and being demolished. There will be the time when the supply is very tight which consequently push the freight rate up to the mean level.

The next property is *heavy-tailed logreturns* which is normally observed for many energy commodities. This may results from extreme volatility and price spike. The electricity prices for instance show heavy tails in return distribution (see Weron [85]). This is parallel with our empirical investigation on the dry-bulk market segments where the tails of the logreturns distribution are far more heavy than normal (see Benth, Koekebakker and Taib [16]). Just to mention that our inspection on the temperatures' residuals (after removing seasonality and continuous autoregressive effect) also show heavy tails in its distribution. This means that the increments of the stochastic process come from a non-Gaussian distribution class and the model based on Brownian motion may not be satisfactory.

Finally, we discuss the possible *seasonal* behaviour. The temperature dynamics is highly dependent on the (deterministic) seasonality with low temperature in the winter period and high temperature in summer. In a country close to Equator like Malaysia (our case study in Chapter 2 and 3) with no summer and winter cycle, the seasonality is still significant. We come back to the issue of seasonality in freight rates. There are mixed results of seasonality test for freight rate time series. A study by Kavussanos and Alizadeh [55] has rejected the stochastic seasonality and found that the dry bulk freight rates exhibit deterministic seasonality at very low level. Their investigation for seasonality in tanker markets is also rejecting the stochastic seasonality but the deterministic seasonality is found to be varying across market segments (see Kavussanos and Alizadeh [56]). However, our simple graphical check using deterministic seasonal functions to two dry bulk freight rates (as reported in Chapter 4) shows that the deterministic seasonality in freight rate dynamics is insignificant. Hence, we do not consider seasonality as a stylized feature of freight markets in this thesis.

All stylized facts discussed above are particularly used to find the best model which can explain very well the dynamics of the prices. In the next section, we will discuss the possible models for spot price based on one factor model.

1.2.2 Spot price modelling

It is not an easy task to propose a *simple but efficient* model for temperature or freight rate dynamics while taking into account all stylized features as discussed above. The proposed model usually has a complex structure and may not be analytically tractable for the purpose of derivatives pricing. In this thesis, we avoid from proposing a too complicated model by adopting a stepwise procedure. This approach will provide us with precise causality of the model rejection or vice versa. Firstly, we introduce a simple but quite famous process used in modelling the stock price called the geometric Brownian motion (see Osborne [65] or Samuelson [70]). Suppose that $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ is a given filtered probability space. Denote $S(t)$ as the price at time $t \geq 0$, evolving as

$$S(t) = S(0)\exp(\mu t + \sigma B(t)), \tag{1.2.1}$$

where μ and $\sigma > 0$ are constants and $B(t)$ is Brownian motion. The price has a continuous trajectory, exponentially increasing or decreasing governed by a stochastic process with normally distributed increments. Moreover, the logarithmic returns (or simply logreturns) will be independent, stationary and normally distributed. In Chapter 4, we will prove empirically our previous claim that the GBM is not satisfactory to model the freight rate dynamics.

We can naturally generalize Brownian motion to the Lévy process, the process with independent and stationary increments to allow for jumps and leptokurtic behaviour of the price dynamics. The model simply takes the form

$$S(t) = S(0)\exp(L(t)), \tag{1.2.2}$$

where $L(t)$ is a Lévy process. Equation (1.2.2) is referred as an exponential Lévy model. We can choose $L(t)$ from various classes of non-Gaussian Lévy processes where the path of the price process is allowed to have jumps at any arbitrary time. A special class of hyperbolic distributions namely the normal inverse Gaussian (NIG), which is connected to a pure jump Lévy process may possibly explain the evolution of prices. With the assumption that the increments of $L(t)$ are distributed according to NIG, the price process will show discontinuity in its path. We will consider such Lévy process in Chapter 3 for the temperature dynamics, and also in

Chapter 4 and 5 for the dry bulk freight rates.

The term $S(0)$ in (1.2.2) can be substituted with a seasonal mean function $\Lambda(t)$ while modelling the dynamics of the seasonal dependent commodity. The process is typically taking the following geometric representation

$$S(t) = \Lambda(t) \exp(X(t)), \tag{1.2.3}$$

where $X(t)$ is the stationary process of Ornstein-Uhlenbeck type defined as

$$dX(t) = -\alpha X(t)dt + \sigma\, dB(t).$$

The model allows for mean reversion, at a speed given by the constant $\alpha > 0$. There is also a possibility to have a two-factor model by adding a factor $Y(t)$ usual to model the spikes (see e.g. Benth, Šaltytė Benth and Koekebakker [22] for the energy spot price model). However, the two-factor or more general multi-factor model is not in the scope of this thesis.

Due to the absence of seasonality feature in freight rates, the $\Lambda(t)$ in this setting is equal to one. This implies that (1.2.3) becomes $S(t) = \exp(X(t))$ or in terms of logarithmic price representation, $\ln S(t) = X(t)$. We let $X(t)$ be equipped with stochastic volatility process. As noted, we will consider the BNS stochastic volatility model in Chapter 4 where $X(t)$ follows the dynamics

$$dX(t) = \{\mu + \beta\sigma^2(t)\}\, dt + \sigma(t)\, dB(t)\,.$$

The volatility process is defined in terms of a *superposition* of independent OU processes with different mean reversion rates. The process is moving up entirely by jumps and decreasing exponentially over time. The log price $X(t)$ is still a continuous process although the volatility process $\sigma^2(t)$ shows jumps on the path.

The more recent stationary process called CARMA is within the class of multi-dimensional OU process. Instead of using $X(t)$, we denote the process as $Y(t)$ where $Y(t) = \mathbf{b}'\mathbf{X}(t)$ and $\mathbf{X}(t)$ is the solution of the following vector-valued stochastic differential equation

$$d\mathbf{X}(t) = A\mathbf{X}(t)\, dt + \mathbf{e}_p \sigma dB(t)\,. \tag{1.2.4}$$

The matrix A contains the different speeds of mean reversion, covering the slow and fast reverting factors in the dynamics. The interesting part of this model is the stochastic process driving the dynamics, $B(t)$. Depending on the property of the process, we can make a generalization from Brownian motion in (1.2.4) to the Lévy process $L(t)$, similarly to the case of exponential Lévy model (1.2.2). This opens up the possibility of capturing the jumps in the process which lead us to a non-Gaussian OU model. Another possibility is using the stochastic volatility dynamics in the model explained by BNS stochastic volatility which is also an OU process. We will explain the CARMA model in Chapter 3, and in Chapter 4 and 5 we go to its subclass called CAR model.

1.2.3 Forward and futures pricing

The price of forward/futures determines the direction of underlying spot in the future. There are two special terminologies used in the commodity markets, namely *contango* and *backwar-*

dation. The former represents the market condition where the price of forward/futures is traded above the expected spot price at maturity, while the latter represents the vice versa. The derivation of forward/futures price for a tradeable asset is simple since the cost-of-carry relationship does hold. We can define the cost of carry as the cost of holding the underlying asset (being purchased in the spot market) until the forward contract matures. This relationship however breaks down for the temperature and freight markets, where the buy and hold hedging strategy obviously cannot be implemented.

We denote $S(t)$ as the spot price at time t, and the forward price with delivery time τ is represented as $f(t, \tau)$. Entering the forward contract is equivalent to locking in the position where payoff function equals $S(\tau) - f(t, \tau)$. We refer to the arbitrage pricing theory (see e.g. Duffie [37]) where the value of derivative is defined under risk neutral pricing measure Q as the present value of its expected payoff. We pay nothing to enter the contract, which implies that the discounted value of the expected payoff,

$$e^{-r(\tau-t)}\mathbb{E}_Q\left[S(\tau) - f(t,\tau) \mid \mathcal{F}_t\right] = 0,$$

where r is the constant risk-free interest rate. The operator $\mathbb{E}_Q$ is the expectation defined under risk neutral measure and $\mathcal{F}_t$ is the filtration encapsulating the revealed market information up to time t.

Since $f(t, \tau)$ is adapted to the filtration $\mathcal{F}_t$, the following spot-forward relationship

$$f(t,\tau) = \mathbb{E}_Q\left[S(\tau) \mid \mathcal{F}_t\right],$$

holds. This relationship resulting of the arbitrage-free dynamics of forward price since the process is martingale under Q. In a similar way, we define the futures price as the expectation of accumulated spot price over the delivery period $[\tau_1, \tau_2]$ measured under risk neutral probability. Mathematically, this can be represented as

$$f(t,\tau_1,\tau_2) = \mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2} S(u)du \mid \mathcal{F}_t\right].$$

Esscher transform for Lévy processes

The martingale property of the process X (we consider such processes as in Subsection 1.2.2) can be obtained by constructing a new probability measure Q which is equivalent to the measure P. The term Q in our context is coined as risk-neutral probability measure, such that the price process becomes martingales after discounting. This applies to all tradeable commodities where the product can be traded in a normal sense. However, since temperature and freight service are nonstorable, the price process may not necessarily be a martingale after discounting. But, the forward prices are martingales under the probability Q. We will use an Esscher transformation, a well-known procedure to construct a density process D (also refer to notation π in Chapter 5), which can be used for construction of the risk neutral measure Q. This is one of the main steps in deriving the forward pricing formula, to be briefly discussed herein. We refer to Benth, Šaltytė Benth and Koekebakker [22] for detailed explanation of the Esscher transform.

Define for a given $\theta_L \in \mathbb{R}$ and $0 \le t \le \tau$, the stochastic process

$$D_L(t) = \exp\left(\int_0^t \theta_L \, dL(u) - \int_0^t \phi_L(\theta_L) \, du \right).$$

The terms ϕ_L and θ_L are respectively the cumulant function of the Lévy process and the market price of risk. The latter is the price charged for the risk of not being able to hedge (see Benth, Šaltytė Benth and Koekebakker [22]). From now on, we assume that the process $D_L(t)$ is a martingale, and hence $\mathbb{E}[D_L(\tau)] = 1$. We can construct a risk neutral measure Q from the density process $D_L(t)$ of the Radon-Nikodym derivative as follows

$$D_L(t) = \frac{dQ}{dP}\Big|_{\mathcal{F}_t}.$$

Hence, L is now the Lévy process under Q-probability. This change of probability measure is known as the Esscher transform. In a similar way, we will introduce the Esscher transform to a stochastic volatility process by assigning a new parameter for the price of volatility risk. We refer the parameter as market price of volatility risk. To be precise, we assume that these two prices of risks are uncorrelated, and the Lévy processes associated to them are independent.

The advantage of Esscher transform is that it preserves the distributional properties of the Lévy process. For the NIG Lévy process, the transformation is just reparameterizing the skewness of the NIG class Lévy measure, that is β being modified to $\beta + \theta_L$ under Q. The process still remains as NIG Lévy process after transformation. Note that the Esscher transform is a generalization of Girsanov-type transformation for more general Lévy processes. The Girsanov transform in particular preserves the normality property of Brownian motion process. The Esscher transform will be used in formulating the temperature futures in Chapter 3 and also the freight forwards in Chapter 5.

1.3 Summary of thesis contributions

Chapter 2 contributes mainly to the innovation of crop insurance where we borrow the financial terminology of derivatives pricing from weather markets. Back to the traditional practice of crop insurance, the *real* losses are prudently counted before claims are paid to the policyholders. Apparently, the traditional crop insurance is not transparent which open for the possibility of moral hazard problem. Furthermore, the loss evaluation will surely require much time which is costly to the policyholder. The weather index insurance is introduced as a new risk management solution to the farmer in lower-income countries. There are several advantages of using weather index insurance such as it requires simpler information and reduces the moral hazard and adverse selection. It also has a transparent structure and quite low administrative cost. The simple payoff calculation as illustrated in many papers along this line (see e.g. Skees [77]) is proportional to the objective index. Our idea is to link the weather-based insurance to the weather derivatives which aims at hedging against weather risk. The weather derivatives is based on an objectively measurable index such as heating degree-day or cooling degree-day for temperature market, as mentioned previously. We propose a CDD-type index for temperature insurance by setting a predefined threshold where the payment starts if temperature moves beyond that, analogous to the terminal point of electricity required for cooling. To the best of our knowledge, this is the first attempt in bridging weather index insurance to the weather derivatives.

In Chapter 3, the concept of half life which was introduced by Clewlow and Strickland [34] is revisited. Their definition of half life is quite simple, computed as $\ln(2)/\alpha$, that is proportionally to the mean reversion rate of OU process namely the parameter α. A high value of α indicates the fast return of OU process towards the mean in contrast with slow return for the small α value. The concept has been widely used in many articles in modelling and pricing of energy markets (see e.g. Benth and Šaltytė Benth [17]). We make several contributions to the literature. Firstly, we extend the half life concept to the continuous-time autoregressive moving average processes with stochastic volatility. We will see that for the simple OU case, our definition is identical to a conventional half life of Clewlow and Strickland. Secondly, we are against Clewlow and Strickland [34] by pointing out that the half life for CARMA process is depending stochastically on the state of the process and volatility, which differs from their definition. This is due to the structure of such processes and the presence of stochastic volatility in the dynamics. In addition, the different persistence rates of the CARMA and volatility process also play the role. Thirdly, the long term stationary mean for CARMA process with stochastic volatility is computed by resorting to the Fourier analysis techniques, which is new in the literature. Thus, the half life of the process will consider the reverting time to this conditional long term average level. Fourthly, we show how the half life concept for CARMA process can be applied to the energy markets by taking the case of temperature. Our discussion on Samuelson's effect (see Samuelson [71]) is more technically presented where we provide evidence of the conjecture of the asymptotically stable futures price in the long end is true.

Starting with geometric Brownian motion, we explore various models describing the dynamics of spot freight rates in Chapter 4. We show how the aforementioned stylized facts can be explained by different stochastic processes, varies from simple to a complicated model in a stepwise procedure. Our contributions mainly in the course of shipping literature. Firstly, our empirical analysis on two dry-bulk market indices, namely the Baltic Panamax Index and Baltic Capesize Index are complimenting the other empirical studies in shipping markets. For example, some investigation of tanker market segments have been done by Kavussanos and Alizadeh [56] and Ådland and Cullinane [3] and also a recent analysis of Baltic Dry Index by Geman and Smith [41]. Interestingly, our analysis are presented along the model description making it easier to see the drawbacks of a particular model and the advantages of other. Secondly, we are dealing with continuous-time modelling framework where much attention in shipping literature are given to the discrete-time case. The works done by Kavussanos [52, 53, 54] and Jing, Hui and Marlow [50] are some examples of such proposed discrete-time models. Thirdly, the implementation of stochastic volatility of Barndorff-Nielsen and Shephard model is among the first in this shipping literature. To test the models performance, we made a calculation of Value-at-Risk using different models in the last part of this chapter.

Chapter 5 comes in the sequence of the empirical findings in Chapter 4. Based on spot-forward relationship, we derive the forward pricing formulas for different models proposed previously. The work contributes to the literature in pricing of shipping forward/futures in conjuction with a study by Prokopczuk [67], but our models are limited to the geometric one-factor model. In Prokopczuk [67], these are extended to two-factor model covering the normal variation and mean reversion. Our approach considers the mean reversion using the continuous autoregressive process and also the dynamics of stochastic volatility in deriving the forward formulas. We are interested in examining the shape of the forward curves contributed by diffferent formulas for various models. Instead of looking at hedging performance in Prokopczuk [67], this chapter investigates the effect of different time to delivery and the maturity effect to the forward dynamics, which is meaningful in the study of forward pricing.

1.4 Discussion on future research

When proposing the temperature index insurance based on temperature derivatives in Chapter 2, we are questioning how the insurance companies could hedge their position since no temperature futures are traded for cities in developing countries. One possibility is using the geographical hedging, that is using other locations which are correlated with the coverage (see Benth and Šaltytė Benth [17]). This issue is quite significant, and one may investigate such hedging possibility for weather index insurance in the future work. One can also look at the example of spatial hedging for temperature derivatives by Barth, Benth and Potthoff [12] when considering the risk for the insured, taking into account the distance between specific temperature station and the place where farmers are located.

Focus on various models proposed for freight rates dynamics in Chapter 4, it would be interesting to see if our models can be applied in a similar manner to the time series of other shipping market segments. We have completed our analysis on two indices for the dry-bulk markets where we find our models captured all stylized facts of the freight rates. Since the freight rate dynamics vary across market segments, the possible future study could be to look at the empirical data of other market segments such as tanker markets consist of Baltic Clean Tanker Index and Baltic Dirty Tanker Index. We may propose our models to be suited with tanker's time series data according to their features suitability. The study by Kavussanov and Alizadeh [56] concerning the seasonality feature of tanker freight rates could be our starting point where we will investigate other reliable features of this particular market.

The sequence of our studies in freight markets are arranged in a stepwise fashion. We first analyse the spot freight rates data, study their stylized facts and propose the suitable models aiming at capturing all the exhibited features. We further extend our study to the derivation of pricing forward formulas and examine the shape of the forward curve. To be in line with Prokopczuk [67], we can also for future research include the two-factor model in the derivation of forward price for dry bulk markets and investigate the hedging performance for all models being considered in Chapter 5. In addition, we may apply Heston [47] model for the stochastic volatility dynamics. This may be possible, but one need to consider the analytical tractability of such model for the derivatives pricing. Next, we may derive the price of options based on our explicit forward formulas. There are nice references in pricing freight rate options by Koekebakker, Ådland and Sødal [62] and valuation of European futures option studied by Tvedt [84].

Based on our theoretical forward price formulas in Chapter 5, it is possible to investigate the difference between theoretical and empirical forward prices. The latter is the price observed in the market. One can represent the difference by a term called *risk premium* (see Benth, Šaltytė Benth and Koekebakker [22] and Benth and Sgarra [23]). Based on experience in some energy commodities, the interesting part is the sign of risk premium. The risk premium should be negative in normal backwardation and positive when the price is in contango. Somehow, the sign of the risk premium may change depending on time horizon. The findings in the study by Cartea and Williams [32] for gas market and Weron [86] for Nord Pool market have verified the claim. We want to investigate the risk premium for freight markets and see if the sign is also changing over time.

1.5 Outline of the thesis

This thesis is based on author's works which are retrieved from four articles. The theoretical parts proposed in this thesis are supported with the empirical (or numerical) analysis to show the applicability and validity of the models. The following two chapters are devoted to the temperature market and the last two chapters are focused on the freight markets.

Chapter 2 covers a topic related to insurance based on temperature index. The calculation of our insurance premium is similar to the pricing of weather derivatives where the proxy is the CDD-type index. We take the temperature data from Malaysia as our empirical case. We consider three different pricing approaches: the classical burn analysis, index modelling and our pricing method of dynamical time series. The two former methods are based on the empirical distribution of the payoffs within the sample data collected and the latter is connected with the time series model of the temperature dynamics. The last part in this chapter complement our analysis where we investigate the profit/loss distribution from the contract, from the perspective of insurance company and the policyholder.

The half life concept is directly connected to the speed of mean reversion in our *autoregressive* model for temperature dynamics in Chapter 2. We extend the half life of Ornstein-Uhlenbeck process to Lévy driven continuous-time autoregressive moving average processes in Chapter 3. In addition, we also consider the stochastic volatility of BNS to the CARMA process. Instead of having the half life which is solely depending on mean reversion rate, our new half life definition is subjected to the state of the process and stochastic volatility. Finally, we implement the half life concept in deriving the futures price of the temperature where in the last part we discuss the Samuelson effect of the futures pricing formula.

The investigation of the BCI and BPI data in Chapter 4 provide us with the properties of freight rates: the mean reversion, stochastic volatility and heavy-tailed logreturns. Hence, we propose different stochastic models which can capture all stylized facts of the freight rates. The most frequently encountered model in stock modelling, the GBM is used as our benchmark. Other models are consisting of NIG Lévy which generalize Brownian motion to Lévy process, the BNS stochastic volatility model which consider the time-varying volatility, and the continuous-time autoregressive model with three different Lévy processes to fit with the mean reversion behaviour. We compare our different models with the benchmark using the calculation of Value-at-Risk.

In Chapter 5, we extend our empirical findings in Benth, Koekebakker and Taib [16] to the derivation of forward price in freight markets. We again introduce our six different models used in the article to prepare for the calculation of forward prices using the spot-forward relationship framework. The pricing is considered under risk neutral measure, where no arbitrage possibility exist. Since the storability property does not hold in freight markets, we do not have the price process which is martingale after discounting. However, the martingale property still applies to the forward price under risk neutral probability Q. Further, we examine the shape of the forward curve for all continuous-time forward pricing formulas. It should be noted that the forward curve is determined by a mixture of fixed and stochastically dependent terms. Finally, we discuss the effect of different time to delivery and the maturity effect to the forward curve.

Chapter 2

Pricing of temperature index insurance*

Che Mohd Imran Che Taib and Fred Espen Benth

Abstract
The aim of this paper is to study pricing of weather insurance contracts based on temperature indices. Three different pricing methods are analysed: the classical burn approach, index modelling and temperature modelling. We take the data from Malaysia as our empirical case. Analysis of Malaysian temperature shows a weak seasonality and its distribution is close to normal. Our results show that there is a significant difference between the burn and index pricing approaches on one hand, and the temperature modelling method on the other. The latter approach is pricing the insurance contract using a seasonal autoregressive time series model for daily temperature variations, and thus provides a precise probabilistic model for the fine structure of temperature evolution. We complement our pricing analysis by an investigation of the profit/loss distribution from the contract, in the perspective of both the insured and the insurer.

2.1 Introduction

Weather index insurance is a class of products targeted to households in developing countries (see Barnett [10, 11, 33], Sakurai and Reardon [69] and Skees [75, 76, 77]). Such insurance contracts have close resemblance with weather derivatives, since the claim is tied to the value of a weather index measured in a specific location. In classical weather-related insurance contracts, the insured must prove that a claim is justified based on damages. The weather index contracts refer to an objective measurement, like for instance the amount of rainfall or the temperature in a specific location. As such, weather index insurance accommodates a transfer of risk for droughts or flooding, say, from households in rural areas in Africa to insurance companies. The premium to pay for buying a weather index insurance is our focus.

Consider a weather index insurance written on a temperature index. For example, we may consider a contract giving protection against unusually high temperatures over a given period in the season for growing crop. If temperatures are above a given limit, then there may be a significant risk of dry conditions leading to a bad harvest. The limit or predefined threshold is the point where payments start. Once the threshold is exceeded, then the payment is calculated

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as how much it goes beyond the limit. We may construct a contract which pays out a certain amount of money according to an index value over the period, for example, based on a CDD-type (cooling-degree day) index.

Suppose, for further concreteness, a contract paying

$$X(\tau_1, \tau_2) = k \times \sum_{s=\tau_1}^{\tau_2} \max(T(s) - c, 0), \tag{2.1.1}$$

that is, an amount k times the CDD index over the time period τ_1 to τ_2, where c is some threshold. Both k and c are positive constants, and c measures the critical temperature level, and k is the conversion factor transforming the weather index into money. Both k and c are contractual parameters, while the temperature is measured at some agreed station. The insured will receive the amount $X(\tau_1, \tau_2)$ of money at time τ_2, against paying a premium for the insurance at time $t \leq \tau_1$. We note that the index and actual experienced losses by the insured are obviously not perfectly correlated. There is a possibility of the insured receiving no indemnity even though experiencing a loss, and in contrast, the insured may also receive the indemnity with having a loss.

Given a reliable temperature model, the insured may assess the distributional properties of $X(\tau_1, \tau_2)$ and determine if the contract provides the protection sought for. A major problem is of course the spatial risk incurred by the location of the temperature measurement station relative to the location where the insured is living. The insurance company offering the protection will most likely wish to settle the contract against an index calculated from an official measurement station, which typically are existing only in major cities. The spatial risk may be significant, and the insurance contract may only provide partial protection against the real temperature risk. In Šaltytė Benth et al. [72] a spatial temperature model is presented, and in Barth et al. [12] questions concerning hedging of spatial risk using weather derivatives are analyzed. The spatial consideration in the pricing of rainfall insurance has been done by Turvey [82].

Important in the assessment of the contract is the premium charged by the insurance company. The standard approach to pricing the contract is by finding the expected value of the claim size $\mathbb{E}[X(\tau_1, \tau_2)]$, adjusted for risk (sometimes called the risk loading), and discount it by some interest rate to obtain the present value,

$$P(t, \tau_1, \tau_2) = \exp(-r(\tau_2 - t))\mathbb{E}[X(\tau_1, \tau_2) \mid \mathcal{F}_t] \,. \tag{2.1.2}$$

Since the money is paid at the end of the measurement period of the index, at time τ_2, we discount by $\exp(-r(\tau_2 - t))$ to get the present value, with $r > 0$ being the discount rate assumed to be a constant. We use continuously compounding discount rates in our analysis. The filtration $\mathcal{F}_t$ denotes all the available information in the market up to time t, which the insurance company will take into account in its pricing.

Our concern is to study the price $P(t, \tau_1, \tau_2)$ using three different approaches. The first approach is the so-called *burn analysis* advocated in Jewson and Brix [49] as the classical method to price weather derivatives. The burn analysis is based on the empirical distribution of the payoff $X(\tau_1, \tau_2)$ within the sample data collected. The price is calculated using the mean value of the observations. Next, we apply the slightly more sophisticated *index modelling* approach presented in Jewson and Brix [49], which amounts in fitting a distribution to the historically observed claims $X(\tau_1, \tau_2)$, and price the contract based on the expected value of the distribution. Lastly, we propose a very detailed modelling approach, where the daily temperature dynamics is modelled by a time series. Based on empirical findings, an *autoregressive*, AR(p), model turns out to be highly suitable for describing the dynamics of temperature evolution. Using this model, one may compute the index $X(\tau_1, \tau_2)$, and find the expectation for pricing the insurance contract. There are several advantages with this approach. First, we obtain a consistent framework for pricing insurance contracts for various given time periods $[\tau_1, \tau_2]$ without having to re-estimate a distribution (as in the index approach), or collecting data (as in the burn analysis). Furthermore, we can use current information on the temperature to price the insurance contract. The index modelling and burn analysis will not use any dynamical model, and hence we cannot take into account current information when pricing. This is an important aspect for the insurance company in their risk assessment of the contracts. As a final remark, a detailed time series model of the temperature dynamics is likely to capture the statistical properties better that simply looking at the historical data (burn analysis), or fit an "arbitrary" distribution to the historical data (index modelling). The two latter approaches also suffer from little available data compared to the situation for the temperature modelling approach, where the amount of information is far better.

We will analyse the profit/loss distribution for both the insurer and the insured. The two factors determining the profit/loss distribution are of course the index $X(\tau_1, \tau_2)$ which settles the payoff, and the price of the contract. The insurance company wants to stay solvent, and thus charges an additional premium on the "fair" expected value. However, the higher premium, the less attractive will these insurance contracts become. We emphasis here that the index-based contracts are very different from traditional insurance, as the insurance company cannot diversify its risk by attracting many clients. In fact, the weather index-based contracts are very similar to financial derivatives. For example, to illustrate matters by a simple case, an insurance company issuing one contract on a temperature index in a given city, will have to pay $X(\tau_1, \tau_2)$. However, if it issues 100 contracts, it must pay 100 times this amount. In traditional non-life insurance, the risk of paying out insurance claims are distributed among the clients, and the company would on average every year have to pay an *expected claim size amount* when having 100 clients, that is, only a fraction of the insured will make a claim. With temperature index contracts, one will risk that *all* clients claim the insurance one year, whereas the next year *none* will claim a payoff. This is parallel to how options functions in financial markets. Hence, in the case of weather index insurance, the insurance company has bigger variations in their claim payoffs as in traditional insurance. This means bigger risk, and higher prices as a consequence thereof.

On the other hand, the insurance company may hedge their risk using financial weather derivatives traded, for example, on the Chicago Mercantile Exchange. Such weather derivatives are not yet being traded on temperature indices in the developing part of the world, like Africa say, but there may be a demand for such products with an increase of weather insurance contracts. The insurance company may also hedge their risk by exploiting weather correlation. From a careful analysis of temperatures in different locations, the insurance company may identify places with independent or negatively correlated weather patterns. This would offer possibilities to spread risk for the insurance company, and thereby lower prices of the contracts.

In our analysis, we shall not analyse such hedging opportunities in more detail, but focus on the effect of charging a risk loading on the contracts. In order to compare the different pricing approaches, we use the a risk loading which is based on the quantiles of the index $X(\tau_1, \tau_2)$, since this is observable in all three approaches.

We use daily temperature data from Malaysia in our analysis. Agriculture is the main economical activity in Malaysia. 15.3 percent of the work force was employed in agriculture 2000, contributing approximately 8.9 percent of the national GDP (Prime Minister's Department [66]). The favourable Malaysian tropical climate spur the production of various crops including the main export articles rubber, palm oil and cocoa. Almost 24 percent of the whole area in Malaysia is allocated for agricultural activity.

The paper is organized as follows. In Section 2.2, we present the temperature data from Malaysia and price empirically the weather insurance contracts based on the three approaches. The next Section 2.3 is devoted to some risk analysis, seen both from the insured and the insurers point of view. Finally, the conclusion is given in Section 2.4.

2.2 Pricing temperature index insurance contracts

In this section we investigate three methods of pricing of a single temperature index insurance contract. The methods include the classical way of pricing weather derivatives using classical burn approach and index modelling (see Jewson and Brix [49]), and a third method of pricing based on a dynamical temperature model adopted from Benth et al. [21, 22, 18].

For all pricing methods, we analyse the CDD-index $X(\tau_1, \tau_2)$ defined in (2.1.1). As we want to perform an empirical analysis of the performance of the different pricing methods, we must choose a threshold level c. Obviously, since these insurance contracts are mainly targeted for farmers, the threshold c will be dependent on the crop grown. For the temperature insurance contract to be attractive for the farmer, the level c must be so that it reflects the harmful threshold for his or her crop. For the sake of illustration, we choose $c = 28$ in our studies, imagining that temperatures above this threshold may imply drought, harming the growth of a specific crop. The money factor k is fixed to RM50 per unit of the contract. We set a low value of the money factor just for ease rather than dealing with any substantial amount. We further concentrate our analysis to January, that is, the time span $[\tau_1, \tau_2]$ means the month of January.

In our analysis of prices, we shall first compute "fair prices", based on the expected payoff of $X(\tau_1, \tau_2)$, appropriately discounted to present values. However, such prices will not compensate the insurer for taking on the risk, and a risk loading will be added in the real pricing of the contract. We will also add a risk loading, based on adding 5% of the 95% quantile of the payoff $X(\tau_1, \tau_2)$. For the insurance company, the 95% quantile of $X(\tau_1, \tau_2)$ means a size of loss which is exceeded with 5% probability.

2.2.1 Description of the data

We have obtained daily average temperature (DATs) data measured in degrees of Celsius from the Malaysian Meteorological Department, which are observed over the period ranging from 1 January 1971 to 31 December 2010. The data have been collected in Petaling Jaya, Malaysia, as the nearest station to the capital city Kuala Lumpur. A number of 14,610 records covering

40 years of DATs data are observed, however, the amount of data is reduced to 14,600 after removing the measurements on February 29 in each leap year to synchronize the length of the years to 365 days in the analysis. A small amount of defective or missing values have been detected in the data, constituting only 0.48% of the total sample. These data observations have been corrected by using the average temperature between the previous and following day observations. The time series of the average DATs is plotted in Figure 2.1. For the purpose of illustration, we just show the last 10 years of the time series data.

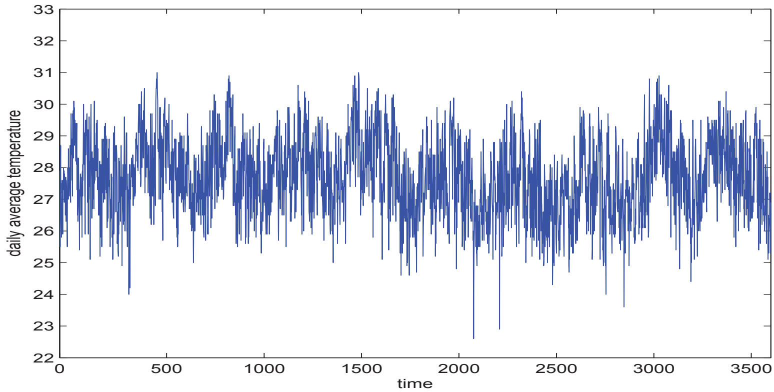

Figure 2.1: Petaling Jaya DATs for the period starting 1 January 2001 to 31 December 2010.

Note that the lowest and highest temperature recorded in the data set are 22.3 and 31.2, respectively, with the mean being equal to 27.4. We observe a rather small variation in the data. In Figure 2.2, we present the histogram of DATs in Petaling Jaya. The skewness coefficient is 0.010, indicating nearly symmetric data. The kurtosis is 3.011, showing that the data are not normally distributed.

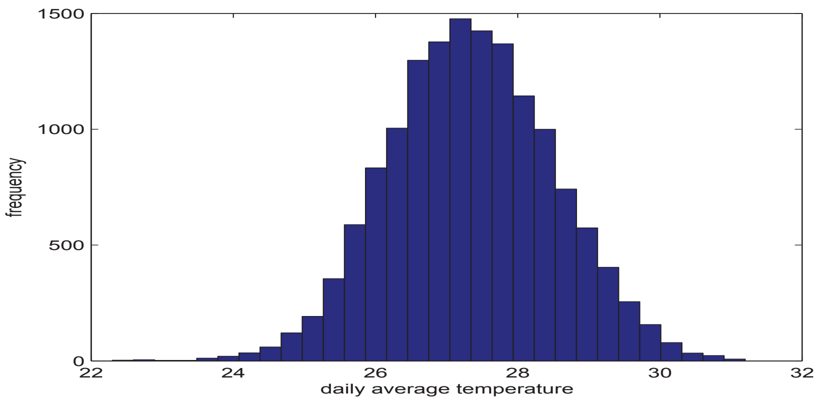

Figure 2.2: Histogram of daily average temperature in Petaling Jaya.

2.2.2 Burn analysis

Burn analysis is a very simple method that is traditionally used for pricing a weather derivatives contracts (see Jewson and Brix [49]). It simply uses the empirically computed mean value of the observed index $X(\tau_1, \tau_2)$. In our case, we computed the index $X(\tau_1, \tau_2)$ for January each year in the data sample, altogether yielding 40 samples of $X(\tau_1, \tau_2)$. Figure 2.3 shows the histogram of $X(\tau_1, \tau_2)$. Based on the 40 observed values, we compute the empirical mean, and discount it by $\exp(-r(\tau_2 - t))$ in order to get the present value at time t of the contract. In the calculation of the premium, we do not include any information $\mathcal{F}_t$ on present or historical temperatures, since this is naturally *not* entering into this pure data driven approach. The conditional expectation is transformed into a standard expectation. As $t \to \tau_1$, the price will converge to the expected claim size $X(\tau_1, \tau_2)$ as estimated from data. The empirical mean was estimated to be 118.25.

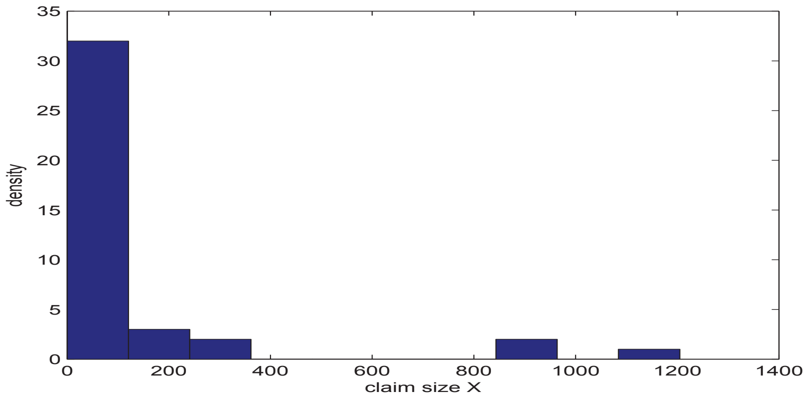

Figure 2.3: Histogram of the claim size X.

2.2.3 Index modelling

Index modelling is a pricing method for weather derivatives based on a distribution statistically modelling the claim size. To the observed claim sizes $X(\tau_1, \tau_2)$, one selects and fits a distribution, and computes the mean of this distribution to find the expected value of $X(\tau_1, \tau_2)$. The advantage of this method is that we may derive statistically information of the claims outside the range of the observed data values, and can make assessments of the probability of extreme events happening. In particular, we may estimate quantiles of the claim $X(\tau_1, \tau_2)$ outside the range of observed data. However, we note that the data backing up the estimation of the distribution is the same as for the burn analysis, which in our case of Malaysian data amounts in only 40 values.

From the histogram of the claim size $X(\tau_1, \tau_2)$ in Figure 2.3, one may propose an exponential distribution in modelling the claims. Recall the density with parameter μ of the exponential distribution as

$$f_{\exp}(x; \mu) = \frac{1}{\mu} \exp\left(\frac{-x}{\mu}\right). \tag{2.2.1}$$

We apply the maximum likelihood (ML) method to estimate the parameter of distribution. We estimated μ to be $\hat{\mu} = 118.25$. As the parameter μ of the exponential distribution is the mean, we find the expected claim size to be 118.25. This estimate coincides with the result of the burn analysis, not unexpectedly as the maximum likelihood estimation in this case will be based on the mean value of the data. We show the empirical density of the claims with the fitted exponential distribution in Figure 2.4. The confidence interval for the estimate $\hat{\mu}$ has upper and lower limit 81.33 and 184.87, respectively, at the 1% significance level. This is a very wide confidence interval, demonstrating clearly the huge statistical uncertainty in this approach to model the claims. The reason is obviously that we have only 40 data points available. We note that the mean claim size estimated for the burn analysis will be infected by the same uncertainty. This is a drawback with these two methods. In a real world application, an insurance company is likely to charge a premium also for this uncertainty, leading to more expensive insurance contracts.

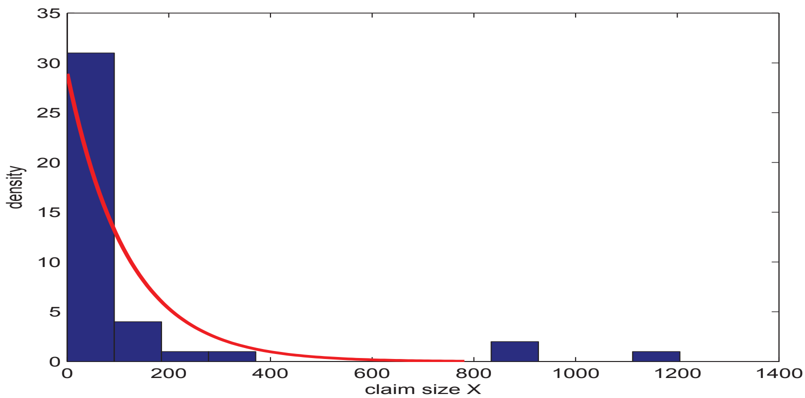

Figure 2.4: The empirical density together with fitted exponential.

We also note that for the index modelling approach, the historical temperature records up to current time t, $\mathcal{F}_t$, do not play any role in the pricing. We do not create a dynamical model of the index, and hence there is no natural definition of the filtration. Therefore, we also in this case compute an expectation rather than a conditional one when assessing the insurance premium of the contract.

2.2.4 Temperature dynamical modelling

As an alternative to the burn analysis and index modelling approach, we propose to model the time dynamics of the temperature evolution. Taking into account 40 years of *daily* data, the amount of information available for such a model is significantly more advantageous than the poor 40 data points for the burn and index model methods.

Suppose that the temperature $T(t)$ at time $t \geq 0$ is given as follows

$$T(t) = S(t) + Y(t), \tag{2.2.2}$$

where $S(t)$ is a deterministic seasonal mean function and

$$Y(t) = \sum_{i=1}^{p} \alpha_i Y(t-i) + \epsilon(t) \,. \qquad (2.2.3)$$

Here, $\epsilon(t)$ are i.i.d normally distributed noise with mean zero. The AR(p)-process $Y(t)$ models the random fluctuations around the seasonal mean, or, in other words, the dynamics of the deseasonalized temperatures, $T(t) - S(t)$.

The seasonal mean function $S(t)$ is defined as

$$S(t) = a_0 + a_1 t + a_2 \sin\left(\frac{2\pi(t-a_3)}{365}\right) \,. \qquad (2.2.4)$$

The constants a_0 and a_1 describe the average level of temperature and slope of a linear trend function, respectively, while the amplitude of the mean is represented by a_2. A constant a_3 is referred to as the phase angle.

From this temperature dynamics, we may compute the temperature index and subsequently the payoff $X(\tau_1, \tau_2)$. Thereafter, we may compute the expected payoff. The advantage now is that we get a very precise model for the temperature and a price which takes current temperature knowledge into account. Moreover, the model is flexible in pricing contracts settled on various periods of the year without having to perform a statistical re-estimation like in the burn approach and index modelling. We efficiently exploit 40 years of daily data to get a detailed statistical description of the claim size distribution.

A look at the ACF of the DATs in Figure 2.5 shows that there are clear seasonal effects in the data, but also apparent signs of mean-reversion. The latter is observed from the decaying ACF for moderate lags. This indicates that our model is appropriate. To estimate it to data, we use a step-by-step procedure, where first we estimate the trend and seasonal component in $S(t)$, and next find the best AR(p) model fitting the deseasonalized temperature data.

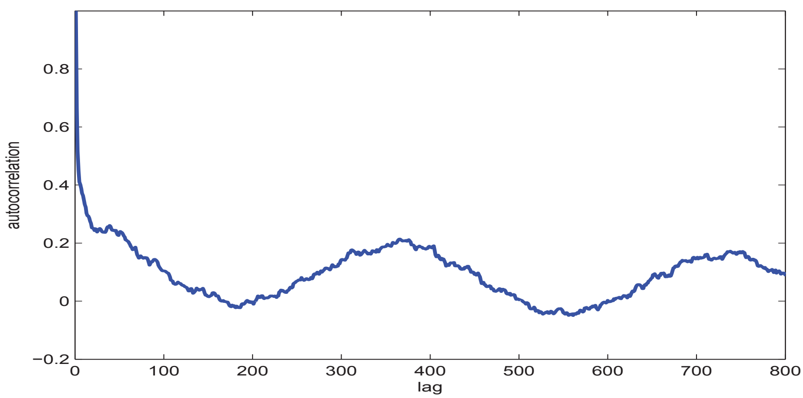

Figure 2.5: Empirical ACF of daily average temperature in Petaling Jaya.

We start by checking for the presence of a linear trend. By simple least squares, we obtain the slope equal to 0.0001 and intercept being 26.8. Although the trend slope seems to be

Table 2.1: Estimated parameters for seasonal fitting

a_0	a_1	a_2	a_3
26.8373	0.0001	0.5673	56.1070

very small, the P-value of 0.0000 validates that it is significant. Next, we fit the data with the complete seasonal function $S(t)$ given by (2.2.4) using least squares method. The estimated parameters are presented in Table 2.1. We find an R^2 value of 18.4%, indicating that there is not much explanatory power in the seasonality function $S(t)$. The DATs for the last 5 years is plotted in Figure 2.6 together with the fitted seasonal function. We conclude that there is not a very pronounced seasonal variation in the data. This is unlike the observations in most of the European countries, which have high temperatures in summer and low in winter. Temperature analysis for Stockholm, Sweden, shows a clear seasonality (see Benth et al. [21, 22]), similar to the findings in USA and Lithuania (see Campbell and Diebold [30] and Šaltytė Benth et al. [72] respectively). Malaysia on the other hand experiences two seasons in general, with a dry season usually ranging from March to October and rainy from November to February whereby June and July are recorded as the driest months of the year. Nevertheless, we find the existence of a small seasonal variation which we include for further analysis.

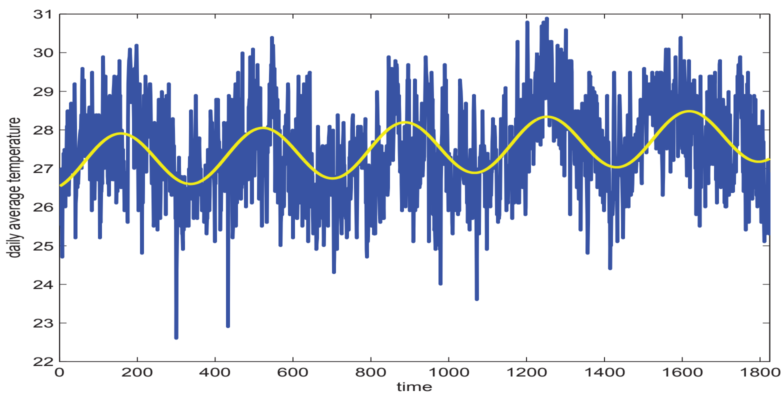

Figure 2.6: DATs in Petaling Jaya with fitted seasonal function.

Next, we eliminate the linear trend and seasonal components by subtracting the estimated $S(t)$ from the original observations and plot the autocorrelation (ACF) of residuals as in Figure 2.7. It shows the positive strong autocorrelations which rapidly decays toward zero. Noteworthy is that we do not observe any strong seasonal variation anymore in the ACF, which, despite a small seasonality pattern $S(t)$, has a clear impact on the ACF. By inspecting the partial ACF (PACF) plot in Figure 2.8, we observe a very high spike in lag 1, thus suggesting AR(1) to be the most preferable model explaining the evolution of the time series.

We estimate the parameter α_1 for the AR(1) process by using a simple linear regression of the detrended and deseasonalized data and found $\alpha_1 = 0.5895$. The α_1 value corresponds to the mean reversion rate where temperature reverts back to its long term mean at this speed. This indicates that the speed of mean reversion is rather fast. As suggested by Clewlow and

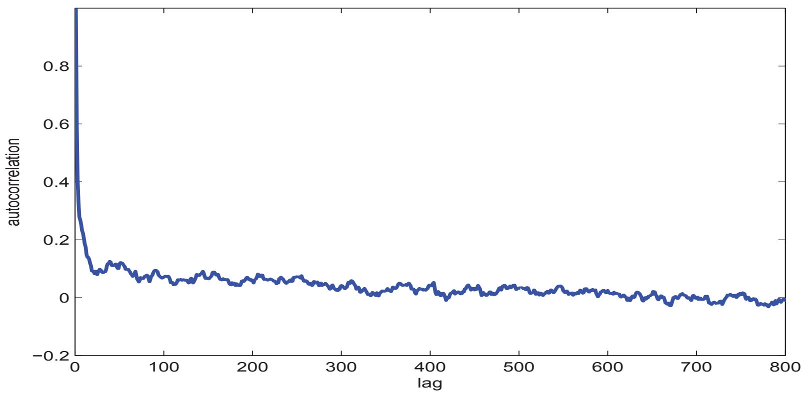

Figure 2.7: The ACF of the residuals of DATs after removing linear trend and seasonal component.

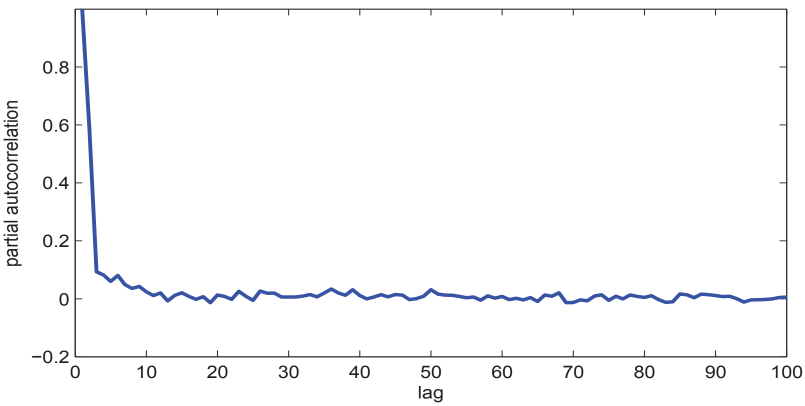

Figure 2.8: The PACF of the residuals of DATs after removing linear trend and seasonal component.

Strickland [34], the half life of Ornstein-Uhlenbeck process with mean reversion β driven by a Brownian motion is given by

$$T_\beta = \frac{\ln(2)}{\beta}. \tag{2.2.5}$$

Converting the speed of mean reversion of our time series dynamics into a continuous time mean reversion, we find $\beta = -\ln(\alpha) \approx 0.5284$. This implies an estimate for the half life of temperature dynamics being $T_\beta = 1.18$, meaning that on average the temperature takes 1.18 days to revert half-way back to its long term level.

Looking at the histogram of residuals in Figure 2.9, we may say that it follows the normal distribution. But the Kolmogorov-Smirnov statistics of 0.022 is significant at the 1% level, and we cannot reject the hypothesis of nonnormality of data. However, taking the large number of data into account, it is very hard to pass through a normality test. We find the normal distribution a satisfactory choice for the residuals. The residuals and squared residuals for the last 10 years are plotted in Fig. 2.10. Looking at the squared residuals, one may suspect the

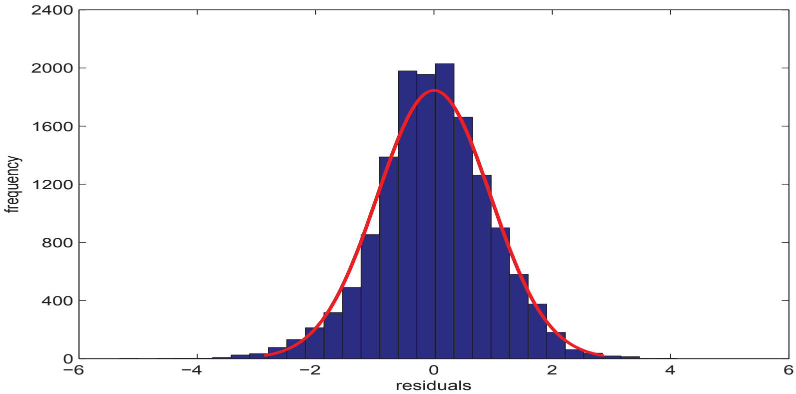

Figure 2.9: Histogram of the residuals of DATs after removing linear trend, seasonal component and AR(1).

existence of clustering, indicating some pattern in the residuals that may be modelled using a seasonal variance in line with Benth et al. [21], or stochastic volatility (see Benth and Saltyte Benth [20]). However, the effects seem to be minor, and we decided not to increase the level of sophistication of our model. The estimated standard deviation for the noise term $\epsilon(t)$ is 0.9571.

Temperature dynamics insurance pricing

From (2.1.2), we have the price of the contract with the conditional expected value, using the filtration generated by the time series $Y(t)$ in the dynamics of $T(t)$. We can simulate this conditional expectation by appealing to the Markov property of $Y(t)$. Thus, to find the price at time t, $P(t,\tau_1,\tau_2)$, we first simulate a path of the temperature $T(s)$ for times $s \le t$ (which means to simulate $Y(s)$, and then to add the seasonality function). Given this $T(t)$, we simulate N paths of $T(s)$ for $t \le s \le \tau_2$, and compute the index $X(\tau_1,\tau_2)$ for each path. Averaging over all the N realizations of $X(\tau_1,\tau_2)$, we obtain an estimate of $P(t,\tau_1,\tau_2)$. In this way, we have a mechanism which allows the insurance company to take current information about the weather into account, and thereby yielding a more accurate and detailed pricing technology.

Figure 2.11 illustrates the price evolution of the contract for January. To obtain this price path, we have conditioned on the actual observed temperatures $T(t)$ at the dates in question. Starting off the path simulations from these observed temperatures, we find the price paths which are wiggling rather than smooth curves. We started at 1 December 2010 and by simulating $n = 10,000$ paths of temperature dynamics, we obtained n indexes of $X(\tau_1,\tau_2)$. The indexes were then averaged and discounted in order to get its present value $P(t,\tau_1,\tau_2)$. This procedure was done for the following dates until 31 December 2010. We used a discounting factor $r = 0.00014$ (corresponding to 5% annual interest rate). This is at the level of the current interest rate in Malaysia. Since the price is calculated with regards to the current information about temperatures, the evolvement is no longer smoothly increasing.

We have plotted the obtained price path together with the price derived from the burn approach for comparison. We can clearly see that $P(t,\tau_1,\tau_2)$ is significantly higher than the prices

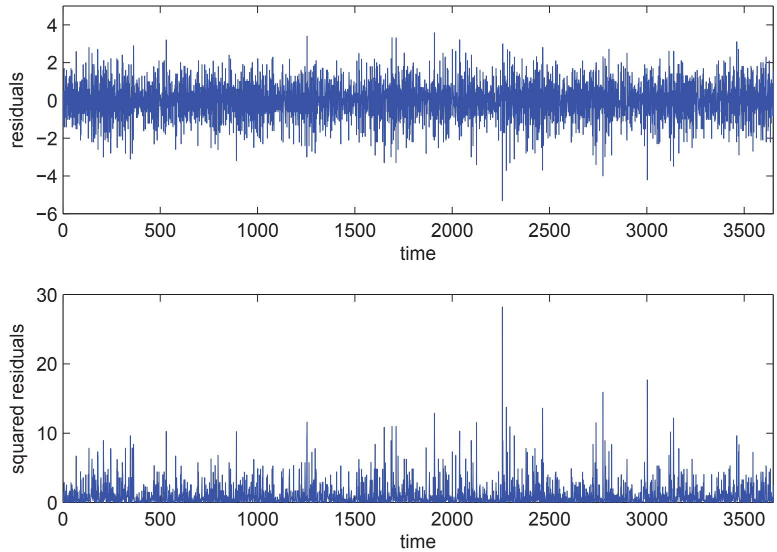

Figure 2.10: Residuals and squared residuals of DATs for the last 10 years after removing linear trend, seasonality component and AR(1).

obtained from the burn analysis. The price at 1 December 2010 computed by temperature modelling is higher than burn approach with a difference of 164.89. At 31 December 2010, the price obtained by temperature modelling deviates about 194.72 above burn. It is to be noted that the premia computed for the temperature approach is prone to Monte Carlo error. We have estimated this by repeating the simulation of premia 100 times for 1 December, 15 December and 31 December 2010 in order to find a numerical estimate of the confidence interval. We found very narrow confidence intervals of $[282.51, 282.77]$, $[294.68, 294.88]$ and $[324.64, 325.21]$ for the respective dates, significant at 5% confidence level.

The histogram of the claim size under temperature modelling is plotted in Figure 2.12 together with the empirical one obtained from burn approach. It is apparent that the claim size distribution resulting from temperature modelling has a mode, and that the exponential distribution seems to be a bad choice. One would rather imagine a lognormal distribution to be more appropriate. This is a first clear sign of the superiority of the temperature modelling approach, since it is able to reveal a much more detailed description of the claim size distribution. The histogram for the claims resulting from the burn analysis has some real big values (around 1000), and a collection of many small. The small values have too big probability compared to with the distribution from the temperature modelling approach, which results in a far lower insurance price. The temperature modelling approach produces an accurate view on the distribution of claims, and is not prone to a high degree of uncertainty. We get far better information on the tail probabilities of the claims, enabling us to get a probabilistic grip on extreme events. Due to the little data supporting the burn analysis estimates, one should put more trust into the temperature modelling.

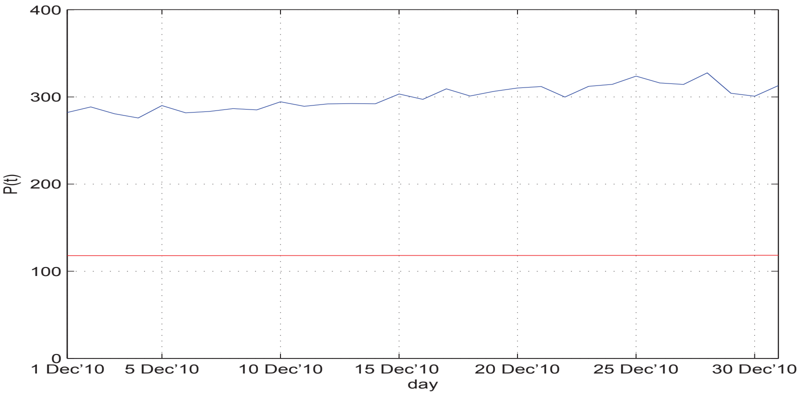

Figure 2.11: The movement of the price P for contract in January. The blue and red curve respectively represent price calculated by temperature modelling and burn approach.

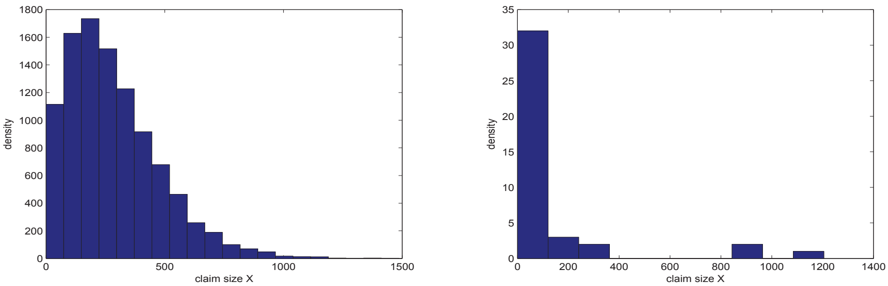

Figure 2.12: Claim size distribution from temperature modelling (left) and burn analysis (right).

2.3 What's in it for the farmer?

Consider a farmer who seeks to insure his crops in a period $[\tau_1, \tau_2]$ against the impact of extreme temperature levels. The decision to buy a weather index insurance will be based upon the size of the premium compared with the actual protection given by the insurance contract. In this section we analyse this protection as a function of the premium.

Based on the prices obtained in the previous section, we can imagine a farmer who has bought the insurance for a certain amount of premium. The premium is an expense that he must pay in advance to the insurer for obtaining the weather index protection. We will look at the distribution for his protection, and the probabilities of getting back money. In parallel, we also wish to find the probability of the money will exceed the premium paid.

The profit (total loss or gain) for the insured is denoted by L and defined by the difference between the total claim sizes $X(\tau_1, \tau_2)$ and the price $P(t, \tau_1, \tau_2)$ at time t. For simplicity, we suppose that $t = 0$, and recall that the premium we have to pay to purchase the insurance today will be $P(0, \tau_1, \tau_2) = \mathrm{e}^{-r\tau_2}\mathbb{E}[X]$. We can express the profit in a more convenient way by

$$L = X(\tau_1, \tau_2) - P(t, \tau_1, \tau_2).$$

We use the same definition for total loss or gain to the insurer but with opposite sign. The loss to the insured can be considered as gain to the insurer and vice versa. We will discuss loss or gain for an individual insured with a single coverage provided by a single contract. In principle, the fair premium holds

$$\mathrm{e}^{-r(\tau_2 - t)}\mathbb{E}[X(\tau_1, \tau_2) - P(t, \tau_1, \tau_2)|\mathcal{F}_t] = 0,$$

which implies that L is equal to zero in expectation. In reality, the insurance company will charge an additional risk loading on the premium, which will make the profit/loss function L have negative expectation (that is, the farmer will on average lose on the insurance contract). The reason is that the insurance contract will add a safety loading to the premium as a compensation for bearing the risk. We have designed some cases to investigate the probability distribution of L under various pricing regimes.

2.3.1 Insurance calculations

Suppose a farmer who wishes to protect his crops for adverse temperature events in the period of January 2011. He is entering a CDD-based weather index insurance contract for January 2011 "today", which we let be 1st of December 2010. At the end of January where the time equals τ_2, the claim for the particular month will be calculated. We use the initial price (the price on the 1st of December 2010) of $P_{burn}(\tau_1, \tau_2) = 117.75$ for burn approach and index modelling and $P_{temp}(\tau_1, \tau_2) = 282.64$ for temperature modelling. These are the 'fair prices' or the prices with no risk loading. With having $n = 10,000$ indexes of expected payoffs, we obtain the distribution of profit as shown in Figure 2.13. Clearly, the distributions have negative values meaning that there is a positive probability for loss.

The probability of loss for the farmer entering the insurance using the burn approach is

$$P\left(X(\tau_1, \tau_2) < P_{burn}(\tau_1, \tau_2)\right) = P\left(X(\tau_1, \tau_2) < 117.75\right) = 0.7958\,.$$

Hence, with an 80% chance, the farmer will receive nothing or less than what he has paid. With only approximately 20% chance he will actually receive more, meaning that if he renews his insurance contract every year, he will only in 1 out of 5 years receive more than he paid in premium that year. In the case of temperature modelling, the probability of loss becomes

$$P\left(X(\tau_1, \tau_2) < P_{temp}(\tau_1, \tau_2)\right) = P\left(X(\tau_1, \tau_2) < 282.64\right) = 0.5642\,.$$

Thus, there is only 56% chance of a loss, and approximately in 3 out of 7 years the farmer will receive a profitable income from the contract. The premium and the loss distribution are much more favourable for him. These considerations emphasize once more the differences in an approach resting on information from temperature series, and the burn analysis which is grounded on very little data. Of course, we need to contrast these probabilities with the actual

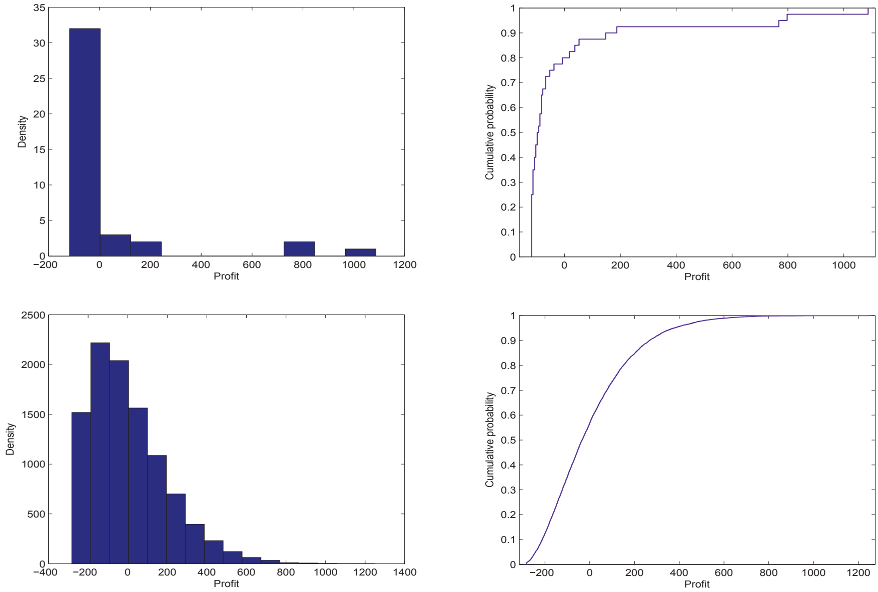

Figure 2.13: Top: Profit distribution and cumulative density for burn approach and index modelling. Bottom: Profit distribution and cumulative density for temperature modelling.

losses incurred in order to get the true picture of the value of this insurance for the farmer.

As already indicated, an insurance company will naturally incur a risk loading to the 'fair premium' as we have calculated above. One may say that the insurance company wants to "insure themselves", and add on to the fair premium such that they can control their risk of having to pay excessive amounts in claims. A standard way to do this is to introduce a safety loading which controls a quantile of their loss distribution. This resembles the value-at-risk concept in finance, and entails in putting a premium on the insurance contract such that their loss distribution from claims is within an acceptable probability.

To be more specific, we find the value-at-risk at a given significance level of the loss distribution for the insurance company under 'fair premium' of the contract. Given this level, we suppose that the insurance company charge a risk loading resembling a certain fraction of this level. Letting the significance level be 5%, we search a level γ such that

$$P(\text{Claim} \geq \gamma) = 0.05 \,.$$

The level γ will only be exceeded with 5% probability. For burn approach, we obtain $\gamma_{burn} = 900.00$ (the same value for index modelling) while for temperature modelling profit distribution, γ_{temp} is equal to 660.77. The insurance company will now add 5% of the estimated γ as risk loading to the fair price (risk loaded premium). With this new price, the probabilities of loss and gain for the farmer are presented in Table 2.2.

A risk loaded premium increases the probability of loss for the farmer, naturally. With the burn approach, the farmer will only receive a gain from the contract with 15% probability,

Table 2.2: Probability of loss and gain for 5% of risk loading

Method	γ	Price $P(\tau_1, \tau_2)$	$P(X < P(\tau_1, \tau_2))$	$P(X > P(\tau_1, \tau_2))$
Burn analysis	900.00	162.75	0.8478	0.1510
Temperature	660.77	315.68	0.6253	0.3729

or approximately once every 7 year. With the temperature modelling approach, the farmer receives a profit with 37% probability, which is slightly less than with the 'fair premium'. It is interesting to note that although the premium is significantly bigger with the temperature modelling approach, the contract seems more attractive than if we solely base our assessment on the burn analysis.

2.4 Conclusions

We have analysed three different pricing approaches for weather index insurance contracts in this paper. Weather index insurance has gained some attention in recent years as a way for farmers, say, in developing coutries to protect their crop. We have focused on contracts settled on temperature indexes in given months, using data collected in Malaysia as our empirical case, and we analyse cooling-degree indexes, which measures excessive temperatures which may harm the crop of a farmer.

As the claims in such weather index contracts are settled on a measurable objective index, and not on actual losses incurred, one may view the insurance contracts as weather derivatives. Motivated by theory on weather derivatives, we have considered the three pricing approaches *burn analysis*, *index modelling* and *temperature modelling*. The first two give similar results, and are based directly on computing historical values of the index in question. We argue that such an approach will rest on very thin data material. The temperature modeling approach, on the other hand, is based on the time series properties of temperature, from which one can compute the index. Usually, one has very rich sets of data for temperature.

We fitted a simple autoregressive time series model with seasonality for Malaysian daily average temperatures, based on 40 years of data. This gives us a very precise information on the dynamics of temperature, and hence, a very detailed description of the temperature index in a given period. In fact, simulating from the model, we can obtain a very detailed probabilistic information on the index, and therefore assess correctly the premium and probabilities of loss and profits.

The burn analysis and index modelling approaches have a high degree of uncertainty in their premium estimates. The premium estimated from the temperature modelling approach is prone to Monte Carlo error, on the other hand. Controlling for this, we find big differences in premia between the approaches. The temperature modelling approach has the additional advantage that it can account for current information of the weather situation, while this is not the case for the two other approaches.

Finally, we analysed the chances of receiving a profit from a weather index insurance contract. The chances are not very high using the burn analysis approach, but far better in the temperature modelling case. This is obviously a result of the very poor data set backing up the results in the burn analysis method. The temperature modelling methodology provides better foundation for making assessments on the probabilities of loss and profit, and for our case the insurance becomes more advantageous for the farmer despite the much higher premium.

As is standard in an insurance contract, the insured must pay a premium upfront in order to obtain a protection. Since farmers in developing countries are rather poor, this may seem like an unfair deal, since information and knowledge of these contracts naturally would be biased towards the insurance company. Since the weather index insurance contracts are closely related to temperature futures traded on the Chicago Mercantile Exchange (CME), the insurance companies may in principle hedge their risk. For the time being, no temperature futures are traded for cities in developing countries. However, one may imagine places where one could partially hedge the risk using other locations (see Barth et al. [12] for spatial hedging of temperature derivatives). This would reduce the risk for the insurance company, and thus the premium, and making such products more attractive. But still, one is left to wonder whether it is better for the farmers to actually save the same amount as the premium in a bank account, controlling the money themselves, rather than entering into swap deals as is the case here.

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