BENEFITS FROM USING HOME EQUITY IN RETIREMENT

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Preface

This thesis represents the completion of my studies at the University of Oslo's Department of Economics, qualifying to the degree Master of Economic Theory and Econometrics.

The experience has been both challenging and inspiring.

This thesis would have never been possible without the assistance and moral support by my supervisor Asbjørn Rødseth, professor at the Department of Economics. I am ever thankful for all the help through the process. I would like to express my gratitude to my girlfriend for helping me through the whole process. Her advice has been of great value to me. I give a big thank you to all my fellow students for five amazing years. To my family, thank you for the unlimited support along the way.

Abstract

This thesis evaluates the benefits and consequences of using home equity in a retirement plan in Norway. Three research questions are answered: “How does home equity affect consumption decisions?” “What level of risk is a pensioner exposed to trying to execute a retirement plan using home equity?” and “What value can one expect to extract from one’s home?” To answer the questions the thesis uses a modified version of a lifecycle model and tests it by simulation. The simulation method uses one thousand scenarios based on a new Keynesian framework of the Norwegian inflation targeting regime to simulate on. The simulation tests two types of cases. One where a pensioner has a high level of home equity and a low level of pension pay out and the other where the pensioner has a low level of home equity and a high level of pension pay out. Analysing the results from the simulation the thesis concludes that there are benefits of using home equity in a retirement plan if it is used correctly and in a moderate amount.

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1 Introduction

The topic for the thesis is retirement. From a newspaper article Barrow(2011) the recent financial crisis has created a new-era of 'HIPpies' that have dawned upon the British Isles. HIP stands for Home In Pension and HIPpies is name for the individuals in the Baby Boomer generation that are planning to use their home equity as a source for pension. Dwindling wealth from stock market losses and low interest rates have hit pension savings hard due to the financial crisis. As regular sources for pension have diminished the HIPpies want to use their home equity as a new source for pension. This is the case in Britain but what if this happens in Norway? In Mikalsen(2011) tells that in the recent years in Norway there has been a rise in availability and use of financial products categorized as home equity release. Home equity release products such as reverse mortgages help pensioners to tap into their home equity. The inspiration for the thesis is my curiosity on how a retirement plan might look like in the future given the recent changes in the soon to be retirees behaviour.

The thesis problem is to evaluate the **"benefits from using home equity in retirement"**. The thesis will use a Norwegian framework to evaluate the benefits and consequences of using home equity. Using lifecycle theory the thesis is going to model, test and analyse different strategies for consuming home equity and discover what risks might come with it. The thesis has a focus on Norway. To keep the problem simple enough to solve, the thesis is avoiding Norwegian taxes and other frictions such as transaction costs, specific laws etc but sticking to how Norwegian housing prices, interest rates and inflation relate to each other under an inflation-targeting regime.

The three main research questions are "how does home equity affect consumption decisions?", "what value can one expect to extract from one's home?" and "what level of risk is a pensioner exposed to trying to execute a retirement plan using home equity?". To answer these questions the thesis uses simulation methods. Programming in Matlab to simulate and using Stata and Excel to analyse the data.

The thesis studies one way to spend home equity. This is done by a pensioner that borrows money on his home's market value to spend on consumption whilst still living there. It's a way to use one's own home as a consumption asset. The household that will be studied is a pensioner who has just retired and makes a retirement plan for the next twenty years. The pensioner has two sources to finance consumption. The first source of income comes from a pension pay out like social security. The second source comes from the home equity of the pensioner's own home. This is the basis for the modified lifecycle model that will be used to analyse the pensioner's consumption behaviour.

The main literature for the theory used in the thesis comes from Romer(2006), Williamson(1999) and Gali(2008). Inspiration to learn simulation methods came from Ayres(2010). The idea to use simulation in the thesis came from Wei et al(2007). To understand the benefits of using lifecycle theory in a retirement plan came from Kotlikoff(2010).

The research for the thesis problem is presented in two parts. Part I explains the theoretical background for consumption, interest rates, inflation and rates of return of housing. The models presented are the basis for the simulation in Part II. Part II is about the simulation method, the results from it and the analysis of the results.

Part I

2 Retirement plan

To model a retirement plan one needs knowledge about the main determinants of a pensioner's consumption choices and savings behaviour. From lifecycle theory the choice between consumption today and savings for tomorrow is determined by income, the rates of return on savings and other constraints that the pensioner faces. The basic result from the Permanent-income Hypothesis is that the individual's lifetime resources are distributed over time based on the individual's preferences for smooth consumption.

A modified version of a basic lifecycle model is used to build a retirement plan model. The theory and solution methods come from Romer(2006) and Williamson(1999). The consumption plan is now a retirement plan and the household is a pensioner. The retirement plan model uses a standard lifetime utility function. The modification is in the budget constraint by adding a home equity asset, debt and an income coming from a pension pay out that follows inflation keeping the pensioner's purchasing power constant. The present value discount rate is the mortgage interest rate as this is the main interest rate the pensioner relates to when deciding to consume. The mortgage interest rate is the central bank's key policy rate plus a risk premium. The risk premium consists of the money market's mark up over the central bank's key rate plus a mortgage lending mark up over the money market rate. The model defines optimal behaviour for the pensioner when he is maximizing lifetime utility considering the budget constraint he faces. The simulation in later chapters is intended to check what can be expected from the optimal behaviour and what results the behaviour gives.

The utility function

Consider an individual who lives a remaining T years and has a lifetime utility U

$$U = E\left[\sum_{t=1}^{T} \beta^t u(C_t)\right] = E\left[\sum_{t=1}^{T} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}\right]$$

The instantaneous utility function $u(C_t)$ is the constant relative risk aversion (CRRA) utility function. t is which year it is. σ describes the amount of risk aversion of having different consumption between the years. β is the discount factor and its value is between zero and one. The lifetime utility function determines the value of consumption per year for the pensioner and is just a standard case of measuring lifetime utility of consumption.

The budget constraint

The individuals expected lifetime budget constraint today is

$$E_1\left[\sum_{t=1}^{T} \frac{P_t C_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right] \leq E_1\left[A_0 \frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0 + \sum_{t=1}^{T} \frac{P_t W_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$

The left hand side is expected lifetime consumption. The right hand side is expected lifetime resources which consist of wealth and income. It states that lifetime consumption must be equal of less than lifetime resources. This is known as the “No Ponzi Scheme” condition. $P_t$is the price level of consumption in year t. r_t^f is the central bank’s key rate[1]. The risk premium μ consists of the money[2] market’s mark up over the central bank’s key rate plus a mortgage lending mark up over the money market rate. The present value discount rate $\frac{1}{\prod_{t=1}^{T}(1+r_t^f+\mu)}$ is based on the interest rate for mortgages$(r_t^f+\mu)$. The background for using interest rate for mortgages is that to consume home equity one must take a loan that bears the interest rate for mortgages. $W_0$is a pension pay out from social security. The pension pay out follows the price level of consumption P_t keeping purchasing power for the pensioner constant each year[3]. A_0 is the value of home equity today[4]. r_t^h is the rate of return of housing. Home equity directly follows the changes in housing prices so $E_1[A_0 \prod_{t=1}^{T}(1+r_t^h)]$ is the expected home equity

[1] For the simulation we will be using the Norwegian central bank’s key rate
[2] For the simulation the money market is the NIBOR
[3] In a regular retirement plan a pension payout would have decreasing purchasing power over time. But I have chosen to model this as a constant to keep the problem simple enough to solve.
[4] $A_0$is only a part of his total home equity due to credit constraints. This is explained in chapter eight.

after T years. D_0 is debt collateralized in home equity and therefore the interest paid on it in year t is the mortgage rate $(1 + r_t^f + \mu)$. Inflation in year t is defined as $P_t = P_{t-1}(1 + \pi_t)$ $\Leftrightarrow \frac{P_t}{P_{t-1}} - 1 = \pi_t$. E_t is the expectation function for year t of the future. This is a simple expected lifetime budget constraint as it only follows expectations of future averages which could lead to that the "No Ponzi Scheme" condition is broken. This happens if the home equity is highly leveraged and there occurs a steep fall in housing prices leaving no wealth left to finance consumption. A full stochastic budget constraint the expectation function would take such an event into account leading to buffer stock saving behaviour[5] to avoid such an event.

Behaviour

The pensioner wants to maximize lifetime utility given the constraints he faces. The budget constraint is solved by equality since marginal utility of consumption is always positive[6]. The maximization problem is

$$\max_{C_t} U = \ E[\beta \frac{C_1^{1-\sigma}}{1-\sigma} + \cdots + \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} + \beta^{t+1} \frac{C_{t+1}^{1-\sigma}}{1-\sigma} + \cdots + \beta^T \frac{C_T^{1-\sigma}}{1-\sigma}]$$

$$\text{s.t } E_1\left[\sum_{t=1}^{T} \frac{P_t C_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right] = E_1\left[A_0 \frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0 + \sum_{t=1}^{T} \frac{P_t W_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$

The problem is solved by using the Euler equation. This is done by inserting the constraint into the utility function. The goal is to find how each year's consumption relates to each other. Using this relation to find which consumption choices maximizes the total lifetime utility.

The first step is to rewrite the budget constraint to get C_{t+1} on its own.

$$E_1\left[\sum_{t=1}^{T} \frac{P_t C_t}{\prod_{s=1}^{t}(1 + r_s^f + \mu)}\right] = E_1\left[A_0 \frac{\prod_{t=1}^{T}(1 + r_t^h)}{\prod_{t=1}^{T}(1 + r_t^f + \mu)} - D_0 + \sum_{t=1}^{T} \frac{P_t W_0}{\prod_{s=1}^{t}(1 + r_s^f + \mu)}\right]$$

[5] The thesis sticks to the simple expected lifetime budget constraint to keep the problem solvable.
[6] $u'(C) = C^{-\sigma} > 0, \forall C > 0$

$$E\left[\frac{P_{t+1}C_{t+1}}{\prod_{s=1}^{t+1}(1+r_s^f+\mu)}\right]$$

$$= E\left[-\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)} + A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0 + \sum_{t=1}^{T}\frac{P_tW_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=t+2}^{T}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=1}^{t-1}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$

$$E[C_{t+1}] = E\left[\frac{\prod_{s=1}^{t+1}(1+r_s^f+\mu)}{P_{t+1}}\left[-\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)} + A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0 + \sum_{t=1}^{T}\frac{P_tW_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=t+2}^{T}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=1}^{t-1}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]\right]$$

and using the relation $\frac{P_{t+1}}{P_t} = (1+\pi_{t+1})$ we get

$$E[C_{t+1}] = E_1\left[-C_t\frac{(1+r_{t+1}^f+\mu)}{(1+\pi_{t+1})} + \frac{\prod_{s=1}^{t+1}(1+r_s^f+\mu)}{P_{t+1}}\theta\right]$$

where θ

$$\theta = A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0 + \sum_{t=1}^{T}\frac{P_tW_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=t+2}^{T}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)} - \sum_{t=1}^{t-1}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}$$

Inserting for C_{t+1}into the maximization problem

$$\max_{C_t} U = E\left[\beta\frac{C_1^{1-\sigma}}{1-\sigma} + \cdots + \beta^t\frac{C_t^{1-\sigma}}{1-\sigma} + \beta^{t+1}\frac{1}{1-\sigma}\underbrace{\left(-C_t\frac{(1+r_{t+1}^f+\mu)}{(1+\pi_{t+1})} + \frac{\prod_{s=1}^{t+1}(1+r_s^f+\mu)}{P_{t+1}}\theta\right)^{1-\sigma}}_{C_{t+1}} + \cdots\right]$$

Then maximizing on C_t to find the consumption relation between C_t and C_{t+1}

$$\frac{\partial U}{\partial C_t} = E\left[\beta^t C_t^{-\sigma} - \beta^{t+1}\frac{(1 + r_{t+1}^f + \mu)}{(1 + \pi_{t+1})}C_{t+1}^{-\sigma}\right] = 0$$

Rewriting the equation gives the per year solution of how the year's consumption relate to each other.

$$E\left[C_t \underbrace{\left(\frac{\beta(1 + r_{t+1}^f + \mu)}{(1 + \pi_{t+1})}\right)^{\frac{1}{\sigma}}}_{\alpha_{t+1}}\right] = E[C_{t+1}]$$

The relation is that consumption is just a mark up α_{t+1} from the previous year's consumption.

Doing the same calculations for all values of $t = 1,2, \dots, T-1, T$ we find

$$C_1 \underbrace{\left(\frac{\beta(1 + r_2^f + \mu)}{(1 + \pi_2)}\right)^{\frac{1}{\sigma}}}_{\alpha_2} = E[C_2]$$

$$E\left[C_2 \underbrace{\left(\frac{\beta(1 + r_3^f + \mu)}{(1 + \pi_3)}\right)^{\frac{1}{\sigma}}}_{\alpha_3}\right] = E[C_3]$$

...

$$E\left[C_{t-1} \underbrace{\left(\frac{\beta(1 + r_t^f + \mu)}{(1 + \pi_t)}\right)^{\frac{1}{\sigma}}}_{\alpha_t}\right] = E[C_t]$$

...

$$E\left[C_{T-1} \underbrace{\left(\frac{\beta(1 + r_T^f + \mu)}{(1 + \pi_T)}\right)^{\frac{1}{\sigma}}}_{\alpha_T}\right] = E[C_T]$$

The relation of the α's tells that the pensioner's preference for consumption is that consumption should follow a consumption path determined by each year's α. So the last year $T$of consumption is just a mark up of the first year's consumption.

$$E[C_T] = [\alpha_T C_{T-1}] = E\left[\alpha_T \underbrace{\alpha_{T-1} C_{T-2}}_{C_{T-1}}\right] = E\left[\alpha_T \alpha_{T-1} \underbrace{\alpha_{T-2} C_{T-3}}_{C_{T-2}}\right] = \cdots = E[\alpha_T \alpha_{T-1} \dots \alpha_2] C_1$$

The pensioner will maximize his utility by following a consumption path determined by the α's which fit his budget constraint.

The mark up α_t determines the consumption path for the pensioner

$$\alpha_t = \left(\frac{\beta(1 + r_t^f + \mu)}{(1 + \pi_t)}\right)^{\frac{1}{\sigma}}$$

and α_t is determined by the real interest rate $\frac{(1+r_t^f+\mu)}{(1+\pi_t)}$, the discount rate β and the amount of risk aversion σ. The interest rate leads to the pensioner preferring consumption later since borrowing is more expensive and savings give a larger wealth later. Inflation makes consumption today cheaper compared to the future leading to the pensioner wanting to use more consumption today. The higher the σ the more equal consumption will be between the years. $\lim_{\sigma \to \infty} \frac{1}{\sigma} = 0$ which leads to $\alpha_t \to 1$. This gives that $C_{t-1} \cdot 1 = C_t$. If $\alpha_t > 1$ then the consumption path is increasing between the years C_{t-1} and C_t. If $\alpha_t < 1$ then consumption is decreasing between the years C_{t-1} and C_t. To determine consumption we must incorporate the preferences into the budget constraint. The pensioner starts by calculating first year consumption. To solve the choice of first year consumption some terms are rewritten.

So from the Euler equation consumption in year t following from the previous results can be rewritten to

$$C_1 = \underbrace{\alpha_1}_{=1} \cdot C_1, \quad C_2 = \alpha_2 \cdot \alpha_1 \cdot C_1 \quad \dots \quad C_t = \alpha_t \cdot \dots \cdot \alpha_2 \cdot \alpha_1 \cdot C_1 \quad \dots \quad , \alpha_1 = 1$$

And the price level can be modified such that inflation is incorporated

$$P_1 = (1 + \pi_1) P_0, \quad P_2 = (1 + \pi_2)(1 + \pi_1) P_0, \quad P_t = (1 + \pi_t) \cdot \dots \cdot (1 + \pi_1) P_0$$

Using the per year relation for consumption and the price level modification to rewrite the budget constraint we can now determine C_1. Starting by rewriting the left hand side of the budget constraint.

$$E_1\left[\sum_{t=1}^{T}\frac{P_t C_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]=E_1\left[\sum_{t=1}^{T}\frac{\prod_{s=1}^{t}(1+\pi_s)\,P_0 C_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$
$$=\left[\sum_{t=1}^{T}\frac{\prod_{s=1}^{t}(1+\pi_s)\,P_0\prod_{s=1}^{t}\alpha_s\,C_1}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]=P_0C_1\underbrace{\left[\sum_{t=1}^{T}\frac{\prod_{s=1}^{t}(1+\pi_s)\prod_{s=1}^{t}\alpha_s}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\gamma_1}$$

And then the right hand side

$$E_1\left[A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0+\sum_{t=1}^{T}\frac{P_t W_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$
$$=\underbrace{E_1\left[A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0+W_0\sum_{t=1}^{T}\frac{P_0\prod_{s=1}^{t}(1+\pi_s)}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\delta_1}$$

The budget constraint has been rewritten to

$$P_0C_1\underbrace{\left[\sum_{t=1}^{T}\frac{\prod_{s=1}^{t}(1+\pi_s)\prod_{s=1}^{t}\alpha_s}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\gamma_1}=\underbrace{E_1\left[A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0+W_0\sum_{t=1}^{T}\frac{P_0\prod_{s=1}^{t}(1+\pi_s)}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\delta_1}$$

Simplifying left hand side with $\gamma_1$and the right hand side with δ_1 we get

$$P_0C_1\gamma_1=\delta_1$$

$$\Leftrightarrow\; C_1=\frac{1}{P_0}\frac{\delta_1}{\gamma_1}$$

The consumption choice for the pensioner in the first year C_1 is determined by the expectation of interest rates, inflation, wages, rates of return of housing and preferences.

After consuming the first year's consumption, what is left to use in the next years? Changes in the economy force the pensioner to recalculate his retirement plan each year. So the calculations have to be done again. The values realised for the next year are the price level$P_2=P_1(1+\pi_2)$, the pension payout W_0P_2, the home equity value $A_1=\ A_0(1+r_1^h)$

and the debt value $D_1 = D_0\left(1 + r_1^f + \mu\right) - P_1 W_0 + P_1 C_1$. Also the changes in future expectations of the interest rates, inflation rates and rates of return of housing need to be accounted for[7]. Using these values the consumption in year two C_2 is

$$P_1 C_2 \underbrace{\left[\sum_{t=2}^{T} \frac{\prod_{s=2}^{t}(1+\pi_s)\prod_{s=2}^{t}\alpha_s}{\prod_{s=2}^{t}(1+r_s^f+\mu)}\right]}_{\gamma_2} = \underbrace{E_1\left[A_1 \frac{\prod_{t=2}^{T}(1+r_t^h)}{\prod_{t=2}^{T}(1+r_t^f+\mu)} - D_1 + W_0 \sum_{t=2}^{T} \frac{P_1 \prod_{s=2}^{t}(1+\pi_s)}{\prod_{s=2}^{t}(1+r_s^f+\mu)}\right]}_{\delta_2}$$

$$C_2 = \frac{1}{P_1}\frac{\delta_2}{\gamma_2}$$

The last year's consumption C_T will be

$$\delta_T = A_{T-1}\frac{(1+r_T^{h)}}{(1+r_T^f+\mu)} - D_{T-1} + W_{T-1}\frac{(1+\pi_T)}{(1+r_T^f+\mu)}$$

$$\gamma_T = \frac{(1+\pi_T)}{(1+r_T^f+\mu)}$$

$$C_T = \frac{1}{P_{T-1}} \frac{A_{T-1}\frac{(1+r_T^{h)}}{(1+r_T^f+\mu)} - D_{T-1} + W_{T-1}\frac{(1+\pi_T)}{(1+r_T^f+\mu)}}{\frac{(1+\pi_T)}{(1+r_T^f+\mu)}}$$

$$= \frac{1}{P_T} A_{T-1}(1+r_T^h) + D_{T-1}(1+r_T^f+\mu) + W_T$$

$$\Leftrightarrow C_T P_T = A_{T-1}(1+r_T^h) - D_{T-1}(1+r_T^f+\mu) + W_T$$

In the last year all of the remaining wealth is consumed.

Home equity process

$$E_1\left[\sum_{t=1}^{T}\frac{P_t C_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right] \le E_1\left[\underbrace{A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)} - D_0}_{home\ equity\ process} + \sum_{t=1}^{T}\frac{P_t W_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$

The way home equity is consumed is incorporated in the budget constraint by the home equity process.

[7] How this works is explained in the chapter 5

$$A_0 \frac{\prod_{t=1}^{T}(1 + r_t^h)}{\prod_{t=1}^{T}(1 + r_t^f + \mu)} - D_0$$

Where the debt changes each year by $D_{t-1}(r_t^f + \mu) - P_t W_0 + P_t C_t$

The consumption in year t that comes from borrowing on home equity C_t^h is

$$C_t^h = \frac{P_t C_t - P_t W_0}{P_t} = C_t - W_0$$

If C_t^h is negative than the pensioner is using parts of his year's consumption C_t to pay down on his debt and if C_t^h is positive then part of his consumption in year t comes from his home equity.

In chapter eight the retirement plan model is used to test home equity consumption by simulation methods.

3 Inflation and interest rates

To determine inflation rates, interest rates and rates of return of housing for the retirement plan it is necessary to understand how these variables relate and how they are expected to behave in the economy.

The model for the economy is based on an inflation-targeting regime. The economy is exposed to supply and demand shocks. Using a new Keynesian framework from Gali(2008) and Larsson(2012) to explain the dynamics of inflation, interest rates and the output gap and how these values relate to each other.

We start off with the New Keynesian Phillips Curve (NKPC)

$$(1) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

It states that inflation today is determined by the discounted future expectations of inflation in the next year plus the output gap and how much it affects inflation is determined by the κ plus a supply shock u_t. The value of κ is derived from how firms price setting react to inflation.

The next equation is the dynamic investment saving curve (DIS)

$$(2) \quad x_t = E_t x_{t+1} - \frac{1}{\sigma}(r_t^f - E_t \pi_{t+1} - \rho) + e_t$$

It states that the output gap is determined by the expectation of the output gap in the next year minus the deviation of the real interest rate from the central bank's real interest rate goal ρ. The affect on the output gap is determined by the amount of risk aversion σ plus a demand shock.

$$(3)\ u_t = \mu_u u_{t-1} + \hat{u}_t, \quad E(\hat{u}_t) = 0$$

The supply shock u_t is modelled as an AR1 process where μ_u tells how much of the shock from the previous year is taken into this year and $\hat{u}_t$is a random normally distributed variable with an expectation of zero.

$$(4)\ e_t = \mu_e e_{t-1} + \hat{e}_t, \quad E(\hat{e}_t) = 0$$

The demand shock e_t is modelled as an AR1 process where μ_e tells how much of the shock from the previous year is taken into this year and $\hat{e}_t$ is a random normally distributed variable with an expectation of zero.

$$(5) r_t^f = \rho + \emptyset_\pi \pi_t + \emptyset_x x_t$$

The last equation is the interest rate rule which describes how the central bank reacts to inflation and the output gap. ρ is the basis real interest rate which the central bank sets as a goal and $\emptyset_\pi$ and $\emptyset_x$are the weights the central bank puts on inflation and the output gap. It is important for an inflation targeting regime that the Taylor principle holds meaning $\emptyset_\pi > 1$ otherwise the central bank might not be able to control inflation.

The equations all relate together and therefore it is needed to see how inflation, the output gap and the interest rate react to supply and demand shocks. To find this the equations must be solved with the method of undetermined coefficients. This is done solve the differential equations by "a lucky guess" Larsson(2012) of how the endogenous variablesπ_t, x_t and the future endogenous variables$E_t\pi_{t+1}$, $E_t x_{t+1}$ relate to shocks.

$$\pi_t = \psi_1 e_t + \psi_2 u_t$$

$$x_t = \alpha_1 e_t + \alpha_2 u_t$$

$$E_t \pi_{t+1} = \psi_1 \mu_e e_t + \psi_2 \mu_u u_t$$

$$E_t x_{t+1} = \alpha_1 \mu_e e_t + \alpha_2 \mu_u u_t$$

Presuming the economy relates in this matter it is necessary to find the values of the four coefficients ψ_1, ψ_2, α_1 and α_2.

Inserting equation (5) into equation (2) and the guesses into equation (1) and (2).

$$(1^*) \quad \psi_1 e_t + \psi_2 u_t = \beta(\psi_1 \mu_e e_t + \psi_2 \mu_u u_t) + \kappa(\alpha_1 e_t + \alpha_2 u_t) + u_t$$

$$\begin{aligned}(2^*) \quad \alpha_1 e_t + \alpha_2 u_t \\ &= \alpha_1 \mu_e e_t + \alpha_2 \mu_u u_t \\ &- \frac{1}{\sigma}(\rho + \emptyset_\pi(\psi_1 e_t + \psi_2 u_t) + \emptyset_x(\alpha_1 e_t + \alpha_2 u_t) - \psi_1 \mu_e e_t - \psi_2 \mu_u u_t - \rho) \\ &+ e_t\end{aligned}$$

Rewriting the hand side of the equations (1^*) and (2^*)

$$\psi_1 e_t + \psi_2 u_t = \underbrace{(\beta\psi_1\mu_e + \kappa\alpha_1)}_{\psi_1} e_t + \underbrace{(\beta\psi_2\mu_u + \kappa\alpha_2 + 1)}_{\psi_2} u_t$$

$$\begin{aligned}&\alpha_1 e_t + \alpha_2 u_t \\ &= \underbrace{(\alpha_1\mu_e - \frac{1}{\sigma}(\emptyset_\pi\psi_1 + \emptyset_x\alpha_1 - \psi_1\mu_e) + 1)}_{\alpha_1} e_t \underbrace{+(\alpha_2\mu_u - \frac{1}{\sigma}(\emptyset_\pi\psi_2 + \emptyset_x\alpha_2 - \psi_2\mu_u)\, u_t}_{\alpha_2}\end{aligned}$$

We now have that

$$\psi_1 = \beta\psi_1\mu_e + \kappa\alpha_1$$

$$\psi_2 = \beta\psi_2\mu_u + \kappa\alpha_2 + 1$$

$$\alpha_1 = \alpha_1\mu_e - \frac{1}{\sigma}(\emptyset_\pi\psi_1 + \emptyset_x\alpha_1 - \psi_1\mu_e) + 1$$

$$\alpha_2 = \alpha_2\mu_u - \frac{1}{\sigma}(\emptyset_\pi\psi_2 + \emptyset_x\alpha_2 - \psi_2\mu_u)$$

After some algebra we get (see appendix for this)

$$\psi_1 = \frac{\kappa\sigma}{\kappa(\emptyset_\pi - \mu_e) + (1 - \beta\mu_e)[\sigma(1 - \mu_e) + \emptyset_x]}$$

$$\psi_2 = \frac{[\sigma(1 - \mu_u) + \emptyset_x]}{\kappa(\emptyset_\pi - \mu_u) + (1 - \beta\mu_u)[\sigma(1 - \mu_u) + \emptyset_x]}$$

$$\alpha_1 = \frac{\sigma(1-\beta\mu_e)}{\kappa(\emptyset_\pi - \mu_e) + (1-\beta\mu_e)[\sigma(1-\mu_e) + \emptyset_x]}$$

$$\alpha_2 = \frac{(\mu_u - \emptyset_\pi)}{\kappa(\emptyset_\pi - \mu_u) + (1-\beta\mu_u)[\sigma(1-\mu_u) + \emptyset_x]}$$

To mimic the Norwegian inflation targeting regime and the Norwegian economy the relations need some values. From Norges Bank(2012) a "normal" interest rate that the Norwegian central bank aims to follow for maintaining financial stability is an interest rate of 4% and the central banks inflation target is 2,5%. This gives that $\rho = 0{,}04 - 0{,}025 = 0{,}015$. Using regular values for simulation from Gali(2008) the rest of the interest rate rule has been set to $\emptyset_\pi = 1{,}5$ and $\emptyset_x = 0{,}125$ and the values for the rest of the economy are $\kappa = 0{,}1$, $\beta = 0{,}99$ and $\sigma = 1$. This gives the coefficients for the inflation and output gap are: $\psi_1 = 0{,}27$, $\psi_2 = 1{,}7$, $\alpha_1 = 1{,}38$, $\alpha_2 = -1{,}37$. For the supply and demand shocks u_t, e_t the $\mu_e = \mu_u = 0{,}5$ and $\hat{u}_t, \hat{e}_t$ are random variables drawn[8] from a normal distribution with zero mean and a standard deviation of 0,01. The simulation later on uses these values plus a basis $\Delta = 2{,}5\%$ on inflation and the interest rate to mimic scenarios that could out fold under a Norwegian inflation targeting regime. The present value discount rate in the retirement plan is the loan rate for mortgages with consisted of the central bank's key rate and a risk premium. The risk premium μ has been set to 1,5% over the central bank rate. The number 1,5% is based on an average difference of 0,6% between the NIBOR and the central bank rate[9] using data from SSB(Statistics Norway) from the years 1998-2011 and an average difference from the year 1998-2012 of 0,95% between the NIBOR rate and the mortgage rate Bache(2012). It has been rounded to 1,5% to keep an even number. The new equations for mimicking the Norwegian inflation targeting regime are

$$\pi_t = \psi_1 e_t + \psi_2 u_t + \Delta = 0{,}27(0{,}5e_{t-1} + \hat{e}_t) + 1{,}7(0{,}5u_{t-1} + \hat{u}_t) + 0{,}025$$

$$x_t = \alpha_1 e_t + \alpha_2 u_t = 1{,}7(0{,}5e_{t-1} + \hat{e}_t) - 1{,}37(0{,}5u_{t-1} + \hat{u}_t)$$

$$\begin{aligned} r_t^f &= \rho + \emptyset_\pi(\pi_t - \Delta) + \emptyset_x x_t + \Delta \\ &= 0{,}015 + 1{,}5\big(0{,}27(0{,}5e_{t-1} + \hat{e}_t) + 1{,}7(0{,}5u_{t-1} + \hat{u}_t)\big) \\ &\quad + 0{,}125\big(1{,}7(0{,}5e_{t-1} + \hat{e}_t) - 1{,}37(0{,}5u_{t-1} + \hat{u}_t)\big) + 0{,}025 \end{aligned}$$

[8] In a more robust model the values for the shocks are most likely different from the ones I have used. This short cut was done because of the lack of time and knowledge to calculate them.

[9] The statistics from SSB(Statistics Norway): Nibor (3 mnd. Effektiv) and Norges Bank foliorente(styringsrenten)

The thesis uses a definition of a "normal state" in the economy. This is when the shocks in the economy are equal to zero. This means that inflation is 2,5%, the output gap is 0% and the central bank's key rate is 4%.

The equations for the mimicking of the Norwegian inflation targeting regime will be used later on in the simulation to create data sets on different scenarios on how the economy evolves over time. This will be discussed more thoroughly in chapter seven.

4 Rates of return of housing

The last undefined variable in the retirement plan is the rates of return of housing. The rates of return of housing have been modelled to be dependent on the inflation rate, interest rate and the output gap. To find the relation between the variables a simple regression[10] is done. The data for the regression is based on data from Norges Bank in the years 1980-2010. The output gap is calculated as the difference from trend growth of GDP over the last thirty one years[11]. The results from the regression are presented in *Table 1 Regression of how rates of return of housings are affected by the output gap, inflation and the deposit rates*.

Table 1: Regression of how rates of return of housings are affected by the output gap, inflation and the deposit rates

```
. regress  house_price output_gap inflation deposit_rates
```

Source	SS	df	MS
Model	.115126464	3	.038375488
Residual	.191010391	27	.007074459
Total	.306136855	30	.010204562

Number of obs	= 31
F(3, 27)	= 5.42
Prob > F	= 0.0047
R-squared	= 0.3761
Adj R-squared	= 0.3067
Root MSE	= .08411

house_price	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
output_gap	1.282161	.8323003	1.54	0.135	-.4255779	2.9899
inflation	2.319679	.6041991	3.84	0.001	1.079964	3.559393
deposit_ra~s	-2.005072	.6911823	-2.90	0.007	-3.423261	-.5868834
_cons	.104524	.0344153	3.04	0.005	.0339095	.1751384

Rates of return of housings are positively correlated with the output gap. If the economy is in a boom the output gap is positive. The economic pressure will lead to higher rates of return of housing. If the economy is in a trough the negative output gap gives lower rates of return of housing. Inflation works positively for the home equity value and the interest rate works negatively. Checking the regression up against a normal state for the Norwegian economy

[10] There is a high level of multicolinearity in this regression but to create a perfect model to determine future rates of return of housing is enough work to write a new thesis on.

[11] This short cut was done because of the lack of time and knowledge to calculate the output gap

meaning that the shocks in the economy are equal to zero and the inflation rate is 2,5% the interest rate is 4% and the output gap is zero we can find the rate of return of housing in a normal state r_n^f.

$$r_n^f = 0{,}104 + 2{,}319 * 0{,}025 - 2{,}005 * 0{,}04 = 0{,}0823$$

So in a normal state year there will be 8,23% rates of return of housing[12] which is close to the average yearly rates of return of housing of 8,4% per year over the last thirty one years. Simulating the rates of return of housing there has been added a random variablev_t. It is defined as the rates of return of housing shock which is a normally distributed variable with an expectation of zero and a standard deviation of 0,01 which is a bit higher than the result from the regression of a standard deviation of 0.007. So the rates of return of housing are determined at each date in the simulation by interest rate, inflation rate and output gap plus a rate of return of housing shock shown in the equation below.

$$r_t^h = \beta_0 + \beta_1 x_t + \beta_2 \pi_t + \beta_3 r_t^f + v = 0{,}104 + 1{,}282 x_t + 2{,}319 \pi_t - 2{,}005 r_t^f + v$$

5 Modelling expectations

In this chapter we will be answering how the variables $E_1\left[\prod_{t=1}^{T}\left(1 + r_t^f + \mu\right)\right], E_1\left[\prod_{t=1}^{T}\left(1 + r_t^h\right)\right]$ and $E_1[\prod_{s=1}^{T}(1 + \pi_s)]$ are determined in the pensioner's budget constraint. The pensioner bases his expectations on the equations from the Norwegian inflation-targeting regime and the rates of return of housing from the beta values from the regression in the previous chapters.

The equations

We introduce a expectation functionE_t. E_t is the expectation function in year t of a future values in years $t + s, s = 1,2,3,$ .... The pensioner starts by finding the expectation of future values of the supply and demand shocks.

Starting with the demand shock

$$u_t = \mu_u u_{t-1} + \hat{u}_t, \quad E(\hat{u}_t) = 0$$

Then the expectation of the demand shock in year $t + s$ is

[12] In the simulation later on the average rate of return of housing is 8,23% and the average inflation rate is 2,5%. A predicted real housing price growth of 5,73% is very high and probably unrealistic but as stated in footnote 20 the calculations are a short cut.

$$E_t u_{t+s} = E_t[\mu_u u_{t+s-1} + \hat{u}_{t+s}] = E_t[\mu_u u_{t+s-1}] + \underbrace{E(\hat{u}_t)}_{=0} = E_t\left[\mu_u\left[\mu_u u_{t+s-2} + \underbrace{\hat{u}_{t+s-1}}_{=0}\right]\right]$$

$$= E_t[\mu_u^2 u_{t+s-2}] = \cdots = {\mu_u}^s u_t$$

Doing the same for the supply shock

$$e_t = \mu_e e_{t-1} + \hat{e}_t, \quad E(\hat{e}_t) = 0$$

$$E_t e_t = E_t[\mu_e e_{t+s-1} + \hat{e}_{t+s}] = E_t[\mu_e e_{t+s-1}] + \underbrace{E(\hat{e}_t)}_{=0} = E_t[\mu_e e_{t+s-1}] = \cdots = {\mu_e}^s e_t$$

The expectation of future demand and supply shocks are just echoes of the shocks today because of the AR1 process. In the distant future the shocks will go to zero since $0 < \mu_e, \mu_u < 1$. Using the equations for the expectations of shocks we can find the expectations for the inflation rate, output gap, the central bank's key rate and the rates if return of housing.

The expectations for the future inflation rate in the years $t + s$

$$\pi_t = \psi_1 e_t + \psi_2 u_t + \Delta$$

$$E_t \pi_{t+1} = \psi_1 \mu_e e_t + \psi_2 \mu_u u_t + \Delta$$

...

$$(6) E_t \pi_{t+s} = \psi_1 {\mu_e}^s e_t + \psi_2 {\mu_u}^s u_t + \Delta$$

The future expectations for the output gap in the years $t + s$

$$x_t = \alpha_1 e_t + \alpha_2 u_t$$

$$E_t x_{t+1} = \alpha_1 \mu_e e_t + \alpha_2 \mu_u u_t$$

...

$$(7) E_t x_{t+t} = \alpha_1 {\mu_e}^s e_t + \alpha_2 {\mu_u}^s u_t$$

The expectations for the central bank's key rate in the years $t + s$

$$r_t^f = \rho + \emptyset_\pi(\pi_t - \Delta) + \emptyset_x x_t + \Delta$$

$$E_t r_{t+1}^f = \rho + \emptyset_\pi(E_t \pi_{t+1} - \Delta) + \emptyset_x E_t x_{t+1} + \Delta$$

...

$$(8)\quad E_t r^f_{t+s} = \rho + \emptyset_\pi (E_t \pi_{t+s} - \Delta) + \emptyset_x E_t x_{t+s} + \Delta$$

And using (6), (7) and (8) to define the expectations for the rates of return of housing in the years $t + s$

$$r^h_t = \beta_0 + \beta_1 x_t + \beta_2 \pi_t + \beta_3 r^f_t + v_1$$

$$E_t r^h_{t+1} = \beta_0 + \beta_1 E_t x_{t+1} + \beta_2 E_t \pi_{t+1} + \beta_3 E_t r^f_{t+1}$$

...

$$E_t r^h_{t+s} = \beta_0 + \beta_1 E_t x_{t+s} + \beta_2 E_t \pi_{t+s} + \beta_3 E_t r^f_{t+s}$$

Example

Using an example to illustrate how expectations work. There is a supply shock u_1=0,01 and a demand shock of $e_1 = 0{,}02$ and a housing price shock of $v_1 = -0{,}03$ today and $\mu_u = \mu_e = 0{,}5$. *Table 2 Example of how demand, supply and rates of return of housing shocks affect the economy* presents the pensioner's expectations. The left hand side of the table is which year. Year 1 denotes today. The rest of the years describe what the pensioner expects future values to be.

Table 2 Example of how demand, supply and rates of return of housing shock affect the economy

	Shocks			Variables			
	Demand shock e	Supply Shock u	Rates of return on housing shock v	Output gap	Inflation rate	Interest rate	Rates of return on housing
1	0,0200	0,0100	-0,03	0,014	0,048	0,077	0,048
2	0,0100	0,0050	0	0,007	0,036	0,059	0,080
3	0,0050	0,0025	0	0,003	0,031	0,049	0,081
4	0,0025	0,0013	0	0,002	0,028	0,045	0,082
5	0,0013	0,0006	0	0,001	0,026	0,042	0,082
6	0,0006	0,0003	0	0,000	0,026	0,041	0,082
7	0,0003	0,0002	0	0,000	0,025	0,041	0,082
8	0,0002	0,0001	0	0,000	0,025	0,040	0,082
9	0,0001	0,0000	0	0,000	0,025	0,040	0,082
10	0,0000	0,0000	0	0,000	0,025	0,040	0,082
11	0,0000	0,0000	0	0,000	0,025	0,040	0,082

12	0,0000	0,0000	0	0,000	0,025	0,040	0,082
13	0,0000	0,0000	0	0,000	0,025	0,040	0,082
14	0,0000	0,0000	0	0,000	0,025	0,040	0,082
15	0,0000	0,0000	0	0,000	0,025	0,040	0,082
16	0,0000	0,0000	0	0,000	0,025	0,040	0,082
17	0,0000	0,0000	0	0,000	0,025	0,040	0,082
18	0,0000	0,0000	0	0,000	0,025	0,040	0,082
19	0,0000	0,0000	0	0,000	0,025	0,040	0,082
20	0,0000	0,0000	0	0,000	0,025	0,040	0,082

The table shows that the initial shocks die out over time and the output gap, inflation rate, interest rate and the rates of return of housing return to their normal state values of 0%, 2.5%, 4.0%, 8.2%. In the budget constraint this determines the values for

$$E_1\left[\prod_{t=1}^{T}\left(1+r_t^f+\mu\right)\right]=\underbrace{(1+0{,}077+0{,}015)}_{t=1}\cdot\underbrace{(1+0{,}059+0{,}015)}_{t=2}\cdot\ldots\cdot\underbrace{(1+0{,}04+0{,}015)}_{t=20=T}$$

$$E_1\left[\prod_{s=1}^{T}(1+\pi_s)\right]=\underbrace{(1+0{,}048)}_{t=1}\cdot\underbrace{(1+0{,}036)}_{t=2}\cdot\ldots\cdot\underbrace{(1+0{,}025)}_{t=20=T}$$

$$E_1\left[\prod_{t=1}^{T}\left(1+r_t^h\right)\right]=\underbrace{(1+0{,}048)}_{t=1}\cdot\underbrace{(1+0{,}080)}_{t=2}\cdot\ldots\cdot\underbrace{(1+0{,}082)}_{t=20=T}$$

The next year the economy is hit by new shocks $\hat{u}_2=-0{,}02$, $\hat{e}_2=0{,}03$ and $v_2=-0{,}01$. The demand shock e_2 and supply shock u_2 in year two are

$$e_2=\mu_e e_1+\hat{e}_2=0{,}5*0{,}02+0{,}03=0{,}04$$

$$u_2=\mu_u u_1+\hat{u}_2=0{,}5*0{,}01-0{,}02=-0{,}015$$

So the pensioner has to recalculate his expectations of the future values for the interest rate, inflation rate and the rates of return of housing.

Part II

6 Why simulation?

The way I have used simulation methods is in two computer programs. The computer programs are written to use the theory presented in chapter two to five. The first program simulates scenarios of the economy that is used in the second program to test how a retirement plan works in the different scenarios.

The reason why I chose simulation methods to test the retirement plan model is that simulation can reveal problems and calculate levels of risk in the model. If the model is often unsuccessful it could be a bad representation of what real behaviour could be. The choice of simulating on “normal” values avoiding extreme outcomes is because these states of nature are rare and would not contribute to the analysis. The simulation provides results that can be analysed to answer the questions that were stated in the introduction.

7 The simulation of the rates of return

I have used the framework from chapters three and four to simulate one thousand different scenarios on how the economy develops over a time path of twenty years. Using the four equations stated below to determine the inflation rate, output gap, interest rate and the rates of return of housing in each year

$$\pi_t = \psi_1 e_t + \psi_2 u_t + \Delta = 0{,}27(0{,}5e_{t-1} + \hat{e}_t) + 1{,}7(0{,}5u_{t-1} + \hat{u}_t) + 0{,}025$$

$$x_t = \alpha_1 e_t + \alpha_2 u_t = 1{,}7(0{,}5e_{t-1} + \hat{e}_t) - 1{,}37(0{,}5u_{t-1} + \hat{u}_t)$$

$$r_t^f = \rho + \emptyset_\pi(\pi_t - \Delta) + \emptyset_x x_t + \Delta$$
$$= 0{,}015 + 1{,}5\big(0{,}27(0{,}5e_{t-1} + \hat{e}_t) + 1{,}7(0{,}5u_{t-1} + \hat{u}_t)\big)$$
$$+ 0{,}125\big(1{,}7(0{,}5e_{t-1} + \hat{e}_t) - 1{,}37(0{,}5u_{t-1} + \hat{u}_t)\big) + 0{,}025$$

$$r_t^h = \beta_0 + \beta_1 x_t + \beta_2 \pi_t + \beta_3 r_t^f + v = 0{,}104 + 1{,}282x_t + 2{,}319\pi_t - 2{,}005r_t^f + v$$

For each scenario the economy was hit each year by demand shocke_t, supply shock u_t and a rate of return of housing shock v_t. In each year three independent normally distributed variables $\hat{e}_t, \hat{u}_t, v_t$ with a zero mean and standard deviation of 0,01 are drawn. Remember[13] that the demand shock and supply shock are an AR1 processes meaning that $\hat{e}_t$ and $\hat{u}_t$ just add to the previous shocks whilst v_t is an independent shock.

Example of a scenario

Figure 1 Variables from a scenario of a simulated path of the economy over twenty years shows the values each year of the output gap, inflation rate, interest rate and the rates of return of housing from one scenario that was simulated. *Figure 2 The associated shocks to the scenario* shows the value of the randomly drawn shocks $\widehat{e_t}, \widehat{u_t}, v_t$ each year that belong to these equations.

$$u_t = \mu_u u_{t-1} + \hat{u}_t = 0{,}5u_{t-1} + \hat{u}_t, \quad E(\hat{u}_t) = 0$$

$$e_t = \mu_e e_{t-1} + \hat{e}_t = 0{,}5e_{t-1} + \hat{e}_t, \quad E(\hat{e}_t) = 0$$

[13] Demand shock $e_t = \mu_e e_{t-1} + \hat{e}_t$, Supply shock $u_t = \mu_u u_{t-1} + \hat{u}_t$,

Figure 1 Variables from a sample of a simulated path of the economy over twenty years

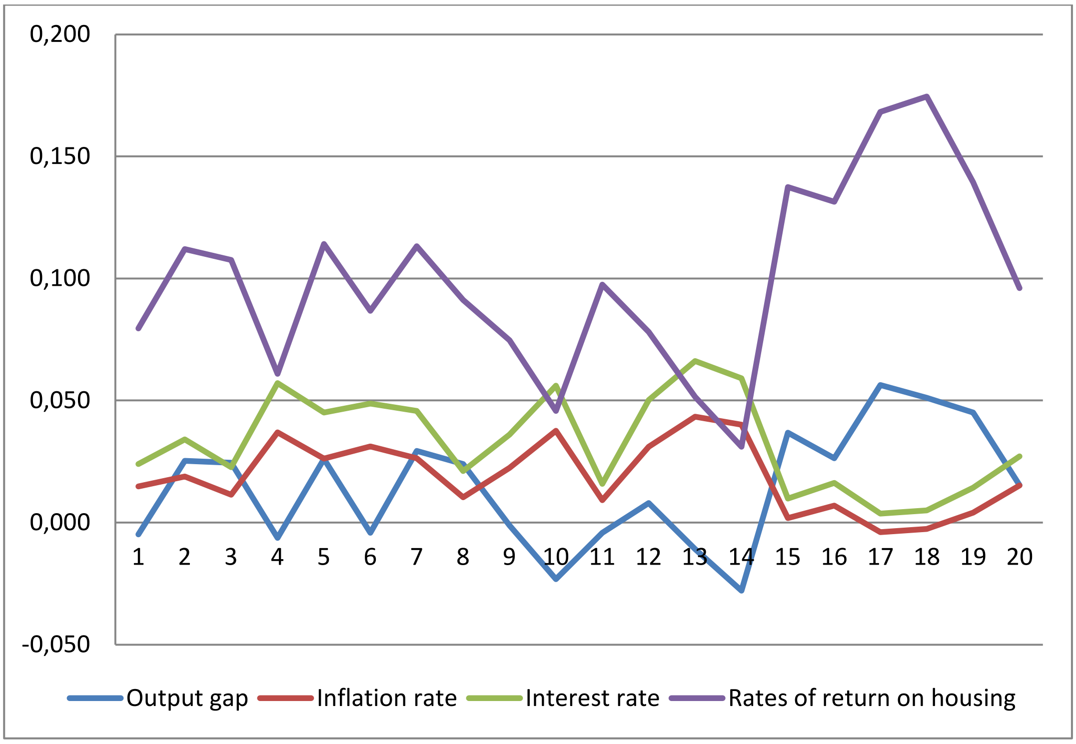
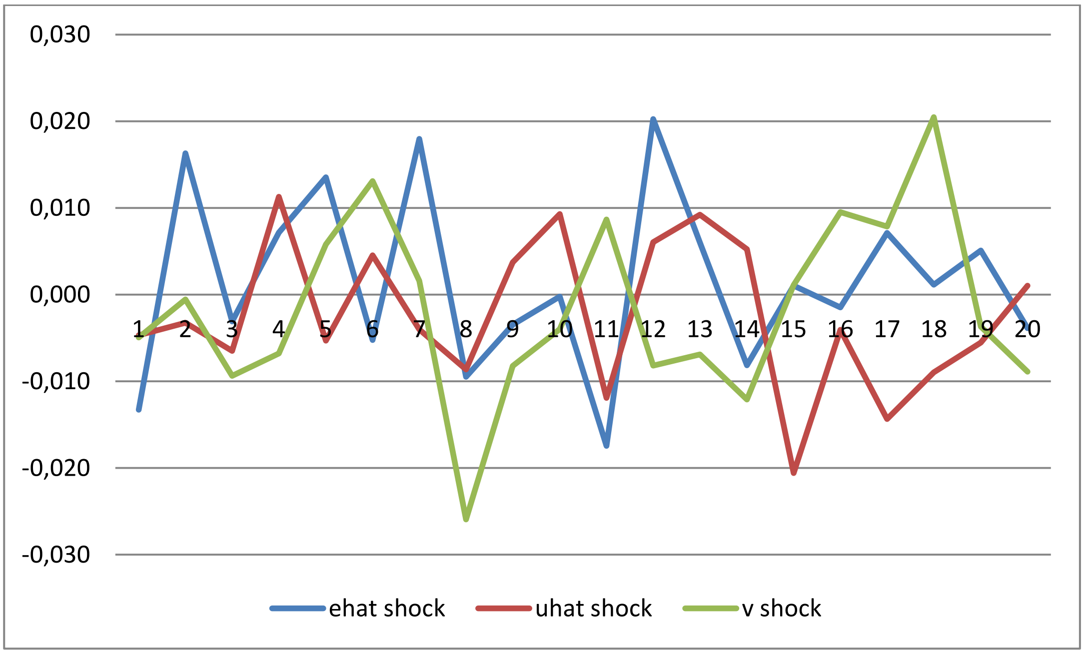


Figure 2 The associated shocks to the scenario



Comparison

To get a perspective on how relevant the simulated data is I have compared the data to data from Norges Bank of historic values of the Norwegian economy from 1980-2010 in a *Table 3: Comparison between the historic and simulated interest rates, inflation rates, the output gap and rates of return of housing*.

Table 3: Comparison between the historic and simulated interest rates, inflation rates, the output gap and rates of return of housing.

Interest rate	**Min**	**Max**	**Average**	**Std**
Simulated	0	0,1438	0,0405	0,0221
Historic 1980-2010	0,0131	0,1106	0,0591	0,0293

Inflation rate	**Min**	**Max**	**Average**	**Std**
Simulated	-0,0414	0,1038	0,0251	0,0173
Historic 1980-2010	0,0044	0,1343	0,0423	0,0337

Output gap	**Min**	**Max**	**Average**	**Std**
Simulated	-0,1194	0,1141	0	0,0308
Historic 1980-2010	-0,0429	0,0328	0	0,0186

Rates of return of housing	**Min**	**Max**	**Average**	**Std**
Simulated	-0,1074	0,2222	0,0814	0,0419
Historic 1980-2010	-0,1367	0,3002	0,0847	0,1010

The historic values and the simulation values are different. The main reason is that Norway has changed monetary regimes during the year 1980-2010 the values are different since the simulation is based on today's inflation targeting regime.

8 The simulation of consumption paths

To test how the retirement plan model performs on the simulated scenarios the thesis uses two different cases. The first case is a pensioner who has a high level of home equity but a low level of pension pay out from social security. The second case is a pensioner who has a low level of home equity but a high level of pension pay out from social security. The first case is defined as "case ARCP" meaning "Asset Rich, Cash flow Poor retirement plan". The second case is defined as "case APCR" meaning "Asset Poor, Cash flow Rich retirement plan". The names of the cases are inspired by the title of Chia et al(2005). The idea behind these two cases is to see if the differences in wealth give dissimilar results from the simulation.

Input values

$$E_1\left[\sum_{t=1}^{T}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]\leq E_1\left[A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0+\sum_{t=1}^{T}\frac{P_tW_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]$$

The budget constraint is in the two cases are different as the they will be using different values of home equity A_0 and pension payout W_0. The value A_0 is not the pensioner's total home equity. The pensioner faces a borrowing constraint of 60% on his total home equity. This comes from that Norwegian banks give on average 60% of the home equity as a maximum credit line Solberg(2010). So when modelling the retirement plan the variable A_0 represents the amount of what percentage the pensioner's home equity he is willing to borrow on. This can be a choice of how much benefit the pensioner wants to leave, the borrowing constraint he faces or keeping a bit of wealth for buffer stock saving if the pensioner lives longer than expected. So have this in mind when the results from the simulation break the "No Ponzi scheme" condition meaning that total lifetime resources are negative[14]. It is not as serious as it seems since there is still some remaining home equity. The consequence of breaking the condition is that since the credit line to the bank has exceeded its limit the pensioner defaults on his debt and has to move and sell his home.

The case ARCP ("Asset Rich, Cash flow Poor") will use the values$A_0 = 60$ for home equity and $W_0 = 10$ for the pension pay out. The case APCR ("Asset Poor, Cash flow Rich") will use the values$A_0 = 40$ for home equity and $W_0 = 15$ for the pension payout. The values for A_0 and W_0 are simplifications. To make the values more tangible we can think of case ARCP that the pensioner has a total home equity value of three million kroner and is willing to use the maximum credit line of 60% and he has an after tax pension pay out of three hundred thousand kroner per year today. Translating these values into the values used in the model using a denominator of thirty thousand we get

$$A_0=\frac{3\ 000\ 000\cdot 0{,}6}{30\ 000}=60,\ W_0=\frac{300\ 000}{30\ 000}=10$$

We can do something similar in case APCR but the point is that it is the ratios of Home Equity over Pension Payout (HEPP) that is studied. In case ARCP the ratio is $\frac{home\ equity}{pension\ payout}=$

[14] Remember that this occurs when housing prices take a steep fall and the debt is so high that all remaining wealth is wiped out.

$\frac{60}{10} = 6$ and in case APCR$\frac{home\ equity}{pension\ payout} = \frac{40}{15} = 2{,}667$. The “Asset Rich, Cash flow Poor” is the pensioner who has a high ratio compared to the “Asset Poor, Cash flow Rich” pensioner who has a low ratio.

$$U = E\left[\sum_{t=1}^{T} \beta^t u(C_t)\right] = E\left[\sum_{t=1}^{T} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}\right] \overset{"\sigma=1,\beta=0{,}99,T=20"}{\cong} E\left[\sum_{t=1}^{20} 0{,}99^t \log(C_t)\right]$$

The pensioner in both cases shares the same utility function. With a value[15] $\sigma = 1, T = 20$ and $\beta = 0{,}99$. When $\sigma = 1$ then the instantaneous utility function is transformed into a log utility function. Sharing the same utility function leads to that they also share the same preferences for their consumption paths which is determined byα_t.

$$\alpha_t = \left(\frac{\beta(1 + r_t^f + \mu)}{(1 + \pi_t)}\right)^{\frac{1}{\sigma}} = \left(\frac{0{,}99 \cdot (1 + r_t^f + \mu)}{(1 + \pi_t)}\right)$$

The consumption per year will be different in the two cases as the pensioner’s lifetime resources are different between the two cases but the consumption paths are predicted to grow in the same way.

Simulation

The next step is to simulate how the pensioner’s consumption paths develop in the one thousand simulated scenarios of how the economy evolves over twenty years. The computer program mimics the pensioner’s behaviour described in chapter two. Each year the economy is hit by shocks. So the pensioner has to recalculate his retirement plan based on the changes in the economy determined by the shocks. I will explain this behaviour step by step.

$$P_0 C_1 \underbrace{\left[\sum_{t=1}^{20} \frac{\prod_{s=1}^{t}(1 + \pi_s) \prod_{s=1}^{t} \alpha_s}{\prod_{s=1}^{t}(1 + r_s^f + \mu)}\right]}_{\gamma_1} = \underbrace{E_1\left[A_0 \frac{\prod_{t=1}^{20}(1 + r_t^h)}{\prod_{t=1}^{20}(1 + r_t^f + \mu)} - D_0 + W_0 \sum_{t=1}^{20} \frac{P_0 \prod_{s=1}^{t}(1 + \pi_s)}{\prod_{s=1}^{t}(1 + r_s^f + \mu)}\right]}_{\delta_1}$$

[15] A comment to the use of a lifetime utility function having a certain time dimension of T is not a realistic case. As the time dimension is chosen on the basis of lifetime expectancy the value T in a more realistic model would take into account that T is expected to increase for each year the pensioner lives. But this process is left out in the model to keep the problem simple enough to solve.

The first year the pensioner knows his home equityA_0, his pension payout W_0, the previous year's price level P_0 and his debtD_0. The starting debt D_0 is equal to zero and P_0 is equal to one. He also knows the first year's inflation rate π_1, rate of return of housing r_t^h and mortgage rate $r_1^f + \mu$ meaning that he also knows what this year's price level $P_1 = (1 + \pi_1)P_0$ is. He also knows the value of the demand shocke_1, supply shock u_t and rates of return of housing shock v_1 today. So the pensioner can decide future expectations on the interest rate, inflation rate and rates of return of housing that was described in chapter five. As stated in chapter two$\alpha_1 = 1$. Using these values the pensioner can decide on first year's consumptionC_1.

$$C_1 = \frac{1}{P_0}\frac{\delta_1}{\gamma_1}$$

After spending the first year's consumption the economy is hit again by a demand shock, supply shock and a rate of return of housing shock and the pensioner needs to recalculate the second year consumption C_2 since the state of the economy is now different.

$$P_1 C_2 \underbrace{\left[\sum_{t=1}^{19} \frac{\prod_{s=1}^{t}(1+\pi_s)\prod_{s=1}^{t}\alpha_s}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\gamma_2} = \underbrace{E_2\left[A_1\frac{\prod_{t=1}^{19}(1+r_t^h)}{\prod_{t=1}^{19}(1+r_t^f+\mu)} - D_1 + W_0\sum_{t=1}^{19}\frac{P_1\prod_{s=1}^{t}(1+\pi_s)}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{\delta_2}$$

His home equity value today in the second year is $A_1 = A_0(1 + r_1^h)$ this is the change in value of the home equity within the first year. His debt today is $D_1 = D_0\left(1 + r_1^f + \mu\right) - P_1 W_0 + P_1 C_1$ this is the previous debt accumulated plus increase by the mortgage interest on the previous debt minus the pension pay out plus the consumption in year one. The price level P_1 gives the nominal value of the pension payout and consumption in the previous year. The price level has been updated to $P_1$and the pensioner's time horizon has been changed to nineteen as there is only nineteen years left in his retirement plan. There is a new demand shock, a new supply shock and a new rate of return of housing shock in the second year. Remember that the demand shock and supply shock are AR1 process so parts of the previous demand and supply still affects the interest rate, inflation rate, output gap and rates of return of housing today. So the pensioner has to calculate future expectations of the interest rate, output gap, inflation rate and rate of return of housing again. The consumption for the second year is

$$C_2 = \frac{1}{P_1}\frac{\delta_2}{\gamma_2}$$

Each year of the twenty years the economy is hit by shocks and the pensioner needs to recalculate his retirement plan. As there are one thousand simulated samples of scenarios of how the economy develops there will be one thousand different results for the consumption paths for the pensioner. Simulating the two cases on the same scenarios and then comparing them I intend to find the behaviour, risk and gains from using home equity in a retirement plan.

9 Tests of the simulation

There were three tests for each of the two cases ARCP and APCR. The first was to test for a benchmark. These results came from a scenario with no shocks where the pensioner would experience a normal state in every year. A normal state is when the output gap = 0, inflation rate =2,5%, interest rate =4% and rates of return of housing = 8,23% in every year. The second test was to check how the model would perform without using home equity in the retirement plan this was done by setting A_0 in both cases to zero but keeping the remaining inputs the same. The third test was to check how the model performed with home equity in the retirement plan this is described in the previous chapter as the main simulation. The intention of the "benchmark test" and the "without using home equity test" was to have scenarios to compare against the "with home equity test". The main results from the simulation of consumption paths are presented in figures and tables in the next chapter.

10 Analysis

Now that the main body of work is done. This chapter is going to present answers to the questions for the main goals of the research that were stated in the introduction by interpreting the results from the simulations and using economic reasoning.

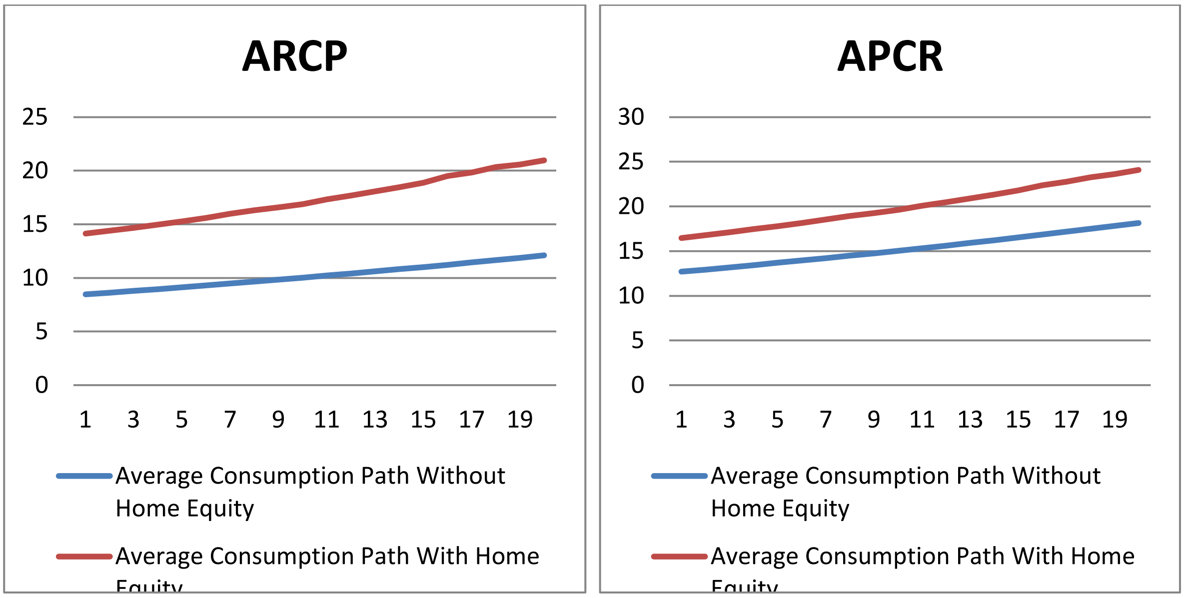
As a reminder the case of an "Asset Rich, Cash flow Poor retirement plan", where $A_0 = 60$ and $W_0 = 10$, is defined as case ARCP. The case of an "Asset Poor, Cash flow Rich retirement plan", where $A_0 = 40$ and $W_0 = 15$, is defined as case APCR.

Consumption decisions

"How does home equity affect consumption decisions?"

Results from the simulation show that using home equity in a retirement plan will lead to an on average higher consumption. This is an obvious result as the pensioner is now using more of his wealth on consumption. This is presented in *Figure 3 ARCP & APCR average consumption paths, with versus without home equity* where on the left axis is the consumption per year and on the bottom axis is denoting the year. The red line shows consumption per year with home equity while the blue line shows consumption per year without using home equity.

Figure 3 ARCP & APCR average consumption paths, with versus without home equity

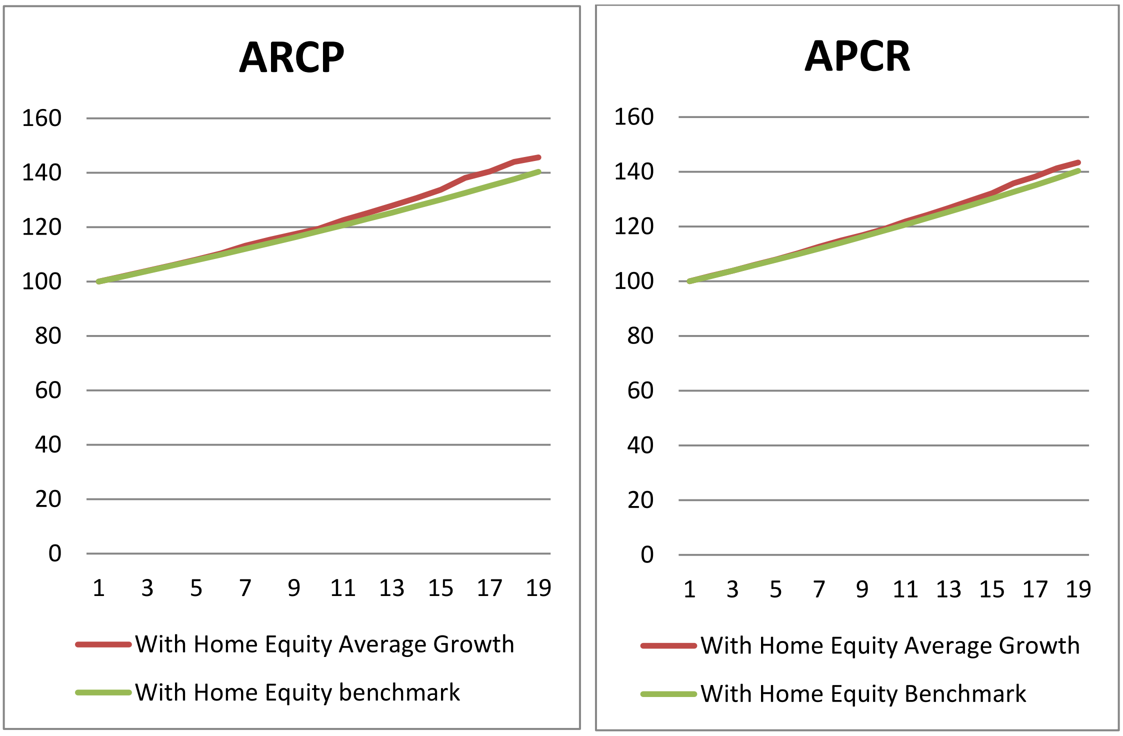


We can see that the average consumption path is increasing in both cases with and without use of home equity. In chapter two α_t described how the consumption path developed. In the simulation the$\alpha_t$was

$$\alpha_t = \left(\frac{\beta(1 + r_t^f + \mu)}{(1 + \pi_t)}\right)^{\frac{1}{\sigma}} = \left(\frac{0{,}99 \cdot (1 + r_t^f + \mu)}{(1 + \pi_t)}\right)$$

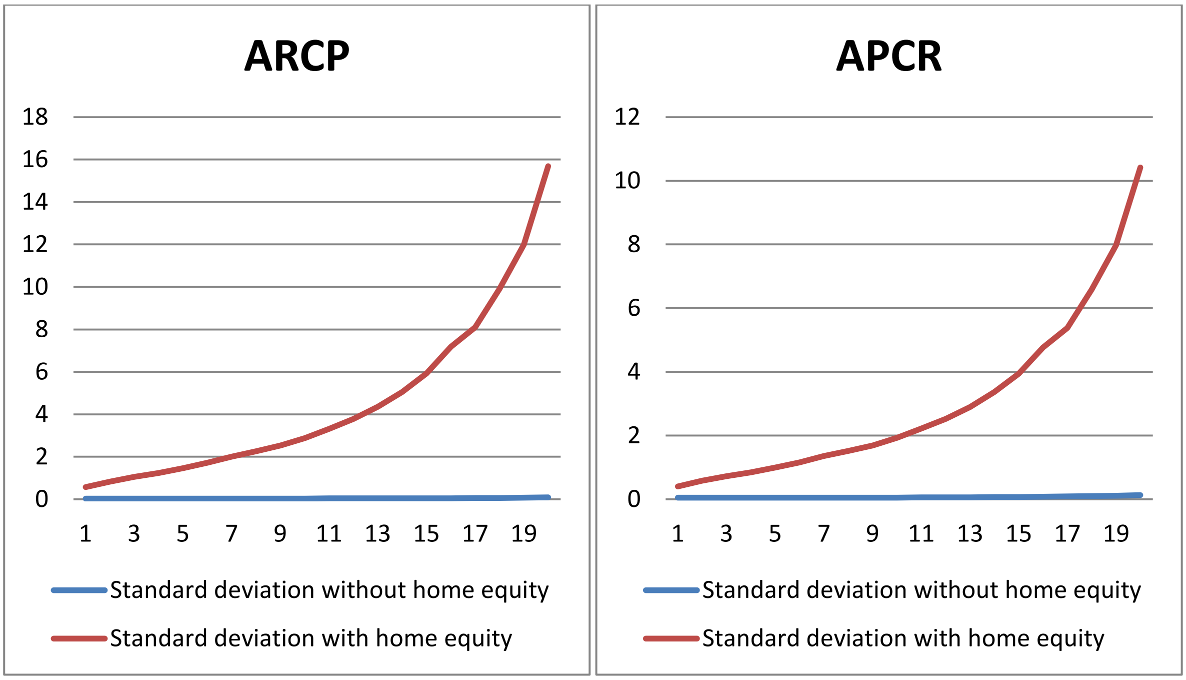
The α's are the same for both the cases since the same preferences and simulated economic scenarios was used. But the results show that when including home equity in a retirement plan the higher the average growth of consumption paths occurred. Comparing against the benchmark test the average consumption path with home equity grew higher than its benchmark. This is depicted in *figure 4 Growth of consumption paths* where the average consumption in the first year is indexed to one hundred. The figure shows how much higher the consumption this year is compared to the first year.

Figure 4 Growth of consumption paths



The use of home equity in a retirement plan also lead to a much higher standard deviation for the consumption paths. *Figure 5 Standard deviation of consumption paths* presents how large the standard deviation of consumption is per year from the different economic scenarios.

Figure 5 Standard deviation of consumption paths



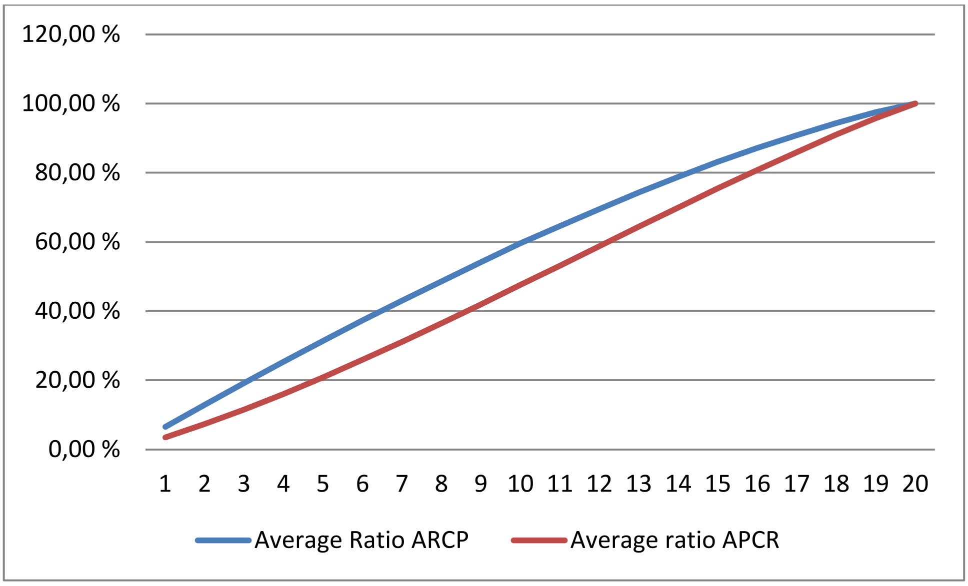
The standard deviation is much higher in the case with home equity than without home equity and over time the standard deviation is growing in a convex way. The reason for the differences in consumption paths in the tests with and without home equity must come from the way home equity is used for consumption since the pension pay outs are the same in both tests. When the pensioner wants to spend more consumption than he has in a pension pay out this year he must borrow money on the home equity. Each year the pensioner takes on more debt. Over time his debt and home equity value increases. As the value of the debt is increasing faster than the home equity value the home equity asset is being leveraged. Leveraging home equity gives higher expected returns explaining the higher consumption growth presented in Figure 4 and leveraging gives larger standard deviation in the consumption paths presented in Figure 5.

The ratio for leveraging is described by the equation

$$\frac{debt\ at\ the\ end\ of\ period\ t}{home\ equity\ at\ the\ end\ of\ period\ t} = \frac{D_t}{A_t} = leverage\ ratio\%$$

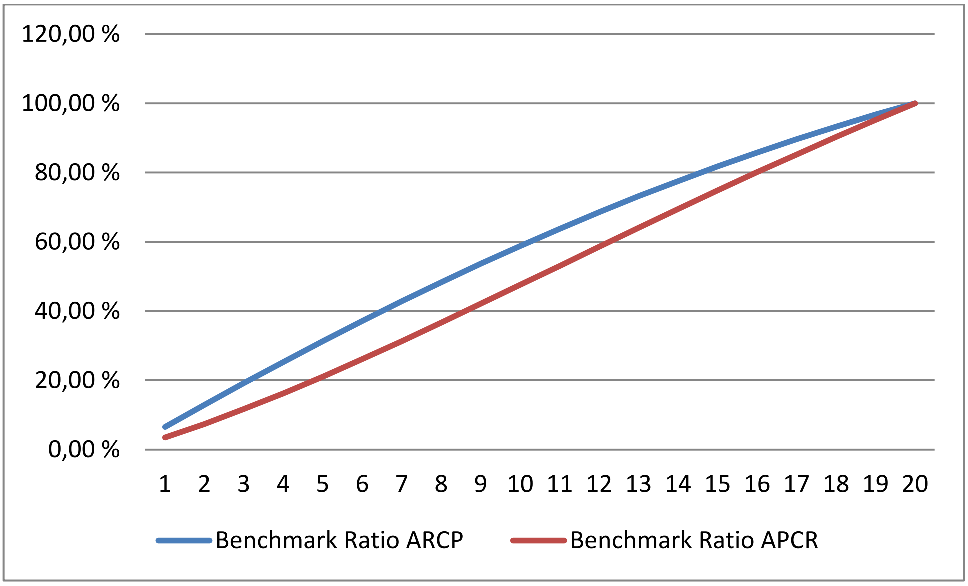
Figure 6 Leverage ratio I presents the development of the average ratios of leverage over time. The left axis shows the ratio in percent and the bottom axis shows which year.

Figure 6 Leverage ratio I



In Figure 6 the average leverage ratios are different in the two cases. In search of an answer I graphed the benchmark tests for both cases in *Figure 7 Leverage ratio II* to check if it was the pensioner's preference that determined the difference. Figure 7 gave a similar result.

Figure 7 Leverage ratio II

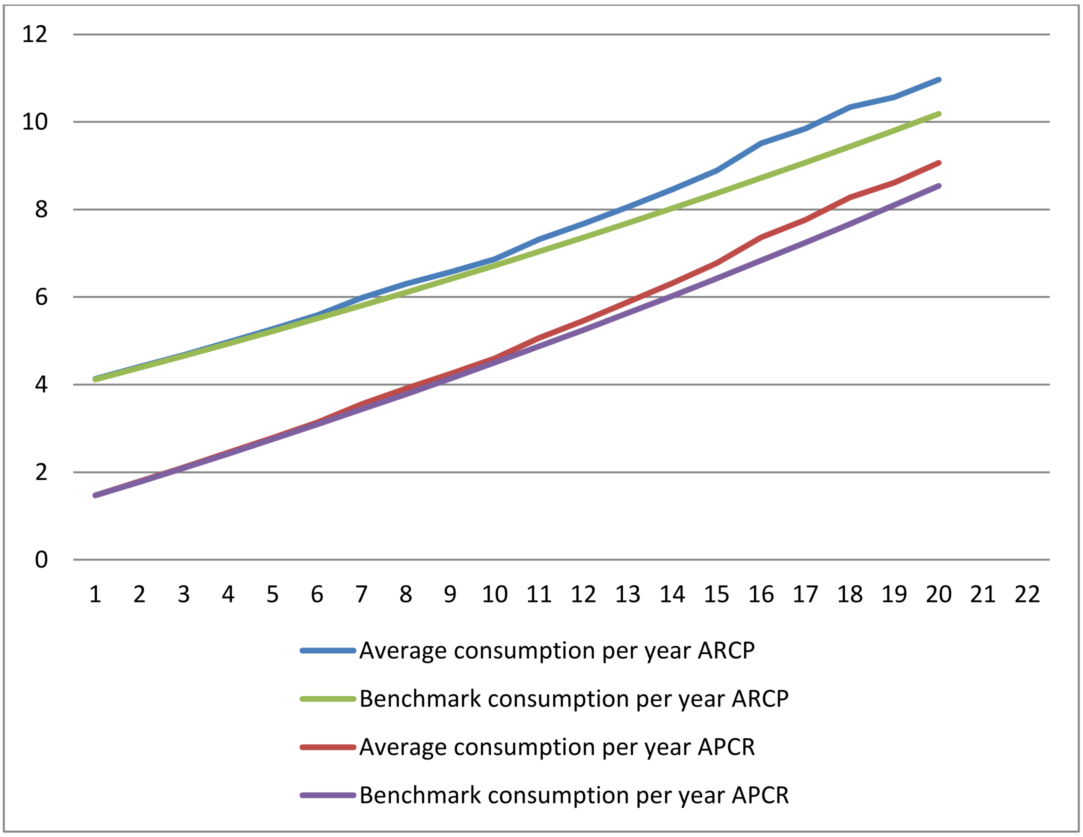


The leverage ratio in case ARCP looks like a concave function whilst APCR looks like a convex function. This behaviour could also be shown if one checked for how much of the year's consumption was based on borrowing on home equity. This is defined in the equation

$$consumption\ based\ on\ borrowing\ on\ home\ equity\ in\ year\ t = \frac{P_t C_t - P_t W_0}{P_t} = C_t - W_0$$

The average and benchmark values for both cases are depicted in *Figure 8 Average consumption paths based on borrowing on home equity*

Figure 8 Average consumption paths based on borrowing on home equity



First years average consumption based on borrowing on home equity in case ARCP is 4,129 and in case APCR is 1,47. If we divide the first year consumption by the first years value of home equity in both cases we get

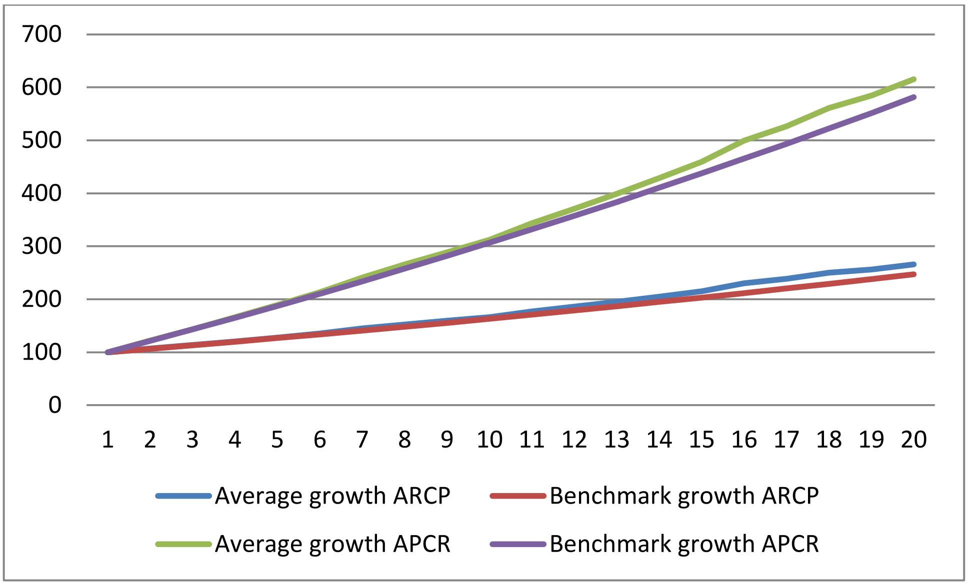
$$Case\ ARCP = \frac{4{,}129}{60} = 0{,}0688167 = 6{,}9\%$$

$$Case\ APCR = \frac{1{,}47}{40} = 00{,}3675 = 3{,}7\%$$

In case ARCP the pensioner starts his consumption path by using a higher percentage of his home equity than in case APCR.

The differences in growth rates of the consumption paths are presented in *Figure 9 Consumption path growth based on borrowing on home equity*. The consumption in the first year is indexed to one hundred in both cases. The figure shows the relative level of consumption in year t compared to the first year.

Figure 9 Consumption path growth based on borrowing on home equity



The figure shows that case APCR grows faster than in case ARCP. This figure seems odd if it is compared to Figure 4. Shouldn't the consumption path growth based on borrowing on home equity be equal if in both cases they share a similar consumption path growth preference of α_t? The reason why case ARCP consumption path based on borrowing on home equity grows slower and starts on a higher level of consumption relative to home equity than in case APCR is because of a *hidden wealth effect* in the budget constraint. Studying the home equity process on the right hand side of the budget constraint described in chapter two.

$$E_1\left[\sum_{t=1}^{T}\frac{P_tC_t}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right] \leq \underbrace{E_1\left[\underbrace{A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0}_{home\ equity\ process}+\sum_{t=1}^{T}\frac{P_tW_0}{\prod_{s=1}^{t}(1+r_s^f+\mu)}\right]}_{lifetime\ resources}$$

Rewriting the home equity process

$$E_1\left[A_0\frac{\prod_{t=1}^{T}(1+r_t^h)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}-D_0\right] = E_1\left[\frac{A_0\prod_{t=1}^{T}(1+r_t^h)-D_0\prod_{t=1}^{T}(1+r_t^f+\mu)}{\prod_{t=1}^{T}(1+r_t^f+\mu)}\right]$$

Since $r_t^h > (r_t^f+\mu)$ most of the time the higher the percentage of A_0 the right hand side of the budget constraint consists of the higher the expected rates of return on the lifetime resources portfolio. And since case ARCP has a higher percentage of A_0 leads to a noticeable wealth effect by choosing to consume more today of home equity and having a flatter consumption path.

Risk

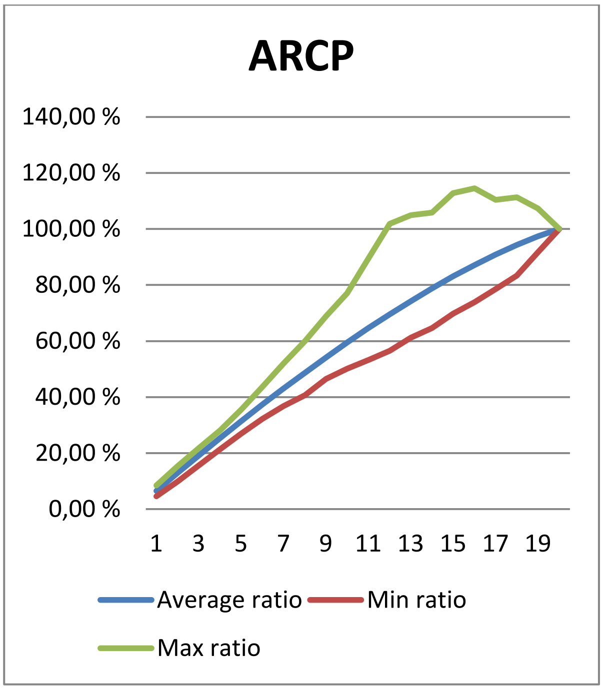
"What level of risk is a pensioner exposed to trying to execute a retirement plan using home equity?"

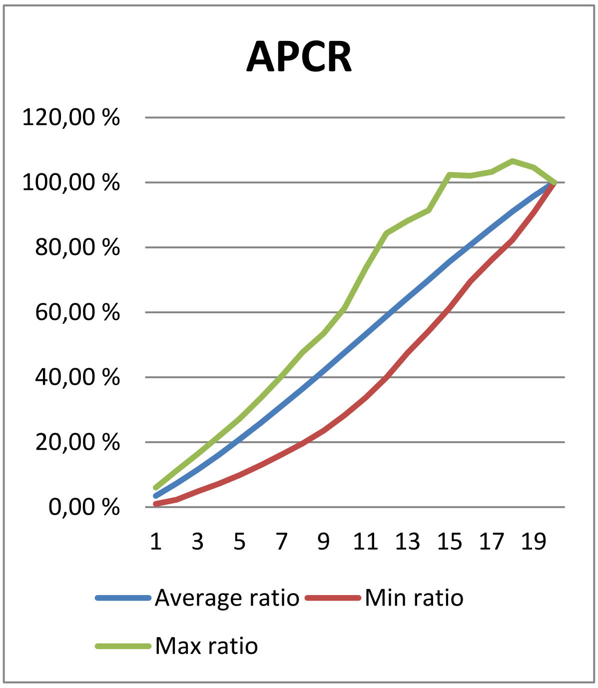
In the retirement plan the pensioner spends his home equity by borrowing money from the bank. This debt process is in a way self financing since the rate of return of housing r_t^h is larger than the mortgage rate $(r_t^f + \mu)$ most of the time, given that the debt D_t is lower than the home equityA_t. So the increase of value of home equity covers the interest payments on the debt most of the time. When the pensioner borrows more each year to finance consumption the home equity asset gets more leveraged. And with all leveraged assets this leads to higher variance in future rates of return of the net value of wealth$A_t - D_t$. In chapter two a weakness in the budget constraint was presented. If home equity value sharply dropped while being highly leveraged would lead to that the "No Ponzi scheme" condition being broken. This in turn would lead to negative consumption as there are no lifetime resources left to spend only debt to be repaid. The consequence as explained in chapter eight is that the pensioner would default on his debt meaning he would have to move and sell his home. Keeping what is left of his home equity after the debt has been repaid.

In the simulations of the test without home equity none of the "No Ponzi scheme" conditions were broken. As the pension pay outs are a secure form for income. When the simulation was tested with home equity the "No Ponzi scheme" condition was broken 5,3% of the time in the ARCP and 0,4% in the APCR case. The reason for the "No Ponzi scheme" condition being broken more often in ARCP case than in the APCR case is that if we study Figure 6 again the ARCP case is on average more leveraged that the APCR case in every year in the retirement plan. This tells us that the higher the Home Equity over Pension Pay Out (HEPP) ratio $\frac{A_0}{W_0}$ meaning that the pensioner's lifetime resources have a higher percentage of home equity in the portfolio the stronger is the hidden wealth effect. The stronger the hidden wealth effect the higher exposure to leverage over time. This again leads to a higher risk of breaking the "No Ponzi scheme" condition.

Other problems that arise because of a high HEPP ratio are revealed studying *Figure 10 Min, Max, Average leverage ratios*. The figure presents the minimum, maximum and average leverage ratios. Notice that the max leverage ratio is above 100% in the last years meaning that $D_t > A_t$.

Figure 10 Min, Max, Average leverage ratios





In several of the consumption paths the leverage level is above 100% in some of years without breaking the No Ponzi scheme condition. The pensioner has to use his pension pay out to finance the debt. This lowers his consumption in those years. The event of having to use the pension pay out to finance the debt is when $W_0 - C_t > 0$ and$D_t > 0$. *Table 3 Occurrence of having to use pension pay outs to finance the debt* presents how often in per cent of the one thousand simulated consumption paths the pensioner has to finance the debt with the pension pay out in the two cases.

Table 4 Occurrence of having to use pension payouts to finance debt

Year	ARCP	APCR
1	0,0 %	0,0 %
2	0,0 %	0,0 %
3	0,0 %	0,0 %
4	0,0 %	0,0 %
5	0,0 %	0,0 %
6	0,0 %	0,0 %
7	0,0 %	0,0 %
8	0,0 %	0,0 %
9	0,0 %	0,0 %
10	0,0 %	0,0 %
11	0,0 %	0,0 %
12	0,1 %	0,1 %
13	0,2 %	0,0 %
14	0,7 %	0,0 %
15	1,5 %	0,5 %
16	4,6 %	0,8 %
17	6,6 %	1,9 %
18	10,4 %	5,3 %
19	17,7 %	10,4 %
20	23,8 %	17,6 %

Case ARCP the pensioner has to finance the debt more often than in the APCR case meaning that the lower HEPP ratio gives a less probability of having to finance the debt with the pension pay out. Having a high HEPP ratio also leads to a larger reduction of consumption as the pensioner has a lower pension pay out to finance the debt combined with that the debt is more likely to be higher which meant the pensioner having higher interest payments.

Having a lower HEPP ratio leads to less risk of breaking the "No Ponzi scheme" condition and reduces the probability and costs of having to finance the debt with the pension pay out.

Home equity value

"What value can one expect to extract from one's home?"

Thinking that there was an easy answer to this question, I was wrong but the question has exposed an interesting relation. Before all the simulations I assumed that the consumption contributed from home equity in the retirement plan would follow the same trend growth rate $\alpha_t$defined in chapter two. But the *hidden wealth effect* gave a different story as the weights on home equity in the portfolio for lifetime resources gave different consumption paths for consumption contributed from home equity. So the results gave no clean answer to *"What value can one expect to extract from one's home?"* but there are many other aspects to the question that need to be discussed.

In the beginning of this chapter the research concluded that on average consumption is higher if one uses home equity in the retirement plan. Utility is a better measure than consumption if one wants to measure the contribution of home equity. Testing the hypothesis "*Using home equity always leads to higher lifetime utility"* is a good starting point.

The simulation calculated the total lifetime utility function $E[\sum_{t=1}^{20} 0{,}99^{t} \log(C_t)]$in the tests with and without home equity and in both cases ARCP and APCR. In case ARCP the lifetime utility was higher with home equity than without home equity 94,7% of time and in case APCR the utility was higher 99,6% of the time. The hypothesis is false. Studying the simulation closer utility was lower only when the pensioner broke the "No Ponzi scheme" condition. When the pensioner breaks the No Ponzi scheme condition consumption becomes negative as the pensioner has to repay his debt. In the log utility function if $C < 0$ the log utility function is undefined. I was expecting to find a different result for the percentages of higher utility since using the log utility function $\lim_{c\to 0} log(C) = -\infty$ and in some cases I thought that consumption would be so low that the percentages of higher utility would take into account these outcomes. But this did not happen. *Table 4 Lifetime utility results from the simulation* present the minimum, maximum, benchmark, average and standard deviation of lifetime utility of the consumption paths that did not break the "No Ponzi scheme" condition.

Table 5 Lifetime utility results from the simulation

	ARCP		APCR	
	Utility With Home Equity	Utility Without Home Equity	Utility With Home Equity	Utility Without Home Equity
Min	43,676	41,559	50,0742058	48,869
Max	60,883	41,701	60,3931507	49,010
Benchmark	50,834		53,6085654	
Average	50,447	41,618	53,5336403	48,928
Standard Deviation	2,913	0,021	1,6396622	0,021

Regression

Two regressions are used to check how the value of lifetime utility in the cases ARCP and APCR are affected by the averages of the interest rate, inflation rate, output gap and rate of return of housing in the one thousand scenarios of how the economy developed. The intention of the regression is to confirm the pensioner's behaviour in the retirement plan. The regression is on the samples where the pensioner did not break the "No Ponzi scheme" condition as lifetime utility is undefined for the samples that did. The results from the regressions are depicted in *Table 6: Regression on lifetime utility cases ARCP* and in *Table 7: Regression on lifetime utility cases APCR*.

Table 6: Regression on lifetime utility case ARCP

Source	SS	df	MS
Model	7700.31206	4	1925.07801
Residual	329.339587	942	.349617396
Total	8029.65165	946	8.48800385

Number of obs =	947
F(4, 942) =	5506.24
Prob > F =	0.0000
R-squared =	0.9590
Adj R-squared =	0.9588
Root MSE =	.59128

Utility	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
i_mean	-17.52685	39.14158	-0.45	0.654	-94.34163	59.28793
pi_mean	-145.4729	54.90661	-2.65	0.008	-253.2263	-37.71951
x_mean	-5.405864	12.4352	-0.43	0.664	-29.80976	18.99803
hp_mean	128.3731	8.971123	14.31	0.000	110.7674	145.9788
_cons	44.32821	.9509343	46.62	0.000	42.46201	46.1944

Table 7: Regression on lifetime utility case APCR

Source	SS	df	MS
Model	2565.21814	4	641.304535
Residual	109.853424	991	.110851084
Total	2675.07157	995	2.68851414

Number of obs =	996
F(4, 991) =	5785.28
Prob > F =	0.0000
R-squared =	0.9589
Adj R-squared =	0.9588
Root MSE =	.33294

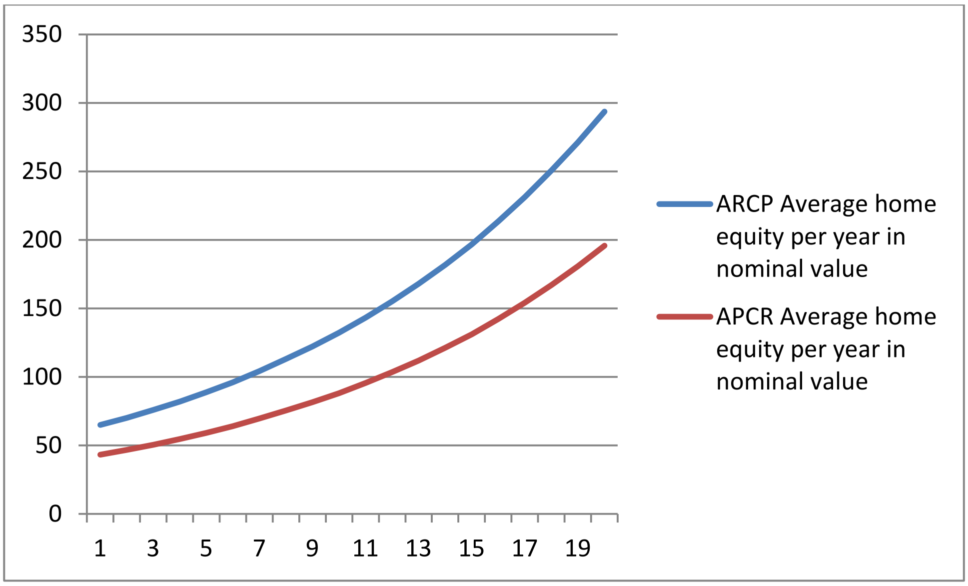
Utility	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
i_mean	-.8827857	20.74824	-0.04	0.966	-41.59832	39.83275
pi_mean	-84.09381	29.00985	-2.90	0.004	-141.0216	-27.16603
x_mean	-5.645778	6.817362	-0.83	0.408	-19.0239	7.732345
hp_mean	77.78604	4.94775	15.72	0.000	68.07677	87.49531
_cons	49.35413	.5240718	94.17	0.000	48.32571	50.38255

There is a high level of multicolinearity in the regressions. The rate of return of housing is determined by the interest rate, inflation rate and the output gap. The interest rate is determined by the inflation rate and the output gap by the central bank's interest rule which reacts to the inflation rate and the output gap. The pensioner's consumption decision is directly affected by the interest rate, inflation rate and the rate of return of housing while the output gap affects consumption indirectly through those variables explaining why the output gap has such a low coefficient in the regression. The confidence intervals show that the estimates are highly uncertain so we can't know for sure that the output gap doesn't affect lifetime utility and that the other variables affect lifetime utility in there estimated values. Taking these problems of multicolinearity and wide confidence intervals into account the regression is only used to confirm what is already known and not to find anything new.

Rates of return of housing

The coefficient of the rates of return of housing is higher in case ARCP than in case APCR. A higher rate of return of housing contributes more lifetime consumption for case ARCP than in case APCR because a higher percentage and value of the lifetime resources in case ARCP consist of home equity. The average home equity per year for the two cases is depicted in *Figure 11 Average home equity per year in nominal values*

Figure 11 Average home equity per year in nominal values



Interest rates

A high average interest rate affects case ARCP more negatively than in case APCR. This is because in case ARCP has an on average higher debt each year compared to case APCR which makes the retirement plan more costly for ARCP if interest rates are high. The average debt per year is depicted in *Figure 12 Average debt per year in nominal values*. The average interest payments per year in nominal values are depicted in *Figure 13 Average interest payments per year in nominal values.*

Figure 12 Average debt per year in nominal values

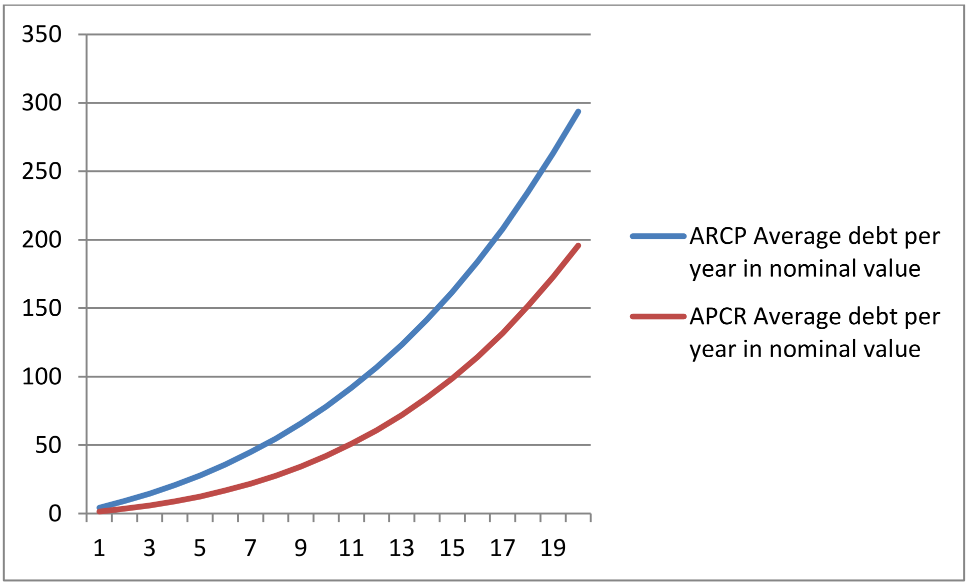
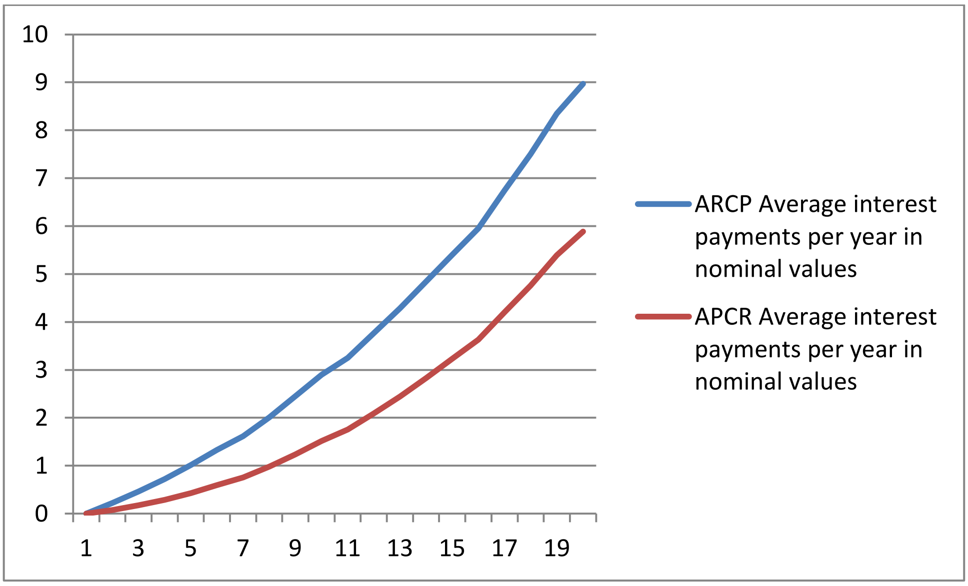


Figure 13 Average interest payments per year in nominal values



Inflation

Inflation is bad for the pensioner. It lowers the value of savings. Savings in the retirement plan mainly consists of home equity. Case APCR has a low level of home equity so the pensioner is less exposed to inflation. He has also a high income from the pension payout which keeps its purchasing power no matter what inflation is. Case ARCP has the opposite with a high savings in home equity and a low pension pay out. High inflation is worse for case ARCP than for case APCR.

11 Conclusion

The thesis was an evaluation of the value and consequences of using home equity in a retirement plan in Norway. Three research questions were answered. "How does home equity affect consumption decisions?" "What level of risk is a pensioner exposed to trying to execute a retirement plan using home equity?" and "What value can one expect to extract from one's home?" To answer the questions the thesis used a modified version of a standard lifecycle model to model a retirement plan. Simulation methods were used to test the retirement plan. To create one thousand scenarios of how the economy will develop a new Keynesian framework of the Norwegian inflation targeting regime was used to create variables on inflation, interest rates and the output gap. The variables for rates of return of housing came from a regression of historic rates of return of housing. Two different cases were tested. One where the pensioner had a high level of home equity and a low pension pay out and the other where the pensioner had a low level of home equity and a high pension pay out. Studying the results from the simulation the research found out that even though the two cases had similar growth in their consumption paths the consumption path for consumption from home equity was different. This was because of a hidden wealth effect due to the different lifetime resource composition of the two cases. The thesis revealed that if a pensioner used a simple budget constraint he was exposed to a risk of default. This happens in a scenario where the home equity is highly leveraged. A steep fall in housing prices leads to the pensioner's wealth becoming negative and the consequence would be that the pensioner would have to move and sell his home. The home equity over pension pay out ratio determined the level of risk. The higher the ratio the more vulnerable the pensioner was to default while a lower ratio avoided this situation as the pensioner's pension pay out had a better ability to finance the debt. The thesis also found out that using home equity in a retirement plan increased utility for the pensioner most of the time. A regression on the results of lifetime utility showed that different levels of home equity over pension pay out ratios led to that the interest rate, inflation rate and the rate of return of housing affect lifetime utility in a different way. These findings gave the answer to the thesis problem. There are benefits to using home equity in a retirement plan if used correctly and in a moderate amount.

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13 Data

Source: Norges Bank, modified from indexes. http://www.norges-bank.no/en/price-stability/historical-monetary-statistics/

Table 8 Data for the regression

Year	Rates of return of housing	GDP annual growth	Deviation from average annual growth of GDP.	Inflation	Deposit rates	The real interest rate
1980	0,0923	0,0449	0,0187	0,1105	0,0713	-0,0392
1981	0,3002	0,0156	-0,0105	0,1343	0,0766	-0,0577
1982	0,2227	0,0012	-0,0250	0,1140	0,0813	-0,0327
1983	0,0552	0,0386	0,0125	0,0846	0,0840	-0,0006
1984	0,0827	0,0590	0,0328	0,0635	0,0876	0,0241
1985	0,0718	0,0535	0,0273	0,0563	0,0901	0,0338
1986	0,2984	0,0403	0,0142	0,0711	0,0957	0,0246
1987	0,2314	0,0178	-0,0084	0,0875	0,1101	0,0226
1988	-0,0038	-0,0016	-0,0278	0,0666	0,1106	0,0440
1989	-0,1367	0,0099	-0,0163	0,0455	0,0907	0,0452
1990	-0,0396	0,0193	-0,0069	0,0410	0,0897	0,0487
1991	-0,0779	0,0310	0,0048	0,0346	0,0866	0,0520
1992	-0,0828	0,0353	0,0092	0,0231	0,0870	0,0639
1993	0,0632	0,0278	0,0016	0,0226	0,0584	0,0358
1994	0,1426	0,0506	0,0244	0,0143	0,0403	0,0260
1995	0,0758	0,0419	0,0157	0,0250	0,0399	0,0149
1996	0,1064	0,0509	0,0247	0,0117	0,0363	0,0246
1997	0,0874	0,0540	0,0279	0,0262	0,0271	0,0009
1998	0,1447	0,0268	0,0006	0,0225	0,0450	0,0225
1999	0,1304	0,0202	-0,0060	0,0230	0,0483	0,0253
2000	0,1517	0,0326	0,0064	0,0313	0,0506	0,0193
2001	0,0770	0,0200	-0,0062	0,0303	0,0580	0,0277
2002	0,0665	0,0150	-0,0112	0,0129	0,0557	0,0428
2003	0,0188	0,0098	-0,0164	0,0245	0,0322	0,0077
2004	0,1231	0,0397	0,0135	0,0044	0,0131	0,0087
2005	0,0909	0,0259	-0,0003	0,0159	0,0143	-0,0016
2006	0,1526	0,0245	-0,0017	0,0226	0,0214	-0,0012
2007	0,1124	0,0265	0,0004	0,0076	0,0370	0,0294
2008	-0,0423	0,0004	-0,0258	0,0379	0,0501	0,0122
2009	0,0271	-0,0167	-0,0429	0,0211	0,0207	-0,0004
2010	0,0827	0,0068	-0,0194	0,0247	0,0210	-0,0037

14 Appendix

Computer programs

The computer programs in the thesis couldn't be presented in a proper manner on paper but I can send a copy of them by email peterborchgrevink@gmail.com upon request.

Finishing the calculations in Chapter 3

We stopped here

$$\psi_1 = \beta\psi_1\mu_e + \kappa\alpha_1$$

$$\psi_2 = \beta\psi_2\mu_u + \kappa\alpha_2 + 1$$

$$\alpha_1 = \alpha_1\mu_e - \frac{1}{\sigma}(\emptyset_\pi\psi_1 + \emptyset_x\alpha_1 - \psi_1\mu_e) + 1$$

$$\alpha_2 = \alpha_2\mu_u - \frac{1}{\sigma}(\emptyset_\pi\psi_2 + \emptyset_x\alpha_2 - \psi_2\mu_u)$$

And jumped over this part

Rewrite ψ_1

$$\psi_1 = \beta\psi_1\mu_e + \kappa\alpha_1$$

$$\psi_1 - \beta\psi_1\mu_e = \kappa\alpha_1$$

$$\psi_1 = \frac{\kappa\alpha_1}{1 - \beta\mu_e}$$

Rewrite ψ_2

$$\psi_2 = \beta\psi_2\mu_u + \kappa\alpha_2 + 1$$

$$\psi_2 - \beta\psi_2\mu_u = \kappa\alpha_2 + 1$$

$$\psi_2 = \frac{\kappa\alpha_2 + 1}{1 - \beta\mu_u}$$

Inserting for ψ_1 in α_1

$$\alpha_1 = \alpha_1\mu_e - \frac{1}{\sigma}\left(\emptyset_\pi \frac{\kappa\alpha_1}{1-\beta\mu_e} + \emptyset_x\alpha_1 - \frac{\kappa\alpha_1}{1-\beta\mu_e}\mu_e\right) + 1$$

$$\alpha_1 - \alpha_1\mu_e + \frac{1}{\sigma}\left(\emptyset_\pi \frac{\kappa\alpha_1}{1-\beta\mu_e} + \emptyset_x\alpha_1 - \frac{\kappa\alpha_1}{1-\beta\mu_e}\mu_e\right) = 1$$

$$\alpha_1 \frac{\sigma(1-\beta\mu_e) - \sigma(1-\beta\mu_e)\mu_e + \emptyset_\pi\kappa + \emptyset_x(1-\beta\mu_e) - \kappa\mu_e}{\sigma(1-\beta\mu_e)} = 1$$

$$\alpha_1 = \frac{\sigma(1-\beta\mu_e)}{\kappa(\emptyset_\pi - \mu_e) + (1-\beta\mu_e)[\sigma(1-\mu_e) + \emptyset_x]}$$

Inserting ψ_2 in α_2

$$\alpha_2 = \alpha_2\mu_u - \frac{1}{\sigma}(\emptyset_\pi \frac{\kappa\alpha_2 + 1}{1-\beta\mu_u} + \emptyset_x\alpha_2 - \frac{\kappa\alpha_2 + 1}{1-\beta\mu_u}\mu_u)$$

$$\alpha_2 - \alpha_2\mu_u + \frac{1}{\sigma}\left(\emptyset_\pi \frac{\kappa\alpha_2}{1-\beta\mu_u} + \emptyset_x - \frac{\kappa\alpha_2}{1-\beta\mu_u}\mu_u\right) = -\frac{1}{\sigma}(\emptyset_\pi \frac{1}{1-\beta\mu_u} - \frac{1}{1-\beta\mu_u}\mu_u)$$

$$\alpha_2 \frac{\sigma(1-\beta\mu_u) - \sigma(1-\beta\mu_u)\mu_u + \kappa\alpha_2\emptyset_\pi + \emptyset_x(1-\beta\mu_u) - \kappa\alpha_2\mu_u}{\sigma(1-\beta\mu_u)} = \frac{(\mu_u - \emptyset_\pi)}{\sigma(1-\beta\mu_u)}$$

$$\alpha_2 = \frac{(\mu_u - \emptyset_\pi)}{\kappa(\emptyset_\pi - \mu_u) + (1-\beta\mu_u)[\sigma(1-\mu_u) + \emptyset_x]}$$

Inserting for α_1 into ψ_1

$$\psi_1 = \frac{\kappa\sigma}{\kappa(\emptyset_\pi - \mu_e) + (1-\beta\mu_e)[\sigma(1-\mu_e) + \emptyset_x]}$$

Inserting for α_2 into ψ_2

$$\psi_2 = \frac{[\sigma(1-\mu_u) + \emptyset_x]}{\kappa(\emptyset_\pi - \mu_u) + (1-\beta\mu_u)[\sigma(1-\mu_u) + \emptyset_x]}$$