*Master thesis for the Master of Environmental and Development Economics degree*

# Economic Theories about the Allocation of Time

*A survey of the literature throughout time and some contributions.*


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**11/11/2007**

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***In loved memory of***
*my* only *uncle* ***Eliseo Sanchis García*** *and*
*my* only *grandfather* ***Eliseo Sanchis Guío***.

*The former tragically died as a consequence of a cancer and the latter died later after having had to bury his only son.*

*Both died during the conception and early developments of this idea, breaking the time line, and producing an impossible gap to fill in our lifetimes.*

# Preface

*"Quod in omni vita facimus in aeternum resonat"*
*"What we do in lifetime, echoes in eternity"*
(Latin proverb)

***…and yours one <u>will</u> echo…***

*"Eternity is in love with the productions of time."*
(William Blake)

***…and yours one <u>already is</u>…***

*"It doesn't take time to change one's destiny."*
(Kashmiri proverb)

***…and yours one <u>tragically did not</u>.***

Be at peace, *Eliseo´*s. Sometime we will meet altogether, in the *Eliseum*, *but not yet, not yet*...I promised you something, something not yet completed; not yet...

It is impossible to thank every gesture to everyone who influenced this work, since everyone I met along all my life has something to do with it, somehow. I can only say the following to everybody, after saying *sorry* for not being able to mention everybody as you deserve:

***Thanks to everybody who has spent some time with me sometime,***

***Thanks to everybody who still is spending some time with me sometime,***

***But overall, infinite thanks to everybody who has done it <u>throughout</u> time, and who will continue doing it, without any demand.***

***"Acquiesce is the mother of Science" (Spanish Proverb, free adaptation), and a lot of you have something to do with my <u>Acquiesce</u>, and vice versa.***

Special mention must be done to my supervisor, **Geir B. Asheim**, for his patience and his wise advices throughout this time. All errors within this thesis are exclusively responsibility of the author, and for any good contribution that might be here included, Geir has a lot to do with it. The same holds for some of the contributions that we decided to not to include within this thesis.

Even a more special one must be done to ***my sister***, **Ana**, since without her working time in my absence this *movie* would not have been *recorded*. Thank you, ***Ana***, as well, for the *inspirations*, the sociological (or *"sociolocos"*) comments and moral support. You will be a very promising "*socioloca"* sociologist very soon, I am sure of it.

Obviously, probably the most special mention is that one to **all my family** and **close friends** in Madrid, Spain, Oslo, Norway and the *uncountable* other parts of the World, simply, **for everything**.

Nevertheless and surely out of any ranking, I must say thanks to those who always have supported me, since the very beginnings, when nobody did: **Julio Segura**, **José Manuel Rey-Simó** , **Francisco Álvarez** and **Alfonso Novales**, my professors and recommenders from the ***Departments of Economics*** at **University Complutense of Madrid**. No other *thanks* would have been possible to say in here to anybody, since simply without your help nothing would have existed. Thank you for the patience, support and wise advices since 2000, and you are probably the only ones who really know the dimension of the cost that all this is still having for me, in general terms. I still remember as if it was yesterday the day in which I was asked *"Have you thought in studying a PhD? There are some good Universities…"*. I have tried three different ones, and still I have not found what I am looking for, exactly. Nothing is perfect and nothing is impossible, as my godmother always told me. However I must say that Oslo is the closer to *reachable perfection* I have been able to find given my possibility set, after almost two years I spent surveying Departments of Economics worldwide, 3 years of PhD studies in two other Universities in Spain and some PhD courses in Oslo. Throughout all this long and tiring way which *99% of people would have already abandoned*, if I am still *alive* and out of that *confidence interval*, this is due to your *confidence* and support, *profes*, which is not only a pleasure, but also an honour.

However, it has existed, yet exists, (who knows if it will continue existing…) and since this time came true, I can never forget the following names below, where the order is not important. Without your support, this long way would have been impossible to be continued, and I am sure that if there is some extra way to be walked, I will never walk alone, either in Oslo, or in Spain, or somewhere else... That is the only important thing for me, once I came to this point, and, actually, probably the only important thing in life.

These names are the following ones:

- **Luis S. Torrente** for the *Best of You*[1], and ***my brother*** **Felipe** for *everything*, during my hard time in my first stage in Oslo in the Autumn 2004, when the burden of dead, and others, ended up with the difficult decision of coming back home, since I could not bear everything.

[1] Without this song composed by the Foo Fighters much of the inspiration would have been gone…

- **Milosz** and **Arvid**, -and the Hygens in general-, for *polish* me and being my *norwegian* family in Oslo. I guess we will always be *Brothers in Arms*[2], Arvid, -knight on the shiny armour-. About you, Milosz…*what the hell! shit happens*, and fortunately it occurred that I met you, first as a neighbour and friend, and in the end as a *brother*.
- **Nachete**, for being there. *"A por las…"*...
- The Son of the Great Britain, Sir **Anthony** Boxall, and the Australian gentleman *Don* **Brandon** Dreyer, for the unforgettable beer times we enjoyed, and for reviewing the English in the chapter 2 (Anthony) and the rest of the thesis and a final review in my final countdown to the deadline (Brandon).
- **Kimberly Fisher** and her mates in the ***Centre for Time Use Research*** *at the* **University of Oxford** for their trust in the worst moment, and making possible my participation in IATUR 2007 Conference at Washington DC. Such a fantastic experience impeded me quitting this idea, and therefore, you have an important part of the *blame* on this thesis. If I have any future in this, this fact will have been just crucial. Thank you for having been the only ones supporting economically this idea.
- **Pilar Moreno Hipólito,** my godmother, who was always there from the hard and very *disappointing* beginnings in the *Department of Economics at the University of Alicante*[3]. She always provided moral support and enlightening discussions, when leaving Madrid to go to a supposedly "*top university*" seemed to have been pointless. As well, she was my "sponsor" facilitating me the access to borrow some money; without that, it would have not been possible to stay in here during the last semester and hence, completing this thesis in particular and my degree in general.
- **And overall**, the ***Department of Economics*** *of the* **University of Oslo**, for its always exceptional dealing with me, letting me go back home 3 years ago with an unconditional possibility of coming back whenever the storm would have past. Apart from **Geir**, already mentioned, I must mention the names of the two Directors of the *Master Programme in Environmental and Development Economics* I met during my two stages in Oslo, **Olav Bjerkholt** (the previous one) and **Finn Førsund** (the current one), for their support; **Knut Sydsæter** and **Atle Seirstad** for their always enlightening general and mathematical comments and the encouragement; the Director of the Department, **Diderik Lund**, and **Hilde Bojer**, for the help in finding me some funding for the last semester, which has been extremely helpful and made possible the enjoyment of a fantastic teaching experience as an assistant of **Arne Strøm**; **Sara Cools**, simply, for everything. And especially, **Kaya Sverre**, since without her help, -not only, but mainly, in the bureaucratic procedures-, a chaotic guy like me would have been wasting a lot of time.

I cannot mention everybody as surely everybody deserves, but these people and institutions could not, and must not, be forgotten here.

***THANK YOU ALL FOR YOUR TIME***


[2] Nothing like a great song composed by the Dire Straits to define it…

[3] This was only worthwhile to learn how I did *not* want to be within this profession, and worthwile for the moments I enjoyed with my classmates, who in an overwhelming proportion run away from there as I did; special mention for the few time we enjoyed in there I would like to make to *termin-Aitor* and *Giovanni Arese* (some of the few still there), *Jaimito* (now in La Nova, Lisboa), and to Mariló Rufete, still the secretary.

# Contents

# ***Abstract***

This thesis attempts to do a survey of the economic literature about the theories of allocation of time throughout the time. We start with the seminal paper written by Becker in 1965, although we make some initial comments about some contributions made previously.

We present, discuss and summarize some theories we selected, considering that this selection is representative of what the economic theories about the allocation of time have been since the beginning until the present day. Furthermore, we provide a more general framework than the one used by Gary S. Becker (1965). Such a theoretical framework allows for joint production within the household production theory. We call this general framework as the *Generalized Economic Theory of Allocation of Time*.

JEL CODES: D00, D01, D10, D11, D13.

# 1. INTRODUCTION

*"It is only a matter of time (...). Certainty of dead, small chance of success…what are we waiting for?"*

(Tolkien, 1955)

The world has changed, is changing, and will continue changing as *time* goes by. Throughout history, one can always find similarities of how the different empires leading the world at each different period of *time* were managing with, fighting for, innovating, exhausting and profiting from their *resources*.

It is obvious how the Egyptian, Greek and Roman Empires dealt with their *resources* in the absence of pressure and scarcity of *them*, with extremely interesting production of their respective leading *time* periods. After the darkness of the Middle Ages, the coming Spanish Empire put some *light* into the world fighting for the gold, which also had some *shameful shadows*. The new golden *good* was supposed to produce wealth. However, Economics started working already during the 16th century (after discovering the New World called *America*[4]) with surely the first documented case of hyperinflation. This golden *resource* was not properly managed, and too much ambition led that Empire to destroy itself, as had already happened to former Empires, continued happening and will probably continue happening throughout *time*. The Spanish Empire is surely not remembered for its way of managing and profiting from its available resources, but is remembered for enormous contributions to Arts; unfortunately not to the art of Science. That task was appointed to the next empire: the British Empire. Newton and many others started with the production of new *scientific ideas* as a new and worthwhile production of their *time*. However, the muscles of the British Empire were the coal and water which were the row materials that led to the inventions, the new machines that allowed the chain production and the specialization of the work, and connected the world with railways and trains even in the far *Oceania*[5]. We will not comment on the present day and on the dominant American Empire here, whose influence is quite global, due to the intrinsic characteristics of the world today. One can easily think of it while walking around the streets in New York and Washington D.C., the capitals of the New Empire, or around Athens, Rome, Paris, Madrid, London or Berlin, the capitals of Old Empires, and currently capitals of one entity: *Europe*[6]. Of course, all this applies for the Occidental World. Expanding the scope to other places, we can talk about the Chinese Empire in the Oriental World, the oldest one in *Asia*[7] and in the world, which has a very interesting and different history. However, the world has been changing, is changing and will continue changing for them throughout *time* also. Paradoxically, one of the regions with more *resources*, *Africa*[8], has never changed much yet. Perhaps this is so because some other regions have never allowed it, or do not want to allow it, creating, allowing or supporting wars for whatever purpose. Hopefully Africa will be able to improve soon.

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[4] Click here to know more about the name *America*.

[5] Click here to know more about the name *Oceania*.

[6] Click here to know more about the name *Europe*.

[7] Click here to know more about the name *Asia*.

[8] Click here to know more about the name *Africa*.

However, everywhere and every *time*, ***many things change, not everything***, as the Old Empires of the Greek and Roman World always kept in mind. Some things are *immutable*. Some of these things are related to the ***main resource*** of any Empire, any country, any household, any human being, or any other being. This resource is ***TIME***, whose ***allocation*** at any period throughout *time* is behind any other economic or non-economic phenomena. It is quite interesting to remember some of the ancient philosophies talking about the *linear* conception of *time*, or the *cyclic* conception of *time*, whenever we think of time use research in general, and the *allocation of time* in particular. Impressively, Economics, since the beginnings as a science *somewhere* within the British Empire, has never dealt *seriously* with it, as a *resource*, as a factor of production, as a source of utility or happiness, or as any other aspect we can imagine. All other phenomena, sources, resources and factors have *surely* been analyzed in detail within Economics, both formally and in discussions. Nevertheless, *time* has never received much attention in Economics.

Fortunately, this does not mean that no attention has been paid to *it* within the field of Economics, and more fortunately, this does not mean that some other fields have never paid attention to *it*, such as Sociology, Psychology and of course Philosophy have been doing. Within this thesis, we try to make a picture (the best possible panoramic we managed to show, given our limited *time*), about what the research related to *time* has been in Economics, from a strictly theoretical perspective.

Bohm-Bawerk[9] (1889) was the first we know who put forward a time-based argument, showing a weakness in one of the most influential economic theories ever written:

> *"In particular, he argued that the Marxist theory of exploitation ignores the dimension of* ***time*** *in production"*
>
> (Wikipedia)

Nevertheless, no formal model has been found by us following Bohm-Bawerk (1889) ideas, though discussions have indeed continued.

On the way to create the first formal model involving time use, we have found interesting contributions. The most famous is Gary S. Becker's contribution in 1965. It is not only the seminal paper but also the undisputed and overwhelmingly dominant theory within the field of time use research in Economics, from a strictly theoretical perspective. His brilliant contribution has inspired a lot of empirical work, however as far as we have discovered throughout this *time*, very little theory has come into the field since Becker. That is not a bad sign, since it is probably a sign of Becker's theory being an exceptional theory and contribution. This is one reason; however perhaps there is another, as pointed out by DeSerpa:

> *"Nevertheless, these difficulties are attributable not to any shortcoming of the theoretical analysis, but to the* ***nature of the beast*** *about which we have been theorising* (: **time**)*."*
>
> (DeSerpa, 1971)

[9] Click here to know more about Bohm-Bawerk.

Whatever the reason, the fact we could check is the *existence of very little strictly theoretical and formal contributions* into the time use research, which is our research interest. We checked this following a very ordinary method: to review *one by one* all the papers that have cited Becker (1965) as a reference, since Becker (1965) represents the *birth* of the Economic Theories about Allocation of Time. That task was boring, and not very productive, although this did not surprise or discourage us. Among the almost 2000 papers citing Becker (1965), about half are within Economics, and the other half are within Sociology. Focusing only on Economics, we found about 900 paper citing Becker (1965). As we pointed out, we went through them one by one (obviously not reading all papers) with the hope of finding papers dealing with theories about *the allocation of time*, which was how Becker titled his theory. After screening everything, and even including some papers that were not strictly theoretical, we ended up with a list of links containing less than 50 papers. Making a second and more detailed screening, we ended with less than 10 papers that we think are showing and can summarize what the research on this field has been. For the selection of these papers we have tried to follow a *chrono*logical principle, keeping the first paper which somehow opened the way of thinking for any relevant theoretical contribution we have been able to find. Our search of papers may be subject to some errors, although we hope not.

For some of the cases, we found difficulties in obtaining the papers, and for some others we unfortunately did not have time to review. However, "surprisingly", the field became so "small" that any new paper in the 1970s (the "golden age" for the theories involving time use in Economics) we find, can easily be presented as a particularization of what, still today, is the dominant theory: Becker's Theory of Allocation of Time in 1965. More recently only a very limited number of new theories have been found, making the 1960s-1970s more important.

Therefore, in this thesis we will attempt to describe all the theories involving *time* we have found and make them readable to anybody. Although there is a lot of mathematics in some chapters, we tried to put words to the equations, for anybody not in the field of Economics to be able to grasp the concepts, the intuitions and the ideas.

In chapter 2 we grasp the concepts, discuss and generalize Becker's Theory of Allocation of Time. This chapter is probably the most important, and therefore, we spent about one third of the thesis on it. It is clear why we did this, since Becker Model is still the dominant model, which we find fascinating.

Chapter 3 is an example of how the theories in the 1970s looked. There are some other papers which might have been worthwhile to include here, such as Evans (1972) and the discussions that Evans (1972) generated. However, as we will show in this chapter, all the theories in the 1970s we found (and in general when we regard static optimization models), can be expressed as particularizations of Becker (1965) with small variations, primarily in the constraints. We use DeSerpa (1971) as a decade example.

Chapter 4 is devoted to one of our favourite papers, the Pollak & Wachter criticism in 1975. They showed the weaknesses in Becker (1965) and in the household production theory in general, with strong results, in the form of theorems.

Chapter 5 is devoted to Gronau. His theory is considered a benchmark within the field. Although the model is, -as all what we read from the 1970s-, a particular case of

Becker (1965), it provides a lot of insights obtained in a very simple way. We will try to go into the essence of the theory, not the details, within this chapter.

Chapter 6 is a very interesting and enlightening reflection on *time* in Economics. We will use it not only to make a flashback (in there is the only place in which we got to know, for example, about Bohm-Bawerk), but also to illustrate the two dimensions of *time*: *time* as something *flowing* continuously, something unstoppable, -i.e., changes *throughout time*-, and *time* as *resource*, as an *input*. This paper is useful for our purpose of connecting Becker-based theories with the few attempts of making dynamic models involving time use.

Chapter 7 is a survey done in 1991 by Juster & Stafford. This survey had a much more general purpose, as can be checked. The theoretical part is acceptable, but too short under our opinion. We used this paper because of the good summing up of the intertemporal models involving time use, which dominated the 1980s, and some other comments.

Chapter 8 is devoted to Fischer (2001), which is another of our favourite contributions, together with Becker (1965) and Pollak & Wachter (1975). And this is so since this paper is the first and only paper we have found that deals with time use as an *exhaustible resource*, using dynamic techniques.

And to finish, chapter 9 is devoted to the conclusions and some concluding remarks.

It must be noted that there are some recent theories not included in this survey involving some game theory in the field of the allocation of time. We got to find out one book written by Beblo (2001)[10], however we could only peruse it once, and not in detail. There was a small part dedicated to game theory within the bargaining over time allocation which she analyzes, though the book was mainly focused on some empirical studies.

To conclude this introduction, we would like to point out two facts we deem to be of importance. On the one hand, although Becker is considered the *birth* of time issues within Economics, we must not forget to stress the following fact: the first person to provide an insight into the importance of time allocation and who also started some empirical studies was Mincer in 1962. On the other hand, although the first person to provide a formal analysis applied to time issues was Becker (bringing to life the first formal Theory of Allocation of Time), the apparatus used by Becker is inspired by Lancaster ideas, which were officially published in his famous paper in 1966. Becker mentions in his paper that Lancaster was a source of inspiration. However, and surprisingly, few people know that those ideas by Lancaster were in essence very similar to a previous work. This work was officially *defended* in 1962, 4 years earlier than Lancaster, in a Doctoral dissertation written by Duncan Ironmonger, who earned his Ph. D in Economics at Cambridge University with it in 1962. Few people are aware of this fact, and we feel obligated to make a reference to this here. Even though it was not specifically related to time issues, Becker was inspired and used Lancaster apparatus, which in fact had already been suggested by Ironmonger in 1962. Therefore, the *genesis* of the apparatus which is used particularly by Becker, applied to *time* issues, can be attributed to Ironmonger, and therefore, 1962 can be considered the year in which time research applied to Economics was conceived from a theoretical point of view, although, as we have noted, its *birth* would come with Becker in 1965.

[10] We must thank Farideh Ramjerdi from the Transportøkonomiks Institutt (TØI) for this reference and her comments, although sadly, we could not manage to include it in this thesis as we believe it deserves.

# 2. BECKER MODEL, 1965-1971: THE MAIN THEORY.

> *"While the growing abundance of goods may reduce the value of additional goods, time becomes more valuable as goods become more abundant."*
>
> (Becker, Nobel lecture, 1992)

a. Idea

Gary S. Becker's[11] paper is motivated by the following observation: *economic development has decreased the number of hours devoted to work*. Even if somebody is working 14 hours per day, 6 days a week, -something quite unrealistic-, only the half of the hours available in the week, i.e. 84 hours, are devoted to work. He highlights how important the study of labour related issues has been,-which can be at most the half of the total time available-, while at the same time he questions why nobody in economics, at that time, wondered about what happened to the nonworking time, -which in the overwhelming majority of cases is more than the half of the total time available-:

> *"Consequently the allocation and efficiency of non-working time may now be more important to economic welfare than that of working time; yet the attention paid by economists to the latter dwarfs any paid to the former."*
>
> (Becker, 1965)

Thus, his intention with this paper, which was subsequently converted into a seminal paper into this field with almost two thousands citations, is simply the following:

> *"Here I attempt to develop a **general** treatment of the allocation of time in all other non-work activities."*
>
> (Becker, 1965)

Later, he argued that the relative importance of the allocation of non-working time had become higher than the one concerning work issues, as already noted. Therefore, its efficiency in the allocation and its impact on welfare would be more determining, although economic theory and literature did not consider this fact very seriously. In support of the fact, Becker provides revealing data: in most of countries the *working time was less than fifty hours per week, which is less than one third of the total time available during the whole week*. It can still be considered that economists and Economics are not dealing very seriously with this, especially with regard to welfare implications.

[11] Click here to know more about the Nobel Prize Winner in 1992: Gary S. Becker

Becker points out several reasons in favour of the decrease in working hours. These are mainly related to increase in education, given the fact that young people in the 1960s were already starting to considerably delay their entry into the job market.

This leads Becker to argue the importance of the forgone earnings, something that as he remarks, was starting to be considered by economists for this topic, and, in general, for all the theories dealing with investment in human capital. However, as he said, this was still focused on working time, and little or no attention, -at least in the same way-, had been paid to non-working time.

Thus, he and some people in Columbia were concerned for a period of time about what he perfectly described as follows:

> *"In the last few years a group of us at Columbia University have been occupied, perhaps initially independently but then increasingly less so, with introducing the cost of time systematically into decisions about non-work activities."*
>
> (Becker, 1965)

And some evidence, result and contribution were provided:

- Mincer showed how the income elasticities of demand tend to be biased when the cost of time is ignored
- Owen analyzed how the demand for leisure can be affected
- Becker then wrote, based and inspired by Mincer and Owen among others, his famous and brilliant Theory of Allocation of Time.

b. Becker Model and our Generalized Theory of Allocation of Time

The main idea revolves around the main argument having an impact on the utility function being the concept of *'commodities'*, something for which an individual has to use inputs in order for commodities to be produced. These inputs are *consumption* goods found in the market and time spent on the *production* of each commodity[12].

Hence, the problem includes time as an input, and requires and states a time constraint in addition to the usual budget constraint.

The Becker model looks like this:

[12] In Becker (1965) the model is presented as we try to show. Nevertheless, it should be pointed out that later on, in the book *Economic Theory* by Becker, a new input is introduced. This input is the set of environmental variables, as he names them. For simplicity, and given that it is not related to the allocation of time directly, we have not included this in our presentation of the model.

$$\max_{x,T} \; U\left(\vec{Z}\right) = U\left(f_1\left(\vec{x}_1, \vec{T}_1\right), ..., f_m\left(\vec{x}_m, \vec{T}_m\right)\right)$$

$$s.t. \quad \sum_i^m \vec{p}_i^T \vec{x}_i \leq I = \vec{w}^T \vec{T}_w + V \qquad (1)$$

$$\sum_i^m \vec{T}_i = \vec{T} - \vec{T}_w \qquad (2)$$

Where:

- $Z_i = f_i\left(\vec{x}_i, \vec{T}_i\right)$, with $Z_i \in \mathbb{R}^m, i = 1, ..., m$ denotes the amount of commodity *i*, for which achievement the vectors $\vec{x}_i$ and $\vec{T}_i$ are needed.
- $\vec{x}_i$: *n*-dimensional vectors of demands of goods, -needed to produce the *i*-th commodity-, whose associated *n*-dimensional price vector is $\vec{p}$.
- $\vec{T}_i$: *p*-dimensional vectors of non-working time inputs, -needed to produce the *i*-th commodity-.
- $\vec{T}$: *p*-dimensional vector of total time available (whose elements must add up to 24 hours a day, 7 days a week, etc)
- $\vec{T}_w$: *p*-dimensional vector of working time into the job market, whose associated *p*-dimensional vector of wages is $\vec{w}$ units of money per unit of working time, for each respective type or aspect of time.
- *V*: other income
- *I*: Total income, earned by working, or stemming from other sources, called *V*.

Although it is never explicitly written in Becker (1965), -however it is commented-, it is trivial that a constraint imposing that any type of time spent in any use has to be nonnegative. We should then always add to the Becker model the following constraint[13]:

$$\vec{T}_i \geq 0, \; \forall i = 1, ..., m \qquad (3)$$

The same trivial fact happens to consumption goods, since of course the amount consumed has to be a nonnegative amount, for obvious physical reasons. A similar constraint[14] can be added to Becker (1965), as follows:

$$\vec{x}_i \geq 0, \; \forall i = 1, ..., m \qquad (3^*)$$

[13] We stress this, even though it may be obvious. LaGrange multipliers associated to all time uses are positive (or exceptionally zero) whenever some of the time uses are zero, and the interpretation of this is just that the multipliers are the *valuation* in terms of utility *of each of the time use that you are not using* (since the amount of time spent on such use is zero) . Such an interpretation is quite curious, and might be very useful for some purposes.

[14] Its consequences in terms of interpretation are parallel to those applicable to time inputs.

We will always keep to such constraints, to be strictly formal, for all the models within this chapter. However, for simplicity the reader can disregard them, since under usual assumptions in microeconomic theory we will be in an interior solution situation.

Moreover, we want to present how the Becker Model looks like for one *aspect* or *type* of time. Becker talks about aspects of time, although to clarify, we will use either the word *aspect* or the word *type* to denote the different *dimensions* of the time vector stated before. The best way to understand what an aspect of time is, as pointed by Becker, is to think of an example like the one he uses. Such an example revolves around two different aspects or types of time being "day time" or "night time". During the considered period of time, whose length we denoted by $T$, then, two aspects of time would be the day time, and the night time, which add up to $T$, the total amount of time available for the period of time considered (24 hours per day, 7 days a week, etc). We want to show here how the Becker Model looks with one aspect of time, since later on in this chapter we will expand Becker model, however we will use the *one type of time* case as a ***sufficient*** illustration for the generalization of the Becker Model. Hence, let us present ***Becker's Theory of Allocation of Time for the case of one aspect or type of time***, which is as follows:

$$\max_{x,T} \; U\left(\vec{Z}\right) = U\left(f_1\left(\vec{x}_1, T_{11}\right), ..., f_m\left(\vec{x}_m, T_{1m}\right)\right)$$

$$s.t. \quad \sum_{i}^{m} \vec{p}_i^T \vec{x}_i \le I = wT_{1w} + V \qquad (1^*)$$

$$\sum_{i}^{m} T_i = T - T_{1w} \qquad (2^*)$$

Where the generic $T_{1m}$ is the *amount* of time spent in the production of commodity $m$ in the subscript, and the *1* in the subscript tells us that there is only one type of time.

As in Becker, we could merge (1*) and (2*) into one single constraint, which was named by Milton Friedman[15] as the ***Full Income Constraint***, as pointed by Becker in 1965. Such a constraint look like this:

$$\sum_{i=1}^{m} \vec{p}^T \vec{x}_i + \sum_{i=1}^{m} wT_{1i} \le wT + V = S \qquad \text{(FIC)}$$

, where $S$ denotes what they named *Full Income*.

In his paper, Becker continues getting closer to the traditional microeconomic model from the model presented earlier, the one with constraints (1*) and (2*), or equivalently the full income constraint (FIC). Then, a quite crucial and discussable assumption comes, where the demands of goods and time inputs for each activity are supposed to be a fixed proportion of the amount of commodities.

[15] Click on here to know more about the Nobel Prize Winner in 1976: Milton Friedman.

We continue the analysis for a single type of time, due to the reasons already noted. Such assumption looks like, formally, as follows:

$$T_{1i} = t_i Z_i \qquad (4)$$

$$\vec{x}_i = \vec{b}_i Z_i \qquad (4^*)$$

Which inserted into the full income constraint in (FIC), yields:

$$\sum_{i=1}^{m} \vec{p}^T \vec{b}_i Z_i + \sum_{i=1}^{m} w t_i Z_i \le wT + V = S \Rightarrow$$

$$\sum_i \left( \vec{p}^T \vec{b}_i + w t_i \right) Z_i \le wT + V = S \Rightarrow$$

$$\Rightarrow \sum_i \pi_i Z_i \le wT + V = S \qquad (5)$$

The model can be then expressed in this way[16]:

$$\max_{\vec{Z}} \; U\left(\vec{Z}\right)$$

$$s.t. \;\; \sum_i \pi_i Z_i \le S \qquad (6)$$

Where:

$$\pi_i = \vec{p}^T \vec{b}_i + w t_i \;\; , \forall i = 1, ..., m \qquad (7)$$

are the **prices for each commodity *i*** expressing both the cost of goods and cost of time (measured by the wage rate as the opportunity cost), and *S* is what Becker calls the full income.

The model brings the standard condition of Marginal Rate of Substitution between each pair of commodities equal to the price ratio.

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[16] Becker provides as well an alternative formulation in terms of what he calls the loss function, *L(Z)=S-I*, what is measuring the *opportunity cost of all the time spent doing all the nonworking activities* in the range, which is measured at the constant wage rate $w$. The alternative formulation can be expressed as follows:

$$\max_{\vec{Z}} \; U\left(\vec{Z}\right)$$

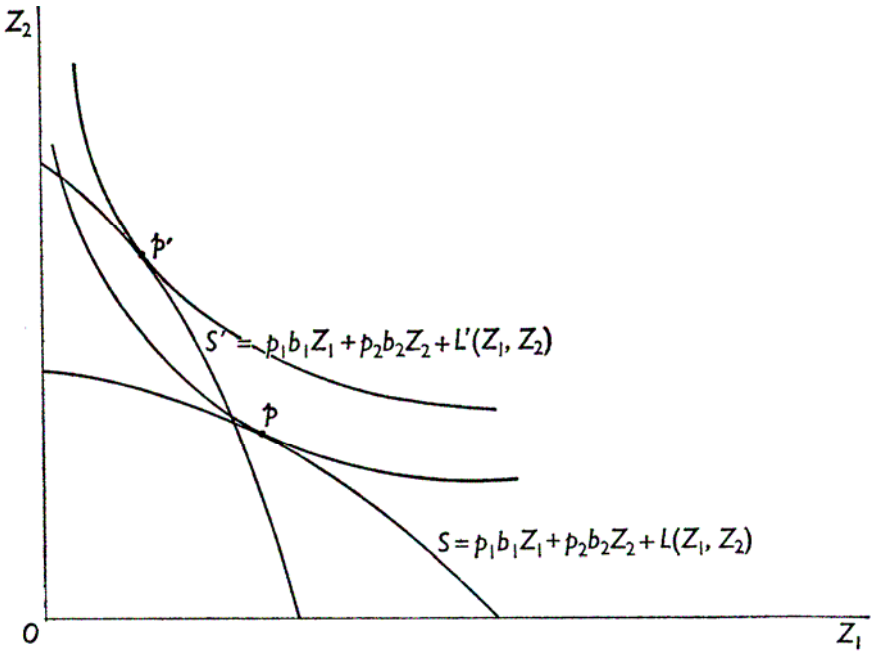
$$s.t. \;\; \sum_i^m \vec{p}^T \vec{b}_i Z_i + L(Z) \le S$$

with $L(Z) = \sum_i w t_i Z_i$ and $S = \vec{w}T + V$

$$\frac{U'_{Z_i}}{U'_{Z_j}} = \frac{\pi_i}{\pi_j} \qquad (8)$$

To get this version we remark on the crucial role of the demands of goods and time inputs for each commodity as fixed proportions of the amount of commodities. This is a bit controversial and it is discussed later in chapter 4, although the clear truth is that is a very strong assumption, as Pollak & Wachter (1975) note.

At this point, Becker gets a model that looks like the textbook model in classical microeconomic theory, and he plays a bit with comparative statics. If we believe that the prices $\pi_i$ are given, and $S$ to be a good measure of your total time available in money (at wage $w$) plus your other income, the model presents no difference with the textbook microeconomic model. Changes in prices for commodities and the wealth $S$ can be represented in the standard graphs, as follows:




*(Figure 1)*[17]

Changes in the constraint can be due to changes in the prices for market goods and to changes in the price of time, which in Becker is measured by the wage as the opportunity cost of non work time.

A change in the price vector is depicted in the graph, and makes no difference with traditional microeconomic interpretations. The budget set changes of shape and the changes in prices brings a change in the tangency conditions and hence, changes in the equilibrium.

[17] Directly copied and pasted from Becker (1965).

Now we proceed to suggesting a more ***general*** framework than in Becker's contribution, which has been very well accepted not only in Economics, but also in other fields such as Sociology, among others, finishing our suggestion by showing how *Becker's Theory of Allocation of Time* is a *particularization* of it.

First, let us expand Becker's model to allow it to present **joint production**. Such a topic of joint production is not allowed in Becker, as the literature throughout time showed[18]. To illustrate how Becker's theory does not allow it, one can easily think that the combination of goods and time used to produce one commodity is exclusively producing that particular commodity, and cannot be used to produce any other commodity. Roughly speaking, this means that if we consider only two commodities, for example, "listening music" and "cooking", we are not allowed in Becker's to *produce* "listening to music" and "cooking" at the same time. *Multitasking* is not possible with Becker's model, and the synonym for *multitasking* in economic vocabulary is *joint production*. The model below allows joint production, as can be seen sequentially from here on:

Hence, again, such specification in Becker (1965) does not allow joint production, which is actually daily life. Another example showing Becker's limitations in this respect is that something as common as "producing cleaning" and "producing enjoyment of singing", -which are two commodities-, cannot be done at the same time. Therefore, if you are cleaning, you just clean, to get a cleaner environment or place, nothing else; if you want to enjoy a moment of singing, you just sing (maybe sitting down? Imagine if you are just dancing, for instance, while cleaning), nothing else. Therefore, to clarify, to stay singing (and/or dancing) while you are cleaning is not allowed in Becker.

To introduce joint production, we just have to expand each production function for each commodity in Becker's to a matricial form, as follows:

$$Z_i = f_i\left(\begin{pmatrix} x_{11} & x_{1q} \\ & \\ x_{n1} & x_{nq} \end{pmatrix}_{n\times q}, \begin{pmatrix} T_{11} & T_{1r} \\ & \\ T_{p1} & T_{pr} \end{pmatrix}_{p\times r}\right) \qquad (9)$$

with $\vec{Z} \in \mathbb{R}^m, i = 1,...,m,$ as commodities,

where $X_{n\times q} = \begin{pmatrix} x_{11} & x_{1q} \\ & \\ x_{n1} & x_{nq} \end{pmatrix}$ is the matrix of types of goods (by rows) used in each different use (by columns) one can use them,

[18] Pollak & Wachter (1975), for example, discuss this. This paper is discussed within this thesis in chapter 4.

and $\Im_{p\times r} = \begin{pmatrix} T_{11} & T_{1r} \\ & \\ T_{p1} & T_{pr} \end{pmatrix}$ is the matrix of types (or aspects) of time inputs (by rows) used in each different use (by columns) one can use them,

Moreover, it is very important to always keep in mind that what we call "types of time" is what Becker called "aspects of time". In Becker's model, "aspects of time" are vaguely presented as being day time and night time, or week time and weekend time. We want to remark this idea, and that is why we again insist on it. Inspired by this idea, but in order to make the theory compatible with the impressively good datasets available in the Multinational Time Use Survey (MTUS from now on) led by the Centre for Time Use Research (CTUR from now on) located at Oxford University, we prefer to think of each type of time being allocated into the 5-minutes time slots in which MTUS and CTUR have divided the day. Their data is very rich, and very detailed. We can consider each slot being a type of time, since it is not the same to perform an activity (let us say, having a beer) at 23.11h at night, than at 08.11h in the morning for breakfast. The production of "quality of beer moments" is radically different in such two time periods of the day (and the same happens if at the same 23.11h in the night, you compare Monday night with Saturday night…). Among each of the types of time, we of course can use our time for whatever use we like, which in the particular example was just to drink beer.

For a parallel interpretation of the matrix of market goods, we have two suggestions, though there could be more. The first suggestion is just to understand, similarly, types of goods as simply the different market goods we find in the market, and the use of them, with regard to, and for example, when are they demanded or consumed, in a similar division into 5-minutes slots. The second suggestion would be replacing the 5-minutes slots division interpretation by simply thinking of the different uses for each particular market good. An example can be whether we use the cell phone just to talk, or just for listening to music on the mp3 (or mp4) application, or even for playing games that every phone has nowadays. This second aspect is something more challenging and difficult to measure, although it might be very interesting to have data about the different uses for each of the market goods. Perhaps the second one is a small utopia.

Then, we can present the ***Generalized (Becker's) Theory of Allocation of Time***, allowing joint production, as follows:

$$\max_{X,\Im} \; U\left(\vec{Z}\right) = U\left(f_1\left(X_{n\times q}, \Im_{p\times r}\right), ..., f_m\left(X_{n\times q}, \Im_{p\times r}\right)\right)$$

$$s.t. \quad \sum_{q=1}^{q} \vec{p}^T \vec{x}_q \leq I = \vec{w}^T \vec{T}_w + V \tag{10}$$

$$\sum_{r=1}^{r} \vec{T}_r = \vec{T} - \vec{T}_w \tag{11}$$

$$\sum_{p=1}^{p} T_p = T \tag{12}$$

$$X_{n\times q} \geq 0_{n\times q}, \; \Im_{p\times r} \geq 0_{p\times r} \tag{13}$$

where:

– $\vec{T}_r$ vector corresponding to the $r$-th column in $\Im_{p\times r}$.

– $\vec{x}_q$ vector corresponding to the $q$-th column in $X_{n\times q}$.

– $\vec{T}$ is a $p$-dimensional vector whose elements, $T_p$, represents the amounts of time available for each type of type $p$.

– $T$ is the immutable total time available (24h, 7 days, etc).

This allows such realistic examples as cooking and listening to music to be commodities (or *activities*, if we prefer to not to follow Becker's vocabulary and use a more naïve terminology) produced at the same instant of time, during/with the same time period/input. This problem could include any extra constraint which might be considered relevant[19], as several papers in the 1970s suggested[20].

The solutions for this problem would be as follows:

$$\vec{x}_q^* = x\left(\vec{p}, w, V\right), \; \forall q \text{ , all possible uses of goods} \tag{14}$$

$$\vec{T}_r^* = T_r\left(\vec{p}, w, V\right), \; \forall r \text{ all possible uses of time} \tag{15}$$

$$H^* = H\left(\vec{p}, w, V\right), \text{ the Indirect Utility function} \tag{16}$$

Although we already commented on the joint production for the case of time inputs, the joint production for inputs of goods is not less interesting. We can

[19] To preserve the generality of the model, any other possible extra constraint included it is included by simply adding this general expression for any extra constraint to the *Generalized Theory of Allocation of Time*:

$$B_{s\times(nq+pr)} Q_{(nq+pr)\times 1} \leq 0_{s\times 1}$$

where:

- $B_{s\times(nq+pr)}$ is a matrix of positive or negative coefficients (all elements equal to zero implies the presence of no extra constraint), and
- $Q^T_{(nq+pr)\times 1} = \left(x_{11}, ..., x_{nq}, T_{11}, ..., T_{pr}\right)$

[20] As it will be commented later, on chapter 3, during the 1970s several papers as Evans (1972) and DeSerpa (1971) studied particularizations of Becker, with some extra constraints, never noted in Becker (1965).

think of the following example: a normal family man finishes the working day. He just opens the door of his car, switches on the engine, turns on the radio which was given as a present by the salesman who sold the car to him, and then starts driving back home. The car is already producing two commodities: "listening to the radio" and "transportation". Suddenly the mobile phone rings, and due to the new technology the car included, he is able to "answer the call" and is talking while driving, without any risk since the speakers of the radio now to allow him to listen to the person calling. It is his 11 year old lovely daughter, asking him whether he can pick up her from the school, since it is too cold to walk the two kilometres home from the school, which is actually on the car route back home. Indeed the father says yes, and after several minutes, they meet at the school and drive back home happily on the same car route, but having also "pick(ed) up the children at the school". A simple market good, a car, is producing several commodities combined with some time inputs: listening to the radio, transportation, answering calls and picking up the children. Such a realistic example is possible under this proposed framework.

However, and for simplicity, we can make things a bit simpler, by assuming that you can always assign the proper amount of every good used into the production of every commodity. Formally, this implies that $q = m$ in our *Generalized Theory of Allocation of Time*, and therefore that only the *m*-th column of the matrix $X_{n\times q}$ is entering into the production of the *m*-th commodity. We will keep to this assumption, for the sake of simplicity, from now on, which is in essence similar to Becker.

It should be noted that Becker's Theory of Allocation of Time is a particular case of the problem stated above, when $q = r = m$ and the production of the *m*-th commodity is only depending on the *m*-th column of both $X_{n\times q}, \Im_{p\times r}$ (and no extra constraints are regarded[21]).

Thus, by reducing the ***Generalized Economic Theory of Allocation of Time*** to the ***case of one and only one type of time***, then the subscript $p$ is such that $p = 1$, and as a by product, we are forced to assume a single wage rate for, at least, each particular individual, instead of a vector. In addition, we assume $q = m$ for simplicity, based on the discussion above after the family man example. Therefore, the model looks like this for such a case:

[21] Such a lack of constraints in Becker's model was used by several authors during the late 1960s and the 1970s to publish new papers. Example of this are DeSerpa (1971) or Evans (1972), being Evans contribution a quite interesting one at this respect, despite of his model being a very particular case of Becker in which ***only*** time inputs are arguments into the utility function ( i.e. $Z_i = T_i,\ \forall i$ ) in his model. It should be clarified that in his model Evans (1972) uses a notation in which $T_i = a_i,\ \forall i$ is the time spent into an activity *i*.

$$\max_{X,\Im} \; U\left(\vec{Z}\right) = U\left(f_1\left(\vec{x}_1, \Im_{1\times r}\right), ..., f_m\left(\vec{x}_m, \Im_{1\times r}\right)\right)$$

$$s.t. \quad \sum_{m=1}^{m} \vec{p}^T \vec{x}_m \leq I = \vec{w}^T T_{1w} + V \qquad (10^*)$$

$$\sum_{r=1}^{r} T_{1r} = T - T_{1w} \qquad (11^*)$$

$$X_{n\times m} \geq 0_{n\times m}, \; \Im_{1\times r} \geq 0_{1\times r} \qquad (13^*)$$

Remembering that $\Im_{1\times r} = \left(T_{11}, ..., T_{1r}\right)$ is the *vector* telling us the amount of time spent in each different use of time, $r$, which is not necessarily coninciding with the amount of commodities, denoted by $m$.

Where as in Becker, we could merge (6*) and (7*) into one single constraint, the so called ***Full Income Constraint*** mentioned earlier. Such a constraint looks like this for the ***case of one aspect of time***:

$$\sum_{q=1}^{q} \vec{p}^T \vec{x}_q + \sum_{r=1}^{r} wT_{1r} \leq wT + V \qquad \text{(FIC*)}$$

Such a constraint can be used without any problem when the problem is considering only a single type of time. Otherwise, some problems may arise leading to unfeasible time inputs for some types of time.

We are now in the case of one single type of time, under the *Generalized Theory of Allocation of Time* presented before, as we already know. It is sufficient to use this case to illustrate how the model allows for joint production and there is no need to impose $r = m$, which means that the range of activities can differ from the different uses in which the individuals can use their time.

However, we also propose the following approach below, based on a 2-step optimization process. Such an approach is based on the following ideas: a) individuals adopt a cost minimization attitude in the sense of classical microeconomic theory of production when they are considered as producers of commodities using time and goods as inputs, and b) individuals adopt a utility maximization attitude in the sense of classical microeconomic theory of consumption when they are considered as consumers of commodities giving them some utility, given the time and budget constraints, and perhaps some others.

Such a 2-step optimization process, for the case of a single type or aspect of time, looks like this:

*First step*:

$$\min_{X_{n\times q},\Im_{p\times r}} \sum_{q=1}^{q} \vec{p}^{T}\vec{x}_q + \sum_{r=1}^{r} wT_{1r}$$

$$s.t.\ f_i\left(X_{n\times q},\Im_{1\times r}\right)\geq Z_i,\ \forall i=1...m \qquad (17)$$

$$X_{n\times q}\geq 0_{n\times q},\ \Im_{1\times r}\geq 0_{1\times r} \qquad (18)$$

which yields:

$$\vec{x}_q^{*}=x\left(\vec{p},w,\vec{Z}\right),\ \forall q \qquad (19)$$

$$T_r^{*}=T_r\left(\vec{p},w,\vec{Z}\right),\ \forall r \qquad (20)$$

and the corresponding Value (Expenditure) Function:

$$E^{*}\left(\vec{p},w,\vec{Z}\right)=\sum_{q=1}^{q}\vec{p}^{T}\vec{x}_q^{*}+\sum_{r=1}^{r}wT_{1r}^{*} \qquad (21)$$

Where $\vec{Z}$ is the amount of commodities, to be determined in the second step.

*Second step:*

$$\max_{\vec{Z}}\ U\left(\vec{Z}\right)$$

$$s.t.\ \ E^{*}\left(\vec{p},w,\vec{Z}\right)\leq wT+V \qquad (22)$$

$$X_{n\times q}\geq 0_{n\times q},\ \Im_{1\times r}\geq 0_{1\times r} \qquad (23)$$

This explains why we can find a concrete functional form for the expenditure function in the first step, -which is precisely the left hand side in the so called Full Income Constraint-, as a difference with the case of multiple types of times. The explanation is, again, stemming from the fact of considering *only one type of time*. Hence there is no possibility of reaching optimal time inputs that might not be feasible, and then, very unrealistic, something that *may* happen for the case of several types of time. This is due to the reduction of the problem to only one type of time case, and we must stress this.

It must be noticed that the wage rate is both a price of one of the factors or inputs (free time, as the opportunity cost of spending some time into free or leisure time) and a source of income, *I*. This makes a difference with traditional microeconomic analysis, and we should be careful and be minded of it.

Hence, we can get the optimal solutions for the second step, -under usual nice properties for the utility function-, which are as follows:

$$\vec{Z}^{*}=Z\left(\vec{p},w,V\right) \qquad (24)$$

$$H^{*}=H\left(\vec{p},w,V\right) \qquad (25)$$

Where $H^*$ is the usual Indirect Utility function, used in classical microeconomic theory. By inserting (24)-(25) into the first step optimal conditions (19)-(21) we get:

$$X^*_{n\times q} = X\left(\vec{p}, w, V\right) \qquad (26)$$

$$\Im^*_{1\times r} = \Im\left(\vec{p}, w, V\right) \qquad (27)$$

$$E^*\left(\vec{p}, w, \vec{Z}^*\right) = E^*\left(\vec{p}, w, V\right) \qquad (28)$$

The problem is then completely solved and all the unknown variables completely determined. For the existence of a solution, usual microeconomic assumptions have to hold, and similarly for uniqueness, applied to our model and not forgetting the warnings we already pointed out before.

Now let us look at the value functions. The value function for the first step is similar to a cost function in classical microeconomic (production) theory, and the value function for the second step is similar too to the indirect utility function in classical microeconomic (consumer) theory.

From the general Cost Minimization Problem (CMP) in classical microeconomics, we recall the expenditure function properties in books like Mas-Colell et al (1995) or Segura (1988)[22] here, adapting some of them to our case in the first step:

- *(Properties of the Expenditure function[23])* Suppose that $E^*\left(\vec{p}, w, \vec{Z}^*\right)$ is the *Expenditure Function* of a multi-output technology with production functions of commodities $f_i\left(X_{n\times q}, \Im_{p\times r}\right)\ \forall i = 1...m$, and $X^*_{n\times q} = X\left(\vec{p}, w, \vec{Z}^*\right)$ and $\Im^*_{1\times r} = \Im\left(\vec{p}, w, \vec{Z}^*\right)$ are the associated conditional demand and time use correspondences, in the first step of the Generalized Theory of Allocation of Time. Assume also that the possibility set is closed and satisfies the free disposal property. Then:

  - 1. $E^*$ is homogeneous of degree 1 in $\left(\vec{p}, w\right)$, nondecreasing in $\vec{Z}$.
  - 2. $E^*$ is concave in $\left(\vec{p}, w\right)$.
  - 3. The conditional demand and time use correspondences $X^*_{n\times q} = X\left(\vec{p}, w, \vec{Z}^*\right)$ and $\Im^*_{1\times r} = \Im\left(\vec{p}, w, \vec{Z}^*\right)$, respectively, are homogeneous of degree zero in $\left(\vec{p}, w\right)$.

[22] His book provides more formal explanations in general and in particular to our case.

[23] See proposition 5.C.2 in Mas-Collell et. al, *Microeconomic Theory*, Oxford University Press 1995. as the problem is parallel to the one considered by them, the properties we highlight are just an adaptation of what it is written by them.

- 4. (Shepard's lemma)
  If $X^*_{n\times q} = X\left(\vec{p}, w, \vec{Z}^*\right)$ and $\Im^*_{1\times r} = \Im\left(\vec{p}, w, \vec{Z}^*\right)$ consist of a single point, then $E^*$ is differentiable with respect to $\left(\vec{p}\right)$ at $\left(\vec{p}^0\right)$ and $\nabla_{\vec{p}} E\left(\vec{p}^0, w, \vec{Z}^*\right) = X^*_{n\times q}$ .

These properties above are exact adaptations of the ones in Classical Microeconomics to our case of a single type of time into the Generalized Economic Theory of Allocation of Time. We only show these four ones above, though in Mas-Colell et al., for example, nine properties are stated. We have chosen just the ones we deem more relevant, though the others hold too, under similar conditions applied to our case. As well, we do it in order to shape our doubts on these properties being holding in our case. If the same thing wants to be generalized for more than one type of time, everything becomes a mess, and we are not sure at this moment about all or some of the properties holding. We leave this for future research. Notice that Shepard's lemma is not valid for getting the time use demands, even for this case of a single type of time, since for all possible uses of time, the "price" or "(opportunity) cost" of each use of time is the same, and equal to the wage rate *w*, which is unique, for the individual. It is not clear what you would get by differentiating the expenditure function at the optimum with respect to the wage rate, though the intuition seems to lead us to the overall demand of leisure time function. Moreover, we must highlight that these properties in Mas-Colell et al. are for the case of one single output. We are dealing with a multi-output case, where our outputs are the different commodities. We must show our limitations at this point, which we hope to get over in future research. We have reproduced them from Mas-Colell et al (1995) or Segura (1988) to comment on them later as we did, showing our limitations.

Similarly, we do the same thing for the Indirect Utility Function in the second step, and we get the following:

- *(Properties of the Indirect Utility Function[24])*. Suppose that $H$ is a continuous utility function representing a locally non satiated preference relation defined on the consumption set. The *Indirect Utility Function* $H^* = H\left(\vec{p}, w, V\right)$ is:
  a) Homogeneous of degree 0 in $\left(\vec{p}, w, V\right)$.
  b) Strictly increasing in $\left(w, V\right)$, and nonincreasing in any element of $\vec{p}$, the price vector.
  c) Quasiconvex in $\left(\vec{p}, w, V\right)$.
  d) Continuous in $\left(\vec{p}, w, V\right)$.

[24] Proposition 3.D.3 in Mas-Collell et al (1995) adapted to our case.

These properties do not seem to present any particular problem compared to classical microeconomic theory, though, again, we will leave a more detailed analysis for the future.

It is out of any doubt the enormous and fantastic contribution made by Becker in 1965, and we have just suggested a more general framework. As it will be discussed in chapter 4, the commodity prices defined by Becker present several problems, which are avoided in our 2-step model, which is aligned to Pollak & Wachter ´s criticisms.

Though in Becker some other analysis or comments are made, related to some applications of this idea, we do not enter in detail, and we just continue summarizing the main findings in Becker (1965).

c. Findings

The contributions of Becker (1965) can be organized as follows:

i. Commodities: the first difference we find with the textbook model in classical microeconomics is that the arguments which are entering directly in the utility function are commodities.
ii. Each commodity is "produced" by demands for market goods (which can be considered as *ingredients*), and time spent on the production of each commodity we enjoy. Hence, the idea of households being both 'mini-firms' *producing* commodities which they are at the same time *consuming* is something to highlight, and the heart of the theory.
iii. Preferences are defined over the set of commodities, and all the commodities are the outputs produced by a household production function, whose inputs are time inputs and market goods.
iv. Time as an input: as pointed out above, time use is something entering for the first time into a formal economic theory.
v. Time constraints: new constraints are introduced, together with the budget constraint. These constraints illustrate the fact that our time is limited and is equal to the immutable amount of time available per period, i.e., 24 hours a day, 7 days a week, etc.
vi. Working time and non-working time: Such a differentiation is made into the time constraints. For the case of one type of time, they present a useful concept called *Full Income*.
vii. Suggestions on and some discussions about several very interesting applications. These applications relate to the hours of work, to the productivity of time, to transport issues, income elasticities and the division of labour within the household. All of them are very

interestingly discussed. As a brief illustration related to transport issues, Becker talks about *dimensions of space* or *environmental conditions* as inputs of the commodity "transportation".

d. Criticisms.

The first critique to Becker's model is mainly that it does not go into more depth about the interpretation of the conditions that actually can be extracted from his model.

Moreover, some specifications as the dimensions of all vectors are not clearly presented, and this fact may be confusing. Concerning these confusions one can say that it is not clearly stated that the spectrum of activities may not be necessarily the same than the spectrum of market goods and time uses, as it is suggested by Becker. This is not only a misspecification but also a limitation, since it is interpreted from the model that what we more generally call uses of both goods and time, in Becker is just coinciding with the spectrum of commodities, which is not necessarily true. Actually we deem that this is a very special case of a more general conception. We have provided insights on this.

The Becker Model does not allow the presence of joint production, since each commodity is exclusively produced by the specific time and good inputs. The use of these cannot imply any other kind of production simultaneously. That restricts human behaviour to a *mono-task*, when in reality human behaviour is quite *multi-task*. We provide examples to illustrate these facts, both for time inputs and demands for goods.

Another criticism, this one from the literature, comes from the fact of the prices of the activities being endogenously determined. This critique about the implicit prices is made by Pollak & Wachter in 1975, and it is relevant when relaxing the fixed proportions assumed by Becker, which is a strong assumption. As we will discuss in chapter 4, it is only under certain conditions that the commodity prices are not endogenously determined.

Now we provide a general criticism of the non observability of the amount of commodities. The whole theory is based on production of commodities, something that is more qualitative than quantitative, and overall, non observable or not very easy to observe in many cases. This can lead to inaccuracy in the measure of welfare, since the utility, -an ordinal measure- would be based on something that is also ordinal, since we may not be able to perfectly define what is exactly an amount of commodity. Although the idea of production of commodities is great, it presents quantitative problems, which we must not forget anytime. While for some commodities it might be reasonable and acceptable (however not always) that you can count the commodity, such as with meals, it seems not so plausible to talk about one cleaning, two cleanings, etc, or even worse, to talk about one social activity, two or three social activities. Whatever the opinion, it is indisputable enough that the output in this production of commodities is something that is not quantitatively observable in all cases, and

the proof of it is that the overwhelming majority of the citations of Becker (1965) are from empirical studies carried out in the last four decades, trying to measure many of them. Hence, there is a risk of distortion in the welfare conclusions. The measure of the main source of utility can be biased and then, -given that the utility is an ordinal measure-, such dependency on the commodity bias may dramatically affect the ranking between two commodities. We will try to illustrate this with the following example:

□*Example*: Assume $U = Z_1^2 Z_2$ , where commodity one is *meals*, and commodity 2 is *cleaning*. Given the optimal amounts of time and goods inputs for situations A and C, and B and D, respectively, imagine that the following combinations of these two outcomes in the production of commodities can be achieved:
- A: you can have 10 meals and 20 cleanings by combining the inputs in a way called *alpha*. This combination yields you a utility of 2000.
- B: you can have 10 meals and 15 cleanings, by combining the inputs in a way called *beta*. This combination yields you a utility of 1500.
- C: you can have 9 meals and 20 cleanings, by combining the inputs in a way called *alpha*. This combination yields you a utility of 1620.
- D: you can have 11 meals and 15 cleanings, by combining the inputs in a way called *beta*. This combination yields you a utility of 1815.

Assume that A and B are the true values, and what we estimate is C and D

By an error of plus/minus one commodity of meals, we actually conclude that the individual prefers *beta* to *alpha* combination, when the truth is the converse. □

If we combine the non-observability of the commodities and the realistic relaxation of the already mentioned assumption about fixed proportions, why insist in commodities being the maximizing arguments in the problem? Why rely on them as arguments for the utility function? Why not to stick with a formulation of the problem where the utility were defined over market demands and time use, letting the utility function absorb all the qualitative factors by means of the parameters? In chapter 4 we will show how Pollak & Wachter (1975) suggest alternative approaches.

Furthermore, we suggest the following argument: Becker assumes that individuals or households are mini-firms producing commodities they later consume. For all the commodities, he assumes that we are utility maximizers. But, is this true, for all types of commodities or activities we perform? We think that probably for some of them, the behaviour could be a cost minimization one (since we agree with the household as a mini-firm). However, for some others (let us say, the market commodities) a utility maximizing approach seems to be realistic. To make this fragmentation would make sense, and it is not considered in Becker (1965).

To conclude, we must say that we strongly believe that Becker is not only a brilliant contribution, but also more fascinating when one spend more and more time thinking of it.

# 3. DeSERPA, 1971: A 1970s DECADE EXAMPLE.

> *"Over the years there have been a number of attempts to modify neoclassical consumer theory to deal with problems of this nature* (time use)*, but none of these works achieved the level of sophistication of the traditional approach, whereby testable properties of demand functions are deduced."*
>
> (DeSerpa, 1971)

After more than four decades since Becker presented a formalized theory about time in Economics, the above citation can still be considered relevant in the present day. We briefly comment on this paper written by DeSerpa in 1971, as an example of how Economics dealt with the production of new theories including time as an input.

The reasons for discussing it now are related to presenting how (during the seventies in particular, but also during the next years from then until the present day) theoretical contributions in the field of time use in Economics have been mere variations of Becker's contributions. These variations revolve around either (a) adding some extra constraints or (b) dropping the commodity-based utilitarian analysis to directly adopt a particular case of it, as it is the case of regarding the utility function to be defined over the market goods and time inputs directly. We illustrate both later on in this chapter.

DeSerpa´s contribution is a clear example of both at the same time, although in his paper he seems to argue, or to try to argue, that his model is a general one. His purpose in the paper is perfectly expressed by himself in the introduction, which we illustrate here with the following citation:

> *"The essential features of the model presented in this paper are: (1) utility is a function not only of commodities*[25] *but also of the time allocated to them; (2) the individual's decision is subject to two resource constraints, a money constraint and a time constraint; and* ***(3)*** *the decision to consume a specified amount of any commodity requires that some minimum amount of time be allocated to it, but the individual may spend more time in that activity if he so desires. Under these specifications, all the implications of neoclassical theory are preserved and many additional results, applicable to situations involving a time dimension, are generated."*

[25] This suggests different understanding between Becker's way of interpreting the term *commodity* and DeSerpa´s one.

(DeSerpa, 1971)

The assertion in his point (1) above shows that he is referring to a particular case of Becker's model. Firstly, we show how for this model in particular, but for any other like this one, the commodity-based approach by Becker is more general, since you can define $i = 1,...,2n$ commodities such that $Z_k = x_k,\ \forall k = 1,...,n$ and $Z_l = T_l,\ \forall l = n+1,...,2n$. However, Becker's approach can have a broader scope if the range of commodities is larger than 2n and is, moreover, allowing combinations of the goods and time inputs. The one in (2) is exactly the same as in Becker's; however it is again a particular case, since it only regards one and only one type of time, and nothing more. The only new factor is the one argued in (3), which is, in essence, the first *big* contribution to the theories of allocation of time made by DeSerpa; however it is again only for one type of time. Further on in the paper, he made a formal analysis using microeconomic theory for this *one type of time*-based model. The analysis is quite good, and we must highlight the graphical analysis and insights he provides which, in our opinion, is the second *big* contribution by DeSerpa, and it is at least as good as the first one. We emphasize the first contribution by showing another citation from DeSerpa´s:

> *"The fact that the consumption of goods generally requires the expenditure of time as well as money does not place an upper bound upon the amount of time an individual may spend consuming the chosen amount of the good. Thus we shall assume that the choice of a positive amount of any $X_i$ places only a lower bound upon the amount $T_i$ consumed."*
>
> (DeSerpa, 1971)

We will not go into more detail, since it is not entirely relevant and we do not have the time or the space within this thesis to give it the dedication it deserves. We will simply show how the model looks, and any reader can easily figure out the formal results which are not very different than in Becker's. However, we show how we only have to redefine commodities in Becker and to add some extra constraints to Becker's model in order to show how DeSerpa is a particular case of Becker. The model by DeSerpa is as follows:

| (DeSerpa) | | | (DeSerpa´s as a particular case of Becker´s) | |
|---|---|---|---|---|
| $\max_{\vec{x},\vec{T}} U(x_1,...,x_n,T_1,...,T_n)$ | | | $\max_{\vec{x},\vec{T}} U(Z_1,...,Z_n,Z_{n+1},...,Z_{n+n})$ | |
| $s.t. \quad \sum_i^n p_i x_i \le I$ | (i) | | $s.t. \quad \sum_{k=1}^{n} p_k Z_k \le wT_w + V = I$ | (i*) |
| $\sum_i^n T_i \le T$ | (ii) | $\Leftrightarrow$ | $\sum_{l=n+1}^{2n} Z_l + T_w = T$ | (ii*) |
| $\sum_i^n a_i x_i \ge T_i, \ \forall i = 1,...,n$ | (iii) | | $\sum_{k=1}^{n} a_k Z_k \ge Z_l, \ \forall l = n+1,...,2n$ | (iii*) |

In DeSerpa, in terms of notation, the only new elements are the $a_i$´s, which, -as DeSerpa *mentions-, "may be interpreted as a technologically or institutionally determined minimum amount of time required to consume one unit of* $X_i$*."* (DeSerpa, 1971).

In DeSerpa´s as a particular case of Becker's, we just define $i=1,...,2n$ , state that there are $2n = m$ commodities such that $Z_k = x_k, \ \forall k = 1,...,n$ and $Z_l = T_l, \ \forall l = n+1,...,2n$ . As well, additional constraints equivalent to (iii) are added, the *time consumption constraints*, following DeSerpa´s vocabulary. Moreover, time devoted to work is regarded, while in DeSerpa income is exogenously given and no working time is in the model.

To conclude, we can attempt to summarize both findings and conclusions reached by DeSerpa by quoting him, as follows:

> *"The single feature which distinguishes this model from others dealing with this problem is the* ***time consumption constraints****, which allow for the fact that the amount of time spent in any activity is partly a matter of choice and partly a matter of necessity. When it becomes a matter of necessity, an additional constraint becomes binding upon the consumer's preferences and this constraint must be made explicit. When it is solely a matter of choice, the constraint is not effective and "time prices" have no effect upon the consumer's decision. The non-linear programming model is the only way to capture both features."*
>
> (DeSerpa, 1971)

We must say that this paper is very well written, it inspired other contributions, and it is on the same line than contributions as the one made by Evans (1972)[26]. Not only Evans,

[26] Last section of his paper is really worthy. Sadly we do not have neither time nor space in this thesis to write at least a similar chapter to this one devoted to DeSerpa, showing how Evans (1972) is as well a particular case of Becker, with extra constraints. We chose DeSerpa in front of Evans just due to chronological reasons. Though his model is only regarding time as arguments into the utility function, the general framework he states is very interesting. The extra constraints are modelled very elegantly in the last pages and his way of doing it inspired our extra constraints in our suggested Generalized Theory of Allocation of Time in chapter 2. We must say thanks to Erika Spissu, -an expert in Evans model-, for providing us such paper, which is not very easy to find.

but some others during the 1970s published some models, but, as we stated at the beginning of this short chapter, they are mere variations of Becker's contribution. We just decided to pick up DeSerpa as an example of it, since it is in the very beginnings of the decade. However, probably the best contribution since Becker's one, -related to time issues under a static perspective-, came four years later than DeSerpa´s one, as we discuss in the coming chapter.

# 4. POLLAK & WACHTER, 1975: A CRITICISM.

> *"The major problem in studying the allocation of time in the household production function model is centred on joint production rather than non-constant returns to scale."*
>
> (Pollak & Wachter, 1975)

a. Idea

The idea of this paper is explained in the following citation:

> *"In this paper we argue that, except in very special cases, the new theory* (referring to household theory and Becker's contribution) *does not provide a satisfactory model of the demand for commodities and the allocation of time as functions of "commodity prices." We identify the cases in which the new theory can do so, and propose alternative approaches for those in which it does not"*
>
> (Pollak & Wachter, 1975)

They argue that if the household technology does not present constant returns to scale or does present joint production properties, then the consequence is that the prices for commodities are depending on the quantities of commodities, which violates traditional microeconomics, where individuals are price takers and prices are exogenously given.

Pollak & Wachter provide very strong results, in the form of theorems, proving this fact in a general framework for all kind of inputs for the household technology. In addition, later on in the paper, they review the particular case of time use as an input, citing Becker and using Becker's ideas. Pollak & Wachter (1975) rule out Becker's assumption about fixed proportions for time and good inputs as being unrealistic. They wrote the following in a footnote:

> *"Although Becker often uses fixed-coefficient production functions as an expositional device, we shall not, since fixed coefficients are not an integral part of the household production function model."*
>
> (Pollak & Wachter, 1975)

Therefore in a more general framework than Becker (for any kind of inputs for the commodities, not only goods or time inputs), Pollak & Wachter show the weaknesses of the already built up household production theory in terms of commodities, by means of the application of his provided theorems. They require no joint production[27] and constant returns to scale in the household production functions, for the household production theory in general, and Becker's one in particular, to hold without problems.

[27] We shall recall the "cooking & listening to music" example to refresh what joint production is.

In concluding, Pollak & Wachter suggest some alternative approaches, such as considering commodities as functions of good prices and discussing the confrontation of commodities-technology versus tastes. We deem this paper to be a very interesting contribution, perhaps not as famous and well known as such a contribution, in our opinion, certainly deserves. .

b. Model

The model attempts to find commodity prices independent of the quantities of commodities. The paper is *obsessed* with it, and the authors forget somewhat to go deeper into the model they are outlining, since in essence they use a 2-step model along for their purpose. With regard to the notation, we would simply remark that we decided to keep the essence of the original notation, for both simplicity and keeping to Pollak & Wachter´s method of writing, which we admire and respect very much.

Thus, he argues that the household minimizes the cost for the commodities subject to the technological constraint for the household.

Let us present the variables and the model in a sketch of the paper.

$- Z = \begin{pmatrix} z_1 \\ ... \\ z_m \end{pmatrix} \equiv$ vector of commodities.

$- y_{rs} \equiv$ amount of good $s$ used to produce commodity $r$.

$- Y_r = \begin{pmatrix} y_{r1} \\ ... \\ y_{rn} \end{pmatrix} \equiv$ vector of goods used to produce commodity $r$.

$- f^r : z_r = f^r\left(Y_r\right) = f^r\left(y_{r1},...,y_{rn}\right) \equiv$ household production function for r-th commodity.

$- X = \sum_{r=1}^{m} Y_r \equiv$ vector of goods.

Pollak & Wachter (1975) argue that the household chooses the least cost expensive collection of goods capable to produce the given $Z$, which will be decided later. We can then write this stage of the model as follows:

$$\min_{X} \; C$$
$$s.t. \quad \left(X,Z\right) \in \Omega \equiv \text{production set}$$

, being C a cost function.

For simplicity we can consider a certain household production function $F$ think in the problem to be like this:

$$\min_{X} \; C = \sum_{k=1}^{n} p_k x_k$$
$$s.t. \quad F(X) \geq Z$$

, being $P$ the $n$-dimensional price vector whose elements are $p_k$

Then the value function can be found:

$$C(P,Z) = \sum_{k=1}^{n} p_k x_k^*(P,Z) \qquad (29)$$

And hence, the commodity prices, $\pi_r$, *could* be calculated as follows:

$$\frac{\partial C(P,Z)}{\partial Z_r} = \pi_r \qquad (30)$$

Therefore, the household maximizes its utility as follows:

$$\max_{Z} \; U(Z)$$
$$s.t. \quad C(P,Z) \leq I \qquad (31)$$

, where $I$ follows the notation by Becker, and the one used in the previous section.

However, the reader must keep in mind that, as we wanted to keep Pollak & Wachter (1975) original notation, our I in chapter 2 represents the same income which Pollak & Wachter (1975) denoted by $\mu$.

Their goal in the paper is to analyze under which conditions the constraint in (31) can be rewritten as follows, for the sake of using classical microeconomic analysis without any problem:

$$\sum_{r=1}^{m} \pi_r z_r \leq I \overset{?}{\Leftrightarrow} \sum_{r=1}^{m} \frac{\partial C(P,Z)}{\partial z_r} \cdot z_r \leq I \qquad (32)$$

, where $I$ denotes the *Income*, as in Becker. We should keep in mind that in Pollak & Wachter (1975) notation $I = \mu$.

In the next section we will show the results by Pollak & Wachter (1975), stating under which conditions over the household technology the problem can be analyzed as the traditional microeconomic problem.

We have not explicitly stated in the formal presentation any reference to time use, since, as in Becker, time use is just considered as an input for the household

production, and the general description provided by Pollak & Wachter (1975) applies to the case of time uses, without loss of generality.

c. Findings

The easy findings to be highlighted are only those serving as a purpose for Pollak & Wachter ´s paper.

They simply provide theorems demonstrating that if we are in the presence of either joint production or non constant returns to scale in the household production function respectively, the prices for commodities are dependent on the choice to be made in the problem concerning the commodities.

*Theorem (Pollak & Wachter, 1975; Theorem 1)*

*Theorem:* If the household's technology exhibits constant returns to scale and no joint production, then the budget constraint in the commodity space is of the form

$$\sum_{s=1}^{m} \pi_s z_s = \mu,$$

where the $\pi$'s depend on goods prices and the household's technology, but not on the commodity bundle consumed.

*(Figure 2)*

The proof of this theorem[28] above is almost straightforward when realizing that constant returns to scale in the household technology imply the following fact for the cost function in the absence of joint production:

$$C^r(P, z_r) = C^r(P, 1)z_r$$

*(Figure 3)*

[28] Theorem 1, sketch of the proof showed in Pollak & Wachter (1975):

*Proof:* With no joint production, we can write the cost function $C(P, Z)$ as

$$C(P, Z) = \sum_{s=1}^{m} C^s(P, z_s),$$

where $C^r(P, z_r)$ is the cost function associated with the production function $z_r = f^r(Y_r)$. Constant returns to scale implies

$$C^r(P, z_r) = C^r(P, 1)z_r.$$

Hence the cost function becomes

$$C(P, Z) = \sum_{s=1}^{m} C^s(P, 1)z_s$$

and

$$\pi_r = C^r(P, 1). \qquad \text{Q.E.D.}$$

*(Figure 4)*

Since, hence, the derivative of C with respect to the *Z*'s yields a commodity-independent expression for the commodity prices:

$$\pi_r = C^r(P,1),\ \forall r \quad (33)$$

*Theorem (Pollak & Wachter, 1975; Theorem 2)*

> *Theorem:* Suppose the household's technology is continuous, and $T(0,\ldots,0,\ 0,\ldots,\ 0) = 0$. Then implicit commodity prices are independent of the commodity bundle consumed only if the household's technology exhibits both constant returns to scale and no joint production.[6]

*(Figure 5)*

It must be clarified that, for this theorem, Pollak assumes a form of representing the household technology different than the one that we decided to present earlier. In our case, we denote technology to be a certain function *F*, while the household technology used in their paper is like the following expression:

$$Z_1 = T(Z_2,\ldots,Z_m,x_1,\ldots,x_n) \quad (34)$$

The statement of the Theorem 2 refers to the expression $0 = T(0,\ldots,0,0,\ldots,0)$ which means that if we have an amount of inputs equal to zero for all inputs, no output is produced, i.e. the amount of output you can get is zero for all outputs.

This theorem is remarkable, since under continuity for the household technology both constant returns to scale and no joint production are needed for the commodity prices to be independent of the commodity bundle consumed.

The formal proof[29] for this theorem is a bit more demanding, and we will not provide any more details than provided by Pollak & Wachter.

[29] Pollak & Wachter (1975), Theorem 2, sketch of the proof showed in Pollak & Wachter (1975):

> *Proof:* If implicit commodity prices are independent of the commodity bundle consumed, then the cost function can be written as
>
> $$C(P, Z) = C^o(P) + \sum_{s=1}^{n} C^s(P)z_s.$$
>
> Since the technology is continuous and $T(0,\ldots,0,\ 0,\ldots,\ 0) = 0$, $C(P,\ 0) = 0$, and, hence, $C^o(P) = 0$. To complete the proof, one must show that $C^r(P)$ is a unit cost function corresponding to some constant-returns-to-scale production function. A formal proof can be found in Diewert (1972) and Hall (1973). Q.E.D.

*(Figure 6)*

*Theorem (Pollak & Wachter, 1975; Theorem 3)*

> *Theorem:* Suppose the household's technology exhibits constant returns to scale. Relative commodity prices are independent of the commodity bundle consumed by the household if and only if the household's technology exhibits no joint production.

*(Figure 7)*

The relevance of this theorem stems from the fact that even if the presumably restrictive assumption of constant returns to scale holds, for the commodity prices to be independent of the commodity bundle consumed, no joint production is fully required.

Again we will not go into the proof[30] in more detail than Pollak & Wachter. In addition, they provide a clear analysis about what happens if there is relaxing of constant returns to scale or joint production. The crucial point is that the analysis in terms of slope is failing, since in the usual ratios to be compared, the amounts of commodities are entering in both sides of the first order conditions yielded by the utility maximization problem, given the dependency of commodity prices with respect to the amount of commodities

[30] Pollak & Wachter (1975), Theorem 3. Proof copied from Pollak & Wachter (1975).

> *Proof:* The "if" assertion is trivial. To establish "only if," suppose that relative prices are independent of $Z$. Then the locus of $z$'s satisfying the feasibility constraint implied by $(P, \mu)$ is given by
>
> $$z_1 = \alpha^0(P, \mu) - \sum_{s=2}^{m} \alpha^s(P, \mu) z_s.$$
>
> By constant returns to scale, if the commodity vector $Z$ is feasible at $(P, \mu)$, then $\lambda Z$ is feasible at $(P, \lambda\mu)$. Hence
>
> $$\lambda z_1 = \alpha^0(P, \lambda\mu) - \sum_{s=2}^{m} \alpha^s(P, \lambda\mu) \lambda z_s$$
>
> for all $\lambda > 0$. Setting $\lambda = 1/\mu$ and solving for $\mu$ yields
>
> $$\mu = \frac{z_1}{\alpha^0(P, 1)} + \sum_{s=2}^{m} \frac{\alpha^s(P, 1)}{\alpha^0(P, 1)} z_s.$$
>
> Hence, the cost function is of the form
>
> $$C(P, Z) = \sum_{s=1}^{m} C^s(P) z_s,$$
>
> and the household's technology exhibits no joint production. Q.E.D.

*(Figure 8)*

Moreover, they provide some alternatives to this problem:

> *"If commodity prices are not analytically useful for studying the demand for commodities when the household's technologies exhibit joint production,* ***where does this leave the household production function model?***
>
> *There are two alternatives.*
>
> *First, one can restrict the application of the model to those cases in which the technology exhibits no joint production. Although no one can object in principle to restricting the model to those cases, the usefulness of the restricted model depends on the prevalence of joint production. In Section 4 we argue that joint production is pervasive when the role of time in the household production process is recognized; this implies that restricting the application of the model in this way is very undesirable.*
>
> *Second, one can argue that commodity prices are not central to the household production function approach. Without commodity prices, we can derive commodity demand functions which depend on goods prices and income. These demand functions are of substantial interest even in the no-joint-production case, since we are often interested in the effects of goods prices on the consumption of commodities; but they must assume a primary role when commodity prices depend on the commodity bundle consumed."*
>
> (Pollak & Wachter, 1975)

As it can be deduced, they outline the weaknesses of the household production theory approach in the case of the allocation of time since under general conditions those assumptions are not plausible

> *"Consequently, the relevancy of the household production function approach to the allocation of time depends crucially on the assumptions of constant returns to scale and no joint production. Under quite general conditions, however, these assumptions are likely to be violated."*
>
> (Pollak & Wachter, 1975)

Some of these conditions are that the same use of time can be a direct source to produce several commodities. A clear example we suggest is when for cooking and listening to music. Under Becker approach individuals are using such time as an input *jointly* to produce two different commodities: cooking and music. Pollak & Wachter (1975) argue that this is a case of joint production, and as pointed out above, this presents problems. They assert there is only one special case in which this household approach could be useful:

> *"In the household, time spent in various activities is often a direct source of satisfaction or dissatisfaction as well as an input into a production activity. This is a case of joint production, and the household production function approach is not a suitable framework for analyzing joint production. We conclude that the household production function model provides a satisfactory account of the*

> *allocation of time only for households which are indifferent among alternative allocations of their time."*
>
> (Pollak & Wachter, 1975)

Thus, they are probably the first ones in stating the identification problem of time use, i.e., to distinguish where tastes are starting, and where production in the household is finishing. However we interpret for this last citation above that they misunderstand joint production with the two different roles played by the individuals, which are consumers and producers at the same time. In our point of view, joint production is one thing (in the style of the cooking & music example), and the fact of the same inputs playing two roles at the same time is another very different thing, which plays a role in the sense of the association between household technology and changes in tastes. They point out that the technology in the household could be used to explain changes in tastes, by directly assuming the commodities to be a function of prices, wages and non labour income instead of the commodities being a function of market demands of goods and time inputs used.

They reflect on the fact of being very tempting to assert that all changes in tastes are due to changes in household technologies, given that the power Economics has to deal with these kinds of changes is better than the power that Economics has to deal with changes in tastes. This is, in our perspective, one of the most important points, in terms of intuitions, Pollak & Wachter (1975) provide in this paper.

These last two paragraphs start connecting with the end of the paper by Pollak & Wachter (1975), who, before finishing and providing conclusions, give us two alternative approaches after considering what the role of household theory was going to be in Economics, as we quoted before. We analyze these two alternative approaches below.

*Alternative approaches by Pollak & Wachter*

Coming from the same criticism proposed in our section on Becker, concerning to the likelihood or not of emphasizing the analysis on the outputs of the household production, -i.e. the commodities-, Pollak & Wachter write the following about Becker approach:

> *"It retains the household production function model's emphasis on commodities while dispensing with commodity prices. But this approach is appropriate only when studying variables which are the outputs of production processes. When household activities can more plausibly be interpreted as directly producing a specific type of utility or satisfaction, the appropriate alternative is to analyze the "inputs"- the allocation of goods and time among household activities-as a function of goods prices, the wage rate, and nonlabor income."*
>
> (Pollak & Wachter, 1975)

*First alternative approach: Commodities as functions of good prices*

Under this approach, what is suggested is that instead of considering the relationship between commodities on the one hand, and demands and time uses on the other hand, inserting the *optimal* relationship between the commodities on the one hand, and prices for inputs and other income, on the other hand, which can be obtained from their model.

> *"The Z = g(P, w, A) demand functions* (where *A* is denoting the ´other income´ in Becker's) *reflect the influence of both technology and tastes, (...)"*
>
> (Pollak & Wachter, 1975)

The model, though mathematically plausible, presents a problem in terms of intuitions, which relate to the second alternative approach.

*Second alternative approach: Commodities – Technology vs. Tastes*

This approach comes back somehow to our criticism in our Becker's section, about welfare issues, as they argue in the following citation, which completes the last citation above, being altogether a paragraph into their original paper:

> *"(...), and, without explicit estimates of the technology, we cannot tell whether variations in consumption are due to variations in tastes or to variations in technology. To argue that all variations in consumption behaviour should be attributed to variations in technology because we have a better theory of technology than of tastes is logically unconvincing. Furthermore, although it may be possible to explain consumption behaviour in terms of either taste change or technical progress, the welfare implications are quite different."*
>
> (Pollak & Wachter, 1975)

How to separate what is what, in terms of technological change in the household and changes in tastes, is a big factor here. It is very easy and tempting to state that either technology is invariant or tastes are invariant, since one has already *identified* the source of change in tastes. Therefore, the important point in our opinion is the *identification* problem have already introduced. The other points they discuss in their paper, though interesting for other purposes, are not relevant in general for our purpose: the theories of allocation of time. In essence, what Pollak & Wachter are suggesting is to simply reduce the role of commodities (which also create a problem of observability) to nothing, and express the model directly in terms of goods and time inputs. They argue, in our opinion correctly, that this requires fewer assumptions but they eliminate the problem by *getting rid of* the household technology:

> *"If this can be done, then the allocation of time and goods among activities can be analyzed without the household production function apparatus"*
>
> (Pollak & Wachter, 1975)

d. Criticisms.

Firstly, we would like to repeat that we strongly believe that Pollak & Wachter ´s contribution in 1975 is very useful and provides very strong results that need to be taken into account.

However Pollak & Wachter placed too much emphasis on the prices for commodities, in order to obtain a model such as in the textbook model for microeconomic theory, and they then come up with the valuable theorems. However their theorems only concern the fact of being able to express the model in terms of commodities, with commodity prices. If you relax this goal, it is indeed plausible to have joint production and other types of returns to scale, not only constant returns to scale. This can be observed in our model, for which there is no need to mention commodity prices, since for the budget constraint the value function is used, (the expenditure function), for the first step.

In Pollak & Wachter's model/critique commodities are again central. The same problems of measurement as in our Becker critiques apply here, however with a very significant difference: Pollak is aware of this, and he suggested some alternative approaches.

Pollak & Wachter propose a plausible cost minimization approach, which is the key point for this theory, leading to strong results in the form of theorems. The presence of no joint production (no multitasking is allowed) and constant returns to scale for the household technology is necessary, something which is quite restrictive.

Therefore the contribution by Pollak & Wachter is very interesting, and we should never forget their insights if we want to deal with household technologies in general, and in particular for the case of the allocation of time. Of course, we could always drop the commodity-based analysis, as they suggest in the second alternative approach. This seems to be not a good alternative, given that the household production theory is quite well accepted, and seems to make sense.

# 5. GRONAU MODEL, 1977: A BENCHMARK.

> *"An increase in the market wage rate is expected to reduce work at home, while its effect on leisure and work in the market is indeterminate."*
>
> (Gronau, 1977)

a. Idea.

In this model consumers maximize their utility again subject to budget and time constrains.

One of the differences now is that he restricts the analysis to only three kind of times (working in the market, at home and leisure) and he distinguishes between consumption at home and in the market, with consumption at home being produced with the working time at home.

The distinction between time working at home and leisure is theoretically discussed at the beginning of his paper. Referring to Mincer, Gronau calls for the Becker model to outline that this distinction is not made in Becker. Of course, he pointed out the reasons, leading to the fact of the difficulty in distinguishing the borders between work at home and leisure. A good example for that is provided with children, for whose purpose Gronau asks the question:

> *"Is playing with a child leisure or work at home?"*
>
> (Gronau, 1977)

Moving on, he mentions that, from the theoretical point of view, the aggregation between both the time working at home and for leisure (made in Becker) into non-market time or home time was done in Becker because of these two following assumptions:

- Both elements are supposed to react similarly to changes in socioeconomic environment, so therefore there is nothing to be earned by the distinction.
- Both elements are said to satisfy the conditions to be composite inputs. Therefore, their relative prices are constant, and this leads the situation to be an uninteresting one for which it makes no sense to study the composition of the aggregate, since changes to it have no impact on the output.

However the empirical research tells us something that contradicts this, as Gronau notes from the beginning. We should mention that the paper starts with a section called "Time Budget Evidence-Data in search of a theory", which clearly

states the motivation of inducing a theory from some particular empirical results. Thus, he asserts the following:

> *"Both assumptions are suspect. Recent time-budget findings have established that work at home and leisure are not affected in the same way by changes in socioeconomic variables, and this paper shows that the composition of the aggregate affects many facets of household behaviour, such as labour supply, specialization in the household, and demand for children."*
>
> (Gronau, 1977).

Then he builds up the model discussion in the next subsection, after some more comments about time use data for households in the USA and Israel.

b. Model.

For the sake of simplicity, he assumes the household to be composed of a single person, who maximizes the production of one single commodity ($Z$), which depends on the goods ($X$) and the consumption time ($L$)

Goods can be purchased or produced at home:

$$X_m + X_h = X \qquad (35)$$

Where the goods produced at home depend exclusively on the time worked at home

$$X_h = f(H) \qquad (36)$$

The household production function $f$ shows the *usual* properties, with decreasing returns to scale.

The utility maximization is a problem which is subject to both budget and time constraints, (very similar to Becker), with the split of non-working time into work at home and leisure.

$$X_m = wN + V \qquad (37)$$
$$N + L + H = T \qquad (38)$$

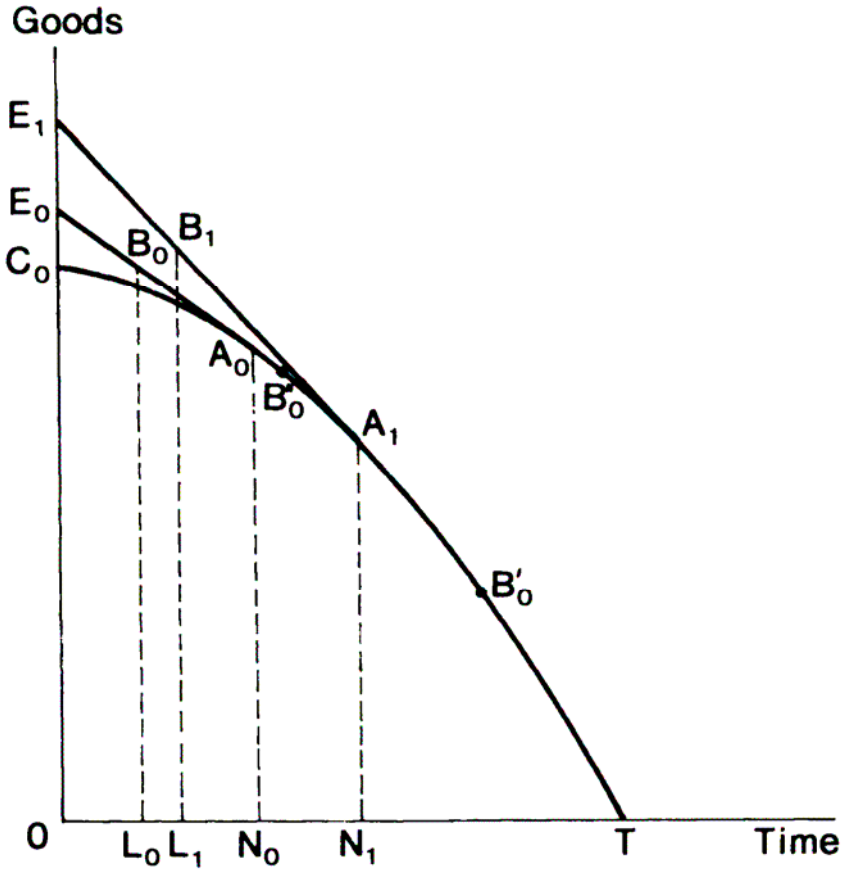
Where $N$ is the working time in the labour market, $L$ is the pure leisure time and $H$ is the working time at home, with $T$ as the total time available.

Finally we can present the Gronau model as follows:

$$\max_{X_m, N, H, L} Z\left(X_m + f(H), L\right)$$
$$s.t. \qquad X_m = wN + V \qquad (39)$$
$$N + L + H = T \qquad (40)$$

, which leads to the condition in which marginal product at home has to equal the wage

$$\frac{df}{dH} = w \qquad (41)$$




*(Figure 9)*

Further on in his paper, Gronau suggest an interesting point related to transaction costs, both in terms of time and goods, linked to situations in which people are employed and have to go to work, spending time and using goods to commute to work.

The model is as follows:

$$\max_{X_m,N,H,L} Z\left(X_m + f(H), L\right)$$

$$s.t. \qquad X_m + \delta C = wN + V \qquad (42)$$

$$N + L + H + \delta t = T \qquad (43)$$

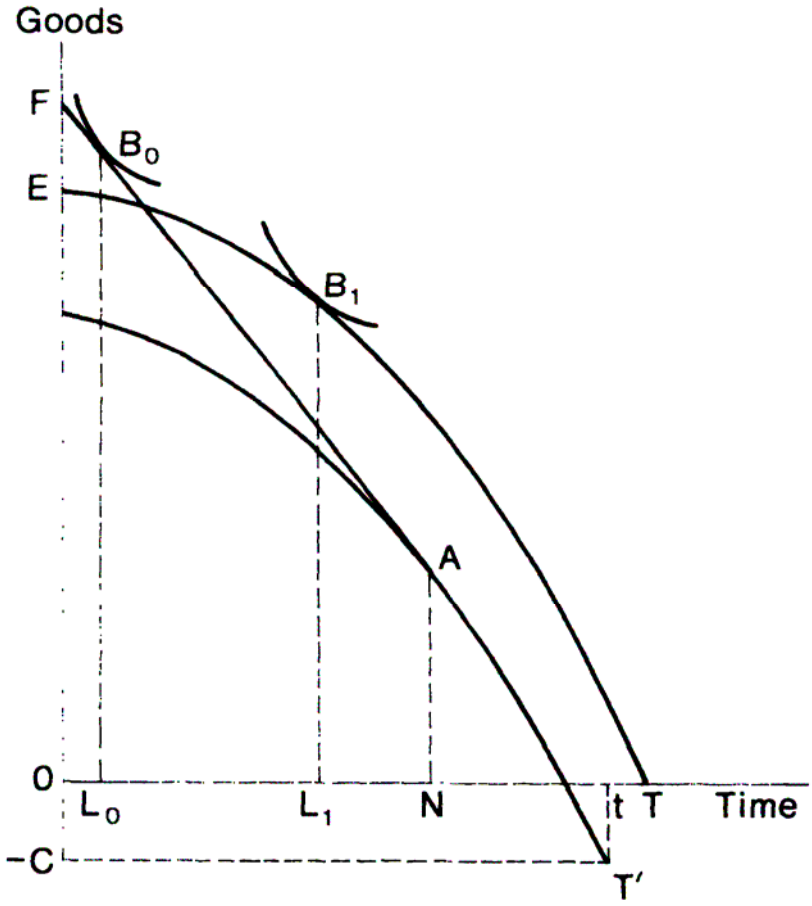
Where

$$\delta = 1, N > 0 \qquad (44)$$

$$\delta = 0, N = 0 \qquad (45)$$

With $C$ as the monetary cost of going to work and $t$ as the cost in time units of going to work.

The model is affected as it is observed in the picture, after the inclusion of the transaction costs for time and goods:




*(Figure 10)*

c. Findings.

- ✓ With this model, criticisms in the sense suggested by Pollak & Wachter (1975) are avoided, since there are no prices in the model, and there is only one commodity, whose production is maximized.
- ✓ By using a graphical analysis, it is observed how working in the market expands the possibilities for consumption of goods, by selling part of the leisure time.
- ✓ Gronau got neat descriptions depicted with nice graphs. Intuitions from the model are making a lot of sense in the real world, and perhaps that is why Gronau´s model became a benchmark.
- ✓ He finds out that an increase in the non labour income is having impact on a desired increase on leisure, at expense of working hours. What is a bit more counterintuitive is that it could happen that such a change has no

impact on working time at home, and this is a result commented by Gronau.

- ✓ Furthermore, the consequences of *changes in wages* are studied, which yields the following conclusion: *working at home is reduced*. The impact on time worked in the job market and leisure time depends on the preferences.
- ✓ An interesting analysis about transaction costs when entering into the job market (both cost in time and goods when going to the job place) is done very neatly. Again, this is done in a simple way providing powerful and realistic interpretations.

d. Criticisms.

Our main criticisms to this model can be summarized as follows:

i. Restricted analysis only to three time uses and one type or aspect of time. Given that the overwhelming majority of the literature has done such a thing, perhaps it is not a big relative disadvantage with respect to others´ contributions. However, with respect to Becker (1965), it is indeed a comparative disadvantage, and consequently, with respect to our Generalized Theory of Allocation of Time.

ii. Restricted analysis to one single commodity. However, Gronau notes that he could extend the model for the case of two commodities. Nevertheless, nothing is explicitly said about being possible for more than two commodities. As noted in (i), again, this is a comparative disadvantage.

iii. *Within* the leisure time, the model does not say anything about how individuals allocate it to different leisure activities. Recalling the relative importance of the leisure time, this is a remarkable criticism.

iv. Consumption is considered in general, as a level of consumption, not making differences between different market goods. As well, it is considered as the monetary value of the consumption, without entering in details about which goods are composing the consumption, and what their prices are. No price-based analysis can be done. This negative part has a positive counterpart, since partly due to this, Gronau avoids problems in the sense of Pollak.

v. The household production function, -i.e., the technology used to produce consumption at home-, only depends on *working time at home*. One can easily think that for your consumption at home you must be endowed with some ingredients, i.e., market goods, and even some tools or capital.

vi. In this model, if the wage is very high, depending on the household technology, there is a chance of no time being devoted to work at home, which is a bit controversial. It can be argued then that the time spent at home is leisure time, but this is not always true, and even in that case, again, the model is not telling anything about how the leisure time is used.

# 6. BOLAND, 1978: A REFLECTION ABOUT TIME.

> *"Several notable writers have recently charged that neoclassical economics is ´timeless´. (...) The proper question to ask is not whether neoclassical economics is timeless but whether its treatment of time is adequate."*
>
> (Boland, 1978)

In Boland's paper, *"Time in economics vs. Economics in time: The ´Hayek[31] Problem´"*, a great analysis is made, whose summary about the different approaches in using time in economics can be read below, regarding time as both the time line (decisions *throughout* time) and the time use (decisions *about how to use/allocate* the time). Although this paper is not dealing exactly with the allocation of time (only when it mentions Becker), we deem this is a very interesting reflection about how the time (in the broad sense, not only regarding the allocation of time) has been modelled in Economics throughout time. We use this paper to motivate the reader to think about variables changing throughout time, since in the two next sections the allocation of time is something to be decided at any period throughout time, i.e. in intertemporal and dynamic contexts.

We summarize the main ideas of this paper as follows:

- ✓ **Time and static models**: even in a standard model in economics, the idea of time is implicit whenever, for example, we make comparative static. The problem is, as Boland argues, this is always a matter of interpretation of the model, and *"Therefore, with respect to any given model, today´s values of the endogenous variables may be shown to be consistent with today´s values of the exogenous variables, but tomorrow their respective values may not be consistent. Since dynamic processes obviously refer to more than one point in time, the explanatory usefulness of a static model would seem rather limited"* (Boland, 1978, pp.2)
- ✓ **Time-based variables**: during the late 50s, Koopmans[32](1957) and Debreu[33](1959) introduced time-based variables, i.e., subscripts making reference to the point in time in which the goods are consumed, suggesting in this way that, as Boland comments, a hamburger is not the same hamburger for the consumer at time *t´* than at time *t´´*. Although very interesting, and quite commonly used, this approach receives a criticism from Boland, since he argues there are no dynamics in this model, since the model is formally

[31] Click here to know more about one of the two Nobel Prize Winners in 1974: Von Hayek.

[32] Click here to know more about one of the two Nobel Prize Winners in 1975: Koopmans.

[33] Click here to know more about the Nobel Prize Winner in 1983: Debreu.

like a static model over all the time range, which only multiplies the number of goods. Nevertheless, this idea is commonly used in economics.

- ✓ **Time preferences or the economics of time:** This approach deals with the idea of including time as a commodity. Bohm-Bawerk (1889) and Becker (1971) are examples of it, and the impact of Becker´s contribution is something to highlight. The former focuses more on production theory, while the latter develops more the consumer side, relating it to several fields. However there are again no dynamics here. Time is an exogenous and static variable. *"Neither Becker´ s nor Bohm-Bawerk´s can avoid the static approach of the givens (the constrains, the tastes, the production functions, time available, etc) […] There is no reason for historical change; hence it cannot be explained"* (Boland,1978, pp.5)
- ✓ **Variable givens or lagged variables:** This is an alternative approach, attempting to determine the time path trajectory of the endogenous variables, and that change is suggested to take place because of either a change in the parameters or in the exogenous variables along time, or both. Hicks´ model (1971) is an example of the former, where he talks about an "autonomous invention", and Kaldor´s growth model is an example of the latter. Of course we are not forced to assume that the period of change for the exogenous variables has to be the same for the endogenous, and an example of this is Von Neumann's balanced growth model. It is worthwhile to read this section, especially the part in which he establishes a parallelism about the dealing that in economics is done to a point in time and what is (in physics) the dealing of a point in space.
- ✓ **Flow variables:** Similarly, this approach is one such approach that is extending a static view of a model to a dynamic view by inserting appropriately differential equations, and examples in the literature can be found in Barro and Grossman (1971) or Arrow[34](1959).
- ✓ **Time, logic and true statements:** this part is related to the discussion about the neoclassical economics to be or not to be timeless, as many authors suggested in the 70s, since some of them say that *"all economical analysis has been merely logical derivation of solutions"*(Boland, 1978) as Boland writes in reference to Georgescu-Roegen (1971) and Shackle (1972). This is a controversial point in Boland and has been comprehensively discussed, so we do not go into the details here.
- ✓ **Time and knowledge: the Hayek problem:** in this section, Boland insisted more than in previous sections that the way in which time is included suggests

[34] Click here to know more about one of the two Nobel Prize Winners in 1972: Arrow.

> that any reliance on only standard general equilibrium theory precludes and discards an explanation for historical changes, since all the causes, motivations and reasons for changes are beyond explanation since they are exogenous to the models. He also points out that Hayek (1937) realized this problem, and that this remains an essential consideration in most Austrian models, such as in Hicks[35] (1973) and in Lachmann (1976). This is what Boland understands as the "Hayek problem", and the same Hayek in 1937 showed and recognized in his very abstract paper his incapability to solve it, however referring in to a number of insights. One of them relates to knowledge. In concluding, Boland suggests that the individual process of acquiring knowledge must be *endogenous*, and that the individual decision and process of learning/adapting must be taken *in real time*.

As we can observe from Boland, his work is a good and quite complete description, introduction, motivation and connexion to any existing research related to how to include time in economic models (understanding time in its widest sense), and the implications it implies in different fields.

[35] Click here to know more about one of the two Nobel Prize Winners in 1972: Hicks.

# 7. JUSTER & STAFFORD, 1991: A SURVEY.

*"How is the stock of household capital to be allocated to the production of various Z's?"*
(Juster & Stafford, 1991)

In 1991, two researchers from the University of Michigan, F. Thomas Juster and Frank P. Stafford completed a paper in which they analyzed the empirical findings, the problems of measurement and what they called the behavioural models, to finish talking a bit about social accounting systems, something related to the way of collecting time use data.

The paper is quite long and exceeds the purpose for this master thesis, which is focused only on theories. Therefore, we will just look at the section in section IV in *The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement* by F. Thomas Juster; Frank P. Stafford at *Journal of Economic Literature*, Vol. 29, No. 2. (Jun., 1991), pp. 471-522. We might make some references to section V as well, since it is a type of extension of section IV. All other sections are devoted to empirical findings and contributions to the field along time, which is not the purpose of this master thesis.

Thus, given that the main part of the effort dedicated to time use research has been put into empirical issues, and not so much into theories, the paper can be reduced considerably for us. And it is even more reduced if we skip all the discussion about Becker's contribution, which we have mentioned in much more detail. Besides, we do not have the highest regard for the discussion about Becker, which is either incomplete or incorrect, since they only partially describe the model. This can be checked when they talk about the budget constrain; if this is so, and what they meant was what Becker calls full income constrain, that is incorrect; if they only want to mention the budget constrain, they are not talking about the time constraint, which is important, and their analysis is therefore incomplete.

When referring to Gronau, they focus only on a particular application devoted to travel time. They do not explicitly mention the posterior paper we analyzed, written in 1977, which provides a more general theoretical framework which is indeed covering the discussion related to the travel time and costs, pointed out earlier, in the extension suggested by Gronau, where transaction costs are included.

However the comments about labour market application of Becker's theory are very illuminating and, in essence, are simply a particularization of Becker's. We will not go into such a particularization and the little analysis they carried out. Simply, we want to mention it in order to show how powerful, in terms of applications, a good theory can be for allocation of time (as Becker's is).

They also mentioned models from economic demography and the intra family allocation of time, as having been important in the time use research. This has generated literature in the sense of the following example which we show as an motivating illustration, in which within a family a type of social welfare function is defined, and

some activities (as for example, childcare) are performed, to maximize the welfare. Implications related to intra family division of labour are important, and is something that is still creating a lot of research. Thus, we want to base our comment on it, to provide the wider panoramic of the spectrum of theories that are being and have been already made.

However, our main interest in this paper is when they analyze and summarize the inter-temporal models used for the sake of building economic theories of allocation of time. As it was perceived in the previous section, the one dedicated to Boland, from there on, the purpose of our thesis is to connect time use research with techniques dealing with variables changing along time. Boland's was a very good reflection of this, although for the sake of the economic theories about allocation of time he did not propose any contribution (for other purposes he indeed does).

Thirteen years later than Boland, Juster & Stafford, by making this survey, collected the new theories that applied to inter-temporal techniques, (mentioned by Boland), to time use research. Again, from Becker's model, particularizing it as follows, and extending it to a dynamic perspective, we get a set of inter-temporal models whose essence is very well described in Juster & Stafford as follows:

Specifically, if $Z_1 = t_1$ and $Z_2 = X$, then the decision problem is to choose "leisure" ($t_1$), training time ($t_2$), and market goods expenditures ($X$) in each period to maximize the present value of discounted utility:

$$\sum_{m=0}^{M} U(t_{1m}, X_m)/(1 + r)^m \qquad (8)$$

subject to

$$R_{m+1} = R_m + (T - t_{1m} - t_{2m})\alpha K_m - pX_m + rR_m \qquad (9)$$

and

$$K_{m+1} = K_m + g(K_m, t_{2m}) - \delta K_m \qquad (10)$$

where $T - t_1 - t_2$ is market work time, $M$ is the planning horizon, and financial assets are $R$ with a discount rate of $r$. The stock of market skills, $K$, produces earnings at a rate $\alpha$ for added market time and depreciates at the rate $\delta$. The production function for skills, $g(\cdot)$, can be thought of as part of the hpm approach, in that time, skill, and (in some specifications) market goods are used to produce increments to the stock of skills.

*(Figure 11)* (Juster & Stafford, 1991)

Such sets of models in this line are connecting with asset accumulation and economics of education, since individuals are assumed to receive education merely for the sake of an investment that, as it is supposed to increase individuals' skills, increases the assets that individuals can accumulate. Time use changes in here as a consequence,

mainly of either changes in assets or changes made for getting more assets. Under our perspective, this vision is very naïve, and not very realistic.

Connecting with these kind of inter-temporal models, we find more recent literature in this research line in **Gonzalez-Chapela (2004)**. In his doctoral dissertation, he proposes a model including consumption of both usual consumption goods and recreational goods, with leisure time and in the presence of asset accumulation. His marginal contribution to the theory is made in a very different sense, however is worth reading, or at least to review his inter-temporal theoretical model[36]. Slightly more interesting is another model suggested by Gonzalez-Chapela in an appendix of his dissertation, since he uses dynamic programming techniques applied to a similar problem, with uncertainty[37]. From our perspective, even though the model is very interesting, time is just an input whose dynamics are explained by the dynamics in the asset formation/accumulation. In addition, time is just reduced to working time and leisure time. Nevertheless, we regard it to be a very good contribution in the line of using dynamic techniques involving time use, which in this case is just leisure time (since working time is, as usual, the remaining time available). The reasons because of such contribution is considered to be a very good one stem from (a) the higher degree of

[36] The intertemporal model by Gonzalez-Chapela looks like this:

$$\max \sum_{t=0}^{T} \frac{1}{(1+\delta)^t} \left( \frac{C_{it}^{*1-\frac{1}{\theta}} - 1}{1-\frac{1}{\theta}} + \psi_{it} \left[ \frac{1}{1-\frac{1}{\gamma}} \left( X_{it}^{1-\frac{1}{\rho}} + \alpha_{it} L_{it}^{1-\frac{1}{\rho}} \right)^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}} \right] \right)$$

subject to

$$\sum_{t=0}^{T} R_t \left( C_{it}^{*} + P_{it} X_{it} + W_{it}^{*} L_{it} \right) = \sum_{t=0}^{T} R_t W_{it}^{*} + A_{i0},$$

$$C_{it}^{*}, X_{it}, L_{it} \geq 0, \quad t = 0, \ldots, T,$$

*(Figure 12)*

Where *R* is the discount rate, *C* is consumption, *X* is recreational goods with price *P*, *L* is leisure time with price *W* and *A* is assets, and all these variables but *A* are larger or equal than zero, with A larger or equal than zero in the last period.

[37] The dynamic programming model suggested in Gonzalez-Chapela is:

$$\mathrm{E}_t \left\{ \sum_{\tau=t}^{T} \frac{1}{(1+\delta)^{\tau-t}} \left( \frac{C_{i\tau}^{*1-\frac{1}{\theta}} - 1}{1-\frac{1}{\theta}} + \psi_{i\tau} \left[ \frac{1}{1-\frac{1}{\gamma}} \left( X_{i\tau}^{1-\frac{1}{\rho}} + \alpha_{i\tau} L_{i\tau}^{1-\frac{1}{\rho}} \right)^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}} \right] \right) \right\}$$

subject to

$$A_{i\tau+1} = (1+r_\tau) A_{i\tau} + W_{i\tau}^{*} (1 - L_{i\tau}) - C_{i\tau}^{*} - P_{i\tau} X_{i\tau}, \quad \tau = t, \ldots, T,$$

*(Figure 13)*

Where *R* is the discount rate, *C* is consumption, *X* is recreational goods with price *P*, *L* is leisure time with price *W* and *A* is assets, and all these variables but *A* are larger or equal than zero, with A larger or equal than zero in the last period.

sophistication for the intertemporal model, (b) the analysis suggested in presence of uncertainty and imperfect financial markets, and (c) the model suggested in the appendix 1.B. within his dissertation, which involves dynamic programming techniques.

To finish our comments on these intertemporal issues, we just want to show this citation, which sheds light on why dynamic techniques can be plausible in time use research:

> It is worth noting that dynamic life cycle models suggest the possibility of large qualitative changes in the lifetime pattern of time allocation in response to relatively small changes in some of the parameters, including initial wealth. For example, it is theoretically possible to observe dramatic changes in career and nonmarket time use of the sort observed for women in industrialized societies as a consequence of small changes in the opportunity set. Similarly, optimizing life-cycle behavior is consistent with dramatic life style changes in response to modest changes in opportunities as an economy begins the transfer from agriculture to industry.

*(Figure 14)* (Juster & Stafford, 1991)

They conclude section IV by referring to Pollak & Wachter's criticisms, and discussing some facts along the line of the alternative approaches which Pollak & Wachter discuss in their paper. In addition, they wonder about several questions which make reference to joint production and how to solve it. Furthermore, the problems concerning both measurement and the definition of what to consider a commodity, and how to do it, are discussed by them too.

Concerning the problems of measurement of commodities, they complain about the time use data and the expenditure data being not very reliable. Concerning the problems for defining commodities, they state the following questions:

> There is the further problem in the hpm of defining the $Z$'s and the inputs ($X$'s and $t$'s) themselves. Is a trip itself

> the Z or is the trip just another intermediate product into the real Z, which might be a visit? Is a prepared meal the Z or is eating the meal the Z, while prepared food is more like an *X* with a shadow price? Does playing sports with a child produce an active leisure Z, a health Z, and a child development Z, or only the latter two for any adult who dislikes playing sports? Is the trip to the theatre an input into a leisure Z or a Z itself? Does that depend on whether it's a family trip or whether the subject enjoys driving per se? How is the stock of household capital to be allocated to the production of various Z's? And so on.

*(Figure 15)* (Juster & Stafford, 1991)

The last question connects with joint production, which carries us to the Pollak criticism, again. Juster & Stafford comment very roughly on Pollak & Wachter's contribution in 1975, although in this case they have the correct point. We have already discussed these issues deeper than in Juster & Stafford, so we will not insist anymore.

As pointed out in the beginning, this paper by Juster & Stafford covers some other empirical discussions and evidence which we will not comment in here. Juster & Stafford (1991) is, in our point of view, a very interesting survey of the literature, however probably better related to empirical findings, measurement issues, and so on, and this is not our purpose. Our purpose is to make a survey of theories about allocation of time, and the survey of theories about allocation of time made by Juster & Stafford (1991) is not too in depth, although it covers the main contributions up to 1991, providing us with the very good summing up concerning the intertemporal models in the 1980s.

# 8. FISCHER, 2001: TIME AS A SCARCE RESOURCE.

*"When a finite work requirement must be completed by a deadline, the remaining time for leisure is an exhaustible resource"*

(Fischer, 2001)

As the quotation is shows, our main reason to include this paper in this survey is a twofold: first, that it is probably the first paper analyzing time as an exhaustible resource, and second, as a consequence of the first reason, it is probably the first paper using dynamic programming techniques in time issues.

Disregarding whether it is or is not the first paper to do this, the fact is that it has been tried very recently. It is impressive how economists are worried about analyzing a wide range of exhaustible resources, however not a lot of them have even thought about dealing with time as one of them, with time being our fundamental resource.

Focusing on Fischer work, it must be introduced in a way as simple as the one she uses: and such a way is just to think and analyze the case of a task having to be completed by a given deadline. Such a task requires an amount *R* of working time, which you have to fulfil within the *T* previous periods to the deadline. As it is common in time use, she splits time in working time (*w*) and leisure time (*24-w*), and she assumes that individuals are exponential discounters (time consistent preferences)[38], with a discount factor delta. Of course, individuals are supposed to have a utility function defined only over the time uses, and they are utility maximizers facing and solving this problem:

$$U(R,T) = \max_{w_t \in [0,24]} \sum_{t=0}^{T-1} u(24 - w_t) \cdot \delta^t - \lambda \cdot \left( R - \sum_{t=0}^{T-1} w_t \right) \qquad (46)$$

There is no time and space within this master thesis to discus this paper in detail as we deem it probably deserves. Easily and very readably (though with much mathematics), Fischer derives and shows the optimal conditions (mainly following

[38] We know about the existence of an extra paper by Fischer which analyzed this same case however regarded *time inconsistent* preferences. Unfortunately we have not been able to find that paper. Regardless, that is a technical discussion which is useful in the sense that it can be compared with the time consistent discussion, and enriches the analysis. However what is important here is the theory and the essence of this. This essence is more than sufficiently captured by this paper.

Hottelling´s rule) and analyzes different cases (choke prices, uncertainty with regard to *task aversion*, and some others). She discusses cases in which penalties and multiple deadlines are included, and finishes by referring to the discounting rates. We will highlight the aspects that are most important for us, for the purpose of this paper.

She starts the paper with a very clear idea as the underlying foundation: modelling time as an exhaustible resource. In the end, she comes up with results which are summarized in the following citation:

> *"The simple model of impatience presented in this paper offers examples of situations where procrastination can be not only dynamically consistent, but also utility maximizing. Rather than contradict the psychological literature, the model of time as an exhaustible resource offers theoretical underpinnings explaining the results of several empirical psychology studies."*
>
> (Fischer, 2001)

Although procrastination is something that is very interesting[39], which interestingly comes up after the analysis she makes, the cause of it is simply because *time is modelled as an exhaustible resource*. The main "*underpinnings*" of the model she analyzes refer to the following very summarized theoretical result: *the closer the deadline, the higher the amount of working time* devoted to the task (a paper, in her setting), and *the higher the valuation of time*.

Even though this work is great, we can point out some weaknesses, under a more general perspective: this model only considers time use (reduced only to leisure and work) as the determinant of the utility function, and there is no other variable in the problem. In addition, she only considers one type or aspect of time. Despite the fact that for her particular purposes the former may be not very interesting, it is indeed important for welfare implications, as pointed out in several chapters within this thesis.

If a dynamic theory of allocation of time under a general framework (allowing us to extract general welfare conclusions as in traditional microeconomics) wants to be achieved, it must include something else, with respect to Fischer (2001) contribution, which we, again, think that is a fantastic contribution concerning to the allocation of time in a dynamic setting.

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[39] *"Modelling time as an exhaustible resource, this paper shows that simple impatience offers a reasonable theoretical explanation of dynamically consistent procrastination."* (Fischer, 2001)

# 9. CONCLUSIONS

*Everything is relative, but Light and Time…*
(Attributed to Albert Einstein)

We are sure everybody has heard about the sentence stated above. Even though we thought it was said by Albert Einstein, after searching extensively for the reference, we found out that it was never proven that Albert Einstein said such a thing. However, it is common knowledge that everybody, -at least most of people I know-, has used this citation at some time, even if they do not know who said it.

Unfortunately, we do not know as much as we would like about Mathematics, in general, -and Physics in particular-, to discuss such statement attributed to a genius like Albert Einstein[40]. However, we feel we *can* comment on how this statement attributed to him may relate to Economics, in light of what we have learnt from this *Survey of Economic Theories about the Allocation of Time along time*.

We know nothing about *Light* in Economics[41], however after learning some things about *Time* within the field of Economics, instead of *"Everything is relative, but Light and Time",* we would rather say that ***everything is relative to the enlightening Time***, and some insights have been remembered, discussed, analyzed and provided within this thesis.

The nature of the *beast* under analysis, -following DeSerpa-, has two faces, or at least these are the only two faces we can see after having been *face to face* with this *beast* for several months. Regardless of how the *beast* faces you, you can always see at least one of these two faces, on the same head. Such faces, as broadly discussed in Philosophy[42] are both the *immutability* of time (since nothing can be done to stop or change the pace of time, and time will always exist) and the *mutability* of time (i.e. the power that we *do* actually have to *control* how to distribute the time that is given to us by the mere fact of existing).

In this thesis, only one of the faces has been regarded: the *mutable* one. How to

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[40] Click here to know more about the Nobel Prize Winner in Physics in 1921: Albert Einstein.

[41] Perhaps Finn Førsund knows, given his *Electricity Economics*…

[42] We must remember and thank in here to our friend, neighbour and soon Master in Philosophy at the University of Oslo, Kristian Bredal, for all the comments, discussions, support and good moments both in Norway and Denmark. Such moments, shared with the *member* of the Austrian School, Martin Schedlbauer, were strictly necessary during the conception of this ideas. Some of them hopefully will be shown in the future, since they are not in this survey.

distribute the time for *a* given period of time corresponds with the concept of Allocation of Time, whose dominant theory was introduced by Becker in 1965, remaining almost *immutable* since then. This does not mean that the *immutable* face described in the previous paragraph is not interesting, since it is probably the most interesting.

In the chapter 2, we showed how Becker's theory looks like by discussing it. It was not only us, but also the literature, that realized the problems that ***joint production*** generated both in Becker's Theory and the household production models in general. This is also discussed by Pollak & Wachter in 1975, and reiterated by Juster & Stafford in 1991. Both contributions have been independently presented and discussed in chapters 4 and 7, respectively. The problem was still there, unsolved, from the strictly theoretical point of view[43]. In the chapter 2 we provide a ***Generalized Theory of Allocation of Time***, which allows for joint production. We show how the theory behaves for the particular case of one type or aspect of time, as a sufficient illustration of the general model. Moreover, we provide, for such a case, an approach in which when the households are considered as producers of commodities, they adopt a cost minimization attitude, and similarly, when households are considered as consumers of commodities, they adopt a utility maximization attitude. Such an approach is, in our opinion, more realistic, given the intrinsic characteristic that household theory endows to households: to be producers of the commodities that they consume. To conclude the chapter, we summarized the findings in Becker (1965) and provided some criticisms.

In chapter 3 we simply showed an example of the 1970s, as an illustration of several theories which were published during what we like to denote as the *golden age* for the Theories about the Allocation of Time. We use DeSerpa (1971) as an example to show how all the theories during the 1970s can be expressed as particularizations of Becker (1965), with some variations. Such variations stem from including mainly new and interesting constraints. However, some other variations come from abandoning the commodity-based analysis. In either of these two cases, Becker is shown as being a general approach, and hence, ours too. DeSerpa (1971) is an example of two such different kinds of variations at the same time, within the same model.

In chapter 4 we presented the very interesting criticism made by Pollak & Wachter in 1975. This one is interesting since it shows the weaknesses of Becker's theory. Such

[43] We must remind that, even though theoretically this is not solved, some attempts when doing empirical jobs have been made. Zhang et al. (2005) for example, analyzes how multi-person households allocate their time. However, this is more related to what they even call *joint activity engagement*, and this is not the same concept as pure **joint production**, i.e., **multitasking**.

weaknesses relate to joint production and non-constant returns to scale for the household technology. Both conditions are needed in Becker, if prices for commodities want to be defined. Such a thing is interesting since by getting it, classical microeconomic theory works, and with it, all the very rich analysis that classical microeconomics provides. Otherwise, classical microeconomics fails. We must note that Pollak & Wachter (1975) prove this by creating theorems, in a general framework for any kind of inputs, i.e. market goods, time inputs or any other. In our opinion, this paper was so good that it stopped new potential contributions to these theories of allocation of time under the case of more than one type of time in static models. Such a case is, in our view, the most interesting one, and given the quality of data existing nowadays[44], our suggested model in chapter 2 might hopefully motivate some new empirical studies. To conclude, we completed the presentation of the paper by discussing some alternative approaches they suggested in their paper in 1975, we summarized the findings and we provided mostly positive critiques of this paper.

In chapter 5 we discussed Gronau´s model in 1977, which is considered a benchmark within the field, together with Becker (1965). Although Gronau only regards one type of time, and one commodity, his analysis provides very good insights in a very easy and understandable way. Surely this is the main reason why his paper became a benchmark. As in chapters 2 and 4, we summarized his findings and provided some criticisms, after presenting and analyzing the model.

Chapter 6 is a different chapter. Although it is not fully related to time allocation, Boland (1978) reflects on what Economics in time had been, and what Time in Economics had been until that date. Both the mutable and the immutable faces we talked about earlier in this chapter are taking place in here, though the allocation of time only enters when Boland mentions Becker. We use this paper as an illustration of how the dynamic techniques have been used in economics, to analyze both the allocation of time itself and changes throughout time. We did that because of what we introduce in the chapters 7 and 8.

In chapter 7, a survey made by Juster & Stafford in 1991, they cover not only the theoretical contributions (as we tried to do with this thesis) but also some other things, as for example empirical findings. We used the part of their survey relating to the theory to deduce what had been new from Gronau until 1991, i.e., mainly the 1980s. We find a new theoretical progress within the field of theories about the allocation of time: intertemporal models. We briefly present an example of an intertemporal model that Juster & Stafford (1991) made, summarizing in it more or less all the variations suggested until that date. We did not discuss them in detail since, as we pointed out ourselves in Boland's chapter, *he*

[44] See MTUS Developer by CTUR at University of Oxford.

*argues there are no dynamics in this model, since the model is formally like a static model over all the time range, which only multiplies the number of goods.* This argument holds for the intertemporal models in general.

Finally, in chapter 8 we briefly presented an interesting paper by Fischer in 2001. She proposed a model in which time is regarded as an exhaustible resource. We included this paper since it is the only one we have found in which (a) time is considered an exhaustible resource that you can exploit to produce something (in her case, a paper to be delivered by a fixed deadline), and (b) dynamic techniques are used (in her case, dynamic programming). Her model yields quite interesting results, which relates to procrastination. Such an interesting topic is currently being studied by some economists[45] within the field of Behavioural Economics and the main reason is because empirical data from psychological studies show that people do actually procrastinate.

As we pointed at several points along the presentation of this thesis, there are some interesting cases, facts, properties and phenomena which escape the purpose of this thesis. However, some of the things that are not included in here are quite developed, although many others escape either our knowledge or our availability of time for the completion of this thesis at this time.

Whatever the case, we hope to have the chance of continuing these very interesting research topics. In our mind is already the idea for (1) a dynamic theory of allocation of time and (2) a general equilibrium model involving time use.

For the first idea, some insights have been given during the presentation we made at IATUR 2007. Such a working paper[46], though has some mistakes we corrected in the relevant parts of this thesis, can be checked as an illustration and a preliminary version of the dynamic theory of allocation of time. For the second idea, a very ambitious one, we hope to have the chance of working on it while pursuing a Doctoral Dissertation somewhere, someday, somehow… The only thing we need is *time*, since, repeating the first words in the first chapter of this thesis:

*It is only a matter of time (...). Small chance of success…what are we waiting for?*

Raúl G. Sanchis.
Oslo (Norway), 11/ 2007

[45] A good example of this is Asheim (2007)

[46] Sanchis (2007): "Time Microeconomics: Optimization Models" IATUR 2007 version.

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