Developing a flexible price version of NEMO

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Abstract

There is no single model that will serve best for all central bank purposes. NEMO (Norwegian Economy Model) is a core model supported by surrounding satellite models which serve certain tasks. Since Norges Bank is an inflation targeting central bank, expectations and the lags with which the monetary policy affects the economy should be paid particular attention, Norges Bank (2006). This is reflected in NEMO, which is a modern DSGE (Dynamic Stochastic General Equilibrium) model, based on the International Monetary Fund's multicountry Global Economic Model (GEM). NEMO is a smaller and simpler model than the GEM, but also modified to better fit the Norwegian economy. NEMO is a two country model, microeconomic founded and builds on the New Keynesian framework, cf. Norges Bank (2006).

The purpose of this thesis is to develop a flexible prices version of NEMO. This is a completely theoretical thesis and it will not give any empirical results.

There are various reasons for why we should care about a flexible price solution of NEMO. The thesis focus on the natural level of production. Woodford (2003) argues that flexible price models should serve as a benchmark for measuring the natural rate of output and the output gap. "*The level of output that would occur in an equilibrium with flexible prices, given current real factors (tastes, technology, government purchases) -what is called the "natural rate" of output, following Friedman (1968)-turns out to be a highly useful concept...*" Woodford (2003, pp.8). Woodford mentions further that Wicksell (1898) discusses "the natural rate of interest", which is the real rate of interest that would be realized in an equilibrium with flexible prices.

"Natural" levels of macroeconomic variables are of highly importance for central banks. Natural level of production and the output gap, which is defined as the gap between the natural level of productio and actual production, are both of high importance in monetary theory, Walsh (2003). The output gap is an indicator for economic pressure and also enters in a central bank's loss function.

The DSGE framework opens for calculations of the natural levels, according to Woodfords definition. This relates the natural level of production to the real shocks in the economy. This will give us a more volatile natural level of output in proportion to other ways of extracting natural levels of production e.g. Hodrick-Prescott filtering. On the other hand, as Neiss, K. S. and Nelson, E. (2005) state, the output gap is no longer a measure of the business cycle. The outputgap is solely related to the nominal rigidities.

In addition to the removal of nominal rigidities, the flexible price model is modified from local currency pricing in NEMO, to producer currency pricing. This is done because it is assumed that domestic households are better off in a model where domestic prices are flexible and prices abroad are sticky, than in a model where all prices are flexible. This assumption is debatable. It is not clear whether prices abroad should be flexible or not. As long as flexible home prices and sticky prices abroad are assumed, then producer currency pricing is needed to avoid monetary policy to have an effect on the real economy.

The flexible price model of NEMO which is developed in this thesis consists of a system of 47 non linear equations and 47 endogenous variables. This include 16 shock processes, where 5 shocks are due to the exogenous foreign country, 4 shocks are preference shocks, 4 are markup shocks, 2 are technology shocks and 1 is public spending shock.

Preface

At the time I was to write my master thesis I was doing an internship in Norges Bank. It was natural for me to write a macroeconomic thesis. With inspiration from courses at Humboldt University, Berlin, a master course at University of Oslo and some recent work for the modelling group in Norges Bank, it was the time for me to plunge into the fishy world of NEMO (Norwegian Economy Model).

Discussions with Kjetil Olsen, Øistein Røisland and Tommy Sveen brought up the idea of deriving a flexible price version of NEMO. Even though NEMO might be a small model in a central bank's perspective, it was a great challenge for me to do the step from simple real business cycle theory models thought at the university to a model designed for policy use.

I want to thank Norges Bank for the opportunity to work in a supporting environment and economic funding. I also want to thank my supervisors Øistein Røisland and Tommy Sveen for their support and critical feedback. Second I will especially thank Tore Anders Husebø for his enthusiasm and for introducing me to the modelling group in the first place. Then I will thank all the others who worked in Norges Bank modelling group during my stay, Leif Brubakk, Kai Halvorsen, Kristine Høegh-Omdal, Kjetil Olsen, Junior Maih and Magne Østnor. Their help and support have been invaluable.

The view in this thesis are those of the author and should not be regarded as those of Norges Bank. Any reminding errors are of course my and only my responsibility.

Oslo, August 2006
Jørgen Bækken

1 Introduction.

There is no single model that will serve best for all central bank purposes. NEMO (Norwegian Economy Model) is a core model supported by surrounding satellite models which serve certain tasks. Since Norges Bank is an inflation targeting central bank, expectations and the lags with which the monetary policy affects the economy should be paid particular attention, Norges Bank (2006). This is reflected in NEMO, which is a modern DSGE (Dynamic Stochastic General Equilibrium) model, based on the International Monetary Fund's multicountry Global Economic Model (GEM). NEMO is a smaller and simpler model than the GEM, but also modified to better fit the Norwegian economy. NEMO is microeconomic founded and builds on the New Keynesian framework, Norges Bank (2006).

Why should we care about a flexible price solution of NEMO? There might be various reasons for that. I will focus on the natural level of production. Woodford (2003) argues that flexible price models should serve as a benchmark for measuring the natural rate of output and the output gap. "The level of output that *would* occur in an equilibrium with flexible prices, given current real factors (tastes, technology, government purchases) -what is called the "natural rate" of output, following Friedman (1968)-turns out to be a highly useful concept..." Woodford (2003). Woodford mentions further that Wicksell (1898) discuss "the natural rate of interest", which is the real rate of interest that would be realized in an equilibrium with flexible prices.

"Natural" levels of macroeconomic variables are of highly importance for central banks. Natural level of production and the output gap, which is defined as the gap between the natural level of production and actual production, are both of high importance in monetary theory, Walsh (2003). The output gap is an indicator for economic pressure and also enters in a central bank's loss function.

The DSGE framework opens for calculations of the natural levels, according to Woodfords definition. This relates the natural level of production to the real shocks in the economy. This will give us a more volatile natural level of output in proportion to other ways of extracting natural levels of production e.g. Hodrick-Prescott filtering. On the other hand, as Neiss, K. S. and Nelson, E. (2005) state, the output gap is no longer a measure of the business cycle. The outputgap is solely related to the nominal rigidities.

Equilibrium with flexible prices is a purely theoretical concept. Unfortunately it is not possible to measure something that is not realized. This is where the flexible price model becomes interesting. Such a model opens for an artificial economy, producing what *would* occur if the prices were flexible. Of course it is not clear how to build such a model and its solutions depends very much on its parametrization.

This thesis will present a suggestion of a flexible price model built on Norges Banks model NEMO. The flexible price model is basically NEMO without nominal rigidities. What is done in this thesis is calculation of the models optimization problems after removal of the nominal rigidities. This gives a set of 46 equations and 46 endogenous variables.

The thesis is structured as follows. Chapter 2 presents the main difference between NEMO and the flexible price model. It is an introduction to the flexible price model, especially for those who know NEMO. Chapter 3 gives an overview of the model and presents and discuss the household sector in the model. Elements from NEMO are presented where there are essential aberrations between the flexible price version and NEMO. Chapter 4 presents the intermediate good production sector. Also in this part are elements from NEMO are presented where there are essential aberrations between the flexible price version and NEMO. Chapter 5 present the final good production sector. Chapter 7 concludes. The appendices give derivation of the log linearization of most of the models equations, derivation of the detrended final good production function, list of variables and parameters and some programming code for the toolbox Dynare for Matlab.

2 Key differences between NEMO and the flexible price version.

Turning NEMO into a flexible price model has aspects other than just making prices flexible. Before I go into the model's equations I will give a short overview of the main differences between NEMO and the flexible price version. See Norges Bank* (2006) and Brubakk, L., Husebø, T. A., Maih, J. and Olsen, K. (2006) for a complete decription of NEMO. The differences I will mention in this chapter are:

- The change from local currency pricing to producer currency pricing.
- The removal of price and wage adjustment costs.

The flexible price model should serve as a benchmark for calculating potential production in the home country. The potential production should then be the best level of production in a welfare perspective. The question is, what prices should be flexible? In NEMO there are two countries, home and abroad. There are prices both home and abroad. Prices should be flexible in the home country since deviation from the flexible price results into distortion in the allocation between consumption and leisure. What about foreign prices, should they also be flexible? Should a domestic monetary policy maker do policy in order to achieve import prices that would appear if foreign prices were flexible? The answer is not clear. The answer is yes if the domestic households get better off when the prices are flexible, and no if the domestic households get worse off. The foreign country suffers when their export price differ from a flexible price, because of distortion in the allocation between consumption and leisure. But the home country does not care about that. The home country cares about domestic household's utility. The domestic households are better off when the price of import decrease. If the foreign markup decreases due to increased competition, the home country is better off if prices are flexible. Then domestic households can enjoy low prices immediately. Vice versa, if the markup changes in opposite direction. If we assume that foreign markup shocks are symmetric around some constant level, meaning that the shocks do not have any upward or downward bias, then flexible foreign prices will give no gain for the domestic households. If we assume that the households prefer predictable prices, then there will be a gain if import prices are sticky. The prices will not change as fast and often as if the prices were flexible.

So far I will conclude that the flexible price model should include flexible prices in the home country and keep the sticky import prices. Still there are two options. Should we keep local currency pricing as it is in NEMO or not? The answer is no. Since the flexible price model will serve as a target for monetary policy we will not let the policy maker affect its target. The potential production should be independent of monetary policy. If we keep local currency pricing, monetary policy will have an effect on the real economy. We do not want that. Monetary policy has an effect on the nominal exchange rate. Since foreign price level is sticky the real exchange rate will also be affected. Therefore monetary policy affect the real economy when there are local currency pricing. On the other hand, monetary policy do not affect the real economy when there are producer currency pricing. Since domestic prices are flexible a change in the nominal exchange rate will not change the real exchange rate. A monetary policy which move the nominal exchange rate are fully passed through to the prices, leaving the real exchange rate and the real economy unchanged.

The new keynesian Philips curve is normally caused by adjustment costs or limited possibilities to adjust prices, Calvo pricing, Calvo, G. A. (1983). The adjustment costs in NEMO build on Rotemberg (1982). These are quadratic adjustment costs to prices and wages. "*The adjustment costs ensure that the model replicates the fairly slow and muted responses of price and wage infation to shocks we tend to see in VAR analysis and in other econometric analysis.*", Norges Bank* (2006). These costs are removed in the flexible price model, but I will present them to emphasise the difference between the models. The costs are:

$$\Gamma_t^W(j) = \frac{\phi^W}{2}\left[\frac{\Pi_t^W(j)}{\Pi_{t-1}^W} - 1\right]^2$$

$$\Gamma_t^{PQ}(h) = \frac{\phi^Q}{2}\left[\frac{\Pi_t^Q(h)}{\Pi_{t-1}^Q} - 1\right]^2 \tag{2.1}$$

and

$$\Gamma_t^{PM}(f) = \frac{\phi^M}{2}\left[\frac{\Pi_t^M(f)}{\Pi_{t-1}^M} - 1\right]^2$$

The first expression is the cost of changing the wage. This is given by type $j's$ change in wages, $\left(\Pi_t^W(j)\right)$, relative to the aggregate change in wages last period, $\left(\Pi_{t-1}^W\right)$. The second expression is the cost of changing the price of the domestically produced and sold intermediate good. This cost depend on firm $h's$ change in prices, $(\Pi_t^Q(h))$, relative to the aggregate change in prices last period, (Π_{t-1}^Q). The last expression is the cost of changing the price of domestic imports facing the intermediate good producer abroad. This cost depends on firm $f's$ change in prices, $(\Pi_t^M(f))$, relative to the aggregate change in prices last period, (Π_{t-1}^M). These costs cause that prices will not be adjusted immediately. Since the costs are convex, the firm is better off if it divides the adjustment over time in stead of doing it in one step.

It can be shown, see Norges Bank* (2006), that the wage Philips cure is given by

$$\pi_t^W = \frac{1}{1+\beta}\pi_{t-1}^W + \frac{\beta}{1+\beta}E_t\pi_{t+1}^W + \kappa(w_t - w^{flex})$$

Wage inflation depends on lagged wage inflation $\left(\pi_{t-1}^W\right)$, expected future inflation $\left(E_t\pi_{t+1}^W\right)$ and the difference between the wage and the wage that would be realised if wages and prices were flexible, $(w_t - w^{flex})$.

3 The household sector.

The model I will present is a variant of the Norges Bank's model, NEMO (Norwegian Economy Model).[1] First of all it differ from NEMO in the absence of nominal rigidities, but it is also modified in other ways. Absence of nominal rigidities results into flexible prices. So this model can be considered more or less as a real business cycle version of NEMO, although it differs from the early RBC models. The model I will present consider the home country as a small open economy, and take therefor the world market prices as given. This corresponds to Norway's position in the world.

3.1 Environment

There are two countries in this model, home and foreign. The structure of the economy is equal in home and foreign, but they differ in size. I will not model the foreign country completely. I rather assume foreign variables follow a certain process. I will get back to this. Home is a small open economy, and therefor take world market prices as given. I assume there is a trend growth in both economies. It is therefor important to detrend relevant variables in order to get stationarity. Small letters refer mostly to real detrended variables and subscript refers to time. There are three different sectors in the economies, the household sector, the intermediate good producer sector and the final good sector.

Households care about consumption and labour. They get utility from consumption and disutility of doing labour. There are two different types of households, spenders and savers. Spenderes are also referred as rule of thumb consumers, they always spend their disposal income at any given time. The savers make all decisions, they set wages, save through domestic and foreign bonds and through capital accumulation. In equilibrium, the spenders always offer the labour demanded at the given wage. Saving make the savers capable of doing intertemporary optimization. The capital is owned by the savers and rented out to the intermediate good producers, which also are owned by the savers. Both households, savers and spenders, pay taxes which always balances the governmental spendings.

Production take place in two stages. First there is a intermediate sector, producing factor inputs to the final good sector by combining labour and capital. The intermediate good producers rent capital from the savers and buy labour from the households. In order to model monopolistic competition in the labour market, each intermediate producer need labour from each household to produce the intermediate good. The amount of labour from each household may differ, depending on the wage the households set. We say that the intermediate good producers produce intermediate goods by bundling a mix of differentiated

[1]Documentation about NEMO is available from Norges Bank.

labour. The intermediate good is sold to the final good producers in the home country or exported to the final good producers in the foreign country. We assume producer currency pricing (PCP) when the intermediate good producers set the price of the intermediate good. This mean that each producer sets two different prices, both in own currency, one for the domestic market and one for the foreign market. In NEMO there is local currency pricing (LCP). The flexible price model is modified to PCP because otherwise would monetary policy have an effect as long as the prices abroad are sticky. Relating to the discussion in chapter 2 it is not clear whether a flexible price model should consider flexible prices abroad or not. But if sticky prices abroad are assumed, then there sholuld be local currency pricing. This is the case of the flexible price version developed in this thesis.

In the second stage, the final good is produced. It is produced by a bundle of domestic- and foreign intermediate goods. The factor inputs are bundled the same way as labour where in production of intermediate goods. This is done to get monopolistic competition in the market for intermediate goods. The final good can either be consumed (privately or publicly) or invested.

There is asumed to be an authority which offer money. We will see that money is redundant. Nominal prices occurs in the derivations, but are absent in the final equations. See figure (3.1) for an overview of the model.

As mentioned earlier, there are two kind of households in the model, spenders and savers. Let $slc \in [0, 1\rangle$ denote the share of the population which is liquidity constrained, the spenders, then $(1 - slc)$ is the share of the total population which are savers. The spenders are liquidity constraint, meaning that they do not have access to the bonds or capital market. This makes them not capable to save over time. They therefor consume after-tax disposal income. We assume the spenders always will offer the demand for labour at the given wage rate. The spenders consumption is then given by

$$P_t C_t^{sp}(i) = L_t(i) W_t - TAX_t(i)$$

where P_t is price of the final good, $C_t^{sp}(i)$ is consumption by spender i, $L_t(i)$ is labour done by spender i, W_t is the wage rate and $TAX_t(i)$ is taxation of spender i. Assuming that all will be similar in equilibrium, we can remove the i notation, and by inserting for the real detrended consumption $c_t^{sp} = \frac{C_t^{sp}}{P_t Z_t}$, real detrended wage rate $w_t = \frac{W_t}{P_t Z_t}$ and real detrended tax $tax_t = \frac{TAX_t}{P_t Z_t}$, we get

$$c_t^{sp} = L_t w_t - tax_t \qquad (c_t^{sp})$$

where P_t is the price of the final good and Z_t is the trend growth.

The others, who are savers, maximize expected discounted lifetime utility. The savers

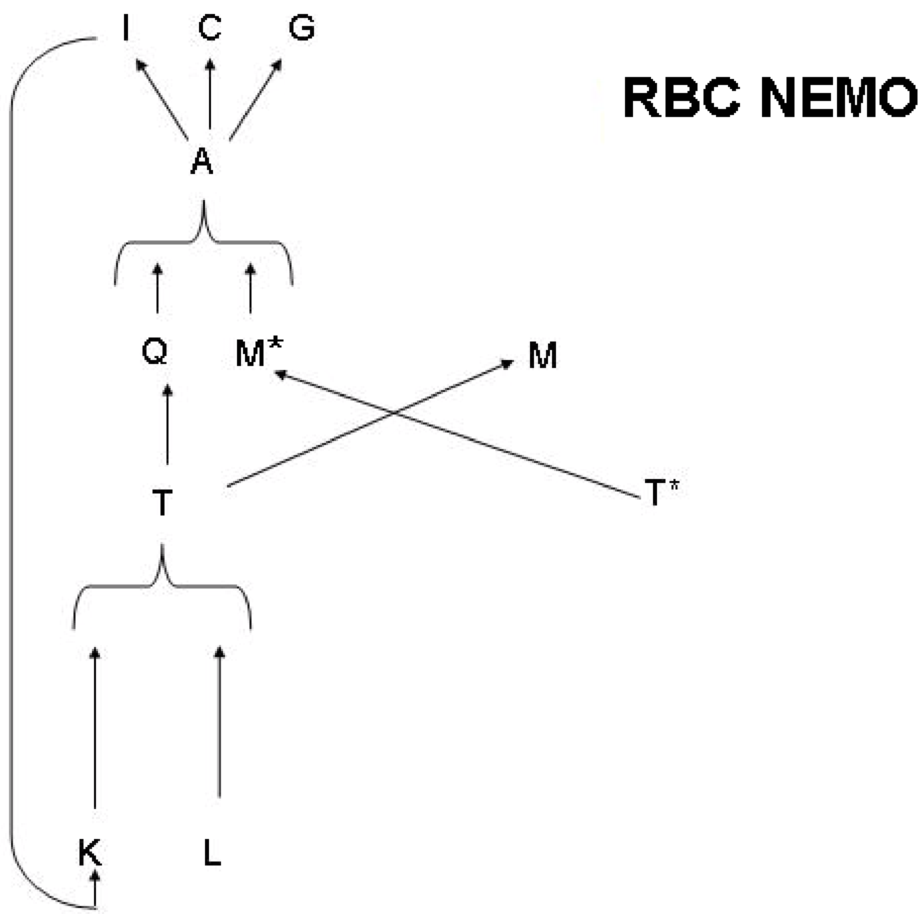

Figure 3.1: ketch of the model. Capital (K) and labour (L) transforms into the intermediate good (T). The intermediate good splits into input in domestic production of the final good (Q) and export (M). The final good (A) is produced from domestic intermediate goods (Q) and imported intermediate goods (M^*). The final good can either be invested (I) into capital, consumed by the households (C) or consumed by the government (G). T^* is foreign intermediate goods.

can save through capital, domestic bonds and foreign bonds. Even though I have removed nominal rigidities from NEMO, there are some real rigidities left. Before I go into the households maximisation problem, I will introduce these real rigidities. These are *financial friction* (transaction costs), *utilization rate of capital*, *cost of changing the utilization rate* and *cost of changing the capital stock.*

The *financial friction,* $\Gamma_t^{B^*}$, is a multiplicative transaction cost home agents face when they take position in the foreign bond market, (foreign bond market is indicated with a star). This cost depends on the nominal exchange rate S_t (home currency / foreign currency), the total holding of foreign bonds $(1-slc)B_t^*$, the gross domestic product Y_t, the price level and a risk premium shock $Z_t^{B^*}$ with expectation one. We adopt the following functional form:

$$\Gamma_t^{B^*} = \exp\left(-\phi^{B1}\frac{S_t(1-slc)B_t^*}{P_tY_t} + \log(Z_t^{B^*})\right)$$

Where $B_t^* \equiv \frac{1}{1-slc}\int_0^{(1-slc)} B_t^*(j)dj$, which is the per savers holding of foreign bonds in period t, $B_t^*(j)$ is what saver j holds of foreign bonds in period t and $\phi_{B_1}$is a parameter which controls the slope of the Γ^{B^*}function. Two important properties of the function is fulfilled. First, in absence of risk aversion shock, $Z_t^{B^*} = 1 \Rightarrow \Gamma_t^{B^*} = 1$ when $B_t^* = 0$, meaning that there are no cost to pay when you do not have any domestic bonds as long as there is no risk aversion shock. Second, the derivative of $\Gamma_t^{B^*}$ with respect to B_t^* is negative, $\frac{\partial\Gamma_t^{B^*}}{\partial B_t^*} < 0$, which states that the cost increase with the amount of bonds you hold.

We assume there are real rigidities in the capital market. The households might not be able to hire out all their capital stock. This is captured by the *utilization rate of capital*, $cu_t(j)$. $cu_t(j)$ is simply the share of type $j's$ capital which type j hires out. This means that the productive capital offered by type j at time t, denoted $\bar{K}_t(j)$, is simply the product of the utilization rate of capital and the physical capital stock offered by type j at time t, that is:

$$\bar{K}_t(j) = cu_t(j)K_t(j)$$

Furthermore, the households can change the utilization rate, but this is costly. This can be interpreted e.g. as advertisement. If you advertise for your kapital, it is likely that you will hire ote more than if you did not. But it is costly to do advertising. We assume the following functional form of the *cost of changing the capital utilization*:

$$\Gamma_t^{\varphi} = \phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_t(j)-1)} - 1)$$

where $\phi_{\varphi 1} \geqq 0$ and $\phi_{\varphi 2} > 0$ are parameters governing the scale and curvature, respectively. For convenience, the utilization rate is normalized to 1 in steady state. This means that there is a cost to pay when the utilization rate deviate from steady state.

In order to simulate realistic investment flows, it is assumed that it is *costly to adjust the capital stock*. This is captured in the way the capital law of motion is specified

$$K_{t+1}(j) = (1-\delta)K_t(j) + \Psi_j(j)K_t(j)$$

where $K_{t+1}(j)$ and $K_t(j)$ are the physical capital stock offered by type j in period $t+1$ and t, respectively. δ is the depreciation rate. $\Psi_t(j)$ is given by

$$\Psi_t(j) = \frac{I_t(j)}{K_t(j)} - \frac{\phi_{I1}}{2}\left(\frac{I_t(j)}{K_t(j)} - \frac{I_{SS}}{K_{SS}}Z_t^I\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}}\right)^2$$

$\Psi_t(j)$ is the rate of capital accumulation in time t. I_{SS} and K_{SS} are the steady state values of the investments and the capital, respectively. Z_t^I is an investment shock with expectation one. The rate of capital accumulation depends, first, on the difference between the actual and steady state investment to capital ratio and, second, the change in investment to capital ratio from last period. The parameters ϕ_{I1} and ϕ_{I2} decide how much each of those two differences affect the rate of capital accumulation, $\Psi_t(j)$. In the case of $\phi_{I1} = 0$ and $\phi_{I2} = 0$, we see that the capital law of motion becomes as in the case without adjustment cost to capital.

Due to the fact that all individuals are equal, inserting for $K_t = Z_t k_t$, $dZ_t = \frac{Z_{t+1}}{Z_t}$ and dividing both sides by Z_t, we get the following stationary capital law of motion

$$dZ_{t+1}k_{t+1} = (1-\delta)k_t + \Psi_t k_t$$

In absence of the type j notation the rate of capital accumulation is given by

$$\Psi_t = \frac{I_t}{K_t} - \frac{\phi_{I1}}{2}\left(\frac{I_t}{K_t} - \frac{I_{SS}}{K_{SS}}Z_t^I\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{I_t}{K_t} - \frac{I_{t-1}}{K_{t-1}}\right)^2$$

In steady state we have that $\Psi_{ss} = \frac{I_{ss}}{K_{ss}}$, inserting into the capital law of motion we get

$$dZ_{ss}k_{ss} = (1-\delta)k_{ss} + i_{ss}$$

Which give

$$\frac{I_{ss}}{K_{ss}} = dZ_{ss} + \delta - 1$$

Substitute into the rate of capital accumulation we get

$$\Psi_t = \frac{i_t}{k_t} - \frac{\phi_{I1}}{2}\left(\frac{i_t}{k_t} - (dZ_{ss} + \delta - 1)Z_t^I\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{i_t}{k_t} - \frac{i_{t-1}}{k_{t-1}}\right)^2$$

The households in the model are representative households for the economy. In real life there are plenty of different households. The cost the households pay are transferred back to the households because of this representative modelling. Fore those who have seen the play " Stones in his pockets[2]", it is easy to imagine you are wearing the carpenters hat in one moment and the next you are wearing the employers hat. Meaning that a representative agent of the carpenter and the employer, would have no net transfers because of the job the carpenter does and the pay the employer gives. Or as it is modelled, the representative household get back their costs, but leaving their decision unchanged.

In each period the savers need to make seven decisions. How much capital they want? How much to consume? How much to work? How much to save in domestic bonds? How much to save in foreign bonds? How much should the capital utilisation be adjusted? How much should the wage be adjusted? There is monopolistic competition in the labour market. This is modelled such that the intermediate good producers need a labour bundle of all household in order to produce. The households take advantage of this and set own wage bearing in mind the demand for labour. Wage setting is therefor a part of the households optimisation problem. In order to set up the savers maximisation problem, we need the demand for labour of type j. The size of the economy is normalized to 1. Each firm h in the intermediate good sector produce intermediate goods by using a mix of differentiated labour, indexed on $j \in [0, 1]$. Let $L_t(h)$ denotes an index of differentiated labour inputs used in production in firm h, which is given by[3]

$$L_t(h) = \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} \, dj \right]^{\frac{\psi_t}{\psi_t - 1}} \tag{3.1}$$

Where $L_t(h,j)$ denote firm $h's$ demand for labour from individual j. Each individual sets wages $W_t(j)$, which is taken as given by the firm. The intermediate firm h seeks to minimize the cost of labour, which can be written as

$$\min_{\{L_t(h,j)\}} \int_0^1 W_t(j) L_t(h,j)$$
$$s.t. \qquad L_t(h) = \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} \, dj \right]^{\frac{\psi_t}{\psi_t - 1}}$$

[2]A play by Marie Jones where over 15 characters are brought to life by only two actors.
[3]This function is taken as given.

The Lagrangian of this minimization problem is

$$\mathcal{L} = -\int_0^1 W_t(j)L_t(h,j)dj - \lambda_t \left[L_t(h) - \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}} \right]$$

The first order conditions become

$$\frac{\partial \mathcal{L}}{\partial L_t(h,j)} = -W_t(j) + \lambda_t \frac{\psi_t}{\psi_t - 1} \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}-1} \frac{\psi_t - 1}{\psi_t} L_t(h,j)^{-\frac{1}{\psi_t}} = 0$$

Rearranging gives

$$W_t(j) = \lambda_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}-1} L_t(h,j)^{-\frac{1}{\psi_t}} \tag{3.2}$$

Multiplying $L_t(h,j)$ on both sides

$$W_t(j)L_t(h,j) = \lambda_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}-1} L_t(h,j)^{1-\frac{1}{\psi_t}}$$

Integrating over all individuals on both sides

$$\int_0^1 W_t(j)L_t(h,j)\,dj = \int_0^1 \left(\lambda_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}-1} L_t(h,j)^{1-\frac{1}{\psi_t}} \right) dj$$

The inner integral and λ_t do not depend on the j's . We can therefore move them out of the integral.

$$\int_0^1 W_t(j)L_t(h,j)\,dj = \lambda_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}-1} \int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj$$

Rearranging the right hand side gives

$$\int_0^1 W_t(j)L_t(h,j)\,dj = \lambda_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}} dj \right]^{\frac{\psi_t}{\psi_t-1}} \tag{3.3}$$

Substitute (3.1) into (3.3)

$$\int_0^1 W_t(j)L_t(h,j)\,dj = \lambda_t L_t(h)$$

Taking the integral over all firms on both sides give us

$$\underbrace{\int_0^1 W_t(j) \underbrace{\int_0^1 L_t(h,j)\,dh}_{\substack{\text{Total labour demand}\\ \text{of type } j}}\,dj}_{\text{Total labour expenditures}} = \lambda_t \underbrace{\int_0^1 L_t(h)\,dh}_{\text{Total labour demand}} \tag{3.4}$$

Since the left hand side of this equation is total labour expenditures and the integral on the right hand side is total labour, then must λ_t represent the average wage, $\lambda_t = \bar{W}_t$. Inserting for $\lambda_t = \bar{W}_t$ into (3.2) gives

$$W_t(j) = \bar{W}_t \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}}\,dj \right]^{\frac{\psi_t}{\psi_t-1}-1} L_t(h,j)^{-\frac{1}{\psi_t}}$$

Multiply both sides with $L_t(h,j)^{\frac{1}{\psi_t}}$, divide by $W_t(j)$ and using the fact $\frac{\Psi_t}{\Psi_t-1} - 1 = \frac{1}{\Psi_t-1}$ gives

$$L_t(h,j)^{\frac{1}{\psi_t}} = \frac{\bar{W}_t}{W_t(j)} \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}}\,dj \right]^{\frac{1}{\psi_t-1}}$$

Raising both sides by the power of ψ_t

$$L_t(h,j) = \left(\frac{\bar{W}_t}{W_t(j)} \right)^{\psi} \left[\int_0^1 L_t(h,j)^{1-\frac{1}{\psi_t}}\,dj \right]^{\frac{\psi}{\psi_t-1}} \tag{3.5}$$

Substitute (3.1) into (3.5)

$$L_t(h,j) = \left(\frac{\bar{W}_t}{W_t(j)} \right)^{\psi} L_t(h)$$

Integrating over all firms gives

$$\int_0^1 L_t(h,j)\,dh = \left(\frac{W_t(j)}{\bar{W}_t} \right)^{-\psi} \int_0^1 L_t(h)\,dh$$

$$L_t(j) = \left(\frac{W_t(j)}{\bar{W}_t} \right)^{-\psi} L_t$$

Where $L_t(j)$ is demand for labour of type j and L_t is total labour demand. We see that the demand for individual $j's$ labour is a function of individual $j's$ wage, the average wage and the aggregate labour demand.

We assume the households have the following separable preferences in consumption and labour

$$
\begin{aligned}
U_t(C_t^{sa}(j), L_t(j)) &= Z_t^U \left(u(C_t^{sa}(j)) - v(L_t(j))\right) \\
u\left(C_t^{sa}(j)\right) &= (1-b^c)\log\left(\frac{C_t^{sa}(j) - b^c C_{t-1}^{sa}(j)}{1-b^c}\right) && (3.6) \\
v\left(L_t(j)\right) &= \frac{1}{1+\varsigma} L_t(j)^{1+\varsigma} && (3.7)
\end{aligned}
$$

with the following derivatives

$$
\begin{aligned}
u\prime\left(C_t^{sa}(j)\right) &= \frac{1-b^c}{C_t^{sa}(j) - b^c C_{t-1}^{sa}(j)} \\
v\prime\left(L_t(j)\right) &= L_t(j)^{\varsigma}
\end{aligned}
$$

where Z_t^U is a over all preference shock affecting both utility of consumption and disutility of labour. $u\left(C_t^{sa}(j)\right)$ is utility of consumption and $v\left(L_t(j)\right)$ is disutility of doing labour. $C_t^{sa}(j)$ is consumption by saver j in time t, b^c and ς are both parameters. There is habit persistence in consumption as long as $b^c \neq 0$. This is done in order to get hump-shaped responses, which matches data.

The savers have the following individual budget constraint

$$
\begin{aligned}
P_t C_t^{sa}(j) + P_t I_t(j) + B_t(j) + S_t B_t^*(j) \leq\ & L_t(j) W_t(j) + R_t^K cu_t(j) K_t(j) - \\
& P_t \Gamma_t^{\varphi} K_t(j) + (1 + r_{t-1}) B_{t-1}(j) + \\
& \left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} S_t B_{t-1}^*(j) + \\
& \Phi_t(j) - TAX_t(j)
\end{aligned}
$$

On the left hand side we have the households expenses. For all variables, the j notation indicates that it is about type j, and the t subsrcipt denotes that it is in period t. The household can spend resources on consumption, $C_t^{sa}(j)$, investment, $I_t(j)$, domestic bonds, $B_t(j)$ or foreign bonds, $B_t^*(j)$. Bonds are bought in local currency and foreign bonds need to be multiplied by the exchange rate such that we get in home currency. For simplicity I will normalize the price of the final good to one, $P_t = 1$. The final good are used for both consumption and investment[4]. On the right hand side of the budget constraint we have income. The household get income of doing labour, $L_t(j)W_t(j)$, where $W_t(j)$ is the wage and $L_t(j)$ is the amount of labour. The capital income is rental rate of capital, R_t^K, multiplied with

[4]We can for instance think on the final good as grain. Grain can either be consumed or used as seed.

the amount of capital that is hired out, $cu_t(j)K_t(j)$, minus the cost of keeping the utilization rate at it's current level, $P_t\Gamma_t^{\varphi}K_t(j)$. The cost of keeping the utilization rate at it's current level is multiplied with the price of the final good in order to get money units. In addition to labour and capital income, the households have position in the bonds market. $B_{t-1}(j)$ and $B_{t-1}^{*}(j)$, is the households holdings of domestic and foreign bonds from period $t-1$. Domestic bonds pay interest rate r_{t-1} on bonds bought in period $t-1$ and foreign bonds pay interest rate r_{t-1}^{*} on bonds bought in period $t-1$. The households also pay the transaction cost $\Gamma_{t-1}^{B^*}$, when they take position in the foreign bonds market. The foreign bonds need to be multiplied with the exchange rate to get in home currency. $\Phi_t(j)$ is profit transferred from the firms and $TAX_t(j)$ is lump-sum taxes paid by the households. The budget constraint is binding in optimum, since the households utility is increasing in consumption.

The households maximize expected future utility with respect to the control variables subject to the budget constraint, that the labour market clears and the capital law of motion. If we substitute the demand for labour into the budget constraint, the households face the following maximisation problem

$$\max_{\{K_{t+1}(j),C_t^{sa}(j),L_t^{sa}(j),B_t(j),B_t^*(j),W_t(j),cu_t(j),I_t(j)\}} E_t\sum_{t=1}^{\infty}\beta^t\left[Z_t^U\left(u(C_t^{sa}(j))-v(L_t(j))\right)\right]$$

$$\begin{aligned} st. \qquad C_t^{sa}(j)+I_t(j)+B_t(j)+S_tB_t^*(j) &= \left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi}L_tW_t(j)+R_t^Kcu_t(j)K_t(j)- \\ &\quad \Gamma_t^{\varphi}K_t(j)+\left(1+r_{t-1}\right)B_{t-1}(j)+ \\ &\quad \left(1+r_{t-1}^*\right)\left(1-\Gamma_{t-1}^{B^*}\right)S_tB_{t-1}^*(j)+ \\ &\quad \Phi_t(j)-TAX_t(j) \\ K_{t+1}(j) &= (1-\delta)K_t(j)+\Psi_j(j)K_t(j) \end{aligned}$$

The Langrangian becomes

$$\mathcal{L} = E_t \sum_{t=1}^{\infty} \beta^t \left[\begin{array}{c} Z_t^U \left(u(C_t^{sa}(j)) - v(L_t(j)) \right) \\ -\lambda_t \left(\begin{array}{c} \left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi} L_t W_t(j) + R_t^K cu_t(j) K_t(j) - \Gamma_t^{\varphi} K_t(j) + \\ \left(1 + r_{t-1}\right) B_{t-1}(j) + \left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} S_t B_{t-1}^*(j) \\ +\Phi_t(j) - TAX_t(j) - B_t(j) - S_t B_t^*(j) - \ C_t(j) - I_t(j) \end{array} \right) \\ -\gamma_t \left(L_t(j) - \left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi} L_t \right) \\ -\omega_t \left((1-\delta) K_t(j) + \Psi_j(j) K_t(j) - K_{t+1}(j) \right) \end{array} \right]$$

After inserting the expressions for the frictions, the Lagrangian look like this

$$\mathcal{L} = E_t \sum_{t=1}^{\infty} \beta^t \left[\begin{array}{c} Z_t^U \left(u(C_t^{sa}(j)) - v(L_t(j)) \right) \\ -\lambda_t \left(\begin{array}{c} \left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi} L_t W_t(j) + R_t^K cu_t(j) K_t(j) - P_t \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_t(j)-1)} - 1) K_t(j) + \\ \left(1 + r_{t-1}\right) B_{t-1}(j) + \\ \left(1 + r_{t-1}^*\right) \exp\left(-\phi^{B1} \frac{S_{t-1} B_{t-1}^*}{P_{t-1} Y_{t-1}} + \log(Z_{t-1}^{B^*}) \right) S_t B_{t-1}^*(j) \\ +\Phi_t(j) - TAX_t(j) - B_t(j) - S_t B_t^*(j) - \ P_t C_t(j) - P_t I_t(j) \end{array} \right) \\ -\gamma_t \left(L_t(j) - \left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi} L_t \right) \\ -\omega_t \left((1-\delta) K_t(j) + \Psi_j(j) K_t(j) - K_{t+1}(j) \right) \end{array} \right]$$

The first order conditions will then be

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}(j)} = \beta^t \omega_t + \beta^{t+1} E_t \left[\begin{array}{c} \lambda_{t+1}\left(P_{t+1}\phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_{t+1}(j)-1)}-1) - R^K_{t+1} cu_{t+1}(j)\right) - \\ \omega_{t+1}\left((1-\delta) + \frac{\partial \Psi_{t+1}(j)}{\partial K_{t+1}(j)} K_{t+1}(j) + \Psi_{t+1}(j)\right) \end{array} \right] = 0 \quad (3.8)$$

$$\frac{\partial \mathcal{L}}{\partial I_t(j)} = E_t \beta^t \left[\lambda_t P_t - \omega_t \frac{\partial \Psi_t(j)}{\partial I_t(j)} K_t(j)\right] = 0 \quad (3.9)$$

$$\frac{\partial \mathcal{L}}{\partial C_t(j)} = E_t \beta^t \left[Z^U_t u\prime(C_t(j)) - \lambda_t(-P_t)\right] = 0 \quad (3.10)$$

$$\frac{\partial \mathcal{L}}{\partial L_t(j)} = E_t \beta^t \left[-Z^U_t v\prime\left(L_t(j)\right) - \gamma_t\right] = 0 \quad (3.11)$$

$$\frac{\partial \mathcal{L}}{\partial W_t(j)} = E_t \beta^t \left[\begin{array}{c} -\lambda_t \left((1-\psi_t)\left(\frac{W_t(j)}{\bar{W}_t}\right)^{-\psi_t} L_t\right) - \\ \gamma_t \left(-(-\psi_t)\left(\frac{1}{\bar{W}_t}\right)^{-\psi_t} W_t(j)^{-1-\psi_t} L_t\right) \end{array} \right] = 0 \quad (3.12)$$

$$\frac{\partial \mathcal{L}}{\partial B_t(j)} = E_t \beta^t \left[-\lambda_t(-1)\right] - \beta^{t+1} E_t \left[\lambda_{t+1}(1+r_t)\right] = 0 \quad (3.13)$$

$$\frac{\partial \mathcal{L}}{\partial B^*_t(j)} = \beta^t E_t \left[-\lambda_t\left(-S_t\right)\right] - \beta^{t+1} E_t \left[\begin{array}{c} \lambda_{t+1}(1+r^*_t) \\ \exp\left(-\phi^{B1} \frac{S_{t-1} B^*_{t-1}}{P_{t-1} Y_{t-1}} + \log(Z^{B^*}_{t-1})\right) S_{t+1} \end{array} \right] = 0 \quad (3.14)$$

$$\frac{\partial \mathcal{L}}{\partial cu_t(j)} = \beta^t E_t \left[-\lambda_t\left(R^K_t K_t(j) - P_t \phi_{\varphi 1} \phi_{\varphi 2} e^{\phi_{\varphi 2}(cu_t(j)-1)} K_t(j)\right)\right] = 0 \quad (3.15)$$

3.2 The marginal rate of substitution between labour and consumption.

Rearrange (3.10) and (3.11) to we get

$$Z^U_t u\prime(C_t(j)) = -\lambda_t P_t$$

$$Z^U_t v\prime\left(L_t(j)\right) = -\gamma_t \quad (3.16)$$

The marginal rate of substitution between labour and consumption is given by

$$MRS^{L,C}_t(j) = \frac{U^2_t(C^{sa}_t(j), L_t(j))}{U^1_t(C^{sa}_t(j), L_t(j))} = \frac{Z^U_t v\prime\left(L_t(j)\right)}{Z^U_t u\prime(C_t(j))} = \frac{\gamma_t}{\lambda_t P_t} \quad (3.17)$$

Inserting for the utility (3.6) and the disutility (3.7) functions

$$MRS^{L,C}_t(j) = \frac{U^2_t(C^{sa}_t(j), L_t(j))}{U^1_t(C^{sa}_t(j), L_t(j))} = \frac{(L_t(j))^{\varsigma}\left(C^{sa}_t(j) - b^c C^{sa}_{t-1}(j)\right)}{(1-b^c)} = \frac{\gamma_t}{\lambda_t P_t}$$

In order to get stationarity insert for $C_t^{sa}(j) = Z_t c_t^{sa}(j)$ and $MRS_t^{L,C} = mrs_t^{L,C} Z_t$

$$mrs_t^{L,C}(j) Z_t = \frac{(Z_t c_t^{sa}(j) - b^c Z_{t-1} c_{t-1}^{sa}(j))}{(1-b^c)} (L_t(j))^{\varsigma}$$

or where $dZ_t = \frac{Z_t}{Z_{t-1}}$, the technology growth from period $t-1$ to t.

$$mrs_t^{L,C}(j) = \frac{(c_t^{sa}(j) - b^c \frac{c_{t-1}^{sa}(j)}{dZ_t})}{(1-b^c)} (L_t(j))^{\varsigma}$$

Due to the fact that each agent is equal, we can drop the j notation

$$mrs_t^{L,C} = \frac{(c_t^{sa} - b^c \frac{c_{t-1}^{sa}}{dZ_t})}{(1-b^c)} (L_t)$$

The marignal rate of substitution depends on today's consumption, lagged consumption, amount labour today, technology growt and the parameters b^c (habit) and ς (labour).

3.3 The stochastic discount rate.

From (3.13) we get

$$\beta^t \lambda_t = \beta^{t+1} E_t [\lambda_{t+1}] (1 + r_t)$$

rearrange and get

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] (1 + r_t) \tag{3.18}$$

Where we define $D_{t,t+1} \equiv E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right]$ as the stochastic discount rate. From (3.10) we get an expression for λ_t

$$\lambda_t = -\frac{Z_t^U u\prime(C_t(j))}{P_t} \tag{3.19}$$

Which also must hold in period $t+1$

$$E_t [\lambda_{t+1}] = -E_t \left[\frac{Z_{t+1}^U u\prime(C_{t+1}(j))}{P_{t+1}} \right] \tag{3.20}$$

Dividing (3.20) by (3.19) and multiplying by β gives this expression for the stochastic discount rate

$$D_{t,t+1} \equiv E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] = \beta E_t \left[\frac{Z_{t+1}^U u\prime(C_{t+1}(j)) P_t}{Z_t^U u\prime(C_t(j)) P_{t+1}} \right]$$

Inserting for the utility function (3.6)

$$
\begin{aligned}
E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right] &= \beta E_t\left[\frac{Z_{t+1}^U\frac{1-b^c}{C_{t+1}^{sa}(j)-b^cC_t^{sa}(j)}P_t}{Z_t^u\frac{1-b^c}{C_t^{sa}(j)-b^cC_{t-1}^{sa}(j)}P_{t+1}}\right] \\
&= \beta E_t\left[\frac{Z_{t+1}^U\left(C_t^{sa}(j)-b^cC_{t-1}^{sa}(j)\right)P_t}{Z_t^U\left(C_{t+1}^{sa}(j)-b^cC_t^{sa}(j)\right)P_{t+1}}\right]
\end{aligned}
$$

In order to get stationarity, insert for $C_t^{sa}(j) = Z_t c_t^{sa}(j)$

$$
E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right] = \beta E_t\left[\frac{Z_{t+1}^U\left(Z_t c_t^{sa}(j)-b^cZ_{t-1}c_{t-1}^{sa}(j)\right)P_t}{Z_t^U\left(Z_{t+1}c_{t+1}^{sa}(j)-b^cZ_tc_t^{sa}(j)\right)P_{t+1}}\right]
$$

Divide with Z_t in the numerator and the denominator on the right hand side

$$
E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right] = \beta E_t\left[\frac{Z_{t+1}^U\left(c_t^{sa}(j)-b^c\frac{Z_{t-1}}{Z_t}c_{t-1}^{sa}(j)\right)P_t}{Z_t^U\left(\frac{Z_{t+1}}{Z_t}c_{t+1}^{sa}(j)-b^cc_t^{sa}(j)\right)P_{t+1}}\right]
$$

Use the definition $dZ_t = \frac{Z_t}{Z_{t-1}}$ and $\Pi_{t+1} = \frac{P_{t+1}}{P_t^A}$

$$
E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right] = \beta E_t\left[\frac{Z_{t+1}^U\left(c_t^{sa}(j)-b^c\frac{1}{dZ_t}c_{t-1}^{sa}(j)\right)}{Z_t^U\left(dZ_{t+1}c_{t+1}^{sa}(j)-b^cc_t^{sa}(j)\right)}\frac{1}{\Pi_{t+1}}\right] \tag{3.21}
$$

The stochastic discount rate depends on four factors. First it depends on the discount factor β. Second it depends on the intertemporal preference shock ratio $\frac{Z_{t+1}^u}{Z_t^u}$, it says that the stochastic discount factor becomes larger the more biased our preferences are toward future consumption. The larger the stochastic discount factor is, the more weights do the household put on future periods, meaning they become more patient. Third it depends on the households relative intertemporal wealth $\frac{u\prime(C_{t+1}(j))}{u\prime(C_t(j))}$. Since $u\prime(C_t(j)) > 0$ and $u\prime\prime(C_t(j)) < 0$, $u\prime(C_t(j))$ will decrease when $C_t(j)$ increase. This mean that the stochastic discount factor decrease if the households expect higher consumption tomorrow than today. In other words, the households become more impatient and therefor consume more today, if they expect better times tomorrow than today. This is consistent with consumption smoothing. Finally, the stochastic discount factor depends on the inflation rate. If the expected price tomorrow raise, the stochastic discount factor decrease, households become more impatient and therefore they consume more today.

If we combine equation (3.21) and (3.18) we get the classic Euler equation

$$1 = \beta E_t \left[\frac{Z^U_{t+1}\left(c^{sa}_t(j) - b^c \frac{1}{dZ_t} c^{sa}_{t-1}(j)\right)}{Z^U_t \left(dZ_{t+1} c^{sa}_{t+1}(j) - b^c c^{sa}_t(j)\right)} \frac{1}{\Pi_{t+1}} \right] (1 + r_t)$$

insert for the real interest rate defined as $\Upsilon_t = \frac{(1+r_t)}{\Pi_{t+1}}$ and due to the fact that each agent is equal, we can drop the j notation

$$1 = \beta E_t \left[\Upsilon_t \frac{Z^U_{t+1}\left(c^{sa}_t - b^c \frac{1}{dZ_t} c^{sa}_{t-1}\right)}{Z^U_t \left(dZ_{t+1} c^{sa}_{t+1} - b^c c^{sa}_t\right)} \right]$$

rearrange

$$E_t \left[\frac{Z^U_t \left(dZ_{t+1} c^{sa}_{t+1}(j) - b^c c^{sa}_t(j)\right)}{Z^U_{t+1}\left(c^{sa}_t(j) - b^c \frac{1}{dZ_t} c^{sa}_{t-1}(j)\right)} \right] = \beta(1 + r_t) E_t \left[\frac{1}{\Pi_{t+1}} \right]$$

or rearrange to get

$$E_t \left[c^{sa}_{t+1}(j) - b^c \frac{c^{sa}_t(j)}{dZ_{t+1}} \right] = \beta E_t \left[\frac{Z^U_{t+1}(1 + r_t)}{Z^U_t \Pi_{t+1} dZ_{t+1}} \right] \left(c^{sa}_t(j) - b^c \frac{1}{dZ_t} c^{sa}_{t-1}(j) \right) \tag{3.22}$$

3.4 The uncovered interest parity condition.

From (3.14) we have

$$\beta^t \lambda_t S_t = \beta^{t+1} E_t \left[\lambda_{t+1}(1 + r^*_t) \exp\left(-\phi^{B1} \frac{S_{t-1} B^*_{t-1}}{P_{t-1} Y_{t-1}} + \log(Z^{B^*}_{t-1}) \right) S_{t+1} \right]$$

Rearranging gives

$$1 = (1 + r^*_t) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \exp\left(-\phi^{B1} \frac{S_{t-1} B^*_{t-1}}{P_{t-1} Y_{t-1}} + \log(Z^{B^*}_{t-1}) \right) \frac{S_{t+1}}{S_t} \right]$$

From equation (3.18) we get $E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \right] = \frac{1}{(1+r_t)}$, substitute and get the uncovered interest parity condition

$$1 = (1 + r^*_t) E_t \left[\frac{1}{(1 + r_t)} \exp\left(-\phi^{B1} \frac{S_{t-1} B^*_{t-1}}{P_{t-1} Y_{t-1}} + \log(Z^{B^*}_{t-1}) \right) \frac{S_{t+1}}{S_t} \right] \tag{3.23}$$

Rearrange and insert for $B^*_{t-1} = Z_{t-1} P^*_t b^*_{t-1}$, $Y_{t-1} = Z_{t-1} y_{t-1}$, where b^*_{t-1} is per savers holding of real detrended foreign bonds, P^*_t is foreign pricel evel, in order to get stationarity and multiply and divide with $\frac{P^*_{t+1}}{P_{t+1}}\frac{P^*_t}{P_t}$ on the right hand side

$$(1+r_t) = (1+r^*_t)E_t\left[\exp\left(-\phi^{B1}\frac{S_{t-1}Z_{t-1}P^*_{t-1}b^*_{t-1}}{P_{t-1}Z_{t-1}y_{t-1}} + \log(Z^{B^*}_{t-1})\right)\frac{S_{t+1}\frac{P^*_{t+1}}{P_{t+1}}\frac{P^*_t}{P_t}}{S_t\frac{P^*_{t+1}}{P_{t+1}}\frac{P^*_t}{P_t}}\right]$$

Insert for the real exchange rate $v_t = \frac{S_t P^*_t}{P_t}$ and inflation $\Pi^*_{t+1} = \frac{P^*_{t+1}}{P^*_t}, \Pi_{t+1} = \frac{P_{t+1}}{P_t}$

$$(1+r_t) = (1+r^*_t)E_t\left[\exp\left(-\phi^{B1}v_{t-1}\frac{b^*_{t-1}}{y_{t-1}} + \log(Z^{B^*}_{t-1})\right)\Pi_{t+1}\frac{1}{\Pi^*_{t+1}}\right]$$

Insert for the real interest rate $\Upsilon_t = \frac{(1+r_t)}{\Pi_{t+1}}$ and $\Upsilon^*_t = \frac{(1+r^*_t)}{\Pi^*_{t+1}}$

$$\Upsilon_t = \Upsilon^*_t E_t\left[\exp\left(-\phi^{B1}v_{t-1}\frac{b^*_{t-1}}{y_{t-1}} + \log(Z^{B^*}_{t-1})\right)\right]$$

This states that the expected payoff in the domestic bond market is equal the expected payoff in the foreign bond market. This rules out arbitrage in the model.

3.5 The individual wage setting equation.

Rearrange equation (3.12) and get

$$-\lambda_t(1-\psi_t) = \gamma_t\psi_t W_t(j)^{-1} \Longleftrightarrow W_t(j) = -\frac{\gamma_t\psi_t}{\lambda_t(1-\psi_t)}$$

From equation (3.17) we have that $\frac{\gamma_t}{\lambda_t} = P_t MRS_t$, insert and get

$$W_t(j) = MRS^{L,C}_t P_t\frac{\psi_t}{(\psi_t-1)}$$

To get in real detrended terms insert for $W_t(j) = P_t Z_t w_t(j)$ and get

$$Z_t w_t(j) = MRS^{L,C}_t\frac{\psi_t}{(\psi_t-1)}$$

simplify and get

$$w_t(j) = mrs^{L,C}_t\frac{\psi_t}{(\psi_t-1)} = \frac{Z^v_t(L_t(j))^\varsigma\left(c_t(j) - b^c\frac{1}{dZ_t}c_{t-1}(j)\right)}{Z^u_t(1-b^c)}\frac{\psi_t}{(\psi_t-1)}$$

Due to the fact that each agent is equal, we can drop the j notation

$$w_t = mrs_t^{L,C}\frac{\psi_t}{(\psi_t - 1)} \tag{3.24}$$

We see that the individual wage depend on the marginal rate of substitution between labour and consumption, today's price level and the agents degree of market power, ψ. The wage will increase when ψ decreases towards the value one.

The wage expression (3.24) differs from the wage expression in NEMO. Brubakk, L, Husebø, T. A. , Maih, J. and Olsen, K. (2006) shows that the wage in NEMO is given by

$$w_t = \psi_t mrs_t \left[\begin{array}{c} (\psi_t - 1)\left(1 - \Gamma_t^W\right) + \phi^W \left[\frac{\Pi_t^W}{\Pi_{t-1}^W} - 1\right]\frac{\Pi_t^W}{\Pi_{t-1}^W} \\ -E_t D_{t,t+1}\left(\Pi_{t+1}^W \frac{L_{t+1}}{L_t}\phi^W \left[\frac{\Pi_{t+1}^W}{\Pi_t^W} - 1\right]\frac{\Pi_{t+1}^W}{\Pi_t^W}\right) \end{array} \right]^{-1}$$

due to a wage adjustment cost, which enters in the budget constraint, given by

$$\Gamma_t^W(j) \equiv \frac{\phi^W}{2}\left[\frac{\Pi_t^W(j)}{\Pi_{t-1}^W} - 1\right]^2$$

where the cost depends on type $j's$ wage inflation $\Pi_t^W(j)$ relative to the past wage inflation for the whole economy Π_{t-1}^W. The parameter ϕ^W determines how costly it is to change the wage inflation rate. We see that when type j set wage equal to the past wage inflation, the adjustment cost will disappear and the wage in NEMO and the flexible price model will be the similar.

3.6 The optimal investment to capital ratio.

From (3.8) we have

$$\omega_t = \beta E_t \left[\begin{array}{c} \omega_{t+1}\left((1-\delta) + \frac{\partial \Psi_{t+1}(j)}{\partial K_{t+1}(j)}K_{t+1}(j) + \Psi_{t+1}(j)\right) \\ -\lambda_{t+1}\left(P_{t+1}\phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_t(j)-1)} - 1) - R_{t+1}^K cu_{t+1}(j)\right) \end{array} \right]$$

From (3.9) we have

$$\lambda_t P_t = \omega_t \frac{\partial \Psi_t(j)}{\partial I_t(j)} K_t(j)$$

Multiplying $\partial \frac{I_{t+1}(j)}{K_{t+1}(j)}$ on both sides of the fraction line in the first equation and $\partial \frac{I_t(j)}{K_t(j)}$ in the latter one

$$\omega_t = \beta E_t \left[\begin{array}{c} \omega_{t+1} \left((1-\delta) + \frac{\partial \Psi_{t+1}(j) \partial \frac{I_{t+1}(j)}{K_{t+1}(j)}}{\partial \frac{I_{t+1}(j)}{K_{t+1}(j)} \partial K_{t+1}(j)} K_{t+1}(j) + \Psi_{t+1}(j) \right) - \\ \lambda_{t+1} \left(\left(P_{t+1} \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_{t+1}(j)-1)} - 1) - R^K_{t+1} cu_{t+1}(j) \right) \right) \end{array} \right] \tag{3.25}$$

$$\lambda_t P_t = \omega_t \frac{\partial \Psi_t(j) \partial \frac{I_t(j)}{K_t(j)}}{\partial \frac{I_t(j)}{K_t(j)} \partial I_t(j)} K_t(j) \tag{3.26}$$

To ease the notation, let us define

$$\frac{\partial \Psi_t}{\partial \frac{I_t}{K_t}} \equiv \Psi'_t = 1 - \phi_{I1} \left(\frac{I_t}{K_t} - \frac{I_{SS}}{K_{SS}} Z^I_t \right) - \phi_{I2} \left(\frac{I_t}{K_t} - \frac{I_{t-1}}{K_{t-1}} \right)$$

Due to the fact that each agent is equal, we can drop the j notation. Two more fact that will be useful is

$$\frac{\partial \frac{I_{t+1}}{K_{t+1}}}{\partial I_{t+1}} = \frac{1}{K_{t+1}}$$

$$\frac{\partial \frac{I_{t+1}}{K_{t+1}}}{\partial K_{t+1}} = -\frac{I_{t+1}}{(K_{t+1})^2}$$

We can now rewrite e.g. (3.25) and (3.26)

$$\omega_t = \beta E_t \left[\begin{array}{c} \omega_{t+1} \left((1-\delta) - \Psi'_{t+1} \frac{I_{t+1}}{K_{t+1}} + \Psi_{t+1} \right) - \\ \lambda_{t+1} \left(P_{t+1} \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_{t+1}-1)} - 1) - R^K_{t+1} cu_{t+1} \right) \end{array} \right]$$

$$\lambda_t P_t = \omega_t \Psi'_t \iff \omega_t = \frac{\lambda_t P_t}{\Psi'_t}$$

Substituting ω_t and ω_{t+1} from the last equation into the first one gives

$$\frac{\lambda_t P_t}{\Psi'_t} = \beta E_t \left[\begin{array}{c} \frac{\lambda_{t+1} P_{t+1}}{\Psi'_{t+1}} \left((1-\delta) - \Psi'_{t+1} \frac{I_{t+1}}{K_{t+1}} + \Psi_{t+1} \right) - \\ \lambda_{t+1} \left(P_{t+1} \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_{t+1}-1)} - 1) - R^K_{t+1} cu_{t+1} \right) \end{array} \right]$$

Divide both sides by P_{t+1} and λ_t

$$\frac{P_t}{\Psi'_t P_{t+1}} = E_t \left[\begin{array}{c} \beta \frac{\lambda_{t+1}}{\lambda_t \Psi'_{t+1}} \left((1-\delta) + \Psi'_{t+1} \frac{I_{t+1}}{K_{t+1}} + \Psi_{t+1} \right) - \\ \beta \frac{\lambda_{t+1}}{\lambda_t} \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_{t+1}-1)} - 1) + \beta \frac{\lambda_{t+1}}{\lambda_t P_{t+1}} R^K_t cu_{t+1} \end{array} \right]$$

Inserting for the real rate of capital $R_t^K = P_t r_t^K$ and rearranging gives

$$\frac{1}{\Psi_t'} = E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\Pi_{t+1}\begin{pmatrix} \frac{1}{\Psi_{t+1}'}\left((1-\delta) - \Psi_{t+1}'\frac{I_{t+1}}{K_{t+1}} + \Psi_{t+1}\right) - \\ \phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_{t+1}-1)} - 1) + r_t^K cu_{t+1} \end{pmatrix}\right] \tag{3.27}$$

Taking into account that each type behave similar, equation (3.21) gives $E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right] = \beta E_t\left[\frac{Z_{t+1}^U\left(c_t^{sa} - b^c\frac{1}{dZ_t}c_{t-1}^{sa}\right)}{Z_t^U\left(dZ_{t+1}c_{t+1}^{sa} - b^c c_t^{sa}\right)}\frac{1}{\Pi_{t+1}}\right]$, insert and get

$$\frac{1}{\Psi_t'} = E_t\left[\beta\frac{Z_{t+1}^U\left(c_t^{sa} - \frac{b^c}{dZ_t}c_{t-1}^{sa}\right)}{Z_t^U\left(dZ_{t+1}c_{t+1}^{sa} - b^c c_t^{sa}\right)}\begin{pmatrix} \frac{1}{\Psi_{t+1}'}\left((1-\delta) - \Psi_{t+1}'\frac{I_{t+1}}{K_{t+1}} + \Psi_{t+1}\right) - \\ \phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_{t+1}-1)} - 1) + r_t^K cu_{t+1} \end{pmatrix}\right]$$

3.7 The optimal utilization rate of capital.

Rearranging and simplifying equation (3.15) gives

$$R_t^K = P_t\phi_{\varphi 1}\phi_{\varphi 2}e^{\phi_{\varphi 2}(cu_t(j)-1)}$$

or inserting for the real interest rate of capital

$$r_t^K = \frac{R_t^K}{P_t}r_t^K = \phi_{\varphi 1}\phi_{\varphi 2}e^{\phi_{\varphi 2}(cu_t(j)-1)}$$

Note that the left hand side is $\frac{\partial\Gamma_t^\varphi}{\partial cu_t(j)}$, therefore we can write

$$r_t^K = \frac{\partial\Gamma_t^\varphi}{\partial cu_t(j)}$$

This express the rental rate of capital as the derivative of the cost of changing the capital utililsation rate with respect to capital utilisation.

4 The intermediate good production sector.

The intermediate foods production sector is introduced in this chapter. First, the environment and the production are presented. Second, the optimal input of labour and capital is obtained. Third, the demand for the intermediate good is derived such that the optimal price of the intermediate good can be obtained, which also is derived.

4.1 Environment and production.

The intermediate good (T_t), is produced by using labour (L_t) and capital ($\bar{K}_t$), as inputs. There is a continuum of intermediate producers indexed $h \in [0, 1]$. The production technology is Cobb Douglas given by

$$T_t(h) = (Z_t Z_t^{LU} L_t(h))^{1-\alpha} \bar{K}_t(h)^{\alpha} \tag{4.1}$$

Where Z_t is the trend growth, Z_t^{LU} is temporary shock to productivity of labour with expectation one, $\bar{K}_t(h)$ is capital rented by intermediate firm h at time t, $L_t(h)$ is labour bought by intermediate firm h at time t, α is parameter which denote the weight on capital in production, $0 < \alpha < 1$.

The intermediate good T is sold both domestic and abroad, denoted Q when it is sold domestic and M when it is sold abroad. The intermediate good is used as input for production of the final good. The intermediate producers set prices in own currency (PCP). Hence the import price the final good producers face, depends on the exchange rate. Figure (4.1)show the pricing procedure.

Labour is, in contrast to other variables like capital which grows over time with rate Z_t, a stationary variable. The amount the households work does not follow the trend. That means that labour fluctuate around some constant level. Since we want stationarity in relevant variables, we must detrend those variables which are nonstationary. As before, small letters represent detrended variables. Inserting for $\bar{k}(h) = \frac{\bar{K}(h)}{Z_t}$ and $t_t(h) = \frac{T_t(h)}{Z_t}$ in (4.1) and taking into account that all firms behave similar in equilibrium, we get the following stationary product function

$$t_t = (Z_t^{LU} L_t)^{1-\alpha} \bar{k}_t^{\alpha} \tag{4.2}$$

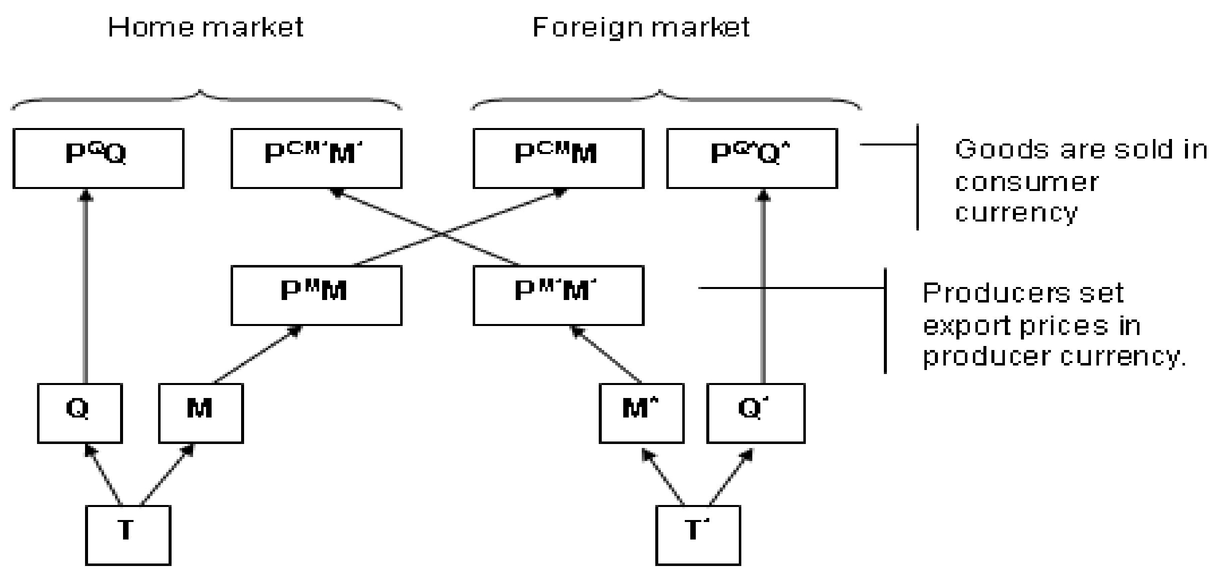

Figure 4.1: The pricing procedure. First, the intermediate good producers set P^M and P^{M^*}. Second, the "consumer price" P^{CM} and P^{CM^*}, depending on the exchange rate, is revealed.

4.2 Finding the optimal mixture of capital and labour in production.

At any level of production, the intermediate good producers minimize their cost. The intermediate producers pay labour cost and capital cost. The capital cost of the intermediate firm is simply the capital rental rate multiplied with the amount of capital the intermediate firm rents. The labour cost can be expressed as the average wage per labour unit times the amount of labour the firm buys. To make this more clear, I will give an example. Let $W_t^h(h)$ denote the average per labour unit wage of firm $h's$ labour bundle. Imagine one firm (firm 1) and two households. The firm's labour bundle at time t will be $L_t^h(1)$. A labour bundle is the firm's total labour demand. This demand include different amount of labour from each household, depending on the wage each household sets. Assume firm 1's labour bundle is 10 labour units from household 1 and 100 labour units from household 2, $L_t(h,j) = L_t(1,1) = 10$ and $L_t(1,2) = 100$. Firm 1's total labour demand will then be 110 labour units, $L_t^h(1) = 110$. Assume further that household 1 has set wage to 100 and that household 2 has set wage to 10, $W_t(1) = 10$ and $W_t(2) = 100$. The average labour unit wage of firm 1's labour bundle will be a weighted average of the wages the households set, $W_t^1(1) = \frac{10}{110}100 + \frac{100}{110}10 = \frac{2000}{110}$. The total labour costs for firm 1 will then be 2000, $W_t^1(1)L_t^1(1) = \frac{2000}{110}110 = 2000$.

The intermediate firm face the following minimization problem.

$$\min_{\{L_t(h)K_t(h)\}} \{W_t^h(h)L_t^h(h) + R_t^K \bar{K}_t(h)\}$$

$$st \qquad T_t(h) = (Z_t Z_t^{LU} L_t^h(h))^{1-\alpha} \bar{K}_t(h)^\alpha \tag{4.3}$$

This give the following Lagrangian

$$\mathcal{L} = -W_t(h)L_t(h) - R_t^K \bar{K}_t(h) - \lambda_t \Big((Z_t Z_t^{LU} L_t(h))^{1-\alpha} \bar{K}_t(h)^\alpha - T_t(h) \Big)$$

This yield following first order conditions

$$\frac{\partial \mathcal{L}}{\partial L_t(h)} = -W_t(h) - \lambda_t (1-\alpha) Z_t Z_t^{LU} (Z_t Z_t^{LU} L_t(h))^{-\alpha} \bar{K}_t(h)^\alpha) = 0 \tag{4.4}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{K}_t(h)} = -R_t^K - \lambda_t \alpha (Z_t Z_t^{LU} L_t(h))^{1-\alpha} \bar{K}_t(h)^{\alpha-1} = 0 \tag{4.5}$$

Equation (4.4) gives:

$$\begin{aligned} W_t(h) &= -\lambda_t(1-\alpha)\left(Z_t Z_t^{LU}\right)^{1-\alpha} L_t(h)^{-\alpha}\bar{K}_t(h)^{\alpha} \\ W_t(h) &= -\lambda_t(1-\alpha)\frac{T_t(h)}{L_t(h)} \end{aligned}$$

Solving for $L_t(h)$ gives:

$$L_t(h) = -\frac{\lambda_t(1-\alpha)T_t(h)}{W_t(h)} \tag{4.6}$$

Equation (4.5)gives:

$$\begin{aligned} R_t^K &= -\lambda_t\alpha(ZZ_t^{LU}L_t(h))^{1-\alpha}\bar{K}_t(h)^{\alpha-1} \\ R_t^K &= -\lambda_t\alpha\frac{T_t(h)}{\bar{K}_t(h)} \end{aligned}$$

Solving for $\bar{K}_t(h)$ gives:

$$\bar{K}_t(h) = -\frac{\lambda_t\alpha T_t(h)}{R_t^K} \tag{4.7}$$

The capital to labour ratio becomes

$$\begin{aligned} \frac{\bar{K}_t(h)}{L_t(h)} &= \frac{-\frac{\lambda_t\alpha T_t(h)}{R_t^K}}{-\frac{\lambda_t(1-\alpha)T_t(h)}{W_t(h)}} \\ &\Updownarrow \\ \frac{\bar{K}_t(h)}{L_t(h)} &= \frac{\alpha}{(1-\alpha)}\frac{W_t}{R_t^K} \end{aligned}$$

Insert $\bar{K}_t(h) = \bar{k}_t(h)Z_t$, $W_t(h) = w_tP_tZ_t$ and $R_t^K = r_t^KP_t$ to get into real detrended terms

$$\begin{aligned} \frac{\bar{k}_t(h)Z_t}{L_t(h)} &= \frac{\alpha}{(1-\alpha)}\frac{w_tP_tZ_t}{r_t^KP_t} \\ &\Updownarrow \\ \frac{\bar{k}_t(h)}{L_t(h)} &= \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t^K} \end{aligned}$$

$\bar{k}_t(h)$ is measured in per savers term. Labour is equally supplied by the savers and the spenders, and therefor in per capita terms. We need to multiply capital by the share of the savers to get in per capita terms. Taking into account that all firms behave in the same way, we can remove the individual firm notation.

$$\frac{(1-slc)\bar{k}_t}{L_t} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t^K} \tag{4.8}$$

This is the optimal capital to labour ratio. It depends on the weight on capital in production, α, and the ratio between wage and rental rate of capital, $\frac{w_t}{r_t^K}$. To no surprise, we see that an increase in wage will increase the capital to labour ratio, and an increase in the rental rate of capital will decrease the capital to labour ratio.

4.3 Deriving the marginal cost.

Plug the expression for the optimal amount of labour (4.6) and the expression for the optimal amount of capital (4.7) into the production function (4.1)

$$T_t(h) = \left(Z_t Z_t^{LU} \frac{\lambda_t(1-\alpha)T_t(h)}{W_t(h)}\right)^{1-\alpha} \left(\frac{\lambda_t \alpha T_t(h)}{R_t^K}\right)^{\alpha}$$

Rearranging gives

$$T_t(h) = \left(Z_t Z_t^{LU}\right)^{1-\alpha} \frac{\lambda_t(1-\alpha)^{1-\alpha}T_t(h)\alpha^{\alpha}}{W_t(h)^{1-\alpha}\left(R_t^K\right)^{\alpha}}$$

Dividing with $T_t(h)$ on both sides

$$1 = \left(Z_t Z_t^{LU}\right)^{1-\alpha} \frac{\lambda_t(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{W_t(h)^{1-\alpha}\left(R_t^K\right)^{\alpha}}$$

Solving for λ_t gives

$$\lambda_t = \frac{W_t(h)^{1-\alpha}\left(R_t^K\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\left(Z_t Z_t^{LU}\right)^{1-\alpha}}$$

Detrend in real terms by inserting $W_t(h) = w_t Z_t P_t$ and $R_t^K = r_t^K P_t$

$$\lambda_t = \frac{\left(w_t Z_t P_t\right)^{1-\alpha}\left(r_t^K P_t\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}\left(Z_t Z_t^{LU}\right)^{1-\alpha}}$$

Rearrange and get

$$\lambda_t = P_t\left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}$$

The interpretation of λ_t is nominal marginal cost of production. λ_t is the Lagrangian's multiplier for the production function. This means if we increase the production by one unit, the cost will raise by λ_t, hence $\lambda_t = MC_t$. Inserting for the real marginal cost, $mc_t = \frac{MC_t}{P_t} =$

$\frac{\lambda_t}{P_t}$ give the following equation

$$mc_t = \left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha} \tag{4.9}$$

This is the real detrended marginal cost function. It depends on the wage level w_t, the weights on capital and labour in production, α and $1-\alpha$, the rental rate of capital r_t^K and the productivity shock to labour.

4.4 Deriving the demand for intermediate goods.

The final good producers use a bundle of domestic and a bundle of foreign intermediate goods in the production of the final goods. Since the intermediate producers set price of the intermediate food, it need the demand fro its goods to be able to set the optimal price. Optimal in perspective of the intermediate producers. The intermediate producers sells, as mentioned earlier, both domestically and abroad. It is therefor interested in the domestic demand and the foreign demand for its products. The domestic intermediate producers need domestic demand for domestic produced intermediate goods and foreign demand for domestic produced intermediates. The domestic final good producers' bundles of domestic intermediate goods and foreign final good producers' bundles of domestic intermediate goods are taken as given and are expressed by

$$Q_t(x) = \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}} dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}}$$

$$M_t(x^*) = \left[\int_0^1 M_t(h,x^*)^{1-\frac{1}{\theta_t^{F*}}} dh\right]^{\frac{\theta_t^{F*}}{\theta_t^{F*}-1}}$$

Where $Q_t(x)$ is domestic final good producer $x's$ demand for domestic intermediate goods, $Q_t(h,x)$ is final good producer $x's$ demand for domestic intermediate goods produced by intermediate producer h. $M_t^*(x)$ is foreign final good producer $x^{*\prime}s$ demand for domestic intermediate goods and $M_t(h,x^*)$ is final good producer $x^{*\prime}s$ demand for domestic intermediate goods produced by domestic intermediate producer h. The parameter $\theta_t^H > 1$ denotes the elasticity of substitution between the differentiated domestic intermediate goods within the domestic final good producers. $\theta_t^{F^*} > 1$ denotes the elasticity of substitution between the differentiated domestic intermediate goods within the foreign final good producers. The demand facing each intermediate firm will depend both on its price and the elasticity of substitution between the differentiated intermediates. A high $\theta_t^H/\theta_t^{F^*}$ indicates that it is relatively easy to substitute between different types of the intermediate goods. The size of these parameters will thus determine the extent of competition in the intermediate sector. We let θ_t^H and θ_t^F be time varying to allow for changes in the competitive environment over time.

To find the demand facing an individual intermediate firm h, a cost minimizing approach is used. Final goods producers (x) seek to minimize the cost of intermediate goods in production subject to how intermediate goods are bundled together in production. This minimization problem can be written as;

$$\min \int_0^1 P_t^Q(h)\, Q_t(h,x)\, dh$$

$$\text{s. t. } Q_t(x) = \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\, dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}}$$

The Lagrangian for this minimization problem is given by;

$$\mathcal{L} = \int_0^1 P_t^Q(h)\, Q_t(h,x)\, dh - \lambda_t\left[\left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\, dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}} - Q_t(x)\right] \tag{4.10}$$

The two first order conditions are given by;

$\frac{\partial \mathcal{L}}{\partial Q_t(h)} = 0 \Rightarrow$

$$P_t^Q(h) - \lambda_t \frac{\theta_t^H}{\theta_t^H - 1}\left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\, dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}-1}\left(1-\frac{1}{\theta_t^H}\right) Q_t(h,x)^{-\frac{1}{\theta_t^H}} = 0$$

$$\Longleftrightarrow P_t^Q(h) = \lambda_t\left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\, dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}-1} Q_t(h,x)^{-\frac{1}{\theta_t^H}} \tag{4.11}$$

$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow$

$$\left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\, dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}} = Q_t(x) \tag{4.12}$$

We start with the first first order condition (4.11) and multiply both sides with $Q_t(h,x)$;

$$P_t^Q(h)\,Q_t(h,x) = \lambda_t \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}-1} Q_t(h,x)^{1-\frac{1}{\theta_t^H}}$$

Integrating over all intermediate firms h, on both sides gives;

$$\int_0^1 P_t^Q(h)\,Q_t(h,x)\,dh = \lambda_t \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}-1} \int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh$$

$$\Longleftrightarrow \int_0^1 P_t^Q(h)\,Q_t(h,x)\,dh = \lambda_t \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}} \tag{4.13}$$

Using the second first order condition (4.12), equation (4.13) can be written as;

$$\int_0^1 P_t^Q(h)\,Q_t(h,x)\,dh = \lambda_t Q_t(x) \tag{4.14}$$

The left hand side of (4.14) represent total cost of differentiated intermediates for final goods firm x. Since the latter part of the right hand side is the aggregate input of intermediates in firm x, λ_t must represent the unit price of that input, hence the price of the bundled good $Q_t(x)$.

Total demand for firm h's good is the integral of the individual demand for firm h's good from all final goods producers x, see equation (4.15). Total demand for differentiated intermediates in the economy is the integral of the demand for the bundled aggregate intermediate from all final goods producers x, see equation (4.16);

$$\int_0^1 Q_t(h,x)\,dx = Q_t(h) \tag{4.15}$$

$$\int_0^1 Q_t(x)\,dx = Q_t \tag{4.16}$$

Integrating (4.14) over all final goods producers x and using demand facing each intermediate firm h (4.15) and the total demand for differentiated intermediates (4.16) we get;

$$\begin{aligned} \int_0^1 P_t^Q(h) \int_0^1 Q_t(h,x)\,dhdx &= \lambda_t \int_0^1 Q_t(x)\,dx \\ \int_0^1 P_t^Q(h)\,Q_t(h)\,dh &= \lambda_t Q_t \end{aligned} \tag{4.17}$$

Now the left hand side of (4.17) represents total cost of intermediates in the economy. The latter part of the right hand side is total demand for intermediate goods. Therefore the interpretation of λ_t changes slightly. λ_t now represent the unit price P_t^Q of an aggregate intermediate good Q_t. Inserting for $\lambda_t = P_t^Q$ in the first first order condition, (4.11) we get;

$$P_t^Q(h) = P_t^Q \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}-1} Q_t(h,x)^{-\frac{1}{\theta_t^H}} \tag{4.18}$$

Multiply both sides with $Q_t(h,x)^{\frac{1}{\theta_t^H}}$, and divide by $P_t^Q(h)$;

$$Q_t(h,x)^{\frac{1}{\theta_t^H}} = \frac{P_t^Q}{P_t^Q(h)} \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{1}{\theta_t^H-1}} \tag{4.19}$$

Raising both sides to the power of θ_t^H;

$$Q_t(h,x) = \left(\frac{P_t^Q}{P_t^Q(h)}\right)^{\theta_t^H} \left[\int_0^1 Q_t(h,x)^{1-\frac{1}{\theta_t^H}}\,dh\right]^{\frac{\theta_t^H}{\theta_t^H-1}}$$

Using the second first order condition (4.12);

$$Q_t(h,x) = \left(\frac{P_t^Q}{P_t^Q(h)}\right)^{\theta_t^H} Q_t(x)$$

Integrating over all firms x gives;

$$\int_0^1 Q_t(h,x)\,dx = \left(\frac{P_t^Q(h)}{P_t^Q}\right)^{-\theta_t^H} \int_0^1 Q_t(x)\,dx \tag{4.20}$$

Inserting for the two demand aggregates, (4.15) and (4.16), into (4.20) we get the demand function for intermediates

$$Q_t(h) = \left(\frac{P_t^Q(h)}{P_t^Q} \right)^{-\theta_t^H} Q_t \tag{4.21}$$

The domestic demand for intermediate firm h's good is a function of the price of that good, $P_t^Q(h)$, the aggregate price, P_t^Q, and total domestic demand for intermediates, Q_t. Demand for firm h's good is increasing in the general price and in total demand for intermediates, and decreasing in the degree of monopolistic power in the intermediate goods market (through the elasticity of substitution).

The foreign demand for domestic products is found equivalently;

$$M_t(h) = \left(\frac{P_t^{CM}(h)}{P_t^{CM}} \right)^{-\theta_t^{F^*}} M_t \tag{4.22}$$

where $P_t^{CM} = \frac{P^M}{S_t}$, P_t^{CM} is the price the foreign final good producer pay in local currency for the domestic intermediate good. Since the intermediate producers set prices in own currency, we need to express the demand as a function of the producer currency price. To do so, substitute $P_t^{CM} = \frac{1}{S_t} P_t^M$ and $P_t^{CM}(h) = \frac{1}{S_t} P_t^M(h)$

$$M_t(h) = \left(\frac{P_t^M(h).}{P_t^M} \right)^{-\theta_t^{F^*}} M_t$$

The foreign demand for intermediate firm h's good is a function of the export price of that good, $P_t^M(h)$, the aggregate export price, P_t^M, and total foreign demand for intermediates, M_t. Demand for firm h's good is increasing in the general export price and in total demand for intermediates, and decreasing in the degree of monopolistic power in the foreign intermediate goods market (through the elasticity of substitution).

4.5 Deriving the domestic price setting equations for the producers in the intermediate goods sector.

The intermediate firm's profit function in period t is

$$\pi_t = \underbrace{(P_t^Q(h) - MC_t(h))Q_t(h)}_{\text{Profits from the domestic market.}} + \underbrace{\left(P_t^M(h) - MC_t(h)\right) M_t(h)}_{\text{Profits from the export market.}} \tag{4.23}$$

Which is simply income minus costs in domestic market and export market. $P_t^Q(h)$ is the price firm h sets for the intermediate good in the home market, $MC_t(h)$ is the marginal cost of production for firm h and $Q_t(h)$ is the quantum firm h sells in the domestic market. $P_t^M(h)$ is the price firm h sets for the intermediate good in the export market and $M_t(h)$ is the quantum firm h sells in the export market. We assume producer currency pricing, meaning that the producers set price for their export in own currency. Therefore does not the profit function depend on the exchange rate.

We have the marginal cost from e.g. (4.9), from e.g. (4.21) and e.g. (4.22) we have the domestic and the foreign demand for intermediate goods respectively. If we plug these three equations into the profit function (4.23) and maximize for all future periods we get

$$\pi = \max_{\left\{P_t^Q(h), P_t^M(h)\right\}} \sum_{t=1}^{\infty} D_{t,t+1} \left[\begin{array}{c} \left(P_t^Q(h) - MC_t(h)\right) \left(\frac{P_t^Q(h)}{P_t^Q}\right)^{-\theta_t^H} Q_t + \\ \left(P_t^M(h) - MC_t(h)\right) \left(\frac{P_t^M(h)}{P_t^M}\right)^{-\theta_t^{F^*}} M_t(h) \end{array} \right]$$

This gives following first order conditions

$$\frac{\partial \pi}{\partial P_t^Q(h)} = 0 \Longleftrightarrow$$

$$(1-\theta_t^H) \left(\frac{1}{P_t^Q}\right)^{-\theta_t^H} Q_t \left(P_t^Q(h)\right)^{-\theta_t^H} - (-\theta_t^H) MC_t(h) \left(\frac{1}{P_t^Q}\right)^{-\theta_t^H} \left(P_t^Q(h)\right)^{-\theta_t^H - 1} Q_t = 0 \quad (4.24)$$

$$\frac{\partial \pi}{\partial P_t^M(h)} = 0 \Longleftrightarrow$$

$$(1-\theta_t^{F^*}) \left(\frac{1}{P_t^M}\right)^{-\theta_t^{F^*}} M_t(h) \left(P_t^M(h)\right)^{-\theta_t^{F^*}} - (-\theta_t^{F^*}) MC_t(h) \left(\frac{1}{P_t^M}\right)^{-\theta_t^{F^*}} \left(P_t^M(h)\right)^{-\theta_t^{F^*} - 1} M_t(h) = 0 \quad (4.25)$$

Rearrange (4.24) and get

$$(1 - \theta_t^H) \left(\frac{P_t^Q(h)}{P_t^Q}\right)^{-\theta_t^H} Q_t = -\theta_t^H MC_t(h) \left(\frac{P_t^Q(h)}{P_t^Q}\right)^{-\theta_t^H} \left(P_t^Q(h)\right)^{-1} Q_t$$

Divide both sides with $(1 - \theta_t^H) \left(\frac{P_t^Q(h)}{P_t^Q}\right)^{-\theta_t^H} Q_t$ and multiply both sides with $P_t^Q(h)$ gives

$$P_t^Q(h) = \frac{\theta_t^H}{(\theta_t^H - 1)} MC_t(h) \quad (4.26)$$

Divide by P_t on both sides to get real terms. We can also get rid of the firm specific notation since all firms behave equally.

$$p_t^Q = \frac{\theta_t^H}{(\theta_t^H - 1)} mc_t \tag{4.27}$$

Which express the optimal price for firm h in the domestic market. It depends on the marginal costs and the parameter θ_t^H.

Rearrange e.g. (4.25) and get

$$(1 - \theta_t^{F^*}) \left(\frac{P_t^M(h)}{P_t^M} \right)^{-\theta_t^{F^*}} M_t(h) = -\theta_t^{F^*} MC_t(h) \left(\frac{P_t^M(h)}{P_t^M} \right)^{-\theta_t^{F^*}} \left(P_t^M(h) \right)^{-1} M_t(h)$$

Divide both sides with $(1 - \theta_t^{F^*}) \left(\frac{P_t^{M^*}(h)}{P_t^{M^*}} \right)^{-\theta_t^{F^*}} M_t(h)$ and multiply both sides with $P_t^M(h)$ gives

$$P_t^M(h) = \frac{\theta_t^{F^*}}{(\theta_t^{F^*} - 1)} MC_t(h) \tag{4.28}$$

Divide by P_t on both sides to get real terms. We can also get rid of the firm specific notation since all firms behave equally.

$$p_t^M = \frac{\theta_t^{F^*}}{(\theta_t^{F^*} - 1)} mc_t \tag{4.29}$$

Which express the optimal real price for firm h in the export market. It depends on the marginal costs and the parameter $\theta_t^{F^*}$.

Both prices p_t^Q and p_t^M are expressed as a markup over marginal cost. As the θ_t parameter in eq. (4.29) and eq.(4.27) increase the price converges towards the marginal cost. In other words, the smaller θ_t is, the biger are the markup and the price.

The price eq.'s (4.29) and (4.27) differ from the price expressions in NEMO. Brubakk, L, Husebø, T. A. , Maih, J. and Olsen, K. (2006) shows that the prices in NEMO is given by

$$p_t^Q = \frac{\theta_t^H}{(\theta_t^H - 1)} mc_t - \frac{1}{(\theta_t^H - 1)\left(1 - \Gamma_t^{PQ}\right)} \left(p_t^Q - mc_t\right) \phi^Q \left[\frac{\Pi_t^Q}{\Pi_{t-1}^Q} - 1 \right] \frac{\Pi_t^Q}{\Pi_{t-1}^Q}$$
$$+ \frac{1}{(\theta_t^H - 1)\left(1 - \Gamma_t^{PQ}\right)} E_t D_{t,t+1} \left(p_{t+1}^Q - mc_{t+1}\right) \Pi_{t+1} \frac{q_{t+1}}{q_t} dz_{t+1} \phi^Q \left[\frac{\Pi_{t+1}^Q}{\Pi_t^Q} - 1 \right] \frac{\Pi_{t+1}^Q}{\Pi_t^Q}$$

and

$$p_t^{MQ^*} \upsilon_t = \frac{\theta_t^{F^*}}{(\theta_t^{F^*} - 1)} mc_t - \frac{1}{(\theta_t^{F^*} - 1)\left(1 - \Gamma_t^{PMQ^*}\right)} \left(p_t^{MQ^*} \upsilon_t - mc_t\right) \phi^{MQ^*} \left[\frac{\Pi_t^{MQ^*}}{\Pi_{t-1}^{MQ^*}} - 1 \right] \frac{\Pi_t^{MQ^*}}{\Pi_{t-1}^{MQ^*}}$$
$$+ \frac{1}{(\theta_t^{F^*} - 1)\left(1 - \Gamma_t^{PMQ^*}\right)} E_t D_{t,t+1} \left(p_{t+1}^{MQ^*} \upsilon_t - mc_{t+1}\right) \Pi_{t+1} \frac{mq_{t+1}^*}{mq_t^*} dz_{t+1} \phi^{MQ^*} \left[\frac{\Pi_{t+1}^{MQ^*}}{\Pi_t^{MQ^*}} - 1 \right] \frac{\Pi_{t+1}^{MQ^*}}{\Pi_t^{MQ^*}}$$

due to price adjustment costs, which enters in the firms profit function, given by

$$\Gamma_t^{P^Q}(h) = \frac{\phi^Q}{2}\left[\frac{\frac{P_t^Q(h)}{P_{t-1}^Q(h)}}{\frac{P_{t-1}^Q}{P_{t-2}^Q}} - 1\right]^2 = \frac{\phi^Q}{2}\left[\frac{\Pi_t^Q(h)}{\Pi_{t-1}^Q} - 1\right]^2$$

and

$$\Gamma_t^{P^{MQ^*}}(h) = \frac{\phi^{MQ^*}}{2}\left[\frac{\frac{P_t^{MQ^*}(h)}{P_{t-1}^{MQ^*}(h)}}{\frac{P_{t-1}^{MQ^*}}{P_{t-2}^{MQ^*}}} - 1\right]^2 = \frac{\phi^{MQ^*}}{2}\left[\frac{\Pi_t^{MQ^*}(h)}{\Pi_{t-1}^{MQ^*}} - 1\right]^2$$

where the firs cost depends on firm $h's$ inflation on domestic sold goods $\Pi_t^Q(h)$ relative to the past inflation for the sector Π_{t-1}^Q, and the second cost depend on firm $h's$ inflation on exported sold goods $\Pi_t^{MQ^*}(h)$ relative to the past inflation for the sector $\Pi_{t-1}^{MQ^*}$. The parameters ϕ^Q and ϕ^{MQ^*}determines how costly it is to change the inflation rates. $p_t^{MQ^*}$ correspons to p_t^M in the flexible price model. We see when firm h set prices equal to the past inflation, the adjustment cost will disappear and the prices in NEMO and the flexible price model will be the same.

5 The final good sector.

In this chapter I will go through the final good production sector. First, the environment and the production are presented. Second, the demand for intermediate goods are derived.

5.1 Environment and production.

The final good, A_t, is produced by combining domestic produced and foreign produced intermediate goods. The following functional form is adopted

$$A_t(x) = \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}} \right]^{\frac{\mu^A}{\mu^A-1}} \tag{5.1}$$

Where the only inputs in production are domestic produced intermediate goods and foreign produced intermediate goods (imports). $A_t(x)$ denotes final good produced by final good producer x, $Q_t(x)$ denotes domestic produced intermediate goods used in production by final good producer x and $M_t^*(x)$ denotes foreign produced intermediate goods used for production by final good producer x. ν and μ^A are both parameters. The stationary version of the production function is derived in Appendix 2, but is given by this expression

$$a_t = \left[\nu^{\frac{1}{\mu^A}} q_t^{1-\frac{1}{\mu^A}} + (1-\nu) m_t^{*^{1-\frac{1}{\mu^A}}} \right]^{\frac{\mu^A}{\mu^A-1}} \tag{5.2}$$

where $a_t \equiv \frac{A_t}{P_t Z_t}$, $q_t \equiv \frac{Q_t}{P_t Z_t}$ and $m_t^* \equiv \frac{M_t^*}{P_t Z_t}$. Because of the same argument as in the other sectors, that all firms are similar in equilibrium, we can drop the x notation.

There are monopolistic competition in both the labour market and the market for intermediate goods. It is assumed to be perfect competition in the final good sector, Hence there will be no profits or dividends in this sector. We can therefore neglect the ownership of the final good producers. The final good sector can be interpreted more as a process than a full production sector.

5.2 Deriving the demand functions for intermediate goods.

The final good producer maximize profit and take the price on the intermediate goods as given. The final good producers problem will then be to minimize the cost subject to a production level

$$\min_{\{Q_t(x), M_t^*(x)\}} P_t^Q Q_t(x) + P_t^{CM^*} M_t^*(x)$$

$$s.t. \qquad A_t(x) = \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}}$$

Changing this into a maximizing problem give the following Lagrangian

$$\mathcal{L} = -P_t^Q Q_t(x) - P_t^{CM^*} M_t^*(x) - \lambda_t \left(A_t(x) - \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}} \right)$$

This gives the following first order conditions

$$\frac{\partial \mathcal{L}}{\partial Q_t(x)} = -P_t^Q + \lambda_t \left[\begin{array}{c} \left(\frac{\mu^A}{\mu^A-1}\right) \left[\begin{array}{c} \nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + \\ (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}} \end{array} \right]^{\frac{\mu^A}{\mu^A-1}-1} \\ \times \quad \nu^{\frac{1}{\mu^A}} \left(1-\frac{1}{\mu^A}\right) Q_t(x)^{-\frac{1}{\mu^A}} \end{array} \right] = 0 \tag{5.3}$$

$$\frac{\partial \mathcal{L}}{\partial M_t^*(x)} = -P_t^{CM^*} + \lambda_t \left[\begin{array}{c} \left(\frac{\mu^A}{\mu^A-1}\right) \left[\begin{array}{c} \nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} \\ +(1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}} \end{array} \right]^{\frac{\mu^A}{\mu^A-1}-1} \\ \times \quad (1-\nu)^{\frac{1}{\mu^A}} \left(1-\frac{1}{\mu^A}\right) M_t^*(x)^{-\frac{1}{\mu^A}} \end{array} \right] = 0 \tag{5.4}$$

Rearranging equations (5.3) and (5.4) give

$$P_t^Q = \lambda_t \left[\begin{array}{c} \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \\ \times \quad \nu^{\frac{1}{\mu^A}} Q_t(x)^{-\frac{1}{\mu^A}} \end{array} \right]$$

$$P_t^{CM^*} = \lambda_t \left[\begin{array}{c} \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \\ \times \quad (1-\nu)^{\frac{1}{\mu^A}} M_t^*(x)^{-\frac{1}{\mu^A}} \end{array} \right] \tag{5.5}$$

The Lagrange's multiplier tells us how much the cost will increase if we increase the production by one unit. In other words, $\lambda_t = MC_t$, where MC_t is the marginal cost. Due to the fact that there is perfect competition in the final good sector, we can replace the marginal cost with the price of the final good, $\lambda_t = MC_t = P_t$.

$$P_t^Q = P_t \left[\begin{array}{c} \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \\ \times \quad \nu^{\frac{1}{\mu^A}} Q_t(x)^{-\frac{1}{\mu^A}} \end{array} \right] \tag{5.6}$$

$$P_t^{CM^*} = P_t \left[\begin{array}{c} \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \\ \times \quad (1-\nu)^{\frac{1}{\mu^A}} M_t(x)^{-\frac{1}{\mu^A}} \end{array} \right] \tag{5.7}$$

Multiplying both sides in (5.6) by $\frac{Q_t(x)^{\frac{1}{\mu^A}}}{P_t^Q}$ and multiplying both sides in (5.7) $\frac{M_t^*(x)^{\frac{1}{\mu^A}}}{P_t^M}$ by yields

$$Q_t(x)^{\frac{1}{\mu^A}} = \frac{P_t}{P_t^Q}\nu^{\frac{1}{\mu^A}}\left[\nu^{\frac{1}{\mu^A}}Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}}M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \tag{5.8}$$

$$M_t^*(x)^{\frac{1}{\mu^A}} = \frac{P_t}{P_t^{CM^*}}(1-\nu)^{\frac{1}{\mu^A}}\left[\nu^{\frac{1}{\mu^A}}Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}}M_t^*(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \tag{5.9}$$

Raise both (5.8) and (5.9) to the power of μ^A, rearrange and insert for the production-function (5.1) and the real prices $p_t^Q = \frac{P_t^Q}{P_t}$ and $p_t^{CM^*} = \frac{P_t^{CM^*}}{P_t}$ to get

$$Q_t(x) = \left(p_t^Q\right)^{-\mu^A}\nu A_t(x)$$

$$M_t^*(x) = \left(p_t^{CM^*}\right)^{-\mu^A}(1-\nu)A_t(x)$$

Due to the fact that all firms are similar in equilibrium we can drop the x notation. To get stationary variables, insert for $Q_t = q_t Z_t P_t$ and $A_t = a_t Z_t P_t$

$$q_t = \nu\left(p_t^Q\right)^{-\mu^A} a_t \tag{5.10}$$

$$m_t^* = (1-\nu)\left(p_t^{CM^*}\right)^{-\mu^A} a_t \tag{5.11}$$

Equation (5.10) express the optimal amount of domestic produced intermediate goods in production of the final good. This depend on the ratio between the price of intermediate goods and final goods, the production function and the parameters ν and μ^A. Equation (5.11) express the optimal amount of foreign produced intermediate goods in production of the final good. This depend on the ratio between the price of domestic produced intermediate goods and the final good, the production-function and the parameters ν and μ^A. In other words, equation (5.10) and (5.11) are the demand-functions for domestic produced intermediate goods and foreign produced intermediate goods.

We assume that the foreign demand for domestic intermediate goods can be derived in the same way, thus

$$m_t = (1-\nu^*)\left(p_t^{CM}\right)^{-\mu^{A^*}} y_t^* Z_t^{DIFF}$$

where $p_t^{CM} = \frac{P_t^{CM}}{P_t^*}$ is the real price of the home country's export in foreign currency, y_t^* is the foreign country's production and Z_t^{DIFF} is an asymmetric technology shock.

6 Equilibrium conditions and the models equations.

In this chapter I will first give the last definitions which closes the model and simplifies the savers budget constraint. Second, the government is introduced. Third, the properties of the foreign country is described. Firth, the shock processes are presented and a list of the model's equations are given.

6.1 Equilibrium conditions.

In order to close the model we need some definitions. y_t denotes the real gross domestic product. Following the national account's definition of the gross domestic product we get

$$y_t = a_t + p_t^M m_t - p_t^{CM^*} m_t^* \tag{6.1}$$

where a_t is the real domestic production, $p_t^M m_t$ is the real exports and $p_t^{CM^*} m_t^*$ is real exports.

The final good can be used for private consumption, public consumption and investment. This is given by

$$a_t = c_t + (1 - slc)i_t + g_t \tag{6.2}$$

where the investment is multiplied whit the share of the savers since there are only savers who can invest. All variables are in real detrended terms.

The intermediate goods are sold domestic and foreign

$$t_t = q_t + m_t$$

The savers budget constraint is also an equation of the model. It might be useful to simplify some. By dividing the households budget constraint by the $Z_t P_t$, we get the real detrended budget constraint.

$$\begin{aligned}
\frac{P_t C_t^{sa}(j)}{Z_t P_t} + \frac{P_t I_t(j)}{Z_t P_t} + \frac{B_t(j)}{Z_t P_t} + \frac{S_t B_t^*(j)}{Z_t P_t} \quad \leq \quad & \frac{L_t(j) W_t(j)}{Z_t P_t} + \frac{R_t^K cu_t(j) K_t(j)}{Z_t P_t} - \\
& \frac{P_t \Gamma_t^{\varphi} K_t(j)}{Z_t P_t} + \frac{(1 + r_{t-1}) B_{t-1}(j)}{Z_t P_t} + \\
& \frac{\left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} S_t B_{t-1}^*(j)}{Z_t P_t} + \\
& \frac{\Phi_t(j)}{Z_t P_t} - \frac{TAX_t(j)}{Z_t P_t}
\end{aligned}$$

Insert for $w_t(j) = \frac{W_t(j)}{Z_t P_t}$ which is type $j's$ real detrended wage, $i_t(j) = \frac{I_t(j)}{Z_t}$ which is detrended real investment by type j, $b_t(j) = \frac{B_t(j)}{P_t Z_t}$ is real detrended domestic bonds hold by type j,

$b_t^*(j) = \frac{B_t^*(j)}{P_t^* Z_t}$ which is the real detrended foreign bonds hold by type j, $v_t = S_t \frac{P_t^*}{P_t}$ which is the real exchange rate, $r_r^K = \frac{R_t^K}{P_t}$ which is the real rental rate of capital, $k_t(j) = \frac{K_t(j)}{Z_t}$ which is the real detrended capital hold by type j, $\Phi_t'(j) = \frac{\Phi_t(j)}{Z_t P_t}$ which is the real detrended transfers to type j and $tax_t(j) = \frac{TAX_t(j)}{Z_t P_t}$ which is the real detrended taxation of type j. Taking into account that all individuals are identical, we can drop the type j notation. Domestic bonds will not be traded. To be able to buy bonds someone need to offer bonds, that can not be the situation when all have the same preferences. Therefore, there will not be traded domestic bonds. The budget constraint is also binding in optimum. Considering all this, we can rewrite the households budget constraint as follows

$$\begin{aligned} c_t^{sa} + i_t + \frac{S_t B_t^*}{Z_t P_t} \frac{P_t^*}{P_t^*} &= L_t w_t + r_t^K cu_t k_t - \Gamma_t^{\varphi} k_t + \\ &\frac{\left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} S_t B_{t-1}^*}{Z_t P_t} \frac{P_t^*}{P_t^*} \frac{P_{t-1}^*}{P_{t-1}^*} v_t + \\ &\Phi_t' - tax_t \end{aligned}$$

Simplify to get

$$\begin{aligned} c_t + i_t + v_t b_t^* &= L_t w_t + r_t^K cu_t k_t - \Gamma_t^{\varphi} k_t + \\ &\frac{\left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} b_{t-1}^*}{\Pi_t^*} v_t + \\ &\Phi_t' - tax_t \end{aligned}$$

The transfers the households get consists of two parts. First the households get the profits the intermediate producers run. This is reasonable since the households own the firms. Second, the households get back the real costs. When the households make the decision of the control variables they do not take into account this transfers, meaning that the transfers do not change their decision.

The intermediate firms run a profit given in real detrended terms by

$$\pi_t = \overbrace{p_t^Q Q_t + p_t^M M_t}^{\text{Income}} - \overbrace{w_t L_t - r_t^K cu_t k_t}^{\text{Costs}}$$

The real detrended cost that will be transferred back to the households is

$$\Gamma_t^{\varphi} k_t + \frac{\left(1 + r_{t-1}^*\right)}{\Pi_t^*} v_t (1 - \Gamma_{t-1}^{B^*}) b_{t-1}^*$$

This give us the real detrended transfer

$$\Phi_t' = p_t^Q Q_t + p_t^M M_t - w_t L_t - r_t^K cu_t \bar{k}_t + \Gamma_t^{\varphi} k_t + v_t(1 - \Gamma_{t-1}^{B^*}) b_{t-1}^* \tag{6.3}$$

The final good firm produce in a full competitive market, hence it will not provide any profits.

$$a_t = p_t^Q Q_t + p_t^{CM^*} M_t^* \tag{6.4}$$

Insert (6.4) into (6.3) and then into the households budget constraint. Use the fact that $g_t = tax_t$.

$$\begin{aligned} c_t + i_t + v_t b_t^* \;=\; & L_t w_t + r_t^K cu_t k_t - \Gamma_t^{\varphi} k_t + \\ & \frac{\left(1 + r_{t-1}^*\right) \Gamma_{t-1}^{B^*} b_{t-1}^*}{\Pi_t^*} v_t + \\ & a_t - p_t^{CM^*} M_t^* + p_t^M M_t - w_t L_t - r_t^K cu_t k_t + \\ & \Gamma_t^{\varphi} k_t + \frac{\left(1 + r_{t-1}^*\right)}{\Pi_t^*} v_t (1 - \Gamma_{t-1}^{B^*}) b_{t-1}^* - g_t \end{aligned}$$

Simplify by using $a_t = c_t + i_t + g$ and inserting for the foreign real interest rate$\Upsilon_{t-1}^* = \frac{\left(1+r_{t-1}^*\right)}{\Pi_t^*}$

$$v_t b_t^* = v_t \Upsilon_{t-1}^* b_{t-1}^* + p_t^M M_t - p_t^{CM^*} M_t^*$$

This states that the value of today's holdings of foreign bonds equals the last periods holdings of foreign bonds multiplied by the real interest rate plus the trade balance.

6.2 The foreign country.

It is assumed that the foreign country follow the same structure as the domestic country, except for sticky prices. Arguments for sticky prices ar given in chapter 2. For simplicity, only the equations that are needed for the home country's decisions are modelled. These equations are described in this section. The remaining foreign variables are assumed to be exogenous. They follow a stochastic process and are described under shocks.

The domestic final good producers' bundle of foreign produced intermediate is parallel to the bundles in section 3.4 and is given by

$$M_t^*(x) = \left[\int_0^1 M_t(h^*, x)^{1 - \frac{1}{\theta_t^F}} dh \right]^{\frac{\theta_t^F}{\theta_t^F - 1}}$$

where $M_t^*(x)$ is domestic final good producer $x's$ demand for foreign produced intermediate goods, $M_t(h^*, x)$ is domestic final good producer $x's$ demand for foreign intermediate goods produced by foreign intermediate good producer h^*. The parameter $\theta_t^F > 1$ denotes the elasticity of substitution between the differentiated foreign intermediate good within the domestic final good producers. By using the procedure in section 3.4, it can be shown that the foreign demand for intermediate goods produced by domestic intermediate producer h is given by

$$M_t^*(h) = \left(\frac{P_t^{CM^*}(h)}{P_t^{CM^*}}\right)^{-\theta_t^F} M_t^*$$

Where $P_t^{CM^*}$ denotes the price level in home currency of abroad produced intermediate goods which is sold on the home market, $P_t^{CM^*}(h)$ denotes the price in home currency after the intermediate firm h^* has set prices in producer currency. M_t^* denotes the total supply of abroad produced intermediate goods sold on the home market. Since the intermediate producers set prices in producer currency, we need to express the demand in producer currency. Do so by inserting for $P_t^{CM^*} = S_t P_t^{M^*}$ and $P_t^{CM^*}(h) = S_t P_t^{M^*}(h)$ and get

$$M_t^*(h) = \left(\frac{P_t^{M^*}(h)}{P_t^{M^*}}\right)^{-\theta_t^F} M_t^*$$

After achieved the domestic demand for foreign produced intermediate goods, the foreign intermediate producers can set the optimal price. They set prise by maximizing profits. The pricesetting is equivalent with the one in NEMO. I will just give the result of the maximization problem, see Brubakk, L, Husebø, T. A., Maih, J. and Olsen, K. (2006) or Norges Bank* (2006) for the derivation. The price of foreign produced intermediate goods sold domestic is given by

$$p_t^{M^*} = \frac{\theta_t^F}{(\theta_t^F - 1)} mc_t^* - \frac{1}{(\theta_t^{F^*} - 1)\left(1 - \Gamma_t^{PMQ^*}\right)} \left(p_t^{M^*} \upsilon_t - mc_t^*\right) \phi^{M^*} \left[\frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}} - 1\right] \frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}}$$
$$+ \frac{1}{(\theta_t^F - 1)\left(1 - \Gamma_t^{PM^*}\right)} E_t D_{t,t+1}^* \left(p_{t+1}^{M^*} \upsilon_t - mc_{t+1}^*\right) \Pi_{t+1} \frac{m_{t+1}^*}{m_t^*} dZ_{t+1} \phi^{M^*} \left[\frac{\Pi_{t+1}^{M^*}}{\Pi_t^{M^*}} - 1\right] \frac{\Pi_{t+1}^{M^*}}{\Pi_t^{M^*}}$$

where

$$\Gamma_t^{PM^*} = \frac{\phi^{M^*}}{2} \left[\frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}} - 1\right]^2$$

Γ_t^{PM} is the quadratic adjustment cost build on Rotemberg (1982). These kind of adjustment costs were discussed in chapter 2. The price of the foreign produced intermediate good in home currency is given by

$$p_t^{CM^*} = v_t p_t^{M^*} \tag{6.5}$$

where $v_t = S_t \frac{P_y^*}{P_t}$ is the real exchange rate.

The maximisation problem for the foreign final good producers is parallel to the problem for the domestic producers, see section 4.2. The foreign final good producers produce in a full competitive market, which means that the price of the final good equal the marginal cost of the final good. The foreign demand for domestic produced intermediate good is slightly modified. The demand is given by

$$m_t = (1 - \nu^*) \left(p_t^{CM}\right)^{-\mu^{A^*}} y_t^* Z_t^{DIFF} \tag{6.6}$$

where $p_t^{CM} = \frac{P_t^{CM}}{P_t^*}$ is the real price of the home country's export in foreign currency, y_t^* is the foreign country's production and Z_t^{DIFF} is an asymmetric technology shock whit mean one. It is opened for that the foreign demand can fluctuate due to some stochastic shock. This might be useful and very realistic, since there is common to believe that two counties can be in different cycles or in different place of a cycle.

6.3 The government.

The government balances the budget in every period. It purchases final goods financed by lump-sum taxing the households. This is given by

$$P_t G_t = TAX_t$$

where G_t is real per capita government spending. Insert $g_t = \frac{G_t}{Z_t}$ and $tax_t = \frac{TAX_t}{P_t Z_t}$ to get a stationar expression

$$g_t = tax_t$$

g_t is then real detrended per capita government spending which is financed by the real detrended lump sum tax, tax_t. The governmental spendings is stochastic and is described under shocks.

6.4 The shock process.

There is shocks in the model. Following Smets and Wouters (2003), we assume all shocks follow a first order autoregresive process (AR(1)) with an i.i.d.-normal error term around some constatn level. The shocks are:

"Preference shock":

$$Z_t^U = (1-\rho^U)Z_{ss}^U + \rho^U dZ_{t-1} + \varepsilon_t^{dZ} \tag{6.7}$$

"Technology shock":

$$dZ_t = (1-\rho^{dZ})dZ_{ss} + \rho^{dZ} dZ_{t-1} + \varepsilon_t^{dZ} \tag{6.8}$$

"Labour productivity shock":

$$Z_t^{LU} = (1-\rho^{LU})Z_{ss}^{LU} + \rho^{LU} Z_{t-1}^{LU} + \varepsilon_t^{LU} \tag{6.9}$$

"Investment shock":

$$Z_t^I = (1-\rho^I)Z_{ss}^I + \rho^I Z_{t-1}^I + \varepsilon_t^I \tag{6.10}$$

"Risk aversion shock":

$$Z_t^{B^*} = (1-\rho^{B^*})Z_{ss}^{B^*} + \rho^{B^*} Z_{t-1}^{B^*} + \varepsilon_t^{B^*} \tag{6.11}$$

"Imported markup shock":

$$\theta_t^{F^*} = (1-\rho^{\theta^{F^*}})\theta_{ss}^{F^*} + \rho^{\theta^{F^*}} \theta_{t-1}^{F^*} + \varepsilon_t^{\theta^{F^*}} \tag{6.12}$$

"Exported markup shock":

$$\theta_t^F = (1-\rho^{\theta^F})\theta_{ss}^F + \rho^{\theta^F} \theta_{t-1}^F + \varepsilon_t^{\theta F} \tag{6.13}$$

"Domestic markup shock"

$$\theta_t^H = (1-\rho^{\theta^H})\theta_{ss}^H + \rho^{\theta^H} \theta_{t-1}^H + \varepsilon_t^{\theta^H} \tag{6.14}$$

"Wage markup shock":

$$\psi_t = (1-\rho^{\psi})\psi_t + \rho^{\psi}\psi_{t-1} + \varepsilon_t^{\psi} \tag{6.15}$$

"Asymetric technology shock"

$$Z_t^{DIFF} = (1-\rho^{Z_t^{DIFF}})Z_t^{DIFF} + \rho^{Z^{DIFF}} Z_{t-1}^{DIFF} + \varepsilon_t^{Z^{DIFF}} \tag{6.16}$$

"Public spending shock"

$$g_t = (1-\rho^g)g_{ss} + \rho^g g_{t-1} + \varepsilon_t^g \tag{6.17}$$

"Foreign real interest rate"

$$\Upsilon_t^* = (1-\rho^{\Upsilon^*})\Upsilon_{ss}^* + \rho^{\Upsilon^*}\Upsilon_{t-1}^* + \varepsilon_t^{\Upsilon^*} \tag{6.18}$$

"Foreign marginal cost"

$$mc_t^* = (1 - \rho^{mc^*})mc_{ss}^* + \rho^{mc^*} mc_{t-1}^* + \varepsilon_t^{mc^*} \tag{6.19}$$

"Foreign production"

$$y_t^* = (1 - \rho^{y^*})y_{ss}^* + \rho^{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{6.20}$$

"Foreign stochastic discountrate"

$$D_{t,t+1}^* = (1 - \rho^{D^*})D_{ss}^* + \rho^{D^*} D_{t-1,t}^* + \varepsilon_t^{D^*} \tag{6.21}$$

"Foreign inflation"

$$\Pi_t = (1 - \rho^{\Pi})\Pi_{ss} + \rho^{\Pi}\Pi_{t-1} + \varepsilon_t^{\Pi} \tag{6.22}$$

Where ρ is the unit root parameter, ε is white noise and subscript ss denote the steady state value of the respective variable.

6.5 List of equations.

All in all the model consist of 47 equations and 47 endogenous variables. In adition to the16 shocks previously presented, the model exist of 31 equations and 31 endogenous variables. These equations are:

Euler

$$1 = \beta E_t \left[\Upsilon_t \frac{Z_{t+1}^U \left(c_t^{sa} - b^c \frac{1}{dZ_t} c_{t-1}^{sa} \right)}{Z_t^U \left(dZ_{t+1} c_{t+1}^{sa} - b^c c_t^{sa} \right)} \right] \qquad c_t^{sa}$$

UIP

$$\Upsilon_t = \Upsilon_t^* E_t \left[\Gamma_t^{B^*} \frac{v_{t+1}}{v_t} \right] \qquad (\Upsilon_t)$$

Wage curve

$$w_t = mrs_t^{L,C} \frac{\psi_t}{(\psi_t - 1)} \qquad (w_t)$$

Investment to capital ratio

$$\frac{1}{\Psi_t'} = E_t \left[\beta \left(\frac{Z_{t+1}^U \left(c_t^{sa} - b^c \frac{1}{dZ_t} c_{t-1}^{sa} \right)}{Z_t^U \left(dZ_{t+1} c_{t+1}^{sa} - b^c c_t^{sa} \right)} \right) \left(\begin{array}{c} \frac{1}{\Psi_{t+1}'} \left((1 - \delta) - \Psi_{t+1}' \frac{i_{t+1}}{k_{t+1}} + \Psi_{t+1} \right) - \\ \phi_{\varphi 1} (e^{\phi_{\varphi 2}(cu_{t+1} - 1)} - 1) + r_{t+1}^K cu_{t+1} \end{array} \right) \right] \qquad (i_t)$$

Optimal utilisation rate

$$r_t^K = \phi_{\varphi 1}\phi_{\varphi 2}e^{\phi_{\varphi 2}(cu_t-1)} \qquad (cu_t)$$

Spenders

$$c_t^{sp} = L_t w_t - g_t \qquad (c_t^{sp})$$

Equilibrium condition

$$v_t\left(1-slc\right)b_t^* = v_t\Upsilon^*\left(1-slc\right)b_{t-1}^* + p_t^M m_t - p_t^{CM^*}m_t^* \qquad (b_t^*)$$

Capital law of motion

$$dZ_{t+1}k_{t+1} = (1-\delta)k_t + \Psi_t k_t \qquad (k_t)$$

Rate of capital accumulation

$$\Psi_t = \frac{i_t}{k_t} - \frac{\phi_{I1}}{2}\left(\frac{i_t}{k_t} - (dZ_{ss}+\delta-1)Z_t^I\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{i_t}{k_t} - \frac{i_{t-1}}{k_{t-1}}\right)^2 \qquad (\Psi_t)$$

Cost of change in capital utilisation

$$\Gamma_t^{\varphi} = \phi_{\varphi 1}(e^{\phi_{\varphi 2}(cu_t-1)} - 1) \qquad (\Gamma_t^{\varphi})$$

Capital utilisation

$$\bar{k}_t = cu_t k_t \qquad (\bar{k}_t)$$

Financial friction

$$\Gamma_t^{B^*} = \exp\left(-\phi^{B1}v_t\frac{b_t^*}{y_t} + \log(Z_t^{B^*})\right) \qquad (\Gamma_t^{B^*})$$

mrs

$$mrs_t^{L,C} = \frac{(c_t^{sa} - b^c\frac{c_{t-1}^{sa}}{dZ_t})}{(1-b^c)}\left(L_t\right)^{\varsigma} \qquad (mrs_t^{L,C})$$

Big psi prime

$$\Psi_t' = 1 - \phi_{I1}\left(\frac{i_t}{k_t} - (dZ_{ss}+\delta-1)Z_t^I\right) - \phi_{I2}\left(\frac{i_t}{k_t} - \frac{i_{t-1}}{k_{t-1}}\right) \qquad (\Psi_t')$$

Prod. function

$$t_t = (Z_t^{LU}L_t)^{1-\alpha}\bar{k}_t{}^{\alpha} \qquad (t_t)$$

Optimal mixture of capital and labour

$$\frac{(1-slc)\bar{k}_t}{L_t} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t^K} \qquad (L_t)$$

Optimal prices

$$p_t^Q = \frac{\theta_t^H}{(\theta_t^H - 1)} mc_t \qquad (p_t^Q)$$

$$p_t^M = \frac{\theta_t^{F^*}}{(\theta_t^{F^*} - 1)} mc_t \qquad (p_t^Q)$$

Marginal cost

$$mc_t = \left(\frac{w_t}{(1-\alpha) Z_t^{LU}} \right)^{1-\alpha} \left(\frac{r_t^K}{\alpha} \right)^{\alpha} \qquad (mc_t)$$

Prod. function

$$a_t = \left[\nu^{\frac{1}{\mu^A}} q_t^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} m_t^{*\,1-\frac{1}{\mu^A}} \right]^{\frac{\mu^A}{\mu^A - 1}} \qquad (a_t)$$

Domestic demand for domestic intermediates

$$q_t = \nu \left(p_t^Q \right)^{-\mu^A} a_t \qquad (q_t)$$

Domestic demand for foreign intermediates

$$m_t^* = (1-\nu) \left(p_t^{CM^*} \right)^{-\mu^A} a_t \qquad (m_t^*)$$

Price for domestic import

$$p_t^{M^*} = \frac{\theta_t^F}{(\theta_t^F - 1)} mc_t^* - \frac{1}{(\theta_t^F - 1)\left(1 - \Gamma_t^{P^{M^*}}\right)} \left(p_t^{M^*} \upsilon_t - mc_t^* \right) \phi^{M^*} \left[\frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}} - 1 \right] \frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}} \qquad (p_t^{M^*})$$
$$+ \frac{1}{(\theta_t^F - 1)\left(1 - \Gamma_t^{P^{M^*}}\right)} E_t D_{t,t+1}^* \left(p_{t+1}^{M^*} \upsilon_t - mc_{t+1}^* \right) \Pi_{t+1} \frac{m_{t+1}^*}{m_t^*} dZ_{t+1} \phi^{M^*} \left[\frac{\Pi_{t+1}^{M^*}}{\Pi_t^{M^*}} - 1 \right] \frac{\Pi_{t+1}^{M^*}}{\Pi_t^{M^*}}$$

Adjustment cost

$$\Gamma_t^{P^{M^*}} = \frac{\phi^{M^*}}{2} \left[\frac{\Pi_t^{M^*}}{\Pi_{t-1}^{M^*}} - 1 \right]^2 \qquad (\Gamma_t^{P^{M^*}})$$

Foreign demand for domestic intermediatess

$$m_t = (1-\nu^*) \left(p_t^{CM} \right)^{-\mu^{A^*}} y_t^* Z_t^{DIFF} \qquad (m_t)$$

Gross domestic product

$$y_t = a_t + p_t^M m_t - p_t^{CM^*} m_t^* \qquad (y_t)$$

Final good

$$a_t = c_t + (1 - slc) i_t + g_t \qquad (v_t)$$

Total consumption

$$c_t = (1 - slc)c_t^{sa} + slc * c_t^{sp} \tag{c_t}$$

Split condition

$$t_t = q_t + m_t \tag{mc}$$

Consumer price and producer price

$$P_t^{M^*} v_t = P_t^{CM^*} \tag{$P_t^{M^*}$}$$

$$P_t^{CM} v_t = P_t^M \tag{P_t^{CM}}$$

7 Conclusion

There are two sources for imperfection in NEMO, nominal and real rigidities. The model presented in this thesis is a flexible price version of Norges Bank's macroeconomic model NEMO (Norwegian Economy Model). The flexible price version deviates from its origin in the absence of nominal rigidities. This gives us a model where prices and wages respond immediately to shocks. It is assumed that the authorities subsidies production in order to correct the distortion due to the monopolistic competition. But still there are imperfection caused by the stochastic markup shocks. However, in steady state is the production in optimum. If we removed the real rigidities as well, we would be left with a pure real business cycle theory model.

The flexible price model keeps sticky prices abroad. Consequently, prices are only flexible in the home country. This leads us into the second notable difference between NEMO and the flexible price model. In the flexible price model there is producer currency pricing, in contrast to local currency pricing in NEMO. This is done because monetary policy would have an effect under local currency pricing and sticky prices abroad. It is not clear whether a flexible price model of NEMO should keep sticky prices abroad or not. I believe this depends much on the substitution between home and abroad produced intermediate goods, but this should bee studied closer.

A property with the flexible price model is that money is redundant. Since the flexible price model will serve as a target for monetary policy, we do not want monetary policy to have any effect on the real economy. In such case monetary policy would be able to influence its goal. Since money do not have any effect, are all variables in the flexible price model are in real terms.

The flexible price model of NEMO which is developed in this thesis consists of a system of 47 non linear equations and 47 endogenous variables. This include 16 shock processes, where 5 shocks are due to the exogenous foreign country, 4 shocks are preference shocks, 4 are markup shocks, 2 are technology shocks and 1 is public spending shock.

Finally, the work can be extended in many ways. First step might be to find the steady state values and solve the model. One way of doing this is using the toolbox Dynare for Matlab. In the appendices I have given the parameter values, which are taken from NEMO, and some Dynare programming codes. Dynare handles both nonlinear systems and linear systems. With a view to program packages that handles only linearized systems, have I derived the linearization of most of the models equations. These derivations are given in the appendices. See Marimon, A. and Scott, A. (1999) and Judd, K. L. (1998) for different solving methods of dynamic models and further references on the subject. Second, the solution of the model opens for more interesting analyses. Impulse response analyses, output gap calculations according to Woodford's definition, Woodford, M. (2003) and real interest rate calculation according to Wicksell K. (1898), are some. An alternative to find the solution of the model derived in this thesis, is to modify the model. One suggestion is to make the foreign country endogenous.

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8 Appendices.

8.1 Appendix A: Some log-linearized equations.

The models equations are linearized in the variables log deviation from the steady state level. A variable's log deviation from steady state is denoted with a hat, $\hat{x}_t = \log\left(\frac{x_t}{x_{ss}}\right) = \log x_t - \log x_{ss}$. The linearization are done by Taylor approximation of first order, see Sydsæter (2000), if $Z = F(x, y)$ then the approximation is

$$Z \approx F(x_0, y_0) + F'_x(x_0, y_0)(x - x_0) + F'_y(x_0, y_0)(y - y_0)$$

Consider this way of express Z_t, $Z_t = Z_{ss}\frac{Z_t}{Z_{ss}} = Z_{ss}e^{\log(\frac{Z_t}{Z_{ss}})} = Z_{ss}e^{\hat{Z}_t}$. In steady state $Z_t = Z_{ss} \Rightarrow \log(\frac{Z_t}{Z_{ss}}) = \hat{Z}_t = 0$. Since we want to linearize around the steady state, is our "starting point" for the approximation $\hat{Z}_0 = 0$. We have then $Z_t = Z_{ss}e^{\hat{Z}_t}$, and we want to linearize around the steady state. Using the formula for the Taylor approximation, we get the following expression for Z_t

$$Z_t \approx \underbrace{Z_{ss}e^0}_{F(\hat{Z}_{ss})} + \underbrace{Z_{ss}e^0}_{F\prime(\hat{Z}_{ss})}\underbrace{(\hat{Z}_t - 0)}_{\left(\hat{Z}_t - \hat{Z}_{ss}\right)} = Z_{ss} + Z_{ss}\hat{Z}_t = Z_{ss}(1 + \hat{Z}_t)$$

We can use this result on the Taylor approximation formula to get a more generalized result. We have that $y \approx y_{ss}(1 + \hat{y})$ and $x \approx x_{ss}(1 + \hat{x})$. Plug this into the Taylor formula

$$Z_{ss}(1 + \hat{Z}_t) = F(x_{ss}, y_{ss}) + F'_x(x_{ss}, y_{ss})(x_{ss}(1 + \hat{x}) - x_{ss}) + F'_y(x_{ss}, y_{ss})(y_{ss}(1 + \hat{y}) - y_{ss}) \quad (8.1)$$

Rearranging gives

$$Z_{ss} + Z_{ss}\hat{Z}_t = F(x_{ss}, y_{ss}) + F'_x(x_{ss}, y_{ss})(x_{ss}\hat{x}) + F'_y(x_{ss}, y_{ss})(y_{ss}\hat{y})$$

Remember that $Z_{ss} = F(x_{ss}, y_{ss})$

$$Z_{ss}\hat{Z}_t = F'_x(x_{ss}, y_{ss})(x_{ss}\hat{x}) + F'_y(x_{ss}, y_{ss})(y_{ss}\hat{y})$$

This result is very useful when doing the log-linearization.

The intermediate goods production function is given by

$$t_t = (Z_t^{LU} L_t)^{1-\alpha} \bar{k}_t^{\alpha}$$

The Taylor approximation is then

$$t_{ss}\hat{t}_t = Z_{ss}^{LU}\hat{Z}_t^{LU}(1-\alpha)\left(Z_{ss}^{LU}L_{ss}\right)^{-\alpha}\bar{k}_{ss}^{\alpha}L_{ss}+L_{ss}\hat{L}_t(1-\alpha)\left(Z_{ss}^{LU}L_{ss}\right)^{-\alpha}\bar{k}_{ss}^{\alpha}Z_{ss}^{LU}+\bar{k}_{ss}\hat{\bar{k}}_t\alpha\left(Z_t^{LU}L_t\right)^{1-\alpha}\bar{k}^{\alpha-1}$$

Rearrange and get

$$t_{ss}\hat{t}_t = Z_{ss}^{LU}\hat{Z}_t^{LU}(1-\alpha)\frac{t_{ss}}{L_{ss}Z_{ss}^{LU}}L_{ss}+L_{ss}\hat{L}_t(1-\alpha)\frac{t_{ss}}{L_{ss}Z_{ss}^{LU}}Z_{ss}^{LU}+\bar{k}_{ss}\hat{\bar{k}}_t\alpha\frac{t_{ss}}{\bar{k}_{ss}}$$

Simplify

$$\hat{t}_t = (1-\alpha)\left(\hat{Z}_t^{LU}+\hat{L}\right)_t+\alpha\hat{\bar{k}}_t$$

The optimal mixture of capital and labour

$$\frac{\bar{k}_t}{L_t} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t^K}$$

Apply (8.1) and get

$$\bar{k}_{ss}\hat{\bar{k}}_t\frac{1}{L_{ss}}+L_{ss}\hat{L}_t\left(-\frac{\bar{k}_{ss}}{L_{ss}^2}\right) = \frac{\alpha}{(1-\alpha)}w_{ss}\hat{w}_t\frac{1}{r_{ss}^K}+\frac{\alpha}{(1-\alpha)}\left(-\frac{w_{ss}}{(r_{ss}^K)^2}\right)r_{ss}^K\hat{r}_t$$

Rearrange and get

$$\frac{\bar{k}_{ss}}{L_{ss}}\left(\hat{\bar{k}}_t-\hat{L}_t\right) = \frac{\alpha}{(1-\alpha)}\frac{w_{ss}}{r_{ss}^K}\left(\hat{w}_t-\hat{r}_t\right)$$

Simplify and get

$$\hat{\bar{k}}_t-\hat{L}_t = \hat{w}_t-\hat{r}_t$$

The marginal cost of production of the intermediate good is

$$mc_t = \left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}$$

Apply (8.1) and get

$$\begin{aligned} mc_{ss}\widehat{mc}_t \quad = \quad & w_{ss}\hat{w}_t(1-\alpha)\left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(\frac{1}{(1-\alpha)Z_t^{LU}}\right) \\ & +Z_{ss}^{LU}\hat{Z}_t(1-\alpha)\left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(-\frac{w_t}{(1-\alpha)\left(Z_t^{LU}\right)^2}\right) \\ & +r_{ss}^K\hat{r}_t^K\alpha\left(\frac{w_t}{(1-\alpha)Z_t^{LU}}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\frac{1}{\alpha} \end{aligned}$$

Rearrange and get

$$\begin{aligned} mc_{ss}\widehat{mc}_t &= w_{ss}\hat{w}_t(1-\alpha)\left(\frac{mc_{ss}}{w_{ss}}\right) \\ &\quad -Z^{LU}_{ss}\hat{Z}_t(1-\alpha)\left(\frac{mc_{ss}}{Z^{LU}_{ss}}\right) \\ &\quad +r^K_{ss}\hat{r}_t\alpha\left(\frac{mc_{ss}}{r^K_{ss}}\right) \end{aligned}$$

Simplify and get

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t - (1-\alpha)\hat{Z}_t + \alpha\hat{r}_t$$

The optimal price for the intermediate good sold in the domestic market is given by

$$p^Q_t = \frac{\theta^H_t}{(\theta^H_t - 1)}mc_t$$

Apply (8.1) and get

$$p^Q_{ss}\hat{p}^Q_t = mc_{ss}\widehat{mc}_t\frac{\theta^H_t}{(\theta^H_t - 1)}$$

Simplify and get

$$\hat{p}^Q_t = \widehat{mc}_t$$

The optimal price for the intermediate good sold in the foreign market is given by

$$p^{M^*}_t = \frac{\theta^{F^*}_t}{S_t(\theta^{F^*}_t - 1)}mc_t$$

Apply (8.1) and get

$$p^{M^*}_{ss}\hat{p}^{M^*}_t = mc_{ss}\widehat{mc}_t\frac{\theta^{F^*}_{ss}}{S_{ss}(\theta^{F^*}_{ss} - 1)} + S_{ss}\hat{S}_t\frac{\theta^{F^*}_{ss}mc_{ss}}{(\theta^{F^*}_{ss} - 1)}\left(-\frac{1}{(S_{ss})^2}\right)$$

Simplify and get

$$\hat{p}^{M^*}_t = \widehat{mc}_t - \hat{S}_t$$

The the final good production function is

$$a_t = \left[\nu^{\frac{1}{\mu^A}}{q_t}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}}{m_t}^{1-\frac{1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}}$$

Apply (8.1) and get

$$\begin{aligned} a_{ss}\hat{a}_t &= q_{ss}\hat{q}_t \left[\nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \nu^{\frac{1}{\mu^A}} {q_{ss}}^{-\frac{1}{\mu^A}} + \\ & m_{ss}\hat{m}_t \left[\nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{-\frac{1}{\mu^A}} \end{aligned}$$

Rearrange and get

$$a_{ss}\hat{a}_t = \left[\nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A-1}} \left(\hat{q}_t \nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + \hat{m}_t (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}\right)$$

Use the fact that $a_{ss}^{\frac{1}{\mu^A}} = \left[\nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}\right]^{\frac{1}{\mu^A}}$

$${a_{ss}}^{1-\frac{1}{\mu^A}} \hat{a}_t = \hat{q}_t \nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + \hat{m}_t (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}$$

Rearrange and get

$$a_{ss}^{\frac{\mu^A-1}{\mu^A}} \hat{a}_t = \hat{q}_t \nu^{\frac{1}{\mu^A}} {q_{ss}}^{1-\frac{1}{\mu^A}} + \hat{m}_t (1-\nu)^{\frac{1}{\mu^A}} {m_{ss}}^{1-\frac{1}{\mu^A}}$$

The optimal level of domestic produced intermediate goods in production of final goods is given by

$$q_t = \nu \left(p_t^q\right)^{-\mu^A} a_t$$

Apply (8.1) and get

$$q_{ss}\hat{q}_t = p_{ss}\hat{p}_t \left(-\mu^A\right) \nu \left(p_{ss}^q\right)^{-\mu^A-1} a_{ss} + a_{ss}\hat{a}_t \nu \left(p_{ss}^q\right)^{-\mu^A}$$

Rearrange and simplify

$$\hat{q}_t = -\mu^A \hat{p}_t + \hat{a}_t$$

The optimal level of foreign produced intermediate goods in production of final goods is given by

$$m_t = (1-\nu) \left(p_t^M\right)^{-\mu^A} a_t$$

Apply (8.1) and get

$$m_{ss}\hat{m}_t = p_{ss}^M \hat{p}_t (1-\nu) \left(-\mu^A\right) \left(p_{ss}^M\right)^{-\mu^A-1} a_{ss} + a_{ss}\hat{a}_t (1-\nu) \left(p_{ss}^M\right)^{-\mu^A}$$

Rearrange and simplify

$$\hat{m}_t = -\mu^A (1-\nu) \hat{p}_t + \hat{a}_t$$

The marginal rate of substitution between consumption and labour is given by

$$\frac{Z_t^u v\prime(L_t)}{Z_t^u u\prime(C_t)} = \frac{\gamma_t}{\lambda_t P_t}$$

The governments budget constraint is given by

$$g_t = tax_t$$

Apply (8.1) and simplify

$$\hat{g}_t = \widehat{tax}_t$$

The Euler equation is given by

$$c_{t+1}^{sa} - b_C \frac{c_t^{sa}}{dZ_{t+1}} = \beta E_t \frac{Z_{t+1}^u}{Z_t^u} \frac{(1+r_t)}{\Pi_{t+1} dZ_{t+1}} \left(c_t^{sa} - b_C \frac{c_{t-1}^{sa}}{dZ_t} \right)$$

Apply (8.1) and get

$$\begin{aligned} c_{ss}\hat{c}_{t+1} - b_c \frac{1}{dZ_{ss}} c_{ss}\hat{c}_t + dZ_{ss} E_t \left[\widehat{dZ}_{t+1} \right] b_c \frac{c_{ss}}{(dZ_{ss})^2} \quad = \quad & Z_{ss}^u E_t \left[\hat{Z}_{t+1}^u \right] \beta \frac{1}{Z_{ss}^u} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[c_{ss} - b_c \frac{c_{ss}}{dZ_{ss}} \right] \\ & - Z_{ss}^u \hat{Z}_t^u \beta \frac{Z_{ss}^u}{(Z_{ss}^u)^2} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[c_{ss} - b_c \frac{c_{ss}}{dZ_{ss}} \right] \\ & + (1+r_{ss}) \widehat{(1+r_t)} \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1}{\Pi_{ss} dZ_{ss}} \left[c_{ss} - b_c \frac{c_{ss}}{dZ_{ss}} \right] \\ & - \Pi_{ss} E_t \left[\hat{\Pi}_{t+1} \right] \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1+r_{ss}}{(\Pi_{ss})^2 dZ_{ss}} \left[c_{ss} - b_c \frac{c_{ss}}{dZ_{ss}} \right] \\ & - dZ_{ss} E_t \left[\widehat{dZ}_{t+1} \right] \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1+r_{ss}}{\Pi_{ss} (dZ_{ss})^2} \left[c_{ss} - b_c \frac{c_{ss}}{dZ_{ss}} \right] \\ & + c_{ss}\hat{c}_t \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \\ & - c_{ss}\hat{c}_{t-1} \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \frac{b_c}{dZ_{ss}} \\ & + dZ_{ss} \widehat{dZ}_t \beta \frac{Z_{ss}^u}{Z_{ss}^u} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} b_c \frac{c_{ss}}{(dZ_{ss})^2} \end{aligned}$$

Divide over all by c_{ss} and simplify

$$
\begin{aligned}
\hat{c}_{t+1} - b_c \frac{1}{dZ_{ss}} \hat{c}_t + E_t\left[\widehat{dZ}_{t+1}\right] b_c \frac{1}{dZ_{ss}} \quad = \quad & E_t\left[\hat{Z}^u_{t+1}\right] \beta \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - \hat{Z}^u_t \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& + (1+r_{ss}) \widehat{(1+r_t)} \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1}{\Pi_{ss} dZ_{ss}} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - E_t\left[\hat{\Pi}_{t+1}\right] \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - E_t\left[\widehat{dZ}_{t+1}\right] \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& + \hat{c}_t \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \\
& - \hat{c}_{t-1} \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} \frac{b_c}{dZ_{ss}} \\
& + \widehat{dZ}_t \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{1+r_{ss}}{\Pi_{ss} dZ_{ss}} b_c \frac{1}{dZ_{ss}}
\end{aligned} \tag{8.2}
$$

The Euler equation reduces in steady state to

$$
\begin{aligned}
c_{ss} - b_C \frac{c_{ss}}{dZ_{ss}} &= \beta \frac{Z^u_{ss}}{Z^u_{ss}} \frac{(1+r_{ss})}{\Pi dZ_{ss}} \left(c_{ss} - b_C \frac{c_{ss}}{dZ_{ss}}\right) \\
1 - \frac{b_C}{dZ_{ss}} &= \beta \frac{(1+r_{ss})}{\Pi dZ_{ss}} \left(1 - \frac{b_C}{dZ_{ss}}\right) \\
1 &= \beta \frac{(1+r_{ss})}{\Pi dZ_{ss}}
\end{aligned}
$$

Use this result in (8.2) and simplify even more

$$\begin{aligned}
\hat{c}_{t+1} - b_c \frac{1}{dZ_{ss}} \hat{c}_t + E_t\left[\widehat{dZ}_{t+1}\right] b_c \frac{1}{dZ_{ss}} \quad = \quad & E_t\left[\hat{Z}^u_{t+1}\right]\left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - \hat{Z}^u_t \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& + (1 + r_{ss}) \widehat{(1 + r_t)} \left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - E_t\left[\hat{\Pi}_{t+1}\right]\left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& - E_t\left[\widehat{dZ}_{t+1}\right]\left[1 - b_c \frac{1}{dZ_{ss}}\right] \\
& + \hat{c}_t \\
& - E_t\left[\hat{c}_{t-1}\right] \frac{b_c}{dZ_{ss}} \\
& + \widehat{dZ}_t \frac{b_c}{dZ_{ss}}
\end{aligned}$$

Rearrange

$$\begin{aligned}
\left(\frac{dZ_{ss} + b_c}{dZ_{ss}}\right) \hat{c}_t \quad = \quad & E_t\left[\hat{c}_{t+1}\right] + \frac{b_c}{dZ_{ss}} \hat{c}_{t-1} - \left(\frac{dZ_{ss} - b_c}{dZ_{ss}}\right)\left(\widehat{(1 + r_t)} - E_t\left[\hat{\Pi}_{t+1}\right]\right) + \\
& E_t\left[\widehat{dZ}_{t+1}\right] - \frac{b_c}{dZ_{ss}} \widehat{dZ}_t - \left(\frac{dZ_{ss} - b_c}{dZ_{ss}}\right)\left(E_t\left[\hat{Z}^u_{t+1}\right] - \hat{Z}^u_t\right)
\end{aligned}$$

Divide $\left(\frac{dZ_{ss}+b_c}{dZ_{ss}}\right)$ on both sides

$$\begin{aligned}
\hat{c}_t \quad = \quad & \frac{dZ_{ss}}{dZ_{ss} + b_c} E_t\left[\hat{c}_{t+1}\right] + \frac{b_c}{dZ_{ss} + b_c} \hat{c}_{t-1} - \left(\frac{dZ_{ss} - b_c}{dZ_{ss} + b_c}\right)\left(\widehat{(1 + r_t)} - E_t\left[\hat{\Pi}_{t+1}\right]\right) \\
& + \frac{dZ_{ss}}{dZ_{ss} + b_c} E_t\left[\widehat{dZ}_{t+1}\right] - \frac{b_c}{dZ_{ss} + b_c} \widehat{dZ}_t - \left(\frac{dZ_{ss} - b_c}{dZ_{ss} + b_c}\right)\left(E_t\left[\hat{Z}^u_{t+1}\right] - \hat{Z}^u_t\right)
\end{aligned}$$

The wage philips curve and the mrs.

$$w_t = mrs_t \left(\frac{\Psi}{\Psi - 1}\right)$$

Apply (8.1) and get

$$\hat{w}_t w_{ss} = \widehat{mrs}_t mrs_t \left(\frac{\Psi}{\Psi - 1}\right)$$

Simplify

$$\hat{w}_t = \widehat{mrs}_t$$

The marginal rate of substitution is given by

$$mrs_t^{L,c} = \frac{(L_t(j))^{\varsigma} \left(c_t^{sa}(j) - \frac{b^c}{dZ_t} c_{t-1}^{sa}(j)\right)}{(1 - b^c)}$$

Apply (8.1) and get

$$\begin{aligned} \widehat{mrs}_t^{L,c} mrs_{ss}^{L,c} &= \hat{L}_t(j) L_{ss}(j) \frac{\varsigma \left(L_{ss}(j)\right)^{\varsigma-1} \left(c_{ss}^{sa}(j) - \frac{b^c}{dZ_t} c_{ss}^{sa}(j)\right)}{(1 - b^c)} + \hat{c}_t^{sa} c_{ss}^{sa} \frac{(L_{ss}(j))^{\varsigma}}{(1 - b^c)} \\ &\quad + \widehat{dZ}_t dZ_{ss} \frac{(L_{ss}(j))^{\varsigma} \frac{b^c}{(dZ_{ss})^2} c_{ss}}{(1 - b^c)} - \hat{c}_{t-1} c_{ss} \frac{(L_{ss}(j))^{\varsigma} \frac{b^c}{dZ_{ss}}}{(1 - b^c)} \end{aligned}$$

Insert for mrs_{ss}

$$\begin{aligned} \widehat{mrs}_t^{L,c} \frac{(L_{ss}(j))^{\varsigma} \left(c_{ss}^{sa}(j) - \frac{b^c}{dZss} c_{ss}^{sa}(j)\right)}{(1 - b^c)} &= \hat{L}_t(j) L_{ss}(j) \frac{\varsigma \left(L_{ss}(j)\right)^{\varsigma-1} \left(c_{ss}^{sa}(j) - \frac{b^c}{dZ_t} c_{ss}^{sa}(j)\right)}{(1 - b^c)} \\ &\quad + \hat{c}_t^{sa}(j) c_{ss}^{sa}(j) \frac{(L_{ss}(j))^{\varsigma}}{(1 - b^c)} + \widehat{dZ}_t dZ_{ss} \frac{(L_{ss}(j))^{\varsigma} \frac{b^c}{(dZ_{ss})^2} c_{ss}^{sa}(j)}{(1 - b^c)} \\ &\quad - \hat{c}_{t-1}^{sa}(j) c_{ss}^{sa}(j) \frac{(L_{ss}(j))^{\varsigma} \frac{b^c}{dZ_{ss}}}{(1 - b^c)} \end{aligned}$$

Simplify

$$\begin{aligned} \widehat{mrs}_t^{L,c} \left(1 - \frac{b^c}{dZ_{ss}}\right) &= \hat{L}_t(j) \varsigma \left(1 - \frac{b^c}{dZ_t}\right) + \hat{c}_t^{sa}(j) \\ &\quad + \widehat{dZ}_t \frac{b^c}{dZ_{ss}} - \hat{c}_{t-1}^{sa}(j) \frac{b^c}{dZ_{ss}} \end{aligned}$$

Divide $(1 - \frac{b^c}{dZss})$ on both sides

$$\widehat{mrs}_t^{L,c} = \varsigma \hat{L}_t(j) + \frac{1}{dZ_{ss} - b^c} \left(dZ_{ss} \hat{c}_t^{sa}(j) - b^c \hat{c}_{t-1}^{sa}(j)\right) + \frac{b^c}{dZ_{ss} - b^c} \widehat{dZ}_t$$

8.2 Appendix B: Derivation of the detrended production function in the final good sector.

The production function is given by

$$A_t(x) = \left[\nu^{\frac{1}{\mu^A}} Q_t(x)^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t(x)^{1-\frac{1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}}$$

Since all firms are the same, we can drop the x notation. Divide $P_t Z_t$ on both sides and use the definition $a_t \equiv \frac{A_t}{P_t Z_t}$

$$a_t = \left[\nu^{\frac{1}{\mu^A}} Q_t{}^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} M_t{}^{1-\frac{1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}} \left(\frac{1}{P_t Z_t}\right)^{\frac{\mu^A}{\mu^A-1}} \left(\frac{1}{P_t Z_t}\right)^{-\frac{1}{\mu^A-1}}$$

Rearrange

$$a_t = \left[\nu^{\frac{1}{\mu^A}} \frac{Q_t}{Q_t^{\frac{1}{\mu^A}}} \left(\frac{1}{P_t Z_t}\right) + (1-\nu)^{\frac{1}{\mu^A}} \frac{M_t}{M_t^{\frac{1}{\mu^A}}} \left(\frac{1}{P_t Z_t}\right)\right]^{\frac{\mu^A}{\mu^A-1}} (P_t Z_t)^{\frac{1}{\mu^A-1}}$$

Use the definitions $q_t \equiv \frac{Q_t}{P_t Z_t}$ and $m_t \equiv \frac{M_t}{P_t Z_t}$ and multiply and divide with $(P_t Z_t)^{\frac{\mu^A}{\mu^A-1}}$ on the right hand side

$$a_t = \left[\nu^{\frac{1}{\mu^A}} \frac{q_t}{Q_t^{\frac{1}{\mu^A}}} + (1-\nu)^{\frac{1}{\mu^A}} \frac{m_t}{M_t^{\frac{1}{\mu^A}}}\right]^{\frac{\mu^A}{\mu^A-1}} (P_t Z_t)^{\frac{\mu^A}{\mu^A-1}} (P_t Z_t)^{\frac{1-\mu^A}{\mu^A-1}}$$

Rearrange

$$a_t = \left[\left(\nu^{\frac{1}{\mu^A}} \frac{q_t}{Q_t^{\frac{1}{\mu^A}}} (P_t Z_t)^{\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} \frac{m_t}{M_t^{\frac{1}{\mu^A}}} (P_t Z_t)^{\frac{1}{\mu^A}}\right) (P_t Z_t)^{\frac{\mu^A-1}{\mu^A}}\right]^{\frac{\mu^A}{\mu^A-1}} (P_t Z_t)^{\frac{1-\mu^A}{\mu^A-1}}$$

Use the definitions $q_t \equiv \frac{Q_t}{P_t Z_t}$ and $m_t \equiv \frac{M_t}{P_t Z_t}$

$$a_t = \left[\left(\nu^{\frac{1}{\mu^A}} q_t^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} m_t^{1-\frac{1}{\mu^A}}\right)\right]^{\frac{\mu^A}{\mu^A-1}} (P_t Z_t)^{\frac{\mu^A-1+1-\mu^A}{\mu^A-1}}$$

Which results into the detrended equation

$$a_t = \left[\left(\nu^{\frac{1}{\mu^A}} q_t^{1-\frac{1}{\mu^A}} + (1-\nu)^{\frac{1}{\mu^A}} m_t^{1-\frac{1}{\mu^A}} \right) \right]^{\frac{\mu^A}{\mu^A - 1}}$$

Variable	**Notation**	**Variable**	**Notation**
Consumption	c	Import	m^*
Savers consumption	c^{sa}	Price of domestic final good input	p^q
Spenders consumption	c^{sp}	Price of export in producer currency	p^m
Real interest rate	Υ	Price of export in consumer currency	p^{cm}
Foreign real interest rate	Υ^*	Price of import in producer currency	p^{m^*}
Rental rate of capital	r^K	Price of import in consumer currency	p^{cm^*}
Investment	i	Excange rate	v
Capital utilisation	cu	Final good	a
Domestic bonds	b	Tax	tax
Foreign bonds	b^*	Marginal cost	mc
Capital	k	Foreign marginal cost	mc^*
Rate of capital accumulation	Ψ	Wage markup	ψ
Cost of change in capital utilisation	Γ^{φ}	Publilc spendings	g
Financial friction	Γ^{B^*}	Asymetric technology shock	Z^{DIFF}
Effective capital	$\bar{k}$	Domestic markup	θ^H
Marginal rate of substitution	mrs	Export markup	θ^F
Big psi prime	$\Psi\prime$	Import markup	θ^{F^*}
Gross domestic product	y	Risk aversion shock	Z^{B^*}
Foreign gross domestic product	y^*	Investment shock	Z^I
Labour	L	Labour productivity shock	Z^{LU}
Wage	w	Technology growth	dZ
Intermediate good	t	Preference shock	Z^U
Domestic final good input factor	q	Labour productivity shock	Z^{LU}
Export	m		

Table 8.1: List of variables
List of varilables

8.3 Appendix C: List of variables and parameters.

Prameter	Value	Prameter	Value
α	0.33	$\rho^{\theta^{F^*}}$	0.85
β	0.9981337	ρ^{ψ}	0.73761
δ	0.015	ρ^{g}	0.966521
b^c	0.85	ρ^{Υ^*}	0.8
μ	0.5	ρ^{MC^*}	0.8
μ^*	0.5	ρ^{y^*}	0.8
ϕ_{B1}	0.59	ν	0.628834738996073
ϕ_{B2}	0.01	ν^*	0.998335573721155
$\phi_{\varphi1}$	1	Z_{ss}^{U}	1
$\phi_{\varphi2}$	$(\Upsilon^*/dZ+\delta)/\phi_{\varphi1}$	Z_{ss}^{dZ}	1.019
ϕ_{I1}	3	Z_{ss}^{I}	1
ϕ_{I2}	120	Z_{ss}^{LU}	1
slc	0	Z_{ss}^{B}	1
ζ	3	Z_{ss}^{DIFF}	0.61573354583011
ρ^{U}	0.77566	θ_{ss}^{F}	6
ρ^{dZ}	0.95	θ_{ss}^{H}	6
ρ^{I}	0.88667	$\theta_{ss}^{F^*}$	6
ρ^{LU}	0.88458	ψ_{ss}	5.5
ρ^{B}	0.94313	g_{ss}	0.3
ρ^{DIFF}	0.96464	Υ_{ss}^*	Z_{ss}^{dZ}/β
ρ^{θ^F}	0.85	MC_{ss}^*	1
ρ^{θ^H}	0.85	y_{ss}^*	1

Table 8.2: List of parameters

8.4 Appendix D: Dynare code.

//This program is written for Toolbox Dynare by Jørgen Bækken

// This is a non-linarized version of the RBC-NEMO

//——————————————————-

// 0. Housekeeping (close all graphic windows)

//——————————————————-

close all;

//——————————————————-

// 1. Defining variables

//——————————————————-

var

//————————Household————————

L, c_sa, c_sp, c, I, K, k_bar, w, B_star, UPSILON, r_K, mrs, big_psi_prime, cu, big_gamma_B_star,

v, big_psi, UPSILON_TP,

//————————Intermed. sec.————————

t, MC_NW, P_Q_NW, P_M_NW, P_CM_NW, P_M_TP, P_CM_TP, MC_TP,

//————————Final good sec.————————-

A_NW, Q_NW, M_NW, y, M_TP, Y_TP,

//————————Shocks————————-

z_U_NW, DZ_NW, z_I, z_LU, z_B, Z_DIFF, THETAF_NW, THETAF_TP, THETAH_NW, small_psi, g_NW;

varexo

e_u e_DZ_NW e_I e_LU e_B e_Z_DIFF e_THETAH_NW e_THETAF_NW e_THETAF_TP e_psi e_g_NW e_UPSILON_TP e_MC_TP e_Y_TP;

parameters

//————————Household————————-

BC_NW BETA_NW DELTA_NW PHI_VARPHI2 PHI_VARPHI1 ZETA_NW PHI_I1_NW PHI_I2 PHI_B1_NW PHI_B2_NW B_bar_H_star

slc

//————————Intermed. sec.————————

ALPHA_NW

//————————Final good sec.————————-

NU_NW MU_NW MU_TP NU_TP

```
//————————————Shocks—————————————-
rho_U_NW rho_DZ_NW rho_I_NW rho_LU_NW rho_B_NW rho_Z_DIFF_NW
rho_THETAF_NW rho_THETAH_NW rho_THETAF_TP rho_small_psi
rho_g_NW rho_UPSILON_TP rho_MC_TP rho_Y_TP
SS_U_NW SS_I_NW SS_B_NW SS_Z_DIFF_NW SS_LU_NW SS_THETAF_NW
SS_THETAH_NW SS_THETAF_TP SS_small_psi SS_DZ_NW SS_g_NW
SS_UPSILON_TP SS_MC_TP SS_Y_TP;
//————————————————————————————-
// 2. Calibration
//————————————————————————————-
ALPHA_NW=0.33;          //PARAMETERVERDIER KOPIERT INN FRA NEMO1GAP.DYN
BC_NW=0.85;
BETA_NW=0.9981337;
DELTA_NW=0.015;
MU_NW=0.5;
MU_TP=0.5;
PHI_B1_NW=0.59;
PHI_B2_NW=0.01;
PHI_VARPHI1=1;
PHI_VARPHI2=0.0369;
PHI_I1_NW=3;
PHI_I2=120;
slc=0;
ZETA_NW=3;
B_bar_H_star=8;
NU_NW=0.628834738996073;
rho_U_NW=0.77566;
rho_DZ_NW=0.95;          //0.80215;
rho_I_NW=0.88667;
rho_LU_NW=0.88458;
rho_B_NW=0.94313;
rho_Z_DIFF_NW=0.96464;
rho_THETAF_NW=0.85;          //rho_THETAF_NW=0.18174;
rho_THETAH_NW=0.85;          //rho_THETAH_NW=0.31423;
rho_THETAF_TP=0.85; //rho_THETAH_TP=0.0;
rho_small_psi=0.73761;
```

```
rho_g_NW=0.966521 ;
rho_UPSILON_TP=0.8;
rho_MC_TP=0.8;
rho_Y_TP=0.8;
NU_TP=0.998335573721155;
SS_LU_NW=1;
SS_U_NW=1;
SS_I_NW=0.1;
SS_B_NW=1;
SS_DZ_NW=1.4850;
SS_Z_DIFF_NW=0.61573354583011;
SS_THETAF_NW=6;
SS_THETAH_NW=6;
SS_THETAF_TP=6;
SS_small_psi=5.5;
SS_g_NW=0.829746309768911;
SS_UPSILON_TP=1.5;
SS_MC_TP=0.8;
SS_Y_TP=10;
//——————————————————————————-
// 3. Model
//——————————————————————————-
model;
//——————————House holds——————————
// euler
BETA_NW*UPSILON*(z_U_NW(+1)*(c_sa-BC_NW*(c_sa(-1)/DZ_NW)))/(z_U_NW*(DZ_NV
BC_NW*c_sa))=1;
// uip
UPSILON=UPSILON_TP*big_gamma_B_star*(v(+1)/v);
// wage curve
w=mrs*(small_psi/(small_psi-1));
// investment to capital ratio
(1/big_psi_prime)=BETA_NW*(z_U_NW(+1)*(c_sa-BC_NW*(c_sa(-1)/DZ_NW)))/(z_U_NW
BC_NW*c_sa))*((1/big_psi_prime)*((1-DELTA_NW)-big_psi_prime(+1)*(I(+1)/K)+big_psi(+1))-
PHI_VARPHI1*(exp(PHI_VARPHI2*(cu(+1)-1))-1)+r_K(+1)*cu(+1));
// optimal utilisation rate
```

(r_K)=PHI_VARPHI1*PHI_VARPHI2*exp(PHI_VARPHI2*(cu-1));

// spenders

c_sp=L*w-g_NW;

// capital law of motion

DZ_NW*K=(1-DELTA_NW)*K(-1)+big_psi*K(-1); // kapitalen er skrevet en periode tilbake, også i de andre ligningene.

// rate of capital accumulation

big_psi=(I/K(-1))-(PHI_I1_NW/2)*((I/K(-1))-(SS_DZ_NW-1+DELTA_NW)*z_I)^2-(PHI_I2/2)*((I/K(-1))-(I(-1)/K(-2)))^2;

//// cost of change in cu

//big_gamma_varphi=PHI_VARPHI1*(exp(PHI_VARPHI2*(cu-1))-1);

//financial friction

//big_gamma_B_star=PHI_B1_NW*((exp(PHI_B2_NW*(v*(1-slc)*B_star-B_bar_H_star*y))-1)/(exp(PHI_B2_NW*(v*(1-slc)*B_star-B_bar_H_star*y))+1))+z_B;

big_gamma_B_star=exp((-PHI_B2_NW*v*B_star/y)+log(z_B));

// mrs

mrs=L^ZETA_NW*(c_sa-(BC_NW/DZ_NW)*c_sa(-1))/(1-BC_NW);

// big_psi_prime!

big_psi_prime=1-PHI_I1_NW*((I/K(-1))-(SS_DZ_NW-1+DELTA_NW)*z_I)-PHI_I2*((I/K(-1))-(I(-1)/K(-2)));

// equilibrium condition

v*(1-slc)*B_star=v*UPSILON_TP*(1-slc)*B_star(-1)+P_M_NW*M_NW-P_CM_TP*M_TP;

// capital utilisation

k_bar=cu*K(-1);

//——————————Intermed sec.—————————

// prod.function intermed. good

t=(z_LU*L)^(1-ALPHA_NW)*k_bar^ALPHA_NW;

// optimal mixture of capital and labour

(1-slc)*(k_bar/L)=(ALPHA_NW/1-ALPHA_NW)*(w/r_K);

// optimal domestic price

P_Q_NW=(THETAH_NW/(THETAH_NW-1))*MC_NW;

// optimal export price

P_M_NW=(THETAF_TP/(THETAF_TP-1))*MC_NW;

// marginal cost

MC_NW=(w/((1-ALPHA_NW)*z_LU))^(1-ALPHA_NW)*(r_K/ALPHA_NW)^ALPHA_NW;

//——————————Final good sec.————————-

```
// prod. function final good
A_NW=(NU_NW^(1/MU_NW)*Q_NW^(1-(1/MU_NW))+((1-NU_NW)^(1/MU_NW))*M_TP
(1/MU_NW)))^(MU_NW/(MU_NW-1));
// domestic demand for domestic intermediates
Q_NW=NU_NW*((P_Q_NW)^(-MU_NW))*A_NW;
// domestic demand for foreign intermediates
M_TP=(1-NU_NW)*((P_CM_TP)^(-MU_NW))*A_NW;
//——————————Foreign sector————————
// optimal import price
P_CM_TP=(THETAF_NW/(THETAF_NW-1))*MC_TP*v;
// foreign demand for domestic intermediates
M_NW= (1-NU_TP)*((P_CM_NW)^(-MU_TP))*Y_TP*Z_diff;
//——————————Equilibrium conditions—————
y=A_NW+P_M_NW*M_NW-P_CM_TP*M_TP;
A_NW=c+(1-slc)*I+g_NW;
// total consumption
c=(1-slc)*c_sa+slc*c_sp;
// split condition for intermediate goods
t=Q_NW+M_NW;
//// inflation
//pi=P_NW/P_NW(-1);
// consumer price and producer price
P_M_TP*v=P_CM_TP;
P_CM_NW*v=P_M_NW;
//——————————shocks———————————-
// preference shock
z_U_NW=(1-rho_U_NW)*SS_U_NW+rho_U_NW*z_U_NW(-1)+e_u;
// technology growth shock
DZ_NW=(1-rho_DZ_NW)*SS_DZ_NW+rho_DZ_NW*DZ_NW(-1)+e_DZ_NW;
//// permanent technology shock
//z=rho_z*z(-1)+e_z;
// investment shock
z_I=(1-rho_I_NW)*SS_I_NW+rho_I_NW*z_I(-1)+e_I;
// labour productivity shock
z_LU=(1-rho_LU_NW)*SS_LU_NW+rho_LU_NW*z_LU(-1)+e_LU;
// risk aversion shock
```

```
z_B=(1-rho_B_NW)*SS_B_NW+rho_B_NW*z_B(-1)+e_B;
// asymetric technology shock
Z_DIFF=(1-rho_Z_DIFF_NW)*SS_Z_DIFF_NW+rho_Z_DIFF_NW*Z_DIFF(-1)+e_Z_DIFF
// imported markup shock
THETAH_NW=(1-rho_THETAH_NW)*SS_THETAH_NW+rho_THETAH_NW*THETAH_NW
1)+e_THETAH_NW;
// domestic markup shock
THETAF_NW=(1-rho_THETAF_NW)*SS_THETAF_NW+rho_THETAF_NW*THETAF_NW(-
1)+e_THETAF_NW;
// foreign markup shock
THETAF_TP=(1-rho_THETAF_TP)*SS_THETAF_TP+rho_THETAF_TP*THETAF_TP(-
1)+e_THETAF_TP;
// wage markup shock
small_psi=(1-rho_small_psi)*SS_small_psi+rho_small_psi*small_psi(-1)+e_psi;
// public spending shock
g_NW=(1-rho_g_NW)*SS_g_NW+rho_g_NW*g_NW(-1)+e_g_NW;
// foreign real interest rate
UPSILON_TP=(1-rho_UPSILON_TP)*SS_UPSILON_TP+rho_UPSILON_TP*UPSILON_TP(-
1)+e_UPSILON_TP;
// foreign marginal cost
MC_TP=(1-rho_MC_TP)*SS_MC_TP+rho_MC_TP*MC_TP(-1)+e_MC_TP;
// foreign production
Y_TP=(1-rho_Y_TP)*SS_Y_TP+rho_Y_TP*Y_TP(-1)+e_Y_TP;
end;
//——————————————————————————-
// 4. Computation
//——————————————————————————-
initval;
A_NW=-0.0246;
B_star=-0.0237;
big_gamma_B_star=0.6813;
//big_gamma_varphi=0;
big_psi=0.035;
big_psi_prime=1;
c=1.67783793285844;
c_sa=1.67783793285844;
```

```
c_sp=0;
cu=1;
I=-0.000061;
K=-0.000122;
k_bar=-0.000122;
L=0.000538;
M_NW=-0.01;
M_TP=1.36027971390027;
MC_NW=0.7379;
mrs=1.0245;
P_CM_NW=1;
P_CM_TP=1.21;
P_M_NW=0.6149;
P_M_TP=1;
P_Q_NW=0.8858;
Q_NW=-0.03;
r_K=0.0369;
t=-0.0236;
UPSILON=1.0219;
v=1.2604;
w=1.2522;
y=-0.94;
z_B=SS_B_NW;
DZ_NW=SS_DZ_NW;
Z_DIFF=SS_Z_DIFF_NW;
THETAF_NW=SS_THETAF_NW;
THETAH_NW=SS_THETAH_NW;
THETAF_TP=SS_THETAF_TP;
small_psi=SS_small_psi;
g_NW=SS_g_NW;
z_LU=SS_LU_NW;
z_U_NW=SS_U_NW;
z_I=SS_I_NW;
UPSILON_TP=SS_UPSILON_TP;
MC_TP=SS_MC_TP;
Y_TP=SS_Y_TP;
```

end;

steady; //(solve_algo=1) // computes the steady state fo the model

//check; // checks if the model has a (rational expectation) solution by the Blanchard-Kahn conditions

//

shocks;

// Estimater fra mars-estimering, se: N:\OKA\Model\Presentations\2006\NZ_Mar2006\Estimation\u

var e_DZ_NW; stderr 0.0002;//0.00076218;0.0002

var e_u; stderr 0.014016;

var e_I; stderr 0.018328;

var e_LU; stderr 0.0091;

var e_psi; stderr 0.53242;

var e_THETAF_NW; stderr 1.4636;

var e_THETAF_TP; stderr 1;

var e_THETAH_NW; stderr 0.53605;

var e_B; stderr 0.0003;

var e_U; stderr 0.0001;//0.003011845; // Gir store utslag, må sjekke hvordan den inngår

var e_g_NW; stderr 0.0043856;

var e_Z_DIFF; stderr 0.0198;

//var e_UPSILON_TP; stderr = 0.01;

//var e_MC_TP; stderr = 0.01;

//var e_Y_TP; stderr = 0.01;

end;

//——————————————————————————-

/