An almost ideal demand system analysis of non-durable consumption categories

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Preface

This thesis was written while I enjoyed a student internship in the Macroeconomics Group at the Research Department of Statistics Norway during the fall of 2011 and spring of 2012. I would like to thank all my colleagues for providing an exciting and stimulating work environment.

This thesis would not have been possible without the assistance of my two supervisors, Eilev Sandvik Jansen and Terje Skjerpen, senior researchers at Statistics Norway. I am grateful for their helpful advice and valuable comments and suggestions, and for the considerable time they have spent proofreading my thesis.

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Furthermore, I wish to thank seminar participants at Statistics Norway for their comments and useful criticism.

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Needless to say, all remaining errors remain my sole responsibility.

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1 Introduction

Estimation of systems of demand functions was at the forefront of applied economic research for large parts of the 20^{th} century. Research was centered around discovering the laws governing consumer preferences and the operations of markets (Brown and Deaton, 1972). Attention was also given to the measurement of elasticities and to the problem of specifying flexible and easily testable functional forms consistent with utility theory. For the past 50 years this literature has grown exponentially, and at this point in time it is therefore virtually impossible to provide a complete historical survey of applications of demand theory. Attention will for that reason be restricted to a few notable contributions. Interested readers are referred to the excellent surveys of consumer demand analysis by Brown and Deaton (1972) and Barten (1977).

The first empirical examination of a system of demand equations is due to Leser (1941), who estimated income and price elasticities for six consumption categories based on U.S. data. More than a decade later, Stone (1954) was the first to estimate the linear expenditure system (LES) proposed by Klein and Rubin (1947-1948), which quickly became the benchmark model for empirical demand analysis. The LES can be derived from a Stone-Geary utility function, $u_t = \sum_i \beta_i log(q_{it} - \phi_i)$, where q_{it} denotes consumption of category $i \in (1, \ldots, n)$ at time $t \in (1, \ldots, T)$ and β_i and ϕ_i are parameters, and can thus be shown to represent a *theoretically consistent* consumer demand system.[1]

Ever since, there has been a continuous flow of research examining alternative and more flexible demand system specifications. In 1965, Henri Theil proposed what has come to be known as the Rotterdam model, which approaches demand analysis in a probabilistic manner (Theil, 1965). The model is linear in parameters and allows theoretical constraints derived from utility theory to be easily imposed and tested. The Rotterdam model has later been criticized on the grounds that it does not in general satisfy *consistency of choice* (cf. e.g. Deaton and Muellbauer (1980*b*, ch. 2.1)). Consistency of choice is only ensured in the Rotterdam model when the utility function is linear logarithmic. Such utility functions are both *additive* and *homothetic*, which implies that the Rotterdam model only satisfies consistency of choice when expenditure shares are constant and elasticities of substitution between all pairs of consumption groups are equal to unity.[2] It has therefore been

[1] A demand function is said to be theoretically consistent (or *integrable*) if it is obtained as a solution to utility maximization.

[2] Preferences are additive if the marginal utility of every good is independent of the quantity consumed of all other goods. Homothetic preferences implies that doubling the

argued in the literature that the Rotterdam model should not be used in applied work if real income and relative prices are subject to more than just trivial variation (Brown and Deaton, 1972).

A decade later, Christensen, Jorgenson and Lau (1975) established one of the two current standards for applied demand analysis, the transcendental logarithmic (translog) demand system. The translog demand system can be derived by applying Roy's identity to a quadratic logarithmic indirect utility function, $log\,V = \alpha_o + \sum_i \alpha_i log\left(\frac{p_{it}}{x_t}\right) + \frac{1}{2}\sum_i \sum_j \beta_{ij} log\left(\frac{p_{it}}{x_t}\right) log\left(\frac{p_{jt}}{x_t}\right)$, where p_{it} and x_t denote the price of good i and total expenditure, respectively, and α_0, α_i and β_{ij} are parameters. This utility function provides a local second-order approximation to any utility function. Unlike linear logarithmic utility functions, quadratic logarithmic utility functions are non-additive and non-homothetic. Hence, unlike the Rotterdam model, consistency of choice is ensured in the translog demand system also under varying budget shares and non-unit elasticities of substitution between the different pairs of consumption categories. Note that the direct translog utility function can be derived from the augmented Johansen additive utility function, $u_t = \sum_i \frac{\beta_i}{\alpha_i}\left(\frac{q_{it}-\gamma_i}{\beta_i}\right)^{\alpha_i} + \frac{1}{2}\sum_i \sum_j \beta_{ij}\frac{\zeta_i\zeta_j}{\delta_i\delta_j}\left(\frac{q_{it}-\varepsilon_i}{\zeta_i}\right)^{\delta_i}\left(\frac{q_{jt}-\varepsilon_j}{\zeta_j}\right)^{\delta_j}$, by imposing $\gamma_i = 0 = \varepsilon_i$ and letting $\alpha_i \to 0$ and $\delta_i \to 0$ for all i, which means the translog demand system is in fact a special case of Johansen (1969) (cf. Barten (1977) for detailed accounts).

The other current standard for applied demand analysis is the almost ideal demand system due to Deaton and Muellbauer (1980*a*), which is the model we will adopt in this analysis (cf. Chapter 3 for the theoretical specification). Its title stems from the six properties associated with the system, which together makes it almost ideal for applied work: (i) it gives an arbitrary first-order approximation to any demand system, (ii) it satisfies the *axioms of choice* exactly (cf. e.g. Deaton and Muellbauer (1980*b*, ch. 2.1)), (iii) it aggregates perfectly over consumers, (iv) it has a functional form which is consistent with household budget data, (v) it is simple to estimate (provided the linear approximation is adopted), and (vi) it can be used to test the theoretical restrictions of *homogeneity* and *Slutsky symmetry* by means of linear restrictions on the parameters (this is further addressed and elaborated upon in Chapter 3 and Chapter 4). Readers interested in a survey of the two current standards for applied demand analysis are referred to the excellent review by Holt and Goodwin (2009).

This analysis gives a number of applications of Deaton and Muellbauer (1980*a*)'s almost ideal demand (AID) system to annual and seasonally unad-

quantity consumed also doubles utility.

justed quarterly household consumption data obtained from the Norwegian national accounts. Attention is restricted to linearized consumer demand systems satisfying Barten (1969)'s *invariance principle*.[3] Parameter estimates from the expenditure systems are utilized to generate Marshallian price and income elasticities for 10 non-durable consumption categories.[4] The analysis compares the explanatory power of the alternative expenditure system specifications by means of the *likelihood dominance criterion* for model selection proposed by Pollak and Wales (1991). The criterion shows that the dynamic linear approximate AID system incorporating habits is preferred to the other specifications (with the exception of the error-correction model, cf. Section A.4). However, the evidence indicates that the homogeneous static linear approximate AID system is the only specification that (i) does not suffer from lack of precisely estimated parameters and (ii) yields results that are interpretable and empirically plausible. Based on static long run solutions we conclude that food, beverages, tobacco, energy, vehicle running costs and public transport, mail & telecommunications are price inelastic necessity goods to Norwegian households, and that clothing & shoes, other products, other services and consumption abroad are price elastic luxuries.

We further evaluate how the linear approximate AID system incorporating habits performs out-of-sample. Despite its excellent data fit prior to the year of prediction, it fails to accurately predict a number of consumption categories only a year or two into the future. We also examine whether simpler expenditure system specifications are more suitable for forecasting. Based on the Diebold-Mariano test proposed by Diebold and Mariano (1995) we conclude that our preferred dynamic specification does not yield more accurate predictions than the dynamic linear approximate AID system incoporating habits without cross-price effects or the random walk model. The evidence also suggest that the focus of dynamic forecast analyses of non-durable consumption categories should be on obtaining accurate price predictions, as prices account for most of the variation in the commodities' expenditure shares.

The rest of the analysis is organized as follows. Chapter 2 gives a description of the data and briefly examines the consumption categories. Chapter 3 discusses the theoretical specification of the non-linear and linear AID system, coupled with a discussion of the theoretical shortcomings of the linearized system. Although emphasis is put on the static model, an alternative dynamic specification incorporating habits is also presented. Chapter 4

[3] Systems satisfying Barten (1969)'s invariance principle are invariant to the omitted equation (cf. Section 4.2 for detailed accounts).

[4] The estimation is performed with the statistical software PcGive 13.0.

presents a number of applications of the linearized AID system to annual and quarterly household consumption data. The chapter gives a detailed treatment of the theoretical constraints derived from utility theory. The chapter further provides diagnostic tests of the residuals and tests of whether the expenditure shares are best viewed as being I(1) or I(0). Income and price elasticities for the alternative static and dynamic specifications are also given. Chapter 5 turns attention to forecasting and out-of-sample performance of our models. Statistical tests are adopted to examine whether certain dynamic specifications give more accurate predictions than others. Chapter 6 concludes and summarizes the analysis. Further details, calculations, regressions and test results are reserved for the appendix.

2 Data

The empirical work is applied to annual and seasonally unadjusted quarterly household consumption data from the Norwegian national accounts for the period 1978-2010 and 1978Q1-2011Q3, respectively. All numbers are net of foreigners' consumption in Norway, which is classified as exports in the national accounts.

Household expenditure can be disaggregated into a number of consumption categories. Every year, Statistics Norway gather data on private travel expenses, housing expenditure, purchase of furniture, etc. They also gather data on considerably more disaggregated groups such as purchase of flowers, lawnmowers, washing powder, refrigerators and membership fees in various organizations. The following analysis will be applied to annual and quarterly data sets containing selections of *non-durable* consumption groups. Estimation results based on the benchmark data set presented in Section 2.1 and Section 2.2 are given in Chapter 4. Findings based on alternative data sets where certain commodities are included and excluded from the expenditure system are briefly summarized in the next section.

2.1 Non-durable Consumption Categories

There are no definite ways to distinguish durable commodities from non-durables.[5] In order to distinguish the two, it will be assumed in this analysis that the utility function satisfies *weak separability* between total consumption of durable and non-durable goods. Weak separability implies that the marginal rate of substitution between any two goods X and Y in a given subset is independent of the value of other goods not included in the subset (Strotz, 1959). Maximization of such a weakly separable utility function will then generate demand functions for the two classes of commodities in which the relative price between durable and non-durable goods is included. However, this relative price turned out to have insignificant effect on all consumption categories listed in Table 2.1, which indicates that the scope for substitution between durable and non-durable commodities is likely to be limited.[6] Due to these findings I will omit durable commodities from the

[5] For analyses of durable goods, cf. e.g. Stone and Rowe (1957)'s stock-adjustment model and the numerous error-correction models (ECM)s that build on their framework. Magnussen (1990) and Magnussen and Skjerpen (1992) give applications to Norwegian data. Unlike non-durable commodities, it is the capital stock rather than the purchase (or, more precisely, the consumption) of a durable good that generates utility to the agent. Purchase of durable goods are therefore often treated as investments in the literature.

[6] Note that a few durable commodities have non-zero effects on the consumption of certain non-durables: The significant effects are: (i) housing expenditure on vehicle run-

expenditure systems in this analysis. Although this can induce *omitted variables bias*, I still find it appropriate because it is likely to improve estimation precision by reducing the number of responses that must be estimated. This is further addressed and elaborated upon in Chapter 4 and Chapter 5, where we turn attention to estimation and forecasting.

Table 2.1 lists the 10 consumption categories in our benchmark data set. Considering that our objective is to estimate an expenditure system for non-durable commodities, I do agree that some of my included groups are questionable. I decided to include both photographic & IT-equipment and telecommunications equipment as part of other products. Although both categories include goods with average life-expectancies of several years, they are still reasonably distinguishable from "pure" durable goods such as vehicles and furniture. The same logic applies to clothing & shoes and explains why also that category has been included in our expenditure system.[7] Finally, I also agree that the inclusion of education as part of other services is questionable as the group might very well be characterized as an investment. Note, however, that unlike a number of Western countries where education is largely a private expense it is almost exclusively paid for by the public sector in Norway. One could of course argue that foregone wages should be added to this group as it represents an indirect cost of educational investment, suggesting that the *true* cost of education is considerably higher than the reported one. However, such an approach would raise at least two problems: (i) opportunity costs differ to a certain extent between individuals, and is therefore almost impossible to measure accurately, and (ii) adding opportunity costs to educational investment means we also have to add similar costs to leisure time activities. I would therefore argue that education, the way it is measured in the national accounts, lacks some of the features necessary for being classified as an investment in Norway, suggesting that it should be part of our non-durable expenditure system. This is also supported by empirical findings, which show that the results are robust to the exclusion of

ning costs, with an estimated cross-price elasticity of 1.31, and (ii) purchase of vehicles and purchase of furniture on consumption abroad with estimated cross-price elasticities of 1.64 and −1.66, respectively (elasticities are calculated according to the formula given in equation (4.17) below). The results are based on a static linearized AID system applied to annual Norwegian national accounts data for the years 1978 to 2010. Detailed accounts are given in Chapter 3 and Chapter 4.

[7] Clothing is characterized as a *semidurable* by Stone and Rowe (1957) (their analysis do not distinguish between clothing and shoes). This suggests including a third group to our analysis. However, as our objective is to estimate an expenditure system for non-durable commodities I have chosen not to include this third consumption category. Then again, analyses that seek to model *both* durable and non-durable commodities should include this third distinction.

Table 2.1: Consumption Categories

Food (C00)[a]	
Beverages (C03)	Alcoholic Beverages[b] Non-Alcoholic Beverages
Tobacco (C04)	Tobacco Narcotics
Clothing & Shoes (C21)[c]	
Energy (C12+C13)	Electricity Fuels & District Heating Coal, Coke, Peat & Wood
Vehicle Running Costs (C14)	Gas & Motor Oil Spare Parts
Other Products (C20)	Books, Flowers, Magazines, Movies, Newspapers & Toys Leisure Time Equipment Photographic & IT-Equipment Telecommunications Equipment Other Personal Commodities[d]
Other Services (C60)	Clothing & Shoe Repair Services Hotel & Restaurant Services Insurance Leisure Time Services Other Services Related to Transportation Repair of Household Appliances & Paid Domestic Work Education Prostitution Other Personal Services[e]
Public Transport, Mail & Telecommunications (C61)	
Consumption Abroad (C66)	

[a] Numbers in parentheses refer to the categorization of goods and services used in KVARTS and MODAG. KVARTS and MODAG are macroeconomic models developed by Statistics Norway for forecasting and policy analyses (cf. Haakonsen and Jørgensen (2007) and Skjerpen and Kolsrud (2008) for detailed accounts).
[b] This includes beer, wine and spirits.
[c] The category also includes yarn and textiles.
[d] Some examples include cosmetics, electric shavers, hair driers and jewelry.
[e] A couple of examples include beauty care and hair cutting.

education.

Neither the benchmark nor the alternative data sets include the following three consumption categories: housing expenditure, health care costs and consumption by non-profit organizations. Housing expenditure is calculated as a fixed share of the housing capital in KVARTS and MODAG (cf. Table 2.1, table note a). The group is therefore excluded from the analysis as we are only interested in non-durable consumption categories. The second group, health care costs, is almost exclusively paid for by the public sector in Norway because medical expenses exceeding a certain limit determined by the government are refundable. Health care expenses are consequently exogenous in KVARTS and MODAG (Jansen, 2009), and can therefore be excluded from the expenditure systems. Finally, the third group, consumption by non-profit organizations, is excluded as we are primarily interested in household consumption.

Financial and legal services (FISIM) was recently taken out of other services in KVARTS and MODAG following the 2011 major revision of the Norwegian national accounts. Including it in the analysis yields implausible elasticity estimates such as strictly *positive* own-price elasticities. Even though this means the elasticities are not robust to the exclusion of FISIM, I have nevertheless chosen not to include the category in the expenditure systems so as to obtain plausible elasticity estimates.

2.2 Budget Shares

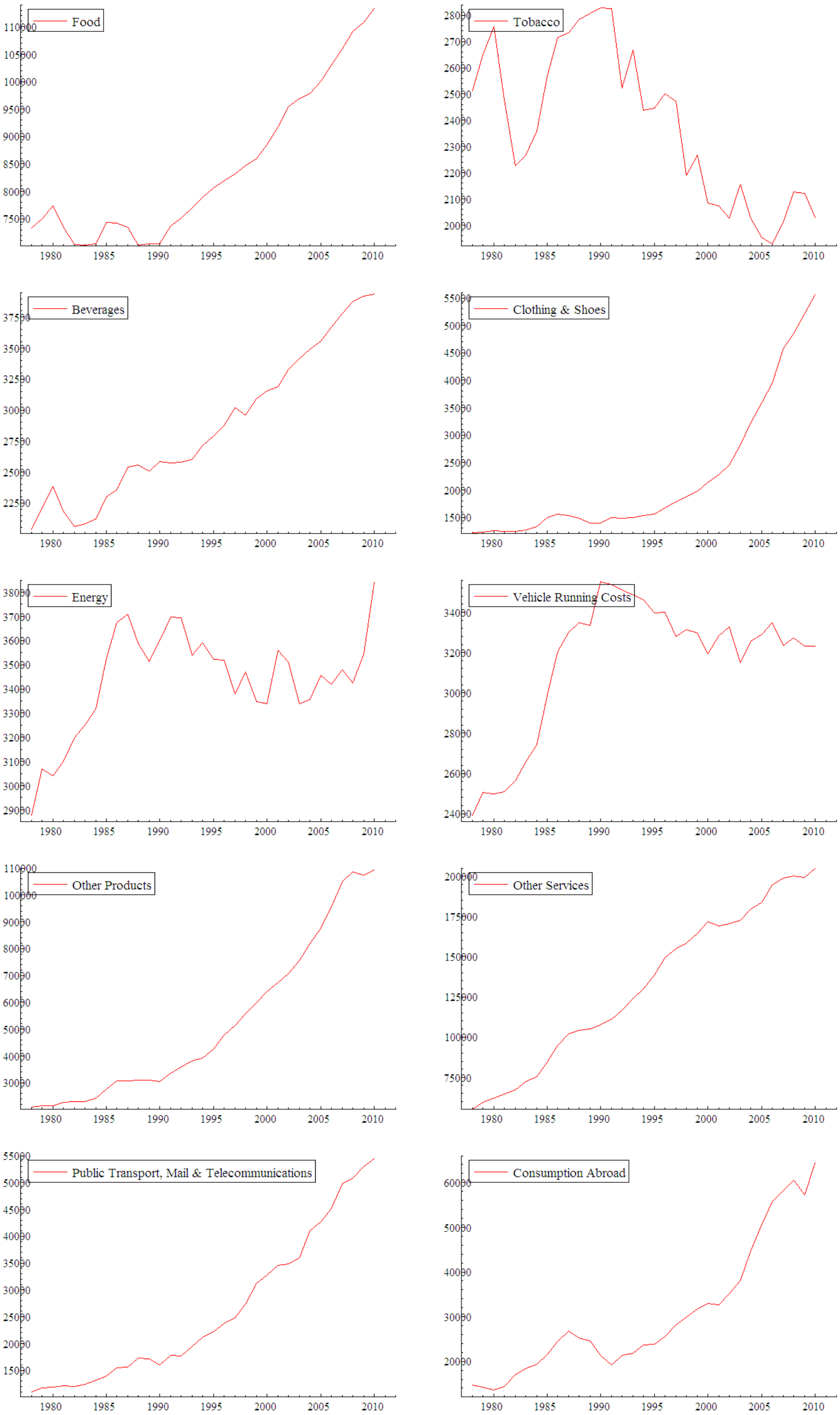
Before we move on to the theoretical specification of the AID system, let us briefly examine our 10 consumption categories' time series (further details are given in Section A.1). Figure 2.1 displays their annual budget shares for the years $1978 - 2010$.[8] Note that a downward-sloping budget share is not equivalent to declining demand for the commodity in question. As is evident from Figure 2.2, demand for all categories but tobacco is considerably higher now than it was three decades ago (y-axes are measured in constant 2009 MNOK). On average, demand for our 10 groups increased by a factor of 2.57 between 1978 and 2010, suggesting an approximate 2.9 percent annual growth in consumption of non-durables.

Food's annual budget share has declined steadily from around 25.6 to 15.1 percent over the sample period. According to *Engel's law*, a commodity whose *income* (*expenditure*) elasticity lies in the $(0, 1)$ interval will experi-

[8]Budget shares are defined by $w_{it} = \frac{q_{it}p_{it}}{c(u,p)}$, where q_{it} and p_{it} denote demand and price for good i at time t, respectively, and $c(\cdot)$ denotes total (non-durable) expenditure (detailed accounts are given in Chapter 3).

Figure 2.1: Annual Budget Shares

Figure 2.2: Annual Consumption



ence a downward-sloping expenditure share as income rises. Goods whose income elasticity exceeds unity, on the other hand, will enjoy increasing budget shares. Commodities whose income elasticity lies in the $(0, 1)$ and above-unit interval are known as *necessity goods* and *luxury goods*, respectively. This means that demand for necessities and luxuries increase less and more than proportional with income, respectively, which cause the former's budget share to decline and the latter's to increase as income rises. Food's downward-sloping expenditure share over the years 1978 to 2010 therefore leads us to suggest that the commodity must be a necessity, a hypothesis that will also be supported by the empirical analysis in Chapter 4.

Although tobacco has occupied a fairly stable share of non-durable expenditures for the last three decades, merely varying by 0.67 percentage points over the entire sample, consumption has still displayed clear downward-sloping tendencies. As is evident from Figure 2.2, it is the only commodity whose demand is now lower than it was in the late 1970s. Much of this reduction can be attributable to the many laws that were passed by the Storting between 1993 and 2004 designed to limit smoking in bars, cafes, hotels and restaurants. These laws determined what portion of restaurant tables, hotel rooms, etc. had to be reserved for non-smokers, eventually culminating with a total ban on smoking in these areas on 1 June 2004.[9] The considerable drop in tobacco consumption from 1980 to 1982, followed by the subsequent surge in the mid-1980s, was largely brought about by commodity tax increases which significantly raised the price of tobacco and strong income growth which radically improved the households' purchasing power, respectively. Note lastly that narcotics was recently added to this group following the 2011 major revision of the Norwegian national accounts (data have been added from 2003).

Like food, beverages and clothing & shoes have also experienced steadily declining budget shares since the late 1970s. Such downward-sloping expenditure shares would normally lead us to suggest that the goods in question must be necessities. However, my estimation results show that only the former has an expenditure elasticity in the (0,1) interval, suggesting that clothing & shoes should be classified as luxuries. Close examination of beverages consumption shows an abrupt drop in the early 1980s. In the fall of 1982, Norway experienced one of its longest post World War II strikes as the State wine and liquor monopoly went on a 14 week-long strike. Demand for beverages consequently plummeted, reaching a 4-year low of 20.6 BnNOK in 1982. However, consumption of beverages has surged ever since, a trend that Statistics Norway predicts will persist for some time.

[9] Cf. "Lov om vern mot tobakksskader (tobakksskadeloven) § 12".

Energy is the most *price inelastic* category in our expenditure system. Goods whose elasticity of demand lies in the $(-1, 0)$ and $(\leftarrow, -1)$ interval are classified as price inelastic and elastic commodities, respectively. The category's low price elasticity of demand indicates that households' energy consumption is considerably insensitive to energy prices. Evidence of this can be found in Figure 2.2 which shows that energy consumption only increased by a factor of 1.33 between 1978 and 2010. However, if one deliberately disregards 2010 from the analysis the number shrinks to 1.23. The Norwegian Meteorological Institute reports that December 2010 was the fourth coldest December registered since 1900, about $4.7\,°C$ below average. In addition, the winter $2009 - 2010$ was the seventh coldest winter registered in Norway since 1957, marking a 24 year low. Together, these atypical events thus explain why energy consumption suddenly soared in 2010.

Vehicle running costs exemplifies another significantly price inelastic good, a characteristic it derives from its key component, gas. This was recently demonstrated by the Institute of Transport Economics, who showed that the elasticity of vehicle use with respect to the price of gas is only -0.16 and -0.33 in the short and long run, respectively (Bekken and Fearnley, 2005). Based on its downward-sloping budget share in Figure 2.1 we further have reasons to suspect vehicle running costs to be income inelastic, a hypothesis that will also be supported by the empirical analysis in Chapter 4.

Data from the national accounts show that demand for other products and other services have grown annually by an average of 5.16 and 4.05 percent since 1978, respectively, well above the 2.9 percent average for the 10 consumption categories. Together the two constituted more than 40 percent of non-durable expenditures in 2010. Although the former's share has dropped recently, both still account for sizeable portions of overall household expenses. The surging budget shares suggest above-unit income elasticities, which will also be supported by the empirical analysis. Note lastly that prostitution was recently added to other services following the 2011 major revision of the Norwegian national accounts (data have been added from 2003).

Public transport, mail & telecommunications have for the last three decades occupied a fairly stable share of Norwegian households' non-durable expenditures, merely varying by 1.63 percentage points over the entire sample. But the category has still experienced a considerable rise in demand, with consumption in 2010 exceeding that of 1978 by as much as 43.5 BnNOK (or 497.36 percent), making it the second fastest growing consumption group in our expenditure system after other services. This surge in demand is largely accounted for by mail & telecommunications, which have improved by nearly a factor of 20 since the late 1970s.[10] Whereas e.g. mobile phones were something out of the ordinary a couple of decades ago, today they are devices each and every one of us carry around in our pockets. Note that there are some connections between public transport, mail & telecommunications, on the one hand, and vehicle running costs, on the other. Recall from our discussion earlier that gas constitutes the key component of the latter group. However, the Institute of Transport Economics shows that gas also influences consumption of the former category, with an estimated cross-price elasticity of demand of roughly 0.2 (Stortingsmelding, 2002). This implies that a 10 percent increase in the price of gas is likely to not only reduce vehicle use by $1.6 - 3.3$ percent, but also to boost demand for public transport by about 2 percent.

Our final category, consumption abroad, was with its 4.59 percent annual growth in demand between 1978 and 2010 the fourth fastest growing consumption group in our expenditure system. Its budget share has also risen markedly and now accounts for nearly a tenth of non-durable household expenses. This analysis demonstrates that consumption abroad is by far the most income elastic group in our expenditure system. Hence, as Norwegians get richer, an increasing share of their income is siphoned away to other countries. It should lastly be stressed that demand for this category is significantly affected by the strength of the Norwegian krone and the economic conditions both at home and abroad. The fact that it is influenced by economic conditions at home can for instance be seen from its reasonably pro-cyclical behavior. It is no coincidence that consumption abroad fell by roughly 7.6 BnNOK between 1987 and 1991. This episode coincided with the Norwegian Banking Crisis, itself brought about by the deregulations of financial markets that took place in $1984 - 1985$, which culminated with unemployment levels suddenly soaring from 2 to 6 percent. Demand also weakened after the collapse of Lehman Brothers 15 September 2008, which marked the start of the recent global financial crisis, but has risen continuously since 2009.

Now that we have presented our selection of non-durable consumption categories, let us move on to the theoretical specification of our econometric model, the almost ideal demand system.

[10] Numbers are based on pre-2011 major revision data.

3 Almost Ideal Demand System

Deaton and Muellbauer (1980*a*)'s AID system is based on a particular class of preferences known as the price-independent generalized logarithmic (PIGLOG) class. These preferences (i) allow exact aggregation across households (Muellbauer, 1975), and (ii) permit the representation of market demand to be the outcome of decisions by a rational agent (Muellbauer, 1976).[11] The PIGLOG cost function can be defined as:

$$log\, c(u,p) = (1-u) log\{a(p)\} + u\, log\{b(p)\} \tag{3.1}$$

where u is utility and p is a price vector. The expenditure function (3.1) thus gives a weighted average of $log\{a(p)\}$ and $log\{b(p)\}$, where the weights are given by $1-u$ and u, respectively.[12] Let us restrict attention to cases where $u \in (0,1)$ and both $a(p)$ and $b(p)$ are concave, which can be shown to be sufficient conditions for concavity of $c(\cdot)$ (Deaton and Muellbauer, 1980*a*). (3.1) then immediately yields an interesting conclusion: total cost varies from $a(p)$ to $b(p)$. $u = 0$ can thus be interpreted as subsistence expenditure, whereas $u = 1$ gives rise to the opposite extreme. In other words, $a(p)$ and $b(p)$ can be thought to represent poverty and affluence expenditure, respectively. In the case of the AID system, $log\{a(p)\}$ and $log\{b(p)\}$ can further be expressed as:

$$log\,\{a(p)\} = \alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} log\, p_k log\, p_j \tag{3.2}$$

and

$$log\,\{b(p)\} = log\,\{a(p)\} + \beta_0 \prod_k p_k^{\beta_k} \tag{3.3}$$

where α_k, β_k and γ^*_{kj} are parameters, p_j's are prices and $k, j \in (1, \ldots, n)$ indicate commodity number.[13] Substitution of (3.2) and (3.3) into (3.1) yields the expenditure function of the AID system:

[11] Note that the AID system only possesses the quality of exact aggregation across households when aggregate income is distributed equally among households and the distribution of real income remains fixed over time. Failure to account for distributional changes in real income will generally bias the estimation results (Muellbauer, 1975). However, most practitioners take the aggregation property of the AID system for granted, an approach that will also be followed in this analysis.

[12] *log* will always refer to natural logarithm in this analysis.

[13] Time-subscript t is omitted from all static expressions for notational simplicity.

$$
\begin{aligned}
log\, c(u,p) &= (1-u)\Big\{\alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj} log\, p_k log\, p_j\Big\} \\
&\quad + u\Big\{\alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj} log\, p_k log\, p_j + \beta_0 \prod_k p_k^{\beta_k}\Big\} \\
&= \alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj} log\, p_k log\, p_j + u\beta_0 \prod_k p_k^{\beta_k}
\end{aligned}
\tag{3.4}
$$

According to Shephard's Lemma, we can obtain the Hicksian (or compensated) demand for good $i \in (1, \ldots, n)$, q_i, by taking the partial derivative of the expenditure function with respect to p_i. This implies that we can obtain commodity i's budget share, w_i, by carrying out the following logarithmic differentiation:

$$
\frac{\partial log\, c(u,p)}{\partial log\, p_i} = \frac{\partial c(u,p)}{\partial p_i}\frac{p_i}{c(u,p)} = \frac{q_i p_i}{c(u,p)} = w_i
$$

where category i's budget share is defined by the last equality. Hence, differentiation of (3.4) with respect to $log\, p_i$ yields:

$$
\begin{aligned}
w_i &= \alpha_i + \frac{1}{2}\sum_j \gamma^*_{ij} log\, p_j + \frac{1}{2}\sum_j \gamma^*_{ji} log\, p_j + \beta_i u \beta_0 \prod_k p_k^{\beta_k} \\
&\equiv \alpha_i + \sum_j \gamma_{ij} log\, p_j + \beta_i u \beta_0 \prod_k p_k^{\beta_k}
\end{aligned}
\tag{3.5}
$$

where $\gamma_{ij} \equiv 1/2(\gamma^*_{ij} + \gamma^*_{ji})$.

Econometrically, (3.5) poses a number of challenges. In addition to being non-linear, it also contains the utility level, u, a variable which is inherently unmeasurable. However, by utilizing the indirect utility function we can rewrite (3.5) in terms of prices and total expenditure, x. We have from (3.4) that total cost is a function of utility and prices, i.e., $x = c(u,p)$. Microeconomic theory then tells us that we can obtain the indirect utility function by inverting $c(\cdot)$, i.e., $x = c(u,p) \rightarrow u = \varphi(x,p)$. From (3.4) we have that $log\, x = log\, c(u,p)$, which yields:

$$
\begin{aligned}
log\, x &= \alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_k\sum_j \gamma^*_{kj} log\, p_k log\, p_j + u\beta_0 \prod_k p_k^{\beta_k} \\
&\equiv logP + u\beta_0 \prod_k p_k^{\beta_k}
\end{aligned}
\tag{3.6}
$$

where P is a non-linear *translog price index* defined implicitly by:

$$logP = \alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\sum_j \sum_k \gamma_{kj} log\, p_k log\, p_j \tag{3.7}$$

and where γ_{kj} is defined as in (3.5). Next, by first solving (3.6) for $log\{x/P\}$, followed by substitution of the derived expression into (3.5), we finally obtain the (non-linear) AID system's demand function on budget share form:[14]

$$w_i = \alpha_i + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/P\} \tag{3.8}$$

3.1 Linear Approximate Almost Ideal Demand System

In his analysis of consumers' expenditure and behavior in the United Kingdom, Sir Richard Stone (1953) proposed a geometric price index that has been employed extensively in applied work for the better part of a century:

$$P^* = \prod_k p_k^{w_k}$$

By taking logarithms on both sides of the equation we obtain what will be referred to as *Stone's price index* for the remainder of the analysis:

$$logP^* = \sum_k w_k log\, p_k \tag{3.9}$$

The price index weights prices by the commodities' respective budget shares. When prices are highly collinear, P and P^* will often be approximately proportional, i.e., $P \cong \zeta P^*$, where ζ is the degree of proportionality.[15] By replacing P with ζP^* in (3.8) we obtain an approximation to the AID system's demand function on budget share form:

$$\begin{aligned} w_i &= \alpha_i + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/\zeta P^*\} \\ &\equiv \alpha_i^* + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/P^*\} \end{aligned} \tag{3.10}$$

where $\alpha_i^* \equiv \alpha_i - \beta_i log\, \zeta$. (3.10) is known in the literature as the linear approximate almost ideal demand (LA/AID) system following Blanciforti and Green (1983).

[14] Banks, Blundell and Lewbel (1997) introduced a quadratic version of the AID system that has become popular in demand analysis. Their model augment (3.8) with a quadratic logarithmic income term, $\frac{\lambda_i}{\prod_i p_i^{\beta_i}}(log\, x/P)^2$, and hence *nests* the classical AID system as the special case of $\lambda_i = 0\, \forall i$.

[15] As will be demonstrated in Section 4.1, the prices in our data set are highly collinear.

3.1.1 Some Remarks on the use of Stone's Price Index

Lack of convergence and failure to improve the likelihood after a certain number of iterations are just two of the problems one frequently encounters when estimating non-linear models like the AID system.[16] Such challenges can sometimes be overcome by utilizing initial values derived from simplified specifications such as the LA/AID system.[17] This means that failure to estimate the AID system might be resolved by running an auxiliary regression of (3.10), where the aim is to obtain appropriate starting values for the parameters in (3.8).

However, this is not to say that Stone's price index is *only* used as a tool to provide starting values for (3.8). The LA/AID system is in fact much more frequently used in applied work than the original non-linear AID system. This is generally accepted as long as prices are highly collinear. However, it is more the exception than the rule that researchers take this condition explicitly into account. Although some researchers follow Deaton and Muellbauer (1980*a*) and estimate both versions, most practitioners simply adopt (3.10) without so much as a comment. This is unfortunate for a number of reasons. First, it is not at all obvious that prices are sufficiently collinear, which implies that P and P^* might be far from proportional. Whenever $P \ncong \zeta P^*$, the LA/AID system loses practical value as it no longer provides an adequate approximation to the AID system. Second, unlike expenditure shares, Stone's price index is not invariant to units of measurement. Changing the units of measurement of all prices will alter the estimation results because the budget shares will apply unchanged weights to the re-scaled prices.[18] Let us demonstrate this by re-scaling the prices in (3.10) with a constant $\phi_i > 0$:

$$\begin{aligned} w_i &= \alpha_i^* + \sum_j \gamma_{ij} log\, \phi_j p_j + \beta_i log\, x - \beta_i \sum_j w_j log\, \phi_j p_j \\ &\equiv \breve{\alpha}_i^* + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/P^*\} \end{aligned} \tag{3.11}$$

where $\breve{\alpha}_i^* \equiv \alpha_i^* + \sum_j \gamma_{ij} log\, \phi_j - \beta_i \sum_j w_j log\, \phi_j$. Theoretically, (3.10) and (3.11) are equivalent demand systems, as the latter is simply a re-scaled ver-

[16] Lack of convergence when estimating AID systems can often be traced to the coefficient associated with subsistence expenditure, α_0 (Capps, Church and Love, 2003).

[17] Unlike the translog price index, Stone's price index does not include unknown parameters and can therefore be employed directly in regressions (this is explained more thoroughly in Section 4.1).

[18] Budget shares remain fixed as the re-scaling is carried out after the realization of w_i. In other words, if the agent originally spends a fraction λ of her budget on commodity i and the remaining share $1 - \lambda$ on the composite good, j, the same must hold also after the re-scaling of prices.

sion of the former. Econometrically, on the other hand, the two systems differ because the intercept term in (3.11) is non-constant and varies with budget shares. Estimation of (3.11) is therefore likely to yield biased estimates of the γ_{ij}'s and β_i's because the $\breve{\alpha}_i^*$'s are treated as constants (Moschini, 1995).[19] Note that the AID system *is* in fact invariant to units of measurement. Let us prove this by re-scaling the prices in (3.8):

$$\begin{aligned} w_i &= \alpha_i + \sum_j \gamma_{ij} log\, \phi_j p_j + \beta_i log\, x \\ &\quad - \beta_i \Big\{ \alpha_0 + \sum_k \alpha_k log\, \phi_k p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} log\, \phi_k p_k log\, \phi_j p_j \Big\} \\ &\equiv \acute{\alpha}_i + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/P\} \end{aligned} \tag{3.12}$$

where $\acute{\alpha}_i \equiv \alpha_i + \sum_j \gamma_{ij} log\, \phi_j - \beta_i \Big\{ \alpha_0 + \sum_k \alpha_k log\, \phi_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} log\, \phi_k log\, \phi_j \Big\}$. Unlike $\breve{\alpha}_i^*$, $\acute{\alpha}_i$ only contains constants and may accordingly also be treated as one, which means that (3.8) and (3.12) are both theoretically and econometrically equivalent demand systems. As the intercept term is practically of very limited interest, we can conclude that both the original and the rescaled version give qualitatively similar results. Put differently, as long as (3.8) yields unbiased estimates of the γ_{ij}'s and β_i's, so will (3.12).

Finally, note that commodity i's budget share appears indirectly on the right-hand side of (3.10). Hence, unlike (3.8), (3.10) does not represent *reduced form* equations (Capps, Church and Love, 2003). Let us try to address this problem by what will turn out to be an unfeasible approach, instrumental variables (IV) estimation.[20] To serve as a valid instrument for w_i in the regression:

$$w_i = \alpha_i^* + \sum_j \gamma_{ij} log\, p_j + \beta_i \Big\{ log\, x - w_i log\, p_i - \sum_{j \neq i} w_j log\, p_j \Big\} + \varepsilon_i$$

where $\varepsilon_i \sim i.i.d\,(0, \sigma^2)$, the IV, z_i, has to satisfy the following criteria:

1. Corr$(w_i, z_i) \neq 0$, i.e., z_i must be correlated with the endogenous explanatory variable, and preferably as highly so as possible.

[19] The fact that the use of Stone's price index makes the parameter estimates inconsistent is also demonstrated by Pashardes (1993) and Buse (1994). Pashardes (1993) shows that the bias is more serious when the empirical analysis is applied to micro rather than aggregate data.

[20] Buse (1994) shows that the IV procedure will always fail to generate consistency in estimation under standard conditions.

2. Corr$(z_i, \varepsilon_i) = 0$, i.e., z_i must only influence the dependent variable through its effect on the instrumented variable.

However, as is evident from the first criterion, this solution will generally fail to produce consistent estimators because valid instruments will also be omitted variables. As Corr$(w_i, z_i) \neq 0$, z_i cannot be left out of the equation for w_i to serve as an IV for the latter variable without simultaneously inducing omitted variable bias (assuming z_i is correlated with at least one of the regressors).

A feasible option, on the other hand, involves rewriting (3.10) so as to eliminate the budget shares from the right-hand side of the equation. Capps, Church and Love (2003) show that the reduced form expenditure shares will then be given by:

$$w_i = \frac{x_i(1 + \sum_{j \neq i} \beta_j log\, p_j) - \beta_i \sum_{j \neq i} x_j log\, p_j}{1 + \sum_i \beta_i log\, p_i} \tag{3.13}$$

where $x_i = \alpha_i^* + \sum_j \gamma_{ij} log\, p_j + \beta_i log\, x$. However, the non-linear functional form of (3.13) severely complicates the analysis and hence do more harm than good.

A final option involves using a re-specified price index:

$$log\tilde{P}_t = \sum_k w_{kt-1} log\, p_{kt} \tag{3.14}$$

with weights given by *lagged* rather than contemporaneous budget shares to avoid simultaneity problems (Eales and Unnevehr, 1993). This third option can have adverse implications for estimation precision if the expenditure system is applied to a small data set because one degree of freedom is lost when generating the price index. According to the likelihood dominance criterion for model selection proposed by Pollak and Wales (1991) it also harms the explanatory power of our expenditure systems, and the price index will therefore not be employed in this analysis.

The adoption of Stone's price index is motivated by the demand for a simplified and more easily testable expenditure system. Most practitioners therefore choose to ignore the index's many shortcomings as the alternative involves using the translog price index, which requires non-linear estimation.

3.2 Dynamic Almost Ideal Demand System

Up until now we have only considered static versions of the AID system. However, time plays an essential role in demand analysis as consumer preferences, prices and expenditure shares are all subject to change. Other potential sources for short run behavior includes, but are not limited to, adjustment costs, habit persistence, incorrect expectations and misinterpretation of real price changes (Anderson and Blundell, 1983), some of which will be addressed in this analysis. This, however, is not to say that (3.8) and (3.10) are no longer of importance. The two can be thought to represent long-run solutions of the AID and LA/AID system, respectively, and will therefore prove crucial when we turn attention to estimation in Chapter 3.

3.2.1 Persistence in Consumption Patterns (Habit Formation)

There are several ways to incorporate dynamic effects in the AID system. This analysis presents four alternative dynamic specifications, three of which will be presented in Section 4.3. Attention will for now be restricted to what will become our preferred dynamic specification, the dynamic linear approximate demand system incorporating habits. The model was originally proposed by Pollak and Wales (1969) who applied a Klein-Rubin linear expenditure system to U.S. postwar data from $1948 - 1965$, and was incorporated in the AID system for the first time by Blanciforti and Green (1983).

Consumers are likely to develop habits for certain commodities. To illustrate, consider for instance the demand for cereals. Consumers that are already familiar with a specific type of cereals generally attach a cost to deviating from their preferred flavor. As a result, they are less likely to opt for new flavors even in the event of a relative price increase of their preferred type. Pollak and Wales (1969) established a way to account for such behavior by means of the following linear habit scheme:

$$\alpha_i = \tilde{\alpha}_i + \varsigma_i q_{it-1} \tag{3.15}$$

where the left-hand side is the original intercept in (3.8), and q_{it-1} denotes demand for commodity i at time $t-1$. By substituting (3.15) into (3.8) we obtain a dynamic almost ideal demand system incorporating habits:

$$w_{it} = \tilde{\alpha}_i + \varsigma_i q_{it-1} + \sum_j \gamma_{ij} log\, p_{jt} + \beta_i log\{x_t/P_t\},\, t = 2, \ldots, T \tag{3.16}$$

The equivalent scheme for the LA/AID system is:

$$\alpha_i^* = \tilde{\alpha}_i^* + \varsigma_i^* q_{it-1} \tag{3.17}$$

where the left-hand side is the intercept in (3.10). Insertion of (3.17) into (3.10) yields the dynamic linear approximate almost ideal demand system incorporating habits:

$$w_{it} = \tilde{\alpha}_i^* + \varsigma_i^* q_{it-1} + \sum_j \gamma_{ij} log\, p_{jt} + \beta_i log\{x_t/P_t^*\},\, t = 2, \ldots, T \qquad (3.18)$$

Our assumption of habit formation now translates into the following easily testable hypotheses: $H_0 : \varsigma_i, \varsigma_i^* = 0$ against $H_1 : \varsigma_i, \varsigma_i^* \neq 0$, with rejection of the null yielding non-rejection of our assumption of habit formation.

4 Estimation

This chapter presents a number of applications of Deaton and Muellbauer (1980*a*)'s AID system. The empirical analysis is applied to the aggregated household consumption data set that was presented in Chapter 2. Chapter 4 is organized as follows. We begin by motivating the use of Stone's price index. Then we move on to present the benchmark model, the static LA/AID system. This section gives a detailed treatment of AID systems, tests for long-run homogeneity and Slutsky symmetry, and calculations of income and price elasticities. Chapter 4 concludes with applications of alternative dynamic expenditure system specifications and a discussion of the rationale behind imposing theoretical constraints in the various models.

4.1 Why opt for a linear approximation?

The LA/AID system does not represent an integrable demand system as the system's demand functions cannot be obtained as solutions to utility maximization. It is nevertheless frequently used in applied work because it greatly simplifies the regressions compared to the theoretically consistent AID system. Stone's geometric price index does not depend on unknown coefficients and can hence be employed directly in regressions. Estimation of the AID system, on the other hand, must be initiated by ordinary least squares (OLS) estimation of P. Fitted values from the auxiliary regression, $\hat{P}$, are then subsequently inserted in (3.8), yielding:

$$w_i = \alpha_i + \sum_j \gamma_{ij} log\, p_j + \beta_i log\{x/\hat{P}\} \tag{4.1}$$

where $log\hat{P} = \hat{\alpha}_0 + \sum_k \hat{\alpha}_k log\, p_k + \frac{1}{2}\sum_j \sum_k \hat{\gamma}_{kj} log\, p_k log\, p_j$. Such an approach is often challenging in practice due to the non-linearity of the translog price index and because of the large number of coefficients that must be estimated. The latter point can be demonstrated by rewriting the last term in (3.7):

$$logP = \alpha_0 + \sum_k \alpha_k log\, p_k + \frac{1}{2}\boldsymbol{\mu}'\boldsymbol{\gamma}\boldsymbol{\mu} \tag{4.2}$$

where $\boldsymbol{\mu} = (log\, p_1, \ldots, log\, p_n)'$ is an $n \times 1$ column vector of (log) prices and $\boldsymbol{\gamma}$ is an $n \times n$ square matrix of coefficients:

$$\boldsymbol{\gamma}_{k,j} = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,n} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n,1} & \gamma_{k,2} & \cdots & \gamma_{n,n} \end{pmatrix}$$

Recall from Section 2.1 that our objective is to estimate an expenditure system for 10 non-durable consumption categories. This implies that $\boldsymbol{\gamma}$ contains 100 unknown parameters that have to be estimated prior to the regression of (3.8) (in addition to the 11 α's).[21] Considering our data availability of 33 years (or 135 quarters), this approach is problematic for an expenditure system of our size due to insufficient degrees of freedom. The rest of this chapter will therefore be devoted to applications of linearized expenditure systems using Stone's share-weighted price index, P^*.

4.2 Static LA/AID System

Recall that the appropriateness of the LA/AID system depends on the degree of proportionality between the two price indices given in (3.7) and (3.9). Therefore, before we move on to the estimation, let us first examine the degree of multicollinearity between the p_j's on the right-hand side of (3.10). This can be done by running single equation regressions where each p_j is regressed on all other prices (and an intercept). R_j^2's will then measure the degree of linear relationship between the prices, with values close to unity indicating a presence of multicollinearity ($R_j^2 = 1$ is ruled out as models that suffer from *perfect collinearity* cannot be estimated). Adjusted R_j^2's from our regressions vary between 0.9575 and 0.9964, suggesting that the two price indices are likely to be close to proportional.

Alternatively, one can examine Table A.2, which gives a correlation matrix of the regressors in (3.10) (with the exception of $log\{x/P^*\}$). After looking at the table there can be no doubt that the prices in our data set are in fact highly collinear, with the matrix demonstrating an average correlation between the regressors of 0.8261.[22] These findings can be interpreted as evidence that the prices in our data set are likely to be sufficiently collinear for the adoption of Stone's geometric price index to be warranted in this analysis.

Let us now move on to the estimation of the static LA/AID system given

[21] Due to *adding-up* and *Slutsky symmetry* $\boldsymbol{\gamma}$ only contains $(1/2)(n-1)(n+1)$ unknown parameters. Cf. equation (4.10) and (4.13) below for detailed accounts.

[22] If one deliberately disregards clothing & shoes from the calculations the number increases to 0.9680. This is further addressed and elaborated upon in Section A.1.

in (3.10), which on matrix form can be expressed as:

$$\underbrace{\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}}_{\mathbf{w}} = \underbrace{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}}_{\boldsymbol{\alpha}} + \underbrace{\begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,n} & \beta_1 \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,j} & \beta_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma_{n,1} & \gamma_{n,2} & \cdots & \gamma_{n,n} & \beta_n \end{pmatrix}}_{\boldsymbol{\Theta}} \underbrace{\begin{pmatrix} log\, p_1 \\ \vdots \\ log\, p_n \\ log\{x/P^*\} \end{pmatrix}}_{\mathbf{z}} \tag{4.3}$$

where $\mathbf{w}$ is an $n \times 1$ column vector of budget shares, $\boldsymbol{\alpha}$ is an $n \times 1$ column vector of intercepts, $\boldsymbol{\Theta}$ is an $n \times (n+1)$ matrix of coefficients and $\mathbf{z}$ is an $(n+1) \times 1$ column vector of explanatory variables. We finally obtain our static regression model by adding an $n \times 1$ column vector of error terms, $\boldsymbol{\varepsilon}$, in (4.3), which captures all other factors affecting the budget shares other than prices and total expenditure:

$$\mathbf{w} = \boldsymbol{\alpha} + \boldsymbol{\Theta} z + \boldsymbol{\varepsilon} \tag{4.4}$$

The disturbance term is assumed to satisfy the following three criteria in our models (including the dynamic expenditure systems that will be presented in Section 4.3):[23]

1. $E[\boldsymbol{\varepsilon}_t] = 0$.
2. $E[\boldsymbol{\varepsilon}_s \boldsymbol{\varepsilon}_t^{'}] = \begin{cases} \mathbf{0}_n & \text{if } s \neq t \\ \boldsymbol{\Sigma}_{n,n} & \text{if } s = t, \end{cases}$ where $\mathbf{0}$ is a 0-matrix of dimension n and $\boldsymbol{\Sigma}$ denotes the variance-covariance matrix.
3. $\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T$ are stochastically independent disturbance terms drawn from a multivariate normal distribution.

Economic theory imposes a number of constraints on our expenditure systems. As consumption behavior is constrained by the agents' consumption possibilities frontier, we have that total expenses cannot exceed their exogenously determined budgets:

$$\sum_i p_i q_i \leq x \tag{4.5}$$

(4.5) will here hold with strict equality if we add the additional assumption of strictly positive marginal utility for all consumption levels. Then, by maximizing the Lagrangian – $\mathcal{L} = U(q_1, \ldots, q_n) - \lambda(\sum_i p_i q_i - x)$ (where

[23] These criteria hold for all t and are conditional on the vector of regressors, $\mathbf{z}_t$. The conditioning has been omitted for notational simplicity.

$U(\cdot)$ is the chosen utility function and λ denotes the Lagrange multiplier) − with respect to each q_i, we obtain a set of Marshallian (or uncompensated) demand functions:

$$q_i = \vartheta(p_1, \ldots, p_n, x) \tag{4.6}$$

which, when inserted into (4.5), yields:

$$\sum_i p_i \, \vartheta(p_1, \ldots, p_n, x) = x \tag{4.7}$$

Here we have utilized the previously mentioned assumption of strictly positive marginal utility for all consumption levels. (4.7) is known as the *adding-up condition*, which plays an essential role in all demand analyses. Let us show that an equivalent constraint is present in AID systems. Deaton and Muellbauer (1980*b*, ch. 1.2) show that the adding-up condition given in (4.7) is equivalent to the following constraint:

$$\sum_i w_i e_i = 1 \tag{4.8}$$

where e_i denotes commodity i's income elasticity, which is given by $e_i = 1 + \beta_i / w_i$ in AID systems (Green and Alston, 1990). Substituting the expression for e_i into (4.8) gives:

$$\sum_i w_i + \sum_i \beta_i = 1 \tag{4.9}$$

This equation tells us that $\sum_i \beta_i = 0$, as logic dictates that the budget shares must sum to unity. Next, in order for both sides of (3.8) (or, alternatively, (3.10)) to be identically equal to unity when we sum over all i, coupled with the fact that adding-up must hold for *all* values and combinations of the regressors, we further have that $\sum_i \alpha_i = 1$ and $\sum_i \gamma_{ij} = 0$. Adding-up is accordingly given by the following condition in AID systems:

$$\sum_i \alpha_i = 1, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0 \tag{4.10}$$

The fact that $\sum_i w_i = 1$ implies that the variance-covariance matrix, $\mathbf{\Sigma}$, is singular and hence non-invertible. This means that we cannot estimate all n equations in (4.4) simultaneously as such a model would suffer from perfect collinearity. However, according to Barten (1969)'s invariance principle, we can still obtain maximum likelihood estimates of all the parameters in (4.4) by means of the following two-step procedure:[24] (i) arbitrarily drop an equation from the expenditure system. The resulting $(n-1) \times 1$ column-vector

[24]Note that Barten (1969)'s invariance principle is conditional on the error term assumptions outlined in this section.

of disturbances yields a non-singular variance-covariance matrix that enables one to obtain parameter estimates for the retained $n-1$ equations. (ii) Utilize adding-up to obtain coefficient estimates from the omitted equation (this is explained more thoroughly below).

Economic theory teaches us that rational agents do not suffer from *money illusion*, which means that relative demand for our 10 consumption categories should remain unaltered if *all* prices suddenly change by a common factor, Φ (or, to put it more technically, the demand functions should be homogeneous of degree 0 in prices and total expenditure). *Homogeneity* is satisfied in the AID system if and only if:

$$\sum_j \gamma_{ij} = 0, \forall\, i \tag{4.11}$$

Utility theory further requires that the cross-price derivatives of the Hicksian demand functions must be symmetric, i.e., $\partial\pi_i(u,p)/\partial p_j = \partial\pi_j(u,p)/\partial p_i$, where $\pi_i(\cdot)$ and $\pi_j(\cdot)$ denote category i's and j's compensated demand function, respectively. This follows from *Young's theorem*, which asserts that $\partial^2 c(\cdot)/\partial p_i \partial p_j$ and $\partial^2 c(\cdot)/\partial p_j \partial p_i$ are identical when the cost function, $c(\cdot)$, is at least twice continuously differentiable in the interior of its effective domain. It will prove useful to form a matrix comprising all these partial derivatives. Let therefore in the following $s_{i,j} \equiv \partial\pi_i(\cdot)/\partial p_j$, which means this matrix can be expressed as:

$$\mathbf{S} = \begin{pmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,n} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n,1} & s_{n,2} & \cdots & s_{n,n} \end{pmatrix} \tag{4.12}$$

(4.12) is the well-known *Slutsky matrix* (or *substitution matrix*) of compensated price effects. Economic theory imposes two constraints on this matrix: $s_{i,j} \equiv s_{j,i}$ and $s_{i,i} \leq 0$ for all $i,j \in (1,\ldots,n)$.[25] The former constraint is known as *Slutsky symmetry*, and follows as we have already seen from Young's theorem. The latter constraint is known as the *negativity constraint* and requires $\mathbf{S}$ to be *negative semidefinite*, which implies that all n (real) eigenvalues must be non-positive (detailed accounts are given in Section 4.4). This requirement that all main diagonal elements of (4.12) must be non-positive means that a price increase of commodity i can only have non-positive effects on the Hicksian demand for good i (or, to put it less technically, compensated demand functions must slope downwards). This is brought about by the fact that utility is kept constant when analyzing price effects on Hicksian

[25] The constraints, $s_{i,j} \equiv s_{j,i}$ and $s_{i,i} \leq 0$, are direct implications of the *axioms of rational choice* (Deaton and Muellbauer, 1980*b*, ch. 2.4).

demand. In other words, we compensate utility for the price effects, thus leaving the agent's utility level unaffected. Slutsky symmetry is captured by the following condition in AID systems:[26]

$$\gamma_{ij} = \gamma_{ji} \tag{4.13}$$

The negativity and symmetry conditions conclude the set of theoretical constraints imposed on our expenditure system.

Table 4.1 displays the result from the estimation of the benchmark model, (4.4) (i.e., the unconstrained static LA/AID system), where quarterly dummies have been included to account for seasonality in the consumption pattern. τ_{i1}, τ_{i2} and τ_{i3} are the coefficients of quarterly dummy D_{i1}, D_{i2} and D_{i3}, respectively (the fourth quarter has been omitted and hence represents the base group). As we still require the system to satisfy adding-up, (4.10) must be replaced by the following constraint when seasonal dummies are included: $\sum_i \alpha_i^* = 1, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0, \sum_i \tau_{is} = 0$, where $s \in (1, 2, 3)$. Although the system has been estimated by full information maximum likelihood (FIML), one would have obtained identical results by means of single equation OLS regressions. This similarity between FIML and OLS is a consequence of the fact that Table 4.1 exemplifies an *unconstrained* system with identical right-hand side variables in *all* equations. It is well-known that simultaneous estimation of all equations in a system of regression equations degenerates to single equation regressions whenever (i) error terms are uncorrelated across equations and/or (ii) every equation has the same set of explanatory variables (cf. e.g. Greene (2003, ch. 14.2)). In other words, nothing is gained in terms of efficiency by treating systems characterized by (i) and/or (ii) as systems of regression equations rather than as single equation regressions. However, this result is conditional on the absence of coefficient restrictions across equations. This means that constrained systems such as the homogeneous & symmetric LA/AID system discussed below cannot be estimated efficiently by means of single equation OLS regressions and hence must be treated as systems of regression equations.

Let us go through the output of Table 4.1 in detail. The first column lists the estimates of the intercept terms of our 10 consumption categories. However, the α_i^*'s are practically of very limited interest, and I will therefore leave this column uncommented. The β_i's determine whether the commodity

[26] Chen (1998) shows that Slutsky symmetry is captured by the following condition in LA/AID systems: $\gamma_{ij} + \sum_{k \neq i}(\beta_k \gamma_{ij} - \beta_i \gamma_{kj}) log\, p_k = \gamma_{ji} + \sum_{s \neq j}(\beta_s \gamma_{ji} - \beta_j \gamma_{si}) log\, p_s$, $\forall i \neq j$. Although it is possible to impose these restrictions econometrically, they destroy the linearity of the LA/AID system. Attention will therefore be restricted to *local* rather than *global* tests for symmetry in this analysis, which means Slutsky symmetry will continue to be captured by (4.13) also for the LA/AID system (cf. Chen (1998) for detailed accounts).

Table 4.1: Unconstrained Parameter Estimates and Test for Homogeneity in the Static LA/AID System [a]

Commodity i	α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	γ_{i10}	τ_{i1}	τ_{i2}	τ_{i3}	$\sum_j \gamma_{ij}$ [b]
Food	1.146	−0.081	0.063	−0.033	−0.014	−0.011	0.022	−0.036	0.046	0.002	−0.028	−0.034	−0.013	−0.007	−0.009	5.57
	(7.5) [c]	(−6.5)	(3.2)	(−2.0)	(−1.9)	(−2.2)	(3.7)	(−3.6)	(2.0)	(0.1)	(−3.0)	(−2.8)	(−6.4)	(−4.2)	(−6.4)	(0.0183) [d]
Beverages	0.198	−0.012	0.001	0.025	−0.007	0.001	−0.006	0.004	−0.010	−0.004	−0.019	0.009	−0.006	0.002	−0.001	1.37
	(2.9)	(−2.1)	(0.2)	(3.4)	(−2.1)	(0.4)	(−2.3)	(0.9)	(−0.9)	(−0.3)	(−4.4)	(1.6)	(−6.2)	(3.0)	(−1.2)	(0.2423)
Tobacco	0.151	−0.010	−0.003	0.003	0.002	−0.002	−0.008	0.000	−0.004	0.003	−0.004	0.020	0.016	−0.017	−0.026	4.48
	(2.2)	(−1.7)	(−0.3)	(0.4)	(0.6)	(−0.8)	(−2.9)	(0.1)	(−0.3)	(0.3)	(−0.8)	(3.6)	(9.1)	(−11.3)	(−21.3)	(0.0342)
Energy	0.203	−0.012	−0.014	0.025	−0.027	0.047	0.004	−0.009	0.019	−0.024	0.010	−0.016	0.016	−0.017	−0.026	2.30
	(1.5)	(−1.1)	(−0.8)	(1.7)	(−4.1)	(9.9)	(0.8)	(−1.0)	(0.9)	(−1.1)	(1.1)	(−1.4)	(9.1)	(−11.3)	(−21.3)	(0.1295)
Clothing & Shoes	−0.345	0.035	0.003	0.001	−0.026	−0.010	−0.009	−0.015	0.033	0.001	0.019	−0.017	−0.017	−0.009	−0.017	9.96
	(−3.4)	(4.3)	(0.2)	(0.1)	(−5.3)	(−2.8)	(−2.3)	(−2.2)	(2.1)	(0.0)	(3.1)	(−2.1)	(−13.2)	(−8.2)	(−18.9)	(0.0016)
Other Products	−0.354	0.043	−0.021	−0.040	0.038	−0.012	−0.010	0.038	−0.026	0.017	−0.017	0.004	−0.015	−0.018	−0.015	15.85
	(−3.1)	(4.5)	(−1.4)	(−3.3)	(7.0)	(−3.2)	(−2.3)	(5.1)	(−1.4)	(1.0)	(−2.5)	(0.4)	(−9.9)	(−15.2)	(−14.5)	(0.0001)
Other Services	0.110	0.013	−0.051	0.049	0.017	0.003	−0.010	0.081	−0.084	0.097	−0.053	0.011	0.024	0.021	0.023	39.33
	(0.7)	(1.0)	(−2.7)	(3.1)	(2.3)	(0.5)	(−1.8)	(8.1)	(−3.5)	(4.2)	(−5.8)	(0.9)	(12.2)	(13.5)	(17.5)	(0.0000)
Vehicle Running Costs	0.709	−0.055	0.000	−0.013	−0.012	−0.002	0.044	−0.007	0.017	−0.011	−0.004	0.024	−0.004	0.004	0.005	65.94
	(10.2)	(−9.6)	(0.0)	(−1.8)	(−3.4)	(−0.9)	(16.4)	(−1.6)	(1.5)	(−1.0)	(−1.0)	(4.3)	(−4.7)	(4.9)	(8.7)	(0.0000)
Public Transport, Mail & Telecommunications	0.156	−0.007	0.018	−0.030	0.019	−0.003	−0.009	0.012	−0.063	0.020	0.031	−0.002	0.009	0.005	0.009	0.93
	(1.5)	(−0.8)	(1.3)	(−2.7)	(3.7)	(−0.8)	(−2.3)	(1.8)	(−3.8)	(1.3)	(4.8)	(−0.2)	(6.7)	(4.7)	(9.2)	(0.3360)
Consumption Abroad	−0.973	0.086	0.004	0.012	0.009	−0.011	−0.017	−0.069	0.072	−0.102	0.065	0.002	0.007	0.018	0.032	9.78
	(−5.5)	(5.9)	(0.2)	(0.6)	(1.1)	(−1.7)	(−2.6)	(−5.9)	(2.6)	(−3.7)	(6.0)	(0.1)	(3.0)	(9.4)	(20.2)	(0.0018)

[a] Estimation is based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3.

[b] Test for homogeneity.

[c] Values in parentheses in columns $1 - 15$ are t-statistics.

[d] Values in parentheses in the final column are p-values.

in question is a luxury or necessity. Recall that good i's expenditure elasticity is given by the following expression in AID systems (Green and Alston, 1990):[27]

$$e_i = 1 + \beta_i / w_i \tag{4.14}$$

Recall further from Section 2.2 that necessities and luxuries are characterized by income elasticities in the $(0, 1)$ and above-unit interval, respectively.[28] As budget shares must be positive we have that $\beta_i > 0$ for luxuries and $\beta_i < 0$ for necessities. Hence, according to Table 4.1, our benchmark system includes three necessities (food, beverages and vehicle running costs) and three luxuries (clothing & shoes, other products, and consumption abroad). The remaining four categories, tobacco, energy, other services and public transport, mail & telecommunications, are insignificant at 5 percent levels and hence cannot be classified. Note that homothetic preferences is captured by $\beta_i = 0\, \forall i$ in AID systems, which implies that all income elasticities are equal to unity and that all expenditure shares are non-varying. Homotheticity is rejected for all expenditure system specifications in this analysis.

Price effects on the budget shares are captured by the γ_{ij}'s in AID systems. These parameters give indications of whether two commodities are likely to be *complements* or *substitutes*. Goods X and Y are said to be complements if the latter's demand is negatively related to the price of the former, whereas the two are said to be substitutes if the opposite holds. However, due to the presence of both an *income* and *substitution effect*, one cannot classify pairs of categories as complements and substitutes based solely on the sign of the γ_{ij}'s. In order to classify the various pairs of consumption groups we need a formula that takes these two (potentially) opposing effects explicitly into account, which means we must derive the expression for the cross-price elasticity of demand. Commodity i's elasticity of demand with respect to the price of good j is defined by $\eta_{i,j} = (\partial q_i / \partial p_j)(p_j / q_i) = \partial log\, q_i / \partial log\, p_j$, where the last equality follows from $\partial log\, q_i = (\partial q_i / q_i)$. By utilizing this formula in (3.8), we find that commodity i's Marshallian elasticity of demand with

[27]Green and Alston (1991) show that the correct expression for commodity i's income elasticity when Stone's geometric price index is used in place of the translog price index is given by: $e_i = 1 + \beta_i / w_i [1 - \sum_j w_j log\, p_j (e_j - 1)]$. However, by treating the budget shares as fixed parameters the expression simplifies to (4.14). This is a standard approach taken in the literature (cf. also footnote 30 and equation (4.17) below).

[28]e_i is *not* restricted to non-negative values. A negative expenditure elasticity implies that the commodity in question is an *inferior* good. However, all our groups are *normal*, i.e., have non-negative income elasticity, which is why the analysis pays no attention to commodities characterized by below-zero expenditure elasticity.

respect to the price of good j can be expressed as:

$$\begin{aligned}\eta_{i,j} &= -\delta_{ij} + \frac{d\,w_i}{w_i\,d\,log\,p_j} \\ &= -\delta_{ij} + \frac{1}{w_i}\left[\gamma_{ij} - \beta_i \frac{d\,logP}{d\,log\,p_j}\right] \\ &= -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \beta_i \frac{\alpha_j}{w_i} - \frac{\beta_i}{w_i}\sum_k \gamma_{kj} log\,p_k\end{aligned} \tag{4.15}$$

where δ_{ij} denotes the Kronecker delta, which takes on the value 0 and 1 when $i \neq j$ and $i = j$, respectively (Green and Alston, 1990).[29] However, as we are primarily interested in the linearized expenditure system, let us also derive the equivalent formula for the LA/AID system:

$$\begin{aligned}\eta_{i,j} &= -\delta_{ij} + \frac{1}{w_i}\left[\gamma_{ij} - \beta_i \frac{d\,logP^*}{d\,log\,p_j}\right] \\ &= -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i}\left[w_j + \sum_k log\,p_k \frac{d\,w_k}{d\,log\,p_j}\right] \\ &= -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \beta_i \frac{w_j}{w_i} - \frac{\beta_i}{w_i}\left[\sum_k w_k log\,p_k(\eta_{kj} + \delta_{kj})\right]\end{aligned} \tag{4.16}$$

where the last equality follows from (4.15) coupled with utilization of $d\,w_k = w_k d\,logw_k$. Green and Alston (1990) demonstrate that the simplified expression for the uncompensated price elasticity used by Chalfant (1987) (where the term in square brackets in the final line of (4.16) is dropped) yields nearly identical results.[30] Marshallian elasticities of demand will therefore be calculated according to the following formula in this analysis:

$$\eta_{i,j} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \beta_i \frac{w_j}{w_i} \tag{4.17}$$

where it is assumed that $d\,logP^*/d\,log\,p_j = w_j$.[31] (4.17) clearly demonstrates the interplay between the income and substitution effect in the LA/AID

[29] Note that Slutsky symmetry has been imposed in (4.15).

[30] Monte Carlo experiments by Alston, Foster and Green (1994) show that (4.16) and the simplified formula (cf. (4.17) below) give consistently more accurate elasticity estimates than (4.15) when the true data generating process is the AID system but the coefficients are obtained from the LA/AID system. They suggest using (4.17) due to its computational simplicity. Slight preference to the simplified formula is also given by Buse (1994).

[31] Hicksian elasticity of demand, $\eta^*_{i,j}$, is obtained from the Slutsky equation: $\eta^*_{i,j} = \eta_{i,j} + e_i w_j$, which yields the following formula: $\eta^*_{i,j} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j$.

system. Due to the presence of the former effect, a positive (negative) γ_{ij} is insufficient for $\eta_{i,j} > 0$ ($\eta_{i,j} < 0$), i.e., it is insufficient for good i and j to be classified as substitutes (complements). Accordingly, although the γ_{ij}'s play a pivotal role in the price elasticity formulas and in the tests for homogeneity and Slutsky symmetry, they are individually of limited interest.

The τ_{is}'s are the coefficients of the quarterly dummies and account for possible seasonality in the consumption pattern. With all but one τ_{is} estimate in the unconstrained static LA/AID system significant at a 5 percent level there can be no doubt that such effects do influence our consumption categories. The table is further seen to support a number of popular beliefs about non-durable expenditure in Norway, with consumption of food, clothing & shoes and other products, on the one hand, and consumption of energy, on the other, reaching its peak in the fourth and first quarter due to Christmas and winter, respectively.

The final column of Table 4.1 displays single equation *Wald tests* for homogeneity. To test the null-hypothesis of absence of money illusion, H_0 : $\mathbf{c}(\boldsymbol{\gamma}_0) = \mathbf{0}$ (where we recall from (4.11) that this multiple linear restriction is given by $\sum_j \gamma_{ij} = 0$, $\forall i$), we form the *Wald statistic*: $W = \mathbf{c}(\hat{\boldsymbol{\gamma}})'(\hat{\mathbf{C}}\hat{\mathbf{V}}\hat{\mathbf{C}})^{-1}\mathbf{c}(\hat{\boldsymbol{\gamma}}) \overset{a}{\sim} \chi^2_q$, where q is the number of restrictions, $\hat{\mathbf{V}}$ is an estimated asymptotic variance-covariance matrix estimator of $\hat{\boldsymbol{\gamma}}$, $\hat{\mathbf{C}} \equiv \mathbf{C}(\hat{\boldsymbol{\gamma}})$, and $\mathbf{C}(\boldsymbol{\gamma})$ is the $n \times n$ Jacobian of $\mathbf{c}(\boldsymbol{\gamma})$ (Wooldridge, 2010, ch. 12.6). Recall from Section 3.2 that the static systems can be thought to represent long-run solutions of the theoretically more plausible dynamic expenditure systems. The homogeneity tests listed in Table 4.1 should therefore be interpreted as tests for absence of money illusion in the *long run*.[32] Perhaps surprisingly, the unconstrained static LA/AID system rejects homogeneity in 7 of 10 equations at a 5 percent level, with absence of money illusion only non-rejected in the equation for beverages, energy and public transport, mail & telecommunications. Qualitatively similar results are obtained by Raknerud, Skjerpen and Swensen (2007) for all but two consumption categories: absence of money illusion for other services and public transport, mail & telecommunications is only rejected by our unconstrained static system and by their general non-homogeneous model, respectively.[33] Let us demonstrate that the model further rejects

[32] Anderson and Blundell (1982) argue that theoretical constraints should only be imposed on the long run structure of the system because there is no reason to assume that the consumers' short run behavior, which is likely to differ from their corresponding long run equilibrium behavior due to adjustment costs, lack of perfect information, etc., will be consistent with utility theory. Attention will therefore be restricted to long run tests for Slutsky symmetry and absence of money illusion in this analysis. Further details are given in Section 4.4.

[33] Raknerud, Skjerpen and Swensen (2007)'s regressions are applied to quarterly Nor-

what I will refer to as *global homogeneity*, where we test for absence of money illusion for *all* consumption categories rather than for individual commodities. In order to test this restriction we form the *likelihood ratio statistic*: $LR = 2(\mathscr{L}_{ur} - \mathscr{L}_r) \overset{a}{\sim} \chi^2_q$, where $\mathscr{L}_{ur}$ and $\mathscr{L}_r$ is the log-likelihood value for the unrestricted and restricted model, respectively, and q is again the number of exclusion restrictions.[34] Table 4.2 lists the log-likelihood value of the four static LA/AID systems that will be analyzed in this section: the unconstrained system (i.e., the benchmark model currently under discussion, cf. Table 4.1 for the estimation results), the homogeneous system (where $\sum_j \gamma_{ij} = 0\,\forall i$, cf. Table A.3 for the estimation results), the symmetric system (where $\gamma_{ij} = \gamma_{ji}$) and the homogeneous & symmetric system (where $\sum_j \gamma_{ij} = 0\,\forall i$ and $\gamma_{ij} = \gamma_{ji}$, cf. Table A.5 for the estimation results). With a test statistic of 125.32 the benchmark system clearly rejects global homogeneity (for comparison, the 1 percent critical value with 9 degrees of freedom is 21.67).

Table 4.2: Log-Likelihood Values of Static LA/AID Systems Applied to Quarterly Norwegian National Accounts Data

	Log-Likelihood	Wald Test Statistic[a]
Unrestricted Static LA/AID System[b]	5534.83	–
Homogeneous Static LA/AID System[c]	5472.17	$\chi^2_9 = 125.32[0.0000]$
Symmetric Static LA/AID System	5390.02[d]	–
Homogeneous & Symmetric Static LA/AID System[e]	5308.14	$\chi^2_{45} = 453.37[0.0000]$

[a] Values in brackets are p-values.
[b] Cf. Table 4.1 for the estimation results.
[c] Cf. Table A.3 for the estimation results.
[d] The value gives the average log-likelihood value of the 10 subsystems listed in Table A.4.
[e] Cf. Table A.5 for the estimation results.

As one equation must be omitted from the expenditure system at the time of estimation, we cannot robustly test Slutsky symmetry without simultaneously imposing homogeneity. In other words, as we are only able to estimate $n - 1$ budget shares simultaneously, we cannot impose symmetry between

wegian national accounts data for the period 1966Q1 to 2001Q4. Their analyses do not distinguish between beverages and tobacco. Note that their general non-homogeneous model fails to reject absence of money illusion for beverages & tobacco.

[34] Recall from our discussion earlier that in order to estimate AID systems we have to delete one equation to avoid perfect collinearity. Unlike coefficient estimates, the Wald test is not invariant to the omitted equation, and is consequently not suited to test for absence of global homogeneity.

the coefficients in the $n-1$ first equations and the omitted one, thus leaving one set of parameters unconstrained.[35] In order to ease this non-invariance problem I ran one regression for each possible subsystem (i.e., I started by excluding the 10^{th} budget share, then the 9^{th}, etc.) until I had obtained log-likelihood values for all different $n-1$ combinations (cf. Table A.4 for the results). Each combination clearly rejects the imposition of Slutsky symmetry, which can be interpreted as evidence that symmetry is rejected also in the entire expenditure system. But as hinted at in the beginning of this paragraph, it is in fact possible to robustly test Slutsky symmetry as long as one simultaneously imposes homogeneity in the system, which ensures that symmetry holds also for the deleted coefficients. Before we continue, let us therefore examine whether simultaneous imposition of these two conditions alter the results we have derived so far. Imposing both conditions in the static LA/AID system yields a log-likelihood value of 5308.14. Compared to the unconstrained value of 5534.83 we can safely conclude that our benchmark model does not support the simultaneous imposition of homogeneity and symmetry.

It is not valid to test for adding-up in a system that automatically imposes it. Estimation of AID systems automatically satisfies this condition as $\sum_i w_i = 1$, which will also hold in practice as budget shares sum to unity by data construction.

Before we move on to the theoretically more plausible dynamic systems, let us utilize (4.14) and (4.17) to obtain income and price elasticities for the static LA/AID systems. Note that both sets of elasticities vary with budget shares in this type of models, which means we initially have to decide *where* to measure the elasticities. All elasticities are calculated at mean budget share values in this analysis, which is the standard approach taken in the literature.[36] Average, maximum and minimum annual budget share values for the years $1978-2010$ for each of our 10 categories is summarized in Table A.1. Table 4.3 lists the uncompensated own-price and income elasticities for three of the static models we have analyzed so far: the unconstrained model (Table 4.1), the homogeneous model (Table A.3) and the homogeneous & symmetric model (Table A.5).[37] As the elasticities are based on static

[35] Due to adding-up the omitted equation is fully determined by the retained system. This implies that *if* the $(n-1)\times(n-1)$ submatrix of (4.12) is symmetric, then the last row and column will also satisfy symmetry from adding-up (Haag, Hoderlein and Pendakur, 2009).

[36] Another conventional approach taken in the literature involves calculating the elasticities at the base year of the data set.

[37] Cf. also Table A.6 for the homogeneous static LA/AID system's Marshallian cross-price elasticities.

models, one should interpret them as responses to permanent shocks. To illustrate, consider e.g. energy's own-price and expenditure elasticity in the unconstrained model. According to the calculations, a 1 percent permanent increase in the price of energy will lower demand for this category by 0.18 percent. Similarly, a 1 percent permanent rise in income will increase demand for energy by 0.79 percent. Hence, according to these findings, energy is a price inelastic necessity good to Norwegian households.

Table 4.3: Own-price and Expenditure Elasticities for 3 Static LA/AID Systems

	Unconstrained		Homogeneous		Homogeneous & Symmetric	
	$\eta_{i,i}$	e_i	$\eta_{i,i}$	e_i	$\eta_{i,i}$	e_i
Food	-0.60	0.58	-0.69	0.50	-0.70	0.14
Beverages	-0.58	0.81	-0.60	0.76	-0.58	0.93
Tobacco	-0.92	0.66	-0.84	0.87	-1.00	1.82
Energy	-0.18	0.79	-0.15	0.94	-0.14	0.59
Clothing & Shoes	-1.20	1.41	-1.17	1.26	-1.02	0.63
Other Products	-1.23	1.32	-1.16	1.18	-1.00	1.15
Other Services	-0.61	1.05	-1.10	1.21	-1.15	1.69
Vehicle Running Costs	-0.21	0.08	-0.30	0.47	-1.35	0.84
Public Transport, Mail & Telecommunications	-0.55	0.90	-0.53	0.84	-0.25	1.17
Consumption Abroad	-1.06	2.36	-0.90	2.00	-1.26	1.22

Notes: Calculations are based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Elasticities are calculated at mean quarterly budget share values for the entire sample.

It is interesting to compare the results in Table 4.3 with those obtained by Raknerud, Skjerpen and Swensen (2007) on quarterly Norwegian national accounts data for the period 1966Q1 to 2001Q4. Care must be exercised when comparing these results as their elasticity estimates are calculated at 1997 budget share values (which represents the base year in their data set), whereas the elasticities in this section are calculated at mean quarterly budget share values for the period 1978Q1 to 2011Q3. Although Table A.7 illustrates a number of qualitative dissimilarities between these two studies, close examination of the results reveals that it is primarily one category that stands out: vehicle running costs. Whereas this group is the least income elastic commodity in our unconstrained and homogeneous static expenditure system, it is in fact the most and second-most income elastic category in their homogeneous and unconstrained system, respectively. This is noteworthy as the differences cannot be a result of the fact that we calculate the elasticities at different budget share values. Even a five-fold increase in the average vehicle running costs budget share would only give rise to an unconstrained expenditure elasticity of 0.82, well below the value of 1.56 obtained by Raknerud, Skjerpen and Swensen (2007). Considering the elasticity formula given in (4.14), this dissimilarity therefore suggests testing for absence of parameter constancy for vehicle running costs. Figure A.2 and Figure A.3 illustrate recursive estimates of the β_i's and break-point Chow-tests for parameter constancy for the unconstrained static LA/AID system, respectively. The latter graph rejects the hypothesis of lack of parameter constancy at a 5 percent level for all consumption categories in our expenditure system, with food, beverages and clothing & shoes even significant at a 1 percent level. These findings call into question the results obtained by Raknerud, Skjerpen and Swensen (2007).[38] Based on the results derived in this section (coupled with the fact that vehicle running costs' budget share has declined steadily since the 1990s, which suggests an income elasticity in the $(0, 1)$ interval), it seems reasonable to classify vehicle running costs as a necessity rather than a luxury to Norwegian households.

Let us now return to Table 4.3, which illustrates some of the points highlighted in Section 2.2. All three models classify food, beverages, energy and public transport, mail & telecommunications as price inelastic commodities, and clothing & shoes as price elastic. Tobacco and other products are *unit elastic* commodities according to the static homogeneous & symmetric model, which means their demand is estimated to respond one-to-one to own-price movements. The three models further agree on the classification of food, beverages, energy and vehicle running costs as necessities, and other products, other services and consumption abroad as luxuries. However, the models also conclude differently on a number of categories. Whereas tobacco is clearly a necessity according to the unconstrained and homogeneous model (though somewhat less so according to the latter model), it is certainly a luxury good according to the homogeneous & symmetric expenditure system specification. Similar results are found for clothing & shoes, other services and vehicle running costs, where our qualitative conclusions differ depending on what model we adopt. To illustrate this last point, imagine that the government wants to collect taxes in the most efficient and least distortive way possible by only taxing clearly price inelastic commodities. According to the unconstrained and homogeneous model, vehicle running costs are almost ideal tax objects, and should consequently face high taxes according to Ramsey's rule for optimal taxation. However, the opposite conclusion would have been drawn if one rather based the analysis on the homogeneous & symmetric model,

[38]The two analyses are not directly comparable as Raknerud, Skjerpen and Swensen (2007) augment their models with stochastic trends. However, augmenting the static LA/AID system with (linear) trends *lowers* the expenditure elasticity of vehicle running costs, which further strengthens the results derived in this section.

which in fact classifies vehicle running costs as reasonably price elastic and should thus face low taxes according to this rule. This demonstrates the potentially detrimental consequences for policy analyses of adopting inappropriate models that are perhaps supported by theory, but not by data. Various calculations and regressions should therefore be preceded by general tests for functional form misspecification. Examples of such tests are given in the next section, where we turn attention to applications of (linearized) dynamic expenditure systems.

4.3 Dynamic Expenditure Systems

The remainder of the analysis draws a distinction between *annual* and *quarterly* models. Annual and quarterly models will refer to expenditure system specifications that are applied to annual and quarterly Norwegian national accounts data, respectively.

It is well-known that significant diagnostic tests are often indications of a misspecified model, suggesting that we should initially interpret them as specification tests. In order to weaken the chance of adopting an inappropriate model, let us initiate this section on dynamic expenditure systems by inspecting the residuals from our systems of regression equations for serial correlation and heteroskedasticity.[39] Table 4.4 displays single equation tests of residuals for autoregressive (AR) serial correlation of order (1-5) for the quarterly unconstrained and homogeneous static model discussed in Section 4.2. For comparison, Table 4.5 gives equivalent tests for the annual static LA/AID systems.[40] With the exception of other services, absence of autocorrelation is rejected at a 5 percent level for all categories in the quarterly unconstrained model, and for the entire quarterly homogeneous expenditure system.

Table 4.5 replicates some of these results for the corresponding annual models. Absence of autocorrelated errors is now rejected at a 5 percent level for clothing & shoes in the annual unconstrained model, and for clothing & shoes, other services, vehicle running costs and consumption abroad in the annual homogeneous model. Then again, all commodities but beverages and public transport, mail & telecommunications are significant at a 10 percent level in the latter system. This complicates the analysis as the presence of serial correlation invalidates our second error term assumption, which means

[39] Cf. Section A.2 for tests for normality of the error terms. Absence of normality is rejected for all consumption categories in the annual homogeneous static LA/AID system.

[40] AR-tests may or may not contain an intercept. t and F-statistics are asymptotically valid either way (Wooldridge, 2009, ch. 12.2). Intercepts were included in both AR(q) regressions presented in Table 4.4 and Table 4.5 but have been omitted from the tables.

Table 4.4: Single Equation Tests of Residuals for AR(1-5) Serial Correlation with Quarterly Data[a]

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	AR(1-5)[b]	AR(1-5)[c]
Food	−0.031	0.014	−0.053	0.264	0.018	2.45	3.09
	(−0.3)[d]	(0.2)	(−0.6)	(3.2)	(0.2)	(0.0380)[e]	(0.0119)
Beverages	0.060	−0.137	0.109	0.241	−0.001	3.49	4.02
	(0.7)	(−1.6)	(1.3)	(2.8)	(0.0)	(0.0057)	(0.0021)
Tobacco	−0.218	−0.077	−0.095	0.352	0.066	5.88	7.96
	(−2.4)	(−0.9)	(−1.1)	(4.1)	(0.7)	(0.0001)	(0.0000)
Energy	0.132	−0.222	−0.081	0.412	−0.142	9.55	10.11
	(1.5)	(−2.8)	(−1.0)	(5.1)	(−1.6)	(0.0000)	(0.0000)
Clothing & Shoes	0.115	0.209	−0.101	0.344	−0.130	8.42	12.28
	(1.3)	(2.5)	(−1.2)	(4.2)	(−1.5)	(0.0000)	(0.0000)
Other Products	0.385	0.063	−0.269	0.497	−0.356	12.17	17.28
	(4.5)	(0.8)	(−3.4)	(5.9)	(−4.1)	(0.0000)	(0.0000)
Other Services	0.103	0.069	0.121	0.128	0.025	1.99	9.16
	(1.1)	(0.8)	(1.3)	(1.4)	(0.3)	(0.0847)	(0.0000)
Vehicle Running Costs	0.342	−0.180	0.145	0.311	−0.174	8.61	37.82
	(3.9)	(−2.0)	(1.7)	(3.6)	(−2.0)	(0.0000)	(0.0000)
Public Transport, Mail & Telecommunications	0.241	0.095	−0.185	0.416	−0.295	9.22	9.65
	(2.8)	(1.2)	(−2.3)	(5.3)	(−3.5)	(0.0000)	(0.0000)
Consumption Abroad	0.065	−0.099	0.020	0.378	−0.110	5.11	6.85
	(0.7)	(−1.2)	(0.2)	(4.4)	(−1.2)	(0.0003)	(0.0000)

[a] Estimation is based on Norwegian national accounts data for the period 1979Q2-2011Q3. The test for AR(1-5) serial correlation is based on the following model: $\varepsilon_t = \rho_1\varepsilon_{t-1} + ... + \rho_5\varepsilon_{t-5} + e_t$, where e_t denotes an error term with classical properties. The null-hypothesis of absence of autocorrelation is given by: $H_0 : \rho_1 = 0, \ldots, \rho_5 = 0$.

[b] $F(5, 115)$-test for AR(1-5) serial correlation in the unconstrained static LA/AID system (cf. Table 4.1 for the estimation results).

[c] $F(5, 115)$-test for AR(1-5) serial correlation in the homogeneous static LA/AID system (cf. Table A.3 for the estimation results).

[d] Values in parentheses in columns $1 - 5$ are t-statistics.

[e] Values in parentheses in the final two columns are p-values.

Barten (1969)'s result no longer applies. Berndt and Savin (1975) show that maximum likelihood estimates and likelihood ratio test statistics will be conditional on the omitted equation in the event of autocorrelation in the errors. They propose using *quasi-differenced data* when tests show sign of serially correlated errors of order one, which for the annual model would be: $\tilde{\mathbf{w}}_t \equiv \mathbf{w}_t - \mathbf{R}\mathbf{w}_{t-1}$ and $\tilde{\mathbf{z}}_t \equiv \mathbf{z}_t - \mathbf{R}\mathbf{z}_{t-1}$, where $\mathbf{R}$ is a square matrix of unknown parameters in the following equation:

$$\boldsymbol{\varepsilon}_t = \mathbf{R}\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\nu}_t,\ t = 2, \ldots, T \tag{4.18}$$

and where $\boldsymbol{\nu}_t$ is an $n \times 1$ column vector of disturbances with classical properties. $\boldsymbol{\varepsilon}_t$ still denotes the error term in (4.4), which means we regain our benchmark model by setting $\mathbf{R} = \mathbf{0}$. Next, subtraction of $\mathbf{R}\mathbf{w}_{t-1} = \mathbf{R}\boldsymbol{\alpha} + \mathbf{R}\boldsymbol{\Theta}z_{t-1} + \mathbf{R}\boldsymbol{\varepsilon}_{t-1}$ on both sides of (4.4), followed by substitution of (4.18) into this rearranged equation, yields:

$$\tilde{\mathbf{w}}_t = \mathbf{R}\mathbf{w}_{t-1} + \boldsymbol{\Theta}z_t - \mathbf{R}\boldsymbol{\Theta}z_{t-1} + \boldsymbol{\nu}_t,\ t = 2, \ldots, T \tag{4.19}$$

Table 4.5: Single Equation Tests of Residuals for AR(1-2) Serial Correlation with Annual Data[a]

	ρ_1	ρ_2	AR(1-2)[b]	AR(1-2)[c]
Food	0.254 (1.4)[d]	−0.326 (−1.8)	2.17 (0.1414)[e]	3.28 (0.0598)
Beverages	−0.054 (−0.3)	−0.097 (−0.5)	0.16 (0.8511)	0.31 (0.7377)
Tobacco	−0.076 (−0.4)	−0.156 (−0.9)	0.71 (0.5056)	3.15 (0.0659)
Energy	−0.064 (−0.3)	−0.244 (−1.4)	1.23 (0.3147)	2.72 (0.0917)
Clothing & Shoes	0.354 (2.1)	−0.454 (−2.7)	4.24 (0.0299)	15.67 (0.0001)
Other Products	−0.018 (−0.1)	−0.380 (−2.1)	2.49 (0.1095)	3.39 (0.0550)
Other Services	0.273 (1.6)	−0.370 (−2.1)	2.51 (0.1079)	7.14 (0.0049)
Vehicle Running Costs	−0.075 (−0.4)	−0.209 (−1.1)	0.61 (0.5542)	19.76 (0.0000)
Public Transport, Mail & Telecommunications	−0.260 (−1.4)	−0.234 (−1.3)	1.39 (0.2735)	1.73 (0.2045)
Consumption Abroad	−0.092 (−0.5)	−0.147 (−0.8)	0.39 (0.6845)	10.10 (0.0010)

[a] Estimation is based on Norwegian national accounts data for the years 1980-2010. The test for AR(1-2) serial correlation is based on the following model: $\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + e_t$, where e_t denotes an error term with classical properties. The null-hypothesis of absence of autocorrelation is given by: $H_0 : \rho_1 = 0, \rho_2 = 0$.
[b] $F(2, 19)$-test for AR(1-2) serial correlation in the unconstrained static LA/AID system.
[c] $F(2, 19)$-test for AR(1-2) serial correlation in the homogeneous static LA/AID system.
[d] Values in parentheses in the first two columns are t-statistics.
[e] Values in parentheses in the final two columns are p-values.

where I have omitted the intercept term for notational simplicity.[41] Berndt and Savin (1975) suggest estimating $\mathbf{R}$ and $\boldsymbol{\Theta}$ by maximum likelihood (where again we have to delete one equation to avoid perfect collinearity). Although this approach yields coefficient estimates and likelihood ratio test statistics that are invariant to the omitted equation whenever the disturbance term takes the form in (4.18), it suffers from some obvious practical limitations. The approach is best suited for annual models with autocorrelated errors of order one. Application of the approach quickly becomes problematic whenever the serial correlation takes on more complex forms. As Table 4.4 made clear, the quarterly static LA/AID system suffers from at least AR(1-5) serial correlation, suggesting that a simple error term replacement of the form $\boldsymbol{\varepsilon}_t = \mathbf{R}\boldsymbol{\varepsilon}_{t-4} + \boldsymbol{\nu}_t$ will be insufficient to correct for the autocorrelation. This conclusion is further supported by Table A.8, which shows sign of autore-

[41] $\mathbf{R}$ can take on all kinds of forms: diagonal, triangular, symmetric, etc. The formula in (4.19) applies equally in all cases.

gressive conditional heteroskedasticity (ARCH) of order 1-4 for half of the consumption categories at a 5 percent level. As hinted at in the beginning of this section, these tests might simply indicate that the static model is misspecified, suggesting that a reformulated dynamic specification incorporating e.g. higher-order lags of the dependent variable might eliminate the presence of autocorrelation. However, correcting for higher-order serial correlation through exaggerated adoption of lags will have adverse consequences for the system's overall degrees of freedom, which can further have detrimental efficiency implications for our estimators. I will therefore leave the quarterly data set for now and devote the rest of this section to applications of dynamic expenditure systems to annual data. Attention will be restricted to systems with identical right-hand side variables in all equations, which means coefficient estimates and likelihood ratio test statistics will remain invariant to the omitted equation. I will further employ heteroskedasticity and autocorrelation consistent standard errors to correct for arbitrary higher-order autocorrelation and heteroskedasticity that might be present in the residuals of the models. We make our inference robust to arbitrary heteroskedasticity not just because of the positive ARCH(1-4) tests referred to earlier, but also because of the White tests for heteroskedasticity reported in Table A.8, which reject the null-hypothesis of homoskedastic errors at a 5 percent level for food, tobacco, energy and clothing & shoes in the quarterly homogeneous model.[42]

Before we present the dynamic LA/AID system in first differences, let us test whether the budget shares' time series are best viewed as being (covariance) stationary or integrated of order one, I(1). This can be tested by augmented Dickey-Fuller (ADF) tests for I(1) against I(0): $\Delta w_{it} = \varpi_i + \mu_i t + \vartheta_i w_{it-1} + \sum_{k=1}^{K} \varrho_{ik} \Delta w_{it-k} + u_{it}$, where t and u_{it} are (linear) trends and white noise error terms, respectively.[43] Rejection of $H_0 : \vartheta_i = 0$ implies that w_{it} is I(0). I(1) is rejected at a 10 percent level for food, beverages, energy, vehicle running costs and public transport, mail & telecommunications, and for all categories in our expenditure system but clothing & shoes and other services at a 15 percent level. Hence, with the possible exception of clothing & shoes

[42]Table A.8 shows that heteroskedasticity is less of a concern in the annual models, with homoskedasticity and absence of ARCH(1) non-rejected for all consumption categories but food in our expenditure system. Heteroskedasticity consistent standard errors are nevertheless employed to correct for arbitrary higher-order heteroskedasticity. Unlike usual t-statistics which have *exact* t-distributions under the null, robust standard errors and t-statistics are justified only asymptotically (Wooldridge, 2009, ch. 8.2).

[43]Critical values are conditional on whether a constant and/or trend is included in the regression, which can optionally be excluded from the ADF-test (cf. e.g. Wooldridge (2009, ch. 18.2) for detailed accounts).

and other services, our budget shares' time series do not appear to be non-stationary. Note, however, that the ADF-test is known to have *low power* for alternatives different from, but close to, the null of unity, which means the test often fails to distinguish near-integrated processes from non-stationary ones (Banerjee et al., 1993, ch. 4). As $0 < w_{it} < 1$ for all i and t, the budget shares will necessarily satisfy asymptotic stationarity. Near-integrated processes can nevertheless behave like I(1) processes for finite sample sizes. Therefore, due to the low power of the ADF-test, coupled with the non-trivial potential for *measurement error* in the explanatory variables e.g. brought about by typographical errors when the data were first recorded, I will continue to treat all dependent variables as covariance stationary for the rest of the analysis.

4.3.1 Dynamic LA/AID System in First Differences

Let us initiate our section on dynamic expenditure systems by analyzing the dynamic LA/AID system in first differences (cf. e.g. Deaton and Muellbauer (1980*a*)). This model can be derived by subtracting $\mathbf{w}_{t-1}$ on both sides of (4.4):

$$\begin{aligned} \mathbf{w}_t - \mathbf{w}_{t-1} = \boldsymbol{\alpha} + \boldsymbol{\Theta}\mathbf{z}_t + \boldsymbol{\varepsilon}_t - \boldsymbol{\alpha} - \boldsymbol{\Theta}\mathbf{z}_{t-1} - \boldsymbol{\varepsilon}_{t-1} \\ \Delta\mathbf{w}_t \equiv \boldsymbol{\Theta}\Delta\mathbf{z}_t + \boldsymbol{\epsilon}_t,\ t = 2, \ldots, T \end{aligned} \tag{4.20}$$

where Δ indicates the first difference and $\boldsymbol{\epsilon}_t \equiv \boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_{t-1}$. It is well-known that when dealing with highly persistent data it is often a good idea to first-difference the model so as to eliminate most of the serial correlation. This alternative dynamic specification might therefore ease the problem of autocorrelation that we encountered in the static models. As is evident from (4.20), the $\boldsymbol{\alpha}$'s drop out when taking first differences, which means the system does not include a vector of intercepts. It is nevertheless frequently included in applied work to capture possible trending behavior, with significant constant terms indicating a presence of linear trends. As Table 4.6 makes clear, only beverages has a significant non-zero estimate of the intercept term at the 5 percent level, suggesting that trending behavior plays a minor role in the expenditure system. However, Wald tests for multiple exclusion restrictions demonstrate that the $\grave{\alpha}_i$ estimates are *jointly* statistically significant with a p-value of 0.0000, and intercept terms have therefore been included in all equations. Before we analyze the output in Table 4.6, it must be noted that economic theory still imposes a number of restrictions on the parameters of the expenditure system. The adding-up condition in (4.10) is replaced by: $\sum_i \grave{\alpha}_i = 0, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0$ for the dynamic LA/AID system in first differences. Homogeneity and Slutsky symmetry, on the other hand, are still as given in (4.11) and (4.13).

Recall from our analysis in Section 4.2 that food, beverages and vehicle running costs were necessities, and clothing & shoes, other products and consumption abroad were luxuries to Norwegian households according to the benchmark model (with the remaining four categories left unclassified due to insignificant β_i estimates, cf. Table 4.1). Nearly identical conclusions can be drawn based on the findings in Table 4.6, where we see that food and vehicle running costs are still classified as necessities, and clothing & shoes, other products and consumption abroad are still categorized as luxuries. Note that beverages is no longer significant at the 5 percent level, although energy is now classified as a necessity good.

Close examination of the estimation result reveals that only energy and vehicle running costs have significant γ_{ii} estimates in the dynamic LA/AID system in first differences, as opposed to all categories but tobacco and other products in the unconstrained static LA/AID system, which indicates an overall drop in estimation precision compared to the benchmark model.[44] This is largely a result of the fact that the two models are applied to different data sets, with the dynamic and static models being fit to annual and quarterly data, respectively.

Tests for long run homogeneity show that absence of money illusion is now only rejected for food in this alternative dynamic expenditure system. Recall that absence of money illusion was rejected at a 5 percent level for all commodities but beverages, energy and public transport, mail & telecommunications in the benchmark model, and that global homogeneity was clearly rejected with a test statistic of 125.32. A test for global homogeneity in the dynamic LA/AID system in first differences, on the other hand, yields a likelihood ratio statistic and corresponding p-value of 12.78 and 0.1729, respectively. These results replicate the findings of Deaton and Muellbauer (1980 *a*), who also saw the sub-sample of non-rejected homogeneous commodities expand after first-differencing the expenditure system. The findings thus clearly supports the imposition of homogeneity in this alternative dynamic expenditure system. Further evidence of this is also found in Table 4.7, where we see that the unconstrained and homogeneous model yield more similar own-price

[44] The variance formula derived under homoskedasticity is not valid when heteroskedasticity is present, which means the t-statistics reported in Table 4.1 cannot be used for inference. However, qualitatively similar results are obtained when heteroskedasticity and autocorrelation consistent standard errors are employed for all categories but consumption abroad (i.e., all categories but tobacco, other products and consumption abroad have significant γ_{ii} estimates at the 5 percent level in the benchmark model when heteroskedasticity and autocorrelation consistent standard errors are employed).

Table 4.6: Unconstrained Parameter Estimates and Test for Homogeneity in the Dynamic LA/AID System in First Differences[a]

Commodity i	$\grave{\alpha}_i$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	γ_{i10}	$\sum_j \gamma_{ij}$[b]
ΔFood	0.003	−0.099	0.040	−0.002	−0.023	−0.012	−0.031	0.023	−0.049	0.010	−0.009	−0.001	5.45
	(1.7)[c]	(−2.6)	(1.3)	(−0.1)	(−2.4)	(−3.0)	(−2.0)	(0.9)	(−1.1)	(1.2)	(−0.4)	(−0.1)	(0.0196)[d]
ΔBeverages	0.002	−0.019	0.014	0.005	−0.011	−0.004	−0.013	0.049	−0.037	−0.001	−0.011	−0.014	2.39
	(2.5)	(−1.7)	(0.7)	(0.5)	(−2.7)	(−1.6)	(−1.6)	(2.3)	(−2.5)	(−0.4)	(−1.4)	(−1.6)	(0.1218)
ΔTobacco	−0.001	−0.012	0.019	−0.012	0.004	0.002	0.004	0.000	0.020	−0.003	−0.018	0.000	1.40
	(−0.5)	(−1.3)	(2.3)	(−1.1)	(0.7)	(0.9)	(0.7)	(0.0)	(1.3)	(−1.9)	(−2.9)	(−0.1)	(0.2366)
ΔEnergy	−0.001	−0.033	−0.015	−0.001	0.000	0.051	−0.013	0.034	−0.013	−0.009	−0.005	−0.003	2.06
	(−0.6)	(−2.4)	(−0.6)	(−0.1)	(0.1)	(7.9)	(−0.5)	(0.7)	(−0.5)	(−1.5)	(−0.3)	(−0.1)	(0.1513)
ΔClothing & Shoes	−0.003	0.061	−0.011	0.004	−0.013	−0.008	−0.002	0.025	0.028	−0.003	−0.004	−0.002	0.38
	(−1.8)	(2.6)	(−0.4)	(0.2)	(−2.0)	(−2.7)	(−0.1)	(0.9)	(0.7)	(−0.5)	(−0.3)	(−0.2)	(0.5358)
ΔOther Products	−0.001	0.047	−0.030	0.036	−0.002	−0.011	0.036	−0.056	0.048	−0.004	−0.019	−0.001	0.00
	(−0.3)	(2.0)	(−0.7)	(1.3)	(−0.2)	(−1.5)	(1.7)	(−1.1)	(1.3)	(−0.3)	(−0.7)	(0.0)	(0.9545)
ΔOther Services	0.002	0.025	−0.003	0.010	0.008	−0.004	0.035	−0.026	0.047	−0.011	−0.015	−0.020	0.22
	(0.7)	(0.6)	(−0.1)	(0.3)	(0.8)	(−0.5)	(1.1)	(−0.5)	(0.7)	(−0.7)	(−0.6)	(−1.5)	(0.6404)
ΔVehicle Running Costs	0.000	−0.042	0.001	−0.019	−0.001	−0.002	0.000	0.023	−0.023	0.045	−0.006	0.003	1.19
	(0.0)	(−2.4)	(0.0)	(−1.5)	(−0.2)	(−0.4)	(0.0)	(1.2)	(−1.0)	(10.8)	(−0.4)	(0.3)	(0.2751)
ΔPublic Transport, Mail & Telecommunications	0.000	−0.016	−0.016	−0.014	−0.001	−0.012	0.026	−0.066	0.073	−0.003	0.027	0.010	0.63
	(0.1)	(−0.5)	(−0.6)	(−1.2)	(−0.1)	(−3.9)	(2.0)	(−2.8)	(1.5)	(−0.7)	(1.5)	(0.9)	(0.4259)
ΔConsumption Abroad	−0.002	0.088	0.002	−0.007	0.040	0.000	−0.041	−0.007	−0.094	−0.019	0.060	0.028	0.98
	(−0.7)	(4.0)	(0.1)	(−0.3)	(3.7)	(0.0)	(−2.1)	(−0.1)	(−2.7)	(−2.0)	(1.6)	(1.4)	(0.3223)

[a] Estimation is based on annual Norwegian national accounts data for the years 1979-2010. The dependent variables are first differenced budget shares, i.e., $\Delta w_{it} = w_{it} - w_{it-1}$.

[b] Test for homogeneity.

[c] Values in parentheses in columns $1 - 12$ are heteroskedasticity and autocorrelation consistent t-statistics.

[d] Values in parentheses in the final column are p-values.

and expenditure elasticities than the corresponding two static systems did in Section 4.2 (cf. Table 4.3). Although both systems yield qualitatively similar results for all but two categories, comparison of the tables nevertheless demonstrates that the own-price elasticities of demand tend to be somewhat larger in absolute value in the model in first differences than in the static systems.[45] Similar results are obtained by Deaton and Muellbauer (1980 *a*) on annual British data from 1954 to 1974. Note lastly that simultaneous imposition of homogeneity and symmetry in the LA/AID system in first differences is with its log-likelihood ratio statistic and corresponding p-value of 65.23 and 0.0256 still rejected by the data.

Table 4.7: Own-price and Expenditure Elasticities for the Dynamic LA/AID System in First Differences

	Unconstrained		Homogeneous	
	$\eta_{i,i}$	e_i	$\eta_{i,i}$	e_i
Food	-0.70	0.49	-0.81	0.56
Beverages	-0.91	0.69	-0.93	0.78
Tobacco	-0.86	0.60	-0.88	0.48
Energy	-0.08	0.43	-0.09	0.32
Clothing & Shoes	-1.08	1.69	-1.07	1.65
Other Products	-1.46	1.35	-1.46	1.36
Other Services	-0.83	1.11	-0.91	1.08
Vehicle Running Costs	-0.22	0.31	-0.23	0.22
Public Transport, Mail & Telecommunications	-0.60	0.77	-0.53	0.69
Consumption Abroad	-0.65	2.40	-0.59	2.54

Notes: Calculations are based on annual Norwegian national accounts data for the years 1979-2010. Elasticities are based on static long run solutions and are calculated at mean annual budget share values for the entire sample.

Before we move on to the dynamic LA/AID system incorporating habits, let us estimate another expenditure system where first differenced budget shares appear as dependent variables. The dynamic specification is inspired by Anderson and Blundell (1983) and follows in the spirit of the ECMs:

$$\Delta \mathbf{w}_t = \mathbf{\Theta} \Delta \mathbf{z}_t - \mathbf{\Omega}[\mathbf{w}_{t-1} - \boldsymbol{\alpha} - \mathbf{\Theta} \mathbf{z}_{t-1}] + \boldsymbol{\varepsilon}_t,\ t = 2, \ldots, T \tag{4.21}$$

where $\mathbf{\Omega}$ is an $n \times n$ square matrix of adjustment parameters and the term in square brackets is an $n \times 1$ column vector of lagged error terms, $\boldsymbol{\varepsilon}_{t-1}$ (cf.

[45] Other services (consumption abroad) is classified as price elastic and inelastic (price inelastic and elastic) in the homogeneous and unconstrained static model, respectively, but price inelastic in both dynamic specifications.

(4.3)). Unfortunately, (4.21) is not estimable as the ε_{it-1}'s on the right-hand side are perfectly collinear. Following Anderson and Blundell (1983), (4.21) will be replaced by:

$$\Delta \mathbf{w}_t^n = \mathbf{\Theta}^n \Delta \mathbf{z}_t - \mathbf{\Omega}^n \boldsymbol{\varepsilon}_{t-1}^n + \boldsymbol{\varepsilon}_t^n,\ t = 2, \ldots, T \tag{4.22}$$

where the superscript indicates that the n^{th} row has been omitted. Although the homogeneity and Slutsky symmetry conditions are as defined earlier, adding-up now implies the following constraints: $\sum_i \breve{\alpha}_i = 0, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0, \sum_i \omega_i = 0$, where ω_i is the coefficient of ε_{it-1} (details have been omitted but are outlined in Anderson (1980) and Anderson and Blundell (1982)).

Unlike log-likelihood values and likelihood ratio statistics, the ω_i estimates are not invariant to the omitted row in (4.22) (although they are invariant to the deleted equation). In my opinion, practitioners should avoid employing non-invariant system estimation techniques such as the one just presented as their (arbitrary) choice of omitted row may yield severely incorrect results, and might in fact both favor and disfavor their hypotheses. Therefore, due to the identification problem and related non-invariance of the ω_i estimates, I will not elaborate further on this alternative dynamic specification. The ECM is mainly included here because it obtained the highest log-likelihood value of all the annual models that were applied to Norwegian aggregated household consumption data in this analysis (cf. Table A.11).

4.3.2 Dynamic LA/AID System Incorporating Habits

Finally, we consider an application of the dynamic LA/AID system incorporating habits. Alessie and Kapteyn (1991) show that Blanciforti and Green (1983)'s model does not satisfy the theoretical requirements of utility theory that were discussed in Section 4.2. They demonstrate that (3.16) either implies habit formation and violation of adding-up, or adding-up and no persistence in consumption patterns (details have been omitted but are outlined in Alessie and Kapteyn (1991)). The model discussed in Section 3.2.1 will therefore be replaced by the following expenditure system:

$$w_{it} = \tilde{\alpha}_i + \sum_j \varsigma_{ij} log\, q_{jt-1} + \sum_j \gamma_{ij} log\, p_{jt} + \beta_i log\{x_t/\breve{P}_t\},\ t = 2, \ldots, T \tag{4.23}$$

where we have augmented the previous equation by the term $\sum_{j \neq i} \varsigma_{ij} log\, q_{jt-1}$.[46] Note that we have also taken logarithms of lagged consumption to better fit

[46](4.23) captures *myopic* rather than *rational* habit formation, as households are assumed to be backward but not forward looking (i.e., present consumption is affected by previous consumption choices but is not assumed to influence future preferences) (Pashardes, 1986).

the data. The homogeneity and Slutsky symmetry constraints are still as given before, though adding-up is now replaced by: $\sum_i \tilde{\alpha}_i = 1, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0, \sum_i \varsigma_{ij} = 0$. Unlike (3.16), (4.23) maintains an identical right-hand side structure in all equations. This simplifies the estimation as FIML degenerates to single equation OLS regressions (cf. Section 4.2), which further ensures that estimates will be automatically invariant to the omitted equation. Finally, tests for habit formation are replaced by the following multiple hypotheses test: $H_0 : \varsigma_{i1} = 0, \ldots, \varsigma_{i10} = 0 \, \forall i$ against H_1 : at least one of $\varsigma_{ij} \neq 0$, with rejection of the null yielding non-rejection of our hypothesis of habit formation.

For analytical reasons it is important that the adopted expenditure system is derived from an explicit cost or utility function.[47] Although a number of models that do not represent integrable demand systems might fit the data better than systems that actually do, they are generally of limited interest due to their unsuitability for forecasting, policy analyses and testing of economic theories. Consider for instance the inclusion of lagged dependent variables as regressors in our models. Although this has been suggested in the literature (cf. e.g. Blanciforti, Green and King (1986)), I do not find it an appropriate alternative for one important reason: agents choose quantities, not budget shares. To illustrate this point, consider the following single equation regression for vehicle running costs:

$$w_{it} = 0.215w_{it-1} - 0.127w_{it-2} + 0.059w_{it-3} + 0.321w_{it-4} - 0.200w_{it-5}$$

where I have omitted the intercept term, prices, deflated income and seasonal dummies for notational simplicity. All lags but w_{it-2} and w_{it-3} are significant at 5 percent levels in this equation, which was applied to quarterly Norwegian national accounts data for the period $1979Q2 - 2011Q3$. Based on these results one might be tempted to conclude that consumption today is affected by consumption 5 quarters ago. But this would be an imprecise statement, as our findings only suggest that the *budget share* today is influenced by the category's expenditure share a little more than a year ago. Because $w_{it} = q_{it}p_{it}/x_t$ we cannot state categorically that demand is affected by previous consumption choices based solely on the results from this regression, as the relationship between w_{it} and, say, w_{it-4}, might simply pick up a relationship between the prices and/or total expenditure at time t and $t-4$.

Before we present the estimation results, let us therefore prove that (4.23) does represent an integrable demand system. Ray (1984) shows that by

[47] Cf. Lau (1977) for an excellent review of the three principal approaches to generating complete systems of consumer demand functions.

replacing (3.2) with:

$$log\,\{a(p)\} = \tilde{\alpha}_0 + \sum_k \varsigma_{jk} log\, q_{kt-1} \sum_k \tilde{\alpha}_k log\, p_{kt} + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} log\, p_{kt} log\, p_{jt} \tag{4.24}$$

followed by substitution of (3.3) and (4.24) into (3.1), we obtain the following cost function:

$$\begin{aligned} log\, c(u,p) = \tilde{\alpha}_0 + \sum_k \varsigma_{jk} log\, q_{kt-1} \sum_k \tilde{\alpha}_k log\, p_{kt} \\ + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} log\, p_{kt} log\, p_{jt} + u_t \beta_0 \prod_k p_{kt}^{\beta_k} \end{aligned} \tag{4.25}$$

Subsequent utilization of Shephard's Lemma, followed by imposition of $\gamma_{kj} \equiv 1/2(\gamma^*_{kj} + \gamma^*_{jk})$ and reformulations of the derived equations (cf. Chapter 3), eventually gives rise to the expression in (4.23) with the price index denoted by $\breve{P}_t$. However, for econometric reasons we will continue to employ Stone's share-weighted price index also in this section. In his analysis, Ray (1984) further augments the price index with lagged aggregate expenditure. He also gave an early example of how to incorporate demographic factors such as family size and composition in the AID system (Ray, 1980). Although neither of these approaches are taken in this analysis, they clearly represent interesting extensions for future research.

Table 4.8 summarizes the results from the unconstrained estimation of the dynamic LA/AID system incorporating habits.[48] Again we see a significant drop in estimation precision compared to the benchmark system, which as mentioned earlier is largely brought about by the fact that we apply the dynamic models to annual rather than quarterly data. Lack of significance of the β_i estimates implies that some of the calculated income elasticities in Table 4.9 may be very misleading, with e.g. categories such as food, beverages, tobacco, energy, other services, vehicle running costs and consumption abroad having expenditure elasticities insignificantly different from unity. Coupled with the imprecise estimation of the γ_{ii}'s, this further suggests that groups such as food, tobacco, other services and consumption abroad might be unit elastic. This general lack of estimation precision calls into question the model's suitability for policy analyses, which is unfortunate considering its superior data fit compared to the other expenditure systems (cf. Section A.4, which shows that the dynamic LA/AID system incorporating habits is

[48]Cross-price effects, $\sum_{j\neq i} \gamma_{ij} log\, p_{jt}$, were included in the regression but have been omitted from Table 4.8.

Table 4.8: Unconstrained Parameter Estimates and Test for Habit Formation and Homogeneity in the Dynamic LA/AID System Incorporating Habits[a]

Commodity i	$\tilde{\alpha}_i^*$	β_i	γ_{ii}	ς_{i1}^*	ς_{i2}^*	ς_{i3}^*	ς_{i4}^*	ς_{i5}^*	ς_{i6}^*	ς_{i7}^*	ς_{i8}^*	ς_{i9}^*	ς_{i10}^*	$\sum_j \gamma_{ij}$[b]	$\sum_j \varsigma_{ij}^*$[c]
Food	1.588	−0.042	0.047	−0.011	0.014	0.002	−0.032	−0.005	0.059	−0.063	−0.021	−0.003	−0.017	1.67	33.19
	(4.5)[d]	(−1.0)	(0.8)	(−0.3)	(0.4)	(0.1)	(−0.6)	(−0.2)	(1.9)	(−1.8)	(−0.9)	(−0.2)	(−0.9)	(0.1959)[e]	(0.0003)
Beverages	0.028	−0.009	0.022	−0.014	0.001	0.015	0.027	−0.024	0.012	−0.001	−0.019	0.011	0.007	0.61	197.02
	(0.1)	(−0.9)	(1.9)	(−1.2)	(0.2)	(1.4)	(1.3)	(−1.8)	(0.8)	(−0.1)	(−2.6)	(1.1)	(0.9)	(0.4344)	(0.0000)
Tobacco	0.298	−0.010	0.001	−0.006	−0.004	0.004	−0.010	0.000	0.005	0.011	−0.003	−0.009	−0.001	0.70	10.58
	(2.2)	(−0.9)	(0.1)	(−0.5)	(−0.3)	(0.4)	(−0.6)	(0.1)	(0.4)	(0.7)	(−0.2)	(−1.7)	(−0.2)	(0.4036)	(0.3916)
Energy	0.906	0.010	0.055	−0.019	−0.015	−0.013	−0.056	0.038	−0.035	0.022	0.002	0.002	−0.022	1.72	72.93
	(4.6)	(0.6)	(12.0)	(−1.1)	(−1.1)	(−1.6)	(−2.5)	(2.8)	(−3.7)	(1.5)	(0.2)	(0.3)	(−3.0)	(0.1892)	(0.0000)
Clothing & Shoes	−0.256	0.078	0.038	0.007	0.002	0.001	−0.043	0.026	0.001	−0.034	0.001	−0.019	−0.005	0.29	71.76
	(−0.9)	(2.7)	(1.2)	(0.2)	(0.1)	(0.0)	(−1.4)	(2.0)	(0.0)	(−0.9)	(0.0)	(−1.2)	(−0.5)	(0.5906)	(0.0000)
Other Products	−0.980	0.046	−0.105	−0.004	0.061	−0.025	0.027	−0.023	0.030	−0.016	0.026	−0.014	−0.013	12.70	176.83
	(−8.1)	(2.9)	(−2.4)	(−0.2)	(3.3)	(−2.7)	(1.2)	(−2.6)	(1.8)	(−0.8)	(2.2)	(−1.6)	(−1.7)	(0.0004)	(0.0000)
Other Services	−1.127	0.007	0.051	0.062	−0.055	0.021	0.035	−0.064	−0.020	0.078	0.012	0.023	0.018	0.04	119.12
	(−2.2)	(0.1)	(0.6)	(1.0)	(−1.3)	(0.8)	(0.6)	(−3.5)	(−0.3)	(1.1)	(0.3)	(0.8)	(0.9)	(0.8492)	(0.0000)
Vehicle Running Costs	0.995	−0.029	0.050	−0.031	0.002	0.005	−0.020	0.013	0.001	−0.031	0.021	−0.004	0.000	4.62	25.00
	(4.5)	(−1.3)	(5.9)	(−1.2)	(0.1)	(0.4)	(−0.8)	(0.6)	(0.1)	(−0.9)	(1.3)	(−0.4)	(0.0)	(0.0316)	(0.0054)
Public Transport, Mail & Telecommunications	0.301	−0.050	0.020	−0.006	0.049	−0.017	0.035	−0.001	−0.013	−0.007	−0.018	0.000	0.019	0.35	45.83
	(1.3)	(−1.8)	(0.9)	(−0.2)	(1.6)	(−0.9)	(0.8)	(0.0)	(−0.6)	(−0.2)	(−0.7)	(0.0)	(1.4)	(0.5561)	(0.0000)
Consumption Abroad	−0.754	0.000	0.074	0.021	−0.054	0.007	0.036	0.040	−0.041	0.041	−0.002	0.013	0.013	4.35	27.13
	(−1.6)	(0.0)	(1.6)	(0.4)	(−2.1)	(0.4)	(0.7)	(1.2)	(−1.5)	(1.0)	(−0.1)	(0.8)	(0.7)	(0.0370)	(0.0025)

[a] Estimation is based on annual Norwegian national accounts data for the years 1979-2010.

[b] Test for homogeneity.

[c] Test for habit formation.

[d] Values in parentheses in columns $1-13$ are heteroskedasticity and autocorrelation consistent t-statistics.

[e] Values in parentheses in the final two columns are p-values.

Table 4.9: Own-price and Expenditure Elasticities for the Dynamic LA/AID System Incorporating Habits

	Unconstrained		Homogeneous	
	$\eta_{i,i}$	e_i	$\eta_{i,i}$	e_i
Food	-0.72	0.78	-0.63	0.76
Beverages	-0.64	0.86	-0.65	0.88
Tobacco	-0.96	0.65	-0.91	0.60
Energy	-0.05	1.18	-0.06	1.14
Clothing & Shoes	-0.65	1.89	-0.68	1.87
Other Products	-1.83	1.34	-1.58	1.38
Other Services	-0.80	1.03	-0.79	1.03
Vehicle Running Costs	-0.13	0.51	-0.21	0.41
Public Transport, Mail & Telecommunications	-0.67	0.29	-0.65	0.26
Consumption Abroad	0.17	1.00	0.06	1.17

Notes: Calculations are based on annual Norwegian national accounts data for the years 1979-2010. Elasticities are based on static long run solutions and are calculated at mean annual budget share values for the entire sample.

preferred to the other specifications (with the exception of the ECM) according to the likelihood dominance criterion for model selection proposed by Pollak and Wales (1991)). Due to these findings I will leave most of the elasticities in Table 4.9 uncommented, as the majority are based on insignificant estimates and hence cannot be trusted. To illustrate this last point, consider for instance consumption abroad's *positive* own-price elasticity. Its sign should not be interpreted as an indication that consumption abroad is a *giffen good* to Norwegian households, but as evidence that the elasticities are severely imprecisely estimated and hence cannot be used for policy analyses.

Judged in terms of a weighted criterion, with weight given to *both* estimation precision and log-likelihood value, it might in situations like these with presence of higher-order autocorrelation in the residuals of the quarterly models − which, as we recall from Section 4.3, complicated the application of dynamic expenditure systems to quarterly data and motivated our use of annual data − make sense to base the elasticity estimates and corresponding policy analyses on the quarterly static models, despite their many theoretical shortcomings and overall inferior data fit. Alternatively, one can apply augmented dynamic expenditure systems incorporating higher-order lags of q_{jt} to quarterly data. These regressions (where varying numbers of lags of q_{jt} were included in the equations) are not reported as they failed to improve estimation precision compared to the dynamic LA/AID system incorporating habits presented here.[49]

Despite the model's lack of significantly estimated parameters, let us nevertheless analyze the two test results reported in Table 4.8. The specification fails to reject absence of money illusion at a 5 percent level for all categories but other products, vehicle running costs and consumption abroad. This replicates some of the results we derived in Section 4.3.1 for the dynamic LA/AID system in first differences, where we also saw the sub-sample of non-rejected homogeneous goods expand once dynamic effects had been accounted for. Test for global homogeneity, on the other hand, is with its log-likelihood ratio statistic of 112.82 still clearly rejected (p-value of 0.0000). Simultaneous imposition of homogeneity and symmetry is also rejected in the system with a test statistic and corresponding p-value of 273.52 and 0.0000, respectively. The final column of Table 4.8 displays single equation Wald tests for habit formation. These results clearly support our hypothesis of habit formation, with lagged consumption only jointly insignificant in the equation for tobacco. Test for simultaneous exclusion of all q_{jt-1}'s in the system further strengthens this with a test statistic of 379.40 (p-value of 0.0000). These findings enable us to conclude that household behavior is influenced by previous consumption choices, which means consumers are likely to develop habits for certain commodities.

The dynamic LA/AID system incorporating habits will be further discussed in Chapter 5, where we turn attention to forecasting and out-of-sample performance of the model. However, before we move on to the next section, let us briefly analyze yet another dynamic expenditure system specification. The results accumulated so far in Section 4.3 suggest that consumers are unlikely to instantaneously adjust to relative price changes, and are hence unlikely to adjust to equilibrium in every period. Although this was indirectly seen in connection with the dynamic LA/AID system in first differences discussed in Section 4.3.1, let us nevertheless highlight this sluggish response to price changes by analyzing a simple distributed lag model, where we augment the benchmark model with lagged prices. With a log-likelihood value of 1819.48 the model fits the data about as well as the dynamic LA/AID system incorporating habits.[50] The inclusion of lagged prices is clearly supported

[49] A number of lags of order $5-10$ are significant at the 5 percent level in the augmented dynamic LA/AID systems incorporating higher-order lags of q_{jt}. The regressions can thus be seen to support our previous hypothesis of presence of higher-order autocorrelation in the residuals of the quarterly models (cf. Table 4.4).

[50] This regression excludes lagged deflated income, x_{t-1}/P^*_{t-1}. Including it improves the log-likelihood value to 1844.73. Test for joint significance of the x_{t-1}/P^*_{t-1}'s yields a test statistic of 34.60, with a corresponding p-value of 0.0001. Similar inclusion of lagged deflated income in the dynamic LA/AID system incorporating habits increases the log-

by data, with a test for simultaneous exclusion of all p_{jt-1}'s in the system yielding a test statistic of 365.96 (p-value of 0.0000). This specification will nevertheless not be elaborated upon in this analysis for three reasons: (i) it does not explain the data better than our preferred dynamic specification, (ii) it complicates the interpretation of the elasticities, and (iii) it prevents us from analyzing persistence in consumption patterns. The third point warrants further explanation. Based on the results derived from the dynamic LA/AID system incorporating habits and the distributed lag model, it would of course have been interesting to analyze a model incorporating lagged values of *both* sets of regressors, i.e., both q_{jt-1} and p_{jt-1}. Unfortunately, such a system is un-estimable due to our limited data availability, which means we cannot robustly test for habit formation in a system that simultaneously includes lagged prices. Due to these findings I have chosen not to elaborate further on this alternative dynamic specification.

4.4 Imposition of Theoretical Constraints in Expenditure Systems

Let us end this chapter with a brief discussion of imposing theoretical constraints such as homogeneity, Slutsky symmetry and negativity in the expenditure systems. Simultaneous imposition of symmetry and absence of money illusion is rejected at a 5 percent level for all expenditure system specifications in this analysis. The fact that systems subject to the two constraints are not supported by the data used in this analysis is likely to explain why those specifications generate elasticity estimates that are empirically implausible (elasticity estimates based on the homogeneous & symmetric dynamic specifications have been omitted but are available upon request).

Negativity is challenging to assess because it cannot be tested by means of linear restrictions on the parameters. Recall that the constraint requires $\mathbf{S}$ to be negative semidefinite (cf. (4.12)). Deaton and Muellbauer (1980 *a*) therefore suggest examining the eigenvalues of the Slutsky matrix, $s^*_{i,j} = \gamma_{ij} + \beta_i\beta_j log\{x/P\} - w_i\delta_{ij} + w_iw_j$ (where $s^*_{i,j} = p_ip_js_{i,j}/x$ and δ_{ij} denotes the Kronecker delta), with presence of positive eigenvalues interpreted as rejection of negativity. As is evident, negativity cannot be imposed globally as $s^*_{i,j}$ varies with budget shares, prices and total (non-durable) expenditure, and must consequently be examined at a particular reference point. This

likelihood value from 1826.20 to 1883.90, with an equivalent test for joint significance of the x_{t-1}/P^*_{t-1}'s yielding a test statistic of 322.42. Including lagged deflated income complicates the interpretation of the elasticities, and the approach has therefore not been followed in this analysis.

approach will not be followed in this analysis, and the negativity constraint will therefore be left unassessed.

Absence of money illusion is rejected at a 5 percent level for all expenditure system specifications but the dynamic LA/AID system in first differences discussed in Section 4.3.1. This is in fact a common finding in the literature. What is noteworthy is that few researchers, despite their numerous statistical tests, have ever concluded that consumers must be suffering from money illusion. The vast majority of practitioners interpret their tests for absence of money illusion as specification tests rather than as falsification or verification tests of consumer demand theory (Keuzenkamp and Barten, 1995), with rejection of homogeneity for instance interpreted as sign of dynamic misspecification. Rejection of homogeneity has further been traced to biased test statistics brought about by large discrepancies between the distribution under the null and the finite sample distribution of the statistic. Ng (1995) also shows that failure to account for time trends can bias the test statistics towards rejection of homogeneity in demand systems when the regressors are non-stationary. The fact that intercept terms were included in all equations in the dynamic LA/AID system in first differences to capture possible trending behavior might thus explain why that system left homogeneity non-rejected. The apparent consensus among economists that rejection of homogeneity should not lead us to question the validity of consumer demand theory is related to the fact that accepting money illusion is tantamount to suggesting that consumers are irrational. It is for that reason problematic to analyze price and income effects in systems that either do not satisfy global homogeneity or do not impose it directly at the time of estimation, as the estimated elasticities will be inherently un-interpretable.

5 Forecasting

Let us end this analysis of non-durable expenditure systems for Norwegian households by an evaluation of our preferred dynamic model's out-of-sample performance. Like the tests for homogeneity discussed in Section 4.4, the tests presented here can also be interpreted as specification tests (i.e., failure to predict the future might be the result of misspecification, and hence does not necessarily call for rejection of the underlying economic theory).[51] Figure 5.1 and Figure 5.2 display actual and fitted values from the estimation of the homogeneous dynamic LA/AID system incorporating habits for the years 1979 to 2008, coupled with *dynamic forecasts* (i.e., where also endogenous explanatory variables are forecasted) for the years 2009 to 2010.[52] The predictions are plotted with approximate 95 percent confidence intervals (shown by the vertical error bars of ±2SE).

Note that despite the system's remarkable data fit prior to 2009, it fails to accurately predict a number of our categories only a year or two into the future. This finding can be related to our previous discussion of the degree of linear relationship between the prices in our data set. Recall from Section 4.1 that the p_{jt}'s are highly collinear. This means that the prices are likely to be subject to the same set of exogenous shocks (or, to put it differently, exogenous shocks are likely to simultaneously affect *all* prices in our systems), which suggests that even trivial exogenous shocks can bring about significant volatility in the models and hence result in considerable prediction uncertainty. Before we continue, it is therefore important to note that one should not necessarily question the validity of the system's underlying economic theories or functional form based solely on its failure to predict the future, as the failure can sometimes be attributed to unforeseen shocks (Clements and Hendry, 2005).

Considering our preferred dynamic system's excellent data fit prior to the year of prediction, it is likely that its moderate forecasting performance is at least partly brought about by exogenous shocks. This finding calls into question the system's suitability for forecasting. Therefore, in order to address the challenge caused by the significant degree of linear relationship between the prices in our data set, I will leave the treatment of systems and devote the rest of the analysis to single equation predictions. Because single equation regressions do not suffer from singular variance-covariance matri-

[51]Note that forecast inaccuracies can also be the result of regime shifts. This applies even if the model is correctly specified.

[52]Due to lack of annual data I was only able to reserve two years for forecasting purposes without encountering lack of convergence problems. Note that the 2010 data are preliminary data, and are therefore potentially subject to future revisions.

Figure 5.1: Fitted Values and Dynamic Forecasts

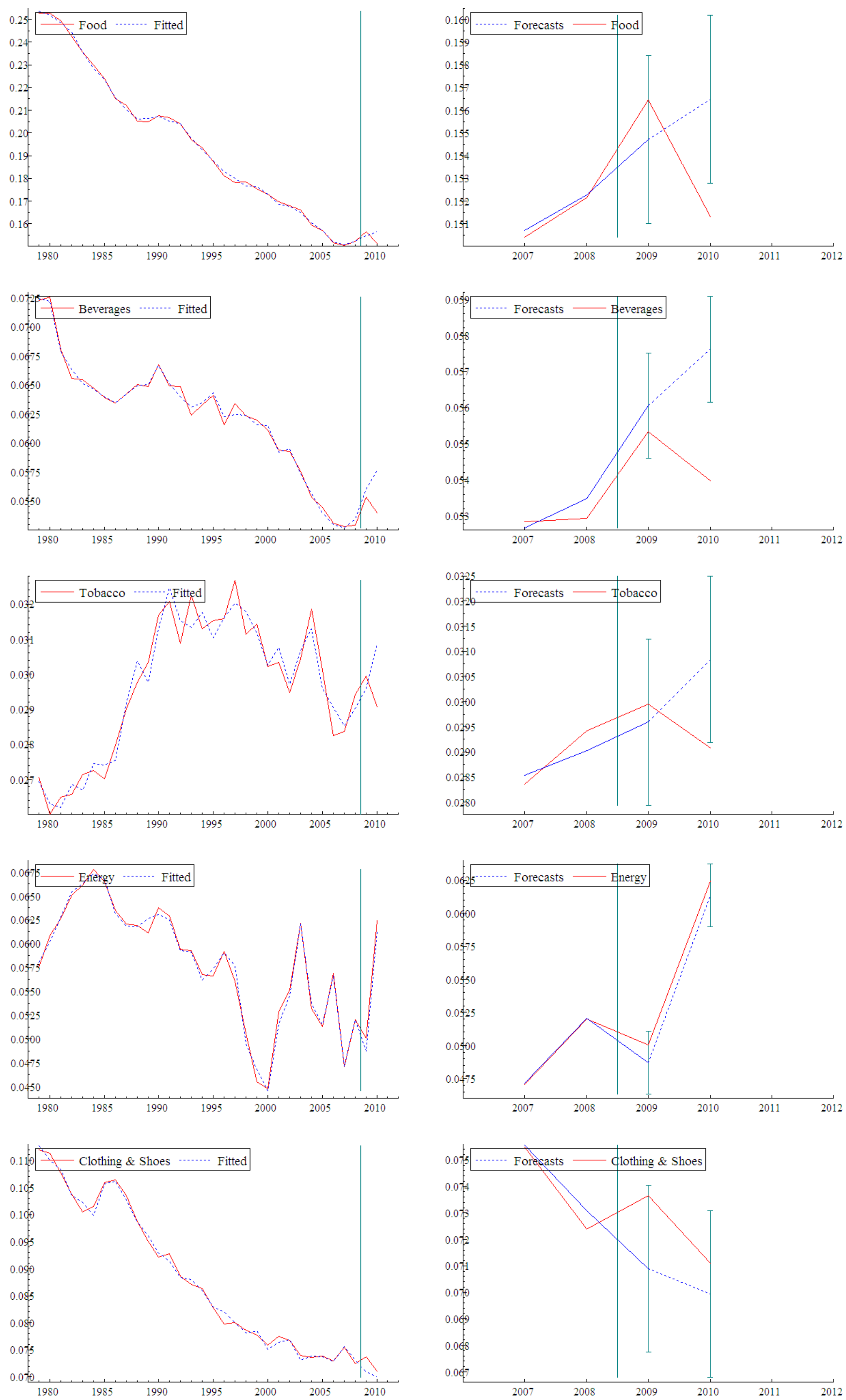
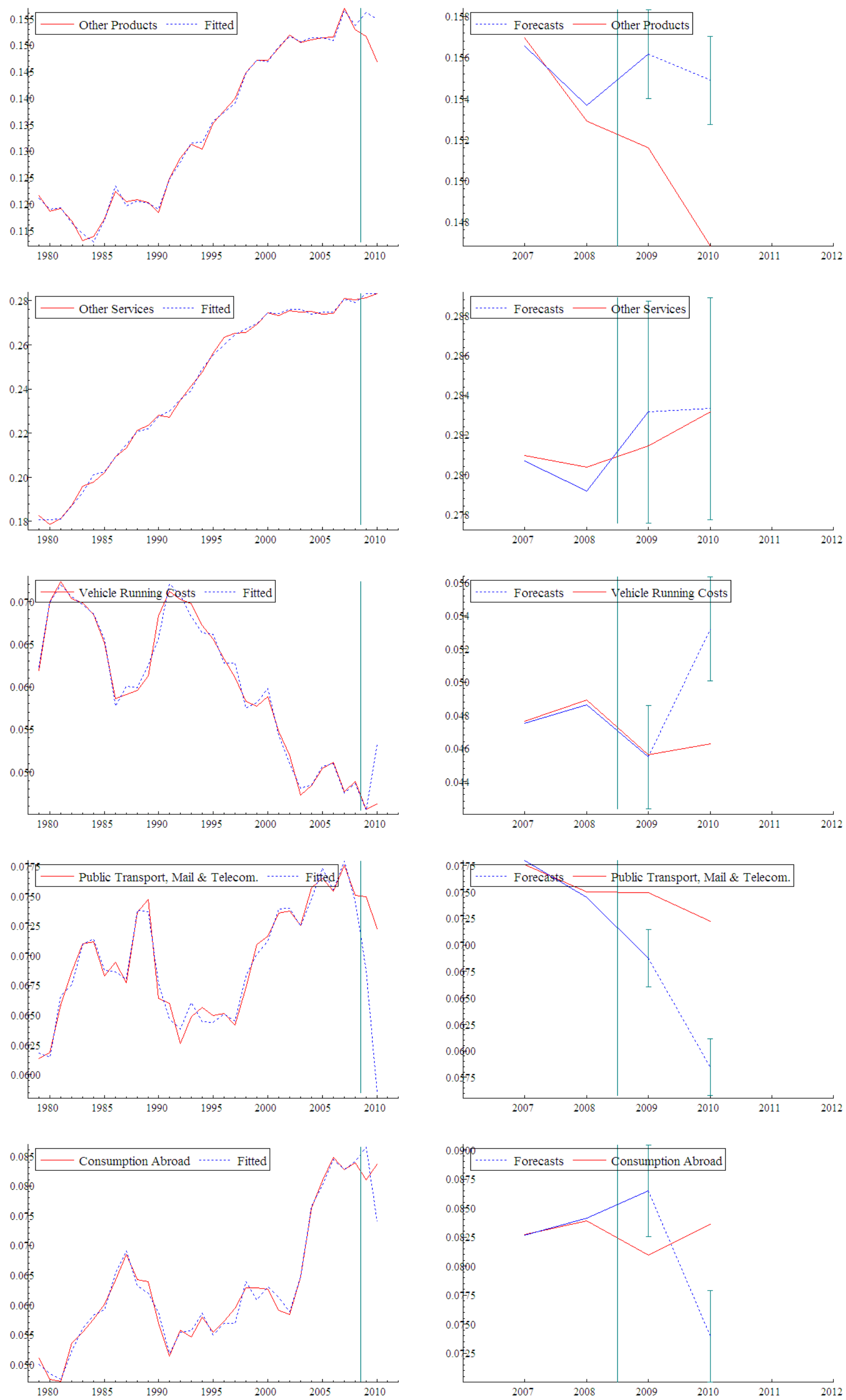


Figure 5.2: Fitted Values and Dynamic Forecasts cont.



ces (as $w_i < 0\,\forall i$), we no longer need to restrict attention to non-flexible functional forms with identical right-hand side variables in all equations to ensure that the maximum likelihood estimates are invariant to the omitted equation.[53] On the other hand, as FIML estimation and single equation OLS regressions only give rise to identical results when the errors are uncorrelated across equations and/or the equations have the same set of regressors (assuming absence of cross-equation coefficient restrictions, cf. Section 4.2), it is possible that the approach will have adverse efficiency consequences for our estimators.

The *ex-ante* (dynamic) forecasts are informally evaluated by means of mean absolute percentage error (MAPE) and root mean square error (RMSE) comparisons, which are common approaches taken in the macroeconomic literature (cf. e.g. Brandt, Freeman and Schrodt (2011) for a review of how forecasts are evaluated in a number of social sciences). The two measures of forecast accuracy are defined by $MAPE = \frac{100}{H}\sum_{h=1}^{H}\left|\frac{y_{T+h}-f_{T+h}}{y_{T+h}}\right|$ and $RMSE = \sqrt{\frac{1}{H}\sum_{h=1}^{H}(y_{T+h}-f_{T+h})^2}$, where y_{T+h}, f_{T+h} and H are the actual values, the forecasts and the forecast horizon, respectively (T still denotes sample length).[54] However, as we are analyzing dynamic forecasts, y_{T+h} must be replaced by its predicted values, $\hat{y}_{T+h}$. The accuracy of the *ex-ante* forecasts are then formally evaluated by means of Diebold-Mariano (DM) tests for equality of prediction accuracy (Diebold and Mariano, 1995). The DM-test statistic for equality of prediction *mean squared errors* can be expressed as $DM = \bar{d}/\hat{se}(\bar{d})$, where $\bar{d} = \frac{1}{B}\sum_{b=1}^{B}[e_{1b}^{2h} - e_{2b}^{2h}]$ is the sample mean loss differential and $\hat{se}(\bar{d}) \approx \sqrt{\frac{1}{B}[\hat{\gamma}_0 + 2\sum_{a=1}^{h-1}\hat{\gamma}_a]}$ is the estimated asymptotic standard error of $\bar{d}$ (Harvey, Leybourne and Newbold, 1997). Here, e_{lb}^{2h} is the squared prediction error from the h-steps ahead forecast, i.e., $(y_{lT+h} - f_{lT+h})^2$ ($l \in (1,2)$ refers to the two competing forecasts), B gives the number of pairs of h-steps ahead forecast errors (here, also equal to the number of observations in the test), and $\hat{\gamma}_0$ and $\hat{\gamma}_a$ denote the estimated variance and k^{th} covariance of d_b, respectively, where $d_b = e_{1b}^{2h} - e_{2b}^{2h}$. The statistic is asymptotically normally distributed under the null-hypothesis of no difference in prediction

[53]Attention is restricted to functional forms that do not yield perfectly collinear explanatory variables. Specifications such as (4.21) (expressed in terms of single equations) where the regressors suffer from exact linear dependence are hence automatically ruled out.

[54]Forecast errors are squared in the RMSE definition because the criterion is based on a quadratic loss function, which captures that large errors are proportionately more serious than small ones. The fact that forecast errors are squared and evaluated in terms of absolute numbers in the RMSE and MAPE definitions also capture that both over- and under-predictions are costly (Clements and Hendry, 1998, ch. 3.2).

accuracy. However, DM-tests are generally improved by using critical values from the Student's t-distribution with $b-1$ degrees of freedom (Harvey, Leybourne and Newbold, 1997), and subsequent p-values are therefore based on this distribution.

The analysis focuses on three competing forecasts (all expressed in terms of single equations): (M1) the homogeneous dynamic LA/AID system incorporating habits, (M2) the dynamic LA/AID system incorporating habits without cross-price effects, $\sum_{j \neq i} \gamma_{ij} log\, p_{jt}$, and (M3) the random walk model. M3 exemplifies what is known as "naive" forecasting, where $\hat{y}_{t+1}$ is set equal to y_t, i.e., where the variable's next period's value is assumed to be equal to its current value. M2 enables us to test the hypothesis that presence of highly collinear prices are detrimental for forecasting accuracy, with statistical support to M2 over M1 interpreted as evidence that prediction accuracy is adversely affected by cross-price effects. Let us elaborate on this last point. On average, own-price effects and deflated income account for 79.40 percent of the variation in the consumption categories' budget shares. Although inclusion of cross-price effects significantly improves the data fit, such effects might still be detrimental for prediction accuracy because they represent new sources of uncertainty in the dynamic forecast models. In other words, it is possible that the improved data fit offered by the inclusion of cross-price effects might be more than offset by the additional uncertainty they introduce in our dynamic forecast models, suggesting that prediction accuracy might in fact be improved by focusing primarily on own-price and income effects.[55]

Recall from footnote 52 that dynamic forecast horizons exceeding two years cannot be examined for the homogeneous dynamic LA/AID system incorporating habits due to lack of convergence problems. The RMSE and MAPE values for the three competing annual dynamic forecasts summarized in Table 5.1 are therefore restricted to a forecast horizon of two years. Note, however, that longer horizons would likely suffer considerably from uncertainty. To illustrate this last point, consider briefly the information set, $\mathcal{I}_{t-1}$, that we condition our predictions on. A large unexpected change in a key macroeconomic variable such as the price of oil is likely to have detrimental implications for forecasting accuracy. Unlike *ex-post* (static) predictions, *ex-ante* forecasts would continue to condition on $\mathcal{I}_{t-1}$ and would hence fail to take these changes into account. Dynamic predictions of non-durable consumption categories are therefore likely to be considerably unreliable for

[55]Chambers (1990) argues that simpler expenditure system specifications are more appropriate for forecasting and should be adopted if the objective is to predict future consumption. He further demonstrates that the LES incorporating habit formation effects is generally superior in terms of forecasting to more advanced dynamic specifications such as the ECM or the vector autoregressive model.

annual forecast horizons exceeding two years.

Table 5.1: RMSE and MAPE for Alternative Annual Dynamic Forecasts

Consumption Category	RMSE M1	RMSE M2	RMSE M3	MAPE M1	MAPE M2	MAPE M3
Food	0.0038	0.0056	0.0031	2.26	3.42	1.65
Beverages	0.0026	0.0056	0.0018	4.04	10.24	3.12
Tobacco	0.0013	0.0011	0.0004	3.62	3.79	1.47
Energy	0.0012	0.0040	0.0075	2.23	4.76	10.31
Clothing & Shoes	0.0021	0.0014	0.0013	2.70	1.75	1.77
Other Products	0.0066	0.0032	0.0044	4.25	1.71	2.50
Other Services	0.0012	0.0060	0.0021	0.33	1.97	0.68
Vehicle Running Costs	0.0049	0.0015	0.0030	7.67	3.24	6.49
Public Transport, Mail & Telecommunications	0.0107	0.0016	0.0020	13.68	2.18	1.99
Consumption Abroad	0.0079	0.0110	0.0021	9.19	12.93	1.95

Notes: Estimation is based on annual Norwegian national accounts data for the years $1979-2010$, with the last two years reserved for forecasting. The three competing forecasts are given by (all expressed in terms of single equations): M1 the homogeneous dynamic LA/AID system incorporating habits, M2 the dynamic LA/AID system incorporating habits without cross-price effects, and M3 the random walk model.

Whereas M1 and M2 achieve the lowest RMSE value for two and three of the consumption categories, respectively, M3 scores the lowest RMSE value for half the groups. Similar results are found in columns $4-6$, which show that M3 achieves the lowest MAPE value for food, beverages, tobacco, public transport, mail & telecommunications and consumption abroad. With a mean of 3.19 percent, M3 also achieves the lowest average MAPE value of the three competing forecasts.

The preliminary test results suggest that the model best suited for predicting non-durable household expenditure is the random walk model. Let us compare these findings with those derived from 1-step ahead DM-tests for equality of prediction mean squared errors. In order to obtain a sample of 1-step ahead prediction mean squared errors I applied each forecasting model to annual Norwegian national accounts data for the years 1979 to $2010-b$, where $b \in (1, \ldots, B)$. This approach resulted in 9 pairs of 1-step ahead prediction mean squared errors (i.e., the DM-tests listed in Table 5.2 are based on $B = 9$ observations). The columns summarize DM-tests for equality of prediction mean squared errors between M1 & M2 and M1 & M3. Note that *none* of these test statistics are significant at conventional levels (the p-values in parentheses below the test statistics are based on critical values from the Student's t-distribution with 8 degrees of freedom). The majority of the tests even fail to reject equality of prediction mean squared errors at 50 percent significance levels.[56] This lack of significance indicates that all three competing predictions are equally unsuited for forecasting, which means that our

[56]Non-rejection of the random walk forecasts indicates that the budget shares might

Table 5.2: Diebold-Mariano Tests for Equality of Prediction Mean Squared Errors[a]

Consumption Category	DM-test of M1 vs. M2	DM-test of M1 vs. M3
Food	0.66 (0.5244)[b]	0.73 (0.4869)
Beverages	0.00 (0.9970)	0.62 (0.5548)
Tobacco	0.65 (0.5336)	0.71 (0.4983)
Energy	0.38 (0.7108)	−0.42 (0.6824)
Clothing & Shoes	0.11 (0.9153)	0.64 (0.5426)
Other Products	−0.34 (0.7415)	−0.06 (0.9529)
Other Services	0.56 (0.5890)	0.72 (0.4919)
Vehicle Running Costs	0.46 (0.6575)	0.53 (0.6109)
Public Transport, Mail & Telecommunications	0.85 (0.4207)	1.16 (0.2808)
Consumption Abroad	0.15 (0.8850)	0.36 (0.7257)

[a] The DM-tests are based on 9 pairs of 1-step ahead prediction mean squared errors obtained from applications of the three competing forecasting models to annual Norwegian national accounts data for the years 1979 to $2010 - b$, where $b \in (1, \dots, 9)$. The three competing forecasts are given by (all expressed in terms of single equations): M1 the homogeneous dynamic LA/AID system incorporating habits, M2 the dynamic LA/AID system incorporating habits without cross-price effects, and M3 the random walk model.

[b] The p-values in parentheses are based on critical values from the Student's t-distribution with 8 degrees of freedom.

previous claim that the random walk model yields more precise predictions than M1 and M2 no longer holds. The evidence further shows that excluding cross-price effects does not necessarily improve prediction accuracy.

On average, prices account for 93.20 percent of the variation in the consumption categories' budget shares. Hence, in order to accurately predict future consumption, appropriate price forecasts must be adopted in the systems. This calls for the employment of a two-step procedure, where price predictions are generated by auxiliary regressions prior to the stage of expenditure share forecasts. Note that such a two-step procedure does not necessarily violate the exogeneity assumption, as prices might be determined by variables that are uncorrelated with $\boldsymbol{\varepsilon}_t$ (cf. Section 4.2). Unfortunately, a

follow difference-stationary time series processes (i.e., I(1)). However, this was rejected by the ADF-tests in Section 4.3 for all consumption categories but clothing & shoes and other services in our expenditure system, and the subject will therefore not be revisited here.

complete forecasting analysis where prices are treated as endogenous rather than as exogenous variables is beyond the scope of this analysis, and the approach will therefore not be followed here.

To conclude, the failure to predict future consumption does not necessarily call for rejection of the dynamic LA/AID system incorporating habits' underlying economic theories or functional form. Nor does it necessarily imply that our preferred dynamic specification is unsuitable for forecasting, as the observed forecast inaccuracies are likely to be at least partly brought about by the fact that predictions are compared with preliminary data that are potentially subject to future revisions. The accumulated evidence does, however, indicate that failure to account for endogenous prices (i.e., failure to account for the supply side) is likely to harm prediction accuracy as prices account for most of the variation in the consumption categories' expenditure shares, which suggests that the focus of dynamic forecast analyses of non-durable commodities should be on obtaining accurate price predictions.

6 Conclusion

This analysis has applied several dynamic and static expenditure system specifications to aggregated household consumption data obtained from the Norwegian national accounts. Their relative explanatory power was compared by means of the likelihood dominance criterion for model selection proposed by Pollak and Wales (1991). Although the ECM was preferred to the dynamic LA/AID system incorporating habits, we decided not to adopt the former as our preferred dynamic specification due to the identification problem and related non-invariance of the estimates to the omitted row associated with the ECM. Note, however, that the homogeneous static LA/AID system was the only specification that (i) did not suffer from lack of precisely estimated parameters and (ii) produced price and income elasticity estimates that were both interpretable and empirically plausible.

Attention was then turned to forecasting and out-of-sample performance of our preferred dynamic specification. We found that despite the dynamic LA/AID system incorporating habits' remarkable data fit prior to the year of prediction, it failed to accurately predict a number of our consumption categories only a year or two into the future. Statistical tests were subsequently employed to test Chambers (1990)'s hypothesis that simpler expenditure system specifications are more appropriate for forecasting. This was not supported by the DM-tests developed by Diebold and Mariano (1995), which failed to reject equality of prediction mean squared errors between the preferred dynamic specification, the dynamic LA/AID system incorporating habits without cross-price effects and the random walk model. Our data also rejected the hypothesis that presence of highly collinear prices are necessarily detrimental for forecasting accuracy. The evidence finally indicated that the focus of dynamic forecast analyses of non-durable consumption categories should be on obtaining accurate price predictions, as prices accounted for most of the variation in the commodities' expenditure shares.

The analysis is not without its limitations. Attention has been restricted to expenditure systems with identical right-hand side variables in all equations to ensure that maximum likelihood estimates are invariant to the omitted equation. More flexible dynamic specifications (preferably applied to quarterly rather than annual data to improve estimation precision) are likely to yield more empirically plausible elasticity estimates. Note, however, that more flexible specifications are generally not invariant to the omitted equation, and care must therefore be exercised when interpreting the results.

Blundell and Robin (1999) show that several popular demand systems (e.g. the non-linear AID system, the quadratic AID (QUAID) system and the translog demand system) possess the property of what they refer to as *conditional linearity*, and can therefore be estimated by *iterated linear least squares*. The estimator can be applied to highly disaggregated data sets containing numerous consumption categories (Blundell and Robin (1999) apply the estimator to a 22-commodity non-linear QUAID system using 20 years of repeated cross-sections from the British Family Expenditure Survey). Although the approach is more suitable for cross-section than for time series data, it would be interesting to compare the results derived in this analysis with those obtained from non-linear expenditure systems using the estimator developed by Blundell and Robin (1999). Another topic for future research would be to augment our models with demographic factors such as family size and composition, which requires estimation on micro rather than aggregate data. Following Alessie and Kapteyn (1991), such effects can be accounted for by augmenting the demand functions on budget share form with the term $\kappa_i log f_{mt}$, where $i \in (1, \ldots, n)$ denotes commodity number and f_{mt} is the number of people in family $m \in (1, \ldots, M)$ at time $t \in (1, \ldots, T)$. Finally, it would be interesting to incorporate wealth effects and credit constraint considerations in the AID system. Such effects are particularly important for durable commodities, but are also likely to influence the consumption of certain non-durable and semidurable consumption groups such as clothing & shoes, photographic & IT equipment and telecommunications equipment.

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A Appendix

The appendix restricts attention to estimation results used for calculation of elasticities and results from tests of parameters, residuals and model selection. Elasticity estimates based on the static long run solution to the homogeneous & symmetric dynamic LA/AID systems discussed in Section 4.3 are available from the author. Other findings and test results referred to in the text are also available upon request.

The appendix is organized in accordance with the rest of the analysis. Section A.1 complements the subjects discussed in Chapter 2 and Section 4.1. Section A.2 and Section A.3 complements the topics addressed in Section 4.2 and Section 4.3, respectively. Section A.4 gives tests for model selection and compares the explanatory power of the alternative expenditure system specifications that are presented in Chapter 4.

A.1 Data

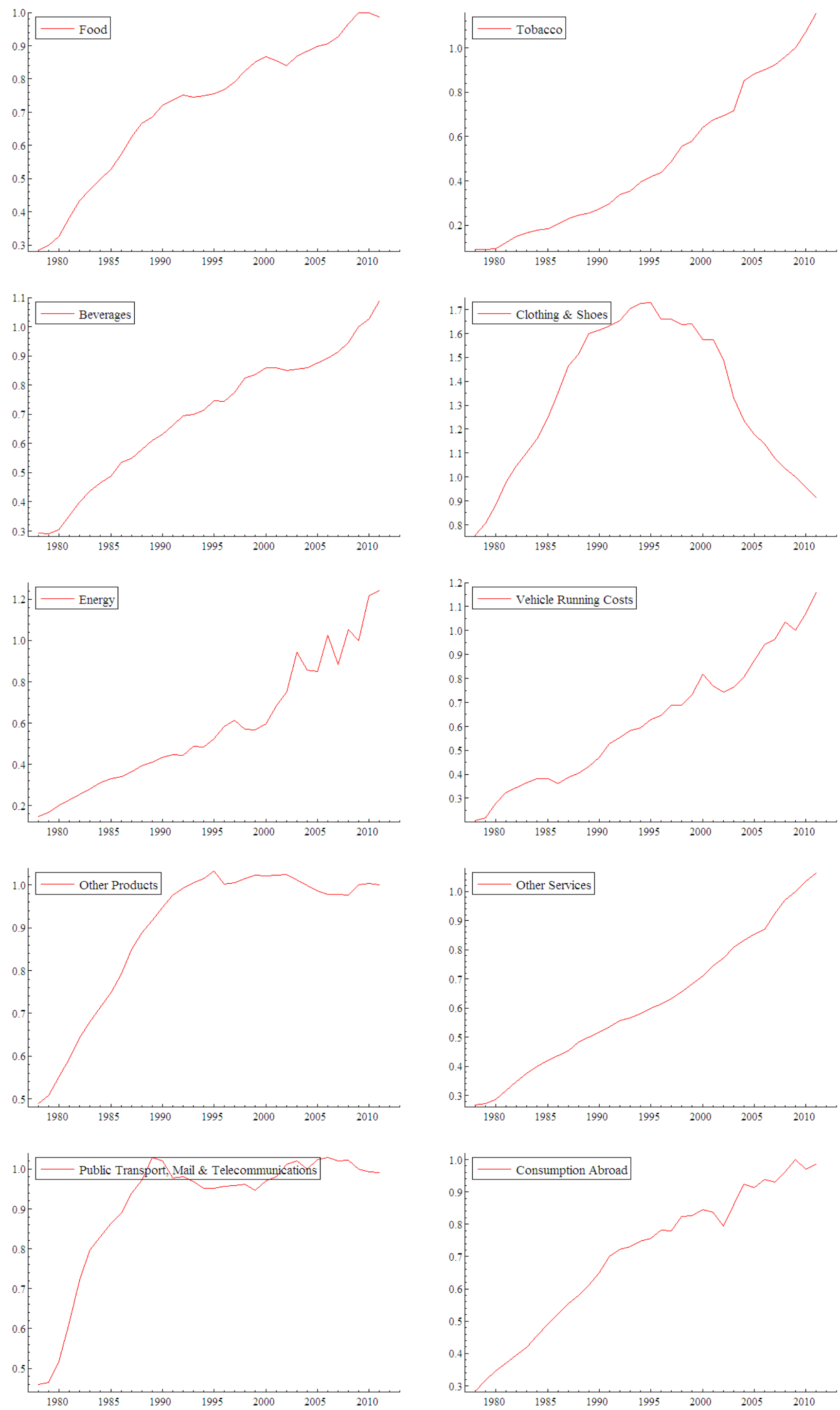
p_{it} is given by the quotient of VC_{it} and C_{it}, where the latter two variables denote the value of consumption of good i at time t expressed in current and constant 2009 prices, respectively. Budget shares are given given by $VC_{it}/\sum_i VC_{it}$, where the denominator denotes total (non-durable) expenditure, x_t. All data transformations have been carried out in OxMetrics 6 (algebra codes are omitted but are available from the author).

Figure A.1 displays the evolution of prices for the years 1978 − 2010. With the exception of clothing & shoes, all categories in our expenditure system have seen their price increase almost monotonically over the sample period. The price decline for clothing & shoes since the mid 1990s has coincided with a drop in import prices, itself brought about by the fact that most of the industry has been relocated to countries like China and Vietnam to take advantage of the abundant supply of inexpensive labor, which has doubled the import's share of textiles, clothing and shoes consumption in Norway since the 1960s. The graph further shows that the price of other products and public transport, mail & telecommunications have remained remarkably stable since the early 1990s. This is largely a result of technological advances, which have enabled prices to remain stable despite surging demand and rising consumer prices.[57]

Table A.1 gives summary statistics for the 10 consumption categories listed in Table 2.1. $\overline{w}_i = (1/T)\sum_{t=1}^{T} w_{it}$ denotes commodity i's mean annual

[57]Norway experienced an average inflation rate of about 2.2 percent between 1990 and 2011.

Figure A.1: Evolution of Prices



budget share for the years $1978-2010$. $\min_w$ and $\max_w$ refer to the category's minimum and maximum budget share value over the sample period, with values in brackets referring to the year of observation. Cf. also Figure 2.1 for the categories' annual budget shares for the years $1978-2010$.

Table A.1: Summary Statistics: Budget Shares

Commodity i	$\overline{w}_i$	$\min_w$	$\max_w$
Food	0.1953	0.1504 [2007]	0.2558 [1978]
Beverages	0.0622	0.0528 [2007]	0.0731 [1978]
Tobacco	0.0296	0.0260 [1980]	0.0327 [1997]
Energy	0.0577	0.0448 [2000]	0.0678 [1984]
Clothing & Shoes	0.0885	0.0711 [2010]	0.1130 [1978]
Other Products	0.1342	0.1130 [1983]	0.1570 [2007]
Other Services	0.2400	0.1786 [1980]	0.2832 [2010]
Vehicle Running Costs	0.0600	0.0456 [2009]	0.0723 [1981]
Public Transport, Mail & Telecommunications	0.0695	0.0613 [1979]	0.0776 [2007]
Consumption Abroad	0.0629	0.0471 [1981]	0.0848 [2006]

Notes: $\bar{w}_i$ denotes commodity i's mean annual budget share for the years $1978-2010$. $\min_w$ and $\max_w$ refer to the category's minimum and maximum budget share value over the sample period, with values in brackets referring to the year of observation. Cf. also Figure 2.1.

A.2 Static LA/AID Systems

Table A.2 gives the correlation matrix of regressors for the unconstrained static LA/AID system (with the exception of $log\{x/P^*\}$). LPC(i) denotes the price of commodity $i \in (00, 03, 04,\text{en}, 14, 20, 21, 60, 61, 66)$ expressed on logarithmic form. The categorization of consumption groups is summarized in the left-hand column of Table 2.1 (Cen is here given by the sum of C12 and C13).

Table A.2: Correlation Matrix of Regressors in the Unconstrained Static LA/AID System

	LPC00	LPC03	LPC04	LPCen	LPC14	LPC20	LPC21	LPC60	LPC61	LPC66
LPC00	1.00	0.99	0.95	0.95	0.95	0.96	0.49	0.96	0.92	0.99
LPC03	0.99	1.00	0.98	0.96	0.97	0.94	0.42	0.98	0.88	0.99
LPC04	0.95	0.98	1.00	0.98	0.99	0.86	0.25	0.99	0.78	0.97
LPCen	0.95	0.96	0.98	1.00	0.97	0.85	0.23	0.99	0.81	0.96
LPC14	0.95	0.97	0.99	0.97	1.00	0.86	0.26	0.98	0.79	0.97
LPC20	0.96	0.94	0.86	0.85	0.86	1.00	0.68	0.87	0.94	0.94
LPC21	0.49	0.42	0.25	0.23	0.26	0.68	1.00	0.24	0.65	0.43
LPC60	0.96	0.98	0.99	0.99	0.98	0.87	0.24	1.00	0.80	0.97
LPC61	0.92	0.88	0.78	0.81	0.79	0.94	0.65	0.80	1.00	0.87
LPC66	0.99	0.99	0.97	0.96	0.97	0.94	0.43	0.97	0.87	1.00

Notes: Calculations are based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. LPC($\cdot$) denotes (log) prices. Cf. Table 2.1 for the categorization of consumption groups (Cen is here given by the sum of C12 and C13).

Table A.3 displays the result from the estimation of the homogeneous static LA/AID system applied to quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. The system has been estimated in PcGive 13 by constrained full information maximum likelihood (CFIML) with the following parameter constraints imposed (batch codes are omitted but are available from the author):

$$\gamma_{i1} = -\gamma_{i2} - ... - \gamma_{i10}, \forall i \in (1, \ldots, n-1) \tag{A.1}$$

with the coefficients of the n^{th} commodity (here, consumption abroad) obtained from adding-up. t-statistics are only available for unconstrained parameter estimates.

The log-likelihood values summarized in the first column of Table A.4 are obtained from 10 different $n-1$ combinations of symmetric static LA/AID systems (cf. Section 4.2 for detailed accounts). The imposition of symmetry is tested by likelihood-ratio tests in the second column, each of which is χ^2 distributed with degrees of freedom equal to the number of parameter restrictions (here, 36, as the regressions include one freely estimated equation, one equation with one restriction, etc.).

Table A.3: Constrained Parameter Estimates with Homogeneity Imposed in the Static LA/AID System

Commodity i	α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	γ_{i10}	τ_{i1}	τ_{i2}	τ_{i3}
Food	1.324	−0.096	0.042	−0.038	−0.020	−0.015	0.024	−0.034	0.052	0.043	−0.027	−0.028	−0.013	−0.007	−0.009
	(9.8)	(−8.7)		(−2.3)	(−2.9)	(−3.0)	(4.1)	(−3.3)	(2.2)	(2.7)	(−2.8)	(−2.3)	(−6.5)	(−4.3)	(−6.2)
Beverages	0.238	−0.015	-0.003	0.024	−0.008	0.000	−0.006	0.005	−0.009	0.006	−0.018	0.010	−0.006	0.002	0.001
	(4.0)	(−3.1)		(3.3)	(−2.7)	(0.1)	(−2.2)	(1.0)	(−0.8)	(0.8)	(−4.3)	(1.8)	(−6.3)	(2.9)	(−1.1)
Tobacco	0.078	−0.004	0.006	0.005	0.005	0.000	−0.008	0.001	−0.006	−0.014	−0.004	0.018	−0.002	0.000	−0.002
	(1.3)	(−0.8)		(0.7)	(1.5)	(−0.1)	(−3.2)	(−0.1)	(−0.5)	(−1.9)	(−1.0)	(3.2)	(−2.1)	(0.5)	(−3.5)
Energy	0.099	−0.003	-0.002	0.028	−0.024	0.049	0.003	−0.011	0.015	−0.048	0.009	−0.019	0.017	−0.016	−0.026
	(0.8)	(−0.3)		(1.9)	(−3.7)	(11.0)	(0.5)	(−1.2)	(0.7)	(−3.4)	(1.0)	(−1.8)	(9.2)	(−11.2)	(−21.3)
Clothing & Shoes	−0.188	0.022	-0.016	−0.004	−0.032	−0.013	−0.007	−0.013	0.038	0.037	0.021	−0.012	−0.017	−0.009	−0.017
	(−2.1)	(3.0)		(−0.3)	(−6.7)	(−3.9)	(−1.8)	(−1.8)	(2.3)	(3.5)	(3.2)	(−1.4)	(−12.9)	(−8.1)	(−18.2)
Other Products	−0.129	0.024	-0.048	−0.046	0.031	−0.017	−0.007	0.041	−0.018	0.069	−0.015	0.011	−0.015	−0.019	−0.015
	(−1.2)	(2.8)		(−3.6)	(5.6)	(−4.5)	(−1.6)	(5.2)	(−1.0)	(5.6)	(−2.1)	(1.1)	(−9.6)	(−14.6)	(−13.6)
Other Services	−0.358	0.051	0.004	0.062	0.033	0.013	−0.016	0.075	−0.098	−0.011	−0.058	−0.004	0.025	0.022	0.023
	(−2.4)	(4.2)		(3.4)	(4.3)	(2.4)	(−2.5)	(6.6)	(−3.7)	(−0.6)	(−5.4)	(−0.3)	(11.0)	(12.1)	(15.1)
Vehicle Running Costs	0.429	−0.032	0.033	−0.005	−0.002	0.004	0.040	−0.011	0.008	−0.076	−0.007	0.015	−0.004	0.004	0.005
	(5.7)	(−5.1)		(−0.6)	(−0.5)	(1.5)	(12.4)	(−1.9)	(0.6)	(−8.6)	(−1.3)	(2.2)	(−3.4)	(4.4)	(6.9)
Public Transport, Mail & Telecommunications	0.206	−0.011	0.012	−0.031	0.017	−0.004	−0.008	0.013	−0.062	0.032	0.032	0.000	0.009	0.005	0.009
	(2.3)	(−1.5)		(−2.8)	(3.6)	(−1.2)	(−2.1)	(1.9)	(−3.8)	(3.0)	(4.9)	(−0.1)	(6.7)	(4.7)	(9.2)
Consumption Abroad	-0.698	0.064	-0.028	0.004	0.000	-0.017	-0.014	-0.065	0.080	-0.038	0.068	0.011	0.006	0.017	0.032

Notes: Estimation is based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Consumption abroad is excluded from the estimation, and its coefficients are obtained from adding-up. Values in parentheses are t-statistics.

Table A.4: Log-Likelihood Values for Symmetric Static LA/AID Systems

Excluded Category	Log-Likelihood	Test Statistic
Food	5383.41	$\chi^2_{36} = 302.84[0.0000]$
Beverages	5389.18	$\chi^2_{36} = 291.29[0.0000]$
Tobacco	5380.48	$\chi^2_{36} = 308.70[0.0000]$
Energy	5384.45	$\chi^2_{36} = 300.74[0.0000]$
Clothing & Shoes	5413.04	$\chi^2_{36} = 243.56[0.0000]$
Other Products	5399.53	$\chi^2_{36} = 270.60[0.0000]$
Other Services	5371.48	$\chi^2_{36} = 326.69[0.0000]$
Vehicle Running Costs	5392.14	$\chi^2_{36} = 285.38[0.0000]$
Public Transport, Mail & Telecommunications	5372.13	$\chi^2_{36} = 325.38[0.0000]$
Consumption Abroad	5414.33	$\chi^2_{36} = 240.99[0.0000]$

Notes: Estimation is based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Values in brackets are p-values.

Table A.5 displays the result from the estimation of the homogeneous & symmetric static LA/AID system applied to quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. The system has been estimated in PcGive 13 by CFIML with the following parameter constraints imposed (in addition to the homogeneity constraints given in equation (A.1)):

$$\begin{aligned} \gamma_{1.2} &= \gamma_{2.1} \\ \gamma_{1.3} &= \gamma_{3.1} \\ &\vdots \\ \gamma_{1.n-1} &= \gamma_{n-1.1} \end{aligned} \tag{A.2}$$

with the coefficients of the n^{th} commodity (here, vehicle running costs) obtained from adding-up.

Table A.6 gives Marshallian cross-price elasticities for the homogeneous static LA/AID system. The elasticities are calculated at mean quarterly budget share values for the period 1978Q1-2011Q3 according to the formula given in (4.17).

Table A.7 compares the own-price and expenditure elasticity estimates listed in Table 4.3 with those obtained by Raknerud, Skjerpen and Swensen (2007) on quarterly Norwegian national accounts data for the period 1966Q1-2001Q4. Attention is restricted to the unconstrained and homogeneous model. Income elasticities are said to be qualitatively similar in the two studies if both classify the good in question as a necessity or luxury, and qualitatively dissimilar if the commodity in question is classified as a neces-

Table A.5: Constrained Parameter Estimates with Homogeneity and Symmetry Imposed in the Static LA/AID System

Commodity i	α_i^*	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	γ_{i10}	τ_{i1}	τ_{i2}	τ_{i3}
Food	2.183	−0.167	0.026	−0.005	0.001	0.004	0.039	−0.003	−0.054	0.034	−0.033	−0.008	−0.023	−0.016	−0.012
	(20.0)	(−18.7)		(−0.9)	(0.3)	(0.9)	(9.1)	(−0.6)	(−5.5)	(4.0)	(−4.5)	(−0.9)	(−11.1)	(−8.7)	(−7.0)
Beverages	0.106	−0.004	-0.005	0.026	−0.013	−0.003	−0.008	0.003	−0.010	0.009	−0.010	0.011	−0.004	0.003	0.000
	(2.2)	(−1.0)			(−5.7)	(−1.5)	(−3.8)	(1.1)	(−1.5)	(1.8)	(−3.1)	(2.4)	(−5.1)	(4.5)	(−0.4)
Tobacco	−0.263	0.024	0.001	-0.013	0.001	−0.011	−0.013	−0.008	0.026	−0.003	0.009	0.011	0.002	0.004	−0.001
	(−5.1)	(5.8)				(−5.8)	(−7.2)	(−3.8)	(6.5)	(−0.8)	(3.5)	(2.9)	(2.6)	(4.6)	(−0.9)
Energy	0.348	−0.024	0.004	-0.003	-0.011	0.049	−0.003	0.001	−0.009	−0.023	0.000	−0.004	0.015	−0.018	−0.027
	(5.7)	(−4.8)					(−1.5)	(0.5)	(−2.3)	(−5.4)	(0.1)	(−0.9)	(11.5)	(−16.0)	(−24.6)
Clothing & Shoes	0.471	−0.032	-0.003	0.003	-0.008	0.001	−0.001	-0.004	0.022	0.028	−0.008	−0.029	−0.025	−0.014	−0.019
	(8.5)	(−7.0)					(−0.7)		(3.7)	(5.3)	(−2.8)	(−6.5)	(−19.9)	(−12.2)	(−17.6)
Other Products	−0.079	0.020	-0.054	-0.010	0.026	-0.009	−0.004	0.022	0.002	0.024	−0.017	0.018	−0.017	−0.019	−0.016
	(−0.8)	(2.6)					(−1.0)			(2.6)	(−2.9)	(2.1)	(−11.8)	(−15.1)	(−14.4)
Other Services	−1.751	0.167	0.034	0.009	-0.003	-0.023	−0.043	0.028	0.024	0.003	−0.018	−0.012	0.041	0.035	0.028
	(−16.9)	(19.6)					(−11.9)				(−3.6)	(−1.5)	(15.7)	(14.4)	(12.1)
Vehicle Running Costs	0.163	-0.010	0.039	-0.008	-0.013	-0.003	-0.021	-0.032	-0.004	-0.043	-0.003	0.002	0.000	0.005	0.007
Public Transport, Mail & Telecommunications	−0.078	0.012	-0.033	-0.010	0.009	0.000	-0.008	-0.017	-0.018	−0.003	0.053	0.027	0.010	0.008	0.010
	(−1.1)	(2.0)								(−1.1)		(4.5)	(8.5)	(7.2)	(10.0)
Consumption Abroad	−0.099	0.014	-0.008	0.011	0.011	-0.004	0.002	-0.029	0.018	-0.012	0.027	-0.016	0.001	0.012	0.030
	(−1.0)	(1.7)					(0.6)						(0.5)	(6.7)	(17.8)

Notes: Estimation is based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Vehicle running costs is excluded from the estimation, and its coefficients are obtained from adding-up. Values in parentheses are t-statistics.

Table A.6: Marshallian Price Elasticities in the Homogeneous Static LA/AID System

	C00	C03	C04	Cen	C14	C20	C21	C60	C61	C66
C00	-0.69	-0.01	0.22	-0.02	0.66	-0.39	-0.23	-0.02	0.20	-0.64
C03	-0.16	-0.60	0.19	0.48	-0.06	-0.35	-0.06	0.24	-0.44	0.01
C04	-0.09	-0.13	-0.84	-0.40	-0.01	0.22	-0.37	0.13	0.25	-0.03
Cen	-0.05	0.02	0.02	-0.15	0.10	-0.14	-0.16	0.04	-0.05	-0.32
C14	0.15	-0.08	-0.28	0.05	-0.30	-0.06	-0.09	-0.08	-0.11	-0.28
C20	0.33	-0.11	-0.18	0.27	0.20	-1.16	0.40	-0.44	-0.86	1.13
C21	-0.13	0.10	0.03	-0.18	-0.14	0.29	-1.17	0.29	0.20	-1.12
C60	0.34	0.15	-0.44	-0.81	-1.14	0.48	0.36	-1.10	0.49	-0.84
C61	-0.10	-0.28	-0.13	0.15	-0.07	-0.13	0.22	-0.25	-0.53	0.99
C66	-0.11	0.18	0.61	-0.33	0.28	0.07	-0.15	-0.03	0.02	-0.90

Notes: Calculations are based on quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Elasticities are calculated at mean quarterly budget share values for the entire sample. Cf. Table 2.1 for the categorization of consumption groups. Cen is here given by the sum of C12 and C13.

sity by one study but as a luxury by the other. Own-price elasticities are said to be qualitatively similar in the two studies if both classify the good in question as price elastic or price inelastic, and qualitatively dissimilar if the commodity in question is classified as price elastic by one study but as price inelastic by the other.

Figure A.2 illustrates recursive estimates of β_i in the unconstrained static LA/AID system applied to quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. Recursive estimation is initialized by direct regresion (here, FIML estimation) over $t = 1, \ldots, R-1$ ($R = 15$ in Figure A.2 and Figure A.3), followed by recursive regression over $t = R, \ldots, T$, where T denotes sample length. The graphs are plotted with approximate 95 percent confidence intervals (shown by the dotted lines of $\pm$2SE).

Figure A.3 illustrates break-point Chow tests for parameter constancy for the unconstrained static LA/AID system applied to quarterly Norwegian national accounts data for the period 1978Q1-2011Q3. The break-point Chow test tests for presence of structural changes in the relationships by comparing the residual sum of squares (RSS) obtained from fitting the budget shares to the entire sample period with the RSS obtained from fitting the expenditure shares to sub-samples of the data. Presence of structural changes is rejected at a 5 percent level for all consumption categories in our expenditure system.

Table A.8 displays single equation tests of residuals for normality, absence of ARCH and absence of heteroskedasticity in the annual and quarterly ho-

Table A.7: Qualitative Comparison of Own-price and Expenditure Elasticities with Raknerud, Skjerpen and Swensen (2007)

	Unconstrained		Homogeneous	
	$\eta_{i,i}$	e_i	$\eta_{i,i}$	e_i
Food	(+)	(+)	(+)	(+)
Energy	(+)	(+)	(+)	(+)
Clothing & Shoes	(+)	(−)	(−)	(+)
Other Products	(−)	(+)	(−)	(−)
Other Services	(+)	(+)	(−)	(+)
Vehicle Running Costs	(+)	(−)	(+)	(−)
Public Transport, Mail & Telecommunications	(+)	(−)	(+)	(+)
Consumption Abroad	(+)	(+)	(−)	(+)

Notes: (+) and (−) denote, respectively, qualitatively similar and dissimilar own-price and expenditure elasticities in Table 4.3 and Raknerud, Skjerpen and Swensen (2007). Beverages and tobacco are excluded from the comparison as Raknerud, Skjerpen and Swensen (2007) do not distinguish between the two categories.

mogeneous static LA/AID system. The ARCH(q) test for commodity i is based on the following model: $E(\varepsilon_{it}^2|\varepsilon_{it-1},\ldots,\varepsilon_{it-q}) = \xi_{i1} + \sum_{j=1}^{q}\xi_{ij}\varepsilon_{it-j}^2$, where $E(\cdot)$ denotes the expectations operator and ε_{it-q} is the i^{th} element of $\boldsymbol{\varepsilon_t}$ lagged q periods. Absence of ARCH(q) (i.e., $\xi_{ij} = 0\,\forall j \in (1,\ldots,q)$) is tested by F-tests. The White test for heteroskedasticity listed in columns 2 and 5 is based on a regression of commodity i's squared residuals, $\hat{\varepsilon}_{it}^2$, on a constant, z_{it} and z_{it}^2, where z_{it} is the i^{th} element of $\mathbf{z}_t$. The null-hypothesis of homoskedastic errors is tested by F-tests. Columns 3 and 6 list Jarque-Bera tests for normality of the errors terms. The test statistic for absence of normality is based on the small sample corrected version proposed by Doornik and Hansen (2008): $r_{i1}^2 + r_{i2}^2 \overset{app}{\sim} \chi_2^2$, where r_{i1}^2 and r_{i2}^2 denote the *transformed* skewness and kurtosis, respectively (detailed accounts are given by Doornik and Hansen (2008)). Non-zero third- and fourth-order central moments is rejected at a 5 percent level for all categories but clothing & shoes, other products and consumption abroad in the quarterly homogeneous static LA/AID system, and for the entire annual homogeneous expenditure system. As the dynamic models are applied to annual data, I will continue to treat $\boldsymbol{\varepsilon}_1,\ldots,\boldsymbol{\varepsilon}_T$ as independently drawn vectors from a multivariate normal distribution, which means that Barten (1969)'s result still applies.

Table A.8: Single Equation Tests of Residuals for ARCH, Heteroskedasticity and Normality[a]

	ARCH(1-4)[b]	Hetero[c]	Normality[d]	ARCH(1)[e]	Hetero[f]	Normality
Food	1.75 (0.1422)	2.24 (0.0023)	0.42 (0.8116)	9.06 (0.0052)	4.06 (0.0131)	0.44 (0.8023)
Beverages	1.26 (0.2892)	0.82 (0.7141)	1.71 (0.4247)	0.00 (0.9492)	0.60 (0.8468)	2.91 (0.2335)
Tobacco	4.25 (0.0029)	2.42 (0.0009)	4.30 (0.1162)	0.01 (0.9119)	1.39 (0.3002)	1.69 (0.4293)
Energy	3.49 (0.0097)	2.26 (0.0021)	1.56 (0.4578)	0.24 (0.6243)	0.76 (0.7180)	1.16 (0.5611)
Clothing & Shoes	12.88 (0.0000)	2.65 (0.0003)	7.72 (0.0210)	0.91 (0.3465)	1.69 (0.1977)	0.52 (0.7699)
Other Products	3.88 (0.0053)	1.52 (0.0725)	7.11 (0.0285)	1.50 (0.2306)	0.41 (0.9601)	0.94 (0.6237)
Other Services	0.76 (0.5539)	1.03 (0.4412)	3.18 (0.2038)	0.00 (0.9627)	0.66 (0.7990)	0.02 (0.9902)
Vehicle Running Costs	2.45 (0.0498)	1.57 (0.0593)	5.15 (0.0760)	1.51 (0.2291)	1.11 (0.4523)	0.49 (0.7843)
Public Transport, Mail & Telecommunications	1.79 (0.1354)	0.98 (0.5050)	0.91 (0.6341)	2.72 (0.1094)	1.41 (0.2912)	1.17 (0.5572)
Consumption Abroad	0.15 (0.9627)	0.76 (0.7786)	6.32 (0.0425)	0.00 (0.9838)	0.51 (0.9090)	0.80 (0.6715)

[a] Columns 1-3 and 4-6 are based on quarterly and annual Norwegian national accounts data for the period 1978Q1-2011Q3 and 1978-2010, respectively. Values in parentheses are p-values.

[b] $F(4, 127)$-test for ARCH(1-4) in the quarterly homogeneous static LA/AID system.

[c] White $F(25, 109)$-test for heteroskedasticity in the quarterly homogeneous static LA/AID system.

[d] Small sample corrected Jarque-Bera χ^2_2-test for normality of the error terms in the quarterly homogeneous static LA/AID system. Identical test is given in the final column for the annual homogeneous static LA/AID system.

[e] $F(1, 31)$-test for ARCH(1) in the annual homogeneous static LA/AID system.

[f] White $F(22, 10)$-test for heteroskedasticity in the annual homogeneous static LA/AID system.

Figure A.2: Recursive Estimates of β_i in the Unconstrained Static LA/AID System

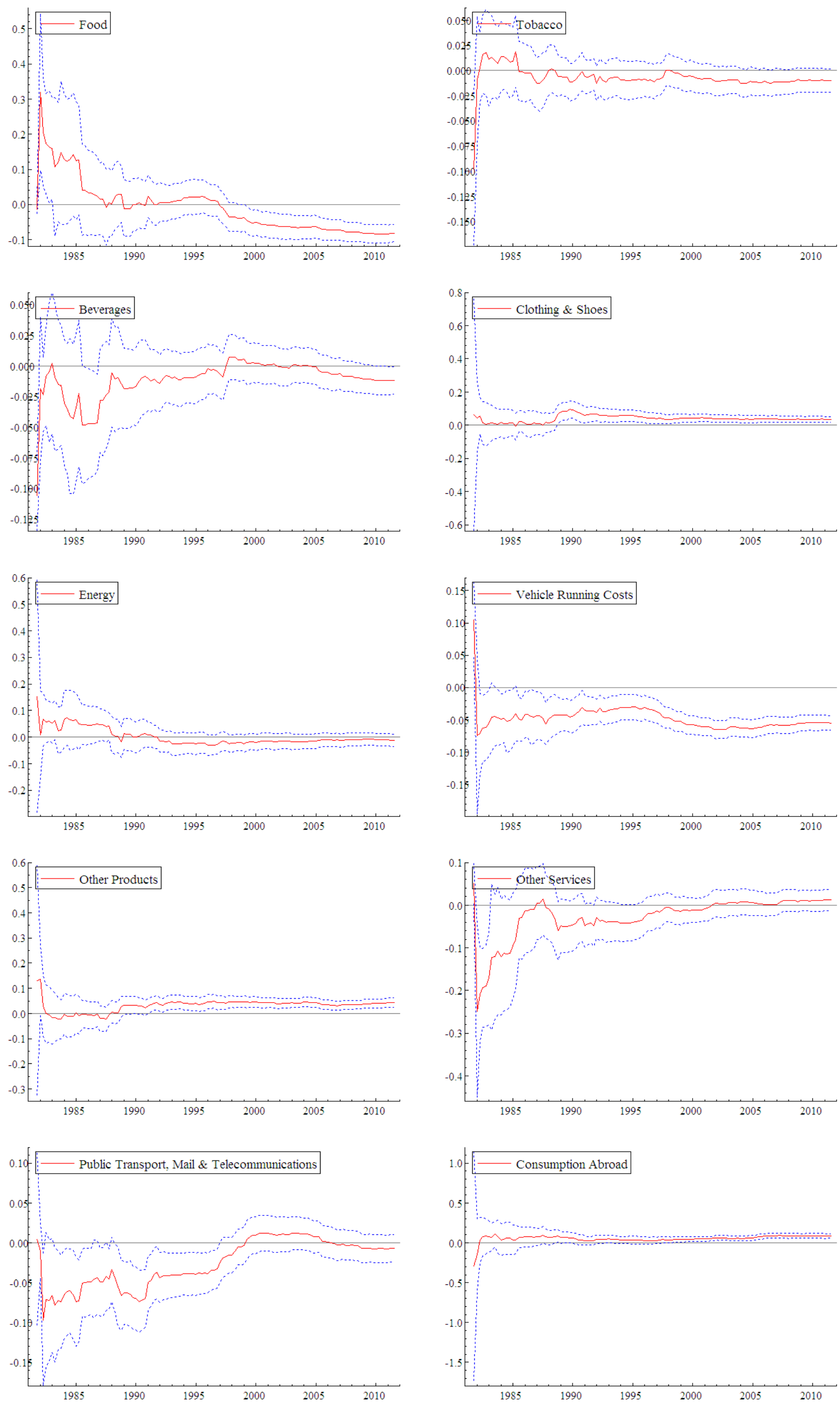


Figure A.3: Break-Point Chow Tests for Parameter Constancy in the Unconstrained Static LA/AID System

A.3 Dynamic LA/AID Systems

Table A.9 displays the result from the estimation of the homogeneous dynamic LA/AID system in first differences applied to annual Norwegian national accounts data for the years $1979 - 2010$. The dependent variables are first differenced budget share, $\Delta w_{it} = w_{it} - w_{it-1}$. The system has been estimated in PcGive 13 by CFIML with the homogeneity constraints given in equation (A.1) imposed. The coefficient estimates from the n^{th} commodity (here, consumption abroad) have been obtained from adding-up.

Table A.10 displays the result from the estimation of the homogeneous dynamic LA/AID system incorporating habits applied to annual Norwegian national accounts data for the years $1979 - 2010$. Cross-price effects, $\sum_{j \neq i} \gamma_{ij} log\, p_{jt}$, were included in the regression but have been omitted from the table. The system has been estimated in PcGive 13 by CFIML with the homogeneity constraints given in equation (A.1) imposed. The coefficient estimates from the n^{th} commodity (here, public transport, mail & telecommunications) have been obtained from adding-up.

Estimation results from the homogeneous & symmetric dynamic LA/AID system in first differences, the homogeneous & symmetric dynamic LA/AID system incorporating habits and the other dynamic specifications referred to in the text are available from the author.

Table A.9: Constrained Parameter Estimates with Homogeneity Imposed in the Dynamic LA/AID System in First Differences

Commodity i	$\grave{\alpha}_i$	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	γ_{i10}
ΔFood	0.000	−0.086	0.021	−0.005	−0.020	−0.010	−0.034	0.047	0.002	0.014	−0.020	0.005
	(−0.1)	(−3.2)		(−0.2)	(−1.7)	(−1.6)	(−1.6)	(1.0)	(0.1)	(1.6)	(−1.0)	(0.3)
ΔBeverages	0.001	−0.014	0.005	0.003	−0.010	−0.003	−0.014	0.060	−0.014	0.000	−0.016	−0.011
	(1.1)	(−1.0)		(0.2)	(−1.7)	(−1.1)	(−1.4)	(2.7)	(−0.9)	(0.1)	(−1.7)	(−1.3)
ΔTobacco	0.000	−0.015	0.024	−0.011	0.003	0.002	0.005	−0.006	0.006	−0.004	−0.015	−0.002
	(0.5)	(−1.5)		(−1.0)	(0.7)	(0.7)	(0.6)	(−0.4)	(0.5)	(−1.3)	(−2.0)	(−0.3)
ΔEnergy	0.001	−0.039	-0.006	0.000	−0.001	0.050	−0.012	0.022	−0.038	−0.011	0.000	−0.006
	(0.6)	(−1.9)		(0.0)	(−0.1)	(10.1)	(−0.7)	(0.6)	(−1.5)	(−1.6)	(0.0)	(−0.4)
ΔClothing & Shoes	−0.002	0.058	-0.006	0.005	−0.014	−0.009	−0.001	0.019	0.015	−0.004	−0.001	−0.003
	(−2.6)	(3.0)		(0.2)	(−1.7)	(−1.9)	(−0.1)	(0.6)	(0.7)	(−0.7)	(−0.1)	(−0.2)
ΔOther Products	−0.001	0.048	-0.031	0.036	−0.002	−0.011	0.036	−0.055	0.050	−0.004	−0.019	−0.001
	(−0.6)	(1.8)		(1.3)	(−0.2)	(−1.8)	(1.7)	(−1.2)	(1.6)	(−0.4)	(−1.0)	(0.0)
ΔOther Services	0.003	0.020	0.005	0.011	0.007	−0.004	0.036	−0.035	0.026	−0.013	−0.010	−0.023
	(1.7)	(0.5)		(0.3)	(0.4)	(−0.5)	(1.2)	(−0.6)	(0.6)	(−1.1)	(−0.4)	(−0.9)
ΔVehicle Running Costs	0.001	−0.047	0.008	−0.018	−0.002	−0.002	0.001	0.014	−0.042	0.043	−0.002	0.001
	(1.5)	(−2.8)		(−1.0)	(−0.3)	(−0.6)	(0.1)	(0.5)	(−2.1)	(8.0)	(−0.1)	(0.0)
ΔPublic Transport, Mail & Telecommunications	0.001	−0.022	-0.008	−0.013	−0.002	−0.013	0.027	−0.076	0.051	−0.005	0.031	0.008
	(1.2)	(−0.8)		(−0.4)	(−0.2)	(−2.0)	(1.2)	(−1.6)	(1.5)	(−0.5)	(1.5)	(0.4)
ΔConsumption Abroad	-0.004	0.097	-0.011	-0.009	0.042	0.001	-0.043	0.010	-0.057	-0.017	0.052	0.032

Notes: Estimation is based on annual Norwegian national accounts data for the years 1979-2010. The dependent variables are first differenced budget shares, i.e., $\Delta w_{it} = w_{it} - w_{it-1}$. Consumption abroad is excluded from the estimation, and its coefficients are obtained from adding-up. Values in parentheses are t-statistics.

Table A.10: Constrained Parameter Estimates with Homogeneity Imposed in the Dynamic LA/AID System Incorporating Habits

Commodity i	$\tilde{\alpha}_i$	β_i	γ_{ii}	ς_{i1}	ς_{i2}	ς_{i3}	ς_{i4}	ς_{i5}	ς_{i6}	ς_{i7}	ς_{i8}	ς_{i9}	ς_{i10}
Food	1.373	−0.047	0.062	−0.012	0.007	−0.002	−0.017	−0.002	0.041	−0.036	−0.013	0.005	−0.023
	(3.6)	(−1.2)		(−0.4)	(0.3)	(−0.1)	(−0.4)	(−0.1)	(1.6)	(−1.0)	(−0.5)	(0.3)	(−1.4)
Beverages	0.089	−0.007	0.021	−0.013	0.003	0.016	0.022	−0.025	0.018	−0.009	−0.021	0.009	0.009
	(0.6)	(−0.5)	(1.7)	(−1.0)	(0.3)	(2.0)	(1.2)	(−2.6)	(1.8)	(−0.6)	(−1.9)	(1.5)	(1.4)
Tobacco	0.236	−0.012	0.002	−0.007	−0.006	0.002	−0.006	0.001	0.000	0.018	0.000	−0.006	−0.003
	(1.5)	(−0.7)	(0.4)	(−0.4)	(−0.6)	(0.3)	(−0.3)	(0.1)	(0.0)	(1.2)	(0.0)	(−1.0)	(−0.4)
Energy	0.797	0.008	0.055	−0.020	−0.018	−0.015	−0.048	0.039	−0.044	0.036	0.007	0.006	−0.025
	(3.5)	(0.3)	(10.0)	(−0.9)	(−1.3)	(−1.2)	(−1.7)	(2.7)	(−3.0)	(1.7)	(0.4)	(0.7)	(−2.6)
Clothing & Shoes	−0.320	0.077	0.035	0.006	0.000	−0.001	−0.038	0.027	−0.005	−0.026	0.003	−0.017	−0.007
	(−1.0)	(2.5)	(1.3)	(0.2)	(0.0)	(0.0)	(−1.0)	(1.3)	(−0.2)	(−0.9)	(0.1)	(−1.4)	(−0.5)
Other Products	−0.746	0.051	−0.071	−0.002	0.068	−0.021	0.011	−0.026	0.050	−0.045	0.017	−0.023	−0.006
	(−3.0)	(2.0)	(−1.3)	(−0.1)	(4.5)	(−1.6)	(0.4)	(−1.6)	(3.1)	(−2.0)	(0.9)	(−2.4)	(−0.6)
Other Services	−1.085	0.007	0.053	0.062	−0.054	0.022	0.032	−0.064	−0.017	0.073	0.011	0.021	0.019
	(−2.1)	(0.1)	(0.7)	(1.3)	(−1.7)	(0.8)	(0.5)	(−1.9)	(−0.5)	(1.5)	(0.3)	(1.0)	(0.8)
Vehicle Running Costs	0.721	−0.036	0.045	−0.034	−0.007	0.000	−0.002	0.017	−0.022	0.003	0.032	0.006	−0.007
	(2.2)	(−1.1)	(5.6)	(−1.1)	(−0.3)	(0.0)	(0.0)	(0.8)	(−1.0)	(0.1)	(1.3)	(0.5)	(−0.5)
Public Transport, Mail & Telecommunications	0.215	-0.052	0.021	-0.006	0.046	-0.019	0.041	0.001	-0.020	0.004	-0.015	0.004	0.016
Consumption Abroad	−0.280	0.011	0.067	0.025	−0.039	0.016	0.004	0.033	0.000	−0.018	−0.020	−0.005	0.026
	(−0.5)	(0.2)	(1.6)	(0.5)	(−1.2)	(0.6)	(0.1)	(0.9)	(0.0)	(−0.4)	(−0.5)	(−0.3)	(1.2)

Notes: Estimation is based on annual Norwegian national accounts data for the years 1979-2010. Public transport, mail & telecommunications is excluded from the estimation, and its coefficients are obtained from adding-up. Values in parentheses are t-statistics.

A.4 Tests for Model Selection

Table A.11 lists the log-likelihood value of the alternative unconstrained, homogeneous and homogeneous & symmetric expenditure system specifications that are discussed in Chapter 4. The table restricts attention to annual models (cf. Table 4.2 for the log-likelihood value of the quarterly static LA/AID systems).

Table A.11: Log-Likelihood Values of Alternative Annual Expenditure Systems

Expenditure System	Log-Likelihood
Unconstrained Static LA/AID System	1636.50
Homogeneous Static LA/AID System	1605.31
Homogeneous & Symmetric Static LA/AID System	1518.88
Unconstrained Dynamic LA/AID System in First Differences	1521.23
Homogeneous Dynamic LA/AID System in First Differences	1514.84
Homogeneous & Symmetric Dynamic LA/AID System in First Differences	1488.59
Unconstrained Dynamic LA/AID System Incorporating Habits	1826.20
Homogeneous Dynamic LA/AID System Incorporating Habits	1769.79
Homogeneous & Symmetric Dynamic LA/AID System Incorporating Habits	1689.44
Unconstrained ECM	1875.62
Unconstrained Dynamic LA/AID System with Distributed Lags	1819.48

Notes: Log-likelihood values and parameter estimates from models with w_{it} and Δw_{it} as dependent variables are comparable as the latter can be reformulated to include w_{it-1} as an explanatory variable with unity-coefficient. Such systems are easily estimated in PcGive 13 by CFIML.

Some of the systems we consider in Chapter 4 are non-nested, which means it is not possible to derive every model from a benchmark (or full) system by means of coefficient restrictions or as a result of a limiting process. Unlike nested models, competing non-nested models cannot be evaluated by likelihood ratio tests because the likelihood ratio statistic does not have a well-defined asymptotic distribution under the null. This analysis uses the likelihood dominance criterion (LDC) for model selection proposed by Pollak and Wales (1991). Unlike most non-nested tests, this criterion is well-suited for testing *multivariate* non-nested hypotheses because the LDC only requires maximum likelihood estimation of the two competing models.[58] The LDC for two non-nested hypotheses H_1 and H_2 can be expressed as (cf. Pollak and Wales (1991) for detailed accounts):

[58]The likelihood ratio test proposed by Vuong (1989) has also been used to test multivariate hypotheses. Cf. e.g. Wang, Halbrendt and Johnson (1996) for a non-nested Vuong-test of the AID system against the translog demand system.

1. H_1 is preferred to H_2 if $\mathscr{L}_2 - \mathscr{L}_1 < [C(k_2+1) - C(k_1+1)]/2$.

2. The criterion is indecisive between H_1 and H_2 if $[C(k_2 - k_1 + 1) - C(1)]/2 > \mathscr{L}_2 - \mathscr{L}_1 > [C(k_2+1) - C(k_1+1)]/2$.

3. H_2 is preferred to H_1 if $\mathscr{L}_2 - \mathscr{L}_1 > [C(k_2 - k_1 + 1) - C(1)]/2$.

$\mathscr{L}_y$ and k_y denote the log-likelihood value and the number of independent parameters in model $y \in (1,2)$, respectively. $C(\iota)$ is the critical value of the χ^2 distribution with ι degrees of freedom (a significance level of 5 percent is used in this analysis).

In the following, the full model (H_2) will refer to the unconstrained dynamic LA/AID system incorporating habits. The unconstrained static LA/AID system is nested in the full model and can be derived by imposing the following coefficient restrictions in H_2: $\sum_j \varsigma_{ij} = 0 \, \forall i$, which yields a total of 90 constraints as only $n-1$ equations can be estimated simultaneously. Imposition of lagged consumption is with its likelihood ratio statistic and corresponding p-value of 379.40 and 0.0000, respectively, clearly supported by data (further details are given in Section 4.3.2).

The unconstrained dynamic LA/AID system in first differences (reexpressed to include w_{it-1} as regressor with unity-coefficient) and H_2 are non-nested. With a 5 percent significance level $C(k_2 - k_1 + 1) = 114.27$ and $C(1) = 3.84$, where $k_1 = 108$ and $k_2 = 198$ refer to the number of independent parameters in the dynamic LA/AID system in first differences and H_2, respectively. According to the LDC, H_2 is preferred to the dynamic LA/AID system in first differences as $\mathscr{L}_2 - \mathscr{L}_1 = 304.97 > 55.22$.

The unconstrained ECM (reexpressed to include w_{it-1} as regressor with unity-coefficient) and H_2 are also non-nested. With a 5 percent significance level $C(k_2+1) = 232.91$ and $C(k_1+1) = 223.16$, where $k_1 = 189$ denotes the number of independent parameters in the unconstrained ECM. According to the LDC, the unconstrained ECM is preferred to H_2 as $\mathscr{L}_2 - \mathscr{L}_1 = -49.42 < 4.88$. However, as we discuss in Section 4.3.1, the ECM is not invariant to the omitted row, and care must therefore be exercised when using the specification in applied work.

The unconstrained dynamic LA/AID system with distributed lags and H_2 are also non-nested.[59] With a 5 percent significance level $\mathrm{C}(\mathrm{k}_2 - k_1 + 1) = 3.84$ and $C(1) = 3.84$, where $k_1 = 198$ denotes the number of independent parameters in the dynamic LA/AID system with distributed lags. According to the LDC, H_2 is preferred to the dynamic LA/AID system with distributed

[59]Vuong (1989) distinguishes between *strictly* and *overlapping* non-nested models. The unconstrained dynamic LA/AID system with distributed lags and H_2 share a set of explanatory variables ($\sum_j \gamma_{ij} log \, p_{jt}$) and are hence examples of the latter type.

lags as $\mathscr{L}_2 - \mathscr{L}_1 = 6.72 > 0$. This result is brought about by the fact that the *dominance ordering* and the LDC always prefer the model with the higher log-likelihood when the two hypotheses contain the same number of parameters (Pollak and Wales, 1991).

To conclude, the LDC indicates that the ECM is preferred to the other expenditure system specifications. However, due to the identification problem and related non-invariance of the estimates to the omitted row associated with the ECM, the second-best choice will be adopted, which means the preferred dynamic specification will be the dynamic LA/AID system incorporating habits in this analysis.