Analysis of a two country version of Eaton-Kortum Model of trade

Yim Hing Ho

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Preface

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All remaining mistakes are mine.

Yim Hing Ho

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Abstracts

This is a analysis of Eaton-Kortum model in two countries version based on the paper written by Fernando Alvarez and Robert E. Lucas "General Equilibrium Analysis of the Eaton-Kortum Model of international trade". The EK Model is a versatile and tractable probabilistic parameterization of the deterministic DFS Ricardian model. Production technology describes by two parameters, θ and λ. In this simple analysis we explore the implications of θ, different variance of individual productivity, by keeping the λ to be the same across country. It shows that gains from trade exist when two countries with same factor endowments trade with each other. Welfare to consumers in both countries will increase in trade situation as long as there is existence of heterogeneity in individual productivity. The bigger is θ, the more gains from trade.

Further more, this analysis shows also that any positive adjustment of trade barriers, for example increasing in transportation cost, will reduce the trade volume by creating a range of non-traded goods. The higher value of θ, which means low dispersion of efficiency across goods, the large will be the reduction in quantity of trade.

Key words: symmetric countries, technology parameters λ and θ, variance of individual productivity, gains of trade, transportation cost.

Contents

List of Figures

1 Introduction

Eaton and Kortum model is a versatile and tractable probabilistic parameterization of the Ricardian model with a continuum of tradeable goods (intermediate goods) indexed on a unit interval $q \in [0, 1]$, and the world is comprised of N countries (regions). In the theory, constant return producers in different countries are subject to idiosyncratic productivity shocks. Buyers of any good search over producers in different countries for the lowest price, and trade assigns production of any good to the most efficient producers, subject to costs of transportation and other impediments like tariff, etc. The gains from trade are larger the larger is the variance of individual productivity, which is θ,the key parameter in the model.

In this model, there are two sections in the economic, the market of a unique final good and the market of intermediate goods. Both final good and intermediate goods are produced under constant returns technology, and labor is the only primary (non-produced) factor to be used in both productions. With the aim of simplification, we assume the non-traded final good is the only good valued by consumers, and we use c for utility as well. The most important assumption of the Eaton and Kortum model is the perfect competition on each market, so it results that price equals marginal cost. In such way, we can just say that the Eaton Kortum model is competitive, involving no fixed costs and no monopoly rents.

In this paper, we analyze a two countries version of the Eaton and Kortum model.

There are two symmetric countries i and j in the economic. Both countries produce intermediate goods $q \in [0, 1]$ .Only intermediate goods, called intermediates, to be traded between two countries subject to some trade barriers like iceberg transportation cost and tariffs. Same labor endowment to both countries, and it is fixed. Full employment, and same wage rate for all workers as because full mobility of labor within own country. But labor is immobile between countries. Same production technologies for all producers in this economic, but individual productivity varies. Country with higher efficiency in producing good x will be end up as net exports of this intermediates, and consumers in both countries have access to the same lowest price for good x. Parameter θ describes the variance of individual productivity. The bigger is θ,the larger is the variance of individual productivity, the more are gains of trade.

Final good c is non-traded good in this model. Price level to final good c will not be the same for both countries. It depends on local wage rate, price level of intermediates and parameter θ. Perfect competition, no market power to any single producer, price equals its marginal cost same as unit cost because of the feature of constant returns to scale in production function. As long as there is only one single final good which is valued by consumers, the utility of consumers measures by consumption to the final good.

In the EK model, production technology describes by two parameters as λ and θ. λ characterizes the overall level of technology of a country (absolute advantage), and the θ (which is common to both countries) reflects the amount of variation within the distribution. In this paper, we explore the implications of different variance of individual productivity. We keep the λ to be the same across country, $\lambda_i = \lambda_j = 1$, let the difference in technology comes from heterogeneity in efficiency. It shows that gains from trade exist when two countries with similar factor endowments trade with each other. Welfare to consumers in countries with similar size will increase in trade situation when there is different in variance of individual productivity across goods. Further more, we explore the implications of increasing in trade barriers, for example increasing of transportation cost. It shows that increasing in transport cost will reduce trade activities. For countries with low dispersion of efficiency across goods, with another word, for countries with economic structure very close to each other, for example Norway and Sweden, increasing in trade barriers will cause large reduction of trade volume, which again will reduce welfare to all consumers in the economic.

The rest of the thesis is organized as follows. Section 2 presents two countries model of trade. In this section, I elaborate the structure of my simple two counties version of Eaton Kortum model by showing the process of finding price index of intermediates and equilibrium price to the non-traded good. I define trade balance condition, and then get reach to labor market clearing and equilibria. Section 3 shows the analysis of the model. I show that bilateral trade of two symmetric countries benefits everyone because of the existence of heterogeneity in efficiency. And I analyze the effect of transport cost on trade. The final section contains summary and conclusions to this simple two countries version of Eaton Kortum model.

2 Two countries model of trade

2.1 Tradeable sector

The definition of "iceberg" transportation cost: *One unit of any tradeable good shipped from j to i results in k_{ij} units arriving in i.* $0 < k_{ij} \leq 1$

For each producer to produce a particular intermediate good x, Cobb-Douglas production technology is applied:

$$q(x) = x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta} \tag{1}$$

where $s(x)$ is the labor input, and $q_m(x)$ is the input of intermediate goods

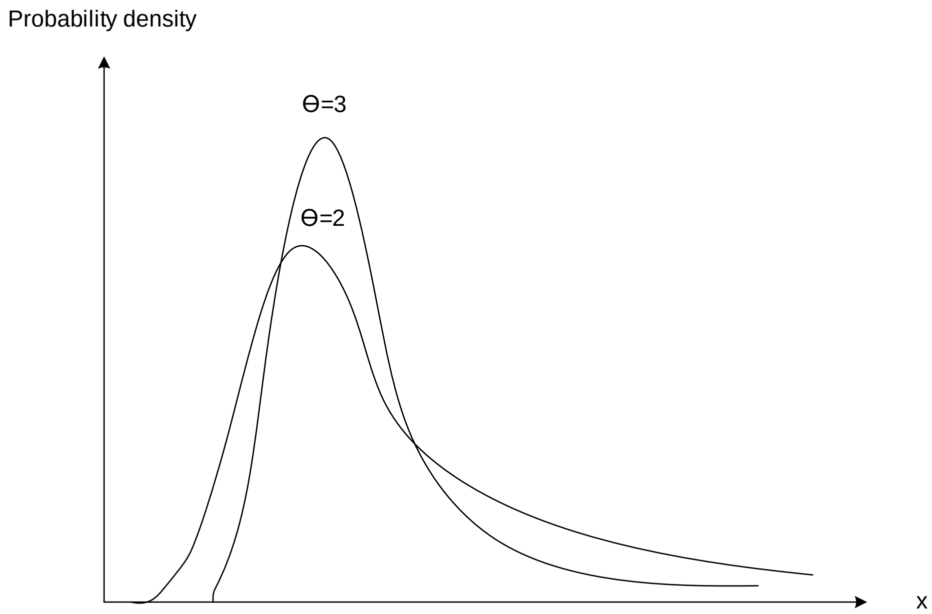

Figure 1: figure of density distribution with different value of θ

(capital input), $x^{-\theta}$ is a stochastic variable and describes the productivity.

Under autarky, the aggregate production function of intermediate goods will be as:

$$q = \left[\int_0^\infty q(x)^{1-\frac{1}{\eta}} \phi(x)\, dx \right]^{\frac{\eta}{(\eta-1)}}$$

For a particular good x, the density ϕ is exponential with parameter λ: $x \backsim \exp(\lambda)$. When $x = (x_1, x_2, ...x_n), \phi(x)$ has Frechet distribution: $\phi(x) = \exp\left(-\lambda x^{-\theta}\right)$.

Figure 1 illustrates the plots of the Frechet density function. The mean of Frechet distribution is $mean = \lambda^{\frac{1}{\theta}} \Gamma\left(1 - \frac{1}{\theta}\right)$. Here λ characterizes the overall level of technology of a country (absolute advantage), and θ (which is common to both countries) reflects the amount of variation within the distribution. A bigger θ implies less variability. However, in the paper of analyzing Eaton-

Kortum Model, written by Robert E. Lucas Jr. and Fernando Alvarez (March, 2006), they use θ for the parameter that Eaton and Kortum call $\frac{1}{\theta}$. Since this simple analyzing of two countries version bases on the paper to Alvarez and Lucas, a larger θ means a larger variance in individual productivity. Therefore, we say that x draws (in equ. 1.1) are then amplified in percentage terms by the parameter θ, and the random variables $x^{-\theta}$ then have a Frechet distribution.

In the case of trade, let x be the vector of technology draws for any given particular tradeable good for those two countries, $x \in R_+^2$. We assume these draws are independent across countries such that the joint density of x is:

$$\phi(x) = (\lambda_1 \lambda_2) \exp(-\lambda_1 x_1 - \lambda_2 x_2)$$

Both countries will produce the full range of intermediate goods. Use $q_i(x)$ for the consumption of tradeable good x in country i, and q_i for consumption in i of the aggregate (home produced + imported from j) intermediates:

$$q_i = \left[\int_{R_+^2} q_i(x)^{1-\frac{1}{\eta}} \ \phi(x)\, dx \right]^{\frac{\eta}{\eta-1}}$$

The labor endowment of the country L_i is allocated over the final good production and production of intermediates. Let s_{fi} and $s_i(x)$ be the fraction of labor force using in production of final good and intermediate goods at country i, the allocation of labor in country i will be as following:

$$s_{fi} + \int_{B_{ri}} s_i(x)\, \phi(x)\, dx \leq 1 \qquad r = i, j.$$

where $B_{ri} \subset R_+^2$ is the set of labor allocation in production of intermediates at country i for both domestic and export market.

All intermediate output will be allocated either in production of final good or in production of intermediate goods, not for direct consumption. Only intermediate goods are tradeable in this model. Total quantity of intermediates in country i comes from domestic production and import from country j, express as following:

$$\int_{B_{ri}} q_{mi}(x)\,\phi(x)\,dx \leq q_i \qquad r = i, j.$$

where $B_{ri} \subset R_+^2$ express the integral interval which included intermediates both from home production and import from j, applying the concept of trade balance: import equals export.

Let $p_i(x)$ be the price paid for tradeable good x by producers in i. If we consider now the case of autarky, for those intermediate goods producers in country i, they will maximize their profit by minimizing cost of production according to assumption of rational behavior of producers. In this case, $p_i(x)$ satisfy: (details in appendix 1)

$$\begin{aligned} p_i(x) &= Bx^{\theta}w^{\beta}p_{mi}^{1-\beta} \\ B &= \beta^{-\beta}(1-\beta)^{-1+\beta} \end{aligned}$$

Where $p_{mi}(w_i)$ is the price for intermediate goods in country i under autarky. It could be expressed as a function of w_i, wage level of country i. (details in appendix 2)

$$p_{mi} = (AB)^{\frac{1}{\beta}}\,\lambda^{-\frac{\theta}{\beta}}w_i \tag{2}$$

Under the situation of trade, there is a new searching area for buyers in both countries to look for even lower prices for intermediate goods. Buyers in country i compare domestic price offers with price offers in country j subject to transportation cost. So, for a particular good x, consumers in country i will compare price offers found in the market as:

$$Bx_i^\theta w_i^\beta p_{mi}^{1-\beta},\ Bx_j^\theta w_j^\beta p_{mj}^{1-\beta}\frac{1}{k_{ij}}$$

All buyers in both countries consume at the same, lowest price. The unit price for good x in country i will be:

$$p_i\left(x\right)=B\min\left[w_i^\beta p_{mi}^{1-\beta}x_i^\theta,\frac{w_j^\beta p_{mj}^{1-\beta}}{k_{ij}}x_j^\theta\right]$$

$$p_i\left(x\right)=B\min_r\left[\frac{w_r^\beta p_{mr}^{1-\beta}}{k_{ir}}x_r^\theta\right]\qquad r=i,j \tag{3}$$

where $p_{mr}=(p_{mi}(w_i),p_{mj}(w_j))$ are nominal price levels in each country. We know that these nominal price levels are functions depending on the local wage level. So we could say that $p_{mr}=(p_{mi},p_{mj})$ could be solved as a function of wage vector $w=(w_i,w_j)$.

The definition of price index: *Price index tells us about how much a producer has to pay for consuming one unit of intermediate goods with the aim of producing his own good.*

We define the price index of intermediate goods in country i to be:

$$p_{mi}=\left[\int_{R_+^2}p_i\left(x\right)^{1-\eta}\phi\left(x\right)dx\right]^{\frac{1}{1-\eta}} \tag{4}$$

After some complicate algebraic process, we can get the equation of price index to intermediate goods in country i as following: (details in appendix 2)

$$p_{mi}\left(w\right)=AB\left[\left(\frac{w_j^\beta p_{mj}(w_j)^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}}\lambda_j+\left(w_i^\beta p_{mi}(w_i)^{1-\beta}\right)^{-\frac{1}{\theta}}\lambda_i\right]^{-\theta} \tag{5}$$

Function (5) is the equation of price index to intermediates for country i under bilateral trade with country j. $p_{mi}(w_i), p_{mj}(w_j)$ depend on the wage vector $w = (w_i, w_j)$ and the value of θ. We also have to notice that (5) catches up the trade cost, k_{ir}, with another word, all the trade costs have been included in the price index of intermediates p_m.

2.2 Non-tradeable, the final good sector

Within Country i, all producers of final good are looking for profit maximization. According to the assumption of labor mobility, wage rate w_i will be the same for both sectors $w_{fi} = w_{mi}$. Trade of intermediate goods gives a opportunity to all producers in country i to consume intermediate goods at the best prices, $p_{mi}(w)$. Now, we try to find out the price of final good in country i.

Denote the wage rate by w_i, the price of final good by p_i. Production technologies of the only final good c, as well as the utility function of good c, are Cobb-Douglas,

$$c_i = s_{fi}^{\alpha} q_{fi}^{1-\alpha}$$

where s_{fi} is the labor input in the production of final good. q_{fi} is input of intermediate goods.

Cost minimizing behavior and the assumption of perfect competition of the final good market ensure that the equilibrium price of final good satisfies:(details in appendix 3)

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha} w_i^{\alpha} p_{mi}(w)^{1-\alpha} \tag{6}$$

Substitute the equation (5), $p_{mi}(w)$ into the price equation for final good

(6), we can get the following:

$$p_i = (AB)^{1-\alpha}\,\alpha^{-\alpha}(1-\alpha)^{-1+\alpha} w_i^{\alpha}\left[\left(\frac{w_j^{\beta} p_{mj}(w_j)^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}}\lambda_j + \left(w_i^{\beta} p_{mi}(w_i)^{1-\beta}\right)^{-\frac{1}{\theta}}\lambda_i\right]^{-\theta(1-\alpha)} \tag{7}$$

We could notice that all these prices, p_i, $p_i(x)$ and p_{mi} are different multiples of the wage vector $w = (w_i, w_j)$. This is a labor theory of value: *Everything is priced according to its labor content.* Say by my own word, all kinds of prices are positive correlated with wage rate, high wage rate, high price level.

p_i depends on w_i (local wage rate) and $p_{mi}(w)$.Perfect competition in final good market ensure that price equals marginal cost. If producers get lower input price of intermediate goods because of the existence of trade, will p_i be smaller. In such way that consumers in country i get higher welfare as the real wage $\frac{w_i}{p_i}$increases. In the section of gain of trade, we will prove that price level to intermediate goods under trade situation will be less or equal to price level under autarky situation, $p_m^T \leq p_m^A$.

2.3 Trade balance condition

First we have to calculate the tradeable expenditure share for country i. We define D_{ij} to be the fraction of country i's per capita spending on tradeable which are imported from country j. The expenditure in country i on intermediates (domestically produced plus imported from country j) is $p_{mi}q_i$. In such way we could define $p_{mi}q_iD_{ij}$ to be the expenditure country i spent on imported tradeable from country j. According to trade balance, country i's expenditure on imported intermediates has to be equal to country j's expenditure spending on intermediates from country i.

$$p_{mi}q_iD_{ij} = p_{mj}q_jD_{ji}$$

As we know, intermediate goods producers in both countries are rivals in the tradeable market. Function (3) says that only the low price vendors for buyers are winners in this competition. Economically, the total spending in i on goods from j is:

$$p_{mi}q_iD_{ij} = \int_{B_{ij}} p_i(x)\, q_i(x)\, \phi(x)\, dx$$

where B_{ij} is the set on which j attains the minimum in (3).

We can prove that the D_{ij} is also the probabilities that for a particular good x, the low price vendors for buyers in i are sellers in j. $D_{ij} = \pi_{ij} = \Pr[p_{ij} \leq p_{ii}]$. (see details in appendix 4).

So, we can describe D_{ij} as the following:

$$\begin{aligned} D_{ij} &= \pi_{ij} = \Pr[p_{ij} \leq p_{ii}] \\ &= \Pr\left[\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}} x_j^{\theta} \leq w_i^{\beta} p_{mi}^{1-\beta} x_i^{\theta}\right] \end{aligned}$$

The left hand side of the inequality above is just: $\varphi_{ij} = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j$, and the random variable on the right hand side of the inequality is exponential with parameter $\varphi_{ii} = \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-\frac{1}{\theta}} \lambda_i$.

Since those two are independent to each other, the property (iii) of exponential distribution

$$x \text{ and } y \text{ are } iid,\ x \sim \exp(\lambda),\quad y \sim \exp(\mu) \quad \Rightarrow \quad \Pr\{x \leq y\} = \frac{\lambda}{\lambda + \mu}$$

implies:

$$D_{ij} = \frac{\varphi_{ij}}{\varphi_{ij} + \varphi_{ii}} = \frac{\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j}{\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j + \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-\frac{1}{\theta}} \lambda_i} \tag{8}$$

Rewrite (5):

$$\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j + \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-\frac{1}{\theta}} \lambda_i = \left(\frac{p_{mi}(w)}{AB}\right)^{-\frac{1}{\theta}}$$

and then replace into the function (8), we get the $i's$ per capita spending on tradeable imported from j as following:

$$\begin{aligned} D_{ij} &= \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j \cdot \left(\frac{p_{mi}(w)}{AB}\right)^{\frac{1}{\theta}} \\ &= (AB)^{-\frac{1}{\theta}} \left(\frac{w_j^{\beta} p_{mj}(w)^{1-\beta}}{p_{mi}(w)\, k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j \end{aligned} \tag{9}$$

We notice that $\sum_r D_{ir} = D_{ij} + D_{ii} = 1$.

If the trade barriers become larger, $\triangle k_{ij} > 0, i's$ per capita spending on tradeable imported from j will increase given that the total imported quantity unchanged.

$$\frac{\partial D_{ij}}{\partial k_{ij}} = \frac{1}{\theta} \lambda_j (AB)^{-\frac{1}{\theta}} k_{ij}^{\frac{1-\theta}{\theta}} \left(\frac{w_j^{\beta} p_{mj}(w)^{1-\beta}}{p_{mi}(w)}\right)^{-\frac{1}{\theta}} > 0$$

The country-specific parameter λ_j governs the location of the distribution. A bigger λ_j implies a high efficiency draw for any good x coming from country j is more likely. We refer to the parameter λ_j as country $j's$ state of technology.

So, bigger λ_j implies that, it's more likely for country i to import intermediates from country j as because country j has higher technology level. In the case of $\lambda_j > \lambda_i$, it means more import from j, more expenditure spending on goods produced in country j.

$$\frac{\partial D_{ij}}{\partial \lambda_j} = (AB)^{-\frac{1}{\theta}} \left(\frac{w_j^{\beta} p_{mj} (w)^{1-\beta}}{p_{mi} (w) k_{ij}} \right)^{-\frac{1}{\theta}} > 0$$

As L_i is the labor endowment in country i, firms in i spend $L_i p_{mi} q_i$ on intermediates. Of this amount will $L_i p_{mi} q_i D_{ij}$ be the payment reaches sellers in country j. Symmetrically, sellers in country i will receive $L_j p_{mj} q_j D_{ji}$ from country i. Trade balance requires that:

$$L_i p_{mi} q_i D_{ii} + L_i p_{mi} q_i D_{ij} = L_i p_{mi} q_i D_{ii} + L_j p_{mj} q_j D_{ji} \tag{10}$$

this says that expenditure equals income. Total expenditure of country i on intermediates, including spending on domestically produced, $L_i p_{mi} q_i D_{ii}$and imported from country j, $L_i p_{mi} q_i D_{ij}$must be equal to total income of country i earned from intermediates.

We can simplify (10) to:

$$L_i p_{mi} q_i D_{ij} = L_j p_{mj} q_j D_{ji}$$

This is the def. of trade balance: *Payments to foreigners equals receipts from foreigners.*

The trade balance condition can be written as:

$$L_i p_{mi} q_i = L_i p_{mi} q_i D_{ii} + L_j p_{mj} q_j D_{ji} \tag{11}$$

The right hand side of (11) describes the total expenditure on tradeable in country i, while the left hand side is divided up into two parts. $L_i p_{mi} q_i D_{ii}$ is the expenditure on domestic produced intermediates, and $L_j p_{mj} q_j D_{ji}$ is the expenditure spending on imported goods. This equation is a budget constrain to country i.

2.4 Labor market clearing

First we turn to find out the share of tradeable in the production of the final good. Production technology of final good is Cobb-Douglas, $c = s_f^{\alpha} q_f^{1-\alpha}$, using the property of Cobb-Douglas function, we can get the following:

$$
\begin{aligned}
q_f &= (1-\alpha)\frac{pc}{p_m} \\
s_f &= \alpha\frac{pc}{w}
\end{aligned}
$$

For country i, $p_i c_i$ equals to the expenditure on final good at i. $L_i p_i c_i = L_i w_i$ describes the fact that GDP equals national income. So, the share of tradeable in the production of the final good is:

$$
\begin{aligned}
1-\alpha &= \frac{p_{mi} q_{fi}}{p_i c_i} \\
1-\alpha &= \frac{L_i p_{mi} q_{fi}}{L_i w_i} \qquad (12)
\end{aligned}
$$

Intermediate goods has Cobb-Douglas technology, (1): $q(x) = x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta}$ for good x. Productivity distribution to x is exponential distribution, $x \sim \exp(\lambda)$. $x = (x_1, x_2, ...x_{n)}$ with iid productivity distribution, we get the share of intermediates in the production of intermediate goods as following:

$$
q_{mi} = (1-\beta)\frac{p_{mi} q_i}{p_{mi}}
$$

where $p_{mi}q_i$ represents the expenditure on tradeable in country i. And we know that,

$$q_{mi} = q_i - q_{fi}$$

So the share of tradeable in the production of intermediate goods is:

$$\begin{aligned} 1-\beta &= \frac{L_i p_{mi}\left(q_i - q_{fi}\right)}{L_i p_{mi} q_i} \\ L_i p_{mi} q_i \left(1-\beta\right) &= L_i p_{mi} q_i - L_i p_{mi} q_{fi} \end{aligned} \tag{13}$$

The fact of trade balance: *the value of tradeable produced must equal to the value of tradeable used in production.*

Combining the equations (12) and (13), after some arrangement, we get:

$$L_i p_{mi} q_i = \frac{(1-\alpha)}{\beta} L_i w_i \tag{14}$$

Equation (14) says that the total expenditure on tradeable in country i takes a fix fraction of total income to country i.This matches the property of Cobb Douglas production function with constants return to scales.

Symmetrically,

$$L_j p_{mj} q_j = \frac{(1-\alpha)}{\beta} L_j w_j \tag{15}$$

Applying (14) and (15) to both sides of the trade balance condition (11), we can obtain:

$$\begin{aligned} \frac{(1-\alpha)}{\beta} L_i w_i &= \frac{(1-\alpha)}{\beta} L_i w_i D_{ii} + \frac{(1-\alpha)}{\beta} L_j w_j D_{ji} \\ L_i w_i &= L_i w_i D_{ii} + L_j w_j D_{ji} \end{aligned} \tag{16}$$

Equation (16) is also a trade balance condition which says that total income of country i equals to total expenditure of country i. This is a budget constrain to country i.

2.5 Equilibrium

From equations (16), (10) and (11), we can develop functions (17) and (18) as below. (see appendix 5 for details).

$$L_i w_i (1 - s_{fi}) = L_i w_i (1 - s_{fi}) D_{ii} + L_j w_j (1 - s_{fj}) D_{ji} \tag{17}$$

where $L_i w_i (1 - s_{fi})$ is the labor income from tradeable in country i. Here we see that the labor income from intermediates for a country can be earned by labor income from producing tradeable for home market, $L_i w_i (1 - s_{fi}) D_{ii}$,and for foreign market, $L_j w_j (1 - s_{fj}) D_{ji}$.

And there is constant labor share in the production of final good c because of Cobb Douglas production technology:

$$s_{fi} = \alpha \tag{18}$$

In our simple two countries version of EK-model, labor is the only primary (non-produced) factor to be used in both productions. There will exist a wage vector where all markets clear. As long as the final good is non-traded, we can say that when the market of intermediates is clear, all markets are clear, i.e., existence of Walrasian equilibria. Therefore, we focus us only in the market of tradeable when we derive excess demand function.

The value of aggregate excess demand function of tradeable to country i will be:

$$w_i Z_i (w) = L_i w_i (1 - s_{fi}) D_{ii} + L_j w_j (1 - s_{fj}) D_{ji} - L_i w_i (1 - s_{fi} (w)) \tag{19}$$

where $w = (w_i, w_j)$, a wage vector. And $Z_i(w)$ is homogeneous of degree zero in w.

The aggregate excess demand function must satisfy a condition known as Walras' law.

Walras's law: *For any price vector w, we have $wz(w) \equiv 0$; i. e., the value of the excess demand is identically zero.*

So that, we have:

$$w_i Z_i(w) \equiv 0$$

We can rearrange the excess demand system to country i, $Z_i(w)$, to get the excess demand of labor in production of tradeable as:

$$Z_i(w) = \frac{1}{w_i}\left[L_i w_i (1 - s_{fi}) D_{ii} + L_j w_j (1 - s_{fj}) D_{ji} - L_i w_i (1 - s_{fi}(w))\right] \quad (20)$$

which equals zero to ensure the **existence of Walrasian equilibria.**

w_j is in positive correlation with $Z_i(w)$. When wages of country j, w_j,increases, the production cost of producing tradeable in country j will increase, such that demand of tradeable from country j will decrease, but the demand of tradeable from country i will increase. So that the labor demand from tradeable section in country i will increase, with another word, when w_j up, $\frac{\partial Z_i(w)}{\partial w_j} > 0$.

$$\frac{\partial Z_i(w)}{\partial w_j} = \frac{L_j}{w_i}(1 - s_{fj}) D_{ji} > 0$$

We sum up all above in the following definition.

Definition: *An equilibrium is a wage vector $w = (w_i, w_j)$ such that $Z_i(w) \equiv 0$, where the function to price index for intermediate goods satisfy* (5)*, the function $D_{ij}(w)$ satisfy* (9)*.and the function $s_{fi}(w)$ satisfy* (18)*.*

Given an equilibrium wage vector w and the price index of the intermediates to country i, $p_{mi}(w)$, we can calculate the equilibrium price to final good p_i (6), and output quantities for country i.

3 Analysis of the model

3.1 Gains of trade

We want to find out the gain of trade by comparing real wage to consumers between autarky situation and the situation of trade. In this model, utility of consumers measures by consumption to the unique final good c, which has the production cost depends on price level to intermediate goods. So, once we can prove the price level of intermediate goods is lower under trade, $p_m^T \leq p_m^A$, according to equ 1,6 and 1,6' (equilibrium price of final good), we get that $\frac{w}{p_c^T} \geq \frac{w}{p_c^A}$. This means that welfare under trade is large than welfare under autarky.

For the aim of simplicity, we assume that there are two symmetric countries, $\lambda_i = \lambda_j = 1, 0 < k_{ij} = k_{ji} = k \leq 1, k_{ii} = k_{jj} = 1$, and $w_i = w_j = w$.

Putting this assumptions into (2) (Price for intermediate goods in autarky), we get the following:

$$p_m^A = (AB)^{\frac{1}{\beta}} w \tag{21}$$

From (5) (Price for intermediate goods under trade), we get:

$$p_m^T = ABw^{\beta} \left(p_m^A\right)^{1-\beta} \left[\left(\frac{1}{k}\right)^{-\frac{1}{\theta}} + 1\right]^{-\theta} \tag{22}$$

Put (21) into (5), the (22) becomes:

$$p_m^T = (AB)^{\frac{1}{\beta}} w \left[\left(\frac{1}{k}\right)^{-\frac{1}{\theta}} + 1\right]^{-\theta} \tag{23}$$

since $0 < k_{ij} = k_{ji} = k \leq 1$ and $\theta > 1$,

$$\left[\left(\frac{1}{k}\right)^{-\frac{1}{\theta}} + 1\right]^{-\theta} < 1 \tag{24}$$

which implies that:

$$p_m^T \leq p_m^A$$

Therefore, we know that welfare under trade is higher than welfare in autarky. Consumers in both countries benefit from trade situation.

Now we set up a table for showing that under same transportation cost, $k = 0.8$, higher value of θ,lower value of (24).

$k = 0.8$	$\theta = 2$	$\theta = 4$	$\theta = 8$	$\theta = 10$
$\left[\left(\frac{1}{k}\right)^{-\frac{1}{\theta}} + 1\right]^{-\theta}$	0.2786	0.0698	0.0044	0.0011

the smaller is the value of (24) the lower will the p_m^T going to be, the higher is the real wages $\frac{w}{p_c^T}$ in trade. θ reflects the amount of variation within the distribution. A larger θ means a larger variance in individual productivity. This observation confirm the assertion that gains from trade are larger the larger is the variance of individual productivity.

At the same time, we notice that the production technologies of good c is Cobb-Douglas in constant return to scale. α is the share of labor input, and $1 - \alpha$ is the share of input of intermediates. So p_i will be smaller the smaller is α,with another word, the smaller is α (larger share of intermediates),the higher will be the real wage $\frac{w_i}{p_i}$,the greater are gains from trade.

3.2 The effect of transport cost on trade

In this part, we want to proof that any kinds of trade barriers, for example transportation cost, will reduce trade activities, and this again will cause reduction of welfare to consumers in both countries.

Since the Eaton Kortum model is in probabilistic formulation, we need to establish a connection between the EK probabilistic formulation and the deterministic DFS model. For simplicity, we will assume that there are two symmetric countries, which means that $\lambda_i = \lambda_j = 1$(for any good x, both country i and country j have the same probability to be drawn into the group of high efficiency), $0 < k_{ij} = k_{ji} = k \leq 1$ (iceberg transportation cost), $k_{ii} = k_{jj} = 1$ (no transportation cost within a country). And we have to assume that labor is the only production factor, $\beta = 1$.

Haberlet (1937) has verified that transport cost give rise to a range of commodities that are non-traded. Non-traded goods are intermediate goods which are produced in both countries for their home market.

First, we have to re-order goods according to the ratio of the relative productivity of labor at country i to the one abroad x_i/x_j. To obtain the new ordering, we define a cutoff value a, and calculate the probability that the relative labor productivity is smaller than a. In our model, there is a continuum of intermediate goods indexed as $q \in [0, 1]$, and the probability measure is normalized to 1. We have to find out the pivotal good $q(a)$, which is the divide for goods with relative productivity level below it (relative productivity level is smaller than a), and goods with relative productivity level above it (relative productivity level is bigger than a).

such pivotal good $q(a)$ is defined as :

$$\begin{aligned} q(a) &\equiv \Pr\left(\frac{X_j}{X_i} \leq a\right) = \Pr\left(X_j \leq aX_i\right) \\ q(a) &= \frac{1}{1 + a^{-\theta}} \end{aligned} \tag{25}$$

(details to (25) states in appendix 6). Inverting (25), we can get the corresponding relative productivity for this pivotal good $q(a)$:

$$a(q) = \left(\frac{1-q}{q}\right)^{-\frac{1}{\theta}} \tag{26}$$

According to (26), the underlying absolute productivity of country i and country j will be as following:

$$\begin{aligned} x_i(q) &= Aq^{-\frac{1}{\theta}} \\ x_j(q) &= A(1-q)^{-\frac{1}{\theta}} \end{aligned}$$

where $A = \frac{\theta-1}{\theta}\Gamma\left(1-\frac{1}{\theta}\right)$, is some constant of proportionality that we can calculate explicitly.

We normalize wages of country i and j to be 1, and we have assumed from the beginning that labor is the only production factor, $\beta = 1$, so the only cost of production is the wages and have been normalized to 1.By the assumption of perfect competition of intermediate goods market, we know the competitive price for a particular good x, $p(x)$ from both countries will equal to its marginal cost.

$$p_{ii}(x) = \frac{1}{x_i} = \frac{1}{Aq^{-\frac{1}{\theta}}} = A^{-1}q^{\frac{1}{\theta}}$$

$$p_{ji}(x) = \frac{1}{k} \cdot \frac{1}{Aq^{-\frac{1}{\theta}}} = k^{-1}A^{-1}q^{\frac{1}{\theta}}$$

$$p_{jj}(x) = \frac{1}{A(1-q)^{-\frac{1}{\theta}}} = A^{-1}(1-q)^{\frac{1}{\theta}}$$

$$p_{ij}(x) = \frac{1}{k} \cdot \frac{1}{A(1-q)^{-\frac{1}{\theta}}} = k^{-1}A^{-1}(1-q)^{\frac{1}{\theta}}$$

Those prices depend on variables k and θ, where k is the transportation cost, and θ is a parameter common to all countries, characterizes the dispersion of efficiency across goods (comparative advantage).

As we have assumed before, market is under perfect competition, and that exists perfect information. Consumers in both countries know about what is the best offer for good x, and every consumer will buy good x in the lowest price appears in the market. So, for a particular good x, country i will import good x from country j if and only if $p_{ij}(x) \leq p_{ii}(x)$. Same argumentation for country j, for a particular good x, country j will import good x from country i if and only if $p_{ji}(x) \leq p_{jj}(x)$.

For the case of country i,

$$\begin{aligned} p_{ij}(x) &\leq p_{ii}(x) \Longrightarrow k^{-1}A^{-1}(1-q)^{\frac{1}{\theta}} = A^{-1}q^{\frac{1}{\theta}} \\ qk^{\theta} + q &= 1 \\ q^{*} &= \frac{1}{1+k^{\theta}} \end{aligned} \tag{1. cutoff}$$

For the case of country j,

$$\begin{aligned} p_{ji}(x) &\leq p_{jj}(x) \Rightarrow k^{-1}A^{-1}q^{\frac{1}{\theta}} = A^{-1}(1-q)^{\frac{1}{\theta}} \\ k^{\theta}(1-q) &= q \\ q^{**} &= \frac{k^{\theta}}{1+k^{\theta}} \end{aligned} \tag{2. cutoff}$$

The 1. cutoff determines the range of goods that are produced by country j for export to country i. The 2. cutoff determines the range of goods that are produced by country i for export to country j. The range of goods in between those two cutoffs are goods produced in both countries, we call this range of goods as non-traded goods. Non-traded goods are produced only for domestic market, not for trade.

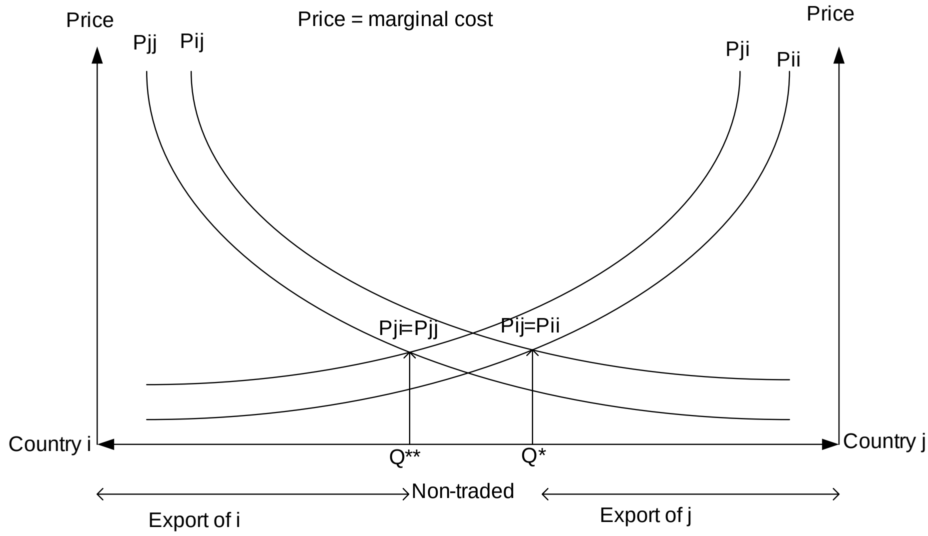

Figure 2: Production ranges for export goods and non-traded goods

When the transportation cost k increases, we get the following:

$$\begin{aligned}\frac{\partial q^*}{\partial k} &= -\left(1+k^\theta\right)^{-2}\theta k^{\theta-1}\\ &= -\frac{\theta k^{\theta-1}}{\left(1+k^\theta\right)^2}<0\end{aligned}$$

It shows that the 1. cutoff moves towards right in the figure (see figure 3) when the transportation cost k increases. This means the range of goods previously are produced in country j for export reduces as because $p_{ij}(x)$ increases alone with increasing of $k.p_{ij}(x)$ up means goods from country j are less competitive / less attractive in the market of country i. At the same time, we can

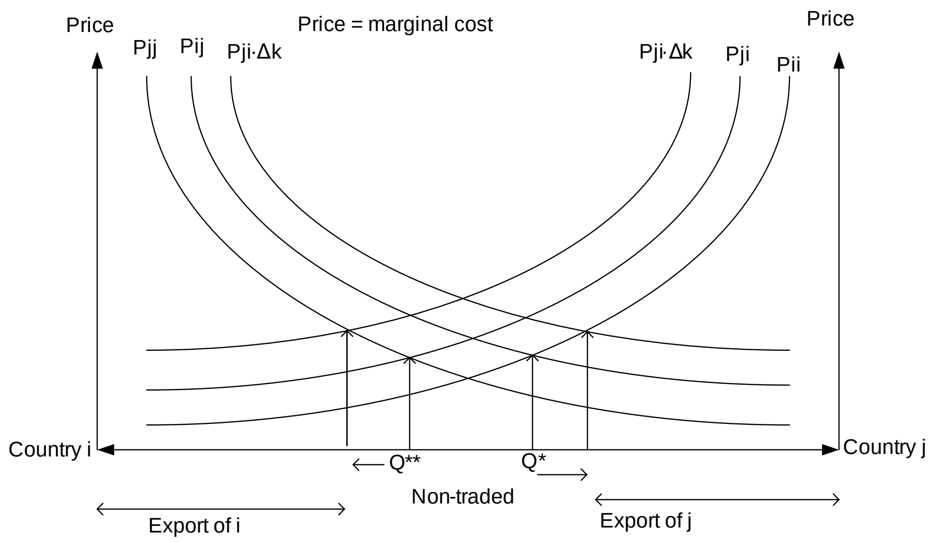

Figure 3: Result of increasing in transportation cost k

find out the effect of increasing k on country i.

$$\begin{aligned}
\frac{\partial q^{**}}{\partial k} &= \theta k^{\theta-1}\left(1+k^{\theta}\right)^{-1} + k^{\theta}(-1)\left(1+k^{\theta}\right)^{-2}\theta k^{(\theta-1)} \\
&= \theta k^{\theta-1}\left(1+k^{\theta}\right)\left[1-k^{\theta}\left(1+k^{\theta}\right)^{-1}\right] \\
&= \frac{\theta k^{\theta-1}}{1+k^{\theta}}\cdot\left[1-\frac{k^{\theta}}{(1+k^{\theta})}\right]
\end{aligned}$$

since $0<k\leq 1$,

$$\frac{\partial q^{**}}{\partial k} = \frac{\theta k^{\theta-1}}{(1+k^{\theta})^{2}} > 0$$

It shows the 2. cutoff moves towards left in the figure when the transportation cost k increases. This means the range of goods previously are produced in country i for export to j reduces. It's because $p_{ji}(x)$ increases when k up, and this makes goods from country i are less attractive for consumers in country j.

And we notice that the range of non-traded goods increases. With another word, both countries produce more goods for only their own home markets.

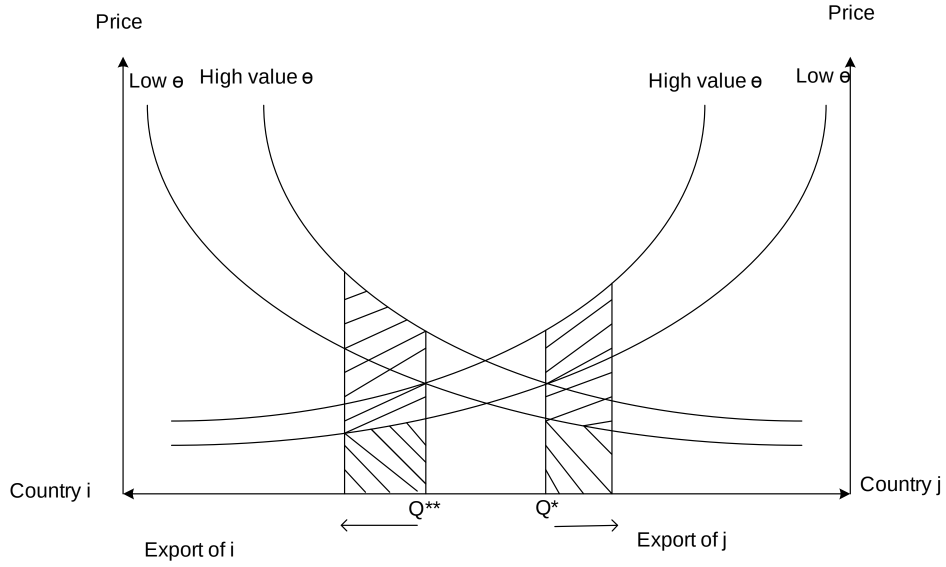

Figure 4: Resulting of increasing in trade barriers on economics in different values of θ

Figure 4 shows that with the same rate of increasing in transportation cost k, $\triangle k > 0$, the economic with lower value of θ, this means large dispersion of efficiency across goods, will experience less reducing in export quantity compare to the economic with higher value of θ, which means smaller dispersion of efficiency.

Under the situation of increasing in transportation cost, with another word, increasing of trade barriers, will force the whole situation towards autarky situation, production of non-traded goods increase in both countries Welfare to consumers in both countries reduces when trade barriers increase. It's because price level to consumption good c will be higher under the situation of less trade activities in intermediate goods, this causes lower real wage $\frac{w}{p_c}$.

4 Summary and conclusions

My analysis of Eaton-Kortum model in two countries version is based on the paper written by Fernando Alvarez and Robert E. Lucas "General Equilibrium Analysis of the Eaton-Kortum Model of international trade". Eaton-Kortum model is a versatile and tractable probabilistic parameterization of the deterministic DFS Ricardian model. I choose Eaton Kortum model to carry out my analysis of bilateral trade between two countries with same factor endowments because this model allows the existence of variance in individual productivity. The assumption of a representative producer with determined productivity in production of a particular good is common used in many models analyzing trade situation. However, in the Eaton Kortum model, there are undetermined many producers with heterogeneity in productivity for producing a particular good. Every producer has same probability to be drawn out, but only the most efficient producers from both countries get the chance to provide the particular good in both markets.

In the EK model, production technology describes by two parameters as λ and θ. λ characterizes the overall level of technology of a country (absolute advantage), and the θ (which is common to both countries) reflects the amount of variation within the distribution. In this paper, we explore the implications of different variance of individual productivity. We keep the λ to be the same across country, $\lambda_i = \lambda_j = 1$, let the difference in technology comes from heterogeneity in efficiency. It shows that gains from trade exist when two countries with similar factor endowments trade with each other. Welfare to consumers in countries with similar size will increase in trade situation when there is different in variance of individual productivity across goods. The key parameter to this model for describing variance of individual productivity is θ, we have showed in the section of gains of trade that the larger is θ,the more gains from trade.

Further more, we explore the implications of increasing in trade barriers, for example increasing of transportation cost. It shows that increasing in transport cost will reduce trade activities by creating a range of non-traded goods. For countries with low dispersion of efficiency across goods, which means large value of θ, increasing in trade barriers will cause large reduction of trade volume, which again will reduce welfare to all consumers in the economic.

So, this analysis shows a possible explanation to the phenomenon that countries with similar factor endowments benefit from trade with each other. The concept of free trade, aiming on reducing trade barriers, will increase trade activities among industry countries in similar size. This will benefit all countries involved in trade, and will result on a higher welfare to all consumers in the economic.

5 References

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6 Appendix

6.1 Appendix 1: Derivation of $p_i(x)$ in autarky.

Under the autarky situation in country i, we apply to the assumption of constant-return-to.scale, the market price of particular good x, $p_i(x)$, equals to its unit cost. With another word, the price $p_i(x)$ equals to the cost of producing one unit of good x, $q_i(x) = 1$.

Problem of the particular good x producers

$$\min w_m s(x) + p(m) q(m)$$

such that

$$x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta} = q(x) = 1$$

Using Lagrangian method:

$$\begin{aligned}
L_{s(x),q(m)} &= w\ s(x) + p(m) q(m) - \lambda \left(x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta} - 1 \right) \\
L'_{s(x)} &= w - \lambda \beta s(x)^{\beta-1} q(m)^{1-\beta} x^{-\theta} = 0 \\
L'_{q(m)} &= p(m) - \lambda (1-\beta) q(m)^{-\beta} x^{-\theta} s(x)^{\beta} = 0 \\
\frac{w}{p(m)} &= \frac{\beta q(m)}{s(x)(1-\beta)} \\
ws(x)(1-\beta) &= p(m) \beta q(m) \\
s(x) &= \frac{\beta p(m) q(m)}{(1-\beta) w}
\end{aligned}$$

So that,

$$\begin{aligned}
x^{-\theta}\left[\frac{\beta p(m)\, q(m)}{(1-\beta)\, w}\right]^{\beta} q(m)^{1-\beta} &= 1 \\
x^{-\theta}\left[\frac{\beta p(m)}{(1-\beta)\, w}\right]^{\beta} q(m) &= 1 \\
q^{*}(m) &= \frac{x^{\theta} w^{\beta}(1-\beta)^{\beta}}{\beta^{\beta} p(m)^{\beta}}
\end{aligned}$$

and,

$$\begin{aligned}
x^{-\theta} s(x)^{\beta}\left[\frac{x^{\theta} w^{\beta}(1-\beta)^{\beta}}{\beta^{\beta} p(m)^{\beta}}\right]^{1-\beta} &= 1 \\
s(x)^{\beta} &= \frac{x^{\theta\beta}\left[\beta p(m)\right]^{\beta(1-\beta)}}{\left[(1-\beta)\, w\right]^{\beta(1-\beta)}} \\
s^{*}(x) &= \frac{x^{\theta}\left[\beta p(m)\right]^{1-\beta}}{\left[(1-\beta)\, w\right]^{1-\beta}}
\end{aligned}$$

As we know, price = unit cost,

$$\begin{aligned}
p_i(x) &= w\frac{x^{\theta}\left[\beta p(m)\right]^{1-\beta}}{\left[(1-\beta)\, w\right]^{1-\beta}} + p(m)\ \frac{x^{\theta} w^{\beta}(1-\beta)^{\beta}}{\beta^{\beta} p(m)^{\beta}} \\
&= p(m)^{1-\beta} w^{\beta} x^{\theta}\frac{(1-\beta)^{\beta-1}}{\beta^{\beta}} \\
&= Bp(m)^{1-\beta} w^{\beta} x^{\theta}
\end{aligned}$$

where,

$$B = \beta^{-\beta}(1-\beta)^{-1+\beta}$$

6.2 Appendix 2: Derivation of p_m^A, price of intermediate goods in autarky.

we define the price index of intermediate goods for country i as:

$$p_{mi} = \left[\int_o^\infty p_i(x)^{1-\eta} \phi(x)\, dx \right]^{\frac{1}{1-\eta}}$$

and we know the density ϕ is exponential with parameter λ: $x \sim \exp(\lambda)$. So, we have $\phi(x) = \lambda \exp(-\lambda x)$.

$$p_{mi} = \left[\lambda \int_o^\infty \exp(-\lambda x)\, p_i(x)^{1-\eta}\, dx \right]^{\frac{1}{1-\eta}}$$

from appendix 1 we get $p_i(x) = Bp(m)^{1-\beta} w^\beta x^\theta$. put it into function for p_{mi}:

$$\begin{aligned} p_{mi} &= \left[\lambda \int_o^\infty \exp(-\lambda x) \left[Bp(m)^{1-\beta} w^\beta x^\theta \right]^{1-\eta} dx \right]^{\frac{1}{1-\eta}} \\ &= Bp(m)^{1-\beta} w^\beta \left[\int_0^\infty \lambda \exp(-\lambda x)\, x^{\theta(1-\eta)} dx \right]^{\frac{1}{1-\eta}} \end{aligned}$$

use substitution method, we set $z = \lambda x, \Rightarrow x = \frac{z}{\lambda}$:

$$\begin{aligned} p_{mi} &= Bp(m)^{1-\beta} w^\beta \left[\int_0^\infty \lambda \exp(-z) \left(\frac{z}{\lambda} \right)^{\theta(1-\eta)} \frac{1}{\lambda} dz \right]^{\frac{1}{1-\eta}} \\ &= Bp(m)^{1-\beta} w^\beta \lambda^{-\theta} \left[\int_0^\infty e^{-z} z^{\theta(1-\eta)} dz \right]^{\frac{1}{1-\eta}} \end{aligned}$$

the Gamma function is $\Gamma(\xi) \equiv \int_0^\infty z^{\xi-1} e^{-z} dz$. We notice that the integral in bracket is Gamma function $\Gamma(\xi)$, evaluated at the argument $\xi = 1 + \theta(1-\eta)$. And convergence of the integral requires

$$1 + \theta(1-\eta) > 0$$

we write $A(\theta, \eta)$ for

$$A(\theta,\eta)=\left[\int_0^{\infty}e^{-z}z^{\theta(1-\eta)}dz\right]^{\frac{1}{1-\eta}}$$

so, the function of p_{mi} can be written as (2):

$$\begin{aligned} p_m &= Bp(m)^{1-\beta}w^{\beta}\lambda^{-\theta}A \\ p_m^{\beta} &= ABw^{\beta}\lambda^{-\theta} \\ p_m &= (AB)^{\frac{1}{\beta}}\lambda^{-\frac{\theta}{\beta}}w \end{aligned}$$

6.3 Appendix 3: Derivation of p_m^T, price of intermediate goods in trade.

we can derive an expression for p_{mi} from (3) and (4) by using two well-known properties of the exponential distribution:

$$x \sim \exp(\lambda) \quad andk>0 \quad \Rightarrow \quad kx\sim \exp\left(\frac{\lambda}{k}\right) \tag{(i)}$$

$$\begin{aligned} x &\sim \exp(\lambda), y\sim \exp(\mu),\ x\ \&\ y \text{ are } iid, dz=\min(x,y) \\ &\Rightarrow z\sim \exp(\lambda+\mu) \end{aligned} \tag{(ii)}$$

From (4), we have:

$$p_{mi}^{1-\eta}=\int_{R_+^2}p_i(x)^{1-\eta}\phi(x)\,dx \tag{27}$$

we have to notice here that the right hand side of (27) is the expected value of the random variable $p_i(x)^{1-\eta}$.

From (3), we get:

$$p_i(x)^{\frac{1}{\theta}}=B^{\frac{1}{\theta}}\left[\left(w_i^{\beta}p_{mi}^{1-\beta}\right)^{\frac{1}{\theta}}x_i,\left(\frac{w_j^{\beta}p_{mj}^{1-\beta}}{k_{ij}}\right)^{\frac{1}{\theta}}x_j\right]$$

Using the property (i) of the exponential distribution, we can find the productivity distribution for tradeable imported from country j to country i will be as following:

let

$$k = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}} \right)^{\frac{1}{\theta}} > 0, \ x_j \sim \exp(\lambda_j)$$

the productivity distribution for tradeable imported to country i is φ_{ij} :

$$\varphi_{ij} = \frac{\lambda_j}{\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}} \right)^{\frac{1}{\theta}}} = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}} \right)^{-\frac{1}{\theta}} \lambda_j$$

Symmetrically, we can find the productivity distribution for intermediates produced domestically in country i as following,

let

$$k = \left(w_i^{\beta} p_{mi}^{1-\beta} \right)^{\frac{1}{\theta}} > 0, \ x_i \sim \exp(\lambda_i)$$

So the productivity distribution for tradeable produced home is φ_{ii} :

$$\varphi_{ii} = \frac{\lambda_i}{\left(w_i^{\beta} p_{mi}^{1-\beta} \right)^{\frac{1}{\theta}}} = \left(w_i^{\beta} p_{mi}^{1-\beta} \right)^{-\frac{1}{\theta}} \lambda_i$$

Since

$$x_j \sim \exp(\lambda_j), x_i \sim \exp(\lambda_i)$$

according to the property (ii) of the exponential distribution,

$$z = \min(x_i, x_j) \Rightarrow z \sim \exp(\lambda_i + \lambda_j)$$

we can get the productivity distribution of country i to be as following:

$$\begin{aligned} z &= \min\left[\left(w_i^\beta p_{mi}^{1-\beta}\right)^{\frac{1}{\theta}} x_i, \left(\frac{w_j^\beta p_{mj}^{1-\beta}}{k_{ij}}\right)^{\frac{1}{\theta}} x_j\right] \\ z &\sim \exp\left(\varphi_{ii} + \varphi_{ij}\right) \end{aligned}$$

So that we can say that $z \equiv \min_r z_r$ is exponentially distributed with parameter $\Phi_i = \varphi_{ii} + \varphi_{ij}$

This parameter Φ_i summarizes how (i) states of technology around the world, (ii) input costs around the world. International trade enlarges each country's effective state of technology with technology available from other countries, discounted by input costs and trade barriers. At one extreme case where no trade barriers, $k_{ij} = k_{ji} = 1, \Phi$ is the same for both countries.

From $p_i(x)^{\frac{1}{\theta}} = B^{\frac{1}{\theta}} \cdot z \sim \exp\left(\varphi_{ii} + \varphi_{ij}\right)$, we set this time $k = B^{\frac{1}{\theta}} > 0$, applying to the property (i), we can prove that $p_i(x)^{\frac{1}{\theta}}$ is exponentially distributed with parameter

$$\begin{aligned} p_i(x)^{\frac{1}{\theta}} &\sim \mu = B^{-\frac{1}{\theta}} \Phi_i \\ p_i(x)^{\frac{1}{\theta}} &\sim \exp(\mu) \end{aligned}$$

Now we set $u = p_i(x)^{\frac{1}{\theta}} \sim \exp(\mu)$, and then $u^\theta = p_i(x)$. Set $\phi(u)$ to be the distribution of u which follows F-distribution, $\phi(u) = \mu \exp(-\mu u)$. From (27) we can get the following:

$$\begin{aligned} p_{mi}^{1-\eta} &= \int_{R_+^2} u^{\theta(1-\eta)} \mu e^{-\mu u} du \\ p_{mi}^{1-\eta} &= \mu \int_{R_+^2} u^{\theta(1-\eta)} e^{-\mu u} du \end{aligned}$$

By using substitution method, let $z = \mu u$, $du = \frac{1}{\mu} dz, u = \left(\frac{z}{\mu}\right)$,

$$\begin{aligned} p_{mi}^{1-\eta} &= \int_{R_{+}^{2}} \mu \left(\frac{z}{\mu}\right)^{\theta(1-\eta)} e^{-z} \frac{1}{\mu} dz \\ p_{mi}^{1-\eta} &= \mu^{-\theta(1-\eta)} \int_{R_{+}^{2}} e^{-z} z^{\theta(1-\eta)} dz \end{aligned}$$

We set $A(\theta, \eta) = \left[\int\limits_{R_{+}^{2}} e^{-z} z^{\theta(1-\eta)} dz \right]^{\frac{1}{1-\eta}}$, following by the Gamma function $\Gamma(\xi) = \int\limits_{0}^{\infty} z^{\xi} e^{-z} dz$, and putting $\xi = 1 + \theta(1-\eta)$. So that the equation above can rewrite to be:

$$\begin{aligned} p_{mi}^{1-\eta} &= \mu^{-\theta(1-\eta)} A^{1-\eta} \\ p_{mi} &= \mu^{-\theta} A \end{aligned}$$

Because of the convergence requirement,

$$1 + \theta(1-\eta) > 0$$

Putting $\mu = B^{-\frac{1}{\theta}} \sum_r \varphi_{ir}$ into the equation above, we can get the following:

$$\begin{aligned} p_{mi}(w) &= A\left(B^{-\frac{1}{\theta}} \Phi_i\right)^{-\theta} \\ &= AB(\Phi_i)^{-\theta} \end{aligned}$$

Since we know that $\Phi_i = \varphi_{ij} + \varphi_{ii}$, we can get the following:

$$\Phi_i = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j + \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-\frac{1}{\theta}} \lambda_i$$

Then, we can get the equation of price index to intermediate goods in country i as following (5):

$$p_{mi}(w) = AB\left[\left(\frac{w_j^{\beta}p_{mj}(w_j)^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}}\lambda_j + \left(w_i^{\beta}p_{mi}(w_i)^{1-\beta}\right)^{-\frac{1}{\theta}}\lambda_i\right]^{-\theta}$$

6.4 Appendix 4: Derivation of equilibrium price to non-traded good (in autarky).

Production function of final good:

$$y = c = s_f^{\alpha}q_f^{1-\alpha}$$

which can be rewritten to:

$$q_f = y^{\frac{1}{1-\alpha}}s_f^{-\frac{\alpha}{1-\alpha}}$$

or,

$$s_f = y^{\frac{1}{\alpha}}q_f^{\frac{\alpha-1}{\alpha}}$$

Problem of the final good producer

$$c(w_f, p_m, y) = \min w_f s_f + p_m q_f$$

subject to,

$$q_f = y^{\frac{1}{1-\alpha}}s_f^{-\frac{\alpha}{1-\alpha}}$$

implicate that

$$w_f s_f + p_m y^{\frac{1}{1-\alpha}}s_f^{-\frac{\alpha}{1-\alpha}}$$

F.o.c. wrt. s_f :

$$\begin{aligned} w_f - \frac{\alpha}{1-\alpha} p_m y^{\frac{1}{1-\alpha}} s_f^{\frac{-1}{1-\alpha}} &= 0 \\ s_f^* &= \left(\frac{1-\alpha}{\alpha} \frac{w_f}{p_m} \right)^{\alpha-1} y \end{aligned}$$

For getting q_f^*, we repeat the same process as above,

$$c(w_f, p_m, y) = \min w_f s_f + p_m q_f$$

subject to,

$$s_f = y^{\frac{1}{\alpha}} q_f^{\frac{\alpha-1}{\alpha}}$$

such that,

$$w_f y^{\frac{1}{\alpha}} q_f^{\frac{\alpha-1}{\alpha}} + p_m q_f$$

F.o.c. wrt. q_f :

$$\begin{aligned} p_m + w_f y^{\frac{1}{\alpha}} \frac{\alpha-1}{\alpha} q_f^{\frac{-1}{\alpha}} &= 0 \\ q_f^* &= \left(\frac{\alpha}{1-\alpha} \frac{p_m}{w_f} \right)^{-\alpha} y \end{aligned}$$

And then, we put s_f^* and q_f^* into cost function:

$$\begin{aligned} c(w_f, p_m, y) &= w_f \left(\frac{1-\alpha}{\alpha} \frac{w_f}{p_m} \right)^{\alpha-1} y + p_m \left(\frac{\alpha}{1-\alpha} \frac{p_m}{w_f} \right)^{-\alpha} y \\ &= w_f^{\alpha} p_m^{1-\alpha} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha-1} + \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] y \end{aligned}$$

Use the constant-returns-to scale assumption, marginal cost = unit cost = price, $y = 1$:

$$\begin{aligned} c(w_f, p_m, y) &= c(w_f, p_m, 1) = p = w_f^{\alpha} p_m^{1-\alpha} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha-1} + \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] \\ p_i &= \alpha^{-\alpha} (1-\alpha)^{\alpha-1} w_f^{\alpha} p_{mi}^{1-\alpha} \end{aligned}$$

where,

$$\begin{aligned} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha-1} + \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] &= \left(\frac{1-\alpha}{\alpha} \right)^{\alpha} \left[\left(\frac{1-\alpha}{\alpha} \right)^{-1} + 1 \right] \\ &= \left(\frac{1-\alpha}{\alpha} \right)^{\alpha} \cdot \frac{1}{1-\alpha} \\ &= \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \end{aligned}$$

6.5 Appendix 5: Prove $D_{ij} = \pi_{ij} = \Pr[p_{ij} \leq p_{ii}]$.

Taking trade barriers into account, the price of delivering a unit of good x produced in country j to country i , p_{ij} , will be:

$$p_{ij}(x) = B x_j^{\theta} w_j^{\beta} p_{mj}^{1-\beta} \frac{1}{k_{ij}}$$

we can rewrite this function to be:

$$p_{ij}(x)^{\frac{1}{\theta}} = B^{\frac{1}{\theta}} \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}} \right)^{\frac{1}{\theta}} x_j$$

We know that $x_j \sim \exp(\lambda_j)$,using the property (i) of exponential distribution:

$$x \sim \exp(\lambda) \ \ and \ k > 0 \quad \Rightarrow \quad kx \sim \exp\left(\frac{\lambda}{k} \right)$$

we can get the following:

$$\varphi_{ij} = \frac{\lambda_j}{\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{\frac{1}{\theta}}} = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{k_{ij}}\right)^{-\frac{1}{\theta}} \lambda_j$$

and,

$$\begin{aligned} p_{ij}(x)^{\frac{1}{\theta}} &\sim y = B^{-\frac{1}{\theta}}\, \varphi_{ij} \\ p_{ij}(x)^{\frac{1}{\theta}} &\sim \exp(y) \end{aligned}$$

The price of delivering a unit of good x produced domestically in country i , p_{ii} , will be:

$$\begin{aligned} p_{ii}(x) &= B w_i^{\beta} p_{mi}^{1-\beta} x_i^{\theta} \\ p_{ii}(x)^{\frac{1}{\theta}} &= B^{\frac{1}{\theta}} (w_i^{\beta} p_{mi}^{1-\beta})^{\frac{1}{\theta}} x_i \\ \varphi_{ii} &= \frac{\lambda_i}{(w_i^{\beta} p_{mi}^{1-\beta})^{\frac{1}{\theta}}} = (w_i^{\beta} p_{mi}^{1-\beta})^{-\frac{1}{\theta}} \lambda_i \\ p_{ii}(x)^{\frac{1}{\theta}} &\sim d = B^{-\frac{1}{\theta}} \varphi_{ii} \\ p_{ii}(x)^{\frac{1}{\theta}} &\sim \exp(d) \end{aligned}$$

Using even one more property of exponential distributions (iii):

$$x \text{ and } y \text{ are } iid,\ x \sim \exp(\lambda)\,, \quad y \sim \exp(\mu) \quad \Rightarrow \quad \Pr\{x \leq y\} = \frac{\lambda}{\lambda + \mu}$$

Now we can get the probability that country j provides a good at the lowest price in country i is simply as following:

$$\begin{aligned}\pi_{ij} &= \Pr\left[p_{ij} \leq p_{ii}\right] = \frac{B^{-\frac{1}{\theta}}\ \varphi_{ij}}{B^{-\frac{1}{\theta}}\ \varphi_{ij} + B^{-\frac{1}{\theta}}\varphi_{ii}} \\ \pi_{ij} &= \frac{\varphi_{ij}}{\varphi_{ij} + \varphi_{ii}}\end{aligned}$$

Since there are a continuum of goods, $x = (x_1, x_2, ...x_n)$,by the law of large numbers, π_{ij} is also the fraction of goods that country i buys from country j. So, $\pi_{ij} = D_{ij}$.

6.6 Appendix 6: Derivation of (17) and (18)

Production technology of final good is Cobb-Douglas $c = s_f^{\alpha} q_f^{1-\alpha}$, the share formulas in final goods production are:

$$w_i s_{fi} = \alpha p_i c_i$$

and

$$p_{mi} q_{fi} = (1-\alpha)\, p_i c_i$$

by doing some simply rearrangement of these two equations, we can easily get:

$$\begin{aligned}p_{mi} q_{fi} &= p_i c_i - \alpha p_i c_i \\ &= p_i c_i - w_i s_{fi} \\ &= \frac{1}{\alpha} w_i s_{fi} - w_i s_{fi} \\ w_i s_{fi} &= \frac{\alpha}{1-\alpha} p_{mi} q_{fi} \qquad \text{(A.1.)}\end{aligned}$$

Starting from the Cobb-Douglas production function of intermediate goods, (1): $q(x) = x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta}$, we can get the labor requirement as:

$$s(x) = \beta \frac{L_i p_{mi} q_i(x)}{L_i w_i}$$

Recall the labor allocation in country i between final good production and intermediate goods production,

$$\begin{aligned} s_{fi} + \int_{B_{ri}} s_i(x)\,\phi(x)\,dx &\leq 1 \qquad r = i, j. \\ \int_{B_{ri}} s_i(x)\,\phi(x)\,dx &= 1 - s_{fi} \end{aligned}$$

and the total quantity of intermediate goods in country i comes from,

$$\int_{B_{ri}} q_{mi}(x)\,\phi(x)\,dx \leq q_i \qquad r = i, j.$$

So that, we rewrite the labor requirement to intermediate goods production in country i as:

$$(1 - s_{fi}) L_i w_i = \beta L_i p_{mi} q_i$$

we know that the total income in intermediates to country i can be earned by selling tradeable goods to home-market plus selling tradeable goods to foreign-market:

$$L_i p_{mi} q_i = L_i p_{mi} q_i D_{ii} + L_j p_{mj} q_j D_{ji}$$

Applying to the concept of trade balance: *import equals export*.

$$L_i p_{mi} q_i D_{ij} = L_j p_{mj} q_j D_{ji}$$

We get the following,

$$L_i p_{mi} q_i = L_i p_{mi} q_i D_{ii} + L_i p_{mi} q_i D_{ij} = L_i p_{mi} q_i \sum_r D_{ir}$$

Putting into $(1 - s_{fi}) L_i w_i = \beta L_i p_{mi} q_i$, we can get:

$$(1 - s_{fi}) L_i w_i = \beta L_i p_{mi} q_i \sum_r D_{ir} = \beta L_i p_{mi} q_i \qquad \text{(A.2.)}$$

since we know that

$$\sum_r D_{ir} = D_{ii} + D_{ij} = 1$$

Recall the share of tradeable in the production of intermediate goods is defined as:

$$\begin{aligned} 1 - \beta &= \frac{L_i p_{mi} (q_i - q_{fi})}{L_i p_{mi} q_i} \\ (1 - \beta) L_i p_{mi} q_i &= L_i p_{mi} (q_i - q_{fi}) \\ (1 - \beta) L_i p_{mi} q_i &= L_i p_{mi} q_{mi} \\ L_i p_{mi} q_{mi} &= (1 - \beta) L_i p_{mi} q_i \sum_r D_{ir} \\ L_i p_{mi} q_{mi} &= (1 - \beta) L_i p_{mi} q_i \qquad \text{(A.3.)} \end{aligned}$$

All intermediates are using either in the production of final good or in the production of intermediate goods:

$$q_i = q_{fi} + q_{mi}$$

such that :

$$\begin{aligned} L_i p_{mi} q_i &= L_i p_{mi} q_{fi} + L_i p_{mi} q_{mi} \\ L_i p_{mi} q_{fi} &= L_i p_{mi} q_i - L_i p_{mi} q_{mi} \end{aligned}$$

By setting (A.3) into the equation, we get:

$$\begin{aligned} L_i p_{mi} q_{fi} &= L_i p_{mi} q_i - \ (1-\beta) L_i p_{mi} q_i \\ L_i p_{mi} q_{fi} &= L_i p_{mi} q_i [1-(1-\beta)] \\ q_{fi} &= q_i [1-(1-\beta)] \qquad \text{(A.4.)} \end{aligned}$$

Then (A.1) and (A.4) imply:

$$w_i s_{fi} = \frac{\alpha}{1-\alpha} p_{mi} q_i [1-(1-\beta)] \qquad \text{(A.5.)}$$

From (A.2), we can have the following:

$$\begin{aligned} (1-s_{fi}) L_i w_i &= \beta L_i p_{mi} q_i \\ w_i (1-s_{fi}) &= \beta p_{mi} q_i \qquad \text{(A.6.)} \end{aligned}$$

So, (A.6.) implicates that:

$$p_{mi} q_i = \frac{w_i (1-s_{fi})}{\beta}$$

and symmetrically,

$$p_{mj} q_j = \frac{w_j (1-s_{fj})}{\beta}$$

Derive (18) by using $\frac{(A.5)}{(A.6)}$:

$$
\begin{aligned}
\frac{w_i s_{fi}}{w_i (1 - s_{fi})} &= \frac{\frac{\alpha}{1-\alpha} p_{mi} q_i \left[1 - (1 - \beta)\right]}{\beta p_{mi} q_i} \\
\frac{s_{fi}}{(1 - s_{fi})} &= \frac{\alpha p_{mi} q_i \left[1 - (1 - \beta)\right]}{1 - \alpha} \cdot \frac{1}{\beta p_{mi} q_i} \\
\frac{s_{fi}}{(1 - s_{fi})} &= \frac{\alpha \left[1 - (1 - \beta)\right]}{(1 - \alpha) \beta} \\
s_{fi} \left[(1 - \alpha) \beta + \alpha (1 - (1 - \beta))\right] &= \alpha \left[1 - (1 - \beta)\right] \\
s_{fi} &= \frac{\alpha \left[1 - (1 - \beta)\right]}{\left[(1 - \alpha) \beta + \alpha (1 - (1 - \beta))\right]} \\
s_{fi} &= \alpha
\end{aligned}
$$

Derive (17):

$$
\begin{aligned}
L_i p_{mi} q_i &= L_i p_{mi} q_i D_{ii} + L_j p_{mj} q_j D_{ji} \\
L_i \frac{w_i (1 - s_{fi})}{\beta} &= L_i \frac{w_i (1 - s_{fi})}{\beta} D_{ii} + L_j \frac{w_j (1 - s_{fj})}{\beta} D_{ji} \\
L_i w_i (1 - s_{fi}) &= L_i w_i (1 - s_{fi}) D_{ii} + L_j w_j (1 - s_{fj}) D_{ji}
\end{aligned}
$$

6.7 Appendix 7: Derive of probability to pivotal good $q(a)$.

Since for any good x, probability distribution in both countries is Frechet distribution, and we set $\lambda_i = \lambda_j = 1$.

$$F_i(x) = \Pr[X_i \le x] = \exp\left(-\lambda_i x_i^{-\theta}\right) = \exp\left(-x_i^{-\theta}\right)$$

$$F_j(x) = \Pr[X_j \le x] = \exp\left(-\lambda_j x_j^{-\theta}\right) = \exp\left(-x_j^{-\theta}\right)$$

solves for $q(a)$:

$$\begin{aligned}
q(a) &= \Pr\left(\frac{X_j}{X_i} \leq a\right) = \Pr\left(X_j \leq aX_i\right) \\
&= \int_0^\infty \exp - (ax_i)^{-\theta} \, dF_i(x_i) \\
&= \int_0^\infty \exp - (ax_i)^{-\theta} \, \theta x_i^{-\theta-1} \exp\left(-x_i^{-\theta}\right) dx_i \\
&= \int_0^\infty \exp\left[x_i^{-\theta}\left(-a^{-\theta}-1\right)\right] \theta x_i^{-\theta-1} dx_i
\end{aligned}$$

we know the probability measure is normalized to 1, as $\int_0^\infty f(x)\,dx = 1$:

$$\begin{aligned}
F(x) &= \exp\left[-x_i^{-\theta}\left(a^{-\theta}+1\right)\right] \\
f(x) &= \exp\left[-x_i^{-\theta}\left(a^{-\theta}+1\right)\right]\left(a^{-\theta}+1\right)\theta x_i^{-\theta-1}
\end{aligned}$$

so,

$$\int_0^\infty \exp\left[-x_i^{-\theta}\left(a^{-\theta}+1\right)\right]\left(a^{-\theta}+1\right)\theta x_i^{-\theta-1} dx_i = 1$$

such that (25):

$$\begin{aligned}
q(a) &= \frac{1}{1+a^{-\theta}} \int_0^\infty \left(1+a^{-\theta}\right) \exp\left[x_i^{-\theta}\left(-a^{-\theta}-1\right)\right] \theta x_i^{-\theta-1} dx_i \\
q(a) &= \frac{1}{1+a^{-\theta}}
\end{aligned}$$