Misallocation and Aggregate TFP in Chile and Norway

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Summary

The productivity differences have been proposed as a main factor of large differences in GDP per capita. Generally speaking, bad aggregate economic performance has been attributed broadly to "government regulations". In particular, Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Alfaro et al. (2008) suggested that resource misallocation affected by shocks is highly related with the aggregate total factor productivity.

Thus in this paper, we have studied on misallocation in Norway and Chile. Norway has strong economic condition which is stable. In the contrast, Chilean economy was heavily regulated, suffered a financial crisis in the beginning of 1980s and implemented in labor and capital market reforms that led to a strong recovery from mid 1980s.

The aim of this paper is to perform analysis on to what extent resources are distorted and how policies relate to aggregate efficiency in Chile and Norway. Using Hsieh and Klenow (2009)'s framework, we build the model that monopolistic competitive firms face distortions on output and capital. Distortions differentiate marginal revenue product across firms and therefore decrease aggregate TFP.

The data used are collected from Instituto National de Estadistica (INE) and World Bank's report for Chile (1980-1996), and from Statistics Norway (1996-2006).

To study the main impact of resource misallocation, we compare detrended aggregate TFP between Chile and Norway and find that the detrended TFP in Norway is fairly stable. We compute TFP relative to efficient TFP by assuming zero distortion and constant wage rate across firms. The high gain from removing distortions in Chile indicates that labor and capital inputs are more distorted in Chile than in Norway. Additionally, the moment of firm size distribution shows that both distorted and efficient size distribution are more dispersed in Chile.

After that we decompose efficiency gain in two ways. First the variance decomposition presents that in Norway the components of efficiency gain are fairly stable. In contrast, the variance of output distortion in Chile is the main component explaining the decreasing efficiency gain. Second, we decompose variances with different productivity quintiles. The

quintile analysis suggests that in Chile the between-group component of output wedges at both end of the distribution mainly explains the change in its variance. Furthermore, the negative correlation between productivity and output wedges concludes that in Chile, the less distortion faced by low productive firms is a stronger driving force to the increasing TFP in Chile since 1986. A possible explanation could be that Chilean reform policies since 1986 were more effective on decreasing distortions from the low productive firms, which would drove falling TFP gain.

The empirical work in this paper is performed using Stata.

Preface

This paper could not have been finished without Alfonso Irarrazabal who served not only as my supervisor but also encouraged and challenged me through the research program.

I would like to extend my heartfelt gratitude to Kaiji Chen for his advice about the theory.

I also would like to acknowledge the support of Centre of Equality, Social Organization and Performance (ESOP). This paper is involved in the ESOP's research project of Aggregate Efficiency, Misallocation and Policies proposed by Alfonso Irarrazabal.

A special thanks to all the helpers who have contributed their time with proof reading.

Great thanks to all my friends in Oslo and Beijing. It is grateful to have them in my life.

The last but not the least, I dedicate this paper to my beloved family for their unconditional love and support in every way.

The ordinary disclaimer applies.

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1 Introduction

The productivity differences have been proposed as a main factor of large differences in GDP per capita (Caselli, 2005; Bergoeing et. al, 2007; Klenow and Rodríguez-Clare, 1997). In particular, Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Alfaro et al. (2008) show that resource misallocation affected by the idiosyncratic shocks is highly related with aggregate total factor productivity (TFP).

Thus it is important to understand the impact of exogenous shocks, like government policies, on aggregate efficiency. Norway has a strong economy with equitable distribution of wealth, high taxes and extensive social welfare system. In the contrast, Chilean economy was heavily regulated, suffered a financial crisis in the beginning of 1980s and implemented in labor and capital market reforms that led to a strong recovery from mid 1980s.

The aim of this paper is to study how resource misallocation relates to aggregate TFP in the manufacturing sector in Norway and Chile. Using Hsieh and Klenow (2009)'s framework, we build the model that monopolistic competitive firms face distortions on output and capital. Distortions differentiate marginal revenue product across firms and, therefore, decrease aggregate TFP.

The firm-level data are collected from Instituto National de Estadistica (INE) and World Bank's report for Chile (1980-1996) and from Statistics Norway (1996-2006) for Norway. To describe the extent of resource misallocation, we compare the moments of firm size distribution and firm productivity distribution between Chile and Norway, and find that the resource allocation in Chile manufacturing is more distorted than in Norway. After that, we decompose the TFP gain from removing distortions in two ways: variance decomposition and quintile analysis. We show that in Chile the variance of output distortion is the main component explaining the decreasing TFP gain, while the between-quintile component of output wedges explain most of the variance change. In contrast, Norway shows stable trend with a small variation.

There are growing literatures studying resource misallocation in manufacturing sector. Restuccia and Rogerson (2008) analyze the distortion and aggregate productivity in production units. They show that policies generate distortions which create different price faced by producer, and thus lead to changes in TFP and aggregate capital accumulation. Neumeyer and Sandleris (2009) test the misallocation in Argentine manufacturing from 1997 to 2002 when the firms within narrowly defined industries face wide dispersion of wedges. They find that the equalizing marginal revenue products results in 60%-80% of efficiency gain. They also observe a positive correlation between capital distortion and productivity of firm. The decomposition of the growth of TFP gives that the within effect is the main component explaining the change in TFP growth. In addition, Midrigan and

Xu (2009) investigate Korean manufacturing data. They evaluate the effects of distorted investment which accounts for 2.5% loss of capital misallocation in financial friction. The low gain of removing distortion may be caused by decreasing return to scale in production function (Moll, 2009)

Moreover, Jones (2009) also discusses the aggregate TFP in the aggregate economy. He demonstrates that the complementary effect across industries can amplify industry-level distortions to have large TFP loss and inefficient resource allocation.

This paper also contributes to the literature related to the asset market. Banerjee and Moll (2009) study the persistence of misallocation in underdevelopment countries by focusing on the asset market. They build the model of agents for profit maximization with credit constrains. They find that the steady state and stabilized interest rate decrease the misallocation. The shock on the assets including distortion on assets and ability creates loss on capital stock, and makes agents be under-invested with increasing marginal product. Then they conclude that capital wedges are the main reasons of persistence.

Additionally, our work relates to the misallocation in financial frictions. Moll (2009) extends his research to the topic of self-financing and capital misallocation in financial friction. He builds the model with heterogeneous firms subject to borrowing constraints and productivity shocks. The result shows that the TFP gain from removing distortions has significant relationship with productivity shocks when financial frictions have impacts on aggregate productivity. He also states that the self-financing can lose the capital misallocation with stable productivity shock. Chen and Song (2009) also build the model for financial frictions. They introduce shock on credit conditions and construct the model without labor input. They introduce asymmetric financing constraint, and test implications by using data for US. Intuitively, they find the variances of financial friction which in turn is the source of transmission mechanism for shocks. It causes changes in aggregate TFP over business cycles.

Hermes and Lensink (1996) study the financial reforms in 1980s in Chile. They work on the firm's investment and finance by using the balance sheet. They show that reforms aiming to reduce intra-conglomerate lending reduce the imperfection in capital market.

Finally, we organize the paper as follow. In Section 2, we introduce the two-firm model with the assumption that only one firm is distorted and then discuss the simple intuitions on aggregate TFP. Section 3 derives the misallocation and performance measures in general case with multiple firms. Section 4 gives the quantitative analysis that we decompose changes in efficient gain in Chile and Norway to identify the most influential component. At last, we give the conclusion.

2 Two-Firms Case

In this section, a model of two firms is created. It is supposed that firm 1 faces an output distortion. The analysis in the following gives the basic model setup. The extended intuition shows that distortion in firm 1 leads to unequalized marginal revenue product and less size dispersion for firms. We also explain the extent to which capital and labor are misallocated in the economy and how a aggregate TFP loss occur.

2.1 The Model Setup

In this section, we build up a model for firms in a monopolistic competition. The aim is to solve for optimal price and marginal revenue product of labor and capital.

The optimization problems are constructed in final industry and intermediate industry. It is assumed that a final good is produced by a representative firm. The final good is a CES aggregator using output produced by monopolistically competitive firms. Firms produce intermediate goods in a monopolistically competitive market by using capital and labor as production input.

Final good produced by a representative producer

Final good Y is a CES function which combines two kind of input Y_1 and Y_2 from firm 1 and 2 with decreasing return to scale. The elasticity of the function equals to σ, where $\sigma > 1$.

$$Y = \left(Y_1^{\frac{\sigma-1}{\sigma}} + Y_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

The industry sells final output Y to plants at numeraire price ($P = 1$) and buys intermediates Y_1 and Y_2 from firms at price P_1 and P_2 in a perfectly competitive market. The problem of the industry is

$$\max_{Y_1,Y_2} \pi = Y - (P_1Y_1 + P_2Y_2)$$

By solving the optimization problem of the firm, the input demand function of intermediate good i with price P_i is

$$Y_i = \frac{1}{P_i^{\sigma}}Y$$

Intermediate good

The production function of each individual firm, Y_i, is Cobb-Douglas with capital and labor inputs.

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha} \tag{1}$$

where A_i is firm's productivity related to firm-level efficiency of production. α is the share of capital input which is same across firms.

With monopolistic competition in the intermediate good market, firms are maximizing profit by hiring labor and capital at fixed factor prices. Firm 1 is supposed to face distorting income tax which accounts for $100\tau\%$ of firm 1's revenue, and $0 < \tau < 1$. This policy does not affect firm 2. Therefore, these two firms will solve the following problems.

$$\begin{aligned}
\max_{P_1,L_1,K_1} \pi_1 &= (1-\tau)P_1 A_1 K_1^{\alpha} L_1^{1-\alpha} - wL_1 - RK_1 \\
s.t. &: Y_1 = \frac{1}{P_1^{\sigma}} Y \\
\max_{P_2,L_2,K_2} \pi_2 &= P_2 A_2 K_2^{\alpha} L_2^{1-\alpha} - wL_2 - RK_2 \\
s.t. &: Y_1 = \frac{1}{P_1^{\sigma}} Y
\end{aligned}$$

The first order condition gives the optimal price for each firm[1].

$$P_1 = \frac{\sigma}{\sigma-1} \underbrace{\left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \frac{1}{A_1(1-\tau)}}_{MC_1} \tag{2}$$

$$P_2 = \frac{\sigma}{\sigma-1} \underbrace{\left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \frac{1}{A_2}}_{MC_2} \tag{3}$$

Notice that, with no distortion, $\tau = 0$, input prices are equalized across firms. Notice that the firm's optimal price is decreasing with firm productivity A_i. In other words, a more productive firm is more competitive and charges lower price. This is because in firm 1 output distortion rises marginal cost, and this leads to higher optimal price. Thus $P_1 > P_1^e$, $P_2 = P_2^e$, where P_i^e is the efficient price with $\tau = 0$ charge by the firms 1 and 2.

The first order condition shows that marginal revenue of products are equalized across firms in the undistorted case with $MRPK_i^e = \overline{MRPK^e}$ and $MRPL_i^e = \overline{MRPL^e}$ for

[1] In Appendix I, we show the deriviation process in details.

$i = 1.2$. In the distorted case, we get

$$
\begin{aligned}
MRPK_1 &\triangleq \alpha\frac{\sigma-1}{\sigma}\frac{P_1Y_1}{K_1} = \frac{R}{1-\tau} > MRPK_1^e \\
MRPK_2 &\triangleq \alpha\frac{\sigma-1}{\sigma}\frac{P_2Y_2}{K_2} = R = MRPK_2^e \\
MRPL_1 &\triangleq (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_1Y_1}{L_1} = \frac{w}{1-\tau} > MRPL_1^e \\
MRPL_2 &\triangleq (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_2Y_2}{L_2} = w = MRPL_2^e
\end{aligned}
$$

where both $MRPK$ and $MRPL$ in firm 1 increase with positive output distortion. The marginal revenue product in firm 1 is always higher than efficient value. Take marginal revenue product of capital as an example. Figure 1 plots both distorted and efficient MRPK in firm 1. The interest rate is set that $R = 0.1$. Figure 1 shows that when $0 < \tau < 1$, the distorted MRPK curve (solid line) lies above its efficient curve(grey dash line). The curvature of $MRPK_1$ depends on the parameter R: the lower interest rate, the more curvature of the distorted $MRPK_1$. In Figure 1, the black dash line with $R = 0.01$ is more curvature than the solid line with $R = 0.1$.

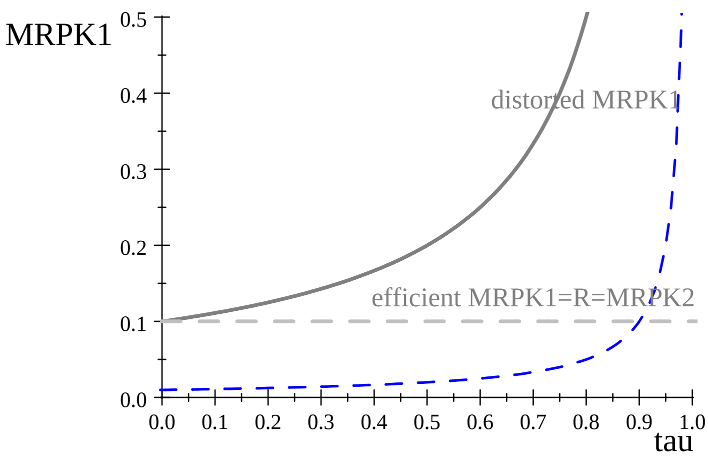

Figure 1: MRPK_1: distorted and efficient case

2.2 Intuition

In this section, we show that distortion causes resource misallocation and decreases size dispersion.

Resource allocation

The social planner always allocate resources to maximize social welfare. In our first best case, resources are appropriate allocated when $MRPK_1^e = MRPK_2^e = \overline{MRPK^e}$ and $MRPL_1^e = MRPL_2^e = \overline{MRPL^e}$. When $\tau > 0$, the output distortion creates wedges in marginal revenue products in firm 1. Thus misallocation of resource leads to suboptimal undesirable resource allocation. In Figure 2, we plot marginal product of capital and labor allocation. When interest rate is distorted with $\tau > 0$ in firm 1, the price per unit of capital increases. Since firm 1 has to pay higher price for capital input and faces an increasing marginal product, the capital demand in firm 1 falls. Thus firm 1 will encounter loss on profit because of capital misallocation.

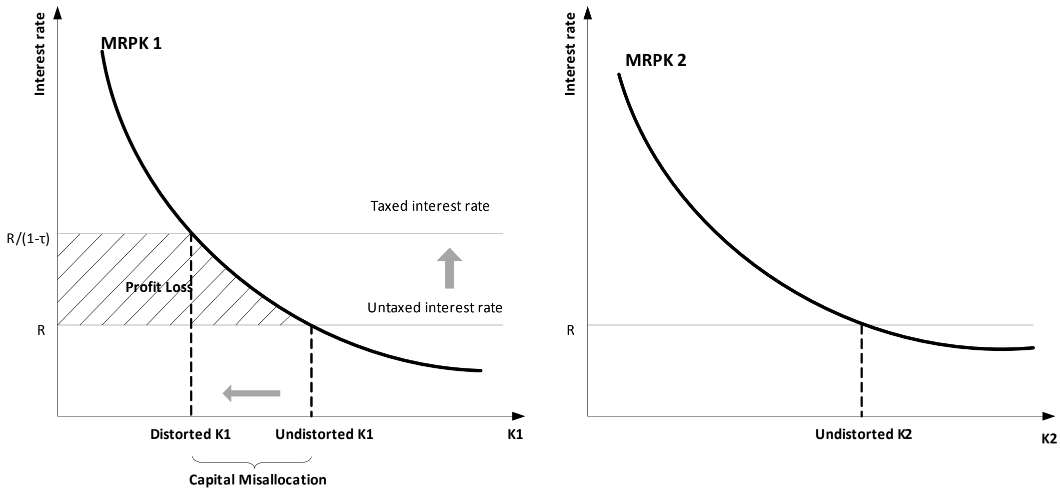

Figure 2: MRPK and Capital Demand

The resource allocation can be derived from optimal problem that $\frac{L_1}{L_2} = \left(\frac{A_1}{A_2}\right)^{\sigma-1}(1-\tau)^\sigma < \frac{L_1^e}{L_2^e}$ and $\frac{K_1}{K_2} = \left(\frac{A_1}{A_2}\right)^{\sigma-1}(1-\tau)^\sigma < \frac{K_1^e}{K_2^e}$. This is consistent with the analysis above. Thus we can conclude that

$$\begin{aligned} L_1 &< L_1^e \quad L_2 = L_2^e \\ K_1 &< K_1^e \quad K_2 = K_2^e \end{aligned}$$

Size dispersion

When $\tau > 0$, the distortion affects revenue size in firm 1.

If firms' productivity are exogenous and are independent from distortion, the production of firm 1 will be lower than efficient output as capital is misallocated. In addition, the optimization gives $\frac{Y_1}{Y_2} = \left(\frac{A_1}{A_2}\right)^{\sigma} (1-\tau)^{\sigma} < \frac{Y_1^e}{Y_2^e}$. Thus (1) indicates

$$
\begin{aligned}
Y_1 &< Y_1^e \\
Y_2 &= Y_2^e
\end{aligned}
$$

As the price ratio equals to $\frac{P_1}{P_2} = \frac{A_2}{A_1(1-\tau)}$ dividing (2) by (3), the revenue ratio between firms is computed as

$$\frac{P_1 Y_1}{P_2 Y_2} = \left(\frac{A_1}{A_2}\right)^{\sigma-1} (1-\tau)^{\sigma-1} < \left(\frac{A_1}{A_2}\right)^{\sigma-1} = \frac{P_1^e Y_1^e}{P_2^e Y_2^e} \tag{4}$$

Equation (4) shows that the revenue ratio decreases with an increase output distortion. As firm 2 sustains efficient price and output, we find that the revenue in firm 1 fall from its efficient level, which gives

$$P_1 Y_1 < P_1^e Y_1^e \tag{5}$$

$$P_2 Y_2 = P_2^e Y_2^e \tag{6}$$

We subtract (5) by (6) to get $P_1Y_1 - P_2Y_2 < P_1^eY_1^e - P_2^eY_2^e$. This shows that higher distortion in firm 1 also leads to less size difference between firms. This situation is intensified when there is a lower elasticity σ ($\sigma > 1$) and $0 < \tau < 1$ in (4).Thus we can conclude that when output distortion works on the firm, and the industry has higher elasticity of substitution of inputs, the size dispersion is less dispersed that the efficient level.

Suppose firm 1 is more productive than firm, when $\tau = 0$, firm 1 demands more capital and labor. Because firm 1 sets a lower price, output in firm 1 is larger, and firm 1 will acquire more revenue from production. Thus the variance of size in efficient case exceeds the level in distorted situation.

TFPR

Here TFPQ is defined as the firm productivity and TFPR as the revenue-based productivity (Hsieh and Klenow, 2009). TFPR is also called real multi-factor productivity in Bartelsman et. al (2008)'s paper.

$$
\begin{aligned}
TFPQ_i &\triangleq A_i \\
TFPR_i &\triangleq P_i A_i
\end{aligned}
$$

TFPR can be written as an weighted average of MRPK and MRPL.

$$TFPR_i \triangleq P_i A_i \propto (MRPK_i)^{\alpha} (MRPL_i)^{1-\alpha}$$

There are two ways to explain the change of TFPR for firm 1. First, the distortion increases marginal revenue products in firm 1, and thus TFPR_1 rises. Second, a positive τ increases the optimal price. Thus distorted TFPR_1 is higher, and TFPR_2 remains at its efficient level.

$$\begin{aligned} TFPR_1 &> TFPR_1^e \\ TFPR_2 &= TFPR_2^e \end{aligned}$$

Recall that in the efficient case, firms have equal TFPR and $TFPR_1^e = TFPR_2^e = \overline{TFPR^e}$.

2.3 Aggregation and TFP

In this section, we discuss the problem of aggregating individual production input to find total labor and capital demand when $\tau > 0$. The aggregate output decreases with distortion. The expression of aggregate TFP is computed which is decreasing with distortion. Finally, it is argued that when more productive firms are distorted then the decrease of TFP is larger

Aggregation

Partial equilibrium is assumed in the capital and labor market. The total capital and labor demand equals to total supply. Thus $L = L_1 + L_2$ and $K = K_1 + K_2$. Since labor and capital demands decrease in firm 1. The aggregate resources are

$$\begin{aligned} L &< L^e \\ K &< K^e \end{aligned}$$

As input demand decreases in firm 1, while it remains in the same level in firm 2. The labor and capital demand ratios are $\frac{L_1}{L_1+L_2} = \frac{1}{1+\frac{L_2}{L_1}}$ and $\frac{K_1}{K_1+K_2} = \frac{1}{1+\frac{K_2}{K_1}}$ which are increasing with $\frac{L_1}{L_2}$ and $\frac{K_1}{K_2}$, thus we have

$$\begin{aligned} \frac{L_1}{L} &< \frac{L_1^e}{L^e} \\ \frac{K_1}{K} &< \frac{K_1^e}{K^e} \end{aligned}$$

Hence the distortion leads to contraction on total capital and labor demand.

Obviously, the total manufacturing output goes down with decreasing output in firm 1 since $Y = Y_1 + Y_2$. We have

$$Y < Y^e$$

Aggregate TFP

By aggregating output, labor and capital (see Appendix I for details), we show that the aggregate TFP calculated by growth-accounting method equals to the TFP formula in Hsieh and Klenow (2009). Thus we have

$$TFP \triangleq \frac{Y}{K^{\alpha}L^{1-\alpha}} = \left[\left(A_1\frac{\overline{TFPR}}{TFPR_1}\right)^{\sigma-1} + \left(A_2\frac{\overline{TFPR}}{TFPR_2}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{7}$$

where

$$\overline{TFPR} = \left[A_1^{\sigma-1}(1-\tau)^{\sigma-1} + A_2^{\sigma-1}\right]^{\frac{1}{\sigma-1}} < \overline{TFPR^e}$$

Furthermore, from (7) we have $TFP < TFP^e$ as distortion decreases $\overline{TFPR}$ and increases TFPR in firm 1.

Then (7) can be written as

$$TFP = \frac{\left[(A_1(1-\tau))^{\sigma-1} + A_2^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{A_1^{\sigma-1}\cdot(1-\tau)^{\sigma} + A_2^{\sigma-1}} \tag{8}$$

By setting $\tau = 0$, the efficient TFP is

$$TFP^e = \left(A_1^{\sigma-1} + A_2^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{9}$$

where efficient TFP is a CES aggregator of individual productivity in firm 1 and 2.

TFP and distortion

Now we discuss the relationship between aggregate TFP and distortion.

According to (8) and (9), we draw curves of TFP against distortion in Figure 3 by assuming $A_1 = 1$, $A_2 = 1$, $\sigma = 3$ and $\alpha = \frac{1}{3}$. The Figure 3 shows that distorted TFP always lies below efficient TFP, and is decreasing with distortion. To illustrate this, we take $\tau = 0.5$ for example in (8) and (9). Then the aggregate TFP in efficient and distorted level are computed respectively, and it gives $TFP^e_{\tau=0.5} \approx 2.236$ and $TFP_{\tau=0.5} \approx 2.124$. Obviously $TFP_{\tau=0.5} < TFP^e_{\tau=0.5}$. So the distorted TFP is lower than efficient TFP when $\tau = 0.5$. The result also gives that the whole industry will obtain 5.3% ($= \frac{2.236}{2.124} - 1$)

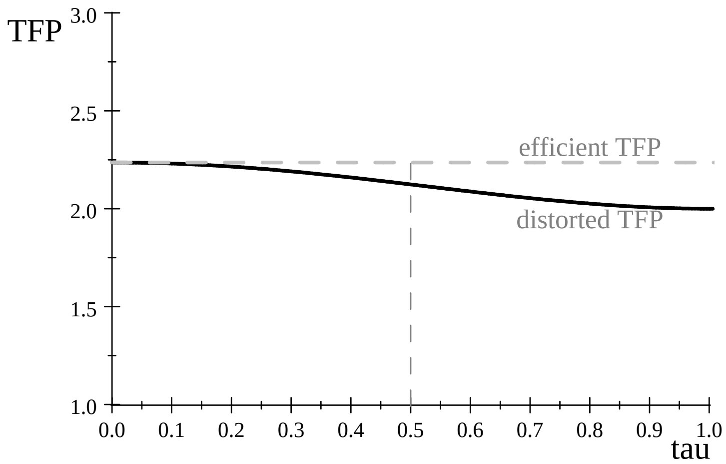

Figure 3: TFP and TFP efficient

more of TFP if distortion decreases from 0.5 to 0 and capital and labor are reallocated to efficient level. The graph also presents that, TFP gain is increasing with positive τ.

Now suppose that firm 1 is more productive than firm2, for example, $A_1 = 2$ and $A_2 = 1$. Again we plot TFP curves with different productivity sets in Figure 4. When firm 1 is a more productive firm being distorted, the aggregate TFP drops faster than when firm 2 is more productive. In this case $TFP_{\tau=0.5} \approx 1.886$. The efficiency gain from removing distortions is about 18.6% ($= \frac{2.236}{1.886} - 1$) which is larger than 5.3% when $A_1 = 1$, $A_2 = 2$. Thus an adjustment of distortion and resource misallocation in a more productive firm 1 results significant growth of aggregate TFP.

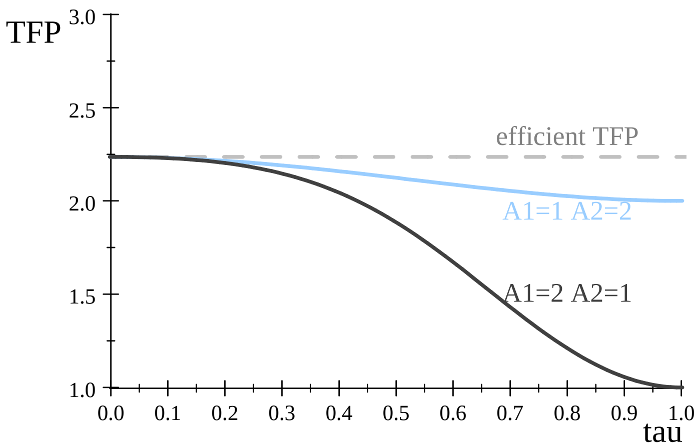

Figure 4: TFP in different productivity parameters

3 Many-Firms Case

In this section, we generalized the previous model to account for many intermediate good. The final good is produced by a representative firm who buy input from M firms. Therefore, each monopolistic competitive firm faces output distortion τ_{Yi} and capital distortion τ_{Ki}. Firm-specific wages are included rather than constant wage for all firms in the last section.

In the following, the general solutions to optimization problems of the final and intermediate sector are presented when firm faces specific distortions. Then we show aggregate labor and capital demand. Finally, the aggregate TFP is expressed as a function of distortions and firm productivity. The TFP gain is derived which relates to TFPR.

3.1 Firm's Optimal Decisions

In this part, we describe the problem of the firm in the final and intermediate sector which is also discussed by Jones(2009). We derive labor and capital demand, optimal price, TFPR, etc as functions of distortions. In addition, the size distribution is discussed. It shows that the firm's distorted size will be less spread out than the efficient size.

Final sector

A single final good is produced by a representative firm in perfectly competitive market. The representative firm takes input price at P_i for each i intermediate good. The final output is a CES aggregator over M differentiated products. The problem of this firm is

$$\begin{aligned} & \max_{Y_i} PY - \sum P_i Y_i \\ st \quad & : \\ Y \quad & = \left(\sum_{i=1}^{M} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{10}$$

This gives demand of output as

$$Y_i = \frac{P_i^{-\sigma}}{\mathbf{P}^{-\sigma}} \cdot Y$$

where $\mathbf{P} = \left(\sum_{k=1}^{M} P_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is the price index, and we set $\mathbf{P}$ as the numeraire.

Intermediate sector

In monopolistic competition, firm i hires labor at firm-specific wage w_i and use capital at a constant interest rate R. The problem of the intermediate producer i is

$$\begin{aligned}
\max_{L_i,K_i,P_i} \pi_i &= (1-\tau_{Yi})P_iY_i - w_iL_i - R(1+\tau_i)K_i \\
st &: \\
Y_i &= A_iK_i^aL_i^{1-\alpha} \qquad (11)\\
Y_i &= \frac{P_i^{-\sigma}}{\mathbf{P}^{-\sigma}} \cdot Y \qquad (12)
\end{aligned}$$

The first order condition gives

$$\begin{aligned}
\frac{K_i}{L_i} &= \frac{\alpha}{1-\alpha} \cdot \frac{w_i}{(1+\tau_{Ki})R} \qquad (13)\\
P_i &= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{Ki})^{\alpha}}{A_i\,(1-\tau_{Yi})} \qquad (14)
\end{aligned}$$

Equation (13) shows that the capital-labor ratio decreases with capital distortion. In the efficient case without output and capital distortions, the capital-labor ratio varies across firms with firm-specific wage. In (14), distortions rise optimal price with increasing marginal cost. In the undistorted case, a low-wage firm expanding production efficiency by increasing A_i decreases optimal price.

Resource allocation

To interpret labor and capital demand, we insert (13) into (11). Then we substitute Y_i and P_i by using (12) and (14). Finally, resource demands are solved from first order condition.

$$\begin{aligned}
L_i &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha(\sigma-1)}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)\sigma+\alpha}\frac{A_i^{\sigma-1}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha(\sigma-1)}}Y \\
K_i &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha(\sigma-1)+1}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)(\sigma-1)}\frac{A_i^{\sigma-1}\,(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha(\sigma-1)+1}}Y
\end{aligned}$$

The resource demand depends on both firm productivity and distortions. Non-zero distortions cause resource misallocation and differentiated marginal revenue of products. From the first order condition, marginal revenue of products are fixed markups over revenue productivity of capital and labor respectively.

$$\begin{aligned}
MRPK_i &\triangleq \frac{\partial P_iY_i}{\partial K_i} = \alpha\frac{\sigma-1}{\sigma}\frac{P_iY_i}{K_i} = R\frac{1+\tau_{Ki}}{1-\tau_{Yi}} \qquad (15)\\
MRPL_i &\triangleq \frac{\partial P_iY_i}{\partial L_i} = (1-\alpha_s)\frac{\sigma-1}{\sigma}\frac{P_iY_i}{L_i} = \frac{w_i}{1-\tau_{Yi}} \qquad (16)
\end{aligned}$$

Equation (15) and (16) show that the gain from revenue earned by firm i from hiring an additional input increases with distortions. When $\tau_{Ki} > 0$ and $0 < \tau_{Lsi} < 1$, Distortions lead to the results that labor and capital demands deviate from social planner's efficient allocation.

Cost and profit share

Equations (15) and (16) show that the share of capital cost and the wage bill decline with distortions as$\frac{RK_i}{P_iY_i} = \alpha\frac{\sigma-1}{\sigma}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}$ and $\frac{w_iL_i}{P_iY_i} = (1-\alpha)\frac{\sigma-1}{\sigma}(1-\tau_{Yi})$. In the efficient case, cost shares are fixed when marginal revenue products are identical across firms. Then we can write the before-distorted profit share as

$$\begin{aligned}\frac{\pi_i^{before\text{-}distorted}}{P_iY_i} &= 1-\frac{w_iL_i}{P_iY_i}-\frac{RK_i}{P_iY_i} \\ &= 1-\frac{\sigma-1}{\sigma}\left[(1-\alpha)(1-\tau_{Yi})+\alpha\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\right]\end{aligned}$$

The profit share before distortion decreases with the level of distortion. While after being distorted, the rate of elasticity σ has negative impact on firm's profit ratio. It means that when the production of final good is more elastic with inputs from firms, the after-distorted profit share for all firms shrinks to a same level $\frac{1}{\sigma}$, as

$$\frac{\pi_i^{after\text{-}distorted}}{(1-\tau_{Yi})P_iY_i} = 1-\frac{1+\tau_{Ki}}{1-\tau_{Yi}}\frac{w_iL_i}{P_iY_i}-\frac{1}{1-\tau_{Yi}}\frac{RK_i}{P_iY_i} = \frac{1}{\sigma}$$

Revenue size dispersion

From the first order condition, we get firm size as

$$P_iY_i = \frac{A_i^{\sigma-1}(1-\tau_{yi})^{\sigma-1}}{(1+\tau_{Ki})^{\alpha(\sigma-1)}}\left[\frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\right]^{-(\sigma-1)} Y \tag{17}$$

with input price $\frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha} = \left[\sum_{i=1}^{M}\left(\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} w_i^{1-\alpha}$.

Intuitively, with $\sigma > 1$ in (17), firm's size is increasing with productivity but decreasing with distortions τ_{Yi} and τ_{Ki}. Now we suppose that TFPQ has positive correlation with τ_{Yi} or τ_{Ki}. As firm's productivity increases with revenue, a high level of efficient revenue of firms implies that firms are more efficient, but face large positive output distortion or high interest rate. This results declined revenue in highly productive firms by involving less capital and labor inputs. Similarly, least productive firms produce more with low distortion. Then revenues in least productive firms increase. Therefore, the revenue gap between efficient and inefficient firms decreases. This is to say, the distorted size distribution has less dispersion than the efficient distribution. A similar result is stated by Bartelsman et. al (2008)

TFPR and TFPQ

Same as in the two-firm case, we define $TFPR_i$ as the revenue-based productivity. Notice that, it depends on distortions all well. Similarly $TFPQ_i$ is defined as the physical productivity.

$$\begin{aligned} TFPR_i &\triangleq P_i A_i = \frac{P_i Y_i}{K_i^{\alpha} L_i^{1-\alpha}} \\ TFPQ_i &\triangleq A_i = \frac{Y_i}{K_i^{\alpha} L_i^{1-\alpha}} \end{aligned} \tag{18}$$

After rearranging (15) and (16), we can rewrite TFPR in (18) as the weighted average of marginal revenue products.

$$TFPR_i = \frac{\sigma}{\sigma-1}\left(\frac{MRPK_i}{\alpha}\right)^{\alpha}\left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha} \tag{19}$$

$$= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{Ki})^{\alpha}}{1-\tau_{Yi}} \tag{20}$$

The efficient TFPR without distortion changes across firm with firm-specific wage. This is different from Hsieh and Klenow(2009)'s model with constant wage across firms.

3.2 Aggregation and TFP

In this section, aggregate TFP is written as a function of distortions. When firms are subject to distortions, TFP is affected by $TFPR_i$ and $\overline{TFPR}$. At the end, the TFP gain from removing distortions is also discussed by comparing distorted TFP with efficient TFP.

Aggregate TFP

The growth accounting expresses aggregate TFP as

$$TFP \triangleq \left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \frac{Y}{K^{\alpha} L^{1-\alpha}} \tag{21}$$

Aggregate output Y in (21) is substituted by inserting (11) into (10). TFP becomes a function of TFPQ and input shares.

$$TFP = \frac{1}{M^{\frac{1}{\sigma-1}}} \left[\sum_{i=1}^{M} \left(A_i \left(\frac{K_i}{K} \right)^{\alpha} \left(\frac{L_i}{L} \right)^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{22}$$

Intuitively, from (22), an industry with firms facing inefficient resource allocation will experience a drop on aggregate TFP. At the same time, TFP increases with TFPQ of each firm.

Now to express TFP in terms of firm's productivity and distortions, the first step is to compute capital and labor ratio. Solving (15) and (16) for capital and labor allocation, we have

$$\begin{aligned} K_i &= \frac{\alpha}{R} \frac{\sigma-1}{\sigma} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} P_i Y_i \\ L_i &= \frac{1-\alpha_s}{w_i} \frac{\sigma-1}{\sigma} \left(1-\tau_{Yi}\right) P_i Y_i \end{aligned}$$

We aggregate L_i and K_i by summing up input demands across firms ($L = \sum_{i=1}^{M} L_i$ and $K = \sum_{i=1}^{M} K_i$). Firm input shares are

$$\frac{K_i}{K} = \frac{\frac{1-\tau_{Yi}}{1+\tau_{Ki}} P_i Y_i}{\sum_{j=1}^{M} \frac{1-\tau_{Yj}}{1+\tau_{Kj}} P_j Y_j} \tag{23}$$

$$\frac{L_i}{L} = \frac{\frac{1-\tau_{Yi}}{w_i} P_i Y_i}{\sum_{j=1}^{M} \frac{1-\tau_{Yj}}{w_j} P_j Y_j} \tag{24}$$

Then (23) and (24) are plugged into (22) which delivers

$$\begin{aligned} TFP &= \left(\frac{\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY}}{\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \frac{P_i Y_i}{PY}} \right)^{\alpha} \frac{\left[\sum_{i=1}^{M} \left(A_i \frac{1-\tau_{Yi}}{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}} \frac{P_i Y_i}{PY} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY}} \left(\frac{1}{M} \right)^{\frac{1}{\sigma-1}} \\ &= \left(\frac{L}{K} \right)^{\alpha} \frac{Y}{L} \left(\frac{1}{M} \right)^{\frac{1}{\sigma-1}} \end{aligned} \tag{25}$$

Although firm-specific wages are used in the model, it is possible to reach a similar conclusion as Neumeyer and Sandleris (2009)'s. The first term in equation (25) represents distortion to capital-labor ratio. The second term plays role on firm's operation scale. Now we substitute P_iY_i by using (17). The growth accounting TFP can be rewritten as

$$
\begin{aligned}
TFP &\triangleq \left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \frac{Y}{K^{\alpha}L_s^{1-\alpha}} \\
&= \frac{\left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}}\left[\sum_{i=1}^{M}\left(\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\left[\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}\right]^{\sigma-1}\right]^{1-\alpha_s}\left[\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\left[\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}\right]^{\sigma-1}\right]^{\alpha}}
\end{aligned} \tag{26}
$$

Appendix II shows that the aggregate TFP derived by Hsieh and Klenow (2009) equals to (26). Thus it can be concluded that

$$
TFP \triangleq \left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \frac{Y}{K^{\alpha}L^{1-\alpha}} = \left[\frac{1}{M}\sum_{i=1}^{M}\left(A_i \cdot \frac{\overline{TFPR}}{TFPR_i}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{27}
$$

Similar to (19), $\overline{TFPR}$ is defined as the geometric average of the $\overline{MRPK}$ and $\overline{MRPL}$.

$$
\begin{aligned}
\overline{TFPR} &= \frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK}}{\alpha}\right)^{\alpha}\left(\frac{\overline{MRPL}}{1-\alpha}\right)^{1-\alpha} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\frac{P_iY_i}{PY}}\right)^{\alpha}\left(\frac{1}{(1-\alpha)\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\frac{P_iY_i}{PY}}\right)^{1-\alpha}
\end{aligned}
$$

$\overline{MRPL}$ and $\overline{MRPK}$ are some "sort" of average of MRPs. Comparing with marginal revenue product of firms, variables in $\overline{MRPL}$ and $\overline{MRPK}$ move in the same direction.

$$
\begin{aligned}
\overline{MRPL} &= \frac{1}{\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\frac{P_iY_i}{PY}} \\
\overline{MRPK} &= \frac{R}{\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\frac{P_iY_i}{PY}}
\end{aligned}
$$

To be noticed, the last equation of TFP in (27) indicates the mean of wedges $1-\tau_{Yi}$ and $1+\tau_{Ki}$ do not have evident effects on aggregate TFP. But the variance of $TFPR$ and the covariance between $TFPR$ and $TFPQ$ will produce impacts on aggregate TFP.

Efficient TFP and TFP Gain from removing distortions

Now we discuss about the implication of misallocation for efficiency. In the efficient case, marginal revenue product of labor and capital are equalized across firms. Finally, $MRPL_i = MRPL_j$ and $MRPK_i = MRPK_j$. From (15) and (16), we get

$$
\begin{aligned}
MRPL_i^e &= MRPL_j^e = \overline{MRPL^e} \Rightarrow \frac{w_i}{1-\tau_{Yi}} = \frac{w_j}{1-\tau_{Yj}} = \overline{w} \\
MRPK_i^e &= MRPK_i^e = \overline{MRPK^e} \Rightarrow \frac{R\,(1+\tau_{Ki})}{1-\tau_{Yi}} = \frac{R\,(1+\tau_{Kj})}{1-\tau_{Yj}} = \overline{R}
\end{aligned}
$$

The above two equations imply that $\overline{TFPR^e} = TFPR_i = TFPR_j$. It gives efficient level of aggregate TFP as

$$
A^e = \left(\frac{1}{M}\sum_{i=1}^{M} A_i^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{28}
$$

Furthermore, the efficient to distorted output ratio could be expressed as the ratio of TFP dividing (28) by (26).

$$
\frac{Y^e}{Y} = \frac{A^e}{TFP} = \left[\sum_{i=1}^{M}\left(\frac{A_i}{A^e}\cdot\frac{\overline{TFPR}}{TFPR_i}\right)^{\sigma-1}\right]^{-\frac{1}{\sigma-1}} \tag{29}
$$

In other words, the TFP gain is the percentage of changing from distorted aggregate TFP to efficient TFP. Then $TFPgain = Ygain = \frac{A^{efficient}}{TFP} - 1$.

Additionally, from (29), the logarithm difference between efficient and actual TFP, $\ln A^e - \ln TFP$, relates to the difference between $\ln \overline{TFPR}$ and $\ln TFPR_i$. Thus when $\ln TFPR_i$ is far from the "average", $\ln \overline{TFPR}$, the difference between $\ln A^e$ and $\ln TFP$ is enlarged. That is to say when the firms are more distorted, TFPR disperses more and the TFP gain from removing distortions increases.

4 Quantitative Analysis of Misallocation

This section compares Chilean to Norwegian manufacturing industry by using firm-level data. The aim is to show to what extent the Chilean manufacturing is distorted by setting Norwegian data as a benchmark case.

First, the data sources are introduced with basic descriptive statistics. Then we will discuss the extent and misallocation, and analyze which particular distortion may have accounted for most of the changing variance of TFPR in Chile and Norway.

4.1 Data and Descriptive Statistics

In this section, the preparation of the data is introduced. First, the sources of Chilean and Norwegian Data are given out, and the perpetual inventory method is explained by which we formulate capital stock in Chilean data. After that, the preliminary cleaning methods are presented for both datasets. At last, the descriptive statistics are presented for Chilean and Norwegian dataset.

4.1.1 Data Sources

Chilean Data

The Chilean data used in this analysis has two main sources: the data from Instituto National de Estadistica (INE) of Chile covers 1980-1986, and the data from World Bank's report covers 1986-1996. The overlapped observations in 1986 from both sources are tested and show a consistent match of the data. The entire dataset, 1980-1996, contains 78,889 observations across years and 9,778 firms before cleaning. Each firm hires 10 or more workers annually. A large number of variables are included, for example four-digit industry code (ISIC), business type, sales of product, material cost, location of producer, etc. The identity test on income, output, and value-added shows high consistency of variables. Most of the data are recorded in nominal price. Variables are in constant 1980 prices deflated by various price deflators on output, capital, intermediate, etc in three digit industry level cited from Liu(1990). For example the real wage of white-collar employees (w_{ist}) in plant i, industry s, year t should be written as:

$$w_{ist} = \frac{PI^t_{is\ 80}}{100} \times W_{ist}$$

where $PI^t_{is\ 80}$ is the price index based on year 1980, and W_{ist} is the nominal wage of white collars in plant i, year t.

The total gross capital includes stock of buildings, machinery, vehicles and other assets. The capital series with one-year lag are defined both forward and backward through perpetual inventory method.

$$K_{ijt} = (1 - \rho_j)K_{ij(t-1)} + I_{ijt}$$

Here K_{ijt} is the real gross capital in j (building, machinery, vehicle or other assets) for plant i in year t. I_{ijt} is the investment on related capital. In Chilean manufacturing, investment is defined as the sum of values on consuming old and new capital goods, selling capital goods, producing capital goods for own use and improving capital goods by a third party. ρ_j is the depreciations rate for capital j. Following Liu(1990)'s assumption, ρ is set to be 5% for buildings, 10% for machinery and 20% for vehicles. Furthermore, since some of the plants get exit the dataset and entry after more than one year, we generate the capital stock of plant with n-year lag as

$$K_{ijt} = (1 - \rho_j)^n K_{ij(t-n)} + I_{ijt}$$

We used either 1980 or 1981 as the base year to construct capital. As many firms have missing capital values in 1980-based data but exist in 1981-based. Liu (1990) suggested a method of capital composition for these two kinds of capital series. Capital equals to 1980-based value if the value exists in the data based on 1980. If not, capital is replaced by the value based on 1981.

Norwegian Data

The Norwegian data is collected from the manufacturing Statistics and accounts statistics within the period 1996-2006. There are 93,578 observations with 16,049 firms included. Only the joint-stock companies are recorded. The measurements include business type, employment, investment, etc. The total investment is the sum of investments in buildings, land and other tangible fixed assets. The capital stocks are given in the dataset which are estimated based on hybrid perpetual inventory method (Raknerud et.al (2007)) All the variable are converted into real values in 1996.

Data cleaning

Several steps for data cleaning are implemented. Firstly, firms with top 0.1% of investment are dropped by pooling across years. Secondly, since we are going to do most of the analysis in logarithmic values, it is necessary to drop firms with missing values and non-positive measurements on capital, value added and wage. Thirdly, following Gourio(2008)'s criteria, we drop firms staying in dataset less than five consecutive year aiming to select a more reliable dataset. In addition, while defining entry and exit, we keep firms who enter or exit no more than twice. We also discard the sole riders who are in the dataset for one year only, unless they appear in 1980 or 1996 in Chilean data and 1996 or 2006 in Norwegian data. All these cleaning processes eliminate firms for all the years (balanced panel). So it will not affect entry and exit analysis in the later section. At last, because it includes firms with less than 10 employees in Norwegian manufacturing statistics, we define those firms exit the dataset when the number of employees is less than 10. Thus we eliminate the firm-year observation when the firm has labors less than 10.

4.1.2 Descriptive Statistics

Before focusing on the productivity analysis, it is of great interest to have a brief discussion on the aggregate value-added, investment-capital ratio and entry-exit analysis.

Figure 5 illustrates the aggregate manufacturing labor productivity detrended by 2.5% per year[2]. It shows that the aggregate labor productivity in Chile decreased dramatically in early 1980s and reached the level of 40% below the trend in 1987. One reason for this is the country experienced severe financial crisis when GDP dropped by 14.1% from 1982 to 1983 (Corbo and Fischer,1994). Chilean labor productivity increases in 1990s and reached the level about 5% above the trend in 1996, whereas the Norwegian labor productivity varied less and it also stopped at the 5% level above the trend in 2006.

Figure 6 presents similar investment-capital ratio in Chile as Fuentes et.al (2006) does. It shows that the Norwegian aggregate investment-capital ratio is much higher which is about 0.5 on average. This is to say Norway has higher investment on each capital good than Chile. Whereas the investment-capital ratio is increasing in Chile but decreasing in Norway.

The measurements of entry and exit are defined following Dunne et. al(1988)'s equation. Measurements are given as

$$\begin{aligned} \text{ER} &= EntryRate_t = \frac{\#Entrant_t}{\#Firm_{t-1}} \\ \text{XR} &= ExitRate_{t-1} = \frac{\#Exiter_{t-1}}{\#Firm_{t-1}} \end{aligned}$$

[2]We focus on the growth accouting between Chile and Norway in Figure 5. We set the average annual growth of productivity in Norway as a bechmark value which is 2.5% from 1996 to 2006. Following Bergoeing et.al (2007), the consturction of detrended labor productivity is according to

$$P_t^{de} = \frac{P_t}{(1+5\%)^{t-t_{initial}}}$$

where $t^{initial}$ is the initial year of the series. It equals to 1980 for Chile and 1996 for Norway.

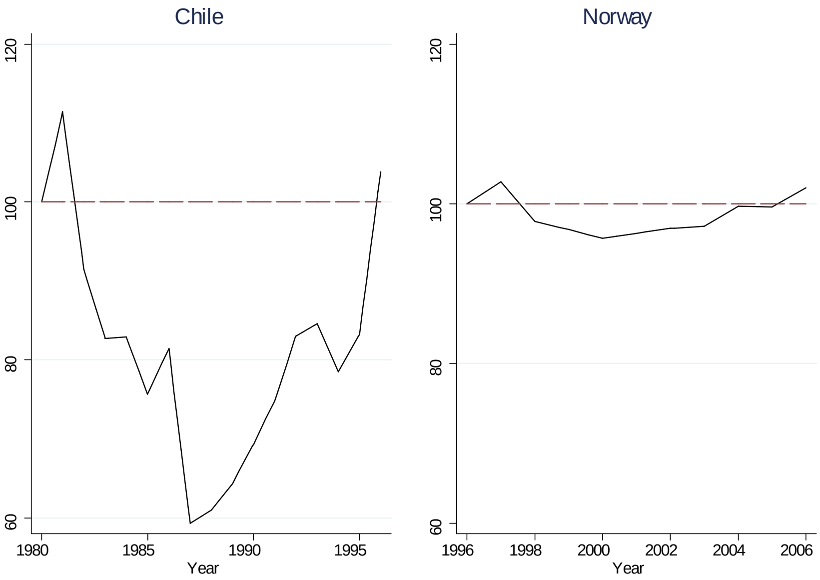

Figure 5: Aggregate Labor Productivity: detrended 2.5% per year

$$\begin{aligned} \text{ESH} &= EntrantShare_t = \frac{OutputEntrant_t}{OutputTotal_t} \\ \text{XSH} &= ExiterShare_{t-1} = \frac{OutputExiter_{t-1}}{OutputTotal_{t-1}} \end{aligned}$$

$$\begin{aligned} \text{ERS} &= EntrantSize_t = \frac{\frac{OutputEntrant_t}{\#Entrant_t}}{\frac{OutputTotal_t - OutputEntrant_t}{\#Firm_t - \#Entrant_t}} \\ \text{XRS} &= ExiterSize_{t-1} = \frac{\frac{OutputExiter_t}{\#Exiter_{t-1}}}{\frac{\#Firm_{t-1} - OutputExiter_{t-1}}{\#Firm_{t-1} - \#Exiter_{t-1}}} \end{aligned}$$

Table 1 shows that in Chile the entry rate increases while it decreases in Norway. The entrants in Chile produce 11% of the manufacturing output on average, and Norwegian entrants produce 26% which is also higher than 15.8% in U.S. (Dunne et.al,1988). Thus the entrants in Norway contribute the most to the manufacturing output. In Chile and Norway the entrant market share is lower than entry rate. In other words, entrants are much smaller than the firms already in the market. Additionally, the average size of entrants in Chile is about 33% of incumbents'. While entrants in Norway has higher entrant relative size towards survivors' average output, which is about 73%.

There is a similar analysis for exiters. The annual exit rate is about 26% in Chile

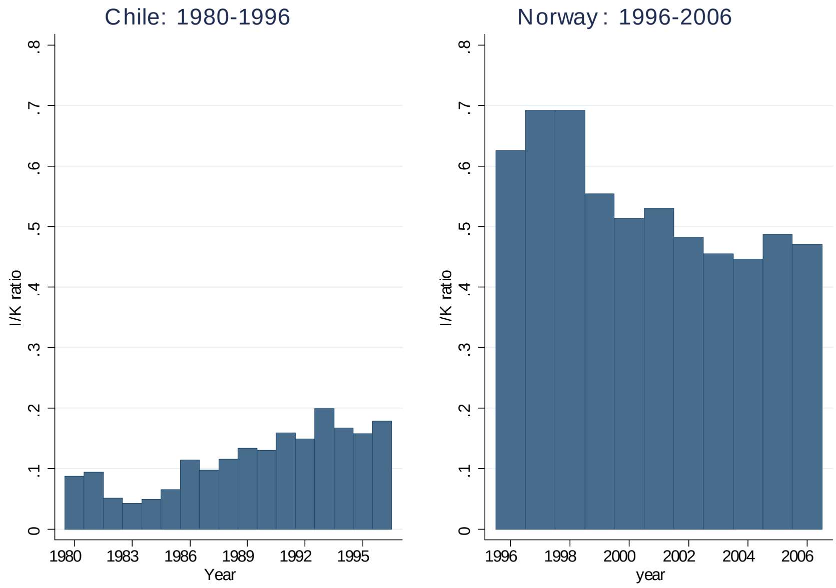

Figure 6: Aggregate Investment-Captial Ratio

and 31% in Norway which are all lower than the measurements in U.S. Exiters in all the countries are small firms with less market share than exit rate. In the "lost period" during 1980-1984 in Chile, there are more firms exit the market who have a high relative size compared to survivors

4.2 Misallocation in Chile and Norway

In this section, we discuss the extent of misallocation in Chile and Norway. First, we calibrate parameters and variables to calculate actual and efficient aggregate TFP. Second, we compare the distribution of productivity in both countries. We find that TFPQ and TFPR are more spread out in Chile than in Norway. At last, we decompose TFP gain and show that output distortion is the main factor to explain resource misallocation in Chile.

4.2.1 Computing Wedges

First of all, we introduce the formula to generate wedges in our data.

In the Chilean data, we include input price of capital for each year from World Bank and Easterly et. al (1994) instead of setting R=0.1 in Hsieh and Klenow(2009). Since capital price is the same across firms in a given year, the time-varied R will not affect the aggregate TFP and variance decomposition. Following Hsieh and Klenow(2005), we set the elasticity in (10) as $\sigma = 3$. Furthermore, since we plan to compare measures in Chile

	Chile					
Year	**ER**	**ESH**	**ERS**	**XR**	**XSH**	**XRS**
1980-1984	0.177	0.077	0.322	0.311	0.094	0.230
1985-1990	0.280	0.123	0.388	0.227	0.045	0.162
1991-1996	0.390	0.127	0.280	0.245	0.080	0.267
	Norway					
Year	**ER**	**ESH**	**ERS**	**XR**	**XSH**	**XRS**
1996-2006	0.350	0.266	0.741	0.285	0.199	0.626
2001-2006	0.308	0.249	0.721	0.328	0.203	0.523
	U.S. (from Dunne et.al, 1988)					
Year	**ER**	**ESH**	**ERS**	**XR**	**XSH**	**XRS**
1963-1967	0.414	0.139	0.271	0.417	0.148	0.247
1967-1972	0.516	0.188	0.286	0.490	0.195	0.271
1972-1977	0.518	0.146	0.205	0.450	0.150	0.221
1977-1982	0.517	0.173	0.228	0.500	0.178	0.226

Table 1: Entry and Exit Statistics

and Norway with the United States, we set the capital shares of production α_s from US data, which is $\alpha_s = \frac{1}{3}$.

As the output Y_i is unobservable from the data, we rearrange equations from the first order condition, and express distortions and productivity of firms in terms of revenue. From (13) and (14), the wedges can be written as

$$1 + \tau_{Ki} = \frac{\alpha}{1-\alpha} \frac{w_i L_i}{R K_i} \tag{30}$$

$$1 - \tau_{Yi} = \frac{\sigma}{\sigma - 1} \frac{w_i L_i}{(1-\alpha) P_i Y_i} \tag{31}$$

Equation (30) shows that a positive capital distortion occurs when wage bill to capital cost with input share ratio is high enough. In equation (31), a positive output distortion occurs when the rate of wage bill to revenue is smaller enough.

From (11) and (12), the firm's physical productivity A_{si} can be obtained

$$TFPQ_i \triangleq A_i = \kappa \frac{(P_i Y_i)^{\frac{\sigma}{\sigma-1}}}{K_i^{\alpha} L_i^{1-\alpha}} \tag{32}$$

$$where\ \kappa = (PY)^{-\frac{1}{\sigma-1}} / P$$

κ is the same across plants and does not affect gain of removing distortions, and $P_i Y_i$ is observable from the data rather Y_i. As a result, we set $\kappa = 1$ as constant for all the industries. In this case, the actual aggregate TFP in our analysis equals to $TFP^{\kappa=1} = \frac{Y}{K^{\alpha} L^{1-\alpha}} \frac{1}{\kappa} = \frac{(PY)^{\frac{\sigma}{\sigma-1}}}{K^{\alpha} L^{1-\alpha}}$.

In order to have a reliable measure of the gain from misallocation, we drop 1% tails of $ln\frac{TFPR_i}{TFPR}$ and $ln\frac{A_i}{A_e}$ by pooling all firm years, and recalculate aggregations and TFP. In this way, trimming eliminates firms across years, it may lead errors when we decompose entry and exit effects on aggregate TFP. However the number of firms dropped is relatively small in both countries. Thus the error is small.

4.2.2 Aggregate Total Factor Productivity

The aggregate TFP is computed according to (26) for Chile and Norway. To compare between countries, we set measures from Norway as benchmark, and detrend aggregate TFP in two countries for 4% off which is the average growth rate of TFP in Norway from data. Figure 7 shows that the Chilean manufacturing TFP was distorted in early 1980s, and TFP declined more than 40% from trend during crisis in 1980s. Since 1987, the Chilean aggregate TFP increased rapidly. It reached the level about 18% above the trend in 1996 . In Norway, the aggregate TFP did not change much from the trend. It came back to the trend in the latest year and stop at the point about 5% below the trend.

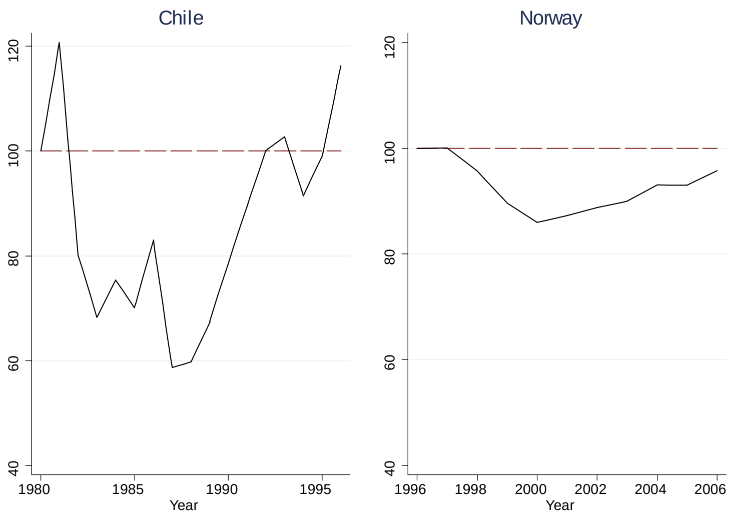

Figure 7: Aggregate Manufacturing TFP: detrended by 4% per year

To figure out the effects of entry and exit on the growth of aggregate TFP, we extend Bartelsman et. al.(2005) and Crespi(2006)'s method (FHK method) to decompose annual growth rate of aggregate TFP as

$$
\begin{aligned}
\frac{\Delta P_{t+1}}{P_t} &= \frac{P_{t+1}-P_t}{P_t} \\
&= \sum_{i\in C_t}\theta_{it}\frac{p_{it+1}-p_{it}}{P_t} + \sum_{i\in C_t}(\theta_{it+1}-\theta_{it})\frac{p_{it}-P_t}{P_t} \\
&\quad + \sum_{i\in C_t}(\theta_{it+1}-\theta_{it})\frac{p_{it+1}-p_{it}}{P_t} \\
&\quad + \sum_{i\in N_t}\theta_{it+1}\frac{(p_{it+1}-P_t)}{P_t} - \sum_{i\in X_t}\theta_{it}\frac{(p_{it}-P_t)}{P_t} \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
p_{it}^{TFP} &= \left(A_{it}\frac{\overline{TFPR_t}}{TFPR_{it}}\right)^{\sigma-1} \\
P_t^{TFP} &= \sum_{i=1}^{M_s}\left(A_{it}\frac{\overline{TFPR_t}}{TFPR_{it}}\right)^{\sigma-1}\theta_{it} \approx TFP_t^{\sigma-1}
\end{aligned}
$$

Here $\theta_{it} = \frac{L_{it}}{L_t}$ is the labor share. P_t is the weighted average of p_{it}. It approximately equals to the squared TFP with $\sigma = 3$. In (33) C_t represents continuing firms who are recorded in year t and year $t+1$. N_t represents entrants who have missing records in year $t-1$ but appear in year t. X_t presents exiting firms who leave the survey in year $t+1$. Following Foster et.al(2001), the first component in (33) captures the growth rate of within-firm effect. The second term represents the growth of between-firm effect which is a (squared) productivity difference weighted by varying labor share. The third term is called the growth of cross effect. It gives covariance between the labor share and productivity (squared). The fourth component shows the growth of entry effect which sums the (squared) productivity change rate of entrants. The last term in (33) is the growth rate of exit effect. It sums the (squared) productivity gap of exiters weighted with labor share.

Figure 8 and Figure 9 graph the decomposition of TFP growth in Chile and Norway according to equation (33)[3]. The net entry effect equals entry effect minus exit. Figure 8 shows that in Chile the annual growth of squared TFP is explained mainly by the within-firm effect. The overall annual growth rate fluctuates from -0.5 to 0.7 approximately. Figure 9 shows that in Norway both within effect and net entry effect have large influence onto the overall growth. However, each growth effects is stable with smaller magnitude, and the overall annual growth rate in Norway varies from -0.02 to 0.17 only.

[3]Decomposition analysis of TFP growth following Foster er. al (2009) gives:
annual growth =within effect +between effect+cross effect+net entry effect

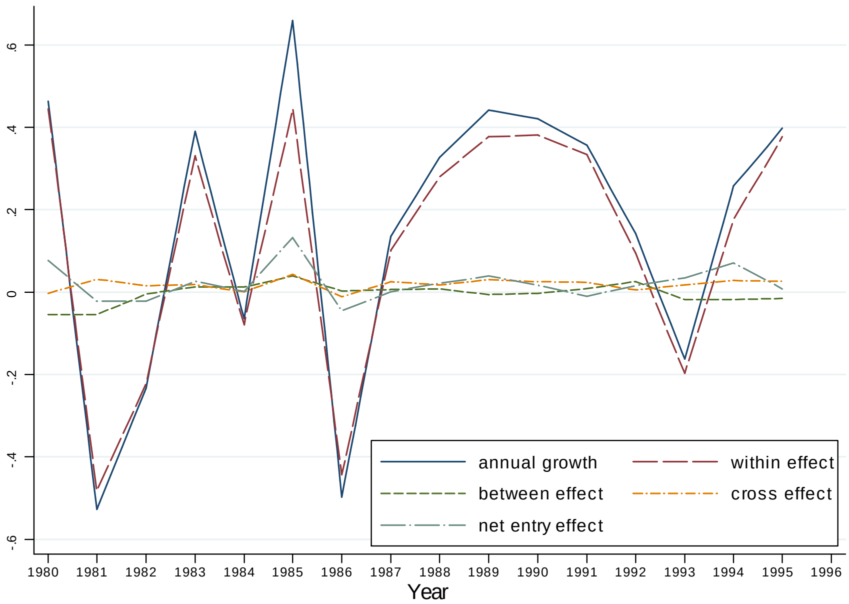

Figure 8: Decomposition of TFP Growth in Chile 1980-1996: $\frac{\Delta P_{t+1}^{TFP}}{P_t^{TFP}}$

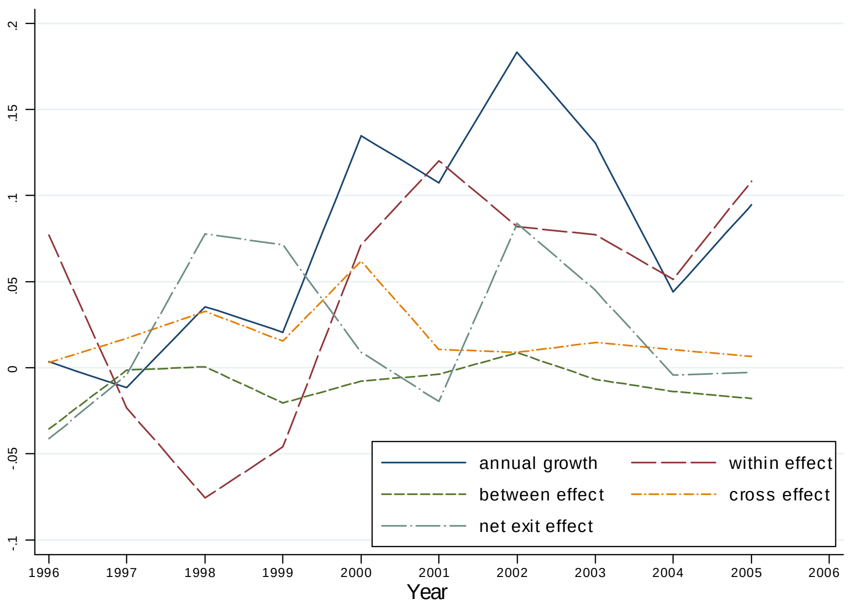

Figure 9: Decomposition of TFP Growth in Norway 1996-2006: $\frac{\Delta P_{t+1}^{TFP}}{P_t^{TFP}}$

4.2.3 Efficiency Gains

The TFP gain from removing distortions is defined as the relative difference between the TFP and its efficient level which can be written as

$$TFPgain = \frac{Y_e}{Y} - 1 = \frac{TFP_e}{TFP} - 1$$

Figure 10 depicts the TFP gain in Chile and Norway. It shows that the efficiency gain from removing misallocation decreases from 154% to 59% in Chile. But the gain in Norway is relatively small. It varies from 24% to 29% which is more stable. Thus Chilean manufacturing industry will obtain more growth of total output by improving industry productivity into efficient level. It also means that the production inputs in Chile is more distorted than in Norway.

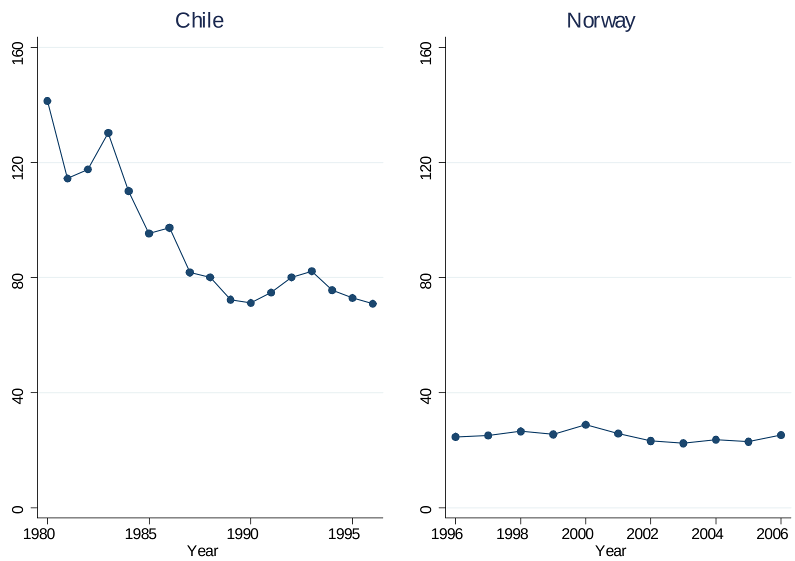

Figure 10: TFP Gain from Reallocation

4.2.4 Size Distribution

According to equation (16), we write the actual and efficient size of firm as

$$P_iY_i = \frac{A_i^{\sigma-1}(1-\tau_{yi})^{\sigma-1}}{(1+\tau_{Ki})^{\alpha(\sigma-1)}}\left[\frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\right]^{-(\sigma-1)} Y \quad (34)$$

$$P_i^eY_i^e = A_i^{\sigma-1}\left[\frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{\overline{w}}{1-\alpha}\right)^{1-\alpha}\right]^{-(\sigma-1)} Y^e \quad (35)$$

Here we set $\overline{w}$ as the mean value of w_i which is constant across firms in the efficient case. In Figure 11, we plot actual and efficient size distribution (adjusted to the mean) in logarithmic value according to equation (34) and (35). Figure 11 shows that for both countries in the latest year, the efficient size distribution is more spread out and has lower density in the mean size. This implies that the efficient resource allocation enlarges size difference among firms. As the size distribution has thicker tails in Chile, the difference of efficient size among firms is larger in Chile than in Norway.

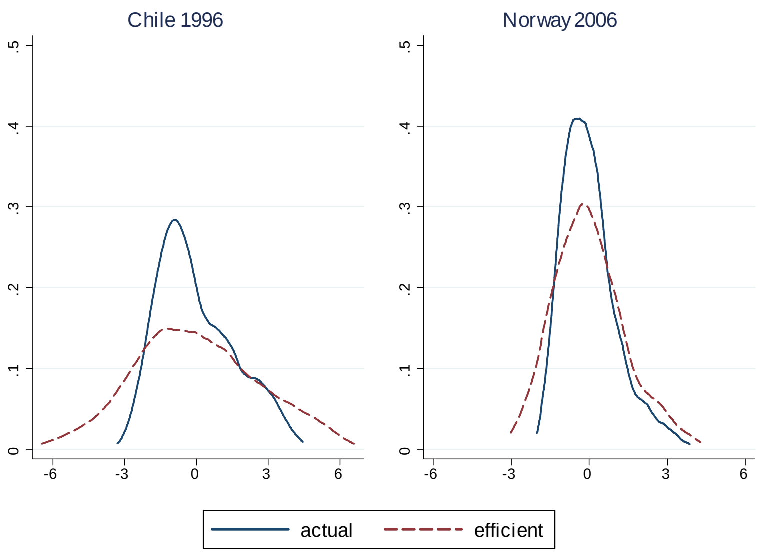

Figure 11: Distribution of Size

4.2.5 TFPR and TFPQ Dispersion

Figure 12 shows the distribution of lnTFPQ, $\ln\frac{A_i}{mean(A_i)}$, for Chile in 1996 and for Norway in 2006. It shows that TFPQ is more dispersed in Chile with thicker tails. It means that there is larger productivity difference between the most and least productive firms in Chilean manufacturing. In contrast, in Norway, the productivity difference is moderate. Table 2 gives a consistent message where Chile has the highest ratio of 75th to 25th percentiles and 90th to 10th percentiles of TFPQ. For Norway, most measures appear fairly stable with less changes. The productivity of firms in Norway is more evenly distributed than in the US. However, the dispersion of TFPQ in the US cited from Hsieh and Klenow(2005) is computed using annual sector average which may be not comparable with our results.

Figure 13 shows the distribution of lnTFPR, $\ln\frac{TFPR_i}{mean(TFPR_i)}$, adjusted by the mean of lnTFPR in the latest year of each country. The TFPR in Chile is more spread-out

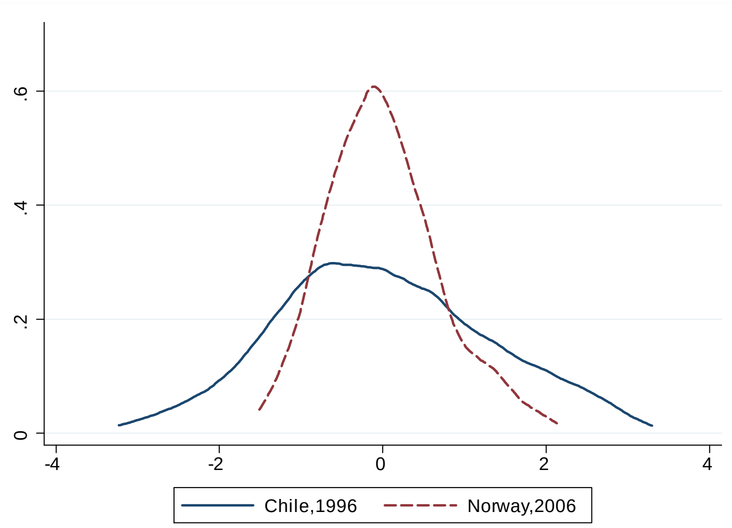

Figure 12: Distribution of lnTFPQ

with heavy tails. Consistent with Figure 13, Table 3 shows lower standard deviation and interquartile range in Norway. The highest standard deviation in Chile indicates that the dispersion of TFPR is larger in Chile. Whereas, in Chile, the standard deviation of TFPR in Table 3 drops 28% between 1980 and 1996 which shows decreasing TFPR dispersion in Chile.

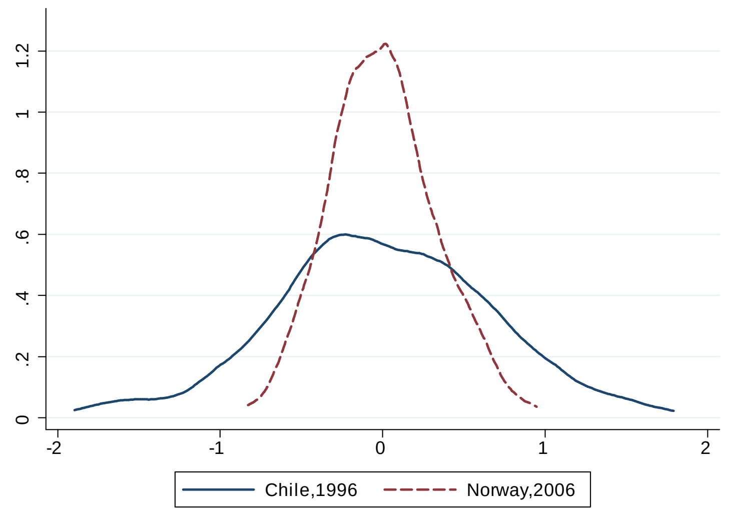

Figure 13: Distribution of lnTFPR

	Chile		
	1980	**1988**	**1996**
SD	1.28	1.27	1.15
p75-p25	1.74	1.81	1.59
p90-p10	3.39	3.33	2.98
	Norway		
	1996	**2000**	**2006**
SD	0.66	0.66	0.69
p75-p25	0.86	0.81	0.90
p90-p10	1.65	1.62	1.78
U.S. (from Hsieh and Klenow, 2009)			
	1977	**1987**	**1997**
SD	0.85	0.79	0.84
p75-p25	1.22	1.09	1.17
p90-p10	2.22	2.05	2.18

Table 2: Dispersion of TFPQ

	Chile		
	1980	**1988**	**1996**
SD	0.81	0.67	0.58
p75-p25	0.99	0.92	0.73
p90-p10	2.05	1.71	1.47
	Norway		
	1996	**2000**	**2006**
SD	0.31	0.29	0.33
p75-p25	0.39	0.38	0.43
p90-p10	0.78	0.74	0.85
U.S. (from Hsieh and Klenow, 2009)			
	1977	**1987**	**1997**
SD	0.45	0.41	0.49
p75-p25	0.46	0.41	0.53
p90-p10	1.04	1.01	1.19

Table 3: Dispersion of TFPR

4.2.6 Variance Decomposition

In this section, we show that there is a similar pattern between ($\ln TFP^e - \ln TFP$) and the variance of lnTFPR. The variables are decomposed in two way: variance decomposition and quintile decomposition. We find that in Chile the variance of output distortion and its between-quintile effect are the main elements of decreasing efficient gain. All the measures in Norwegian data are fairly stable with small magnitude.

TFPR and TFP gain

Equation (27) shows that the aggregate TFP is expressed as a CES function of the "average" TFPR and firm's TFPR. Then we plot the variance of TFPR with lnTFP gain ($\ln TFP^e - \ln TFP$) in Figure 14. It shows that the Chilean TFP gain decreases from 0.85 in 1983 to 0.55 in 1990. The trend of lnTFP gain tend to have a similar pattern as the decreasing var(lnTFPR) from 1986. The financial reform during 1983-1992 in Chile can explain the increase of aggregate TFP. The tax reform in 1984-1985 and the trade policy since 1985 are the reasons of decreasing var(lnTFPR). The tax reform removes the high tax on firms' profit from 50% to 10%, and the trade policy decreases tariffs to 25% (Brock, 2009). However, in Norway, lnTFP gain and var(lnTFPR) are stable and with lower magnitude. The variance of lnTFPR maintains at the level of 0.1 from 1996 to 2006.

From equation (20), we can decompose the variance of lnTFPR as

$$\begin{aligned} var\left(\log TFPR_i\right) &= \alpha^2 var\left(\log MRPK_i\right) + (1-\alpha)^2 var\left(\log MRPL_i\right) \\ &\quad +2\alpha\left(1-\alpha\right) cov\left(\log MRPK_i, \log MRPL_i\right) \end{aligned} \tag{36}$$

In Figure 15, we plot components of var(lnTFPR) according to (36). It shows that in Chile the main component explaining the change in var(lnTFPR) is var(lnMRPL), while cov(lnMRPL lnMRPK) also has similar pattern. The var(lnMRPL) component decreases from 1986 to 1994. It accounts for 60% of the var(lnTFPR) on average, while the var(lnMRPK) component is about 28%. The cov(lnMRPL, lnMRPK) changes to negative after 1991. In Norway, each component appears steady trend at low magnitude. However, the var(lnMRPK) component accounts for more than 60% to the variance of lnTFPR. This means the variance of lnMRPK in Norway has more impact onto the change of var(lnTFPR) than that in Chile. Moreover, the marginal revenue products are negative correlated during 1996-2006 in Norway. Thus in Norway, the high MRPL decreases MRPK.

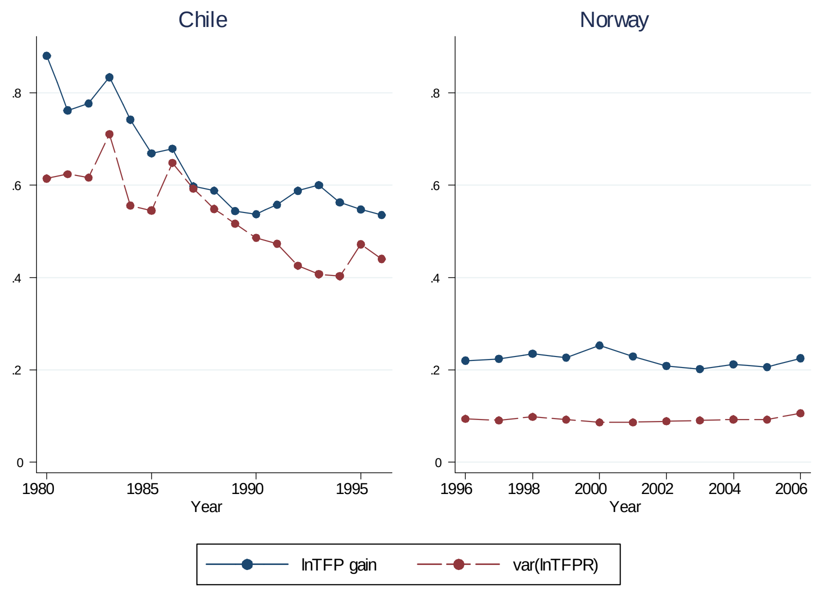

Figure 14: lnTFP Gain and var(lnTFPR$_{si}$)

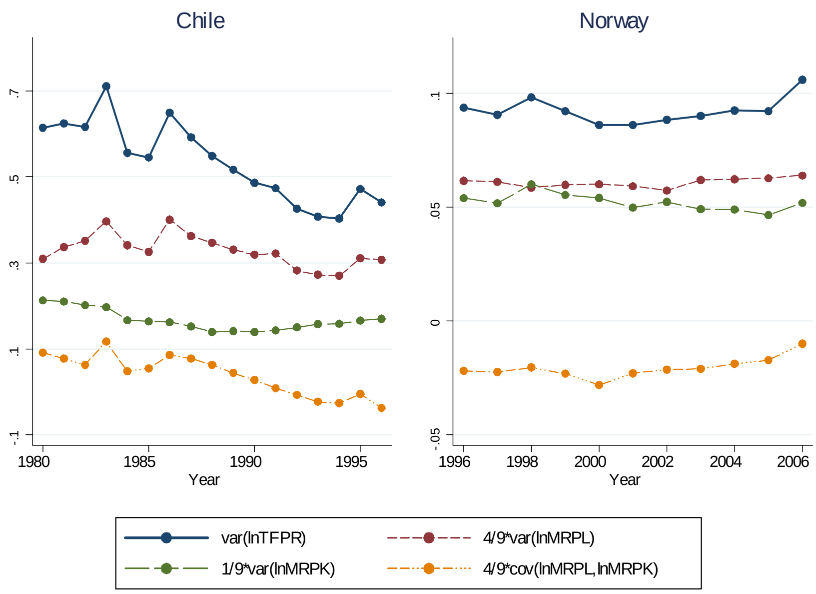

Figure 15: Decomposition of var(lnTFPR)

Next, we extends Davis and Haltiwanger (1991)'s method to decompose variance of TFPR for different lnA_i quintiles (see Appendix III for details). The var(lnTFPR) equals

$$
\begin{aligned}
var(lnTFPR_i) &= \underbrace{\frac{1}{N}\sum_{q}^{Q}\sum_{i}^{N_q}\left(\ln TFPR_{qi} - \overline{\ln TFPR}\right)^2}_{over\ all\ variation} \\
&= \underbrace{\sum_{q}^{Q}\frac{N_q}{N}var(\ln TFPR)_q}_{within-group\ component} + \underbrace{\sum_{q}^{Q}\frac{N_q}{N}\left(\overline{\ln TFPR_q} - \overline{\ln TFPR}\right)^2}_{between-group\ component} \quad (37)
\end{aligned}
$$

where $\overline{\ln TFPR}$ is the overall mean of $TFPR_i$. Here $var(\ln TFPR)_q$ is the variance for lnTFPR in each quintile q. $\overline{\ln TFPR_q}$ is the mean of lnTFPR in quintile q. N is total number of firm. N_q is the number of firm in the qth quintile. As we group lnA_i into quintiles, $\frac{N_q}{N} = \frac{1}{5}$. We define the first term in (37) as within-group component which is the sum of quintile variances weighted by observation rate in each quintile group. The second component is called between-group component which adds up squared difference between quintile mean and overall mean weighted by the observation rate in each quintile.

Figure 16 plots the decomposition of var(lnTFPR) for different lnAi quintiles according to equation (37). In Chile, the between-group component accounts for most of the change in var(lnTFPR). In contrast, in Norway, the figure shows that the within-group component is larger. But since each component in Norway is relatively steady and has small magnitude, it is ambiguous to tell the which component represents large proportion.

As TFPR is price times TFPQ, higher TFPR implies higher productivity in the firm. Also because the between-group component explains the "variance" of mean, the high(low) productivity firm with high(low) TFPR value implies large difference between top (bottom) quintile mean and overall mean. Thus in Figure 17, the 1st and 5th quintile of productivity explain the most change of between group variance in var(lnTFPR) for Chile and Norway. In addition, in both Chile and Norway the mean difference in lower end of the lnA_i distribution always lies above the upper end. This indicates that the least productive firm has higher TFPR difference with the average TFPR, whereas the most productive firm has less TFPR difference comparing to the mean.

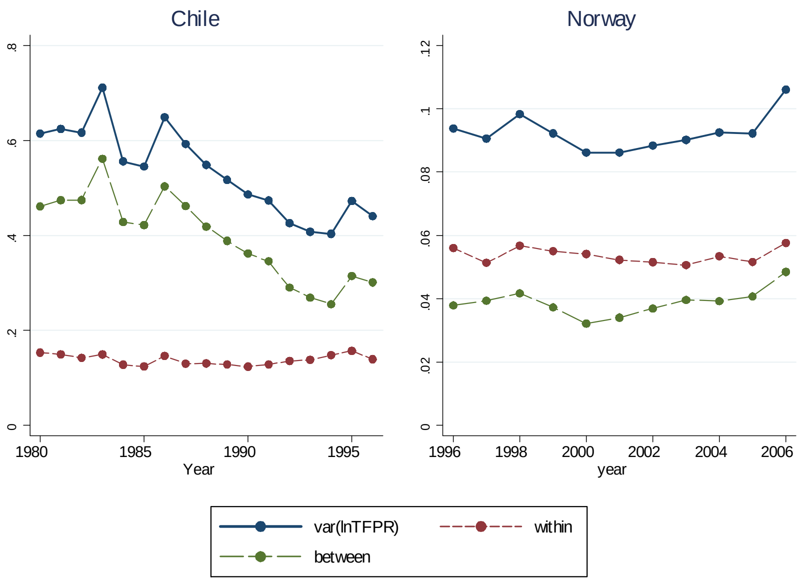

Figure 16: var(lnTFPR): $\ln A_{si}$ quintiles

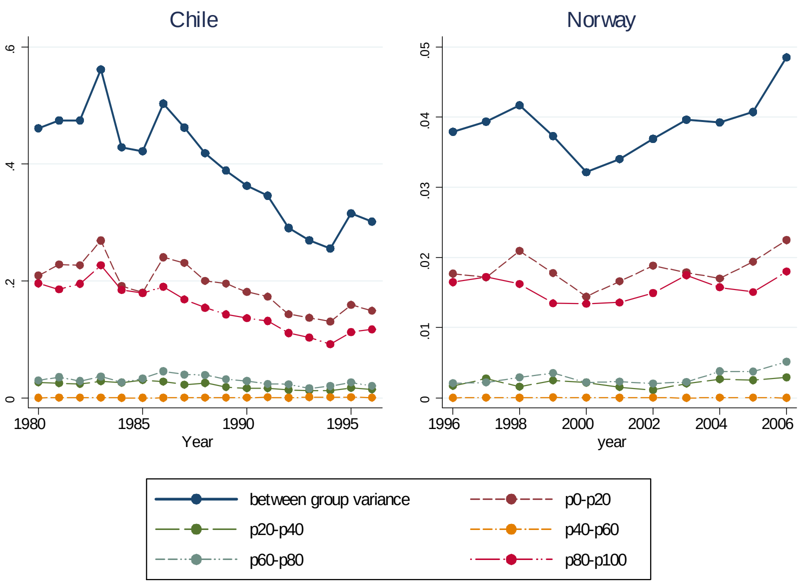

Figure 17: var(lnTFPR): Between-Group Component

MRPL

To understand why the variance of lnTFPR has decreased in Chilean manufacturing, we decompose variance of lnMRPL according to (16).

$$var\left(\log MRPL_i\right) = var\left(\log w_i\right) + var\left(\log\left(1-\tau_{Yi}\right)\right) - 2cov\left(\log w_i, \log\left(1-\tau_{Yi}\right)\right) \tag{38}$$

Figure 18 shows that Norway has more steady trend and lower level of each component in var(lnMRPL). In contrast, in Chile the main component explaining the decreasing of var(lnMRPL) in 1986-1994 is the variance of output wedges. The $var(\ln(1-\tau_{yi}))$ accounts for more than 50% of the var(lnMRPL) on average. One possibility of decreasing var(lnMRPL) can be explained by the elimination of wage indexation policy since 1982 which decreases real wages (Corbo and Fischer,1994 ;Bergoeing et.al, 2007).

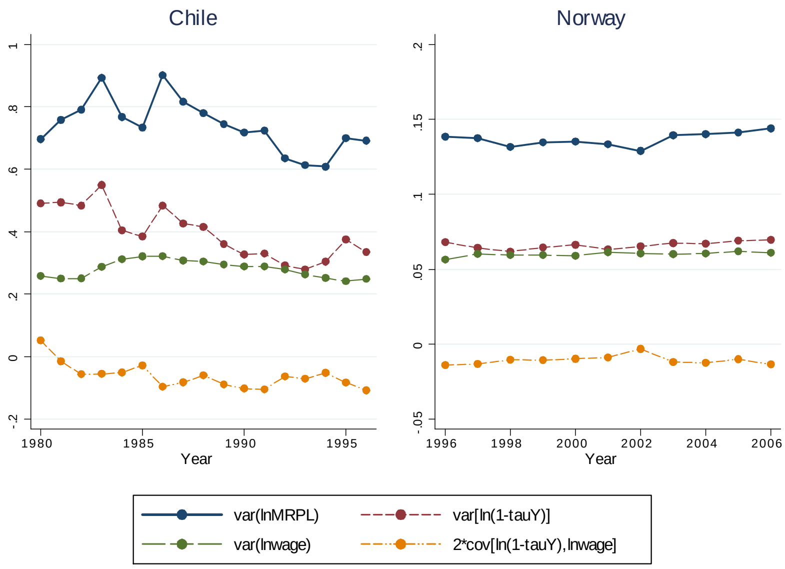

Figure 18: Decomposition of var(lnMRPL)

Similarly, we study quintile analysis of var(lnMRPL). The result shows that in Chile the between-group component represents 70% of var(lnMRPL) on average which mainly explains the drop in var(lnMRPL). In Norway, the flat trend and low value of components result less change in lnMRPL from 1996 to 2006. Moreover, in both Chile and Norway, the changes of the mean lnMRPL in the lower and upper end of the productivity distribution are the main factors explaining the trend in the between-group component of var(lnMRPL).

cov(lnMRPL, lnMRPK)

Recall from previous analysis, cov(lnMRPL, lnMRPK) has similar pattern as the variance of lnTFPR. Now we use equation (15) and (16) to decompose cov(lnMRPL, lnMRPK) as

$$cov\left(\ln MRPK_i, \ln MRPL_i\right) = var\left(\ln\left(1-\tau_{Yi}\right)\right) - cov\left(\ln\left(1+\tau_{Ki}\right), \ln\left(1-\tau_{Yi}\right)\right) + cov\left(\ln w_i, \ln\left(1+\tau_{Ki}\right)\right) - cov\left(\ln w_i, \ln\left(1-\tau_{Yi}\right)\right) \quad (39)$$

Figure 19 presents that in Chile, the main driving force of decreasing cov(lnMRPK, lnMRPL) from 1986 is $var\left(\ln\left(1-\tau_{Yi}\right)\right)$. In Norway, the high and positive covariance between wedges is the main reason of negative cov(lnMRPK, lnMRPL).

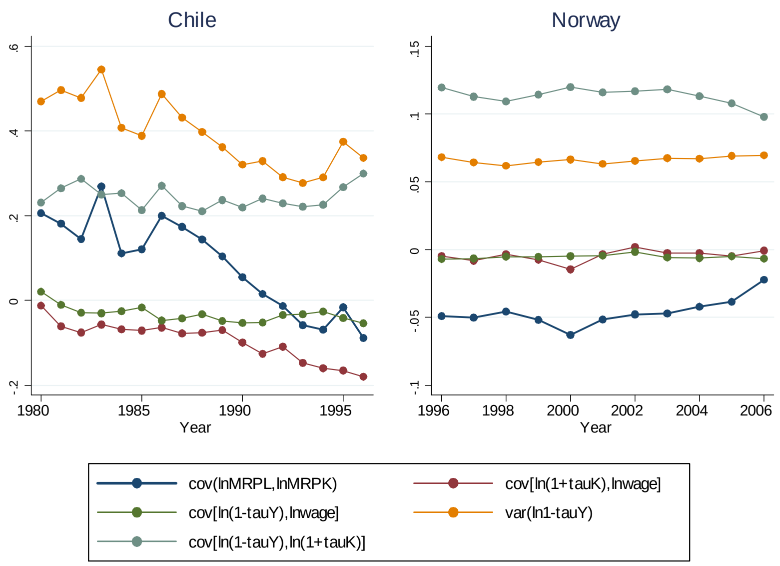

Figure 19: Decomposition of cov(lnMRPL,lnMRPK)

Output distortion

Finally, as we shown that the $var\left(\ln\left(1-\tau_{Yi}\right)\right)$ in Chile is the main component explaining the changes in var(lnMRPL) and cov(lnMRPL, lnMRPK), we decompose $var\left(\ln\left(1-\tau_{Yi}\right)\right)$ into between- and within-group effects in Figure 20. In Chile, both within- and between-group component explain the the trend of the var($\ln\left(1-\tau_{Yi}\right)$). But the between-group effect has stronger driving force to the decreasing var($\ln\left(1-\tau_{Yi}\right)$) during 1986-1993. However, in Norway, the between-group component varies slightly around 0.01 which represents weak driving force to the variance of output distortion.

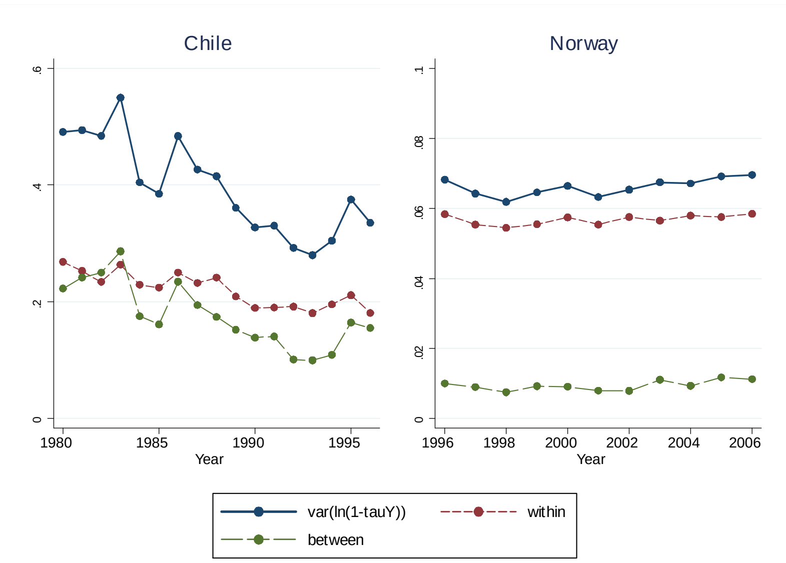

Figure 20: $var\left(\ln\left(1-\tau_{Yi}\right)\right)$: $\ln A_{si}$ quintiles

Figure 21 illustrates that the changes of $\overline{\ln\left(1-\tau_{Yi}\right)_q}$ (the quintile mean) in the 1st and 5th quintile are the dominating effects explaining the trend of between-group component which has a dramatic drop in Chile and an insignificant rise in Norway. Relating to policies in Chile, the least and most productive firms are more sensitive to the tax reform and trade policy since 1985 which decrease the high marginal income tax and tariffs respectively (Brock, 2009).

Furthermore, in Chile and Norway the $cov(\ln\left(1-\tau_{Yi}\right), \ln TFPQ_i)$ is stable and negative in the most and least productive firms from the data. The overall covariance retains the level at about -0.7 in Chile and -0.4 in Norway. That is to say, the output wedges have negative correlation with firm productivity in the top and bottom productivity quintiles. Then the mean of $\ln\left(1-\tau_{Yi}\right)$ in lower quintile is higher than the overall mean. Additionally, we find that the overall mean $\overline{\ln\left(1-\tau_{Yi}\right)}$ in Chile increases during 1986-1993. Thus in Figure 21 in Chile, when the between-group component of $\ln\left(1-\tau_{Yi}\right)$ in the lower end quintile falls rapidly during 1986-1993, the lower end quintile mean of $\ln\left(1-\tau_{Yi}\right)$ approaches to the overall mean in a faster speed. Hence we can conclude that in Chile the fast approaching of quintile mean to the overall mean of output wedges in low productive firms mainly explains the change in var$(\ln\left(1-\tau_{Yi}\right))$. In other words, less distortion in low productive firm is a strong driving force to the increasing TFP in Chile since 1986. One possible explanation could be that the Chilean reform policies since 1986 were more effective on decreasing distortions from the low productive firms, which drove falling TFP gain.

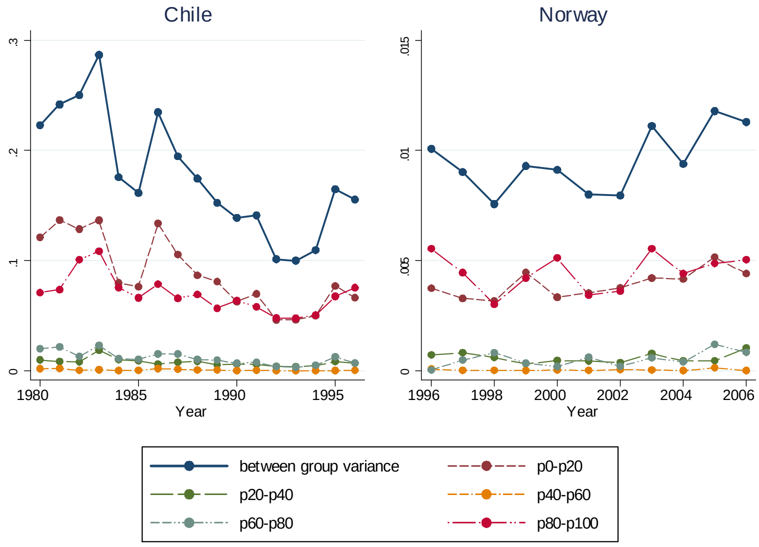

Figure 21: $var\left(\ln\left(1-\tau_{yi}\right)\right)$: Between-Group Decomposition

5 Conclusion

This paper uses Hsieh and Klenow (2009)'s accounting methodology to describe the pattern of the misallocation and aggregate TFP. We calibrate the model and compare firm-level data for Chile during 1980-1996 and Norway during 1996-2006.

We compare detrended aggregate TFP between Chile and Norway. The study shows that the detrended TFP in Norway is fairly stable. The TFP growth in Norway is mainly driven by between effect and net exit effect. In contrast, Chile experiences big swings in aggregate TFP which is explained mainly by the within effect.

We compute TFP relative to efficient TFP by assuming zero distortion and constant wage rate across firms. The result shows that the gain from removing distortions in Chile decreases rapidly from 154% to 59% during 1980-1990. While the gain in Norway is fairly stable around 26%. It also means that the production inputs in Chile is more distorted than in Norway. In addition, we compare the size distribution. In Chile and Norway the equalizing TFPR gives more spread-out of size distribution. Both distorted and efficient size distribution are more dispersed in Chile. Moreover, the distribution of TFPQ adjusted to the mean gives thicker tails for the latest year in Chile which means firms in Chile had larger efficiency difference in 1996, while the Norwegian productivity was almost evenly distributed across firms in 2006.

As TFP can be expressed as the relative ratio between "average" TFPR and TFPR, the variance of lnTFPR appears a similar trend as lnTFP^e-lnTFP in our data. That is to say the var(lnTFPR) explains most of the changes in the efficient gain. The variance decomposition shows that in Chile the main component explaining the decreasing var(lnTFPR) is var(lnMRPL), which can be explained by the trend in the var($\ln(1-\tau_{Yi})$). However in Norway, the components of var(lnTFPR) are stable, and the variance of MRPK accounts for higher proportion of the change in var(lnTFPR).

After that, the variances are decomposed into different productivity quintiles. The quintile analysis shows that in Norway, the subgroup variance (within-group) of $\ln(1-\tau_{Yi})$ explains more of the trend in var($\ln(1-\tau_{Yi})$). Whereas, in Chile, falls in the mean value of $\ln(1-\tau_{Yi})$ relative to the overall mean (between-group) at both end of the distribution explain mainly the fall in var($\ln(1-\tau_{Yi})$). Thus the output distortions in both less and most productive firms are the main factors explaining decreasing variance of output wedges. Furthermore, since we find increasing overall mean of $\ln(1-\tau_{Yi})$ and negative cov($\ln(1-\tau_{Yi})$, lnTFPQ$_i$) , the large drop of $\overline{\ln(1-\tau_{Yi})_q}-\overline{\ln(1-\tau_{Yi})}$ in low productive firm indicates less decreasing output distortion in inefficient firms. This suggests that in Chile, the less distortion faced by low productive firms is a stronger driving force to the increasing TFP in Chile since 1986.

We conjecture that in Chile the dramatic drop in TFP gain is caused by a series of reform policies after the crisis which includes the bankruptcy law in 1982, the market-determined interest rate since 1982 and the tax reform in 1984. These reforms drop the output distortion in Chile and result rising manufacturing aggregate TFP while the TFP gain decreases.

There are some limitations in our result. The dataset used for two countries are in different timeline. In addition, we can not depart distortion on labor from the recent model. Thus in future research we could introduce τ_{Li} by adding intermediates as input, and discuss how the resource misallocation affects aggregate TFP.

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6 Appendix I. Derivation of Two-firm Case

This part give the derivation of equations in the two-firm case.

6.1 Final Good

$$\begin{aligned} \max_{Y_1,Y_2}.\pi &= Y - (P_1Y_1 + P_2Y_2) \\ s.t. &: \\ Y &= \left(Y_1^{\frac{\sigma-1}{\sigma}} + Y_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{40}$$

First order condition gives

$$Y_i = \frac{1}{P_i^\sigma}Y \tag{41}$$

where $P = 1$

6.2 Intermediate Good

The production function for firms are

$$Y_i = A_i K_i^\alpha L_i^{1-\alpha} \tag{42}$$

Profit maximization for firm 1

$$\max_{L_1,K_1,P_1} \pi_1 = (1-\tau)P_1A_1K_1^aL_1^{1-\alpha} - wL_1 - RK_1 \tag{43}$$

$$s.t. \quad : \quad Y_1 = P_1^{-\sigma} \cdot Y \tag{44}$$

for firm 2

$$\max_{L_2,K_{21},P_2} \pi_2 = P_2A_2K_2^aL_2^{1-\alpha} - wL_2 - RK_2 \tag{45}$$

$$s.t. \quad : \quad Y_2 = P_2^{-\sigma} \cdot Y \tag{46}$$

The Lagrangians are

$$\mathcal{L}_1 = (1-\tau)\, P_1A_1K_1^aL_1^{1-\alpha} - wL_{si} - RK_{si} - \lambda_1\left(A_1K_1^aL_1^{1-\alpha} - Y\left[\frac{1}{P_1}\right]^{-\sigma}\right) \tag{47}$$

$$\mathcal{L}_2 = P_2A_2K_2^aL_2^{1-\alpha} - wL_{si} - RK_{si} - \lambda_2\left(A_2K_2^aL_2^{1-\alpha} - Y\left[\frac{1}{P_2}\right]^{-\sigma}\right) \tag{48}$$

The first order conditions from (47) and (48) give

$$\frac{K_1}{L_1} = \frac{w}{R}\frac{\alpha}{1-\alpha} = \frac{K_2}{L_2} \tag{49}$$

and

$$P_1 = \frac{\sigma}{\sigma-1}\frac{\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}{A_1}\frac{1}{1-\tau} \tag{50}$$

$$P_2 = \frac{\sigma}{\sigma-1}\frac{\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}}{A_2} \tag{51}$$

We express L_i and K_i in terms of Y, by inserting (49) into (42) then combing with (41). Labor demands are

$$L_1 = A_1^{\sigma-1}(1-\tau)^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)}\left(\frac{w}{1-\alpha}\right)^{\alpha(\sigma-1)-\sigma} Y \tag{52}$$

$$L_2 = A_2^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)}\left(\frac{w}{1-\alpha}\right)^{\alpha(\sigma-1)-\sigma} Y \tag{53}$$

Dividing (52) by (53), labor demand ratio is

$$\frac{L_1}{L_2} = \frac{A_1^{\sigma-1}(1-\tau)^{\sigma}}{A_2^{\sigma-1}}$$

Capital demands are

$$K_1 = A_1^{\sigma-1}(1-\tau)^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)-1}\left(\frac{w}{1-\alpha}\right)^{(\alpha-1)(\sigma-1)} Y \tag{54}$$

$$K_2 = A_2^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)-1}\left(\frac{w}{1-\alpha}\right)^{(\alpha-1)(\sigma-1)} Y \tag{55}$$

Dividing (54) by (55), capital demand ratio is

$$\frac{K_1}{K_2} = \frac{A_1^{\sigma-1}(1-\tau)^{\sigma}}{A_2^{\sigma-1}}$$

We also compute Y_1 and Y_2 by rewrite (42) as

$$Y_{si} = A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s} L_{si}$$

Inserting (49) (52) and (53), firm output is

$$\begin{aligned} Y_1 &= A_1 \left[\frac{w}{R}\frac{\alpha}{1-\alpha}\right]^{\alpha} L_1 \\ &= A_1^{\sigma}(1-\tau)^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{w}\right)^{\sigma(1-\alpha)} Y \qquad (56) \\ Y_2 &= A_2^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{w}\right)^{\sigma(1-\alpha)} Y \qquad (57) \end{aligned}$$

Now we insert (56) and (57) into (40)

$$Y = Y\left[\frac{\sigma-1}{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha}\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\right]^{\sigma}\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}} \qquad (58)$$

(58) gives factor price equals

$$\frac{\sigma}{\sigma-1}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha} = \left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \qquad (59)$$

We define $MRPK_1 = \frac{\partial P_1 Y_1}{\partial K_1}$ and $MRPL_2 = \frac{\partial P_2 Y_2}{\partial L_2}$. Accordingly,

$$\begin{aligned} MRPL_i &= (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_i Y_i}{K_i} \Longrightarrow L_i = (1-\alpha)P_i Y_i MRPL_i^{-1}\frac{\sigma-1}{\sigma} \qquad (60) \\ MRPK_i &= \alpha\frac{\sigma-1}{\sigma}\frac{P_i Y_i}{K_i} \Longrightarrow K_{si} = \alpha P_i Y_i MRPK_i^{-1}\frac{\sigma-1}{\sigma} \qquad (61) \end{aligned}$$

We can also have marginal revenue products as function of distortion and input prices by plugging (50) (51) (42) and (49) into (60) and (61).

$$\begin{aligned} MRPK_1 &= \frac{R}{1-\tau} \\ MRPK_2 &= R \\ MRPL_1 &= \frac{w}{1-\tau} \\ MRPL_2 &= w \end{aligned}$$

The revenue based TFP could be rewritten with marginal revenue of products as

$$\begin{aligned} TFPR_i &= \frac{P_i Y_i}{(K_i)^{\alpha}(L_i)^{1-\alpha}} \\ &= \frac{\sigma}{\sigma-1}\left(\frac{MRPK_i}{\alpha}\right)^{\alpha}\left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha} \qquad (62) \end{aligned}$$

Then

$$TFPR_1 = \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\frac{1}{(1-\tau)} \quad (63)$$

$$TFPR_2 = \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha} \quad (64)$$

We define $\overline{TFPR}$ as a kind of average of $TFPR_i$

$$\overline{TFPR} = \frac{\frac{\sigma}{\sigma-1}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}}{(1-\tau)\frac{P_1Y_1}{Y}+\frac{P_2Y_2}{Y}} \quad (65)$$

Inserting (41) (50) and (51) into (65)

$$\overline{TFPR} = \frac{\left[\frac{\sigma}{\sigma-1}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}\right]^{\sigma}}{A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}} \quad (66)$$

We insert factor price (59) into (66)

$$\overline{TFPR} = \left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \quad (67)$$

6.3 Aggregate TFP

On one hand, we use growth accounting to derive TFP as

$$TFP^{aggregates} = \frac{Y}{K^{\alpha}L^{1-\alpha}} \quad (68)$$

Adding (52) and (53) up, aggregate labor is

$$\begin{aligned} L &= L_1 + L_2 \\ &= \left[A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}\right]\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)}\left(\frac{w}{1-\alpha}\right)^{\alpha(\sigma-1)-\sigma} Y \end{aligned} \quad (69)$$

From (54) and (55), aggregate capital is

$$\begin{aligned} K &= K_1 + K_2 \\ &= \left[A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}\right]\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{R}{\alpha}\right)^{\alpha(1-\sigma)-1}\left(\frac{w}{1-\alpha}\right)^{(\alpha-1)(\sigma-1)} Y \end{aligned} \quad (70)$$

Inserting (69) and (70) into (68)

$$
\begin{aligned}
TFP^{aggregates} &= \frac{\left[\frac{\sigma}{\sigma-1}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}\right]^{\sigma}}{\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\alpha}\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{1-\alpha}} \\
&= \frac{\left[\frac{\sigma}{\sigma-1}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}\right]^{\sigma}}{A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}} \qquad (71)
\end{aligned}
$$

We substitute factor price in (71), then $TFP^{aggregates}$ becomes

$$
TFP^{aggregates} = \frac{\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}} \qquad (72)
$$

On the other hand we find another method to compute TFP by using TFPR from (63) and (64).

$$
\begin{aligned}
TFP^{TFPR} &= \left[\left(A_1\frac{\overline{TFPR}}{TFPR_1}\right)^{\sigma-1}+\left(A_2\frac{\overline{TFPR}}{TFPR_2}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \\
&= \frac{\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}} \qquad (73)
\end{aligned}
$$

(72) and (73) illustrate the same result. Thus

$$
\begin{aligned}
TFP &= \frac{Y}{K^{\alpha}L^{1-\alpha}} = \left[\left(A_1\frac{\overline{TFPR}}{TFPR_1}\right)^{\sigma-1}+\left(A_2\frac{\overline{TFPR}}{TFPR_2}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \\
&= \frac{\left[A_1^{\sigma-1}(1-\tau)^{\sigma-1}+A_2^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}}{A_1^{\sigma-1}(1-\tau)^{\sigma}+A_2^{\sigma-1}} \qquad (74)
\end{aligned}
$$

7 Appendix II. Derivation of Multiple-Firm Case

Here we show the derivations in the multiple-firm case in Section 3.

7.1 Final Sector

For each industry, output is a CES aggregator over M_s differentiated products

Each industry decides

$$\max_{Y_i} PY - \sum P_i Y_i$$
$$st \quad :$$
$$Y = \left(\sum_{i=1}^{M_s} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{75}$$

Solve this problem and show optimally condition.

$$\mathcal{L} = P \cdot \left(\sum_{i=1}^{M} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum P_i Y_i$$

First order condition gives

$$\frac{\partial \mathcal{L}}{\partial Y_i} \quad \Rightarrow \quad P_i = P \cdot \left(\sum_{i=1}^{M} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \cdot Y_i^{\frac{\sigma-1}{\sigma}-1} \tag{76}$$

$$\frac{\partial \mathcal{L}}{\partial Y_j} \quad \Rightarrow \quad P_j = P \cdot \left(\sum_{i=1}^{M_s} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \cdot Y_j^{\frac{\sigma-1}{\sigma}-1} \tag{77}$$

Dividing (76) by (77), we get

$$\frac{P_i}{P_j} = \left(\frac{Y_j}{Y_i} \right)^{\frac{1}{\sigma}} \tag{78}$$

Then rearrange (78),

$$
\begin{aligned}
Y_i^{\frac{1}{\sigma}} &= P_i^{-1} \cdot P_j \cdot Y_j^{\frac{1}{\sigma}} \\
\sum_{i=1}^{M_s} Y_i^{\frac{\sigma-1}{\sigma}} &= \sum_{i=1}^{M_s} \left(P_i^{1-\sigma} \cdot P_j^{\sigma-1} \cdot Y_j^{\frac{\sigma-1}{\sigma}} \right) \\
\left(\sum_{i=1}^{M_s} Y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} &= \left(P_j^{\sigma-1} \cdot Y_j^{\frac{\sigma-1}{\sigma}} \cdot \sum_{i=1}^{M_s} P_i^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \\
Y &= P_j^{\sigma} \cdot Y_j \cdot \left(\sum_{i=1}^{M} P_i^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Show Y_j as

$$Y_j = \frac{P_j^{-\sigma}}{\left(\sum_{i=1}^{M} P_i^{1-\sigma}\right)^{-\frac{\sigma}{1-\sigma}}} \cdot Y$$

Rewrite output of each firm Y_i

$$Y_i = \frac{P_i^{-\sigma}}{\mathbf{P}^{-\sigma}} \cdot Y \tag{79}$$

Define price index as

$$\mathbf{P} = \left(\sum_{k=1}^{M} P_k^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{80}$$

7.2 Intermediate Sector

Within the manufacturing, producers produce differentiates goods that are monopolistically competitive. For each i producer, the production technology is Cobb-Douglas

$$Y_i = A_i K_i^a L_i^{1-\alpha} \tag{81}$$

$$\begin{aligned} \max_{L_i, K_i, P_i} \pi_i &= (1-\tau_{Yi}) P_i A_i K_i^a L_i^{1-\alpha} - w_i L_i - R(1+\tau_i) K_i \\ st &: \\ Y_i &= \frac{P_i^{-\sigma}}{\mathbf{P}^{-\sigma}} \cdot Y \end{aligned} \tag{82}$$

The Lagrangian gives

$$\mathcal{L} = (1-\tau_{Yi}) P_i A_i K_i^\alpha L_i^{1-\alpha} - w_i L_i - (1+\tau_{Ki}) R K_i - \lambda \left[A_i K_i^\alpha L_i^{1-\alpha} - Y \left(\frac{P}{P_i}\right)^{-\sigma} \right]$$

First order condition gives

$$(1-\tau_{Yi}) Y_i - \sigma \lambda Y \frac{P_i^{\sigma-1}}{P^{-\sigma}} = 0 \tag{83}$$

$$\alpha \left[(1-\tau_{Yi}) P_i - \lambda \right] A_i K_i^{\alpha-1} L_i^{1-\alpha} - (1+\tau_{Ki}) R = 0 \tag{84}$$

$$(1-\alpha) \left[(1-\tau_{Yi}) P_i - \lambda \right] A_i K_i^{\alpha} L_i^{-\alpha} - w_i = 0 \tag{85}$$

Equation (84) and (85) implies

$$\frac{K_i}{L_i} = \frac{w_i}{R} \frac{\alpha}{1-\alpha} \frac{1}{1+\tau_{Ki}} \tag{86}$$

Plugging equation (86) into (81), we get

$$L_i = A_i^{-1}\left[\frac{\alpha}{1-\alpha}\cdot\frac{w_i}{(1+\tau_{Ki})R}\right]^{-\alpha} Y_i \tag{87}$$

$$K_i = A_i^{-1}\left[\frac{\alpha}{1-\alpha}\frac{w_i}{(1+\tau_{Ki})R}\right]^{1-\alpha} Y_i \tag{88}$$

The unit cost function is

$$\begin{aligned} c &= \frac{w_i L_{si} + R\left(1+\tau_{ksi}\right)K_{si}}{Y_{si}} \\ &= \left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{Ki})^{\alpha}}{A_i} \end{aligned} \tag{89}$$

Equation (83) gives

$$\lambda = \frac{\left(1-\tau_{Yi}\right)P_i}{\sigma} \tag{90}$$

Plugging (86) and (90) into (85), we get

$$\begin{aligned} P_i &= \frac{\sigma}{\sigma-1}\frac{1}{1-\tau_{Yi}}\cdot c \\ &= \frac{\sigma}{\sigma-1}\frac{1}{1-\tau_{Yi}}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w_i}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{Ki})^{\alpha}}{A_i} \end{aligned} \tag{91}$$

The optimal price is a markup over the unit cost, multiplied by the distortion.

7.3 L_i, K_i, and Y_i

Insert (91) for P_i into (79), we get the function of output as

$$\begin{aligned} Y_i &= \frac{P_i^{-\sigma}}{\mathbf{P}^{-\sigma}}Y \\ &= \left(\frac{\sigma}{\sigma-1}\frac{1}{1-\tau_{Yi}}MC\right)^{-\sigma}P^{\sigma}Y \\ &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)\sigma}\frac{A_i^{\sigma}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha\sigma}}P^{\sigma}Y \end{aligned} \tag{92}$$

From (92), the output level has a common parameter among different firms, then

$$Y_i \propto \frac{A_i^{\sigma}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\sigma\alpha}} \tag{93}$$

Then insert (92) into (75), we have

$$\begin{aligned}
Y &= \left[\sum_{i=1}^{M_s} Y_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[\sum_{i=1}^{M}\left(\frac{A_i^{\sigma}\left(1-\tau_{Yi}\right)^{\sigma}}{\left(1+\tau_{Ki}\right)^{\alpha\sigma}}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{W}\right)^{\sigma(1-\alpha)} Y\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= Y\left[\frac{\sigma-1}{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha}(1-\alpha)^{1-\alpha}\right]^{\sigma}\left[\sum_{i=1}^{M}\left(A_i\frac{1-\tau_{Yi}}{w_i^{1-\alpha}\left(1+\tau_{Ki}\right)^{\alpha}}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

which gives

$$\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}=\left[\sum_{i=1}^{M}\left(A_i\frac{1-\tau_{Yi}}{w_i^{1-\alpha}\left(1+\tau_{Ki}\right)^{\alpha}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{94}$$

Insert (92) into (87), **labor demand** is

$$\begin{aligned}
L_i &= A_i^{-1}\left[\frac{\alpha}{1-\alpha}\frac{w_i}{(1+\tau_{Ki})R}\right]^{-\alpha} Y_i \\
&= A_i^{-1}\left[\frac{\alpha}{1-\alpha}\frac{w_i}{(1+\tau_{Ki})R}\right]^{-\alpha}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)\sigma} \\
&\quad \frac{A_i^{\sigma}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha\sigma}}P^{\sigma}Y \\
&= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha(\sigma-1)}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)\sigma+\alpha}\frac{A_i^{\sigma-1}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha(\sigma-1)}}P^{\sigma}Y
\end{aligned} \tag{95}$$

then

$$L_{si} \propto \frac{A_{si}^{\sigma-1}(1-\tau_{Ysi})^{\sigma}}{(1+\tau_{Ksi})^{\alpha_s(\sigma-1)}} \tag{96}$$

The same process for K_i, it can be achieved by inserting (92) into (88)

$$\begin{aligned}
K_i &= A_i^{-1}\left[\frac{\alpha}{1-\alpha}\frac{w_i}{(1+\tau_{Ki})R}\right]^{1-\alpha} Y_i \\
&= A_i^{-1}\left[\frac{\alpha}{1-\alpha}\frac{w_i}{(1+\tau_{Ki})R}\right]^{1-\alpha}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\alpha}{R}\right)^{\alpha\sigma}\left(\frac{1-\alpha}{w_i}\right)^{(1-\alpha)\sigma} \\
&\quad \frac{A_i^{\sigma}(1-\tau_{Ysi})^{\sigma}}{(1+\tau_{Ki})^{\alpha\sigma}}P^{\sigma}Y \\
&= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\cdot\left(\frac{\alpha}{R}\right)^{\alpha(\sigma-1)+1}\left(\frac{1-\alpha}{w_{si}}\right)^{(1-\alpha)(\sigma-1)}\frac{A_i^{\sigma-1}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha(\sigma-1)+1}}P^{\sigma}Y
\end{aligned} \tag{97}$$

then

$$K_i \propto \frac{A_i^{\sigma-1}(1-\tau_{Yi})^{\sigma}}{(1+\tau_{Ki})^{\alpha(\sigma-1)+1}} \tag{98}$$

7.4 MRPL and MRPK

The distortions τ_{Yi} and τ_{Ki} result differences in $MRPL$ and $MRPK$ across firms. Thus the allocation of resources across firms are different.

We rearrange (79)

$$P_i = \mathbf{P}Y^{\frac{1}{\sigma}}Y_i^{-\frac{1}{\sigma}}$$

Then write revenue as

$$R = P_i Y_i = Y_i^{\frac{\sigma-1}{\sigma}}\mathbf{P}Y^{\frac{1}{\sigma}} \tag{99}$$

Differentiate **(99) to labor and solve for** $MRPL_i$

$$\begin{aligned} MRPL_i &= \frac{\partial P_i Y_i}{\partial L_i} = \frac{\sigma-1}{\sigma}Y_i^{-\frac{1}{\sigma}}\mathbf{P}Y^{\frac{1}{\sigma}}(1-\alpha)A_i K_i^{\alpha_s}L_i^{-\alpha} \\ &= (1-\alpha)\frac{\sigma-1}{\sigma}P_i A_i K_i^{\alpha}L_i^{-\alpha} \\ &\triangleq (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_i Y_i}{L_i} \end{aligned} \tag{100}$$

Insert (87) and (91) into (100), and get $MRPL_i$ as

$$MRPL_i = w_i \frac{1}{1-\tau_{Yi}} \tag{101}$$

Differentiate **(99)** to capital and solve for $MRPK_i$

$$\begin{aligned} MRPK_i &= \frac{\partial P_i Y_i}{\partial K_i} = \frac{\sigma-1}{\sigma}Y_i^{-\frac{1}{\sigma}}\mathbf{P}Y^{\frac{1}{\sigma}}\alpha A_i K_i^{\alpha-1}L_i^{1-\alpha} \\ &= \alpha\frac{\sigma-1}{\sigma}P_i A_i K_i^{\alpha-1}L_i^{1-\alpha} \\ &\triangleq \alpha\frac{\sigma-1}{\sigma}\frac{P_i Y_i}{K_i} \end{aligned} \tag{102}$$

Insert (88) and (91) into (102), and get $MRPK_i$ as

$$MRPK_i = R\frac{1+\tau_{Ki}}{1-\tau_{Yi}} \tag{103}$$

7.5 TFPR and TFPQ

Firm's revenue productivity is a measure of firm-specific distortions ($TFPR_i$). Hiseh and Klenow (2009) define the total factor productivity revenue $TFPR_i$ as

$$
\begin{aligned}
TFPR_i &\triangleq P_i A_i = \frac{P_i Y_i}{K_i^{\alpha} L_i^{1-\alpha}} \qquad (104)\\
TFPQ_i &\triangleq A_i = \frac{Y_i}{K_i^{\alpha} L_i^{1-\alpha}}
\end{aligned}
$$

Rearrange (100) and (102)

$$
\begin{aligned}
L_i &= (1-\alpha)\frac{\sigma-1}{\sigma} P_i Y_i MRPL_i^{-1} \qquad (105)\\
K_i &= \alpha\frac{\sigma-1}{\sigma} P_i Y_i MRPK_i^{-1} \qquad (106)
\end{aligned}
$$

Insert (105) and (106) into (104) solve for $TFPR_i$ which is a geometric average of marginal revenue product of capital and labor

$$
\begin{aligned}
TFPR_i &= \frac{P_i Y_i}{\left(\alpha\frac{\sigma-1}{\sigma} P_i Y_i MRPK_i^{-1}\right)^{\alpha}\left[(1-\alpha)\frac{\sigma-1}{\sigma} P_i Y_i MRPL_i^{-1}\right]^{1-\alpha}}\\
&= \frac{\sigma}{\sigma-1}\left(\frac{MRPK_i}{\alpha}\right)^{\alpha}\left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha} \qquad (107)
\end{aligned}
$$

Insert (101) and (103) into (107) to get $TFPR_{si}$,

$$
\begin{aligned}
TFPR_i &= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}}{1-\tau_{Yi}} \qquad (108)\\
&= \left[\sum_{i=1}^{M}\left(A_i\frac{(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}\frac{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}}{1-\tau_{Yi}} \qquad (109)
\end{aligned}
$$

Where the last equality is derived using 94.

7.6 Aggregate Labor and Capital

Rearranging (100) and (101), we get

$$
L_i = (1-\alpha)\frac{\sigma-1}{\sigma}\frac{1-\tau_{Yi}}{w_i} P_i Y_i \qquad (110)
$$

Sum over all i in (110)

$$\begin{aligned} L &= \sum_{i=1}^{M_s} L_i = \sum_{i=1}^{M_s} (1-\alpha) \frac{\sigma-1}{\sigma} \frac{1-\tau_{Yi}}{w_i} P_i Y_i \\ &= \frac{\sigma-1}{\sigma} (1-\alpha) \sum_{i=1}^{M} \frac{(1-\tau_{Yi})}{w_i} P_i Y_i \end{aligned} \tag{111}$$

Multiply and divide by $P_s Y_s$ on right side of (111)

$$L = PY \frac{\sigma-1}{\sigma} (1-\alpha) \sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY} \tag{112}$$

Similar for capital, we rearrange (102) and (103), and get

$$K_i = \alpha \frac{\sigma-1}{\sigma} \frac{1-\tau_{Yi}}{R(1+\tau_{Ki})} P_i Y_i \tag{113}$$

Then sum over all i in (113)

$$\begin{aligned} K &= \sum_{i=1}^{M} K_i = \sum_{i=1}^{M} \alpha \frac{\sigma-1}{\sigma} \frac{1-\tau_{Yi}}{R(1+\tau_{Ki})} P_i Y_i \\ &= \frac{\alpha}{R} \frac{\sigma-1}{\sigma} \sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} P_i Y_i \\ &= PY \frac{\sigma-1}{\sigma} \frac{\alpha}{R} \sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \frac{P_i Y_i}{PY} \end{aligned} \tag{114}$$

7.7 Definition of $\overline{MRPL}_s$, $\overline{MRPK}_s$ and $\overline{TFPR}_s$

We define

$$\begin{aligned} \overline{MRPL} &= \frac{1}{\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY}} \\ \overline{MRPK} &= \frac{R}{\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \frac{P_i Y_i}{PY}} \end{aligned}$$

and $\overline{TFPR}$ is the geometric average of the $\overline{MRPK}$ and $\overline{MRPL}$.

$$\begin{aligned}\overline{TFPR} &= \frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK}}{\alpha}\right)^{\alpha}\left(\frac{\overline{MRPL}}{1-\alpha}\right)^{1-\alpha} \\ &= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\frac{P_iY_i}{PY}}\right)^{\alpha}\left(\frac{1}{(1-\alpha)\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\frac{P_iY_i}{PY}}\right)^{1-\alpha}\end{aligned} \tag{115}$$

7.8 Aggregate TFP

Here we are going to compute TFP_s by using two methods. The first method is to derive TFP as a function of TFPR in Hiseh and Klenow (2009). The second is to use the growth accounting with aggregate inputs. The aim of doing this is to see whether these two methods can derive the same result of TFP.

Method 1: TFP$_s$ as a function of TFPR

In here, we follow the definition of TFP_s of Hsieh and Klenow at equation (15)

$$TFP = \left[\frac{1}{M}\sum_{i=1}^{M}\left(A_i\frac{\overline{TFPR}}{TFPR_i}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{116}$$

Plugging equation (108) and (115) into (116), we get

$$\begin{aligned}TFP &= \left[\frac{1}{M}\sum_{i=1}^{M}\left(A_i\frac{\frac{1-\tau_{Yi}}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}}{\left[\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\frac{P_iY_i}{PY}\right]^{1-\alpha}\left[\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\frac{P_iY_i}{PY}\right]^{\alpha}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \\ &= \frac{\left[\frac{1}{M}\sum_{i=1}^{M}\left(A_i\frac{1-\tau_{Yi}}{(1+\tau_{Ki})^{\alpha}w_i^{1-\alpha}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\left(\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{w_i}\frac{P_iY_i}{PY}\right)^{1-\alpha}\left(\sum_{i=1}^{M}\frac{1-\tau_{Yi}}{1+\tau_{Ki}}\frac{P_iY_i}{PY}\right)^{\alpha}}\end{aligned} \tag{117}$$

Method 2: TFP$_s$ as a function of aggregate inputs

We use growth accounting.

$$
\begin{aligned}
TFP &= \frac{Y}{(M)^{\frac{1}{\sigma-1}} K^{\alpha} L^{1-\alpha}} \\
&= \frac{\left[\sum_{i=1}^{M} \left(A_i K_i^{\alpha} L_i^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{(M)^{\frac{1}{\sigma-1}} K^{\alpha} L^{1-\alpha}} \\
&= \frac{1}{(M)^{\frac{1}{\sigma-1}}} \left[\sum_{i=1}^{M} \left(A_i \left(\frac{K_i}{K}\right)^{\alpha} \left(\frac{L_i}{L}\right)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \qquad (118)
\end{aligned}
$$

We divide (110) by (112), and (113) by (114), we get input shares are

$$
\frac{K_i}{K} = \frac{\frac{1-\tau_{Yi}}{1+\tau_{Ki}} P_i Y_i}{\sum_{j=1}^{M} \frac{1-\tau_{Yj}}{1+\tau_{Kj}} P_j Y_j} \qquad (119)
$$

$$
\frac{L_i}{L} = \frac{\frac{1-\tau_{Yi}}{w_i} P_i Y_i}{\sum_{j=1}^{M} \frac{1-\tau_{Yj}}{w_j} P_j Y_j} \qquad (120)
$$

Plugging (119) and (120) into (118), we get

$$
TFP = \left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \frac{\left[\sum_{i=1}^{M} \left(\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}} \frac{P_i Y_i}{PY}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \frac{P_i Y_i}{PY}\right)^{\alpha} \left(\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY}\right)^{1-\alpha}} \qquad (121)
$$

To solve for $\frac{P_i Y_i}{PY}$, we rearrange (79)

$$
\begin{aligned}
\frac{P_i Y_i}{PY} &= \left(\frac{Y_i}{Y}\right)^{\frac{\sigma-1}{\sigma}} \\
&= \left[\frac{A_i(1-\tau_{Yi})}{(1+\tau_{ki})^{\alpha}} \left(\frac{1-\alpha}{w_i}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha} \frac{\sigma-1}{\sigma}\right]^{\sigma-1} \qquad (122)
\end{aligned}
$$

which delivers.

$$\begin{aligned} A_i \frac{(1-\tau_{Yi})}{(1+\tau_{ki})^{\alpha}} \frac{P_i Y_i}{PY} &= \left(A_i \frac{(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} w_i^{\sigma-1} \left[(1-\alpha)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha} \frac{\sigma-1}{\sigma} \right]^{\sigma-1} \\ &= \left[\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} \right]^{\sigma-1} \left[\sum_{i=1}^{M} \left(\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} \right]^{-1} \end{aligned} \tag{123}$$

We insert (94) in the last equality. Then Plugging (123) into the numerator of 121, we get

$$\begin{aligned} & \left[\sum_{i=1}^{M} \left(\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} P_i Y_i \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ = & \left[\sum_{i=1}^{M} \left(\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} \right]^{\frac{\sigma}{\sigma-1}} \left[\sum_{i=1}^{M} \left(\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} \right]^{-1} \\ = & \left[\sum_{i=1}^{M} \left(\frac{A_i (1-\tau_{Yi})}{w_i^{1-\alpha} (1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \end{aligned} \tag{124}$$

Plug (124) back to (121), we get

$$TFP = \frac{\left[\frac{1}{M} \sum_{i=1}^{M} \left(\frac{A_i(1-\tau_{Yi})}{w_i^{1-\alpha}(1+\tau_{Ki})^{\alpha}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}}{\left(\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \frac{P_i Y_i}{PY} \right)^{1-\alpha} \left(\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \frac{P_i Y_i}{PY} \right)^{\alpha}} \tag{125}$$

Equation(125) is the same as (117). Thus

$$TFP \triangleq \left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \frac{Y}{K^{\alpha} L^{1-\alpha}} = \left[\frac{1}{M} \sum_{i=1}^{M} \left(A_i \cdot \frac{\overline{TFPR}}{TFPR_i} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}$$

Then we insert (122) and (94) into (125), to solve for aggregate TFP, which equals to

$$TFP = \frac{\left(\frac{1}{M}\right)^{\frac{1}{\sigma-1}} \left[\sum_{i=1}^{M} \left(\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}} \right)^{\sigma-1} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{w_i} \left[\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}} \right]^{\sigma-1} \right]^{1-\alpha} \left[\sum_{i=1}^{M} \frac{1-\tau_{Yi}}{1+\tau_{Ki}} \left[\frac{A_i(1-\tau_{Yi})}{(1+\tau_{Ki})^{\alpha} w_i^{1-\alpha}} \right]^{\sigma-1} \right]^{\alpha}}$$

8 Appendix III. Variance decomposition

For firm i, in quintile q, we can write

$$\begin{aligned}\sum_q^Q\sum_i^{N_q}\left(t_{qi}-\bar{t}\right)^2 &= \sum_q^Q\sum_i^{N_q}\left[\left(t_{qi}-t_q\right)+\left(t_q-\bar{t}\right)\right]^2 \\ &= \sum_q^Q\sum_i^{N_q}\left(t_{qi}-t_q\right)^2+\sum_q^Q\sum_i^{N_q}\left(t_q-\bar{t}\right)^2 \\ &\quad +2\sum_q^Q\sum_i^{N_q}\left(t_{qi}-t_q\right)\left(t_q-\bar{t}\right) \end{aligned} \tag{126}$$

where N_q is the number of firms in quintile group q, Q is the number of quintile group.

Here we define t_q as the average t within quintile group $_q$, and $\bar{t}$ is the total average among all t (across quintiles).

$$t_q \triangleq \frac{1}{N_q}\sum_i^{N_q} t_{qi} \tag{127}$$

$$\bar{t} \triangleq \frac{1}{N}\sum_i^{N} t_{qi} \tag{128}$$

where

$$N=\sum_q^Q N_q$$

For the last term of (126) with out parameter, we find that

$$\begin{aligned}\sum_q^Q\sum_i^{N_q}\left(t_{qi}-t_q\right)\left(t_q-\bar{t}\right) &= \sum_q^Q\sum_i^{N_q}\left[t_{qi}\left(t_q-\bar{t}\right)-t_q\left(t_q-\bar{t}\right)\right] \\ &= \sum_q^Q\sum_i^{N_q}t_{qi}\left(t_q-\bar{t}\right)-\sum_q^Q\sum_i^{N_q}t_q\left(t_q-\bar{t}\right) \\ &= \sum_q^Q\left[\left(t_q-\bar{t}\right)\left(\sum_i^{N_q}t_{qi}\right)\right]-\sum_q^Q N_q t_q\left(t_q-\bar{t}\right) \\ &= \sum_q^Q\left(t_q-\bar{t}\right)\left(\sum_i^{N_q}t_{qi}-N_q t_q\right) \end{aligned} \tag{129}$$

Insert (128) into (129), and get

$$\sum_{q}^{Q}\sum_{i}^{N_q}\left(t_{qi}-t_q\right)\left(t_q-\bar{t}\right)=0$$

Then we rewrite $\sum_{q}^{Q}\sum_{i}^{N_q}\left(t_{qi}-\bar{t}\right)^2$ as

$$\begin{aligned}
\sum_{q}^{Q}\sum_{i}^{N_q}\left(t_{qi}-\bar{t}\right)^2 &= \sum_{q}^{Q}\sum_{i}^{N_q}\left(t_{qi}-t_q\right)^2+\sum_{q}^{Q}N_q\left(t_q-\bar{t}\right)^2 \\
&= \sum_{q}^{Q}N_q\sum_{i}^{N_q}\frac{\left(t_{qi}-t_q\right)^2}{N_q}+\sum_{q}^{Q}N_q\left(t_q-\bar{t}\right)^2 \\
&= \sum_{q}^{Q}N_qVar_q(t_{qi})+\sum_{q}^{Q}N_q\left(t_q-\bar{t}\right)^2
\end{aligned}$$

dividing both side by N, the total number of firms ($N=\sum_{q}^{Q}N_q$)

$$\begin{aligned}
\underbrace{\frac{1}{N}\sum_{q}^{Q}\sum_{i}^{N_q}\left(t_{qi}-\bar{t}\right)^2}_{over\text{-}all\ variation} &= \underbrace{\frac{1}{N}\sum_{q}^{Q}N_qVar_q}_{within-group\ component} + \underbrace{\frac{1}{N}\sum_{q}^{Q}N_q\left(t_q-\bar{t}\right)^2}_{between-group\ component} \\
Var(t_i) &= \sum_{q}^{Q}\frac{N_q}{N}Var_q(t_{qi})+\sum_{q}^{Q}\frac{N_q}{N}\left(t_q-\bar{t}\right)^2 \qquad (130)
\end{aligned}$$

Thus take the TFPR as an example. TFPR is grouped into productivity quintiles, then (130) becomes

$$\begin{aligned}
var(lnTFPR_i) &= \underbrace{\frac{1}{5}\sum_{q}^{Q}\sum_{i}^{N_q}\left(\ln TFPR_{qi}-\overline{\ln TFPR}\right)^2}_{over\ all\ variation} \\
&= \underbrace{\sum_{q}^{Q}\frac{1}{5}var(\ln TFPR)_q}_{within-group\ component}+\underbrace{\sum_{q}^{Q}\frac{1}{5}\left(\overline{\ln TFPR_q}-\overline{\ln TFPR}\right)^2}_{between-group\ component}
\end{aligned}$$

where $\overline{\ln TFPR}$ is the overall mean of TFPR$_i$. $var(\ln TFPR)_q$ is the variance of lnTFPR in each quintile q. $\overline{\ln TFPR_q}$ is the mean of lnTFPR in quintile q.