Pension liabilities: Should wage gowth and exit rates be age dependent?

by

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Preface

Characteristic age patterns in future wage growth and job shifts are not considered in today's actuarial calculations of defined benefit pension plans. Via the archives of the pensionconsulting firm GablerWassum, models for this has been established showing a considerable effect on future pension liabilities. Age patterns also varies from one branch of the economy to the other, leading to corresponding differences in the liabilites. Today, the data amount these historical model calibrations depend on is decreasing and the uncertainty in parameter estimates is increasing. That is, both for the wage growth models and job shift models. The question is how big the data amount needs to be in order to keep the uncertainty in estimated liabilities on a satisfactory level.

Acknowledgements

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I have been blessed with a wonderful family and I am grateful for each one of you and your support. Especially, I would like to show my appreciation towards my grandparents Fred Midtsem and Berit Midtsem who encouraged me to choose science as a career path. To my father Fred Oscar Midtsem, his wife Elin Brenne Midtsem, my mother Katarzyna E. Andresen and her spouse Jarle Djuvsland thank you for your encouragement and love. I could not have done this without your support.

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Chapter 1

Introduction

1.1 The problem

Pension liabilities are dependent on a number of different elements. This thesis seeks to explore how age-dependent wage growth and exit rates affect a firm's future liabilities and their uncertainty. It is based on the unpublished note *The GablerWassum wage models*, Bølviken (2009). Wage modelling is discussed in chapter 2. The main objective will be to see how different simulation criteria influence the results. Job shift rates with similar assumptions are discussed in chapter 4. Accordingly the key criterion is how the age profile looks like in different populations. When the main model was derived, age profiles from the GablerWassum[1] archives were used, how males and females distribute is shown in figure 1.1 on the following page. A thing to take notice of is the age most males and females are, there is in fact an age difference up to 15 years with a peak at 40 for females and 55 for males. The number of observations is also less for females. This will be a source of uncertainty explored in chapter 3, where simulations with two age profiles will be done and the number of observations will be given different values. The code to derive age profiles is in appendix A and they are displayed in figure 1.2 on page 7 with a population size of ten thousand.

The retirement age in Norway today is 67 years. Accumulation of pension starts from the age of 16 and all Norwegian citizens are entitled to receive pension upon retirement, NIS (2010). The size of a pension, the part from the National Insurance Scheme(NIS) and the company pension, is dependent on a number of different elements such as wage today, wage growth, G-adjustment[2] done by the government and choice of pension plan. The pension plan considered here is defined benefit which is based on a predetermined rate, usually between 60% to 70% of the wage at the time of

[1] `http://www.gablerpartners.no/`

[2] G stands for *grunnbeløpet* and the NIS uses it as a basic amount to calculate pension. It is determined in May each year and is p.t. 75 641 NOK

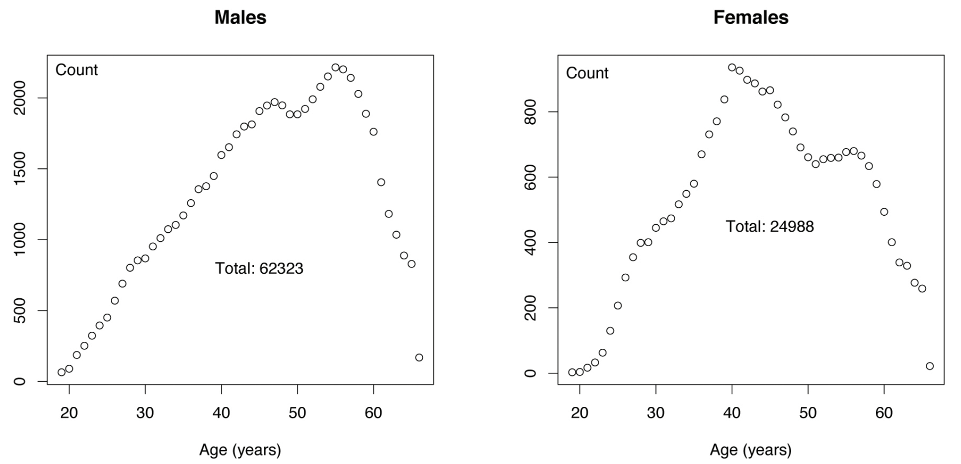


Figure 1.1: *The age distribution of the GablerWassum historical data.*

retirement. It is also called a gross pension plan. When a person retires the difference between the pension recieved from the NIS and wage(times the predetermined rate) is paid out by the plan. More background information on post-employee benefit plans is written in section 2.1.

The rules deciding the size of the retirement pension from the NIS is for instance how long the person has been in the labour market and how long the membership in the NIS has been. To recieve a full basic retirement pension, something which is independent of wage, a person needs to be a member for 40 years to prevent that the basic pension is scaled down. Supplementary pension on the other hand is dependent on wage and somewhat simplified based on the 20 years with highest income. In connection with this *Ny fleksibel alderspensjon*[3] is worth mentioning although it will not be used in the calculations here. Increased life expectancy and the fact that the working population is decreasing to the number of retired persons led to the establishment of the Pensioncommittee in 2001. When the NIS was established in 1967 there were 4 working persons for each retired. Today this number is 2,6 and it is predicted to be 1,8 in year 2050, see Ministry of Labour (2007). They have made a change to *Lov om folketrygd* so that people benefit from choosing to work longer than age 67. For more information see Ministry of Labour (2009) and note that this will also force a redefining of the pension plans mentioned in chapter 2.

Projections of wage growth is treated in chapter 3 and deals with deterministic forecasting and parameter uncertainty. The question is how big the amount of data needs to be to keep the uncertainty in the approximated liabilities at an acceptable level. A pension fund with a young age profile in a branch with a wage growth above average might cause a much higher

[3] New flexible retirementpension

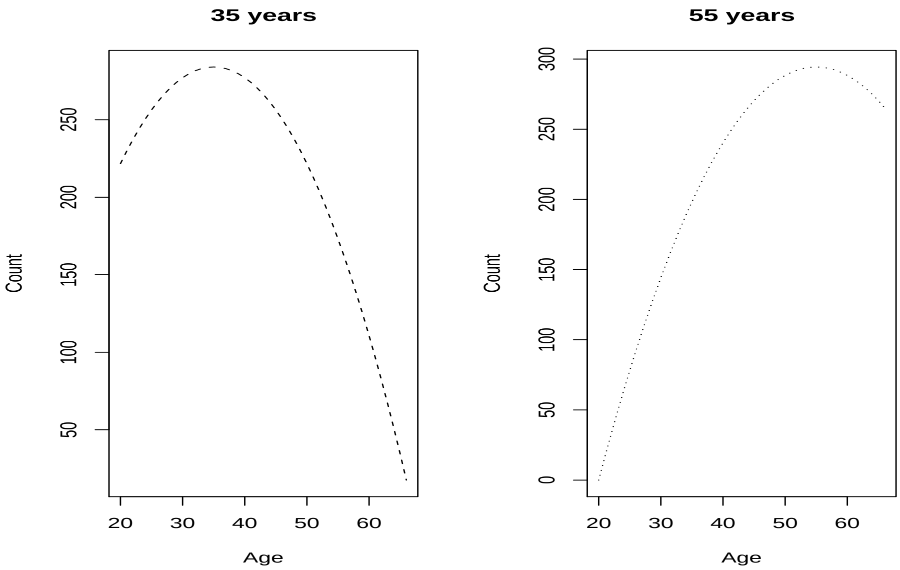


Figure 1.2: *Distribution of the population in a young and old age profile with a population size of ten thousand.*

obligation for the employer than a fund with a different age distribution or growth curve. The challenge will be to project the wage for different ages ahead in time and calculate the Projected Benefit Obligation (PBO). A PBO is an estimate of pension liability based on the assumption that the employee will continue to work until retirement. It takes into account future increases in pension contribution caused by an employee's future wage increase. The equation used is simplified, but adequate for this purpose.

$$\mathrm{PBO} = \frac{2}{3} \times W_{67} \times \int_{n}^{\infty} v^t {}_t p_x dt \times v^{67-t}, \tag{1.1}$$

where W_{67} is wage at age 67, a product of current wage and growth factor. The integral represents a deferred life annuity and it is used to scale the PBO so that the owner of the pension plan is secured the right benefit at retirement and until death. It is dependent on the age x, gender, distribution of future lifetime and discount factor v. The PBO is the amount of money the pension fund needs to have at hand at the time the person reaches pension age. If it is not big enough the fund will not be able to pay the former employee what he or she is promised each year until death. Since the PBO is based on wage at retirement, this is not the annual pension cost for the firm. The actual cost is called the service cost and this is the value used in the current accounting year.

There are many sources of uncertainty connected to this calculation. For example how the coefficients are reestimated from the original GablerWas-

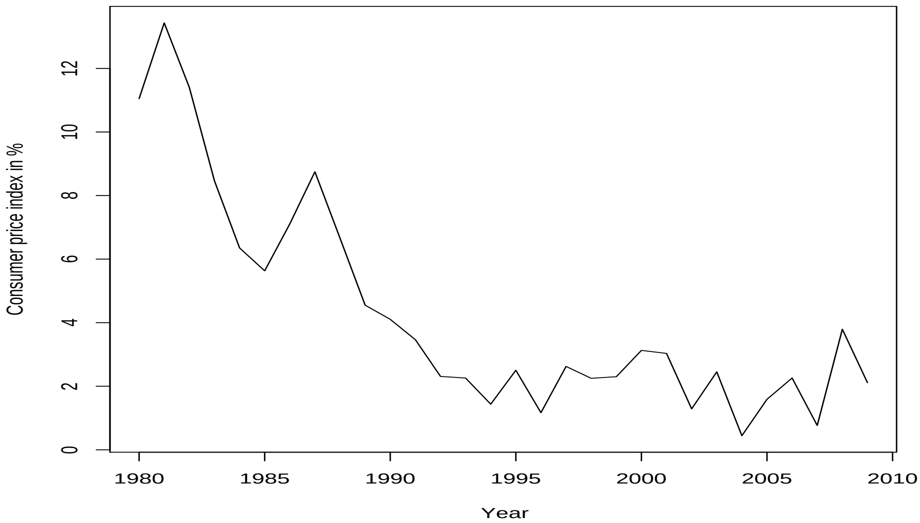


Figure 1.3: *The consumer price index in Norway, 1980-2009.*

sum wage model and the estimation of future inflation, something which practically is not possible to model in a good way. The Norwegian Central Bureau of Statistics offer records of the consumer price index from 1980-2009[4] and it is evident that it fluctuates a lot, see figure 1.3. Mortality on the other hand is a area with more solid knowledge and the Gompertz-Makeham model is sufficient for this purpose even though it has its disadvantages discussed in section 1.2. Regarding reestimation of the coefficients from the original model, a lot of different techniques may be used. The method used here is polynomial regression combined with Monte Carlo simulation and bootstrapping. A brief description is given in section 1.3 and 1.4.

In chapter 4 the model for exit rates is analysed. The data shows evidence of higher job mobility for young employees than old employees. When an employee is getting closer to retirement, the exit rate is converging to 0 %. Analysis of the relationship between exit rates and wage growth shows how the curves can possibly explain each other. This also affects the pension liabilities a firm has. Differences compared to the model used today are discussed continously through all the chapters. In chapter 5, combinations of the rules today and assumption made under the GablerWassum are analysed. The results are summarized in chapter 6. The remainings of this chapter is an introduction to basic life insurance theory and estimation technique.

[4]`http://statbank.ssb.no/statistikkbanken/`

1.2 Life insurance and the Gompertz-Makeham model

Consider an individual today, from the insurer's view an employee can stay in the same job, die, become disabled or leave current job. Job shift is the topic in chapter 4 and disability will not be considered. To deduce the probability of death, notation from Gerber (1997) is used.

The probability distribution function $G(t)$ of the future lifetime T is assumed to be continuous and is expressed as

$$G(t) = Pr(T \leq t), \quad t \geq 0. \tag{1.2}$$

This means the probability density is $g(t) = G'(t)$ and the probability that death will occur in the infinitesimal time interval from t to $t + dt$ is

$$g(t)dt = Pr(t < T < t + dt). \tag{1.3}$$

With this notation the probability that a life aged x will die within t years is

$${}_tq_x = G(t), \text{ or survive t years } \quad {}_tp_x = 1 - G(t). \tag{1.4}$$

From equation 1.3 and 1.4 force of mortality can now be explained. Force of mortality is a different way to express the probability of dying in the infinitesimal interval mentioned above and is defined by

$$\mu_{x+t} = \frac{g(t)}{1 - G(t)} = -\frac{d}{dt}\ln[1 - G(t)] = -\frac{d}{dt}\ln {}_tp_x. \tag{1.5}$$

Solving 1.5 with respect to the survival probability leads to

$${}_tp_x = e^{-\int_0^t \mu_{x+s}ds}. \tag{1.6}$$

The analytical distribution of T is chosen to be Gompertz-Makeham, but because of late-life mortality deceleration it is important to remember that the model is not ideal, it does not reflect human mortality in a realistic way for ages above 80. 1-year probability death rates and the model are found in Bølviken and Moe (2008) and displayed in table 1.1.

Table 1.1: *Gompertz-Makeham model for one year mortality rates q_x.*

Gompertz Makeham: $q_x = 1 - e^{-\theta_0 - \theta_1 e^{\theta_2 x}}$			
Female	$\theta_0 = 0.000204$	$\theta_1 = 0.0000068$	$\theta_2 = 0.110118$
Male	$\theta_0 = 0.000309$	$\theta_1 = 0.0000219$	$\theta_2 = 0.100047$

1.3 Regression

Standard theory on this topic is found in Rice (1995). It is basically about fitting a straight line to a data set and explaining the relationship between the variation in the response variable y with the explanatory variable x. A form of linear regression called polynomial regression is a special case of this theory and multiple regression. With this approach the main idea is to explain the relationship in the data modeled as an nth order polynomial something which allows the fitted line to curve. This relationship is given in the general polynomial regression model as

$$y = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots + b_nx^n + \epsilon, \tag{1.7}$$

where the dependent variable y is expressed as a linear combination of the independent variable x and the coefficients b_k, $k = 0, ..., n$ and $\epsilon \sim N(0, 1)$. The task is to estimate these coefficients and analyse the results based on different assumptions. The coefficient in a polynomial regression does not have an easy interpretation like β_0(intercept) and β_1(slope) have in an ordinary linear regression. Since the underlying items are highly correlated, it is generally more informative to consider the fitted regression function as a whole.

Logistic regression is a different type of regression. The response variable y is now a categorical variable, that is a variable with finite numbers of possible values for the response. An example of this can for instance be a survey where the goal is to figure out the reason some people have internet at home or not. y is now either *yes* or *no*, with multiple explanatory variables $\mathbf{x}$ i.e. age, gender, location, income and so on. In statistical terms this means y has a binomial distribution and the probability can be expressed as $\pi(\mathbf{x}) = P(y = yes|\mathbf{x})$. In terms of regression $\pi(\mathbf{x})$ can still be expressed as a linear predictor, the combination of coefficients b_k, $k = 0, ..., n$, but there is a problem with this since $\pi(\mathbf{x})$ is not necessarily a number between 0 and 1. To deal with this a link function may be used. It provides a relationship between the linear predictor and the mean of the binomial distribution. With logistic regression this link function is called a *logit* link and is expressed as

$$logit\, \pi(\mathbf{x}) = log\, \frac{\pi(\mathbf{x})}{1 - \pi(\mathbf{x})} = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots + b_nx^n \tag{1.8}$$

where

$$\pi(\mathbf{x}) = \frac{1}{1 + e^{-(b_0+b_1x+b_2x^2+b_3x^3+\cdots+b_nx^n)}}. \tag{1.9}$$

Note that these equations are derived with polynomial regression in mind. The theory is from de Jong and Z. Heller (2008).

Connected to regression the correlation between coefficients is interesting. As mentioned, it is not possible to explain causality with this, but how the coefficients interact is visibile in the correlation matrix. A coefficient is always perfectly correlated with itself, hence ones in the diagonal of the matrix. If the correlation is closer to -1 or 1 the degree of relationship between two coefficients is very strong. If the correlation between two coefficients is 0, there is no correlation.

1.4 Monte Carlo simulations and Bootstrapping

Bootstrapping is a resampling method where the idea is to estimate an unknown parameter based on observations from an unknown distribution. Using notation from Storvik (2005), θ is the unknown parameter and the objective is to estimate θ with $\hat{\theta} = \hat{\theta}(\mathbf{x})$ where $\mathbf{x} = (x_1, \cdots, x_n)$ is the observed values. Next, two possible approaches can be used to decide the unknown distribution, either by a parametric or non-parametric bootstrap. When using a parametric approach more assumption is made on the unknown distribution, for example a normal distribution where maximum likelihoods are used as estimates for mean and standard deviation. A non-parametric bootstrap on the other hand makes the least assumptions on the distribution and an empirical distribution function can be used. Typically questions connected with a bootstrap estimate are whether $\hat{\theta}$ is unbiased or not, what the uncertainty is and if it is possible to make a confidence interval for the true value of θ.

Monte Carlo simulation is a very wide concept and it is not a specific method but rather a guideline for the approach to simulations. The core of Monte Carlo simulations and bootstrapping is the fact that when the number of observations approaches infinity the simulation error decreases and the estimates become closer to the true value.

Chapter 2

Wage models

2.1 Different models

The liabilities associated with pension are of interest to most firms, especially if the firm has a defined benefit pension plan for its employees. There are two different pension plans in the Norwegian market today and the defined benefit plan has been the most common pension plan so far, both in the private and public sector of the economy. Defined contribution plan on the other hand is a relativly new product. It came on the market in 2001 and since 2006, when *Lov om obligatorisk tjenestepensjon*[1], came into force the number of defined contribution plans have increased considerably. This means that in addition to the pension provided by the NIS, all firms are obliged to maintain an additional pension plan for its employees. For a firm, having a defined contribution plan is an advantage since the cost related to pension is a fixed amount of the employee's wage. The defined benefit plan on the other hand is influenced by several elements, but most importantly the wage at retirement. Finding a way to model the wage will make the future liabilities for a firm with a defined benefit plan more reliable.

To be able to model the wage, knowledge about the variables influencing the wage is necessary. Perhaps the first thing most people think of when it comes to wages are the differences between men and women. Different studies have shown what is evident in figure 2.1 on the next page, namely that women earn less than men. It is therefore reason to believe that there is a difference in the wage growth as well. Why there exists such a difference is a comprehensive study and it will not be discussed here, but for futher reading on the Norwegian labour market, see Ministry of Children, Equality and Social Inclusion (1997). Three models are elaborated in the article; female, male and gender-neutral. The work here will focus on the gender-neutral model, but the discussion is transferable to both genders. Two other explanatory variables which are partially correlated are the length of educa-

[1] LOV 2005-12-21 nr 124: Lov om obligatorisk tjenestepensjon

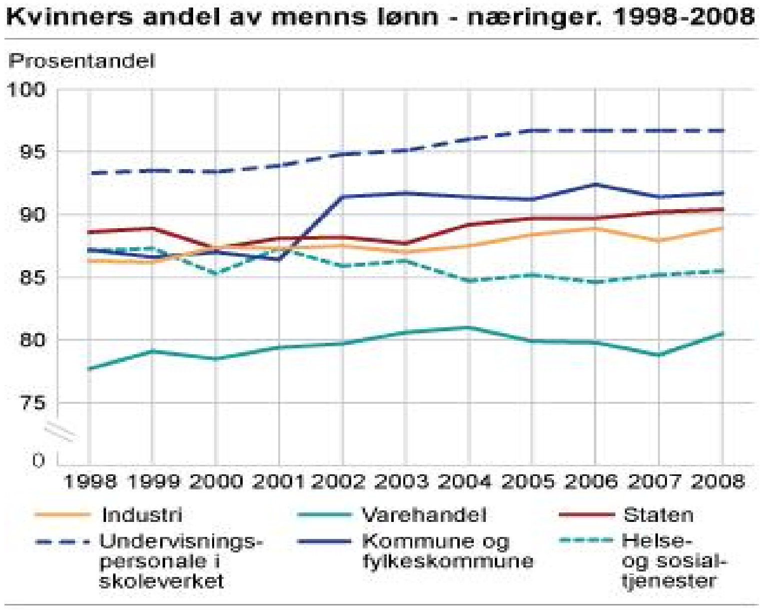


Figure 2.1: *Women's wages in precent of men's on different sectors, SSB (2010).*

tion and the amount of experience. One can debate the possibility that in some cases a longer education may have a negative effect on the wage compared to the wage for those who finish a grade earlier. They gain experience instead. Of course human capital is gained through education, but in some situations on-the-job training may be of bigger value to a firm, something that could motivate a higher salary. A challenge with all the variables mentioned, except for the gender, is to gather sufficient amounts of data. This brings forth the main variable for further analysis which may be seen in context to the other variables already mentioned and is more easily observed, namely the age. Observing the wage at different ages makes it possible to figure out wage growth and hence project wages ahead in time. Today when pension liabilities are calculated, a fixed wage growth is assumed throughout the entire career, it remains to figure out if that is a reasonable assumption. The Norwegian Accounting Standards Board[2] (NASB) determines the real growth rate of wages to be used for the current accounting year. Currently[3] this value is expected to be 1,75% which clearly differs from the GablerWassum model in figure 2.2 on page 17, NASB (2009a).

It is also of interest to narrow the analysis down to different sectors of the economy. Intuitively, differentiating between industrial workers and office workers makes sense. Some jobs are more prestigious or the demand

[2]http://www.regnskapsstiftelsen.no/
[3]November 2010

for workers can be increasing. In that case, an employee could bargain for a higher salary than normal since his or her expertise is sought-after. The economic situation in Norway and the world in general is of huge importance regarding this. Higher unemployment leads to lower wages and wage growth, something observed over the last year in the wake of the recent finance crisis.

Inflation is affecting the wage growth as well, this macroeconomic factor is influencing several areas of the economy. Thus it can not be left out of the analysis when the time period considered can be more than half a century. This is particularly important to think about since the results are based on historical data. If the inflation in that time period was unusual, this might cause incorrect conlusions and the results will not apply to other time periods. Because of this, the real wage growth will be explored in chapter 3 since exterior factors will not play a part here. In chapter 5 on the other hand deterministic inflation is incorporated.

2.2 Today

In the article NASB (2009a) a generel guidance on how to use pension assumptions and estimate liabilities is described. In Norway these main assumptions are the discount factor, yield on accumulated pension assets and average wage growth. The discount factor can be observed in the interest rate market at all times, average wage growth on the other hand needs to be estimated. Some of the assumptions are also required to be consistent to each other. A general theory in a normal economy assumes that the real interest rate is higher than the real wage growth. It is important that the estimates for future wage growth is unbiased. Independent sources like The Central Bank of Norway[4] and The Norwegian Central Bureau of Statistics[5] have done research on this and the forecasts are expected to be approximately on the same level as the expected real interest rate. With knowledge of the current way of looking at wage growth, it is possible to compare the results from the GablerWassum model.

2.3 Foundation

Finding an adequate model thus means to decide how complex it should be. Exploring some of the literature on the topic gives ideas to possible approaces. Battochio and Menoncin (2004) use a stochastic differential equation to explain the labour income at different ages. By doing this they are able to explore how risk sources like interest rates and stocks affect the salaries. Another type of model is presented in Borjas (1981) where the main focus is how job mobility and investment in human capital affect the wage

[4]Monetary Policy Report 2/09

[5]Economic Survey 4/2009

Table 2.1: *The GablerWassum wage growth coefficients.*

	b_0	b_1	b_2	b_3
Male	0.1863	-0.0077	0.0141	-0.0095
Female	-0.0128	0.0059	-0.0154	0.0109
Both	0.1394	-0.0045	0.0071	-0.0047

over time. This approach is partially used in Carriere and Shand (1998) as well, but the explanatory variables here are inflation and merit. As mentioned, gathering and finding enough data is difficult and for the dataset used here only age is available.

The nominal wage of an individual aged x one year from now will be as follows, where I is the rate of inflation and g_x is the expected real growth at age x

$$W_{x+1} = (1 + g_x)(1 + I)W_x. \tag{2.1}$$

With this formula the wage at any time in the future can be calculated recursively. For now, the variable of interest is g_x in the factorized form

$$g_x = \alpha\hat{g}_x \tag{2.2}$$

where α is introduced to make it possible to trust the result based on the historical data. As discussed in the article Bølviken (2009) section 2.1, α gives the opportunity to choose a *future* growth level. How it was decided is carefully described in Bølviken (2009) section 2.3. Thus the curve always have the same shape, but α can be used to shift the average growth level up or down. $\hat{g}_x$ is a regression estimate obtained from the archives of Gabler-Wassum. It is described by a polynomial function of order 3, a standard model in labour economics.

$$\hat{g}_x = e^{b_0+b_1x+(b_2/100)x^2+(b_3/100^2)x^3} - 1. \tag{2.3}$$

The coefficients are given in table 2.1, and the real growths are plotted together in figure 2.2 on the facing page. As expected, there exists a difference among men and women, mainly at a young age in the beginning of the career. As age increases, the growth seems to move towards the same level, although there is a difference up until the age around 40 for women. This is likely caused by pregnancy. The reason wage growth is higher for women in the thirties and forthies might be an effect of compensating for the lower growth in the twenties.

Next, analysis will be done on different branches of the economy with main categories Finance, Energy, Shipping, Industrial workers, Office workers and Academics/engineers. This narrows down the dataset considerably and how the different employees distribute among the branches is shown in figure 2.3 on page 18. Especially the category Shipping is suffering from

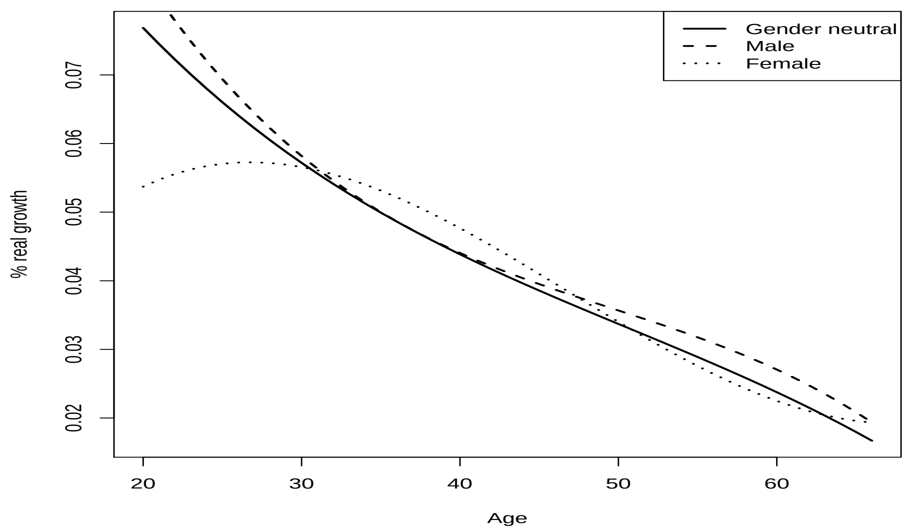


Figure 2.2: *The GablerWassum real wage growth curves with* $\alpha = 1$.

a sparse number of employees to base the calculations on. This source of error is affecting all the categories where women is under-represented or the dataset too small, something which is evident in most categories except for Finance and Office workers. The reason for different results on sectors can then be from both having a too small dataset and the fact that a job in for example Finance has a higher wage growth than in Energy. A summary of the coefficients based on the GablerWassum archives on branches is listed in table C.1 in appendix C. To illustrate the relationship based on sectors of the economy, figure 2.4 on page 19 shows how the real growth in percent is for the gender-neutral model. The pattern repeats itself, showing that the wage growth for all branches is highest at a young age and declining as the age increases. Other things to take notice of is the fact that Office workers have the highest wage growth throughout their entire career while people from the shipping industry has the lowest wage growth recorded. This again might originate from the size of data available in this particular branch.

2.4 Uncertainty

Dividing the uncertainty in two, one part is the fact that it is not possible to know for sure what the true wage model with corresponding coefficients in table 2.1 really is for a random group of employees. It is necessary to account for a certain amount of error in the modelling based on this model.

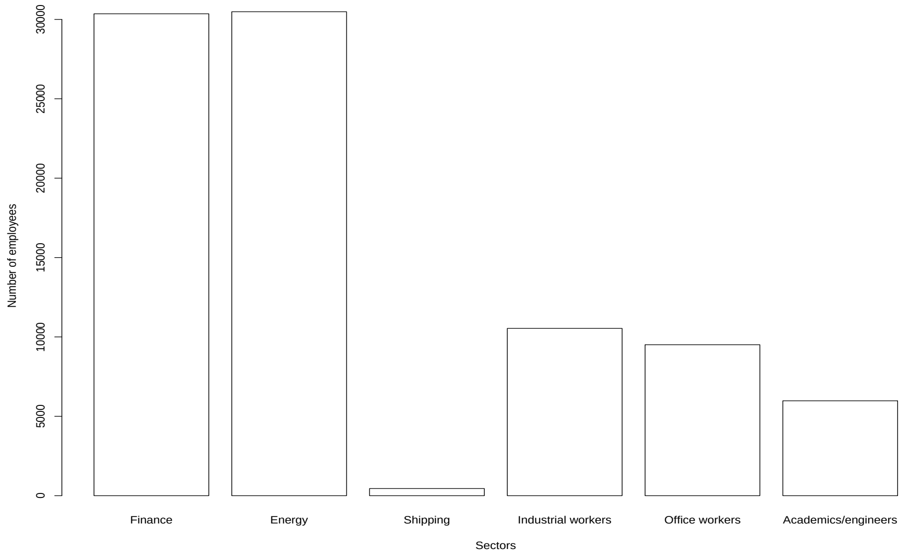


Figure 2.3: *Number of employees distributed on sectors.*

A way of doing this is to expand the equation with an error term so that

$$\hat{g}_x = e^{b_0 + b_1 x + (b_2/100)x^2 + (b_3/100^2)x^3 + \sigma\epsilon} - 1 \tag{2.4}$$

where ϵ is normal distributed with expectation 0 and standard deviation 1. $\sigma = 0.15$ and is given a value based on the data from the archives[6]. Part two of the uncertainty is found in the wage path. There is no reason to believe that true real wage will be a smooth curve as the model predicts. The work will focus mainly on the uncertainty related to the coefficients.

How this affect the calculations will be explored further in chapter 3.

[6]with reference to Professor Erik Bølviken.

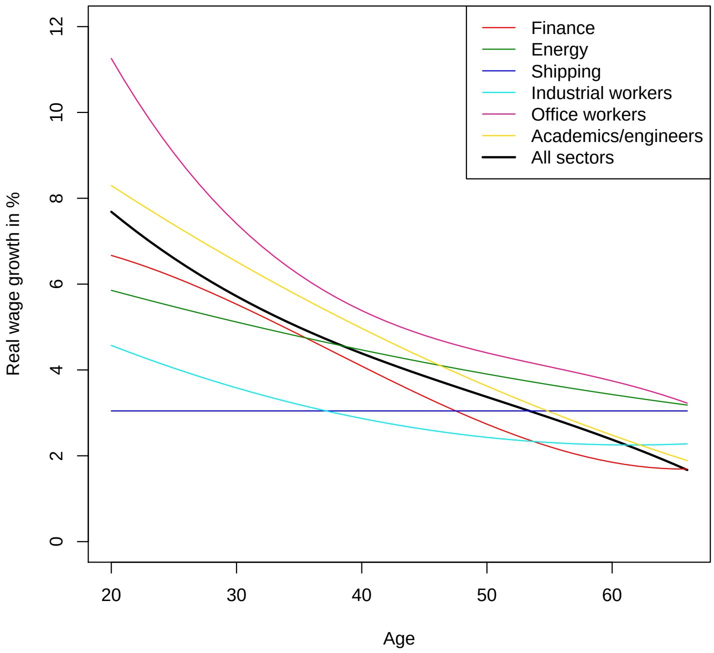


Figure 2.4: *The GablerWassum wage growth model on sectors.*

Chapter 3

Projecting present values

To analyse the model and the effects of different simulation criteria, the present value of future pension liabilities needs to be calculated. In the light of this, a young and old age profile with ten thousand employees are defined as the test portfolio. Since an NRS calculation is done each year, the test portfolios are also followed over a period of one year. The number of employees in the test portfolios is not to be confused with M, which is the total number of the population the historical data for reestimation is built on. Multiplying M with the age profile gives the result N_x, number of employees each age x. The next sections will explore these aspects with simulation in R, R Development Core Team (2008). How accurate the estimates should be are dependent on the best of one's judgement, and it will be discussed continuously throughout the chapter. The code is found in appendix A, references will be given. For ease of notation a billion is abbreviated as $\mathrm{B} = 10^9$.

3.1 Wage path uncertainty

Implementing uncertainty in the wage path is done by using equation 2.4 on page 18 with deterministic model coefficients. In the prediction done here the initial GablerWassum coefficients from table 2.1 on page 16 are used. By adding noise, this reflects the uncertainty in the wages at each age. Although the model predicts a certain level of wage growth, what the individual employee is left with will differ. The pension liabilities are calculated with equation 1.1 and the results are displayed in figure 3.1 on the next page, see appendix A for technical details. Both curves are close to normal, but a little right-skewed. The mean is thus positioned to the right in the density curve, and will serve as the foundation for the analysis, see the vertical lines.

The wage path uncertainty can be approached from two different angles. When time to retirement is long, uncertainty can be reduced because the

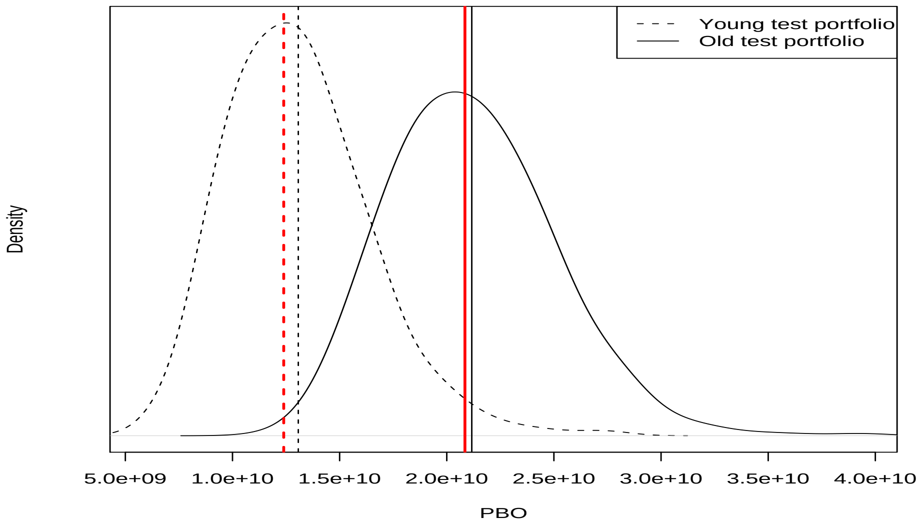


Figure 3.1: *The PBO with mean and corresponding values based on today's wage growth assumptions(red lines).*

fluctuations at each age evens out when time goes by. When an employee recieves a wage rise close to retirement this will cause a sudden jump in the pension liabilities. On the other hand, you can say that uncertainty in the wage caused by time to retirement is less in an old population because the probability of a wage rise is lower. A useful measure of risk in this case is **the coefficient of variation(CV)** defined as the *standard devation/mean*. It has the advantage of beeing unitless, an absolute advantage since the calculation is simplified[1]. The CV is reduced when the old age profile is analysed, but the data set is still relatively noisy. From the numbers in the table it is clear that including a stochastic error term in the wage path modelling results in higher pension liabilities than using the original GablerWassum model.

Table 3.1: *The PBO calculated with different assumptions.*

Age profile	With error term (sd)	CV	GablerWassum	Today
35	13.071 B(3.329 B)	0.255	11.512 B	12.390 B
55	21.167 B(3.860 B)	0.182	19.546 B	20.848 B

The assumptions made for the model used today are simplified in the sense that inflation is left out. A more thorough comparison and explaination is done in chapter 5. A short description of the assumption used in this

[1]the benefits from the National Insurance Scheme is left out

chapter is as follows:

- wage growth young test portfolio: 2%
- wage growth old test portfolio: 1.5%
- a career supplement of 0.25% is included for employees under the age of 45 for both portfolios.

In figure 3.1 the vertical red lines represent the results calculated on the same test portfolios with wage growth as described above. Compared to constant growth the difference is not that big. With these assumptions the GablerWassum model demands a lower pension liability than today's model, a result we will allow to stand and use later.

3.2 Parameter uncertainty

To understand where the results originate from, exploring the algorithm for these simulations can be useful. In algorithm 1, see appendix B, the input parameters are coefficients from table 2.1, σ and ϵ as explained in section 2.4, age profiles of the shape in figure 1.2 on page 7 and $M = 5000, 10000, 100000, 1000000$. Output with different values of M is given in table 3.2 with standard deviation in parenthesis.

A first glance at the results shows that there is not a huge difference from the model coefficients and the reestimated coefficients. One trend is visible though. From the law of large numbers it is clear that reestimation from an increasing population produces more accurate coefficients. Although the variations are small, it remains to analyse in what way these variations influence the results. Because the error is multiplied many times, sometimes up to 47 times, even small deviations can have an effect. Reestimation from M equal five thousand and ten thousand shows standard deviations for b_0

Table 3.2: *Reestimated wage growth coefficients.*

Young coefficients				
M	5000	10000	100000	1000000
$\hat{b}_0$	0.1374(0.1156)	0.1359(0.0786)	0.1397(0.0258)	0.1397(0.0081)
$\hat{b}_1$	-0.0044(0.0093)	-0.0042(0.0063)	-0.0045(0.0021)	-0.0045(0.0007)
$\hat{b}_2$	0.0067(0.0238)	0.0065(0.0162)	0.0071(0.0053)	0.0072(0.0017)
$\hat{b}_3$	-0.0044(0.0192)	-0.0042(0.0131)	-0.0047(0.0043)	-0.0048(0.0013)
Old coefficients				
M	5000	10000	100000	1000000
$\hat{b}_0$	0.1454(0.1689)	0.1372(0.1193)	0.1384(0.0376)	0.1406(0.0123)
$\hat{b}_1$	-0.0049(0.0118)	-0.0043(0.0084)	-0.0044(0.0027)	-0.0046(0.0009)
$\hat{b}_2$	0.0081(0.0265)	0.0066(0.0190)	0.0070(0.0060)	0.0073(0.0019)
$\hat{b}_3$	-0.0054(0.0190)	-0.0043(0.0137)	-0.0046(0.0043)	-0.0048(0.0014)

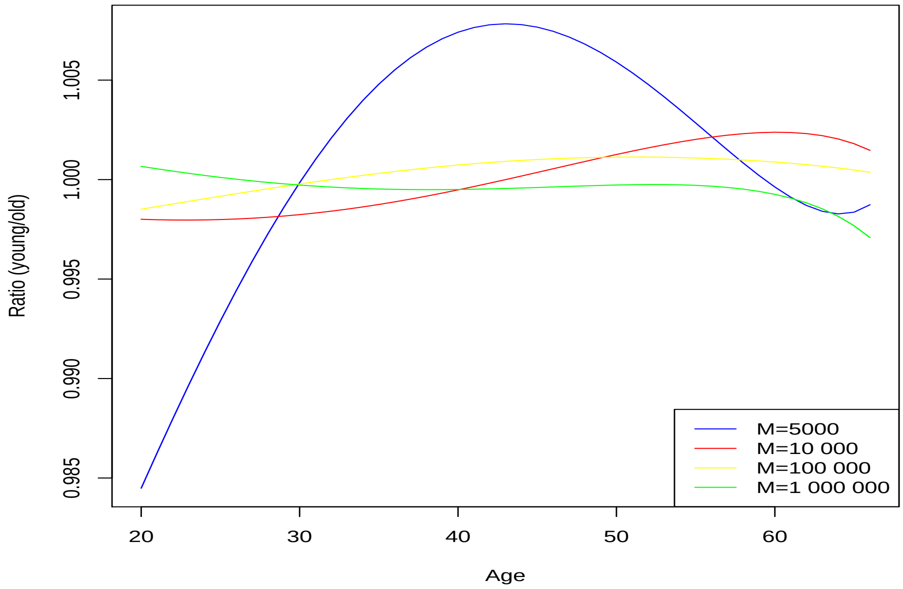


Figure 3.2: *Graphical summary of reestimated wage growth coefficients.*

close to the size of the estimate itself and more than twice the size of the other coefficients. Too much uncertainty in the coefficients can mean a great deal for further calculations. Possible consequences will in that case appear when looking at pension liabilities later. There is also a difference when comparing young and old coefficients against each other. An interpretation of this is based on what is observed at each age. For young employees the change in wage growth from one year to another is bigger. Since the noise is equal and independent of the difference for each age, this gives a better foundation for doing the regression. When the changes are small, they disappear in the noise, causing the old coefficients to be less accurate.

In figure 3.2 the deviation between young and old coefficients is analysed. As M increases, the ratio converges to 1. Only M equal five thousand shows immediate evidence of a possible significant difference. However, it is important to take notice of the y-axis and the scale before drawing a conclusion. Even though the ratio with five thousand stands out as the one with the highest deviation from 1, the deviation in itself is not very far from 1. If, and in that case how this influences the pension liabilities is explored in more depth later. Since the data observed is coming from a polynomial regression where the coefficients are correlated, caution must be made when

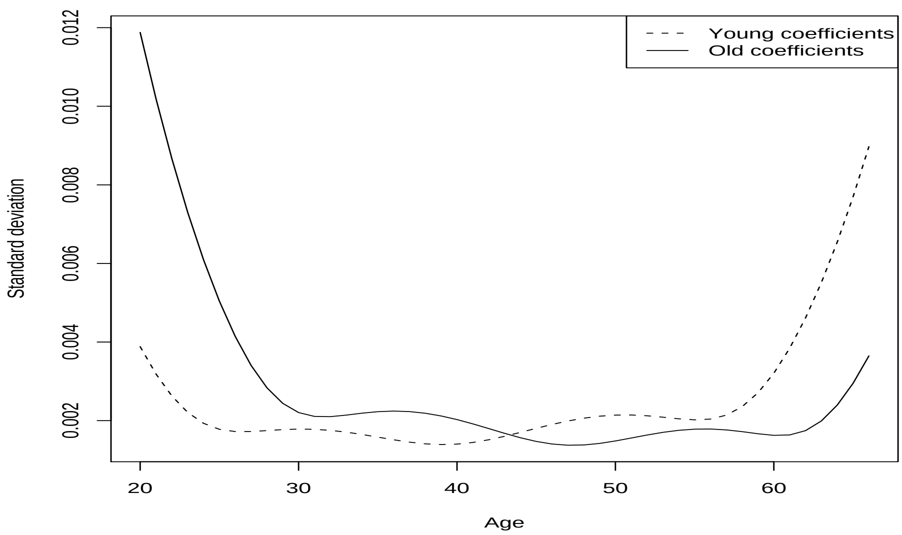


Figure 3.3: *Standard deviation distributed on age with M equal five thousand.*

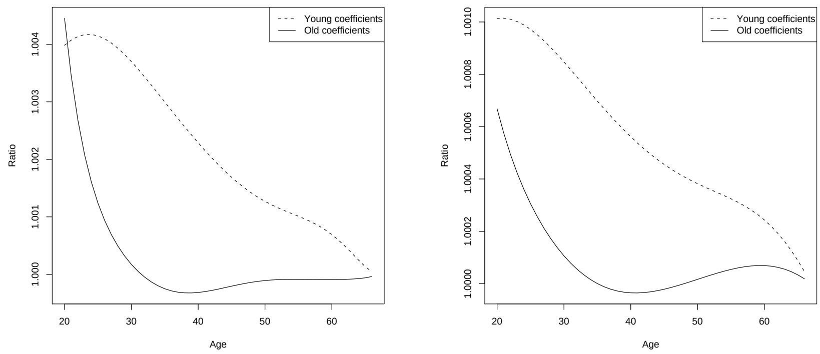
interpreting the results, see section 1.3. The correlation matrix is as follows

	b_0	b_1	b_2	b_3
b_0	1	−0.9969	0.9895	−0.9798
b_1	−0.9969	1	−0.9977	0.9921
b_2	0.9895	−0.9977	1	−0.9982
b_3	−0.9798	0.9921	−0.9982	1

and shows high correlation among the coefficients. This is obvious since there is only one explenatory variable and the other coefficients are a product of it.

How the standard deviation in the young and old coefficients actually affect the results is connected to the age profile they are used on. When the reestimation is from a young age profile, the data amount for older ages is smaller and vice versa for the old age profile. To support this statement the standard deviation of the wage growth for each age is plotted in figure 3.3 using the reestimated coefficients.

To show the significance of uncertainty in historical data, the present value of pension liabilities are calculated. Based on equation 1.1, figure 3.5(a) on the following page is produced. It shows the factor each age, with corresponding wage, is multiplied with. W_{67} is left out of this calculation because it is more convenient to keep it unit-less. How this factor depends on the choice of M is illustrated in figure 3.4(a) on the next page. It is evident



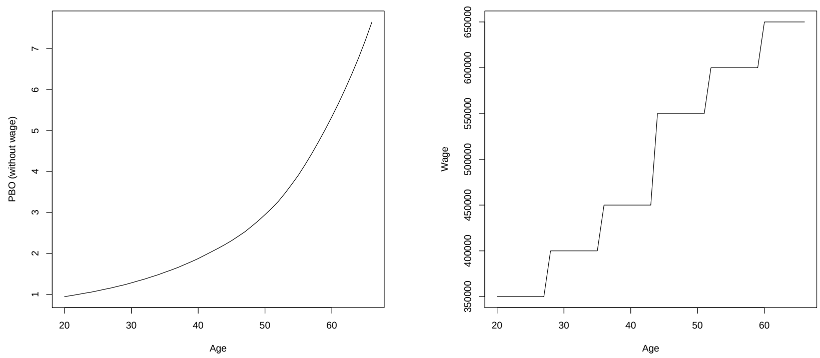
(a) *The ratio between the curves with M equal five thousand and one million.*

(b) *The ratio between the curves M equal one hundred thousand and one million.*

Figure 3.4: *Graphical summary.*

that young coefficients use more time to converge to 1, while the old coefficients fluctuate around 1 from the late thirties. This is also the case in figure 3.4(b), but on an even more accurate scale. To see how the pension liabilites are affected under the different simulation criteria, some additional assumptions are necessary. The test portfolios already mentioned consist of ten thousand emloyees distributed in a certain way. Thus, the wages at given ages are necessary to calculate the exact PBO. Each simulation of wage growth is multiplied with a vector consisting of a probable wage for each age. Figure 3.5(b) shows the wage distributed as a function of age.

Uncertainty in historical data is shown in figure 3.6 on the facing page,



(a) *The PBO factor based on old coefficients and M equal one hundred thousand.*

(b) *The chosen wage distribution.*

Figure 3.5: *The PBO factor and wage distribution.*

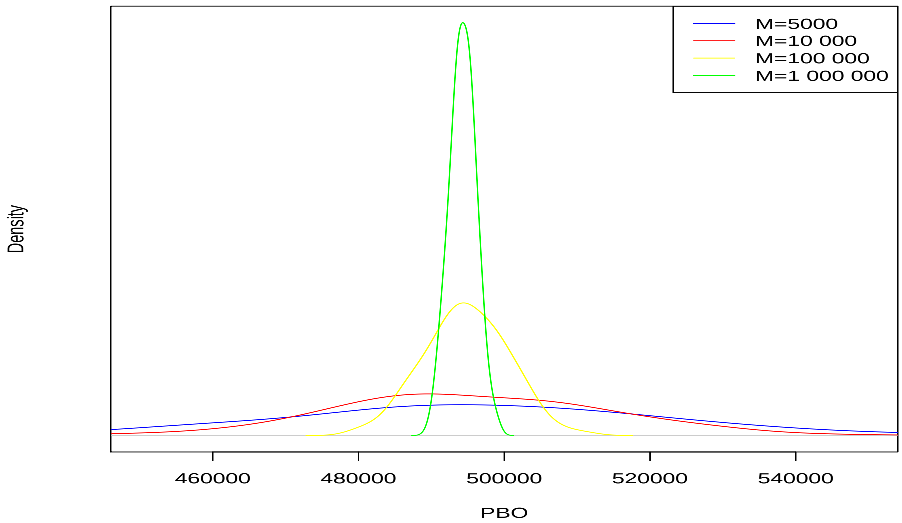


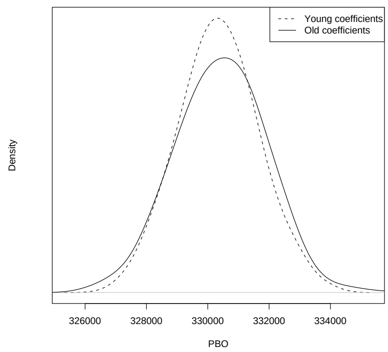
Figure 3.6: *The PBO based on age 30, young coefficients and different values of M.*

where the present value of future pension liabilities is plotted. From the figure the standard deviations with different values of M are evident. Here, an employee aged 30 is considered, to get an impression of how the uncertainty behaves. Both M equal five thousand and ten thousand produces somewhat uncertain results. To analyse this further, the mean and standard deviation with the corresponding CV are given in table 3.3. The CV is at most approximately 5%, but the standard deviation still makes out an amount of more than 20 000 for each employee with M equal five thousand. Even though some NRS assumptions are left out of the calculations, the consequences if this number is multiplied with the total number of employees a firm has can be grave.

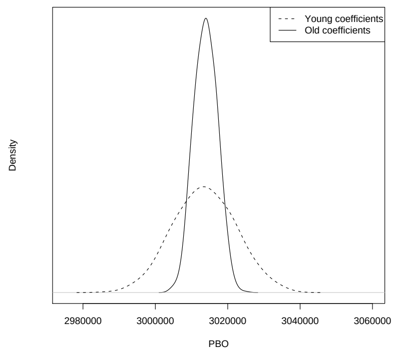
The densities in figure 3.7(a) and 3.7(b) on the following page gives even stronger evidence that there is a difference when estimating from young and

Table 3.3: *Pension liabilites with different values of M.*

Pension liabilities for a person aged 30 years			
M	Mean	Sd	CV
5000	496 192	25 367	0.0511
10 000	495 405	18 292	0.0369
100 000	494 736	5 793	0.0117
1 000 000	494 303	1 806	0.0037



(a) *The PBO based on an employee aged 20 and M equal one million.*



(b) *The PBO based on an employee aged 59 and M equal one million.*

Figure 3.7: *The PBO with both young and old coefficients.*

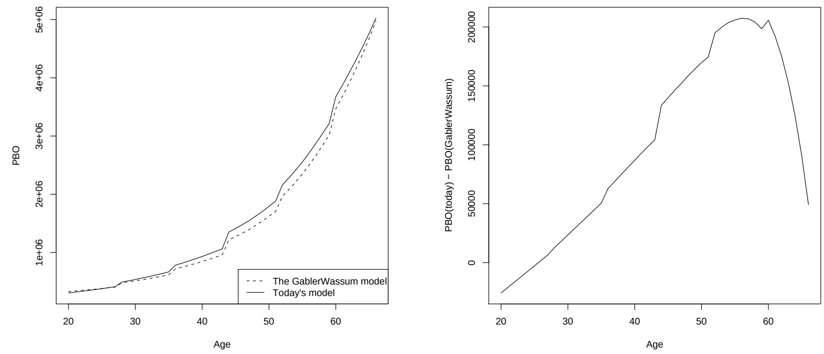
old coefficients. Figure 3.3 explains the deviation in the figures. Looking at the PBO for a employee aged 20, it is clear that the deviation evens out over a long time period. Also, the accumulated deviation over time is slightly higher for the old coeffcents, explaining the difference between young and old coefficients in the figure. For an employee aged 59, the curves are different. Since they are only depending on a few years to retirement the deviation in the coefficients have a larger effect.

To get an idea of how the the results differ from the original GablerWassum wage model and today's NRS assumptions, the numbers in table 3.4 can be compared to each other. Here, it is visible that today's model demands a higher PBO than the GablerWassum model. Also, it is important to keep in mind that this is only for one employee. Multiplying the difference between the original GablerWassum model and today's model with the total number of employees in a firm, even a small difference is significant. The PBO compared to the simulated values shows that they are relatively similar. Both M equal one hundred thousand and one million are satisfactorily close to the original model, together with a low standard deviation.

To get an idea how the PBO behaves for different ages, figure 3.8(a) on the facing page shows results from today's model and the GablerWassum model together. Except for a short time period in an employee's early twenties, the PBO calculated with respect to today's model demands a higher

Table 3.4: *Pension liabilites with deterministic model coefficients.*

Pension liabilities for a person aged 30 years	
Original GablerWassum	Today
494 470	512 616



(a) *The PBO distributed on age.*

(b) *The difference between the PBO today and the PBO with respect to the GablerWassum model.*

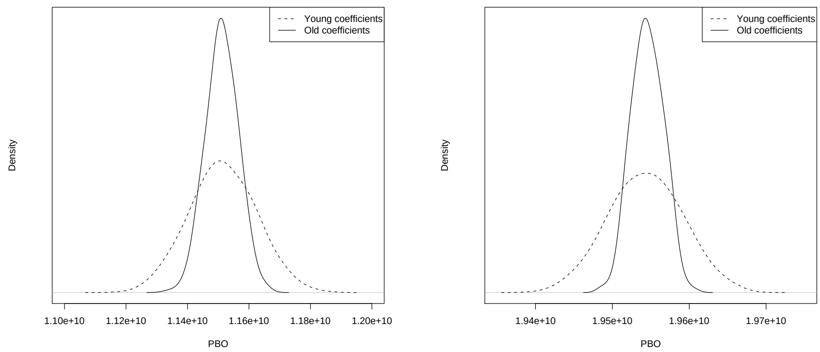
Figure 3.8: *The relationship between today's model and the GablerWassum model.*

amount than with the GablerWassum model. An even better view of this is given in 3.8(b). Here, it is evident that the difference is increasing with age and that the biggest difference is for employees in their fifties. This is because of the development of the wage from age 20 and onwards.

With these findings in mind, the accumulated pension liability for both test portfolios can be analysed. The estimated PBOs have a higher standard deviation when calculated from young coefficients. How much deviation should be allowed? At figure 3.9(a) and 3.9(b) on the following page the uncertainty in population size is plotted. Assuming a wage distribution like figure 3.5(b) for the members of the two test portfolios, the PBOs are given in table 3.5 on page 31. With the CV at 4.1% and the possibility of the sum of expected pension liabilities reaching the billions, this makes out a considerable amount. Using M equal one hundred thousand and a million on the other hand produces more reasonable results. Even M equal ten thousand produces a CV at an acceptable level when the old coefficients are used. Another thing to take note of is the difference between the PBO with young and old coefficients. Old coefficients result in a slightly lower PBO than when calculated with young coefficients. This effect is related to the previous discussion about deviation above.

Based on the results from this section, a reasonable curve to base the further calculations on is chosen. Since the population is aging and available data amounts in most cases are sparse, the red line in figure 3.10 on page 31 is calculated from old coefficients and M equal ten thousand.

All the results are based on equation 1.1 on page 7 and the R code for this section is given in appendix A.



(a) *The PBO for the young test portfolio and M equal one hundre thousand.*

(b) *The PBO for the old test portfolio and M equal one million.*

Figure 3.9: *The PBO calculated with different assumptions.*

3.3 Different branches

The analysis is now divided into six main branches, namely Finance, Energy, Shipping, Industrial workers, Office workers and Academics/engineers. The coefficients are found in table C.2 in the appendix. Using models on different sectors of the economy will affect the pension liabilities, and how these results deviate from the model on all sectors and today's model is the question of interest. This section will deal with the gender neutral model for each branch and analyse these in a similar way as done in the sections above. With reference to the previous results, using M equal ten thousand will serve as a satisfactory middle course.

Following the same procedure as in section 3.1 gives the results in table 3.6 on page 32. Several patterns are visible in the results here. The standard deviation is higher for the old population and the liabilities are obviously higher. The differences between the standard deviations are not that big though. Relating that to the CV, the dispersion in the results is less when an old population is considered. It is also the case that with the stochastic term, the results are higher than the GablerWassum model without error term for all sectors. Looking at the difference between the sectors, this can be connected to figure 2.4 on page 19. Office workers with the highest expected wage growth leads to the highest liabilities, the line is above the line *all sectors* for every age. After Office workers, employees in the branch Energy represent the highest liabilities. Although this line is not above *all sectors* for every age, workers in this branch have a higher expected wage growth after their thirties. This way of interpreting the results in the table and corresponding lines in the graph can be applied to all branches. It is also worth noting how the results relate to today's model in table 3.1. All

Table 3.5: *Present value of future pension liabilities based on young and old coefficients.*

Young test portfolio						
	Young coefficients			Old coefficients		
M	Mean	Sd	CV	Mean	Sd	CV
5000	11.529 B	0.475 B	0.041	11.511 B	0.238 B	0.020
10 000	11.524 B	0.348 B	0.030	11.516 B	0.164 B	0.014
100 000	11.514 B	0.106 B	0.009	11.512 B	0.052 B	0.004
1 000 000	11.509 B	0.033 B	0.002	11.511 B	0.016 B	0.001

Old test portfolio						
	Young coefficients			Old coefficients		
M	Mean	Sd	CV	Mean	Sd	CV
5000	19.563 B	0.684 B	0.035	19.543 B	0.294 B	0.015
10 000	19.561 B	0.501 B	0.025	19.550 B	0.202 B	0.010
100 000	19.549 B	0.152 B	0.007	19.546 B	0.064 B	0.003
1 000 000	19.543 B	0.047 B	0.002	19.545 B	0.026 B	0.001

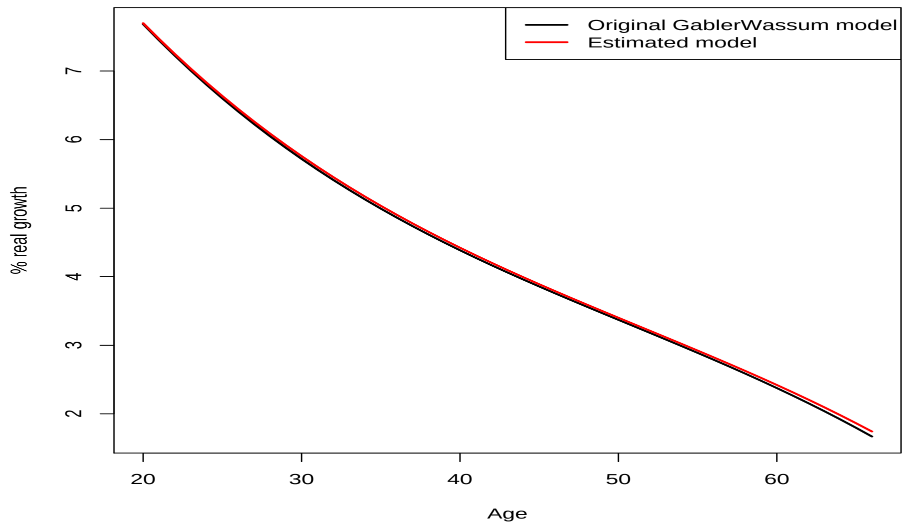


Figure 3.10: *The estimated curve from old coefficients and M equal ten thousand.*

Table 3.6: *Present value of future pension liabilities on sectors.*

Young test portfolio			
Branch	PV(Sd)	CV	GW
Finance	12.250 B(3.173 B)	0.259	11.083 B
Energy	13.585 B(3.834 B)	0.282	12.268 B
Shipping	12.696 B(3.252 B)	0.256	11.546 B
Industrial workers	12.109 B(3.025 B)	0.249	10.988 B
Office workers	14.325 B(3.865 B)	0.269	12.866 B
Academics/engineers	13.003 B(3.398 B)	0.261	11.778 B
Old test portfolio			
Branch	PV(Sd)	CV	GW
Finance	20.330 B(3.830 B)	0.188	19.120 B
Energy	21.839 B(3.948 B)	0.181	20.532 B
Shipping	21.192 B(3.997 B)	0.189	19.950 B
Industrial workers	20.381 B(3.691 B)	0.181	19.254 B
Office workers	22.430 B(4.322 B)	0.192	20.964 B
Academics/engineers	21.018 B(3.981 B)	0.189	19.763 B

sectors except for *Office workers* are estimated with a lower present value in both the young and old test portfolio.

When it comes to parameter uncertainty divided on branches, these results are given in table 3.7 on the facing page. Common for all the reestimated coefficients on branches is the fact that the standard deviations are less than in the model on all sectors. This is again explained by how the data is distributed on each age and how the wage growth is at the ages with more data.

How the uncertainty in historical data affects the pension liabilities is shown in table 3.8 on the next page. Compared to the results in table 3.5 on the preceding page a repeating pattern is how Finance and Industrial workers are lower than the values for *all sectors* and the remaining higher. This is valid for both the young and old test portfolio. Branches with pension liabilities lower than the *all sectors* model is hence overestimated if this model is used, and the other way around for Energy, Shipping, Office workers and Academics/engineers. The picture looks a whole lot different if the results are compared to today's model on *all sectors*. All the estimated PBO's are lower, except for office workers which is approximately 4 % higher. The same result applies to the old test portfolio, but the estimated value is only approximately 2.5% higher for this group. With these results in mind it is possible to adjust the wage growth model such that the pension liabilities are more accurately assessed.

Table 3.7: *Reestimated wage growth coefficients on sectors.*

Young coefficients				
Branch	b_0	b_1	b_2	b_3
Finance	0.0620(0.0219)	0.0013(0.0018)	-0.0071(0.0047)	0.0062(0.0039)
Energy	0.0732(0.0068)	-0.0009(0.0004)	0.0004(0.0005)	-
Shipping	0.0300(0.0021)	0.0000(0.0001)	-	-
Industrial workers	0.0716(0.0068)	-0.0016(0.0004)	0.0013(0.0005)	-
Office workers	0.2511(0.0233)	-0.0104(0.0018)	0.0178(0.0048)	-0.0108(0.0039)
Academics/engineers	0.1178(0.0069)	-0.0021(0.0004)	0.0009(0.0005)	-
Old coefficients				
Branch	b_0	b_1	b_2	b_3
Finance	0.0618(0.0391)	0.0013(0.0027)	-0.0071(0.0062)	0.0062(0.0044)
Energy	0.0736(0.0108)	-0.0009(0.0005)	0.0004(0.0005)	-
Shipping	0.0301(0.0028)	0.0000(0.0001)	-	-
Industrial workers	0.0712(0.0107)	-0.0016(0.0005)	0.0013(0.0005)	-
Office workers	0.2520(0.0383)	-0.0104(0.0027)	0.0179(0.0060)	-0.0109(0.0043)
Academics/engineers	0.1181(0.0109)	-0.0021(0.0005)	0.0009(0.0005)	-

Table 3.8: *Present value of future pension liabilities based on M equal ten thousand.*

Young test portfolio						
	Young coefficients			Old coefficients		
Branch	Mean	Sd	CV	Mean	Sd	CV
Finance	11.106 B	0.320 B	0.028	11.088 B	0.157 B	0.014
Energy	12.277 B	0.388 B	0.029	12.273 B	0.179 B	0.015
Shipping	11.552 B	0.334 B	0.028	11.544 B	0.176 B	0.015
Industrial workers	10.990 B	0.335 B	0.030	10.994 B	0.158 B	0.014
Office workers	12.872 B	0.385 B	0.029	12.869 B	0.193 B	0.015
Academics/engineers	11.802 B	0.371 B	0.031	11.782 B	0.171 B	0.014
Old test portfolio						
	Young coefficients			Old coefficients		
Branch	Mean	Sd	CV	Mean	Sd	CV
Finance	19.155 B	0.467 B	0.024	19.124 B	0.196 B	0.010
Energy	20.544 B	0.526 B	0.025	20.536 B	0.217 B	0.010
Shipping	19.957 B	0.415 B	0.020	19.947 B	0.226 B	0.011
Industrial workers	19.253 B	0.473 B	0.024	19.259 B	0.202 B	0.010
Office workers	20.971 B	0.535 B	0.025	20.971 B	0.228 B	0.010
Academics/engineers	19.793 B	0.503 B	0.025	19.769 B	0.208 B	0.010

Chapter 4

Job shift model

Another aspect of the labour market and pension liability calculations is the probability of an employee quitting their current job. There are many possible reasons why people may choose to leave their job. The data from GablerWassum contains information about voluntary exits, probably motivated by a pay rise. Connecting the observations makes it possible to get closer to an adequate and correct conclusion. This model is also narrowed down on different sectors of the economy, see section 4.3. Turnover connected to retirement is left out of this analysis.

An employee's path through the labour market can be translated into a Markov chain. Theory on stochastic processes is based on the book by J.S. Allen (1991). With the assumptions of this thesis, figure 4.1 on the next page shows what the possible paths are in this case. The probability of all possible outcomes is 1, the exit rate thus needs to be incorporated in the expression for the PBO, see explanation below.

When employees quit their job, employers are no longer responsible for paying the premium in the defined benefit plan. If the employee has worked in the firm for more than a year, the firm is obliged to issue a paid-up policy to the former employee[1]. This policy represents the value of the accumulated pension saved during the working years in a specific firm. Value of paid-up policy =

$$\frac{2}{3} \times W_{\text{resignation}} \times \int_{n}^{\infty} v^t {}_t p_x dt \times \kappa, \tag{4.1}$$

where κ equals *years in firm/period of service*. Compulsory period of service is determined by the employer. This period has to be a minimum of 30 years, but not more than 40 years. If a person starts working in a firm and the time to retirement is longer than the compulsory time, period of service is equal to time to retirement. Various combinations of the age distributions together with how the exit rates fluctuate are important to keep in mind when analysing the results. High turnover in a young portfolio

[1] `http://www.lovdata.no/all/tl-20000324-016-014.html`

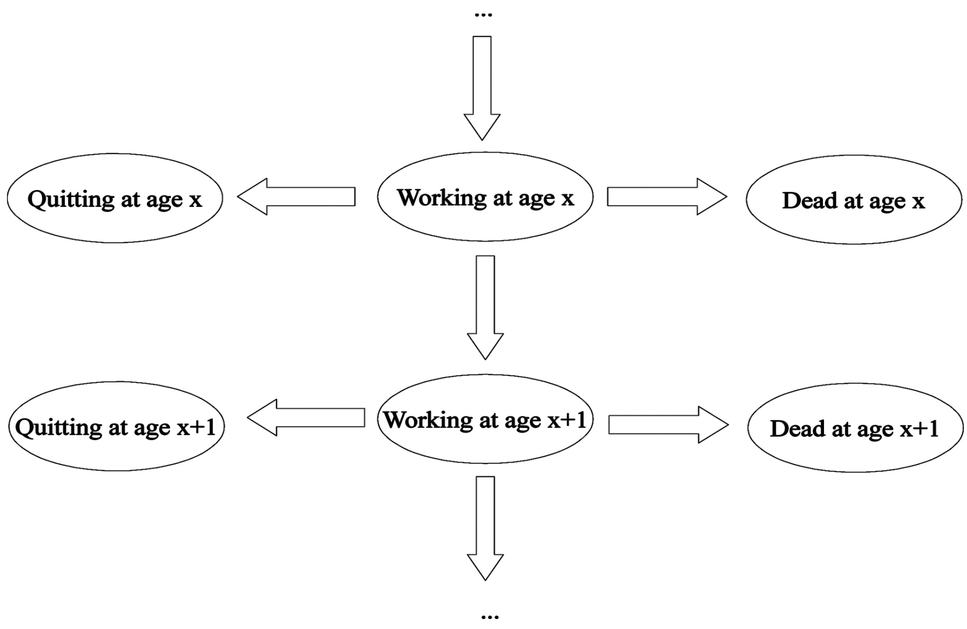


Figure 4.1: *A Markov chain of the labour market.*

represents a smaller expenditure than expenses calculated with an old portfolio. The premium is generally higher for older employees because time to retirement is shorter. This means shorter time to provide money if the wage increases. Turnover will in all cases result in reduced pension liabilities. For the specific pension liability calculations, equation 1.1 is multiplied with the corresponding exit rate probabilities.

4.1 The model

The data from GablerWassum provides numbers on voluntary job change and a model was fitted with logistic regression. A similar pattern found in the wage data is visible in figure 4.2 on the facing page as well. Young people stand out and are the ones changing jobs most frequently. As employees grow older and time to retirement decreases, this rate decreases too. Based on the theory in section 1.3 the annual probability that an employee with age x will exit his or her current job voluntarily is

$$\omega_x = \frac{1}{1 + e^{-(b_0 + b_1 x + (b_2/100)x^2)}}. \tag{4.2}$$

With the coefficients in table 4.1 on the next page, it is possible to show how the exit rate model is for male, female and both genders in figure 4.3 on page 38. Since the dataset consists of 2/3 male the gender neutral model is closer to this curve. Females differ from the others in the way that the exit

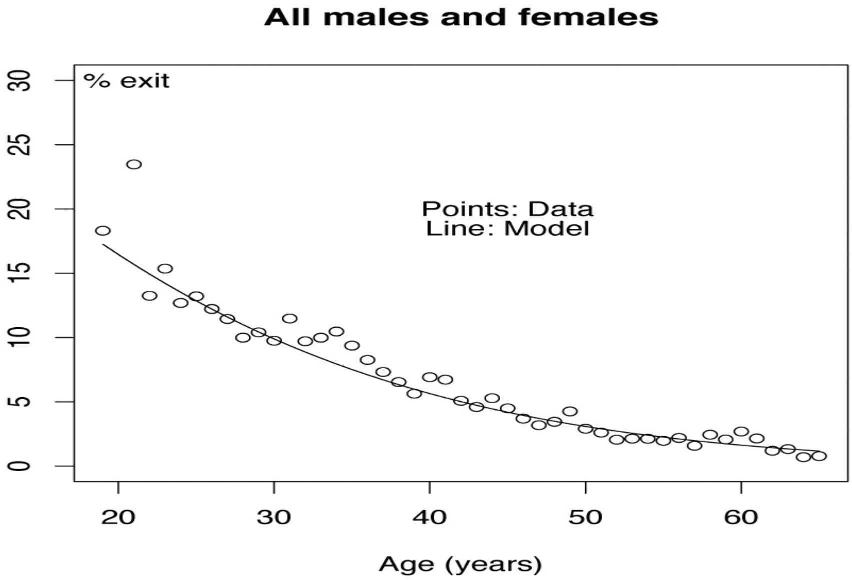


Figure 4.2: *Gender neutral exit rate data with the fitted model from Bølviken (2009).*

rate is less for young ages but slightly higher for older ages. Looking at this figure in context to 2.2 on page 17, it is easy to see the connection between high exit rates and high wage growth for young ages. A thing to take notice of, though, is the age profile and number of employees at the young ages. The GablerWassum dataset is small for young ages and this leads to less reliable results for this part of the population.

Today's model is described more carefully in chapter 5. It is a partial constant function based on the rates in table 5.3 on page 52.

4.2 Parameter uncertainty

Algorithm 2 in appendix B shows how the exit rate coefficients are reestimated. The results with different population sizes are summarized in table 4.2 on page 39. Based on $M = 10000, 100000, 1000000$, the coefficients do not deviate particularly from the original coefficients. This can be observed

Table 4.1: *The GablerWassum exit rate coefficients.*

	b_0	b_1	b_2
Male	-0.23219	-0.06552	-0.00056
Female	-1.29612	-0.02041	-0.03820
Both	-0.52260	-0.05281	-0.01138

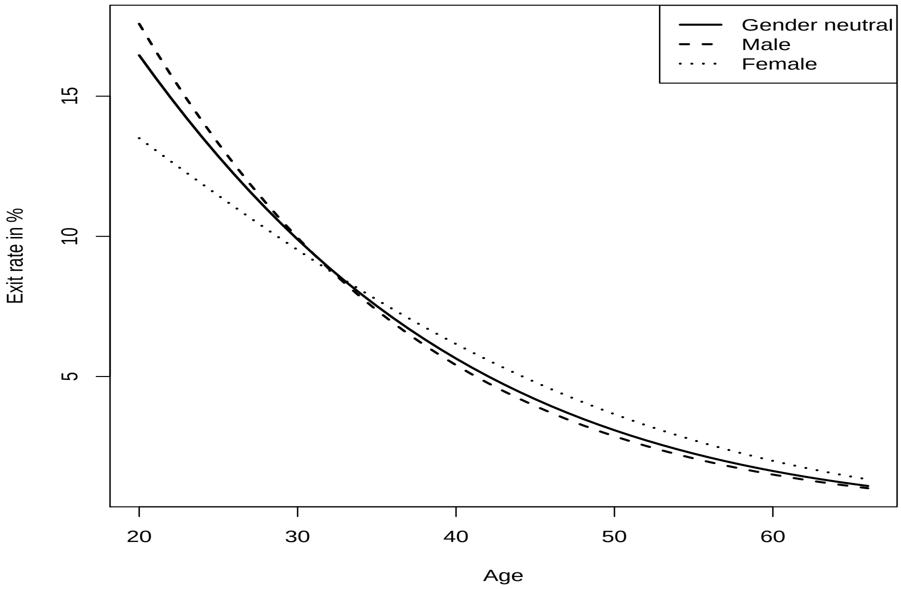


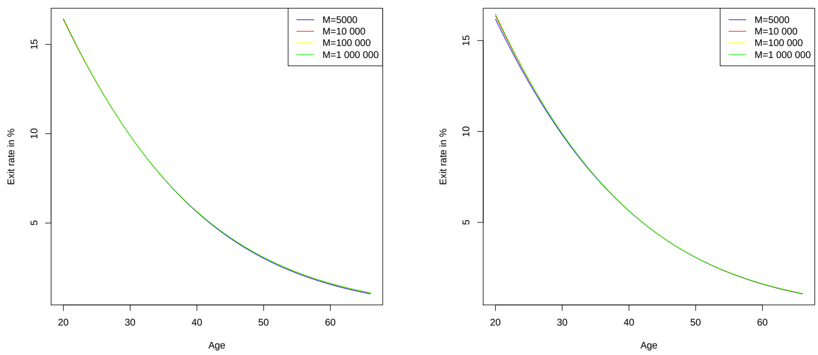
Figure 4.3: *The GablerWassum exit rate model.*

for both the young and old age profile. As M increases, the standard deviation decreases. Also, here the reestimation from an old population produces more volatile results. Looking at figure 4.4(a) and 4.4(b) on the facing page, the effects on the curves are visible. From this it is clear that the population size does not matter much when reestimating. In figure 4.5 on page 40 the ratio between young and old reestimated coefficients are given. From the range of the y-axis it is clear that the differences are marginal. This figure can also be interpreted with figure 4.7 on page 41 in mind. The figure shows how the deviation distributes for each age. The old coefficients have a higher average standard deviation than the young coefficients. Though it is barely visible, this deviation is evident in figure 4.4(b) for young ages. Doing the reestimation with different values of M shows that a population size as small as five thousand is sufficient in order to get satisfactory results in this case.

How these different settings affect the cost related to pension liabilities for firms is discussed next. The starting wage follow a similar distribution as in chapter 3. Since job shift is the topic for this chapter, employees are assumed to follow the same wage growth model independent of which job shift model is being analysed. Thus, the results in this chapter are analysed with respect to job shifts only. A combination of the two models is analysed further in chapter 5.

Table 4.2: *Reestimated exit rate coefficients with corresponding standard deviation in parentheses.*

Young coefficients				
M	5000	10000	100000	1000000
$\hat{b}_0$	-0.57587(0.68356)	-0.52119(0.45197)	-0.52874(0.14558)	-0.52322(0.04489)
$\hat{b}_1$	-0.04956(0.03903)	-0.05293(0.02569)	-0.05245(0.00838)	-0.05279(0.00256)
$\hat{b}_2$	-0.01621(0.05254)	-0.01127(0.03430)	-0.01188(0.01128)	-0.01141(0.00342)
Old coefficients				
M	5000	10000	100000	1000000
$\hat{b}_0$	-0.58306(0.96450)	-0.54748(0.65566)	-0.52164(0.20960)	-0.52301(0.06594)
$\hat{b}_1$	-0.04988(0.04719)	-0.05134(0.03218)	-0.05285(0.01037)	-0.05275(0.00319)
$\hat{b}_2$	-0.01511(0.05517)	-0.01343(0.03771)	-0.01138(0.01215)	-0.01148(0.00370)



(a) Young age profile. (b) Old age profile.

Figure 4.4: *The exit rate models with different values of M.*

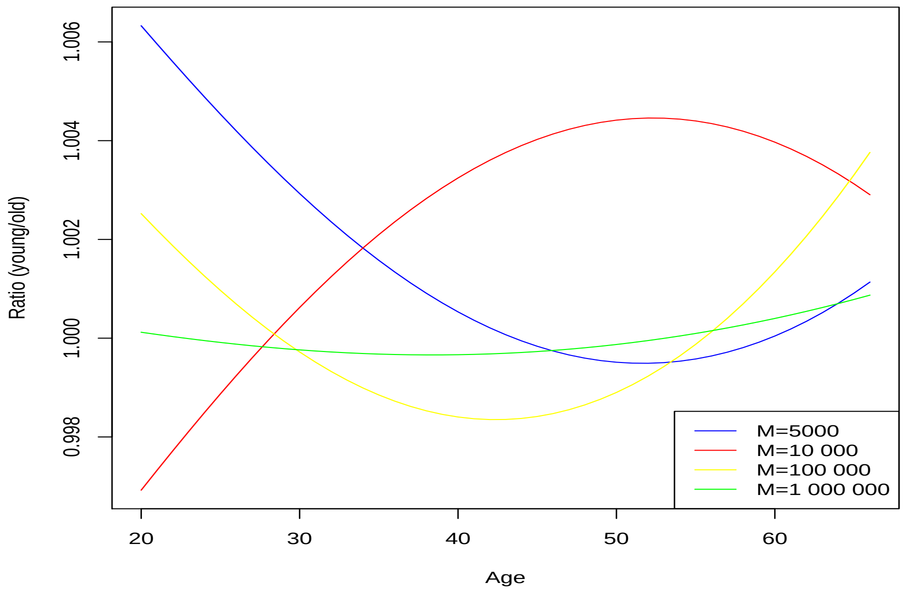


Figure 4.5: *Graphical summary of reestimated job shift coefficients.*

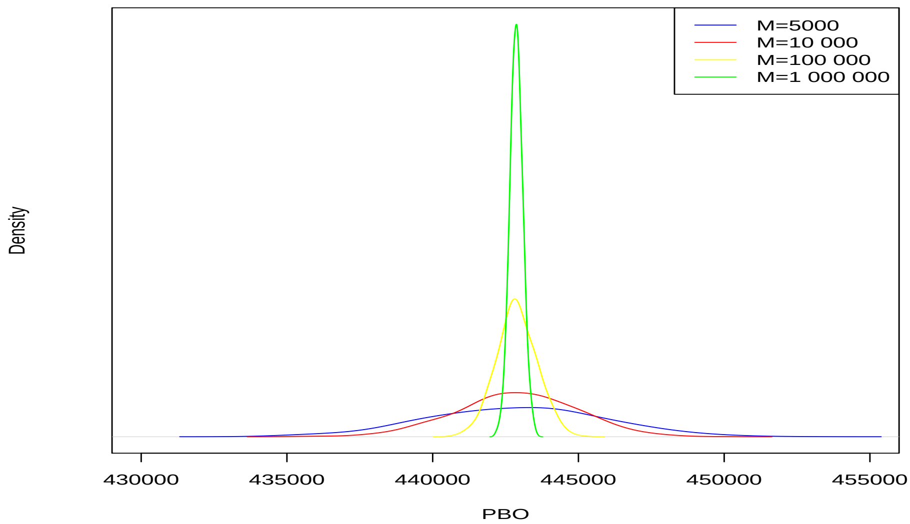


Figure 4.6: *The PBO based on age 30, young coefficients and different values of M.*

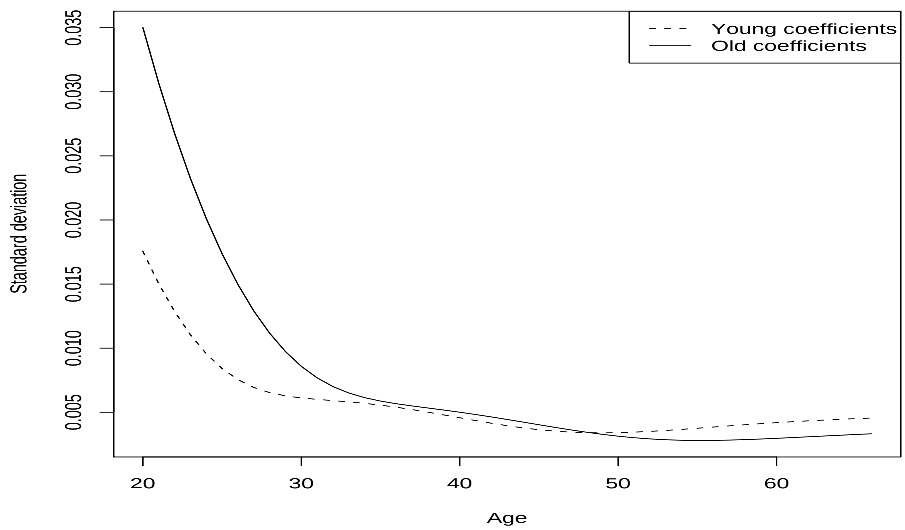


Figure 4.7: *Standard deviation of ω_x distributed on age, with M equal five thousand.*

To see how the calculated PBOs behave under different values of M, the densities of a PBO for an employee aged 30 is displayed in figure 4.6 on the preceding page. Looking at this figure, it confirms the observations above. Although M equal five and ten thousand produces volatile results compared to M equal one hundred thousand and one million, the dispersion is less than what was observed in figure 3.6 on page 27. In this particular case, table 4.3 shows the precise numbers. For a person with this age, the liabilites are 8.85 % higher with today's assumptions. Looking at the bigger picture, table 4.4 on the next page shows the PBOs for the test portfolios.

Pension liabilitites calculated in the old test portfolio represents a higher cost for a firm compared to the young test portfolio. The standard deviation under these assumptions is higher too. Even though the differences between the CVs are not that significantly large, the curves in figure 4.6 are an example of how the deviations play a part. There are small differences between the results, they seem to be independent of the age distribution used when reestimating the coefficients. Looking at the numbers, the CV

Table 4.3: *Pension liabilites with deterministic model coefficients.*

Pension liabilities for a person aged 30 years	
Original GablerWassum	Today
442.872	482.108

Table 4.4: *Total value of the paid-up policies based on different values of M.*

Young test portfolio						
	Young coefficients			Old coefficients		
M	Mean	Sd	CV	Mean	Sd	CV
5000	11.004 B	0.033 B	0.003	11.003 B	0.034 B	0.003
10 000	11.001 B	0.022 B	0.002	11.002 B	0.023 B	0.002
100 000	11.002 B	0.007 B	0.001	11.002 B	0.007 B	0.001
1 000 000	11.002 B	0.002 B	0.000	11.002 B	0.002 B	0.000

Old test portfolio						
	Young coefficients			Old coefficients		
M	Mean	Sd	CV	Mean	Sd	CV
5000	19.022 B	0.067 B	0.004	19.020 B	0.047 B	0.002
10 000	19.018 B	0.045 B	0.002	19.020 B	0.031 B	0.001
100 000	19.020 B	0.014 B	0.000	19.020 B	0.010 B	0.000
1 000 000	19.020 B	0.005 B	0.000	19.020 B	0.003 B	0.000

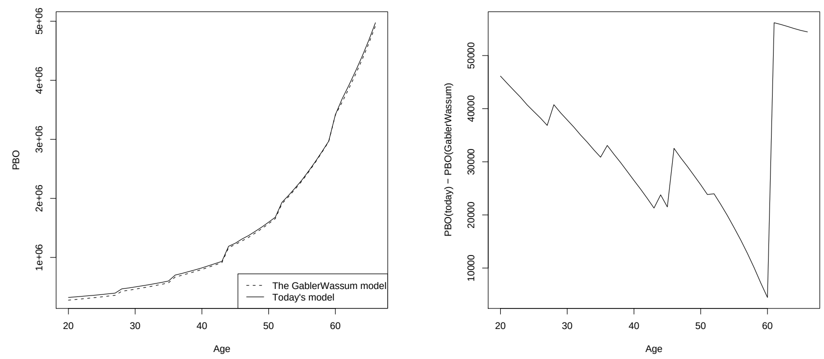
decrease as M increases, but for all M these results are satisfactory low. As mentioned earlier the numbers are not realistic because the benefit from the National Insurance Scheme is left out of the calculations, but the ratio is proportional and valid in this analysis.

Doing the calculations without the stochastic parts of the expression results in the values in table 4.5. The estimated values are close to these results, but something which is more interesting is how these results are compared to the assumption used in today's NRS calculations. The assumption today's numbers are based on are explained more carefully in chapter 5. For the young test portfolio today's value is 2.86% higher and 1.60% higher in the old test portfolio. Since the difference between exit rates ist the greatest for young employees the effect is highest for the young test portfolio. Independent of the size of a firms portfolio, the effect of implementing this model is a reduction in the pension liabilities. Looking at the ratio between the PBO today without exit rates incorporated and the GablerWassum PBO with job shift, it is 12.61 % higher. This surely represents a reduction in pension liabilities any firm would benefit from.

To see how the two models develop together, the PBO with respect to

Table 4.5: *Pension liabilities for the two test portfolios with today's model and the GablerWassum model.*

Pension liabilities		
Age profile	GablerWassum	Today
35	11.002 B	11.317 B
55	19.020 B	19.325 B



(a) *The PBO distributed on age.* (b) *The difference between the PBO today and the PBO with respect to the GablerWassum model.*

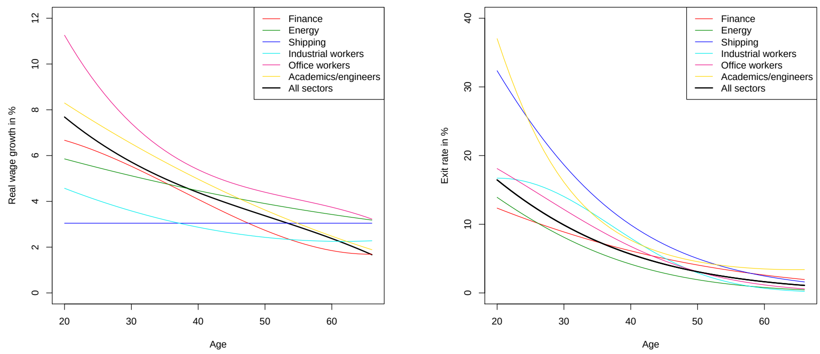
Figure 4.8: *The relationship between today's model and the GablerWassum model.*

age is given in figure 4.8(a). The GablerWassum predicted PBO is below today's predicted PBO for all ages. This is a result more visible in figure 4.8(b) where the difference is calculated for each age. The difference is highest at the point where today's model assumes 0% exit rate, that is at age 60. For young employees the effect of implementing the job shift model is also high and decreasing as the GablerWassum model converges towards 0.

The code for the results in this section is found in appendix A.

4.3 Different branches

As done with the entire population in the previous sections, the analysis can be broken down on sectors also in this case. Figure 4.9(b) on the following page is based on the coefficients from table C.2. Except for Shipping and Industrial workers, the two figures correspond realtively well. When wage growth is high, movement in the job market is high too. How the age profile differs within the branches is something not taken into account. As mentioned earlier, this adds further uncertainty to the reestimation. Academics/engineers and Shipping are the groups which stand out the most with an exit rate above 20% for employees in their twenties. Since these are the two groups with the smallest observations initially this makes especially the results from shipping difficult to trust. There does not seem to be any connection between exit rates and wage growth for this branch. It is reasonable to believe that some branches have a higher turnover than others. From the previous results M equal five thousand gave numbers with acceptable



(a) *Real wage growth on sectors with* $\alpha = 1$.

(b) *Exit rates on sectors.*

Figure 4.9: *Graphical summary.*

standard deviations, hence the value used in the further calculations.

Parameter uncertainty is given in table 4.6 on the next page and a repeating pattern is how the old coefficients have a larger standard deviation than the young coefficients. Compared to the original coefficients in table C.2 on page 74, the errors are marginal.

By looking at the numbers in table 4.7 on the next page and curves in 4.9(b) one can see how the results are connected. First, the results are compared to the original GablerWassum model without branches. Using a model for branches with exit rates higher than the original model results in lower pension liabilities. This is the case for all branches. How big the difference is, however, differs from one branch to the other. For example firms in the shipping industry will have approximately 93 % of the expenses compared to the original model without exit rate probabilities. If this branch is to be compared to today's model, the expenses is reduced to 85 %. Energy, the branch with lowest exit rates is the branch with a PBO closest to today's PBO. The estimated values for each branch are very close to the GablerWassum estimates. With a CV lower than 0.50 % for each branch, these results are at a satisfactory level.

The data from the GablerWassum archives does not contain information about the wage an employee gets after quitting their current job, but it is natural to believe that these processes are correlated.

Table 4.6: *Reestimated exit rate coefficients in different branches.*

Young coefficients			
Branch	b_0	b_1	b_2
Finance	-1.31467(0.14790)	-0.02918(0.00823)	-0.01549(0.01072)
Energy	-0.90244(0.16947)	-0.03627(0.00985)	-0.04864(0.01351)
Shipping	0.73234(0.03465)	-0.07350(0.00099)	-
Industrial workers	-2.47342(0.14256)	0.08603(0.00834)	-0.21329(0.01165)
Office workers	-1.10365(0.14141)	-0.00267(0.00820)	-0.08822(0.01125)
Academics/engineers	2.55496(0.10926)	-0.18240(0.00622)	0.14087(0.00820)
Old coefficients			
Branch	b_0	b_1	b_2
Finance	-1.32237(0.20844)	-0.02880(0.00988)	-0.01596(0.01118)
Energy	-0.90154(0.25303)	-0.03628(0.01277)	-0.04866(0.01533)
Shipping	0.73631(0.04665)	-0.07359(0.00112)	-
Industrial workers	-2.47720(0.22710)	0.08616(0.01154)	-0.21338(0.01416)
Office workers	-1.09583(0.21160)	-0.00308(0.01054)	-0.08769(0.01258)
Academics/engineers	2.54910(0.15635)	-0.18207(0.00757)	0.14048(0.00865)

Table 4.7: *Total value of pension liabilities related to exit rates. Calculated for each sector with M equal five thousand.*

Young test portfolio							
	Young coefficients			Old coefficients			GW
	Mean	Sd	CV	Mean	Sd	CV	
Finance	10.949 B	0.039 B	0.003	10.950 B	0.035 B	0.003	10.949 B
Energy	11.144 B	0.026 B	0.002	11.146 B	0.029 B	0.003	11.145 B
Shipping	10.615 B	0.041 B	0.004	10.612 B	0.043 B	0.004	10.613 B
Industrial workers	10.921 B	0.032 B	0.003	10.921 B	0.034 B	0.003	10.923 B
Office workers	10.949 B	0.033 B	0.003	10.952 B	0.035 B	0.003	10.951 B
Academics/engineers	10.667 B	0.043 B	0.004	10.669 B	0.042 B	0.004	10.668 B
Old test portfolio							
	Young coefficients			Old coefficients			GW
	Mean	Sd	CV	Mean	Sd	CV	
Finance	18.859 B	0.087 B	0.004	18.862 B	0.054 B	0.003	18.860 B
Energy	19.213 B	0.045 B	0.002	19.215 B	0.033 B	0.002	19.214 B
Shipping	18.682 B	0.064 B	0.003	18.676 B	0.055 B	0.003	18.679 B
Industrial workers	19.058 B	0.043 B	0.002	19.058 B	0.035 B	0.002	19.060 B
Office workers	19.029 B	0.056 B	0.003	19.035 B	0.040 B	0.002	19.035 B
Academics/engineers	18.615 B	0.101 B	0.005	18.617 B	0.065 B	0.003	18.619 B

Chapter 5

Comparison with today's model

This chapter is based on Norsk RegnskapsStandard 6[1] (NRS 6), NASB (2009a). It reflects how pension liabilities are managed today versus pension liabilities managed under the GablerWassum wage models in a more detailed way. This accounting standard's objective is to make sure pension liabilities are incorporated correctly while accumulated. That is, distributed in a reasonable way during the contribution time. Conditions regarding future wage growth are dealt with, but the question is how comprehensive these conditions are and if they are comprehensive enough.

Independent of what the wage growth is caused by, it still needs to be part of the calculations. In NRS 6, inflation in general and other circumstances which influence the real wage for example productivity and individual bonuses are mentioned. Other economical assumptions that pension liabilities depend on are expected return on the pension assets, regulation of running pension expenses and regulation of the NIS G-value. These assumptions will not be considered here, but the discount factor and wage growth are economical assumptions which obviously cannot be left out. Assumption labeled as actuarial are demographic factors, for example mortality and voluntary resignation. The discount factor should be independent of the firms' economical situation and settled based on the riskless long term interest rate. This can be difficult when the time horizon of the pension liabilities are longer than the interest rate which is possible to observe in the market. A possible solution is to extrapolate the interest rate curve based on available data of the Norwegian swap rate. An example of this is found in NASB (2009b) appendix I. A macro economical view of the economy is also important. How the economy develops in the future will affect the interest rate level, development in prices and economic growth in general.

As an extention to NRS 6, the guide *Veiledning Pensjonsforutsetninger*

[1] Norwegian Accounting Standard 6

NASB (2009b) is a more accurate description of all the assumptions mentioned above. The following sections will go deeper into the conditions for wage growth and job shifts.

5.1 Wage growth

Wage growth is defined as the sum of inflation and real wage development. Estimates on expected wage growth over a longer time period is connected to a considerable level of uncertainty. Norwegian monetary policy is to keep the long-term inflation at 2.50 %, but it will fluctuate, and how much is dependent on the terms of the pension agreement. For further calculations, expected inflation at 2.25 % is used. A prediction of the expected development in real wage is 1.75 %. This is for all employees, all ages and in addition to 1.75 %, employees up to the age of 45 get a 0.50 % career supplement(or an average addition of 0.25 % for all employees). In NASB (2009b), firms with a different age profile than an average at 45 years are encouraged to take it into consideration and raise/reduce the level of expected real wage growth development. When calculating pension liabilitites, only wage development on an aggregated level is usually considered.

Since both of the age profiles used in the calculations are 10 years below/above 45, this should technically result in some regulation of the expected real wage growth. For example, 2.00 % for a young employee and 1.50 % for an old employee. To keep things simple, 1.75 % is used for all employees. Adding the components together makes expected average wage growth for an employee 4.00 %. In the GablerWassum model $\alpha = 0.4512$ and the inflation is 2.00 %. As a basis for the discount factor, numbers from NASB (2009b) appendix I is used.

To compare the two models, pension liabilities connected to an individual with a certain age and time to retirement is considered. What kind of expenses do employers need to calculate with if this person is employed? How does age play a part in this? The young employee is 35 years and

Table 5.1: *Estimated pension liabilites.*

	Employee age 35	Employee age 55
PBO today	1.509.304	3.873.335
PBO GW	1.293.405	3.465.301
Ratio	16.692 %	11.177 %

recieves a salary of 450 000. The old employee is 55 years old and is currently receiving 700 000. From table 5.1 one can see how the PBO's are under these assumptions. With today's assumptions, the results observed in the previous chapters correspond. The PBO is higher when the NRS guidelines are used, and for a young employee, the expenses is 16 % higher when compared to

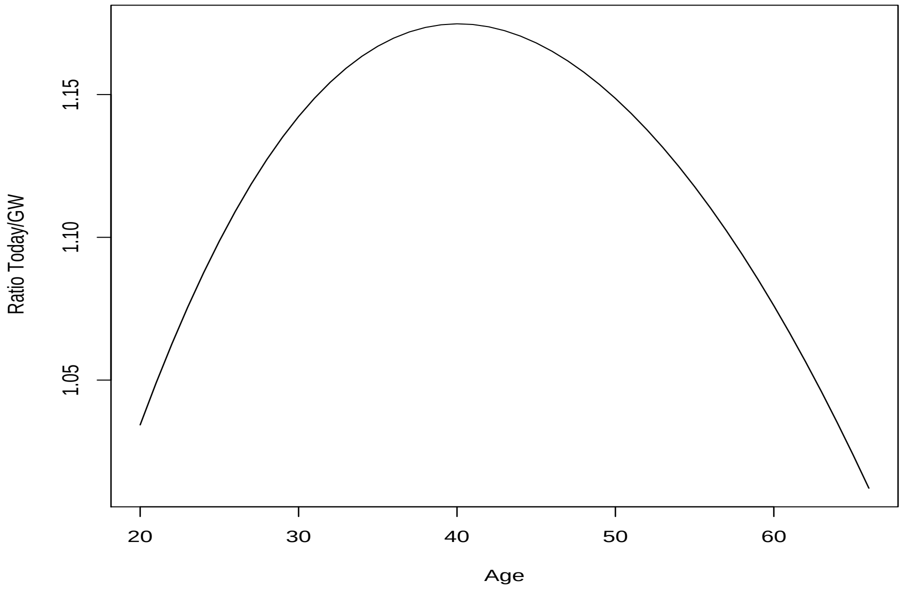


Figure 5.1: *Ratio between pension liabilites estimated with today's model and the GablerWassum model.*

the GablerWassum PBO. This ratio is displayed for all ages in figure 5.1. With a peak at age 42, that is when the GablerWassum wage growth is equal today's wage growth, one can se how the effect of implementing this model is for different ages. A young employee, under the age of 42, has a higher wage growth than today's model in the beginning of his or her career, see figure 2.2 on page 17. Since the GablerWassum wage growth after the age of 42 is lower than today's model, the total average for a young employee is more similar to today's model. Despite this, the GablerWassum model is still estimating lower pension liabilities than today's model. On the other hand, if an employee with an age close to 42 is to be considered on the other hand, the results are even more in favour of the GablerWassum model. After the age of 42, the GablerWassum wage growth is at all times lower than today's wage growth, resulting in lower pension liabilities. The effects of implementing the GablerWassum model is thus greatest for the age groups around 42, but all age groups will benefit from this. This is good news, since the average age for Norwegian firms' employees are just above 40.

It is also of interest to see how the pension liabilites for different branches are affected. With reference to the section above, an employee aged 42 with wage 600 000 is considered. In table 5.2 on the following page one can

Table 5.2: *Estimated pension liabilites.*

	PBO today	PBO GW	Ratio
Finance	2.344.242	1.891.645	23.926 %
Energy	2.344.242	2.170.849	7.987 %
Shipping	2.344.242	2.015.874	16.289 %
Industrial workers	2.344.242	1.877.921	24.831 %
Office workers	2.344.242	2.273.409	3.115 %
Academics/engineers	2.344.242	2.048.568	14.433 %

see that the consequences are somewhat scattered but extensive for certain branches, for example Industrial workers. Depending on the branch, the consequences of implementing the GablerWassum model for each branch can be analysed further by looking at figure 5.2 on the next page. Here, the ratios between today's PBO and the GablerWassum PBO are plotted for each branch. Finance has the same shape of the curve as the original GablerWassum model, where the highest ratio is for age 39. Also, Energy is similar with a peak at age 42 but with an overall lower ratio for all ages. The model for Shipping is linear and has in general low wage growth for all ages, resulting in a high ratio for young employees. Industrial workers are the employees with the lowest average wage growth and it is similar to the Shipping ratios, both are high. Office workers and Academics/engineers are different from the other branches in the way that they have ratios for young employees lower than 1. In these cases, today's model actually estimates too low pension liabilities. Although a firm in one of these branches will not gain by implementing this model, it is also important to make sure that the firm is able to meet future expenses. Also, the age distribution of the firm will have an effect on the grand total.

A ratio deviating from 1, that is higher or lower is in both cases an indication of the need for a more adequate wage growth model. To tailor a model depending on the branch is what seems to be necessary.

5.2 Job shift

When a person quits a job, a paid-up policy is issued if the conditions for this are satisfied. As seen in equation 4.1 on page 35 this value is based on wage at resignation, not retirement as in equation 1.1 on page 7. When exit rates are included, it is still the wage at retirement which is considered. Exit rates are treated in the same way as mortality in this case. In NASB (2009b) section 5.4 voluntary job shift is briefly discussed. How a firm wishes to incorporate this is based on the particular state of the firm. A suggestion in the guide is to analyse job shifts in the last 5 to 10 years. With background in historical data it is then possible to say something about job

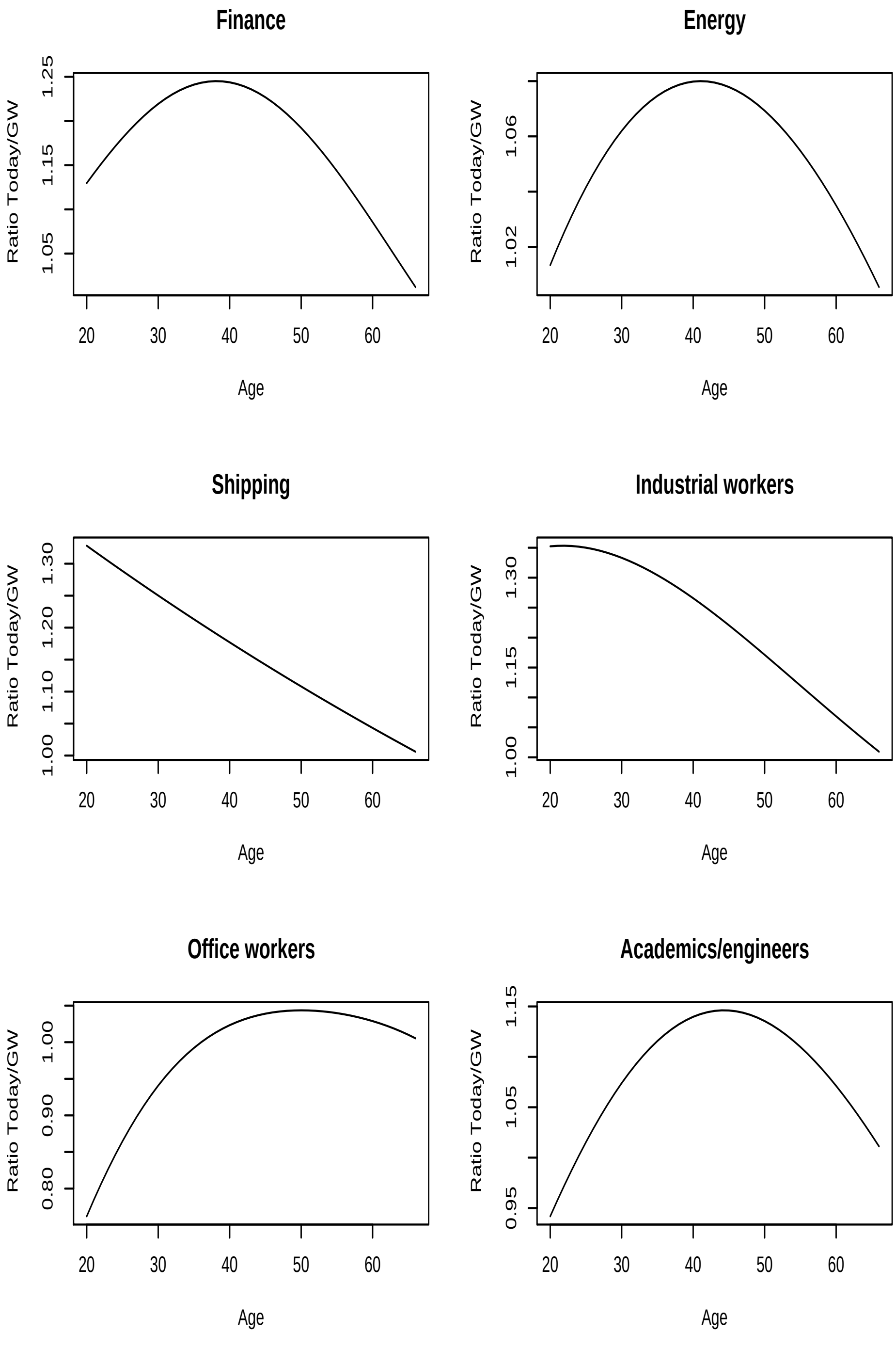


Figure 5.2: *Ratio between pension liabilites estimated with today's model and the GablerWassum model.*

Table 5.3: *Rates from NASB used in today's calculations.*

Exit rates		
15-45	46-60	61-67
2.5 %	1.5 %	0 %

shift for different age groups.

Today's model uses constant rates divided on three age intervals. At first glance, the difference between the numbers in table 5.3 and figure 4.3 on page 38 are quite extensive.

From the results in chapter 4 it is obvious that with a young age profile in the firm, this means a greater reduction in the costs than with an old age profile. With similar assumptions as those in the section above, but with exit rates in addition, the results are those in table 5.4. The consequence of not implementing exit rates for a young employee is 5.42 % higher costs. With a ratio above 0 % there is no doubt that the effects are evident, though highest for the young employees. The effect which still remains at the age of 55 is only 0.72 %. In figure 5.3 on the next page, the entire range for all ages is visible. With a ratio above 15 % for an employee at the age of 20, it is obvious that the younger the age distribution in the firm, the greater the gain of implementing this model.

Dividing these results on branches gives the results in table 5.5 on the facing page. Here, both a young and an old employee is considered to get an idea how age plays a part. Because of the way exit rates distribute with age, pension liabilities for young employees experience a greater effect. Age definitely plays an important part here. Also, the liabilities for older employees have a positive effect on this, though not to the same extent. A plot of the ratios for each branch is given in figure 5.4 on page 54. The same trend is visible for all branches, the ratios are converging to 1, but some faster than others. For exampel Academics/engineers have higher exit rates than Shipping for young ages, but since Academics/engineers is converging faster than Shipping, the average exit rate for Shipping is higher. Compared to the original GablerWassum model, all branches except Shipping and Academics/engineers are relatively similar. That is, both in shape and average exit rates for all ages.

Table 5.4: *Estimated pension liabilites with exit rate probabilities.*

	Employee age 35	Employee age 55
PBO today	1.120.951	2.925.704
PBO GW	1.063.311	2.903.421
Ratio	5.42 %	0.76 %

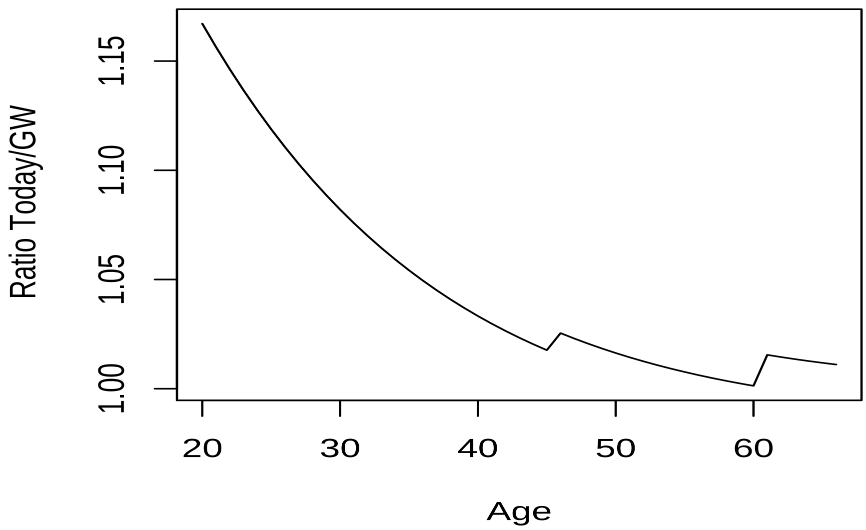


Figure 5.3: *Ratio between pension liabilites estimated with today's model and the GablerWassum model.*

Table 5.5: *Estimated pension liabilites.*

	Employee age 35			Employee age 55		
	PBO today	PBO GW	Ratio	PBO today	PBO GW	Ratio
Finance	1.120.951	1.064.495	5.30 %	2.925.704	2.873.276	1.82 %
Energy	1.120.951	1.081.764	3.62 %	2.925.704	2.933.055	-0.26 %
Shipping	1.120.951	992.129	12.98 %	2.925.704	2.865.639	2.09 %
Industrial workers	1.120.951	1.021.467	9.73 %	2.925.704	2.926.085	-0.02 %
Office workers	1.120.951	1.042.799	7.49 %	2.925.704	2.912.342	0.45 %
Academics/engineers	1.120.951	1.024.577	9.40 %	2.925.704	2.855.676	2.45 %

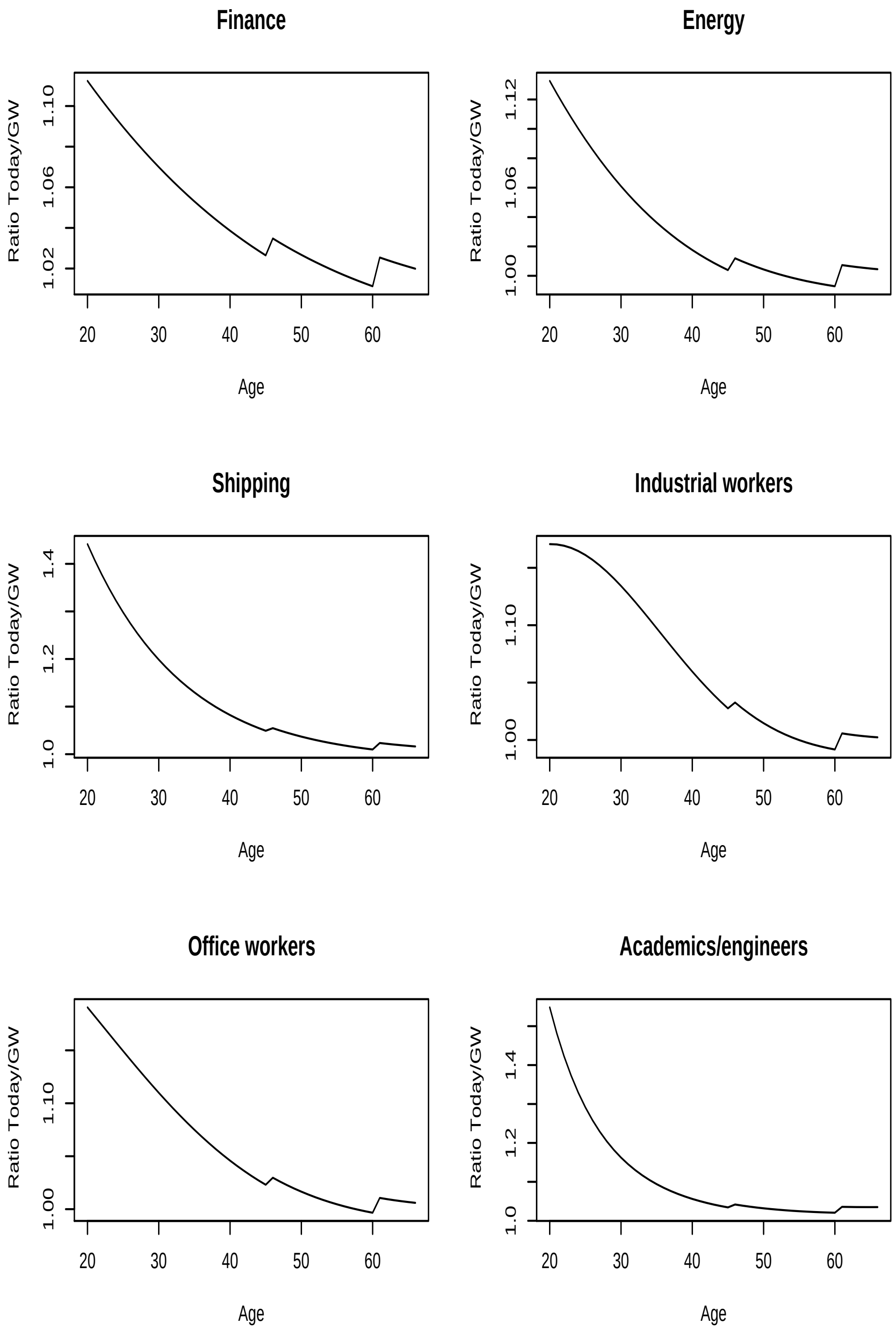


Figure 5.4: *Ratio between pension liabilites estimated with today's model and the GablerWassum model.*

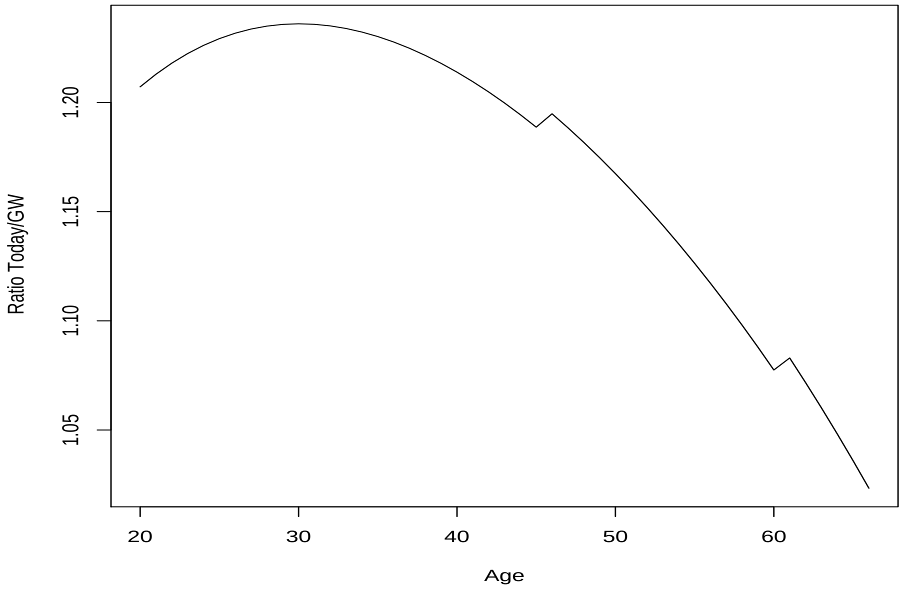


Figure 5.5: *Ratio between pension liabilites estimated with today's model and the GablerWassum model.*

5.3 Comments

Combining both the exit rate model and wage growth model, the ratios between today's model and the original GablerWassum model on age are those in figure 5.5. Here, the age with the highest reduction is 31 and the curve has a similar shape as the curves in section 5.2 caused by the stepwise construction of today's exit rates. With a ratio of approximately 24 % for this age, a firm would save considerably if the age distribution is young. Compared to figure 5.1 the greatest effect of adding exit rates is an increase in the ratio for young ages.

Again, it is important to remember that the NRS calculations are just a snapshot and only valid at the time of calculation. The market will change instantly.

Chapter 6

Conclusions

The main purpose with this thesis was to explore how different population sizes affected the calibration of the GablerWassum wage growth model and job shift model. To analyse this, the pension liabilities with corresponding uncertainties were computed and compared to each other. It was also of interest to analyse how great these effects were, compared to today's model.

In chapter 3 wage growth was analysed. Even though the coefficients seemed to be very uncertain, the effects on the wage growth curve itself was of less importance. Also, there was a difference between old and young coefficients. With this in mind, it is interesting to draw the parallel to the new national insurance shceme. The Norwegian population is currently 4.9 millions[1] and aging. It is also a fact that the young people in the population are using more time before they start their career. If one was dependent on a population size of one million to get a satisfactory level of uncertainty, this could be a problem with a Norwegian population in the current state. From the results, this was not necessary and a curve based on an old population of ten thousand proved to be good enough. Compared to today's assumptions, we also observed a reduction in pension liabilites when the GablerWassum model was assesed. Looking at the wage path uncertainty compared to the uncertainty when reestimating from historical data, we can see that there is a much higher deviation connected to the wage growth a person recieves than the deviation we observe when calibrating the model.

The uncertainty in chapter 4 was much lower compared to the deviation observed in chapter 3. This is visible in the pension liabilites with corresponding deviations. Also, there is a difference in the pension liabilites using the two age distributions. Although old coefficients have a higher deviation than young coefficients similar to the wage growth coefficients, the correlation influences the curves such that the job shift curves are very similar. A population size of five thousand proved to be sufficient. With these results, the pension liabilities were less than today's liabilites. The

[1] `http://www.ssb.no/befolkning/`

difference between the two models were relatively large, both how the exit rates distribute and the level of exits.

In both the wage growth and exit rate discussion, dividing the analysis on branches showed how the uncertainty affected pension liabilites with different coefficients. With the assumptions done in chapter 3 and 4, the uncertainty was acceptable with a population size of ten thousand and five thousand. With a more accurate evaluation, firms are able to know the future expenses more certainly. This is in line with the new regulatory requirements of Solvency II which will be implemented at the end of 2012. The GablerWassum model will help identify, measure, and manage risk levels in a better way.

Based on the analysis in chapter 5, we are able to say something about the effects of implementing the GablerWassum model. With knowledge about a firm's age distribution and affiliated branch of the economy, the firm can adapt the most suitable model.

Over the last year, a lot of countries have experienced some effects of the recent finance crisis. While most countries had a negative growth in wages and increase in the unemployment rate, Norway was not affected by the crisis as seriously as other countries[2]. An increase in the number of bankruptcies and unemployment have been reported along with the lowest interest rate level in Norwegian history ever. This is also the general trend in the world where the interest rate level is even lower and unemployment is still increasing. Norway holds a unique position because of the oil wealth. Because of this, a special behavior is sometimes visible in the Norwegian economy even when the world economy is unstable.

With this in mind, there are several interesting possibilities for further research. Looking at historical data from the past two years would be a good way to see how well the model estimates wage growth and exit rates after a finance crisis. We can also use the data from an existing population further back in time and see how wage growth and exit rates behave under normal circumstances in the economy. One could also add the amount from the NIS for realistic numbers in the analysis.

From the note *The GablerWassum wage models* and the analysis done in this thesis we know that the pension liabilitites are more accurately evaluated when age is taken into account. Accurately is a word which can imply both higher and lower pension liabilities. Thus, the intention of implementing this model is in the best interest of both the firm and its employees.

[2] Ministry of Finance, press release no.:48/2009, 15.05.2009

Appendix A

R code

Age profiles

```
############################################################################
##Criteria to find a valid age profile:
##- integrate to 1
##- maximum point on curve = peak age
##- can not have a negative population
############################################################################
equa=matrix(nrow=3,ncol=3)
b.mat=c(1,0,0)

peak=35
from=20
to=66

equa[1,1]=((to^3)/3)-((from^3)/3);equa[1,2]=((to^2)/2)-((from^2)/2);equa[1,3
                                                                   ]=to-from
equa[2,1]=2*peak;equa[2,2]=1;equa[2,3]=0
equa[3,1]=(to+1)^2;equa[3,2]=(to+1);equa[3,3]=1
coeff=solve(equa)%*%b.mat

c.a=coeff[1,1]
c.b=coeff[2,1]
c.c=coeff[3,1]
age=from:to
ageP=(c.a*(age^2))+(c.b*age)+c.csplit.screen(c(1,2))
screen(1)
matplot(age,ageP*50000,xlim=c(from,to),type='l',xlab='Age',ylab='Count', mai
                                                   n='35 years',lwd=3,lty=2)
integrand1 <- function(x) { (c.a*(x^2))+(c.b*x)+c.c }
EX.2 <- integrate(integrand1, from, to)
EX.2 <- EX.2$value
```

Projecting present values: Parameter uncertainty

```
##Number of simulations
```

```
Msim=1000

##Total size of population
M=5000

##Number of individuals distributed on age
n.indi35=ageP35*M
n.indi55=ageP55*M
n.indi35=round(n.indi35)
n.indi55=round(n.indi55)

testport=10000
rows35=max(round(n.indi35))
rows55=max(round(n.indi55))

sigma=0.15
##Gender neutral model GW
n.b0=0.1394;n.b1=-0.0045;n.b2=0.0071;n.b3=-0.0047

DB.level=2/3
alpha=0.4512
wage=350000

interest=c(2.05,2.31,2.41,2.48,2.56,2.65,2.73,2.82,2.91,3.01,3.10,3.19,3.24,
3.30,3.35,3.36,3.36,3.37,3.37,3.38,3.36,3.35,3.33,3.31,3.29,3.28,3.28,3.27,3
.26,3.26,3.25,3.24,3.24,3.23,3.23,3.22,3.22,3.22,3.21,3.21,3.20,3.20,3.20,3.
                                                      19,3.19,3.19,3.19)/100
n.g=alpha*(exp(n.b0+(n.b1*age)+((n.b2/100)*(age^2))+((n.b3/(100^2))*(age^3))
                                                                         )-1)
today=rep(0.0175,length(age))
############################################################################
## Initialize
############################################################################

b0s.young=rep(0,Msim)
b1s.young=rep(0,Msim)
b2s.young=rep(0,Msim)
b3s.young=rep(0,Msim)

b0s.old=rep(0,Msim)
b1s.old=rep(0,Msim)
b2s.old=rep(0,Msim)
b3s.old=rep(0,Msim)

matrix.young=matrix(nrow=length(age),ncol=Msim)
matrix.old=matrix(nrow=length(age),ncol=Msim)
matrix.GW=matrix(nrow=length(age),ncol=Msim)
matrix.today=matrix(nrow=length(age),ncol=Msim)

prodByAge<-function(vec){
vec.tmp=rep(0,length(vec))
for(i in 1:length(age)){
age.tmp=prod(1+vec[i:length(vec)])
vec.tmp[i]=age.tmp
```

```
}
return(vec.tmp)
}#end prodByAge

############################################################################
## Main loop
############################################################################

for(sim in 1:Msim) {

n.ghat.sim.young=matrix(data=NA,nrow=rows35,ncol=length(age))
n.ghat.sim.old=matrix(data=NA,nrow=rows55,ncol=length(age))
temp.nghatsim.young=rep(0,length(age))
temp.nghatsim.old=rep(0,length(age))

for(i in 1:length(age)) {

if(n.indi35[i]>=1) {
n.ghat.sim.young[1:n.indi35[i],i]=n.b0+(n.b1*age[i])+((n.b2/100)*(age[i]^2))
                     +((n.b3/(100^2))*(age[i]^3))+(sigma*rnorm(n.indi35[i]))
}#end if

if(n.indi55[i]>=1) {
n.ghat.sim.old[1:n.indi55[i],i]=n.b0+(n.b1*age[i])+((n.b2/100)*(age[i]^2))+(
                       (n.b3/(100^2))*(age[i]^3))+(sigma*rnorm(n.indi55[i]))
}#end if

##Mean, each age
temp.nghatsim.young[i]=mean(n.ghat.sim.young[1:n.indi35[i],i])
temp.nghatsim.old[i]=mean(n.ghat.sim.old[1:n.indi55[i],i])
}#end for

##The foundation for each regression line is 47 points
temp.data.young=as.data.frame(cbind(temp.nghatsim.young,age,age^2,age^3))
temp.data.old=as.data.frame(cbind(temp.nghatsim.old,age,age^2,age^3))

temp.data.old=temp.data.old[-1,]
names(temp.data.young)<-c('value','age','age2','age3')
names(temp.data.old)<-c('value','age','age2','age3')

resultat.young=lm(value~age+age2+age3,data=temp.data.young)
resultat.old=lm(value~age+age2+age3,data=temp.data.old)

##Model free estimates
sim.b0.young=summary(resultat.young)$coefficients[1]
sim.b1.young=summary(resultat.young)$coefficients[2]
sim.b2.young=summary(resultat.young)$coefficients[3]*100
sim.b3.young=summary(resultat.young)$coefficients[4]*(100^2)
b0s.young[sim]=sim.b0.young
b1s.young[sim]=sim.b1.young
b2s.young[sim]=sim.b2.young
b3s.young[sim]=sim.b3.young

sim.b0.old=summary(resultat.old)$coefficients[1]
```

```
sim.b1.old=summary(resultat.old)$coefficients[2]
sim.b2.old=summary(resultat.old)$coefficients[3]*100
sim.b3.old=summary(resultat.old)$coefficients[4]*(100^2)
b0s.old[sim]=sim.b0.old
b1s.old[sim]=sim.b1.old
b2s.old[sim]=sim.b2.old
b3s.old[sim]=sim.b3.old

##Pension liabilities, this simulation
reserve.young=rep(NA,length(age))
reserve.old=rep(NA,length(age))

reserve.GW=rep(NA,length(age))
reserve.today=rep(NA,length(age))

n.simhat.young=alpha*(exp(b0s.young[sim]+(b1s.young[sim]*age)+((b2s.young[si
                     m]/100)*(age^2))+((b3s.young[sim]/(100^2))*(age^3)))-1)
n.simhat.old=alpha*(exp(b0s.old[sim]+(b1s.old[sim]*age)+((b2s.old[sim]/100)*
                               (age^2))+((b3s.old[sim]/(100^2))*(age^3)))-1)

for(i in 1:length(age)){

part2.young=DB.level*singlePlot[age[i]]*prodByAge(n.simhat.young)[i]*((1/(1+
                                               interest[48-i]))^(67-age[i]))
part2.old=DB.level*singlePlot[age[i]]*prodByAge(n.simhat.old)[i]*((1/(1+inte
                                                   rest[48-i]))^(67-age[i]))

part2.GW=DB.level*singlePlot[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]
                                                         ))^(67-age[i]))
part2.today=DB.level*singlePlot[age[i]]*prodByAge(today)[i]*((1/(1+interest[
                                                        48-i]))^(67-age[i]))

reserve.young[i]=part2.young
reserve.old[i]=part2.old
reserve.GW[i]=part2.GW
reserve.today[i]=part2.today

}#end for

matrix.young[,sim]=reserve.young
matrix.old[,sim]=reserve.old
matrix.GW[,sim]=reserve.GW
matrix.today[,sim]=reserve.today


}#end for sim


## PBO CALCULATIONS

wage.vec=c(rep(350000,8),rep(400000,8),rep(450000,8),rep(550000,8),rep(60000
                                                       0,8),rep(650000,7))

TEST5matrixPBO.young=matrix(nrow=length(age),ncol=Msim)
```

```
TEST5matrixPBO.old=matrix(nrow=length(age),ncol=Msim)
TEST6matrixPBO.young=matrix(nrow=length(age),ncol=Msim)
TEST6matrixPBO.old=matrix(nrow=length(age),ncol=Msim)
TEST7matrixPBO.young=matrix(nrow=length(age),ncol=Msim)
TEST7matrixPBO.old=matrix(nrow=length(age),ncol=Msim)
TEST8matrixPBO.young=matrix(nrow=length(age),ncol=Msim)
TEST8matrixPBO.old=matrix(nrow=length(age),ncol=Msim)

##Values based on known parameters
PBO.GW=rep(NA,length(age))
PBO.today=rep(NA,length(age))

for(i in 1:Msim){

TEST5matrixPBO.young[,i]=TEST5matrix.young[,i]*wage.vec
TEST5matrixPBO.old[,i]=TEST5matrix.old[,i]*wage.vec
TEST6matrixPBO.young[,i]=TEST6matrix.young[,i]*wage.vec
TEST6matrixPBO.old[,i]=TEST6matrix.old[,i]*wage.vec
TEST7matrixPBO.young[,i]=TEST7matrix.young[,i]*wage.vec
TEST7matrixPBO.old[,i]=TEST7matrix.old[,i]*wage.vec
TEST8matrixPBO.young[,i]=TEST8matrix.young[,i]*wage.vec
TEST8matrixPBO.old[,i]=TEST8matrix.old[,i]*wage.vec

}#end for

PBO.GW=TEST5matrix.GW[,1]*wage.vec
PBO.today=TEST5matrix.today[,1]*wage.vec

## TEST PORTFOLIO

TOTALreserve35.today=rep(NA,Msim)
TOTALreserve55.today=rep(NA,Msim)

TOTALreserve35.GW=rep(NA,Msim)
TOTALreserve55.GW=rep(NA,Msim)

TOTALreserve35.young=rep(NA,Msim)
TOTALreserve55.young=rep(NA,Msim)
TOTALreserve35.old=rep(NA,Msim)
TOTALreserve55.old=rep(NA,Msim)

new.n.indi35=ageP35*testport
new.n.indi55=ageP55*testport

matrix.young=TEST8matrix.young
matrix.old=TEST8matrix.old

for(sim in 1:Msim){

reserve35.today=rep(NA,length(age))
reserve55.today=rep(NA,length(age))

reserve35.GW=rep(NA,length(age))
reserve55.GW=rep(NA,length(age))
```

```
reserve35.young=rep(NA,length(age))
reserve55.young=rep(NA,length(age))
reserve35.old=rep(NA,length(age))
reserve55.old=rep(NA,length(age))

for(i in 1:length(age)){

reserve35.today[i]=new.n.indi35[i]*wage.vec[i]*matrix.today[i,sim]
reserve55.today[i]=new.n.indi55[i]*wage.vec[i]*matrix.today[i,sim]

reserve35.GW[i]=new.n.indi35[i]*wage.vec[i]*matrix.GW[i,sim]
reserve55.GW[i]=new.n.indi55[i]*wage.vec[i]*matrix.GW[i,sim]

reserve35.young[i]=new.n.indi35[i]*wage.vec[i]*matrix.young[i,sim]
reserve55.young[i]=new.n.indi55[i]*wage.vec[i]*matrix.young[i,sim]
reserve35.old[i]=new.n.indi35[i]*wage.vec[i]*matrix.old[i,sim]
reserve55.old[i]=new.n.indi55[i]*wage.vec[i]*matrix.old[i,sim]

}# end for

TOTALreserve35.today[sim]=sum(reserve35.today)
TOTALreserve55.today[sim]=sum(reserve55.today)

TOTALreserve35.GW[sim]=sum(reserve35.GW)
TOTALreserve55.GW[sim]=sum(reserve55.GW)

TOTALreserve35.young[sim]=sum(reserve35.young)
TOTALreserve55.young[sim]=sum(reserve55.young)
TOTALreserve35.old[sim]=sum(reserve35.old)
TOTALreserve55.old[sim]=sum(reserve55.old)

}#end for
```

Projecting present values: Wage path uncertainty

```
##Number of simulations
Msim=1000

M=10000
sigma=0.15
n.b0=0.1394;n.b1=-0.0045;n.b2=0.0071;n.b3=-0.0047

DB.level=2/3
wage.vec
interest
matrix.nsimhat=matrix(ncol=Msim,nrow=length(age))

pliab35=rep(NA,Msim)
pliab55=rep(NA,Msim)

pliab35.true=rep(NA,Msim)
```

```
pliab55.true=rep(NA,Msim)

for(sim in 1:Msim) {

for(l in 1:length(age)){

eps=sigma*rnorm(1)
n.simhat=alpha*(exp(n.b0+(n.b1*age[l])+((n.b2/100)*(age[l]^2))+((n.b3/(100^2
                                                    ))*(age[l]^3))+(eps))-1)
matrix.nsimhat[l,sim]=n.simhat
}#end for

##Original GablerWassum
n.hat=alpha*(exp(n.b0+(n.b1*age)+((n.b2/100)*(age^2))+((n.b3/(100^2))*(age^3
                                                                      )))-1)
##Pension liabilites, this simulation

pboGW35=rep(NA,length(age))
pboGW55=rep(NA,length(age))

##Today's model
pboGW35.true=rep(NA,length(age))
pboGW55.true=rep(NA,length(age))

new.n.indi35=ageP35*M
new.n.indi55=ageP55*M

for(j in 1:length(age)){

part2=DB.level*singlePlot[age[j]]*wage.vec[j]*prodByAge(matrix.nsimhat[,sim]
                                  )[j]*((1/(1+interest[48-j]))^(67-age[j]))

pboGW35[j]=new.n.indi35[j]*part2
pboGW55[j]=new.n.indi55[j]*part2

#Original GablerWassum
part2.true=DB.level*singlePlot[age[j]]*wage.vec[j]*prodByAge(n.hat)[j]*((1/(
                                             1+interest[48-j]))^(67-age[j]))

pboGW35.true[j]=new.n.indi35[j]*part2.true
pboGW55.true[j]=new.n.indi55[j]*part2.true

}#end for

pliab35[sim]=sum(pboGW35)
pliab55[sim]=sum(pboGW55)

pliab35.true[sim]=sum(pboGW35.true)
pliab55.true[sim]=sum(pboGW55.true)

}#end for sim

############################################################################
##Today's model
```

```
##############################################################################
today.y=rep(0.02, length(age))
today.y[1:26]=0.025
today.o=rep(0.015, length(age))
today.o[1:26]=0.02
today.factor.y=prodByAge(today.y)
today.factor.o=prodByAge(today.o)

today.factor=prodByAge(rep(0.0175,length(age)))

pboTO35=rep(NA,length(age))
pboTO55=rep(NA,length(age))

for(j in 1:length(age)){
part2.y=DB.level*singlePlot[age[j]]*wage.vec[j]*today.factor[j]*((1/(1+inter
                                                   est[48-j]))^(67-age[j]))
part2.o=DB.level*singlePlot[age[j]]*wage.vec[j]*today.factor[j]*((1/(1+inter
                                                   est[48-j]))^(67-age[j]))

pboTO35[j]=new.n.indi35[j]*part2.y
pboTO55[j]=new.n.indi55[j]*part2.o

}#end for

TOpliab35=sum(pboTO35)
TOpliab55=sum(pboTO55)
WAGE5TOpliab35=TOpliab35
WAGE5TOpliab55=TOpliab55
```

Job shift model: Parameter uncertainty

```
n.g=alpha*(exp(n.b0+(n.b1*age)+((n.b2/100)*(age^2))+((n.b3/(100^2))*(age^3))
                                                                         )-1)
n.b0=0.1394;n.b1=-0.0045;n.b2=0.0071;n.b3=-0.0047

#b0=-0.52260;b1=-0.05281;b2=-0.01138 # gender neutral
#b0=-1.31463;b1=-0.02913;b2=-0.01560 # finance
#b0=-0.89927;b1=-0.03647;b2=-0.04834 # energy
#b0=-2.47359;b1=0.08602;b2=-0.21325  # industrial workers
#b0=-1.09911;b1=-0.00293;b2=-0.08785 # office workers
#b0=2.54743;b1=-0.18200;b2=0.14039   # academics engin
b0=0.73283;b1=-0.07351               # shipping

hx=b0+(b1*age)#+((b2/100)*age^2)
wx=1/(1+(exp(-hx)))

Msim=1000
M=5000

DB.level=2/3
wage.vec=c(rep(350000,8),rep(400000,8),rep(450000,8),rep(550000,8),rep(60000
                                                        0,8),rep(650000,7))
```

```
N35=round(ageP35*M,0)
N55=round(ageP55*M,0)
wx.35=matrix(ncol=Msim,nrow=length(age))
wx.55=matrix(ncol=Msim,nrow=length(age))

b0.35=rep(NA,Msim)
b0.55=rep(NA,Msim)
b1.35=rep(NA,Msim)
b1.55=rep(NA,Msim)
b2.35=rep(NA,Msim)
b2.55=rep(NA,Msim)

Jpliab35.35=matrix(nrow=length(age),ncol=Msim)
Jpliab55.35=matrix(nrow=length(age),ncol=Msim)

Jpliab35.55=matrix(nrow=length(age),ncol=Msim)
Jpliab55.55=matrix(nrow=length(age),ncol=Msim)

new.n.indi35=round(ageP35*10000)
new.n.indi55=round(ageP55*10000)

for(sim in 1:Msim){

##Binomial data
quits35=rep(NA,length(age))
cont35=rep(NA,length(age))
quits55=rep(NA,length(age))
cont55=rep(NA,length(age))

for(i in 1:length(age)){

quits35[i]=rbinom(1,N35[i],wx[i])
cont35[i]=N35[i]-quits35[i]

quits55[i]=rbinom(1,N55[i],wx[i])
cont55[i]=N55[i]-quits55[i]
}#end for

#logistisk regresjon
expl=matrix(nrow=length(age),ncol=2)
expl[,1]=age
expl[,2]=age^2

logistfit35<-glm(cbind(quits35,cont35)~expl,family=binomial(link=logit))
logistfit55<-glm(cbind(quits55,cont55)~expl,family=binomial(link=logit))


b0.35[sim]=logistfit35$coefficients[1]
b1.35[sim]=logistfit35$coefficients[2]
b2.35[sim]=logistfit35$coefficients[3]*100
b0.55[sim]=logistfit55$coefficients[1]
b1.55[sim]=logistfit55$coefficients[2]
b2.55[sim]=logistfit55$coefficients[3]*100
```

```
hx.35=b0.35[sim]+(b1.35[sim]*age)+((b2.35[sim]/100)*age^2)
hx.55=b0.55[sim]+(b1.55[sim]*age)+((b2.55[sim]/100)*age^2)

wx.35[,sim]=1/(1+(exp(-hx.35)))
wx.55[,sim]=1/(1+(exp(-hx.55)))

###########################################################################
## Annuities
###########################################################################

tmp35.35=rep(NA,length(age)-4)
tmp55.35=rep(NA,length(age)-4)
tmp35.55=rep(NA,length(age)-4)
tmp55.55=rep(NA,length(age)-4)

for(i in 1:length(age)){

tmp35.35[i]=new.n.indi35[i]*(1-wx.35[i,sim])*DB.level*wage.vec[i]*singlePlot
            m[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

tmp55.35[i]=new.n.indi55[i]*(1-wx.35[i,sim])*DB.level*wage.vec[i]*singlePlot
            m[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

tmp35.55[i]=new.n.indi35[i]*(1-wx.55[i,sim])*DB.level*wage.vec[i]*singlePlot
            m[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

tmp55.55[i]=new.n.indi55[i]*(1-wx.55[i,sim])*DB.level*wage.vec[i]*singlePlot
            m[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

}#end for

Jpliab35.35[,sim]=tmp35.35
Jpliab55.35[,sim]=tmp55.35

Jpliab35.55[,sim]=tmp35.55
Jpliab55.55[,sim]=tmp55.55

}#end for hovedlkke

###########################################################################
## Today's model
###########################################################################
Jpliab.stat35=rep(NA,length(age))
Jpliab.stat55=rep(NA,length(age))
new.wx=c(rep(0.025,26),rep(0.015,15),rep(0,6))

for(i in 1:length(age)){

Jpliab.stat35[i]=new.n.indi35[i]*(1-new.wx[i])*DB.level*wage.vec[i]*singlePl
          otm[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

Jpliab.stat55[i]=new.n.indi55[i]*(1-new.wx[i])*DB.level*wage.vec[i]*singlePl
          otm[age[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))
```

```
}#end for

############################################################################
### Original GW model
############################################################################

Jpliab.GW35=rep(NA,length(age))
Jpliab.GW55=rep(NA,length(age))

for(i in 1:length(age)){

Jpliab.GW35[i]=new.n.indi35[i]*(1-wx[i])*DB.level*wage.vec[i]*singlePlotm[ag
                e[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))
Jpliab.GW55[i]=new.n.indi55[i]*(1-wx[i])*DB.level*wage.vec[i]*singlePlotm[ag
                e[i]]*prodByAge(n.g)[i]*((1/(1+interest[48-i]))^(67-age[i]))

}#end for
sum(Jpliab.GW35)
sum(Jpliab.GW55)
```

Appendix B

Algorithms

Algorithm 1 Reestimation of coefficients: Wage growth model

input: $b_0, b_1, b_2, b_3, \sigma, M, \epsilon$ and age profile {age profile: young, old}
initial: $N = M \times$ age profile
for i=0,...,1000 **do**
  initial: $\hat{g}_x^{sim}, \bar{g}_x^i$
  for j=20,...,66 **do**
    $\hat{g}_j^{sim} = b_0 + b_1 j + (b_2/100)j^2 + (b_3/100^2)j^3 + \sigma\epsilon \times N$
    $\bar{g}_j^i = mean(\hat{g}_j^{sim})$
  end for
  {polynomial multiple regression:}
  $\bar{g}_x^i \sim age + age^2 + age^3$
  output: $\hat{b}_0^i, \hat{b}_1^i, \hat{b}_2^i, \hat{b}_3^i$
end for
output: $mean(\hat{b}_0), mean(\hat{b}_1), mean(\hat{b}_2), mean(\hat{b}_3)$

Algorithm 2 Reestimation of coefficients: Job shift model

input: $b_0, b_1, b_2, \sigma, M, \omega_x$ and age profile {age profile: young, old}
initial: $N = M \times$ age profile
for i=0,...,1000 **do**
  initial: quitsAt35, contAt35
  for j=20,...,66 **do**
    $quitsAt35_j \sim Bin(N35_j, \omega_{x,j})$
    $contAt35_j = N35_j - quitsAt35_j$
  end for{logistic regression:}
  $\bar{\omega}_x \sim age + age^2$
  output: $\hat{b}_0^i, \hat{b}_1^i, \hat{b}_2^i$
end for
output: $mean(\hat{b}_0), mean(\hat{b}_1), mean(\hat{b}_2)$

Appendix C

Tables and figures

Table C.1: *The GablerWassum wage growth coefficients based on different branches.*

	b_0	b_1	b_2	b_3
Finance				
Male	0.1743	-0.0055	0.0058	-0.0016
Female	-0.0188	0.0058	-0.0146	0.0100
Both	0.0620	0.0013	-0.0071	0.0062
Energy				
Male	0.0558	0.0004	-0.0028	0.0027
Female	0.0614	0.0001	-0.0009	-
Both	0.0733	-0.0009	0.0004	-
Shipping				
Male	0.0430	-0.0003	-	-
Female	0.0317	0.0001	-	-
Both	0.0300	0.0000	-	-
Industrial workers				
Male	0.1617	-0.0092	0.0200	-0.0146
Female	0.0676	-0.0008	0.0001	-
Both	0.0715	-0.0016	0.0013	-
Office workers				
Male	0.2730	-0.0119	0.0216	-0.0137
Female	0.1631	-0.0046	0.0061	-0.0033
Both	0.2518	-0.0104	0.0179	-0.0109
Academics/engineers				
Male	0.2394	-0.0102	0.0181	-0.0119
Female	0.0731	-0.0003	-0.0004	-
Both	0.1181	-0.0021	0.0009	-

Table C.2: *The GablerWassum exit rate coefficients based on different branches.*

	b_0	b_1	b_2
Finance			
Male	-0.67171	-0.05985	0.02083
Female	-2.08179	0.00840	-0.06129
Both	-1.31463	-0.02913	-0.01560
Energy			
Male	-0.93391	-0.03619	-0.04910
Female	-0.47829	-0.05614	-0.01881
Both	-0.89927	-0.03647	-0.04834
Shipping			
Male	1.30339	-0.08654	-
Female	-1.14535	-0.03387	-
Both	0.73283	-0.07351	-
Industrial workers			
Male	-1.84956	0.05937	-0.19261
Female	0.22777	-0.06597	-
Both	-2.47359	0.08602	-0.21325
Office workers			
Male	-0.98234	-0.00796	-0.08823
Female	-1.36772	0.00407	-0.06952
Both	-1.09911	-0.00293	-0.08785
Academics/engineers			
Male	2.73682	-0.18883	0.14858
Female	1.23735	-0.11432	0.04638
Both	2.54743	-0.18200	0.14039

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