Switched Surplus-Based Distributed Security Dispatch for Smart Grid with Persistent Packet Loss

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***Abstract*—Communication network failure, e.g., persistent packet loss, may considerably affect the safe and stable operation of smart grids. This may degrade the performance of various components and applications including energy management and economic dispatch. We propose a switched surplus-based distributed security dispatch approach to cope with the persistent packet loss under unreliable communication network environment. First, we jointly consider the packet loss sequence and the dynamic triggering sequence to define actual affected periods caused by the persistent packet loss. Then, we outline an incentive scheme, integrate primal-dual analysis and eigenvalue perturbation theory to design the switched surplus-based distributed security dispatch algorithm. Further, we design a dynamic triggering mechanism that enables the proposed algorithm to dynamically switch to different modes according to the change in network state. With those components, the proposed method offers strong robustness against persistent packet loss. In addition, we provide the convergence and optimality proofs of the algorithm. Finally, simulation results are provided to validate the proposed method, and to demonstrate its effectiveness.**

***Index Terms*—Economic dispatch, persistent packet loss, dynamic trigger, distributed optimization.**

I. Introduction

As the next generation electric power system, smart grid has been envisioned to achieve optimal, flexible, secure, and reliable system operations [1]. Internet of Things (IoT) is one of the promising technologies that facilitates the transformation from traditional power systems to smart grids, thereby promoting their development. For instance, IoT enables smart meters to establish flexible communication among themselves, thereby facilitating distributed operations within the smart grid, such as distributed economic dispatch and distributed frequency control. In this paper, we focus on studying the economic dispatch problem (EDP). The EDP is an important research topic in smart grid, which has gained extensive interest from the research community in the recent years[2]. It studies how to allocate energy generation devices or loads to minimize the cost to the system, while satisfying the supply-demand balance and local boundary constraints. In essence, EDP is a system decision problem that requires designing effective optimization algorithms.

Currently, the algorithms for EDP can be roughly classified as centralized algorithms and distributed algorithms. The traditional centralized algorithms, including genetic algorithm [3], Lagrange multiplier method [4]-[5] and gradient descent method [6], etc., have been thoroughly studied in the past. However, the major drawback associated with these algorithms is the centralized execution that requires a centralized controller to collect all information, and to compute the optimal solutions. With the high integration of IoT and smart grids, there is an increasing trend to offload the heavy computing task to the edge [7], [8]. Meanwhile, the distributed participants are unwilling to expose personal date to the centralized controller for both security and profit. In this situation, the conventional centralized algorithms may not be relevant for many applications in the smart grid including energy management and economic dispatch.

To tackle this challenge, the distributed algorithms have been proposed, mainly based on the blockchain technology, distributed alternating direction method of multipliers (ADMM) and consensus-based method. First of all, the essence of the blockchain technology is to use only one ledger/database [9], [10]. The decentralized characteristic makes blockchain technology more advantageous in data transmission and consistency operation. However, this method requires public storage and sharing. Secondly, the distributed ADMM aims to utilize the dual-descent technique to optimize both the original variable and the dual variable alternatively [11]. For instance, the distributed ADMM was employed to solve demand-side EDP with reduced energy, where the combine heat and power (CHP) unit is taken into account. By incorporating dynamic average consensus variables into ADMM, [12] presented a distributed consensus-ADMM algorithm to achieve peer-to-peer energy trading without knowing the number of system nodes. Thirdly, the major concept of consensus-based method is to establish distributed Lagrangian dual variables, and to enable them to converge to the same value to fulfil the optimization conditions. Based on this concept, multiple types of consensus-based algorithms have been developed, e.g., distributed Newton descent methods [13], [14] and distributed neurodynamic methods [15], [16]. Recently, the surplus-based distributed dispatch method was proposed in [17], [18]. This method is designed by introducing an auxiliary variable (called surplus) instead of decision variable to perform distributed calculation, which possesses high privacy attribute.

The aforementioned literature provide solutions for EDP

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TABLE I: Comparative Technical Features of Previous Researches

Ref.	operation	Reduced communication	packet loss	economic dispatch
[3-5]	centralized			✓
[7, 9-13,16-18]	distributed			✓
[14-15]	distributed	✓		✓
[19-20]	distributed	✓	✓	
[21-24]	distributed		✓	✓
This paper	distributed	✓	✓	✓

under safe communication network. Note that the IoT network serves as the infrastructure for data transmission and information interaction within smart grids, which frequently encounter various safety-related concerns, one of which is the data packet loss [19]. The data packet loss is an unavoidable occurrence resulting from communication line uncertainties and random noise. This may drastically affect the operation performance for smart grid [20].

There is an increasing interest to investigate the effects of packet loss on EDP, and then to design corresponding resistance mechanisms. For instance, a Newton-consistent optimization algorithm was proposed in [21], for better robustness against bounded packet loss. However, the method is only suitable for unconstrained optimization problem. For constrained optimization problem, a robust distributed economic dispatch algorithm was proposed in [22], which takes the packet-dropping communication links into consideration. However, this method is built upon the assumption that the distributed communication network is strongly connected. To avoid this assumption, [23] found that the errors caused by the packet loss were accumulated for the λ-consensus algorithm. A compensation mechanism was presented to reduce the impact of packet loss. Most recently, literature [24] employed the distributed robust chance constrained optimization method to solve EDP with considerations of both packet loss and uncertainty in wind power generation. The existing studies [21], [22], [23], [24] have obtained good results to deal with the problem of packet loss. Nevertheless, they mainly focus on one-step or short-term packet loss. It is still a challenge to handel the persistent packet loss under worse communication channels.

We propose a switched surplus-based distributed security dispatch approach for the smart grid in an unreliable communication network. Compared to [3]-[5] that are centralized methods, this paper belongs to a distributed method. In contrast to [7], [9]-[13] and [16]-[18] that primarily focus on distributed optimization, this paper emphasizes on strategy making when communication failures occur and reduction of the communication cost. Although [14]-[15] use the event triggering strategies to reduce the frequency of communication, they cannot handle network failures caused by packet loss. Despite the contribution of [19]-[20] in reducing communication and handling packet loss, their approach cannot be directly applied to distributed economic dispatch. In comparison to the methods presented in [21]-[24], which suffer from high communication costs, our approach leverages system switching and event-triggered techniques to reduce communication expenses while ensuring fault-tolerant system operation. The distinctions between this paper and existing studies are summarized in Table I.

1) We define the actual safe and unsafe periods that take the coexistence of persistent packet loss sequence and event-triggering sequence into account, thus offering an analytic model to study the persistent packet loss, which is used designing the distributed dispatch approach.

2) We propose a switched surplus-based distributed security dispatch approach for smart grid under unreliable communication network. Compared to the existing surplus-based distributed dispatch approach [17]-[18], the proposed approach enables the utilization of switched system dynamics and estimation to defend persistent packet loss. Meanwhile, the high privacy can be maintained due to the surplus variable.

3) We embed an event-triggered based communication strategy into the proposed distributed dispatch approach. To reduce the communication expenditure, each agent only needs to asynchronously share information with its neighbors at discrete time if necessary. Moreover, theoretically analytic results are offered to verify the global convergence and optimality of the proposed dispatch approach with the event-triggered communication strategy.

The rest of this paper is organized as follows. In Section II, the persistent packet loss model is introduced and the EDP is formulated. The structure of the switched surplus-based distributed security dispatch approach is introduced in detail, and the proof of convergence and optimality is provided in Section III. In Section IV, simulation results are provided to validate the proposed method, and to demonstrate its effectiveness. Section V concludes this paper. Some frequently used notations are provided in Table II for quick reference.

II. Problem Formulation

A. Economic Dispatch: Problem Formulation

We consider that a distributed energy system consists of multiple distributed generators (DGs) and loads, each of which can be seen as an agent. For convenience of expression, $\mathbb{R}^{\aleph}$ and $\mathbb{R}^{+}$ denote the $\aleph$-dimensional real numbers and positive real numbers, respectively. $|z|$ is the modulus of z. $\|h\|_i$ denotes the i-norm of vector $h \subseteq \mathbb{R}^n$ or matrix $h \subseteq \mathbb{R}^{n\times n}$. $1_n(0_n)$ denotes an n-dimensional column vector of ones (zeros). The EDP focuses on finding the total operation cost (1), while meeting the global supply-demand balance constraint (2) and local operation constraints (3-4).

$$\min \quad Obj = \sum\nolimits_{i=1}^{n} f_i(x_i(t)) \tag{1}$$

$$s.t. \quad \sum\nolimits_{i=1}^{n} x_i(t) = \sum\nolimits_{i=1}^{n} \varsigma_i, \tag{2}$$

$$x_i^{\min} \le x_i(t) \le x_i^{\max}, \tag{3}$$

$$|x_i(t+1) - x_i(t)| \le x_i^{ramp}, \tag{4}$$

where $x_i \in \mathbb{R}$ and $\varsigma_i \in \mathbb{R}$ represent the power generation and local load of DG i, respectively; $f_i(x_i(t))$ is the cost function of DG i. In this paper, we only require that $f_i(x_i(t))$ is convex without any specific form. $x_i^{\min}$, x_i^{max} and x_i^{ramp} represent the lower bound, upper bound and ramp rate of $x_i(t)$, respectively.

TABLE II: Frequently Used Notation

Notation	Description	Notation	Description
$\mathbb{R}^{\aleph}$,$\mathbb{R}^{+}$	$\aleph$-dimensional real numbers and positive real numbers	t ,i, π_i	Indices of operation time, distributed generator, and inequality constraint for generator i
n, γ_i	Total number of distributed generator and the number of inequality constraint for generator i	x_i, x	Index and vector form of power output
x_i^{min}, x_i^{max}, x_i^{ramp}	Lower bound, upper bound and ramp rate of power output	ς_i	Local load of distributed generator i
λ_i, λ	Index and vector form of dual variable	y_i, y	Index and vector form of auxiliary variable
Ω_i, Ω	Index and vector form of feasible operation region	Θ_i, Θ	Index and vector form of dual variable feasible operation region
$f(\bullet)$, $g(\bullet)$, $h(\bullet)$	Indices of cost, inequality and equality function	a, b, c	Coefficients of cost function
T_1	The settling time of Lyapunov function V_1 and V_2	Ξ_{loss}, Ξ_{safe}	The total packet loss time, total safe time
$\bar{\Xi}_{loss}$, $\bar{\Xi}_{safe}$	The actual total packet loss time, actual total safe time	$\Gamma_{loss}(k)$, $\Gamma_{safe}(k)$	The start and end time of kth packet loss
$\bar{\Gamma}_{loss}(k)$, $\bar{\Gamma}_{safe}(k)$	The first trigger time and system returns time	$\Psi_{loss}(k)$, $\Psi_{safe}(k)$	The packet loss period and safe period
$\bar{\Psi}_{loss}(k)$, $\bar{\Psi}_{safe}(k)$	The actual packet loss period and actual safe period	k	Index of packet loss number
t_i^m	Index of mth triggering time inactual period	t_i^l	Index of last triggering time before packet loss occurs
k_1, k_2, e, u, v	Coefficients of the switched surplus-based algorithm	$sign(\bullet)$, $P_\Omega(\bullet)$, $\lvert\bullet\rvert$	Sign function, projection operation, and absolute value

To simplify notations, we define $h(x) = [h_1(x_1), h_2(x_2), \ldots, h_n(x_n)]$ and

$$\sum\nolimits_{i=1}^{n} h_i(x_i) = \sum\nolimits_{i=1}^{n} x_i(t) - \sum\nolimits_{i=1}^{n} \varsigma_i. \tag{5}$$

Further, (3) and (4) can be re-written in the form of a common inequality constraint, i.e.,

$$g_{\pi_i}(x_i(t)) \leq 0, \tag{6}$$

where $g_{\pi_i}(x_i(t))$ is the function of the inequality constraint and $\pi_i = 1, 2, \ldots, \gamma_i$ represents the index of inequality constraint for x_i. The expression of (6) has been widely used in the existing studies, e.g., [12], [14] and [17]. For the studied EDP, equation (6) comes from equations (3) and (4). Thus, we have $\gamma_i = 4$. The detailed mathematical expressions are given by

$$g_1(x_i(t)) = x_i(t) - x_i^{\max} \leq 0, \tag{7}$$
$$g_2(x_i(t)) = x_i^{\min} - x_i(t) \leq 0, \tag{8}$$
$$g_3(x_i(t)) = x_i(t+1) - x_i(t) - x_i^{ramp} \leq 0, \tag{9}$$
$$g_4(x_i(t)) = x_i(t) - x_i(t+1) - x_i^{ramp} \leq 0. \tag{10}$$

Note that our proposed method is capable of solving universal inequality constraint, i.e., equation (6), rather than being limited to the specific expressions as shown in equations (7-10). Next, we can get the feasible operation region of $x_i(t)$, defined as $\Omega_i = \{x_i | g_{\pi_i}(x_i) \leq 0\}$. The Lagrangian function of problem(1-4) is given by

$$\mathcal{L}(x,\lambda) = \sum_{i=1}^{n} \mathcal{L}_i(x_i,\lambda) = \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n} \lambda_i h_i(x_i), \tag{11}$$

where $x = [x_1, x_2, \ldots, x_n]$; $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]$ is the Lagrangian multiplier.

We denote λ_i^* as the optimal solution of λ_i, $x^* = [x_1^*, \ldots, x_n^*]$ is the saddle point of problem (1-4). The physical meaning of λ_i is the power price. Note that the power price is positive. Thus, $\lambda_i^* > 0$. The following Assumption 1 implies that x^* satisfies that the Slater condition as a Slater vector.

Assumption 1 (Slater condition)[4]: There exists a vector $x^* \in x$ such that $g_\pi(x^*) < 0$, *for all* $\pi = 1, 2, \ldots, \gamma$.

According to Lemma 1 in [4], λ_i^* lies in the compact set

$$\Theta_i = \{\lambda_i^* \in \mathbb{R}^+ | \|\lambda_i^*\|_2 \leq \frac{\sum_{i=1}^{n} f_i(x_i^*) - \bar{q}}{\bar{r}}\}, \tag{12}$$

where $\bar{r} = \min\{-g_{\pi_i}(x_i^*(t)) | 1 \leq i \leq n, 1 \leq \pi_i \leq \gamma_i\}$, $\bar{q} \leq \min_{x\in\Omega}\mathcal{L}(x,\lambda)$ and $\Theta = [\Theta_1, \Theta_2, \ldots, \Theta_n]$ is the feasible region of the Lagrangian multiplier. The significance of (12) is to show that the Laplace multiplier λ is positive and bounded in Θ.

To handle the inequality constraint (6), the projection operation is applied. The projection operation refers to a mathematical procedure that projects a given point onto a feasible set defined by a set of constraints. The purpose of the projection operation is to find the closest feasible point to the given point. The projection operation of x on $\Omega = \{x \in \mathbb{R} | x^{min} \leq x \leq x^{max}\}$ comes from $P_\Omega(x) = \arg\min_{\tilde{x}\in\Omega} \|x - \tilde{x}\|$.

B. Communication Network and Persistent Packet Loss Model

In this paper, the communication network is defined as a undirected graph $\mathcal{G} = (\mathcal{V}, \xi, \mathcal{A})$. Therein, $\mathcal{V} = \{1, 2, \ldots, n\}$ is the node set of the graph $\mathcal{G}$, each of which represents a DG.

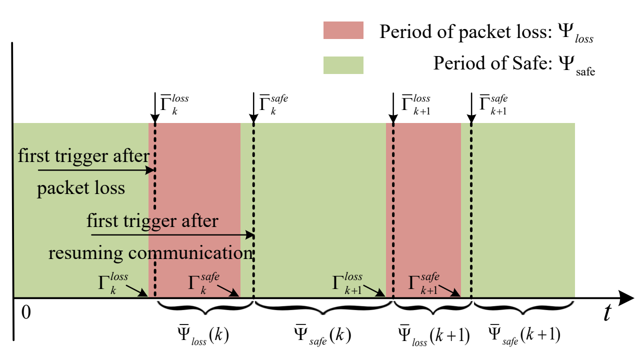

Fig. 1: Packet loss period and trigger period

$\xi \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of communication lines among the nodes. $\mathcal{A} = [a_{ij}] \subseteq \mathbb{R}^{n\times n}$ is the adjacency matrix, where a_{ij} is ith row and jth column element of $\mathcal{A}$. If node i can receive the information from node j, then there exists an edge $(j,i) \in \xi$ and $a_{ij} = 1$; otherwise, $a_{ij} = 0$. $N_i = \{j|(j,i) \in \xi\}$ is the neighbors of node i and $|N_i|$ denotes the number of neighbors. We denote $D = diag\{d_1, d_2, \ldots, d_n\}$ as the degree of node i, where $d_i = \sum_{j=1,i\neq j}^{n} a_{ij}$. And Laplace matrix can be represented as $L = D - \mathcal{A}$.

There are many factors (e.g., malicious cyberattacks) that may cause persistent packet loss, resulting in failure or even blackout. For the time period $[t_1, t_2]$, we define $\Xi_{loss}(t_1, t_2)$ and $\Xi_{safe}(t_1, t_2)$ as the total packet loss time and total safe time of $[t_1, t_2]$, respectively. As shown in Fig. 1, the start and end time of kth packet loss are defined as $\Gamma_{loss}(k)$ and $\Gamma_{safe}(k)$. Under this definition, each packet loss period and each safe period are further denoted as $\Psi_{loss}(k) = [\Gamma_{loss}(k), \Gamma_{safe}(k))$ and $\Psi_{safe}(k) = [\Gamma_{safe}(k), \Gamma_{loss}(k+1))$, respectively. Then, we can get $\Xi_{loss}(t_1, t_2) = \cup\Psi_{loss}(k) \cap [t_1, t_2)$ and $\Xi_{safe}(t_1, t_2) = \cup\Psi_{safe}(k) \cap [t_1, t_2)$. Since we adopt an event-triggered communication strategy to avoid unnecessary communication interaction among DGs, each DG suspends communication before an event is triggered. In fact, the failure or recovery of the network does not affect the communication network immediately. This is because each agent only shares information with its neighbors at the triggering time [19]. This implies that there exists the first trigger time $\bar{\Gamma}_{loss}(k)$ after $\Gamma_{loss}(k)$. Similarly, the system returns to normal state if an event is triggered at $\bar{\Gamma}_{safe}(k)$ after $\Gamma_{safe}(k)$. Therefore, the actual unsafe period caused by the packet loss and the actual safe period are $\bar{\Psi}_{loss}(k) = [\bar{\Gamma}_{loss}(k), \bar{\Gamma}_{safe}(k))$ and $\bar{\Psi}_{safe}(k) = [\bar{\Gamma}_{safe}(k), \bar{\Gamma}_{loss}(k+1))$, respectively. Then, to avoid misunderstanding, we further define $\bar{\Xi}_{loss}(t_1, t_2) = \cup\bar{\Psi}_{loss}(k) \cap [t_1, t_2)$ and $\bar{\Xi}_{safe}(t_1, t_2) = \cup\bar{\Psi}_{safe}(k) \cap [t_1, t_2)$.

In order to enable the objective function (1-4) to be solvable, the packet loss should neither last too long nor occur too frequently. Following from [25], we consider the following Assumption 2 to limit the duration $\Xi_{loss}(t_1, t_2)$ and number $N(t_1, t_2)$ of the packet loss.

Assumption 2 [25]: For duration $0 \leq t_1 \leq t_2$, there exists $\rho \in [0, 1]$ and $N_0, T_0, \sigma \in R^+$, such that

$$\Xi_{loss}(t_1, t_2) \leq \Xi_0 + \rho(t_2 - t_1), \tag{13}$$
$$N(t_1, t_2) \leq N_0 + \sigma(t_2 - t_1), \tag{14}$$

where ρ and σ are the constraint coefficients; $N(t_1, t_2)$ represents the number of switching from the safe period to the packet loss period in time period $[t_1, t_2]$; Ξ_0 and N_0 are the maximum persistent packet loss time and the maximum frequency that the system can tolerate, respectively. Meanwhile, the packet loss constraints can be separated into the time-ratio constraint, i.e., equation (13) and the dwell-time constraint, i.e., equation (14). Note that the performance of the distributed algorithm is related to the frequency and duration of the packet loss directly.

III. Switched Surplus-Based Distributed Economic Dispatch Strategy

A. Algorithm Design

Inspired by [17], we introduce an auxiliary variable, i.e., surplus, is considered designing the subsequent distributed dispatch algorithm, which is denoted as $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)] \in \mathbb{R}^n$. It locally records the state changes of power generation and communicates with neighbor nodes instead of the decision variable, i.e., power generation $x_i(t)$. By making use of dual variable $\lambda(t) = [\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t)] \in \mathbb{R}^n$ and incentive projection operation $k_1 sig(P_\Omega(x_i(t)) - x_i(t))^u$, we propose a switched surplus-based distributed security dispatch strategy to solve the EDP under persistent packet loss. The proposed algorithm encompasses dynamics $P1$ and dynamics $P2$ that are performed during $t \in \bar{\Psi}_{safe}$ and $t \in \bar{\Psi}_{loss}$, respectively.

$$P1: \begin{cases} \dot{x}_i(t) = -\alpha(t)(\nabla f_i(x_i(t)) + \lambda_i(t)) \\ \qquad + |-\alpha(t)(\nabla f_i(x_i(t)) + \lambda_i(t))| \cdot \\ \qquad sign(P_{\Omega_i}(x_i(t)) - x_i(t)) \\ \qquad + k_1 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^u \\ \qquad + k_2 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^v, & (15a) \\ \dot{\lambda}_i(t) = \Lambda_i(t) + \varepsilon y_i(t) + \alpha(t)(x_i(t) - \varsigma_i) \\ \qquad + \left|\tilde{\Lambda}_i(t) + \varepsilon y_i(t) + \alpha(t)(x_i(t) - \varsigma_i)\right| \cdot \\ \qquad sign(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t)) \\ \qquad + k_1 sig(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t))^u \\ \qquad + k_2 sig(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t))^v, & (15b) \\ \dot{y}_i(t) = \sum_{j\in N_i} a_{ij}(y_j(t_j^m) - y_i(t_i^m)) \\ \qquad -\varepsilon y_i(t) - \Lambda_i(t), & (15c) \\ \Lambda_i(t) = \sum_{j\in N_i} a_{ij}(\lambda_j(t_j^m) - \lambda_i(t_i^m)), & (15d) \end{cases}$$

$$P2: \begin{cases} \hbar\dot{x}_i(t) = -\alpha(t)(\nabla f_i(x_i(t)) + \lambda_i(t)) \\ \qquad + |-\alpha(t)(\nabla f_i(x_i(t)) + \lambda_i(t))| \cdot \\ \qquad sign(P_{\Omega_i}(x_i(t)) - x_i(t)) \\ \qquad + k_1 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^u \\ \qquad + k_2 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^v, & (16a) \\ \hbar\dot{\lambda}_i(t) = \tilde{\Lambda}_i(t) + \varepsilon y_i(t) + \alpha(t)(x_i(t) - \varsigma_i) \\ \qquad + \left|\tilde{\Lambda}_i(t) + \varepsilon y_i(t) + \alpha(t)(x_i(t) - \varsigma_i)\right| \cdot \\ \qquad sign(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t)) \\ \qquad + k_1 sig(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t))^u \\ \qquad + k_2 sig(P_{\Theta_i}(\lambda_i(t)) - \lambda_i(t))^v, & (16b) \\ \hbar\dot{y}_i(t) = \sum_{j\in N_i} a_{ij}(y_j(t_j^l) - y_i(t_i^l)) \\ \qquad -\varepsilon y_i(t) - \tilde{\Lambda}_i(t), & (16c) \\ \tilde{\Lambda}_i(t) = \sum_{j\in N_i} a_{ij}(\lambda_j(t_j^l) - \lambda_i(t_i^l)), & (16d) \end{cases}$$

where $\alpha(t)$ is a non-increasing gain variable satisfying $\int_0^\infty \alpha(t)dt = \infty$, and $\int_0^\infty \alpha^2(t)dt < \infty$. For example, we can choose $\alpha(t) = 10/(1+t)$. The function as $sig(a)^b =$

$[sign(a_1)|a_1|^b, \ldots, sign(a_n)|a_n|^b]^T$ comes from [17], where $sign(a)$ is the sign function with $sign(a) < 0$ if $a < 0$, $sign(a) = 0$ if $a = 0$ and $sign(a) > 0$ if $a > 0$. $P_{\Omega_i}(x_i)$ and $P_{\Theta_i}(\lambda_i)$ represent the projections of x_i and λ_i in the local constraint interval Ω_i and set Θ_i, respectively. $k_1 > 0$, $k_2 > 0$, $0 < \mu < 1$, and $v > 1$ are control variables. ε and $\hbar$ are positive constants. t_i^m represents mth triggering time in actual period. t_i^l refers to the last triggering time before packet loss occurs.

In the proposed algorithm, we design dynamics $P1$ and $P2$ to deal with the persistent packet loss, which can be adaptively switched according to the change of the network state. The decision variable x_i is calculated locally, which can protect the sensitive information well. The item $-\alpha(t)(\nabla f_i(x_i(t)) + \lambda_i(t))$ in (15a) is obtained by calculating the descent direction of x_i related to the dual problem of (1) with equality constraint 2. To further deal with inequality constraints (3), we employ the incentive projection operation. Specifically, the second term in (15a) enables the infeasible points projecting into the feasible region Ω_i. The items of $k_1 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^u$ and $k_2 sig(P_{\Omega_i}(x_i(t)) - x_i(t))^v$ enable the system converging to the feasible region in a fixed time. This will be described in detail in the next subsection. $\lambda_i(t)$ is the Lagrangian dual multiplier associated with equality constraint (2). The functionality of (15b) is to enable all local Lagrangian dual multipliers converging to the same value. Meanwhile, the design of $x_i(t) - \varsigma_i$ further enables the system to achieve global supply-demand balance after converging. Moreover, the incentive projection operation is applied in (15b) to enable λ_i operating in feasible region Θ_i. In (15c), the design of surplus variable $y_i(t)$ is to locally record the state changes of power generation. It converges to zero eventually, which will be introduced subsequently in Lemma 1. Further, owing to the utilization of consensus protocols (i.e., $\sum_{j\in N_i} a_{ij}(y_j(t_j^m) - y_i(t_i^m))$ and $\Lambda_i(t)$), we only need to share λ_i and y_i with neighbors and perform local calculation to solve problems (1-4). Regrading the differences between $P1$ and $P2$, $\Lambda_i(t)$ and $\sum_{j\in N_i} a_{ij}(y_j(t_j^m) - y_i(t_i^m))$ in (15) enable utilizing the information from neighbor nodes at the triggering time to update the system dynamics. Considering the packet loss, dynamics (16) is designed by making use of the last received information, e.g., $\lambda_i(t_i^l)$ and $y_i(t_i^l)$, to estimate the surplus mismatch $\sum_{j\in N_i} a_{ij}(y_j(t_j^l) - y_i(t_i^l))$ and $\tilde{\Lambda}_i(t)$.

Next, to reduce the communication cost, the event-triggered communication strategy is considered for $t \in \bar{\Psi}_{safe}$. In this scenario, each agent establishes communication interaction with neighbors and transmits the sharing variables (λ_i, y_i) in the triggering time. The triggering functions take the form:

$$\begin{aligned}\mathcal{C}_1(\lambda_i(t_i), y_i(t_i)) =& a_3(\varpi_1\|\lambda_i(t_i^m) - \lambda_i(t_i)\|^2 \\ &+ \varpi_2\|y_i(t_i^m) - y_i(t_i)\|^2 \\ &- \frac{1}{2a_1}\sum_{i\in N_i} a_{ij}\left\|\lambda_i(t_i^m) - \lambda_i(t_j^m)\right\|^2),\end{aligned} \tag{17}$$

$$\mathcal{C}_2(\lambda_i(t_i)) = |\lambda_i^m(t) - \lambda_i(t)| - \alpha(t)\pi e^{-\sigma t}, \tag{18}$$

$$\mathcal{C}_3(y_i(t_i)) = |y_i^m(t) - y_i(t)| - \alpha(t)\pi e^{-\sigma t}, \tag{19}$$

$$\mathcal{C} = \mathcal{C}_1(\lambda_i, y_i) + \mathcal{C}_2(\lambda_i) + \mathcal{C}_3(y_i), \tag{20}$$

where $\mathcal{C}_1(\lambda_i(t_i), y_i(t_i))$ is the triggering function related to both of $\lambda_i(t_i)$ and $y_i(t_i)$; $\mathcal{C}_2(\lambda_i(t_i))$ is the triggering function related to $\lambda_i(t_i)$ only; $\mathcal{C}_3(y_i(t_i))$ is the function related to $y_i(t_i)$ only; $\mathcal{C}$ refers to the overall triggering function, i.e., the sum of $\mathcal{C}_1(\lambda_i(t_i), y_i(t_i))$, $\mathcal{C}_2(\lambda_i(t_i))$, and $\mathcal{C}_3(y_i(t_i))$. $\varpi_1 = \left[\frac{2(a_1-1)}{a_1} + a_2\right]|N_i|$; $\varpi_2 = a_2|N_i|$; a_1, a_2, a_3, π, and σ are positive constants.

The motivation of (17)-(20) is to utilize the principles of event control to trigger state updates when necessary. For the triggering function (17), when the event is triggered as $t_i = t_i^m$, both $|\lambda_i(t_i^m) - \lambda_i(t_i)|^2$ and $|y_i(t_i^m) - y_i(t_i)|^2$ are reset to zero. Thus, at the instant t_i^m, it is evident that $\mathcal{C}1(\lambda i(t_i^m), y_i(t_i^m))= -\frac{1}{2a_1}\sum i \in N_i aij\left|\lambda_i(t_i^m) - \lambda_i(t_j^m)\right|^2 < 0$. As time progresses, the system operates without being triggered. As t_i increases, the difference between $\lambda_i(t_i^m)$ and $\lambda_i(t_i)$, as well as $y_i(t_i^m)$ and $y_i(t_i)$, changes gradually. When $\mathcal{C}_1(\lambda_i(t_i), y_i(t_i)) > 0$, information is exchanged using event triggering. The functionality of the triggering functions (18)-(19) are similar to (17). In this paper, since algorithm (15)-(16) is designed to be controlled by the three triggering functions simultaneously, we define the overall triggering function (20) as the sum of (17)-(19). Information exchange among neighbors occurs once $\mathcal{C} \geq 0$.

Based on (17-20), the next triggering time is determined as

$$t_i^{m+1} = \max\{t \geq t_i^m | \mathcal{C}_4(\lambda_i(t_i), y_i(t_i)) \leq 0\}. \tag{21}$$

When $t \in \bar{\Psi}_{loss}$, the event-triggering mechanism is out of work caused by the packet loss. To address the issue, we introduce a virtual-trigger strategy which makes each agent periodically attempt to communicate with its neighbor nodes. Once the communication succeeds, the network is considered to restore from packet loss state to safe state, that is $\bar{\Psi}_{loss} \to \bar{\Psi}_{safe}$. The virtual triggering time is determined by

$$t_i^{l+1} = t_i^l + \Delta T, \tag{22}$$

where ΔT is the virtual-triggering interval.

To clearly show the implementation procedure, the pseudocode of the proposed switched surplus-based distributed security dispatch algorithm with communication strategy (21-22) is summarized in Algorithm 1.

Next, a complexity analysis based on the switched surplus-based distributed security dispatch algorithm is presented. There are four main elements that affect the time complexity: (i) Calculation of the cost function $f_i(x_i)$ in (1). The dimension of $f_i(x_i)$ is $\aleph$ by $x_i \in \mathbb{R}^{\aleph}$, where $f_i(x_i)$ is equal to $a_i {x_i}^2(t) + b_i x_i(t) + c_i$. The time complexity for calculating the cost function $f_i(x_i)$ is $O(\aleph^2)$; (ii) Derivative of the cost function $f_i(x_i)$ in (15a) and (16a). The time complexity for the derivative of $f_i(x_i)$ is $O(\aleph^2)$; (iii) Calculation of y_i and λ_i in (15b-15c) and (16b-16c). The dimension of y_i and λ_i are both equal to x_i. The time complexity for calculating y_i and λ_i is $O(\aleph^2)$; (iv) The operation time t. We define the iteration step as Δt, then the number of iterations is $t/\Delta t$. With those components, the final time complexity of the proposed method is $O(\frac{t}{\Delta t}\aleph^2)$, which is consistent with the traditional economic dispatch algorithms as literature [17], [25].

Algorithm 1: Switched surplus-based algorithm

Input: Local operation constraints Ω_i, boundary constraints Θ_i of λ_i, local loads ς_i and Lagrange matrix L of communication network. Any admissible values of $x_i(0)$, $y_i(0)$, $\lambda_i(0)$ and parameters of $f_i(x_i)$.

Output: The optimal output of each power device.

```
some description;
while non-convergence do
    only if t = 0;
    Each power device exchange the information of
      y_i(t_0) and λ_i(t_0) to its neighbour nodes. Each
      node computes y_i and λ_i according to P1;
    if communication network safe then
        if C > 0 then
            1. The nodes complete the information
              exchange.;
            2. Calculate the decision variable x_i(t)
              using the moment information by P1.
        else
            The decision variable is calculated using
              the information from the last successful
              information exchange by P1.
        end
    end
    if communication network occurs persistent packet
      loss then
        The decision variable x_i(t) is calculated using
          the information of the last successful
          information exchange before the packet loss
          occurred by P2.
    end
end
```

Remark 1: Our proposed method can be used to solve a class of distributed optimization, i.e., equations (1), (2), and (6), with consideration of persistent packet loss. Note that both of distributed optimization and packet loss are typical problems in IoT-based systems, e. g., IoT-based smart grid considered in this paper. Thus, our proposed method can be easily employed to solve relevant optimization problems considering persistent packet loss in other IoT-based systems, if the studied problem can be abstracted in the form of equations (1), (2), and (6).

B. Optimality and Convergence Analysis

In this section, we propose three Lemmas to analyze the state valuation of the proposed algorithm during the actual safe period and the actual packet loss period. Then, the global optimality and convergence are provided in Theorem 1. To increase the readability of paper, $x_i(t)$ is abbreviated as x_i, $\lambda_i(t)$ as λ_i, $y_i(t)$ as y_i in the later.

Specifically, We define the Lyapunov function as $V_1 = (x_i - P_{\Omega_i}(x_i))^2$. Since $2(x_i - P_{\Omega_i}(x_i))(-\alpha(t)(\nabla f_i(x_i) + \lambda_i)) - 2\left|x_i - P_{\Omega_i}(x_i)\right|\left|-\alpha(t)(\nabla f_i(x_i) + \nabla h_i^T(x_i)\lambda_i)\right| \leq 0$ [31], we can get that Lie derivative of V_1 is

$$\begin{aligned}\dot{V}_1 =& 2(x_i - P_{\Omega_i}(x_i))\dot{x}_i \\ =& 2(x_i - P_{\Omega_i}(x_i))(-\alpha(t)(\nabla f_i(x_i) + \lambda_i)) \\ & - 2\left|x_i - P_{\Omega_i}(x_i)\right|\left|-\alpha(t)(\nabla f_i(x_i) + \nabla h_i^T(x_i)\lambda_i)\right| \\ & - 2k_1|x_i - P_{\Omega_i}(x_i)|^{u+1} \\ & - 2k_2|x_i - P_{\Omega_i}(x_i)|^{v+1} \\ \leq& - k_1 V_1^{\frac{u+1}{2}} - k_2 V_1^{\frac{v+1}{2}}. \end{aligned} \tag{23}$$

Recalling the studied problem (1-4), it is not diffcult to verify that $f_i(x_i)$, $g_i(x_i)$ are strongly convex and Lipschitz continuous. Then, it can be derived from [26] that V_1 converges to 0 in the fixed time when the settling time is defined as $T_1 \leq \frac{1}{k_1(1-\mu)} + \frac{1}{k_2(1-v)}$. Further, we can see that $x_i = P_{\Omega_i}(x_i)$ for $t \geq T_1$. Therefore, the projection part of the algorithm can be ignored and the decision variable x_i can converge to the constraint set in the fixed time T_1. Similarly, we choose the Lyapunov function as $V_2 = (\lambda_i - P_{\Theta_i}(\lambda_i))^2$. Since V_1 and V_2 hold the same structure. The proof process of V_1 can be applied to V_2. Thus, we can get the local Lagrangian multiplier λ_i can converge to the constraint set in the fixed time T_1, i.e., $\lambda_i = P_{\Theta_i}(\lambda_i)$.

Based on the separation principle [27], the fixed-time projection can be achieved. Then, for $t = \{t | t > T_1, t \in \bar{\Psi}_{safe}\}$, dynamics $P1$ can be rewritten as

$$P3: \begin{cases} \dot{x}_i = -\alpha(t)(\nabla f_i(x_i) + \lambda_i), \\ \dot{\lambda}_i = \tilde{\Lambda}_i + \varepsilon y_i + \alpha(t)(x_i - \varsigma_i), \\ \dot{y}_i = \sum\limits_{j \in N_i} a_{ij}(y_j(t_j^m) - y_i(t_i^m)) \\ \qquad -\varepsilon y_i - \tilde{\Lambda}_i \\ \tilde{\Lambda}_i = \sum\limits_{j \in N_i} a_{ij}(\lambda_j(t_j^m) - \lambda_i(t_i^m)), \end{cases} \tag{24}$$

We denote x, λ, λ^m, y, y^m $\nabla f(x)$, and ς as the column vector forms of x_i, λ_i, $\lambda_i(t_i^m)$ y_i, $y_i(t_i^m)$, $\nabla f_i(x_i)$, and ς_i, respectively. I is the Identity matrix.

By the definition of the adjacency matrix $\mathcal{A}$, the degree matrix D and the Laplace matrix L, we can get

$$\begin{aligned}\tilde{\Lambda}_i =& \sum_{j \in N_i} a_{ij}(\lambda_j(t_j^m) - \lambda_i(t_i^m)) \\ =& \sum_{j \in N_i} a_{ij}(\lambda_j(t_j^m)) - \sum_{j \in N_i} a_{ij}(\lambda_i(t_i^m)) \\ =& \mathcal{A}_i \lambda^m - D_i \lambda^m \\ =& -L_i \lambda^m, \end{aligned} \tag{25}$$

where $\mathcal{A}_i$, D_i and L_i represent ith row of $\mathcal{A}$, D and L, respectively. Similarly, it is not difficult to obtain $\sum_{j \in N_i} a_{ij}(y_j(t_j^m) - y_i(t_i^m)) = -L_i y^m$. We denote $\tilde{\Lambda}$ as the column vector form of $\tilde{\Lambda}_i$. Thus, we can get

$$\tilde{\Lambda} = \begin{bmatrix} \tilde{\Lambda}_1 \\ \tilde{\Lambda}_2 \\ \dots \\ \tilde{\Lambda}_n \end{bmatrix} = \begin{bmatrix} -L_1 \lambda^m \\ -L_2 \lambda^m \\ \dots \\ -L_n \lambda^m \end{bmatrix} = -L\lambda^m. \tag{26}$$

The second item in P3 can be further rewritten as

$$\begin{aligned}\dot{\lambda} = \begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \cdots \\ \dot{\lambda}_n \end{bmatrix} &= \begin{bmatrix} \tilde{\Lambda}_1 + \varepsilon y_1 + \alpha(t)x_1 - \varsigma_1 \\ \tilde{\Lambda}_2 + \varepsilon y_2 + \alpha(t)x_2 - \varsigma_2 \\ \cdots \\ \tilde{\Lambda}_n + \varepsilon y_n + \alpha(t)x_n - \varsigma_n \end{bmatrix} \\ &= \begin{bmatrix} \tilde{\Lambda}_1 \\ \tilde{\Lambda}_2 \\ \cdots \\ \tilde{\Lambda}_n \end{bmatrix} + \varepsilon \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix} + \alpha(t) \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} - \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \\ \cdots \\ \varsigma_n \end{bmatrix} \\ &= -L\lambda^m + \varepsilon I y + \alpha(t)(x - \varsigma). \end{aligned} \tag{27}$$

The third item in P3 can be further rewritten as

$$\begin{aligned}\dot{y} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \cdots \\ \dot{y}_n \end{bmatrix} &= \begin{bmatrix} -L_1 y_1^m \\ -L_2 y_2^m \\ \cdots \\ -L_n y_n^m \end{bmatrix} - \varepsilon \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix} - \begin{bmatrix} \tilde{\Lambda}_1 \\ \tilde{\Lambda}_2 \\ \cdots \\ \tilde{\Lambda}_n \end{bmatrix} \\ &= -Ly^m - \varepsilon I y + L\lambda^m \end{aligned} \tag{28}$$

So the second and third items in P3 are rewritten as

$$\begin{cases} \dot{\lambda} = -L\lambda^m + \varepsilon I y + \alpha(t)(x - \varsigma), \\ \dot{y} = -Ly^m - \varepsilon I y + L\lambda^m. \end{cases} \tag{29}$$

By appending $-L\lambda + L\lambda$ and $L\lambda - L\lambda - Ly + Ly$ to the right side of (29), respectively. We can get

$$\begin{cases} \dot{\lambda} = -L\lambda + \varepsilon I y + \alpha(t)(x - \varsigma) - L\lambda^m + L\lambda, \\ \dot{y} = L\lambda - Ly - \varepsilon I y + L\lambda^m - L\lambda - Ly^m + Ly. \end{cases} \tag{30}$$

We rewrite (30) as the following matrix form,

$$\begin{aligned}\begin{bmatrix} \dot{\lambda} \\ \dot{y} \end{bmatrix} &= \begin{bmatrix} -L & \varepsilon I \\ L & -L - \varepsilon \end{bmatrix} \begin{bmatrix} \lambda \\ y \end{bmatrix} + \alpha(t) \begin{bmatrix} x - \varsigma \\ 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} -L & 0 \\ L & -L \end{bmatrix} \begin{bmatrix} \lambda^m - \lambda \\ y^m - y \end{bmatrix}. \end{aligned} \tag{31}$$

First, we propose Lemma 1 to show the performance of λ_i and y_i during the actual safe period.

Lemma 1: We define $Z_i(t) = \lambda_i(t)$, $Z_{i+n}(t) = y_i(t)$ and $\hat{Z}(t) = \frac{1}{n}\sum_{i=1}^{2n} Z_i(t)$, $i = 1, \ldots, 2n$. When $t \in \bar{\Psi}_{safe}$, we can get: (i) $\lambda_i(t) \to \hat{Z}(t)$ as $t \to \infty$; (ii) $y_i(t) \to 0$ as $t \to \infty$.

Proof: For sake of expression, we define $h_i(x_i) = x_i - \varsigma_i$ and denote the column vector of $h_i(x_i)$ as $h(x)$. We also define that $p_i(t) = \lambda_i(t_i^m) - \lambda_i(t)$, $q_i(t) = y_i(t_i^m) - y_i(t)$, and $W(t) = [p_1(t), p_2(t), \ldots, p_n(t), q_1(t), q_2(t), \ldots, q_n(t)]$. Therein, the mathematical expression of matrixes M and $\bar{M}$ is given by

$$M = \begin{bmatrix} -L & \varepsilon I \\ L & -L - \varepsilon I \end{bmatrix}, \qquad \bar{M} = \begin{bmatrix} -L & 0 \\ L & -L \end{bmatrix}. \tag{32}$$

Equation (27) can be rewritten as

$$\begin{aligned}\begin{bmatrix} \dot{\lambda}_1 \\ \cdots \\ \dot{\lambda}_n \\ \dot{y}_1 \\ \cdots \\ \dot{y}_n \end{bmatrix} &= \begin{bmatrix} -L & \varepsilon I \\ L & -L - \varepsilon I \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \cdots \\ \lambda_n \\ y_1 \\ \cdots \\ y_n \end{bmatrix} + \alpha(t) \begin{bmatrix} x - \varsigma \\ 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} -L & 0 \\ L & -L \end{bmatrix} \begin{bmatrix} \lambda_1(t_1^m) - \lambda_1 \\ \cdots \\ \lambda_n(t_n^m) - \lambda_n \\ y_1(t_1^m) - y_1 \\ \cdots \\ y_n(t_n^m) - y_n \end{bmatrix}. \end{aligned} \tag{33}$$

Recalling the definitions of $Z(t)$, $W(t)$, $h(x)$, M and $\bar{M}$, equation (33) can be rewritten as

$$\dot{Z}(t) = MZ(t) + \bar{M}W(t) + \begin{bmatrix} \alpha(t)h(x(t)) \\ 0 \end{bmatrix}. \tag{34}$$

We consider that there is 1-order differential equation as $\dot{\chi}(t) = A\chi(t) + B(t)$. According to the solution of 1-order differential equation [32], we have $\chi(t) = e^{A(t-T_1)}\chi(T_1) + \int_{T_1}^{t} e^{A(t-\tau)}B(\tau)dt$. Note that equation (34) is also a 1-order differential equation. Thus, the solution of $Z(t)$ has the same form as the one of $\chi(t)$. Then, we replace $\chi(t)$ to $Z(t)$, A to M, and $B(t)$ to $\bar{M}W(t) + \begin{bmatrix} \alpha(t)h(x(t)) \\ 0 \end{bmatrix}$. The solution of (34) is given by

$$\begin{aligned} Z(t) =& e^{M(t-T_1)}Z(T_1) + \int_{T_1}^{t} e^{M(t-\tau)} \begin{bmatrix} \alpha(\tau)h(x(\tau)) \\ 0 \end{bmatrix} d\tau \\ &+ \int_{T_1}^{t} \bar{M}e^{M(t-\tau)}W(\tau)d\tau. \end{aligned} \tag{35}$$

1_n refers to the n-dimensional column vector, where each element is equal to 1. Recalling the definition of the Laplace matrix L, one obtains $1_n^T L = 0$. Then, recalling the define of M and $\bar{M}$, we can get

$$\begin{aligned} \frac{1}{n}(1_n^T, 1_n^T)M &= \frac{1}{n}(1_n^T, 1_n^T) \begin{bmatrix} -L & \varepsilon I \\ L & -L - \varepsilon I \end{bmatrix} \\ &= \frac{1}{n}\left[-1_n^T L + 1_n^T L \quad \varepsilon 1_n^T L - 1_n^T L - \varepsilon 1_n^T L \right] \\ &= \frac{1}{n}[0 \quad 0] = 0, \end{aligned} \tag{36}$$

$$\begin{aligned} \frac{1}{n}(1_n^T, 1_n^T)\bar{M} &= \frac{1}{n}(1_n^T, 1_n^T) \begin{bmatrix} -L & 0 \\ L & -L \end{bmatrix} \\ &= \frac{1}{n}\left[-1_n^T L + 1_n^T L \quad -1_n^T L \right] \\ &= \frac{1}{n}[0 \quad 0] = 0, \end{aligned} \tag{37}$$

$$(1_n^T, 1_n^T)\alpha(t) \begin{bmatrix} x - \varsigma \\ 0 \end{bmatrix} = 1_n^T \alpha(t)(x - \varsigma) = 1_n^T \alpha(t) h(x). \tag{38}$$

By multiplying $\frac{1}{n}(1_n^T, 1_n^T)$ on both sides of the (34), it follows from (36)-(38) that

$$\frac{1}{n}(1_n^T \dot{\lambda} + 1_n^T \dot{y}) = \frac{\alpha(t)}{n} 1_n^T h(x(t)). \tag{39}$$

Then, by calculating the integration of (39) from T_1 to t, we have

$$\int_t^{T_1} \frac{1}{n}(1_n^T \dot{\lambda}(\tau) + 1_n^T \dot{y}(\tau))d\tau = \int_t^{T_1} \frac{\alpha(\tau)}{n} 1_n^T h(x(\tau))d\tau \tag{40}$$

$$\Rightarrow \frac{1}{n}1_n^T\lambda\Big|_t^{T_1} + \frac{1}{n}1_n^T y\Big|_t^{T_1} = \int_t^{T_1}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau \tag{41}$$

$$\Rightarrow \frac{1}{n}1_n^T(\lambda(T_1)-\lambda(t)+y(T_1)-y(t)) = \int_t^{T_1}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau \tag{42}$$

According to the definition of $\hat{Z}(t)$, we have

$$\begin{aligned}\hat{Z}(t) =& \frac{1}{n}\sum\nolimits_{i=1}^{2n} Z_i(t)\\ =& \frac{1}{n}(Z_1(t)+Z_2(t)+\ldots+Z_n(t)\\ &+Z_{1+n}(t)+Z_{2+n}(t)+\ldots+Z_{2n}(t))\\ =& \frac{1}{n}(\lambda_1(t)+\lambda_2(t)+\ldots+\lambda_n(t)\\ &+y_1(t)+y_2(t)+\ldots+y_n(t))\\ =& \frac{1}{n}(1_n^T\lambda(t)+1_n^T y(t)).\end{aligned} \tag{43}$$

Thus, it follows from (42) that

$$\frac{1}{n}(1_n^T\lambda(T_1)+1_n^T y(T_1)) - \frac{1}{n}(1_n^T\lambda(t)+1_n^T y(t)) = \int_t^{T_1}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau, \tag{44}$$

$$\Rightarrow \hat{Z}(T_1)-\hat{Z}(t) = \int_t^{T_1}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau, \tag{45}$$

$$\Rightarrow \hat{Z}(t) = \hat{Z}(T_1) - \int_t^{T_1}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau, \tag{46}$$

$$\Rightarrow \hat{Z}(t) = \hat{Z}(T_1) + \int_{T_1}^{t}\frac{\alpha(\tau)}{n}\sum_{i=1}^n h_i(x_i(\tau))d\tau. \tag{47}$$

Let $[e^{Mt}]_i$ and $[e^{Mt}]_{ij}$ represent the ith row and the element in ith row and jth column of matrix e^{Mt}, respectively. According to the definitions of $Z(t)$ and $\hat{Z}(t)$, we can write $Z_i(t)=\lambda_i(t)$ and $\hat{Z}(T_1)=\frac{1}{n}(1_n^T,0_n^T)Z(T_1)$ for $i=1,\ldots,n$. Then, we have

$$\begin{aligned}&\left|\lambda_i(t)-\hat{Z}(t)\right|\\ \le& \left|([e^{M(t-T_1)}]_i - \frac{1}{n}(1_n^T,0_n^T))Z(T_1)\right|\\ &+\int_{T_1}^t \alpha(t)\left|([e^{M(t-\tau)}]_i-\frac{1}{n}(1_n^T,0_n^T))\begin{bmatrix} h(x(\tau))\\ 0_n\end{bmatrix}\right|d\tau\\ &+\int_{T_1}^t \bar{M}\left|[e^{M(t-\tau)}]_i\right|\cdot\|W(\tau)\|_1 d\tau\\ \le& \max_{1\le j\le 2n}\left|[e^{M(t-T_1)}]_{ij}-\frac{1}{n}\right|\|Z(T_1)\|\\ &+\int_{T_1}^t\alpha(\tau)\max_{1\le j\le 2n}\left|[e^{M(t-\tau)}]_{ij}-\frac{1}{n}\right|\|h(x(\tau))\|\,d\tau\\ &+\int_{T_1}^t\bar{M}\left|[e^{M(t-\tau)}]_i\right|\cdot\|W(\tau)\|_1 d\tau.\end{aligned} \tag{48}$$

According to (18), if $\mathcal{C}_2(\lambda_i(t_i))\le 0$, we have

$$|\lambda_i^m(t)-\lambda_i(t)|\le\alpha(t)\pi e^{-\sigma t}. \tag{49}$$

Recalling the definition of $W(t)$, for $i=1,\ldots,n$, it is not difficult to obtain that

$$\|W(\tau)\|_1\le n\alpha(t)\pi e^{-\sigma t}. \tag{50}$$

Thus, we can get

$$\begin{aligned}&\int_{T_1}^t\bar{M}\left|[e^{M(t-\tau)}]_i\right|\cdot\|W(\tau)\|_1 d\tau\\ &\le n\pi\bar{M}\int_{T_1}^t\left|[e^{M(t-\tau)}]_i\alpha(\tau)e^{-\sigma\tau}\right|d\tau.\end{aligned} \tag{51}$$

Since x_i is within the corresponding constraint set for $t\ge T_1$, there exists a constant ℓ satisfying $\|h(x)\|_1\le\ell$. Based on Lemma 4 in [17], we can get $\max_{1\le j\le 2n}\left|[e^{M(t-\tau)}]_{ij}-\frac{1}{n}\right|\le\gamma e^{-\beta(t-\tau)}$, where $\gamma,\beta\in\mathbb{R}^+$. According to (51), it follows from (48) that

$$\begin{aligned}\left|\lambda_i(t)-\hat{Z}(t)\right|\le&\gamma e^{-\beta(t-T_1)}\|Z(T_1)\|_1\\ &+l_1\gamma\int_{T_1}^t\alpha(\tau)e^{-\beta(t-\tau)}d\tau\\ &+n\pi\bar{M}\int_{T_1}^t\left|[e^{M(t-\tau)}]_i\alpha(\tau)e^{-\sigma\tau}\right|d\tau.\end{aligned} \tag{52}$$

Multiplying the term $\alpha(s)$ on both sides of (52) and integrating it from T_1 to ∞, one has

$$\begin{aligned}&\int_{T_1}^\infty\alpha(s)\left|\lambda_i(s)-\hat{Z}(s)\right|ds\\ \le&\int_{T_1}^\infty\alpha(s)\gamma e^{-\beta(s-T_1)}\|Z(T_1)\|_1 ds\\ &+\ell\gamma\int_{T_1}^\infty\alpha(s)\int_{T_1}^s\alpha(\tau)e^{-\beta(s-\tau)}d\tau ds\\ &+n\pi\bar{M}\int_{T_1}^\infty\alpha(s)\int_{T_1}^s\left|\alpha(\tau)[e^{M(s-\tau)}]_i e^{-\sigma\tau}\right|d\tau ds.\end{aligned} \tag{53}$$

Since $\alpha(t)$ is non-increasing and $\int_0^\infty\alpha^2(t)dt<\infty$, (53) satisfies

$$\begin{aligned}&\int_{T_1}^\infty\alpha(s)\gamma e^{-\beta(s-T_1)}\|Z(T_1)\|_1 ds\\ &\le\alpha(T1)\|Z(T_1)\|_1\int_{T_1}^\infty\gamma e^{-\beta(s-T_1)}ds\\ &=\frac{\gamma\alpha(T1)\|Z(T_1)\|_1}{\beta}\le\infty,\end{aligned} \tag{54}$$

$$\begin{aligned}&\ell\gamma\int_{T_1}^\infty\alpha(s)\int_{T_1}^s\alpha(\tau)e^{-\beta(s-\tau)}d\tau ds\\ &\le\ell\gamma\int_{T_1}^\infty e^{-\beta\theta}\int_{T_1}^\infty\alpha(s-\theta)\alpha(s-\theta)dsd\theta\\ &\le\frac{l_1\gamma}{\beta}\int_{T_1}^\infty\alpha^2(s)ds\le\infty,\end{aligned} \tag{55}$$

$$n\pi\bar{M}\int_{T_1}^{\infty}\alpha(s)\int_{T_1}^{s}\left|\alpha(\tau)[e^{M(s-\tau)}]_i e^{-\sigma\tau}\right|d\tau ds \leq n\pi\alpha(T_1)\frac{\bar{M}}{M_{ij}+\sigma}\int_{T_1}^{\infty}\alpha^2(s)ds \leq \infty. \quad (56)$$

Eqns. (54-56) imply that $\int_{T_1}^{\infty}\alpha(s)\left|\lambda_i(s)-\bar{Z}(s)\right|ds \leq \infty$ for $i=1,\ldots,n$. Thus, $\lambda_i(t) \to \hat{Z}(t)$, as $t\to\infty$. Similarly, since $Z_i(t)=y_i(t)$ as $i=n+1,\ldots,2n$, we have $\int_{T_1}^{\infty}\alpha(s)\left|y_i(s)\right|ds \leq \infty$. Thus, $y_i(t)\to 0$, as $t\to\infty$. The proof is completed. ■

Next, in Lemmas 2 and 3, we analyze the dynamics behavior of the proposed algorithm for $t\in\bar{\Psi}_{safe}$ and $t\in\bar{\Psi}_{loss}$, respectively. The equilibrium point of dynamics (15-16) is denoted as (x^*,λ^*,y^*), which means that

$$\begin{cases} 0=-\alpha(t)(\nabla f(x^*)+\lambda^*), \\ 0=L\lambda^*, \\ 0=y^*. \end{cases} \quad (57)$$

We further design five auxiliary variables, i.e.,

$$\begin{aligned}&\theta(t)=R^T(x(t)-x^*), \quad \eta(t)=R^T(\lambda(t)-\lambda^*),\\ &\delta(t)=R^T(y(t)-y^*), \quad e(t)=\eta(t^m)-\eta(t),\\ &z(t)=\delta(t^m)-\delta(t), \end{aligned} \quad (58)$$

where $R\in\mathbb{R}^{n\times n}$ satisfies $R^TR=RR^T=I_n$.

Note that $\|\theta(t)\|^2$, $\|\eta(t)\|^2$, $\|\delta(t)\|^2$ are continuous and differentiable, we define the following Lyapunov function

$$V_3(t)=\frac{1}{2}(\|\theta(t)\|^2+\|\eta(t)\|^2+\|\delta(t)\|^2). \quad (59)$$

Lemma 2: There exists a positive constant φ_1 such that for any $t\in\bar{\Psi}_{safe}$, dynamics P1(15) with the triggering strategy (21) enables

$$\dot{V}_3(t)\leq V_3(\bar{\Gamma}_k^{safe})\exp(-\varphi_1(t-\bar{\Gamma}_k^{safe})). \quad (60)$$

Proof: On the basis of Lemma 1 and (57), we can obtain

$$\begin{aligned}\dot{V}_3(t)=&\theta^T(t)\dot{\theta}(t)+\eta^T(t)\dot{\eta}(t)+\delta^T(t)\dot{\delta}(t)\\ =&\theta^T(t)R^T(-\alpha(t)\nabla f(x(t))-\alpha(t)\lambda(t))\\ &+\eta^T(t)R^T(-L\lambda(t^m)+\varepsilon y(t)+\alpha(t)(x(t)-\varsigma))\\ &+\delta^T(t)R^T(-Ly(t^m)-\varepsilon y(t)+L\lambda^m)\\ =&\alpha(t)\theta^T(t)R^T(-\nabla f(x(t))+\nabla f(x^*))\\ &-\alpha(t)\theta^T(t)\eta(t)-\eta^T(t)R^TL(\lambda(t^m)-\lambda^*)\\ &+\varepsilon\eta^T(t)\delta(t)+\alpha(t)\eta^T(t)\theta(t)\\ &-\delta^T(t)R^TL(y(t^m)-y^*)-\varepsilon\delta^2(t)\\ &+\delta^T(t)R^TL(\lambda(t^m)-\lambda^*). \end{aligned} \quad (61)$$

From the definition of convex function, we can get $\alpha(t)\theta^T(t)R^T(-\nabla f(x(t))+\nabla f(x^*)) \leq -a(t)\theta^T(t)\zeta\theta(t)$, where $\zeta=diag\{\zeta_i\}$ is the convex coefficient. Since $\mathcal{G}$ is connected, we have $\eta^T(t)L\eta(t)\geq\kappa_2\|\eta(t)\|^2$, $\lambda^T(t)L\lambda(t)\geq\kappa_2\|\lambda(t)\|^2$. According to the properties of the Laplace matrix, we can obtain $e^T(t)Le(t)\leq 2\sum_{i=1}^n|N_i|\cdot\|\lambda_i(t_i^m)-\lambda_i(t_i)\|^2$ and $z^T(t)Lz(t) \leq 2\sum_{i=1}^n|N_i|\cdot\|y_i(t_i^m)-y_i(t_i)\|^2$. We choose $a_1\geq\frac{1-\varepsilon}{2\kappa_2}$ and $a_2\geq\frac{\kappa_2}{\varepsilon}$, where κ_2 is the smallest nonzero eigenvalue of the Laplacian matrix. Then, we have

$$\begin{aligned}&-\delta^T(t)R^TL(y(t^m)-y^*)\\ \leq&\sum_{i=1}^n|N_i|\cdot\|y_i(t_i^m)-y_i(t_i)\|^2-\frac{1}{2}\delta^T(t)L\delta(t), \end{aligned} \quad (62)$$

$$\begin{aligned}&-\eta^T(t)R^TL(\lambda^m-\lambda^*)\\ =&\frac{a_1-1}{a_1}(\lambda(t^m)-\lambda(t))TL(\lambda(t^m)-\lambda(t))\\ &-\frac{1}{a_1}\lambda^T(t^m)L\lambda(t^m)-\frac{a_1-1}{a_1}\eta^T(t)L\eta(t)\\ \leq&\frac{2(a_1-1)}{a_1}\sum_{i=1}^n|N_i|\cdot(\lambda_i(t_i^m)-\lambda_i(t_i))^2\\ &-\frac{1}{2a_1}\sum_{i=1}^n\sum_{j\in N}a_{ij}\left\|\lambda_i(t_i^m)-\lambda_j(t_j^m)\right\|^2\\ &-\frac{a_1-1}{a_1}\eta^T(t)L\eta(t). \end{aligned} \quad (63)$$

Applying Young's inequality [30], we have the following facts that

$$\varepsilon\eta^T(t)\delta(t)\leq\frac{\varepsilon}{2}\|\eta(t)\|^2+\frac{\varepsilon}{2}\|\delta(t)\|^2, \quad (64)$$

$$\begin{aligned}&\delta^T(t)R^TL(\lambda(t^m)-\lambda^*)\\ =&\delta^T(t)R^TL(e(t)+\eta(t))\\ \leq&\frac{1}{2a_2}\delta^T(t)L\delta(t)+a_2\sum_{i=1}^n|N_i|\cdot\|\lambda_i(t_i^m)-\lambda_i(t_i)\|^2\\ &+\frac{1}{2}\delta^T(t)L\delta(t)+\frac{1}{2}\eta^T(t)L\eta(t). \end{aligned} \quad (65)$$

By substituting (62-65) in (61) , we further get

$$\begin{aligned}\dot{V}_3(t)\leq&-\alpha(t)\zeta\|\theta(t)\|^2\\ &-\left[(\frac{a_1-2}{2a_1})\kappa_2-\frac{\varepsilon}{2}\right]\|\eta(t)\|^2\\ &-\left[\frac{\varepsilon}{2}-\frac{\kappa_2}{2a_2}\right]\|\delta(t)\|^2\\ &+a_3(\varpi_1\|e(t)\|^2+\varpi_2\|z(t)\|^2\\ &-\frac{1}{2a_1}\sum_{i=1}^n\sum_{j\in N}a_{ij}\left\|\lambda_i(t_i^m)-\lambda_j(t_j^m)\right\|^2. \end{aligned} \quad (66)$$

According to the triggering strategy (21) and the triggering function (17), we have

$$\begin{aligned}&a_3(\varpi_1\|e(t)\|^2+\varpi_2\|z(t)\|^2\\ &-\frac{1}{2a_1}\sum_{i=1}^n\sum_{j\in N}a_{ij}\left\|\lambda_i(t_i^m)-\lambda_j(t_j^m)\right\|^2\\ =&\sum_{i=1}^n\mathcal{C}_1(\lambda_i(t_i),y_i(t_i))\leq 0. \end{aligned} \quad (67)$$

Based on (66) and (67), we can get

$$\begin{aligned}\dot{V}_3(t)\leq&-\alpha(t)\zeta\|\theta(t)\|^2\\ &-\left[(\frac{a_1-2}{2a_1})\kappa_2-\frac{\varepsilon}{2}\right]\|\eta(t)\|^2\\ &-\left[\frac{\varepsilon}{2}-\frac{\kappa_2}{2a_2}\right]\|\delta(t)\|^2. \end{aligned} \quad (68)$$

By selecting $\varphi_1 = \min\{\alpha(t)\zeta, (\frac{a_1-2}{2a_1})\kappa_2 - \frac{\varepsilon}{2}, \frac{\varepsilon}{2} - \frac{\kappa_2}{2a_2}\}$, (68) can be further organized as

$$\dot{V}_3(t) \le -\varphi_1 V_3(t), \tag{69}$$

which implies that (60) is satisfied for any $t \in \bar{\Psi}_{safe}$. The proof is thus completed. ■

Lemma 3: There exists a positive constant φ_2 such that for any $t \in \bar{\Psi}_{loss}$, dynamics P2 enables

$$\dot{V}_3(t) \le V_3(\bar{\Gamma}_k^{loss}) \exp(-\varphi_2(t - \bar{\Gamma}_k^{loss})). \tag{70}$$

Proof: The same variables and parameters in Lemma 2 are employed. we let t_i^l denote the moment of packet loss. The Lyapunov function (59) is applied for $t \in \bar{\Psi}_{loss}$. Then,

$$\begin{aligned}
\dot{V}_3(t) \le & -\alpha(t)\zeta\|\theta(t)\|^2 - \left[(\frac{a_1-2}{2a_1})\kappa_2 - \frac{\varepsilon}{2}\right]\|\eta(t)\|^2 \\
& - \left[\frac{\varepsilon}{2} - \frac{\kappa_2}{2a_2}\right]\|\delta(t)\|^2 \\
& + \sum_{i=1}^n \varpi_1 \left\|\lambda_i(t_i^l) - \lambda_i(\bar{\Gamma}_k^{loss}) + \lambda_i(\bar{\Gamma}_k^{loss}) - \lambda_i(t_i)\right\|^2 \\
& + \sum_{i=1}^n \varpi_2 \left\|y_i(t_i^l) - y_i(\bar{\Gamma}_k^{loss}) + \lambda_i(\bar{\Gamma}_k^{loss}) - \lambda_i(t_i)\right\|^2 \\
& - \sum_{i=1}^n \sum_{i \in N_i} \frac{1}{2a_1} \left\|\lambda_i(t_i^l) - \lambda_j(t_j^l)\right\|^2 \\
\le & -\alpha(t)\zeta\|\theta(t)\|^2 - \left[(\frac{a_1-2}{2a_1})\kappa_2 - \frac{\varepsilon}{2}\right]\|\eta(t)\|^2 \\
& - \left[\frac{\varepsilon}{2} - \frac{\kappa_2}{2a_2}\right]\|\delta(t)\|^2 \\
& + \sum_{i=1}^n \varpi_1 \left\|\lambda_i(t_i^l) - \lambda_i(\bar{\Gamma}_k^{loss})\right\|^2 \\
& + \sum_{i=1}^n \varpi_2 \left\|y_i(t_i^l) - y_i(\bar{\Gamma}_k^{loss})\right\|^2 \\
& - \frac{1}{2a_1} \sum_{i=1}^n \sum_{i \in N_i} a_{ij} \left\|\lambda_i(t_i^l) - \lambda_j(t_j^l)\right\|^2 \\
& + \varpi_1 \left\|\eta(\bar{\Gamma}_k^{loss})\right\|^2 + \varpi_1\|\eta(t)\|^2 \\
& + \varpi_2 \left\|\delta(\bar{\Gamma}_k^{loss})\right\|^2 + \varpi_2\|\delta(t)\|^2.
\end{aligned} \tag{71}$$

Since t_i^l belongs to the packet loss period, we can get (72) in light of (22), i.e.,

$$\begin{aligned}
\sum_{i=1}^n \varpi_1 \left\|\lambda_i(t_i^l) - \lambda_i(\bar{\Gamma}_k^{loss})\right\|^2 + \sum_{i=1}^n \varpi_2 \left\|y_i(t_i^l) - y_i(\bar{\Gamma}_k^{loss})\right\|^2 \\
- \sum_{i=1}^n \sum_{i \in N_i} \frac{1}{2a_1} \left\|\lambda_i(t_i^l) - \lambda_j(t_j^l)\right\|^2 \le 0.
\end{aligned} \tag{72}$$

By substituting (72) in (71) , we further get

$$\begin{aligned}
\dot{V}_3(t) \le & -\alpha(t)\zeta\|\theta(t)\|^2 + \varpi_3\|\eta(t)\|^2 + \varpi_4\|\delta(t)\|^2 \\
& + \varpi_1 \left\|\eta(\bar{\Gamma}_k^{loss})\right\|^2 + \varpi_2 \left\|\delta(\bar{\Gamma}_k^{loss})\right\|^2 \\
\le & \varphi_2 \max\{V_3(t), V_3(\bar{\Gamma}_k^{loss})\},
\end{aligned} \tag{73}$$

where $\varpi_3 = \varpi_1 - \left[(\frac{a_1-2}{2a_1})\kappa_2 - \frac{\varepsilon}{2}\right]$, $\varpi_4 = \varpi_2 - \left[\frac{\varepsilon}{2} - \frac{\kappa_2}{2a_2}\right]$ and $\varphi_2 = \max\{\alpha(t)\zeta, \varpi_1, \varpi_2, \varpi_3, \varpi_4\}$. From (73), we can easily obtain function (70) such that for any $t \in \bar{\Psi}_{loss}$. The proof is thus completed. ■

Finally, we prove the global convergence and optimality of proposed algorithm in Theorem 1.

Theorem 1: Suppose that $f_i(x_i(t))$ and $g_i(x_i(t))$ are strongly convex and Lipschitz continuous. If the system packet loss frequency and duration satisfy $\varphi_1((\sigma+\rho\Delta T)-1)+\varphi_2(\sigma+\rho\Delta T) < 0$, the switched surplus-based distributed economic dispatch strategy (15-16) can exponentially converge to the global optimal solution.

Proof: For $t \in [\bar{\Gamma}_{k-1}^{safe}, \bar{\Gamma}_k^{loss}]$, the proposed strategy enables $V_3(t)$ satisfying

$$\begin{aligned}
\dot{V}_3(t) \le & \exp(-\varphi_1(t - \bar{\Gamma}_{k-1}^{safe}))V(\bar{\Gamma}_{k-1}^{safe}) \\
\le & \exp(-\varphi_1(t - \bar{\Gamma}_{k-1}^{safe})) \\
& \exp(-\varphi_2(\bar{\Gamma}_{k-1}^{safe} - \bar{\Gamma}_{k-1}^{loss}))V(\bar{\Gamma}_{k-1}^{safe}) \\
& \dots \\
\le & \exp(-\varphi_1(t - t_0 - \bar{\Xi}_{loss}(t_0, t)) \\
& + \varphi_2 \bar{\Xi}_{loss} V(t_0)).
\end{aligned} \tag{74}$$

where t_0 is the initial time.

Similarly, for $t \in [\bar{\Gamma}_k^{loss}, \bar{\Gamma}_k^{safe}]$, we can obtain,

$$\begin{aligned}
\dot{V}_3(t) \le & \exp(-\varphi_1(t - \bar{\Gamma}_k^{safe}))V(\bar{\Gamma}_k^{loss}) \\
\le & \exp(-\varphi_1(t - \bar{\Gamma}_k^{safe})) \\
& \exp(-\varphi_2(\bar{\Gamma}_k^{loss} - \bar{\Gamma}_{k-1}^{safe}))V(\bar{\Gamma}_{k-1}^{safe}) \\
& \dots \\
\le & \exp(-\varphi_1(t - t_0 - \bar{\Xi}_{loss}(t_0, t)) \\
& + \varphi_2 \bar{\Xi}_{loss} V(t_0)).
\end{aligned} \tag{75}$$

Recalling the definition of $\bar{\Xi}_{loss}$, we can get $\bar{\Xi}_{loss}(t_0, t) \le \Xi_{loss}(t_0, t) + N(t_0, t)\Delta T$. It further follows from (74-75) that

$$\begin{aligned}
\dot{V}_3 \le & \exp(-\varphi_1(t - t_0) + (\varphi_1 + \varphi_2)(T_0 + \sigma(t - t_0)) \\
& + \Delta T(N_0 + \rho(t - t_0))))V(t_0)
\end{aligned}$$

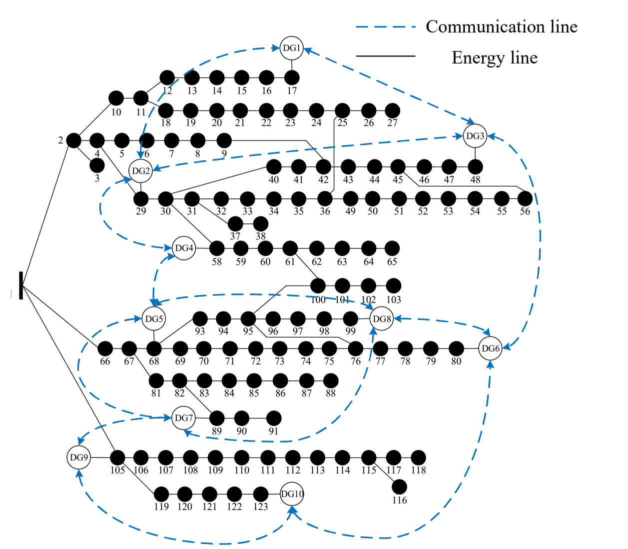

Fig. 2: The physical topology and communication network of the 119-bus test system

$$\le V_3(t_0)\exp((\varphi_1+\varphi_2)(T_0+\Delta T N_0))$$
$$\exp(\varphi_1((\sigma+\rho\Delta T)-1)$$
$$+\varphi_2(\sigma+\rho\Delta T)(t-t_0)). \tag{76}$$

Due to $\varphi_1((\sigma+\rho\Delta T)-1)+\varphi_2(\sigma+\rho\Delta T)<0$, the proposed algorithm is exponentially convergent.

According the Lemma 1 and (57), we can get

$$\sum_{i=1}^{n} x_i^* = \sum_{i=1}^{n} \varsigma_i,$$
$$\lambda_i^* = \lambda_j^*,$$
$$\sum_{i=1}^{n} f_i(x_i^*) = \mathcal{L}(x^*,\lambda^*). \tag{77}$$

Based on the Karush-Kuhn-Tucker (KKT) conditions, we can conclude that x^* is the global optimal solution. The proof is thus completed. ■

IV. Simulation Results and Experiment

To evaluate the performance of, and to demonstrate the effectiveness of the switched surplus-based distributed economic dispatch strategy considering the persistent packet loss, we conduct two studies on the IEEE 119-bus system [28]. The physical topology and communication network of the IEEE 119-bus system are shown in Fig. 2. Each load bus can send its load information to the nearest DGs. This system contains ten DGs, each of which has the cost function with the form of $f_i(x_i)=a_i{x_i}^2+b_ix_i+c_i$. Each DG is capable of exchanging information with neighbors through the distributed communication network and performing local computing. The parameters of the cost functions and constraints are listed in Table III from [29].

A. Optimality and Convergence Analysis without Packet Loss

In this section, we focus on demonstrating the optimality and convergence of the proposed method without packet loss. The local loads of 10 DGs are randomly set as $[1.9;1.8;2.2;2.4;1.5;3.8;1.7;1.9;3.1;2.1]$. Moreover, we set the gain variable to $\alpha(t)=10/(t+1)$, and several control parameters to $\varepsilon=0.03$, $v=0.5$ and $u=2$ [17]. We compare the performance of our proposed method for EDP, with a centralized cuckoo optimization algorithm [5] and a surplus-based distributed algorithm [17]. The final results are listed in Table IV. The performances of the three algorithms are comparable. In Table IV, the first three algorithms, i.e., the centralized cuckoo optimization algorithm [5], the surplus-based distributed algorithm [17], and our proposed algorithm, are used to solve the EDP without any packet loss. It can be observed that the power generations of DG1-DG10 by using the three algorithms are very similar. To be specific, compared the centralized cuckoo optimization algorithm with our proposed method, DG3 has the largest deviation that is 2.463-2.467=0.004. The total calculation error is 0.23%. Similarly, compared the surplus-based distributed algorithm with our proposed method, the DG with the largest deviation is also DG3. That is 2.463-2.467=0.004. The total calculation error is 0.29%. Those imply that the calculation results are highly similar with small calculation errors, thereby verifying the optimality of the proposed algorithm when the packet loss is not taken into account.

To clearly observe the convergence processes, the trajectories of the variables (event triggering sequence, x_i, λ_i, y_i and power mismatch) are shown in Fig. 3-4. Specifically, Fig. 3 shows the event-triggered sequence of each DG. Each triggering time is represented as "†". On the right side of Fig. 3, the period between 58 and 62 is enlarged for the sake of observation. It can be seen that each DG only needs to exchange information with its neighbors at discrete time, which reduces the frequency of communication, and also the communication cost. Fig. 4(a), (b) and (c) show the power

TABLE III: Parameters of Cost Functions and Constraints

-	a	b	c	$p_i^{g,\min}$	$p_i^{g,\max}$	$p_i^{g,ramp}$
DG1	0.021	7.88	460	0	25	5
DG2	0.010	7.85	510	0	35	4.5
DG3	0.022	7.82	130	0	36	3.3
DG4	0.031	7.8	310	0	40	5.5
DG5	0.025	7.82	500	0	50	3
DG6	0.019	7.87	375	0	30	6.5
DG7	0.012	7.79	210	0	80	4.5
DG8	0.021	7.78	260	0	66	5.2
DG9	0.041	7.81	250	0	79	3.5
DG10	0.029	7.90	170	0	60	5.9

TABLE IV: Result Comparison of Different Algorithms

The centralized cuckoo optimization algorithm [5] without persistent packet loss	DG1	DG2	DG3	DG4	DG5
	1.152	3.919	2.463	2.070	2.167
	DG6	DG7	DG8	DG9	DG10
	1.536	5.766	1.390	1.443	0.489
The surplus-based distributed algorithm [17] without persistent packet loss	DG1	DG2	DG3	DG4	DG5
	1.153	3.921	2.463	2.070	2.167
	DG6	DG7	DG8	DG9	DG10
	1.535	5.766	1.391	1.443	0.489
The switched surplus-based distributed algorithm without persistent packet loss	DG1	DG2	DG3	DG4	DG5
	1.152	3.921	2.467	2.072	2.167
	DG6	DG7	DG8	DG9	DG10
	1.537	5.766	1.392	1.443	0.489
the switched surplus-based distributed algorithm with persistent packet loss	DG1	DG2	DG3	DG4	DG5
	1.154	3.919	2.463	2.071	2.167
	DG6	DG7	DG8	DG9	DG10
	1.537	5.767	1.392	1.444	0.490

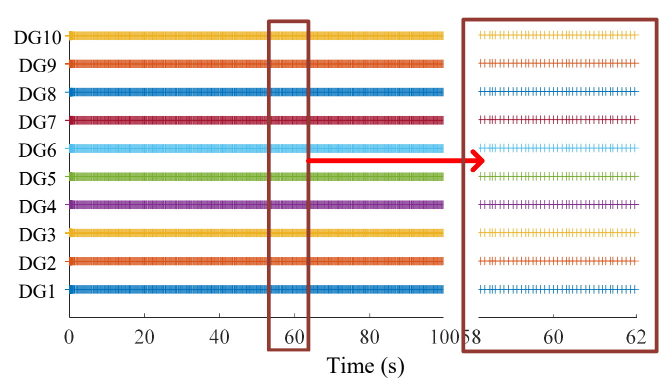

Fig. 3: Event triggering sequence

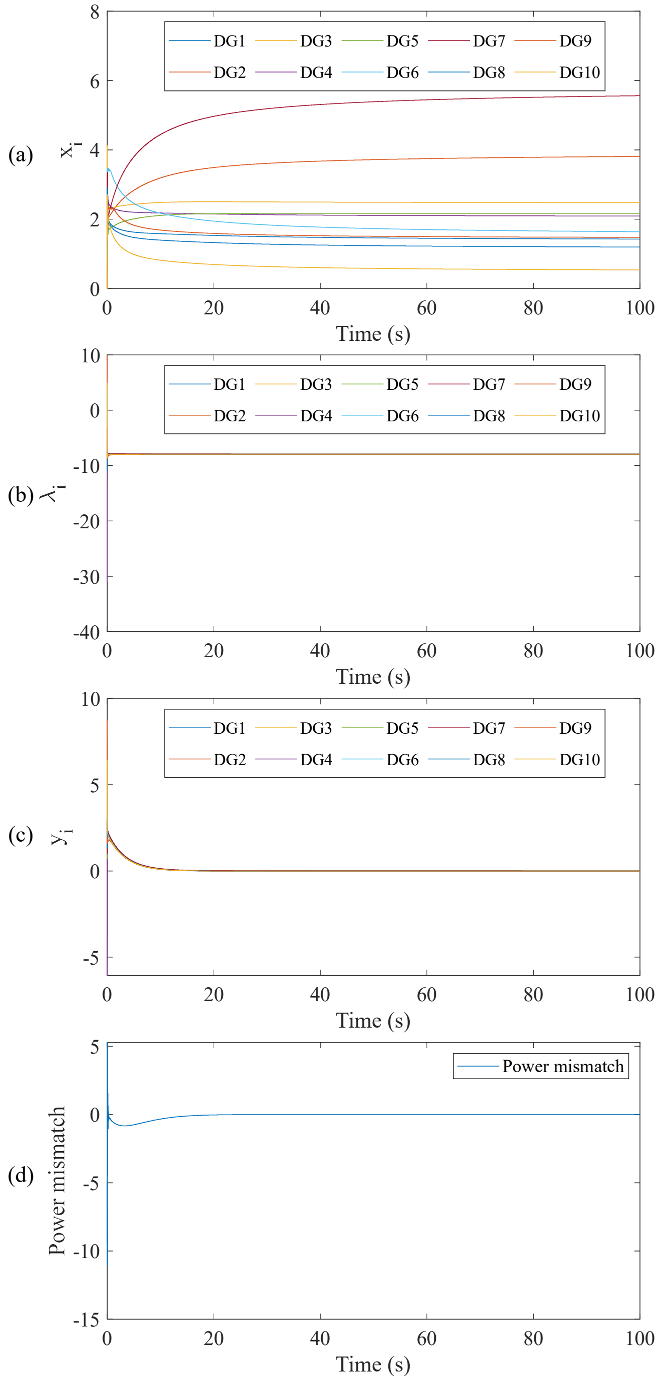

Fig. 4: Convergence results by using the proposed method without persistent packet loss: (a) trajectory of x_i, (b) trajectory of λ_i, (c) trajectory of y_i, (d) trajectory of power mismatch.

generation x_i, Lagrangian multiplier λ_i and auxiliary variable y_i, respectively. It is clear that each of them converges. In addition, the power mismatch of all generators and loads are shown in fig. 4(d). The power mismatch eventually converges to 0. This implies that the supply and demand are balanced, i.e., $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \varsigma_i$.

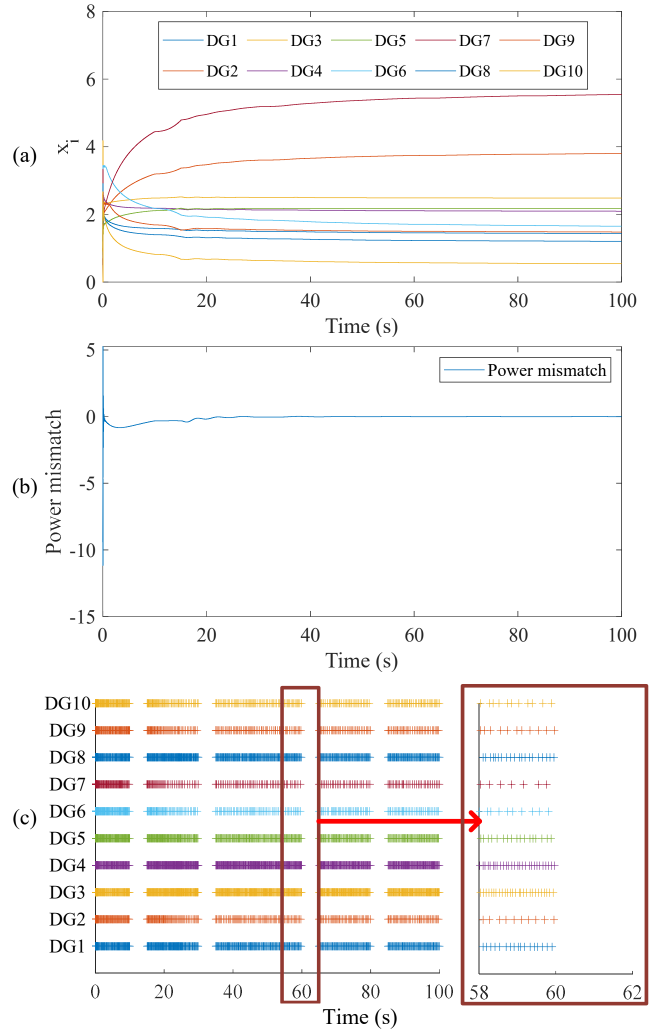

Fig. 5: Convergence results by using the proposed method with persistent packet loss: (a) trajectory of x_i, (b) trajectory of power mismatch, (c) event triggering sequence.

B. Performance Analysis under Persistent Packet Loss

This case study focuses on demonstrating the performance of the proposed algorithm under persistent packet loss by comparing with the state-of-the-art distributed algorithm, i.e., the surplus-based distributed algorithm [17]. The same system setting is employed as the first case study. We consider that the persistent packet loss happens at time interval [10, 15), [30, 35), [60, 65) and [80, 85). ΔT is set to $0.02s$. The simulation results of the power generation, power mismatch, and event triggering sequence for our proposed method are shown in Fig. 5. Fig. 5(a) and Fig. 5(b) imply that the proposed method enables each x_i being convergent and the global power mismatch to be zero, even if there exists persistent packet loss. It can be seen from Fig. 5(c) that communication strategy can be adaptively switched to (21) and (22), respectively. To be specific, the feature of asynchronous event triggering is maintained during safe period. Meanwhile, each DG attempts to periodically restore communication during the packet loss period. The finally convergent results are reported in Table IV. It can be seen that those results are very similar to the case

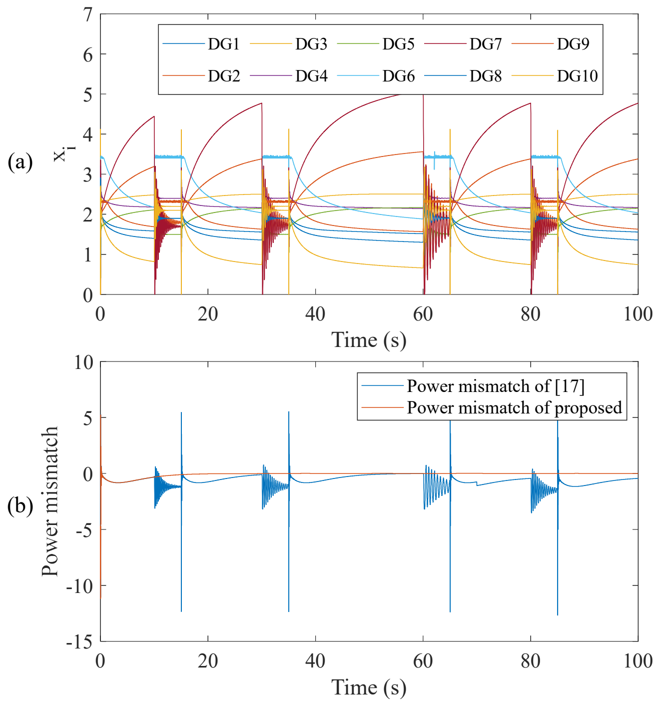

Fig. 6: Comparison results between this algorithm and the surplus-based distributed algorithm from [17]: (a) trajectory of x_i form [17] with persistent packet loss, (b) comparison results of energy mismatch.

without packet loss. To further evaluate the optimality of the proposed method considering packet loss, the corresponding calculation results are also listed in Table IV. Notably, these results remain highly similar to the first three algorithms that do not consider packet loss. The calculation errors between the proposed method with packet loss and the three algorithms without packet loss, i.e., the packet loss and the centralized cuckoo optimization algorithm, the surplus-based distributed algorithm, and the proposed method, are 0.32%, 0.72%, and 0.58%, respectively. All these calculation errors are low, demonstrating that the proposed method can still achieve optimal solutions even in the presence of packet loss.

Next, to further demonstrate the robustness of the proposed method, we test the performance of the switch surplus-based distributed algorithm with the same system setting. The simulation results are reported in Fig. 6. It can be seen that the surplus-based distributed algorithm [17] is sensitive to packet loss. The estimated power generation of each DG fails to converge; meanwhile, the global power generation and demand is unbalanced. This is because the surplus-based distributed algorithm does not take any resistance strategy into account to deal with the packet loss. On the contrary, our proposed method is capable of resisting the persistent packet loss well, thus making it of high value for practical applications.

V. Conclusion

In this paper, we investigated the economic dispatch problem considering persistent packet loss, and proposed a novel switched surplus-based distributed economic dispatch approach to find the global optimal solutions. The proposed algorithm is designed with switched system dynamics and dynamic trigger mechanism, which can effectively reduce the impact of persistent packet loss while maintaining the asynchronous communication capability. In addition, we do not require that the communication topology is strongly connected or connected when packet loss occurs, which considerably broadens the practical value of our proposed scheme for more generic applications. Finally, though theoretical analysis and simulation results, we validated the performance of our proposed scheme and demonstrated the effectiveness of our proposed method.

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