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Master's thesis

Exploring The Impact Of Dimension-8 Operators In A Higgs Effective Field Theory Framework

Significance of Dimension-8 Operators in a New Effective Lagrangian and Impact on Established Wilson Coefficients

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Abstract

This master's thesis utilizes a Higgs effective field theory framework to contribute to the advancement of research regarding the inclusion of dimension 8 operators in effective field theories. By integrating dimension 8 operators into the Higgs effective field theory framework, an updated Wilson Coefficient dictionary is presented, offering a potentially enhanced parameterization of the Higgs coupling to Standard Model matter fields. The investigation identifies various dimension 8 operators that influence the Higgs normalization and fermion mass shifts, accompanied by the introduction of necessary field and mass redefinitions. The thesis concludes by discussing the potential implications and relevance of these findings for future research in the field.

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Preface

Acknowledgement

I would like to express my gratitude to my supervisor, Alexander Lincoln Read, for guiding me through this master's thesis and to my friends for their valuable insights. I also appreciate my parents patiently listening to my struggles with complex concepts. Although they could not give me any more insight, they did, however, relive me of some stress. Thank you all for your support.

Acronyms

BSM Beyond Standard Model.

EOM Equations of Motion.

EWPM Electroweak Precision Measurements.

EWSB Electroweak Symmetry Breaking.

HEFT Higgs Effective Field Theory.

HL-LHC High-Luminosity Large Hadron Collider.

LHC Large Hadron Collider.

NP New Physics.

OPE Operator Product Expansion.

PC Power Counting.

SHIL Strongly Interacting Light Higgs.

SM Standard Model.

SMEFT Standard Model Effective Field Theory.

SSB Spontaneous Symmetry Breaking.

VEV Vacuum Expectation Value.

WC Wilson Coefficients.

Part I

Introduction

Chapter 1

Prelude

With the upcoming launch of the High-Luminosity Large Hadron Collider (HL-LHC) located at CERN, it is imperative that we improve upon already existing Effective Field Theory (EFT) models to be able to have better predictive capabilities with the influx of new data. The Large Hadron Collider (LHC) is the world's most powerful particle accelerator, and the upgrade of the HL-LHC aims to boost the integrated luminosity by a factor of 10 beyond the original LHC luminosity. With the increased luminosity, rare and elusive physics phenomena will hopefully become more commonly observed and will allow scientists to delve deeper into fundamental questions about particle physics. Outstanding questions in particle physics include the properties of the Higgs boson, and the existence of dark matter, or potential new particles and forces beyond those currently described by the Standard Model (SM) of particle physics.

The primary motivation behind this master's thesis stems from the immense potential of future data influx to facilitate a more comprehensive exploration of the Higgs boson. By delving into the detailed study of its properties concerning potential deviations from the predictions of the SM, we aim to gain deeper insights. These deviations can be effectively characterized by employing an EFT framework.

Currently, EFTs are limited to operators of mass dimension 6 [9]. However, it is valuable to investigate the possibility of extending the EFT framework to include dimension 8 operators, enabling a more accurate parameterization of potential deviations in the Higgs boson coupling to SM matter fields. This enhanced framework not only has the potential to improve the predictive capabilities of EFTs for the existing data from the LHC, but also becomes indispensable when considering the forthcoming HL-LHC with its anticipated higher integrated luminosity.

Dimension 8 operators have already demonstrated their impact on Drell-Yan angular distributions, energy and angular dependencies in β- decay, and they introduce a new class of flavor diagonal CP-odd operators that break time reversal, but not parity [2][19]. The objective of this master thesis is to expand upon an existing EFT framework by integrating dimension 8 operators that describe the coupling of the Higgs boson to SM matter fields. By doing so, we anticipate obtaining an updated EFT Lagrangian that provides a more refined set of Wilson coefficients, facilitating a more precise parameterization of deviations from the SM. This updated framework holds potential not only for the current luminosity of the LHC but also for future applications, particularly with the more sensitive HL-LHC.

1.1 Problem Statement

EFTs with Power Counting (PC) rules that restrict operators to mass dimension 6, along with dimension 6 operators, have already received extensive coverage in scientific literature, as evident from references such as [9], [21], and [23]. In contrast, research on EFTs capped at dimension 8 remains relatively sparse. The scarcity of research on the effects of dimension 8 operators poses a challenge for this master thesis. This lack of exploration can be attributed to two primary factors: the extensive nature of the dimension 8 operator set and the limited sensitivity of the current particle collider datasets to high-dimension operators.

The vastness of the dimension 8 operator set presents a formidable task, making it complex to comprehensively study and understand the effects associated with these operators within the EFT framework. Moreover, the current limited sensitivity of particle colliders pose practical constraints on probing the specific signatures and manifestations of dimension 8 operators.

However, this master thesis aims to propose a viable framework in which the Effective Lagrangian incorporates the most relevant dimension 8 operators, carefully selected to accurately parameterize both present and future deviations from the SM. By addressing these research gaps, this thesis intends to contribute to the understanding and advancement of EFTs with dimension 8 operators, shedding light on their significance in effectively characterizing deviations from the SM.

This master thesis hopes to answer the three following research questions:

Question 1: Is it possible to expand current EFTs with dimension 8 operators, and obtain a Wilson Coefficient (WC) dictionary better suited to parameterize new physics (NC)?

Question 2: Will the updated EFT framework and Updated WC dictionary contribute to a better parametrization of new physics?

Question 3: What are the implications and applications of such an updated EFT framework?

In order to best answer these research questions, the following five main objectives are proposed:

Objective 1: Present a comprehensive description of the current EFTs capped at dimension 6 and the dimension 6 Warsaw Basis[23].

Objective 2: Expand the constraints on the EFT operator set to also include the most relevant dimension 8 operators for Higgs physics.

Objective 3: Derive the effective Lagrangian after spontaneous symmetry breaking (SSB).

Objective 4: Use the effective Lagrangian after SSB to derive an updated WC dictionary.

Objective 5: Use the updated WC dictionary and Pesking-Takeuchi parameters [38] to explore the potential of dimension 8 operators and the potential for future work.

The purpose of these five objectives is twofold: to provide answers to the research question posed and to assist the reader in efficiently understanding and evaluating the findings of this master's thesis.

1.2 Scope and Limitations

This master thesis encounters several limiting factors that warrant consideration. The primary constraint is the scarcity of research surrounding the inclusion of dimension 8 operators in effective field theories. Although some articles, [18] and [33], provide initial insights into the potential of dimension 8 operators, the body of work in this specific area remains relatively limited. Notably, there is currently no dimension 8 equivalent to the well-established irreducible dimension 6 operator basis, known as the Warsaw basis. However, [33] serves as the closest available reference and has been invaluable in informing and guiding this master thesis's progress.

The scope of this master thesis will be significantly constrained as it does not involve matching the derived theoretical framework with experimental data. This limitation primarily arises from time constraints associated with the challenging task of matching such a large effective Lagrangian with potential unknown interactions, which would pose difficulties in modeling within software packages like **FeynRules** and **MadGraph**. Consequently, this master thesis does not have the opportunity to empirically validate whether the mathematical framework developed herein represents an improvement over previous effective theories that adhere to Power Counting rules restricting operators to mass dimension 6.

1.3 Thesis outline

This section provides a concise overview of the structure of this master thesis, along with an explanation of the rationale behind its organization.

Part 1: Introduction, Problem Statement, and Thesis outline focus on establishing why there is a need for an EFT with PC rules restricting the operators to mass dimension 8, and the outlining of the master thesis.

Part 2: Part 2 of the thesis focuses on providing the necessary background of the theoretical framework employed to achieve the goals outlined in the Problem Statement 1.1.

The chapter titled *Effective Operators* offers a comprehensive background on the fundamental building blocks utilized to expand the Standard Model. It delves into the theoretical foundations and principles behind effective operators, exploring their significance and role in extending the existing framework.

The subsequent chapter, *Exploring the Effective Lagrangian Before and After Electroweak Symmetry Breaking*, aims to construct the physical effective dimension 8 Lagrangian. This chapter details the process of incorporating dimension 8 operators into the Lagrangian and investigates the implications of these operators both before and after the electroweak symmetry breaking.

Part 3: Part 3 aims to answer the questions and objectives laid out in section 1.1. The main chapters here are:

Results: The results and findings will be presented here.

Discussion: The implications and synthesis of the results will be presented

Conclusion: Here the reader will be presented with a final summary of the findings, as well as a foundation for future work and applications.

As this master thesis primarily focuses on deriving a theoretical framework, the results presented will mainly consist of a discussion regarding the usefulness of the theoretical framework introduced in **Part 2**, specifically in Section 6 (Effective Lagrangian Before and After Electroweak Symmetry Breaking). It is important to note that since the framework is not applied or matched to experimental data, the results primarily exist at a theoretical level.

The results chapter summarizes the findings and discoveries made throughout **Part 2**. It will provide a comprehensive overview of the implications and potential significance of the derived theoretical framework. While the results are predominantly theoretical in nature, they contribute to the understanding and advancement of the field, paving the way for future experimental verification and validation of the framework. Due to the theoretical nature of the results, the background in itself does answer some of the research questions. These results will be more succinctly summarized in Chapter 8.

The final part of the thesis is dedicated to the Appendices. The Appendices are further divided into two segments. The appendices **A**, **B**, and **C** serve as a compilation of various derivations of mathematical statements utilized throughout the master thesis. It includes detailed explanations and calculations that support the theoretical framework and analysis presented in the main body of the thesis.

On the other hand, appendices **E, D** and **F** specifically focus on the derivation of all relevant dimension 6 and dimension 8 operators and the SM terms that undergo modification under the phenomenon of electroweak symmetry breaking (EWSB). This section provides a comprehensive and systematic account of the derivation process, offering insight into the theoretical and mathematical formulations underpinning the research.

Part II

Background

Chapter 2

The Standard Model

2.0.1 Foundational Terminology: An Introduction to Key Concepts and Vocabulary

This section serves as a brief introduction to clarify the meanings of challenging or complex terms used in this master's thesis. It provides a general understanding to help readers build intuition and readiness for extensive usage.

Throughout this master thesis, **Operator** is extensively employed to describe contractions of various fields in the SM. Generally, operator refers to higher-order contractions of fields, surpassing mass dimension four while maintaining gauge and Lorentz invariance. Occasionally, "term" and "operator" are used interchangeably. "Term" primarily pertains to the regular SM field contractions equal to or lower than mass dimension 4. These contractions of SM fields constitute kinetic or interaction terms. It is important to note that certain operators when the covariant derivative is expanded, yield multiple contributions to several interactions.

The term **Mass Dimension** essentially describes the number of fields contracted with each other. When an operator or an SM term has a low mass dimension, it implies it involves a small number of fields. Depending on the specific fields being contracted, it could be a kinetic term or an interaction term. In common literature, mass dimension is often described more generally in terms of units of power or inverse mass, given that units of $c = \hbar = 1$ are used, resulting in quantities being measured in power units. For example, the mass dimension of a quantity A would be expressed in terms of power units E^x, where four-momentum and mass have energy E to the power of $+1$, while position has power $+1$. Although this latter description will not be extensively elaborated upon in this master thesis, the mass dimension of an operator becomes important when deriving the building blocks for constructing higher-order operators and employing PC rules to impose restrictions on the mass dimension of operators.

A **Lagrangian** is a model used to describe the dynamics and interactions of particles. It comprehensively represents the physics governing a system, incorporating kinetic energy, potential energy, and particle interactions. In everyday scenarios, a Lagrangian can be employed to describe simple systems like a pendulum. In this case, the Lagrangian captures information about the pendulum's motion, position (height), and external forces, deriving equations that describe its swinging behavior. In particle physics, the Lagrangian serves a similar purpose by encompassing a wide range of information. However, in this context, the Lagrangian must also account for various physics phenomena such as quantum numbers, symmetries, conservation laws, and more.

Group describes operators that satisfy specific selection criteria. These criteria could include the mass dimension of the operator, its field content, and other relevant factors. The operators within the group are characterized as one-point disconnected vertex diagrams. Even if operators within the group contain Covariant Derivatives or expanded flavor or generation indices, they are still referred to as operators within the group, even though they can be seen as a collection of operators when expanded.

Hypercharge
Hypercharge is a quantum number briefly discussed, primarily in how conservation laws associated with hypercharge constrain the structure of specific operators. Hypercharge characterizes the electric charge of particles and is closely connected to the $U(1)$ symmetry group. It is also related to electric charge.

Wilson Coefficients are the parameters that quantify the contributions of different terms in the operator product expansion, i.e, the expansion of the SM Lagrangian with higher order operators. WCs are fundamental constants that govern the strength of interactions between different fields or particles.

2.1 Introduction to the Standard Model

In the SM of particle physics context, numerous valid approaches exist for defining the SM Lagrangian. In this master's thesis, inspiration is drawn from the articles[9], [11], and [23]. The SM Lagrangian is a fundamental mathematical formulation encompassing the elementary particles' dynamics and interactions. Throughout this master thesis, the SM will be defined as (2.1)

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q,u,d,\ell,e}\bar{\psi}i\not{D}\psi \\ &+ (D_\mu H)^\dagger (D^\mu H) - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 + \left(\frac{1}{2}\Psi_L^T C h H \Psi_L + \text{ h.c. }\right).\end{aligned} \tag{2.1}$$

The notation used here aligns with the references mentioned above. The symbols $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$ represent the gauge field strength tensors associated with the respective symmetry groups. The indices A and I take on values within specified ranges, such as $I = 1, 3, 4$ and $A = 1, \ldots, 8$, corresponding to the generators of their corresponding symmetry group. These gauge bosons are spin-1 particles and are associated with the gauge group of the SM, namely $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here, the subscript C denotes color, L denotes weak isospin, and Y represents the $U(1)_Y$ hypercharge generator. Additionally, H signifies the Higgs field, ψ represents fermions characterized by spin-$\frac{1}{2}$, v is the vacuum expectation value (VEV) of the Higgs field which acts to break electroweak symmetry and is defined as $\left\langle H^\dagger H\right\rangle = v^2/2$, with $v = 256$GeV. C is the charge conjugation matrix.

The covariant derivative associated with a local $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry is

$$D_\mu = \partial_\mu - ig_1 Y \mathbf{B}_\mu - ig_2 \mathbf{W}_\mu - ig_3 \mathbf{G}_\mu.$$

The field strength tensors, denoted by boldface symbols, possess a specific mathematical structure that aligns with that of a compact Lie group. This distinction arises because the generators of an abelian group, wherein the generators commute, can be represented simply as numerical quantities. However, in the case of non-abelian groups such as $SU(2)$, the field strength tensors exhibit a more intricate framework. Notably, the irreducible generators of the $SU(2)$ group are represented by a set of hermitian 2×2 matrices known as the Pauli matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Within the framework of the $SU(3)$ group, the generators are represented by a set of hermitian 3×3 matrices known as the Gell-Mann matrices. Similar to the Pauli matrices in $SU(2)$, these matrices play a crucial role in characterizing the transformation properties and dynamics associated with the $SU(3)$ symmetry group.

The field tensors $\mathbf{W}\mu$ and $\mathbf{G}\mu$ are composed of three and eight vector fields, respectively. These vector fields correspond to each generator's respective group, pointing in different directions within the abstract space associated with the group[39]. The field tensors can be seen as linear combinations of these vector fields, encapsulating the gauge bosons and their interactions governed by the $SU(3)$ symmetry.

It is worth noting that the Gell-Mann matrices, along with the associated field tensors, serve as essential tools in the study of Quantum Chromodynamics, which describes the strong interaction among quarks and gluons within the framework of the Standard Model[39].

$$\mathbf{G}_\mu = G^a_\mu t^a, t^a = \frac{\lambda^a}{2}, \qquad \mathbf{W}_\mu = W^a_\mu t^a, t^a = \frac{\sigma^a}{2}, \qquad B_\mu = B_\mu Y.$$

The fermionic sector of the SM comprises three generations of leptons (e_i and ν_i) and quarks (u_i and d_i), where the index i runs over the three generations, namely $i = 1, 2, 3$. Within the fermion sector, the three-component generation indices are contracted with the fermion mass matrix, while h is the matrix of Yukawa couplings.

In terms of their representation under the symmetries of the Standard Model, the right-handed quarks and leptons are singlet states, while the left-handed quarks and leptons form $SU(2)$ doublet states. Consequently, the complete Lagrangian of the Standard Model encompasses implicit sums over the 3-component generation indices, color indices, the 2-component Pauli matrices associated with $SU(2)$ doublet states, the field tensors $\mathbf{W}_\mu$, and the Higgs doublet H. The matter contents are more compactly summarized in table ref 2.1.

The Yang-Mills field strength tensors, denoted by $F = \mathbf{W}_{\mu\nu}, \mathbf{G}_{\mu\nu}$, along with the abelian field strength tensor $B_{\mu\nu}$, and their respective covariant derivatives, are formulated in the equations 2.2[23]

$$\begin{aligned}
\mathbf{G}_{\mu\nu} &= \partial_\mu \mathbf{G}_\nu - \partial_\nu \mathbf{G}_\mu + ig_3 \left[\mathbf{G}_\mu, \mathbf{G}_\nu\right], & D_\rho \mathbf{G}_{\mu\nu} &= \partial_\rho \mathbf{G}_{\mu\nu} + ig_3 \left[\mathbf{G}_\mu, \mathbf{G}_\nu\right], \\
\mathbf{W}_{\mu\nu} &= \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + ig_2 \left[\mathbf{W}_\mu, \mathbf{W}_\nu\right], & D_\rho \mathbf{W}_{\mu\nu} &= \partial_\rho \mathbf{W}_{\mu\nu} + ig_2 \left[\mathbf{W}_\mu, \mathbf{W}_\nu\right], \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, & D_\rho B_{\mu\nu} &= \partial_\rho B_{\mu\nu}.
\end{aligned} \tag{2.2}$$

An important relation that will be utilized in deriving several operators in the unitary gauge is the commutation of two covariant derivatives, which yields the field strength tensor $A_{\mu\nu}$ of the gauge field multiplied by the coupling constant.

$$[D_\mu, D_\nu]\psi(x) = ig(A_{\mu\nu})\psi. \tag{2.3}$$

The derivation of Eq 2.3 can be found in Appendix F Section 11.6.2.

This same definition holds for non-abelian theories as well. However, in non-abelian theories, the generators of the symmetry group do not commute. This arises from the non-commutativity of the group generators $[\mathbf{A}_\mu, \mathbf{A}_\nu] = [A^a_\mu t^a, A^b_\nu t^b] = A_\mu A_\nu [t^a t^b]$, and we get an extra factor of $[t^a, t^b] = if^{abc}t^c$. f is the structure constant of the symmetry group, i.e., constants are coefficients that characterize the non-commutative behavior of generators in a Lie algebra. The inclusion of structure constant belonging to the symmetry group of the field is important, as shown in Eq 2.4.

$$[D_\mu, D_\nu] = gf^{abc}A^b_{\mu\nu}t^c. \tag{2.4}$$

Writing the fields in component form proves to be advantageous in several aspects. Firstly, it provides a clearer understanding of how the fields transform under gauge transformations. By explicitly expressing the components, it becomes evident how the fields change under the symmetries imposed by the gauge group.

Additionally, representing the fields in component form facilitates the examination of their coupling patterns. It becomes easier to discern how different fields are coupled to each other, mainly when dealing with higher-order operators involving multiple fields.

Moreover, when deriving the equations of motion (EOM) for the gauge fields and the Higgs field, the various equations 2.5, 2.6 and 2.7 are useful[23].

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu, \qquad (D_\rho G_{\mu\nu})^a = \partial_\rho G^a_{\mu\nu} - g_s f^{abc} G^b_\rho G^c_{\mu\nu}, \tag{2.5}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu, \qquad (D_\rho W_{\mu\nu})^a = \partial_\rho W^a_{\mu\nu} - g f^{abc} W^b_\rho W^c_{\mu\nu}, \tag{2.6}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad D_\rho B_{\mu\nu} = \partial_\rho B_{\mu\nu}. \tag{2.7}$$

The numbers f^{abc} aforementioned structure constants. These structure constants satisfy the Jacobi identity [39]. In the case of the $SU(2)$ symmetry group, the structure constants correspond to the Levi-Civita symbol.

2.2 Hypercharge Constraints, Group Representations, and Matter Content

This section briefly explains the underlying principles that justify the inclusion of certain higher-order operators. It is important to note that all higher-order operators in the

Higgs Effective Field Theory (HEFT) adhere to the same constraints and possess the same matter content as the SM, which is required for being a part of the SMEFT framework.

By adhering to the same matter content and symmetry group structure as the SM, HEFT ensures consistency and compatibility with the fundamental principles of the underlying theory. Table 2.1[31][20] provides an overview of how the fields in the SM under different symmetry group transformations.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_l \times SU(2)_r$
q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	$\left(\frac{1}{2}, 0\right)$
u	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	$\left(\frac{1}{2}, 0\right)$
d	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	$\left(\frac{1}{2}, 0\right)$
l	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$\left(\frac{1}{2}, 0\right)$
e	$\mathbf{1}$	$\mathbf{1}$	1	$\left(\frac{1}{2}, 0\right)$
H	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	$(0, 0)$
G	$\mathbf{8}$	$\mathbf{1}$	0	$(1, 0)$
W	$\mathbf{1}$	$\mathbf{3}$	0	$(1, 0)$
B	$\mathbf{1}$	$\mathbf{1}$	0	$(1, 0)$

Table 2.1: Transformations of SM fields under different symmetry group transformations

The representations indicated in the table, such as **3**, **2**, and **1**, signify how a field transforms within the corresponding symmetry group. Specifically, a field represented by a triplet (**3**) transforms under a symmetry group as a triplet, a field defined by a doublet (**2**) transforms as a doublet and a field represented by a singlet (**1**) remains invariant under the given symmetry group transformation.

Understanding these field representations is crucial when justifying the structure of various operators. For instance, the operators must adhere to specific constraints related to hypercharge. By examining Table 2.1, one can observe the transformations of the fields and identify the constraints associated with the operators.

It is worth noting the distinction between the field q and the individual up and down quarks (u and d) in Table 2.1. While u and d are also quarks, the table clearly illustrates that the up and down quarks transform as singlets, meaning they remain invariant under $SU(2)_L$ transformations. In contrast, the remaining quarks transform as left-handed doublets. This distinction is significant and reflects why the SM Lagrangian and various calculations throughout the master thesis differentiate between these quarks.

Chapter 3

Introduction to Effective Field Theory

EFT is a theoretical framework used to describe physical phenomena at a specific energy scale or distance scale.

EFTs are useful both in particle physics and fields such as condensed matter physics, as it is often not practical or even possible to directly calculate the behavior of a system using its fundamental microscopic laws. EFT offers a more tractable alternative by providing a simpler, effective theory, which captures the essential physics at the energy or distance scale of interest while ignoring the microscopic details that are not relevant.

Constructing an EFT involves identifying the relevant degrees of freedom and the symmetries that govern their interactions. These degrees of freedom can be elementary particles, collective excitations, or other entities that effectively describe the system's behavior. The symmetries, such as conservation laws, gauge symmetries, or spacetime symmetries, provide essential constraints on the form of the effective theory.

Once the relevant degrees of freedom and symmetries are identified, one can write down a Lagrangian or an action that describes the system's dynamics. This Lagrangian contains a set of parameters known as coupling constants, which encode the strength of the interactions between the degrees of freedom.

The coupling constants, known as the WCs, need to be determined through experimental measurements by matching the effective theory to the underlying more fundamental theory. This ensures that the effective theory correctly captures the physics of the underlying theory within a given energy or distance scale.

One important note is that since EFTs aim to capture the low-energy phenomena of a fundamental UV theory, it is only valid up to a particular energy scale. For the EFT to remain valid, the dynamics and interactions it describes must occur at energies involved much smaller than the scale at which new degrees of freedom or interactions become relevant.

3.1 SMEFT vs HEFT

One important clarification is needed to make sure the distinction between Standard Model Effective Field Theory (SMEFT) and Higgs Effective Field Theory (HEFT) is clear. HEFT shares similarities with SMEFT in using WCs as parameters in the Lagrangian to determine the strength of higher-order operators. These coefficients, similar to those in SMEFT, can be constrained through experimental measurements. The critical difference lies in the contexts in which these two EFTs are applied.

SMEFT is chosen when studying broad aspects of the SM and exploring potential new interactions that may affect low-energy phenomena within a particular energy scale. On the other hand, if the focus is on the properties of the Higgs boson, such as its production rates, decay rates, and interactions with other SM particles (i.e., the Higgs sector), the most appropriate framework to describe these properties is HEFT. The difference between the two lies in their scopes and emphasis.

3.2 Higgs Effective Field Theory

HEFT is a framework utilized to describe low-energy phenomena within the Standard Model that pertain to the Higgs sector, and any Beyond Standard Model (BSM) states associated with the Higgs boson.

The HEFT is constructed out of higher-dimensional operators, where the operators are built from only SM fields. These operators describe interactions between SM particles and potential new heavy particles. The assumed Higgs boson is embedded in an $SU_L(2)$ Higgs doublet. HEFT is a subset of SMEFT, and since this thesis aims to parameterize the Higgs boson coupling to SM fields, the entirety of SMEFT is unnecessary.

The mathematical formulation of the HEFT Lagrangian is given in Eq 3.1[10]:

$$\mathcal{L}_{HEFT} = \mathcal{L}_{SM} + \sum_{d=5}^{\infty} \sum_{i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}. \tag{3.1}$$

$\mathcal{L}_{SM}$ is the SM Lagrangian. Although the $\mathcal{L}_{SM}$ is denoted "SM" it is not quite correct to say that $\mathcal{L}_{SM}$ is equal to the SM Lagrangian, as the higher order operators from the operator product expansion(OPE) would affect it. Section 6.5 on fermion mass shift, Higgs renormalization and Z mass shift shows how the OPE necessitates field and mass redefinitions. The OPE is summed over every mass dimension d, and every possible operator allowed by the symmetries of the theory. The coefficient in front of each operator, c_i is the Wilson coefficient divided by the energy scale. $\mathcal{O}$ is the effective operator.

Since new particles are expected to be heavier than the VEV, they do not affect current collider experiments. These Beyond Standard Model (BSM) states are integrated out of the theory since they do not represent propagating particles. The effects of these heavy new states are encoded into the coefficients of the higher dimensional operators, which are determined through a process known as "matching". Matching essentially matches the HEFT onto a complete theory where the BSM states are included. Matching allows the HEFT to make predictions regarding the low-energy behavior, independently of the details of the underlying physics. Section 10.3 briefly outlines the matching theory and how one would apply it. While the matching procedure is essential for determining the WCs and quantifies the importance of the new physics, it is not the focus of this thesis.

The HEFT Lagrangian is required to follow the same rules governing the SM. This requirement mandates that the effective operators must also be invariant under the SM gauge group $SU_c(3) \times SU_L(2) \times U_Y(1)$, follow baryon/lepton conservation, and Lorentz invariance.

$\mathcal{L}^5$ and $\mathcal{L}^7$ consists only of operators of odd mass dimension, which are not allowed since the operators must contain only interactions conserving lepton(L) and baryon(B) number as is a requirement in the SM. Operators of even mass dimension are the only ones that are of interest since they do not violate B and L conservation). As the operators increase in mass dimension, they are more and more suppressed by the energy scale Λ. $[\mathcal{O}^6]$ is suppressed by a factor of $\frac{1}{\Lambda^2}$ and $[\mathcal{O}]^8$ are suppressed by a factor of $\frac{1}{\Lambda^4}$.

When deciding the contents of $\mathcal{L}^6$ and $\mathcal{L}^8$ there are three rules.

(1) Power Counting

(2) Symmetry requirements

(3) Particle content

The most crucial rule is power counting. This rule limits the number of possible operators the EFT can have by imposing that the operators be restricted by their dimensionality. From just symmetry requirements we can build an infinite number of possible operators. PC rules allow us to neglect operators severely suppressed by factors of $\frac{1}{\Lambda}$. The consensus is to use operators of mass dimension 6. However, with the potential for future high-energy collider experiments, the effects of higher-order operators, such as those of mass dimensions eight and above, may become increasingly important to consider. As such, there is value in exploring their potential already.

With the number of operators we can have limited by the total mass dimension of $d = 8$, there are also symmetry requirements to be met. Symmetry requirements limit the number of possible ways the effective operators can be structured. The symmetries of the standard model have been measured with high precision. Hence, any violation of these symmetries has to happen at extremely high energies or are minimal violations of the SM [6] [15]. Therefore it is highly motivated that these symmetries should also govern the structure of the effective operators.

Lastly, one has to define the particle content of the HEFT. The particles in the HEFT are the fields with dynamic degrees of freedom. All particles with mass $m << \Lambda$ are included as dynamical degrees of freedom, while all particles with $m \approx \Lambda$ are integrated out of the theory and can not propagate. These non-propagating states are the BSM states we integrated out to obtain the expression for the effective Lagrangian. The fields making up the effective Lagrangian are the four gauge bosons $W^{\pm}, Z$, which carry the weak interaction, gluons G, which carry the strong interactions, and the photon γ carrying the electromagnetic interactions. The unique force carrier is the scalar Higgs boson H, which differs from the other by being a scalar field with spin-0 while the others have spin-1. The gauge bosons, Higgs scalar field, and fermions make up the matter content of both SM and effective operators.

The challenge will be to use all three rules to find and group all the different combinations of allowed effective operators.

The different effective operators have already been extensively covered in the literature. Two well-known sets of operators called the HISZ-set[1] and the Strongly Interacting Light Higgs (SIHL)-set, are often referred to as basis but are, in-fact in-complete sub-sets (HISZ) or over-complete sets (SIHL) of operators in $\mathcal{L}^6$[11]. It took over 20 years after the first complete set of 80 operators was published [13] by W. Buchmuller and D. Wyler before it was realized that not all 80 operators are linearly independent and could be reduced using EOM and field re-definitions. The first full and non-redundant operator basis for $\mathcal{L}^6$ (disregarding flavor structure and hermitian conjugation as in the work by W. Buchmuller and D. Wyler) were published in [23]. The full and non-redundant operator basis found by B. Grzadkowski is now known as the Warsaw basis[11]. These different bases are equivalent in that you can transform them into each other through EOM and field re-definitions. However, they put different constraints on the Wilson coefficients resulting in a slight deviation from SM experimental data for the bases.

Both the Warsaw basis and the SHIL basis are used to derive the final effective Lagrangian.

3.3 Advanced EFT (

This section hopes to highlight the various advanced aspects of EFT, which are not discussed in this master thesis. One important way of determining the relevance of your EFT is "Running the Renormalization Group Equation." This is not done in this thesis, as this is difficult. Running the RGE involves solving the statement that the bare[2] Lagrangian does not depend on the energy scale[12], as shown in Eq 3.2.

$$0 = \Lambda \frac{d}{d\Lambda} \mathcal{L}^{(n)\ \text{bare}} \,. \tag{3.2}$$

From this statement, one can derive the *beta-function*[12], which encodes how the EFT Lagrangian's parameters depend on the energy scale Λ.

$$\Lambda \frac{d}{d\Lambda} \mathcal{L}^{(n)} = \mathcal{B}^{(n)} \tag{3.3}$$

The *beta* function captures the effects of quantum corrections and the renormalization process. By solving Eq 3.3, we can determine how the couplings and parameters evolve from high-energy to lower-energy scales. In the context of EFTs, 3.3 is important for understanding the scale dependence of the WCs. As the β-function 3.3 encodes how the parameters of the effective Lagrangian depend on scale, if $\beta > 0$, then you have a coupling constant that grows in strength and becomes strongly coupled at high energies. The opposite is true for $\beta < 0$. The former is known as a "running coupling," where the coupling grows as the energy scale increases. Diving deeper into scale dependence would be an exciting adventure, but this master thesis focuses on relating the EFT to a more fundamental theory by establishing the coefficients of the EFT operators without considering its scale dependence.

[1] Named after Hagiwara K, Ishihara S, Szalapski R, Zeppenfeld D, who first used it in the article [24]
[2] Bare is a reference to a Lagrangian, which has not had its heavy fields integrated out

Integrating out heavy fields

Integrating out heavy states in an EFT refers to the process of eliminating the degrees of freedom associated with high-energy or heavy particles from the theory. These heavy particles do not appear in the final effective Lagrangian. The "integrating out" step is done by creating an effective action Eq 3.4[3], S

$$S_{\Lambda}^{\text{eff}}\left[\phi_L\right] = -\ln\left[\int \mathcal{D}\phi_H e^{-S_{\Lambda_0}[\phi_L+\phi_H]}\right] \tag{3.4}$$

By rewriting the action as Eq 3.4, a low-energy description is obtained where the fields ϕ are separated into low-energy source fields ϕ_L to the high-energy fields ϕ_H.

[3]This is taken from Problem Set 9, in the course FYS-5120 Advanced Quantum Field Theory, held by Lasse Lorentz Braseth, Jonas Eidesen

Chapter 4

Effective Operators

4.1 Introduction

Let's clarify our objectives before selecting the dimension-6 and dimension-8 operators to incorporate into our final effective Lagrangian. Specifically, our goal is to identify the relevant operators from the set of dimension-6 and dimension-8 operators, $\mathcal{L}_6 + \mathcal{L}_8$, that are most effective in characterizing deviations in the Higgs coupling to SM matter fields. To do this, we must first define the various operators and their fundamental building blocks.

4.2 Mass Dimension of Matter Content in the Standard Model

To define the various effective operators in HEFT, it is necessary first to establish the mass dimension of the "building blocks" that make up the operators. These building blocks are the matter content of the SM Lagrangian, which includes various fields representing particles such as quarks, leptons, and gauge bosons. Knowing the mass dimension of these building blocks, we can construct higher-order operators that capture the interactions between these fundamental particles at energy scales beyond the SM.

The action, $S = \int \mathcal{L} d^4x$ must be dimensionless, meaning that the Lagrangian density must be of dimension $(mass)^4$. To determine the mass dimensions of the different fields and operators in the Lagrangian, we can use the kinetic terms of QED as a reference. Using the kinetic terms of the QED Lagrangian, we can determine that the gauge boson A^μ and the partial derivative must have $[A^\mu] = [\partial_\mu] = 1$. This allows us to establish the mass dimensions of other fields and operators that appear in the Lagrangian.

To determine the mass dimensions of fields in interacting theories, we can begin by considering the free Lagrangian of scalar fields. $(\partial\phi)^2$ is the free Lagrangian for scalar fields, from which it is clear that $[\phi] = 1$. Similarly for the free theory of fermions, $\mathcal{L}_{Dirac} = \bar{\psi}(i\partial - m)\psi$ it is clear that fermions must have $[\psi] = \frac{3}{2}$. These mass dimensions are a starting point for determining the mass dimensions of more complex interacting theories. For an interacting theory involving only scalars, the allowed interaction terms are[39]

$$\mu\phi^3, \lambda\phi^4.$$

μ is a coupling constant with mass dimension $[\mu] = 1$ whilst λ is dimensionless coupling constant. We are only considering renormalizable interactions where the coupling constants are dimensionless or have $[\mu] > 0$. Non-renormalizable interactions, on the other hand, have coupling constants with negative mass dimension, making the theory increasingly sensitive to higher energy scales and less predictive[39]. Renormaliazability is required for a theory to predict observables with precision, which is also why there is a cut-off scale to our effective theory to ensure our theory is valid.

Fermions cannot self-interact because it would violate Lorentz invariance and lead to a mass dimension of $\frac{9}{2}$. Therefore, the only allowed interaction for fermions is through the Yukawa term:

$$g\bar{\psi}\psi\phi.$$

g is a coupling constant. There exist similar interactions which can be constructed out of Weyl and Majorana spinors[39]. We now have all the necessary building blocks we need to construct our higher-order dimensional terms. The

$$[\psi] = \frac{3}{2}, [A_\mu] = 1, [A_{\mu\nu}] = 2, [\phi] = 1, [\partial_\mu] = 1, [D_\mu] = 1.$$

Here, A_μ represent the different gauge bosons, while their respective field strength tensors are denoted by $A_{\mu\nu}$. The fermionic fields, which include the leptonic and quark fields, are collectively represented by the symbol ψ.

Before identifying potential dimension 8 operators, we will start by classifying all possible dimension 6 operators. We will use the same notation. When defining the operators $\mathcal{O}_{xxxx}$, the different x's represent either fermions ψ (which could be *up*, *down*, *top*, e, l, ν etc), the different gauge bosons G, B, W, the Higgs field H or the covariant derivative D.

Beginning with the easiest dimension 6 operators, we can combine the scalar gauge singlet $H^\dagger H$ with any dimension-4 operator from the SM. The only dimension-4 operators in the SM are the kinetic terms for the gauge bosons

$$-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu}, \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu}, \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

the Yukawa interaction terms

$$\left(\frac{1}{2}\Psi_L^T ChH\Psi_L + \text{ h.c. }\right),$$

and lastly the four point scalar interaction term

$$\frac{1}{2}(H^\dagger H)(H^\dagger H).$$

These operators combined with the scalar gauge singlet $H^\dagger H$ gives the various dimension-6 terms

$$\begin{aligned}
\mathcal{O}_{H^\dagger H} &= \left(H^\dagger H\right)^3, \quad \mathcal{O}_{DDH^\dagger H} = \left(H^\dagger H\right)(D_\mu H)^\dagger (D^\mu H) \\
\mathcal{O}_{H^\dagger HG} &= \left(H^\dagger H\right) G^a_{\mu\nu}G^{\mu\nu a}, \quad \mathcal{O}_{H^\dagger HB} = \left(H^\dagger H\right) B_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{H^\dagger HW} &= \left(H^\dagger H\right) W^k_{\mu\nu}W^{\mu\nu k}, \quad \mathcal{O}_{H^\dagger H} = \left(H^\dagger H\right) \psi_L^T H\psi_R + \text{ h.c.}
\end{aligned}$$

The normalization has not been accounted for. These are only a handful. Before writing down the specific structure of each operator, the general structure of the group will be derived. But before we write the various groups, we will first see how equations of motion (EOM) help simplify them.

4.3 SM Equations of motion

Equations of motion are vital as they will be used to reason why several of the upcoming dimension 6 and dimension 8 groups are empty. Using these equations of motion, derived from the principle of least action, several operators are reduced to a simpler form. A simpler form follows the same reasoning as in [23], which is by reducing the number of covariant derivatives or gauge bosons corresponding to a lower operator class or group.

The classical equations of motion are derived by varying the action with respect to the different fields in the theory. Since we are only interested in the EOM for the SM fields, there are four different EOMs to be obtained from varying the action with respect to the four Standard Model fields

$$\frac{\delta S}{\delta B_i} = 0. \tag{4.1}$$

Where $B_i \in \{H, W, G, B\}$, and $i = \{1 \dots 4\}$ is the index which runs over the different fields. From the principle of least action, we obtain the Euler-Lagrange equations by expanding the above condition[39]

$$\frac{\delta S}{\delta B_i} = \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial B_i} \delta B_i - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu B_i \right)} \right) \delta B_i + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu B_i \right)} \delta B_i \right) \right\} = 0. \tag{4.2}$$

The last term is just a surface integral, leaving us with the Euler-Lagrange equations of a field

$$\frac{\partial \mathcal{L}}{\partial B_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu B_i \right)} \right) = 0. \tag{4.3}$$

Plugging in the different SM fields, we obtain three equations of motion for the gauge fields, and one for the Higgs field. We will use the gauge fields EOM to reduce the operators in H^2D^4, H^2AD^2, and A^2D^2. See Appendix 11.1 for a closer look at how they are derived.

We will not take into account higher-order terms when varying the SM with respect to the different fields. Accounting for higher-order terms when varying the SM with respect to a field would lead to complex calculations which is not the focus of this thesis. For a brief discussion regarding this issue see section **??**. The various equations of motion we obtain after varying the Standard Model with respect to each field are[23]

$$\begin{aligned}
(D^\mu D_\mu H)^j &= m^2 H^j - \lambda \left(H^\dagger H \right) H^j - \bar{e}\Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k \Gamma_u u - \bar{d}\Gamma_d^\dagger q^j , \\
(D^\rho G_{\rho\mu})^A &= g_s \left(\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d \right), \\
(D^\rho W_{\rho\mu})^I &= \frac{g}{2} \left(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q \right), \\
\partial^\rho B_{\rho\mu} &= g' Y_\varphi \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi + g' \sum_{\psi \in \{q,l\}} Y_\psi \bar{\psi}\gamma_\mu \psi .
\end{aligned} \tag{4.4}$$

From the above equations, we see how we are able to reduce the groups H^2D^4, H^2AD^2 and A^2D^2 by replacing DA or DDH in the groups with the definitions given in 4.4, transferring them to other fermionic subgroups by removing the number of covariant derivatives and gauge fields.

4.4 Covariant Derivatives and Field Combinations of Dimension-6 Operators

To list all possible operators, we must consider their power counting first. We will later justify the specific structure of the operators based on their compliance with gauge invariance, Lorentz invariance, and lepton and baryon conservation principles. For the various fields, we will use the following notation: field strength tensors (A), fermions (ψ), covariant derivatives (D), and the scalar field (H). The possible operators will be grouped, with various allowed combinations of fields and covariant derivatives restricted by power counting. All the possible groups of dimension 6 operators are listed in Table 4.1[23]

A^3	$A\psi^2H$	D^2H^4	AD^4	H^2D^4
A^2H^2	ψ^2H^3	ψ^2H^2D	A^2D^2	ψ^2AD
AH^4	$\psi\psi\psi\psi$	H^6	H^2D^2A	

Table 4.1: All possible dimension 6 groups which only obey PC rules

Dimension-5 operators are excluded due to their failure to conserve B and L. Furthermore, purely bosonic dimension-5 terms are strictly prohibited since they require uncontracted covariant derivatives or an odd number of scalar fields, which would violate $SU(2)_L$ tensor product constraints. This is because $[A] = 2$ and $[D] = [H] = 1$, and any dimension-5 operator would need to contain an odd number of these fields or derivatives, which cannot be contracted to form a Lorentz-invariant term. [23].

In Table 4.1, not every allowed combination of fields and covariant derivatives contains any operators. For example, AD^4 and AH^4 do not include operators. Although AH^4 has an even number of scalar fields, the antisymmetric nature of the field strength tensors and the absence of any object that can contract with the lone field strength tensor without making it a higher than dimension 6 term breaks Lorentz invariance.

AD^4 has no elements simply because all the elements can be simplified by using a commutator between covariant derivatives to promote all the operators to the D^2A^2 group. As shown in eq 2.3, one way of defining the field strength tensor is through the covariant derivative:

$$[D_\mu, D_\nu] \sim A_{\mu\nu}.$$

The commutator above, therefore, promotes every single AD^4 operator to the A^2D^2 by turning two derivatives into one bosonic field[23].

The groups H^2D^4, H^2AD^2, A^2D^2, and ψ^2AD are also empty since every single operator can be redefined/reduced using classical EOM. The groups are reduced to either a purely fermionic group, or to one of the four following groups

$$A^3, A^2H^2, H^6, H^4D^2.$$

4.5 Systematic Classification of Dimension 6 Effective Operators (Explain assumption of gauge invariance)

The structure of this section is inspired by [23]. After seeing how various operators in the above groups can be reduced, we are left with the groups A^3, H^6, H^4D^2, ψ^2H^3, A^2H^2, $\psi^2AH, H^2\psi^2D$, and $\psi\psi\psi\psi$.

We have already found several different operators by adding the scalar gauge singlet $H^\dagger H$ to all dimension 4 terms. These operators are just 6 out of 59 established independent dimension-6 operators from [23].

To better understand the operators, i.e., why they are structured the way they are, the dimension-6 groups will be presented in tables listing their respective operators. Under each table, the operator's structure will be explained. In contrast, the vast number of dimension-8 operators precludes us from explicitly enumerating them. Nonetheless, systematically breaking down each group enhances our understanding of the operators' validity. Such knowledge becomes especially crucial when dissecting complex dimension-8 operators.

We start by examining the A^3 group, which comprises only gauge fields. Since there are an odd number of gauge fields, the field indices must be contracted with the structure constant of the symmetry group. Below are all the possible A^3 operators.

$$\begin{aligned}
\mathcal{O}_{GGG} &= f^{abc}G_\mu^{a\nu}G_\nu^{b\rho}G_\rho^{c\mu} \\
\mathcal{O}_{\tilde{G}GG} &= \varepsilon^{abc}\tilde{G}_\mu^{a\nu}G_\nu^{b\rho}G_\rho^{c\mu} \\
\mathcal{O}_{WWW} &= f^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu} \\
\mathcal{O}_{\tilde{W}WW} &= \varepsilon^{abc}\tilde{W}_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}
\end{aligned}$$

Table 4.2: A^3.

These are all possible three boson operators allowed for by PC. All the operators also have contracted indices, meaning they are Lorentz invariant. The absence of any A^3 operator constructed out of only B fields is due to the inability to build any Lorentz invariant object using its structure constant. It is important to note that not all tensors with contracted indices are Lorentz invariant. In this case, they are Lorentz invariant as both $G_{\mu\nu}$ and $W_{\mu\nu}$ are constructed out of gauge fields that follow Lorentz transformation rules.

The field strength tensors contracted with each other give a Lorentz invariant quantity. Two tensors contracted with respect to Lorentz indices will always give a Lorentz invariant quantity, i.e., a Lorentz scalar. The term with a tilde above its field strength tensor is a dual-tensor, which follows the same convention as in [23].

Next up is the group A^2H^2 as shown in Table 4.3. We can form the group A^2H^2 by combining one scalar gauge singlet with the three gauge-invariant kinetic energy terms in the Standard Model Lagrangian. By doing so, we can obtain all the acceptable terms under this group. As we will see, other operators can contain such a gauge singlet, but these operators belong to other operator groups.

In A^2H^2, one operator also mixes the field strength tensors $W^a_{\mu\nu}$ and $B_{\mu\nu}$. W-field and B-field can mix as they belong to the electroweak symmetry group. $G_{\mu\nu}$, however, belongs to another gauge group and therefore can not mix with $B_{\mu\nu}$ and $W_{\mu\nu}$.

$\mathcal{O}_{H^\dagger HG} = H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$
$\mathcal{O}_{H^\dagger H\tilde{G}} = H^\dagger H \widetilde{G}^a_{\mu\nu} G^{a\mu\nu}$
$\mathcal{O}_{H^\dagger HW} = H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}$
$\mathcal{O}_{H^\dagger H\tilde{W}} = H^\dagger H \widetilde{W}^a_{\mu\nu} W^{a\mu\nu}$
$\mathcal{O}_{H^\dagger HB} = H^\dagger H B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{H^\dagger H\tilde{B}} = H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{HHWB} = H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{HH\tilde{W}B} = H^\dagger t^a H \widetilde{W}^a_{\mu\nu} B^{\mu\nu}$

Table 4.3: A^2H^2.

Since the operators in A^2H^2 are constructed from gauge-invariant terms from the Standard Model Lagrangian, gauge invariance is preserved in all operators. Additionally, there are no uncontracted Lorentz indices in the operators, ensuring that Lorentz invariance is preserved.

The groups H^6 and H^4D^2 are small and only contain one and two operators, respectively. The group H^6 can only have the operator $(H^\dagger H)^3$, since any other combination of H-fields where two and two Higgs fields are not contracted in a scalar gauge singlet would break Lorentz-invariance. The only different combination is $H^\dagger t^a H$.

The structure of the Higgs field is that of an $SU(2)$ doublet with representation $(\frac{1}{2}, 1)$ with $\frac{1}{2}$ being third component of weak isospin and 1 being weak hypercharge. The fermion fields always come in pairs, with one being conjugated so that total hypercharge vanishes[23]. All other combinations of fields, dual-fields, or complex conjugated fields, would break hypercharge constraints. Hypercharge constraint requires that the sum of the hypercharges of the fields in any interaction must be conserved. Therefore the combination of non-zero hypercharge fields in Table 4.3 is the only combination that does not violate hypercharge conservation.

The two operators in H^4D^2 are rather complex, but both of these two operators can be derived using a Fierz identity and an EOM. As with all the other operators up until now, these operators are constructed out of already-known SM operators. Taking a scalar gauge singlet and adding the operator from the SM containing two derivatives, i.e., the kinetic term for the scalar Higgs field, we get Table 4.4[23].

$$\left(\varphi^\dagger \tau^I \varphi\right) \left[(D_\mu\varphi)^\dagger \tau^I (D^\mu\varphi)\right]$$
$$\left(\varphi^\dagger \varphi\right) \left[(D_\mu\varphi)^\dagger (D^\mu\varphi)\right]$$

Table 4.4: H^4D^2.

These operators are already gauge- and Lorentz invariant by the same reasoning as the previous group A^2H^2. From these two operators we are able to obtain the operators in Table 4.5. These operators are derived using the Fierz-identity

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn},$$

and the EOM for the hypercharge $U(1)$ field. See Appendix 11.2 for the derivation of the two operators.

Fierz identities are a helpful tool for eliminating redundant operators. In Table 4.4, we can see that only two operators are present, as all other potential operators can be obtained using the Fierz identity or an EOM on these two operators. Fierz identities greatly simplify calculations.

$$\mathcal{O}_{\Box H^\dagger H} = \left(H^\dagger H\right) \Box \left(H^\dagger H\right)$$
$$\mathcal{O}_{2DH^\dagger H} = \left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right)$$

Table 4.5: $H^4 D^2$.

The next three groups introduce fermion currents mixed with scalar and bosonic fields: $\psi^2 AH$, $\psi^2 H^3$, and $\psi^2 H^2 D$.

The group $\psi^2 AH$ consists of one fermion current in the form of $\bar{\psi}_p \sigma^{\mu\nu} \psi_r$ (where p and r are generation indices), and σ represents the antisymmetric product of the Dirac matrices, along with the Higgs field and one gauge boson. The sigma matrix is needed to combine the Lorentz indices of the two fermionic fields and to ensure that the operator has the correct Lorentz transformation properties. The fields must be structured similarly to the Yukawa interactions in the Standard Model to conserve hypercharge. For dimension-six operators involving scalar and tensor fermionic currents, the number of associated Higgs fields is always odd. This implies that the fermionic currents must be arranged in isospin doublets. The way these currents are written are either $\bar{\psi}_1 \psi_2$ and $\bar{\psi}_1 \sigma_{\mu\nu} \psi_2$. The form of the fermion current must be $\bar{\psi}_1 \sigma_{\mu\nu} \psi_2$, for the sigma to be contracted with a Higgs or a gauge field.

The operators can include any of the three gauge bosons as long as the bosons are contracted with their respective isospin triplets and color octets. Groups with scalar and tensor fermionic currents are always associated with an odd number of Higgs fields due to power counting constraints. Consequently, groups with an odd number of Higgs fields force the fermion currents to be isospin doublets. [23]

There are three different currents that can be contracted with three different bosons, but the gluonic field cannot create leptonic fields. On the other hand, both the $B_{\mu\nu}$ and $\mathbf{W}_{\mu\nu}$ bosons can couple to leptonic fields and quark fields. Table 4.6 lists all possible operators that can arise from these combinations. In the subscripts of the operators, the fermion current is denoted as either e, u, or d to indicate the type of current that it represents.

$$\mathcal{O}_{eHW} = \left(\bar{l}_p \sigma^{\mu\nu} e_r\right) \tau^a H W^a_{\mu\nu}$$
$$\mathcal{O}_{eHB} = \left(\bar{l}_p \sigma^{\mu\nu} e_r\right) H B_{\mu\nu}$$
$$\mathcal{O}_{u\tilde{H}G} = \left(\bar{q}_p \sigma^{\mu\nu} u_r\right) t^a \tilde{H} G^a_{\mu\nu}$$
$$\mathcal{O}_{u\tilde{H}W} = \left(\bar{q}_p \sigma^{\mu\nu} u_r\right) \tau^a \tilde{H} W^a_{\mu\nu}$$
$$\mathcal{O}_{u\tilde{H}B} = \left(\bar{q}_p \sigma^{\mu\nu} u_r\right) \tilde{H} B_{\mu\nu}$$
$$\mathcal{O}_{dHG} = \left(\bar{q}_p \sigma^{\mu\nu} d_r\right) t^a H G^a_{\mu\nu}$$
$$\mathcal{O}_{dHW} = \left(\bar{q}_p \sigma^{\mu\nu} d_r\right) \tau^a H W^a_{\mu\nu}$$
$$\mathcal{O}_{dHB} = \left(\bar{q}_p \sigma^{\mu\nu} d_r\right) H B_{\mu\nu}$$

Table 4.6: $\psi^2 AH$.

Again, all operators maintain Lorentz invariance as there are no uncontracted Lorentz indices.

The next group, $\psi^2 H^3$, combines Yukawa interactions with a scalar gauge singlet. As with the group of Yukawa-like interaction, $\psi^2 AH$, the current in $\psi^2 H^3$ must be an isospin doublet and a color singlet on the form $\bar{\psi}_1\psi_2$.

The scalar fields in the $\psi^2 H^3$ group are also subject to hypercharge constraints. As we know, the scalar gauge singlet $H^\dagger H$ has a hypercharge of zero, as the hypercharge of the Higgs fields cancels, while the remaining scalar field should be either conjugated or un-conjugated based on the hypercharge of the fermion current. Table 4.7 shows the possible operators in the $\psi^2 H^3$ group.

$\mathcal{O}_{e3H} = \left(H^\dagger H\right)\left(\bar{l}_p e_r H\right)$
$\mathcal{O}_{u3H} = \left(H^\dagger H\right)\left(\bar{l}_p u_r H\right)$
$\mathcal{O}_{d3H} = \left(H^\dagger H\right)\left(\bar{l}_p d_r H\right)$

Table 4.7: $\psi^2 H^3$.

The operators listed in the table are not all possible operators in the $\psi^2 H^3$ group due to the constraint imposed by combining scalar gauge singlet into isospin doublets, which is necessary for maintaining gauge invariance. The scalar gauge singlet has a total hypercharge of zero, and any additional scalar field combined with it must also have a total hypercharge of zero to conserve hypercharge. Therefore, the possible combinations of scalar fields with the scalar gauge singlet are restricted, and only a subset of potential operators are allowed. The creation of doublet scalar fields is unique, and combinations such as $H^\dagger \widetilde{H}$ are zero[1].

The next group of operators we will examine is $\psi^2 H^2 D$. This group is characterized by two scalar fields, two fermion fields, and one covariant derivative.

If the covariant derivative acts on the fermion current, then using classical equations of motion, every operator in this group can be transformed into another group. Thus, the covariant derivative must act on one of the two scalar fields to maintain uniqueness.

It is essential to emphasize that the operators must be Hermitian. According to Weinberg [43], the action must be real, and thus, Lagrangian operators must satisfy the constraint of being hermitian. This constraint ensures that the observables are real and physically meaningful. To simplify the notation and account for the hermiticity of operators, we use the symbol $\leftrightarrow D_\mu$, which includes the hermitian conjugate of the operator in the scalar part of the Lagrangian.

Without gauge bosons, the fermion current cannot change the type or generation of fermions. As a result, only three distinct fermion currents can appear in the $\psi^2 H^2 D$ group of operators. The complete list of operators in this group is presented in table 4.8. It should be noted that the current notation used in the previous group is also applicable here, except for the last operator in the current, which involves mixing between up and top quarks.

Four-fermion operators are of no value when considering HEFT, so they will not be listed. Next up is the classification of dimension 8 operators. Dimension 8 operators

[1] $H_j^\dagger \tilde{H}^j = \varepsilon^{ja}\left(\varphi_a\right)^\star \varepsilon_{jk}\left(\varphi^k\right)^\star = 0$

$$
\begin{aligned}
\mathcal{O}_{2Hl} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{l}_p \gamma^\mu l_r\right) \\
\mathcal{O}_{2Hl} &= \left(H^\dagger i \overleftrightarrow{D}^a_\mu H\right)\left(\bar{l}_p t^a \gamma^\mu l_r\right) \\
\mathcal{O}_{2HL} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{e}_p \gamma^\mu e_r\right) \\
\mathcal{O}_{2Hq} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p \gamma^\mu q_r\right) \\
\mathcal{O}_{2Hq} &= \left(H^\dagger i \overleftrightarrow{D}^a_\mu H\right)\left(\bar{q}_p t^a \gamma^\mu q_r\right) \\
\mathcal{O}_{2Hu} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{u}_p \gamma^\mu u_r\right) \\
\mathcal{O}_{2Hd} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p \gamma^\mu d_r\right) \\
\mathcal{O}_{2Hud} &= \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{u}_p \gamma^\mu d_r\right)
\end{aligned}
$$

Table 4.8: $\psi^2 H^2 D$.

consist of a vast set of operators and will not be discussed at the same length as the dimension-6 case.

4.6 Covariant Derivatives and Field Combinations of Dimension-8 Operators

The SM, on its own, consists of 14 operators. If we limit the PC rules to dimension 6, there are 59 additional independent operators[23]. Furthermore, considering PC rules restricted dimension-8 operators, the total number of independent operators reaches a staggering 44,807 (disregarding flavor variations for both operator sets)[33]. The effective Lagrangian can include these additional operators to describe various physical phenomena better and provide insights into fundamental physics beyond the SM. Four-fermion operators are useless when considering HEFT, so they will not be listed. Next up is the classification of dimension 8 operators. Dimension 8 operators consist of a vast set of operators and will not be discussed at the same length as the dimension-6 case.

It is unnecessary to write down all 44807 operators, as we are only interested in improving the parametrization of the deviation in the Higgs coupling to SM matter fields. Of those 44807 operators, as we will see later, there are only a handful of relevant bosonic ones. As with dimension 6 operators, most operators, including flavor variations, are unimportant when describing Higgs physics.

The dimension-8 operators that contribute to the Higgs coupling involve interactions between the Higgs field and some SM fields. When exploring the extensive set of dimension-8 operators, it is crucial that we carefully consider only those that are relevant to the task at hand. Including irrelevant operators in our analysis would lead to unnecessary complexity in our calculations when calculating the Lagrangian in unitary gauge and would ultimately not improve parametrization. But before delving into the structure of the various dimension-8 operators and their significance, it is essential to ask: what is the motivation for including them in our analysis?

Several interesting interactions can only be described by dimension 8 operators. A few of these include light-by-light scattering and the contribution dimension 8 operators can have to electroweak precision measurements (EWPM)[33]. Dimension-8 operators can also contribute to improving the U-parameter, which is a parameter introduced by Peskin and Takeuchi to describe new physics in the electroweak sector and quantify electroweak radiative corrections [38]. Section 7.1 introduces electroweak radiative corrections in greater detail.

The article by Murphy [33] was consulted for the discussion of different dimension-8 operators.

When classifying the different dimension-8 operators, we can get a couple of apparent groupings by just expanding some existing ones with simple dimension two operators, such as the scalar Higgs singlet $H^\dagger H$.

Adding the scalar gauge singlet to the group H^6 results in the group H^8. Using the same arguments as for H^6, it becomes apparent that there can be only one operator in H^8, namely $(H^\dagger H)^4$. This operator represents the self-interaction of the Higgs field. However, even at order $\frac{1}{\Lambda^2}$, the Higgs self-interaction operator ($(H^\dagger H)^3$) is considered to be of little practical use because current experimental precision is not sensitive enough to detect it [9].

Table **??** illustrates how the addition of various objects to the existing dimension-6 groups creates dimension-8 groups

We will discuss every dimension-8 group below. The eight tree diagrams below show how one can obtain a dimension-8 group by adding one gauge boson A (blue), two covariant derivatives D (green), or two Higgs fields H (red) to a dimension-6 group. The tree diagram provides a general overview of all possible dimension-8 groups, some of which make significant contributions to the dimension-8 Lagrangian.

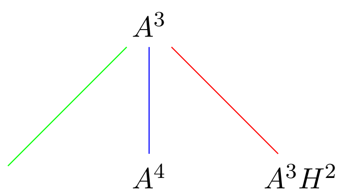

Figure 4.1: Expanding group A^3. Notably one branch is empty

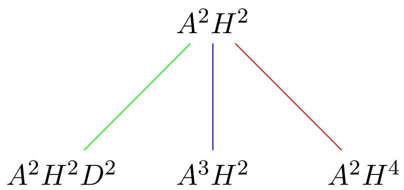

Figure 4.2: Expanding A^2H^2

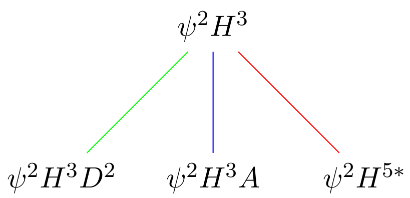

Figure 4.3: Expanding ψ^2H^3

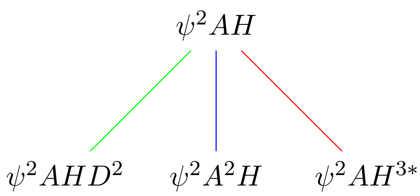

Figure 4.4: Expanding ψ^2AH. One branch is overlaping with ψ^2H^3

Not all the dimension 8 groups are presented here. Some groups are expanded by one scalar field and one covariant derivative, which is not covered by the diagrams. Additionally, it is impossible to include two covariant derivatives in the A^3 group, as

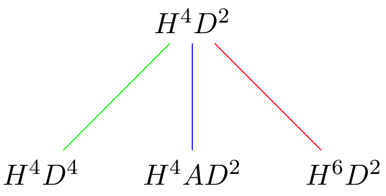

Figure 4.5: Expanding H^4D^2

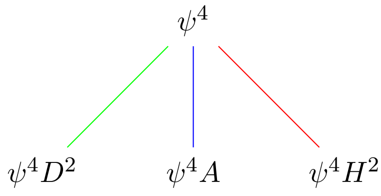

Figure 4.6: Expanding ψ^4

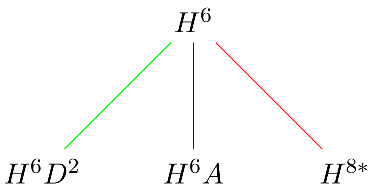

Figure 4.7: Expanding H^6

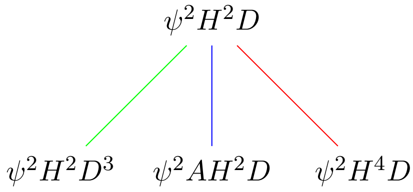

Figure 4.8: Expanding ψ^2H^2D

such terms can be reduced to other groups using EOM. The use of EOM to demonstrate the reduction of different groups was previously demonstrated in the classification of the dimension 6 operators.

Two groups are not included in the tree diagrams, namely ψ^4HD and ψ^2A^2D. The former group is obtained by adding one covariant derivative and one gauge field to the four-fermion group. The latter is obtained by combining a fermion current with two gauge fields and one covariant derivative.

Some groups can lead to the same operators when adding either a Higgs field or a gauge field. For example, the H^6D^2 group can be obtained by adding two covariant derivatives to the dimension 6 H^6 group or two Higgs fields to the H^4D^2 group. This overlap results from exploring different ways to obtain the dimension-8 groups by adding various combinations of fields/derivatives with dimension 2.

The groups marked with an asterisk indicate that the only addition to the dimension 6 operators is the scalar gauge singlet $(H^\dagger H)$. While this article will not provide a systematic grouping and explanation of all the dimension 8 operators, similar to what was done with the dimension 6 operators, as this is beyond the scope and goal of the article. Instead, we focus on the relevant dimension 8 operators, drawn mainly from [33], and further explanation will be provided for those pertinent operators to the parametrization of the Higgs coupling to matter fields.

4.7 Classifying Dimension-6 Operators

We will begin by choosing the dimension 6 operators for our effective Lagrangian. In this regard, we will adopt the same dimension 6 Lagrangian as utilized in the work of [43]. However, we will provide a more detailed explanation for the specific selection of operators than presented in [43]. Furthermore, we will demonstrate the complete calculations of the Lagrangian in unitary gauge. Every higher-order operator in unitary gauge can be found in Appendix H 11.6 .

The work of [43], which has inspired this article, relied on various key assumptions in selecting an effective Lagrangian for calculating constraints on the Higgs couplings.

A contrast between their approach and ours is the exclusion of dimension-8 operators. It is yet to be determined whether including such operators can enhance the parametrization of the deviation in Higgs coupling to SM matter fields. Moreover, they made additional assumptions, such as disregarding the operator $(H^\dagger H)^3$, as the experimental precision is not yet refined enough to account for the Higgs self-coupling.

The operators in $\mathcal{L}_6$ that will contribute to the parametrization involve at least one Higgs field. Initially, the relevant groups containing Higgs fields are identified as H^6, H^4D^2, ψ^2AH, ψ^2H^2D, and ψ^2H^3. However, it is important to note that not all operators containing a Higgs field will necessarily contribute to the parametrization.

The dynamics of EWSB have been experimentally investigated by LEP1 (Large Electron-Positron Collider), which has provided strong evidence that the EWSB dynamics are weakly-coupled, thus indicating the existence of a light Higgs boson [21]. This light Higgs boson is responsible for the weakly-coupled Higgs self-interaction dynamics [21]. The EFT of the SIHL has already been extensively studied, and the low-energy effective Lagrangian corresponding to SILH is the same as the previously mentioned SILH Lagrangian.

The SILH Lagrangian, along with the inclusion of the terms Δ_{F_1} and Δ_{F_2} containing 2-fermion vertex and 2-fermion dipole operators, respectively, has proven to be valuable for comparing to the Warsaw basis since it solely contains operators describing Higgs physics [21]. One notable distinction is the reformulation of the operators in the H^2AD^2 group in the Warsaw basis into the two groups H^4D^2 and ψ^2H^2D. The H^4D^2 group poses no issue since the operators in the Warsaw and SILH bases are the same. However, the reduction of the H^2AD^2 group in the Warsaw basis introduces vertex corrections and changes to the Fermi constant due to the presence of fermion currents [16].

As the corrections introduced by the reduced operators in the H^2AD^2 group can be cumbersome, it is preferable to employ the original operators. While other bases may be utilized, modifying Fermi constants requires incorporating a four-fermion operator, as demonstrated in [4].

Next, we will go over every group to decide which operators can be cut for the final description of the effective Lagrangian.

The H^6 group comprises the Higgs self-coupling operator $(H^\dagger H)^3$. However, detecting Higgs self-interaction is already a challenging task [27], and detecting a six-fold interaction is not feasible with the current LHC luminosity. Thus, omitting this operator from the final effective Lagrangian description is reasonable.

$\boldsymbol{H^4D^2}$: The group H^4D^2 contains only two operators: $\left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right)$ and $\frac{1}{2}\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$. We will employ a different structure of the operator $\frac{1}{2}\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$. This operator can be rewritten according to a field-redefinition as [21] lays out, to the SHIL operator $\frac{1}{2}\partial_\mu\left(H^\dagger H\right)\partial^\mu\left(H^\dagger H\right)$[2]. The SHIL version is

[2]The Warsaw operator and the SHIL operator both stem from the same operator before EOM in the Warsaw case and a field redefinition in the SHIL case.

preferable, as it is easier to see how the operator would affect the canonical kinetic Higgs term. $\frac{1}{2}\partial_\mu\left(H^\dagger H\right)\partial^\mu\left(H^\dagger H\right)$ effects also the Higgs self-interactions[22], while $\left(H^\dagger D^\mu H\right)^\star\left(H^\dagger D_\mu H\right)$ is responsible for interactions between the Higgs field and gauge bosons.

$\boldsymbol{\psi^2 H^3}$ **:** The operators in this group are also relevant, as the Higgs field is coupled to Standard Model fermions. Although EWPM heavily constrains fermion-currents coupled to gauge bosons[9], not allowing for any Higgs physics, these operators do not have any gauge-fermion coupling. These operators are, therefore, theoretically helpful, as they modify the Yukawa interactions and can affect the Higgs couplings to fermions. However, the current LHC luminosity may limit the experimental sensitivity to these operators. Therefore, their relevance depends on future colliders' experimental reach and sensitivity.

The Higgs boson is responsible for giving mass to the fermions through the Yukawa interaction, which couples the Higgs field to the fermions. The strength of the coupling is proportional to the mass of the fermion, which means that the Higgs interaction is most prominent with heavy fermions. This makes the top quark particularly relevant for Higgs physics since it is the heaviest of the quarks. As the mass of the fermion decreases, the likelihood of producing a Higgs boson also decreases.

The operators in this group and their dimension 8 equivalents induce a shift in fermion mass. This shift will be examined in 6.5.

$\boldsymbol{A^2 H^2}$: This group is essential to include as it impacts the Higgs boson's coupling to all Standard Model gauge fields.

$\boldsymbol{\psi^2 A H}$: The operators in this group involve fermion currents coupled to gauge bosons, which are already tightly constrained by EWPM. Consequently, the effect of adding higher order operators of this form is negligible [7]. It is not reasonable to assume that entire groups of operators can be cut just due to them being constrained by EWPM. One motivating factor for the inclusion of dimension 8 operators is that they increase experimental sensitivity to the new physics, which could be found even with the stringent constraints on the electroweak sector. However, relaxing the assumption that fermion currents coupled to gauge bosons could allow for new Higgs physics would open up too many operators that could be included in the final Lagrangian. Assuming that the fermion currents coupled to gauge bosons are overly constrained, even for dimension 8 operators, is a reasonable assertion supported by the precedent set in [7]. Additionally, for practical reasons, constructing a Lagrangian that covers all groups with fermion currents would result in a large and too complex expression to put into unitary gauge.

$\boldsymbol{\psi^2 H^2 D}$ **:** As previously explained, the incorporation of fermion currents instead of the original group H^2AD^2 before reduction by the equations of motion results in an undesired change of the Fermi constant. This necessitates the introduction of a four-fermion operator to compensate for the change. To circumvent the need for the four-fermion operator, it is more convenient to utilize the original form without fermion currents.

$\boldsymbol{A^3}$: This group consists solely of gauge fields, representing boson propagators and boson self-interactions that do not affect Higgs physics. In this thesis, our focus is not on purely bosonic operators or those that result in an interaction involving three or more gauge bosons after being subjected to unitary gauge.

In the next section, we will explore various dimension 8 operators and, in the end, settle for a dimension 8 Lagrangian.

4.8 Key Considerations and Selection Criteria for Dimension-8 Operators

Although the anticipated yield of incorporating dimension 8 operators with current LHC data is low, their utility is expected to increase with the advent of the HL-LHC. As the HL-LHC becomes operational, the collision rate of protons in the collider will increase, leading to more precise measurements and greater sensitivity to new physics phenomena. As a result, dimension 8 operators are anticipated to become more valuable.

A complete dimension-8 basis has been established in existing literature [33], but the quantitative impact of dimension-8 operators still needs to be discovered [18]. Nonetheless, the inclusion of dimension-8 operators is essential to obtain reliable and accurate results in certain instances. For instance, calculating the physical cross-section over suitable phase space necessitates the involvement of dimension-8 operators as the $\mathcal{O}(\frac{1}{\Lambda^2})$ contributions cancel, thereby relying on the contributions of dimension-8 operators to evaluate the cross-section [35]. Furthermore, dimension-8 operators serve a crucial role in describing the U-parameter.

As mentioned earlier, new physics phenomena that initially occur at dimension-8, such as photon-photon scattering or new physics effects such as $ZZ\gamma$ coupling, represent further examples where dimension-8 operators may play an essential role [18]. However, exploring whether dimension-8 operators may contribute to the parametrization of the Higgs to SM matter fields constitutes an exciting expedition.

Selecting relevant dimension-8 operators poses two main challenges. The primary and most crucial challenge involves determining which operators should be included among the hundreds available to parametrize the Higgs coupling to SM matter effectively. Even when disregarding flavor symmetries, choosing a limited number of dimension-8 operators remains daunting.

The second issue pertains to the impact of these dimension-8 operators on canonical renormalization, fermion mass terms, and various corrections, including the modification to the Z-boson mass, which is already present at dimension-6. These matters will be addressed in the subsequent section. When selecting the various dimension-8 operators, existing constraints from dimension-6 operators will also be extended to encompass dimension-8 operators.

It is reasonable to assume that if any of the dimension 6 groups presented in section 4.6 are excluded, the corresponding sister groups in the tree diagrams should also be excluded for similar reasons. The inclusion of too many dimension-8 operators can lead to unnecessary complexity in calculations. Therefore, the group of dimension-8 operators used will be carefully selected based on their potential impact on Higgs physics and their level of constraint. The less constrained operators with the greatest potential to affect Higgs physics will be prioritized. The restraints we put on dimension-6 operators were

Constraint 1: Detection of Higgs self-interactions poses a significant challenge due to their small cross-section.

Constraint 2: The current experimental precision at the LHC may not be sufficient to reveal the effects of higher-order operators.

Constraint 3: At least one Higgs field is required in order for the operator to parametrize Higgs physics.

Constraint 4: Interactions heavily constrained by EWPM, limit the potential impact of higher-order operators where fermion currents are coupled to gauge bosons.

These assumptions are crucial in determining the relevant dimension-8 operators to include. However, additional constraints must be considered when selecting these operators. Certain heavily constrained operators can potentially be transformed into less constrained ones using equations of motion and field redefinitions.

Given that the term $(H^\dagger H)^3$ has already been removed due to experimental precision, the same is done to $(H^\dagger H)^4$. Therefore, the Higgs field's VEV remains unaltered. Consequently, the Higgs potential in its current form is deemed valid without requiring any further modifications

To achieve our ultimate objective of parameterizing the constraints on BSM physics using the STU-parameters, it is crucial to recognize that the U-parameter is solely influenced by dimension-8 operators. On the other hand, dimension-6 operators provide the primary contributions to S and T [33]. We will employ the traditional notation for the STU parameters, as defined in [9].

Building upon previous restrictions, the following groups have been excluded: H^8, all two-fermion, and four-fermion operators(except Yukawa-like operators). This leaves us with a bunch of bosonic operators to consider. Another reason fermionic operators are excluded is that the dictionary we are expanding upon does not have any fermionic interactions other than those provided by the SM Lagrangian.

Bosonic operators are more manageable to consider than fermionic ones, and we already know which interactions to look for. These interactions will be defined in chapter 6. Due to time constraints not every single bosonic operator will be considered. Only bosonic operators of similar structure to ones already used in the effective Lagrangian will be used not to make calculations too complex.

This leaves us with the following four groups to pick our dimension-8 operators from A^2H^4, AH^4D^2, H^6D^2, and ψ^2H^5+ h.c. . The group $A^2H^2D^2$ was cut, as you would only get propagators or three gauge bosons interactions from the group. Groups that are purely or almost completely bosonic are also excluded.

The possible operators are three from the group A^2H^4, three from the group ψ^2H^5+ h.c. and lastly, two from the group AH^4D^2. The two operators in group H^6D^2 contribute to a new redefinition of the Higgs field, while the operators in ψ^2H^5+ h.c. contribute to a broader shift in fermion mass. It is pretty fascinating how we started with 44,807 [9] operators and narrowed it down to only 89 relevant bosonic operators, plus three fermionic operators without gauge bosons. Of these 89 operators, only 18 of them are relevant in this analysis. All the other operators have been excluded based on containing too many gauge bosons (W, G, or B). This accounts for 67 of the 89 bosonic operators, which are purely bosonic or contain three or more gauge bosons. Of the 22 operators left, one is the pure Higgs self-coupling term, which is excluded, and three more are excluded to prevent another correction to the Z-mass. This leaves us with 18 bosonic operators + 3 Yukawa-like operators to build our Lagrangian from. Operators in dual space are not shown in the effective Lagrangian.

The dimension-8 operators that are most likely to affect the Higgs coupling to standard model matter fields are those included in the effective Lagrangian 4.5

$$
\begin{aligned}
\mathcal{L}_8 = {} & c_r \left(H^\dagger H\right)^2 \left(D_\mu H^\dagger D^\mu H\right) + c_q \left(H^\dagger H\right) \left(H^\dagger \tau^I H\right) \left(D_\mu H^\dagger \tau^I D^\mu H\right) \\
& + \frac{\left(H^\dagger H\right)^2}{v^2} \left(\left(c_l \bar{l}_p e_r H\right) + \left(c_u \bar{q}_p u_r \widetilde{H}\right) + \left(c_d \bar{q}_p d_r H\right)\right) \\
& + \left(H^\dagger H\right)^2 G^A_{\mu\nu} G^{A\mu\nu} + \left(H^\dagger H\right)^2 W^I_{\mu\nu} W^{I\mu\nu} \\
& + \left(H^\dagger H\right)^2 B_{\mu\nu} B^{\mu\nu} + \left(H^\dagger H\right) \left(D^\mu H^\dagger \tau^I D^\nu H\right) W^I_{\mu\nu} \\
& + \left(H^\dagger \tau^I H\right) \left(H^\dagger \tau^J H\right) W^I_{\mu\nu} W^{J\mu\nu} + \left(H^\dagger H\right) \left(H^\dagger \tau^I H\right) W^I_{\mu\nu} B^{\mu\nu} \\
& \left(H^\dagger H\right) \left(D^\mu H^\dagger D^\nu H\right) B_{\mu\nu}
\end{aligned} \tag{4.5}
$$

The first line is included as these dimension-8 operator will alter the canonical renormalization of the Higgs field again. The second line will cause an even broader shift in the fermion mass. Correct normalization is given in the next chapter.

Chapter 5

Higgs Mechanism

5.1 Understanding the Significance of the Higgs Mechanism

The Higgs mechanism is a process whereby the symmetry of the $SU(2)_L \times U(1)Y$ group is spontaneously broken, resulting in the emergence of a residual symmetry in $U(1)_{em}$. Although the overall symmetry remains preserved, the ground state, or vacuum state, no longer adheres to this symmetry and instead adopts a non-zero VEV. The requirement of invariance under $SU(2)_L \times U(1)_Y$ and renormalizability necessitates that the Higgs potential takes the specific form[32]

$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 .$$

A non-zero VEV occurs when $\mu^2 > 0$, where λ represents the quartic coupling strength. When $\mu^2 > 0$, the vacuum expectation energy of the scalar field is given by $\langle H \rangle = \sqrt{\frac{\mu^2}{2\lambda}}$. To break the symmetry, obtaining a symmetry-breaking VEV for the scalar field is necessary. This can be achieved through the use of the unitary gauge

$$\langle H \rangle \rightarrow \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} .$$

The choice of gauge used in the Higgs mechanism is not inherently significant. The choice of gauge introduces a minimal number of scalar degrees of freedom by eliminating the Goldstone boson through the gauge transformation in 5.1

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \chi. \tag{5.1}$$

Considering the general gauge field A as having "absorbed" the Goldstone boson χ is conceptually helpful. In the $SU(2)_L \times U(1)_Y$ theory, three Goldstone bosons correspond to the three broken generators of the gauge group. As a result, all three Goldstone bosons are absorbed and contribute to the mass of the three heavy fields, $W^\pm$, and Z, with only one remaining massless photon.

The final phenomenological Lagrangian will necessarily include the $W^\pm$ and Z fields, as opposed to the W and B fields present before symmetry breaking. This is because the specific gauge chosen, the unitary gauge, generates massive fields via symmetry breaking, resulting in the emergence of the physical $W^\pm$ and Z fields in the final Lagrangian. The following section will delve into the physics of electroweak symmetry breaking, exploring how the Higgs mechanism takes our effective Lagrangian from an unphysical one to a physical one which can be used to further explore and define the WCs.

Chapter 6

Exploring the Effective Lagrangian Before and After Electroweak Symmetry Breaking

6.1 Electroweak Symmetry Breaking: A Key Component of the Standard Model

This chapter investigates the impact of the unitary gauge and EWSB on dimension 6 and dimension 8 operators and the SM Lagrangian. The dimension 6 and dimension 8 operators will be presented as comprehensive Lagrangians denoted by $\mathcal{L}_6$, $\mathcal{L}_8$, $\mathcal{L}_{6,h}$, $\mathcal{L}_{8h}$, depending on whether they consist solely of dimension 6 operators or dimension 8 operators, and if they are dependent on the Higgs field expansion h.

Initially, the Lagrangians will be introduced in their "pure" or unphysical form, which refers to the previously discussed operators in Chapter 4, but with the correct coefficients preceding each operator, as per the conventions established in [9]. Subsequently, each Lagrangian will undergo calculations and be presented in the unitary gauge in the subsequent chapter. The computations in the unitary gauge can be found in Appendices 11.6.3, 11.6.7, and 11.3, with the latter addressing the mechanisms through which the SM Higgs sector and fermions acquire mass via EWSB.

Towards the end of the chapter, the effects of certain operators and necessary field re-definitions will be presented.

6.2 Effective Lagrangian Pre-Electroweak Symmetry Breaking

The WCs and the energy scale will be restored before each operator to provide a complete notation. For conciseness, only the matter content will be listed as subscripts for each Wilson coefficient. To avoid redundancy, subscripts for repeated operators will not be capitalized. After omitting the gauge group A^3 and the four-fermion group $\psi\psi\psi\psi$, and rewriting the group $\psi^2 AH$, the resulting dimension-6 effective Lagrangian, excluding CP-violating parts, can be expressed as follows:

$$
\begin{aligned}
\mathcal{L}_6 = & \frac{\bar{c}_T}{2\Lambda^2}\left(H^\dagger D^\mu H\right)^\star\left(H^\dagger D_\mu H\right) + \frac{\bar{c}_h}{2\Lambda^2}\partial_\mu\left(H^\dagger H\right)\partial^\mu\left(H^\dagger H\right) \\
& + \left[\left(H^\dagger H\right)\frac{\bar{c}_{Hle}}{\Lambda^2}\left(\bar{l}_p e_r H\right) + \frac{\bar{c}_{Hqu}}{\Lambda^2}\left(H^\dagger H\right)\left(\bar{q}_p u_r \widetilde{H}\right) + \frac{\bar{c}_{Hqd}}{\Lambda^2}\left(H^\dagger H\right)\left(\bar{q}_p d_r H\right) + h.c\right] \\
& + \frac{\bar{c}_{HB}}{\Lambda^2}\left(H^\dagger H\right)B_{\mu\nu}B^{\mu\nu} + \frac{\bar{c}_{HG}}{\Lambda^2}\left(H^\dagger H\right)G^a_{\mu\nu}G^{a\mu\nu} + \frac{\bar{c}_{HW}}{\Lambda^2}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)\left(D^\nu W_{\mu\nu}\right)^i \\
& + \frac{\bar{c}_{Hb}}{\Lambda^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(\partial^\nu B_{\mu\nu}\right) + \frac{\bar{c}_w}{\Lambda^2}\left(D^\mu H\right)^\dagger\sigma^i\left(D^\nu H\right)W^i_{\mu\nu} \\
& + \frac{\bar{c}_b}{\Lambda^2}\left(D^\mu H\right)^\dagger\left(D^\nu H\right)B_{\mu\nu}.
\end{aligned}
\tag{6.1}
$$

Lagrangian 6.2 uses the normalization convention of [9] which adopts the convention of [16], where the new-physics scale is absorbed into the Wilson-coefficient. The kinetic term is also altered from the Warsaw basis and the SILH basis in [16][21] has been used. The SHIL basis is again more useful here, as it is easier to calculate the SHIL operator in unitary gauge.

$\mathcal{L}_6$ is presented in 6.2 with the convention where the energy scale cut-off is the weak-scale VEV and the W-boson. Only operators that contain "light" fields, or electroweak fields, are cut-off at the electroweak scale of the Higgs field's VEV. All other operators are cut-off at the mass of the heavy, weak force mediating W-boson.

$$
\begin{aligned}
\mathcal{L}_6 = & \frac{\bar{c}_T}{2v^2}\left(H^\dagger D^\mu H\right)^\star\left(H^\dagger D_\mu H\right) + \frac{\bar{c}_h}{2v^2}\partial_\mu\left(H^\dagger H\right)\partial^\mu\left(H^\dagger H\right) \\
& + \left[\left(H^\dagger H\right)\frac{\bar{c}_{Hle}}{v^2}\left(\bar{l}_p e_r H\right) + \frac{\bar{c}_{Hqu}}{v^2}\left(H^\dagger H\right)\left(\bar{q}_p u_r \widetilde{H}\right) + \frac{\bar{c}_{Hqd}}{v^2}\left(H^\dagger H\right)\left(\bar{q}_p d_r H\right) + h.c\right] \\
& + \frac{\bar{c}_{HB}g_1^2}{m_W^2}\left(H^\dagger H\right)B_{\mu\nu}B^{\mu\nu} + \frac{\bar{c}_{HG}g_3^2}{m_W^2}\left(H^\dagger H\right)G^a_{\mu\nu}G^{a\mu\nu} + \frac{\bar{c}_{HW}g_2^2}{m_W^2}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)\left(D^\nu W_{\mu\nu}\right)^i \\
& + \frac{\bar{c}_{Hb}g_1}{2m_w^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(\partial^\nu B_{\mu\nu}\right) + \frac{\bar{c}_w g_2}{2m_W^2}\left(D^\mu H\right)^\dagger\sigma^i\left(D^\nu H\right)W^i_{\mu\nu} \\
& + \frac{\bar{c}_b g_1}{m_W^2}\left(D^\mu H\right)^\dagger\left(D^\nu H\right)B_{\mu\nu}.
\end{aligned}
\tag{6.2}
$$

There are a few CP-violating operators that do contain Higgs physics which are not included in the Lagrangian above. CP-violation is an inherent property of the SM, as it is one of the Sakharov conditions required for baryogenesis[34][41]. Therefore, it is reasonable to assume that there would also be dimension-6 operators that violate CP.

It can be shown that every operator constructed out of dual gauge fields is CP-odd. This can be attributed to dual gauge fields giving rise to a non-zero axial current J_5^μ with non-zero divergence [36]. Further, the divergence, along with the correct boundary conditions used by 't Hooft, demonstrated that $U(1)_A$ is not a true symmetry of QCD, and that the QCD vacuum is more complex [36]. As a result, operators with dual tensors are CP-odd. Table **??** presents various CP-violating operators that can affect Higgs physics. This table is identical to the one provided in [23].

A^2H^2	H^2AD^2(Originally ψ^2HA Warsaw basis)
$H^\dagger H\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$(D^\mu H)^\dagger \sigma^a (D^\nu H)\tilde{W}^a_{\mu\nu}$
$H^\dagger H\widetilde{W}^a_{\mu\nu}W^{a\mu\nu}$	$(D^\mu H)^\dagger (D^\nu H)\tilde{B}_{\mu\nu}$
$H^\dagger H\widetilde{B}_{\mu\nu}B^{\mu\nu}$	
$H^\dagger\tau^a H\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$	

Table 6.1: Dimension-6 CP violating operators

In the Warsaw basis, dual gauge tensors do not exist in the group ψ^2HA due to the Bianchi identity ($D^\rho\widetilde{A}_{\rho\mu} = 0$), which eliminates all of them. However, as we are not using the Warsaw basis but the SHIL basis[16][21] for this particular group, it is essential to note that they have a different form before the reduction. The form used in the table above is H^2AD^2, where the original CP-even operators with a non-dual gauge field have been replaced with a dual gauge tensor, resulting in them being CP-odd.

Two operators from the SIHL basis are also eliminated by the Bianchi identity, leaving us with only two CP-odd operators from this group. Out of these CP-odd operators, two of them are not included in the table above. With this information, we are now able to construct the complete Lagrangian, consisting only of dimension-6 terms, including both CP-violating and CP-conserving terms.

$$
\begin{aligned}
\mathcal{L}_6 = & \frac{\bar{c}_T}{2v^2}\left(H^\dagger D^\mu H\right)^\star\left(H^\dagger D_\mu H\right) + \frac{\bar{c}_h}{2v^2}\partial_\mu\left(H^\dagger H\right)\partial^\mu\left(H^\dagger H\right) \\
& + \left[\left(H^\dagger H\right)\frac{\bar{c}_{Hle}}{v^2}\left(\bar{l}_p e_r H\right) + \frac{\bar{c}_{Hqu}}{v^2}\left(H^\dagger H\right)\left(\bar{q}_p u_r\widetilde{H}\right) + \frac{c_{Hqd}}{v^2}\left(H^\dagger H\right)\left(\bar{q}_p d_r H\right) + h.c\right] \\
& \frac{\bar{c}_{HB}g_1^2}{m_W^2}\left(H^\dagger H\right)B_{\mu\nu}B^{\mu\nu} + \frac{\bar{c}_{HG}g_3^2}{m_W^2}\left(H^\dagger H\right)G^a_{\mu\nu}G^{a\mu\nu} + \frac{\bar{c}_{HW}g_2^2}{m_W^2}\left(H^\dagger\sigma^a\overleftrightarrow{D^\mu}H\right)\left(D^\nu W_{\mu\nu}\right)^a \\
& + \frac{\bar{c}_{Hb}g_1}{2m_w^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(\partial^\nu B_{\mu\nu}\right) + \frac{\bar{c}_w g_2}{2m_W^2}\left(D^\mu H\right)^\dagger\sigma^a\left(D^\nu H\right)W^a_{\mu\nu} \\
& + \frac{\bar{c}_b g_1}{m_W^2}\left(D^\mu H\right)^\dagger\left(D^\nu H\right)B_{\mu\nu} \\
& + \frac{i\widetilde{c}_{HG}g_3^2}{m_W^2}H^\dagger H\widetilde{G}^a_{\mu\nu}G^{a\mu\nu} + \frac{i\widetilde{c}_{HB}g_1^2}{m_W^2}H^\dagger H\widetilde{B}_{\mu\nu}B^{\mu\nu} + \frac{i\widetilde{c}_w g_2}{m_W^2}\left(D^\mu H\right)^\dagger\sigma^a\left(D^\nu H\right)\tilde{W}^a_{\mu\nu} \\
& + \frac{i\widetilde{c}_b g_1}{m_W^2}\left(D^\mu H\right)^\dagger\left(D^\nu H\right)\tilde{B}_{\mu\nu}.
\end{aligned}
\tag{6.3}
$$

The dimension-8 operators that are most likely to affect the Higgs coupling to standard model matter fields are those included in Lagrangian $\mathcal{L}_8$ 6.4

$$
\begin{aligned}
\mathcal{L}_8 = & \frac{\bar{c}_r}{2v^4}\left(H^\dagger H\right)^2\left(D_\mu H^\dagger D^\mu H\right) + \frac{\bar{c}_q}{2v^4}\left(H^\dagger H\right)\left(H^\dagger \tau^I H\right)\left(D_\mu H^\dagger \tau^I D^\mu H\right) \\
& + \frac{\left(H^\dagger H\right)^2}{v^2}\left(\left(\bar{c}_l \bar{l}_p e_r H\right) + \left(\bar{c}_u \bar{q}_p u_r \widetilde{H}\right) + \left(\bar{c}_d \bar{q}_p d_r H\right)\right) \\
& + \frac{\bar{c}_g g_3^2}{m_W^4}\left(H^\dagger H\right)^2 G^A_{\mu\nu} G^{A\mu\nu} + \frac{\bar{c}_g g_2^2}{m_W^4}\left(H^\dagger H\right)^2 W^I_{\mu\nu} W^{I\mu\nu} \\
& + \frac{\bar{c}_g g_1^2}{m_W^4}\left(H^\dagger H\right)^2 B_{\mu\nu} B^{\mu\nu} + \frac{i\bar{c}_{DHW} g_2}{2m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger \tau^I D^\nu H\right) W^I_{\mu\nu} \\
& + \frac{\bar{c}_{HWW} g_2^2}{m_w^4}\left(H^\dagger \tau^I H\right)\left(H^\dagger \tau^J H\right) W^I_{\mu\nu} W^{J\mu\nu} + \frac{\bar{c}_{WB} g_1 g_2}{m_w^4}\left(H^\dagger H\right)\left(H^\dagger \tau^I H\right) W^I_{\mu\nu} B^{\mu\nu} \\
& + \frac{i\bar{c}_g g_1^2}{m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger D^\nu H\right) B_{\mu\nu}.
\end{aligned}
\tag{6.4}
$$

The first line of dimension-8 operators is included to account for their effects on the canonical re-normalization of the Higgs field. Similarly, the second line of dimension-8 operators leads to significant shifts in the fermion mass. Therefore, it is crucial to consider both types of operators in our analysis. The normalization convention follows that of the dimension 6 Lagrangian [9] which follows the convention of [16]. [21] is also useful in determining correct normalization.

6.3 Effective Lagrangian Post-Electroweak Symmetry Breaking

In this section, we will consider our Lagrangian in unitary gauge, which corresponds to choosing the Higgs field to be only real. This choice eliminates mixing terms and ensures that the fields appearing in the Lagrangian after SSB are the actual physical fields. We will expand our Lagrangian as a power series around the physical Higgs fields h, using only linear expansions for simplicity. Another reason to use a linear Higgs expansion is that we aim to derive our updated WC dictionary using the same Lagrangian in [9]. [9] includes the Higgs field expansion up to order $\mathcal{O}(h^2)$ due to the LHC experiments not being sensitive to multi-Higgs production.

First, let us examine how EWSB introduces the three massive weak force fields and one massless photon field. After SSB, the massive electroweak sector takes on the form:

$$
\mathcal{L}_{mass} = \frac{1}{2}m_W^2\left[\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2\right] + \frac{1}{2}m_Z^2\left(c_W W^3_\mu - s_W B_\mu\right)^2 \tag{6.5}
$$

In the electroweak sector after SSB, the W, and Z bosons acquire mass through the Higgs mechanism, with mass given by $\frac{gv}{2}$ and $\frac{v}{2}\sqrt{g^2+g'^2}$, respectively. Appendix 11.3 provides a detailed explanation of how these mass terms are derived.

The kinetic part looks the same, however since we have done the following field re-definitions, represented as a rotation in the original vector boson plane[1],

$$
\begin{pmatrix} Z_\mu \\ \gamma_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \tag{6.6}
$$

[1] γ_μ is used instead of A_μ to keep in line with the notation in [9]

and

$$W_\mu^{\pm} = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}. \tag{6.7}$$

Where c_w and s_w are shorthand for sine and cosine of the weak mixing angle θ_W. Fields appearing in the Lagrangian are the charged $W^{\pm}$ bosons and the neutral Z boson. The photon field in the physical will be The Higgs mechanism also gives mass to fermions. However, due to chiral symmetry, left-handed and right-handed fermions transform differently under the $SU(2)$ gauge group, making a standard mass term forbidden before SSB. After SSB, fermions can appear as standard mass terms in the Lagrangian.

We will consider the effective Lagrangian after SSB in three parts. First, we will write down the SM Lagrangian 6.8, then the dimension-6 Lagrangians 6.9 and 6.10, and lastly, the dimension-8 Lagrangians 6.10 and 6.11. This stepwise approach facilitates understanding how the Standard Model Lagrangian and the higher-order Lagrangian change when using the unitary gauge. It is worth noting that the higher-order operators may become considerably intricate when evaluated in the unitary gauge.

We will also write down the Higgs-dependent parts of $\mathcal{L}_6$, 6.10, and $\mathcal{L}_8$, 6.11, independently before combining them into a single Higgs-dependent Lagrangian, denoted as $\mathcal{L}_h$ 6.12. Note that $\mathcal{L}_h$ will have new Wilson coefficients that encapsulate the contributions of the Wilson coefficients belonging to $\mathcal{L}_{6,h}$ and $\mathcal{L}_{8,h}$. These relations will be used later in the updated WC dictionary.

$$\begin{aligned}\mathcal{L}_{SM} =& -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \overline{\psi_L}^i i \not{D}\psi^i_L + \overline{\psi_R}^i i \not{D}v^i_R \\ &+ \frac{m_W^2}{2}W^+_\mu W^{-\mu} + \frac{m_Z^2}{2}\left(1-\bar{c}_T\right)Z_\mu Z^\mu - (m_u\bar{u}_L u_R + m_d\bar{d}_L d_R + m_e\bar{l}_L l_R + h.c).\end{aligned} \tag{6.8}$$

The strong sector of the SM is unaffected by EWSB and thus remains unchanged. As for the fermion terms, they have been expressed as a linear combination of left-handed and right-handed fields, as is always possible for a chiral theory[39]. The mixing terms between left and right-handed fields vanish as $P_L P_R = 0$. Furthermore, the mass terms have been written out explicitly.

The next step is to consider the dimension-6 Lagrangian in unitary gauge. The explicit calculations of dimension 6 operators in unitary gauge can be found in appendices 11.6.5 and 11.6.4. The dimension 6 Lagrangian after SSB is then expressed as [9]

$$\begin{aligned}\mathcal{L}_6 =& \, 2\bar{c}_{HB}\tan^2\theta_W\left(s_w^2 Z_{\mu\nu}Z^{\mu\nu} + c_w^2\gamma_{\mu\nu}\gamma^{\mu\nu} - 2s_w c_w Z_{\mu\nu}\gamma^{\mu\nu}\right) + 2\bar{c}_{HG}\frac{g_S^2}{g^2}G_{\mu\nu}G^{\mu\nu} + \text{ CP-Odd} \\ &+ \bar{c}_{Hb}Z^\mu\partial^\nu\left(\tan^2\theta_W Z_{\mu\nu} - \tan\theta_W\gamma_{\mu\nu}\right) + \text{ CP-Odd} \\ &+ \bar{c}_{HW}\left(\tan\theta_W Z^\mu\partial^\nu\gamma_{\mu\nu} + Z^\mu\partial^\nu Z_{\mu\nu} + W^\mu D^\nu W^\dagger_{\mu\nu} + \text{ h.c. }\right) + \text{ CP-Odd }.\end{aligned} \tag{6.9}$$

Omitted from this Lagrangian are the purely bosonic contributions stemming from the WCs c_b and c_w. Furthermore, each operator presented above also possesses a CP-odd counterpart, in which a dual-field strength tensor replaces one of the gauge bosons. The portion of the dimension-6 Lagrangian that is reliant on the Higgs field can be expressed as

$$
\begin{aligned}
\mathcal{L}_{6,h} = & \bar{c}_{HW}\frac{h}{v}\left(\tan\theta_w Z^\mu \partial^\nu \gamma_{\mu\nu} + Z^\mu \partial Z_{\mu\nu}\right) - \bar{c}_{HW}\frac{h}{v} W^\mu D^\nu W^\dagger_{\mu\nu} \\
& + 4c_{HG}\frac{g_3^2}{g_2^2}\left(\frac{h}{v}\right) G^a_{\mu\nu}G^{a\mu\nu} + 4\bar{c}_{HB}\left(\frac{h}{v}\right)\left(s_w^2\gamma^2_{\mu\nu} - 2s_w^2\frac{s_w}{c_w}\gamma_{\mu\nu}Z^{\mu\nu} + \frac{s_w^4}{c_w^2}Z^2_{\mu\nu}\right) \\
& - \bar{c}_{Hb}\frac{h}{v}\left(\tan\theta_w Z^\mu\partial^\nu\gamma_{\mu\nu} - \tan\theta_w^2 Z^\mu\partial^\nu Z_{\mu\nu}\right) - \frac{2\bar{c}_w}{v}\frac{h}{v}\left(\tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} + Z^{\mu\nu}Z_{\mu\nu}\right) \\
& - 2\bar{c}_w\frac{h}{v}\left(\tan\theta_w Z^\mu\partial^\nu\gamma_{\mu\nu} + Z^\mu\partial^\nu Z_{\mu\nu}\right) + 2\bar{c}_w\frac{h}{v}W^\mu D^\nu W^\dagger_{\mu\nu} \\
& - 2\bar{c}_b\frac{h}{v}\left(\frac{s_w}{c_w}Z^{\mu\nu}\gamma_{\mu\nu} - \frac{s^2}{c_w^2}Z^{\mu\nu}Z_{\mu\nu}\right) - 2\bar{c}_b\frac{h}{v}\left(\frac{s_w}{c_w}Z^\mu\partial^\nu\gamma_{\mu\nu} - \frac{s_w^2}{c_w^2}Z^\mu\partial^\nu Z_{\mu\nu}\right).
\end{aligned}
$$

The dimension-8 Lagrangian after SSB can be expressed as 6.10

$$
\begin{aligned}
\mathcal{L}_8 = & 4\bar{c}_{HHG}\frac{g_3^2}{g_2^4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{\bar{c}_{WW}}{g_2^2}\left(W_{\mu\nu}W^{\mu\nu} + s_w^2\gamma^2_{\mu\nu} + s_w c_w \gamma_{\mu\nu}Z^{\mu\nu} + Z^2_{\mu\nu}\right) \\
& + \bar{c}_{HHB}\left(4\frac{s_w^2}{g_2^2}\gamma^2_{\mu\nu} - 8\frac{\tan^2\theta_W c_W s_w}{g_2^2}c_w s_w \gamma_{\mu\nu}Z^{\mu\nu} + 4\frac{\tan^2\theta_W s_W^2}{g_2^2}Z^2_{\mu\nu}\right) \\
& + 2\frac{\bar{c}_{DHW}}{g_2^2}\left(\tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} + Z^{\mu\nu}Z_{\mu\nu}\right) \\
& + 16\frac{\bar{c}_{HWW}}{g_2^2}\left(2\gamma_{\mu\nu}Z^{\mu\nu}c_w s_w + c_w^2 Z_{\mu\nu}Z^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu}\right) \\
& - \frac{4\bar{c}_{HWB}}{g_2^2}\left(s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu} + \tan\theta_w\left(c_w^2 - s_w^2\right)Z_{\mu\nu}\gamma^{\mu\nu} - s_w^2 Z_{\mu\nu}Z^{\mu\nu}\right) + \text{ CP-Odd }.
\end{aligned} \tag{6.10}
$$

The last term "CP-odd" refers to the CP-odd part of the entire Lagrangian. As every single operator can be represented in the same way by its dual counterpart (Except operators with the structure $A^\mu D^\nu \tilde{A_{\mu\nu}}$ as these dual tensors cancel by the Bianchi identity. It is easier just to write CP-odd at the end. The Higgs-dependent part of the dimension-8 Lagrangian is expressed as 6.11

$$
\begin{aligned}
\mathcal{L}_{8,h} = {} & 8\bar{c}_{HHG}\frac{g_3^2}{g_2^4}\frac{h}{v}G^a_{\mu\nu}G^{a\mu\nu} \\
& + 8\bar{c}_{HHB}\frac{h}{v}\left(\frac{s_w^2}{g_2^2}\gamma_{\mu\nu}\gamma^{\mu\nu} - 16\frac{\tan\theta_W s_w^2}{g_2^2}\gamma_{\mu\nu}Z^{\mu\nu} + \frac{\tan^2\theta_W s_w^2}{g_2^2}Z_{\mu\nu}Z^{\mu\nu}\right) \\
& + 4\frac{\bar{c}_{WW}}{g_2^2}\frac{h}{v}\left(W_{\mu\nu}W^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu} + 2s_w c_w\gamma_{\mu\nu}Z^{\mu\nu} + Z_{\mu\nu}Z^{\mu\nu}\right) \\
& + 8\frac{\bar{c}_{DHW}}{g_2^2}\frac{h}{v}\left(\tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} + Z^{\mu\nu}Z_{\mu\nu} + \tan\theta_w\left(Z^\mu\partial^\nu\gamma_{\mu\nu} + Z^\mu\partial^\nu Z_{\mu\nu}\right)\right) \\
& + 32\frac{\bar{c}_{DHW}}{g_2^2}\frac{h}{v}\left(WDW^3\right) \\
& - 16\frac{\bar{c}_{DHB}}{g_2^2}\frac{h}{v}\left(\tan\theta_w Z^\mu D^\nu\gamma_{\mu\nu} + \tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} - \tan^2\theta_w Z^\mu D^\nu Z_{\mu\nu} - \tan^2\theta_w Z^{\mu\nu}Z_{\mu\nu}\right) \\
& + 4^5\frac{i\bar{c}_{HWW}}{g^2}\frac{h}{v}\left(2c_w s_w\gamma_{\mu\nu}Z^{\mu\nu} + c_w^2 Z_{\mu\nu}Z^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu}\right) \\
& + \frac{16\bar{c}_{HWB}}{g_2^2}\frac{h}{v}\left(s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu} + \tan\theta_w\left(c_w^2 - s_w^2\right)Z_{\mu\nu}\gamma^{\mu\nu} - s_w^2 Z_{\mu\nu}Z^{\mu\nu}\right).
\end{aligned}
\tag{6.11}
$$

For a detailed derivation of dimension-6 and dimension 8 operators in unitary gauge, see appendix 11.6. The Higgs-dependent Lagrangian denoted as $\mathcal{L}_h$, is constructed to ensure that each interaction appears only once, and all contributions are aggregated into a new WC for that specific interaction.

Our objective is to strengthen the current parametrization on coupling the Higgs to SM matter fields by utilizing not only dimension 6 operators but also dimension 8 operators. To achieve this goal, we will adopt the same Higgs-dependent Lagrangian employed in [9].This is to prevent the introduction of any novel interactions that can only be described by contributions from dimension 8 operators. We employ the same Lagrangian with identical interactions. While it may be reasonable to speculate whether introducing new interactions could improve parameterization, time constraints necessitate our focus on existing interactions.

The final Higgs dependent Lagrangian, incorporating the effects of both dimension 6 and 8 operators, and integrating their contribution into novel WCs, is presented in 6.12

$$
\begin{aligned}
\mathcal{L}_h = {} & \frac{h}{v}(2c_W m_W^2 W_\mu^\dagger W^\mu + c_Z m_Z^2 Z_\mu Z^\mu - \sum_{f=u,d,l} m_f\bar{f}\left(c_f + i\gamma_5\widetilde{c_f}\right)f \\
& - \frac{1}{2}c_{WW}W^\dagger_{\mu\nu}W^{\mu\nu} - \frac{1}{4}c_{ZZ}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}c_{\gamma\gamma}\gamma_{\mu\nu}\gamma^{\mu\nu} - \frac{1}{2}c_{Z\gamma}\gamma_{\mu\nu}Z^{\mu\nu} + \frac{1}{4}c_{gg}G^a_{\mu\nu}G^{a\mu\nu} + \text{CP-Odd} \\
& - \left(\kappa_{WW}W^\mu D^\nu W^\dagger_{\mu\nu} + \text{ h.c. }\right) - \kappa_{ZZ}Z^\mu\partial^\nu Z_{\mu\nu} - \kappa_{Z\gamma}Z^\mu\partial^\nu\gamma_{\mu\nu}).
\end{aligned}
\tag{6.12}
$$

The final Lagrangian encompasses all operators that parameterize the Higgs coupling in our theory. To derive this Lagrangian, we have collected all contributions to a specific interaction, which are now represented by the newly defined Wilson coefficients: c_{WW}, c_{ZZ}, c_{FF}, c_w, c_z, c_γ, $c_{Z\gamma}$, c_{GG}, κ_W, κ_Z, and $\kappa_{Z\gamma}$. The fermion mass parameter m_f is important to remember is now redefined according to equation 11.26. To establish the relationship between the new and old coefficients, we have compared the Lagrangians $\mathcal{L}_{6,h}$ 6.10, $\mathcal{L}_{8,h}$6.11 to the same Lagrangians but before SSB.

However, the updated dictionary would not be complete without incorporating dimension-8 operators. To avoid creating two separate dictionaries, as previously mentioned, only dimension-8 operators that contribute to the pre-existing interactions have been incorporated. Nevertheless, this requirement imposes an arbitrary threshold on which dimension-8 operators are included, which warrants further review in future research.

The new WCs represent the cumulative contribution of other WCs to a specific interaction, i.e., the operators that contribute to each $Z^{\mu\nu}Z_{\mu\nu}, Z^{\mu\nu}\gamma_{\mu\nu}, \gamma^{\mu\nu}\gamma_{\mu\nu}, W^{\dagger}_{\mu\nu}W^{\mu\nu}, Z^{\mu}\partial^{\mu}Z_{\mu\nu}, Z^{\mu}\partial^{\nu}\gamma_{\mu\nu},$ and the conventional SM terms. While these interactions do not encompass the entirety of the effective field theory, other interactions are simply modified standard model terms resulting from the inclusion of dimension-8 operators or the unitary gauge.

The relationship between the various dimension-8 WCs and the newly defined WCs outlined above is depicted in the next section.

6.4 WC Dictionary

The dictionary of WCs, frequently referenced in this thesis, refers to the comprehensive collection of WCs that encapsulate all contributions to specific physical interactions. By "physical," it simply denotes the interaction obtained after implementing the unitary gauge. In addition to the SM interaction terms, the effective operators yield higher-order interactions such as $Z^{\mu\nu}Z_{\mu\nu}$, $Z^{\mu\nu}\gamma_{\mu\nu}$, $\gamma^{\mu\nu}\gamma_{\mu\nu}$, $W^{\dagger}\mu\nu W^{\mu\nu}$, $Z^{\mu}\partial^{\mu}Z\mu\nu$, $Z^{\mu}\partial^{\nu}\gamma_{\mu\nu}$, and $W^{\mu}D^{\nu}W^{\dagger}_{\mu\nu}$, as mentioned in the previous section. Together with the SM interactions, these interactions constitute the entirety of the dictionary.

The original version of the dictionary, which does not include any dimension 8 interactions, can be found in the seminal article [9].

The new dictionary will expand the already established Wilson coefficients in [9]. All WCs before SSB are labeled with an overhead bar, while WCs after SSB are not.

In a general overview, the first three interactions $Z^{\mu\nu}Z_{\mu\nu}$, $Z^{\mu\nu}\gamma_{\mu\nu}$, $\gamma^{\mu\nu}\gamma_{\mu\nu}$ could potentially capture deviations relating to the scattering and productions of Z bosons and photons. The next two interactions can capture low-energy phenomena in the EWSB sector, related to the production of Higgs and Higgs decays. The last operator describes the kinetic mixing between Z and γ bosons. The kinetic mixing term is important as it can modify the coupling strengths of Z and γ bosons to fermions, which could lead to deviations from the standard model. This mixing operator is also important for the production and decay of the Higgs bosons in loop diagrams involving fermions. In the description of the STU parameters, there are no loop corrections involving fermions, as we are only interested in the description of electroweak oblique corrections to gauge bosons. Another more obscure consequence of this last operator is its ability to affect dark matter searches, as dark-matter particles can interact with the Z and γ bosons through Z-γ mixing. However, the last consequence, although exciting, will not be further explored in this thesis[42].

In this master thesis, we are looking to further expand the dictionary in [9] by adding contributions from dimension 8 operators. The new and updated dictionary, 6.14, will not contain any new interactions. This is mostly done to better compare the two dictionaries in future work, and introducing new interactions with only contributions from dimension 8 operators could give an unreliable result. There is one missed opportunity by excluding potential valuable new interactions, which is the opportunity to describe the U parameter better, as the U parameter is heavily affected by higher order operators[33]. The dictionary 6.13 shows only the contribution from dimension 8 WCs.

$$
\begin{aligned}
c_W &= -\frac{\bar{c}_q + \bar{c}_r}{4} \\
c_f &= -\frac{\bar{c}_q + \bar{c}_r}{4} + \mathrm{Re}\,(\bar{c}_f) \\
\tilde{c_f} &= \mathrm{Im}\,(\bar{c}_f) \\
c_Z &= -\frac{\bar{c}_q + \bar{c}_r}{4} \\
c_{WW} &= \frac{4}{g_2}\bar{c}_{WW} \\
c_{ZZ} &= 8\frac{\tan^2\theta_W s_w^2}{g_2^2}\bar{c}_{HHB} + \frac{4}{g_2^2}\bar{c}_{WW} + \frac{8}{g_2^2}\bar{c}_{DHW} + \frac{16}{g_2^2}\tan\theta_w\bar{c}_{DHB} + \frac{4^5}{g_2^2}c_w^2\bar{c}_{HWW} + \frac{16}{g_2^2}s_w\bar{c}_{HWB} \\
c_{\gamma\gamma} &= 8\frac{s_w^2}{g_2^2}\bar{c}_{HHB} + 8\frac{s_w^2}{g_2^2}\bar{c}_{WW} + 4^5\frac{s_w^2}{g_2^2}\bar{c}_{HWW} + 16\frac{s_w^2}{g_2\bar{c}_{HWB}} \\
c_{Z\gamma} &= 2^7\frac{\tan\theta_w s_w^2}{g_2^2}\bar{c}_{HHB} + 8\frac{s_w c_w}{g_2}\bar{c}_{WW} + 8\frac{\tan\theta_w}{g_2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} + 2\times 4^5\frac{s_w c_w}{g_2^2}\bar{c}_{HWW} \\
c_{gg} &= 8\frac{g_3^2}{g_2^4}\bar{c}_{HHG} \\
\kappa_{Z\gamma} &= 8\frac{\tan\theta_w}{g_2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} \\
\kappa_{ZZ} &= 8\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} \\
\kappa_{WW} &= \frac{32}{g_2^2}\bar{c}_{DHW}
\end{aligned}
\tag{6.13}
$$

The revised and enhanced dictionary, now accounting for contributions from both dimension-6 and dimension-8 operators, is presented in 6.14[2].

[2]The dictionary in the original article incorporates a minus sign in front of the WCs of order operators. Therefore some minus signs are flipped in comparison to the appendix. The original article also normalizes the WCs which have not been accounted for in this updated dictionary

$$
\begin{aligned}
c_W &= 1 - \frac{1}{2}\bar{c}_h + \frac{1}{4}\left(\bar{c}_q + \bar{c}_r\right) \\
c_f &= 1 - \frac{1}{2}\bar{c}_h + \frac{1}{4}\left(\bar{c}_q + \bar{c}_r\right) \\
\tilde{c_f} &= 2\,\mathrm{Im}\left(\bar{c}_f\right) \\
c_Z &= 1 - \frac{1}{2}\bar{c}_h + \frac{1}{4}\left(\bar{c}_q + \bar{c}_r\right) - \bar{c}_T \\
c_{WW} &= 4\bar{c}_{HW} - \frac{4}{g_2}\bar{c}_{WW} \\
c_{ZZ} &= 4\left(\bar{c}_{HW} + \frac{s_w^2}{c_w^2}\bar{c}_{HB} - 4\frac{s_w^4}{c_w^2}\bar{c}_\gamma\right) \\
&\quad + \frac{8}{g_2}\left(-\frac{s_w^4}{c_w^2}\bar{c}_{HHB} - \frac{1}{2}\bar{c}_{WW} - \bar{c}_{DHW} - 2\tan\theta_w\bar{c}_{DHB} - 2^7 c_w^2\bar{c}_{HWW} + 2s_w\bar{c}_{HWB}\right) \\
c_{\gamma\gamma} &= -16 s_w^2\bar{c}_\gamma + 8\frac{s_w^2}{g_2}\left(-\bar{c}_{HHB} - \bar{c}_{WW} - 2^7\bar{c}_{HWW} - 2\bar{c}_{HWB}\right) \\
c_{Z\gamma} &= 2\frac{s_w}{c_w}\left(\bar{c}_{HW} - \bar{c}_{HB} + 8s_w^2\bar{c}_\gamma\right) \\
&\quad + \frac{8}{g_2^2}\left(2^3\frac{s_w^4}{c_w^2}\bar{c}_{HHB} - s_w c_w\bar{c}_{WW} - \tan\theta_w\bar{c}_{DHW} + 2\tan\theta_w\bar{c}_{DHB} - 4^4 s_w c_w\bar{c}_{HWW}\right) \\
\kappa_{Z\gamma} &= -2\frac{s_w}{c_w}\left(\bar{c}_{HW} + \bar{c}_W - \bar{c}_{HB} - \bar{c}_B\right) + 8\frac{\tan\theta_w}{g_2^2}\left(-\bar{c}_{DHW} + 2\bar{c}_{DHB}\right) \\
\kappa_{ZZ} &= -2\left(\bar{c}_{HW} + \bar{c}_W + \frac{s_w^2}{c_w^2}\bar{c}_{HB} + \frac{s_w^2}{c_w^2}\bar{c}_B\right) + 8\frac{\tan\theta_w}{g_2^2}\left(-\bar{c}_{DHW} - 2\bar{c}_{DHB}\right) \\
\kappa_{WW} &= -2\left(\bar{c}_{HW} + \bar{c}_W\right) - \frac{32}{g_2^2}\bar{c}_{DHW}
\end{aligned}
\tag{6.14}
$$

The WCs representing the equivalent CP-odd operators, which differ by the field exchange $A_{1\mu\nu}A_2^{\mu\nu} \to A_{1\mu\nu}\widetilde{A}_2^{\mu\nu}$, is left out to reduce clutter as they appear exactly the same [3]. One notable point is that all the dimension 8 contributions, as a consequence of the factor $\frac{1}{\Lambda^4}$, introduce a factor $\frac{1}{g_2^2}$. The consequences of this factor and whether or not it restricts or reduces the importance of the dimension 8 contributions will be discussed in section 8.2. Other aspects of this updated dictionary, such as its potential impact on the STU parameters and future research, are discussed in section 8.4 and 10.4.

6.5 Exploring the Effects of Higher Order Operators on Fermion Mass, Higgs Field, and the Z-Mass

When introducing higher-order terms to the SM Lagrangian, it is important to note that the first two lines in the dimension 6 and dimension 8 Lagrangians, as shown in equations 6.3 and 6.4, have significant consequences on Higgs renormalization, Z boson mass, and fermion mass. Specifically, we will examine how the Wilson coefficients c_h, c_q, and c_r impact the canonical renormalization of the Higgs, how the Wilson coefficient c_T affects the Z mass (though this contribution is ultimately already negligible at order

[3] $\tilde{c}_f$ is kept as this is the only coefficient which is not similar after changing to dual fields

$\frac{1}{\Lambda^2}$), and how the higher order Yukawa-like terms cause a shift in the masses of the fermions. Appendices 11.5, 11.4 and 11.6.6 provide greater insight into the calculations behind these effects.

Impact of Higher Order operators on the canonical normalization of the kinetic Higgs term

To begin, let us examine the impact of the operator $\frac{\bar{c}_h}{2v^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right)$ on the canonical renormalization of the kinetic Higgs term. To restore canonical renormalization, it is necessary to rescale the field expansion around the VEV to account for this higher-order term. When expressed in the unitary gauge, the term c_h takes the form

$$\frac{\bar{c}_h}{2v^2}\partial_\mu(\frac{1}{2}v^2+hv+\frac{1}{2}h^2)\partial^\mu(\frac{1}{2}v^2+hv+\frac{1}{2}h^2)=\frac{1}{2}\bar{c}_h(\partial_\mu h)^2\ldots.$$

Where the ellipsis represents higher-order terms. This term contributes a factor of $\frac{1}{2}c_h$ to the kinetic term in the Lagrangian, which becomes $\frac{1}{2}(\partial_\mu h)^2+\frac{c_h}{2}(\partial_\mu h)^2$. To remove this contribution, a field redefinition of the form $h\to\frac{h}{\sqrt{1+c_h}}$ is performed. This rescaling affects the decay widths of all Higgs particles, as noted in [9].

The dimension 8 operator $c_r\left(H^\dagger H\right)^2\left(D_\mu H^\dagger D^\mu H\right)$ and $c_q\left(H^\dagger H\right)\left(H^\dagger\tau^I H\right)\left(D_\mu H^\dagger\tau^I D^\mu H\right)$ necessitates another re-scaling of the Higgs field.

The kinetic part of the Higgs Lagrangian now gets in addition to c_h, the contributions c_r and c_q as:

$$\mathcal{L}_{kin}=\frac{1}{2}\left(\partial_\mu h\right)^2+\frac{c_h}{2}\left(\partial_\mu h\right)^2+\frac{c_q}{8}\left(\partial_\mu h\right)^2+\frac{c_r}{8}\left(\partial_\mu h\right)^2 \tag{6.15}$$

Which is also re-scaled away by a similar field redefinition for the Higgs field:

$$h\to\frac{h}{\sqrt{1+c_h+\frac{1}{4}\left(c_q+c_r\right)}} \tag{6.16}$$

Once more, the rescaling of the Higgs field will contribute to an even broader decay width of the Higgs particle. Additionally, it will result in a slight modification of the dictionary, where the WCs c_q and c_r will be added to all the WCs that parameterize the masses of the fermions[4], Z and W bosons.

Impact of Higher Order operators on Fermion Mass

The Yukawa-like terms induce a shift in the mass of the fermions as described in [9].

The WCs, which are complex numbers, are often split into their real and imaginary parts to understand better the physical implications of the resulting mass shift in fermions. It is important to note that the contribution to the mass shift may not be equally distributed between the real and imaginary parts. The real part of the Wilson coefficient describes the magnitude of the coupling strength between the effective

[4] With the exception of $\tilde{c}_f$

field theory operators and SM particles. In contrast, the imaginary part characterizes the CP-violating phase of the coupling. Although in some cases, such as the specific calculations described below, the contribution to the mass shift from both the real and imaginary parts of the Wilson coefficient may be equal, the contributions may generally differ depending on the specific model. Therefore, it is crucial to scrutinize the model to determine the correct contributions from each part. Nonetheless, even when the contributions are equal, it remains essential to split the Wilson coefficient into its real and imaginary parts to understand the underlying physics of the model fully. Equation 6.17 shows the SM Yukawa terms as well as the higher-order Yukawa-like operators introduced by the OPE

$$y_f \overline{f_L} H f_R + \frac{H^\dagger H}{v^2} \bar{c}_f y_f \overline{f_L} H f_R + \left(\frac{H^\dagger H}{v^2}\right)^2 \bar{c}_f y_f \overline{f_L} H f_R + \text{ h.c. } . \tag{6.17}$$

When the Higgs field is set to its vacuum expectation value, it causes an additional shift in the fermion mass, which is given by $\frac{3}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \, \mathrm{Im}\left(\bar{c}_f\right)\right]$. This results in a new effective fermion mass 6.18

$$m_f^* = m_f \left[1 + \frac{3}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \, \mathrm{Im}\left(\bar{c}_f\right)\right]\right] . \tag{6.18}$$

In comparison to [9], the dimension 8 operators have caused an additional shift in the fermion mass by a factor of $\frac{1}{2}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \, \mathrm{Im}\left(\bar{c}_f\right)\right]]$. To incorporate this new effective fermion mass, modifications must be made to the effective Lagrangian to account for m_f^*. After this change in mass, the effective Lagrangian contains Yukawa-like terms that appear after EWSB as 6.19

$$\rightarrow m_f^* \overline{f_L} f_R + \frac{h}{v} m_f^* \bar{f} \left[1 + 2\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \, \mathrm{Im}\left(\bar{c}_f\right)\right]\right] f + \mathcal{O}\left(\frac{h^2}{v^2}\right) . \tag{6.19}$$

The shift in fermion mass will have an impact on the relationships between various Wilson couplings listed in the dictionary of WCs.

Impact of Higher Order operators on the Z-boson mass

The mass of the Z boson is affected by the dimension 6 operator $\frac{\bar{c}_T}{2v^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right)$. Similar to the previously discussed operators c_h, c_q, and c_r, the mass term in the Lagrangian is altered by $-\frac{1}{2} m_Z^2 c_T Z^\mu Z_\mu$. However, due to the strict constraints on the S and T parameters at current experimental limits, it is necessary to set c_T to zero, as noted in [9].

Since the up-to-date experimental limits already motivate setting c_T to zero at dimension 6, there is no need to examine further if any dimension 8 could further alter the Z boson mass term. This will be further discussed in the conclusion part of this thesis.

Chapter 7

Peskin-Takeuchi Parameters

This chapter will briefly cover the *STU* parameters. The Peskin-Takeuchi parameters, also known as the electroweak precision observables, are a set of quantities used to measure the agreement between theoretical predictions and experimental observations. They were introduced by Mark E. Peskin and Tatsu Takeuchi in the early 1990s and have since played a crucial role in testing the validity of the electroweak theory[28][37].

7.1 Exploring the STU-parameters

The Peskin-Takeuchi parameters quantify deviations of a theory from the tree-level predictions of the Standard Model in the electroweak sector. These parameters arise primarily from quantum corrections at the loop level, which contribute to the self-energies of the weak vector and scalar bosons. Therefore, a nonzero value of the Peskin-Takeuchi parameters would indicate the presence of new physics beyond the SM in the electroweak sector.

Measurements of the *STU* parameters play an important role in testing the validity of the SM and searching for new physics phenomena, as large *STU* parameters indicate large deviations from expected measurements.

Specifically, the *STU* parameters constrain the self-energies of the W, Z, and photon bosons[9]. In the limit where the scale of new physics M_{NP} is much greater than the electroweak scale M_{EW}, and the gauge group remains $SU(2)_L \times U(1)_Y$, oblique corrections become the dominant corrections resulting from new physics[9][37]. In a HEFT, these criteria are always satisfied, as an effective field theory on the electroweak scale would break down if the scale of new physics is close to the electroweak scale.

The *STU* parameters are defined as the difference in the first-order correction to the two-point functions of the W, Z, and photon bosons. Specifically, they are defined as the differences between $\Pi_{\gamma\gamma}$, $\Pi_{\gamma Z}$, Π_{ZZ}, and Π_{WW}, as given by the equations 15-17 in [28].

$$S \equiv -\frac{8\pi}{M_Z^2}\left[\Pi_{3Y}\left(M_Z^2\right) - \Pi_{3Y}(0)\right] \tag{7.1}$$

$$T \equiv \frac{4\pi}{c^2 s^2 M_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right] \tag{7.2}$$

$$U \equiv \frac{16\pi}{M_W^2}\left[\Pi_{11}\left(M_W^2\right) - \Pi_{11}(0)\right] - \frac{16\pi}{M_Z^2}\left[\Pi_{33}\left(M_Z^2\right) - \Pi_{33}(0)\right]. \tag{7.3}$$

The self-energy is the leading-order correction to the propagator, and it represents the effects of virtual particles that can interact with the propagating particle. In the context of the STU parameters, the "two-point functions" refer specifically to the self-energies of the W, Z, and photon bosons, represented as loop diagrams.

The loop diagrams below represent these self-energies, which are the leading corrections defined by the STU parameters. These corrections are essential because their magnitude indicates the presence of new physics beyond the Standard Model.

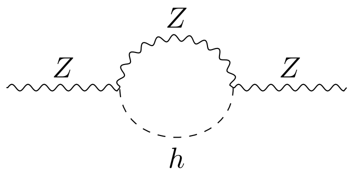

Figure 7.1: $Z - Z/h - Z$

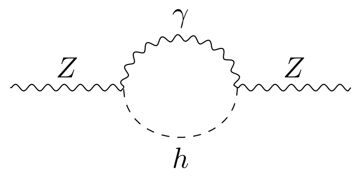

Figure 7.2: $Z - \gamma/h - Z$

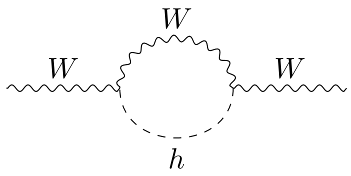

Figure 7.3: $W - W/h - W$

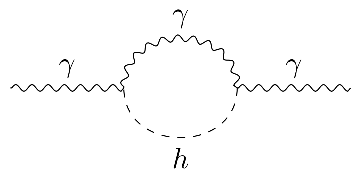

Figure 7.4: $\gamma - \gamma/h - \gamma$

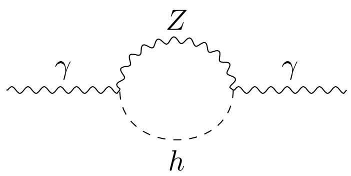

Figure 7.5: $\gamma - Z/h - \gamma$

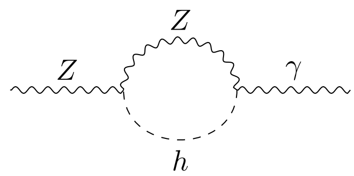

Figure 7.6: $Z - Z/h - \gamma$

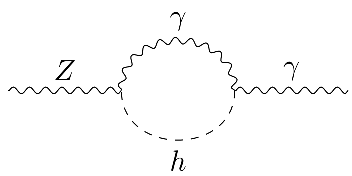

Figure 7.7: $Z - \gamma/h - \gamma$

S measures the disparities between hypercharge and third weak isospin. These disparities occurs at $q^2 = M_Z$ and $q^2 = 0$. q^2 is the momentum transfer squared in the two-fermion scattering process(Two-fermion scattering generates the gauge boson for which the loops are corrections to). The disparity the S-parameter measures is that from the theoretical value of the mixing angle. A large S would, therefore, signify a large disparity of mixing between hypercharge and weak isospin, meaning there is potential for new physics (i.e., new interactions or particles).

T measures the strength of the interaction between the W and Z boson, and by extension the deviation from the ρ-parameter. The ρ-parameter is important to determine the structure of the Higgs-sector[26]. At tree level, the experimental value is unity, signifying the deeper custodial symmetry in the Higgs sector. This custodial symmetry, represented by the ρ-parameter, which is a ratio between the masses of the W mass and Z mass, and the strength of the interaction between them, is an exact symmetry when the ρ-parameter is close to unity. This symmetry is a residual symmetry left over after the spontaneous symmetry breaking of the electroweak symmetry group, and it prevents higher-order radiative corrections. That is why it is significant if the ρ deviates from unity, as this would suggest that the custodial symmetry is broken and allows for potential new physics in the higher-order radiative corrections. These higher-order radiative corrections would be the loop diagrams above. If T deviates from unity, it would suggest that the symmetry between W and Z is broken. If T is positive, the broken symmetry favors the W-boson[26].

T depends strongly on the number of Higgs doublets and their hypercharges.

U, although similar to T, U quantifies the contribution caused by differences in mass by the weak force carrying W and Z to the weak isospin symmetry[26].

7.2 STU-parameters and the WC dictionary

One way of using the updated WCs would be to get a better description of observables, such as cross-section and branching ratio.

The interesting observables would be the **relative decay widths** and **relative cross-section production** of Higgs into two vector bosons, known as Vector Bosons Fusion (VBF), or to a fermion-anti-fermion pair at tree level. The STU-parameters, with its updated description using contributions from dimension 8 operators, would hopefully constrain the value of the Higgs boson couplings to vector bosons at one loop level, with their respective diagrams given in section 7.1.

The formulas for relative cross-section production and the on-shell decay rate of the Higgs boson as given in [9], are

$$\frac{\mathrm{Br}(h \to XX)}{\mathrm{Br}(h \to XX)_{SM}} = \frac{\Gamma_{XX}}{\Gamma_{XX,SM}} \frac{\Gamma_{tot,SM}}{\Gamma_{tot}} \quad \textit{relative CS production}, \tag{7.4}$$

$$\hat{\mu}^{YH}_{XX} = \frac{\sigma_{YH}}{\sigma^{SM}_{YH}} \frac{\mathrm{Br}(h \to XX)}{\mathrm{Br}(h \to XX)_{SM}} \quad \textit{Higgs Signal Strength}. \tag{7.5}$$

Equations 7.4 and 7.5 show the branching ratio, $\mathrm{Br}(h \to XX)$ of Higgs decaying into some final state XX. The final state is either a fermion-antifermion pair or a vector boson. The denotation SM refers to the branching ratio in the SM, absent of any new physics effect. Γ_{XX} is the decay width. It quantifies the rate at which the Higgs boson decays into the final state XX. Γ_{tot} is the total decay width of the Higgs boson, considering all possible decay channels. Equation 7.4 compares the branching ratios and decay widths in the presence of new physics, to those predicted by the SM. $\hat{\mu}^{YH}_{XX}$ is the signal strength ratio for the process YH, which is equal to unity in the SM. σ_{YH} is the cross-section for YH, i.e., the probability for a final state YH. The effect of higher-order operators is to shift the Higgs decay rates and production cross-sections from their SM values, allowing for the parameters of our effective Lagrangian to be constrained by comparing the theoretical rates to the SM rates. This method is known as the matching procedure, where we match higher-order operators' parameters (Wilson coefficients) onto the underlying theory, which would be the SM.

Part III

Final Thoughts

Chapter 8

Results

At the outset of this master's thesis, when the subject of effective field theories was largely unfamiliar, the ultimate objective was somewhat ambiguous. However, as the study progressed and the vastness of the field was explored, a clear goal emerged: the production of updated Wilson coefficients that could serve as a framework for improving predictions by matching the theory with the latest experimental data. This goal materialized in the three research questions introduced in the introduction. Also, in the introduction, four main objectives were created to answer the proposed research questions sufficiently. Throughout this study, Wilson coefficients have been an intriguing topic, and delving further into their intricacies has proven challenging and fascinating. This result section will present the findings of the four main objectives in an effort to try and answer the three research questions.

I am gratified to have produced an updated and comprehensive dictionary 6.4 of effective field theory that encompasses the most relevant contributions from dimension 6 and, most significantly, dimension 8 operators within the boundaries presented in this thesis. Along the way, this study has uncovered several dimension 8 operators that impact the canonical normalization of the Higgs field and contribute to a shift in fermion mass. Though these findings may appear minor in the overall context, the ability to calculate the necessary field and mass redefinitions required to account for these operators was both surprising and satisfying. Although the significance of these results may seem relatively modest, the calculations presented in Section 6.5 are nevertheless fascinating, particularly given the apparent complexity of the dimension 8 operators set at the outset of this study.

While it would be highly rewarding to continue this study and apply the updated dictionary framework to analyze the STU parameters and implement the matching procedure, such endeavors lie beyond the scope of this master's thesis

8.1 Updated Dictionary with Dimension 6 and 8 Coefficients

The principal findings of this master thesis, are the novel contributions to the Wcs in Section 6.4. Specifically, we extended the analysis beyond the previously considered dimension-6 operators, incorporating the effects of dimension-8 operators. This novel inclusion enables us to identify previously unexplored contributions that affect the behavior of the theory at higher energy scales.

8.2 Influence of $\frac{1}{g_2}$ on Dimension 8 Contributions

One interesting discovery is that since dimension 8 operators are suppressed quadratically by the energy scale, as we can see in equation 3.1, a factor of $\frac{1}{g_2}$ or $\frac{1}{g_2^2}$[1] emerges before each WC associated with a dimension 8 operator. In contrast, no such factor appears in front of any WC corresponding to a dimension 6 operator. This factor, representing the inverse coupling strength, is a distinct characteristic found in the WC dictionary and contributes to the enhanced influence of dimension 8 operators.

The presence of the inverse coupling strength factor, as observed in the analysis, leads to an amplification of the contribution from dimension 8 operators within the overall theoretical framework. This observation presents an intriguing and somewhat paradoxical finding since dimension 8 contributions are typically expected to be suppressed by the energy scale rather than indirectly enhanced by it. The implications of this unexpected phenomenon will be thoroughly examined and critically discussed in the discussion section. It raises an intriguing point of contention that warrants further investigation and analysis to better understand the underlying mechanisms.

8.3 Field redefinition result

Another interesting discovery made during this research is how various dimension 8 operators also affect canonical Higgs normalization and fermion mass. At the start of this thesis, how various dimension 8 operators could affect aspects such as canonical renormalization, fermion, and Z mass normalization was a mystery, as it was believed there were so many to choose from. However, it was quickly realized that although the overall difference in the size of the dimension 6 and dimension 8 operator set was extreme, the groups which contributed to changes in normalization and mass were not expanded.

Failing to account for the contributions arising from field redefinitions in the case of the Higgs field and mass redefinition for the fermion can lead to several significant issues. Among the consequences is the potential impact on the accuracy of predictions for specific physical observables, including the production rate of Higgs bosons in high-energy collisions. Furthermore, the neglected field redefinitions can also influence the interactions between the Higgs field and other particles, causing modifications in their masses and couplings.

The effects of field redefinitions are not mere formalities, and their inclusion is necessary for a complete and accurate understanding of the underlying physics. Neglecting these contributions may lead to inconsistencies in the theoretical framework, such as divergences in perturbative calculations or violations of unitarity. Therefore, it is vital to consider the necessary field redefinitions and mass redefinitions, to ensure reliable and consistent predictions of physical observables.

This is because the higher-order operators introduce new degrees of freedom (new

[1] This factor comes from the definition of m_W

non-propagating heavy particles) that need to be properly accounted for in the renormalization procedure.

8.4 Impact of Updated Wilson Coefficient Dictionary on STU Parameters

It is difficult to say how well the updated WCs would impact the STU parameters without performing the matching procedure and data analysis. However, we will try to speculate somewhat on the impact it may have, and specifically on the U-parameter,which is proportional to the ratio of the Higgs mass to the Z boson mass [cite] , as this is "extra" sensitive for dimension 8 operators.

Dimension 8 operators contribute to the U-parameter through loop corrections. These corrections would involve either top quarks or heavy new particles running in the loop. These corrections would create slight deviations U from its value in the SM.

What specific deviation one would get from our theoretical framework and choice of dimension 8 operators

8.5 Summary

In this chapter, we have presented the outcomes obtained from the incorporation of dimension 8 operators into an effective field theory Lagrangian that describes Higgs physics. The outcomes encompass an enhanced Lagrangian denoted as $\mathcal{L}_h$, which incorporates updated WCs. Additionally, we will provide a concise overview of the field re-definitions, the revised WC dictionary, and the discoveries pertaining to dimension 8 operators, along with the accompanying factor $\frac{1}{g^2}$. The significance and pertinence of these findings will be thoroughly examined and discussed in the subsequent chapter.

During the investigation of dimension 8 operators for the incorporation of $\mathcal{L}_h$, a crucial field re-definition was implemented for the Higgs field, accompanied by a mass shift to accommodate the inclusion of dimension 8 operators. In summary, the revised definitions for the mass and the Higgs field can be stated as follows:

$$h \to \frac{h}{\sqrt{1 + c_h + c_q + c_r}}$$

and for the fermion mass

$$m_f^* = m_f \left[1 + \frac{3}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right].$$

The WC dictionary was also updated with these new contributions from dimension 8 operators:

$$
\begin{aligned}
c_W &= -\frac{\bar{c}_q + \bar{c}_r}{2} \\
c_f &= -\frac{\bar{c}_q + \bar{c}_r}{2} + \mathrm{Re}\,(\bar{c}_f) \\
\tilde{c_f} &= \mathrm{Im}\,(\bar{c}_f) \\
c_Z &= -\frac{\bar{c}_q + \bar{c}_r}{2} \\
c_{WW} &= \frac{4}{g_2}\bar{c}_{WW} \\
c_{ZZ} &= 8\frac{\tan^2\theta_W s_w^2}{g_2^2}\bar{c}_{HB} + \frac{4}{g_2^2}\bar{c}_{WW} + \frac{8}{g_2^2}\bar{c}_{DHW} + \frac{16}{g_2^2}\tan\theta_w\bar{c}_{DHB} + \frac{4^5}{g_2^2}c_w^2\bar{c}_{HWW} + \frac{16}{g_2^2}s_w\bar{c}_{WB} \\
c_{\gamma\gamma} &= 8\frac{s_w^2}{g_2^2}\bar{c}_{HB} + 8\frac{s_w^2}{g_2^2}\bar{c}_{WW} + 4^5\frac{s_w^2}{g_2^2}\bar{c}_{HWW} + 16\frac{s_w^2}{g_2\bar{c}_{WB}} \\
c_{Z\gamma} &= 2^7\frac{\tan\theta_w s_w^2}{g_2^2}\bar{c}_{HB} + 8\frac{s_w c_w}{g_2}\bar{c}_{WW} + 8\frac{\tan\theta_w}{g_2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} + 2\times 4^5\frac{s_w c_w}{g_2^2}\bar{c}_{HWW} \\
c_{gg} &= 8\frac{g_3^2}{g_2^4}\bar{c}_{HG} \\
\kappa_{Z\gamma} &= 8\frac{\tan\theta_w}{g_2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} \\
\kappa_{ZZ} &= 8\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHW} + 16\frac{\tan\theta_w}{g_2^2}\bar{c}_{DHB} \\
\kappa_{WW} &= \frac{32}{g_2^2}\bar{c}_{DHW}
\end{aligned}
$$

The factor $\frac{1}{g^2}$ emerges as a coefficient preceding each WC belonging to a dimension 8 operator. This factor holds significant meaning in terms of determining the relevance of incorporating dimension 8 operators in relation to dimension 6 operators. It elucidates how the inclusion of dimension 8 operators is influenced by and compared to the effects of dimension 6 operators.

Chapter 9

Discussion

In this chapter, we will discuss whether or not we sufficiently accomplished our main objectives, in order to best answer our desired research questions. In addition to discussing the research questions and objectives, we will also discuss some of the choices made throughout the master thesis, such as the validity of dimension 8 operator constraints, which ones we decided to cut, and the rationale for our dimension 8 operator choice. At the very end, we will summarize various choices that could be improved in hindsight, and how this could impact future work.

9.1 Scaling of Wilson Coefficients with inverse coupling

This result is rather strange and hard to fit with the rest of the theory, as WCs with more and more factors of $\frac{1}{g_2}$ actually become more and more relevant. Intuitively this does not make sense, as in equation 3.1, the higher dimension operators are suppressed by the energy scale. However, when that energy scale is the electroweak energy scale, factors of $\frac{1}{g_2}$ are introduced when converting the energy scale according to the mathematical formulation for m_W in Section 11.6.2 into essentially factors of $\frac{1}{g_2}$.

This does not square with the intuition that the operators of higher mass dimension, i.e. interactions which involve more fields and therefore have more powers of energy or momentum in their expressions compared to lower-dimensional operators, should become more and more suppressed.

But due to the inverse coupling being a number larger than one, the contributions from higher order operators actually grow and grow. Also, it is important to consider the concept of running couplings[1], where the values of coupling constants evolve with the energy scale. As the energy scale increases, the inverse coupling constant becomes larger and larger. Consequently, at higher energy scales, the contribution from dimension 8 operators becomes increasingly relevant despite their inherent higher suppression at lower energies. For deeper insight into this issue, one would have to solve and study the scaling behavior of the coupling constant, by solving the β-function 3.3.

The presence of dimension 8 contributions in the WC dictionary with factors of inverse coupling strength, which seem to enhance these contributions even at lower energies, raises a question beyond the sole influence of running couplings. While running

[1] The concept of running couplings have not been extensively covered in this thesis, but briefly touched upon in the section on more advanced concepts of EFTs

couplings explain the increased relevance of dimension 8 operators at higher energy scales, they fall short in explaining the specific observation of inverse coupling strength factors amplifying dimension 8 contributions at seemingly lower energies.

9.2 Rationale for Dimension 8 Operator Choice

One of the primary challenges encountered in this master thesis was the vast number of available dimension 8 operators to choose from. At the outset, there was a concern that identifying the correct or optimal operator for Higgs physics would be akin to finding a needle in a haystack. Therefore, it was of utmost importance to establish the crucial constraint that fermion currents coupled to the electroweak gauge bosons impose significant limitations, rendering many effective operators irrelevant.

In order to overcome this challenge and ensure the selection of meaningful operators, the extension of constraints from dimension 6 operators to also encompass dimension 8 operators became a key consideration. The validity and applicability of such constraints will be thoroughly discussed in 9.3, where an evaluation of the validity of constraints for dimension 6 operators in the context of dimension 8 operators will be presented.

The specific constraint applied, focusing on neglecting operators with fermion currents coupled to the electroweak gauge bosons, proved to be instrumental in streamlining the analysis within this master thesis. By excluding this subset of operators, a significant portion of the dimension 8 operator space was effectively rendered negligible, allowing the research to primarily focus on bosonic operators. The reduction in complexity brought about by this constraint made the selection process more manageable and facilitated a more targeted investigation.

Among the remaining set of dimension 8 operators, several groups were identified as particularly relevant and deserving of further exploration. These groups include A^2H^4, ψ^2H^5+ h.c, AH^4D^2, and H^6D^2. By selecting these specific groups, the analysis could concentrate on operators involving combinations of Higgs fields, gauge bosons, and derivatives. Other operators that either involved an excessive number of gauge bosons or introduced entirely new interactions not already encompassed by the existing dictionary were deemed irrelevant for the purposes of this analysis.

Among the valid groups of operators, certain operators were excluded from further analysis for specific and evident reasons. One such example is the Higgs sextuple operator, which contributes to the decay of a Higgs boson into five other Higgs bosons. This decay process is currently considered to be highly improbable to detect, both at present and in the foreseeable future. Consequently, the operators belonging to this group were deemed impractical to investigate within the scope of this research.

Another example is the group of operators denoted as H^4D^4. Although mathematically intriguing, as this group has the potential to alter the canonical mass of the Z boson, further examination was not pursued. This decision was based on the fact that the corresponding WC c_T, which represent the dimension 6 equivalent operator, was set to zero. As a result, no additional exploration was conducted to ascertain whether the inclusion of dimension 8 operators would modify this aspect.

Following the application of these specific exclusions, the resulting set of dimension 8 operators proved to be reasonably manageable. This was particularly advantageous considering that the effective Lagrangian needed to be computed in unitary gauge. The manageable nature of the remaining operators allowed for a thorough calculation of each pertinent dimension 8 operator in unitary gauge. This comprehensive calculation, coupled with an updated Wilson coefficient dictionary, forms a crucial aspect of this thesis and serves as one of its focal points.

9.3 Evaluating the Validity of Constraints(Mangler mye)

This section focuses on investigating the diverse constraints that have been employed to restrict the number of dimension 8 operators, as well as the critical decisions that have been made in this regard. We will delve into several pivotal choices. Furthermore, we will explore the exclusion of operators affecting the Z-mass, the rationale behind disregarding operators involving three or more gauge bosons, and the structural organization of the dictionary. By conducting a thorough examination of these choices and constraints, our aim is to provide a comprehensive understanding of the fundamental principles that have guided the development of this specific study.

The key assumption made in order to reduce the number of relevant dimension 8 operators was to extend the existing constraints that determine the relevant dimension 6 operators. However, the validity of this approach in applying the same set of constraints to both dimension 6 and dimension 8 operators raises important questions. Can we confidently assert that the constraints governing dimension 6 operators should also govern dimension 8 operators?

This is a critical aspect that requires careful evaluation. While there may be some overlap in the constraints that apply to both dimension 6 and dimension 8 operators, it may not be guaranteed that the same set of constraints will be applicable to both cases. The higher-dimensional operators may introduce new phenomena or exhibit different behavior that necessitates distinct constraints.

Constraint 1: The detection of the sole Higgs self-interaction term, $(H^\dagger H)^3$, already poses a significant challenge with the current detection equipment at the LHC[9]. This term represents the trilinear interaction of the Higgs boson. Given this difficulty, it is reasonable to assume that detecting and studying higher-order interactions, such as the sole dimension 8 Higgs sextuple interaction term $(H^\dagger H)^4$, may be even more challenging. Therefor the operator $(H^\dagger H)^4$ is also excluded from the theory.

Constraint 2: Constraint 2 in this master thesis is a general constraint based on experimental precision. In the referenced EFT [9], the Lagrangian is limited to operators of mass dimension 6. In this thesis, the constraint is expanded to include operators of mass dimension 8 in the effective Lagrangian. Currently, the scientific consensus primarily focuses on EFTs limited to operators of mass dimension 6, with only a few articles exploring specific dimension 8 operators.

Constraint 3: Constraint 3 focuses on including only higher-order operators that are relevant for describing Higgs physics, and hence, only operators containing Higgs fields are relevant.

Constraint 4: The exclusion of fermionic operators or operators with fermion currents coupled to electroweak gauge bosons is a critical constraint in this analysis. The decision to exclude these operators is motivated by the recognition that their contributions would be tightly constrained by the wealth of existing electroweak precision data. Expanding this constraint to also cover dimension 8 operators is reasonable, as dimension 8 operators that contain fermion currents coupled to gauge bosons would also be subject to the same electroweak precision constraints. There are however some considerations.

The inclusion of higher-order operators, such as dimension 8 operators, in an EFT introduces new constraints on the theory as demonstrated in the article referenced as [19]. While it is reasonable to assume that there may exist dimension 8 operators that could be fermionic in nature but not constrained by existing electroweak precision measurements, it is challenging to identify these operators without matching the theory to experimental data.

One can also argue that the presence of such constraints indicates the need for operators that are more sensitive, as they would be able to capture smaller deviations from the SM. While the contributions from dimension 6 operators may be too large to fit within the existing constraints, the inclusion of more sensitive operators, such as certain dimension 8 operators, could potentially parametrize these smaller deviations.

It is worth considering whether it is necessary to include these more sensitive operators or if the focus should be on parametrizing sectors where there is a greater potential for deviation and the emergence of new physics. This decision depends on the specific research goals and the extent to which the chosen operators can effectively capture the desired phenomena.

Excluding three gauge bosons and structure of the WC dictionary: The reason why three gauge bosons interactions or more does not appear is because the original dictionary in [9] also excluded interactions with three or more gauge bosons. Therefore the updated dictionary presented in this thesis has also excluded interaction terms with three or more gauge bosons as we did not want to include any new interactions other than the ones in the original article.

The interactions in the original WC dictionary, $Z^{\mu v} Z_{\mu v}, Z^{\mu v} \gamma_{\mu v}, \gamma^{\mu v} \gamma_{\mu v} W^{\dagger}_{\mu v} W^{\mu v}$, $Z^{\mu} \partial^{\mu} Z_{\mu v}, Z^{\mu} \partial^{v} \gamma_{\mu v}$, and $W^{\mu} D^{v} W^{\dagger}_{\mu v}$, along with the SM terms, encompass all relevant dynamics and interactions associated with the Higgs boson.

9.4 Omitted Dimension 8 Operators

The decision to exclude the group of dimension 8 operators capable of modifying the canonical Z-mass in a similar manner to the Higgs canonical renormalization and fermion mass shift was based on several factors. First, the existing EFT framework described in the article [9] already includes an operator that supplements the coefficient c_T to the Z-mass. While this operator is present, it is assumed to have a negligible impact.

Furthermore, the exclusion of these dimension 8 operators can be justified by considering the tree-level constraints resulting from the current experimental limits on the S and T parameters. These parameters are quantities used to quantify potential deviations from the SM in EWPM. The experimental constraints on the S and T parameters set bounds on the contribution of new physics effects to the Z-mass, and therefore, operators that significantly modify the Z-mass would likely be in conflict with these constraints.

The referenced article [9] provides evidence to support the aforementioned assumption. The underlying rationale is that the corrections stemming from dimension 8 operators would also be subject to the most recent experimental limits for S and T. Nonetheless, this may not be entirely valid, given that the S and T parameters are parameterized around a fixed U value at the tree level, which happens to be zero. However, as mentioned earlier, the U parameter obtains specific contributions from different dimension 8 operators. This is a consequence of the U parameter's objective to quantify the impact of new particles, which is precisely what we aim to detect through higher-order operators, or at the very least, the low-energy effects of these heavier new particles.

It is possible that if the U-parameter is not fixed at zero, then non-propagating heavy particles which contributed to the coupling between the Z boson and fermions, could loosen the constraints around the T-parameters, which could be parametrized by the set of operators which we just justified the exclusion of. However, this is hard to say without knowing exactly how dimension 8 operators would alter the U-parameter, both at current experimental data, and future data.

9.5 SHIL vs Warsaw basis

In retrospect, it would have been advantageous to utilize the operator structure of the SHIL basis from the outset, particularly considering the main objective of this master thesis, which is to parameterize Higgs physics. The SHIL basis consists of operators designed explicitly to describe Higgs physics. However, due to the incompleteness of the SIHL basis [23] and our intention to extend the power counting rules to encompass dimension 8 operators, which the SHIL basis does not encompass, it proved valuable to embark on a comprehensive exploration of the various potential dimension 6 operators and their possible extensions to dimension 8, as elaborated upon in Section 4.6. By adopting a systematic classification in the form of diagrams, we were able to avoid the potential oversight of significant dimension 8 operators that might have been missed had we solely expanded a subset of operators to dimension 8.

9.6 Normalization convention

When deciding the correct coefficient in front of each operator in 6.4 and 6.3 both [21] and [16] were consulted. The challenge was deciding the correct coefficient in front of the dimension 8 operators, as [33] only discussed the Lorentzian and gauge structures. In the end, the Lagrangian 6.4 is normalized by adding the energy scale, $\frac{1}{\Lambda^4}$(Λ could be either v or m_W depending on the fields involved), and one coupling constant for each respective field. There could be some inconsistencies regarding whether a factor of $\frac{1}{2}$ should be included or not. In the end, the dimension 8 Lagrangian 6.4 tries to emulate the normalization convention of the dimension 6 Lagrangian 6.3,

9.7 Look back at the Problem Statement

When embarking on this master thesis, three central research questions were formulated to provide a clear direction for the study. Additionally, five main objectives were established to guide the research and ensure its successful completion. In this section, the focus is on evaluating whether these research questions were effectively addressed and if the objectives were successfully achieved.

9.7.1 Questions 1 and 2

The initial inquiry pertains to the feasibility of augmenting current EFTs with operators of dimension 8. In chapter 6, our study successfully expands the existing EFT by incorporating dimension 8 operators. Initially, identifying the pertinent dimension 8 operators proved to be a challenge. However, Chapter 6 offers a potential set of operators that, upon application of constraints, appear to be justifiable. For an evaluation of the validity of these constraints, please refer to Section 9.3. Notably, Chapter 6 solely provides a theoretical framework that could potentially yield improvements through the implementation of the matching procedure. However, due to time limitations, the matching procedure was omitted, as explained in Section 1.2. Objectives one through four were established to address the primary question, and all objectives were successfully accomplished. The first objective was fulfilled in Section 4, while the second objective spanned multiple sections. Section 4.8 introduced the proposed constraints, and presented the outcomes of applying these constraints to the set of dimension 8 operators.

The third objective entailed establishing the expanded Lagrangian of the EFT after SSB. The primary challenge here involved deriving each effective operator following the system's SSB and expressing the effective Lagrangian in unitary gauge. The comprehensive collection of higher-order operators in unitary gauge can be found in Appendix 11.6. While [9] presents the final effective Lagrangian up to dimension 6 in unitary gauge, it lacks explicit calculations for each individual operator, which are provided in Appendix 11.6.3. Calculating each dimension 8 operator in unitary gauge proved to be a more arduous task. The explicit calculations of dimension 8 operators in unitary gauge can be found in Section 11.6.7. The resulting EFT Lagrangian is presented in Section 6.3 as a theoretical framework with potential implications for future research.

Objective four aims to utilize the effective Lagrangians $\mathcal{L}_6$, $\mathcal{L}_{6,h}$, $\mathcal{L}_8$, and $\mathcal{L}_{8,h}$, as presented in Section 6.3, to establish an expanded dictionary incorporating contributions from dimension 8 operators. This was accomplished by comparing the various Lagrangians $\mathcal{L}_6$, $\mathcal{L}_{6,h}$, $\mathcal{L}_8$, and $\mathcal{L}_{8,h}$ in a manner similar to [9]. In Section 6.3, a new effective Lagrangian $\mathcal{L}_h$ (Equation 6.12) is presented, encompassing the contributions from each Lagrangian $\mathcal{L}_6$, $\mathcal{L}_{6,h}$, $\mathcal{L}_8$, and $\mathcal{L}_{8,h}$ through new WCs, namely c_W, c_Z, c_f, cWW, c_{ZZ}, $c_{\gamma\gamma}$, $c_{Z\gamma}$, c_{gg}, κ_{WW}, $\kappa_{W\gamma}$, and κ_{ZZ}. These coefficients represent the interactions $Z^{\mu v}Z_{\mu v}$, $Z^{\mu v}\gamma_{\mu v}$, $\gamma^{\mu v}\gamma_{\mu\nu}$, $W^{\dagger}_{\mu\nu}W^{\mu v}$, $Z^{\mu}\partial^{\mu}Z_{\mu v}$, $Z^{\mu}\partial^{v}\gamma_{\mu v}$, and $W^{\mu}D^{v}W^{\dagger}_{\mu\nu}$. While $\mathcal{L}_h$ is the same Higgs-dependent Lagrangian presented in [9], it now incorporates contributions from dimension 8 operators in its WCs. It is based on these updated WC coefficients that we present the revised dictionary in Section 6.4, either as an equation or a table (e.g., 6.14). For further discussion on the relevance of the chosen dimension 8 operator set, the validity of constraints, and the omitted dimension 8 operators, please refer to Sections 4.8, 9.3, and 9.4, respectively.

Objectives one through four have successfully achieved their aim of addressing the primary research question. However, the second research question inquires whether this accomplishment ultimately leads to an improved parametrization of the Higgs coupling to SM matter fields. This specific question remains unanswered within this master's thesis, as explained in Section 1.2. To adequately address this question, the matching procedure must be conducted, which involves establishing a connection between the theoretical framework presented here and experimental data. An approach to accomplishing this task is outlined in Section 10.2.

Upon the completion of this master's thesis, the final objective, objective five, remains incomplete. Objective five aimed to utilize the derived framework in this thesis to not only construct an updated WC dictionary but also establish a match between the theoretical framework and experimental data. The potential of this framework is discussed in Section 10.2 as part of the future directions and further research possibilities.

9.7.2 Question 3

Given the scope and limitations of this master's thesis, the task of bridging the gap between theory and experiment was not undertaken. However, it is crucial to emphasize that outlining the uses and implications of the presented theoretical framework remains a central focus of this thesis. To address this objective, the final research question is proposed in order to provide the best possible answer. Objective five primarily revolves around addressing this last question, and one approach to parametrize new physics and the Higgs boson coupling to SM matter fields is through the utilization of the STU-parameters, as discussed in chapter 7. In chapter 7, a brief introduction to the basics of the Peskin-Takeuchi parameters is provided, albeit without an extensive theoretical background on the STU-parameters. Furthermore, Section 8.4 presents the final results related to the third research question. Nevertheless, due to the scope and limitations of this master's thesis, the answer to this final research question remains unanswered, with only a surface-level discussion regarding the potential impact of the theoretical framework.

Two notable outcomes of the theoretical framework, although not central to the research questions, are presented in sections 8.3 and 8.2. These results were serendipitous discoveries made during the course of the research and provide intriguing insights.

Section 8.3 discusses the significance of incorporating field redefinitions and mass shifts when including dimension 8 operators to achieve accurate predictive results. Failing to consider these field redefinitions neglects the potential influence of higher-order operators on the Higgs normalization and fermion mass. The exclusion of changes in the Z-boson mass, as stated in [9], is attributed to the already stringent constraints imposed by the STU parameters, which do not allow for deviations in the Z-boson mass. However, as the HL-LHC is launched in the future, reassessing this exclusion may become necessary. It is worth noting that the exclusion of changes in the Z-boson mass in this master's thesis could be seen as over-extending the existing constraints on dimension 6 operators to also encompass dimension 8 operators. Nonetheless, considering the current constraints on the STU parameters outlined in [9], as well as the precision of current experimental data from which these constraints are derived, it appears to be a reasonable choice.

9.8 Summary

The investigation revealed that despite the initial size of the dimension 8 operator set, a significant portion of operators were ultimately excluded based on the constraints proposed in Section 4.8. This outcome played a critical role in the remainder of the master's thesis, as it enabled the completion of comprehensive calculations for the dimension 8 operators in unitary gauge. The selection criteria for dimension 8 operators were based on the rationale presented in [9], while acknowledging that expanding the criteria in this manner presents remaining challenges. However, determining the accuracy and validity of this expansion necessitates the completion of the matching procedure outlined in Section 10.2.

The validity of certain constraints raises questions, given the inherent challenge of definitively determining which EFT and dimension 8 operators can effectively parametrize deviations, and which dimension 8 operators may become more constrained in specific sectors with the introduction of new data. The expansion of constraints to encompass dimension 6 operators, as proposed in [9], is a matter that warrants reconsideration in future research, as new data may necessitate the inclusion of operators containing, for example, dipole fermion couplings. Conversely, certain exclusions appear entirely reasonable, such as the dimension 8 Higgs sextuple coupling, which is unlikely to be discovered even with the implementation of the HL-LHC or any subsequent upgrades.

It is crucial to emphasize that the incorporation of dimension 8 operators must be driven by empirical evidence. The assessment of their significance and validity hinges upon the comparison between theoretical predictions that incorporate these operators and experimental data. Only through this iterative process of refinement and verification can we comprehensively evaluate the true impact and sensitivity of dimension 8 operators.

The exclusion of certain dimension 8 operators aligns with the summary of the rationale behind the selection of dimension 8 operators and does not warrant repetition. Dimension 8 operators capable of modifying the Z-boson mass were ultimately disregarded, as even at dimension 6, such operators faced significant constraints imposed by the STU-parameters, particularly the T-parameter. However, the validity of this exclusion in light of future data remains uncertain and subject to reassessment.

In regards to the research questions, question one was sufficiently answered, and a comprehensive theoretical framework has been sufficiently laid out in sections 6.3, 6.4.

Question 2 is open-ended, and requires a deep dive into various Mathematica packages presented in the section on Future Directions, 10.2, which would require going beyond the scope of this master thesis.

Question 3 is answered in the sense that it gives a theoretical way of applying the framework in section 10.2. The results as presented in 8.3 and 8.4 also lays out how the results found are necessary and important in future research. Objective 5 was intended to complement research question 3 and provide a comprehensive answer. However, this master's thesis primarily focused on presenting the theoretical framework rather than its direct application. As a result, the actual utilization of the framework, along with its potential future applications, remains an area for further exploration.

Chapter 10

Conclusion

In this chapter, we will present the remaining work and obstacles, and reflect on the results. A novel way of how one could solve the remaining work and unanswered questions will also be presented.

10.1 Summary

The conclusion will commence by restating and address the research objectives outlined in section 1.1. Additionally, the conclusion will provide insights on potential avenues for applying the presented framework as discussed in sections 10.2, 10.3, and 10.4.

Objective 1: *Present a comprehensive description of the current EFTs capped at dimension 6 and the dimension 6 Warsaw Basis.*

A comprehensive depiction of dimension 6 operators and the Warsaw basis was provided in Sections 4.7. This discussion was built upon the theoretical foundation established in Sections 4.2, 4.3, and 4.4. Having established the classification and rationale for these operators, the EFT Lagrangian, $\mathcal{L}_h$, originally presented in [9], was subsequently recapitulated in Section 6.2.

Objective 2: *Expand the constraints on the EFT operator set to also include the dimension 8 operators most relevant for Higgs physics.*

Section 4.7 provides a comprehensive analysis of dimension 6 operators, applying the selection criteria outlined in [9]. Building upon this foundation, the selection criteria were expanded to encompass the dimension 8 operators discussed in Section 4.8. The resulting dimension 8 Lagrangian, presented as operators in Equation 4.5 in Section 4.8, effectively covers the most pertinent operators for parametrizing Higgs physics, as determined by the extended operator selection criteria.

Objective 3: *Derive the effective Lagrangian after SSB*

The final Higgs-dependent effective Lagrangian, represented by Equation 6.12 in Section 6.3, captures the contributions from higher-order operators specifically pertaining to the interaction under consideration in our WC dictionary. It is important to note that the Lagrangian 6.12 may warrant future revision to encompass additional types of interactions beyond those included in our current framework. Furthermore, it is crucial to emphasize that this final EFT Lagrangian, $\mathcal{L}_h$, solely represents the higher-order extensions and should be combined with $\mathcal{L}_{SM}$, as $\mathcal{L}_h$ already incorporates all the necessary operators for obtaining the updated WC dictionary.

Objective 4: *Use the effective Lagrangian after SSB to derive an updated WC dictionary.*

The updated WC dictionary, presented in Section 6.4, was derived by utilizing the post-SSB effective Lagrangians $\mathcal{L}_{6,h}$ (Equation 6.10) and $\mathcal{L}_{8,h}$ (Equation 6.11). By combining all the contributions to the interactions outlined in Section 6.4, we obtained a final effective Lagrangian denoted as $\mathcal{L}_h$ 6.12. This updated effective Lagrangian $\mathcal{L}_h$ builds upon the framework presented in the article [9], incorporating the additional contributions from $\mathcal{L}_{8,h}$.

Objective 5: *Use the updated WC dictionary and Peskin-Takeuchi parameters to explore the potential of dimension 8 operators, and the potential for future work.*

The updated WC dictionary was not matched onto experimental data, however, the impact and potential work were discussed in Section 10.2.

10.2 Future Directions

A promising approach to advance further would be to establish a correlation between the revised STU-parameters and empirical evidence. This would enable us to determine whether integrating dimension 8 operators in Higgs physics parameterization would be beneficial. In the following section, a succinct outline of the methodology that can be employed will be laid out, along with a description of the potential implications for my master's thesis.

10.3 Bridging the Gap between Theory and Experiment

"Bridging the gap" between theory, and data in the context of this master thesis, would be to use the updated Wilson coefficients to get a better description of the observables as the novel description in Section 7.2 outlines. This section will briefly go over how one could do this using various numerical tools.

Useful tools to numerically evaluate and generate these decay widths include implementation of the effective model in **FeynRules** [3], where we could use the Mathematica package for the generation and visualization of Feynman diagrams and amplitudes called **FeynArts** [25]. The tree-level and one-loop diagrams could then be read by another Mathematica package **FormCalc**[1], which calculates and returns data suited for further analysis. The latest versions of these packages are the **FeynArts version 3**, along with **FormCalc version 9**.

Exploring the use of these packages could be an intriguing endeavor, albeit challenging given their complexity and size. Mastery of these packages is essential to utilize them effectively.

The utilization of **MadGraph**[5] and **FeynRules** allows for the derivation of precise descriptions pertaining to the decay rates and cross-section production of VBF processes. These descriptions can be expressed through the use of equations 10.1 and 10.2. It is important to note that the presented formulas serve as a mere exemplification of the use of various Mathematica packages, and as such, they are not all-encompassing. Specifically, the contributions stemming from dimension 6 and dimension 8 operators are excluded, given that we cannot currently match our theoretical models to experimental data, and consequently, we are unable to accurately weigh the contributions from these operators.

To further elaborate, the integration of **MadGraph** and **FeynRules** provides a powerful computational tool that permits the analysis of various high-energy physics phenomena, including those related to VBF processes. The resulting formulas are constructed through the use of complex mathematical expressions that account for various contributing factors, including cross sections and decay rates. However, in the absence of matching our theory to experimental data, it is currently impossible to accurately determine the precise contributions of certain dimension 6 and dimension 8 operators. As a result, these contributions are not included in the presented formulas.

The matching procedure involves aligning the Ultra-Violet(UV) theory with the full underlying theory or the infrared (IR) theory in the low energy spectra. High energy operators that describe new physics appear as deviations from the SM observables. The relative cross-section and decay rate formulas, given as 10.1 and 10.2 provide a general overview of the final expression. The success of the matching procedure depends on the ability to identify and integrate these contributions with precision through a comprehensive understanding of the underlying physics and experimental data

$$\left(\frac{\sigma}{\sigma_{SM}}\right) = c_{SM} + c_{d=6} + c_{d=8}, \tag{10.1}$$

$$\left(\frac{\Gamma}{\Gamma_{SM}}\right) = c_{SM} + c_{d=6} + c_{d=8}. \tag{10.2}$$

The purpose of equations 10.1 and 10.2 is to demonstrate that, following the application of the theoretical framework and the comparison and matching of the theory to comprehensive underlying data obtained from experiments, the resulting observables are described using the WCs associated with higher-order operators.

10.4 Significance and Applications

The specific goal of this thesis was to lay the groundwork of how one could integrate dimension 8 operators into an effective field theory, and how these dimension 8 operators would alter the various Wilson coefficients already established [9].

The value and impact of this work, as well as the contributions of dimension 8 operators, can only be fully assessed by fitting the theoretical framework to experimental data. Until this fitting is completed, the precise extent of the contributions of dimension 8 operators and the implications of this work for particle physics research remains uncertain. However, the integration of dimension 8 operators into the theoretical framework holds significant potential. Thus, this work represents a valuable step forward in the ongoing effort to develop more comprehensive models describing low-energy phenomena. Ultimately, the impact of this work will depend on the ability to successfully integrate dimension 8 operators into the theoretical framework and to rigorously test these predictions against experimental data.

An intriguing area of study that I aimed to contribute to through my work is the determination of the energy scales at which the inclusion of dimension 8 operators becomes necessary. While there are indications from theoretical results that contributions from these operators may be suppressed, a definitive determination of the energy scale at which such contributions become necessary requires a detailed analysis of the weighting of the parameters in the effective Lagrangian following fitting to experimental data. Without such an analysis, it is impossible to state with certainty when the inclusion of dimension 8 operators is required for a comprehensive understanding of the underlying physical processes.

Chapter 11

Appendices

11.1 Appendix A

11.1.1 EOM in the SM

The EOM for the Higgs field follows the same procedure. Although when deriving the EOM for the Higgs field, it is easier to use the Euler-Lagrange equations which incorporate the covariant derivative. As shown [30], the E-L equations maintain their familiar structure but with the partial derivative has been replaced by a covariant one:

$$\frac{\partial \mathcal{L}}{\partial H} = D_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(D_\mu H \right)} \right) \tag{11.1}$$

The Higgs part of the SM Lagrangian is where the Yukawa interaction term is split into quarks and leptons[14]

$$\begin{aligned} \mathcal{L}_{Higgs} = &|D_\mu H|^2 - \lambda \left(H^\dagger H - \frac{1}{2}v^2 \right)^2 \\ &+ \left(h_{uij}\bar{u}_{Ri}q_{Lj}H + h_{dij}\bar{d}_{Ri}q_{Lj}\epsilon_{ab}H_b^* + h_{eij}\bar{e}_{Ri}l_{Lj}\epsilon_{ab}H_b^* + h_{nij}\bar{n}_{Ri}l_{Lj}H + h.c \right). \end{aligned} \tag{11.2}$$

Varying the Lagrangian above with respect to the Higgs field gives:

$$-\lambda(H^\dagger H)H + \bar{q}^j Y_u^\dagger u\epsilon_{jk} + \bar{d}Y_d q_k + \bar{e}Y_e l_k + m^2 H^\dagger H = D^\mu D_\mu H, \tag{11.3}$$

$$D^\mu D_\mu H = \bar{q}^j Y_u^\dagger u\epsilon_{jk} + \bar{d}Y_d q_k + \bar{e}Y_e l_k + m^2 H^\dagger H - \lambda(H^\dagger H)H. \tag{11.4}$$

One way of deriving the EOM for non-abelian fields is by using the conserved currents of the system. The conserved currents for non-abelian terms, with a covariant derivative, looks like this

$$D^\mu A_{\mu\nu} = g \cdot j_\nu. \tag{11.5}$$

Where the different SM currents of fields are given as[8]

$$j_\mu^A = \sum_{\psi=u,d,q} \bar{\psi}t^A\gamma_\mu\psi, \tag{11.6}$$

$$j_\mu^I = \frac{1}{2}\bar{q}t^I\gamma_\mu q + \frac{1}{2}\bar{\ell}t^I\gamma_\mu\ell + \frac{1}{2}H^\dagger i \overleftrightarrow{D}_\mu^I H, \tag{11.7}$$

$$j_\mu = \sum_{\psi=u,d,q,e,\ell} \bar{\psi}\mathrm{y}_i\gamma_\mu\psi + \frac{1}{2}H^\dagger i \overleftrightarrow{D}_\mu H. \tag{11.8}$$

Equation 11.6 is the current belonging to the gluonic field, 11.7 is the current belonging to the weak field, and the last current 11.8 is for the hypercharge field. Combining these currents with equation 11.5, gives the remaining EOM the gauge fields[23]

$$(D^\rho G_{\rho\mu})^A = g_3 \left(\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d\right), \tag{11.9}$$

$$(D^\rho W_{\rho\mu})^I = \frac{g_2}{2} \left(H^\dagger i \overleftrightarrow{D}^I_\mu H + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right), \tag{11.10}$$

$$\partial^\rho B_{\rho\mu} = g_1 Y_H \varphi^\dagger i \overleftrightarrow{D}_\mu H + g' \sum_{\psi\in\{l,e,q,u,d\}} Y_\psi \bar{\psi}\gamma_\mu \psi. \tag{11.11}$$

11.2 Appendix B

11.2.1 Derivation of operators in group H^4D^2

Using the Fierz-identity [23] we can "split" the operator into two operators. To see this more clearly, the indies are written out explicitly :

$$\begin{aligned}
\left(H^\dagger t^a H\right)\left[(D_\mu H)^\dagger t^a (D^\mu H)\right] &= \left(H_i^\dagger t_{ij}^a H^j\right)\left[(D_\mu H)_k^\dagger t_{kl}^a (D^\mu H)_l\right] \\
&= \left(H_i^\dagger H^j\right)\left[(D_\mu H)_k^\dagger t_{ij}^a t_{kl}^a (D^\mu H)_l\right] \\
&= \left(H_i^\dagger H^j\right)\left[(D_\mu H)_k^\dagger (2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}) (D^\mu H)_l\right] \\
&= \left(H_i H^j\right)(D_\mu H)_k (2\delta_{il}\delta_{kj}) (D^\mu H)_l - \left(H_i H^j\right)(D_\mu H)_k (\delta_{ij}\delta_{kl}) (D^\mu H)_l \\
&= 2\left(H_i H^k\right)(D_\mu H)_k (\delta_{il}) (D^\mu H)_l - \left(H_i H^i\right)(D_\mu H)_k (\delta_{kl}) (D^\mu H)_l \\
&= 2\left(H_l H^k\right)(D_\mu H)_k (D^\mu H)_l - \left(H_i H^i\right)(D_\mu H)_k (D^\mu H)_k \\
&= 2\left(H_l H^k\right)(D_\mu H)_k (D^\mu H)_l - \left(H_i H^i\right)(D_\mu H)_k (D^\mu H)_k \\
&= 2H_l (D_\mu H)_l H^k (D^\mu H)_k - \left(H_i H^i\right)(D_\mu H)_k (D^\mu H)_k \\
&= 2\left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right) - \left(H^\dagger H\right)\left[(D_\mu H)^\dagger (D^\mu H)\right]
\end{aligned}$$

And as we can see, both operators $\left(H^\dagger t^a H\right)\left[(D_\mu H)^\dagger t^a (D^\mu H)\right]$ and $\left(H^\dagger H\right)\left[(D_\mu H)^\dagger (D^\mu H)\right]$ are just one Fierz transformation away.

To establish the relationship between the operators $\frac{1}{2}\left(H^\dagger H\right) D_\mu H^\dagger D^\mu H$ and $\frac{1}{2}\left(H^\dagger H\right)(D_\mu D^\mu) H^\dagger H$, we can use the Leibniz rule. The application of the Leibniz rule proceeds as follows:

$$\begin{aligned}
\frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H &= \frac{1}{2}(H^\dagger H)D_\mu((D^\mu H^\dagger)H + H^\dagger(D^\mu H)) \\
\frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H &= \frac{1}{2}(H^\dagger H)D_\mu(D^\mu H^\dagger)H + \frac{1}{2}(H^\dagger H)D_\mu(H^\dagger(D^\mu H)).
\end{aligned}$$

Proceeding from this point, I will now derive the expression for the operator I initially began with, namely $\frac{1}{2}\left(H^\dagger H\right) D_\mu H^\dagger D^\mu H$, by solving the aforementioned equation. The steps involved in obtaining the desired expression are as follows:

$$\frac{1}{2}(H^\dagger H)D_\mu H^\dagger D^\mu H = \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)D_\mu((D^\mu H^\dagger)H). \tag{11.12}$$

(11.13)

We can utilize the EOM for the Higgs field, which is provided in the article referenced as [9] and derived in Appendix A

$$(D^\mu D_\mu H)^j = m^2 H^j - \lambda\left(H^\dagger H\right)H^j - \bar{q}^j h_u^\dagger u\epsilon_{jk} + \bar{d}h_d q_k + \bar{e}Y_e l_k \tag{11.14}$$

By substituting 11.14 into the relation 11.12, we obtain the desired operator 11.15 and various additional terms[23].

$$
\begin{aligned}
&\frac{1}{2}(H^\dagger H)D_\mu H^\dagger D^\mu H = \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)D_\mu((D^\mu H^\dagger)H)\\
&= \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)D_\mu(D^\mu H^\dagger)H - \frac{1}{2}(H^\dagger H)(D_\mu H^\dagger)D^\mu H\\
&= \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)D_\mu(D^\mu H^\dagger)H + \frac{1}{2}(H^\dagger H)(H^j - \lambda\left(H^\dagger H\right)H^j\\
&- \bar{q}^j h_u^\dagger u\epsilon_{jk} + \bar{d}h_d q_k + \bar{e}h_e l_k)H\\
&= \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)D_\mu(D^\mu H^\dagger)H + \frac{1}{2}(H^\dagger H)H^j H - \lambda\frac{1}{2}(H^\dagger H)\left(H^\dagger H\right)H^j H\\
&- \frac{1}{2}(H^\dagger H)\psi^2 H\\
&= \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \frac{1}{2}(H^\dagger H)(D^\mu(D_\mu H^\dagger)H) + \frac{1}{2}(H^\dagger H)H^j H\\
&- \lambda\frac{1}{2}(H^\dagger H)\left(H^\dagger H\right)H^j H - \frac{1}{2}(H^\dagger H)\psi^2 H\\
&= \frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H - \psi^2 H^3 + H^6 + m^2 H^4 + E
\end{aligned}
\tag{11.15}
$$

It is worth noting the steps taken in this process[1]:

- In transitioning from the first line to the second line, the Leibniz rule was applied to the second term of the equation.
- From the second line to the third line, the EOM for the Higgs field was employed.
- The final line presents a potential approach for resolving the remaining operator. Using the Leibniz rule again on the second-to-last line makes it possible to obtain two equal operators and one residual operator. After canceling the two equal operators in the second last line, we obtain the desired operator $\frac{1}{2}(H^\dagger H)(D_\mu D^\mu)H^\dagger H$, along with other various operators and terms. E is operators which vanish due to the EOM[23]

[1] Some sloppy interpretation of parenthesis may have occurred during the calculation of this process

11.3 Appendix C

11.3.1 The basics of EWSB

The fields $W^{\pm}$ and Z acquire mass thru the symmetry breaking of the electroweak group. The mass terms come from the kinetic part of the Higgs Lagrangian:

$$\begin{aligned}|\,(D_\mu H)\,|^2 &= \frac{1}{2^3}\left|(2\partial_\mu + ig_1B_\mu + ig_2\mathbf{W}_\mu)\begin{pmatrix}0\\ v\end{pmatrix}\right|^2 = \frac{1}{2^3}|i|^2\left|\begin{pmatrix}vg_2^2(W_\mu^1 - iW_\mu^2)\\ v(g_1B_\mu - ig_2W_\mu^3)\end{pmatrix}\right|^2\\ &= \frac{1}{2^3}v^2\begin{pmatrix}g_2^2(W_\mu^1 - iW_\mu^2)^2\\ (g_1B_\mu - ig_2W_\mu^3)^2\end{pmatrix} = \frac{1}{2^3}v^2\begin{pmatrix}g_2^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2)\\ (g_1B_\mu - ig_2W_\mu^3)^2\end{pmatrix}. \end{aligned} \quad (11.16)$$

The field expansion around the VEV is set to zero, as we are only interested in studying how the fields acquire mass. We recognize the mass term for a scalar field with a complex conjugate $H^\dagger H$ namely the two charged weak force mediating bosons $W_\mu^{\pm} \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$, and the neutral gauge boson defined as $Z_\mu \equiv \frac{1}{\sqrt{g_1^2+g_2^2}}\left(g_2W_\mu^3 - g_1B_\mu\right)$. The mass of these two fields is given by the coefficient in front of the mass term, $\frac{1}{2}m_W = \frac{1}{2}(\frac{(gv)^2}{2^2})$, and the same for the heavy neutral field Z, $m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$. There is a fourth gauge field of-course, the photon field with mass $m_\gamma = 0$, and structure $\gamma_\mu \equiv \frac{1}{\sqrt{g_1^2+g_2^2}}\left(g_1W_\mu^3 + g_2B_\mu\right)$. The relation $\frac{g^i}{\sqrt{g_1+g_2}}, i = 1, 2$ is known as the Weinberg angle. The Weinberg angle often written in shorthand as just $\cos\theta_W$ for $i = 2$, or $\sin\theta_W$ for $i = 1$. We have broken the symmetry by implementing the unitary gauge, creating 3 Goldstone bosons. These Goldstone bosons comes from the three scalar degrees of freedom, which end up giving the weak gauge bosons their longitudinal component [17] [29] [40].

The Higgs mechanism also gives rise to fermion masses. Fermion mass terms without the Higgs mechanism mixes right and left handed fields(remembering that chiral fields can always be written as a sum of a right and left handed field)[39]

$$\bar{\psi}\psi = \psi^\dagger P_L\gamma_0P_R\psi + \psi^\dagger P_R\gamma_0P_L\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L. \quad (11.17)$$

The right and left handed fields change under different $SU(2)$ representations and have distinct $U(1)$ charges. This mismatch causes the mass term to not be invariant. In order to fix this, we have to use Yukawa interaction terms where an left-handed fermion $SU(2)$ doublet is contracted with the Higgs $SU(2)$ doublet:

$$\mathcal{L}_{Yuk} = -y_f\left(\bar{\psi}_LH\psi_R + \bar{\psi}_RH^\dagger\psi_L\right) \quad (11.18)$$

Where ψ_L is some left handed doublet. The electron mass for example, after symmetry breaking:

$$\begin{aligned}\mathcal{L}_m &= -\frac{y_f}{\sqrt{2}}\left[\begin{pmatrix}\bar{\nu}_e & \bar{e}\end{pmatrix}_L\begin{pmatrix}0\\ v\end{pmatrix}e_R + \bar{e}_R\begin{pmatrix}0 & v\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}\right]\\ &= -m_e\left(\bar{e}_Le_R + \bar{e}_Re_L\right) = -m_e\bar{e}e \end{aligned} \quad (11.19)$$

Where the electron mass is recognized as $m_e = \frac{y_fv}{\sqrt{2}}$. The electron mass term is now invariant in our effective Lagrangian, along with the mass terms of the other fermions.

11.4 Appendix D

11.4.1 Derivation of Fermion Mass Shift

All Yukawa-like operators induce a shift in the masses of the fermions as per the procedure outlined in [9]. By adding the dimension 8 Yukawa operators to the SM Yukawa operators, and dimension 6 Yukawa operators, the Yukawa interaction part of our EFT can now be represented as

$$y_f \overline{f_L} H f_R + \frac{H^\dagger H}{v^2} \bar{c}_f y_f \overline{f_L} H f_R + \left(\frac{H^\dagger H}{v^2}\right)^2 \bar{c}_f y_f \overline{f_L} H f_R + \text{ h.c. } . \tag{11.20}$$

Setting the Higgs field to its VEV, the resulting shift in fermion mass can be attributed to the Higgs field. The WCs, which are complex numbers, can be decomposed into their real and imaginary parts, as shown below. The WCs are split into real and imaginary parts in order to interpret better the physical meaning of the mass shift better. The real and imaginary part have different physical interpretations, and the contribution to the shift in fermion mass is not equally split between the real and imaginary part. The real part of the Wilson coefficent describe the magnitude of the coupling strength between the EFT operators and SM particles, while the imaginary part describes the CP-violating phase of the coupling.

$$\begin{aligned}
&\rightarrow \frac{1}{\sqrt{2}} y_f \overline{f_L} f_R (v+h) + \frac{1}{2\sqrt{2}} y_f \overline{f_L} \left[\mathrm{Re}(\bar{c}_f) + i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right] f_R \frac{(v+h)^3}{v^2} \\
&+ \frac{1}{4\sqrt{2}} y_f \overline{f_L} \left[\mathrm{Re}(\bar{c}_f) + i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right] f_R \frac{(v+h)^5}{v^4} + \text{ h.c.} \qquad (11.21) \\
&\rightarrow \frac{1}{\sqrt{2}} y_f v \overline{f_L} \left(1 + \frac{\left[\mathrm{Re}(\bar{c}_f) + i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right]}{2} + \frac{\left[\mathrm{Re}(\bar{c}_f) + i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right]}{4}\right) f_R \\
&+ \frac{h y_f}{\sqrt{2}} \bar{f} \left[1 + \frac{3}{2}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right] + \frac{5}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] f + \mathcal{O}\left(\frac{h^2}{v^2}\right) \qquad (11.22) \\
&\rightarrow \frac{1}{\sqrt{2}} y_f v \overline{f_L} \left[1 + \frac{3}{4}\left[\mathrm{Re}(\bar{c}_f) + i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right]\right] f_R \\
&+ \frac{h y_f}{\sqrt{2}} \bar{f} \left[1 + \frac{11}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] f + \mathcal{O}\left(\frac{h^2}{v^2}\right) \qquad (11.23)
\end{aligned}$$

The mass of a fermion is determined by the interaction-strength between the fermion and the Higgs field, as represented by the Yukawa coupling constant ${y_f}^2$ for that particular fermion. The mass term without including any effective operators looks like $m_f = \frac{y_f v}{\sqrt{2}}$, however when including effective operators, the mass shifts with and extra $\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)$ in comparison to [9]. This gives a total shift in the fermion mass

$$m_f^* = m_f \left[1 + \frac{3}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] \tag{11.24}$$

The equation above is then rewritten to account for the shift in fermion mass, by expressing it in terms of m_f^*. In order to make it easier to see the change between the

[2] One notational error was discovered here upon the final reading. y_f should be h.

two masses m_f and m_f^*, the above equation will be explicitly written out in terms of m_f, before shifting to m_f^*

$$\rightarrow m_f \overline{f_L} \left[1 + \frac{3}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] f_R$$
$$+ \frac{h}{v} m_f \bar{f} \left[1 + \frac{11}{4}\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] f + \mathcal{O}\left(\frac{h^2}{v^2}\right) \tag{11.25}$$

Rewriting the above expression to express it in terms of the effective mass m_f^*, which incorporates the shift, for use in the final effective Lagrangian(Here there is one unsolved problem. After the article rewrites the expression, it seem they are left with a factor $\frac{\left[\mathrm{Re}(\bar{c}_f)+i\gamma_5 \,\mathrm{Im}(\bar{c}_f)\right]^2}{2}$ to much. I the below calculation, I rewrite it how i see it correct, I.e not changing much about the $\frac{h}{v}$ term. Also state somewhere that γ_5 allows to keep track of the chiral properties of the interactions in the theory.):

$$\rightarrow m_f^* \overline{f_L} f_R + \frac{h}{v} m_f^* \bar{f} \left[1 + 2\left[\mathrm{Re}\left(\bar{c}_f\right) + i\gamma_5 \,\mathrm{Im}\left(\bar{c}_f\right)\right]\right] f + \mathcal{O}\left(\frac{h^2}{v^2}\right) \tag{11.26}$$

11.5 Appendix E

11.5.1 Higgs Canonical Renormalization

This appendix provides an exposition on the impact of the two dimension-8 operators, comprising solely of two covariant derivatives and the remaining Higgs fields, on the canonical renormalization of the Higgs.

$$
\begin{aligned}
&\frac{c_r}{2v^4}\left(H^\dagger H\right)^2\left(D_\mu H^\dagger D^\mu H\right)+\frac{c_q}{2v^4}\left(H^\dagger H\right)\left(H^\dagger\tau^I H\right)\left(D_\mu H^\dagger\tau^I D^\mu H\right)\\
&=\frac{c_r}{8v^4}(v^2+2vh+h^2)^2(\partial_\mu h\partial^\mu h)+\frac{c_q}{8v^4}(v^2+2vh+h^2)\left((v+h)\tau^3(v+h)\right)(\partial_\mu h\tau^3\partial^\mu h)\\
&=\frac{c_r}{8v^4}(v^4\dots)^2(\partial_\mu h\partial^\mu h)+\frac{c_q}{8v^4}(v^2\dots)\left(-(v+h)^2\right)(-\partial_\mu h\partial^\mu h)\\
&=\frac{c_r}{8v^4}v^4(\partial_\mu h\partial^\mu h)+\frac{c_q}{8v^4}v^4(\partial_\mu h\partial^\mu h)=\frac{1}{8}\left(c_r v^4(\partial_\mu h)^2+c_q v^4(\partial_\mu h)^2\right)\\
&=\frac{v^4}{8v^4}[c_r(\partial_\mu h)^2+c_q(\partial_\mu h)^2]=\frac{1}{8}[c_r+c_q](\partial_\mu h)^2. \qquad (11.27)
\end{aligned}
$$

The kinetic part of the Higgs now Lagrangian get in addition to c_h, the contributions c_r and c_q

$$
\begin{aligned}
\mathcal{L}_{kin}&=\frac{1}{2}(\partial_\mu h)^2+\frac{c_h}{2}(\partial_\mu h)^2+\frac{c_q}{8}(\partial_\mu h)^2+\frac{c_r}{8}(\partial_\mu h)^2\\
&=\frac{1}{2}\left(1+c_h+\frac{1}{4}(c_q+c_r)\right)(\partial_\mu h)^2 \qquad (11.28)
\end{aligned}
$$

Which is also re-scaled away by a similar field redefinition for the Higgs field, by using a Maclaurin series expansion, $\frac{1}{\sqrt{1+x}}=1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}\dots$ up to order $\mathcal{O}(\vec{c}^2)$

$$
h\to\frac{h}{\sqrt{1+c_h+\frac{1}{4}(c_q+c_r)}}\approx\frac{1}{2}\left(1+c_h+\frac{1}{4}(c_q+c_r)\right)h. \qquad (11.29)
$$

11.6 Appendix F

11.6.1 In-Depth Calculations of Operators in Unitary Gauge

This appendix presents a comprehensive summary of all calculations for each operator in unitary gauge. The appendix is organized in a chronological order in reference to their occurrence in the main text. Each operator in the principal Lagrangians, namely, $\mathcal{L}_6$, $\mathcal{L}_8$, $\mathcal{L}_{6,h}$, and $\mathcal{L}_{8,h}$, will be presented in this section in unitary gauge. $\mathcal{L}_{SM}$ only contains the kinetic part of the Higgs field, and the Higgs self-interaction terms responsible for EWSB. These terms in unitary gauge are explained and calculated in the chapter Higgs mechanism, and its corresponding appendix.

11.6.2 Useful Mathematical Formulas and Assumptions

Prior to delving into complex calculations, it is beneficial to first introduce a series of useful mathematical relations and assumptions. These relations will greatly facilitate our calculations in unitary gauge, simplifying the process and enabling a more efficient approach.

Two assumptions will be made. Firstly, non-linear Higgs operators may be disregarded, thereby excluding every operator containing h^2. This simplification significantly reduces the complexity of the calculations. Secondly, it will be assumed that operators with an even number of covariant derivatives may be transformed into the field strength tensor $Z_{\mu\nu}$, as established in eq 2.3.

We will also use these important mathematical relations:

$$m_Z = \frac{m_W}{c_W} = \frac{g_2 v}{2c_W} \tag{11.30}$$

$$m_W = \frac{g_2 v}{2} \tag{11.31}$$

$$\frac{g_1}{g_2} = \frac{\sin\theta_W}{\cos\theta_W} = \tan\theta_W \tag{11.32}$$

$$\cos\theta_{\mathrm{W}} = \frac{g_2}{\sqrt{g_2^2 + g_1^2}} \tag{11.33}$$

$$\left(H^\dagger H\right) = \frac{1}{2}\begin{bmatrix}0 & v+h\end{bmatrix}\begin{bmatrix}0 \\ v+h\end{bmatrix} = (v+h)^2 = \frac{1}{2}(v^2 + 2vh + h^2) \tag{11.34}$$

$$\left(H^\dagger H\right)^2 = \frac{1}{4}(v^4 + 2v^3h + 2v^2h^2 + 4v^2h^2 + 4vh^3 + h^4) \tag{11.35}$$

$$\begin{aligned} D^\mu &= \partial^\mu + ig_1B^\mu + ig_2W^at^a = \ldots \frac{i}{\sqrt{g_1^2+g_2^2}}(g_1B^\mu + g_2W^3) \\ &= i\sqrt{g_1^2+g_2^2}Z^\mu \end{aligned} \tag{11.36}$$

$$\frac{\sqrt{g_1^2+g_2^2}}{g_2} = \frac{1}{c_W} \tag{11.37}$$

Along with the proof for equation 2.3

$$
\begin{aligned}
[D_\mu, D_\nu]\psi(x) &= D_\mu(D_\nu\psi) - D_\nu(D_\mu\psi) \\
&= (\partial_\mu + igA_\mu)(\partial_\nu\psi + igA_\nu\psi) - ((\partial_\nu + igA_\nu)(\partial_\mu\psi + igA_\mu\psi)) \\
&= \partial_\mu(\partial_\nu\psi) + ig(\partial_\mu A_\nu\psi) + igA_\mu\partial_\nu\psi - g^2A_\mu A_\mu \\
&- (\partial_\nu(\partial_\mu\psi) + ig(\partial_\nu A_\mu\psi) + igA_\nu\partial_\mu\psi - g^2A_\nu A_\nu) \\
&= (\partial_\mu(\partial_\nu\psi) - \partial_\nu(\partial_\mu\psi)) + ig((\partial_\nu A_\mu\psi) - A_\nu\partial_\mu\psi) \\
&+ g^2(-A_\mu A_\nu + A_\nu A_\mu)
\end{aligned}
\tag{11.38}
$$

The first and last term in the last line cancels as partial derivatives commute, and fields in distinct positions in space also commute. Therefore are left with the definition of a field strength tensor 11.39 (in this case, just a general one, which will primarily be used for the Z-boson when calculating Lagrangians in unitary gauge), times an imaginary number and a coupling constant

$$
[D_\mu, D_\nu]\psi = ig(A_{\mu\nu})\psi, \tag{11.39}
$$

The last two relations Eq 11.36 and Eq 11.37 along with the definition of the mass of the W boson, 11.31 are used extensively throughout this appendix. One important note is that the relationship between the physical Z and γ fields and unphysical W and B fields, derived in appendix 11.3 , is used to obtain the desired interactions in the dictionary.

Each operator will be calculated twice, first without the Higgs field and then with it. As some of the operators are similar, there will be redundant calculations, which will be performed once and subsequently referenced where necessary. Since each operator can contribute to a different interaction, in cases where only a specific part is of interest, the remainder of the equation will be temporarily disregarded and represented by the ellipsis symbol .

Given that the Higgs self-coupling terms from both dimension 6 and dimension 8 have been excluded, the unmodified Higgs Vacuum Expectation Value of 246 GeV is utilized as the scale for the light fields below. Conversely, for the heavier bosons, the energy scale is defined as m_W.

There are only six possible interactions, excluding the terms that contribute to altering various Standard Model terms, such as the operators changing fermion mass, Z mass, and Higgs renormalization. These six interactions form the basis of our dictionary, namely $Z^{\mu\nu}Z_{\mu\nu}$, $Z^{\mu\nu}\gamma_{\mu\nu}$, $\gamma^{\mu\nu}\gamma_{\mu\nu}$, $W^\dagger_{\mu\nu}W^{\mu\nu}$, $Z^\mu\partial^\mu Z_{\mu\nu}$, $Z^\mu\partial^\nu\gamma_{\mu\nu}$, and $W^\mu D^\nu W^\dagger_{\mu\nu}$. Therefore, we have a clear understanding of the type of interaction that we want our higher-order operators to contribute to.

11.6.3 Dimension 6 operators

11.6.4 $\mathcal{L}_6$ operators without Higgs field

$$
\boldsymbol{c_{HW}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)(D^\nu W_{\mu\nu})^i}
$$

$$
\begin{aligned}
&\frac{i\bar{c}_{HW}g_2}{2m_W^2}\left(H^\dagger\sigma^i D^\mu H + (D^\mu H\sigma^i)^\dagger H\right)(D^\nu W_{\mu\nu})^i \\
&= \frac{i\bar{c}_{HW}g_2}{4m_W^2}\left(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \sigma^i D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \sigma^i)\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^i \\
&= \frac{i\bar{c}_{HW}g_2}{4m_W^2}\left[\left(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}0&1\\1&0\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}0&1\\1&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^1\right. \\
&+ i\left(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}0&-i\\i&0\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}0&-i\\i&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^2 \\
&\left.+ i\left(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}1&0\\0&-1\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}0\\v+h\end{bmatrix}^\dagger \begin{bmatrix}1&0\\0&-1\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)\right](D^\nu W_{\mu\nu})^3 \\
&= \frac{i\bar{c}_{HW}g_2}{4m_W^2}\left(\begin{bmatrix}v+h&0\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}v+h&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^1 \\
&+ i\left(\begin{bmatrix}-i(v+h)&0\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}-i(v+h)&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^2 \\
&+ i\left(\begin{bmatrix}0&-(v+h)\end{bmatrix} D^\mu \begin{bmatrix}0\\v+h\end{bmatrix} + D^{\mu\dagger}(\begin{bmatrix}0&-(v+h)\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^3 \\
&= \frac{i\bar{c}_{HW}g_2}{4m_W^2}\left[\left(\begin{bmatrix}v+h&0\end{bmatrix} \begin{bmatrix}0\\D^\mu(v+h)\end{bmatrix} + (\begin{bmatrix}D^{\mu\dagger}(v+h)&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^1\right. \\
&+ i\left(\begin{bmatrix}-i(v+h)&0\end{bmatrix} \begin{bmatrix}0\\D^\mu(v+h)\end{bmatrix} + (\begin{bmatrix}-iD^{\mu\dagger}(v+h)&0\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)(D^\nu W_{\mu\nu})^2 \\
&\left.+ i\left(\begin{bmatrix}0&-(v+h)\end{bmatrix} \begin{bmatrix}0\\D^\mu(v+h)\end{bmatrix} + (\begin{bmatrix}0&-D^{\mu\dagger}(v+h)\end{bmatrix})\begin{bmatrix}0\\v+h\end{bmatrix}\right)\right](D^\nu W_{\mu\nu})^3 \\
&= \frac{i\bar{c}_{HW}g_2}{4m_W^2}\left(-(v+h)D^\mu(v+h) - (v+h)D^{\mu\dagger}(v+h)\right)(D^\nu W_{\mu\nu})^3
\end{aligned}
$$

Collecting all terms containing no Higgs field

$$
= \frac{i\bar{c}_{HW}g_2}{4m_W^2}(v^2 D^\mu - v^2 D^{\mu\dagger})\,(D^\nu W_{\mu\nu})^3
$$

To calculate the contributions to the interactions $Z^\mu\partial^\mu Z_{\mu\nu}$ and $Z^\mu\partial^\nu\gamma_{\mu\nu}$, it is necessary to expand the first covariant derivative and substitute it with the Z-field using the field definition introduced in the appendix. We also use the equation $m_W = \frac{g_2 v}{2}$ in addition to our definition of the W-field, which is provided in the appendix pertaining to the fundamentals of EWSB, throughout this appendix.

$$
\begin{aligned}
&= \frac{i\bar{c}_{HW}}{g_2}(D^\mu)\left(D^\nu W_{\mu\nu}\right)^3 \\
&= \frac{i\bar{c}_{HW}}{g}(D^\mu)(D^\nu(s_W\gamma_{\mu\nu} + c_W Z_{\mu\nu})) \\
&= \frac{i\bar{c}_{HW}}{g}(\partial^\mu + ig_1 B^\mu + ig_2 W^a t^a)(s_W D^\nu\gamma_{\mu\nu} + c_W D^\nu Z_{\mu\nu}) \\
&= \frac{i\bar{c}_{HW}\sqrt{g_1^2+g_2}}{g_2^2}((\ldots \frac{i}{\sqrt{g_1^2+g_2^2}}(g_1 B^\mu + g_2 W^3) - D^{\mu\dagger})(s_W D^\nu\gamma_{\mu\nu} + c_W D^\nu Z_{\mu\nu})) \\
&= -\frac{\bar{c}_{HW}}{c_w}((\ldots Z^\mu)(s_W D^\nu\gamma_{\mu\nu} + c_W D^\nu Z_{\mu\nu})) = -\bar{c}_W(Z^\mu)(\tan\theta_w D^\nu\gamma_{\mu\nu} + D^\nu Z_{\mu\nu})) \\
&= \bar{c}_{HW}(\tan\theta_w Z^\mu D^\nu\gamma_{\mu\nu} + Z^\mu D^\nu Z_{\mu\nu}) \\
&= \bar{c}_{HW}(\tan\theta_w Z^\mu \partial^\nu\gamma_{\mu\nu} + Z^\mu \partial^\nu Z_{\mu\nu})
\end{aligned}
$$

From the second to last line, only the partial derivative term remains, as we are not interested in the three-gauge boson interaction.

The interaction $W^\mu D^\nu W^\dagger_{\mu\nu}$ arises from the contribution of the complex conjugated covariant derivative. Calculating this term involves certain subtleties. In particular, in accordance with the established set of relations in [9], the final interaction involves the complex conjugation of the W-field strength tensor. At present, it is merely assumed that the complex conjugation can be flipped over to the field strength tensor.

$$
\begin{aligned}
&-\frac{i\bar{c}_{HW}g_2}{4m_W^2}\left(v^2 D^{\mu\dagger}\right)\left(D^\nu W_{\mu\nu}\right)^3 = -i\bar{c}_{HW}\left(\ldots - ig_2 W^\mu\right)^\dagger D^\nu W_{\mu\nu} \\
&= \bar{c}_{HW} W^\mu D^\nu W^\dagger_{\mu\nu}
\end{aligned}
$$

In the above equation, we used $m_W = \frac{gv}{2}$.

It is also a mystery as to why we allow for three gauge bosons here(as the covariant derivative also contains a W).

$$\boldsymbol{c_{HG}\left(H^\dagger H\right) G^a_{\mu\nu}G^{a\mu\nu}}$$

$$
\begin{aligned}
\frac{c_{HG}g_3^2}{2m_W^2}\left(H^\dagger H\right) G^a_{\mu\nu}G^{a\mu\nu} &= \frac{c_{HG}g_3^2}{2m_W^2}(v^2+2vh+h^2)G^a_{\mu\nu}G^{a\mu\nu} \equiv \frac{c_{HG}g_3^2}{2m_W^2}(v^2+2vh)G^a_{\mu\nu}G^{a\mu\nu} \\
&= \frac{c_{HG}g_3^2v^2}{\frac{2v^2g^2}{2^2}}G^a_{\mu\nu}G^{a\mu\nu} = 2c_{HG}\frac{g_3^2}{g^2}G^a_{\mu\nu}G^{a\mu\nu}
\end{aligned}
$$

$$\boldsymbol{\bar{c}_{HB}\left(H^\dagger H\right) B_{\mu\nu}B^{\mu\nu}}$$

$$\begin{aligned}\frac{\bar{c}_{HB}g_1^2}{m_W^2}\left(H^\dagger H\right)B_{\mu\nu}B^{\mu\nu} &= \frac{\bar{c}_{HB}g_1^2}{2m_W^2}(v^2+2vh+h^2)(c_W\gamma_{\mu\nu}-s_WZ_{\mu\nu})^2\\ &= \frac{\bar{c}_{HB}g_1^2}{2m_W^2}(v^2+2vh+h^2)(c_W^2\gamma_{\mu\nu}^2-2\gamma_{\mu\nu}s_WZ^{\mu\nu}+s_W^2Z_{\mu\nu}^2)\\ &= \frac{\bar{c}_{HB}g_1^2v^2}{2m_W^2}(c_W^2\gamma_{\mu\nu}^2-2\gamma_{\mu\nu}s_WZ^{\mu\nu}+s_W^2Z_{\mu\nu}^2)\\ &= \frac{2\bar{c}_{HB}g_1^2}{g_2^2}(c_W^2\gamma_{\mu\nu}^2-2\gamma_{\mu\nu}s_WZ^{\mu\nu}+s_W^2Z_{\mu\nu}^2)\\ &= 2\bar{c}_{HB}\tan^2\theta_W(c_W^2\gamma_{\mu\nu}^2-2\gamma_{\mu\nu}s_WZ^{\mu\nu}+s_W^2Z_{\mu\nu}^2)\end{aligned}$$

Here we used the first instance of the relation $\frac{g_1}{g_2}=\tan\theta_W$.

$$\boldsymbol{\bar{c}_{Hb}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)(\partial^\nu B_{\mu\nu})}$$

$$\begin{aligned}&\frac{i\bar{c}_{Hb}g_1}{2m_W^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)(\partial^\nu B_{\mu\nu})\\ &= \frac{i\bar{c}_{Hb}g_1}{4m_W^2}\left(\begin{bmatrix}0 & v+h\end{bmatrix}D^\mu\begin{bmatrix}0\\ v+h\end{bmatrix}+(D^{\mu\dagger}\begin{bmatrix}0 & v+h\end{bmatrix})\begin{bmatrix}0\\ v+h\end{bmatrix}\right)(\partial^\nu B_{\mu\nu})\\ &= \frac{i\bar{c}_{Hb}g_1}{4m_W^2}\left(v^2D^\mu+v^2D^{\mu\dagger}+vhD^\mu+vD^{\mu\dagger}h\right)(\partial^\nu B_{\mu\nu})\end{aligned}$$

Considering only the covariant derivative which is not complex conjugated:

$$\begin{aligned}&= \frac{i\bar{c}_{Hb}g_1}{4m_W^2}\left(v^2D^\mu\right)(\partial^\nu B_{\mu\nu}) = \frac{i\bar{c}_Bg_1}{4m_W^2}v^2D^\mu(\partial^\nu(c_w\gamma_{\mu\nu}-s_WZ_{\mu\nu}))\\ &= \frac{i\bar{c}_{Hb}g_1}{g_2^2}\left(\partial^\mu-ig_1YB^\mu-ig_2\mathbf{W}^\mu\right)(\partial^\nu)\left(c_W\gamma_{\mu\nu}-s_WZ_{\mu\nu}\right)\end{aligned}$$

Remembering that we have defined the Z^μ field as $Z_\mu\equiv\frac{1}{\sqrt{g^2+g^2}}\left(gW_\mu^3-g'B_\mu\right)$

$$\begin{aligned}&= \frac{i\bar{c}_{Hb}g_1\sqrt{g_2+g_2^2}}{g_2^2}(\ldots\frac{i}{\sqrt{g_2+g_2^2}}\left(g_2W_\mu^3-g_1B_\mu\right))(\partial^\nu)\left(c_W\gamma_{\mu\nu}-s_WZ_{\mu\nu}\right)\\ &= -\frac{\bar{c}_{Hb}\tan\theta_w}{c_w}(\ldots Z^\mu)(\partial^\nu)\left(c_w\gamma_{\mu\nu}-s_wZ_{\mu\nu}\right)\\ &= \bar{c}_{Hb}Z^\mu\partial^\nu\left(\tan\theta_w\frac{s_w}{c_w}Z_{\mu\nu}-\tan\theta_w\frac{c_w}{c_w}\gamma_{\mu\nu}\right)\\ &= \bar{c}_{Hb}Z^\mu\partial^\nu\left(\tan\theta_w^2Z_{\mu\nu}-\tan\theta_w\gamma_{\mu\nu}\right)\end{aligned}$$

Where we have used the relations $\frac{\sqrt{g_1^2+g_2^2}}{g_2}=\frac{1}{c_w}$, and $\frac{g_1}{g_2}=\tan\theta_w$.

$$\boldsymbol{c_w \left(D^\mu H\right)^\dagger \sigma^i \left(D^\nu H\right) W^i_{\mu\nu}}$$

$$\begin{aligned}
\frac{ic_w g_2}{m_W^2}\left(D^\mu H\right)^\dagger \sigma^i \left(D^\nu H\right) W^i_{\mu\nu} &= \frac{ic_w g_2}{2m_W^2}\begin{bmatrix}0 & D^{\mu\dagger}(v+h)\end{bmatrix}\sigma^i\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}W^i_{\mu\nu}\\
&= \frac{ic_w g_2}{2m_W^2}\begin{bmatrix}0 & D^{\mu\dagger}(v+h)\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&+ \begin{bmatrix}0 & D^{\mu\dagger}(v+h)\end{bmatrix}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&+ \begin{bmatrix}0 & D^{\mu\dagger}(v+h)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&= \frac{ic_w g_2}{2m_W^2}\begin{bmatrix}D^{\mu\dagger}(v+h) & 0\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&+ \begin{bmatrix}iD^{\mu\dagger}(v+h)\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&+ \begin{bmatrix}0 & -D^{\mu\dagger}(v+h)\end{bmatrix}\begin{bmatrix}0\\ D^\mu(v+h)\end{bmatrix}\\
&= \frac{ic_w g_2}{2m_W^2}(v^2 D^{\mu\dagger}D^\nu + vD^{\mu\dagger}D^\nu h + vD^{\mu\dagger}hD^\nu)\\
&= \frac{ic_w g_2}{2m_W^2}(v^2 D^{\mu\dagger}D^\nu)W^3_{\mu\nu}
\end{aligned}$$

Three gauge bosons interactions are of no interest. However, it is important to note that the WC c_w does contribute when we do not ignore the Higgs field.

$$\boldsymbol{\bar{c}_b \left(D^\mu H\right)^\dagger \left(D^\nu H\right) B_{\mu\nu}}$$

$$\begin{aligned}
\frac{i\bar{c}_b g_1}{m_W^2}\left(D^\mu H\right)^\dagger \left(D^\nu H\right) B_{\mu\nu} &= \frac{i\bar{c}_b g_1}{m_W^2}(v^2 D^{\mu\dagger}D^\nu + vD^{\mu\dagger}D^\nu h + vD^{\mu\dagger}hD^\nu)B_{\mu\nu}\\
&= \frac{i\bar{c}_b g_1}{m_W^2}(v^2 D^{\mu\dagger}D^\nu)B_{\mu\nu}
\end{aligned}$$

As we only end up with the interaction term of three bosons again, we deem it appropriate to disregard them. However, this Wilson coefficient, along with C_w, will become significant when we also take into account the Higgs-field later on.

11.6.5 $\mathcal{L}_6$ operators with one Higgs field

In this section, we shall keep the expansion of the Higgs field, denoted by the symbol h, while neglecting any terms beyond $\mathcal{O}(h^2)$. It is worth noting that the VEV is a scalar quantity and, since no partial derivatives act on h, we can extract a common factor of $\frac{h}{v}$ in front of the operators. Although there are terms where the partial derivative acts on h these are not relevant to our theory and can be disregarded.

$$\boldsymbol{c_{HW}\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)(D^\nu W_{\mu\nu})^i}$$

$$\begin{aligned}
&\frac{i\bar{c}_{HW}g_2}{4m_W^2}\left(\left(-(v+h)D^\mu(v+h)-(v+h)D^{\mu\dagger}(v+h)\right)(D^\nu W_{\mu\nu})^3\right.\\
&=-\frac{i\bar{c}_{HW}g_2}{4m_W^2}(hvD^\mu+vD^{\mu\dagger}h)(D^\nu W_{\mu\nu})^3\\
&=-\frac{i\bar{c}_{HW}}{v^2g_2}(hvD^\mu+vD^{\mu\dagger}h)(D^\nu W_{\mu\nu})^3=-\frac{i\bar{c}_{HW}}{g_2}\frac{h}{v}(D^\mu(D^\nu W_{\mu\nu})^3)+\dots
\end{aligned}$$

The ellipsis symbol denotes the terms that involve the complex conjugated covariant derivative, which will be employed subsequently. Similar to the reasoning presented in the preceding section, it can be demonstrated that the covariant derivative simplifies to a partial derivative:

$$\begin{aligned}
&=-i\bar{c}_{HW}\frac{\sqrt{g_1^2+g_2^2}}{g_2}\frac{h}{v}Z^\mu D^\nu(s_w\gamma_{\mu\nu}+c_w z_{\mu\nu})\\
&=-i\bar{c}_{HW}\frac{h}{v}Z^\mu D^\nu(\tan\theta_w\gamma_{\mu\nu}+z_{\mu\nu})\\
&=-i\bar{c}_{HW}\frac{h}{v}(\tan\theta_w Z^\mu\partial^\nu\gamma_{\mu\nu}+Z^\mu\partial Z_{\mu\nu})
\end{aligned}$$

We also require $WDW^\dagger$ terms. To acquire a complex conjugated W-field, we must examine the term containing the complex conjugated covariant derivative. The methodology for obtaining $WDW^\dagger$ terms is similar to that outlined in the preceding section for the same operator, but now with a $\frac{h}{v}$ factor in front.

$$\begin{aligned}
\frac{i\bar{c}_{HW}g}{4m_W^2}(vD^{\mu\dagger}h)(D^\nu W_{\mu\nu})^3&=i\frac{\bar{c}_{HW}}{g_2}\frac{h}{v}(\dots g_2W^\mu)^\dagger D^\nu W_{\mu\nu}\\
&=-\bar{c}_{HW}\frac{h}{v}W^\mu D^\nu W_{\mu\nu}^\dagger
\end{aligned}$$

$$\boldsymbol{c_{HG}\left(H^\dagger H\right)G^a_{\mu\nu}G^{a\mu\nu}}$$

$$\frac{c_{HG}g_3^2}{2m_W^2}(2vh)\,G^a_{\mu\nu}G^{a\mu\nu}=4c_{HG}\frac{g_3^2}{g_2^2}\left(\frac{h}{v}\right)G^a_{\mu\nu}G^{a\mu\nu}$$

$$\boldsymbol{\bar{c}_{HB}\left(H^\dagger H\right)B_{\mu\nu}B^{\mu\nu}}$$

$$\frac{\bar{c}_{HB} g_1^2}{m_W^2} \left(H^\dagger H\right) B_{\mu\nu} B^{\mu\nu} = \frac{\bar{c}_{HB} g_1^2}{2m_W^2} (2vh) \left(c_w^2 \gamma_{\mu\nu}^2 - 2c_w s_w \gamma_{\mu\nu} Z^{\mu\nu} + s_w^2 Z_{\mu\nu}^2\right) \quad (11.40)$$

$$= 4\bar{c}_{HB} \frac{g_1^2}{g_2^2} \left(\frac{h}{v}\right) (c_W^2 \gamma_{\mu\nu}^2 - 2c_w s_W \gamma_{\mu\nu} Z^{\mu\nu} + s_W^2 Z_{\mu\nu}^2) \quad (11.41)$$

$$= 4\bar{c}_{HB} \left(\frac{h}{v}\right) (s_w^2 \gamma_{\mu\nu}^2 - 2s_w^2 \frac{s_w}{c_w} \gamma_{\mu\nu} Z^{\mu\nu} + \frac{s_w^4}{c_w^2} Z_{\mu\nu}^2) \quad (11.42)$$

$\bar{c}_{Hb} \left(H^\dagger \overleftrightarrow{D^\mu} H\right) (\partial^\nu B_{\mu\nu})$

$$\frac{i\bar{c}_{Hb} g_1}{4m_W^2} \left(vhD^\mu + vD^{\mu\dagger} h\right) (\partial^\nu B_{\mu\nu})$$
$$= \frac{4i\bar{c}_{Hb} g_1}{4m_W^2} (vhD^\mu + vD^{\mu\dagger} h) \partial^\nu (c_w \gamma_{\mu\nu} - s_w Z_{\mu\nu})$$

Interactions that involve contracting the photon-field and a partial derivative with either $Z_{\mu\nu}$ or $\gamma_{\mu\nu}$ are prohibited as they violate $U(1)$-symmetry[9]. However, these are not the sole interactions that can be obtained from this operator. It is still possible to obtain interactions that involve $Z^\mu \partial^\nu$ contracted with either a $\gamma_{\mu\nu}$ or $Z_{\mu\nu}$. In this context, we will focus solely on the non-complex conjugated covariant derivative.

$$\frac{i\bar{c}_{Hb} g_1}{4m_W^2} (vhD^\mu) \partial^\nu (c_w \gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= -\frac{\bar{c}_{Hb} g_1 \sqrt{g_1^2 + g_2^{22}}}{g_2^2} \frac{h}{v} (\ldots Z^\mu) \partial^\nu (c_w \gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= -\bar{c}_{Hb} \frac{h}{v} \left(\tan\theta_w Z^\mu \partial^\nu \gamma_{\mu\nu} - \tan\theta_w^2 Z^\mu \partial^\nu Z_{\mu\nu}\right)$$

$c_w \left(D^\mu H\right)^\dagger \sigma^i \left(D^\nu H\right) W_{\mu\nu}^i$

The complexity of this operator stems from its reliance on both complex conjugated covariant derivatives and covariant derivatives squared. Specifically, this operator contributes to six distinct interactions, as demonstrated in the previous section where the calculation of $DHDH$ was presented:

$$\frac{i\bar{c}_w g_2}{2m_W^2} \left(vD^{\mu\dagger} D^\nu h + vD^{\mu\dagger} h D^\nu\right) W_{\mu\nu}^3$$
$$= \frac{2i\bar{c}_w}{g_2} \frac{h}{v} \left(D^{\mu\dagger} D^\nu + D^{\mu\dagger} D^\nu\right) W_{\mu\nu}^3$$
$$= -\frac{2\bar{c}_w \sqrt{g_1^2 + g_2^2}}{g_2} \frac{h}{v} (Z^{\mu\nu}) W_{\mu\nu}^3 = \frac{i2\bar{c}_w}{} \frac{h}{v} (Z^{\mu\nu}) (\tan\theta_w \gamma_{\mu\nu} + Z_{\mu\nu})$$
$$= -\frac{2\bar{c}_w}{} \frac{h}{v} (\tan\theta_w Z^{\mu\nu} \gamma_{\mu\nu} + Z^{\mu\nu} Z_{\mu\nu})$$

Of the two $D^{\mu\dagger}D^{\nu}$ terms, one makes a notable contribution to both the $Z\gamma$ and ZZ interactions.

$$\frac{2i\bar{c}_w}{g_2}\frac{h}{v}\left(D^{\mu\dagger}D^{\nu}\right)W^3_{\mu\nu}$$

The operator $\left(H^\dagger\sigma^i\overleftrightarrow{D^\mu}H\right)(D^\nu W_{\mu\nu})^i$ shares the same computational characteristics as this, with the exception of the complex conjugation present on one of its covariant derivatives. At first glance, this distinction may suggest that the operator would contribute to a distinct interaction; however, since we have neglected the contribution of three gauge boson interactions, the covariant derivative reduces to a partial derivative anyways.

$$\begin{aligned}
&= -\frac{2\bar{c}_W\sqrt{g_1^2+g_2^2}}{g_2}\frac{h}{v}\left((\ldots Z^\mu)\left(s_W D^{\nu\dagger}\gamma_{\mu\nu}+c_W D^{\nu\dagger}Z_{\mu\nu}\right)\right)\\
&= -2\bar{c}_w\frac{h}{v}\frac{\sqrt{g_1^2+g_2^2}}{g_2}(Z^\mu)\left(D^{\nu\dagger}\gamma_{\mu\nu}+c_w D^{\nu\dagger}Z_{\mu\nu}\right)\\
&= -2\bar{c}_w\frac{h}{v}\left(\tan\theta_w Z^\mu D^{\nu\dagger}\gamma_{\mu\nu}+Z^\mu D^{\nu\dagger}Z_{\mu\nu}\right)\\
&= -2\bar{c}_w\frac{h}{v}\left(\tan\theta_w Z^\mu\partial^\nu\gamma_{\mu\nu}+Z^\mu\partial^\nu Z_{\mu\nu}\right)
\end{aligned}$$

While the calculations presented may not make it immediately evident, the complex conjugated covariant derivative does not act upon the unconjugated derivative, and interchange of their positions does not alter the expression. To maintain consistency with the notation presented in [9], the indices were also swapped when the covariant derivatives were interchanged. This swapping of indices is acceptable, as they are summed over.

$$\begin{aligned}
&\frac{2i\bar{c}_w}{g_2}\frac{h}{v}\left(D^{\mu\dagger}D^{\nu}\right)W^3_{\mu\nu} = \frac{2i\bar{c}_w}{g_2}\frac{h}{v}(\ldots+ig_2W^\mu)^\dagger D^\nu W_{\mu\nu}\\
&= 2\bar{c}_w\frac{h}{v}W^\mu D^\nu W^\dagger_{\mu\nu}
\end{aligned}$$

$\bar{c}_b\,(D^\mu H)^\dagger\,(D^\nu H)\,B_{\mu\nu}$

In the preceding section, $DHDH$ was computed. This operator bears close resemblance to the preceding one, with the difference being the B-field instead of the W-field. The interactions contained within this operator are identical to the previous one, save for WDW and WW as a result of the substitution of the fields. The output of the preceding operator will be utilized.

$$\frac{2i\bar{c}_b g_1}{g_2^2}\frac{h}{v}\left(D^{\mu\dagger}D^{\nu}+D^{\mu\dagger}D^{\nu}\right)B_{\mu\nu}$$
$$=-\frac{2\bar{c}_b g_1\sqrt{g_1^2+g_2^2}}{g_2^2}\frac{h}{v}(\ldots Z^{\mu\nu})(c_w\gamma_{\mu\nu}-s_w Z_{\mu\nu})$$
$$=-2\bar{c}_b\frac{h}{v}(\frac{s_w}{c_w}Z^{\mu\nu}\gamma_{\mu\nu}-\frac{s^2}{c_w^2}Z^{\mu\nu}Z_{\mu\nu})$$

The remaining contributions to κ are

$$=-\frac{2\bar{c}_b g_1\sqrt{g_1^2+g_2^2}}{g_2^2}\frac{h}{v}\left((\ldots Z^{\mu})\left(c_w D^{\nu\dagger}\gamma_{\mu\nu}-s_w D^{\nu\dagger}Z_{\mu\nu}\right)\right)$$
$$=-\frac{2\bar{c}_b g_1\sqrt{g_1^2+g_2^2}}{g_2^2}\frac{h}{v}\left(c_w Z^{\mu}D^{\nu\dagger}\gamma_{\mu\nu}-s_w Z^{\mu}D^{\nu\dagger}Z_{\mu\nu}\right)$$
$$=-2\bar{c}_b\frac{h}{v}\left(\frac{s_w}{c_w}Z^{\mu}D^{\nu\dagger}\gamma_{\mu\nu}-\frac{s_w^2}{c_w^2}Z^{\mu}D^{\nu\dagger}Z_{\mu\nu}\right)$$
$$=-2\bar{c}_b\frac{h}{v}\left(\frac{s_w}{c_w}Z^{\mu}\partial^{\nu}\gamma_{\mu\nu}-\frac{s_w^2}{c_w^2}Z^{\mu}\partial^{\nu}Z_{\mu\nu}\right)$$

11.6.6 Correction to the Z mass

Correction to the Z Boson Mass Arising from the Interaction Term $\bar{c}_T\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(H^\dagger\overleftrightarrow{D_\mu}H\right)$

$$\frac{\bar{c}_T}{2v^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(H^\dagger\overleftrightarrow{D_\mu}H\right)=\frac{\bar{c}_T}{4v^2}\left(\begin{bmatrix}0 & v+h\end{bmatrix}D^\mu\begin{bmatrix}0\\ v+h\end{bmatrix}+(D_\mu\begin{bmatrix}0 & v+h\end{bmatrix})\begin{bmatrix}0\\ v+h\end{bmatrix}\right)^2$$
$$=\frac{\bar{c}_T}{4v^2}\left(2v^2D^\mu+vhD^\mu+vD_\mu h\right)^2$$
$$=\frac{\bar{c}_T}{4v^2}(2v^2D^\mu)^2=2v^2\bar{c}_T(D^\mu)^2$$
$$=v^2\bar{c}_T(\partial^\mu-ig_1YB^\mu-ig_2\mathbf{W}^\mu)(\partial_\mu-ig_1YB_\mu-ig_2\mathbf{W}_\mu)$$
$$=-v^2(g_1^2+g_2^2)\bar{c}_T(\ldots Z^\mu)(\ldots Z_\mu)$$

To achieve a correction to the Z-mass, it is necessary for the coefficient in front to be equivalent to $m_Z=\frac{m_W}{c_w}=\frac{g_2 v}{2c_w}$. This can be obtained by employing the definition $(g_1^2+g_2^2)=\frac{g_2}{c_w}$

$$=-\frac{v^2g_2^2}{c_w}\bar{c}_T(\ldots Z^\mu)(\ldots Z_\mu)=-m_Z^2c_TZ^\mu Z_\mu$$

11.6.7 Dimension 8 operators

Similar to the calculation of dimension 6 operators in unitary gauge, the dimension 8 operators will also be divided into two sections. In the first section, the contribution of the Higgs field is neglected, while in the subsequent section, it is taken into account.

11.6.8 $\mathcal{L}_8$ operators without a Higgs field

$$\boldsymbol{c_{HHG}\left(H^{\dagger}H\right)^{2}G^{a}_{\mu\nu}G^{a\mu\nu}}$$

$$\frac{c_{HHG}g_3^2}{2m_W^4}\left(H^{\dagger}H\right)^{2}G^{a}_{\mu\nu}G^{a\mu\nu}=\frac{c_{HG}g_3^2v^4}{\frac{4v^4g^4}{2^4}}G^{a}_{\mu\nu}G^{a\mu\nu}=4c_{HHG}\frac{g_3^2}{g_2^4}G^{a}_{\mu\nu}G^{a\mu\nu}$$

$$\boldsymbol{\bar{c}_{HHB}\left(H^{\dagger}H\right)^{2}B_{\mu\nu}B^{\mu\nu}}$$

$$\begin{aligned}\frac{\bar{c}_{HHB}g_1^2}{4m_W^4}\left(H^{\dagger}H\right)^{2}B_{\mu\nu}B^{\mu\nu}&=4\bar{c}_{HHB}\frac{\tan^2\theta_W}{g_2^2}\left(c_W^2\gamma_{\mu\nu}^2-2c_ws_w\gamma_{\mu\nu}Z^{\mu\nu}+s_W^2Z_{\mu\nu}^2\right)\\&=\bar{c}_{HHB}\left(4\frac{s_w^2}{g_2^2}\gamma_{\mu\nu}^2-8\frac{\tan^2\theta_Wc_Ws_w}{g_2^2}c_ws_w\gamma_{\mu\nu}Z^{\mu\nu}+4\frac{\tan^2\theta_Ws_W^2}{g_2^2}Z_{\mu\nu}^2\right)\end{aligned}$$

$$\boldsymbol{\bar{c}_{WW}\left(H^{\dagger}H\right)^{2}W^{I}_{\mu\nu}W^{I\mu\nu}}$$

$$\begin{aligned}\frac{\bar{c}_{WW}g_2^2}{4m_W^4}\left(H^{\dagger}H\right)^{2}W^{I}_{\mu\nu}W^{I\mu\nu}&=\frac{\bar{c}_{HW}g_2^2}{16m_W^4}\left(v^4+2v^3h\ldots\right)W^{I}_{\mu\nu}W^{I\mu\nu}\\&=\frac{\bar{c}_{WW}g_2^2}{16m_W^4}\left(v^4+2v^3h\right)(W^1_{\mu\nu}W^{1\mu\nu}+W^2_{\mu\nu}W^{2\mu\nu}+W^3_{\mu\nu}W^{3\mu\nu})\\&=\frac{\bar{c}_{WW}g_2^2v^4}{16m_W^4}(W^1_{\mu\nu}W^{1\mu\nu}+W^2_{\mu\nu}W^{2\mu\nu}+W^3_{\mu\nu}W^{3\mu\nu})\\&=\frac{\bar{c}_{WW}}{g_2^2}(W_{\mu\nu}W^{\mu\nu}+(s_w\gamma_{\mu\nu}+c_wZ_{\mu\nu})^2\\&=\frac{\bar{c}_{WW}}{g_2^2}(W_{\mu\nu}W^{\mu\nu}+s_w^2\gamma_{\mu\nu}^2+s_wc_w\gamma_{\mu\nu}Z^{\mu\nu}+Z_{\mu\nu}^2)\end{aligned}$$

$$\boldsymbol{\bar{c}_{DHW}\left(H^{\dagger}H\right)\left(D^{\mu}H^{\dagger}\tau^{I}D^{\nu}H\right)W^{I}_{\mu\nu}}$$

$$\frac{i\bar{c}_{DHW}g_2}{2m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger \tau^I D^\nu H\right)W^I_{\mu\nu} = \frac{i\bar{c}_{DHW}g_2}{8m_W^4}(v^2+2vh\dots)\left(-D^\mu(v+h)D^\nu(v+h)\right)W^3_{\mu\nu}$$
$$= -\frac{i\bar{c}_{DHW}g_2v^4}{8m_W^4}D^\mu D^\nu W^3_{\mu\nu} - \frac{i\bar{c}_g g_2 v^3 h}{2m_W^4}D^\mu D^\nu W^3_{\mu\nu}\dots$$
$$= -\frac{i\bar{c}_{DHW}g_2v^4\sqrt{g_1^2+g_2^2}}{8m_W^4}Z^{\mu\nu}(s_w\gamma_{\mu\nu}+c_wZ_{\mu\nu})$$
$$= 2\frac{\bar{c}_{DHW}}{g_2^2}(\tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu}+Z^{\mu\nu}Z_{\mu\nu})$$

$$\boldsymbol{\bar{c}_{DHB}\left(H^\dagger H\right)\left(D^\mu H^\dagger D^\nu H\right)B_{\mu\nu}}$$

$$\frac{i\bar{c}_{DHB}g_1}{2m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger D^\nu H\right)B_{\mu\nu} = \frac{i\bar{c}_g g_1}{2m_W^4}(v^2+2vh\dots)(D^\mu D^\nu(v^2+2vh\dots))B_{\mu\nu}$$
$$= \frac{i\bar{c}_{DHB}g_1v^4}{2m_W^4}D^\mu D^\nu B_{\mu\nu} = 8\frac{i\bar{c}_g g_1^2}{g_2^2}D^\mu D^\nu(c_w\gamma_{\mu\nu}-s_wZ_{\mu\nu})$$
$$= 8\frac{i\bar{c}_{DHB}g_1}{g_2^2}D^\mu D^\nu(c_w\gamma_{\mu\nu}-s_wZ_{\mu\nu})$$

Once again, we obtain three gauge bosons or propagators, which are not relevant to our current analysis. However, the operator becomes significant when the Higgs field is incorporated into the system.

$$\boldsymbol{\bar{c}_{HWW}\left(H^\dagger \tau^I H\right)\left(H^\dagger \tau^J H\right)W^I_{\mu\nu}W^{J\mu\nu}}$$

This operator contributes to the interaction WW by the same logic as previously, and $\gamma\gamma, \gamma Z$, ZZ .

$$\frac{\bar{c}_{HWW}g_2^2}{m_W^4}\left(H^\dagger\tau^I H\right)\left(H^\dagger\tau^J H\right)W^I_{\mu\nu}W^{J\mu\nu} = \frac{\bar{c}_{HWW}g_2^2}{m_W^4}\left(-(v+h)\right)^4 W^3_{\mu\nu}W^{3\mu\nu}$$

The above expression involves expanding the sums, yet only the term where $I=J=3$ can survive. Otherwise, if either I or J does not correspond to the index that yields the third Pauli matrix, then the product of the Higgs doublets and Pauli matrices would evaluate to zero.

$$\left(H^\dagger\tau^{I=1,2}H\right)\left(H^\dagger\tau^{J=1,2}H\right)=0, \begin{bmatrix}0 & v+h\end{bmatrix}\begin{bmatrix}0 & 1/(-i)\\ 1(i) & 0\end{bmatrix}\begin{bmatrix}0\\ v+h\end{bmatrix} = \begin{bmatrix}1/(-i)(v+h) & 0\end{bmatrix}\begin{bmatrix}0\\ v+h\end{bmatrix}$$
$$=0.$$

The W^3W^3 term enables us to derive the interactions between gamma particles and Z particles.

$$= 16\frac{\bar{c}_{HWW}}{g_2^2 v^4}\left(-(v^4 + 4v^3h\dots)\right)^4((s_w\gamma_{\mu\nu} + c_w Z_{\mu\nu})^2)$$
$$= 16\frac{\bar{c}_{HWW}}{g_2^2}((s_w\gamma_{\mu\nu} + c_w Z_{\mu\nu})^2) = 16\frac{\bar{c}_{HWW}}{g_2^2}(2\gamma_{\mu\nu}Z^{\mu\nu}c_w s_w + c_w^2 Z_{\mu\nu}Z^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu})$$

$\boldsymbol{c_{HWB}\left(H^\dagger H\right)\left(H^\dagger \tau^I H\right)W^I_{\mu\nu}B^{\mu\nu}}$

$$\frac{c_{HWB}g_1 g_2}{m_W^4}\left(H^\dagger H\right)\left(H^\dagger\tau^I H\right)W^I_{\mu\nu}B^{\mu\nu} = -\frac{c_{HWB}g_1 g_2}{m_W^4}(v^2 + 2vh + h^2)^2(s_w\gamma_{\mu\nu} + c_w Z_{\mu\nu})(c_w\gamma^{\mu\nu} - s_w Z^{\mu\nu}$$
$$= -\frac{c_{HWB}g_1 g_2}{m_W^4}(v^2 + 2vh + h^2)^2(s_w c_w\gamma_{\mu\nu}\gamma^{\mu\nu} - s_w^2\gamma_{\mu\nu}Z^{\mu\nu} + c_w^2 Z_{\mu\nu}\gamma^{\mu\nu} - c_w s_w Z^{\mu\nu 2})$$
$$= -\frac{4c_{HWB}}{g_2^2}\frac{g_1}{g_2}(s_w c_w\gamma_{\mu\nu}\gamma^{\mu\nu} + (c_w^2 - s_w^2)Z_{\mu\nu}\gamma^{\mu\nu} - c_w s_w Z_{\mu\nu}Z^{\mu\nu})$$
$$= -\frac{4c_{HWB}}{g_2^2}(s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu} + \tan\theta_w(c_w^2 - s_w^2)Z_{\mu\nu}\gamma^{\mu\nu} - s_w^2 Z_{\mu\nu}Z^{\mu\nu})$$

$\boldsymbol{\epsilon^{IJK}\left(H^\dagger\tau^I H\right)\left(D^\mu H^\dagger\tau^J D^\nu H\right)W^K_{\mu\nu}}$

$$\epsilon^{IJK}\left(H^\dagger\tau^I H\right)\left(D^\mu H^\dagger\tau^J D^\nu H\right)W^K_{\mu\nu}$$

The term in question is complex in nature, since it incorporates a third-order tensor known as the Levi-Civita permutation symbol. Specifically, the value of the permutation symbol is positive (+1) for even permutations, negative (-1) for odd permutations, and zero when any of the indices are equal[3].

$$= \epsilon^{123}a^1b^2c^3 - \epsilon^{132}a^1b^3c^2$$
$$+ \epsilon^{213}a^2b^1c^3 - \epsilon^{231}a^2b^3c^1$$
$$+ \epsilon^{312}a^3b^1c^2 - \epsilon^{321}a^3b^2c^1$$

$$= \epsilon^{123}\left(H^\dagger\tau^1 H\right)^1\left(D^\mu H^\dagger\tau^2 D^\nu H\right)W^3_{\mu\nu} - \epsilon^{132}\left(H^\dagger\tau^1 H\right)\left(D^\mu H^\dagger\tau^3 D^\nu H\right)W^2_{\mu\nu}$$
$$+ \epsilon^{213}\left(H^\dagger\tau^2 H\right)\left(D^\mu H^\dagger\tau^1 D^\nu H\right)W^3_{\mu\nu} - \epsilon^{231}\left(H^\dagger\tau^2 H\right)\left(D^\mu H^\dagger\tau^3 D^\nu H\right)W^1_{\mu\nu}$$
$$+ \epsilon^{312}\left(H^\dagger\tau^3 H\right)\left(D^\mu H^\dagger\tau^1 D^\nu H\right)W^2_{\mu\nu} - \epsilon^{321}\left(H^\dagger\tau^3 H\right)\left(D^\mu H^\dagger\tau^2 D^\nu H\right)W^1_{\mu\nu}$$

Most of the terms in the equation evaluate to zero, owing to the off-diagonal elements of the first two Pauli matrices. The only non-zero terms are those that involve the third Pauli matrix.

[3] This operator is not included in the final Lagrangian, but is kept in the appendix due to to it evaluating to zero, which I found odd

$$
\begin{aligned}
&= \epsilon^{123}\left(H^\dagger\tau^1H\right)\left(D^\mu H^\dagger\tau^2D^\nu H\right)W^3_{\mu\nu} - \epsilon^{132}\left(H^\dagger\tau^1H\right)\left(D^\mu H^\dagger\tau^3D^\nu H\right)W^2_{\mu\nu}\\
&+ \epsilon^{213}\left(H^\dagger\tau^2H\right)\left(D^\mu H^\dagger\tau^1D^\nu H\right)W^3_{\mu\nu} - \epsilon^{231}\left(H^\dagger\tau^2H\right)\left(D^\mu H^\dagger\tau^3D^\nu H\right)W^1_{\mu\nu}\\
&+ \epsilon^{312}\left(H^\dagger\tau^3H\right)\left(D^\mu H^\dagger\tau^1D^\nu H\right)W^2_{\mu\nu} - \epsilon^{321}\left(H^\dagger\tau^3H\right)\left(D^\mu H^\dagger\tau^2D^\nu H\right)W^1_{\mu\nu} = 0?
\end{aligned}
$$

11.6.9 $\mathcal{L}_8$ operators with a Higgs field

$$\boldsymbol{\bar{c}_{HHG}\left(H^\dagger H\right)^2 G^a_{\mu\nu}G^{a\mu\nu}}$$

$$\frac{c_{HHG}g_3^2}{2m_W^4}\left(H^\dagger H\right)^2 G^a_{\mu\nu}G^{a\mu\nu} = 8c_{HHG}\frac{g_3^2}{g_2^4}\frac{h}{v}G^a_{\mu\nu}G^{a\mu\nu}$$

$$\boldsymbol{\bar{c}_{HHB}\left(H^\dagger H\right)^2 B_{\mu\nu}B^{\mu\nu}}$$

$$\frac{\bar{c}_{HHB}g_1^2}{4m_W^4}\left(H^\dagger H\right)^2 B_{\mu\nu}B^{\mu\nu} = 8\bar{c}_{HHB}\frac{h}{v}\left(\frac{s_w^2}{g_2^2}\gamma^2_{\mu\nu} - 16\frac{\tan\theta_W s_w^2}{g_2^2}\gamma_{\mu\nu}Z^{\mu\nu} + \frac{\tan^2\theta_W s_w^2}{g_2^2}Z^2_{\mu\nu}\right)$$

$$\boldsymbol{\bar{c}_{WW}\left(H^\dagger H\right)^2 W^I_{\mu\nu}W^{I\mu\nu}}$$

$$
\begin{aligned}
\frac{\bar{c}_{WW}g_2^2}{4m_W^4}\left(H^\dagger H\right)^2 W^I_{\mu\nu}W^{I\mu\nu} &= \frac{\bar{c}_{WW}g_2^2}{16m_W^4}\left(v^4 + 2v^3h\dots\right)W^I_{\mu\nu}W^{I\mu\nu}\\
&= \frac{\bar{c}_{WW}g_2^2}{16m_W^4}\left(v^4 + 2v^3h\right)(W^1_{\mu\nu}W^{1\mu\nu} + W^2_{\mu\nu}W^{2\mu\nu} + W^3_{\mu\nu}W^{3\mu\nu})\\
&= 4\frac{\bar{c}_{WW}}{g_2^2}\frac{h}{v}(W_{\mu\nu}W^{\mu\nu} + (s_w\gamma_{\mu\nu} + c_wZ_{\mu\nu})^2\\
&= 4\frac{\bar{c}_{WW}}{g_2^2}\frac{h}{v}(W_{\mu\nu}W^{\mu\nu} + s_w^2\gamma^2_{\mu\nu} + s_wc_w\gamma_{\mu\nu}Z^{\mu\nu} + Z^2_{\mu\nu})
\end{aligned}
$$

$$\boldsymbol{\bar{c}_{DHW}\left(H^\dagger H\right)\left(D^\mu H^\dagger\tau^I D^\nu H\right)W^I_{\mu\nu}}$$

$$
\begin{aligned}
&\frac{i\bar{c}_{DHW} g_2}{2m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger \tau^I D^\nu H\right) W^I_{\mu\nu} \\
&= \frac{i\bar{c}_{DHW} g_2}{8m_W^4}(v^2 + 2vh \ldots)\left(-D^\mu(v+h) D^\nu(v+h)\right) W^3_{\mu\nu} \\
&= -\frac{i\bar{c}_{DHW} g_2 v^4}{8m_W^4} D^\mu D^\nu W^3_{\mu\nu} - \frac{i\bar{c}_g g_2 v^3 h}{2m_W^4} D^\mu D^\nu W^3_{\mu\nu} \ldots \\
&= -\frac{i\bar{c}_{DHW} g_2 v^3 h}{2m_W^4}(D^\mu D^\nu + D^\mu D^\nu) W^3_{\mu\nu} \ldots \\
&= \frac{\bar{c}_{DHW} g_2 v^3 h}{2m_W^4}(\sqrt{g_1^2+g_2^2}(Z^{\mu\nu}) + (\sqrt{g_1^2+g_2^2} Z^\mu D^\nu))(s_w \gamma_{\mu\nu} + c_w Z_{\mu\nu})
\end{aligned}
$$

The reason the D^ν was turned into a Z and not a γ, is because γ would break $U(1)$ symmetry [9].

$$
\begin{aligned}
&= \frac{\bar{c}_{DHW} g_2 v^3 h}{2m_W^4}(\sqrt{g_1^2+g_2^2}(Z^{\mu\nu}) + (\sqrt{g_1^2+g_2^2} Z^\mu (\partial^\nu \ldots))(s_w \gamma_{\mu\nu} + c_w Z_{\mu\nu}) \\
&= \frac{\bar{c}_{DHW} g_2 v^3 h}{2m_W^4}\sqrt{g_1^2+g_2^2}\,(s_w Z^{\mu\nu}\gamma_{\mu\nu} + c_w Z^{\mu\nu} Z_{\mu\nu} + (s_w Z^\mu \partial^\nu \gamma_{\mu\nu} + c_w Z^\mu \partial^\nu Z_{\mu\nu})) \\
&= 8\frac{\bar{c}_{DHW}}{g_2^2}\frac{h}{v}\frac{\sqrt{g_1^2+g_2^2}}{g_2}\,(s_w Z^{\mu\nu}\gamma_{\mu\nu} + c_w Z^{\mu\nu} Z_{\mu\nu} + (s_w Z^\mu \partial^\nu \gamma_{\mu\nu} + c_w Z^\mu \partial^\nu Z_{\mu\nu})) \\
&= 8\frac{\bar{c}_{DHW}}{g_2^2}\frac{h}{v}\,(\tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} + Z^{\mu\nu} Z_{\mu\nu} + (\tan\theta_w Z^\mu \partial^\nu \gamma_{\mu\nu} + Z^\mu \partial^\nu Z_{\mu\nu}))
\end{aligned}
$$

As outlined in the reference [9], the dictionary suggests that this operator plays a role in determining the value of κ_{WW}, as it bears similarity to the operator c_{HW} under unitary gauge.

Further contributions to κ_{WW} can also be acquired. To obtain these contributions, we must utilize the remaining terms of $(D^\mu D^\nu) W^3_{\mu\nu}$. By 'remaining', we refer to the fact that we have already utilized the W^3 particle in the construction to construct Z^μ thereby yielding the Z^μ particle, to obtain contributions to κ_{ZZ}, $\kappa_{Z\gamma}$, $c_{Z\gamma}$ and c_{ZZ}. However, in order to derive additional contributions, we will employ either the W^1 or W^2 (or both, which we will collectively denote as W), and use these particles to form WDW terms.

$$
\frac{i\bar{c}_{DHW} g_2}{2m_W^4}(-4v^3 h(ig_2 W \ldots) DW^3) = 32\frac{\bar{c}_g}{g_2^2}\frac{h}{v}(WDW^3)
$$

$$
\boldsymbol{\bar{c}_{DHB}\left(H^\dagger H\right)\left(D^\mu H^\dagger D^\nu H\right) B_{\mu\nu}}
$$

$$\frac{i\bar{c}_{DHB}g_1}{2m_W^4}\left(H^\dagger H\right)\left(D^\mu H^\dagger D^\nu H\right)B_{\mu\nu} = \frac{i\bar{c}_g g_1}{2m_W^4}(v^2+2vh\ldots)(D^\mu D^\nu(v^2+2vh\ldots))B_{\mu\nu}$$
$$= \frac{i\bar{c}_{DHB}g_1 v^4}{2m_W^4}D^\mu D^\nu B_{\mu\nu} = 8\frac{i\bar{c}_g g_1^2}{g_2^2}D^\mu D^\nu(c_w\gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= 8\frac{i\bar{c}_{DHB}g_1}{g_2^2}D^\mu D^\nu(c_w\gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= 4\frac{i\bar{c}_{DHB}g_1 v^3 h}{2m_W^4}D^\mu D^\nu B_{\mu\nu} = 16\frac{i\bar{c}_g g_1}{g_2^4}\frac{h}{v}(D^\mu D^\nu + D^\mu D^\nu)(c_w\gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= 16\frac{i\bar{c}_{DHB}g_1}{g_2^4}\frac{h}{v}(i\sqrt{g_1^2+g_2^2}(Z^\mu D^\nu + Z^{\mu\nu})c_w\gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= -16\frac{\bar{c}_{DHB}}{g_2^2}\tan\theta_w\frac{\sqrt{g_1^2+g_2^2}}{g_2^2}\frac{h}{v}((Z^\mu D^\nu + Z^{\mu\nu})(c_w\gamma_{\mu\nu} - s_w Z_{\mu\nu})$$
$$= -16\frac{\bar{c}_{DHB}}{g_2^2}\tan\theta_w\frac{h}{v}((Z^\mu D^\nu + Z^{\mu\nu})(\gamma_{\mu\nu} - \tan\theta_w Z_{\mu\nu})$$
$$= -16\frac{\bar{c}_{DHB}}{g_2^2}\tan\theta_w\frac{h}{v}(Z^\mu D^\nu\gamma_{\mu\nu} + Z^{\mu\nu}\gamma_{\mu\nu} - \tan\theta_w Z^\mu D^\nu Z_{\mu\nu} - \tan\theta_w Z^{\mu\nu}Z_{\mu\nu})$$
$$= -16\frac{\bar{c}_{DHB}}{g_2^2}\frac{h}{v}(\tan\theta_w Z^\mu D^\nu\gamma_{\mu\nu} + \tan\theta_w Z^{\mu\nu}\gamma_{\mu\nu} - \tan^2\theta_w Z^\mu D^\nu Z_{\mu\nu} - \tan^2\theta_w Z^{\mu\nu}Z_{\mu\nu})$$

$\bar{c}_{HWW}\left(H^\dagger\tau^I H\right)\left(H^\dagger\tau^J H\right)W^I_{\mu\nu}W^{J\mu\nu}$

Following similar logic to that employed on the previous operator, this operator makes a contribution to the WW interaction, as well as to the interactions between $\gamma\gamma$, γZ, and ZZ particles.

$$\frac{\bar{c}_{HWW}g_2^2}{m_W^4}\left(H^\dagger\tau^I H\right)\left(H^\dagger\tau^J H\right)W^I_{\mu\nu}W^{J\mu\nu} = \frac{\bar{c}_{HWW}g_2^2}{m_W^4}\left(-(v+h)\right)^4 W^3_{\mu\nu}W^{3\mu\nu}$$
$$= 16\frac{\bar{c}_{HWW}}{g_2^2 v^4}\left(-(v^4+4v^3h\ldots)\right)^4((s_w\gamma_{\mu\nu}+c_w Z_{\mu\nu})^2)$$
$$= 4^5\frac{i\bar{c}_{HWW}g_2^2 v^3 h}{g^4 v^4}(2\gamma_{\mu\nu}Z^{\mu\nu}c_w s_w + c_w^2 Z_{\mu\nu}Z^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu})$$
$$= 4^5\frac{i\bar{c}_{HWW}v^3 h}{g^2}\frac{h}{v}(2\gamma_{\mu\nu}Z^{\mu\nu}c_w s_w + c_w^2 Z_{\mu\nu}Z^{\mu\nu} + s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu})$$

$c_{HWB}\left(H^\dagger H\right)\left(H^\dagger\tau^I H\right)W^I_{\mu\nu}B^{\mu\nu}$

$$\frac{c_{HWB}g_1g_2}{m_W^4}\left(H^\dagger H\right)\left(H^\dagger\tau^I H\right)W^I_{\mu\nu}B^{\mu\nu} = -\frac{c_{HWB}g_1g_2}{m_W^4}(v^2+2vh+h^2)^2(s_w\gamma_{\mu\nu}+c_wZ_{\mu\nu})(c_w\gamma^{\mu\nu}-s_wZ^{\mu\nu})$$
$$= -\frac{c_{HWB}g_1g_2}{m_W^4}(v^2+2vh+h^2)^2(s_wc_w\gamma_{\mu\nu}\gamma^{\mu\nu}-s_w^2\gamma_{\mu\nu}Z^{\mu\nu}+c_w^2Z_{\mu\nu}\gamma^{\mu\nu}-c_ws_wZ^{\mu\nu 2})$$
$$= \frac{16c_{HWB}}{g_2^2}\frac{g_1}{g_2}\frac{h}{v}(s_wc_w\gamma_{\mu\nu}\gamma^{\mu\nu}+(c_w^2-s_w^2)Z_{\mu\nu}\gamma^{\mu\nu}-c_ws_wZ_{\mu\nu}Z^{\mu\nu})$$
$$= \frac{16c_{HWB}}{g_2^2}\frac{h}{v}(s_w^2\gamma_{\mu\nu}\gamma^{\mu\nu}+\tan\theta_w(c_w^2-s_w^2)Z_{\mu\nu}\gamma^{\mu\nu}-s_w^2Z_{\mu\nu}Z^{\mu\nu})$$

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