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SIMULATION BASED ANALYSIS AND AN APPLICATION TO AN OFFSHORE OIL AND GAS PRODUCTION SYSTEM OF THE NATVIG MEASURES OF COMPONENT IMPORTANCE IN REPAIRABLE SYSTEMS

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Abstract

In the present paper the Natvig measures of component importance for repairable systems, and its extended version are analysed for two three component systems and a bridge system. The measures are also applied to an offshore oil and gas production system. According to the extended version of the Natvig measure a component is important if both by failing it strongly reduces the expected system uptime and by being repaired it strongly reduces the expected system downtime. The results include a study of how different distributions affect the ranking of the components. All numerical results are computed using discrete event simulation. In a companion paper [4] the advanced simulation methods needed in these calculations are described.

KEYWORDS: Importance measures; Repairable systems; Discrete event simulation; Birnbaum; Barlow-Proschan; Natvig

1 Basic ideas, concepts and results

Intuitively it seems that components that by failing strongly reduce the expected remaining system lifetime are very important. This is at least true during the system development phase. This is the motivation for the Natvig measure [5] of component importance in nonrepairable systems. In [6] a stochastic representation of this measure was obtained by considering the random variable:

$$Z_i = Y_i^1 - Y_i^0, \tag{1}$$

where:

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$Y_i^0 =$ The remaining system lifetime just *after* the failure of the ith component.

$Y_i^1 =$ The remaining system lifetime just *after* the failure of the ith component, which, however, immediately undergoes a minimal repair; i.e., it is repaired to have the same distribution of remaining lifetime as it had just before failing.

Thus, Z_i can be interpreted as the increase in system lifetime due to a minimal repair of the ith component at failure. The Natvig measure of importance of the ith component is then defined as:

$$I_N^{(i)} = \frac{EZ_i}{\sum_{j=1}^n EZ_j}, \tag{2}$$

tacitly assuming $EZ_i < \infty$, $i = 1, \ldots, n$. Obviously

$$0 \leq I_N^{(i)} \leq 1, \qquad \sum_{i=1}^n I_N^{(i)} = 1. \tag{3}$$

For repairable systems we consider a time interval $[0, t]$ and start by introducing some basic random variables ($i = 1, \ldots, n$):

$T_{ij} =$ The time of the jth failure of the ith component, $j = 1, 2, \ldots$,

$S_{ij} =$ The time of the jth repair of the ith component, $j = 1, 2, \ldots$,

where we define $S_{i0} = 0$. Let ($i = 1, \ldots, n$ and $j = 1, 2 \ldots$):

$U_{ij} = T_{ij} - S_{ij-1} =$ The length of the jth lifetime of the ith component.

$D_{ij} = S_{ij} - T_{ij} =$ The length of the jth repair time of the ith component.

We assume that U_{ij} has an absolutely continuous distribution $F_i(t)$ with density $f_i(t)$ letting $\bar{F}_i(t) \stackrel{d}{=} 1 - F_i(t)$. Furthermore, D_{ij} is assumed to have an absolutely continuous distribution $G_i(t)$ with density $g_i(t)$ letting $\bar{G}_i(t) \stackrel{d}{=} 1 - G_i(t)$. $EU_{ij} = \mu_i$, $ED_{ij} = \nu_i$ and all lifetimes and repair times are assumed independent.

Parallel to the nonrepairable case we argue that components that by failing strongly reduce the expected system uptime should be considered as very important. In order to formalize this, we introduce ($i = 1, \ldots, n$ and $j = 1, 2, \ldots$):

$T'_{ij} =$ The fictive time of the jth failure of the ith component after a fictive minimal repair of the component at T_{ij}.

$Y_{ij}^0 =$ System uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ assuming that the ith component is failed throughout this interval.

$Y_{ij}^1 =$ System uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ assuming that the ith component is functioning throughout this interval as a result of the fictive minimal repair.

In order to arrive at a stochastic representation similar to the nonrepairable case, see Eq.(1), we introduce the following random variables ($i = 1, \ldots, n$):

$$Z_{ij} = Y_{ij}^1 - Y_{ij}^0, \qquad j = 1, 2, \ldots. \tag{4}$$

Thus, Z_{ij} can be interpreted as the fictive increase in system uptime in the interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ as a result of the ith component being functioning instead of failed in this interval. Note that since the minimal repair is fictive, we have chosen to calculate the effect of this repair over the entire interval $[\min(T_{ij}, t), \min(T'_{ij}, t)]$ even though this interval may extend beyond the time of the real repair, S_{ij}.

In order to summarize the effects of all the fictive minimal repairs, we have chosen to simply add up these contributions. Note that the fictive minimal repair periods, i.e., the intervals of the form $[\min(T_{ij}, t), \min(T'_{ij}, t)]$, may sometimes overlap. Thus, at a given point of time we may have contributions from more than one fictive minimal repair. This is efficiently dealt with by the simulation methods presented in [4]. Taking the expectation, we get:

$$E\Big[\sum_{j=1}^{\infty} I(S_{ij-1} \leq t) Z_{ij}\Big] \stackrel{d}{=} EY_i(t), \tag{5}$$

where I denotes the indicator function. The time dependent Natvig measure of the importance of the ith component in the time interval $[0, t]$ in repairable systems can then be defined as:

$$I_N^{(i)}(t) = \frac{EY_i(t)}{\sum_{j=1}^{n} EY_j(t)}. \tag{6}$$

We now also take a dual term into account where components that by being repaired strongly reduce the expected system downtime are considered very important. Introduce ($i = 1, \ldots, n$ and $j = 1, 2, \ldots$):

S'_{ij} = The fictive time of the jth repair of the ith component after a fictive minimal failure of the component at S_{ij}.

X^0_{ij} = System downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ assuming that the ith component is functioning throughout this interval.

X^1_{ij} = System downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ assuming that the ith component is failed throughout this interval as a result of the fictive minimal failure.

We then introduce the following random variables parallel to Eq.(4) ($i = 1, \ldots, n$):

$$W_{ij} = X^1_{ij} - X^0_{ij}, \qquad j = 1, 2, \ldots. \tag{7}$$

In this case W_{ij} can be interpreted as the fictive increase in system downtime in the interval $[\min(S_{ij}, t), \min(S'_{ij}, t)]$ as a result of the ith component being failed instead of functioning in this interval.

Now adding up the contributions from the repairs at S_{ij}, $j = 1, 2, \ldots$, and taking the expectation, we get:

$$E\Big[\sum_{j=1}^{\infty} I(T_{ij} \leq t) W_{ij}\Big] \stackrel{d}{=} EX_i(t). \tag{8}$$

The time dependent dual Natvig measure of the importance of the ith component in the time interval $[0, t]$ in repairable systems can then be defined as:

$$I_{N,D}^{(i)}(t) = \frac{EX_i(t)}{\sum_{j=1}^{n} EX_j(t)}. \tag{9}$$

An extended version of Eq.(6) is given by:

$$\bar{I}_N^{(i)}(t) = \frac{EY_i(t) + EX_i(t)}{\sum_{j=1}^{n}[EY_j(t) + EX_j(t)]} \,. \tag{10}$$

In [7] it is shown that:

$$\begin{aligned} &P(T'_{ij} - S_{ij-1} > t) \\ &\quad = \bar{F}_i(t) + \int_0^t f_i(t-u)\frac{\bar{F}_i(t)}{\bar{F}_i(t-u)}du \\ &\quad = \bar{F}_i(t)[1 - \ln \bar{F}_i(t)]. \end{aligned} \tag{11}$$

Hence, applying Eq.(11) we get:

$$\begin{aligned} &\int_0^\infty \bar{F}_i(t)(-\ln \bar{F}_i(t))dt \\ &\quad = \int_0^\infty \bar{F}_i(t)[1 - \ln \bar{F}_i(t)]dt - \int_0^\infty \bar{F}_i(t)dt \\ &\quad = E(T'_{ij} - S_{ij-1}) - E(T_{ij} - S_{ij-1}) \\ &\quad = E(T'_{ij} - T_{ij}) \stackrel{d}{=} \mu_i^p. \end{aligned} \tag{12}$$

Accordingly, this integral equals the expected prolonged lifetime of the ith component due to a minimal repair. Completely parallel we have:

$$\int_0^\infty \bar{G}_i(t)(-\ln \bar{G}_i(t))dt = E(S'_{ij} - S_{ij}) \stackrel{d}{=} \nu_i^p. \tag{13}$$

Let $A_i(t)$ be the availability of the ith component at time t, i.e., the probability that the component is functioning at time t. The corresponding stationary availabilities are given by:

$$A_i = \lim_{t\to\infty} A_i(t) = \frac{\mu_i}{\mu_i + \nu_i}, \qquad i = 1, \ldots, n. \tag{14}$$

Introduce $\boldsymbol{A}(t) = (A_1(t), \ldots, A_n(t))$ and $\boldsymbol{A} = (A_1, \ldots, A_n)$. Now the availability of the system at time t is given by $h(\boldsymbol{A}(t))$, where h is the system's reliability function.

The Birnbaum measure [2] at time t is given by:

$$I_B^{(i)}(t) = h(1_i, \boldsymbol{A}(t)) - h(0_i, \boldsymbol{A}(t)), \tag{15}$$

which is the probability that the ith component is critical for system functioning at time t. The corresponding stationary measure is given by:

$$I_B^{(i)} = \lim_{t\to\infty} I_B^{(i)}(t) = h(1_i, \boldsymbol{A}) - h(0_i, \boldsymbol{A}), \tag{16}$$

with standardized version

$$\hat{I}_B^{(i)} = \frac{I_B^{(i)}}{\sum_{j=1}^{n} I_B^{(j)}}. \tag{17}$$

The Barlow and Proschan stationary measure [1] is given by:

$$I_{B-P}^{(i)} = \frac{I_B^{(i)}/(\mu_i + \nu_i)}{\sum_{j=1}^{n} I_B^{(j)}/(\mu_j + \nu_j)}, \tag{18}$$

which is the stationary probability that the failure of the ith component is the cause of system failure, given that a system failure has occurred. Note that if $(\mu_j + \nu_j)$, $j = 1, \dots, n$ are all equal, Eq.(18) reduces to Eq.(17).

In [8] the following stationary versions of Eqs. (6), (9) and (10) are arrived at:

$$I_N^{(i)} = \lim_{t\to\infty} I_N^{(i)}(t) = \frac{[I_B^{(i)}/(\mu_i + \nu_i)]\mu_i^p}{\sum_{j=1}^n [I_B^{(j)}/(\mu_j + \nu_j)]\mu_j^p}. \tag{19}$$

$$I_{N,D}^{(i)} = \lim_{t\to\infty} I_{N,D}^{(i)}(t) = \frac{[I_B^{(i)}/(\mu_i + \nu_i)]\nu_i^p}{\sum_{j=1}^n [I_B^{(j)}/(\mu_j + \nu_j)]\nu_j^p}. \tag{20}$$

$$\bar{I}_N^{(i)} = \lim_{t\to\infty} \bar{I}_N^{(i)}(t) = \frac{[I_B^{(i)}/(\mu_i + \nu_i)](\mu_i^p + \nu_i^p)}{\sum_{j=1}^n [I_B^{(j)}/(\mu_j + \nu_j)](\mu_j^p + \nu_j^p)}. \tag{21}$$

Note that if μ_j^p, $j = 1, \dots, n$ are all equal, which by Eq.(12) is the case when all components have the same lifetime distribution, then Eq.(19) reduces to Eq.(18). Similarly, if ν_j^p, $j = 1, \dots, n$ are all equal, which by Eq.(13) is the case when all components have the same repair time distribution, then Eq.(20) reduces to Eq.(18). Similarly, if both μ_j^p, $j = 1, \dots, n$ are all equal and ν_j^p, $j = 1, \dots, n$ are all equal, which by Eqs. (12) and (13) is the case when all components have the same lifetime distribution and the same repair time distribution, then Eq.(21) reduces to Eqs.(18) and (17).

Now consider the special case where the lifetime and repair time distributions are Weibull distributed; i.e.,

$$\begin{aligned} \bar{F}_i(t) &= e^{-(\lambda_i t)^{\alpha_i}}, \qquad \lambda_i > 0,\ \alpha_i > 0, \\ \bar{G}_i(t) &= e^{-(\gamma_i t)^{\beta_i}}, \qquad \gamma_i > 0,\ \beta_i > 0. \end{aligned}$$

We then have:

$$\begin{aligned} &\int_0^\infty \bar{F}_i(t)(-\ln \bar{F}_i(t))dt \\ &\quad = \frac{1}{\alpha_i}\frac{1}{\lambda_i}\int_0^\infty u^{1/\alpha_i+1-1}e^{-u}du \\ &\quad = \frac{1}{\alpha_i}\frac{1}{\lambda_i}\Gamma(\frac{1}{\alpha_i}+1) = \frac{\mu_i}{\alpha_i}. \end{aligned}$$

Hence, Eq.(21) simplifies to:

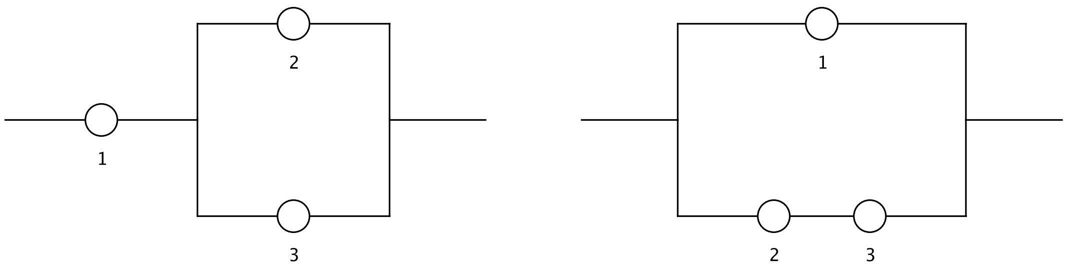
$$\bar{I}_N^{(i)} = \frac{[I_B^{(i)}/(\mu_i + \nu_i)](\mu_i/\alpha_i + \nu_i/\beta_i)}{\sum_{j=1}^n [I_B^{(j)}/(\mu_j + \nu_j)](\mu_j/\alpha_j + \nu_j/\beta_j)}. \tag{22}$$

Now assume that α_i is increasing and λ_i changing in such a way that μ_i is constant. Hence, according to Eq.(14) the availability A_i is unchanged. Then $\bar{I}_N^{(i)}$ is decreasing in α_i. This is natural since a large $\alpha_i > 1$ corresponds to a strongly increasing failure rate and the effect of a minimal repair is small. Hence, according to $\bar{I}_N^{(i)}$ the ith component is of less importance. If on the other hand $\alpha_i < 1$ is small, we have a strongly decreasing failure rate and the effect of a minimal repair is large. Hence, according to $\bar{I}_N^{(i)}$ the ith component is of higher importance. A completely parallel argument is valid for β_i.

In the present paper the Natvig measures of component importance for repairable systems, given by Eqs.(6), (9) and (10) are in Section 2 analysed for two three component systems and in Section 3 for the bridge system. In Section 4 the measures are applied to an offshore oil and gas production system. Some concluding remarks are given in Section 5. In a companion paper [4] the advanced simulation methods needed in the calculations are described. In [8] a more thorough theoretical presentation of the Natvig measures for repairable systems and their stationary versions is given.

2 Component importance in two three component systems

In this section we will simulate the component importance in two systems with three components. Figure 1 shows the systems we will be looking at.



(a) System A

(b) System B

Figure 1: Systems of three components.

The life- and repair times are assumed to be gamma distributed. We will first see how an increasing variance in the lifetime distribution of one of the components influences the component importances. The effect of a decreasing mean time to repair of one of the components will be investigated next.

Let first the components have the following life- and repair time distributions:

Component 2: $\bar{F}_2(t) \sim \text{gamma}(\frac{4}{k}, 3k)$, $\bar{G}_2(t) \sim \text{gamma}(4, \frac{1}{2})$,

Component j: $\bar{F}_j(t) \sim \text{gamma}(8, 1)$, $\bar{G}_j(t) \sim \text{gamma}(4, \frac{1}{2})$, $j = 1, 3$,

where k is a positive number. The mean lifetimes are $\mu_2 = 12$ for component 2 and $\mu_j = 8$, $j = 1, 3$. All components have mean time to repair equal to $\nu_i = 2$, $i = 1, 2, 3$. The variance associated with the lifetime distribution of component 2 is $36k$, while the variance associated with the lifetime distribution of components 1 and 3 is 8. The variances in the repair time distributions are 1 for all components. Table 1 displays the results from the simulations for all three versions of the Natvig measure and for $\hat{I}_B^{(i)}$ and $I_{B-P}^{(i)}$.

We first note that for both systems $I_{N,D}^{(i)}(t)$ and $I_{B-P}^{(i)}$ are practically equal. Since stationarity is reached and the repair time distributions are the same for all three components, this is in accordance with the results of Section 1. Furthermore, component 2's importance is increasing in k both for the $I_N^{(i)}(t)$ and the extended measure. Hence, according to these measures the increased uncertainty associated with an increasing variance leads to increased importance of a component. This behavior is present in both systems.

		System A					System B				
k	i	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$
	1	0.772	0.809	0.782	0.780	0.810	0.477	0.523	0.488	0.487	0.524
1/2	2	0.137	0.095	0.126	0.128	0.095	0.265	0.191	0.244	0.248	0.190
	3	0.091	0.096	0.092	0.092	0.095	0.261	0.286	0.267	0.265	0.286
	1	0.729	0.809	0.749	0.780	0.810	0.429	0.524	0.451	0.487	0.524
1	2	0.185	0.095	0.163	0.128	0.095	0.337	0.190	0.304	0.248	0.190
	3	0.086	0.095	0.088	0.092	0.095	0.234	0.286	0.246	0.265	0.286
	1	0.674	0.810	0.705	0.780	0.810	0.373	0.524	0.404	0.487	0.524
2	2	0.247	0.095	0.212	0.128	0.095	0.422	0.190	0.375	0.248	0.190
	3	0.079	0.095	0.083	0.092	0.095	0.204	0.286	0.221	0.265	0.286
	1	0.612	0.809	0.654	0.780	0.810	0.318	0.523	0.355	0.487	0.524
4	2	0.316	0.096	0.269	0.128	0.095	0.508	0.191	0.451	0.248	0.190
	3	0.072	0.095	0.077	0.092	0.095	0.174	0.285	0.194	0.265	0.286
	1	0.544	0.810	0.596	0.780	0.810	0.265	0.523	0.305	0.487	0.524
8	2	0.392	0.096	0.334	0.128	0.095	0.590	0.191	0.529	0.248	0.190
	3	0.064	0.095	0.070	0.092	0.095	0.145	0.286	0.167	0.265	0.286
	1	0.520	0.810	0.574	0.780	0.810	0.247	0.524	0.287	0.487	0.524
10	2	0.419	0.095	0.359	0.128	0.095	0.618	0.190	0.556	0.248	0.190
	3	0.061	0.095	0.067	0.092	0.095	0.135	0.286	0.157	0.265	0.286

Table 1: *Simulations of System A and B with varying variance in the lifetime distribution of component 2. Components 1 and 3 have identical life- and repair time distributions. The time horizon is* $t = 20000$.

k	System A	System B
1/2	$1 > 2 > 3$	$1 > 3 > 2$
$1 - 2$	$1 > 2 > 3$	$1 > 2 > 3$
$4 - 10$	$1 > 2 > 3$	$2 > 1 > 3$

Table 2: *The ranks of the extended measure of component importance corresponding to the results in Table 1.*

We will then look at how the extended measure ranks the components which is shown in Table 2. For system A components 2 and 3 are equally important according to $I_{B-P}^{(i)}$ irrespective of k. Since the variance of the lifetime distribution of component 2 is greater than the corresponding one of component 3 for all k, the former component is more important than the latter according to the extended measure. However, component 1 is not challenged as the most important for this measure being in series with the rest of the system and by far the most important according to $\hat{I}_B^{(i)}$.

For system B component 3 is more important than component 2 according to $I_{B-P}^{(i)}$ irrespective of k. This is also true for the extended measure for $k = 1/2$. However, as the variance of the lifetime distribution of component 2 increases, this component gets increasingly more important according to the extended measure, and finally the most important one.

We will now turn our attention to the case where one of the components experiences a decreasing mean time to repair (MTTR). First we will assume that this is the case for component 1, and that components 2 and 3 have identical life- and repair time distributions. Then the roles of components 1 and 2 are interchanged.

More specifically, let the components have the following life- and repair time distributions:

Component 1: $\bar{F}_1(t) \sim \text{gamma}(24, 1)$, $\bar{G}_1(t) \sim \text{gamma}(\frac{24}{k^2}, k)$,

Component j: $\bar{F}_j(t) \sim \text{gamma}(24, 1)$, $\bar{G}_j(t) \sim \text{gamma}(4, \frac{1}{2})$, $j = 2, 3$,

where k is a positive number. Component 1's mean time to repair is $\nu_1 = \frac{24}{k}$, while components 2 and 3 have constant mean times to repair equal to $\nu_i = 2$, $i = 2, 3$. The mean times to failure are equal for all components, thus $\mu_i = 24$, $i = 1, 2, 3$. The availabilities of the components are $A_1 = \frac{k}{k+1}$ for component 1 and $A_i = \frac{12}{13}$, $i = 2, 3$ for components 2 and 3. The variances are constant in all distributions. In the lifetime distributions the variances are 24 for all components. This is also the variance in the repair time distribution of component 1. The variance in the repair time distributions of components 2 and 3 equals 1.

	System A					System B				
k	$I_N^{(1)}(t)$	$I_{N,D}^{(1)}(t)$	$\bar{I}_N^{(1)}(t)$	$\hat{I}_B^{(1)}$	$I_{B-P}^{(1)}$	$I_N^{(1)}(t)$	$I_{N,D}^{(1)}(t)$	$\bar{I}_N^{(1)}(t)$	$\hat{I}_B^{(1)}$	$I_{B-P}^{(1)}$
1	0.875	0.970	0.920	0.928	0.875	0.080	0.291	0.126	0.138	0.080
2	0.875	0.971	0.921	0.906	0.875	0.148	0.457	0.225	0.194	0.148
3	0.875	0.972	0.922	0.896	0.875	0.207	0.564	0.306	0.243	0.207
4	0.876	0.972	0.923	0.890	0.875	0.258	0.635	0.372	0.286	0.258
6	0.875	0.972	0.922	0.886	0.875	0.345	0.734	0.472	0.359	0.342

Table 3: *Simulations of System A and B with decreasing MTTR of component 1. Components 2 and 3 have identical life and repair time distributions. The time horizon is* $t = 20000$.

The simulation results are shown in Table 3, where the importance of component 1 is listed for all three versions of the Natvig measure and for $\hat{I}_B^{(1)}$ and $I_{B-P}^{(1)}$. Since components 2 and 3 have interchangeable positions both in System A and B, and identical life- and repair time distributions, they have the same importance for each of the five measures. These importances are easily found given the ones for component 1. Now we note for both systems that $I_N^{(1)}(t)$ and $I_{B-P}^{(1)}$ are practically equal. Since stationarity is reached and the lifetime distributions are the same for all three components, this is in accordance with the results in Section 1.

For system A all measures are practically constant in k for component 1 except $\hat{I}_B^{(1)}$ which is decreasing in k. The latter fact follows since the component is in series with the rest of the system and its availability, A_1, increases due to the decreasing mean time to repair, ν_1. For $I_{B-P}^{(1)}$ it follows from Eq.(18) that the increase in the asymptotic failure rate $1/(\mu_1 + \nu_1)$ as ν_1 decreases, compensates for the decrease in $\hat{I}_B^{(1)}$.

For system B component 1 is in parallel with the rest of the system and the increase in A_1 as ν_1 decreases leads to an increasing $\hat{I}_B^{(1)}$. Since $1/(\mu_1 + \nu_1)$ increases as well, $I_{B-P}^{(1)}$ also increases. As for system A $I_N^{(1)}(t)$ behaves like $I_{B-P}^{(1)}$.

Now, we consider the case where components 1 and 3 are assumed to have identical life-and repair time distributions. More specifically, we assume that the components have the following life- and repair time distributions:

Component 2: $\bar{F}_2(t) \sim \text{gamma}(8,1)$, $\bar{G}_2(t) \sim \text{gamma}(\frac{6}{k^2}, \frac{k}{2})$,

Component j: $\bar{F}_j(t) \sim \text{gamma}(8,1)$, $\bar{G}_j(t) \sim \text{gamma}(6, \frac{1}{2})$, $j = 1, 3$,

where k is once again a positive number. In this example all components have identical lifetime distributions with a mean time to failure $\mu_i = 8$, $i = 1, 2, 3$. Moreover, the variances associated with the lifetime distributions are all equal to 8. The repair time distributions are identical for components 1 and 3 as well. The mean time to repair of these components are $\nu_j = 3$, $j = 1, 3$, while the mean time to repair of component 2 is $\nu_2 = \frac{3}{k}$. The variances in the repair time distributions are $\frac{3}{2}$ for all components. The availabilities of the components are $A_2 = \frac{8k}{8k+3}$ and $A_j = \frac{8}{11}$, $j = 1, 3$.

		System A					System B				
k	i	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$
	1	0.653	0.656	0.654	0.634	0.654	0.652	0.655	0.653	0.623	0.652
1/2	2	0.116	0.112	0.115	0.142	0.115	0.174	0.170	0.173	0.211	0.174
	3	0.231	0.232	0.231	0.224	0.231	0.174	0.175	0.174	0.166	0.174
	1	0.682	0.683	0.682	0.673	0.682	0.586	0.589	0.587	0.575	0.586
3/4	2	0.136	0.134	0.135	0.147	0.136	0.206	0.204	0.206	0.222	0.207
	3	0.182	0.183	0.182	0.180	0.182	0.207	0.208	0.207	0.203	0.207
	1	0.700	0.700	0.700	0.700	0.700	0.542	0.542	0.542	0.543	0.543
1	2	0.150	0.150	0.150	0.150	0.150	0.229	0.229	0.229	0.229	0.229
	3	0.150	0.150	0.150	0.150	0.150	0.229	0.229	0.229	0.229	0.229
	1	0.722	0.719	0.721	0.733	0.722	0.490	0.487	0.489	0.501	0.489
3/2	2	0.167	0.170	0.168	0.154	0.167	0.255	0.258	0.256	0.238	0.255
	3	0.111	0.111	0.111	0.113	0.111	0.255	0.255	0.255	0.261	0.255
	1	0.734	0.731	0.733	0.753	0.735	0.456	0.454	0.455	0.475	0.458
2	2	0.177	0.181	0.178	0.156	0.176	0.272	0.277	0.274	0.243	0.271
	3	0.089	0.088	0.089	0.090	0.088	0.271	0.269	0.271	0.282	0.271
	1	0.745	0.740	0.743	0.767	0.744	0.438	0.432	0.436	0.458	0.437
5/2	2	0.183	0.188	0.184	0.158	0.183	0.282	0.288	0.284	0.247	0.282
	3	0.072	0.072	0.072	0.075	0.073	0.281	0.279	0.280	0.295	0.282

Table 4: *Simulations of System A and B with decreasing MTTR of component 2. Components 1 and 3 have identical life- and repair time distributions. The time horizon is* $t = 20000$.

k	System A	System B
1/2 – 3/4	$1 > 3 > 2$	$1 > 3 > 2$
1	$1 > 3 \approx 2$	$1 > 3 \approx 2$
3/2 – 5/2	$1 > 2 > 3$	$1 > 2 > 3$

Table 5: *The ranks of the extended measure of component importance corresponding to the results in Table 4.*

Table 4 shows the results from the simulations. As for the previous case $I_N^{(i)}(t)$ and $I_{B-P}^{(i)}$ are practically equal since the lifetime distributions are the same for all three components. When $k = 1$, all components have identical life- and repair time distributions. Hence, in accordance with the results of Section 1 all measures give the same results in each system in this case. Irrespective of k the results are very similar for the three Natvig importance measures and $I_{B-P}^{(i)}$ in each system. For System A for all measures the importance of component 3 is decreasing in k, while the other components get increasingly more important. For System B for all measures the importance of component 1 is decreasing in k, while the other components get increasingly more important. According to $\hat{I}_B^{(i)}$ this behavior can be explained by comparing the structures of the two systems. In System A components 2 and 3 are connected in parallel. Thus, when the availability of component 2 increases, the importance of component 3 decreases. In System B component 1 is in parallel with the rest of the system. The availability of the branch containing components 2 and 3 improves as component 2 gets stronger. As a result, the importance of component 1 decreases.

We now turn to the ranking of the components according to the extended measure which is shown in Table 5. We see that component 1 is ranked on top in both systems for all values of k, so we focus on the ranking of components 2 and 3. As seen from Table 5 the rankings between these two components are identical for both systems. To explain this, however, we consider each system separately.

We first consider System A where the ranking is the same for $\hat{I}_B^{(i)}$. When $k < 1$, component 2 has a lower availability than component 3. Since these components are connected in parallel, according to $\hat{I}_B^{(i)}$ component 3 is ranked before component 2. As soon as k gets larger than 1, the roles of component 2 and 3 change. Now component 2 has the higher availability of the two, thus according to $\hat{I}_B^{(i)}$ this component is ranked before component 3.

We then turn to System B. Here the ranking of the Natvig measures does not follow $\hat{I}_B^{(i)}$ rather $I_{B-P}^{(i)}$ since the factor $1/(\mu_2 + \nu_2)$ is increasing in k due to ν_2 is decreasing in k.

3 Component importance in the bridge system

In this section we will investigate the bridge system depicted in Figure 2. As in the previous section the life- and repair times are assumed to be gamma distributed. We will again first see how an increasing variance in the lifetime distribution of one of the components influences the component importances. More specifically, we assume that the components have the following life- and repair time distributions:

Component 1: $\bar{F}_1(t) \sim \text{gamma}(\frac{12}{k}, k)$, $\bar{G}_1(t) \sim \text{gamma}(2, 1)$,

Component j: $\bar{F}_j(t) \sim \text{gamma}(3, 2)$, $\bar{G}_j(t) \sim \text{gamma}(2, 1)$, $j = 2, 3, 4$,

Component 5: $\bar{F}_5(t) \sim \text{gamma}(12, 1)$, $\bar{G}_5(t) \sim \text{gamma}(2, 1)$,

where k is a positive number. The mean lifetimes of components 1 and 5 are $\mu_j = 12$, $j = 1, 5$ which is twice the size of the mean lifetimes of the other components. Thus, $\mu_j = 6$, $j = 2, 3, 4$. All the components have the same repair time distributions with expectation $\nu_j = 2$, $j = 1, \ldots, 5$. The resulting availabilities are $A_j = \frac{6}{7}$, $j = 1, 5$ and $A_j = \frac{3}{4}$, $j = 2, 3, 4$. The variances in the lifetime distributions are 12 for all components except for the lifetime distribution of component 1 which has a variance of $12k$. In the repair time distributions the variances are 2. The results of the simulations are shown in Table 6, while the ranks of the extended measures are shown in Table 7. As for the case presented in Table 1 all

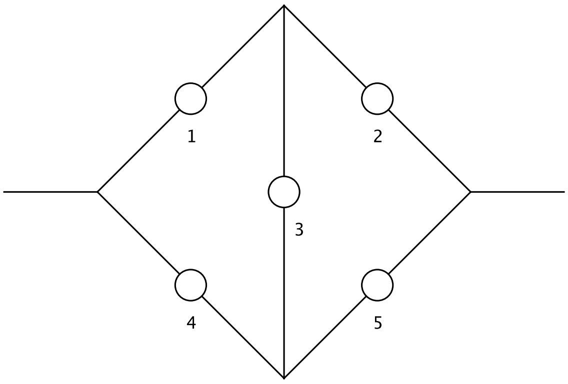


Figure 2: The bridge system.

components have the same repair time distribution. Hence, in accordance with the results in Section 1, $I_{N,D}^{(i)}(t)$ and $I_{B-P}^{(i)}$ are practically equal.

Parallel to the case mentioned above component 1's importance is increasing in k both for the $I_N^{(i)}(t)$ and the extended measure. Hence, again according to these measures the increased uncertainty associated with an increasing variance leads to increased importance of a component.

The ranks of the extended measure shown in Table 7 changes a lot as for System B in the case mentioned above. As k increases, and component 1 becomes more uncertain, it also becomes more important. Already when $k = 1$ components 1 and 5 have swapped places in the ranking, and component 1 becomes the most important component when $k \geq 6$.

We will now look at how a decreasing mean time to repair of one of the components influences the importance measures. More specifically, we let the life- and repair time distributions be as follows:

Component 1: $\bar{F}_1(t) \sim \text{gamma}(8, 1)$, $\bar{G}_1(t) \sim \text{gamma}(\frac{6}{k^2}, \frac{k}{2})$,

Component j: $\bar{F}_j(t) \sim \text{gamma}(8, 1)$, $\bar{G}_j(t) \sim \text{gamma}(6, \frac{1}{2})$, $j = 2, 3, 4, 5$,

where k as above is a positive number. In this example the lifetime distributions are equal for all components, thus all components have mean time to failure $\mu_i = 8$, $i = 1, \ldots, 5$. The variance in this distribution is also equal to 8. Components $2, \ldots, 5$ have mean time to repair equal to 3. The mean time to repair of component 1 is $\frac{3}{k}$, which is decreasing in k. The variance in the repair time distributions are $\frac{3}{2}$ for all components, while the availabilities are $A_1 = \frac{8k}{8k+3}$ and $A_j = \frac{8}{11}$, $j = 2, \ldots, 5$.

The results of the simulations are shown in Table 8. As for the cases presented in Tables 3 and 4 all components have the same lifetime distributions. Hence, in accordance with the results in Section 1 $I_N^{(i)}(t)$ and $I_{B-P}^{(i)}$ are practically equal. When $k = 1$, all components have identical life- and repair time distributions. Hence, in accordance with the results in Section 1 all measures give the same results in this case. As is seen from the table, the results are very similar for all three Natvig importance measures and $I_{B-P}^{(i)}$. The importance of components 1 and 2 are increasing in k, while the ones of components 3 and 4 are decreasing in k.

k	i	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$
	1	0.147	0.202	0.164	0.271	0.202
	2	0.277	0.258	0.271	0.197	0.257
1/2	3	0.089	0.082	0.087	0.063	0.082
	4	0.276	0.257	0.270	0.197	0.257
	5	0.211	0.202	0.208	0.271	0.202
	1	0.198	0.203	0.200	0.271	0.202
	2	0.260	0.256	0.259	0.197	0.257
1	3	0.083	0.082	0.083	0.063	0.082
	4	0.261	0.257	0.260	0.197	0.257
	5	0.198	0.202	0.199	0.271	0.202
	1	0.235	0.203	0.226	0.271	0.202
	2	0.249	0.256	0.251	0.197	0.257
3/2	3	0.079	0.082	0.080	0.063	0.082
	4	0.248	0.256	0.250	0.197	0.257
	5	0.189	0.203	0.193	0.271	0.202
	1	0.262	0.201	0.245	0.271	0.202
	2	0.240	0.257	0.245	0.197	0.257
2	3	0.077	0.082	0.078	0.063	0.082
	4	0.239	0.256	0.244	0.197	0.257
	5	0.182	0.203	0.188	0.271	0.202
	1	0.386	0.202	0.342	0.271	0.202
	2	0.198	0.256	0.212	0.197	0.257
6	3	0.063	0.082	0.068	0.063	0.082
	4	0.200	0.258	0.214	0.197	0.257
	5	0.153	0.203	0.165	0.271	0.202

Table 6: *Simulations of the bridge system with increasing variance in the lifetime distribution of component 1. The time horizon is* $t = 20000$.

k	Rank
1/2	$2 \approx 4 > 5 > 1 > 3$
1	$2 \approx 4 > 1 \approx 5 > 3$
3/2	$2 \approx 4 > 1 > 5 > 3$
2	$1 \approx 2 \approx 4 > 5 > 3$
6	$1 > 2 \approx 4 > 5 > 3$

Table 7: *The ranks of the extended measure of component importance corresponding to the results in Table 6.*

The same is true for $\hat{I}_B^{(i)}$. The resulting ranks, which are identical for all the three Natvig measures, are shown in Table 9. As in the previous case, component 3 is always the least important component. For all the importance measures there is a turning point at $k = 1$. For this k all components have identical life- and repair time distributions leaving components 1, 2, 4 and 5 equally important.

A more in depth analysis of the results in this and the previous section is given in [3]. As a conclusion the results of the present section very much parallel the ones of the previous one.

k	i	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$	$\hat{I}_B^{(i)}$	$I_{B-P}^{(i)}$
1/2	1	0.174	0.170	0.173	0.211	0.174
	2	0.198	0.199	0.198	0.189	0.198
	3	0.074	0.074	0.074	0.071	0.074
	4	0.323	0.323	0.323	0.308	0.322
	5	0.232	0.234	0.233	0.222	0.232
1	1	0.233	0.233	0.233	0.233	0.233
	2	0.234	0.234	0.234	0.233	0.233
	3	0.066	0.066	0.066	0.066	0.066
	4	0.233	0.234	0.234	0.233	0.233
	5	0.234	0.233	0.234	0.233	0.233
3/2	1	0.264	0.268	0.265	0.246	0.264
	2	0.250	0.249	0.250	0.257	0.251
	3	0.063	0.062	0.063	0.064	0.062
	4	0.189	0.187	0.188	0.193	0.188
	5	0.234	0.233	0.234	0.240	0.234
5/2	1	0.295	0.302	0.297	0.259	0.294
	2	0.269	0.267	0.268	0.283	0.269
	3	0.058	0.057	0.058	0.061	0.059
	4	0.143	0.141	0.142	0.150	0.143
	5	0.235	0.233	0.234	0.247	0.235

Table 8: *Simulations of the bridge system with decreasing MTTR of component 1. The time horizon is* $t = 20000$.

k	Rank
1/2	$4 > 5 > 2 > 1 > 3$
1	$4 \approx 5 \approx 2 \approx 1 > 3$
3/2	$1 > 2 > 5 > 4 > 3$
5/2	$1 > 2 > 5 > 4 > 3$

Table 9: *The common ranks of the three measures of component importance corresponding to the results in Table 8.*

4 Application to an offshore oil and gas production system

We will now look at a West-African production site for oil and gas based on a memo [9]. For this real life example we need to do some simplifications. Originally this is a multi-state system, which means that it has several functioning levels. In this paper, however, we are only considering binary systems. Thus a simplified definition of the system will be used. There are several different possible definitions, but we will use the following:

The oil and gas production site is said to be functioning if it can produce some amount of both oil and gas. Otherwise the system is failed.

Oil and gas are pumped up from one production well along with water. These substances are separated in a separation unit. We will assume this unit to function perfectly.

After being separated the oil is run through an oil treatment unit, which is also assumed to function perfectly. Then the treated oil is exported through a pumping unit.

The gas is sent through two compressors which compress the gas. When both compressors are functioning, we get the maximum amount of gas. However, to obtain at least *some* gas production, it is sufficient that at least one of the compressors is functioning. If this is the case, the uncompressed gas is burned in a flare, which is assumed to function perfectly. The compressed gas is run through a unit where it is dehydrated. This is called a TEG (Tri-Ethylene Glycol) unit. After being dehydrated, the gas is ready to be exported. Some of the gas is used as fuel for the compressors.

The water is first run through a water treatment unit. This unit cleanses the water so that it legally can be pumped back into the wells to maintain the pressure, or back into the sea. If the water treatment unit fails, the whole production stops.

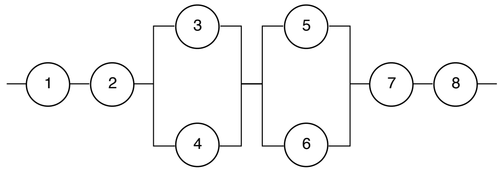


Figure 3: Model of oil and gas production site.

The components in the system also need electricity which comes from two generators. At least one generator must function in order to produce some oil and gas. If both generators are failed, the whole system is failed. The generators are powered by compressed and dehydrated gas.

Thus, the simplified production site considered in the present paper, consists of the following 8 relevant components, which are assumed to operate independently:

1. Well: A production well where the oil and gas come from.
2. Water cleanser: A component which cleanses the water which is pumped up from the production well along with the oil and gas.
3. Generator 1: Generator providing electricity to the system.
4. Generator 2: The same as Generator 1.
5. Compressor 1: A compressor which compresses the gas.
6. Compressor 2: The same as Compressor 1.
7. TEG: A component where the gas is dehydrated.
8. Oil export pump: An oil export pump.

The structure of the system is shown in Figure 3. The components 1, 2, 7 and 8 are all in series with the rest of the system, while the two generators, 3 and 4, operate in parallel with each other. Similarly the two compressors, 5 and 6, operate in parallel with each other.

Component	Failure rate	ν_i	μ_i
1	$2.736 \cdot 10^{-4}$	7.000	3654.97
2	$8.208 \cdot 10^{-3}$	0.167	121.83
3 & 4	$1.776 \cdot 10^{-2}$	1.167	56.31
5 & 6	$1.882 \cdot 10^{-2}$	1.083	53.11
7	$1.368 \cdot 10^{-3}$	0.125	730.99
8	$5.496 \cdot 10^{-4}$	0.125	1819.51

Table 10: *Failure rates, mean repair times and mean lifetimes of the components in the oil and gas production site.*

Table 10 shows the given failure rates, mean repair times and mean lifetimes of the components in the system. The time unit is *days*. The mean lifetimes are considerably larger than the mean repair times. For some components (the well, the TEG unit and the oil export pump) the mean lifetimes are actually several years.

4.1 Exponentially distributed life- and repair times

In this section we assume that the components have exponentially distributed life- and repair times. The failure rates in the lifetime distributions are the inverses of the mean lifetimes, while the repair rates are the inverses of the mean repair times. Thus, all the parameters needed in the simulations can be derived from Table 10. The time horizon t is set to 100000 days.

In Table 11 we see that $I_N^{(i)}(t)$ is equal to its extended version $\bar{I}_N^{(i)}(t)$. This is because $E[Y_i(t)]$ is very large compared to $E[X_i(t)]$ for all components. Hence, the contributions of the latter terms in Eq.(10) are too small to make any difference.

The reason for this is that the repair times of the components are much shorter than the corresponding lifetimes. Hence, the fictive prolonged repair times of the components due to the fictive minimal failures are much shorter than the fictive prolonged lifetimes due the fictive minimal repairs. Especially, the fictive prolonged repair times will, due to the much longer lifetimes, mostly end long before the next real repair. Hence, it is very unlikely that the fictive minimal failure periods will overlap. As a conclusion it is very sensible for this case study that $I_N^{(i)}(t)$ is equal to $\bar{I}_N^{(i)}(t)$.

We also observe from Table 11 that for the two equal measures the components 1, 2, 7 and 8 that are in series with the rest of the system have approximately the same importance. This can be seen by the following argument. Since $t = 100000$ days we have reached stationarity. Furthermore, for the exponential lifetime distribution $\mu_i^p = \mu_i$. If components i and j both are in series with the rest of the system, by conditioning on the state of component j and applying Eq.(14), the numerator of Eq.(19) equals $h(1_i, 1_j, \boldsymbol{A})A_iA_j$. By a parallel argument this is also the numerator of $I_N^{(j)}$.

Note also that the remaining components that are parts of parallel modules are much less important than the ones in series with the rest of the system. This is due to the very small unavailability $(1 - A_i)$ that appears as a common factor when factoring the numerator of Eq.(19). Indeed, in the exponential case we have, if $i = 3$ or 4 and $j = 5$ or 6, or vice versa, that this numerator equals

$$A_1A_2A_7A_8(1 - (1 - A_j)^2)(1 - A_i)A_i,$$

where all factors except $(1 - A_i)$ are close to 1. Furthermore, from Table 10 we see that all components 3, 4, 5 and 6 have almost identical unavailabilities, explaining why these components have identical importances.

The ranks of the component importance for the three versions of the Natvig measure are given in Table 12. We suggest to apply the common ranking based on the measures $I_N^{(i)}(t)$ and $\bar{I}_N^{(i)}(t)$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.244	0.371	0.244
2	0.249	0.267	0.249
3 & 4	0.005	0.080	0.005
5 & 6	0.005	0.077	0.005
7	0.247	0.033	0.246
8	0.241	0.013	0.241

Table 11: *Component importance using exponential distributions.*

Measure	Rank
$I_N^{(i)}(t)$	$2 > 7 > 1 > 8 > 3 \approx 4 \approx 5 \approx 6$
$I_{N,D}^{(i)}(t)$	$1 > 2 > 3 \approx 4 > 5 \approx 6 > 7 > 8$
$\bar{I}_N^{(i)}(t)$	$2 > 7 > 1 > 8 > 3 \approx 4 \approx 5 \approx 6$

Table 12: *The ranks of the component importance for the three versions of the Natvig measure according to the results given in Table 11.*

4.2 Gamma distributed life- and repair times

In this section we assume instead that the components have gamma distributed life- and repair times. More specifically, we assume that for $i = 1, \ldots, 8$, the lifetimes of the ith component have the densities:

$$f_i(t) = \frac{1}{(\beta_i)^{\alpha_i}\Gamma(\alpha_i)} t^{\alpha_i - 1} \exp(-t/\beta_i),$$

while the repair times of the ith component have the densities:

$$g_i(t) = \frac{1}{(\beta_i')^{\alpha_i'}\Gamma(\alpha_i')} t^{\alpha_i' - 1} \exp(-t/\beta_i').$$

Thus, for $i = 1, \ldots, 8$ and $j = 1, 2, \ldots$, we have:

$$\begin{aligned} E[U_{ij}] &= \mu_i = \alpha_i \beta_i, \\ Var[U_{ij}] &= \alpha_i (\beta_i)^2, \\ E[D_{ij}] &= \nu_i = \alpha_i' \beta_i', \\ Var[D_{ij}] &= \alpha_i' (\beta_i')^2, \end{aligned}$$

where $\mu_1, \ldots, \mu_8$ and $\nu_1, \ldots, \nu_8$ are given in Table 10.

By choosing different values for the density parameters it is possible to alter the variances in the lifetime distributions and still keep the expectations fixed. In order to see the effect of this on the importance measures, we focus on component 1 where we consider five different parameter combinations for the lifetime distribution. For all these combinations, the expected lifetime is 3654.97 days, but the variance varies between $1.827 \cdot 10^3$ and $1.170 \cdot 10^6$. Table 13 lists these parameter combinations. For the remaining gamma densities we use the parameters listed in Table 14 and Table 15. All parameters are chosen such that the expectations in the life- and repair time distributions match the corresponding values given in Table 10. We also use the same time horizon $t = 100000$ days as in the previous section.

Set	α_1	β_1	Variance
1	7309.940	0.500	$1.827 \cdot 10^3$
2	550.033	6.645	$2.429 \cdot 10^4$
3	101.493	36.012	$1.316 \cdot 10^5$
4	45.687	80.000	$2.924 \cdot 10^5$
5	11.422	319.994	$1.170 \cdot 10^6$

Table 13: *Parameter sets for the lifetime distribution of component 1.*

Component	α_i	β_i	Variance
2	30.000	4.062	$4.950 \cdot 10^2$
3 & 4	30.000	1.877	$1.057 \cdot 10^2$
5 & 6	10.000	5.311	$2.821 \cdot 10^2$
7	179.958	4.062	$2.969 \cdot 10^3$
8	218.219	8.338	$1.517 \cdot 10^4$

Table 14: *Parameters in the lifetime distributions of components* $2, \ldots, 8$.

Component	α'_i	β'_i	Variance
1	3.500	2.000	$1.400 \cdot 10^1$
2	0.668	0.250	$4.175 \cdot 10^{-2}$
3 & 4	3.000	0.389	$4.540 \cdot 10^{-1}$
5 & 6	1.500	0.722	$7.819 \cdot 10^{-1}$
7	1.000	0.125	$1.563 \cdot 10^{-2}$
8	1.000	0.125	$1.563 \cdot 10^{-2}$

Table 15: *Parameters in the repair time distributions of components* $1, \ldots, 8$.

Tables 16, 17, 18, 19 and 20 display the results obtained from simulations using the parameters listed in Tables 13, 14 and 15. As for the case with exponentially distributed life- and repair times, $I_N^{(i)}$ is equal to its extended version $\bar{I}_N^{(i)}$.

We now observe that for these two equal measures the components 1, 2, 7 and 8 that are in series with the rest of the system have different importances as opposed to the case with exponentially distributed life- and repair times. However, the remaining components that are parts of parallel modules are still much less important.

Furthermore, we see that the extended component importance of component 1 is increasing with increasing variances, and decreasing shape parameters α_1, all greater than 1, in its lifetime distribution. Since we have reached stationarity, this observation is in accordance with the discussion following Eq.(22) concerning the Weibull distribution.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.031	0.246	0.034
2	0.521	0.419	0.520
3 & 4	0.010	0.059	0.011
5 & 6	0.018	0.081	0.019
7	0.202	0.043	0.200
8	0.188	0.017	0.186

Table 16: *Component importance using gamma distributions. Variance of component 1 lifetimes:* $1.827 \cdot 10^3$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.107	0.244	0.109
2	0.477	0.415	0.476
3 & 4	0.009	0.059	0.010
5 & 6	0.017	0.082	0.017
7	0.194	0.043	0.193
8	0.169	0.018	0.168

Table 17: *Component importance using gamma distributions. Variance of component 1 lifetimes:* $2.429 \cdot 10^4$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.213	0.248	0.213
2	0.420	0.415	0.420
3 & 4	0.008	0.058	0.009
5 & 6	0.015	0.081	0.015
7	0.166	0.042	0.164
8	0.156	0.017	0.155

Table 18: *Component importance using gamma distributions. Variance of component 1 lifetimes:* $1.316 \cdot 10^5$.

Table 21 displays the ranks of the components according to the extended measure. Along with the increased importance, according to the extended measure, of component 1 as α_1 decreases, we observe from this table a corresponding improvement in its rank. All the other components are ranked in the same order for every value of α_1. This is as expected from Eq.(21) since the ordering is determined by its numerator. For all components except component 1 the numerator depends on the life- and repair time distributions of this component only through A_1, which is kept fixed when varying α_1. We also see that the components

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.301	0.248	0.300
2	0.375	0.414	0.376
3 & 4	0.007	0.058	0.008
5 & 6	0.013	0.081	0.014
7	0.149	0.049	0.148
8	0.134	0.018	0.133

Table 19: *Component importance using gamma distributions. Variance of component 1 lifetimes:* $2.924 \cdot 10^5$.

Component	$I_N^{(i)}(t)$	$I_{N,D}^{(i)}(t)$	$\bar{I}_N^{(i)}(t)$
1	0.476	0.239	0.475
2	0.279	0.421	0.280
3 & 4	0.006	0.058	0.006
5 & 6	0.010	0.082	0.010
7	0.111	0.043	0.111
8	0.102	0.017	0.101

Table 20: *Component importance using gamma distributions. Variance of component 1 lifetimes:* $1.170 \cdot 10^6$.

that are in series with the rest of the system are ranked according to the shape parameter α_i, such that components with smaller shape parameters are more important.

Table	Rank
16	$2 > 7 > 8 > 1 > 5 \approx 6 > 3 \approx 4$
17	$2 > 7 > 8 > 1 > 5 \approx 6 > 3 \approx 4$
18	$2 > 1 > 7 > 8 > 5 \approx 6 > 3 \approx 4$
19	$2 > 1 > 7 > 8 > 5 \approx 6 > 3 \approx 4$
20	$1 > 2 > 7 > 8 > 5 \approx 6 > 3 \approx 4$

Table 21: *The ranks of the extended component importance according to the results given in Tables 16, 17, 18, 19 and 20.*

5 Concluding remarks

In the present paper first a review of basic ideas, concepts and theoretical results, as treated in [8], for the Natvig measures of component importance for repairable systems, and its extended version, has been given. Then two three component systems and the bridge system were analysed. We saw that an important feature of the Natvig measures is that they reflect the degree of uncertainty in the life- and repair time distributions of the components. Finally, the theory was applied to an offshore oil and gas production system which is said to be functioning if it can produce some amount of both oil and gas. First life- and repair times were assumed to be exponentially distributed and then gamma distributed both in accordance with the data given in the memo [9]. The time horizon was set at 100000 days so stationarity is reached.

A finding from the simulations of this case study is that the results for the original Natvig measure and its extended version, also taking a dual term into account, are almost identical. This is perfectly sensible since the dual term vanishes because the fictive prolonged repair times are much shorter than the fictive prolonged lifetimes. The weaknesses of this system are linked to the lifetimes and not the repair times.

Component 1 is the well being in series with the rest of the system. For this component we see that the extended component importance, in the gamma case is increasing with increasing variances, and decreasing shape parameters, all greater than 1, in the lifetime distribution. This is in accordance with a theoretical result for the Weibull distribution. Along with this increased importance we also observed a corresponding improvement in its ranking.

As a conclusion we feel that the presented Natvig measures of component importance for repairable systems on the one hand represent a theoretical novelty. On the other hand the case study indicates a great potential for applications, especially due to the simulation methods developed, as presented in the companion paper [4].

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