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HYDROCARBON PRODUCTION OPTIMIZATION IN FIELDS WITH DIFFERENT OWNERSHIP AND COMMERCIAL INTERESTS

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Abstract

A main field and satellite fields consist of several separate reservoirs with gas cap and/or oil rim. A processing facility on the main field receives and processes the oil, gas and water from all the reservoirs. This facility is typically capable of processing only a limited amount of oil, gas and water per unit of time. In order to satisfy these processing limitations, the production needs to be choked. The available capacity is shared among several field owners with different commercial interests. In this paper we focus on how total oil and gas production from all the fields could be optimized. The satellite field owners negotiate processing capacities on the main field facility. This introduces additional processing capacity constraints (booking constraints) for the owners of the main field. If the total wealth created by all owners represents the economic interests of the community, it is of interest to investigate whether the total wealth may be increased by lifting the booking constraints. If all reservoirs may be produced more optimally by removing the booking constraints, all owners may benefit from this when appropriate commercial arrangements are in place. We will compare two production strategies. The first production strategy optimizes locally, at distinct time intervals. At given intervals the production is prioritized so that the maximum amount of oil is produced. In the second production strategy a fixed weight is assigned to each reservoir. The reservoirs with the highest weights receive the highest priority.

1 Introduction

Optimization is an important element in the management of multiple-field oil and gas assets, since many investment decisions are irreversible and finance is committed for the long term. Optimization of oil and gas recovery in petroleum engineering is a considerable research field, see Bittencourt & Horne (1997), Horne (2002) or Merabet & Bellah (2002). Another important research tradition focuses on the problem of modelling the entire hydrocarbon value chain, where the purpose is to make models for scheduling and planning of hydrocarbon field infrastructures with complex objectives, see van den Heever et al. (2001), Ivyer & Grossmann (1998) or Neiro & Pinto (2004). Since the entire value chain is very complex, many aspects of it needs to be simplified to be able to construct such a comprehensive model.

The purpose of the present paper is to focus on the problem of optimizing production in an oil or gas field consisting of many reservoirs, which constitutes an important component

*At the time employed by DNV.

in the hydrocarbon value chain. By focusing on only one important component we are able to develop a framework that provides insight into how an oil or gas field should be produced. The optimization methods developed here can thus be used in the broader context of a total value chain analysis. The present paper applies an already developed model framework for hydrocarbon production optimization of an oil and gas field development project. More specifically, the methodology developed in Haavardsson & Huseby (2007), Huseby & Haavardsson (2008) and Haavardsson & Huseby (2008) will be used.

We assume that state-of-the-art production profile models based on reservoir simulation models exist for every reservoir. Simplified *production profile models* can then be constructed, as described in Haavardsson & Huseby (2007). In the present paper we will utilize such production profile models in production optimization where several reservoirs share the same processing facilities. These facilities are only capable of processing limited amounts of oil, gas and water per unit of time. In order to satisfy these processing limitations, the production needs to be choked according to a production strategy. Each reservoir produces a primary hydrocarbon phase - oil or gas. In addition to the primary phases, most reservoirs also produce associated phases; gas in oil reservoirs, condensate in gas reservoirs and water.

Huseby & Haavardsson (2008) is a theoretical paper, where the problem of optimizing production strategies with respect to various types of objective functions is considered. It is shown that the solution to the optimization problem depends on certain key properties, e.g., convexity or concavity, of the objective function and of the potential production rate functions. An algorithm for finding the best production strategy and two main analytical results are presented.

Haavardsson & Huseby (2008) focuses on applied multi-reservoir production optimization, and an alternative approach to production optimization is proposed. By introducing a parametric class of production strategies the best production strategy is found using standard numerical optimization techniques.

We close this section listing the main interests of the present paper:

- The main focus of the paper is the modelling approach and the basic principles for a modelling tool for general use in examination of production strategy effects on multi-reservoir fields, with different and varying hydrocarbon phases, with individual production constraints and priorities, different owners and with the functionality to extend and cover multi fields integration in a regional / processing hub evaluation.

- The article also highlights the importance of being aware of local and global production optimization effects and the importance booking constraints may have. To study this two different production strategies are presented.

- As an illustration a case study based on real data[1] will be presented. Thus, the case study serves as a tool for the investigation of the general issues listed above. We seek knowledge that is valid beyond the numerical results obtained in the case study.

[1]In the present paper the case study is un-named and the data are made anonymous to reduce the ability to derive commercial values.

2 Model framework

2.1 Production profile model framework

The reservoir simulation output available from the reservoir simulator Eclipse [2] is used to construct simplified production profile models for each well. See Appendix A for a broad-brush introduction to simplified production profile models, or Haavardsson & Huseby (2007) for details.

To model multiple phases of production we assume that the production of each associated hydrocarbon phase can be expressed as a function of the cumulative production of the primary hydrocarbon phase. If the primary hydrocarbon phase is oil, we denote the cumulative production $Q(t)$, while $G(t)$ is used analogously for gas.

A fundamental model assumption is that the *potential production rate* of the primary hydrocarbon phase from a reservoir, can be expressed as a function of the remaining producible volume, or equivalently as a function of the volume produced. Thus, if $Q(t)$ denotes the cumulative production of the primary hydrocarbon phase at time $t \geq 0$, and $f(t)$ denotes the potential production rate at the same point in time, we assume that $f(t) = f(Q(t))$. This assumption implies that the total producible volume from a reservoir does not depend on the production schedule. In particular, if we delay the production from a reservoir we can still produce the same volume at a later time. We refer to the function f as the *potential production rate function* or *PPR-function* of the reservoir. If a reservoir is produced without any production constraints from time $t = 0$, the cumulative production function will satisfy the following autonomous differential equation:

$$\frac{dQ(t)}{dt} = f(Q(t)),$$

with the boundary condition $Q(0) = 0$.

2.1.1 A single production well

We assume that we are given a ratio expressing the units of the associated hydrocarbon phase that is produced depending on the units produced of the primary hydrocarbon phase. We refer to this function as the *associated ratio* and denote it $\psi(Q(t))$ or $\gamma(G(t))$ depending on whether oil or gas is the primary hydrocarbon phase. Although we can handle any finite number of associated phases we will assume in this paper that there is only one associated phase. Thus, we are not concerned with water production in this application. If the primary hydrocarbon phase is oil, the associated ratio expresses the Gas-Oil-Ratio (GOR). If the primary hydrocarbon phase is gas, the associated ratio expresses the Condensate-Gas-Ratio (CGR).

To model $\psi(Q(t))$ we choose to use the following representation

$$\psi(Q(t)) = \psi(0) + (\psi(V) - \psi(0)) \cdot R(t)^{P_\psi}, \tag{2.1}$$

where $R(t) = \frac{Q(t)}{V}$ denotes the fraction produced, $R(t) \in [0, 1]$, where V denotes producible volume of the primary hydrocarbon phase. The parameter P_ψ is assumed to be positive. The parameters $\psi(0), \psi(V)$ and P_ψ are estimated using the output from the reservoir simulator. Typically $\psi(Q(t))$ is increasing in $Q(t)$, reflecting the increasing quantity of gas produced per unit produced oil as the reservoir is produced.

[2]For details on Schlumberger's Eclipse Reservoir Engineering Software, see `www.slb.com.`

For $\gamma(G(t))$ we use the same representation, i.e.,

$$\gamma(G(t)) = \gamma(0) + (\gamma(V) - \gamma(0)) \cdot R(t)^{P_\gamma}, \tag{2.2}$$

where $P_\gamma > 0$. Typically $\gamma(G(t))$ is decreasing in $G(t)$, so that typically $\gamma(0) > \gamma(V)$. This reflects the decreasing quantity of condensate produced per unit produced gas as the reservoir is produced. Furthermore, we will typically choose $P_\gamma < 1$.

2.1.2 Multiple production wells

We consider oil and gas production from N wells that share a processing facility with a constant oil processing capacity K_o and a constant gas capacity K_g.

Let $\boldsymbol{I} = (I_1, \ldots, I_N)$ be the vector expressing the type of primary hydrocarbon phase of each well, so that

$$I_i = \begin{cases} 1 & \text{if the primary hydrocarbon phase of well } i \text{ is oil,} \\ 0 & \text{if the primary hydrocarbon phase of well } i \text{ is gas,} \end{cases} \tag{2.3}$$

for $i = 1, \ldots, N$. Let $\mathcal{O} = \{i \,|I_i = 1\}$ and $\mathcal{G} = \{i \,|I_i = 0\}$, so that $\mathcal{O}$ contains the indices of the oil wells and $\mathcal{G}$ contains the indices of the gas wells.

We introduce

$$P_i(t) = \begin{cases} Q_i(t) & \text{if } i \in \mathcal{O}, \\ G_i(t) & \text{if } i \in \mathcal{G}. \end{cases} \tag{2.4}$$

and assume that the PPR-functions can be written as

$$f_i(t) = \begin{cases} f_i(Q_i(t)) & \text{if } i \in \mathcal{O}, \\ f_i(G_i(t)) & \text{if } i \in \mathcal{G}, \end{cases} \tag{2.5}$$

for $i = 1, \ldots, N$. Then $\boldsymbol{P}(t) = (P_1(t), \ldots, P_N(t))$ represents the vector of cumulative primary hydrocarbon phase production functions for the N wells, and $\boldsymbol{f}(t) = (f_1(t), \ldots, f_N(t))$ the corresponding vector of PPR-functions. Thus, f_i represents the PPR-function of well i. Note that the formulation (2.5) implies that the potential production rate of one well does not depend on the volumes produced from the other wells.

A *production strategy* is defined by a vector valued function $\boldsymbol{b} = \boldsymbol{b}(t) = (b_1(t), \ldots, b_N(t))$, defined for all $t \geq 0$, where $b_i(t)$ represents the *choke factor* applied to the ith well at time t, $i = 1, \ldots, N$. We refer to the individual b_i-functions as the *choke factor functions* of the production strategy. The *actual oil production rates* from the wells, after the production is choked is given by:

$$\boldsymbol{q}(t) = (q_1(t), \ldots, q_N(t)),$$

where

$$q_i(t) = \begin{cases} b_i(t) f_i(Q_i(t)) & \text{if } i \in \mathcal{O}, \\ b_i(t)\gamma_i(G_i(t)) f_i(G_i(t)) & \text{if } i \in \mathcal{G}, \end{cases} \tag{2.6}$$

so that $q_i(t)$ either expresses the actual oil rate from an oil well or the actual condensate rate from a gas well. The *actual gas production rates* from the wells are similarly denoted

$$\boldsymbol{g}(t) = (g_1(t), \ldots, g_N(t)),$$

where

$$g_i(t) = \begin{cases} b_i(t) f_i(G_i(t)) & \text{if } i \in \mathcal{G}, \\ b_i(t)\psi_i(Q_i(t)) f_i(Q_i(t)) & \text{if } i \in \mathcal{O}, \end{cases} \tag{2.7}$$

so that $g_i(t)$ either expresses the actual gas rate from a gas well or the actual associated gas rate from an oil well.

We also introduce the total oil production rate function $q(t) = \sum_{i=1}^N q_i(t)$ and the total cumulative oil production function $Q(t) = \sum_{i=1}^N Q_i(t)$. The total gas production rate function is analogously denoted $g(t) = \sum_{i=1}^N g_i(t)$, while the total cumulative gas production function is denoted $G(t) = \sum_{i=1}^N G_i(t)$.

To satisfy the physical constraints of the wells and the processing facility, we require that for a hydrocarbon phase, the actual well production rate cannot exceed its potential production well rate. Moreover, the total well production rate cannot exceed the production capacity. These requirements imply that

$$\begin{aligned} 0 &\leq q_i(t) \leq f_i(Q_i(t)), & t \geq 0, \quad i \in \mathcal{O}, \\ 0 &\leq q_i(t) \leq \gamma_i(G_i(t)) f_i(G_i(t)), & t \geq 0, \quad i \in \mathcal{G}, \\ 0 &\leq g_i(t) \leq \psi_i(Q_i(t)) f_i(Q_i(t)), & t \geq 0, \quad i \in \mathcal{O}, \\ 0 &\leq g_i(t) \leq f_i(G_i(t)), & t \geq 0, \quad i \in \mathcal{G}, \end{aligned} \tag{2.8}$$

for $i = 1, \dots, N$ and that

$$\begin{aligned} q(t) &= \sum_{i=1}^N q_i(t) \leq K_o, \quad t \geq 0, \\ g(t) &= \sum_{i=1}^N g_i(t) \leq K_g, \quad t \geq 0. \end{aligned} \tag{2.9}$$

Expressed in terms of the production strategy $\boldsymbol{b}$, this implies that

$$0 \leq b_i(t) \leq 1, \quad t \geq 0, \quad i = 1, \dots, N, \tag{2.10}$$

and that

$$\begin{aligned} &\sum_{i \in \mathcal{O}} b_i(t) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} b_i(t) \gamma_i(G_i(t)) f_i(G_i(t)) \leq K_o, \quad t \geq 0, \\ &\sum_{i \in \mathcal{O}} b_i(t) \psi_i(Q_i(t)) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} b_i(t) f_i(G_i(t)) \leq K_g, \quad t \geq 0. \end{aligned} \tag{2.11}$$

The constraint (2.10) implies that the actual production rate cannot be increased beyond the potential production rate at any given point in time, while the constraint (2.11) states that the actual, total production rates cannot exceed the capacities of the processing facility. Let $\mathcal{B}$ denote the class of production strategies that satisfy the physical constraints (2.10) and (2.11). We refer to production strategies $\boldsymbol{b} \in \mathcal{B}$ as *valid production strategies.*

We need to specify how the choke factors are determined. In this paper we will determine the choke factors sequentially. A sequential approach only produces one of the phases at the plateau level. First the choke factors are determined so that the constraint of the primary hydrocarbon phase is not exceeded. Then, if the constraint of the associated hydrocarbon phase is exceeded the choke factors are modified accordingly.

Definition 2.1 *We say that* $\boldsymbol{x} > \boldsymbol{y}$ *if* $x_i \geq y_i \quad \forall i$ *and* $\exists\, j \in \{1, \ldots, n\}$ *such that* $x_j > y_j$. *Let* $\boldsymbol{b}, \boldsymbol{b}' \in \mathcal{B}$ *be two production strategies. If* $\boldsymbol{b}'(t) > \boldsymbol{b}(t)$ *implies that either*

$$\sum_{i \in \mathcal{O}} b_i'(t) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} b_i'(t) \gamma_i(G_i(t)) f_i(G_i(t)) > K_o$$

or

$$\sum_{i \in \mathcal{O}} b_i'(t) \psi_i(Q_i(t)) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} b_i'(t) f_i(G_i(t)) > K_g,$$

then $\boldsymbol{b}$ *is an admissible production strategy. We denote the class of admissible production strategies* $\mathcal{B}'$.

2.2 Production strategies and objective functions

2.2.1 Strategy for local production optimization

Consider a production strategy that optimizes production locally at predefined, discrete time intervals. The production is prioritized so that the oil production is accelerated following an argument that the oil has the highest value and accelerated production would be beneficial from a net present value perspective if disregarding other potentially overriding effects as e.g. gas capacity utilization and oil price assumptions. This is obtained by assigning the highest priority to the well with the highest oil to gas production ratio, meaning first sorting the oil wells after the GOR in ascending order followed by the gas wells sorted after CGR in descending order. The production at time t is thus prioritized strictly according to $\boldsymbol{\pi} = (\pi(1), \ldots, \pi(N))$. At the next decision point, i.e., at time $t + \delta$, the procedure is repeated.

To be a bit more precise we start by dividing a finite time horizon $[0, T]$ into S intervals. Thus we obtain a partition $[0, \delta, 2\delta, \ldots, (S-1)\delta, T]$, where $\delta = T/S$. Let ϕ^l denote the objective function of the local production strategy. At time $t = 0$ ϕ^l is initialized so that $\phi^l = 0$. At time t the following algorithm is used:

Algorithm 2.2 STEP 1. *Sort the wells by any predefined order given by commercial agreements or other priorities. Sort the remaining wells by first sorting the oil wells after GOR in ascending order followed by the gas wells sorted after CGR in descending order. Denote the resulting permutation vector* $\boldsymbol{\pi}$.

STEP 2. *Find the number of producing wells* $i_c = 1 + min(i_q, i_g)$ *where* i_q *and* i_g *are the largest integers that fulfill*

$$\begin{aligned}
\sum_{j \leq i_q, \pi(j) \in \mathcal{O}} f_{\pi(j)}(Q_{\pi(j)}(t)) + \sum_{j \leq i_q, \pi(j) \in \mathcal{G}} \gamma_{\pi(j)}(G_{\pi(j)}(t)) f_{\pi(j)}(G_{\pi(j)}(t)) &\leq K_o \\
\sum_{j \leq i_g, \pi(j) \in \mathcal{G}} f_{\pi(j)}(G_{\pi(j)}(t)) + \sum_{j \leq i_g, \pi(j) \in \mathcal{O}} \psi_{\pi(j)}(Q_{\pi(j)}(t)) f_{\pi(j)}(Q_{\pi(j)}(t)) &\leq K_g
\end{aligned}$$

Note that if $min(i_q, i_g) = N$ *choking is not necessary. We let* $\boldsymbol{b} = \mathbf{1}$ *in this case. If* $min(i_q, i_g) < N$ *the (time-dependent) choke factors are given as*

$$b_{\pi(i)} = \begin{cases} 1, & i < i_c, \\ b_c, & i = i_c, \\ 0, & i > i_c, \end{cases} \tag{2.12}$$

where $b_c = min(b_q, b_g)$ *and*

$$b_q = \begin{cases} \frac{K_o - \sum_{j<i_c} q_{\pi(j)}(t)}{f_{\pi(i_c)}(Q_{\pi(i_c)}(t))}, & \pi(i_c) \in \mathcal{O}, \\ \frac{K_o - \sum_{j<i_c} q_{\pi(j)}(t)}{\gamma_{\pi(i_c)}(G_{\pi(i_c)}(t)) f_{\pi(i_c)}(G_{\pi(i_c)}(t))}, & \pi(i_c) \in \mathcal{G}, \end{cases} \tag{2.13}$$

and

$$b_g = \begin{cases} \frac{K_g - \sum_{j<i_c} g_{\pi(j)}(t)}{\psi_{\pi(i_c)}(Q_{\pi(i_c)}(t)) f_{\pi(i_c)}(Q_{\pi(i_c)}(t))}, & \pi(i_c) \in \mathcal{O}, \\ \frac{K_g - \sum_{j<i_c} g_{\pi(j)}(t)}{f_{\pi(i_c)}(G_{\pi(i_c)}(t))}, & \pi(i_c) \in \mathcal{G}. \end{cases} \tag{2.14}$$

STEP 3. *Update* ϕ^l*, so that*

$$\phi^l = \phi^l + \int_t^{t+\delta} \sum_{i=1}^{N} \{q_i(u) + \alpha g_i(u)\} e^{-ru} du.$$

Algorithm 2.2 is repeated at every grid point in the partition $[0, \delta, 2\delta, \ldots, (S-1)\delta]$. The parameter α converts a unit of gas into an oil unit equivalent. Thus, we are capable of comparing the energy amount in gas versus oil. In this paper we use $\alpha = 0.001$, as stated by the Norwegian Petroleum Directorate [3].

Due to the nature of the local production strategy, the production rates of some of the individual wells might fluctuate in periods. The fluctuation occurs when it is equally beneficial to produce from two or more wells, so that when the wells compete for capacities they will alternate between being produced in one period and choked the next. The primary purpose of the local production strategy is to give decision support to project teams. The focus can for example be the assessment of different infrastructure investment alternatives. Hence, we are interested in the resulting cash flows of these different alternatives so that we can ultimately select and recommend one of the alternatives. The purpose is not to give the obtained production strategy as an input for long-term production planning to a field manager. In the case of fluctuating production it would not be advisable to produce the wells exactly as prescribed by the local production strategy.

2.2.2 Strategy for fixed-weight production optimization

The following production strategy is introduced in Haavardsson & Huseby (2008). Consider the set

$$\mathcal{Q} = [0, V_1] \times \cdots \times [0, V_N], \tag{2.15}$$

where $V_1, \ldots, V_N$ are the recoverable volumes of the primary hydrocarbon phase from the N reservoirs. We then introduce the subset $\mathcal{M}_o \subseteq \mathcal{Q}$ given by:

$$\mathcal{M}_o = \{\boldsymbol{Q} \in \mathcal{Q} : \sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \gamma_i(G_i(t)) f_i(G_i(t)) \geq K_o\}, \tag{2.16}$$

so that $\mathcal{M}_o$ the points in $\mathcal{Q}$ where the oil production rate can be sustained at its plateau level. Furthermore we introduce the *oil plateau length* defined as

$$T_{K,o} = T_{K,o}(\boldsymbol{b}) = \sup\{t \geq 0 : \sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \gamma_i(G_i(t)) f_i(G_i(t)) \geq K_o\}. \tag{2.17}$$

[3] `http://www.npd.no/English/Om+OD/Nyttig/Olje-ABC/maaleenheter_oljeoggass.htm`

First we explain intuitively how the fixed-weight strategy was constructed in single-phase production, for the moment neglecting the gas constraint expressed in (2.11). Then we will explain how the fixed-weight strategy can be modified to handle two-phase production.

A simple production strategy can always be constructed using the same choke factor for all the reservoirs. That is, we let $b_i(t) = c(t)$, $i = 1, \dots, N$. For such a production strategy to be admissible $c(t)$ must satisfy the following:

$$\sum_{i \in \mathcal{O}} c(t) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} c(t) a_i(G_i(t)) f_i(G_i(t)) = \min\{K_o, \sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} a_i(G_i(t)) f_i(G_i(t))\}. \tag{2.18}$$

Thus, for $0 \leq t \leq T_{K,o}$, we have:

$$c(t) = \frac{K}{\sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \gamma_i(G_i(t)) f_i(G_i(t))}, \tag{2.19}$$

while $c(t) = 1$ for all $t > T_{K,o}$, neglecting for the moment the gas constraint expressed in (2.11). Note that since $\sum_{i \in \mathcal{O}} c(t) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} c(t) a_i(G_i(t)) f_i(G_i(t)) \geq K_o$ for $0 \leq t \leq T_{K,o}$, the common choke factor, $c(t)$ will always be less than or equal to 1. A production strategy defined in this way, will be referred to as a *symmetry strategy*. We observe that when a symmetry strategy is used, the available production capacity is shared proportionally among the reservoirs such that none of the reservoirs are given any kind of priority. The idea now is to expand this class so that some reservoirs can be prioritized before others. To facilitate this we start out by considering production strategies where for $0 \leq t \leq T_{K,o}$ the choke factors are given by:

$$b_i(t) = w_i c(t), \qquad i = 1, \dots, N, \tag{2.20}$$

where $w_1, \dots, w_N$ are positive real numbers representing the relative priorities assigned to the N reservoirs, and where $c(t)$ is chosen so that the strategy is admissible. For $t > T_{K,o}$, we define $b_i(t) = 1$, $i = 1, \dots, N$. Note that if $w_1 = \cdots = w_N$ we get a symmetry strategy.

In order to ensure admissibility, $c(t)$ must be chosen so that:

$$\begin{aligned} & \sum_{i \in \mathcal{O}} w_i c(t) f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} w_i c(t) \gamma_i(G_i(t)) f_i(G_i(t)) \\ = \; & \min\{K, \sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \gamma_i(G_i(t)) f_i(G_i(t))\}. \end{aligned}$$

Thus, for $0 \leq t \leq T_{K,o}$ the choke factors are given by:

$$b_i(t) = w_i c(t) = \frac{w_i K}{\sum_{j \in \mathcal{O}} w_j f_j(Q_j(t)) + \sum_{j \in \mathcal{G}} w_j \gamma_j(G_j(t)) f_j(G_j(t))}, \qquad i = 1, \dots, N. \tag{2.21}$$

Unfortunately, this definition does not guarantee that the choke factors are less than or equal to 1. To fix this problem, we instead let:

$$b_i(t) = \min\{1, w_i c(t)\}, \qquad i = 1, \dots, N. \tag{2.22}$$

While this ensures that the resulting production strategy is valid, it makes the calculation of $c(t)$ slightly more complicated. To ensure admissibility, $c(t)$ must now be chosen so that:

$$\begin{aligned} & \sum_{i \in \mathcal{O}} \min\{1, w_i c(t)\} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \min\{1, w_i c(t)\} \gamma_i(G_i(t)) f_i(G_i(t)) \\ = \; & \min\{K, \sum_{i \in \mathcal{O}} f_i(Q_i(t)) + \sum_{i \in \mathcal{G}} \gamma_i(G_i(t)) f_i(G_i(t))\}, \end{aligned}$$

see Haavardsson & Huseby (2008) for details on how $c(t)$ is calculated in single-phase production. Then, if the gas constraint K_g is exceeded with this choice of $c(t)$, the choke vector $\boldsymbol{b}(t)$ is modified so that

$$\sum_{i\in\mathcal{O}} \min\{1, w_i c(t)\}\psi_i(Q_i(t))f_i(Q_i(t)) + \sum_{i\in\mathcal{G}} \min\{1, w_i c(t)\}f_i(G_i(t)) = K_g. \tag{2.23}$$

By varying the weights $w_1, \ldots, w_N$ in $\mathbb{R}_+^N$ a whole range of admissible production strategies is obtained. We will refer to such production strategies as *fixed-weight strategies*. It is straight-forward to show that

$$\boldsymbol{b}(\boldsymbol{w}) = \boldsymbol{b}(\lambda \boldsymbol{w}) \tag{2.24}$$

for any $\lambda > 0$. Thus, to avoid over-parametrization, the dimension of the search space is reduced by fixing the value of one of the weights, e.g., by letting $w_N = 1$, see Haavardsson & Huseby (2008) for details.

A numerical algorithm is used to maximize the following objective function

$$\phi_{C,r}(\boldsymbol{b}) = \int_0^\infty I\{q(u) \geq C\}\{q(u) + \alpha g(u)\}e^{-ru}du, \quad r \geq 0 \tag{2.25}$$

with respect to the vector of weights $\boldsymbol{w} = (w_1, \ldots, w_N)$, see Haavardsson & Huseby (2008) for details. We denote the vector of weights that maximizes $\phi_{C,r}$ in (2.25) $\boldsymbol{w}^*$. The parameter r may be interpreted as a discount factor, while the parameter C represents a threshold value for total production, i.e. , all wells are shut down when the total production is below this total field production rate. As in Section 2.2.1, the parameter α converts one unit of gas into one oil unit equivalent and is set equal to 0.001. $\phi_{C,r}$ in (2.25) expresses *discounted total production.*

The fixed-weight production strategy can be used in decision support, as the local production strategy defined in Section 2.2.1. Using the fixed-weight strategy for production planning and forecasting we avoid the fluctuations we might experience using the local strategy as discussed in Section 2.2.1, which is clearly an advantage. However, the weights assigned to each reservoir are fixed over the life of the field, which is clearly a disadvantage if the chosen fixed-weight production strategy is not optimal. If it can be proved that an optimal production strategy can always be found within the parametric class of fixed-weight strategies, this does not represent a problem. In Haavardsson & Huseby (2008) it is explained that in single-phase production optimization an optimal production strategy can always be found within the parametric class of fixed-weight strategies. A forth-coming paper will extend the framework to two-phase production and examine the optimality properties of the parametric class in two-phase production.

3 Description of the case study

In the case considered two parties referred to as the *main field* and the *satellite field* are involved in offshore[4] oil and gas production. The main field consists of separate reservoirs containing gas or gas cap with oil rim, as illustrated in Figure 1. In reservoirs with gas cap and oil rim, the oil must be produced before the gas cap to avoid significant loss in oil

[4] In principle the problems considered also apply to onshore fields; however in this specific case offshore fields are considered.

recovery due to pressure depletion. The oil and gas are processed to export specification on a central production facility.

The satellite field consists of one gas reservoir and one oil reservoir, with associated condensate and gas, respectively. The satellite field is developed with two gas production wells and one oil production well. The oil and gas of the satellite field are sent to the main field in pipelines, where it is being processed at the processing facility of the main field. The main field and the satellite field have different owners and hence different commercial interests regarding production optimization.

Relating the case study to the notation and model framework of Section 2 the number of wells is 16. Thus, $N = 16$ and the vector $\boldsymbol{I}$ expressing the type of hydrocarbon phase of each well is $\boldsymbol{I} = (I_1, \ldots, I_N) = (I_1, \ldots, I_{16}) = (1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0)$.

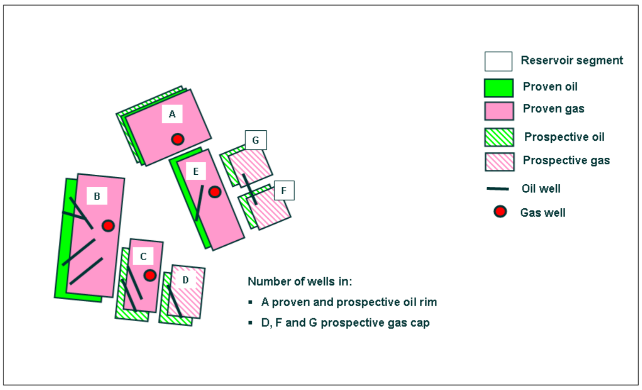

Figure 1: *An overview of the seven reservoirs of the main field. Oil is proven in the reservoirs A, B and E, while gas is proven in the reservoirs A, B, C and E. There are oil prospects in the reservoirs A, C, D, F and G. There are gas prospects in the reservoirs D, F and G.*

4 Optimization of total production under booking constraints

The satellite field and the main field have agreed to allocate a share of the main field processing capacities to the satellite field. The allocated capacities are in the following called booking constraints. Table 1 lists the booking constraints of the satellite field in percent of the processing capacities of the main field. The main field will thus use the remaining capacities for its own production, as long as its processing capacities are not exceeded. [5] Note that these booking constraints necessitate modification in the capacity constraints introduced in (2.9), yielding different capacity constraints for each year the booking constraints apply.

Since the satellite field has booked the capacities specified in Table 1 it is in its self-interest to exploit this capacity. We are interested in analyzing the effect of lifting the booking constraints, since different owners with potentially conflicting commercial interests

[5] In the implementation the main field uses the total capacity minus booked capacity. In reality one would expect that the total field would use total capacity minus the capacity actually used by the satellite field.

	Year												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Gas	21.5	23.2	21.5	21.5	21.5	19.8	18.5	14.9	11.6	9.9	8.9	7.3	6.6
Oil	19.2	14.8	12.2	10.6	8.8	6.3	5.6	3.7	3.4	2.8	3.0	2.3	1.5

Table 1: *Booking constraints of the satellite field stated in percent of the processing capacities at the main platform. The main field uses the remaining capacities to process its own hydrocarbons.*

are excepted to have different preferences. If total discounted production increases when the booking constraints are lifted, both owners may benefit from this. If both owners benefit when the booking constraints are lifted, it is sensible to do so. If one owner benefits and the other suffers, it may still be beneficial to lift the constraints, if the gain of the profiting owner exceeds the loss of the suffering owner. In this case the profiting owner may buy out the suffering owner, compensating him for his loss. This way all owners benefit if total production increases. If both owners suffer when the booking constraints are lifted, or the gain of the beneficiary owner does not exceed the loss of the losing owner, it is not sensible to lift the booking constraints. However, if this is the case, there may exist booking constraints that increase total production. Hence, a new optimization problem arises, where the total discounted production of the owners is maximized. The booking constraints are the free parameters in this optimization problem. We leave this optimization problem for future research.

Lifting the booking constraints may result in a radical change in the production rates of each party. Reservoir and production engineering considerations often play an important role in production optimization. High production rates may have negative effect on reservoir behaviour. Such effects are not addressed in this paper. In real life such reservoir considerations need to be taken into account.

The local and fixed-weight production strategy described in the sections 2.2.1 and 2.2.2 will be compared. Since we are interested in the effect of lifting the booking constraints, we calculate the production for the satellite field and the main field with and without booking constraints. The satellite field production *with* booking constraints is calculated optimizing the production strategies as described in the sections 2.2.1 and 2.2.2. The booking constraints specified in Table 1 are used. The production for the main field with booking constraints is calculated analogously. In the local optimization calculations, two gas wells, i.e. , well 4 and 5, have received fixed priority in the production phasing. This is done as there is an underlying assumption in the applied reservoir simulation results that there will be early gas production from the respective two reservoirs. If this assumption is not accounted for, the results will not reflect the expected physical performance of the reservoirs.

The satellite and main field production *without* booking constraints is calculated optimizing the production strategies as described in the sections 2.2.1 and 2.2.2. In the local optimization calculations, the two gas wells still received fixed priority in the production phasing. The production rates of the remaining wells are found using Algorithm 2.2. For the fixed-weight production strategy we use $\phi_{C,r}(\boldsymbol{b})$ specified in (2.25) as an objective function. We denote this production strategy $\boldsymbol{b}_C^*$, where the subscript C denotes *Combined*. The satellite and main field production without constraints is then found by aggregating the oil and gas production from all the satellite and main wells, respectively, using strategy $\boldsymbol{b}_C^*$.

Having inspected individual gas well rates, the fixed-weight production strategy assigns a fair amount of gas production from day one from well 4 and 5 which is in accordance with some of the main assumptions in and results from the reservoir simulation.

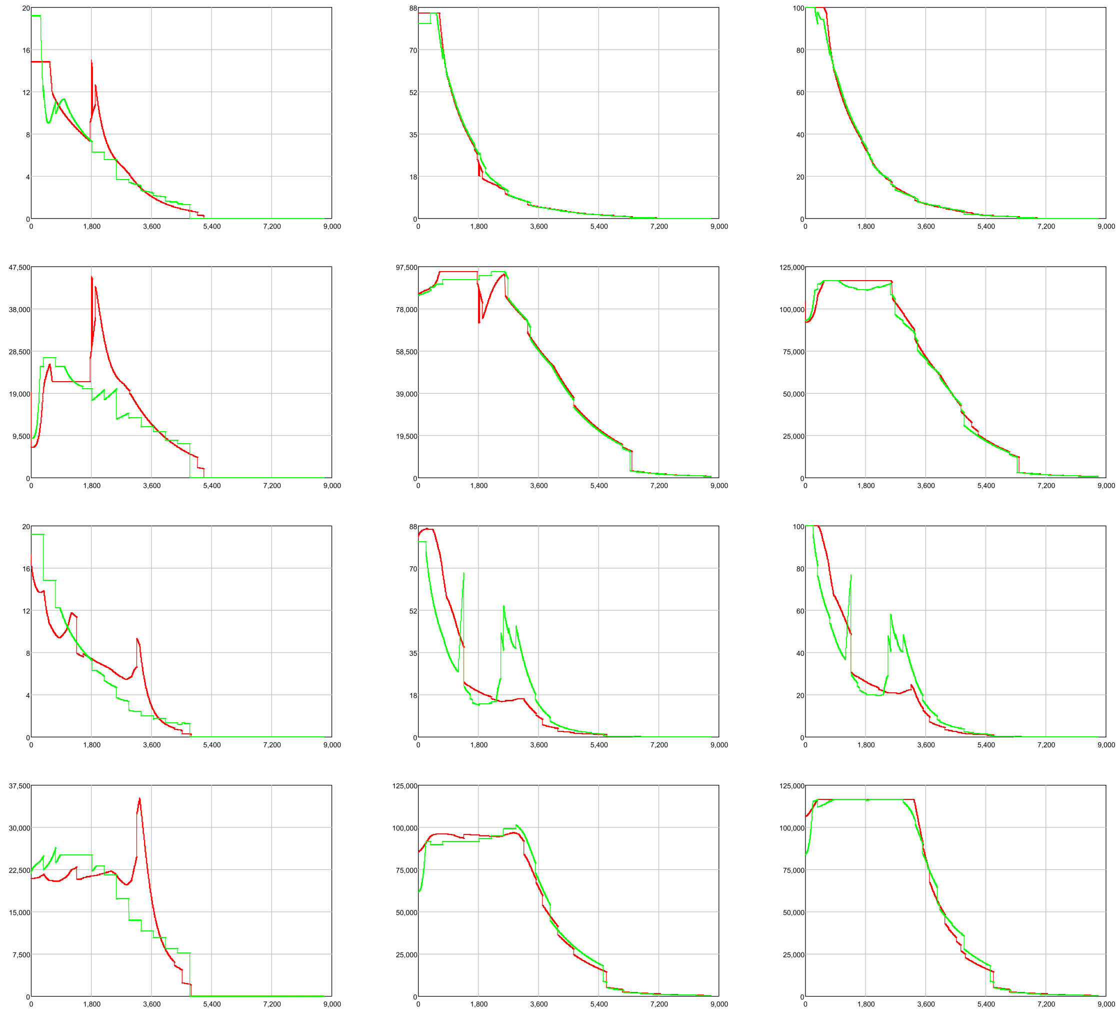

Figure 2: *In every row the satellite field, the main field and the total are displayed in the left, middle and right panel, respectively. The red and green graph display the production rates without and with booking constraints, respectively. The upper two rows show the production rates for the local strategy, while the lower two rows show the production rates for the fixed-weight strategy. The first and third rows display oil rates, while the second and fourth rows display gas rates. A coarser and standardized scale is used in the plots.*

Table 2 summarizes the results of the calculations. The results indicate that it is beneficial to lift the constraints with both strategies. With the local strategy the discounted production increases with 1.5% when the booking constraints are lifted, while the corresponding increase with the fixed-weight strategy is 1.7%. However, with the local strategy the satellite field benefits far more than the main field from lifting the constraint, while it is the other way around with the fixed-weight strategy. To understand this we take a look at the actual production rates. Figure 2 shows the resulting total production rates of oil and gas with

Main field						
Production strategy	Oil	Gas	Total with booking	Oil	Gas	Total without booking
Local	22.6	58.0	80.6	22.7	58.0	80.7
Fixed-weight	21.3	58.5	79.8	22.5	59.1	81.7
Satellite field						
Production strategy	Oil	Gas	Total with booking	Oil	Gas	Total without booking
Local	4.9	11.8	16.8	5.2	13.1	18.3
Fixed-weight	5.2	13.2	18.5	5.1	13.2	18.3
Main field and satellite field combined						
Production strategy	Oil	Gas	Total with booking	Oil	Gas	Total without booking
Local	27.5	69.9	97.4	27.8	71.1	98.9
Fixed-weight	26.6	71.8	98.3	27.6	72.4	100.0

Table 2: *Results with and without booking constraints. All numbers are discounted production of oil equivalents, stated in* kSm^3*, as measured in percent using total discounted production of main field and satellite field combined without booking constraints with the fixed-weight strategy as a base case.* $\alpha = 0.001$ *has been used to convert gas into oil equivalents.*

and without booking constraints for the two production strategies.

For the local strategy we observe that lifting the booking constraints has a large impact on the discounted gas production of the satellite field. The gas can be produced far more efficiently when the constraints are lifted for this field. Without the constraints the main field manages to maintain its gas plateau level for approximately 3.5 years, i.e. ,from approximately 600 days until 1,800 days. Then the gas plateau level cannot be sustained anymore and the satellite field is given an increasing share of the production capacity. In fact, for a long period from approximately 1,800 days until 2,700 days, the local production strategy without constraints assigns far less gas production to the main field than it obtained with the quotas. As a result, the satellite field can now produce far more than it could with the booking constraints in place. This advantage is held for several years. This positive effect on the discounted production is reduced by heavier discounting due to the delay in time, but the advantage of the increased production by far outweighs the disadvantage represented by the delay.

With the fixed-weight strategy it is the main field that benefits from lifting the constraints. The main field is able to sustain a very high gas production for a very long time, almost 3,000 days. The satellite field suffers from this and is allocated a relatively low gas production in this period. For the main field the oil is produced far more efficiently with the fixed-weight strategy when the booking constraints are lifted. Again, this efficient oil recovery is at the expense of the resulting lower share the satellite field receives. The satellite field can produce more when the main field goes into decline, as we observed with the local strategy. However, since the main field is able to maintain a high gas production rate for a very long time, the heavy discounting reduces the advantage of this unrestricted production. Furthermore, the efficient main field oil and gas production the first 3,000 days leads to relatively low satellite field production in this period. The satellite field needs a large increase in later production to balance out the loss earlier on. From Table 2 we see that the satellite field experiences a loss of 1% when the constraints are lifted with the fixed-weight strategy, i.e., the reduction in total discounted production from 18.5 % to 18.3 % relative to the base case.

Comparing the local strategy and the fixed-weight strategy without booking constraints, the total discounted production of the fixed-weight strategy is 1.1% larger than total discounted production of the local strategy.

5 Conclusions

This paper has analyzed production of oil and gas fields with different ownership and commercial interests. Satellite field booking constraints are negotiated due to different ownerships in field and an important issue is to assess the effects imposed by these constraints. Two different production strategies have been compared, with respect to performance measured in discounted production of oil equivalents.

The modelling results highlight the importance of the booking constraints. In particular the results obtained in the case study indicate that the total wealth expressed in discounted production of oil equivalents created from the satellite field and main field combined can increase when the booking constraints are lifted using both production strategies. The gain for the society as a whole thus increases. Using terminology from *game theory*, see Myerson (1991), both production strategies mimic the behaviour of a positive sum game since total discounted production increases in both cases when the booking constraints are lifted. Producing with the local strategy the satellite field receives the lion's share of the gain. Since the main field does not sustain gas plateau level for a very long time when the constraints are lifted, and the main field subsequently for a long period receives a lower share of the production capacity than it received with the booking constraints in place, the gas of the satellite field can be produced far more efficiently. Selecting the fixed-weight strategy the main field is able to sustain a high gas plateau level for a substantial amount of time. When the gas production of the satellite field is let in, it happens so late that the discounting effect outweighs the advantage of being able to produce unrestrictedly. Furthermore, the satellite field has to produce effectively and fast later in the production period to offset the loss in discounted production it suffered early in the production period. This loss is caused by the high proportion of the production capacity the main field received in this period. Thus, with the fixed-weight strategy it is the main field that benefits from lifting the constraints.

A A brief introduction to multi-segmented production profiles using ordinary differential equations

Single Arps curves, introduced by Arps (1945) model the production rate function and the cumulative production function mathematically through a one-way, causal relation. In Haavardsson & Huseby (2007) this approach is extended to multiple segments so that a combination of Arps curves may be used to get a satisfactory fit to a specific set of production data.

To also take into account various production delays, the dynamic two-way relation between the production rate function and the cumulative production is modelled in terms of a differential equation. The relation between the production rate function, q, and the cumulative production function, Q, should be of the following form:

$$q(t) = f(Q(t)), \qquad \text{for all } t \geq 0, \tag{A.1}$$

with $Q(t_0) = 0$ as a boundary condition.

The differential equation approach can also be extended to the more general situation where the production rate function consists of s segments. For each segment we assume that we have fitted a model in terms of a differential equation on the form given in (A.1). In order to connect these segment models, we need to specify a *switching rule* describing when to switch from one segment model to the next one. We define a switching rule based on the produced volume. By using this switching rule, we obtain a model for the combined differential equation.

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