

# Computational Analysis of Geometric Nonlinear Effects in Adhesively Bonded Single Lap Composite Joints

Alfred Andersen

Harald Osnes \*

Mechanics Division, Dept. of Mathematics,  
University of Oslo, P.O. Box 1053 Blindern,  
N-0316 Oslo, Norway

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## Abstract

It is well-known that geometric nonlinear effects have to be taken into account when the ultimate strength of single lap composite joints are studied. In the present paper we investigate for which level of loads or prescribed end displacements nonlinear effects become significant and how they appear. These aspects are studied by comparing finite element results obtained from geometric nonlinear models with the results from the linear ones. The well-known software package *ANSYS* is applied in the numerical analysis together with a self-implemented module in the C++ library *Diffpack*.

The joints examined are made of cross-ply laminates having 0 or 90 degree surface layers. A combined cross-ply/steel joint and an isotropic joint made of steel are also studied. All the models except the all-steel one are assembled with adhesives, while the latter is welded.

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\*Corresponding author. E-mail: osnes@math.uio.no, Fax: +47 22 85 43 49

Through the investigation a considerable departure from linear behaviour has been detected for a large regime of prescribed end displacements or external loads. Geometric nonlinear effects begin to develop for external loads that produces stresses which are far below ultimate strength limits and for average longitudinal strains that are less than 0.5 percent. It has also been detected that the distribution of materials within the joint has some influence on the nonlinear behaviour. Thus, geometric nonlinear methods should always be applied when single lap (or other non-symmetric) composite joints are analyzed.

**Keywords:** *Single lap joint; geometric nonlinear effects; composites*

## Introduction

The use of fibre composites has shown a tremendous growth in many fields during the last decades. The application of composite materials range from trivial, industrial products such as boxes and covers produced in enormous numbers each day to pipelines and large, crucial, load bearing parts of constructions. Composites are also extensively used in the aerospace and marine industries [14, 15]. Important reasons for this popularity are the high strength (and stiffness) to weight ratio, the possibility of controlling the anisotropy and the fact that fibre composites are resistant to corrosion. The possibility of making products of almost any geometry is also of great advantage. As a result of this, composites have been used more and more frequently in various combinations and situations over the last years [8].

Due to the comprehensive use, the need for joining composite components to construction details made of composites or other materials is obvious. These joints can be constructed in numbers of ways, such as bolted or other mechanically fastened joints or as bonded joints. Adhesively bonded joints have an advantage over traditional mechanically fastened joints in that they avoid drilled holes, consequently broken fibres, and stress concentrations [12]. There are also benefits to be achieved from using adhesives instead of welding in numerous occasions. Adhesively bonded joints have therefore been adopted when joining metal parts such as aluminum in a number of high performance constructions, e.g. sports cars. One can mention several reasons for this - better fatigue behaviour, greater construction stiffness, lower weight and the possibility of joining different

materials [4, 8]. These joints are also esthetically superior to bolted and welded joints as they can easily be made nearly invisible. Since it is very difficult to find out whether an adhesively bonded joint is good or bad without damaging the joint in a test, these joints have to be made with a higher degree of accuracy than welded or bolted joints. Moreover, hidden errors in the adhesive-adherent region are of great danger as they can be hard to detect. There is also some uncertainty regarding their behaviour over a long period of time. Furthermore, joints can be symmetric (e.g. double lap joints) or non-symmetric (e.g. single lap joints) [16]. The single lap joint is the least capable of these two because the eccentricity of this type of geometry generates significant bending of the adherents that magnifies the peel stresses [4, 8, 9] and increases the longitudinal normal stresses. On the other hand, in some applications the single lap joint is the only applicable type of joint and therefore the only alternative. There are of course a lot of possibilities regarding modification of the single lap geometry. Hildebrand discusses several modified geometries in [3]. We have chosen a relatively simple single lap geometry with a 45 degrees fillet slope. This type of joint is not among the strongest available but it has the advantage that the plates do not have to be modified anyway in the joining process. In this paper we will keep our main focus at this specific type of single lap joints.

Adhesively bonded joints have been investigated in the literature for some time. In some papers, e.g. Hildebrand [3] and Kairouz and Matthews [5], it is claimed that geometric nonlinear effects have to be taken into account when the ultimate strength of non-symmetric composite joints is being studied. However, they do not investigate for which values of applied loads or prescribed end displacements nonlinear effects become significant.

The practice of taking geometric nonlinear effects into account may seem unnecessary since the average longitudinal strains are small, as a matter of fact less than 0.6 percent for loads near the ultimate strength limit. Consequently, we wish to pose the following questions, which the forthcoming investigation will address.

- i) For which levels of load are geometric nonlinear effects significant?
- ii) Are the geometric nonlinear effects due to the non-symmetric geometry?
- iii) Which role does the distribution of materials play?

In the present study we will focus on these aspects by comparing finite element

results from geometric nonlinear models with linear simulations. In order to provide a clean analysis in which geometric nonlinear effects related to single lap joints are in focus, all the materials are assumed to behave linearly. This assumption is also made by, e.g., Tong et.al. [13] and Kairouz and Matthews [5].

## Numerical analysis

The single lap joints are assumed wide compared to the thickness of the plates and the length of the overlap region. Furthermore, each laminated plate consists of plies with fibre directions 0 and 90 degrees. The adhesive is uniformly distributed in the overlap and has a uniform thickness. Thus, out-of-plane bending is avoided, and the joint can be investigated as a 2-D plane strain problem. The well-known software package *ANSYS* [1, 10] is applied in the analysis. Additionally, the relatively complicated expressions defining relations between stresses and strains (including modifications related to the geometric nonlinear model from Zienkiewicz and Taylor [7]) for laminated composites are derived and implemented in a general elasticity module using the commercial C++ library *Diffpack* [2]. This combination of the use of a relatively robust method in *ANSYS* with a self-implemented module in *Diffpack*, in which all programming details are known, has been very fruitful and has provided valuable insight into important modeling aspects (for example, it is observed that certain 2-D anisotropic problems have to be treated with extreme care if *ANSYS* is used, cf. Tsai et.al. [6]). The agreement between the numerical results obtained by *ANSYS* and *Diffpack* is excellent for the problems in the present investigation although the finite element meshes are not identical.

The numerical, 2-D domain is shown in figure 1, which also defines the boundary conditions applied. As the figure shows, the  $x = 0$  line is prevented from moving in the  $x$  direction while the upper boundary from  $x = 0$  to  $x = 5$  and the lower boundary from  $x = 45$  to  $x = 50$  is prevented from moving in the  $y$  direction. A prescribed displacement is then applied at the right hand side of the model. This is a more realistic way of simulating a test machine than applying nodal forces for each layer, because the stresses in each layer depend on the stiffness of the layers. The same procedure is followed by Kairouz and Matthews in [5].

The laminated plates consist of eight plies with thickness 0.125 mm. Furthermore, the thickness of the adhesive layer is 0.1 mm as shown in figure 1. Notice

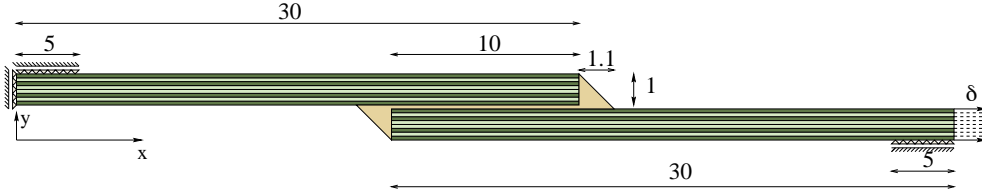


Figure 1: The 2-D domain for the numerical simulations. (Not to scale)

the different length scales along the  $x$  and  $y$  direction in the illustration. Two different stacking sequences are investigated in the first place:  $[0/90]_{2s}$  and  $[90/0]_{2s}$ . Later we will study the effects of other materials used as an additament to the composites. The meshes, on the other hand, consist mainly of four-noded bilinear elements. In the *ANSYS* models the PLANE13 elements are used with standard shape functions and displacement degrees of freedom. Consequently the elements have the same properties as those used in the *Diffpack* models. There are three elements through the thickness of each ply, and through the thickness of the adhesive layer there are four elements in the *ANSYS* model and eight in the *Diffpack* model. This is more than required to capture the stress distribution in the joint. In addition to the bilinear elements there are some three-noded triangular elements (CST) in the spew fillet. The total number of elements is approximately 10000. Two mesh examples are shown in figure 2. The *ANSYS* mesh at the left hand side has a free-meshed spew fillet while the *Diffpack* mesh on the right hand side has a very stringent mesh compared to the latter. The results are, however, not significantly affected by this.

The linear material properties, which are listed in the tables 4 and 5, are picked from Kairouz and Matthews [5], where the adherents are made of *XAS/914C* carbon fibre/epoxy resin and the adhesive used is Ciba-Geigy Redux 308A.

## Results and discussion

### General features

Firstly, we investigate the behaviour of the single lap joint with the  $[0/90]_{2s}$  lay-up in the adherents. A typical distribution of the von Mises equivalent stress

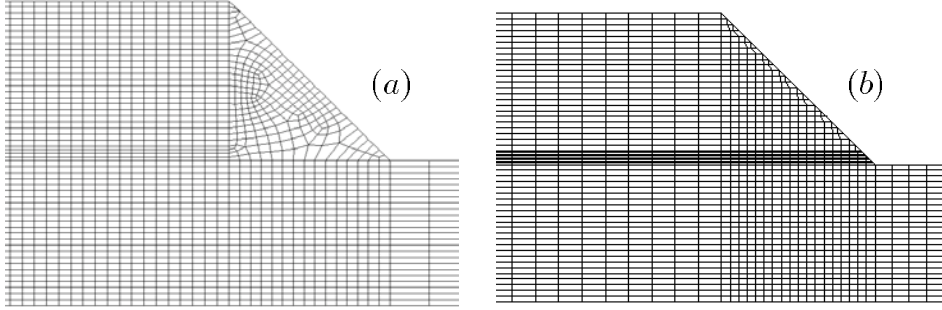


Figure 2: *The ANSYS (a) and Diffpack (b) grids around the fillet tip.*

resulting from the geometric nonlinear method is depicted in figure 3 for a crucial region of the (deformed) geometry. Here, the prescribed displacement,  $\delta$ , of the

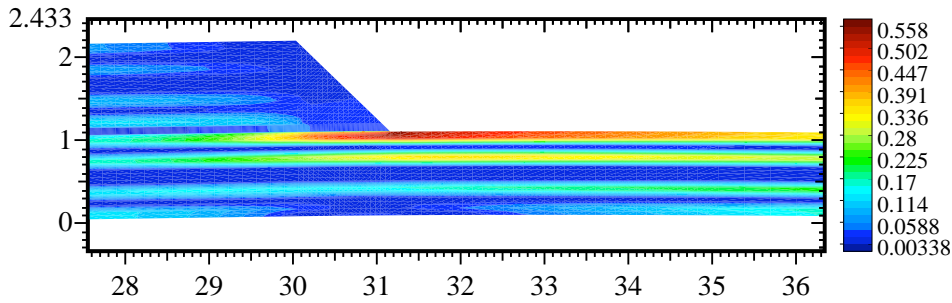


Figure 3: Distribution of the von Mises equivalent stress (GPa) near the fillet tip for  $\delta = 0.1$  (mm) and  $[0/90]_{2s}$  stacking sequence for the adherents.

right hand side boundary is 0.1 mm which leads to stresses and strains far below critical ply crack limits (approximately one third). The maximum value for the equivalent stress (as well as for  $\sigma_x$  and  $\sigma_y$ ) is observed in the upper layer of the lower right laminate near the fillet tip (figure 3 and 4 (c)), while the maximum shear stress occurs in the adhesive layer near  $x = 30$  (figure 4 (a)). Similar behaviour and values are obtained around the spew fillet at  $x = 20$ . As seen in figure 3 most of the external load is captured by the upper 0-layer near the spew fillet. The stress magnitude then falls rapidly as one moves to the left in the model. Consequently, the adhesive layer transfers the load from the lower plate to the upper by shear as illustrated in figure 4 (b). In addition to this

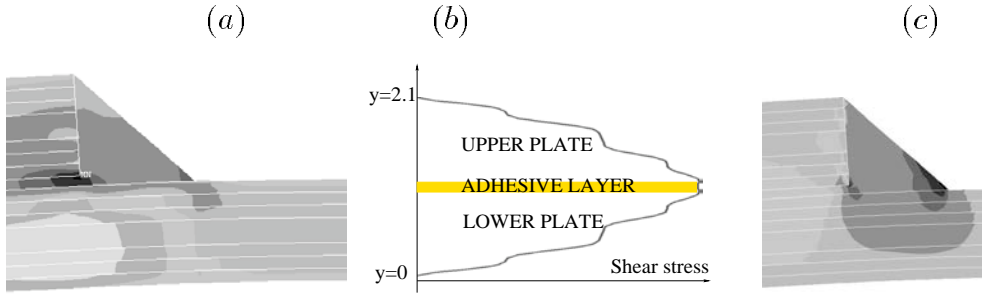


Figure 4: The black regions indicate where the maximum values for the shear stress (a) and the transverse normal stress (c) occur. The distribution of the shear stress at  $x=25$  is shown in (b).

observe that the equivalent stress in the 90-layers, as figure 3 shows, is very small compared to the observed values in the 0-layers.

The claimed geometric nonlinear effects have all been studied at the locations where their respective maximum values appears. Figure 4 (a) shows the location for the maximum shear stresses, and figure 4(c) indicates where the maximum transverse normal stresses occur. On the other hand, the von Mises equivalent stress, illustrated in figure 3, is investigated in the upper 0-layer next to the fillet tip, and the transverse displacement is studied at the fillet tip.

## Composite joints - effects of geometry

The investigations reveal several interesting results. The first parameter to be considered is the horizontal reaction force obtained by integrating the longitudinal normal stress across the thickness of the plate at the left hand side of the model. The geometric nonlinear solution shows that the reaction force varies proportionally with the end displacement, but that the values, nevertheless, are clearly influenced by the nonlinear geometry. This is occurring since the real joint is much stiffer than what is predicted by the linear solution. Consequently, the linear solution gives a reaction force that is about 15% smaller than the one obtained when geometric nonlinearities are taken into account. This is pictured in figure 5.

Figure 6(a) shows the vertical displacement at the fillet tip as a function of  $\delta$ , while the equivalent stress is depicted as a function of the external load (per unit width) in figure 6(b). The results from the nonlinear model are compared with

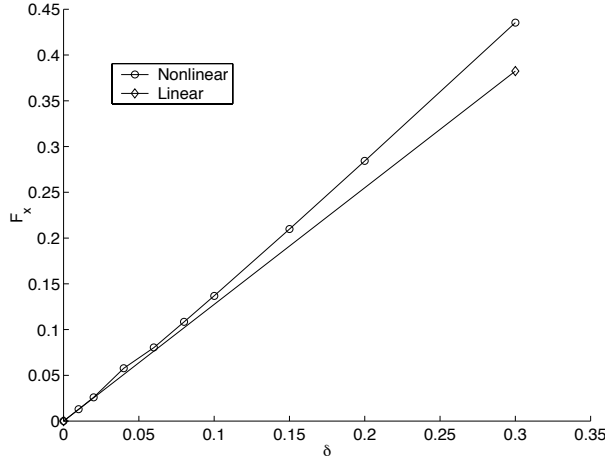


Figure 5: The horizontal reaction force (kN/mm) at the left side,  $[0/90]_{2s}$  laminate.

solutions obtained by a linear model.

For the present material behaviour, ply cracks begin to develop when  $\delta \approx 0.3$  as claimed in [5], and it is seen that nonlinear effects are significant for end displacements (or corresponding external loads) above approximately 25% of that value. Especially the vertical displacement shows considerable nonlinearities. However, also the equivalent stress is clearly modified. This is quite remarkable, taking into account the small end displacement and the domain geometry.  $\delta = 0.1$  corresponds to an average longitudinal strain on the order of 0.2%.

The tendencies are exactly the same for a  $[90/0]_{2s}$  stacking, and it is therefore convenient to continue the study to find out whether this behaviour is due to the material properties or is intrinsic in the nature of the non-symmetric geometry of a single lap joint. It is, of course, also possible that the answer is a combination of these two explanations.

Consequently, a symmetric double lap joint model was made for further investigations. The model is pictured in figure 7. The left end is suppressed from moving and at the right end a prescribed displacement is used to simulate a test machine as for the previous model.

Each laminated plate consists of eight plies with the same  $[0/90]_{2s}$  stacking as for the single lap model. The dimensions are also the same, except that a new adhesive layer and a lower laminate are added to get the double lap geometry.

We perform the same investigations as for the single lap joint and observe that



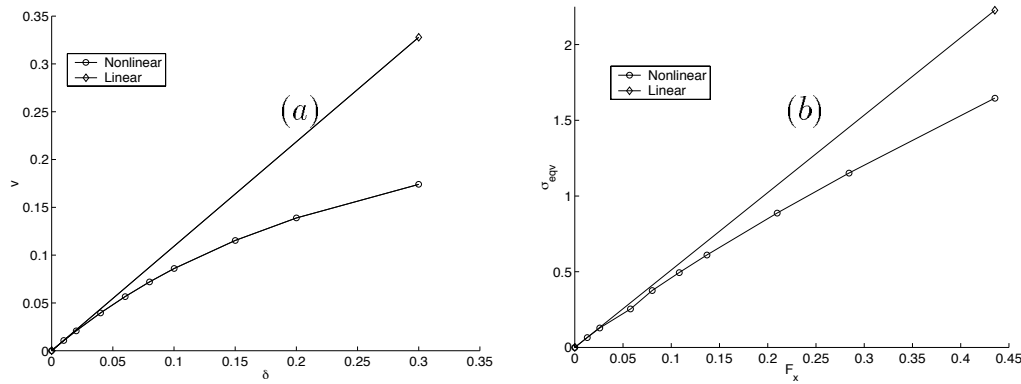


Figure 6: Comparison of geometric nonlinear (o) and linear (solid line) results for the  $[0/90]_{2s}$  laminate. Figure (a): Vertical displacement (mm) at the fillet tip as a function of joint end displacement (mm). Figure (b): von Mises equivalent stress (GPa) at the fillet tip as a function of external load (kN/mm).

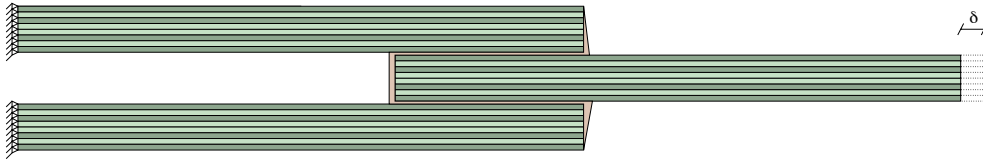


Figure 7: The 2-D domain for the numerical simulations of the double lap joint. (Not to scale)

the double lap joint behaves linearly for all levels of load, as shown in figure 8. This indicates that the non-symmetric geometry of the single lap joint is a major reason for the strong nonlinear behaviour. However, this does not exclude the possibility that the material distribution may have an influence on the matter.

## Various joints - effects of material distribution

Now, we expand the examination with three new non-symmetric single lap joints, all of which contain steel. The isotropic material properties for steel are shown in table 6. The steel-adhesive-steel model consists of two steel plates joined together with adhesive (of lower stiffness than steel), while the all steel model is supposed to illustrate a weld-bonded joint [11]. The last joint is assembled by a  $[0/90]_{2s}$  composite plate and a plate of steel. The reason for the latter model is to detect any effects due to a combination of non-symmetric material and geometric

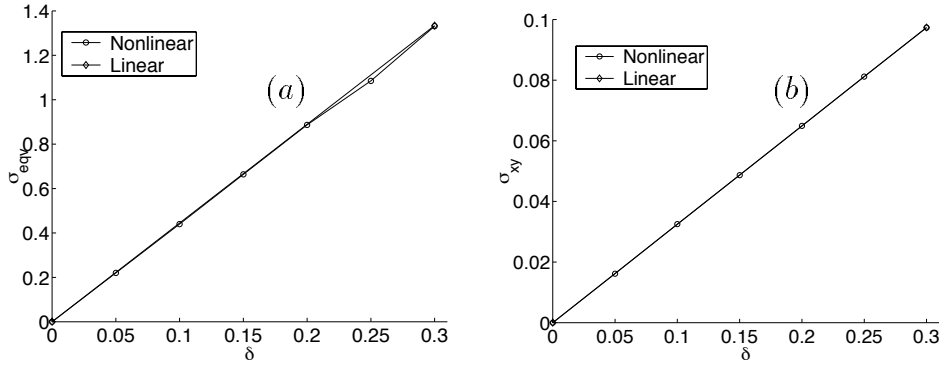


Figure 8: Comparison of geometric nonlinear ( $\circ$ ) and linear (solid line) results for the symmetric double lap joint with lay-up  $[0/90]_{2s}$ . Figure (a): The von Mises equivalent stress (GPa) at the upper fillet tip as a function of joint end displacement (mm). Figure (b): The shear stress (GPa) in the adhesive layer near  $x=30$  as a function of joint end displacement (mm).

distributions. The geometry of the joints is shown in figure 9. For all models, except the mixed composite-adhesive-steel joint, the physical behaviour is similar around the spew fillets at  $x = 20$  and  $x = 30$ . This is obviously not the case for the latter joint. Therefore, the fillets are treated separately for the mixed model. The lower left fillet is called *A* while the upper right is called *B*. Figure 10 pictures the stresses around the two fillets, from which it is easily seen that the stress distributions are very different.

Figures 11, 12, 13, 16 and 17 show geometric nonlinear results (von Mises equivalent stress, transverse normal stress, vertical displacement, horizontal reaction force and shear stress, respectively) at critical locations as a function of the end displacement and divided by the corresponding linear results for end displacement equal to 0.3. Each plot will be discussed and evaluated separately in the forthcoming text. Furthermore, the curves for all the joints are included for every parameter.

Figure 11 shows that all the joints behave very similarly. The nonlinearity of the von Mises equivalent stress is obvious due to the fact that the maximum values are approximately 80% of the linear values. As figure 11 clearly shows, there is no considerable difference between the models, and this may indicate that the observed nonlinear stress distribution is due to the non-symmetric geometry and that the material distribution has no influence.

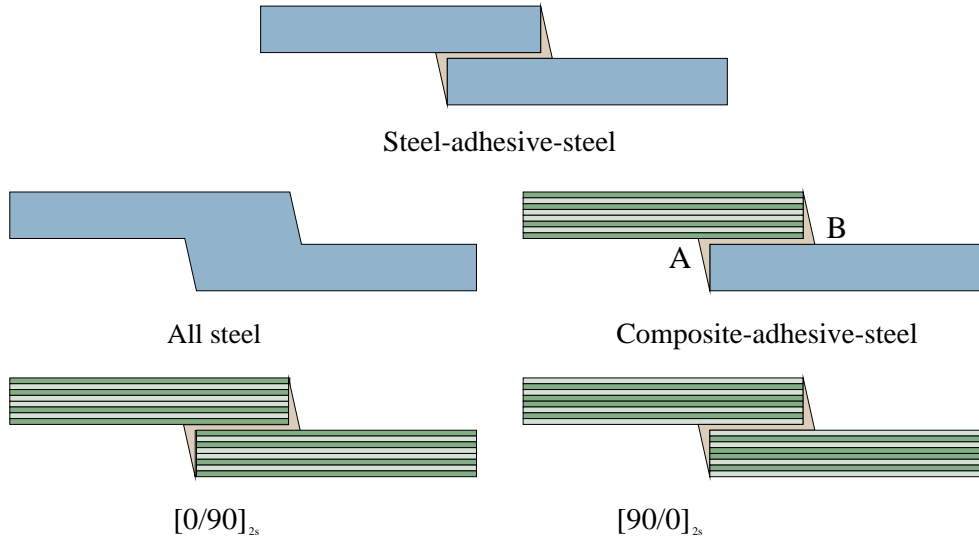


Figure 9: Five non-symmetric single lap models. (Not to scale)

The next parameter to be studied, the transverse normal stress, shows a behaviour that is very similar to the von Mises equivalent stress. The stress values for the different joints are, however, more spread in this case. As a matter of fact, the nonlinear results vary between, approximately, 60% and 80% of the linear values for  $\delta = 0.3$ . In other words, the nonlinear effects are slightly more excessive for the transverse normal stresses than for the equivalent stress. It should be noticed that the model with the strongest nonlinear behaviour is the one containing steel-adhesive-steel. Figure 12 shows the complete picture which clearly does not make it easier to detect any particular reason for the nonlinear effects apart from the geometry.

Figure 13 shows the transverse displacement for the different models. It is seen that the nonlinear results vary between 35% and 55% of the corresponding linear displacements for  $\delta = 0.3$ . Especially the results for the steel-adhesive-composite model deviates from the rest of the joints. This indicates that the material distribution has an important effect on the nonlinear behaviour for the transverse displacement. To summarize, a linear solution technique gives transverse displacements that are almost three times the real ones. In figure 13 the results from the fillet tip B is left out for the composite-adhesive-steel model. While all the other joints behave similarly at the fillet tips A and B, the latter joint

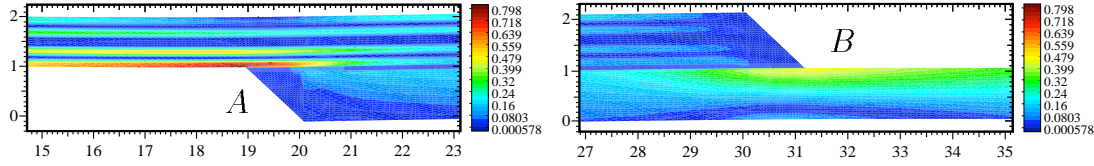


Figure 10: The von Mises equivalent stress (GPa) around the fillets  $A$  and  $B$  of the composite-adhesive-steel joint.

moves a larger distance at  $A$  than at  $B$ . This is sketched in figure 14 (a) and (b). As figure 15 (a) shows, point  $A$  moves down twice as much as  $B$  moves up for  $\delta = 0.3$  when geometric nonlinearities are taken into account. The reason for this behaviour is that the composite plate deforms much easier than the plate of steel because it has a lower bending stiffness than the latter. Consequently, it is easier to bend the composite plate out of shape than the plate made of steel. If, on the other hand, the materials are of similar quality the plates have the same bending stiffness and will therefore move the same distance. A linear analysis for the same end displacement as above shows that  $A$  moves approximately three times more than predicted by the nonlinear model, and that  $B$  almost does not move at all. To be exact, the linear analysis suggests that  $A$  moves around 11 times more than  $B$  - a disastrous exaggeration of the previous observed and discussed behaviour. Figure 15 (b) shows the nonlinear transverse displacement for the composite-adhesive-steel model divided by the linear solution for  $\delta = 0.3$ . The figure clearly pictures that the scaled displacement at fillet tip  $B$  is non-typical as  $V_N/V_{L,max} \gg 1$ . Consequently, the plot for point  $B$  becomes non-representative and is left out in the previous discussion. The result is however not without interest. On the contrary, it clearly illustrates the great importance of applying a geometric nonlinear solution method.

For all the three parameters studied so far (see the figures 11, 12 and 13) the  $[90/0]_{2s}$  model shows a larger amount of nonlinear behaviour than observed for the  $[0/90]_{2s}$  laminate. The reason for this might be connected to the fact that the 0-layers closest to the adhesive carry the major part of the external load for both models. Consequently, one will have a greater eccentricity and therefore a stronger generation of bending and nonlinear effects for the laminates with the  $[90/0]_{2s}$  stacking than for the  $[0/90]_{2s}$  joints. The differences are almost

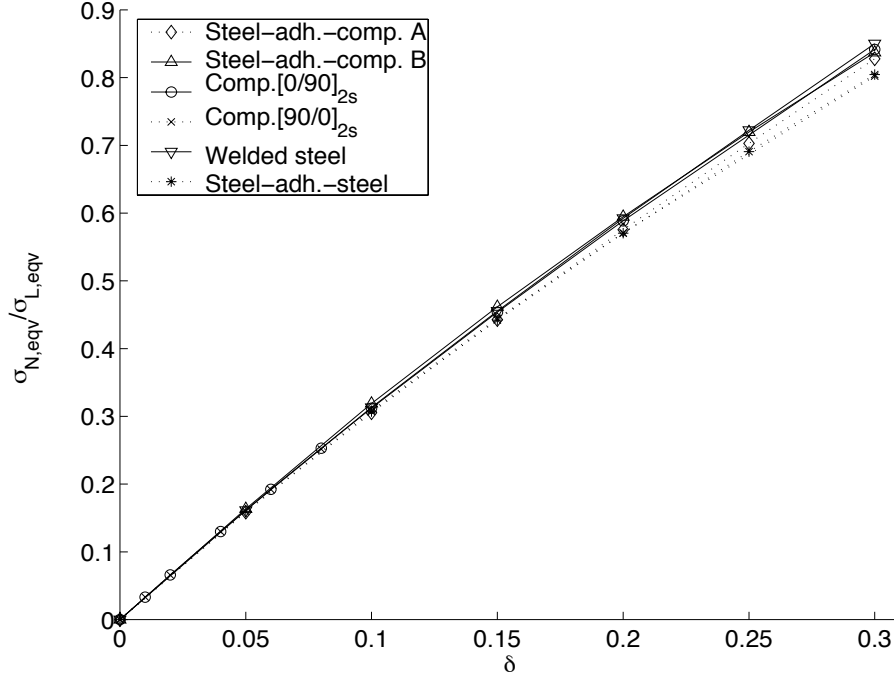


Figure 11: The von Mises equivalent stress (at the fillet tip) from the geometric nonlinear model scaled with linear results for  $\delta = 0.3$  (mm).

negligible for the von Mises equivalent stress and the transverse normal stress. However, the transverse displacements deviate significantly for the two stacking sequences. It should also be observed that the composite-adhesive-steel model shows a stronger nonlinear behaviour than the  $[90/0]_{2s}$  and  $[0/90]_{2s}$  models for both transverse stresses and transverse displacements, while it lays between them when it comes to the equivalent stresses. The above discussion clearly illustrates that, in addition to the geometry, the distribution of the materials has a considerable influence on the nonlinear behaviour of single lap joints.

The reaction force on the left hand side of the model is a parameter that shows a very linear sort of behaviour, which is shown in figure 16. However, a more careful look at the figure tells us that the nonlinear effects are important because a linear solution will give reaction forces that are too small. This is also depicted in figure 5, but only for a  $[0/90]_{2s}$  stacking. The fact that the bending stiffness of the single lap joint is lower for the linear model than for the geometric nonlinear one gives an obvious explanation of this result. The nonlinear values are between

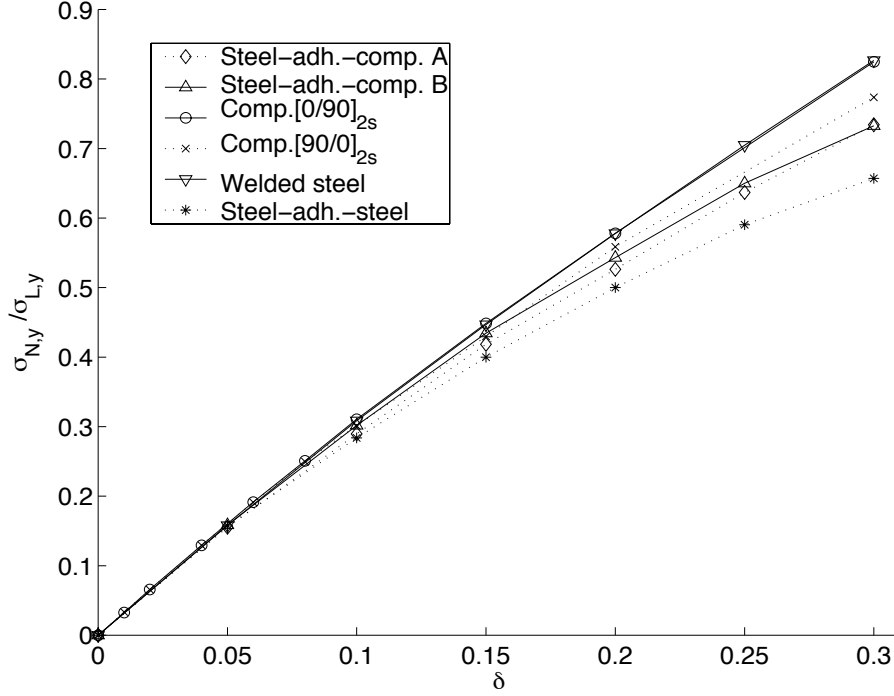


Figure 12: The transverse normal stress (at the fillet tip) from the geometric nonlinear model scaled with linear results for  $\delta = 0.3$  (mm).

15-30 % higher than the linear ones, and the  $[90/0]_{2s}$  stacking shows the strongest influence of the geometry while the welded steel joint and the  $[0/90]_{2s}$  joint are less influenced.

The last parameter to be studied is the shear stress, which is depicted in figure 17. The figure reveals a very linear behaviour, and the similar behaviour of all the examined joints should be noticed. However, certain joints are left out from the figure because their shear distribution diverts from the rest. The all steel model has, for example, a totally different stress pattern than the others due to the uniform material property of that model. Consequently, a direct comparison is irrelevant.

## Generality of results

To investigate the generality of the results and conclusions drawn so far, a modified single lap joint with a  $[90/0]_{2s}$  stacking is investigated. The length of the new laminates are 50 mm compared to 30 mm in the previous single lap joint. The

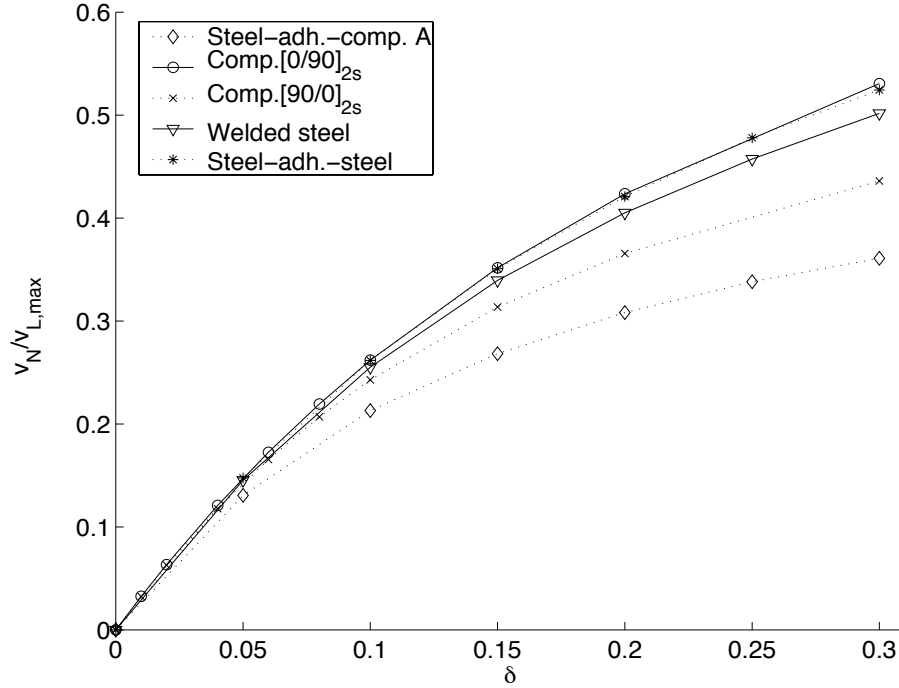


Figure 13: The transverse displacement (at the fillet tip) from the geometric nonlinear model scaled with linear results for  $\delta = 0.3$  (mm).

overlap region and the boundary conditions are unchanged. In other words, the distance from the fillet tips to the ends of the plates is approximately doubled. The maximum applied displacements on the right hand side of the new model is adapted to the new dimensions to cause stresses of the same magnitude as for the previous models. The results obtained from these nonlinear solutions are then compared to the old results.

These investigations show the same degree of nonlinear behaviour as for the shorter model for both the transverse normal stress and the von Mises equivalent stress at the fillet tip as well as the shear stress just behind the spew fillet. To be more precise, the results are within a 2.5 – 5% difference - which is very similar. The other parameters, the transverse displacement at the fillet tip and the reaction forces at the left hand side of the model, show a behaviour that makes the necessity of using geometric nonlinear methods even stronger than indicated in the previous model. This very strong influence of geometric nonlinear effects is clearly illustrated in figure 18. The magnitude of the transverse displacement at

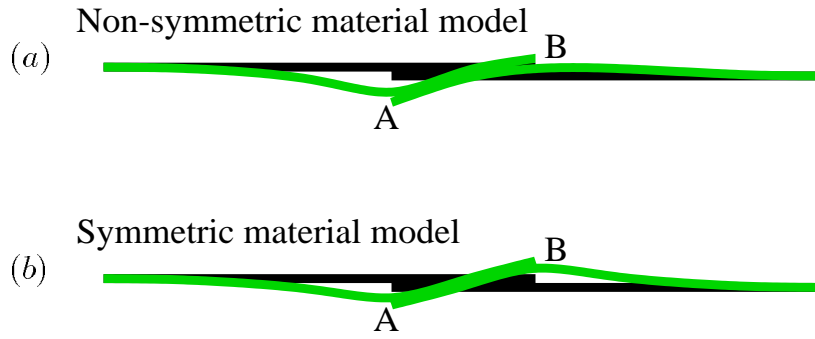


Figure 14: Sketch of typical deformations for single lap joints with non-symmetric and symmetric material distributions.

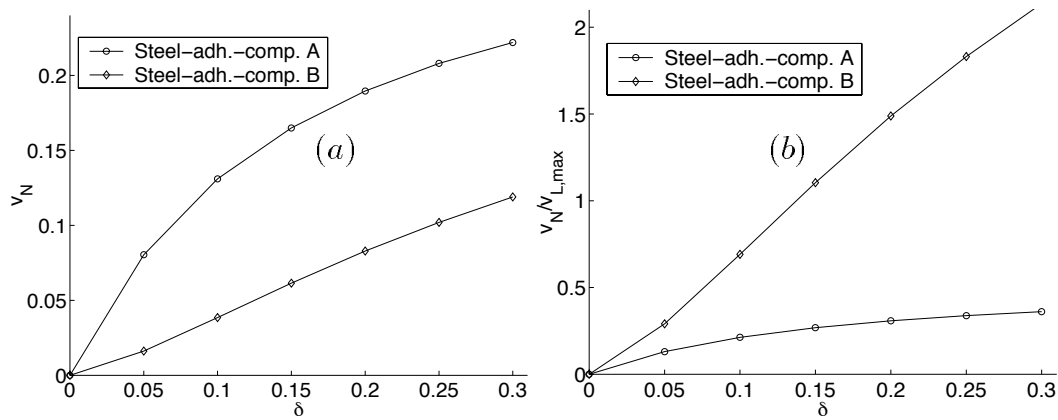


Figure 15: Transverse nonlinear displacements at the fillet tips  $A$  and  $B$  (figure (a)). Corresponding results scaled with linear values for  $\delta = 0.3$  (mm) (b).

the fillet tip obtained from the nonlinear analysis is only 25% of the displacement obtained from the linear analysis. This strengthens the claimed need for taking geometric nonlinearities into account when these kinds of joints are examined. When it comes to the horizontal reaction forces at the left hand side of the model, the 30 % overshoot from the shorter model has grown to 60%, and, consequently, the need for a geometric nonlinear solution has grown, also for this parameter.



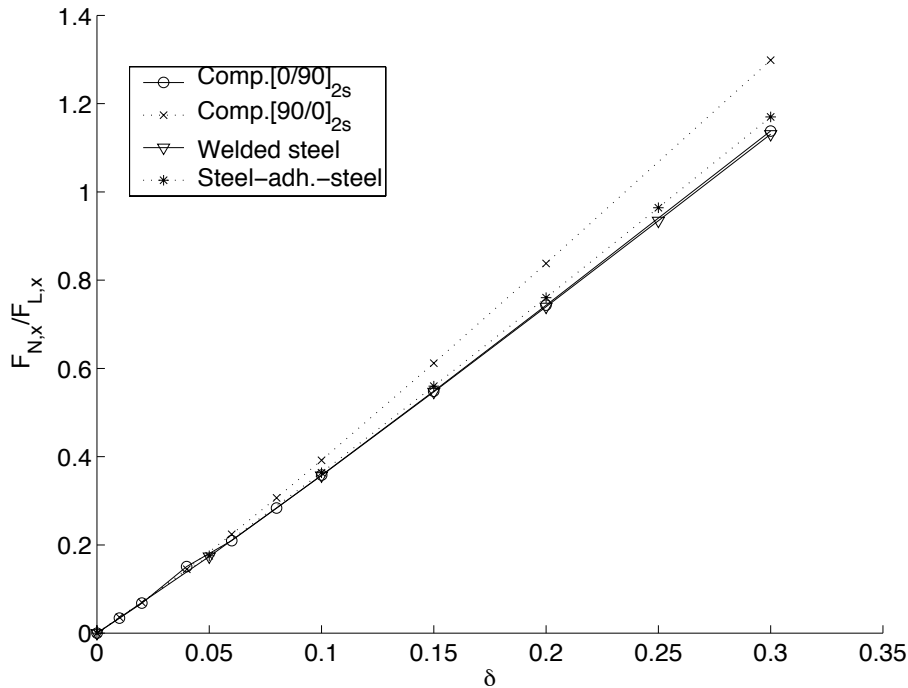


Figure 16: Reaction force (on the left hand side of model) from the geometric nonlinear model scaled with linear results for  $\delta = 0.3$  (mm).

## Concluding remarks

### General

The examinations made in the present paper confirm the fact that geometric nonlinear effects are of great importance when single lap joints are studied, although the average longitudinal strains are very small and the stresses are far below ply crack levels. Consequently, the practice of taking them into account [3, 5] is correct and necessary.

### Transverse displacement of fillet tip

Especially the transverse displacements show a kind of nonlinear behaviour that makes linear analysis completely unsuitable. A linear analysis of joints with a symmetric distribution of materials will produce transverse displacements at the fillet tips that are at least two or three times the magnitude of the real displacements for loads near ply crack level. This is due to the fact that the non-symmetric geometry of the single lap joint will bend when a horizontal dis-

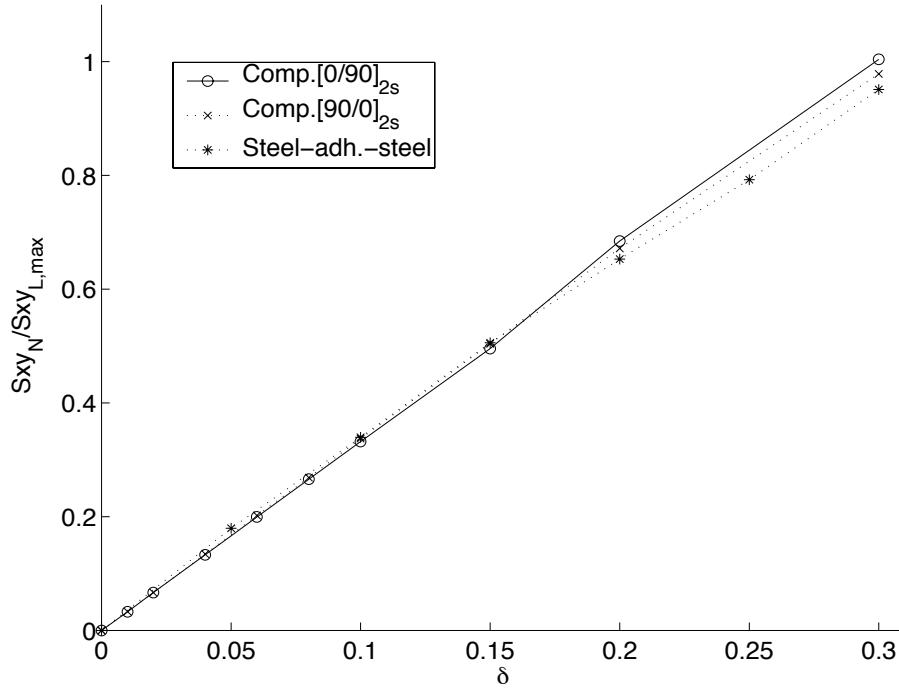


Figure 17: Shear stress from the geometric nonlinear model scaled with linear results for  $\delta = 0.3$  (mm).

placement is applied on one of the ends, and consequently, as a result of the increased bending stiffness due to the geometric nonlinear solution, it will displace less. Furthermore, if the joint has a non-symmetric material distribution, the transverse displacement pattern from a linear analysis is even worse than the one achieved for a joint with a symmetrical distribution of materials, and, therefore, far from reality. If the plates get very long, the effect of the geometry will get even stronger, and the need of taking geometric nonlinear effects into account will be strengthened. The exact displacement magnitude will in each case mainly depend on the length of the joint, but also on the overlap length, the material properties and the distribution of them within the joint. If the transverse displacement is of great importance in a construction, as for example as illustrated with the two single lap joints in figure 19, this knowledge is of vital interest. A linear analysis will give a too large minimum vertical distance  $V_{min}$  between the joints.

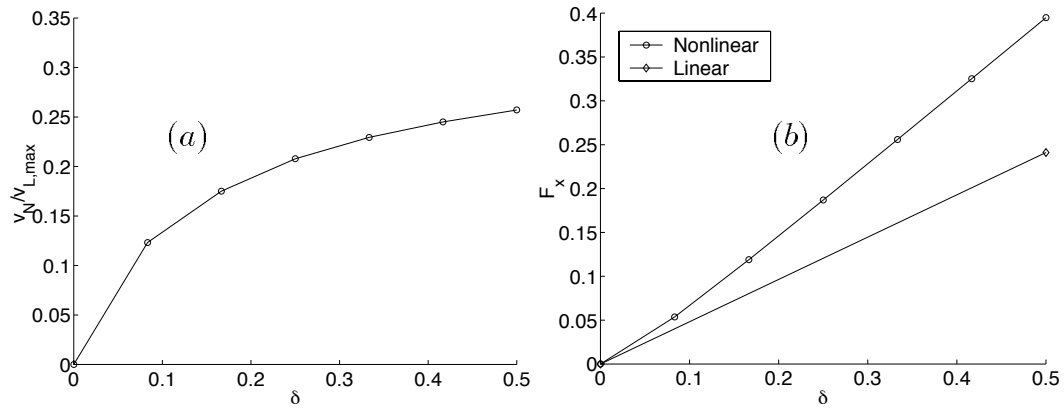


Figure 18: Figure (a): Vertical displacement at the fillet tip - geometric nonlinear results scaled with the linear value for  $\delta = 0.3$  (mm). Figure (b): The horizontal reaction force (kN/mm) as a function of end displacement (mm). Comparison of geometric nonlinear (o) and linear (solid line) results.

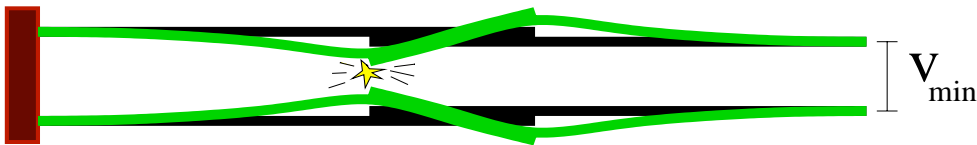


Figure 19: Sketch of a possible deformation for a construction containing two single lap joints.

## Various stress components

The transverse normal stress and von Mises equivalent stress at the fillet tips also show a significant nonlinear behaviour for loads that produce stresses below ply crack limit. Linear analysis will offer stress values that are too large compared with results from a geometric nonlinear solution technique. Shear stresses, on the other hand, are linear in their appearances, and they are not significantly affected by the choice of analysis. Furthermore, the main nonlinear behaviour of these parameters are independent of the change of the domain geometry.

## Reaction forces

The values of the reaction forces on the left hand side of the model depend strongly on the geometric nonlinear effects. Interestingly, they are also very dependent on the geometry of the joint. The effect of the increased bending stiffness one gets from the geometric nonlinear solution method is, as for the transverse displacement, clearly illustrated and to be considered as the main reason of this behaviour.

## Assessment

An assessment of these results therefore indicates answers to the first and second question posed in the introduction of this paper:

- i) Nonlinear effects are very important even for load levels that produces stresses far below ply crack limit.
- ii) The nonlinear geometry is the main reason for this behaviour

It is hereby settled that the non-symmetric geometry is an important reason for the nonlinear behaviour, but it is not the only one. If the materials used are distributed non-symmetrically as, e.g., the composite-adhesive-steel joint, an even greater nonlinear appearance is observed for the transverse displacement pattern. For the other examined parameters there is not observed any pattern that links material distributions and nonlinear behaviour, except that the stacking sequence has some influence. Consequently, the answer to the last question posed is not obvious. It should therefore be divided and answered separately for each parameter.

- iii) Regarding vertical displacement there is little doubt that material distribution matters. A non-symmetric material distribution will result in increased nonlinear behaviour for this parameter. Furthermore, a stacking sequence that has its 0-layers far from the bond will generate an increased amount of eccentricity and therefore a stronger degree of nonlinear behaviour than a sequence that has the 0-layer closer to the bond. This is due to the fact that the largest amount of loads is captured by the 0-layers near the bond. For the other parameters, von Mises equivalent stress, transverse normal stress and shear stress, the material distribution has no major observed influence on the matter.

As a result of these examinations one might therefore conclude that the geometric nonlinear method is of great importance and should always be taken into account when dealing with non-symmetric joints, regardless the joint dimensions, material distribution and the amount of strain.

Longitudinal elastic modulus, $E_1$ (GPa)	138
Transverse elastic modulus, $E_2$ (GPa)	9.4
In-plane shear modulus, $G_{12}$ (GPa)	6.7
Major Poisson's ratio, $\nu_{12}$	0.32

Table 1: Mechanical properties of *XAS/914C* composite.

Elastic modulus, $E$ (MPa)	3000
Shear modulus, $G$ (MPa)	1145
Poisson's ratio, $\nu$	0.31

Table 2: Mechanical properties of Redux 308A adhesive.

Elastic modulus, $E$ (GPa)	210
Poisson's ratio, $\nu$	0.30

Table 3: Mechanical properties of steel.

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