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ABSTRACT

Several aspects of nonslender (short) column limits are considered. Such limits, below which it is acceptable to ignore local second-order effects that may cause maximum moment to develop between member ends, are generally confined to columns of braced frames, but are relevant for sway-restricted columns of unbraced frames also. Local and global second-order effects are reviewed in this context, and the effect of global second-order effects (sway-modified moments) is discussed, and it is concluded that the end moment ratios of approximate moment magnifier expressions and slenderness limits should be expressed in terms of sway-modified end moments. The need for providing separate slenderness limits, such as in the ACI 318-02 code, for columns of nonsway and sway frames, is questioned. The ACI limits are critically reviewed and several inconsistencies pointed out with respect to their differences and different basis on which they were derived. They are compared with a more comprehensive limit, that may replace both of the ACI limits, and comparisons are also made with a summary of nonlinear analysis results.

KEYWORDS

Axial forces; Columns; Concrete, reinforced; Frame design; Sidesway; Slenderness ratio; Slenderness limit; Structural stability.

INTRODUCTION

Second-order load effects in typical frames are due to local effects in individual members and global effects due to sidesway of the frame as such. If the compression members of the frame can be considered "nonslender" (or "short"), second-order effects will become negligible and the structural analysis can be simplified. Most design codes for reinforced concrete structures give nonslender member limits. An overview of a number of these is given in Hellesland (2005).

The various limits may give widely different predictions, and are given either for columns that are free to sway or for columns that are braced against sidesway. Columns in unbraced and flexibly braced frames will deflect laterally with the frame, and are therefore neither free to sway nor completely prevented against it. Most codes do not provide, or are at best very ambiguous with regard to, limits for the local slenderness effects that may cause development of maximum moment between ends in such cases. An exception is the ACI 318 code. Since the 1995 revision of the code, separate lower slenderness limits have been given for local slenderness effects in columns of socalled "nonsway" and "sway" frames.

The objective of this paper is threefold. 1) First, the mechanics of column behavior and moment formulations for braced and unbraced frames are briefly reviewed in order to clarify some aspects that seemingly is not always well understood, and in order to provide a proper framework for discussing effects of moment gradients on the development of maximum moments between ends and on slenderness limit formulations. 2) Second, a critical review of the two relevant ACI 318-02 slenderness limits is carried out with respect to their differences and basis on which they were derived, and of the justification given by the code writers for given two separate limits for a problem that in principle is the same (development of maximum moment between ends). The main focus is on possible inconsistencies and inaccuracies that may detract from the rationale of the code provisions. 3) Third, an alternative, more comprehensive slenderness limit, defined in terms of a normalized slenderness parameter, is briefly reviewed. This limit may replace both of the two present ACI 318 limits, and may allow local slenderness effects to be ignored in considerably more cases than the present limits. Comparisons are made with a summary of nonlinear analysis results and with the ACI limits.

FRAME MECHANICS – REVIEW

Frame and member behavior

Structures and compression members are defined as *braced* or *unbraced* depending upon whether the overall lateral stability is provided by some kind of external bracing system (lateral supporting device, shear walls, truss, etc.), or not. Unlike the term *fully braced*, which is normally used to denote a frame or column of which the lateral displacement is completely prevented, the term braced does not per see exclude some limited lateral displacement that must be accommodated by the columns. Indeed, for practical bracing stiffnesses, this will normally be the case. In an unbraced frame with stiff and flexible columns interconnected by a beam, floor etc., one may have both braced and "bracing" (unbraced) columns. For lateral stability of such frames, the stiffer columns will interact with, and provide lateral bracing to the more flexible columns, that without this bracing might become laterally unstable. Such columns can be considered braced at an imposed lateral sway, restricted to that of the frame.

Second-order effects due to axial loads affect interconnected members of a framed structure in two ways: (1) in an overall, or global sense, due to sidesway of the frame system as such, and (2) in a individual member, or local sense, due to axial loads acting on the deflections away from the chord between member ends and thereby giving rise to nonlinear (curved) moment distributions. In approximate analyses, this subdivision into global and local effects is very common (and sometimes referred to as $P\Delta$ and $P\delta$ effects, respectively).

There is an interaction between global and local effects. Sway due to global effects affect the moment gradients along individual columns, and therefore the development of curved moment distributions and maximum moments between ends. Local effects (curved moment distributions) in individual, axially loaded columns affect the lateral displacement Δ and, consequently, the sidesway moments. This effect, which can be accounted for reasonably well (Hellesland 1976, 2000; LeMessurier 1977), is normally small and is often neglected.

First-order moments

First-order column moments in a frame can be given, at each cross section, by either of the two moment sums defined by

$$M = M_b^* + M_s^* \quad \text{or} \quad M = M_{ns} + M_s \tag{1}$$

In the first sum, M_b^* is the fully "braced" moment, obtained from an analysis in which the frame is considered fully braced (by a fictitious holding force), and M_s^* is the "sway" moment due to the sway caused by all loads on the frame (i.e., due to the total holding force in the opposite direction). This subdivision was common in earlier hand calculation procedures. In computer analyses, in which moments are computed for individual load cases, the second sum is more practical. There, M_s is *due to lateral (sideways) loads*, H, and M_{ns} is *due to all other, "no-sideways-acting" loads.* The same subscripts are used in ACI 318, but there with a somewhat different meaning as will be discussed later.

Sway-modified first-order moments

Global second-order effects of the vertical loading acting on the laterally displaced locations of the joints (Δ), increase the relative translation of laterally interacting columns from the first-order value $\Delta_s^* = \Delta_{ns} + \Delta_s$ (corresponding to M_s^*) to $\delta_s \Delta_s^*$, where δ_s is the sidesway magnification factor. This leads to a similar increase in the column moments due to sidesway. Thus,

$$M = M_b^* + \delta_s M_s^* \quad \text{or} \quad M = M_{ns} + \delta_s' M_s \tag{2}$$

In the second formulation, M_{ns} includes the effect of the first order sidesway Δ_{ns} due to the "ns-loading" (or due to the portion of the holding force corresponding to the "ns-loading"). The moment M_s is due to the first-order sidesway Δ_s caused by the lateral load H only. In a multistory frame, H is the story shear and the Δ values refer to the displacement of the top relative to the bottom of the story. Noting that $M_b^* = M_{ns} - M_s f$ and $M_s^* = M_s + M_s f$, where $f = \Delta_{ns}/\Delta_s$, it can readily be shown, by requiring the two sums in Eq. (2) to be equal, that the modified sidesway magnification factor δ_s' can be written

$$\delta_s' = \delta_s + (\delta_s - 1)\Delta_{ns}/\Delta_s \tag{3}$$

It is tacitly assumed here that the relative displacement is the same in all axes with laterally interacting columns. This is the most common case. The general case with unequal displacements in the various column axes, caused by shortenings or elongations of the connecting beams (due to temperature, prestress, creep, etc.), or by unequal inclinations (out-of-plumbs), are covered in Hellesland (1976). Equal out-of-plumbs can be accounted for by adding it to Δ_{ns} in Eq. (3).

Strictly, Δ_{ns} and a portion of M_s^* above should have been calculated using the holding force for the "ns-loading" obtained from second order theory. This is impractical, and the first order value is in normal applications sufficiently accurate.

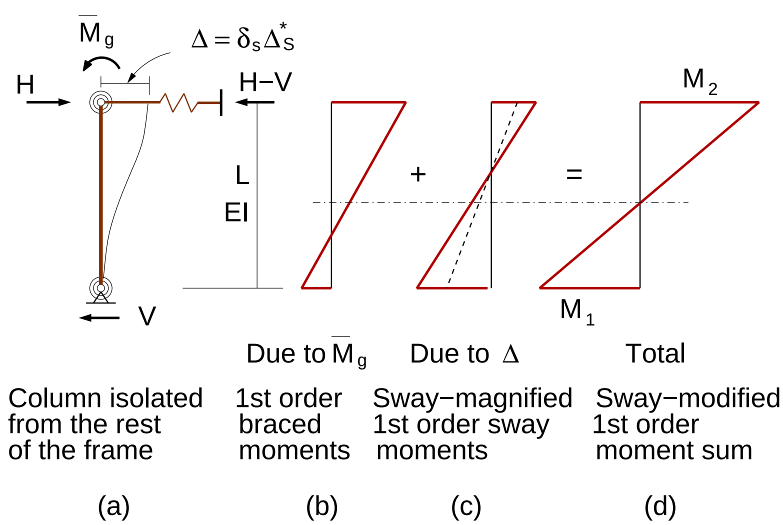

Figure 1: Restrained, axially not-loaded column, with global second order effects.

Typical moment distributions for a column with no axial load, for which Eq. (2) applies, is illustrated in Fig. 1. The column is part of a plane frame that is subjected to both sideways loading H, and to gravity loads giving unbalanced joint moments $\overline{M}_g$ that result in the braced column moments M_b^*. Lateral interaction with other columns in the frame (on the same level) are reflected by the lateral springs and rotational interaction by the rotational springs.

The term $\delta_s M_s^*$ in Eq. (2) is simply the "*sway-magnified first-order sway moment*". The moment sum may conveniently be referred to as the "*sway-modified first-order moment*" for the sake of distinguishing it from the conventional first-order moment sum in Eq. (1). For a column without axial load, and therefore with no local slenderness effects (when assuming invariant rotational restraints), it is irrelevant whether sidesway is due to a lateral load or due to a global second-order effect. These moments are correct provided δ_s is correctly determined. Fairly accurate δ_s expressions are available (e.g., Hellesland 1976, LeMessurier 1977), but more approximate ones are accepted by codes (e.g., ACI 2002, AISC 2005).

Local effects – Maximum moment

For framed columns with axial loads, local second-order effects ($P\delta$) affect end moments and introduce a nonlinear moment distribution as illustrated in Fig. 2 (full lines) for a double and a single curvature bending case. The fully braced Case (c) has identical local second-order (member stability) effects to Case (a), provided rotationally restraints are identical and the first-order moments are equal to the sway modified first-order moments in Case (a).

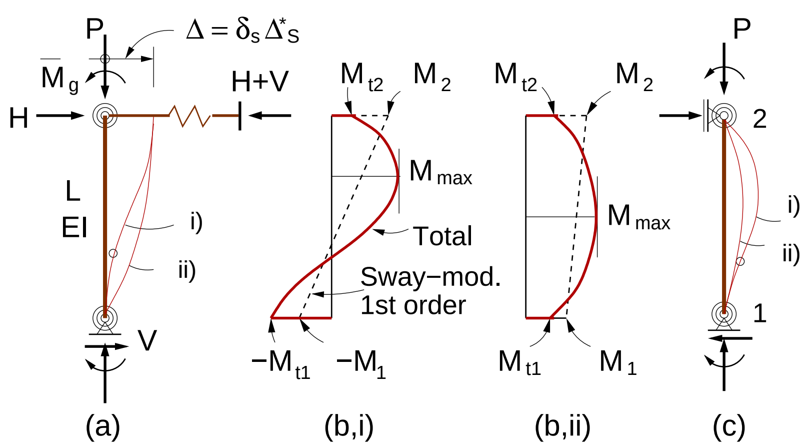

Figure 2: (a) Restrained, axially loaded column with global and local second order effects; (b) typical double and single curvature moment distributions; (c) fully braced column.

In the example, the maximum total moment has developed between the member ends and can be defined by

$$M_{max} = \delta_{max}\,(M_b^* + \delta_s\,M_s^*)_2 \;=\; \delta_{max}\,(M_{ns} + \delta_s'\,M_s)_2 \tag{4}$$

where δ_{max} is the individual column's maximum moment magnifier and the moment sum in the parentheses is the larger of the two sway-modified end moments (M_1, M_2)

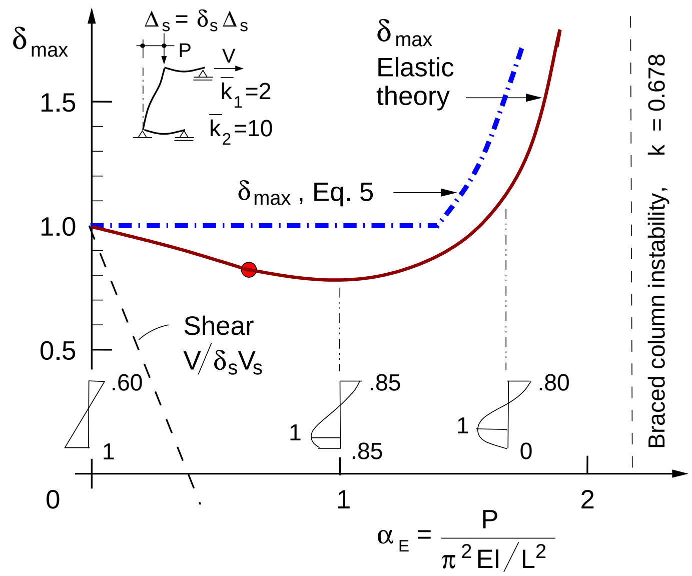

Figure 3: Maximum moment magnification factor vs. axial load level for a restrained column with an imposed lateral end displacement ($\bar{k} = k_\theta/(EI/L)$, k_θ=rotational stiffness).

The inclusion of magnified sway moments in the moment sums will generally increase the moment gradient (as in Fig. 1). This is beneficial, since the development of a maximum moment between ends will be increasingly delayed with increasing moment gradient. This is illustrated by the exact δ_{max} in Fig. 3, computed using second order elastic theory for a restrained column in significant first order double curvature bending. The column might be part of a multibay frame subjected to a translation $\delta_s\Delta_s$ due to lateral loading and global second-order effects. It is held (by the rest of the frame) at this displacement by the shear V, which is also shown (dashed line). For low axial load levels, V is positive. It becomes negative at higher axial loads at which the column requires bracing to remain laterally stable. In this case, with no gravity load moments, $M_{max} = \delta_{max}\delta_s M_{2s}$.

The approximate maximum moment magnifier expression defined by

$$\delta_{max} = \frac{C_m}{1 - P/P_{cr}} \geq 1.0 \quad ; \quad C_m = 0.6 + 0.4\mu_o \geq 0.4 \tag{5}$$

is also shown in the figure (dash-dot line). Here, $P_{cr} = \pi^2 EI/(kL)^2$ is the elastic critical load of the column considered braced, kL is the effective length, C_m is a factor that accounts for unequal end moments and μ_o is a ratio defined here between sway-modified first-order end moments:

$$\mu_o = \frac{M_1}{M_2} = \frac{M_{1b}^* + \delta_s M_{1s}^*}{M_{2b}^* + \delta_s M_{2s}^*} = \frac{M_{1ns} + \delta_s' M_{1s}}{M_{2ns} + \delta_s' M_{2s}} \tag{6}$$

This ratio, between the smaller (M_1) and larger (M_2) end moment, is taken positive for members bent in single curvature by these moments, and negative otherwise.

Except for the definition of the end moment ratio by Eq. (6), Eq. (5) is a most common maximum moment magnifier and adopted by many codes, e.g., ACI 318-02, Eurocode 2 (CEN 2004), and, with C_m defined without the limitation 0.4, by AISC (2005). The two latter codes, and many similar codes, define the end moment ratio with first-order end moments of the column considered braced. In view of the foregoing presentation, it is clear that the ratio should be between sway-modified first-order end moments. This is also how the ratio is defined in its most general form, in the sway frame provisions, in ACI 318-02.

Code formulations

The double magnifier expression $M_{max} = \delta_{max}\delta_s M_{2s}$, obtained from Eq. 4 in the case with lateral loading only, was presented and discussed by Hellesland and MacGregor (1982) in conjunction with the hearing of the ACI 318-83 revision proposal. The rational extension to the general case, Eq. 4, was presented by Lai and MacGregor (1983). The year after, it was incorporated into the Canadian code (CSA 1984) and in 1995 into the *sway frame provisions* of ACI 318-95 in the form

$$M_c = \delta_{ns}\,(M_{ns} + \delta_s\,M_s)_2 \tag{7}$$

where M_{ns} is the first-order moment "*due to loads that cause no appreciable sidesway*", and M_s is "*due to loads that cause appreciable sidesway*". The maximum moment magnifier, denoted δ_{ns} ($=\delta_{max}$, Eq. (5)), is to be computed with a sway-modified first order end moment ratio similar to the second expression in Eq. (6), but with δ_s' replaced by the more approximate δ_s. It should be noted that these moment definitions are more limited than the definition of moments with the same subscripts in Eq. (1). However, they are suitable for typical, reasonably symmetrical frames, in which M_{ns} is due to vertical (gravity) loads and M_s is due to sideways loads. In such cases, the sidesway caused by the gravity loading is normally negligible and the approximation $\delta_s \approx \delta_s'$ is acceptable.

Eq. (7) replaced a more approximate formulation, proposed by Ford et al. (1981), that was incorporated into the ACI 318-83 edition, and that may be given by

$$M_c = \delta_{ns}\,M_{2ns} + \delta_s\,M_{2s} \tag{8}$$

where δ_{ns} ($=\delta_{max}$) was defined with the moment ratio $\mu_o = M_{1ns}/M_{2ns}$. Except for differences in moment definitions, this form was also adopted by AISC and is still retained today (AISC 2005). It is, from a column mechanics point of view, less accurate than Eq. (7). It will generally, but not always, be more conservative than Eq. (7).

When introduced, Eq. (8) represented a major improvement from earlier design provisions in which a single multiplier expression was used, giving

$$M_c = \delta\,(M_{ns} + M_s)_2 \tag{9}$$

Here, δ was to be taken as the braced column multiplier (δ_{ns}) for columns "braced against sidesway" and as the greater value of the braced column multiplier or the story multiplier (here denoted δ_s) for columns " in frames not braced against sidesway" (cfr ACI 318-71). Several codes are still based on similar moment formulations.

Unlike in the last formulation, it is not necessary be able to classify frames as braced or unbraced in formulations that include the two magnification factors, δ_s (δ'_s) and δ_{max}. The same moment formulation applies to columns in both frames and therefore unifies the treatment of columns in different frames.

NONSLENDER MEMBER LIMITS

Despite a moment formulation that allows for a unified treatment, design provisions in ACI 318-95 and later editions are presented separately for socalled "nonsway" and "sway" frames and columns. A column or story are designated as "nonsway" or "sway" exclusively depending on their sensitivity to second-order sidesway effects. They are "sway" if the sway magnifier δ_s is greater than about 1.05. Otherwise, they are "nonsway", and second order system sway can be neglected ($\delta_s = 1$). Terms like braced and unbraced are consistently avoided in ACI 318-02. For effective length factors, which is dependent on restraint conditions, the term nonsway is tacitly used in the meaning of braced. Separate slenderness limits are given for each category. Both are presented and discussed below.

ACI. Nonsway frames. The code permits slenderness effects to be ignored in *"compression members in non-sway frames"* when the slenderness is less than

$$\frac{kL}{r} = 34 - 12\,\frac{M_1}{M_2} \quad \leq 40 \tag{10}$$

Here, L is the member length (l_u in ACI 318), k is the braced effective length factor, and r is the radius of gyration of the gross section. The moment ratio (positive when single curvature bending) is between factored design end moments calculated by conventional first-order analysis (without imperfections included).

The code does not specify M_1 and M_2 in terms of the components M_{ns} and M_s. In nonsway frames that are unbraced, axial forces may not be sufficiently high to cause maximum moments to develop between ends. Therefore, the most relevant definition, and the only one possible in ACI 318-83, is believed to be

$$\frac{M_1}{M_2} = \frac{M_{1ns}}{M_{2ns}} \tag{11}$$

ACI. Sway frames. For "*individual compression members*" in sway frames, it is allowed in the code to neglect individual slenderness effects when L/r is less than

$$\frac{L}{r} = \frac{35}{\sqrt{P_u/f_c'A_g}} \tag{12}$$

For greater values, a maximum moment may develop between ends. This limit was first introduced in the ACI 318-95 code version.

PREMISES AND DISCUSSION

Why two separate limits?

Both Eq. (10) and Eq. (12) deal with local second-order (member slenderness) effects that may cause the development of maximum moments between ends. So why give two limits for essentially the same phenomenon? In a reply to a question on this issue (Hellesland 1995), ACI Committee 318 (Closure 1995) simply states that whereas Eq. (10) "allow one to disregard slenderness effects altogether" when the slenderness is less than this limit, columns with slenderness less than Eq. (12) "may have magnified moments, but the maximum moment will be at the ends of the column". This explanation does not provide any rationale for giving two greatly different limits. It is discussed further in the sections below.

Criteria

According to the ACI 318-02 Commentary, the limits above are based on the premise that the maximum moment between member ends may be 5% greater than the largest end moment. It is questioned, however, whether this criterion is the one really used in the derivation of the first limit (Eq. (10)). According to the original presentation (MacGregor, Breen and Pfrang 1970), the limit was derived using the moment magnifier expression to "compute the slenderness ratios (kl_u/r) corresponding to a slender column strength equal to 95% of the cross-sectional strength", i.e., $P_{long}/P_{short} = 0.95$ with the notation used in the 1970 paper. Results based on this axial load capacity criterion may deviate considerably from results based on a 5% increase in moments, which, for a given axial load, implies a moment capacity reduction of about the same magnitude.

This is illustrated schematically in the axial load–moment ($P-M$) interaction diagram in Fig. 4. The curves labeled a, b and c correspond to criteria related to a specified percentage reduction
(a) in moment capacity for an applied constant axial load;
(b) in axial load capacity (and moment capacity) for an applied constant axial load eccentricity;
(c) in axial load capacity for an applied constant moment.

Slender member strengths based on these three criteria, and corresponding slenderness results, may become very different. This is especially so at higher axial load levels where Criterion (b) and (c) are seen to be considerably more generous than Criterion (a). This will be discussed further in a later section (Comparisons. ACI limits).

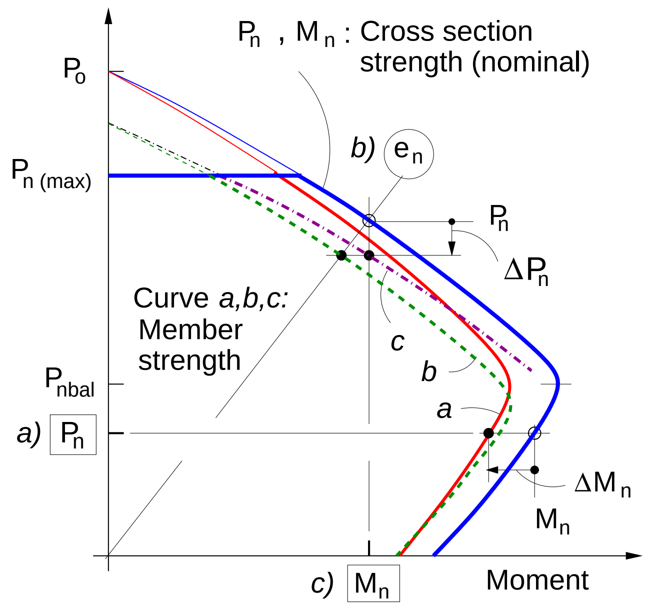

Figure 4: Illustration of capacity reductions (10%) according to various slenderness limit criteria.

Restraint independence

Unlike the first limit (Eq. (10)), the second limit, Eq. (12), is not a function of the effective length factor. In a discussion of the 1995-revision proposal, this issue was brought up (Hellesland 1995). In the closure to the discussion (ACI Committee 318, 1995) it is stated that Eq. (12) "was based on the equation for the elastic curve of a beam-column which does not include the effective length factor". This reply is not entirely relevant. It should be recalled that the equation referred to is given in terms of total (second-order theory) end moments. End restraints are consequently reflected in the moments themselves. In conventional analysis and design, which is not based on second-order theory, other means, like effective length factors, are necessary to reflect end restraints.

The maximum moment magnifier expression that is applicable when the maximum moment forms between member ends, can be obtained from the differential equation for the elastic curve (e.g., Galambos 1968) and expressed by

$$\delta_{t,max} = \frac{M_{max}}{M_{t2}} = \frac{\sqrt{1 + \mu_t^2 - 2\mu_t \, cos(PL^2/EI)^{1/2}}}{sin(PL^2/EI)^{1/2}} \tag{13}$$

where $\mu_t = M_{t1}/M_{t2}$ is the ratio between *total end moments*, i.e., with second-order member slenderness effects included, and M_{t2} is taken as the larger of the two end moments. The ratio is positive when the moments at each end act in opposite directions, and negative otherwise. Slenderness results an elastic moment magnifier $\delta_{t,max} = 1.05$ are shown in Fig. 5 versus the total moment ratio μ_t. An approximation is also included. A similar figure were used in the derivation of Eq. (12) (MacGregor 1993). A problem with the figure, in addition

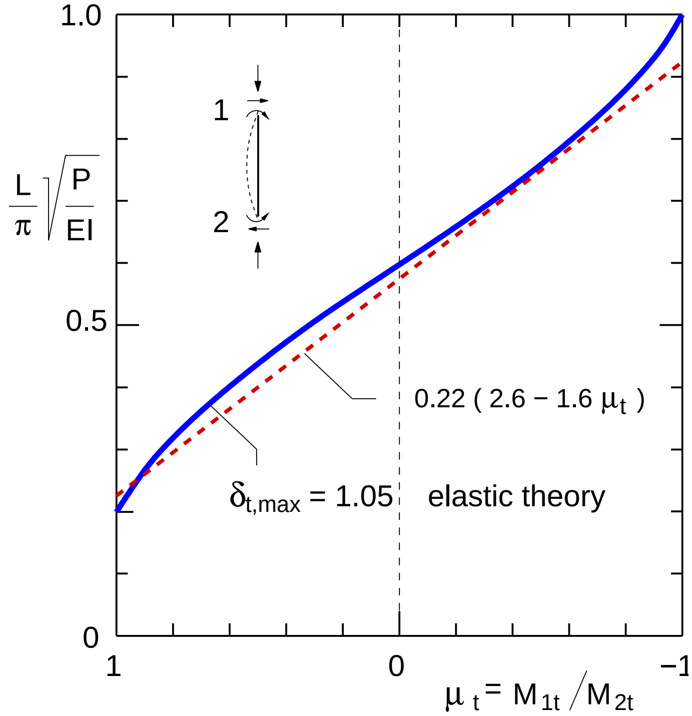

Figure 5: Slenderness limits vs. ratio between total end moments at 5% increase in moments beyond the larger total end moment.

to be given in terms of total end moment ratios, is that it is limited to cases with $P < P_E$, where P_E is the Euler buckling load of a column hinged at both ends. In terms of first-order moments, the expression above and Fig. 5 are strictly relevant only for unrestrained (hinged) columns in which case total end moments are equal to the applied end moments.

For restrained columns, it is necessary to consider specific end restraints in the calculations. Such results have been calculated and are shown in Fig. 6 for the two braced columns defined by the inserts labeled a and b in the figure. The slenderness values, given in terms of $(P/P_{cr})^{1/2}$, are at a moment magnification of 5% beyond the larger first-order end moment ($\delta_{max} = M_{max}/M_2 = 1.05$), and are shown versus the first-order end moment ratios ($\mu_o = M_1/M_2$). The inclusion of effective length factors k are seen to reflect different end restraints well for nearly uniform bending ($\mu_o = 1$), in which cases the different curves are close together, but not as well for more non-uniform bending.

Curve (a), for the column hinged at both ends (k=1.0), is reasonably representative also for restrained columns with nearly equal restraints at both ends. Curve (b) is for a member with one end hinged and the other end rotationally restrained by a very stiff beam (k=0.7). The larger moment is applied, most unfavorably, at the end with the smallest restraint, i.e., at the hinge (where there is no moment relief). Curve (b) can for all practical purposes be considered a reasonable lower bound on results for any end restraint combination. The straight

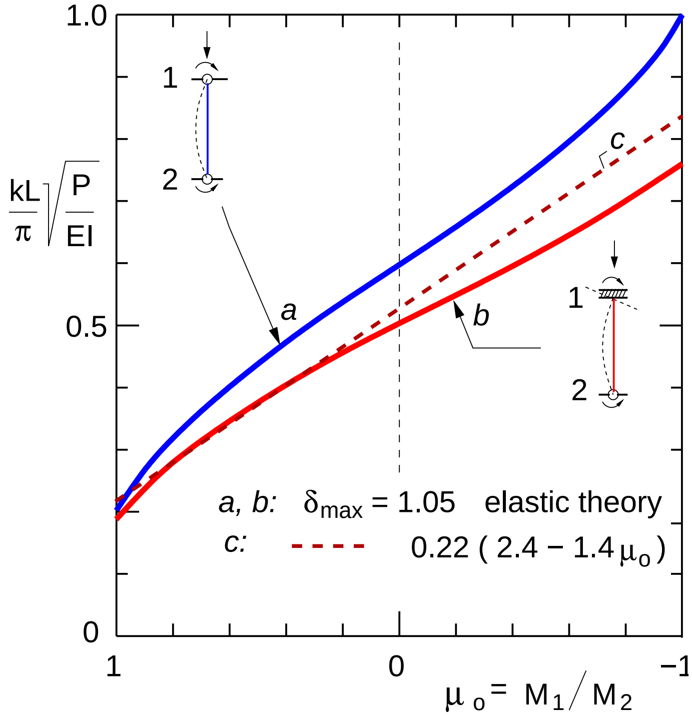

Figure 6: Slenderness limits vs. ratio between first-order (or sway-modified first-order) end moments at 5% increase in moments beyond the larger (sway-modified) first-order end moment.

line approximation labeled (c) in Fig. 6, located between curve (a) and (b), is a reasonable lower bound for cases with less extreme, but still very significant differences in end restraints ($k_{\theta 1} = 8k_{\theta 2}$). For reasonably equal end restraints, a steeper approximation ($0.22(2.6 - 1.6\mu_o)$) is acceptable. However, in such cases, the approximation should not be allowed to exceed the value of curve (c) at μ_o=-1, in order to avoid possible unwinding (unwrapping) problems.

From the above, it is clear that it is appropriate to include the effective length factor k in Eq. (12). Conservatively, one could adopt $k = 1$. However, this would not be more rational than taking $k = 1$ in the nonsway limit.

Moment gradient independence

In the derivation of Eq. (12), an end moment ratio of -0.5 was assumed on the basis that ratios of interest for sway frames was considered to be approximately -0.5 to -1.0 (MacGregor 1993). This choice will probably be conservative in most cases. However, in some cases, nonsway moments may become significant, and the combination of nonsway and sway moments may produce greater, and even positive ratios (corresponding to single curvature bending). The absence of an end moment dependence limits the applicability of Eq. (12) in such cases..

The range of end moment ratios mentioned above does not make sense if they are based on total end moments (Eq. (13)). They must be based on (sway-modified) first-order moments, which is an indication that the second slenderness limit does not evolve from the equation of the elastic curve of a general column, but rather, contrary to Committee 318's explanation in the closure (1995), from the special case of a hinged column ($k = 1$).

Modification of existing limits

Tacitly, the message conveyed by present code provisions is that the two slenderness limits discussed above may pertain to somewhat different phenomena. This is not the case. One and the same limit can be used for both nonsway and sway frames. This could for instance be Eq. (10) with an end moment ratio defined in the general case with the sway-modified first-order moments, Eq. (6). However, in order to reflect the influence of other important parameters than the end moment ratio, alternative, more comprehensive limits of the type presented below are recommended.

ALTERNATIVE LIMIT (Hellesland)

Slenderness parameter

For restrained, elastic compression members with negligible shear, it is seen above that $kL(P/EI)^{1/2}$ is the appropriate slenderness parameter (Fig. 5). A nondimensional form that is suitable for reinforced concrete members, and that may appropriately be labeled *normalized slenderness* in order to distinguish it from other slenderness parameters, has been proposed previously (Hellesland 2005) and can be expressed by

$$\lambda_{no} = \frac{kL}{r}\sqrt{\frac{\nu}{S\,(1 + k_t\,\omega_t)}} \tag{14}$$

in which

$$\nu = \frac{P_u}{\phi f'_{cn} A_g} \quad ; \quad \omega_t = \frac{f_y\, A_{st}}{f'_{cn} A_g} \quad ; \quad k_t = \frac{4.3}{1000\,\varepsilon_y}\left(\frac{r_s}{r}\right)^2 \tag{15}$$

Here, ν, ω_t and k_t, all in a form suitable for the resistance factor (ϕ) safety philosophy of ACI 318, is the factored design axial load level (nominal), the total mechanical reinforcement ratio, and a relative reinforcement contribution factor, respectively. Further, f_y and ε_y are the steel yield strength and strain, f'_{cn} is the nominal structural concrete compressive strength (often denoted f''_c or $k_3 f'_c$ in the literature), r and r_s are the radii of gyration of the gross section (A_g) and the total area of the longitudinal reinforcement (A_{st}), respectively, both about the centroidal axis of the gross section. In the partial safety factor approach (e.g., CEN 2004), ϕ in Eq. (17) should be deleted and f'_{cn}, f_y and ε_y replaced by the respective design values f_{cd}, f_{yd} and ε_{yd}.

S is a factor included to reflect some overall effect of axial load levels on stiffness. It is generally acceptable to take S=1. However, in the ACI resistance factor format, the λ_{no} formulation can be simplified, and improved, by assuming

S to be inversely proportional to the strength reduction factor ϕ. By introducing the stiffness factor $S = \phi_S/\phi$, then

$$\lambda_{no} = \frac{kL}{r}\sqrt{\frac{(P_u/f'_{cn}A_g)}{\phi_S(1+k_t\omega_t)}} \tag{16}$$

where ϕ_S is a constant. The present stiffness parameter (the parenthesis in the denominator) is chosen such as to be most representative at higher axial load levels. Consequently, a physically motivated choice of ϕ_S is ϕ_S=ϕ_{min}=0.65. Then a trilinear S variation is obtained, with ϕ_S=0.72 at low load levels, 1.0 at load levels above the balanced point, and a linear variation in between. These numbers are based on present ϕ values for tied sections (ϕ= 0.9 for tension-controlled sections and 0.65 for compression-controlled sections (ACI 2002)).

Although no longer transparent, Eq. (16) still embodies the basic safety factor format of ACI 318 (where design loads are at most ϕ times nominal capacities, $P_u = \phi P_n$, $M_u = \phi M_n$). The constant ϕ_S is in principle different from the socalled "stiffness reduction factor" ϕ_k in the magnifier expression (δ_{ns}) of ACI 318-02, where ϕ is directly replaced by ϕ_k=0.75 based on other arguments than those leading to ϕ_S above. However, consistency with that formulation is an argument for giving ϕ_S the same value as ϕ_k. This is expected to be compensated for by the conservativeness of using the same ϕ factor and material properties of the most critical section along the whole member length in typical analyses.

There is room for simplifications. In lieu of more accurate values, it is considered acceptable to take f'_{cn}=0.8f'_c for all concrete strengths. Further, k_t may normally be approximated by $k_t = 2(r_s/r)^2$. The term $(r_s/r)^2$, reflecting the cross-section shape and the reinforcement distribution and location, becomes $2(h'/h)^2$ for a rectangular or circular section with evenly distributed reinforcement. This approximation is found to be reasonable for a wide range of cross-sections (Hellesland 2005). Further, by adopting $h'/h = 0.7$, then $(r_s/r)^2 \approx 1$ and $k_t \approx 2$.

The product $k_t\omega_t$ can alternatively be given directly. It is not given dependent on steel grade, and can, with f'_{cn} in the range 0.8-0.85f'_c, be rounded down to

$$k_t\omega_t = \frac{1000\,\rho_t}{f'_{c,MPa}}\left(\frac{r_s}{r}\right)^2 = \frac{145\,\rho_t}{f'_{c,ksi}}\left(\frac{r_s}{r}\right)^2 \tag{17}$$

for the case with f'_c given in MPa and ksi, respectively, and where $\rho_t = A_{st}/A_g$. In Eq. (17), concrete grade could alternatively have been reflected through the concrete modulus E_c. However, it is found (Hellesland 2005) that the slenderness limit results at 5% reduction are not much affected by the slope of the first part of the ascending portion of the concrete stress-strain diagrams.

Slenderness limit

A load and reinforcement dependent lower slenderness limit, at which local second-order load effects can be ignored, is defined by

$$\lambda_{no} = 24 - 14\mu_o \tag{18}$$

It was initially proposed in the context of braced columns (Hellesland 2002b, 2005), which in codes sofar generally are associated with braced frames only. However, is it not limited to braced frames. It may applied to individual columns in any frame. However, in all cases, the braced effective length factor is to be used. In the general case, the moment ratio μ_o must be defined by the ratio of sway-modified end moments, Eq. (6). Unintentional imperfections or uncertainties in load eccentricities, may be included in the moment ratio, or by reducing the gradient of the limit itself if considered necessary (Hellesland 2005).

The limit will not be exceeded in other than braced (sway-restricted) columns, whether these are part of braced or unbraced frames. For instance, for bracing columns in unbraced frames, maximum design moments will always be at an end (δ_{max}=1). However, since it may not be obvious beforehand which columns are braced, the limit can be applied summarily to all columns of the frame.

The limit allows for differences in restraints at the two ends. The main premise for the limit was that local slenderness effects, including normal sustained load (creep) effects, should not reduce a member's load-carrying capacity by more than 5% (below the critical cross-section capacity, or "nonslender member strength"). A combination of Criterion (*a*) and (*c*) was considered acceptable, as the appropriate load-carrying capacity is considered to be the moment capacity for a member with low to intermediate axial load levels (with significant load eccentricities), and as the axial load capacity for intermediate to high axial load levels (with smaller load eccentricities). The limit is found to satisfy Criterion (*a*) (at most 5% reduction in moment capacity) in most practical cases. It will in such cases be quite conservative at high axial load levels relative to Criterion (*c*) results (at most 5% reduction in axial load capacity for constant moment), thereby implying a capacity for additional creep effects at high axial load levels.

A similar limit (proposed by the author) was adopted by the Norwegian Standard NS 3473 (NSF 1989). Also Eurocode 2 (CEN 2004) adopted a closely similar limit, but there written in a different form, and with a specific creep parameter included. In both cases, end moment ratios are defined (incomplete) with first-order moments.

COMPARISONS

Nonlinear analysis

The slenderness limit, Eq. (18), is compared in Fig. 7 to nonlinear analysis results at a specified reduction of 5% in a member's moment capacity (Criterion (*a*)). The results are given in terms of λ_{no} versus first order moment ratios, and with ω_t and k_t as defined by Eq. 15. The results are given as bands on individual results presented previously (Hellesland 2002, 2005) in terms of a slightly differ-

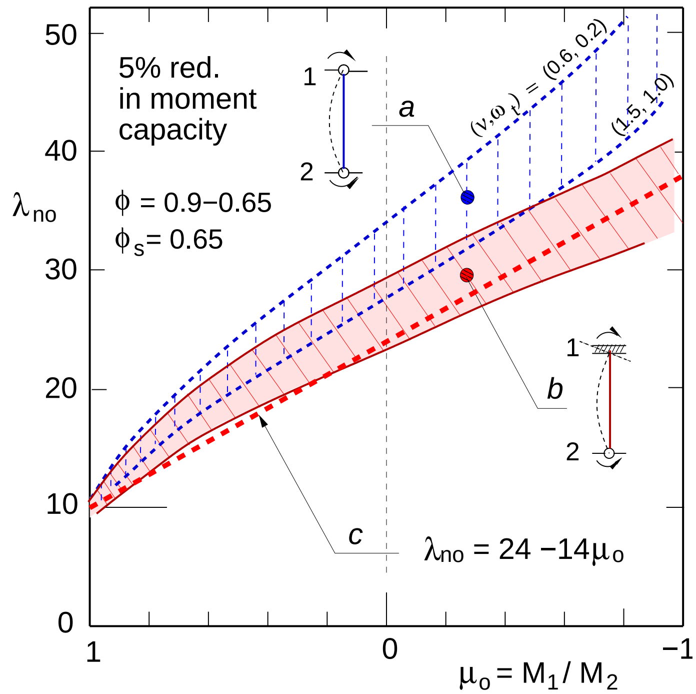

Figure 7: Normalized slenderness vs. (sway-modified) first-order end moment ratio. Bands (a) and (b): from nonlinear analysis of RC columns at 5% moment capacity reduction. Curve (c): alternative slenderness limit.

ent normalized slenderness parameter (λ_n for $S = 1$). The reinforcement levels are between ω_t=0.2 and 1.0, i.e., between about 1-1.5% to about 6-8%. Axial load levels vary between about 0.1 to 0.75 times the axial load capacity at zero eccentricity $(1 + \omega_t)$. The approximate combinations of ν and ω_t at which outer limits of the bands have been obtained are indicated in the figure.

The columns considered were initially straight, uniform, symmetrically reinforced and subjected to constant axial loads and to moments applied at member ends. They were either (a) hinged at both ends, or (b) hinged at one end and rotationally restrained by a very stiff restraint at the other end. The usual braced, elastic effective length factors of k=1.0 and k=0.7 were adopted in the presentation, respectively, for these two cases. The first-order moment ratios in the latter, statically indeterminate case, were those obtained with the nonlinear material properties. This complicates any direct use of the case (b) results, but they still serve as a good indication of a lower bound on results.

The analyses were carried out using a tailor made computer program based on an iterative finite difference approach. It included both nonlinear geometric effects, and nonlinear material effects through computed moment–curvature relationships for given sections, reinforcement and given nominal axial loads $(P_u/\phi = P_n)$. Nonmechanical strains (creep, shrinkage) were not included. For additional details, see (Hellesland 2005).

This figure represents the reinforced concrete equivalent to Fig. 6 for elas-

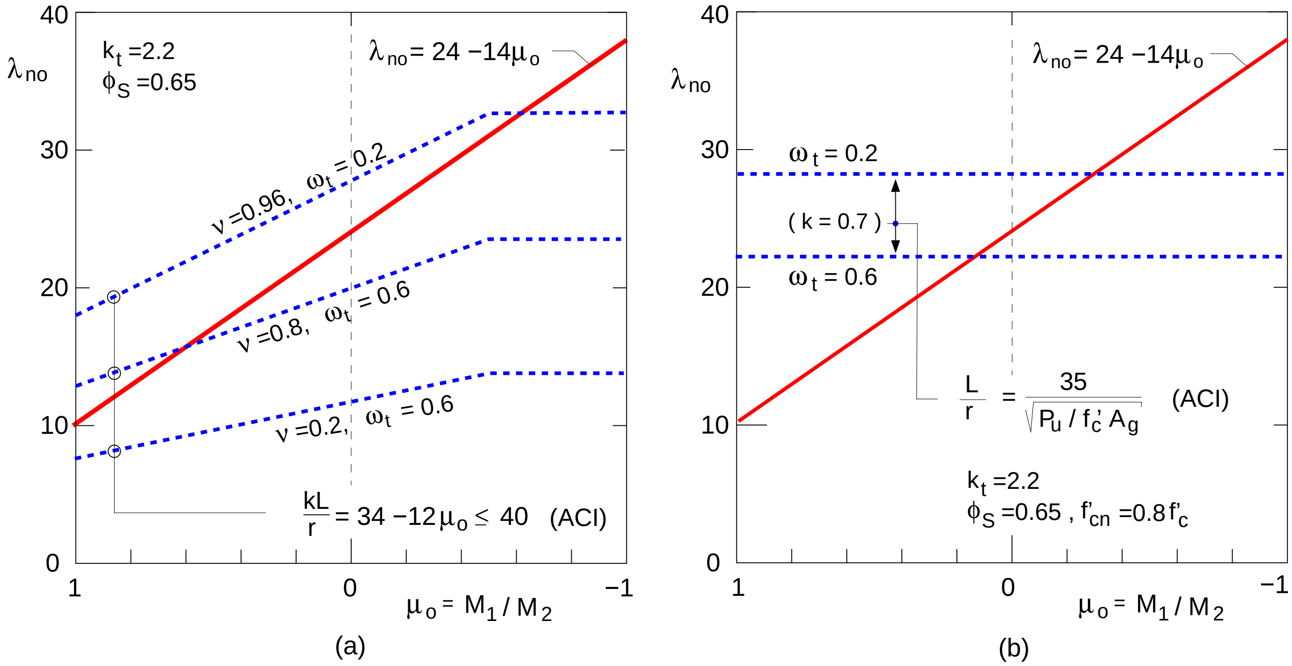

Figure 8: Comparison of the ACI slenderness limits with the alternative limit; (a) ACI limit for nonsway frames. (b) ACI limit for sway frames.

tic cases. Considering that the lower band in Fig. 7 will be rotated upward somewhat for more realistic differences in end restraints, Eq. (18) represents a fairly conservative limit for the given short-term results at 5% moment capacity reduction (Criterion (*a*)). Sustained load effects will reduce the conservativeness, however, in particular at high axial load levels.

By comparison with results of fully nonlinear, time dependent finite element analysis of both unrestrained and restrained columns with small eccentricities (Mari and Hellesland 2005), it has been found (Hellesland 2005) that Eq. (18) allows for normal creep effects (linear creep factor of about 2.0 and sustained loads as high as 50 to 60% of sectional capacities) even for Criterion (*a*). There is an additional margin to Criterion (*c*) (5% axial load capacity reduction for constant moment). This was also confirmed by comparison with results of an early study of framed columns with very high creep effects by Manuel and MacGregor (1967), for which Criterion (*c*) was the relevant criterion.

ACI limits

In Fig. 8(a), the ACI "nonsway" limit, Eq. (10), is compared to the alternative, load and reinforcement dependent limit, Eq. (18). All results are presented in terms of the normalized slenderness λ_{no}, Eq. (16). The three ACI curves are obtained with k_t=2.2 and ϕ_S=0.65. With ϕ_S=0.75, the curves will be lowered about 7%, but not affect general conclusions.

The curves cover a reasonably practical range. The lowest is obtained for a combination of an intermediate reinforcement ratio and low axial load level

($\nu = 0.2 = 0.17(1+\omega_t)$). At such combinations, the ACI limit is seen to be very conservative. The middle curve, for the same reinforcement, but with an intermediately high axial load level ($\nu = 0.8 = 0.5(1+\omega_t)$), which is more representative of typical columns, is less, but still quite conservative for most moment gradients.

The upper curve, for a combination of a low reinforcement ratio and a very high axial load level ($n = 0.96 = 0.8(1+\omega_t)$), represents a reasonably upper bound. It is seen to be generally very unconservative compared to the alternative limit for columns in single curvature, and it is quite clear that it does not meet Criterion (a) for such cases. But is it still acceptable with respect to an axial load capacity criterion?

In Hellesland (2005), comparisons of various criteria are made based on the conventional moment magnifier expression (Eq. (5)), which was also used in deriving Eq. (10) (ACI 318-02 Commentary). For highly compressed columns in nearly uniform bending, it was found that an axial load capacity reduction criterion (Crit. (b), (c)) may give about twice the slenderness obtained with a moment capacity criterion (Crit. (a)). The difference decreases with increasing moment gradients. The upper ACI curve in Fig. 8(a) may consequently seem acceptable relative to an axial load capacity criterion.

As mentioned earlier, it is believed that it is such a criterion, rather than a moment capacity criterion, that was used in deriving Eq. (10). Further, isolated comparisons in this study with the fully nonlinear analysis results at high axial loads mentioned in the section above, indicate that the ACI limit allows for normal to high sustained load effects relative to Criterion (c).

A similar comparison is shown in Fig. 8(b) between the second ACI limit, Eq. (12), and the alternative limit. One horizontal line is obtained for each combination of effective length factor k and reinforcement ω_t. Restrained columns in sway frames that may develop maximum moments between ends are likely to be very flexible. As such they are likely to receive substantial rotational end restraint, and consequently have effective length factors well below 1.0. In the comparison, k=0.7 is used. This is a reasonably realistic value for a column restrained at both ends. For double curvature cases, which are expected to be the normal situation in sway frames, and which was assumed in the derivation of the limit ($\mu_o = -0.5$), the ACI limit is seen to be generally conservative relative to the alternative limit. However, for members in single curvature, which may be a case to consider also in sway frames, the limit may become grossly unconservative. The applicability of the limit need to be increased by including additional influencing factors.

SUMMARY AND CONCLUSIONS

The mechanics of columns in braced and unbraced frames, and the effect of local and global second-order effects on moment formulations in such frames, have been reviewed. To account properly in approximate methods for local second-order effects on the development of maximum moment between ends of slender compression members, it is concluded that it is necessary to include moments due to global second-order effects in the moments, and to define the end moment ratio with sway-modified end moments.

Similarly, nonslender column limits defining when the development of maximum moments between ends can be ignored, should also be defined in terms of sway-modified end moments. The present practice of defining such ratios with first-order end moments only, is conceptually incorrect.

The need for giving two separate slenderness limits for one and the same phenomenon, such as in ACI 318-02 for nonsway and sway frames, has been questioned. The quality of the code provisions would be enhanced if only one slenderness limit is given to deal with what is one and the same phenomenon. Also it has been questioned why the two limits are functions of different parameters, and why they seem to be based on premises (criteria) that appears to be, contrary to information given in the ACI 318-02 Commentary, significantly different at high axial load levels. This detracts from the rationality of the code.

The ACI "nonsway-limit" in Eq. (10), defined with sway-modified end moments, will provide safe nonslender member estimates for columns in both nonsway and sway frames, also at very high axial load levels within Criterion (c). However, since the limit does not reflect but one major influencing parameter (moment gradient) in addition to the geometrical slenderness (kL/r), the estimates vary widely with other major parameters and will be very conservative for a great many columns in typical structures. The limit has been in use since 1971, and has in periods been adopted in several codes. At this time, alternatives should probably be considered.

An alternative, more comprehensive nonslender column limit formulation is reviewed. It is derived based on rational principles as a function of geometrical slenderness and three major case dependent, influencing factors (moment gradient, axial load and reinforcement). Extensive comparisons with nonlinear analysis results document the reliable of the limit. At the initial design stage, it requires estimates or assumptions of axial load and reinforcement, which may be replaced by more correct values at a more advanced design stage. This added complexity, which is minor, is compensated for by increased reliability and by reduced conservativeness, and thus reduced design efforts, in a great many cases.

NOTATION

A_g, A_{st} = area of gross section and of total longitudinal reinforcing steel
I_g, I_s = second moment of area of gross section and of total reinforcing steel
L = length of compression member (column, strut, etc.)
M_1, M_2 = smaller and larger factored first-order, or sway-modified first-order, end moment
M_b^* = column moment in a fully braced frame

M_s^* = column moment due to sway caused by all loads on the frame
M_{ns} = column moment due to all loads except lateral loads
(ACI 318 definition: due to loads that cause no appreciable sidesway)
M_s = column moment due to lateral (sideways) loads
(ACI 318 definition: due to loads that cause appreciable sidesway)
P_n, P_u = nominal axial load capacity and ultimate factored (design) axial load
f'_c, f'_{cn} = cylinder and nominal structural compressive strength of concrete
f_y = yield strength of reinforcing steel
h, h' = section depth and distance between reinforcement in opposite faces
k = effective length factor of compression member
$P_E = \pi^2 EI/L^2$ = Euler buckling load of a pin-ended, uniform column
$r = (I_g/A_g)^{1/2}$ = radius of gyration of gross cross section
$r_s = (I_s/A_{st})^{1/2}$ = radius of gyration of total reinforcing steel
$\varepsilon_y = f_y/E_s$ = yield strain of reinforcing steel
ϕ, ϕ_k = strength and stiffness reduction factor
ϕ_S = stiffness scaling factor
λ_{no} = normalized slenderness
$\mu_o = M_1/M_2$ = first-order, or sway-modified first-order, end moment ratio (positive when single curvature bending)
ν = relative axial load
ω_t = total mechanical reinforcement ratio

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